A MIXED INTEGER MODEL FOR RESOURCE ALLOCATION IN CONSTRUCTION MANAGEMENT

by

Rawle Nari Ramlogan

A thesis
presented to the
University of Manitoba
in partial fulfillment of the
Requirements for the degree of
MSc in Civil Engineering
in
Department of Civil Engineering

Winnipeg, Manitoba, 1987

ISBN 0-315-37136-6

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A thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

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ABSTRACT

A 'multiobjective' optimization approach has been developed for the solution of scheduling problems in construction. The model uses a mixed integer optimization procedure for the resource levelling within the scheduling problem. The starting point for the model is the results of a critical path analysis of the project. The project is scheduled by the resource levelling model within the constraints of the free float of the activities. 0-1 integer variables are used to ensure the allocation of integer resource requirements in each time period. Besides meeting the resource requirements of the activities, the model also constrains the activities to be scheduled on consecutive days. The model has three global objectives - (1) the overall resource levelling on the project, (2) the resource levelling of individual activities (internal levelling), and (3) the minimization of the width of the windows (number of consecutives days). The model is demonstrated by application to two example projects using the LINDO optimization package.

ACKNOWLEDGEMENTS

The author would like to thank Dr. I.C. Goulter for his ideas and guidance in the development of this thesis. Also the kind assistance of Dr. R.B. Pinkney in developing the computer program for generating the equations.

I would like to thank my parents and brothers for their support in my educational endeavours, I could not have accomplished all that I have without their help.

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Chapter I

INTRODUCTION

The Critical Path Method (CPM) was developed by the DuPont Company and has become one of the best network modelling techniques. It is used extensively for the planning, scheduling and control of projects consisting of many independent activities which must be completed in a defined sequence. The CPM identifies the activities which make up the critical path as well as the floats of the non-critical activities. The model described in this paper utilizes the free float of non-critical activities because it allows the scheduling of the activity without causing any delay in the following activities.

Resource levelling is the scheduling of the activities such that the variation on the resource requirement in each time period is minimized. By itself resource levelling can be a tedious exercise if done manually, and becomes more complicated with consideration of the two other objectives of the model. The model presented in this paper represents one approach for solving the problem by the use of operation research techniques.

The CPM approach provides data on early start and late start dates for each activity of the project. However, because the analysis is done independently of the resource requirements of the activities these schedules can and often lead to uneconomic allocations of resources committed to the project. These schedules usually exhibit widely fluctuating resource requirements with isolated peak demands. Efficient scheduling of the project requires consideration be given to some resource levelling. Resource levelling can be performed on any type of resource, e.g., manpower. Each resource would have to be levelled separately with the resource having the highest priority being done first. Other resources can then be levelled within the limits of the resource requirements imposed by the levelling of prior resources.

The models described in this paper consider the levelling of only one resource at a time. The development of models to handle more than one type of resource simultaneously is left to future expansion of the model. The general approach outlined is likely to provide the basis for these future models.

The model itself is based on a 0-1 integer variable which ensures that the resource allocated in any period for each activity is an integer, most resources are considered in whole units. The allocation of 1/2 man or 1/3 bulldozer for a day is not realistic. The use of 0-1 variables also allows the other global objectives and the consecutive days constraint to be easily incorporated into the model. The consecutive days constraint is introduced into the model by allowing the model to select of one of several 'windows' in the allowable time period. A window is a set of consecutive days between the early start and late finish, as defined by CPM, for each activity. The actual number of days in the window is chosen by the model. This allows the scheduled duration of the activity to be determined by the model and not preset from the critical path analysis.

The second objective is the levelling of the resource requirement for each activity within its window. This is achieved by considering each activity as a small project within itself and applying the same general approach used for the whole project.

The third objective of the model is the minimization of the window width of the activities. This objective is included to offset the flattening effect of the internal levelling objective. The minimization of the window width tends to compress the duration of the individual activities. These two objectives can be thought of as constraining the resource allocation of the activity in the horizontal and vertical directions.

These three global objectives are handled simultaneously in the model by placing mathematically statements of each within the objective function. Variations in relative emphasis on the three objectives is obtained by placing weights on the appropriate objective.

Chapter II

LITERATURE REVIEW

Since the introduction of PERT[1]/CPM procedures, network models have proven to be a useful means of formulating a variety of activity planning and scheduling problems. The main limitation of these procedures is that they assume unlimited availability of resources, which is not realistic.

Davis [1973] reviewed the existing procedures (up to 1973) of project scheduling under resource constraints. He classified the procedures available at the time into two general headings - Heuristic Procedures involving some 'rule of thumb', and Optimal Procedures involving some form of mathematical programming. The main problem faced by current researchers in the field appears to be the lack of an efficient integer program. While this problem has not been overcome, with the accelerating advancements in computer technology, it can be expected that a suitably efficient integer program will be available in the future.

Smith et al. [1983] presented a simple method for resource allocation. The method involved a quadratic objective function. The method was not developed into a

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[1] Program Evaluation and Review Technique

full fledged programming model but used to evaluate different trial solutions obtained by a manual iterative analysis. Ramlogan [1985] used this quadratic objective function to build a programming model for the resource levelling and project scheduling of a project. The quadratic programming model was later converted through application of the Kuhn-Tucker conditions to a linear formulation for solution by LINDO.

While the results of this model were similar to the results of a manual analysis, the major disadvantage of the model was the non-integer allocation of resources. To overcome the non-integer problem, the model was then converted to a mixed integer approach with 0-1 integer variables used to impose non-linear penalties on deviations from the mean resource requirement of the project (Goulter and Ramlogan [1987]). This model did provide an integer allocation, but as noted in the conclusions to the paper, the model does not restrict that an activity be completed on consecutive days. Furthermore, the above approach by Ramlogan [1985] and Goulter and Ramlogan [1987] has only one objective - ie. the resource levelling of the whole project subject to the constraints imposed by the critical path analysis of the network.

Lee and Olson [1985] approached the problem of project scheduling for multiple objectives by using a 0-1 integer based goal programming technique. While the model presented in this paper does not consider multiple resources and therefore cannot be compared directly with that of Lee and Olson [1985], it is interesting to note that these authors suggest the use of LINDO for dealing with larger models by solving "a sequence of linear programming models reflecting the preemptive structure of the goal programming."[2] Lee and Olson [1985] built into their model constraints of fixed duration and amount of the resource that can be allocated in each time period for each activity. The model presented by Lee and Olson [1985] schedules the project with each activity being assigned a duration as initially specified for the critical path analysis. Their model does not provide the flexibility of shortening the duration of the activity by utilizing more resource in a given time period, or vice versa.

A considerable amount of material has been written on multiobjective analysis theory and its application to a wide range of fields. The use of multiobjective programming techniques are used when the problem has several objective which may be in partial conflict with each other. Cohn and Marks [1975] in their paper on multiobjective programming techniques have concluded that "when there are fewer than four objectives, a generating

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[3] Lee et al., "Project Scheduling for Multiple Objectives", Project Management: Methods and Studies. edited by Burton V. Dean, Elversier Science Publishers B.V. (North-Holland), 1985, pg 130.

technique such as the weighting method or the constraint method should be used in order to capture the essence of the multiobjective problem."[3] The model to be presented in this paper deals with three objectives, and consequently the weighting method is used to solve the problem.

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[3] Cohn et al., "A Review and Evaluation of Multiobjective Programming Techniques". Water Resources Research. Vol. 11, No. 2, 1975, pg 218.

Chapter III

MODEL DEVELOPMENT

The model presented in this paper is based on the modification of the quadratic programming model developed by Ramlogan [1985]. In a later paper by Goulter and Ramlogan [1987] the quadratic programming model was converted to a mixed integer formulation to overcome the problem of non-integer results. A review of both the quadratic programming model and the mixed integer formulation is included to provide a basis for understanding the development of the present model.

3.1 INFORMATION REQUIREMENTS

Before describing the model it is necessary to define the data requirements upon which the model is built. As discussed earlier the starting point for the model is the set of results obtained from a critical path analysis. The other piece of information is the resource requirement (e.g., manpower days or machine hours) for each activity. The resource requirement and the estimated duration of each activity is usually based on the past experience of the project manager in dealing with similar projects.

3.2 REVIEW OF THE QUADRATIC PROGRAMMING MODEL

The quadratic programming model achieved the resource levelling by minimizing the square of the deviations of the total daily resource requirement from the average resource requirement of the project. The reason for squaring the deviations is to ensure that positive deviations do not cancel with negative deviations. It also places greater penalties on the objective function for larger deviations. Therefore the model would schedule the project in a way which minimizes these deviations. Smith et al. [1983] showed that the minimization of the square of the deviations is the same as minimizing the sum of the square of the total daily resource requirement. The reason for this is that the average resource requirement for the project is a constant. Consequently the objective function is represented as :

$$\min \sum_{j=1}^{ND} Y_j^{\,2} \tag{1}$$

where Y_j = resource allocated in time period j.

ND = number of time period in the project.

The constraints used with this objective function come from the results of the critical path analysis which gives the early start and late start time for each activity, as well as the associated floats. The total float is the total amount of time that an activity may be delayed without affecting the total project time. The free float is the total amount of time an activity may be delayed without affecting subsequent activities or the total project time. In the scheduling problem the free float (for non-critical activities) and the estimated duration of the activity (critical and non-critical) is used to determine the allowable period in which the activity must be completed so as to not prolong the overall duration of the project. The free float is used because it can be considered as local slack for that activity. This means that the activity can be delayed up to this time without delaying the starting time of the following activities.

This allowable time period (the estimated duration plus any free float) impose what is termed within the model as the activity requirements. This is the most crucial constraint in project scheduling. The constraint ensures that sufficient resources are allocated for each activity within this time period. This constraint can be written mathematically as:

$$\sum_{j \in T_i} X_{ij} = TR_i \qquad \forall\ i \qquad (2)$$

where X_{ij} = amount of resource allocated to activity i in time period j.

TR_i = total resource requirement for activity i.

T_i = allowable time period for activity i.

The last constraint of the model identifies the total resource allocated in a given time period. This constraint is defined as follows :

$$\sum_{i \in A_j} X_{ij} - Y_j = 0 \qquad \forall j \qquad (3)$$

where A_j = the set of activities which are permitted to occur in time period j.

Y_j = the total resources allocated in period j.

3.3 REVIEW OF THE MIXED INTEGER APPROACH

In the above quadratic programming approach, the main weakness was related to the non-integer results. The quadratic programming model was attractive however in that it imposed penalties on deviations from the average project resource usage in a non-linear fashion. This meant that larger deviations from the average resource were penalized more. In converting to the mixed integer approach it was felt necessary to still maintain this non-linear penalty while forcing the resource allocated to any activity in any time period to be an integer value.

To ensure that the resource allocated to activity i in time period j in Equation 2 take on an integer value the following sets of constraints are used:

$$\sum_{n=0}^{MR_i} nXI_{nij} - X_{ij} = 0 \qquad \forall ij \qquad (4)$$

$$\sum_{n=0}^{MR_i} XI_{nij} = 1 \qquad \forall ij \qquad (5)$$

where XI_{nij} = 1 when n units of resource are allocated to activity i in time period j.

= 0 when n units of resource are not allocated to activity i in time period j.

MR_i = maximum possible resource allocated for activity i. MR_i is some realistic amount of the resource that would be allocated in any given time period. At the limit MR_i could be the total resource requirement of the associated activity.

Equation 3 specifies that X_{ij} can take on any value n, (n ranging from 0 to MR_{ij}) because XI_{nij} can only take on values of 0 or 1. Equation 4 ensures that only one of the XI_{nij} will be 1 for a particular combination of i and j, and the rest of the variables in the equation will be zero. The actual value to X_{ij} will depend on which of the XI_{nij} variables is set to one. An additional benefit of this formulation is that it is possible to impose upper and lower limits on the amount of resource allocated to any activity in any given time period. This is done by changing the limits of n in Equations 3 and 4 so that n goes from the lower limit to the upper limit.

The next issue in the development of the mixed integer approach is to modify the objective function given. In the quadratic programming model the objective function was the minimization of the sum of the squares of the total resource allocated in any given time period (Equation 1). The formulation with integer restrictions uses the same concept for the scheduling.

Due to the integer restrictions in the resource allocations, three sets of additional constraints are required. The non-linear penalizing of the deviations for a particular period j from the average level of the resources requirement throughout the project is achieved by using the following constraints sets.

$$\sum_{m=0}^{MD_j} mYI_{mj} - Y_j = 0 \qquad \forall\ j \tag{6}$$

$$\sum_{m=0}^{MD_j} YI_{mj} = 1 \qquad \forall\ j \tag{7}$$

$$\sum_{m=0}^{MD_j} m^2 YI_{mj} - Z_j = 0 \qquad \forall\ j \tag{8}$$

where YI_{mj} = 1 when m units of resources are allocated in time period j.

= 0 when m units of resources are not allocated in time period j.

MD_j = maximum possible resource requirement allocated in time period j. Like MR_j, MD_j is chosen to be some realistic level.

Z_j = square of the total resource allocated in time period j.

Equations 6 and 7 are used to identify the total resource that is allocated in a given time period. These two constraints fulfill a similar purpose of mathematically defining the variables as do Equations 3 and 4. Equation 8 squares the values of the Y_j which is then used in the used in the objective function.

With the constraints defined as above the objective function which minimizes the sum of the square of the total resource allocated in a given period over the duration of the project can then be written as :

$$\min \sum_{j=1}^{ND} Z_j \tag{9}$$

where ND = total duration of the project.

The model presented so far represents a complete integer programming model for project scheduling using one resource. When the schedule obtained from this model is graphed it was observed that still better results are possible. This led to further expansion of the model which is described below. The actual reasons for the expansion are described in an example given later in the thesis.

3.4 FURTHER EXPANSIONS TO THE MODEL

3.4.1 Scheduling on Consecutive Days (time periods)

The first expansion of the model was to impose the constraint that the activities be completed on consecutive days. That is, once work on an activity has started, there are no stoppages until the activity is completed. This may appear too restrictive, but it is actually realistic. If intermittent stoppages are necessary, this can be accomplished by representing the activity as a series of smaller activities.

In order to provide as much flexibility in the model while still maintaining the consecutive days constraint, the model uses a variable width moving window for each activity. The flexibility is achieved by allowing the model to select the scheduled duration of the activity, instead of being specified before. Each activity must be scheduled within an allowable time period determined from the critical path analysis. The variable width moving window represents different sets of consecutive days for the scheduling of the activity. For example, if an activity must be scheduled within days 5 to 7 inclusive, the set of possible consecutive days for the activity will contain the windows: day 5; day 6; day 7; days 5 and 6; days 6 and 7; days 5, 6 and 7. The use of windows for the activity is achieved by first specifying a window width and then moving the window within the allowable time period.

The constraint representing these variables width moving windows can be mathematically written as:

$$\sum_{j \in cs} X_{ij} - TR_i XW_{ics} \geq 0 \qquad \forall\ i,c,s \qquad (10)$$

$$\sum_{c=1}^{T_i} \sum_{s \in T_i} XW_{ics} = 1 \qquad \forall\ i \qquad (11)$$

where XW_{ics} = 1 when activity i is scheduled in the window defined by cs.

= 0 when activity i is not scheduled in the window defined by cs.

c = width of the window.

s = starting time period of window.

Equation 10 generates all possible window options in which to schedule a given activity. In essence this constraint states that the total resources allocated in any window for activity i must be greater than or equal to the resource requirement of that activity. The greater than or equal condition is necessary because only one window will be chosen. If this constraint set is made equal to zero it will lead to infeasibility because it would require that all X_{ij} be equal to zero which is not possible for any activity. Equation 11 ensures that only one window for each activity is chosen.

Equation 10 and 11 enforces the choice of one window from the window set for each activity. However to ensure that the window represents a set of consecutive days for the activity, some resource must be allocated to all days within the window. This condition is introduced into the model as follows:

$$X_{ij} - \sum_{ij \in cs} XW_{ics} \geq 0 \qquad \forall\ i,j \qquad (12)$$

Equation 12 specified that if X_{ij} belongs to the window chosen for activity i, then X_{ij} must be greater than or equal to one. This ensures that the constraint of consecutive days is fulfilled. Equation 12 is required because Equation 10 says only that the total resource allocated to activity i must meet the activity requirement, while Equation 12 says that in all windows, the minimum resource allocated is one unit.

3.4.2 Internal Levelling of Individual Activities

The next expansion of the model was to impose penalties for variation in the resource allocated for each activity over its duration. The logic of imposing the penalty is the same as the overall levelling of the project; the activity is considered a small project; hence the reason for calling this objective 'internal levelling'. It should be noted that the internal levelling is still an objective of the model and not a constraint. The objective itself attempts to schedule the activities and allocated resources such that the amount of resource allocated is relatively constant over the duration of each activity. The objective can be represented by an equation similar to Equation 8 and is stated mathematically as follows:

$$\sum_{n=0}^{MD_j} n^2 XI_{nij} - A_{ij} = 0 \qquad \forall\ i,j \tag{13}$$

where A_{ij} = the square of the resource allocated to activity i in time period j.

The model now has two objectives to achieve. Both of these objectives can be represented in the objective function by using the following sets of equations:

$$\sum_{j=1}^{ND} Z_j - ZT = 0 \tag{14}$$

$$\sum_{i=1}^{NA} \sum_{j \in T_i} A_{ij} - AT = 0 \tag{15}$$

where ZT = represents the objective of overall levelling of the project.

AT = represents the objective of internal levelling of the activities.

NA = number of activity in the project.

The use of Equations 14 and 15 permits a concise objective function to be obtained by defining both ZT and AT in the constraints. With the above two equations defined, the objective function can be simply written as:

$$\min \quad k_z ZT + k_a AT \tag{16}$$

where k_z and k_a = represent the relative weights on the overall and internal levelling objectives respectively.

The actual weighting assigned to the two objectives will depend on the relative priority ranking. The trade-off in the objective function with variations in the relative weighting will be discussed later in the paper.

3.4.3 Minimization of the Width of the Windows

The final expansion of the model is the minimization of the widths of the windows used in the scheduling process. Since the model uses a variable width window technique, a penalty is imposed for not choosing the smallest width given the other objectives and their relative ranking. Internal levelling tends to flatten and spread out the allocated resources for the activity, whereas the minimization of the window width tends to compress the duration of the activity. These two objectives work against each other and therefore some tradeoff analysis is required. The minimization of the window widths is an objective and consequently it is handled similar to that of the internal levelling. It is expressed by the following equations :

$$\sum_{cs \in T_i} c^3 XW_{ics} - W_i = 0 \qquad \forall\ i \tag{17}$$

$$\sum_{i=0}^{NA} W_i - WT = 0 \tag{18}$$

where W_i = the penalty for scheduling activity i in a window of width c (starting in time period s).

WT = represents the mathematical statement of the minimization of the window widths.

Equation 17 cubes the width of the window selected for each activity. This equation imposing a non-linear penalty, similar to Equations 8 and 13, on this last objective for selecting larger window widths. The reason for cubing the width is that in the model development squaring the width did not achieve the desired results. Therefore a stiffer penalty was imposed, i.e., the cubing the window width of the individual activities. As this last expansion is an objective it can be included in the total objective function as follows:

$$\min \quad k_z ZT + k_a AT + k_w WT \tag{19}$$

where k_w = represents the relative weight assigned to the window width objective.

The above model defined by this objective function (Equation 19) and the corresponding constraint sets (Equations 2-8, 10-15, 17, 18) represents a project scheduling and resource allocation method which levels the resource usage without increasing the duration of the project as defined by the critical path analysis.

Chapter IV

DISCUSSION OF MODEL - EQUATIONS

As developed the model is complex and would theoretically give an optimal project schedule within the constraints and objectives imposed. However there are several changes to the model which have to, or should be included in order for a solution to be found.

4.1 REDUCTION IN THE NUMBER OF WINDOW CONSTRAINTS

Equations 10 and 11 defined the selection of one of the variable width moving windows for each activity. Due to the requirement of only one window for each activity, the XW_{ics} is defined as a 0-1 integer variable. The more integer variables there are present in the model, the more complex and computationally intensive the branch-and-bound procedure becomes. Reduction in the number of integer variables greatly helps the efficiency of the model solution. Considering activities which can only be scheduled over a maximum of two days (mainly critical activities with a duration of 2 days), there is no need to impose the constraint of scheduling on consecutive days. This observation allows for the model to be reduced by 3 constraints and two integer variables for each such activity.

4.2 NON-DEFINITION OF YI AS AN INTEGER VARIABLE

As stated above the reduction of the number of integer variables would make the model more efficient. The YI_{mj} integer variables are used in the model to facilitate the squaring of the total daily resource requirement (Equation 8). Because the form of Equations 3, 6, and 7, it turns out that the model does not require that YI_{mj} be defined as an integer variable. In Equation 3, Y_j will always be an integer as long as X_{ij} takes on integer values. Equation 6 defines Y_i in terms of YI_{mj} and because of the form of this equation together with Equation 7, ensures that YI_{mj} will be a 0-1 integer variable without having to be defined explicitly as such within the model.

4.3 NON-DEFINITION OF XI AS AN INTEGER VARIABLE

When the XI variables are defined explicitly in the model as integer variables, the computational efficiency of the model is significantly reduced. In fact for the example project which is described later in the paper, sixty minutes of CPU time was used without obtaining one possible integer solution. Another project resulted in 'no integer solution found' after 25 minutes of CPU time. However like the YI variable, it was found that the XI variables do not have to be defined as integers. The reasons for this is not fully understood, but is suspected to be related to the construction of model and specifically the constraint defining the window objective. With only the XW variables defined as an integer variable, the solution for the example project is obtained in approximately 6 minutes of CPU time.

4.4 NEED FOR EQUATION 2

As stated earlier, Equation 2 is the most crucial constraint in project scheduling. However with the addition of the constraint of scheduling activities on consecutive days, Equation 10 causes Equation 2 to be redundant. In effect Equation 2 is really embedded in Equation 10. However computational analysis shows that with the inclusion of Equation 2 in the model, the same schedule is obtained as without it. The model containing Equation 2 requires less CPU time to reach the final answer. Equation 2 appears to restrict the branch and bound procedure leading to the optimal schedule in less CPU time.

Chapter V

DISCUSSION OF MODEL - APPLICATION

The model was developed and expanded through application to an example project. The project used is represented by the network shown in Figure 1. It consists of eight activities with project duration of 13 days as determined by the critical path analysis (see Appendix A for the results of the critical path analysis). The manpower requirements are given in Table 1.

Table 1

Description of Activities in Example Project

Activity	Manpower Requirements/day	Total Man-days
1	8	8
2	4	16
3	2	2
4	6	18
5	4	16
6	3	9
7	4	12
8	4	2

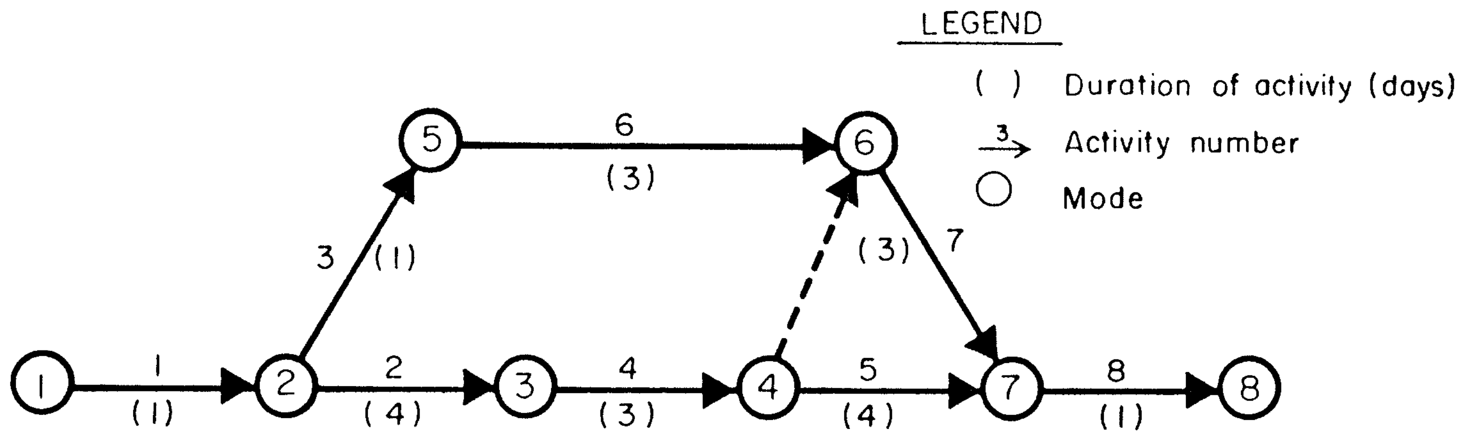

Figure 1. Network for Example Project

If the project is scheduled at the early start time of the activities the resource allocation shown in Figure 2 is obtained. With this schedule the maximum manpower requirement is 10 unit over days 6-8. As can be seen this schedule does not meet the objectives of resource levelling, which is usually required to prevent frequent hirings and firings and also because of limited resources available.

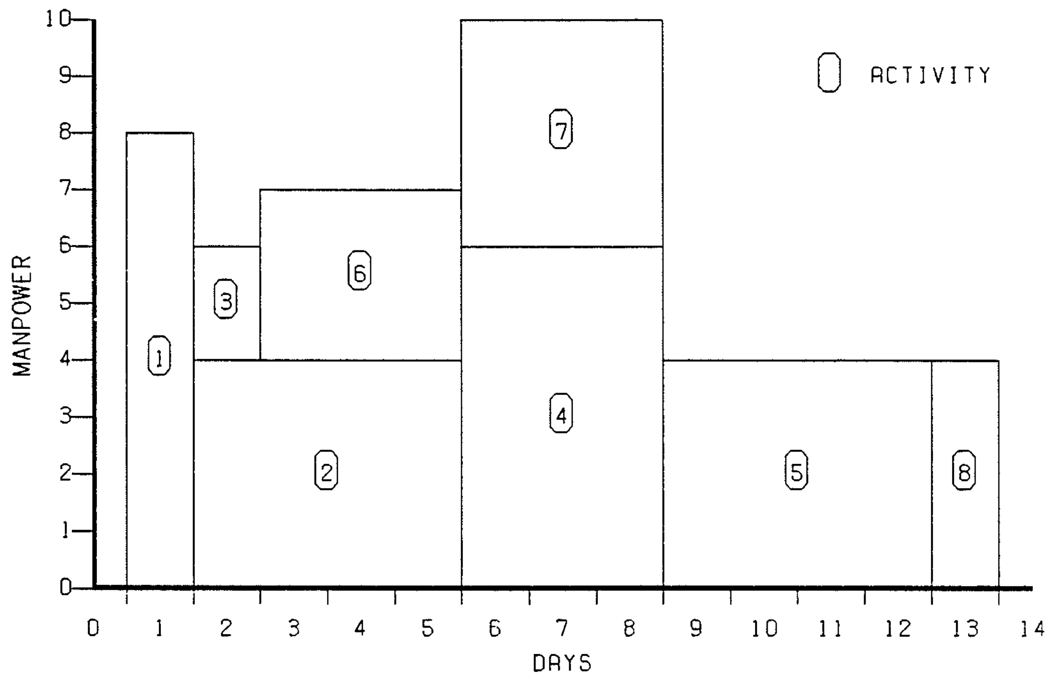

FIGURE 2 SCHEDULE USING EARLY START TIMES

5.1 OVERALL RESOURCE LEVELLING FORMULATION

As stated before, the major weakness of the quadratic programming model was the non-integer results. The solution to this problem was achieved by converting the model to a mixed integer model. This first model, with resource levelling as the single objective, is defined by

Equation 9 and is subject to constraints defined by Equations 2-8. When this model is applied to the project (Figure 1), the schedule shown in Figure 3 is obtained.

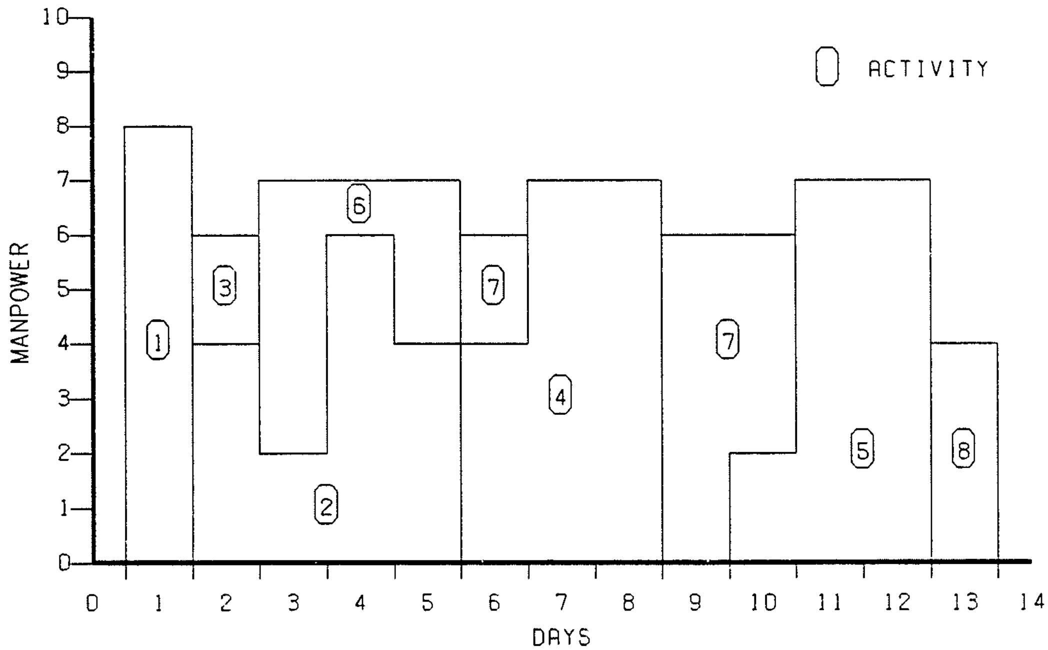

FIGURE 3 SCHEDULE WITH THE OVERALL LEVELLING FORMULATION

This model has achieved the objective of resource levelling. Over the duration of the project, the manpower requirement is relatively constant, fluctuating between 6 and 7, with no isolated peaks. The maximum resource requirement is 8 units compared to 10 units from the early start schedule. However an inspection of the schedule shows that better solutions are possible. For example the two units of manpower allocated to activity 7 on day 6 could better be scheduled as one unit on days 9 and 10. Activity 4 could then be scheduled for days 6-8 with 6 units of manpower on each day. In order for the model to automatically make these adjustments the additional constraint of scheduling the activities on consecutive days is required.

5.2 OVERALL LEVELLING / CONSECUTIVE DAYS FORMULATION

The requirement of scheduling activities on consecutive days required the addition of three sets of constraints (Equations 10-12). The objective function remains the same because scheduling on consecutive days is a constraint, not an objective. The schedule obtained with these additional constraints is shown in Figure 4.

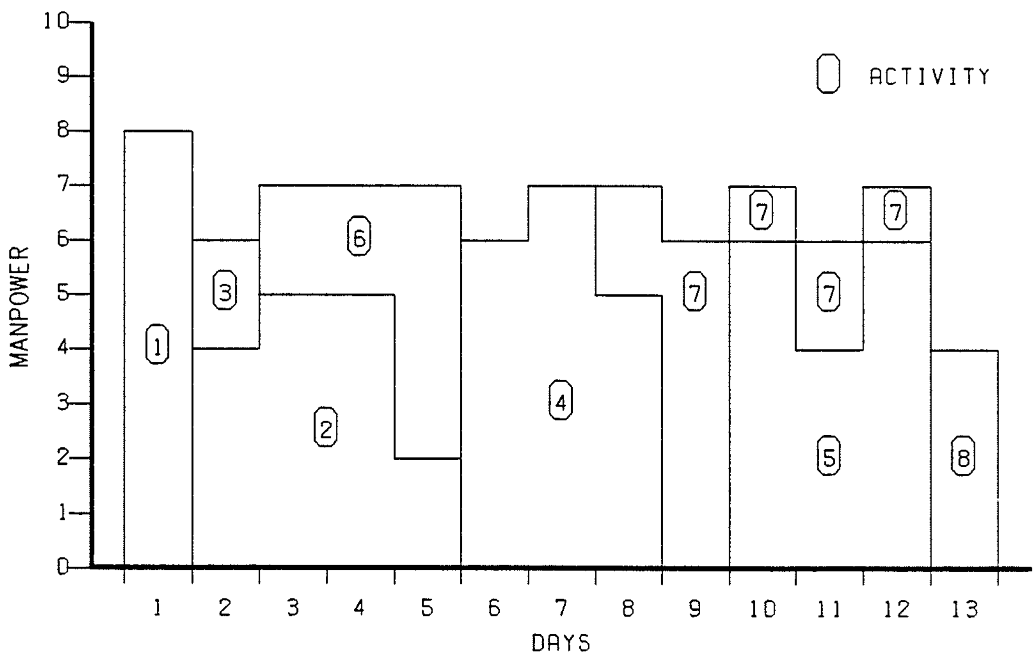

FIGURE 4 SCHEDULE WITH THE OVERALL LEVELLING / CONSECUTIVE DAYS FORMULATION

As can be seen from this schedule, all the activities are scheduled on consecutive days. Again an inspection of the resource allocation shows that while it is acceptable it can nevertheless be further improved. The main problem with this schedule is that activity 7 is spread out over five days with a resource requirement that fluctuated between 1 and 6 units. Also activities 2 and 6 can be rescheduled so that their manpower requirements are constant over their duration, i.e., activity 2 can be scheduled with 4 units of resource on days 2-5, and activity 6 with 3 units of resource over days 3-5. This problem of fluctuating resource demand for individual activities led to the next expansion of the model.

5.3 OVERALL LEVELLING / CONSECUTIVE DAYS / INTERNAL LEVELLING FORMULATION

The problem of fluctuating resource allocations to the activities is handled by considering each activity as a small project. This objective of the internal levelling is to try to remove fluctuations in the resource allocation over the duration of the activity if it is possible, given the other objectives and constraints. When this objective is added to the model and applied to the example project, the schedule shown in Figure 5 is obtained.

Figure 5 shows that the model has achieved the objective of internal levelling as well as achieving the other objective of resource levelling, and the constraint of scheduling the activity on consecutive days. In fact there

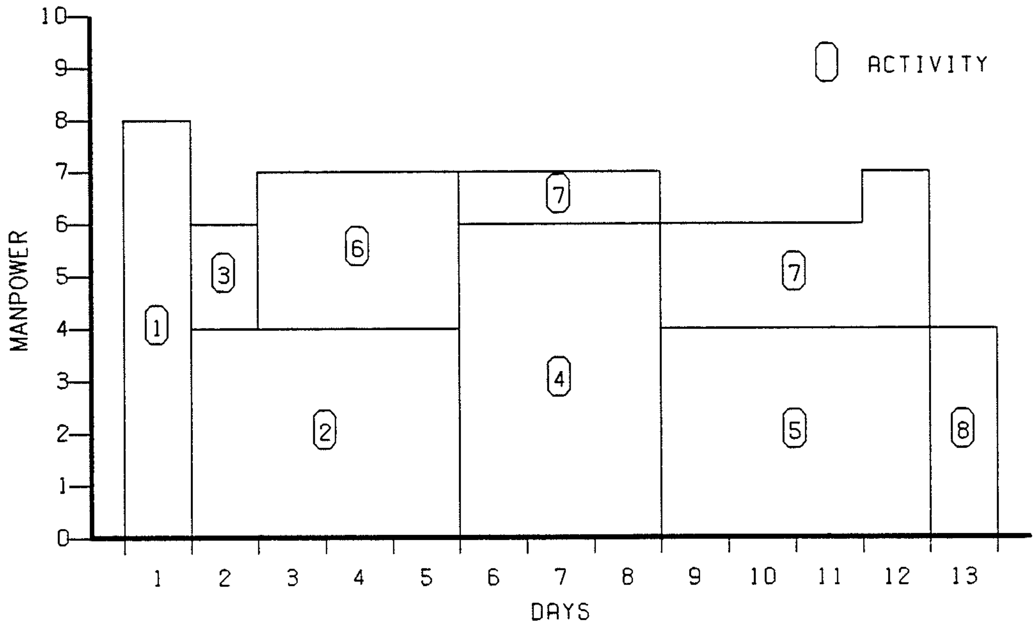

FIGURE 5 SCHEDULE WITH THE OVERALL LEVELLING / CONSECUTIVE DAYS / INTERNAL LEVELLING FORMULATION

appears to be only one improvement to the schedule which can be made. One unit of resource is allocated to activity 7 on days 6-8. A better and more reasonable schedule would be obtained if these resource allocations were rescheduled on days 9-11. The reason that the model did not select the revised schedule is that the internal levelling objective tends to force the model to spread out the resource allocations of the activities where possible. In order to balance this flattening of the resource allocation the final objective to the model was included.

5.4 OVERALL LEVELLING / CONSECUTIVE DAYS / INTERNAL LEVELLING / WINDOW WIDTH MINIMIZATION FORMULATION

When the final objective of minimization of the width of the windows was added, and the model applied to the example project the resource allocation that resulted is shown in Figure 6. The schedule was obtained after varying the relative weights of the three objectives in the objective function of the model. The tradeoff analysis of the relative weight is described later in the paper.

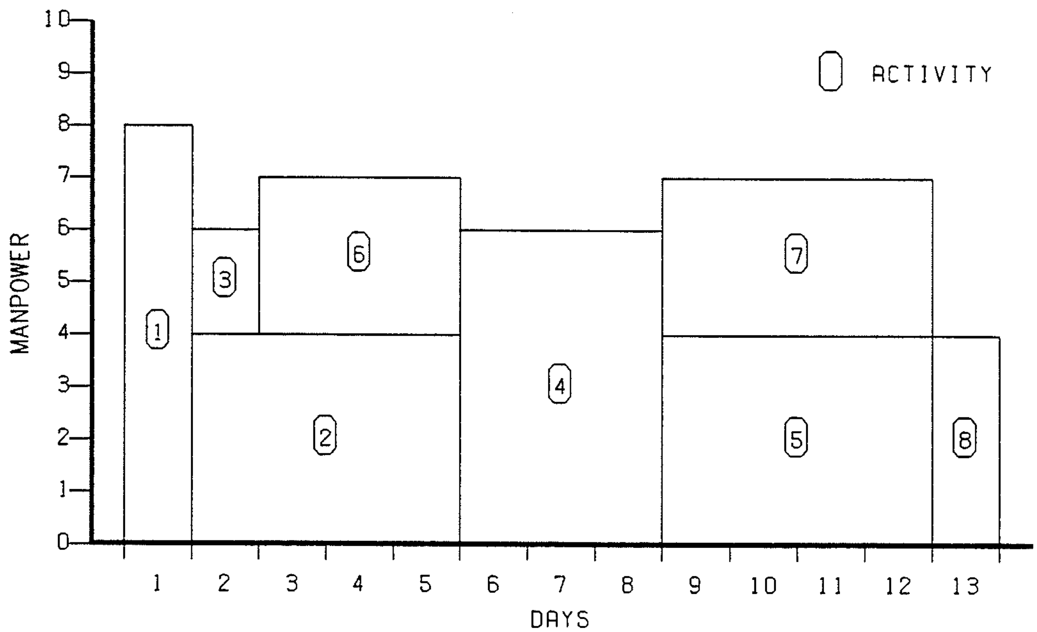

FIGURE 6 SCHEDULE WITH THE OVERALL LEVELLING / CONSECUTIVE DAYS / INTERNAL LEVELLING / WINDOW WIDTH MINIMIZATION FORMULATION

The schedule shows that all the objectives of the model have been achieved to the best possible level. No further improvements to the schedule can be made. The schedule obtained by the model is essentially the same as that obtained by Smith et. al., [1983]. The main difference between the schedules is that Smith et al. schedules activity 7 over its specified duration of 3 days, i.e., 4 units of resource are allocated each day. However the model presented in this paper, schedules activity 7 over 4 days with 3 units of resource per day. This shows the flexibility of the model in selecting the duration of each activity which can vary between one day to a maximum given by the estimated duration (used in the critical path analysis) plus any free float.

Chapter VI

'MULTIOBJECTIVE' ANALYSIS

The model described in the paper has three objectives and consequently the solution is not as straightforward as for normal linear programming models which simply maximizes or minimizes the single objective. As stated earlier, the problem with multiple objective models is that the objectives are often in conflict with each other.

The methods used for dealing with multiple objective problems include goal programming approaches, the constraint methods and weighting methods. Goal programming essentially consists of ranking the objectives, and then satisfying the objectives in the order of the ranking. The constraint method replaces all but the most important objectives by constraints so the problem becomes a single objective problem. The weighting method maintains all the objectives, but places a relative ranking on the objectives by applying different weights in the objective function.

As stated earlier the weighting method is used for dealing with the three objectives of the model presented. Since the objective function of the model is the minimization of weighted values of the three objectives it follows that the relative weights would be highest for the objective with the highest priority. That is, the overall levelling of the resource allocations should have a higher relative weight compared to the other two objectives.

The relative weights used to obtain the schedule shown in Figure 6 are: $k_w = 1$ (window width minimization), $k_a = 2$ (internal levelling), and $k_z = 3$ (overall levelling). This 1-2-3 weighting combination was originally selected on an intuitive basis, as it corresponds (in reverse order) to the ranking of the objectives. However to justify these weights a tradeoff analysis was performed by varying the weights applied to the three objectives. The analysis was limited to varying each weight between 1 and 3 for all possible combinations, this resulted in 27 possible solutions to the project. However 27 unique solutions were not obtained as some weighting combinations gave the same resource allocation. The results of the analysis is shown in Table 2, and reveal that the weights from Figure 6 result in a solution where the model is most stable, i.e., WT=246, AT=383, and ZT=567.

When the results are plotted as shown in Figure 7, a series of contours concave to the origin is obtained. These contours show how the levels of the various objectives interact, i.e., how much of each of two objectives must be given up in order to give improvements in the level of the other objective. It is interesting to note that the 'stable' point discussed above lies on the

Table 2

Multiobjective Tradeoff Analysis

Coefficient			Value		
WT	AT	ZT	WT	AT	ZT
1	1	1	116	463	573
1	1	2	116	471	569
1	1	3	116	475	567
1	2	1	209	397	573
1	2	2	246	383	567
1	2	3	246	383	567
1	3	1	246	383	567
1	3	2	246	383	567
1	3	3	246	383	567
2	1	1	78	523	573
2	1	2	78	523	573
2	1	3	97	495	569
2	2	1	116	463	573
2	2	2	116	463	573
2	2	3	116	463	573
2	3	1	209	395	579
2	3	2	209	397	573
2	3	3	172	423	571
3	1	1	78	523	573
3	1	2	78	527	571
3	1	3	78	527	571
3	2	1	97	489	575
3	2	2	97	491	571
3	2	3	97	491	571
3	3	1	116	461	573
3	3	2	116	463	573
3	3	3	116	463	573

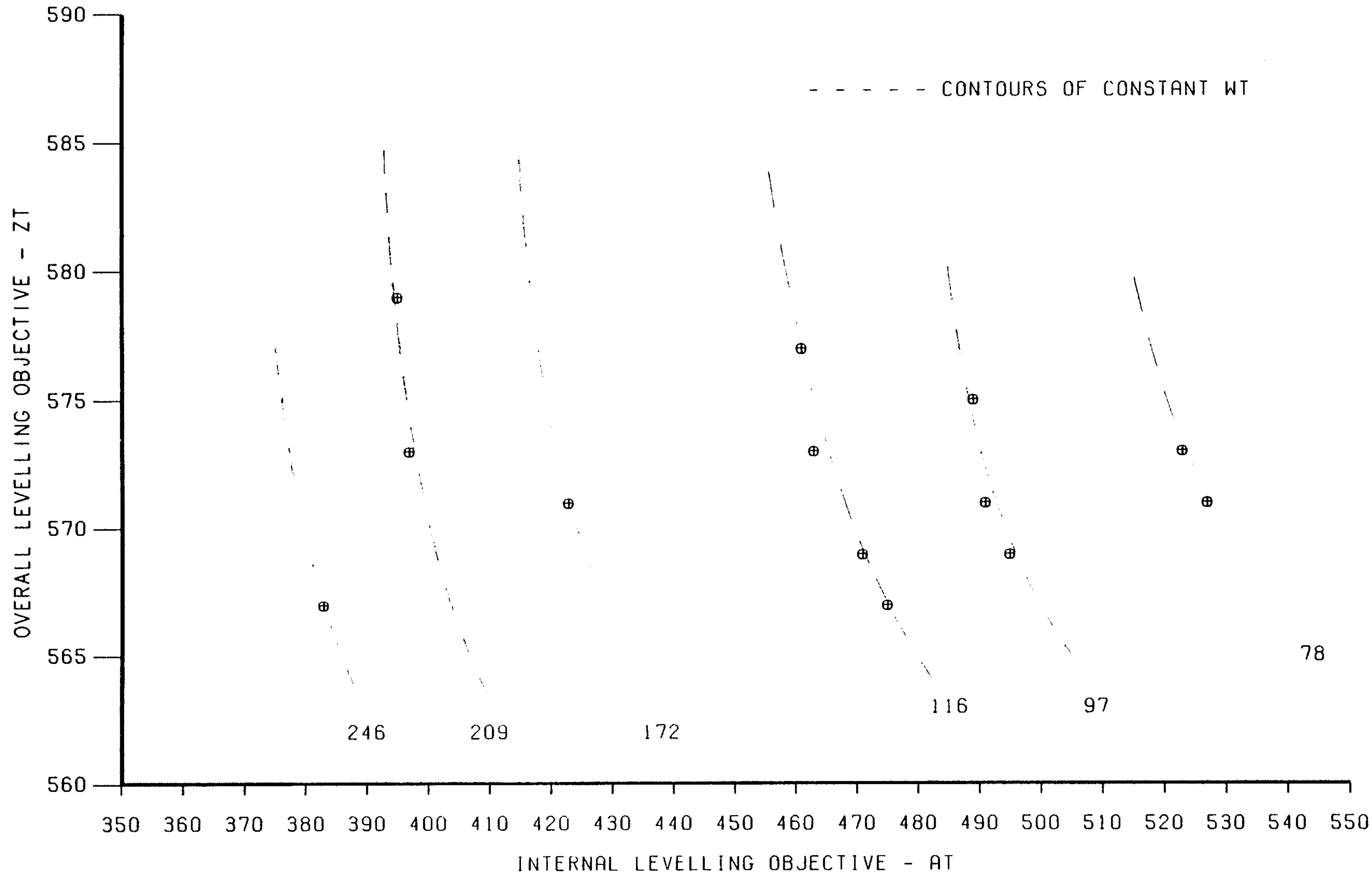

FIGURE 7 TRADEOFF ANALYSIS OF ZT, AT, WT

contour with the highest WT level. The exact combination of the three objectives to be used in a analysis depends on the relative preferences of the model user for the three objectives.

There is, however , further information to be derived from the analysis. Consider for example the 'stable' point discussed before. A computational analysis shows that the weights $k_w = 1$, $k_a = 2$, and $k_z = 3$, are not the most efficient computationaly. All the weighting combinations which define the point WT=246, AT=383, and ZT=567 gives the schedule shown in Figure 6. Therefore the combination which uses the least CPU time would be the most efficient. The CPU times used by these combinations are shown in Table 3. The combination $k_w = 1$, $k_a = 3$, and $k_z = 3$ uses the least CPU time to obtain the schedule,

Table 3

Computational Analysis

for First Example

Coefficient			CPU
WT	AT	ZT	(min)
1	2	2	6.67
1	2	3	6.31
1	3	1	5.56
1	3	2	5.21
1	3	3	3.01

indicating that these should be the weights used in the model for this example. A similar analysis can be applied to other points.

For the second example, which is presented in the next section, the 1-2-3 weights used slightly less CPU time than the 1-3-3 weights (32 min. 25 sec. vs 39 min. 44 sec.). However the results show that the schedule corresponding to the 1-3-3 weights lie on a WT contour which is closer to the origin than the 1-2-3 combination and therefore other issues become involved.

The exact combination of weights to be used will be problem dependent, and if the 'correct' combination is needed then an analysis similar to that described above must be performed.

Chapter VII

A SECOND EXAMPLE

Since the model was developed by application to an example project, it was felt necessary to apply it to a second example to see if it would achieve the stated objectives. Because restrictions on the computer facilities at the University of Manitoba, it was not possible to solve a large project, as CPU times in excess of one hour would be required. However a slightly larger project than the first was solved. The project, shown in Figure 8, consists of 10 activities and has a total duration of 14 days. The resource requirements are given in Table 4.

The early start schedule is shown in Figure 9 and the result of the model is shown in Figure 10. These figures shown that the model has achieved its objectives of overall resurce levelling, internal levelling and window width minimization subject to the constraint of scheduling activties on consecutive days. The weights used to obtain the schedule in Figure 10 are : $k_w = 1$, $k_a = 3$, $k_z = 3$. It can be seen that the model works equally effective for the second case.

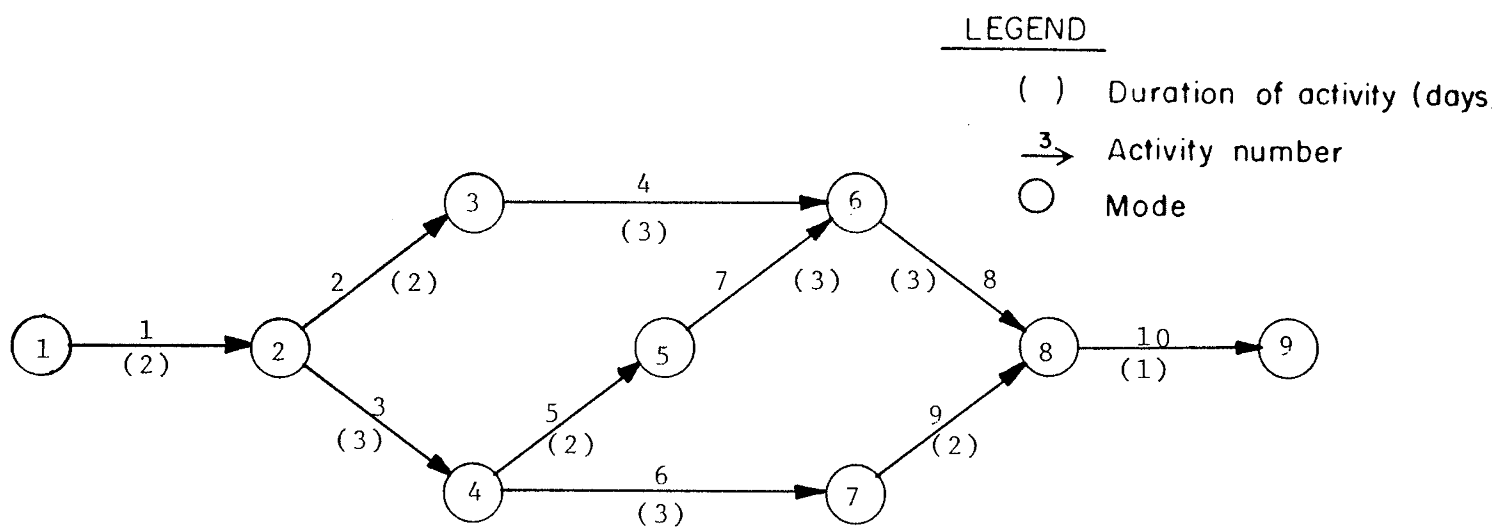

Figure 8. Network for Second Example Project

Table 4

Description of Activities for Second Example

Activity	Manpower Requirements/day	Total Man-days
1	7	14
2	3	6
3	4	12
4	3	9
5	3	6
6	2	6
7	5	15
8	6	18
9	3	6
10	6	6

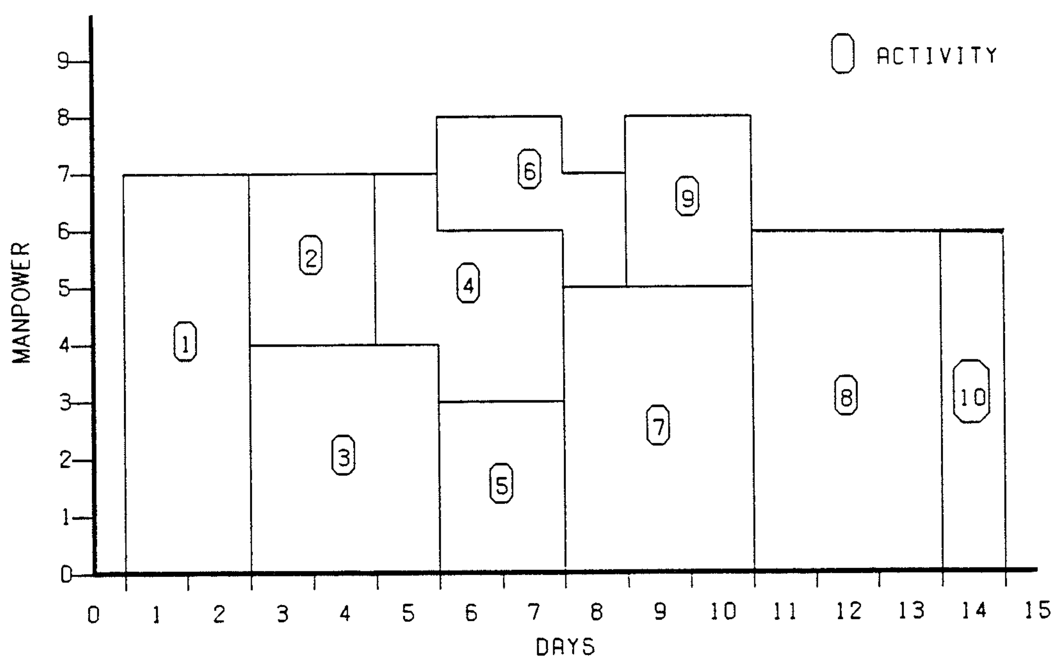

FIGURE 9 EARLY START SCHEDULE FOR SECOND EXAMPLE

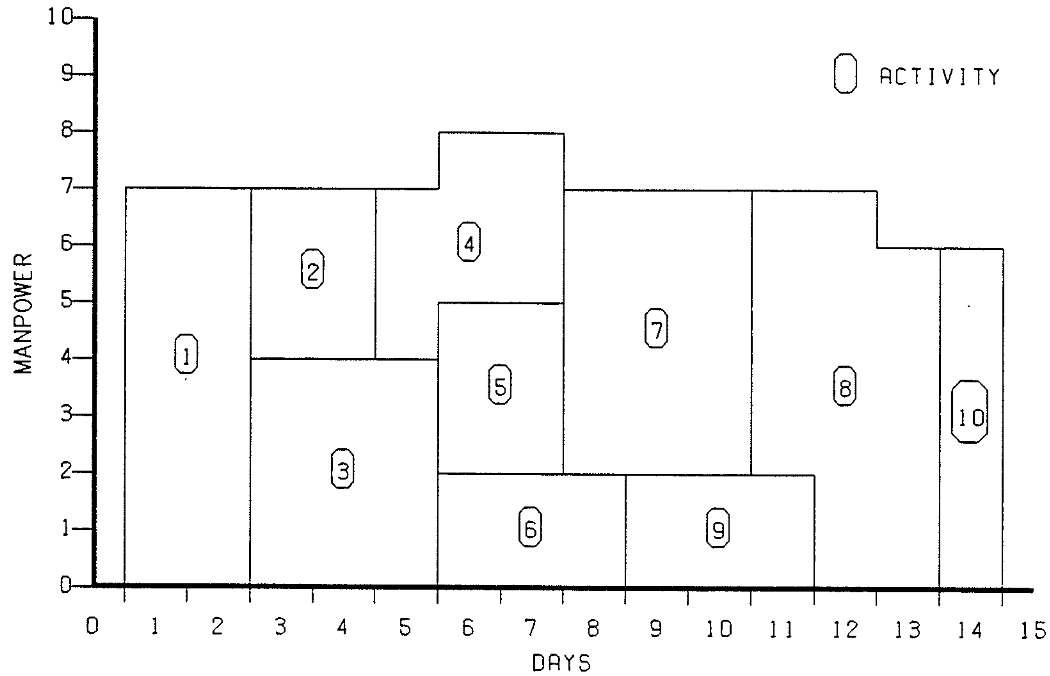

FIGURE 10 FINAL SCHEDULE FOR SECOND EXAMPLE

Chapter VIII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

A mixed integer optimization model has been developed for the allocation of a single resource in scheduling of projects. Although the model was developed for application in the construction industry, it can be applied to other project management areas. The model is based on the results of the critical path analysis of the project along with the estimated resource requirements of the individual activities.

The two examples described in the paper show that the model is capable of scheduling projects as defined by the objectives, and subject to the constraints imposed. The model schedules the project such that the minimum duration as defined by the critical path analysis is not increased. However the estimated duration of the activities used in the critical path analysis are not a constraint in the model. The scheduled duration of the activities is determined by the model, thus increasing the flexibility of the model. The other constraints in the model are that sufficient resources must be allocated to each activities, and the activities must be scheduled on consecutive days.

The three objectives of the model are handled by applying different weights to them in the objective function. A limited tradeoff analysis of the weights show that the weightings of 1-2-3 or 1-3-3 are acceptable when applied to the objectives - window width minimization, internal levelling and overall levelling respectively.

Practical application of the model is presently restricted because of the extensive amount of computer time necessary to obtain the final solution to the model. However it is hoped that when more efficient branch and bound algorithms are developed, the model will have some practical applications. Although the model theoretically has three sets of integer variables, only one set (XW's) are explicitly defined as integers thus significantly reducing the amount of branch and bound necessary for solution.

Recommendations

The three objective are incorporated into the model by applying different weights to them in the objective function. Further research on the model could involve a more extensive investigation of the weightings to be used in the objective function. Also, an investigation of why the XI variables cannot be explicitly defined as integers is needed. The reason may result in a method of writing the equations which reduces the number of integer variables to be included in the model, therefore resulting in a more efficient model.

The model was developed by considering only a single resource. However projects usually require scheduling giving consideration to multiple resources. Therefore the next major expansion to the model would be ability of the model to schedule projects with multiple resource demands.

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APPENDIX A

Results of the Critical Path Analysis of Example Projects

First Example Project

Activity	Node		Duration of Activity (days)	Starting Time		Float	
	From	To		Earliest (days)	Latest (days)	Total	Free
* 1	1	2	1	1	1	0	0
* 2	2	3	4	2	2	0	0
3	2	5	1	2	6	4	0
* 4	3	4	3	6	6	0	0
* 5	4	7	4	9	9	0	0
6	5	6	3	3	7	4	0
7	6	7	3	6	10	4	4
* 8	7	8	1	13	13	0	0

Second Example Project

Activity	Node From	Node To	Duration of Activity (days)	Starting Time Earliest (days)	Starting Time Latest (days)	Float Total	Float Free
* 1	1	2	2	1	1	0	0
2	2	3	2	3	6	3	0
* 3	2	4	3	3	3	0	0
4	3	6	3	5	8	3	3
* 5	4	5	2	6	6	0	0
6	4	7	3	6	9	3	0
* 7	5	6	3	8	8	0	0
* 8	6	8	3	11	11	0	0
9	7	8	2	9	12	3	3
*10	8	9	1	14	14	0	0

APPENDIX B

Equations of Model for First Example Project

```
MIN 1WT+  2AT +  3ZT

S.T.

Z01+Z02+Z03+Z04+Z05+Z06+Z07+Z08+Z09+Z10+Z11+Z12+Z13-ZT=0

A0101+A0202+A0203+A0204+A0205+A0302+A0406+A0407+A0408+A0509
+A0510+A0511+A0512+A0603+A0604+A0605+A0706+A0707+A0708+A0709
+A0710+A0711+A0712+A0813-AT=0

X0101= 8
X0202+X0203+X0204+X0205=16
X0302= 2
X0406+X0407+X0408=18
X0509+X0510+X0511+X0512=16
X0603+X0604+X0605= 9
X0706+X0707+X0708+X0709+X0710+X0711+X0712=12
X0813= 4

-Y01+X0101=0
-Y02+X0202+X0302=0
-Y03+X0203+X0603=0
-Y04+X0204+X0604=0
-Y05+X0205+X0605=0
-Y06+X0406+X0706=0
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-Y08+X0408+X0708=0
-Y09+X0509+X0709=0
-Y10+X0510+X0710=0
-Y11+X0511+X0711=0
-Y12+X0512+X0712=0
-Y13+X0813=0

-X0101+00XI000101+01XI010101+02XI020101+03XI030101
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     +016XI040709+025XI050709+036XI060709+049XI070709
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-A0710+000XI000710+001XI010710+004XI020710+009XI030710
     +016XI040710+025XI050710+036XI060710+049XI070710
     +064XI080710=0
-A0711+000XI000711+001XI010711+004XI020711+009XI030711
     +016XI040711+025XI050711+036XI060711+049XI070711
     +064XI080711=0
-A0712+000XI000712+001XI010712+004XI020712+009XI030712
     +016XI040712+025XI050712+036XI060712+049XI070712
     +064XI080712=0
-A0813+000XI000813+001XI010813+004XI020813+009XI030813
     +016XI040813+025XI050813+036XI060813+049XI070813
     +064XI080813=0

-Y01+00YI0001+01YI0101+02YI0201+03YI0301+04YI0401+05YI0501
    +06YI0601+07YI0701+08YI0801=0
-Y02+00YI0002+01YI0102+02YI0202+03YI0302+04YI0402+05YI0502
    +06YI0602+07YI0702+08YI0802=0
-Y03+00YI0003+01YI0103+02YI0203+03YI0303+04YI0403+05YI0503
    +06YI0603+07YI0703+08YI0803=0
-Y04+00YI0004+01YI0104+02YI0204+03YI0304+04YI0404+05YI0504
    +06YI0604+07YI0704+08YI0804=0
-Y05+00YI0005+01YI0105+02YI0205+03YI0305+04YI0405+05YI0505
    +06YI0605+07YI0705+08YI0805=0
-Y06+00YI0006+01YI0106+02YI0206+03YI0306+04YI0406+05YI0506
    +06YI0606+07YI0706+08YI0806=0
-Y07+00YI0007+01YI0107+02YI0207+03YI0307+04YI0407+05YI0507
    +06YI0607+07YI0707+08YI0807=0
-Y08+00YI0008+01YI0108+02YI0208+03YI0308+04YI0408+05YI0508
    +06YI0608+07YI0708+08YI0808=0
-Y09+00YI0009+01YI0109+02YI0209+03YI0309+04YI0409+05YI0509
    +06YI0609+07YI0709+08YI0809=0
-Y10+00YI0010+01YI0110+02YI0210+03YI0310+04YI0410+05YI0510
    +06YI0610+07YI0710+08YI0810=0
-Y11+00YI0011+01YI0111+02YI0211+03YI0311+04YI0411+05YI0511
    +06YI0611+07YI0711+08YI0811=0
-Y12+00YI0012+01YI0112+02YI0212+03YI0312+04YI0412+05YI0512
    +06YI0612+07YI0712+08YI0812=0
-Y13+00YI0013+01YI0113+02YI0213+03YI0313+04YI0413+05YI0513
    +06YI0613+07YI0713+08YI0813=0
```

```
+YI0001+YI0101+YI0201+YI0301+YI0401+YI0501+YI0601+YI0701
       +YI0801=1
+YI0002+YI0102+YI0202+YI0302+YI0402+YI0502+YI0602+YI0702
       +YI0802=1
+YI0003+YI0103+YI0203+YI0303+YI0403+YI0503+YI0603+YI0703
       +YI0803=1
+YI0004+YI0104+YI0204+YI0304+YI0404+YI0504+YI0604+YI0704
       +YI0804=1
+YI0005+YI0105+YI0205+YI0305+YI0405+YI0505+YI0605+YI0705
       +YI0805=1
+YI0006+YI0106+YI0206+YI0306+YI0406+YI0506+YI0606+YI0706
       +YI0806=1
+YI0007+YI0107+YI0207+YI0307+YI0407+YI0507+YI0607+YI0707
       +YI0807=1
+YI0008+YI0108+YI0208+YI0308+YI0408+YI0508+YI0608+YI0708
       +YI0808=1
+YI0009+YI0109+YI0209+YI0309+YI0409+YI0509+YI0609+YI0709
       +YI0809=1
+YI0010+YI0110+YI0210+YI0310+YI0410+YI0510+YI0610+YI0710
       +YI0810=1
+YI0011+YI0111+YI0211+YI0311+YI0411+YI0511+YI0611+YI0711
       +YI0811=1
+YI0012+YI0112+YI0212+YI0312+YI0412+YI0512+YI0612+YI0712
       +YI0812=1
+YI0013+YI0113+YI0213+YI0313+YI0413+YI0513+YI0613+YI0713
       +YI0813=1

-Z01+000YI0001+001YI0101+004YI0201+009YI0301+016YI0401
    +025YI0501+036YI0601+049YI0701+064YI0801=0
-Z02+000YI0002+001YI0102+004YI0202+009YI0302+016YI0402
    +025YI0502+036YI0602+049YI0702+064YI0802=0
-Z03+000YI0003+001YI0103+004YI0203+009YI0303+016YI0403
    +025YI0503+036YI0603+049YI0703+064YI0803=0
-Z04+000YI0004+001YI0104+004YI0204+009YI0304+016YI0404
    +025YI0504+036YI0604+049YI0704+064YI0804=0
-Z05+000YI0005+001YI0105+004YI0205+009YI0305+016YI0405
    +025YI0505+036YI0605+049YI0705+064YI0805=0
-Z06+000YI0006+001YI0106+004YI0206+009YI0306+016YI0406
    +025YI0506+036YI0606+049YI0706+064YI0806=0
-Z07+000YI0007+001YI0107+004YI0207+009YI0307+016YI0407
    +025YI0507+036YI0607+049YI0707+064YI0807=0
-Z08+000YI0008+001YI0108+004YI0208+009YI0308+016YI0408
    +025YI0508+036YI0608+049YI0708+064YI0808=0
-Z09+000YI0009+001YI0109+004YI0209+009YI0309+016YI0409
    +025YI0509+036YI0609+049YI0709+064YI0809=0
-Z10+000YI0010+001YI0110+004YI0210+009YI0310+016YI0410
    +025YI0510+036YI0610+049YI0710+064YI0810=0
-Z11+000YI0011+001YI0111+004YI0211+009YI0311+016YI0411
    +025YI0511+036YI0611+049YI0711+064YI0811=0
-Z12+000YI0012+001YI0112+004YI0212+009YI0312+016YI0412
    +025YI0512+036YI0612+049YI0712+064YI0812=0
```

```
-Z13+000YI0013+001YI0113+004YI0213+009YI0313+016YI0413
    +025YI0513+036YI0613+049YI0713+064YI0813=0

+X0202-16XW020102=>0
+X0202+X0203-16XW020202=>0
+X0202+X0203+X0204-16XW020302=>0
+X0202+X0203+X0204+X0205-16XW020402=>0
+X0203-16XW020103=>0
+X0203+X0204-16XW020203=>0
+X0203+X0204+X0205-16XW020303=>0
+X0204-16XW020104=>0
+X0204+X0205-16XW020204=>0
+X0205-16XW020105=>0
+X0406-18XW040106=>0
+X0406+X0407-18XW040206=>0
+X0406+X0407+X0408-18XW040306=>0
+X0407-18XW040107=>0
+X0407+X0408-18XW040207=>0
+X0408-18XW040108=>0
+X0509-16XW050109=>0
+X0509+X0510-16XW050209=>0
+X0509+X0510+X0511-16XW050309=>0
+X0509+X0510+X0511+X0512-16XW050409=>0
+X0510-16XW050110=>0
+X0510+X0511-16XW050210=>0
+X0510+X0511+X0512-16XW050310=>0
+X0511-16XW050111=>0
+X0511+X0512-16XW050211=>0
+X0512-16XW050112=>0
+X0603- 9XW060103=>0
+X0603+X0604- 9XW060203=>0
+X0603+X0604+X0605- 9XW060303=>0
+X0604- 9XW060104=>0
+X0604+X0605- 9XW060204=>0
+X0605- 9XW060105=>0
+X0706-12XW070106=>0
+X0706+X0707-12XW070206=>0
+X0706+X0707+X0708-12XW070306=>0
+X0706+X0707+X0708+X0709-12XW070406=>0
+X0706+X0707+X0708+X0709+X0710-12XW070506=>0
+X0706+X0707+X0708+X0709+X0710+X0711-12XW070606=>0
+X0706+X0707+X0708+X0709+X0710+X0711+X0712-12XW070706=>0
+X0707-12XW070107=>0
+X0707+X0708-12XW070207=>0
+X0707+X0708+X0709-12XW070307=>0
+X0707+X0708+X0709+X0710-12XW070407=>0
+X0707+X0708+X0709+X0710+X0711-12XW070507=>0
+X0707+X0708+X0709+X0710+X0711+X0712-12XW070607=>0
+X0708-12XW070108=>0
+X0708+X0709-12XW070208=>0
+X0708+X0709+X0710-12XW070308=>0
```

```
+X0708+X0709+X0710+X0711-12XW070408=>0
+X0708+X0709+X0710+X0711+X0712-12XW070508=>0
+X0709-12XW070109=>0
+X0709+X0710-12XW070209=>0
+X0709+X0710+X0711-12XW070309=>0
+X0709+X0710+X0711+X0712-12XW070409=>0
+X0710-12XW070110=>0
+X0710+X0711-12XW070210=>0
+X0710+X0711+X0712-12XW070310=>0
+X0711-12XW070111=>0
+X0711+X0712-12XW070211=>0
+X0712-12XW070112=>0

+XW020102+XW020202+XW020302+XW020402+XW020103+XW020203
          +XW020303+XW020104+XW020204+XW020105=1
+XW040106+XW040206+XW040306+XW040107+XW040207+XW040108=1
+XW050109+XW050209+XW050309+XW050409+XW050110+XW050210
          +XW050310+XW050111+XW050211+XW050112=1
+XW060103+XW060203+XW060303+XW060104+XW060204+XW060105=1
+XW070106+XW070206+XW070306+XW070406+XW070506+XW070606
          +XW070706+XW070107+XW070207+XW070307+XW070407
          +XW070507+XW070607+XW070108+XW070208+XW070308
          +XW070408+XW070508+XW070109+XW070209+XW070309
          +XW070409+XW070110+XW070210+XW070310+XW070111
          +XW070211+XW070112=1

X0203-XW020302-XW020402=>0
X0204-XW020303-XW020402=>0
X0407-XW040306=>0
X0510-XW050309-XW050409=>0
X0511-XW050310-XW050409=>0
X0604-XW060303=>0
X0707-XW070306-XW070406-XW070506-XW070606-XW070706=>0
X0708-XW070307-XW070406-XW070407-XW070506-XW070507-XW070606
     -XW070607-XW070706=>0
X0709-XW070308-XW070407-XW070408-XW070506-XW070507-XW070508
     -XW070606-XW070607-XW070706=>0
X0710-XW070309-XW070408-XW070409-XW070507-XW070508-XW070606
     -XW070607-XW070706=>0
X0711-XW070310-XW070409-XW070508-XW070607-XW070706=>0

-W02+   1XW020102+  8XW020202+ 27XW020302+ 64XW020402
    +   1XW020103+  8XW020203+ 27XW020303+  1XW020104
    +   8XW020204+  1XW020105=0
-W04+   1XW040106+  8XW040206+ 27XW040306+  1XW040107
    +   8XW040207+  1XW040108=0
-W05+   1XW050109+  8XW050209+ 27XW050309+ 64XW050409
    +   1XW050110+  8XW050210+ 27XW050310+  1XW050111
    +   8XW050211+  1XW050112=0
-W06+   1XW060103+  8XW060203+ 27XW060303+  1XW060104
    +   8XW060204+  1XW060105=0
```

```
-W07+  1XW070106+  8XW070206+ 27XW070306+ 64XW070406
    +125XW070506+216XW070606+343XW070706+  1XW070107
    +  8XW070207+ 27XW070307+ 64XW070407+125XW070507
    +216XW070607+  1XW070108+  8XW070208+ 27XW070308
    + 64XW070408+125XW070508+  1XW070109+  8XW070209
    + 27XW070309+ 64XW070409+  1XW070110+  8XW070210
    + 27XW070310+  1XW070111+  8XW070211+  1XW070112=0

-WT+W02+W04+W05+W06+W07=0

END

INTEGER XW020102
INTEGER XW020202
INTEGER XW020302
INTEGER XW020402
INTEGER XW020103
INTEGER XW020203
INTEGER XW020303
INTEGER XW020104
INTEGER XW020204
INTEGER XW020105
INTEGER XW040106
INTEGER XW040206
INTEGER XW040306
INTEGER XW040107
INTEGER XW040207
INTEGER XW040108
INTEGER XW050109
INTEGER XW050209
INTEGER XW050309
INTEGER XW050409
INTEGER XW050110
INTEGER XW050210
INTEGER XW050310
INTEGER XW050111
INTEGER XW050211
INTEGER XW050112
INTEGER XW060103
INTEGER XW060203
INTEGER XW060303
INTEGER XW060104
INTEGER XW060204
INTEGER XW060105
INTEGER XW070106
INTEGER XW070206
INTEGER XW070306
INTEGER XW070406
INTEGER XW070506
INTEGER XW070606
INTEGER XW070706
```

```
INTEGER XW070107
INTEGER XW070207
INTEGER XW070307
INTEGER XW070407
INTEGER XW070507
INTEGER XW070607
INTEGER XW070108
INTEGER XW070208
INTEGER XW070308
INTEGER XW070408
INTEGER XW070508
INTEGER XW070109
INTEGER XW070209
INTEGER XW070309
INTEGER XW070409
INTEGER XW070110
INTEGER XW070210
INTEGER XW070310
INTEGER XW070111
INTEGER XW070211
INTEGER XW070112
LEAVE
```

APPENDIX C

Program for Generating the Equations

```
C     THIS PROGRAM GENERATES THE EQUATIONS REQUIRED BY THE
C     MODEL PRESENTED.  THE VARIABLES AND EQUATIONS CORRESPOND
C     TO THOSE DEFINED IN THE PAPER.
C
C     *** NOTE *** THE OBJECTIVE FUNCTION WILL HAVE TO BE
C                  MODIFIED DEPENDING ON THE WEIGHTS USED.
C
C     ES = EARLY START TIME OF ACTIVITY
C     LF = LATE FINISH TIME OF ACTIVITY
C     TR = TOTAL RESOURCE REQUIREMENT OF ACTIVITY
C     NA = NUMBER OF ACTIVITIES
C     ND = DURATION OF PROJECT
C     MAX = MAXIMUM RESOURCE ALLOCACTED PER PREIOD
C
C
C
      CHARACTER*80 BUFFER
      INTEGER ES, LF, TR, A1, A2, D1, D2, C1, C2, C3
      INTEGER CS1, CS2, CS3, CD, S1, S2, D, CSQD
      DIMENSION ES(99), LF(99), TR(99)
      READ (5,*) NA, ND, MAX, K1, K2
      DO 100 L=1,NA
         READ (5,*) I, ES(L), LF(L), TR(L)
  100 CONTINUE
C
C OBJECTIVE FUNCTION : EQN 19
C
      BUFFER=' '
      BUFFER(1:14) = 'MIN 3ZT+2AT+WT'
      WRITE(4,2000)BUFFER
      BUFFER= ' '
      BUFFER(1:4)='S.T.'
      WRITE(4,2000) BUFFER
      BUFFER=' '
C
C OBJECTIVE FUNCION EQUATION : EQN 14
C
      I1=1
      I2=3
      DO 200 L=1,ND
         D1=L/10
         D2=L-10*D1
         WRITE (BUFFER(I1:I2), '(''Z'',2I1)') D1, D2
         IF(L.EQ.ND) THEN
           BUFFER(I2+1:I2+5)='-ZT=0'
           GOTO 210
         ELSE
           BUFFER(I2+1:I2+1)='+'
           I1=I1+4
           I2=I2+4
         ENDIF
```

```
          IF(I2.LE.60) GO TO 200
  210     WRITE(4,2000) BUFFER
          BUFFER=' '
          I1=1
          I2=3
  200 CONTINUE
C
C INTERNAL LEVELLING OF ACTIVITY RESOURCE REQUIREMENT : EQN 15
C
      I1=1
      I2=6
      DO 1400 L=1,NA
      A1=L/10
      A2=L-10*A1
      DO 1410 L1=ES(L),LF(L)
         D1=L1/10
         D2=L1-10*D1
         WRITE(BUFFER(I1:I2),'(''A'',4I1,''+'')')A1,A2,D1,D2
         I1=I1+6
         I2=I2+6
         IF(L1.EQ.LF(L)) GO TO 1410
         IF(I2.LE.60) GO TO 1410
            WRITE(4,2000) BUFFER
            BUFFER=' '
            I1=1
            I2=6
 1410 CONTINUE
 1400 CONTINUE
 2001 FORMAT(5I10)
 2000 FORMAT(A80)
      BUFFER(I1-1:I1+3)='-AT=0'
      WRITE(4,2000) BUFFER
      BUFFER=' '
C     WRITE(BUFFER(1:7),'(''AT<'',I4)')K1
C     WRITE(4,2000) BUFFER
C     BUFFER=' '


C
C ACTIVITY REQUIREMENTS : EQN 2
C
      DO 300 L=1,NA
      A1=L/10
      A2=L-10*A1
      I1=1
      I2=5
      DO 320 L1=ES(L),LF(L)
         D1=L1/10
         D2=L1-10*D1
         WRITE (BUFFER(I1:I2), '(''X'',4I1)') A1, A2, D1, D2
         IF (L1.NE.LF(L)) THEN
```

```
          BUFFER(I2+1:I2+1)='+'
        ELSE
          BUFFER(I2+1:I2+1)='='
          WRITE (BUFFER(I2+2:I2+4),'(I2)') TR(L)
          GO TO 310
        ENDIF
        I1=I1+6
        I2=I2+6
        IF(I2.LE.60) GO TO 320
 310    WRITE(4,2000) BUFFER
        BUFFER=' '
        I1=1
        I2=5
 320 CONTINUE
 300 CONTINUE
C
C TOTAL DAILY RESOURCE REQUIREMENT : EQN 5
C
     DO 400 L=1,ND
     D1=L/10
     D2=L-10*D1
     WRITE (BUFFER(1:4),'(''-Y'',2I1)') D1, D2
     I1=5
     I2=10
     DO 410 L1=1,NA
        A1=L1/10
        A2=L1-10*A1
        IF((L.LT.ES(L1)).OR.(L.GT.LF(L1))) GO TO 420
          WRITE (BUFFER(I1:I2),'(''+X'',4I1)') A1, A2, D1, D2
          I1=I1+6
          I2=I2+6
 420      IF(L1.NE.NA) GO TO 430
            BUFFER(I1:I1+1)='=0'
            GO TO 440
 430      IF (I2.LE.60) GO TO 410
 440        WRITE(4,2000) BUFFER
            BUFFER=' '
            I1=5
            I2=10
 410 CONTINUE
 400 CONTINUE
C
C RANGE OF INTEGER VALUES FOR X(I,J) : EQN 3
C
     DO 500 L=1,NA
     A1=L/10
     A2=L-10*A1
     DO 510 L1=ES(L),LF(L)
        D1=L1/10
        D2=L1-10*D1
        WRITE (BUFFER(1:6),'(''-X'',4I1)') A1, A2, D1, D2
```

```
        I1=7
        I2=17
        DO 520  L2=0,MAX
           C1=L2/10
           C2=L2-10*C1
           WRITE(BUFFER(I1:I2),'(''+'',2I1,''XI'',6I1)') C1,C2,
     $C1,C2,A1,A2,D1,D2
           I1=I1+11
           I2=I2+11
           IF(L2.NE.MAX) GO TO 530
             BUFFER(I1:I1+1)='=0'
             GO TO 540
  530      IF(I2.LE.60) GO TO 520
  540        WRITE(4,2000) BUFFER
             BUFFER = ' '
             I1=7
             I2=17
  520     CONTINUE
  510 CONTINUE
  500 CONTINUE
C
C SELECTING ONE INTEGER VALUE FOR X(I,J) : EQN 4
C
      DO 600 L=1,NA
      A1=L/10
      A2=L-10*A1
      DO 610 L1=ES(L),LF(L)
         D1=L1/10
         D2=L1-10*D1
         I1=1
         I2=9
         DO 620  L2=0,MAX
            C1=L2/10
            C2=L2-10*C1
            WRITE(BUFFER(I1:I2),'(''+XI'',6I1)') C1, C2, A1,
     $A2, D1, D2
            I1=I1+9
            I2=I2+9
            IF(L2.NE.MAX) GO TO 630
              BUFFER(I1:I1+1)='=1'
              GO TO 640
  630       IF(I2.LE.60) GO TO 620
  640         WRITE(4,2000) BUFFER
              BUFFER = ' '
              I1=10
              I2=18
  620     CONTINUE
  610 CONTINUE
  600 CONTINUE
C
C INTERNAL LEVELLING OF ACTIVITY : EQN 13
```

```
C
      BUFFER=' '
      DO 1300 L=1,NA
      A1=L/10
      A2=L-10*A1
         DO 1310 L1=ES(L),LF(L)
         D1=L1/10
         D2=L1-10*D1
         WRITE(BUFFER(1:6),'(''-A'',4I1)')A1,A2,D1,D2
         I1=7
         I2=18
         DO 1310 L2=0,MAX
            M1=L2*L2
            CS1=M1/100
            M2=M1-100*CS1
            CS2=M2/10
            CS3=M2-10*CS2
            C1=L2/10
            C2=L2-10*C1
            WRITE(BUFFER(I1:I2),'(''+'',3I1,''XI'',6I1)')CS1,CS2,
     $CS3,C1,C2,A1,A2,D1,D2
            I1=I1+12
            I2=I2+12
            IF(L2.NE.MAX) GO TO 1330
              BUFFER(I1:I1+1)='=0'
              GO TO 1340
 1330       IF (I2.LE.60) GO TO 1320
 1340         WRITE(4,2000) BUFFER
              BUFFER=' '
              I1=7
              I2=18
 1320    CONTINUE
 1310 CONTINUE
 1300 CONTINUE
C
C RANGE OF INTEGER VALUES FOR Y(J) : EQN 6
C
      DO 700 L=1,ND
      D1=L/10
      D2=L-10*D1
      WRITE(BUFFER(1:4),'(''-Y'',2I1)') D1,D2
      I1=5
      I2=13
      DO 710 L1=0,MAX
         C1=L1/10
         C2=L1-10*C1
         WRITE(BUFFER(I1:I2),'(''+'',2I1,''YI'',4I1)') C1,C2,
     $C1,C2,D1,D2
         I1=I1+9
         I2=I2+9
         IF(L1.NE.MAX) GO TO 720
```

```
           BUFFER(I1:I1+1)='=0'
           GO TO 730
  720    IF (I2.LE.60) GO TO 710
  730      WRITE(4,2000) BUFFER
           BUFFER=' '
           I1=5
           I2=13
  710 CONTINUE
  700 CONTINUE
C
C SELECTING ONE INTEGER VALUE FOR Y(J) : EQN 7
C
      DO 800 L=1,ND
      D1=L/10
      D2=L-10*D1
      I1=1
      I2=7
      DO 810 L1=0,MAX
         C1=L1/10
         C2=L1-10*C1
         WRITE(BUFFER(I1:I2),'(''+YI'',4I1)')C1,C2,D1,D2
         I1=I1+7
         I2=I2+7
         IF(L1.NE.MAX) GO TO 820
           BUFFER(I1:I1+1)='=1'
           GO TO 830
  820    IF(I2.LE.60) GO TO 810
  830      WRITE(4,2000) BUFFER
           BUFFER=' '
           I1=8
           I2=15
  810 CONTINUE
  800 CONTINUE
C
C SQUARING THE COEFFICIENT OF Y(J) : EQN 8
C
      DO 900 L=1,ND
      D1=L/10
      D2=L-10*D1
      WRITE(BUFFER(1:4),'(''-Z'',2I1)') D1,D2
      I1=5
      I2=14
      DO 910 L1=0,MAX
         M1=L1*L1
         CS1=M1/100
         M2=M1-100*CS1
         CS2=M2/10
         CS3=M2-10*CS2
         C1=L1/10
         C2=L1-10*C1
         WRITE(BUFFER(I1:I2),'(''+'',3I1,''YI'',4I1)')CS1,CS2,
```

```
     $CS3,C1,C2,D1,D2
         I1=I1+10
         I2=I2+10
         IF(L1.NE.MAX) GO TO 920
           BUFFER(I1:I1+1)='=0'
           GO TO 930
  920    IF (I2.LE.60) GO TO 910
  930      WRITE(4,2000) BUFFER
           BUFFER=' '
           I1=5
           I2=14
  910 CONTINUE
  900 CONTINUE
C
C WINDOW CONSTRAINT : EQN 10
C
      BUFFER=' '
      DO 1000 L=1,NA
      CD=LF(L)-ES(L)+1
      IF(CD.LT.3) GO TO 1000
      A1=L/10
      A2=L-10*A1
      DO 1010 L1=ES(L),LF(L)
         S1=L1/10
         S2=L1-10*S1
         DO 1020 L2=1,CD
            I1=1
            I2=6
            C1=L2/10
            C2=L2-10*C1
            DO 1030 L3=1,L2
               D=L1+L3-1
               D1=D/10
               D2=D-10*D1
               WRITE(BUFFER(I1:I2),'(''+X'',4I1)')A1,A2,D1,D2
               I1=I1+6
               I2=I2+6
               IF(L3.NE.L2) GO TO 1040
               WRITE(BUFFER(I1:I2+8),'(''-'',I2,''XW'',6I1,''=>0'
     $')')TR(L),A1,A2,C1,C2,S1,S2
               GO TO 1050
 1040          IF(I2.LE.48) GO TO 1030
 1050           WRITE(4,2000) BUFFER
                BUFFER=' '
                I1=7
                I2=12
 1030       CONTINUE
 1020    CONTINUE
         CD=CD-1
 1010 CONTINUE
 1000 CONTINUE
```

```
C
C WINDOW CONSTRAINT : EQN 11
C
      DO 1100 L=1,NA
      I1=1
      I2=9
      CD=LF(L)-ES(L)+1
      IF(CD.LT.3) GO TO 1100
      A1=L/10
      A2=L-10*A1
      DO 1110 L1=ES(L),LF(L)
         S1=L1/10
         S2=L1-10*S1
         DO 1120 L2=1,CD
            C1=L2/10
            C2=L2-10*C1
            WRITE(BUFFER(I1:I2+8),'(''+XW'',6I1)')A1,A2,C1,C2,
     $S1,S2
            I1=I1+9
            I2=I2+9
            IF(LF(L).NE.L1) GO TO 1130
              BUFFER(I1:I1+1)='=1'
              GO TO 1140
 1130       IF(I2.LE.60) GO TO 1120
 1140         WRITE(4,2000) BUFFER
              BUFFER=' '
              I1=10
              I2=18
 1120     CONTINUE
          CD=CD-1
 1110 CONTINUE
 1100 CONTINUE
C
C SETTING THE INTERNAL DAYS GREATER THAN 1 IN WINDOWS : EQU 12
C
      BUFFER=' '
      DO 1200 L=1,NA
      CD=LF(L)-ES(L)+1
      IF(CD.LT.3) GO TO 1200
      A1=L/10
      A2=L-10*A1
      DO 1210 L1=ES(L)+1,LF(L)-1
         D1=L1/10
         D2=L1-10*D1
         WRITE(BUFFER(1:5),'(''X'',4I1)')A1,A2,D1,D2
         I1=6
         I2=14
         DO 1220 L2=3,CD
            C1=L2/10
            C2=L2-10*C1
            DO 1230 L3=ES(L),LF(L)-L2+1
```

```
              L4=L3+L2-1
              IF(L4.GT.LF(L)) GO TO 1230
              IF((L1.LE.L3).OR.(L1.GE.L4)) GO TO 1230
               S1=L3/10
               S2=L3-10*S1
               WRITE(BUFFER(I1:I2),'(''-XW'',6I1)')A1,A2,C1,
     $C2,S1,S2
               I1=I1+9
               I2=I2+9
               IF(L2.LT.CD)    GO TO 1240
                 BUFFER(I1:I1+2)='=>0'
                 GO TO 1250
 1240          IF(I2.LE.60) GO TO 1230
 1250            WRITE(4,2000)BUFFER
                 BUFFER=' '
                 I1=6
                 I2=14
 1230       CONTINUE
 1220    CONTINUE
 1210 CONTINUE
 1200 CONTINUE
C
C WINDOW CONSTRAINT : EQN 17
C
      DO 1600 L=1,NA
      CD=LF(L)-ES(L)+1
      IF(CD.LT.3) GO TO 1600
      A1=L/10
      A2=L-10*A1
      WRITE(BUFFER(1:4),'(''-W'',2I1)')A1,A2
      I1=5
      I2=18
      DO 1610 L1=ES(L),LF(L)
         S1=L1/10
         S2=L1-10*S1
         DO 1620 L2=1,CD
            CSQD=L2*L2*L2
            C1=L2/10
            C2=L2-10*C1
            WRITE(BUFFER(I1:I2),'(''+'',I5,''XW'',6I1)')
     $CSQD,A1,A2,C1,C2,S1,S2
            I1=I1+14
            I2=I2+14
            IF(LF(L).NE.L1) GO TO 1630
              BUFFER(I1:I1+1)='=0'
              GO TO 1640
 1630       IF(I2.LE.40) GO TO 1620
 1640         WRITE(4,2000) BUFFER
              BUFFER=' '
              I1=5
              I2=18
```

```
 1620     CONTINUE
          CD=CD-1
 1610 CONTINUE
 1600 CONTINUE
C
C WINDOW CONSTRAINT : EQN 18
C
      BUFFER(1:3)='-WT'
      I1=4
      I2=7
      DO 1700 L=1,NA
         CD=LF(L)-ES(L)+1
         IF(CD.LT.3) GO TO 1730
         A1=L/10
         A2=L-10*A1
         WRITE(BUFFER(I1:I2),'(''+W'',2I1)')A1,A2
         I1=I1+4
         I2=I2+4
 1730    IF(L.LT.NA) GO TO 1720
           BUFFER(I1:I1+1)='=0'
           GO TO 1710
 1720    IF(I2.LE.60) GO TO 1700
 1710    WRITE(4,2000) BUFFER
         BUFFER=' '
         I1=4
         I2=7
 1700 CONTINUE
C     WRITE(BUFFER(1:7),'(''WT<'',I4)')K2
C     WRITE(4,2000) BUFFER
C     BUFFER=' '
C
C DEFINING INTEGER VARIABLES XI(N,I,J)
C
C     BUFFER='END'
C     WRITE(4,2000) BUFFER
C     BUFFER='  '
C     DO 3000 L=1,NA
C     A1=L/10
C     A2=L-10*A1
C     DO 3010 L1=ES(L),LF(L)
C        D1=L1/10
C        D2=L1-10*D1
C        DO 3020 L2=0,MAX
C           C1=L2/10
C           C2=L2-10*C1
C           WRITE(BUFFER(1:16),'(''INTEGER XI'',6I1)')C1,C2,A1,
C    $A2,D1,D2
C           WRITE(4,2000) BUFFER
C           BUFFER=' '
C3020    CONTINUE
C3010 CONTINUE
```

```
C3000 CONTINUE
C
C DEFINING INTEGER VARIABLES YI(M,J)
C
C     DO 4000 L=1,ND
C     D1=L/10
C     D2=L-10*D1
C     DO 4010 L1=0,MAX
C        C1=L1/10
C        C2=L1-10*C1
C        WRITE(BUFFER(1:14),'(''INTEGER YI'',4I1)')C1,C2,D1,D2
C        WRITE(4,2000) BUFFER
C        BUFFER=' '
C4010 CONTINUE
C4000 CONTINUE
C
C DEFINING INTEGER VARIABLES XW(I,C,S)
C
 6000 DO 5000 L=1,NA
      CD=LF(L)-ES(L)+1
      IF(CD.LT.3) GO TO 5000
      A1=L/10
      A2=L-10*A1
      DO 5010 L1=ES(L),LF(L)
         S1=L1/10
         S2=L1-10*S1
         DO 5020 L2=1,CD
            C1=L2/10
            C2=L2-10*C1
            WRITE(BUFFER(1:16),'(''INTEGER XW'',6I1)')A1,A2,C1,
     $C2,S1,S2
            WRITE(4,2000) BUFFER
            BUFFER=' '
 5020    CONTINUE
         CD=CD-1
 5010 CONTINUE
 5000 CONTINUE
      BUFFER='LEAVE'
      WRITE(4,2000)BUFFER
      END
```

APPENDIX D

Notes on Running LINDO

Due to the large amount of computer time necessary to solve the model, it is advisable to operate LINDO in batch mode.

Because of the extensive amount of branch and bounding necessary, LINDO would normally exceed it default pivot limit and branch count, therefore the additional amount of pivots and branchs allowed must be entered. It is recommended that an additional amount be entered for both limits regardless of if it required. If these additional pivots and branchs are not required LINDO will respond with 'invalid command' until it encounters the 'QUIT' command, this does not affect the solution.

The output should be diverted to an output file. A very large temporary dataset should be allocated as LINDO generates pages and pages of output. The required results of the resource allocations can then be transfered to a smaller permanent dataset.