# IDENTIFICATION OF THE PARAMETERS OF A LUMPED EQUIVALENT CIRCUIT AND CALCULATION OF THE IMPULSE VOLTAGE DISTRIBUTION OF POWER TRANSFORMER WINDINGS

A

Thesis presented to

The Faculty of Graduate Studies

and Research

The University of Manitoba

In Partial Fulfillment

of the Requirements for the Degree

Master of Science

in

Electrical Engineering

by

Robert John Hamlin

October 1966

# A B S T R A C T

The high voltage winding of a transformer is a system of distributed resistances, inductances and capacitances that can be represented by lumped parameters. These paramaters can be identified by measurement techniques and some of them can be calculated from dimensions. The lumped elements are used to make up an equivalent circuit for the entire winding. The general analysis of back-turn and drop-down strip and drop-down disc windings for any transient input is presented and the equations are solved using the Runge-Kutta method. The detailed analysis of a five section strip winding is performed and compared with experimental results.

ACKNOWLEDGEMENTS

Grateful acknowledgement is extended to my advisor, Prof. R. A. Johnson, for suggesting the study proposed by Pioneer Electric, his continual interest, valued guidance throughout the study and in the preparation of this thesis.

The writer wishes to express his sincere thanks to Pioneer Electric for the financial assistance and research facilities without which this study could not have proceeded.

Thanks are due to Messrs. E. A. Dillon and J. K. Holland of Pioneer Electric for their many helpful discussions on the practical aspects of the design of power transformer windings.

The writer expresses his appreciation to Mr. A. F. James, a summer student employed by Pioneer Electric, for the photographic work required in this study.

Thanks are also due to the computer staff for the efficiency with which they handled the programs, without which the theory could not have been verified.

Finally, sincere appreciation is extended to Mrs. W. B. Jones for the typing of this thesis.

## PREFACE

Upon close examination a transformer is a system of distributed resistances, inductances, and capacitances, but there is something that is periodic in the construction. There are portions referred to as disc coils, strip coils or sections that can be represented as lumped units. These elements have equivalent parameters which may be used to form a lumped equivalent circuit for the entire winding. The impulse voltage response of the total equivalent circuit can then be calculated.

It was first necessary to study the single disc or strip coil and to obtain a method of finding the significant parameters. This was done in two ways, by direct measurement and by an indirect method utilizing the frequency response of a single coil. The direct current inductance of a coil can be calculated very accurately and the inter-turn capacitance of a two-turn coil can be obtained from the dimensions.

There are two basic types of windings that can be made using either disc or strip coils; the drop-down and the back-turn. Each has slightly different parameters and is discussed separately. The parameters of the single disc or strip coil are obtained and used as part of the equivalent circuit of the total winding. The additional parameters that arise in the complete equivalent circuit of a winding are capacitance between adjacent discs and mutual inductance between each and every other coil. The general analysis of the complete winding using the Runge-Kutta method of solving the differential equations is demonstrated and the detailed analysis of a five section strip winding is performed and compared with experimental results.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

# CHAPTER I

## INTRODUCTION TO THE PROBLEM

The purpose of this thesis is to study the high voltage winding of power transformers and to obtain a method of calculating the impulse voltage distribution of the winding.

### I. The Reason For Studying Transformer Windings

The calculation of the impulse response of a transformer winding is of primary importance because the voltage stresses on impulse test are usually much higher, although of shorter duration, than the stresses due to the normal operating voltage. It is, therefore, the voltage stresses under impulse conditions that the insulation must withstand. A method of calculating the impulse stresses would be extremely useful because the minimum necessary amount of insulation could be determined. A more economical transformer can be constructed if this insulation is confined to a minimum.

There have been a great many papers[1] written on the subject of impulse phenomena in power transformers. The earliest papers assumed uniform windings and neglected many details in order to arrive at a

[1] Abetti, P. A. "Bibliography on the Surge Performance of Transformers and Rotating Machinery," Transactions, American Institute of Electrical Engineers. Paper 58, December, 1958, pp. 1150-64.

tractable solution. Some of the approaches have been to consider: the winding in terms of distributed parameters, the total winding as a finite number of lumped elements, a geometrical scale model with external capacitive networks and scaling problems, a pure capacitive network and the initial distribution, and finally an electromagnetic model with analysis using Maxwell's Equations. All the earlier papers examined the response to a step input of voltage. Recently the actual impulse wave shape has been mentioned, but all of the papers referenced by the author use a rectangular input wave for calculation purposes. The most complete solutions to the problem have been obtained with either an analog[2] or a digital[3] computer.

## II. Previous Methods of Analysis

Rudenberg[4] analyzed a uniform single-layer helical transformer winding by defining distributed parameters as linear functions of length. He then proceeded to set up differential equations for voltages and currents between turns neglecting only the resistance and a small portion of the mutual-inductance. His equations predict an infinite number of natural frequencies in the windings, whereas in actual experi-

---

[2]Waldvogel, P. and Rouxel, R. "A New Method of Calculating the Electrical Stresses in a Winding Subjected to a Surge Voltage", Brown Boveri Review. Vol. 43, 1956, pp. 206-13.

[3]Dent, B. M., Hartill, E. R. and Miles, J. G. "A Method of Analysis of Transformer Impulse Voltage Distribution Using a Digital Computer," The Proceedings of the Institute of Electrical Engineers. Vol. 105, Part A, No. 23, October, 1958, pp. 445-59.

[4]Rudenberg, R. "Performance of Travelling Waves in Coils and Windings," Transactions, American Institute of Electrical Engineers. Vol. 59, 1940, pp. 1031-40, pp. 1257-62.

mentation the natural frequencies are finite in number. His method of defining capacitance and inductance make them difficult to calculate and to visualize.

Norris[5] does an extension of Rudenberg's method by explaining non-uniform windings. His experimental work is good and provides more information on impulse stresses in non-uniform windings.

Lewis[6] presented a paper in which he defined the winding to be a system of lumped parameters in which the mutual-inductance was included in an equivalent self-inductance. He solved for the response to a step input for a uniform winding, but more accurate results can be obtained by making fewer assumptions.

Abetti[7] used a scale model to predict the design requirements of much larger transformers. By scaling the model appropriately and using an external capacitance network the voltage distribution can be measured and scaled to obtain the correct insulation level in the actual transformer. This method is rather limited and expensive because a scale model must be constructed for each type of transformer.

A method that utilized lumped parameters and included mutual-inductances was set up by Waldvogel and Rouxel.[8] Their method uses an

---

[5]Norris, E. T. "The Lightning Strength of Power Transformers," Journal, Institute of Electrical Engineers, Vol. 95, Part II, 1948, pp. 389-406.

[6]Lewis, T. J. "The Transient Behavior of Ladder Networks of the Type Representing Transformer and Machine Windings," Proceedings Institution of Electrical Engineers, Vol. 101, Part II, 1954, pp. 541-53.

[7]Abetti, P. A. "Transformer Models for the Determination of Transient Voltages," Transactions American Institute of Electrical Engineers. Vol. 72, Part III, June 1953, pp. 468-80.

[8]Waldvogel, P. and Rouxel, R., loc. cit.

analog computer to solve for the unit step voltage response. Although their analysis seems to provide satisfactory answers the use of an analog computer drastically limits the number of sections that can be analyzed.

The method presented by Dent, Hartill and Miles[9] is the most complete and accurate procedure that has been referenced by the author. They set up the matrix equations for a general non-uniform winding with a general input voltage. Their lumped equivalent circuit included all the significant parameters and neglected only the damping. They solved for the step voltage response of a uniform winding using the Runge-Kutta method of solving differential equations. Their solutions seem very realistic and they can easily vary the parameters and recalculate the response to determine the change in voltage distribution.

With so many different methods it seems there is no absolutely correct method of analyzing the winding. Some are more accurate, some are easier to use and others are not suitable. An understanding of the complete winding is necessary. After a satisfactory concept and analysis of the transformer winding are evolved, modifications may then be attempted and tested to see whether or not they are valid.

---

[9] Dent, B. M., Hartill, E. R., and Miles, J. G., loc. cit.

## CHAPTER II

## METHOD OF ANALYSIS

In order to facilitate the study of high voltage windings certain simplifications are made. The standard impulse test of a transformer is performed by applying the impulse to the high voltage terminal with all other terminals of transformer either directly grounded or grounded through resistors. In this manner the low voltage winding behaves very nearly as a ground plane and in theoretical and experimental work it is assumed that it is permissible to replace the low voltage winding with a cylindrical ground shield consisting of a sheet of core steel. The time variations of the impulse wave are very rapid and almost no flux will penetrate the laminated iron core. The principal flux will therefore be the air flux, and it will not be necessary to consider the iron core when theoretical calculations and corroborating experiments are performed. After the method of analysis is proven, the changes, if any, due to an iron core can be analyzed. These simplifications will enable the consideration of the winding as an isolated coil, with an inner ground shield.

If the winding is examined closely all the capacitances and inductances are distributed, yet there is something that is periodic about the four types of windings as can be noted in Figures 1, 2, 3 and 4.

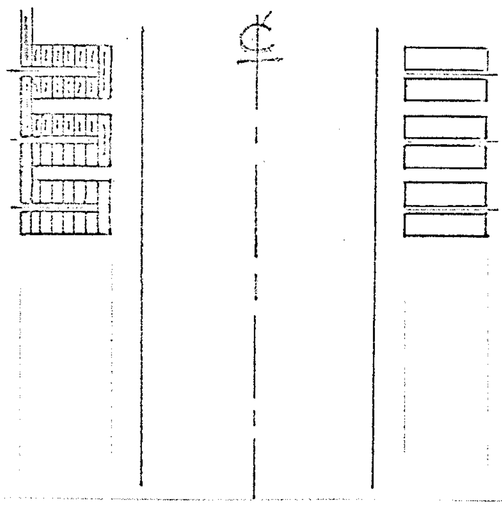


FIGURE 1

BACK-TURN DISC COIL WINDING

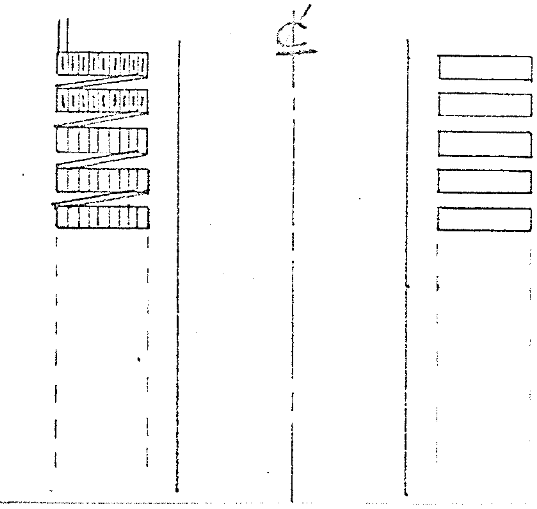


FIGURE 2

DROP-DOWN DISC COIL WINDING

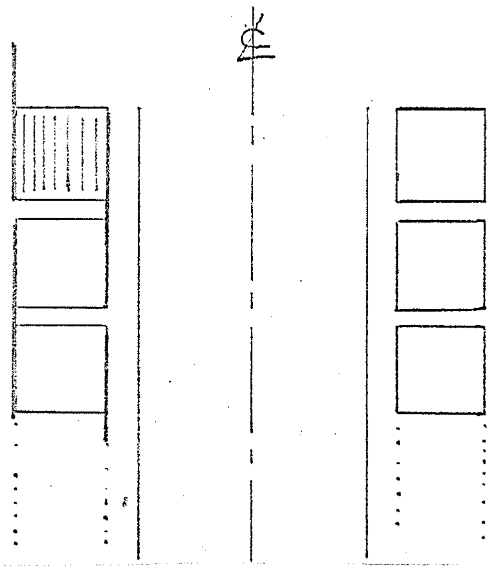


FIGURE 3

BACK-TURN STRIP COIL WINDING

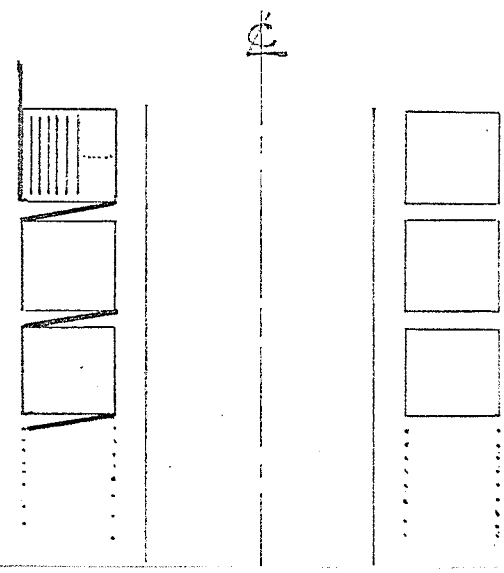


FIGURE 4

DROP-DOWN STRIP COIL WINDING

Each coil can be represented by a lumped circuit and a set of these circuits with the addition of the mutual parameters can be used to represent the transformer winding. The inter-turn, inter-disc and ground capacitances, self- and mutual-inductances and resistances are all included in the circuit. The first step is to find an equivalent circuit for the coil and then a method of determining the parameters.

An individual coil may be considered to have an equivalent circuit consisting of an inductance in series with a resistance all in parallel with a capacitance. These parameters can be obtained by direct measurement. The disc coils are subject to the skin effect and consequently the inductance and resistance change with frequency. As the frequency increases the inductance decreases while the resistance increases. The inductance of strip coils is fairly independent of frequency and the resistance is dependent on frequency. The low frequency inductance can be measured with the Anderson Bridge and the distributed capacitance, referred to as the inter-turn capacitance, can if it is not too large, be measured with the Q-meter. The measured frequency response of the single coil provided an alternative method of obtaining the parameters and a comparison for the calculated frequency response. The direct current inductance can be calculated very accurately. As a final check on the equivalent circuit the impulse response of a single coil was measured and compared to the theoretical impulse response.

The next step is to consider what happens when two coils are placed near to each other and the additional parameters that may come into effect. Both types of disc winding have an inter-disc capacitance that can be measured or calculated using an empirical constant. The strip coil windings have a very small capacitance between coils and it is assumed to be negligible. The direct current mutual-inductance can be accurately calculated. The frequency response of a pair of adjacent discs was measured and calculated to show the validity of lumped circuit elements for a more complicated system. It now remains to be shown that the technique of using lumped circuit elements is a good method of representing the entire transformer winding.

The back-turn and drop-down strip windings and the drop-down disc winding have the same equivalent circuit but with different parameters. The circuit equations are written in general matrix form and the matrices manipulated to produce a result that can be solved using the Runge-Kutta method. The equivalent circuit of the back-turn disc winding has an equivalent circuit that is more complex and as yet the circuit has not been analyzed. The complete analysis of a five section strip winding was undertaken in order to obtain a comparison between the measured and the theoretical impulse response.

A knowledge of the frequency response of an isolated transformer winding can provide information on the impulse voltage distribution.

The winding is considered as a filter and the frequency components of the impulse wave are observed to see whether the transformer will pass or attenuate the frequency components of the test wave.

The intent of this thesis is to develop an understanding, to demonstrate the significant parameters and to calculate the impulse response of the high voltage winding of power transformers. The analysis presented provides a more accurate and complete solution of the impulse distribution than any of the other solutions that the author has referenced.

# CHAPTER III

## THE EQUIVALENT CIRCUIT AND PARAMETERS OF A SINGLE COIL

In order to analyze the individual coil an equivalent circuit was assumed as shown in Figure 5. It was discovered that the inductance, resistance and capacitance could be obtained by direct measurement. To show that the circuit model described the physical behavior, the frequency response of several isolated coils was calculated and measured to provide a comparison between the theoretical and experimental results. A method was formulated to obtain the high frequency inductance and capacitance from measurements of critical points on the frequency response curves. The direct current inductance can be accurately calculated, but as yet a satisfactory method of calculating the inter-turn capacitance has not been developed. The impulse response of a single coil as compared to the calculated response lends further support to the concept of using lumped elements to represent a distributed system. The basic unit of a transformer winding is the single coil, and it has in the frequency range important to impulse response behavior, a simple equivalent lumped circuit.

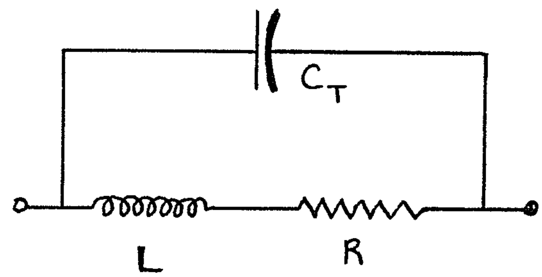



FIGURE 5

THE EQUIVALENT CIRCUIT OF AN INDIVIDUAL COIL

The coils are constructed from relatively large cross-sectional area wire that is very susceptible to skin effect. Consequently the inductance and resistance of a coil change with frequency because of the different current distribution. The resistance of the disc and strip coils changes by several orders of magnitude while the self-inductance of a disc coil decreases by as much as forty per cent. The strip coil is constructed of a wide strip of copper that is much less susceptible to the skin effect, with the result that the inductance changes very little.

I. Direct Measurement of Parameters

The Anderson Bridge provided a very reliable method of measuring the inductance of the coils at low frequencies. Table I presents the measured low frequency inductance, high frequency inductance as obtained from the frequency response curves, and the calculated direct current inductance.

TABLE I

LOW FREQUENCY, HIGH FREQUENCY AND CALCULATED SELF-INDUCTANCES OF COILS

| WIRE SIZE | NO. OF TURNS | INSIDE RADIUS | RADIAL BUILD | LOW ($\mu h$) | HIGH ($\mu h$) | CALCULATED ($\mu h$) |
|---|---|---|---|---|---|---|
| .072" x .182" | 19 | 9.87" | 1.75" | 402 | 352 | 407 |
| .102" x .289" | 15 | 9.87" | 1.75" | 253 | 200 | 248 |
| 3.0" x .020" | 50 | 5.50" | 1.50" | 1069 | 952 | 951 |

The Q-meter is a very satisfactory instrument for measuring the distributed capacitance of coils, provided that the unknown capacitance is not too large. A precaution that must be taken is to ensure that one end of the coil is connected directly to a terminal of the Q-meter and as short a lead as possible is used for the other connection. Table II presents the measured value of inter-turn capacitance and the value obtained from the frequency response curves.

TABLE II

INTER-TURN CAPACITANCE OF COILS OBTAINED BY DIRECT MEASUREMENT AND BY AN INDIRECT METHOD USING FREQUENCY RESPONSE CURVES

| WIRE SIZE | NO. OF TURNS | INSIDE RADIUS | RADIAL BUILD | CAPACITANCE (pf.) DIRECT | INDIRECT |
|---|---|---|---|---|---|
| .072" x .182" | 19 | 9.87" | 1.75" | 33 | 33 |
| .102" x .289" | 15 | 9.87" | 1.75" | 39 | 38 |
| 3.0' x .020" | 50 | 5.50" | 1.50" | Too Large | 177 |

II. The Equivalent Circuit

The justification of the configuration of the equivalent circuit is completed by a comparison between a calculated and a measured frequency response. The frequency response of a coil was measured using the test circuit of Figure 6. The circuit requires a high frequency oscillator and an oscilloscope with either a dual beam or a dual channel chopped beam. The ratio of the magnitude of the output voltage to the input voltage is measured over the frequency range from 10 KHZ to 5 MHZ. This is the frequency range that provides information about the coils measured.

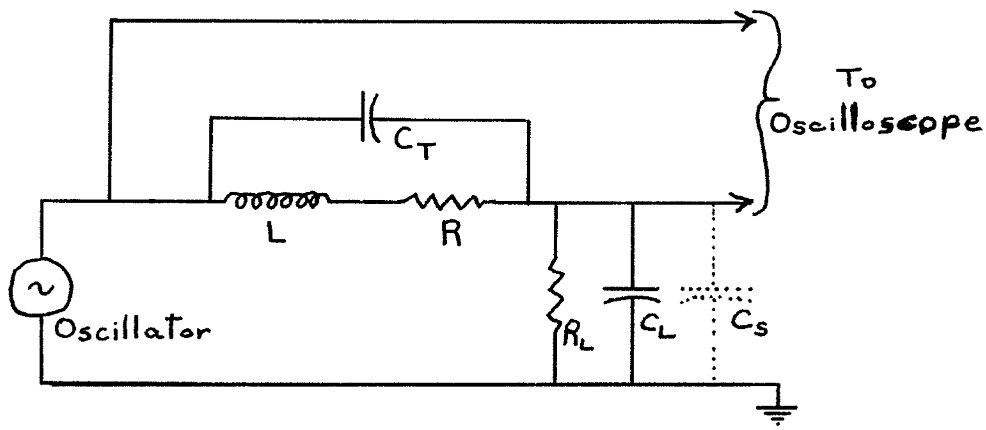



FIGURE 6

TEST CIRCUIT FOR A SINGLE COIL

The load resistor ($R_L$) and capacitor ($C_L$) were used to stabilize the impedance of the oscilloscope leads and the stray ground capacitance. Two tests were performed on each coil; the first with a load resistance of 1 K$\Omega$ and the second with a load resistance of 1 M$\Omega$. The 1 K$\Omega$ resistor damped out the series resonance of the inductance with the inter-turn and ground capacitance. This test indicated the lower half-power frequency $F_1$ and the parallel resonant frequency ($F_3$) of the inductance with the inter-turn capacitance. Figure 7 indicates the general shape of the curve.

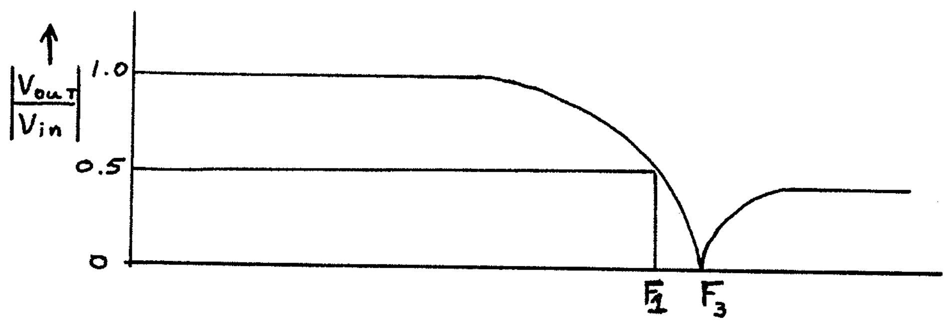



FIGURE 7

GENERAL SHAPE OF THE FREQUENCY RESPONSE

OF A SINGLE COIL WITH $R_L$ = 1 K$\Omega$

The second test with load resistance of 1 M $\Omega$ shows the frequency $F_2$ at which the inductance resonates with the inter-turn, load and stray ground capacitances. The general shape of the response from this test is shown in Figure 8.

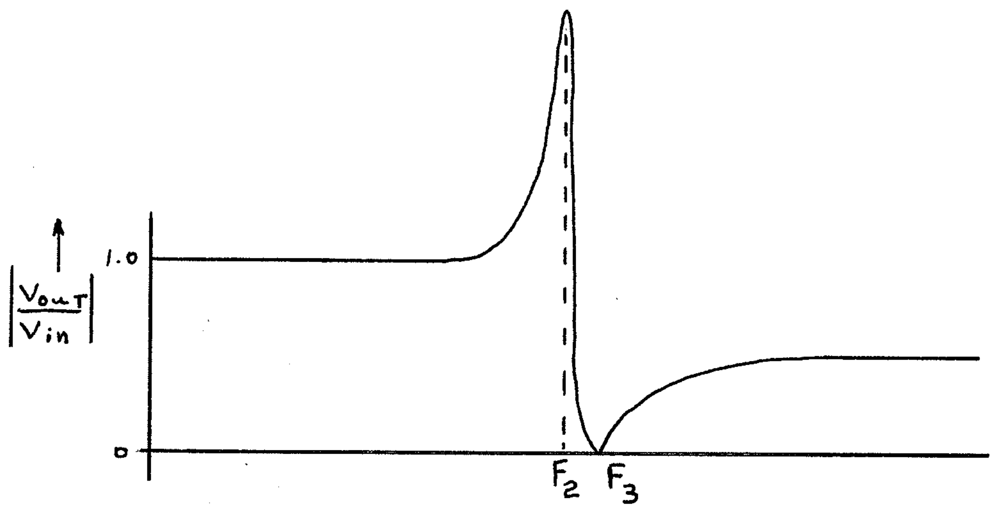



FIGURE 8

GENERAL SHAPE OF FREQUENCY RESPONSE OF A
SINGLE COIL WITH $R_L$ = 1 M $\Omega$

The measured and calculated frequency responses are plotted in Figures A-1, A-2 and A-3 in Appendix A. The high frequency parameters for the calculated response were obtained from the respective measured response by a method described in the following section. Figure A-4 compares the computed responses using high frequency and direct current inductance. The calculated graphs are essentially the same as the measured graphs and the equivalent circuit chosen may be considered to be an accurate model of a single coil. The only significant difference is the amplitude of the peak which is determined by the damping of the coil. The resistance changes through several orders of magnitude over the frequency range and is rather difficult to calculate due to its dependance on frequency, but fortunately it is a second order effect in impulse response study and does not need to be specified accurately. Appendix B contains the analysis and the program that executes and plots the frequency response.

III. Frequency Response Method of Obtaining Parameters

A method to obtain the parameters of the coil from the frequency response curve can be formulated. There are three principal parameters that are unknown: the inductance, the inter-turn capacitance and the ground capacitance. These parameters can be determined using the equations (1), (2) and (3). The derivation of these equations is given in Appendix C. The frequency response curve provides the most accurate and reliable method of obtaining the inductance and inter-turn capacitance of a coil and in addition it produces the value of inductance at the frequency that is most significant to the behavior of the impulsed winding.

$$L = \frac{R_L}{2\pi F_1}\left[3 - 8\left[\frac{F_1}{F_3}\right]^2 + 2\left[\frac{F_1}{F_2}\right]^2 - \left[\frac{F_1}{F_2}\right]^4 + 4\left[\frac{F_1}{F_3}\right]^4\right]^{\frac{1}{2}} \qquad (1)$$

$$C_T = \frac{1}{(2\pi F_3)^2 L} \qquad (2)$$

$$C_g = \frac{1}{(2\pi F_2)^2 L} - C_T \qquad (3)$$

## IV. The Direct Current Inductance

The direct current inductance can be calculated quite accurately with formula[10] (4). The dimensions are in inches and N is the number of turns, $R_b$ is the radial build, b is the axial length, $R_m$ is the mean radius and

$$R = 0.2235\left[R_b + b\right]$$

$$L = 4\pi R_m N^2 2.54 \times 10^{-9}\left[\log_e \frac{8R_m}{R}\left(1 + \frac{3R^2}{16R_m^2}\right) - \left(2 + \frac{R^2}{16R_m^2}\right)\right] \text{henries} \qquad (4)$$

The significant parameters in the formula are the number of turns and the mean radius. Winding techniques will have very little effect on the dimensions and consequently the inductance of different coils manufactured of the same number of turns, wire size, and mean radius will be very nearly the same. The calculated values of direct current inductance that appear in Table I were obtained with this formula.

## V. The Inter-Turn Capacitance

The inter-turn capacitance of a single coil is a distributed capacitance created by the proximity of the turns. The capacitance of

---

[10] Rosa, E. B. and Grover, F. W. "Formulas and Tables For The Calculation of Mutual and Self Inductance," Bureau of Standards Bull. 8, January, 1912, p. 136.

a coil consists of two portions: the capacitance between turns and the capacitance of the inner turn with itself. For two reasons only the capacitance between turns need be considered. When the winding is tested the inner shield used to simulate the low voltage winding converts the distributed capacitance of the inner turn with itself to a ground capacitance. The capacitance created by the inner turn with itself is very small.

It is first necessary to point out some of the differences between parallel plate capacitance and distributed capacitance. Reference[11,12] has been made of this, but no experimental data has been submitted. Consider the capacitance between two unconnected pieces of copper much longer in one direction than the other, as in Figure 9. This is a parallel plate capacitor and can be measured with the Schering Bridge.

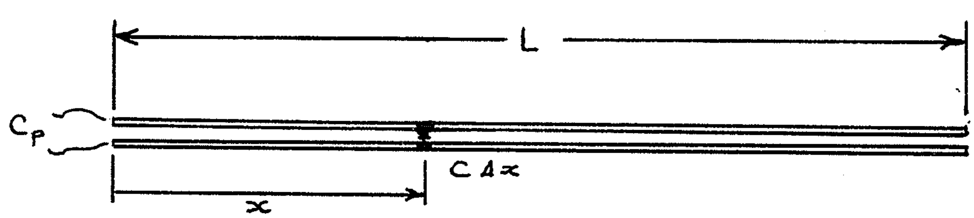



FIGURE 9

PARALLEL PLATE CAPACITOR

---

[11] Crowhurst, N. H. "Winding Capacitance," Electronic Engineering. November, 1949, pp. 417-31.

[12] Waldvogel, P. and Rouxel, R., *loc. cit.*, p. 208.

If the two extreme ends of the parallel wire capacitor are joined to form a single folded wire as in Figure 10, the capacitance becomes a distributed capacitance that is one-third of the parallel wire capacitance. To verify this an experiment was set up utilizing rectangular wire to form the parallel wire capacitor which could then be joined at one end to represent the folded wire. The parallel unconnected wires provided a very satisfactory method of obtaining a constant that took both the dielectric constant and the fringing effect into account. The constant used in all the distributed capacitance formulae represents the combined dielectric and fringing constant.

The capacitance of the parallel unconnected wires was measured using the Schering Bridge and then the combined dielectric and fringing constant was calculated. The wires were then joined at one end to create a folded wire and the distributed capacitance was measured with the Q-meter. The data in Table 3 show that the distributed capacitance of a folded wire is in effect one-third of the parallel wire capacitance.

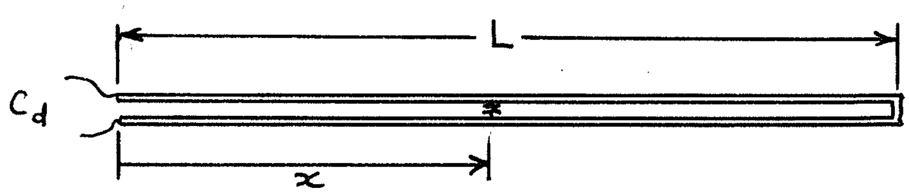



FIGURE 10

DISTRIBUTED CAPACITANCE OF A FOLDED WIRE

TABLE III

COMPARISON OF FOLDED WIRE CAPACITANCE WITH

PARALLEL WIRE CAPACITANCE

| WIRE SIZE | INSULATION THICKNESS COPPER TO COPPER | COMBINED DIELECTRIC & FRINGING CONSTANT | LENGTH | CAPACITANCE (pf.) PARALLEL WIRE | FOLDED WIRE |
|---|---|---|---|---|---|
| .091" x .204" | .080" | 2.70 | 25.5" | 39.4 | 14.0 |
| .091" x .229" | .080" | 1.89 | 29.2" | 68.0 | 21.2 |
| .072" x .182" | .020" | 1.76 | 33.2" | 120.0 | 39.8 |
| .102" x .289" | .080" | 2.96 | 34.7 | 83.8 | 27.1 |

The parallel wire and folded wire capacitance is considered to be an effect measured and calculated at the terminals of the wire. The voltage at the terminals is considered to be fixed and an equivalent capacitance occurs between the terminals. The energy stored in this equivalent capacitance is simply one-half of the product of the terminal capacitance and the square of the voltage occurring across the terminals. The terminal capacitance can be calculated by considering the voltage distribution along the wire, the voltage across each elemental section and the sum of each infinitesimal of stored energy over the length of the wire. The derivation of this from energy concepts is given in Appendix D.

The concept of inter-turn capacitance is studied in the same manner.

Figure 11 shows the distributed capacitance that occurs between the turns of a two-turn coil.

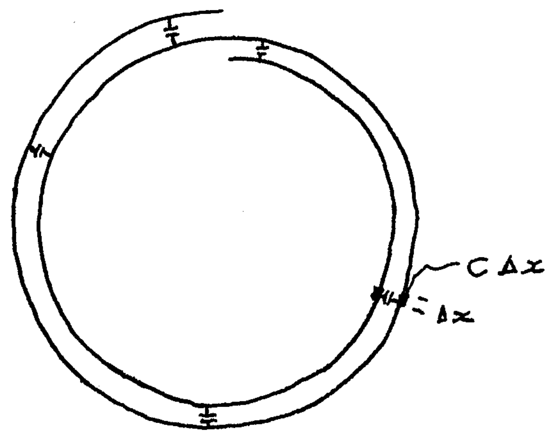



FIGURE 11

DISTRIBUTED CAPACITANCE OF A

TWO-TURN COIL

The voltage distribution of a two-turn coil is linear along the wire. For a coil of three or more turns there is one or more turns with turns on either side and this makes the voltage distribution non-linear. Because the voltage distribution on a two-turn coil is linear the distributed inter-turn capacitance can be calculated by assuming that the voltage occurring across any elemental section is one-half the applied voltage and the energy storage in an elemental section is

$$dE = \frac{1}{2} C \left[\frac{V}{2}\right]^2 dx$$

where C is the capacitance per unit length.

The length of the capacitance is taken to be:

$$L = 2\pi R_m$$

The total energy is given by:

$$E = \frac{1}{2}\left[\frac{C\,2\pi R_m}{4}\right]V^2 = \frac{1}{2}C_{eq}V^2$$

The inter-turn capacitance of a two-turn coil is therefore

$$C_{T2} = C_{eq} = \frac{1}{4}K\epsilon_0 2\pi R_m \frac{w}{d} \qquad (5)$$

where W is the width of the wire, d is the separation copper to copper and K is the combined dielectric and fringing constant. Table IV shows the measured and calculated capacitance of various two-turn coils. The combined dielectric and fringing constant for each wire, spacing and insulation must be obtained by measuring the capacitance of two parallel unconnected wires and calculating the constant.

TABLE IV

DISTRIBUTED CAPACITANCE OF TWO-TURN COILS

| WIRE SIZE | INSULATION THICKNESS COPPER TO COPPER | COMBINED DIELECTRIC AND FRINGING CONSTANT | MEAN DIAMETER | DISTRIBUTED CAPACITANCE (pf.) MEASURED | CALCULATED |
|---|---|---|---|---|---|
| .091" x .229" | .040" | 1.89 | 3.81" | 7.0 | 7.1 |
| .091" x .229" | .040" | 1.89 | 9.06" | 19.0 | 17.3 |
| .102" x .289" | .080" | 2.96 | 10.82" | 21.3 | 20.2 |
| .102" x .289" | .080" | 2.96 | 23.02" | 45.0 | 43.5 |
| .072" x .182" | .020" | 1.76 | 11.42" | 32.2 | 32.4 |

The extension of this theory to a multi-turn coil must take into account the non-linear voltage distribution of the coil. If the voltage distribution were known the inter-turn capacitance could be found simply by considering the capacitance of the N-turn coil to be N-1 capacitors each with its own voltage gradient and summing the energy as was done for a two-turn coil.

## VI The Ground Capacitance

When the coil is part of a winding there is a capacitance between the inner turn and the inner ground shield. This capacitance can be calculated by assuming the shield and inner turn of the coil to be concentric cylinders between which insulation of different dielectric constants may be placed. The ground capacitance can then be calculated with formula[13] (6).

$$C_g = \frac{2\pi\epsilon_0 L\, 2.54\times10^{-2}}{\left[\frac{1}{K_1}\ln\left(\frac{b_1}{a}\right) + \frac{1}{K_2}\ln\left(\frac{b_2}{b_1}\right) + \frac{1}{K_i}\ln\left(\frac{b_{i+1}}{b_i}\right) + \cdots + \frac{1}{K_n}\ln\left(\frac{b}{b_n}\right)\right]} \qquad (6)$$

The dimensions are in inches and L is the length of the copper cylinder, a is the inner radius, $b_1$ is the radius to the first dielectric change, $b_i$ is the radius to the i'th dielectric change, b is the outer radius and $K_i$ is the i'th dielectric constant.

## VII The Impulse Response of a Single Coil

The impulse response of a single coil was measured using the

[13] Knowlton, A. E. "Standard Handbook for Electrical Engineers", McGraw-Hill Book Company, 1941, p. 67.

same circuit as was used for the frequency response test with a load resistance of 1 M$\Omega$. The impulse wave is represented[14] by equation (7).

$$e(t) = E\left(e^{-at} - e^{-bt}\right) \qquad (7)$$

Table V shows the constants for three impulse waves. Figure A-5 shows a measured impulse wave and mathematical calculation of two different waves.

TABLE V

PARAMETERS FOR THE REPRESENTATION OF

STANDARD IMPULSE WAVES

| WAVE | E | a | b |
|---|---|---|---|
| 0.9 X 55 $\mu$sec | 1.015 | $0.01296 \times 10^{+6}$ | $7.04 \times 10^{+6}$ |
| 1.5 X 40 $\mu$sec | 1.036 | $0.01833 \times 10^{+6}$ | $3.558 \times 10^{+6}$ |
| 2.0 X 40 $\mu$sec | 1.038 | $0.01825 \times 10^{+6}$ | $2.425 \times 10^{+6}$ |

The voltage transfer function with an input impulse wave was analyzed using the Runge-Kutta Method and the program used in computing the responses is presented in Appendix E.

Figures A-6, A-7, A-8 and A-9 compare the measured and computed response of single coils. The computed response of a disc coil using the high frequency inductance value is compared to the measured response in Figure A-6 and A-7. Figure A-8 indicates the measured and computed response of a strip coil. These curves agree very well both in amplitude

[14]The British Electrical and Allied Industries Research Association, "Surge Phenomena," Seven Years' Research for the Central Electricity Board (1933-1940), pp. 154-6, 1941.

and frequency of response. Figure A-9 compares the measured response with the computed response using the D.C. inductance and although the amplitude is basically the same the frequency of response does not agree as well as when the high frequency inductance was used in Figure A-7. The inductance of the disc coil therefore depends upon the frequency components by which it is excited. The lumped circuit model has now been shown to be valid for impulse waves.

Two methods of obtaining coil parameters have been indicated. The most satisfactory method is to obtain the parameters by measuring the critical frequencies of the frequency response curves. The direct measurement of the parameters is reliable and produces the low frequency inductance and the value of distributed capacitance provided it is not too large. The calculation procedure is not complete and produces a value of direct current inductance and an accurate value of inter-turn capacitance for only a two-turn coil. The model of a single coil has been confirmed and proven to give a reliable description of the physical behavior of the coil in the frequency range important to impulse response behavior.

## CHAPTER IV

## MUTUAL INDUCTANCE AND THE ADDITIONAL PARAMETERS OF DISC WINDINGS

In a transformer winding the coils are wound closely together and are separated by either a duct or a collar. Because they are near to each other a large mutual-inductance is created. The effect of separation on low frequency mutual-inductance was measured with the Anderson Bridge. Figure A-10 indicates that it is a significant parameter and that the measured values compare very well with the calculated direct current mutual-inductance. For such coils each turn is considered to be a circular current filament having a mutual-inductance with every turn of the opposite coil as in Figure 12. The sum of these mutual-inductances produces the total mutual-inductance between the coils.

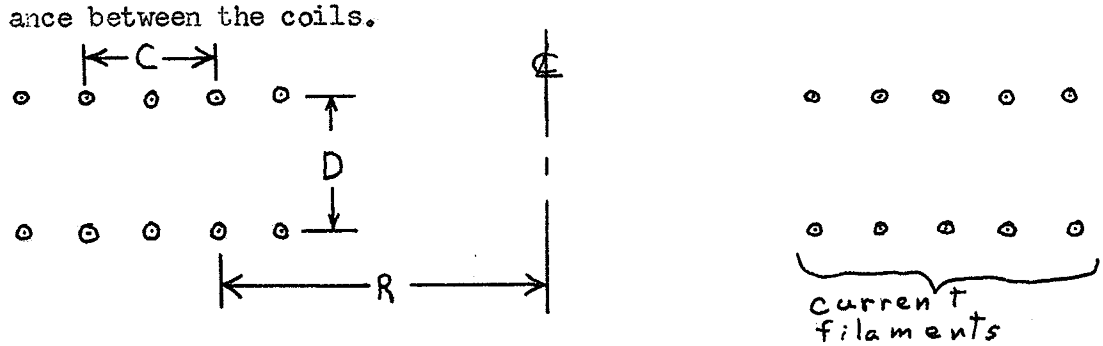



FIGURE 12

REPRESENTATION OF TURNS BY CURRENT FILAMENTS

Two formulae, which have overlapping ranges, are used for the calculations. Formula[15] (8) is to be used for coils that are separated by less than the mean radius.

$$M = 4\pi\, 2.54 \times 10^{-9} R \left[ \log_e \frac{8R}{D^2+C^2}\left(1 + \frac{C}{2R} + \frac{C^2+3D^2}{16R^2} - \frac{C^3+2CD^2}{32R^2}\right) - \left(2 + \frac{C}{2R} - \frac{C^2+3D^2}{16R^2} + \frac{C^3+3CD^2}{32R^2}\right) \right] \quad (8)$$

The dimensions are in inches, M is the mutual inductance in henries R is the radius to a current filament, C is the perpendicular distance from the radius R to the desired current filament and D is the separation between coils. For coils at a greater separation than half the mean radius Formula[16] (9) is valid and for distances greater than twice the mean radius Formula (9) can be simplified to Formula (10). The dimensions are in inches, M is the mutual-inductance in henries, $R_m$ is the mean radius, and $N_1$ and $N_2$ the number of turns of coil 1 and 2 respectively.

$$M = 16\pi^2 N_1 N_2 R_m \sqrt{Q^3}\,(1+\epsilon)\, 2.54 \times 10^{-9} \quad (9)$$

$$M = 16\pi^2 N_1 N_2 R_m \sqrt{Q^3}\, 2.54 \times 10^{-9} \quad (10)$$

where:

$$K' = D/\sqrt{4R_m^2 + D^2}$$

$$Q = \frac{1}{2}\left[\frac{1-\sqrt{K'}}{1+\sqrt{K'}}\right] + 2\left[\frac{1}{2}\cdot\frac{1-\sqrt{K'}}{1+\sqrt{K'}}\right]^5 + 15\left[\frac{1}{2}\cdot\frac{1-\sqrt{K'}}{1+\sqrt{K'}}\right]^9$$

$$\epsilon = 3Q^3 - 4Q^6 + 9Q^8 - 12Q^{10}$$

---

[15]Rosa, E. B. and Grover, F. W., loc. cit., p. 13.

[16]Rosa, E. B. and Grover, F. W., loc. cit., p. 11.

A computer program, presented in Appendix F, was written to calculate the mutual-inductance between two identical coils at any separation. For separations up to 0.9 $R_m$ it can be assumed that each turn is a circular current filament and at greater distances all the filaments can be assumed to occur at the mean radius of the coil.

The distributed capacitance occurring between adjacent coils of a back-turn or a drop-down disc winding is referred to as the inter-disc capacitance. In strip windings this capacitance is very small in comparison to the inter-turn capacitance and is neglected. The physical construction and the distributed capacitance of a pair of discs is indicated in Figure 13. For a back-turn winding the capacitance can be represented as a lumped capacitance $C_D$ occurring between the two terminals as in Figure 14. The pair of discs have the same voltage distribution as the folded wire discussed in Chapter II. The distributed capacitance is, therefore, one third the capacitance measured between two unconnected discs.

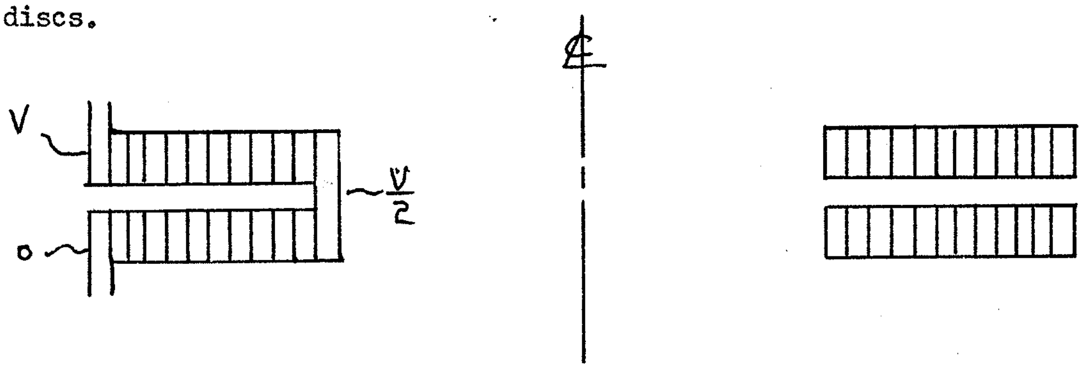



FIGURE 13

CONSTRUCTION AND VOLTAGE DISTRIBUTION OF A PAIR OF BACK-TURN CONNECTED DISCS

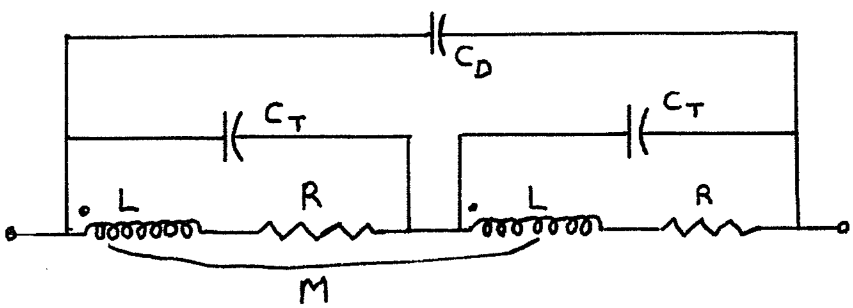



FIGURE 14

THE EQUIVALENT CIRCUIT OF A PAIR OF BACK-TURN CONNECTED DISCS

The voltage distribution of a pair of coils drop-down connected, as in Figure 15, is the same as a coil of two turns and the distributed inter-turn capacitance is one-quarter of the capacitance measured between the two unconnected discs. In the equivalent circuit, as in Figure 16, the distributed inter-disc capacitance is included by adding one-half the distributed inter-disc capacitance to the inter-turn capacitance of each coil.

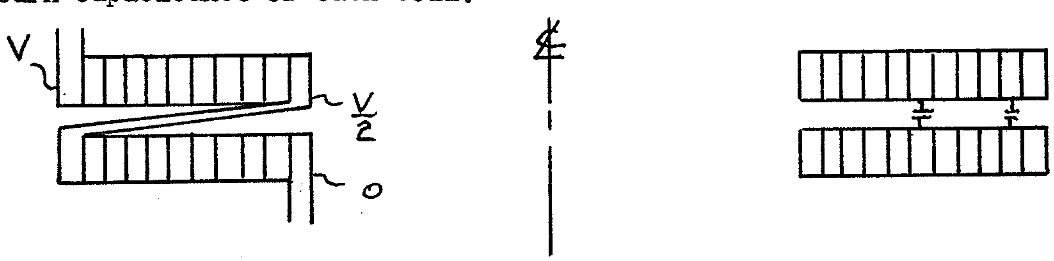



FIGURE 15

CONSTRUCTION AND VOLTAGE DISTRIBUTION OF A PAIR OF DROP-DOWN CONNECTED DISCS

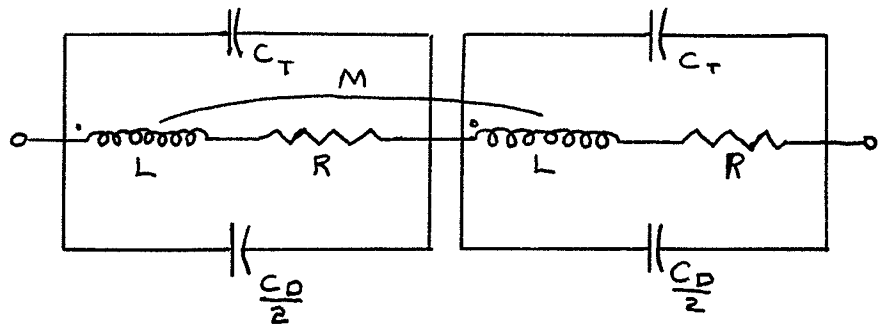



FIGURE 16

THE EQUIVALENT CIRCUIT OF A PAIR OF DROP-DOWN CONNECTED DISCS

The separation between discs is usually such that there is considerable fringing of the electric field and the capacitance between diagonally opposite turns becomes significant. The dielectric is made up of solid insulation and air and may be considered as an effective dielectric. Because of these irregularities in the electric field and variations in the dielectric it is very difficult to calculate the capacitance between discs and an empirical constant to account for the fringing and the effective dielectric must be used for the specific configuration under consideration. The approximate formulae are presented in Appendix G. Figures A-11 and A-12 show the variation of capacitance between unconnected discs with solid insulation and air dielectrics.

In order to demonstrate that the equivalent circuit in Figure 14 is a proper representation, the frequency response of a pair of back-turn connected discs was calculated and measured in the same manner as for a single coil. The frequency response was calculated from the voltage transfer function of the circuit in Figure 17. The capacitance $C_g$ was included to represent any stray capacitance between the two inner turns and ground. Appendix H contains the calculation and the computer program used to compute and plot the response.

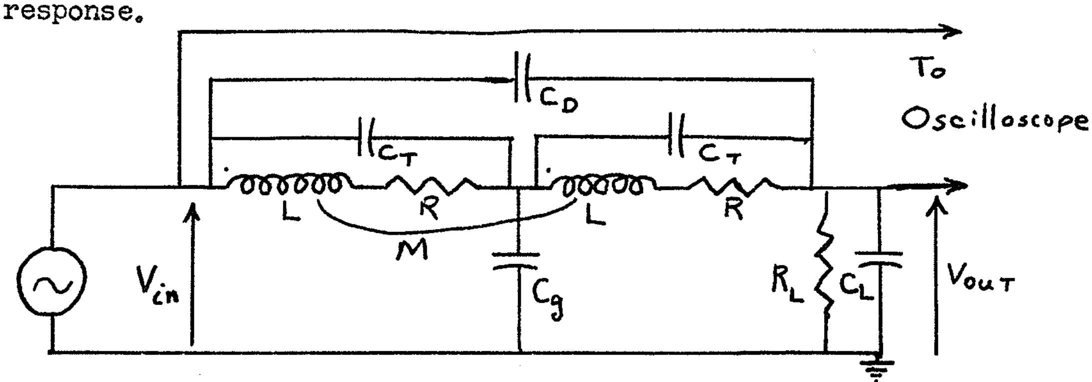



FIGURE 17

TEST CIRCUIT FOR A PAIR OF BACK-TURN CONNECTED DISCS

The self-inductance and resistance of disc coils changes a great deal over the frequency range. When the frequency response is calculated it is extremely difficult to take this change into account and it is necessary to assume a constant inductance and resistance. For coils close together the mutual-inductance will change with frequency and this change must be allowed for in frequency response calculations. The ratio of low frequency to high frequency inductance is used to adjust the measured low frequency mutual-inductance to a value suitable at high frequencies. The frequency response was calculated using the high frequency inductance values. The measured frequency response is very characteristic and compares favorably with the calculated response as Figure A-13 indicated. The peak and null frequencies do not agree exactly, but this is due to the variation of inductance with frequency. The lower peak and null, which most effect the impulse response behavior, are quite accurate.

The performance of a pair of back-turn connected discs may then be described by this equivalent circuit which may be used to represent a pair of discs in a complete winding. The significant parameters of the four types of windings have been identified and now the equivalent circuit of a complete winding may be examined.

CHAPTER V

# ANALYSIS OF THE EQUIVALENT CIRCUIT OF DROP-DOWN DISC, BACK-TURN AND DROP-DOWN STRIP WINDINGS

Three types of windings are analyzed and each has the same equivalent circuit. They differ in the parameter values and how each element is specified. The circuit equations are written in matrix form and any non-uniform winding may be solved, provided only that the parameters may be specified. The current equations, which are obtained from the matrix equations, are solved using the Runge-Kutta Method. The voltage at each crossover is obtained by using the ground capacitance and the difference of two loop currents. The voltage between coils may be calculated by differencing the respective voltages to ground.

The equivalent circuit of a drop-down disc winding has been previously identified by Dent, Hartill and Miles,[17] but they neglected the coil resistance and solved the equations for a simplified input of a rectangular wave. The schematic of the winding is indicated in Figure 18 in the same configuration as its physical construction.

[17]Dent, B. M., Hartill, E. R. and Miles, J. G., loc. cit., p. 447.

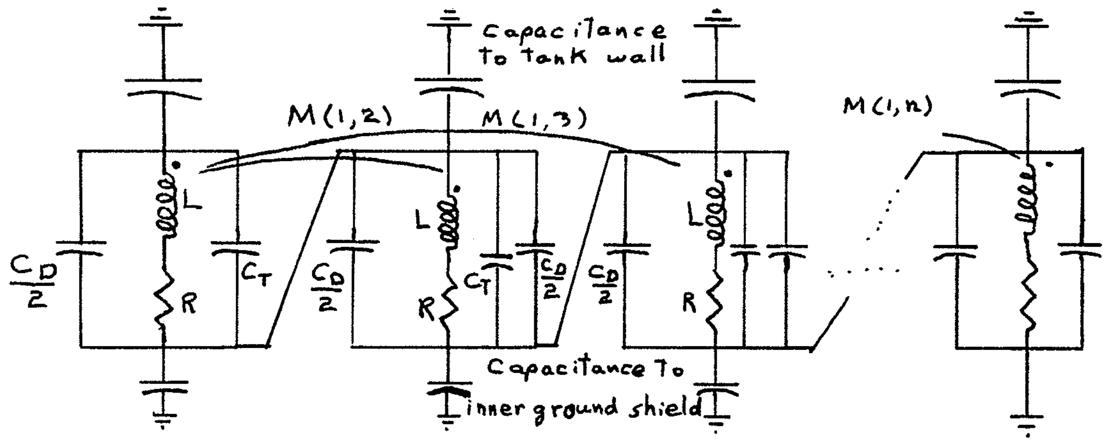



FIGURE 18

SCHEMATIC OF A DROP-DOWN DISC WINDING

The equivalent circuit in Figure 21 can be used to represent the drop-down disc winding and the parameters are:

$C_g$ (1) = capacitance to tank of first coil

$C_g$ (K) = capacitance to tank of (K + 1)' th coil plus capacitance to inner shield of the K'th coil( $K \neq 1$).

$C_T$ (K) = inter-turn capacitance of K'th coil plus one-half of distributed capacitance between the K'th and (K-1)'th coil plus one-half of the distributed capacitance between the K'th and the (K + 1)'th coil.

The drop-down strip winding schematic is shown in Figure 19.

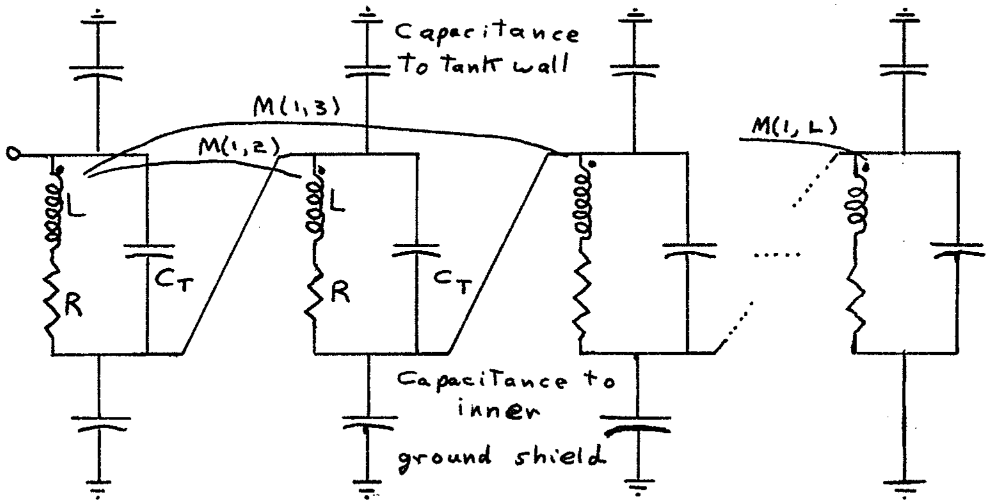



FIGURE 19

SCHEMATIC OF DROP-DOWN STRIP WINDING

The parameters of the equivalent circuit in Figure 21 are:

$C_T$ (K) = inter-turn capacitance of the K'th coil

$C_g$ (1) = capacitance to tank of first coil

$C_g$ (K) = capacitance to tank of (K + 1)'th coil plus capacitance to the inner shield of K'th coil

The back-turn strip winding has the same equivalent circuit as the back-turn disc and drop-down strip winding, but the ground capacitance is slightly different. Figure 20 shows the schematic of a back-turn strip winding.

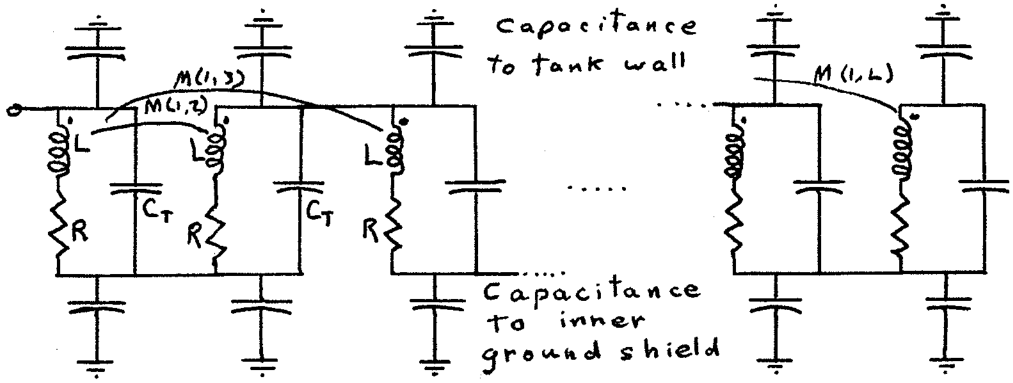



FIGURE 20

SCHEMATIC OF A BACK-TURN STRIP WINDING

The equivalent circuit is drawn in Figure 21 and the parameters are as follows:

$C_T$ (K) = inter-turn capacitance of K'th coil

$C_g$ (1) = capacitance to tank wall of first coil

$C_g$ (K) = (for K even) capacitance to the inner shield of the K'th and the (K + 1)'th coil, (for K odd) the capacitance to the tank wall of the (K-1)'th and the K'th coil.

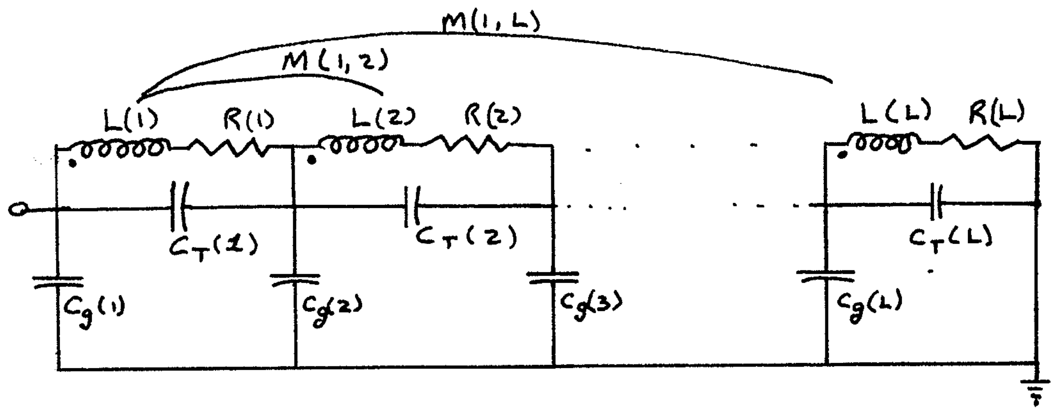



FIGURE 21

THE EQUIVALENT CIRCUIT OF THE THREE TYPES OF WINDINGS

The equivalent circuit in Figure 21 is used to represent the three types of windings. Appendix I contains the circuit analysis and the matrix manipulation necessary to obtain a form that can be readily solved with the Runge-Kutta method. Two programs, which are given in Appendix J, were written to perform the computation. The first program reads the parameters, inverts and multiplies the matrices and punches the coefficients of the differential equations in a form that can be solved by the Runge-Kutta method. The second program, which was written for the IBM 360, performs the computation of the voltage to ground at each crossover.

A series of tests were performed on a strip winding consisting of five coils. The frequency response of one of the coils is given in Figure A-3. The winding was drop-down connected and the impulse response at each crossover was measured with and then without an inner ground shield. The coil was changed to a back-turn connected winding and the tests re-run.

The measured and calculated impulse voltage response (Figure A-14) of a drop-down connected winding with an inner ground shield compares very well in amplitude, but the frequencies of response do not coincide. In order for the computed wave to compare more favorably a ground capacitance of about 20 pf. should be used. The measured response (Figure A-15) of the drop-down connected winding with no inner ground shield has oscillations of larger amplitude than the calculated response. The reason they do not compare more favorably is that the oscilloscope leads add a stray capacitance that is not taken into account by the calculations. In Chapter III on the subject of frequency response of a single coil, the oscilloscope probe and stray ground capacitance had to be taken into account. A better comparison could be obtained if the parameters were to account for the ground capacitance added by the oscilloscope probe.

The measured and calculated impulse response (Figure A-16) of the back-turn strip winding with an inner ground shield compares favorably in amplitude, but again the frequencies do not coincide. A better comparison could be obtained by using a smaller ground capacitance.

The measured and calculated impulse response (Figure A-17) of the back-turn winding with no inner ground shield does not compare very well for the same reasons as the drop-down winding with no inner shield.

A short comparison of the effects of parameter change can be noted by examining the calculated impulse voltages in Figures A-18, A-19, and A-20. The resistance has an effect on the damping at all times. Figure A-18 indicates that the voltages are insensitive to a change of resistance. An increase in the ground capacitance increases the magnitude of the voltage to ground and decreases the frequencies of response. Figure A-19 shows the effect of varying to ground capacitance.

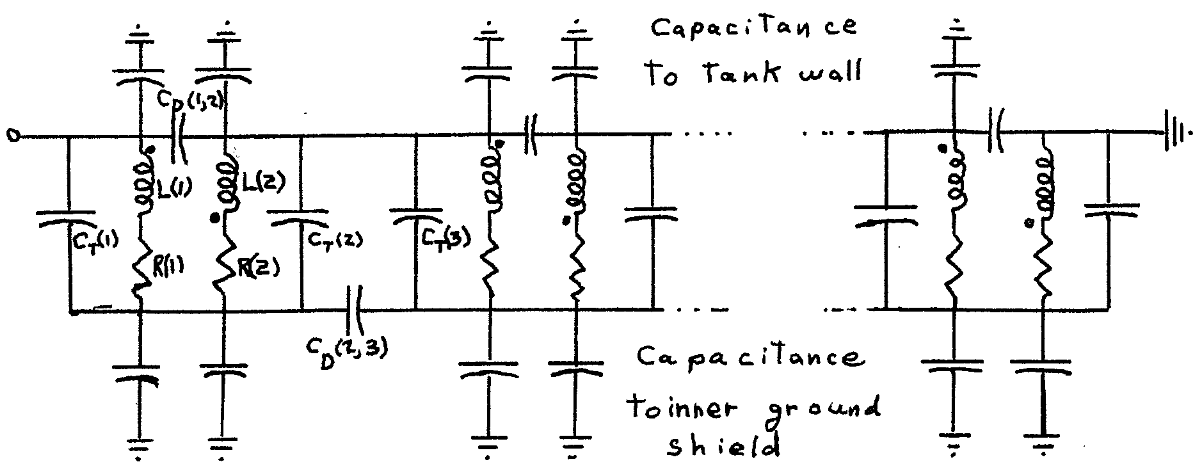
For strip windings a simplifying approximation can be made by lumping the self- and mutual-inductances into an equivalent self-inductance. This decreases the frequencies of oscillation, increases the magnitude of the voltages and increase the time to the first maximum. Figure A-20 shows the calculated responses with the same total inductance, but with mutuals considered on one computation and the mutuals lumped into an equivalent self-inductance in the other computation.

The concept of using lumped elements to represent the distributed parameters of a coil is very useful for describing the behavior of the winding. This method provides a very accurate model with which to study the windings.

## CHAPTER VI

## THE EQUIVALENT CIRCUIT OF A BACK-TURN DISC WINDING

The back-turn disc winding has an equivalent circuit that is different and more complicated than the three types previously discussed. The schematic, Figure 22, of the winding is obtained in the same manner as for the other three windings.

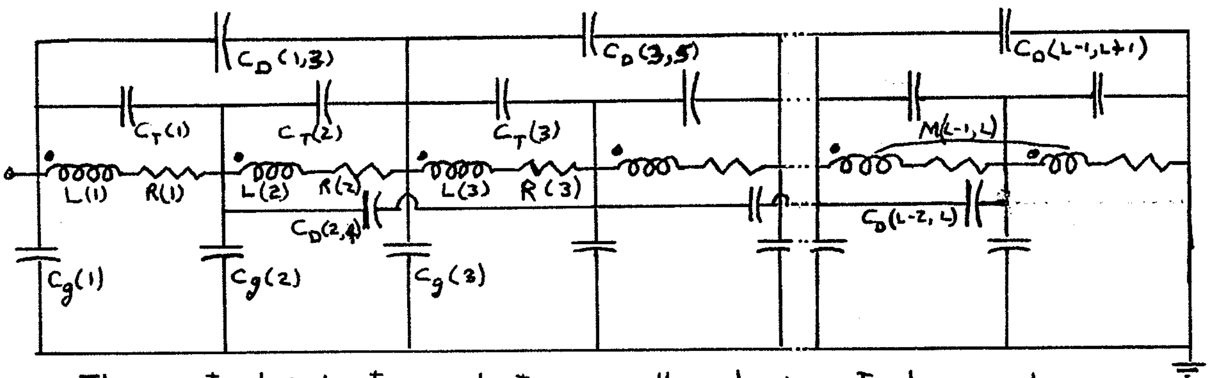



The mutual-inductance between all coils is not shown above

FIGURE 22

SCHEMATIC OF A BACK-TURN DISC WINDING

The equivalent circuit is given in Figure 23.




The mutual-inductance between all coils is not shown above

FIGURE 23

THE EQUIVALENT CIRCUIT OF A BACK-TURN

DISC WINDING

The parameters are:

$C_g$ (1) = capacitance to tank of first coil

$C_g$ (K) = (K even) capacitance to inner shield or low voltage winding of K'th and (K + 1)'th coil

$C_g$ (K) = (K odd), capacitance to tank wall of K'th and (K + 1)'th coil

$C_T$ (K) = inter-turn capacitance of K'th coil

R (K) = resistance of K'th coil

$C_D(K, K+2)$ = capacitance between discs (across a collar for K odd, K = 1 to L − 1.

$C_D(K, K+2)$ = capacitance between discs (across a duct) for K even, K = 2 to L − 2.

L (K) = self-inductance of K'th coil.

M (K, m) = mutual-inductance between K'th and m'th coil for K = 1 to L and m = 1 to L.

## CHAPTER VII

## THE FILTER CONCEPT OF A TRANSFORMER WINDING

A further understanding of the transformer winding can be obtained by examining it from the point-of-view of filter theory. The input wave is fixed and composed of many different frequencies. The frequency components of a 1 x 50 microsecond impulse wave have been calculated and it has few significant frequencies above 0.1 MHZ.[18] If the winding has a comparatively low cut-off frequency then most of the frequency components of the input will be stopped by the line end of the winding. The voltage distribution will be non-uniform, with comparatively high voltages between sections and high voltages to ground at the line end. On the other hand, if the winding has a high cut-off frequency most of the input frequencies will be passed by the winding and the voltage distribution will be very nearly linear with turns.

The frequency response, Figure A-21, of an isolated strip winding with no ground shield was measured using the same circuit as for a single coil. The impulse response of the winding with an inner shield was then measured and the maximum inter-section voltages plotted in Figure A-22. Since the cut-off frequency of the winding was comparatively high the voltage distribution is very nearly linear as the theory would predict.

---

[18] Miles, J. G. "Frequency Spectra of Standard Impulse Waveshapes", <u>The Metropolitan-Vickers Gazette</u>, Vol. 25, September, 1954, pp. 367-69.

The same tests were performed on a disc winding (Figures A-23 and A-24). The cut-off frequency of the winding is rather low and a poor impulse voltage distribution would be expected. The theory is supported by the experimental evidence, because the voltage distribution of Figure A-24 is very non-uniform.

The concept of filter theory provides another method of predicting and understanding the impulse response behavior of a transformer winding.

# CHAPTER VIII

## CONCLUSIONS AND FURTHER RESEARCH

The parameters of high voltage transformer windings have been identified and a model that accurately describes the winding under impulse conditions has been developed. The accuracy with which the impulse voltages may be predicted is probably limited principally by the accuracy to which the parameters are known. The idea of using lumped elements to generate an equivalent circuit to replace a system of distributed parameters has been shown to be a very satisfactory method of analyzing the problem. The mechanisms of distributed capacitance have been indicated and illustrated by measurements. Frequency response methods have been used to identify parameters that are used in impulse response study. Finally filter theory has contributed a greater understanding of the winding behavior.

Further study using the model and equivalent circuit presented in this thesis can be pursued. The circuit equations of a back-turn disc winding can be analyzed with the help of the large storage, high speed computer now available. The sensitivity of the impulse response due to parameter change can be studied with sensitivity functions.

The determination of inter-turn capacitance of a multi-turn coil could be investigated and the inter-disc capacitance could be specified more accurately.[19] The effect of an iron core and low voltage winding which the ground shield represented could be investigated. Approximations to the exact model can be studied to find a simpler method of determining the impulse voltage distribution. A further investigation is needed to clarify the term "initial voltage distribution".

---

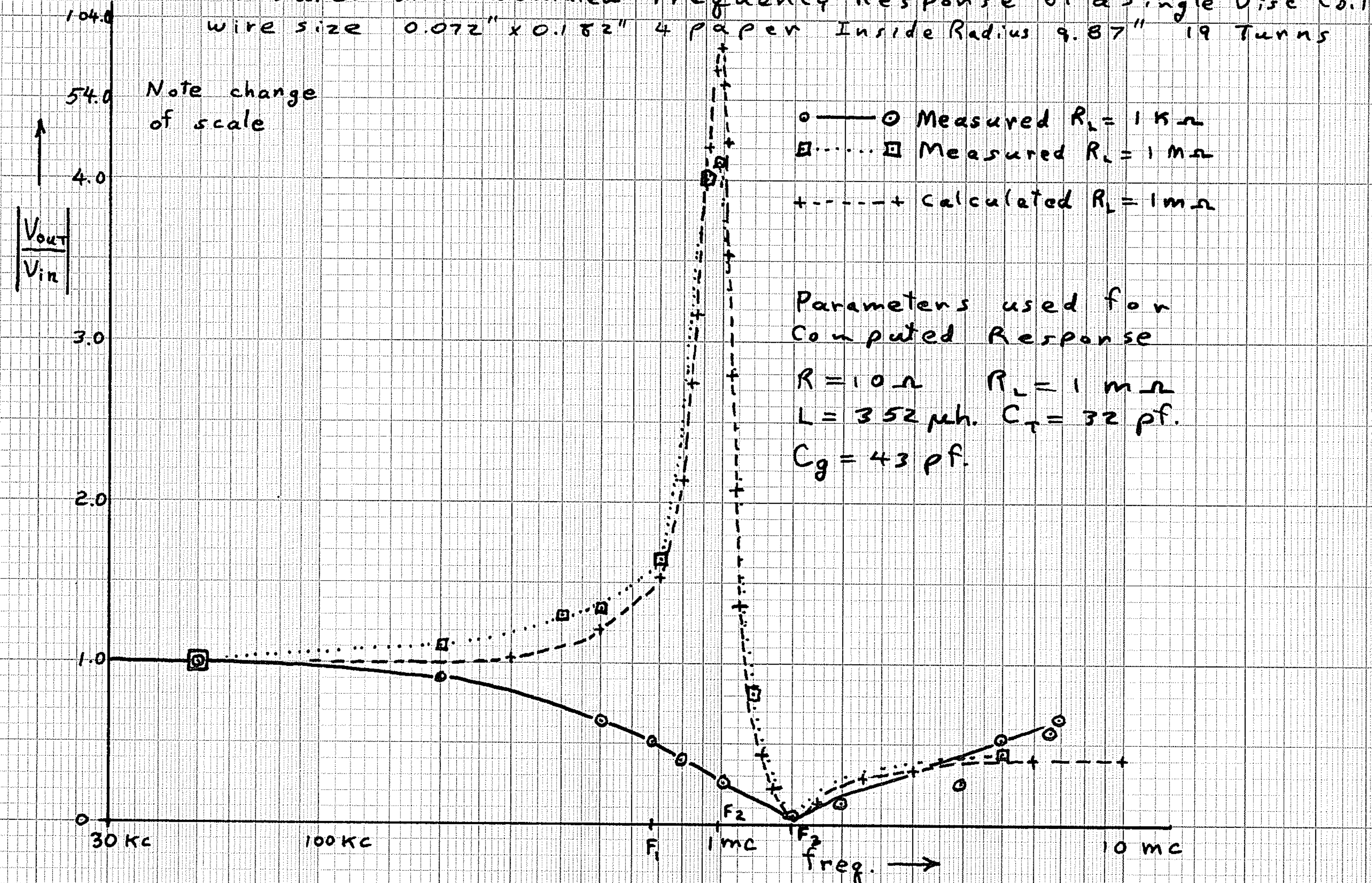
[19] Veverka, A. and Hon, A. "Disc-Coil Winding on Extra-High Voltage Transformers", Electrotech. Obzor. (Czechoslovakia), Vol. 54, No. 7, pp. 304-9 (1965). In Czech.

APPENDICES

APPENDIX A

# FIGURE A-1

Measured and Calculated Frequency Response of a Single Disc Coil
wire size 0.072" x 0.182" 4 paper Inside Radius 9.87" 19 Turns



# FIGURE A-2

Measured and Calculated Frequency Response of Single Disc Coil
wire size 0.102" x 0.289" 4 paper Inside Radius 9.87" 15 Turns

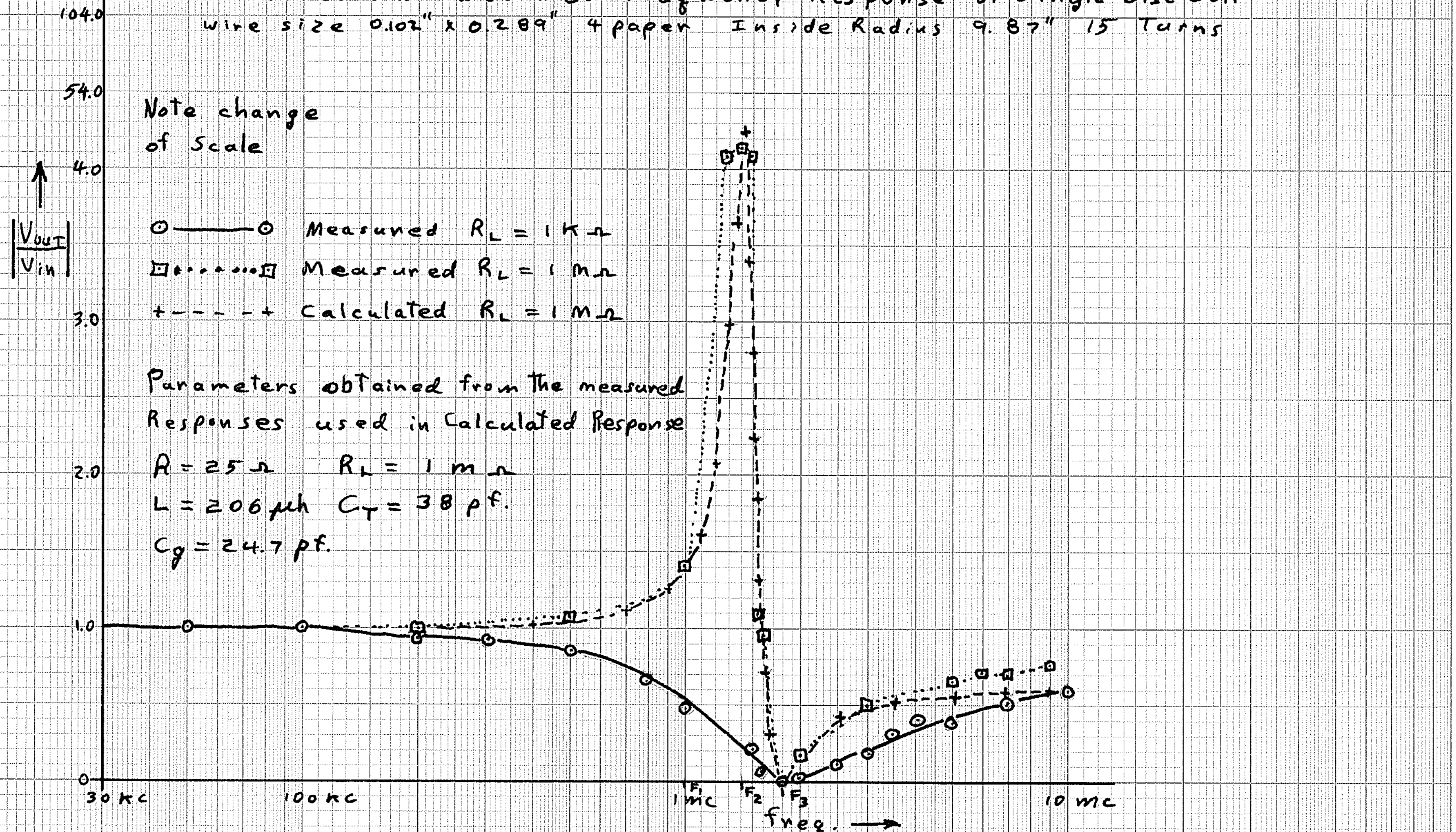
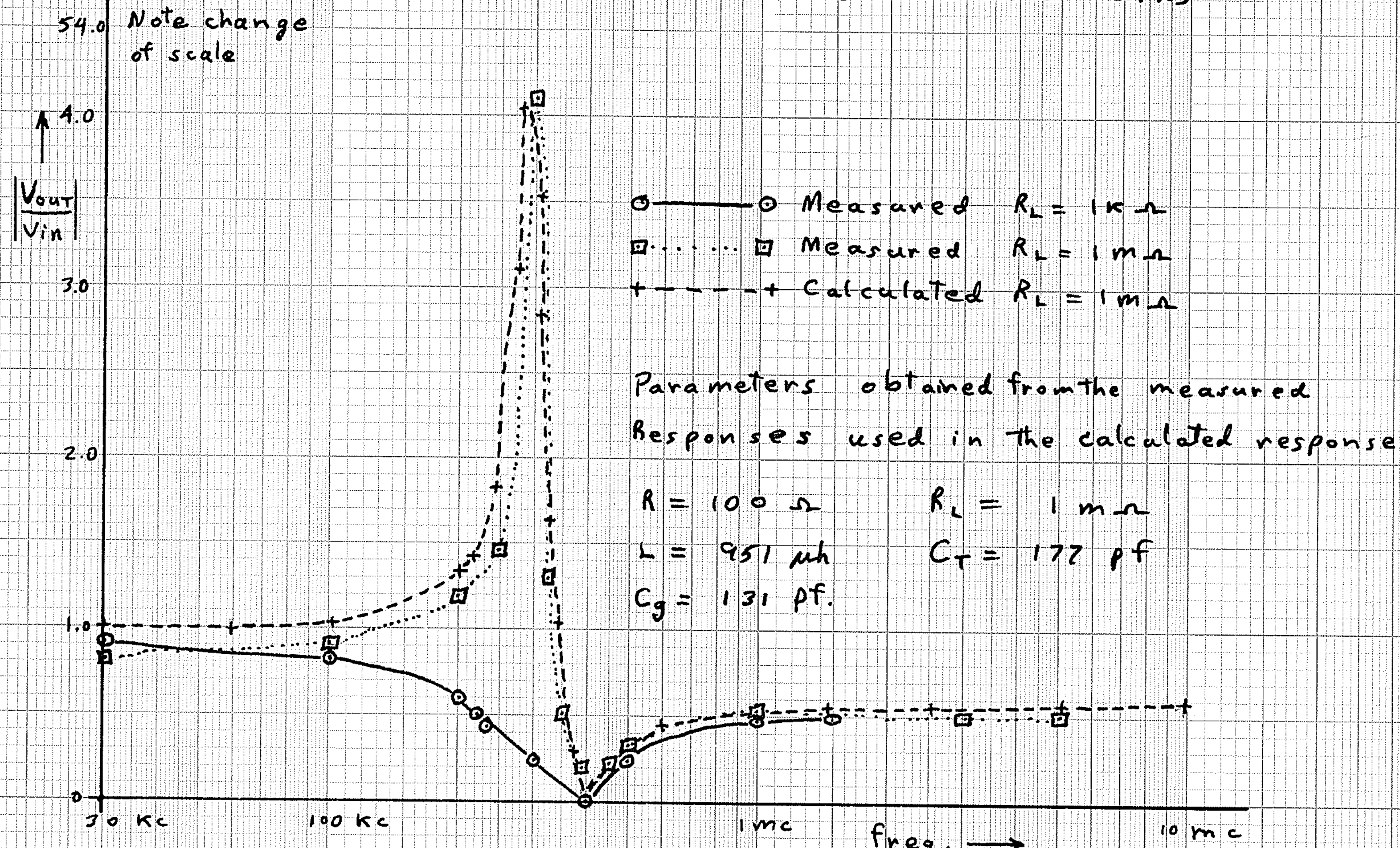


FIGURE A-3

Measured and Calculated Frequency Response of a Single Strip Coil

Wire Size 3.0" x 0.020" with 0.005" insulation copper to copper

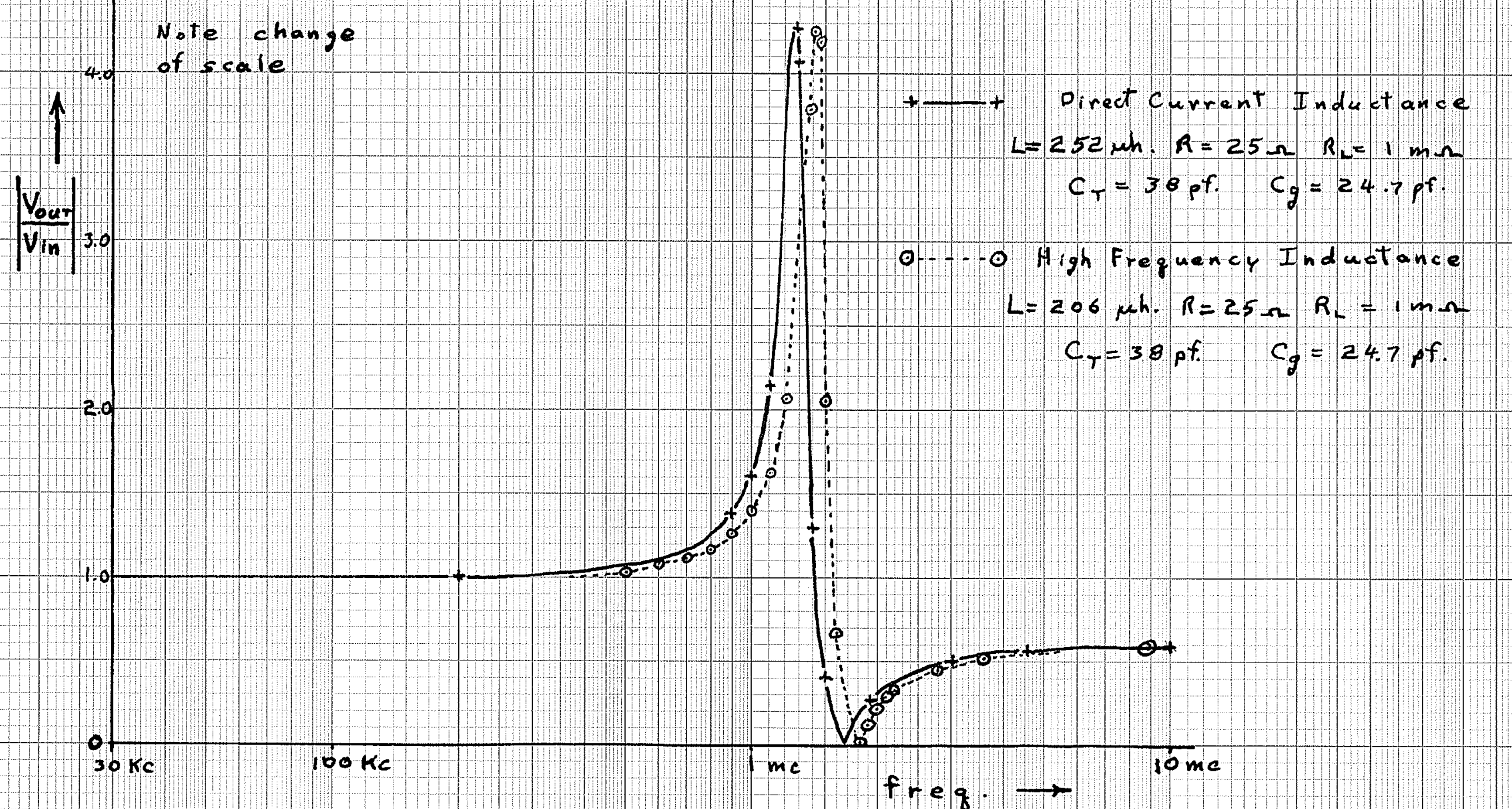
Inside Radius 5.5" 50 Turns



# FIGURE A-4

Comparison of Computed Frequency Response of a Single Coil using High Frequency Inductance and Direct Current Inductance

Wire Size 0.102" x 0.289 4 paper Inside Radius 9.87" 15 Turns




rithmic
to the inch

FIGURE A-5

IMPULSE WAVESHAPES

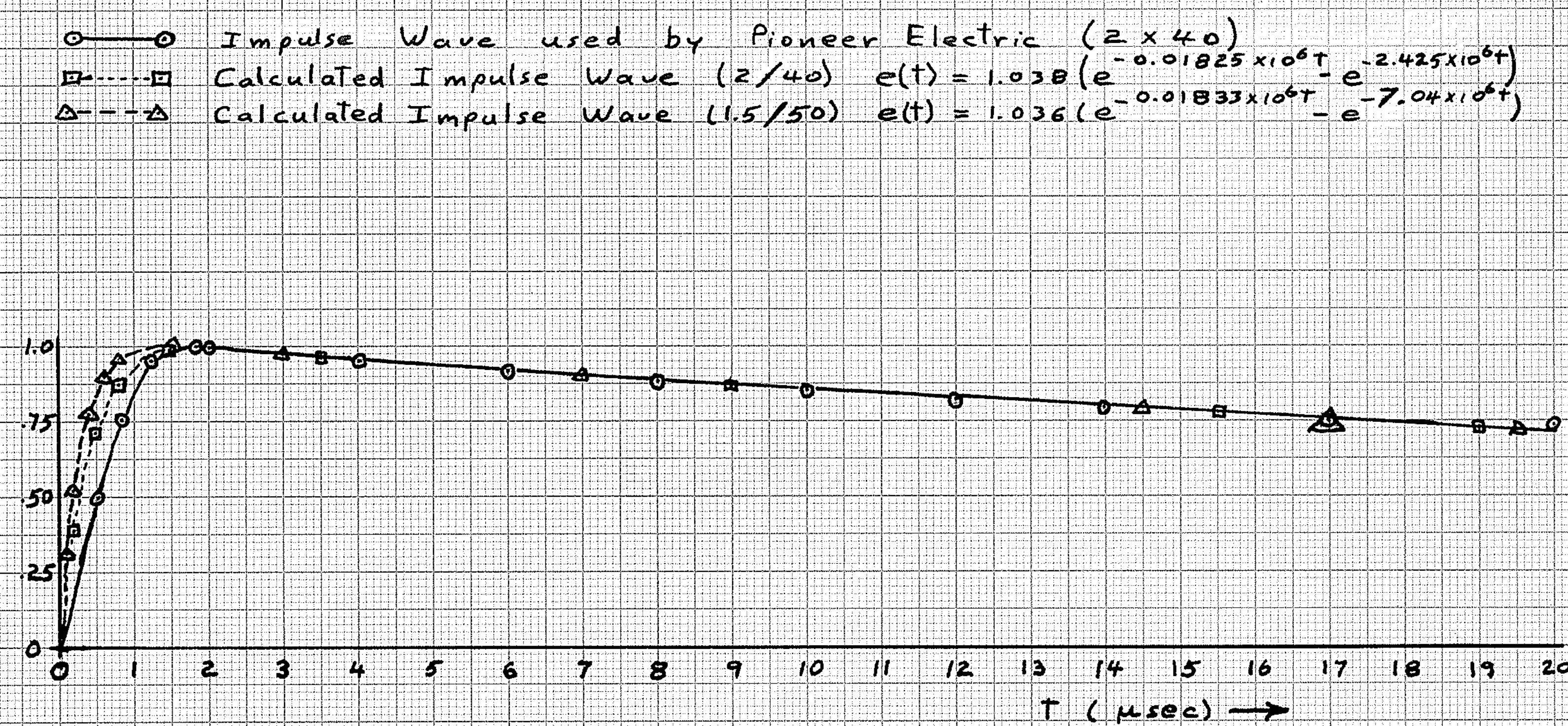
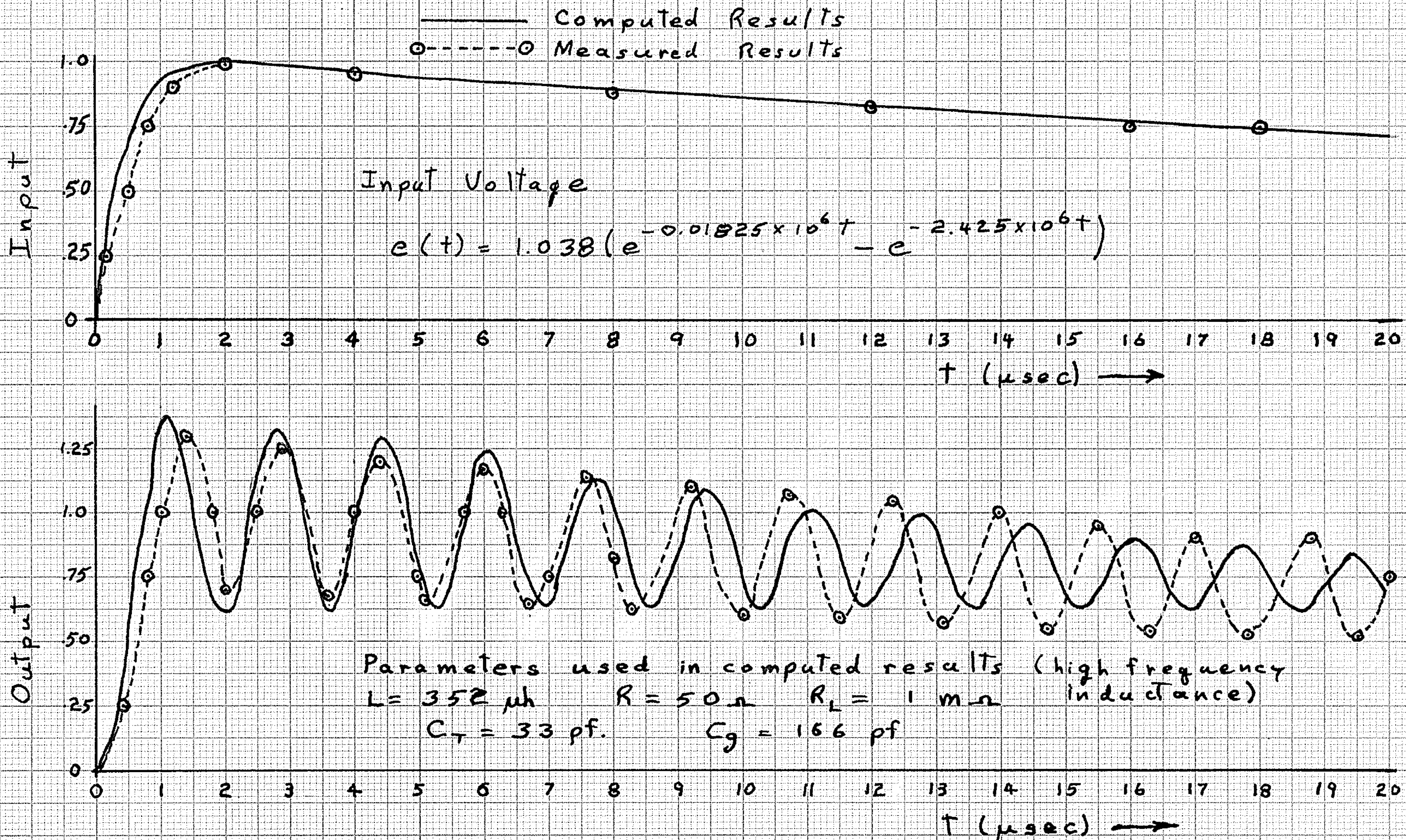


FIGURE A-6

Measured and Calculated Impulse Response of a Single Disc Coil

Wire Size 0.072" x 0.182" 4 paper Inside Radius 9.87" Radial Build 1.75"

19 Turns



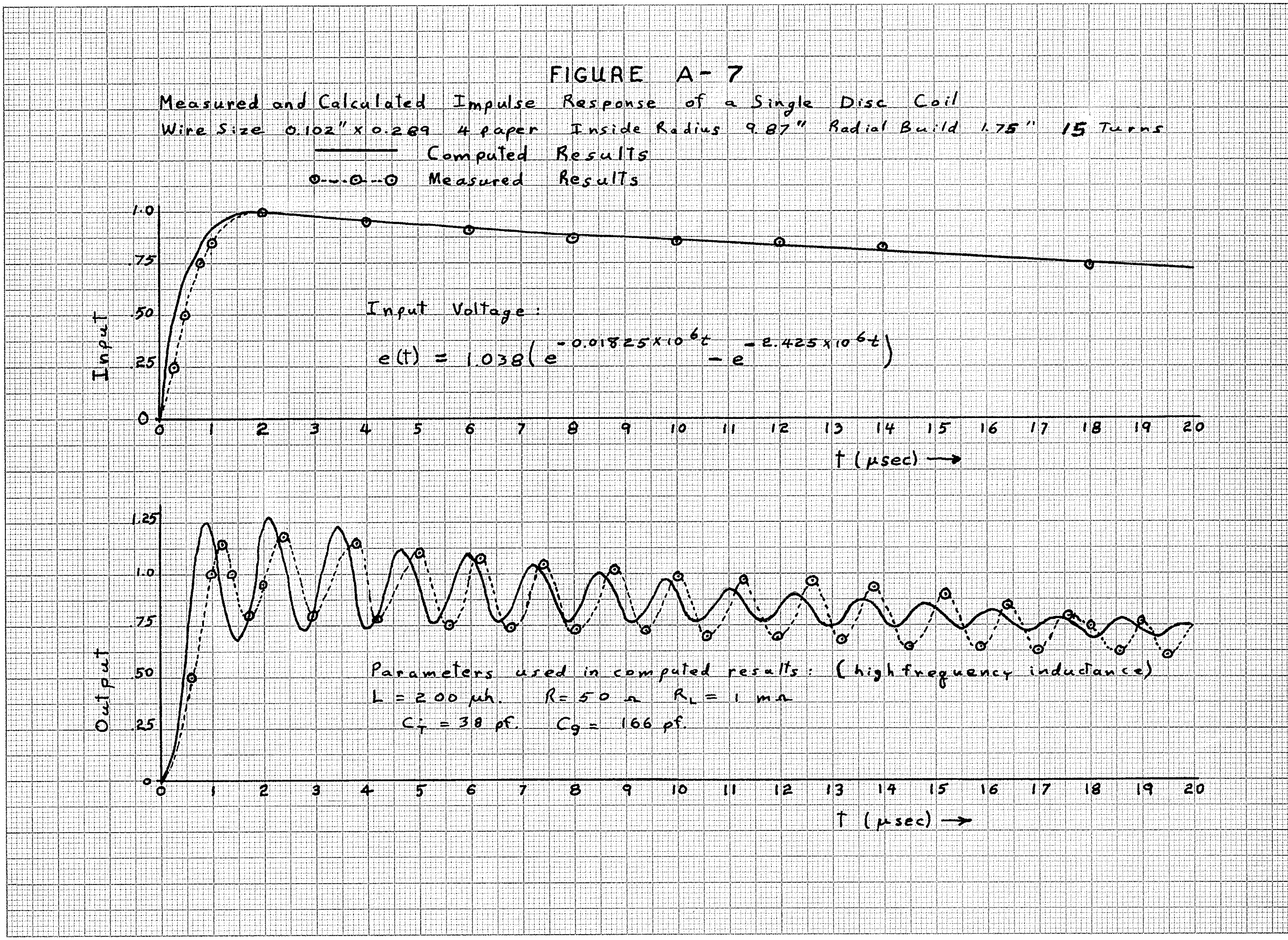


FIGURE A-7
Measured and Calculated Impulse Response of a Single Disc Coil
Wire Size 0.102" x 0.289 4 paper Inside Radius 9.87" Radial Build 1.75" 15 Turns
Computed Results
Measured Results
Input
1.0
.75
.50
.25
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
t (μsec) →
Input Voltage:
$e(t) = 1.038\left(e^{-0.01825\times10^{6}t} - e^{-2.425\times10^{6}t}\right)$
Output
1.25
1.0
.75
.50
.25
0
Parameters used in computed results: (high frequency inductance)
L = 200 μh. R = 50 Ω R_L = 1 mΩ
C_T = 38 pf. C_g = 166 pf.
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
T (μsec) →

# FIGURE A-8

Measured and Calculated Impulse Response of a Single Strip Coil

Wire Size 3.0" X 0.020 0.005" Insulation Copper to Copper Inside Radius 5.5"

Radial Build 1.5" 50 Turns

——— Computed Results

O- - - -O Measured Results

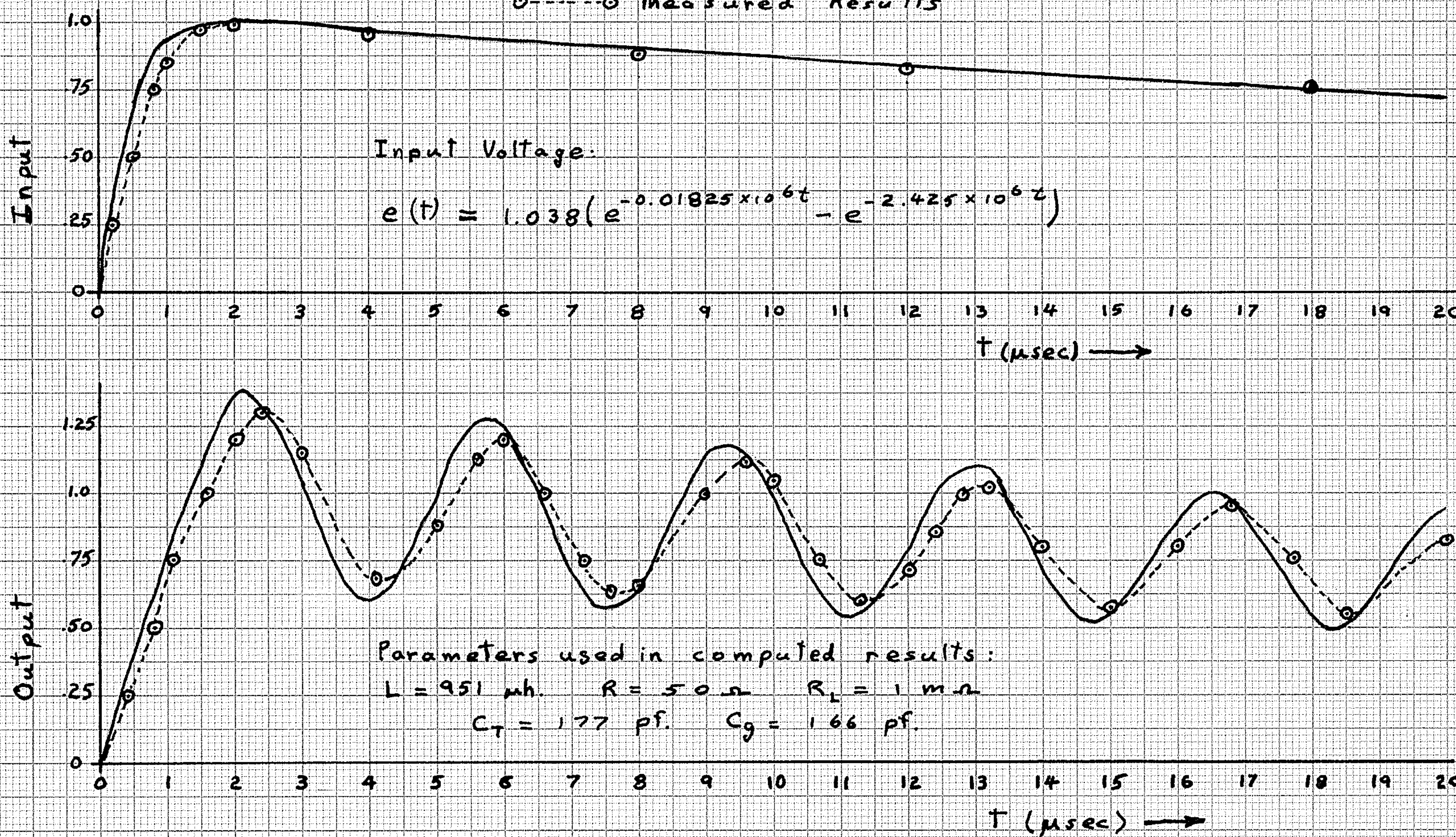
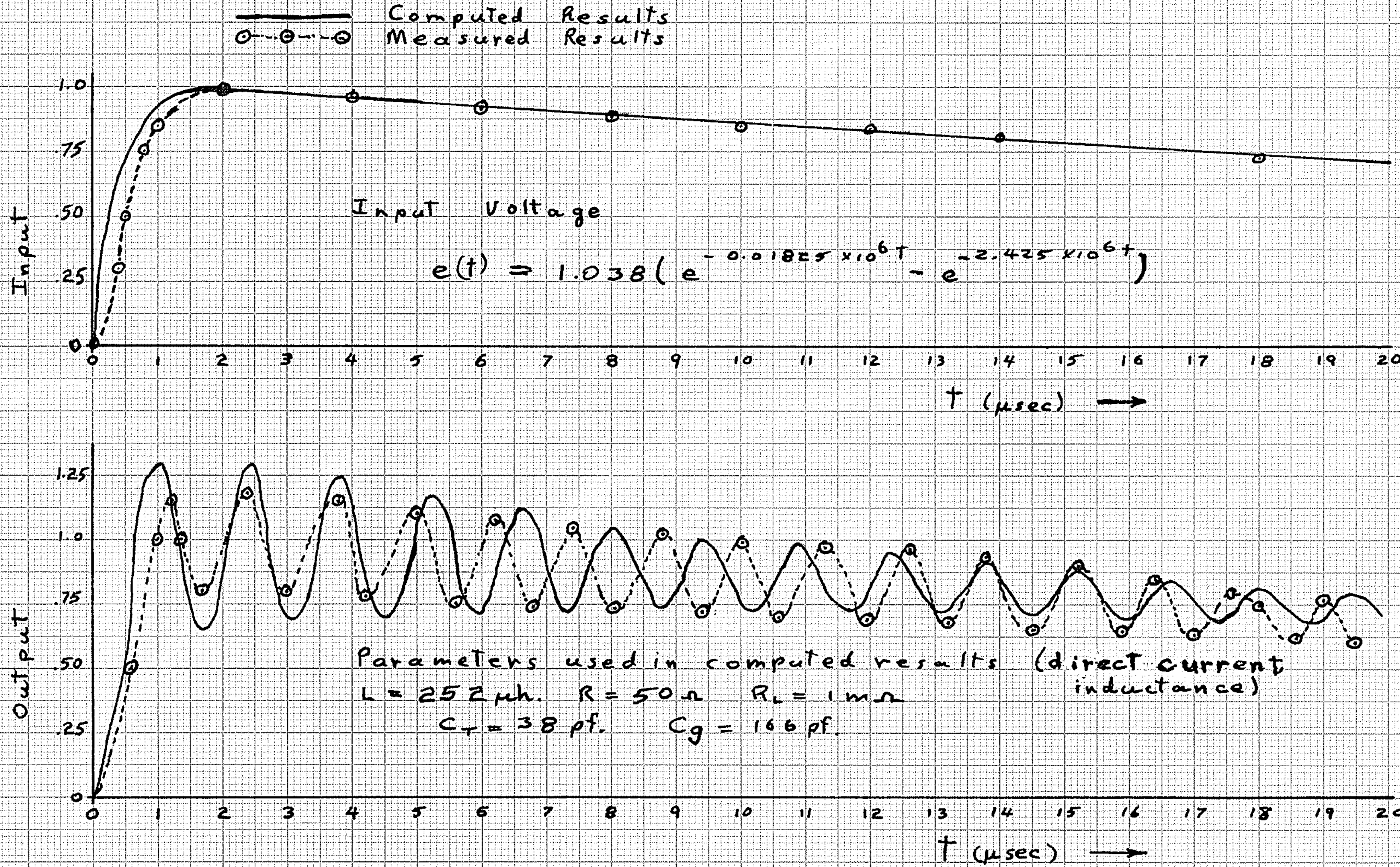


FIGURE A- 9

Measured and Calculated Impulse Response of a Single Disc Coil

Wire Size 0.102" x 0.289" 4 paper Inside Radius 9.87" Radial Build 1.75" 15 Turns




eters to the Centimeter

# FIGURE A-10

Measured and Calculated Mutual Inductance Between Two Coils

Wire Size 0.072" x 0.182" 4 paper Inside Radius 9.87" 19 Turns

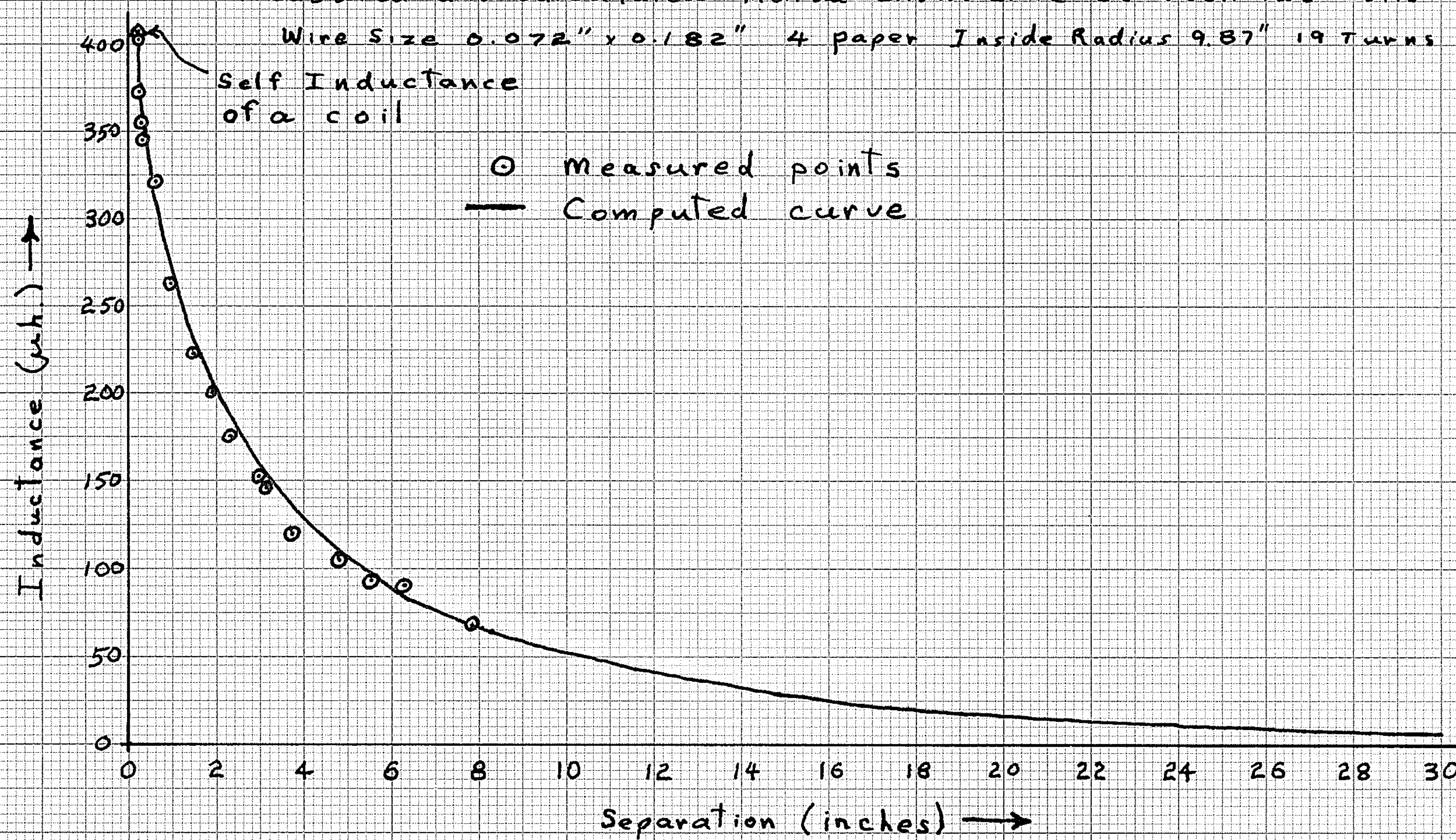
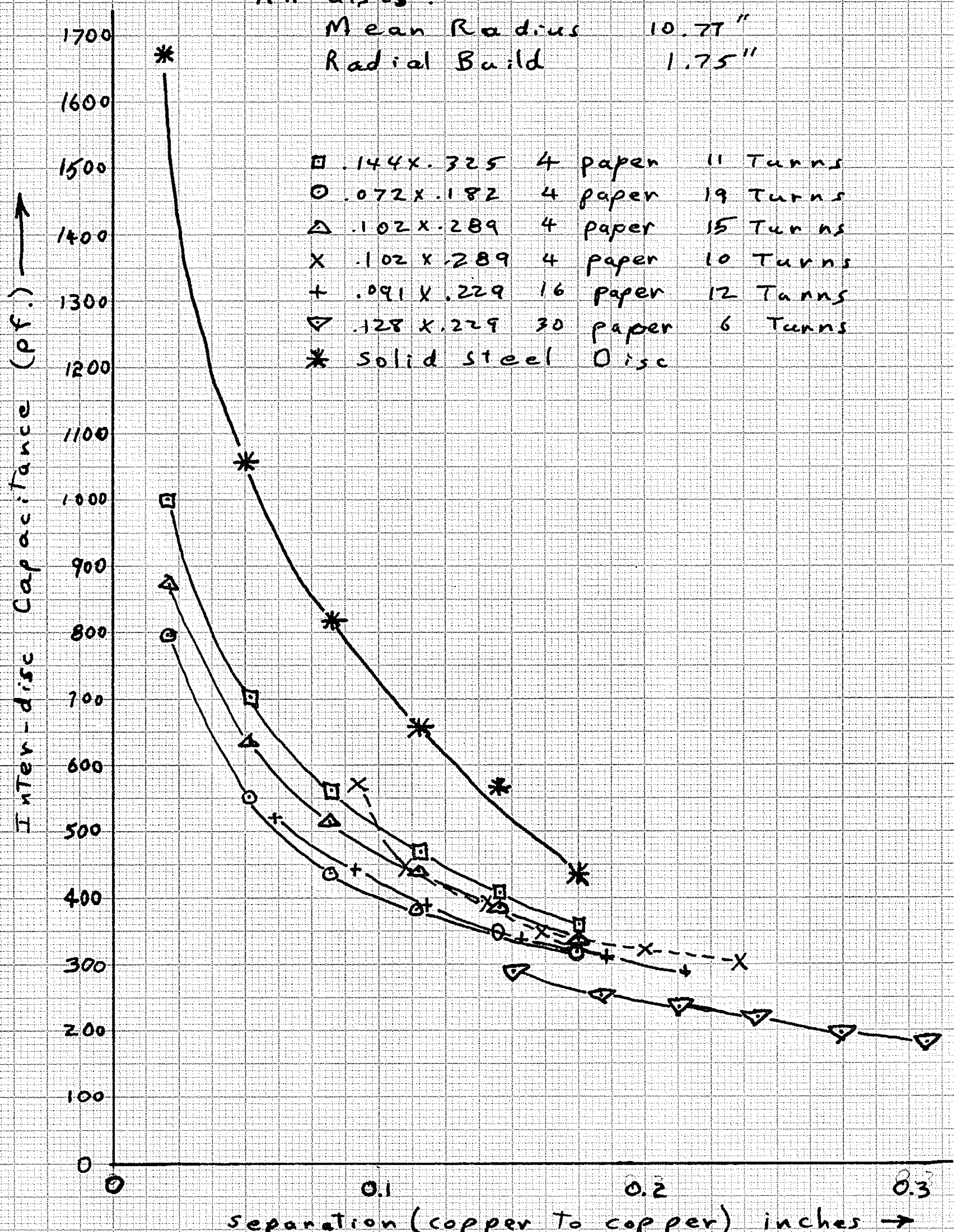


FIGURE A-11

Measured Inter-disc Capacitance
Solid Insulation Dielectric
(No connection between discs)
All discs:
Mean Radius 10.77"
Radial Build 1.75"



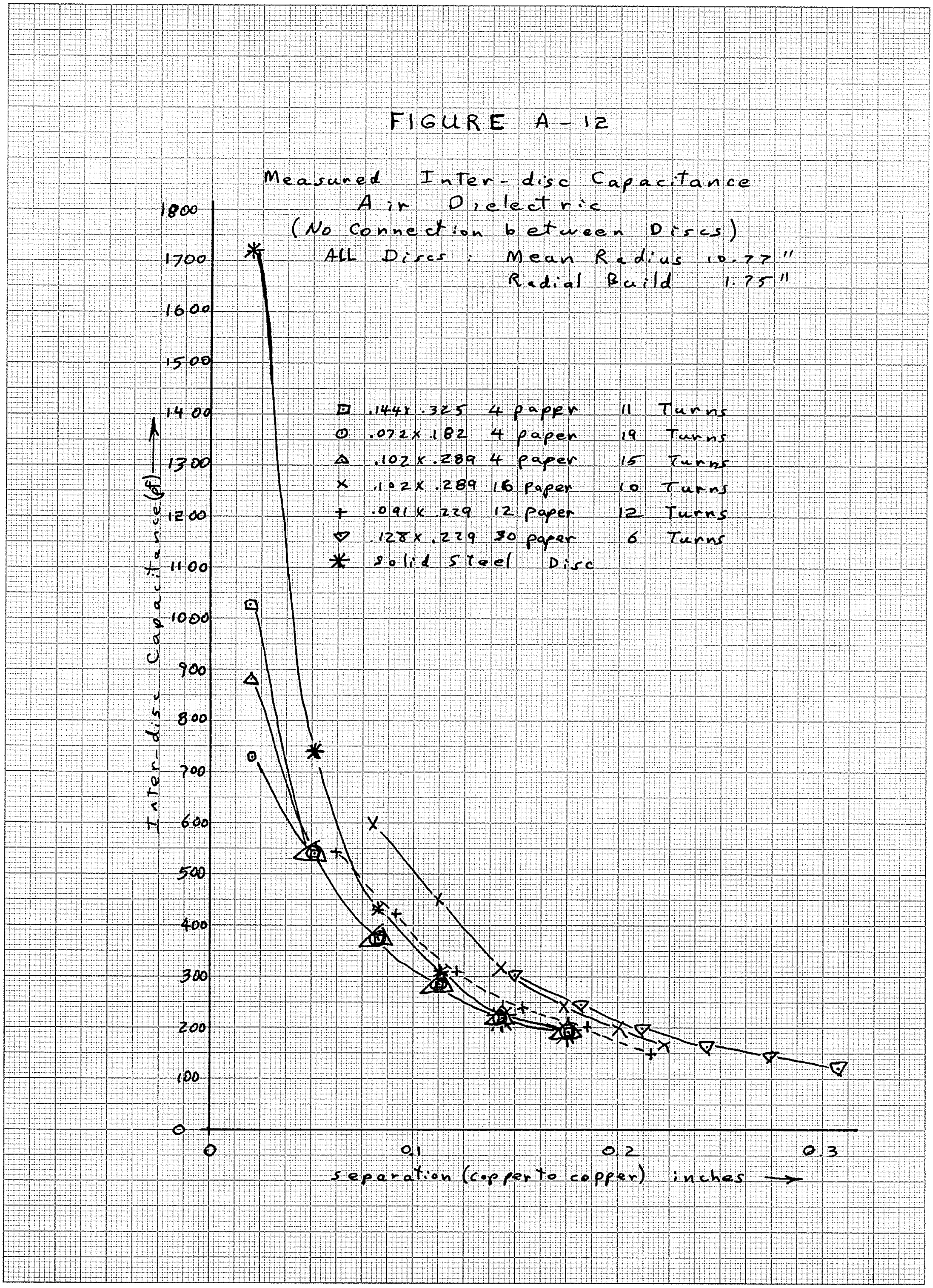
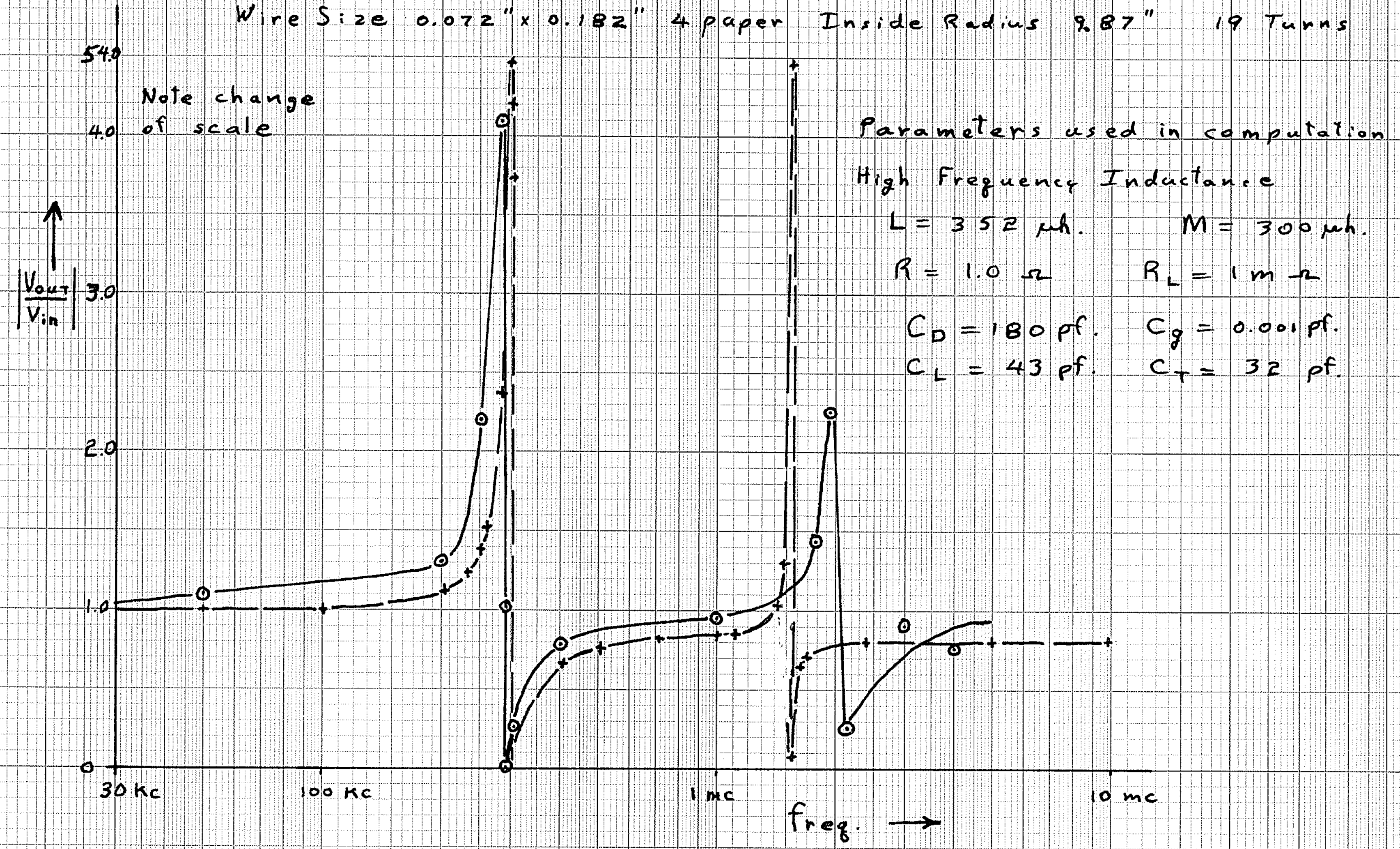


FIGURE A-12
Measured Inter-disc Capacitance
Air Dielectric
(No Connection between Discs)
ALL Discs : Mean Radius 10.77"
Radial Build 1.75"
.144x.325 4 paper 11 Turns
.072x.182 4 paper 19 Turns
.102x.289 4 paper 15 Turns
.102x.289 16 paper 10 Turns
.091x.229 12 paper 12 Turns
.128x.229 30 paper 6 Turns
Solid Steel Disc
Inter-disc Capacitance (pf)
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
0
0.1
0.2
0.3
separation (copper to copper) inches

FIGURE A- 13

Calculated and Measured Frequency Response of a Pair of Disc Coils

Wire Size 0.072" x 0.182" 4 paper Inside Radius 8.87" 19 Turns




rithmic
to the inch

# FIGURE A-14

Calculated and Measured Impulse Response of a Five Section Drop-Down Strip Winding With Inside Ground Plane 1/2" from inner turns

Inside Radius 5.5" Wire Size 3.0" x 0.020" 0.005" Insulation 50 Turns 1/2" Duct

Parameters used in computation:

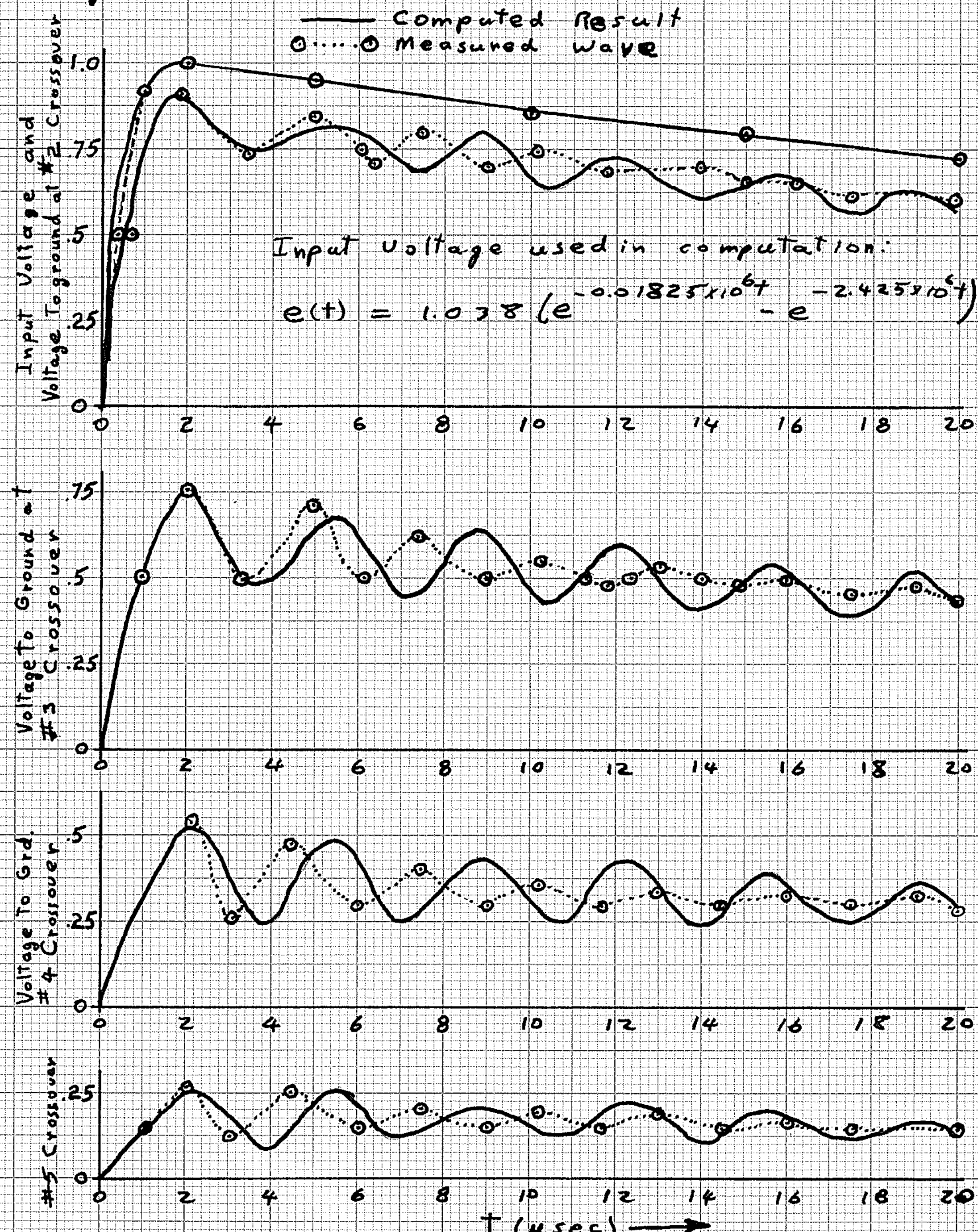
$C_g(1) = 30.0\,pf$ $C_T(1) = 177.\,pf$ $R(1) = 100\,\Omega$ $L(1,1) = 951\,\mu h$

$C_g(2) = 30.0\,pf$ $C_T(2) = 177\,pf.$ $R(2) = 100\,\Omega$ $M(1,2) = 391\,\mu h$

$C_g(3) = 30.0\,pf$ $C_T(3) = 177\,pf.$ $R(3) = 100\,\Omega$ $M(1,3) = 164\,\mu h$

$C_g(4) = 30.0\,pf$ $C_T(4) = 177\,pf$ $R(4) = 100\,\Omega$ $M(1,4) = 80\,\mu h$

$C_g(5) = 30.0\,pf$ $C_T(5) = 177\,pf.$ $R(5) = 100\,\Omega$ $M(1,5) = 65\,\mu h$



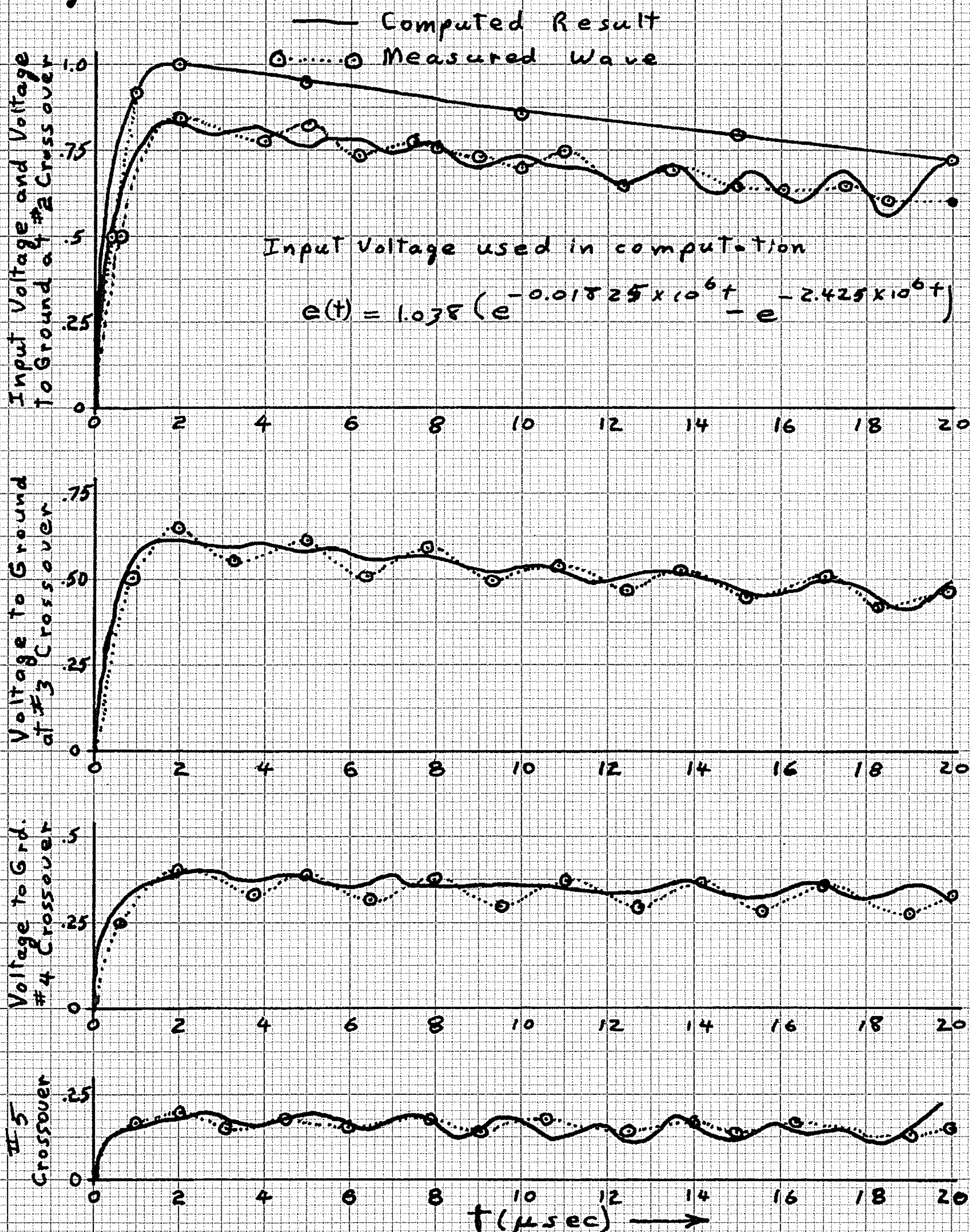
# FIGURE A-15

Calculated and Measured Impulse Response of a Five Section Drop-Down Winding with no Ground Planes

Inside Radius 5.5" Wire Size 3.0" x 0.020" 0.005" Insulation 50 Turns $\frac{1}{2}$" Duct

Parameters used in computation:

| | | | |
|---|---|---|---|
| Cg(1) = 1.0 pf. | $C_T$(1) = 177 pf. | R(1) = 0.022 Ω | L(1,1) = 951 μh |
| Cg(2) = 1.0 pf. | $C_T$(2) = 177 pf. | R(2) = 0.022 Ω | M(1,2) = 391 μh |
| Cg(3) = 1.0 pf. | $C_T$(3) = 177 pf. | R(3) = 0.022 Ω | M(1,3) = 164 μh |
| Cg(4) = 1.0 pf. | $C_T$(4) = 177 pf. | R(4) = 0.022 Ω | M(1,4) = 80 μh |
| Cg(5) = 1.0 pf. | $C_T$(5) = 177 pf | R(5) = 0.022 Ω | M(1,5) = 65 μh |



# FIGURE A- 16

Calculated and Measured Impulse Response of a Five Section Back-Turn Strip Winding with an Inner Ground Plane 1/2" from inner Turns

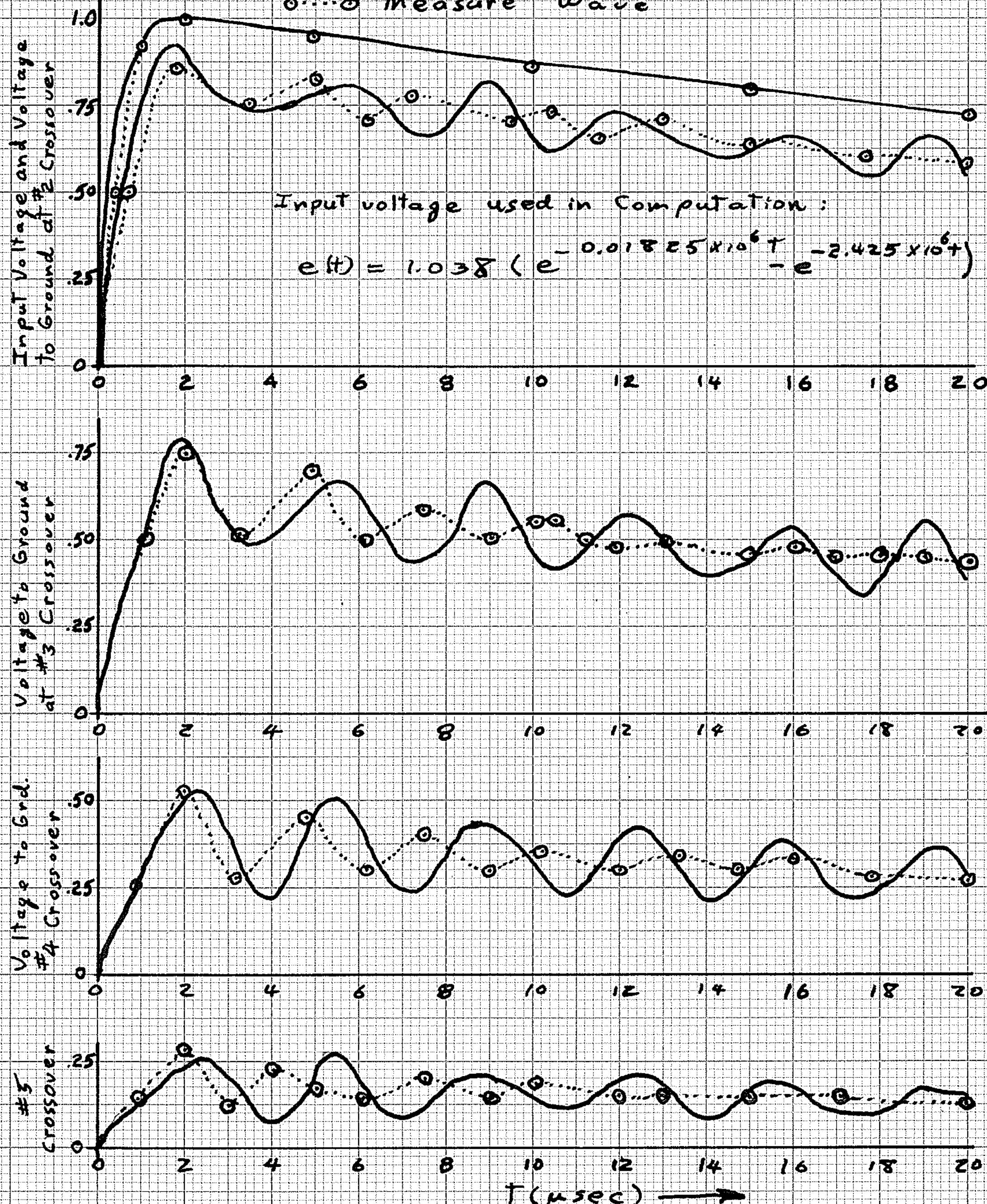
Inside Radius 5.5" Wire Size 3.0" x 0.020" 0.005" Insulation 50 Turns $\frac{1}{2}$" Duct

Parameters used in computation:

| | | | |
|---|---|---|---|
| $C_g(1) = 1.0$ pf. | $C_T(1) = 177$ pf. | $R(1) = 100.0\ \Omega$ | $L(1,1) = 951\ \mu h$ |
| $C_g(2) = 60.0$ pf | $C_T(2) = 177$ pf | $R(2) = 100.0\ \Omega$ | $M(1,2) = 391\ \mu h$ |
| $C_g(3) = 1.0$ pf | $C_T(3) = 177$ pf | $R(3) = 100.0\ \Omega$ | $M(1,3) = 164\ \mu h$ |
| $C_g(4) = 60.0$ pf | $C_T(4) = 177$ pf | $R(4) = 100.0\ \Omega$ | $M(1,4) = 80\ \mu h$ |
| $C_g(5) = 1.0$ pf | $C_T(5) = 177$ pf | $R(5) = 100.0\ \Omega$ | $M(1,5) = 65\ \mu h$ |

—— Computed Result

o···o Measure Wave



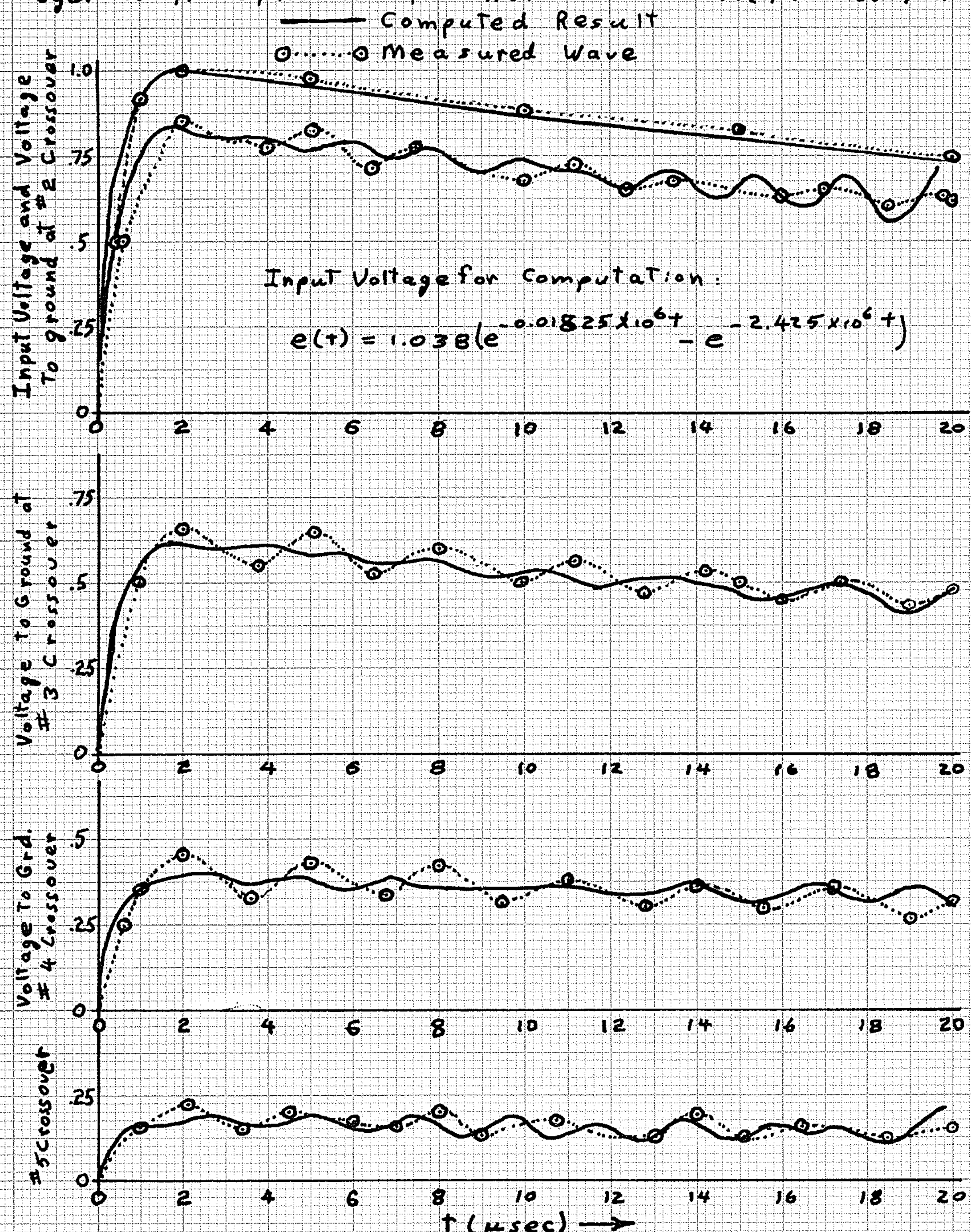
# FIGURE A – 17

Calculated and Measured Impulse Response of a Five Section Back-Turn Strip Winding with no ground plane

Inside Radius 5.5" Wire Size 3.0" x 0.020" 0.005" Insulation 50 Turns 1/2" Duct

Parameters used in computation:

| | | | |
|---|---|---|---|
| $C_g(1) = 1.0$ pf | $C_T(1) = 177$ pf | $R(1) = 0.022\ \Omega$ | $L(1,1) = 951\ \mu h.$ |
| $C_g(2) = 1.0$ pf | $C_T(2) = 177$ pf | $R(2) = 0.022\ \Omega$ | $M(1,2) = 391\ \mu h.$ |
| $C_g(3) = 1.0$ pf | $C_T(3) = 177$ pf | $R(3) = 0.022\ \Omega$ | $M(1,3) = 164\ \mu h.$ |
| $C_g(4) = 1.0$ pf | $C_T(4) = 177$ pf | $R(4) = 0.022\ \Omega$ | $M(1,4) = 80\ \mu h.$ |
| $C_g(5) = 1.0$ pf | $C_T(5) = 177$ pf | $R(5) = 0.022\ \Omega$ | $M(1,5) = 65\ \mu h.$ |

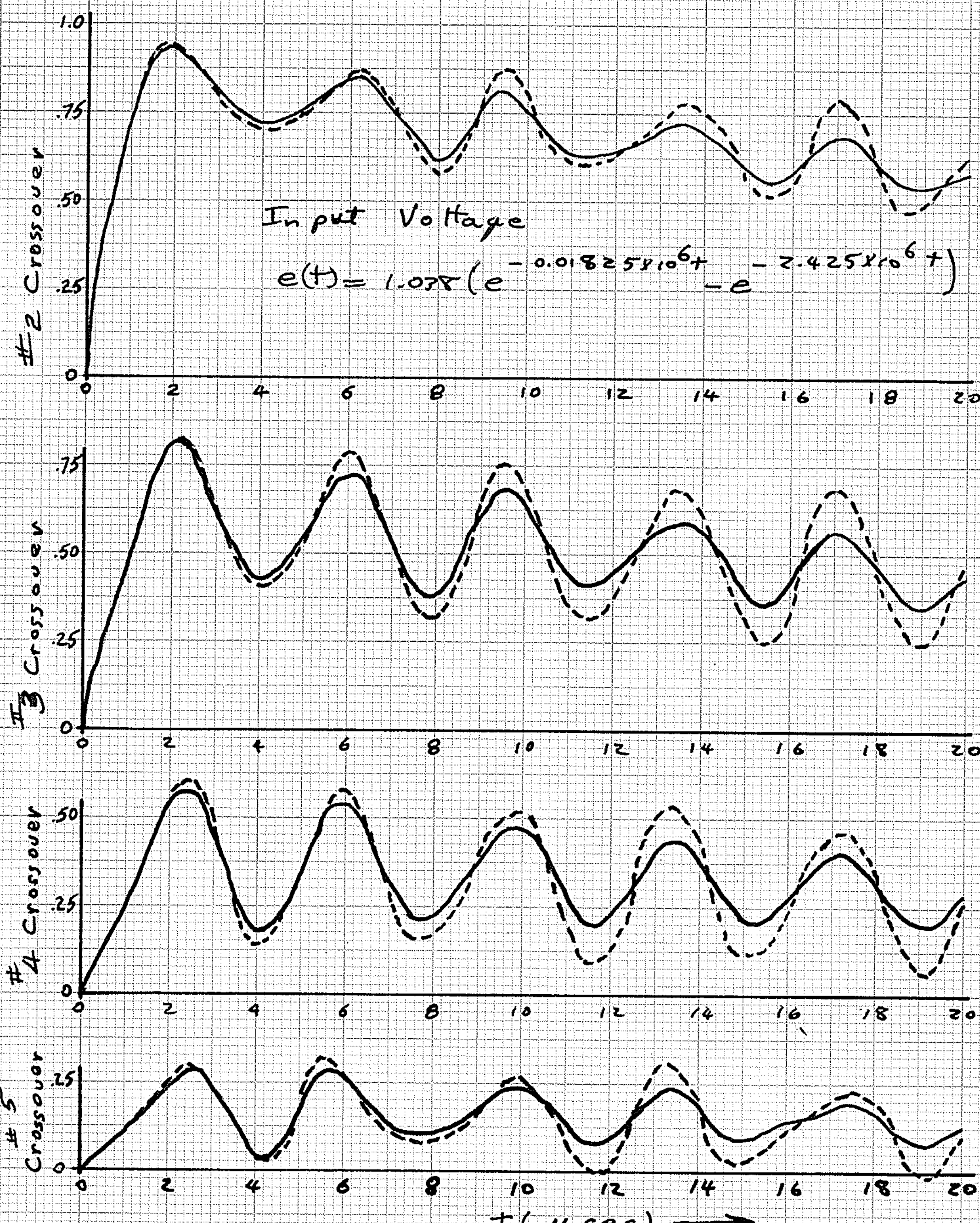


# FIGURE A-18

Effect of Resistance Change on computed Impulse Response of a Five Section Drop-Down Strip Winding

Fixed Parameters used in Computation:

| | | | —— Curve | - - - Curve |
|---|---|---|---|---|
| $C_g(1) = 50.$ pf | $C_T(1) = 177.$ pf | $L(1,1) = 951.$ μh | $R(1) = 100.\Omega$ | $R(1) = 0.0222\,\Omega$ |
| $C_g(2) = 50.$ pf | $C_T(2) = 177.$ pf | $M(1,2) = 391.$ μh | $R(2) = 100.\Omega$ | $R(2) = 0.0222\,\Omega$ |
| $C_g(3) = 50.$ pf | $C_T(3) = 177.$ pf | $M(1,3) = 184.$ μh | $R(3) = 100.\Omega$ | $R(3) = 0.0222\,\Omega$ |
| $C_g(4) = 50$ pf | $C_T(4) = 177.$ pf | $M(1,4) = 80.$ μh | $R(4) = 100.\Omega$ | $R(4) = 0.0222\,\Omega$ |
| $C_g(5) = 50$ pf. | $C_T(5) = 177.$ pf. | $M(1,5) = 65.$ μh | $R(5) = 100.\Omega$ | $R(5) = 0.0222\,\Omega$ |

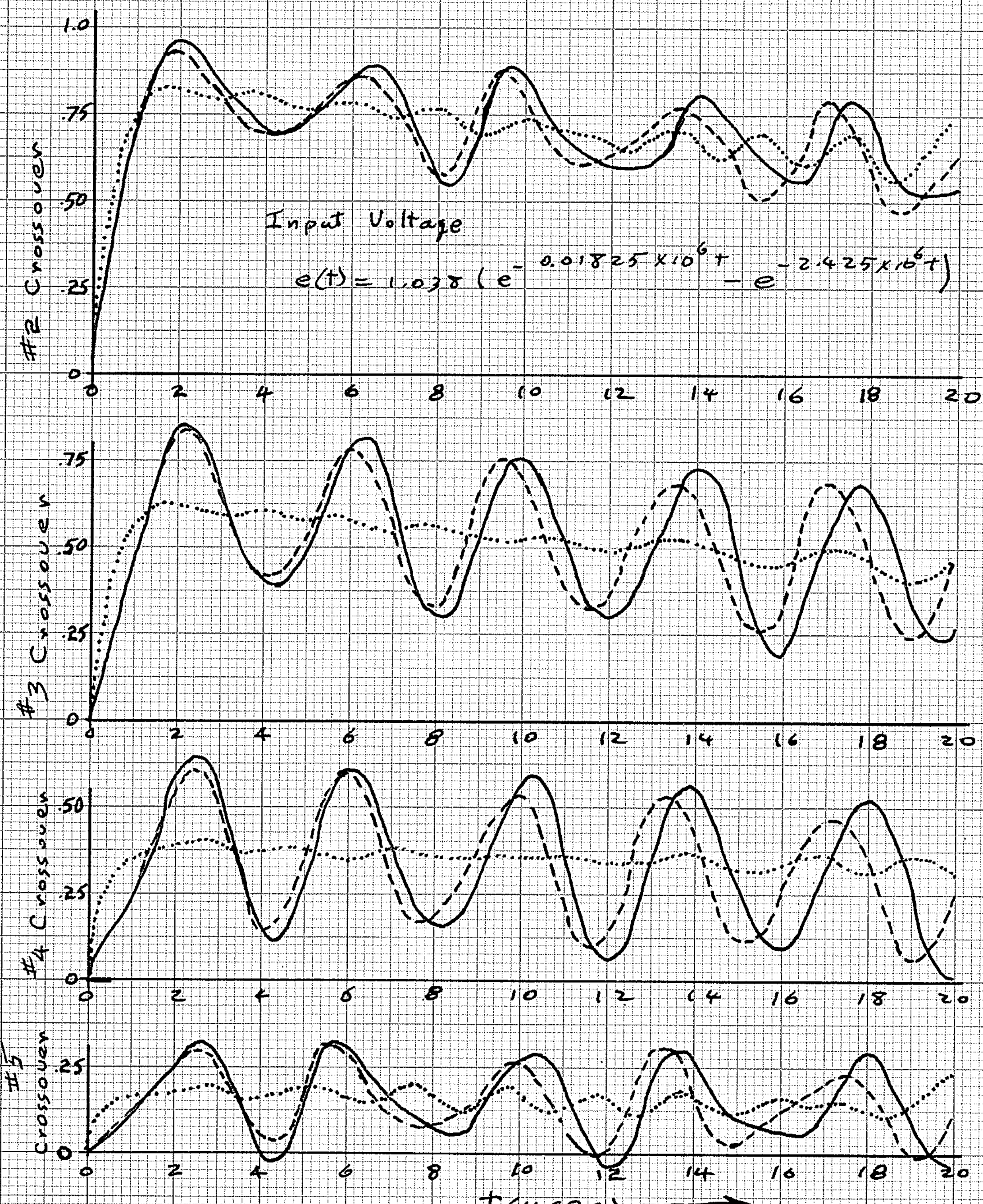


## FIGURE A- 19

Effect of a change in Ground Capacitance on the Calculated Impulse Response of a Five Section Drop-Down Strip winding.

Fixed Parameters used in Computation:

| | | | — curve | --- curve | ···· curve |
|---|---|---|---|---|---|
| $C_T(1) = 177$ pf | $R(1) = 0.0222\,\Omega$ | $L(1,1) = 951\,\mu h$ | $C_g(1) = 60$ pf | $C_g(1) = 50$ pf. | $C_g(1) = 1.$ pf |
| $C_T(2) = 177$ pf | $R(2) = 0.0222\,\Omega$ | $M(1,2) = 391\,\mu h$ | $C_g(2) = 60$ pf. | $C_g(2) = 50$ pf. | $C_g(2) = 1.$ pf |
| $C_T(3) = 177$ pf | $R(3) = 0.0222\,\Omega$ | $M(1,3) = 164\,\mu h$ | $C_g(3) = 60$ pf. | $C_g(3) = 50$ pf | $C_g(3) = 1.$ pf |
| $C_T(4) = 177$ pf | $R(4) = 0.0222\,\Omega$ | $M(1,4) = 80\,\mu h$. | $C_g(4) = 60$ pf | $C_g(4) = 50$ pf | $C_g(4) = 1.$ pf |
| $C_T(5) = 177$ pf | $R(5) = 0.0222\,\Omega$ | $M(1,5) = 65\,\mu h$ | $C_g(5) = 60$ pf | $C_g(5) = 50$ pf | $C_g(5) = 1.$ pf |

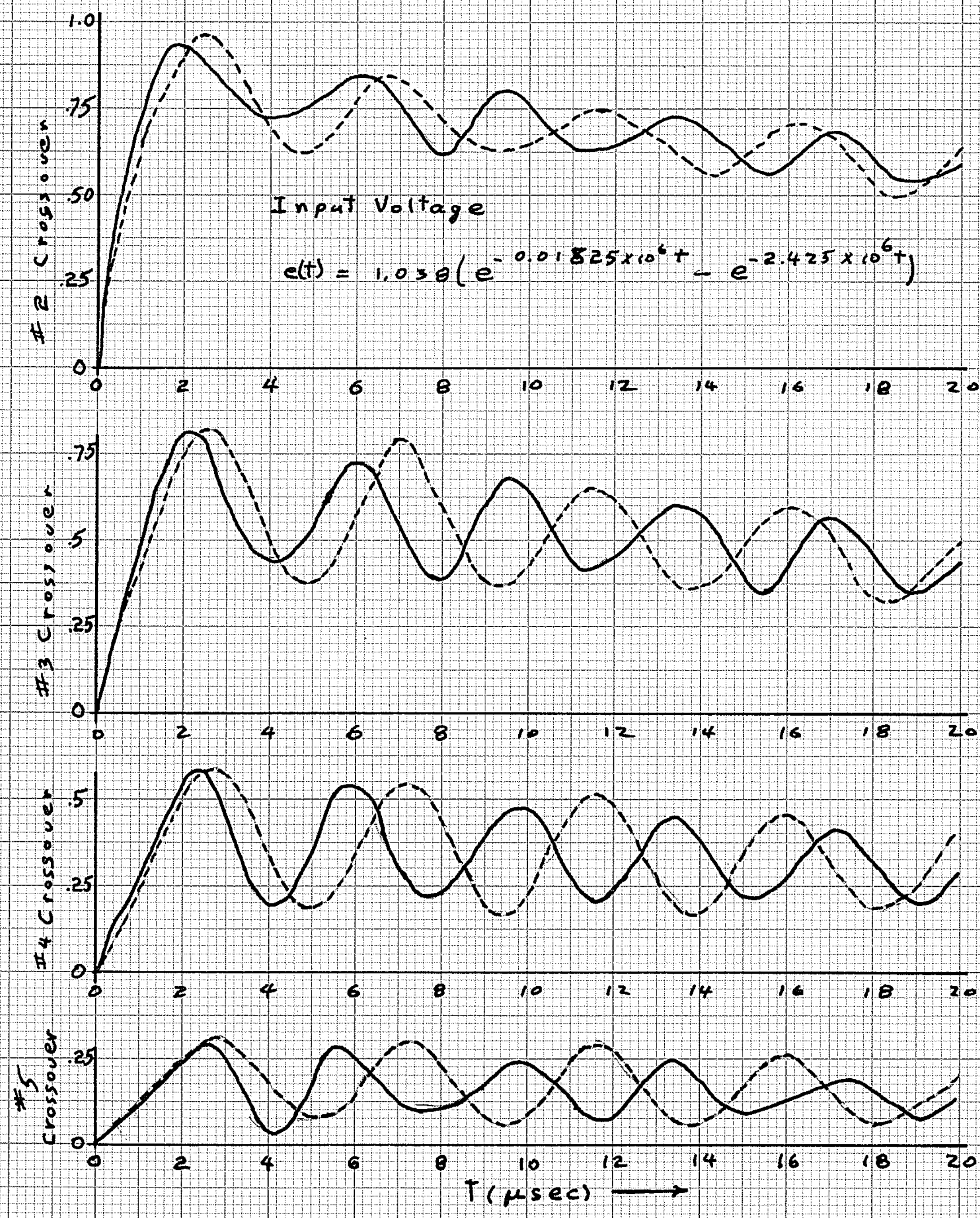


# FIGURE A- 20

Effect of Mutual Inductance on Calculated Impulse Response of a Five Section Drop-Down Strip Winding. The mutual inductance is added to produce an Equivalent Self-Inductance.

Parameters used in computation:

| | | | Inductance used for: ___ curve | ---- curve |
|---|---|---|---|---|
| $C_g(1) = 50.0$ pf | $C_T(1) = 177$ pf. | $R(1) = 100\,\Omega$ | $L(1,1) = 951\ \mu h$ | $L(1,1) = 1651\ \mu h$ |
| $C_g(2) = 50.0$ pf | $C_T(2) = 177$ pf. | $R(2) = 100\,\Omega$ | $M(1,2) = 391\ \mu h$ | $M(1,2) = 1.\ \mu h$ |
| $C_g(3) = 50.0$ pf | $C_T(3) = 177$ pf. | $R(3) = 100\,\Omega$ | $M(1,3) = 164\ \mu h$ | $M(1,3) = 1.\ \mu h$ |
| $C_g(4) = 50.0$ pf | $C_T(4) = 177$ pf. | $R(4) = 100\,\Omega$ | $M(1,4) = 80\ \mu h$ | $M(1,4) = 1.\ \mu h$ |
| $C_g(5) = 50.0$ pf. | $C_T(5) = 177$ pf | $R(5) = 100\,\Omega$ | $M(1,5) = 65\ \mu h$ | $M(1,5) = 1.\ \mu h$ |




to the Inch

FIGURE A-21

Measured Frequency Response of an Isolated Strip Winding

Wire Size 3.0" x 0.020" Insulation Thickness 0.005" 50 Turns

Inside Radius 5.5" No. of Sections or Coils 5

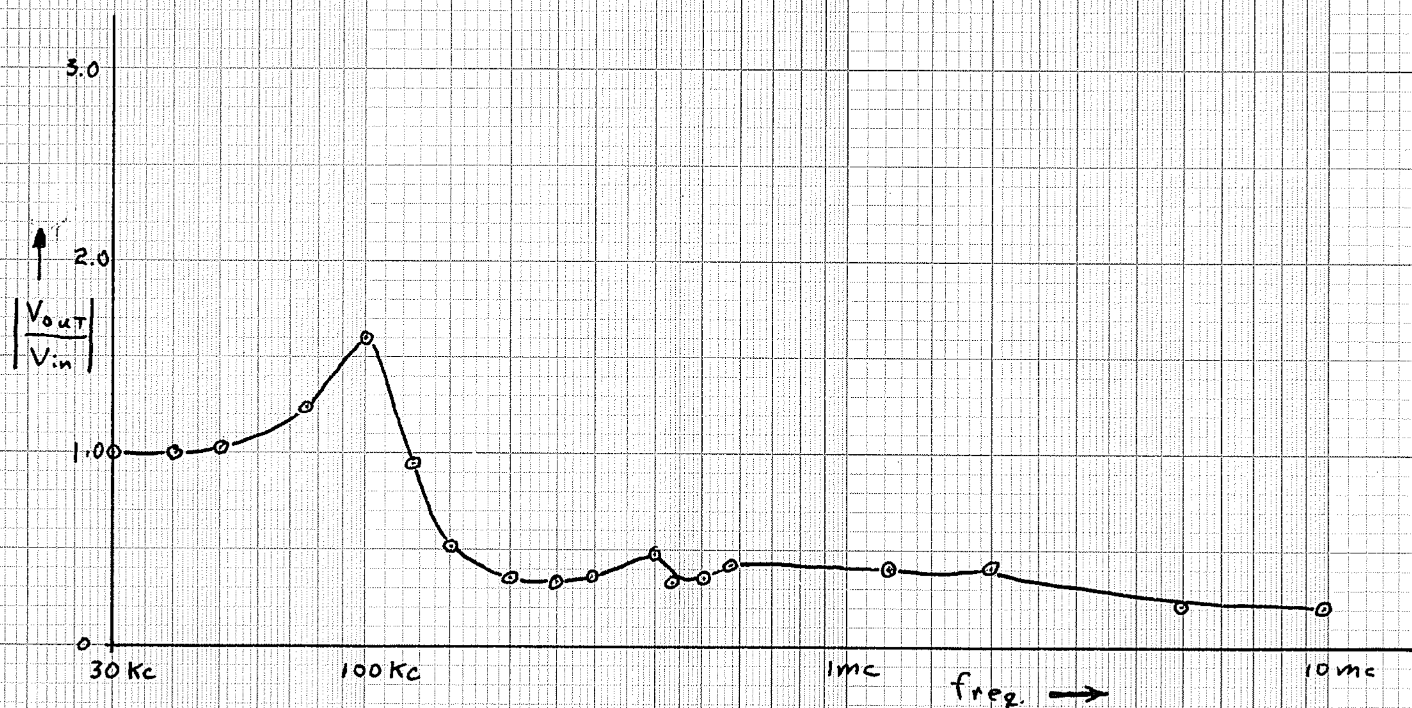
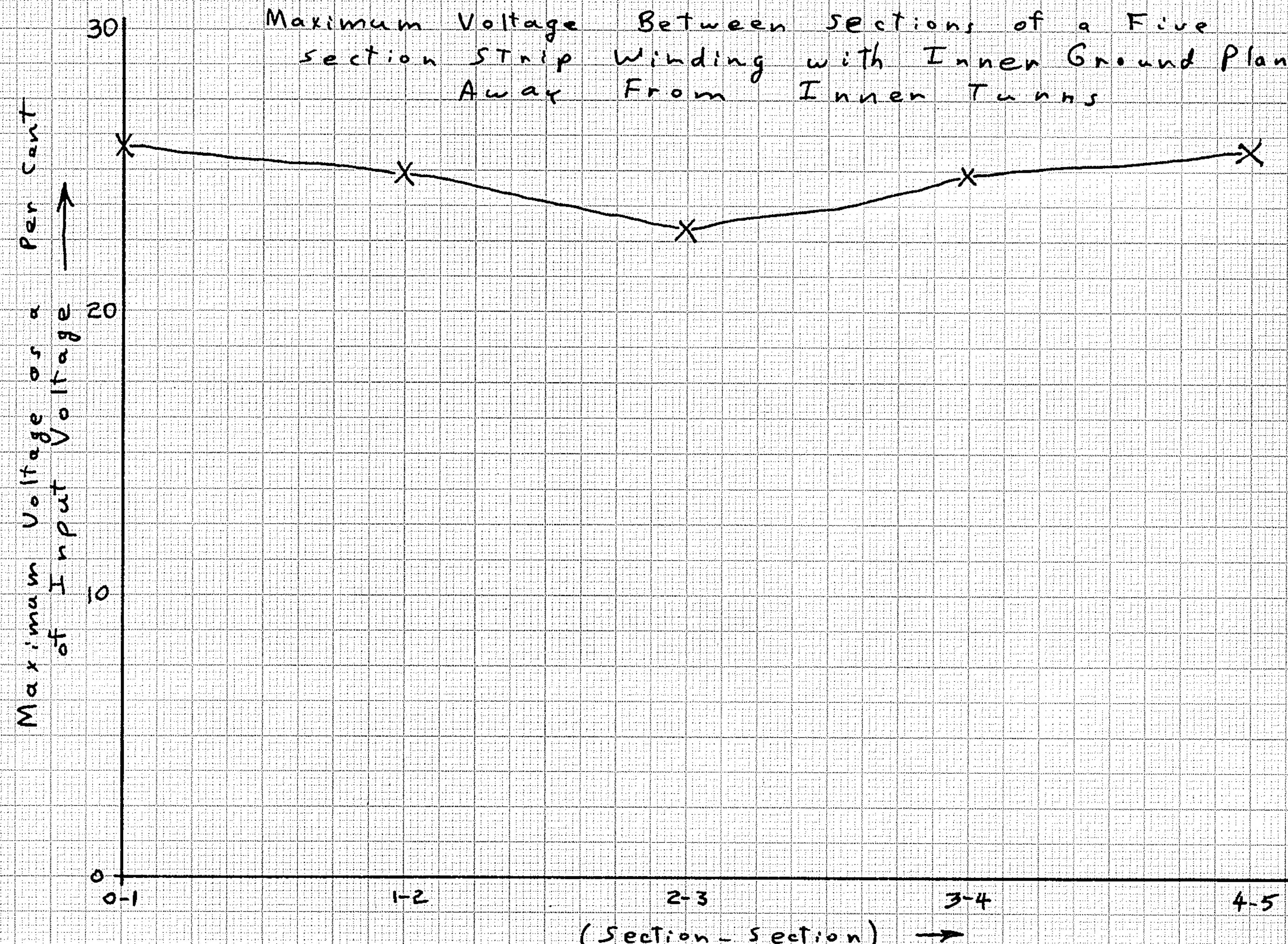


FIGURE A-22

Maximum Voltage Between sections of a Five section Strip Winding with Inner Ground Plane 1" Away From Inner Turns



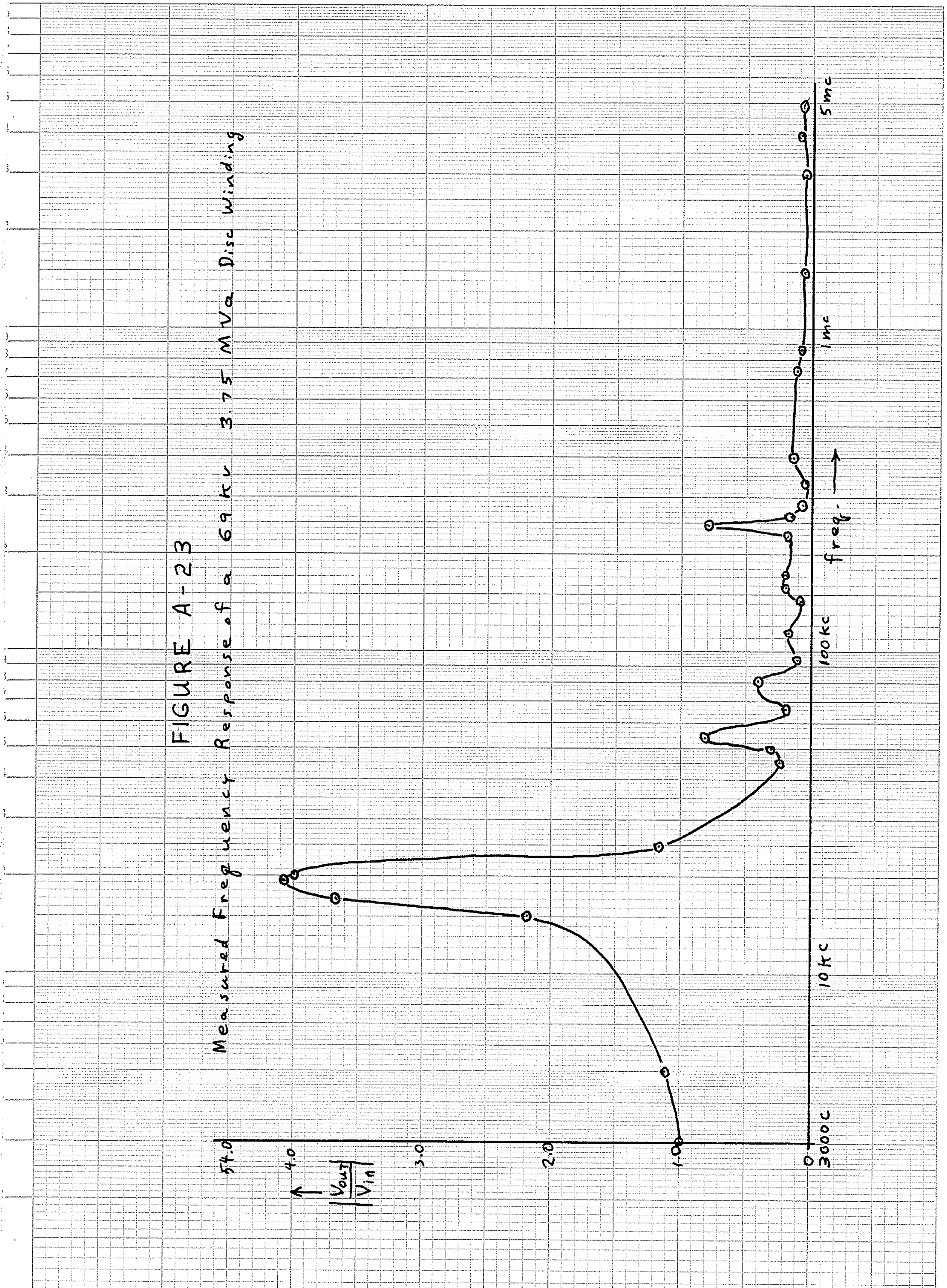


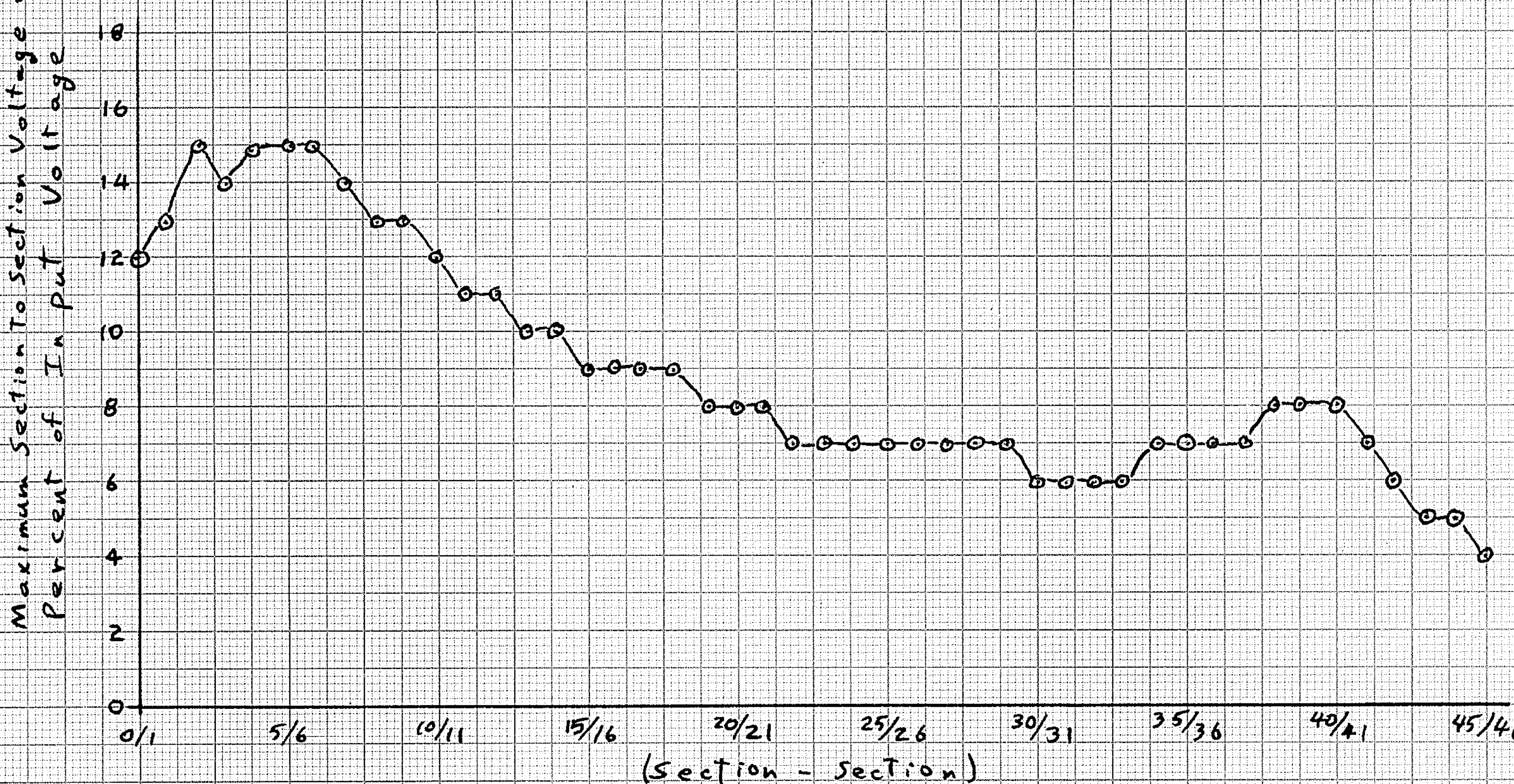

FIGURE A-23

Measured Frequency Response of a 69 kv 3.75 MVa Disc Winding

rithmic
to the inch

FIGURE A-24

Measured Maximum Voltage between Sections of a 69 kV 3.75 Mva Winding with a Ground Shield at radius of a Low Voltage Winding



APPENDIX B

## APPENDIX B

## CALCULATION OF THE FREQUENCY RESPONSE OF A SINGLE DISC

The ratio of the magnitude of the output voltage to the input voltage is the voltage transfer function of the test circuit.

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\left[R_L + s R R_L C_T + s^2 R_L L C_T\right]}{\left[R_L + R + s(L + R_L R (C_T + C_g)) + s^2 R_L L (C_T + C_g)\right]}$$

If the input voltage is sinusoidal, of radial frequency $\omega$, the magnitude of the voltage transfer function is written:

$$\left|\frac{V_{out}}{V_{in}}\right| = \sqrt{\frac{R_L^2\left[(1 - \omega^2 L C_T)^2 + (\omega R C_T)^2\right]}{\left[(R_L + R - \omega^2 L R_L (C_T + C_g))^2 + (\omega(L + R R_L (C_T + C_g)))^2\right]}}$$

This function evaluated over the frequency range produces the frequency response of the single disc.

# APPENDIX B

## CALCULATION OF FREQUENCY RESPONSE OF A SINGLE DISC

```
ZZJOB 5                          4200024G                          HAMLIN
ZZFOR 52
*POBJP4
*LDISK
      SUBROUTINE SDISC
      COMMONRL2,ALCT,RCT,RLPR,RLLCTG, RLRCTG, FQ,PHASE,XINCR,GAINO,AL
      FQ =FQ + XINCR
      W = 2.0*3.14159265*FQ
      W2 = W*W
      DR = 1.0 - W2 * ALCT
      DI = W*RCT
      ANR = RLPR -W2*RLLCTG
      ANI =  W*(AL + RLRCTG)
      GAINM = SQRTF((RL2*(DR*DR+DI*DI))/(ANR*ANR+ANI*ANI))
      IF(DR) 301,302,302
  302 TH1 = ATANF(DI/DR)
      GO TO 305
  301 TH1 =-ATANF(DI/ABSF(DR))
  305 IF(ANR) 303,304,304
  304 TH2 = ATANF(ANI/ANR)
      GO TO 306
  303 TH2 = - ATANF(ANI/ABSF(ANR))
  306 TH = TH1 - TH2
      PHASE = 180.0*TH/3.14159265
      IF(GAINM-4.0)311,311,310
  310 Y = GAINM/100. + 4.
      GO TO 312
  311 Y = GAINM
  312 X = LOGF(FQ)
      CALL PLOT(9,X,Y)
      IF(ABSF(GAINO - GAINM) - 0.05) 307,307,308
  308 PUNCH 100,FQ,GAINM,PHASE
```

```
  307 CONTINUE
      GAINO = GAINM
  100 FORMAT (F14.2,E15.5,F10.5)
      RETURN
      END
ZZFORX52
*POBJP4
C   CALCULATION OF  FREQ RESP  OF  SINGLE DISC WITH CG AND RL
      COMMONRL2,ALCT,RCT,RLPR,RLLCTG, RLRCTG, FQ,PHASE,XINCR,GAINO,AL
      PRINT 103
      PAUSE
  200 READ 201,A,R,RL,AL,CT,CG,FQ,DELTA,FINAL
  202 PUNCH  203,A,R,RL,AL,CT,CG,FQ,DELTA,FINAL
      CALL PLOT (101,9.21034,18.42068,10.,2.30259,0.0,9.0,9.0,1.0)
      GAINO = 1.0
      PHASEO = -0.002
      CTG = CT + CG
      RL2 = RL*RL
      ALCT = AL*CT
      RCT = R*CT
      RLPR = RL + R
      RLRCTG = RL*R*CTG
      RLLCTG = RL*AL*CTG
    3 IF(FQ-FINAL) 4,13,13
  204 IF(A) 2,200,2
   13 PRINT 1101
      CALL PLOT(7)
      GO TO 204
    4 XINCR = DELTA
      IF(SENSE SWITCH 1) 1001,1000
 1001 XINCR = XINCR*10.
      GO TO 1000
 1000 HOLD = FQ
      CALL SDISC
      FREQ = FQ
```

```
 15 PROD1 = PHASEO*PHASE
    IF(PROD1)16,18,18
 18 GO TO 24
 16 FQ = HOLD
    STORE = PHASE
    IF(PHASE) 41,24,42
 41 PHASEO = ABSF(PHASE)
    GO TO 40
 42 PHASEO =  - PHASE
 40 XINCR =XINCR/10.
    IF(XINCR-0.25) 43,44,44
 43 FQ = FREQ
    PHASEO = STORE
    GO TO 3
 44 CONTINUE
    SIM = XINCR
 17 CALL SDISC
    HERE = FQ
 19 TOLS = .001
 20 IF(ABSF(PHASE)-TOLS)50,50,22
 22 TOLL = 88.0
 29 IF(ABSF(PHASE)-TOLL)30,50,50
 30 PROD2 = PHASEO*PHASE
    IF(PROD2)26,28,28
 28 XINCR = SIM
    GO TO 17
 26 FQ = HERE -SIM
    GO TO 40
 50 PHASEO = STORE
    GO TO 24
 24 FQ = FREQ
    GOTO 3
101 FORMAT (8E10.2)
102 FORMAT (3F10.0,I3)
103 FORMAT (40H THIS PROGRAM REQUIRES PLOTTER AND PUNCH)
201 FORMAT(1F2.0,1F8.0,1F10.0,3E10.3,3F10.0)
```

```
  203 FORMAT(1F2.0,1F8.0,1F10.0,3E10.3,3F10.0)
 1101 FORMAT(21H READ NEW SET OF DATA)
    2 CALL EXIT
      END
  1.0      1000000.0 206.00E-0638.000E-1224.700E-120.0        100000.0  10000000.0
ZZJOB                           2400042G                  HAMLIN
ZZDUP
*DELETSDISC
ZZZZ
```

APPENDIX C

## APPENDIX C

### DERIVATION OF COIL PARAMETERS FROM FREQUENCY RESPONSE CURVES

The inter-turn capacitance, the ground capacitance and the inductance of a coil can be calculated by measuring the frequencies $F_1$, $F_2$ and $F_3$ and using equations C-2, C-3 and C-7. The voltage transfer function for the lossless case is equation C-1.

$$\left|\frac{V_{out}}{V_{in}}\right| = \sqrt{\frac{\left[R_L(1 - \omega^2 L C_T)\right]^2}{\left[R_L - \omega^2 L R_L (C_T + C_g)\right]^2 + \omega^2 L^2}} \qquad (C-1)$$

From equation C-1 it will be noted that the frequency at which the zero occurs is:

$$F_3 = \frac{1}{2\pi\sqrt{L\,C_T}} \qquad (C-2)$$

and the frequency at which the maximum occurs is approximately

$$F_2 = \frac{1}{2\pi\sqrt{L\,(C_T + C_g)}}. \qquad (C-3)^*$$

There are now two equations, but three unknowns. The third equation is obtained by deriving the lowest frequency at which the magnitude of the voltage transfer function is one-half, when a load resistor of 1 K$\Omega$ is used.

*Provided $L \ll 2 R_L^2 (C_T + C_g)$

$$\left|\frac{V_{OUT}}{V_{in}}\right| = \frac{1}{2} = \sqrt{\frac{R_L^2\left(\left(1-\omega_1^2 L C_T\right)^2 - \left(\omega_1 R C_T\right)^2\right)}{\left(R_L + R - \omega_1^2 R_L L (C_T + C_g)\right)^2 + \left(\omega_1 \left(L - R R_L (C_T + C_g)\right)\right)^2}} \quad (C-4)$$

Equation C-4 can be altered to C-5.

$$\omega_1^4 R_L^2 L^2\left((C_T + C_g)^2 - 4C_T^2\right) + \omega_1^2\left[L R_L^2 (8C_T - 2(C_T + C_g)) + L^2 + R^2 R_L^2\left((C_T + C_g)^2 - 4C_T^2\right)\right] + R^2 + 2RR_L - 3R_L^2 \quad (C-5)$$

The following approximations are used:

$$R^2 + 2RR_L < 3R_L^2$$

$$R^2 R_L\left((C_T + C_g)^2 - 4C_T^2\right) < L^2 \text{ or } L R_L\left(8C_T - 2(C_T + C_g)\right)$$

Then C-5 is reduced to C-6:

$$\omega_1^4 R_L^2 L^2\left((C_T + C_g)^2 - 4C_T^2\right) + \omega_1^2\left(R_L^2 L C_T + L^2\right) - 3R_L^2 = 0 \quad (C-6)$$

Substituting equation C-2 and C-3 in C-6, one gets

$$L^2 = \frac{10^6}{(2\pi F_1)^2}\left[3 - 8\left[\frac{F_1}{F_3}\right]^2 + 2\left[\frac{F_1}{F_2}\right]^2 - 2\left[\frac{F_1}{F_2}\right]^4 + 4\left[\frac{F_1}{F_3}\right]^4\right]$$

$$(C-7)$$

APPENDIX D

## APPENDIX D

## DERIVATION OF THE DISTRIBUTED CAPACITANCE OF A FOLDED WIRE

The capacitance between two parallel connected or unconnected wires can be calculated from the stored energy. The stored energy available at the terminals of the two parallel unconnected wires is determined by V, the applied terminal voltage, which is constant across the wires at all points, C the capacitance per unit length, and L the length of a single wire. The total stored energy is

$$E = \frac{1}{2}\int_0^L C V^2 d\ell = \frac{1}{2} C L V^2 = \frac{1}{2} C_{eq} V^2$$

The equivalent capacitance is:

$$C_{eq} = CL = K \epsilon_0 \frac{w}{d} L$$

In which,

K is the dielectric constant and w is the width of copper and d is the separation from copper to copper.

When the wires are joined at one end to form a folded wire the voltage ($V_x$), across any elemental capacitance, is a function of position ($x$) along the wire.

$$V_x = V - \frac{Vx}{L}$$

the total stored energy is

$$E = \frac{1}{2}\int_0^L C\left(V - \frac{Vx}{L}\right)^2 dx = \frac{1}{2} C_{eq} V^2$$

$$= \frac{1}{2} \frac{CL}{3} V^2$$

The equivalent capacitance is one-third of a parallel wire capacitance.[20]

20. Harris, F.K. "Electrical Measurements," John Wiley and Sons, Inc., 1962, p. 219.

# APPENDIX E

## APPENDIX E

## IMPULSE RESPONSE OF A SINGLE COIL

The voltage transfer function is:

$$\frac{V_{OUT}(s)}{V_{in}(s)} = \frac{R_L + sRR_LC_T + s^2LR_LC_T}{R_L+R + s(L + R_LR(C_T+C_g)) + s^2R_LL(C_T+C_g)} \quad (E-1)$$

The differential equations that are solved by Runge-Kutta method are:

$$\frac{dv_1(t)}{dt} = \frac{D(1)}{D(3)}v_0(t) - \frac{D(2)}{D(3)}v_1(t) + \frac{a}{D(3)}\left(BOe^{-bt} - COe^{-ct}\right)$$

$$\frac{dv_0}{dt} = v_1(t)$$

where the coefficients are obtained from (E-1).

# APPENDIX E

## CALCULATION OF IMPULSE RESPONSE OF A SINGLE DISC

```
ZZJOB                            2400042G                    HAMLIN
ZZFORX52
C     CALCULATION OF IMPULSE RESPONSE OF A SINGLE DISC
      DIMENSION D(3),XN(2)
C THE INPUT VOLTAGE HAS BEEN MANIPULATED INTO NORMALIZED EQUATION
    5 DERV1F(VO,V1) = D13*VO + D23*V1 + AD3*(DRIV1*BO + DRIV2*CO)
C READ IN PARAMETERS
    1 READ 101 ,R,RL,AL,CT,CG
    2 READ 102,A,B,C,DT,N,M
      PUNCH 101,R,RL,AL,CT,CG
      PUNCH 102,A,B,C,DT,N,M
C CALCULATE COEFFICIENTS OF DIFF. EQ. IN NORMALIZED FORM
      CTG = CG + CT
      D(1) = RL + R
      D(2) = AL + RL*R*CTG
      D(3) = RL*AL*CTG
      XN(1) = R*RL*CT
      XN(2) = RL*AL*CT
      DRIV1 = RL - XN(1)*B*1.E+06 + XN(2)*B*B*1.E+12
      DRIV2 = - RL + XN(1)*C*1.E+06 - XN(2)*C*C*1.E+12
      D13 = -D(1)/D(3)
      D23 = -D(2)/D(3)
      AD3 = A/D(3)
C INITIAL VALUES OF V1 AND VO AND T
      T = 0.0
      VO = 0.0
      V1 = A*CT/CTG*(C-B)*1.E+06
      H = DT*1.E-06
    6 DO 40 I = 1,N
      BO = EXPF(-B*T)
      CO = EXPF(-C*T)
```

```
      X = A*(BO -CO)
    9 D1V1 = DERV1F(VO,V1)*H
      D1VO = V1*H
      BO = EXPF(-B*(T + DT/2.))
      CO = EXPF(-C*(T + DT/2.))
      D2V1 = DERV1F(VO + D1VO/2.,V1 + D1V1/2.)*H
      D2VO =(V1 + D1V1/2.)*H
      D3V1 = DERV1F(VO + D2VO/2.,V1 + D2V1/2.)*H
      D3VO =(V1 + D2V1/2.)*H
      BO = EXPF(-B*(T + DT))
      CO = EXPF(-C*(T + DT))
      D4V1 = DERV1F(VO + D3VO,V1 + D3V1)*H
      D4VO =(V1 + D3V1)*H
      PUNCH 103,VO,V1,X,T
      DELVO = (D1VO + 2.*D2VO + 2.*D3VO + D4VO)/6.
      DELV1 = (D1V1 + 2.*D2V1 + 2.*D3V1 + D4V1)/6.
      VO = VO + DELVO
      V1 = V1 + DELV1
   40 T = T + DT
   41 IF(M) 42, 1,42
   42 CALL EXIT
  101 FORMAT (F10.2,F10.0,3E10.2)
  102 FORMAT (4F10.5,2I5)
  103 FORMAT (2E15.5,2F10.3)
      END
50.        1000000.  951.20E-06177.00E-12166.00E-12
     1.038    0.01825       2.425         0.1   200
50.0       1000000.  407.00E-0633.000E-12166.00E-12
     1.038    0.01825       2.425         0.1   200
50.0       1000000.  352.00E-0633.000E-12166.00E-12
     1.038    0.01825       2.425         0.1   200
50.0           1000000.252.00E-0638.000E-12166.00E-12
     1.038    0.01825       2.425         0.1   200
50.0           1000000.200.00E-0638.000E-12166.00E-12
     1.038    0.01825       2.425         0.1   200
ZZZZ
```

# APPENDIX F

# APPENDIX F

## CALCULATION OF SELF AND MUTUAL INDUCTANCE

```
ZZJOB 5                        2400042G             HAMLIN
ZZFORX52
C  PROGRAM FOR CALCULATING SELF AND MUTUAL INDUCTANCE OF IDENTICAL COILS
C  AT ANY SPACING
C PROGRAM USES A SPECIAL TECHNIQUE FOR COILS CLOSE TOGETHER
      DIMENSION D(100),AMO(100),PART(100),R1(100),R12(100)
    1 READ 101,RI,W,TC,TI,NCOILS,NTURN,MSTOP
      NC1 = NCOILS - 1
    2 READ 102,(D(I),I = 1,NC1)
      T2 = NTURN*NTURN
      PI = 3.14159265
    3 T = TC + 2.0*TI
      TURN = NTURN
      RB = T*TURN
      RM = RI + RB/2.
      R = 0.2235*(W + RB)
      RA =R/RM
      RA2 = RA*RA
C CALCULATION OF INDUCTANCE OF COIL
   50AL = 4.*PI*RM*2.54E-09*T2*(LOGF(8./RA*(1. + 3.*RA2/16.))
    1 -   (2. + RA2/16.))
   10 DO 51 I = 1,NC1
      PRINT 110,I
      D2 = D(I)*D(I)
   11 IF(D(I) -.9*RM) 12,23,23
C FOR COILS CLOSE TOGETHER
   12 R1(1) = RI + T/2.
      SUM = 0.0
C CALCULATE MUTUAL INDUCTANCE OF TURNS OPPOSITE EACH OTHER
   13 DO 15 NT = 1,NTURN
      R12(NT) = R1(NT)*R1(NT)
```

```
      D216A2 = D2/(16.*R12(NT))
  140PART(NT) =   R1(NT)*(LOGF(8.*R1(NT)/D(I)*(1. + 3.*D216A2))
     1   -  (2. + D216A2))
      R1(NT + 1) = R1(NT) + T
   15 SUM = SUM + PART(NT)
C     CALCULATE MUTUAL INDUCTANCE OF TURNS DIAGONALLY AWAY FROM EACH OTHER
      INT = NTURN - 1
      C = T
   17 DO 19 KR  = 1,INT
      C2 = C*C
      DIAG = SQRTF(C2 + D2)
      ALPHA = C/(2.*R1(KR))
      BETA  = (C2 + 3.*D2)/(16.*R12(KR))
      GAMMA = (C2*C +3.*D2*C)/(32.*R12(KR)*R1(KR))
  180PART(KR)  =  2.*R1(KR)*(LOGF(8.*R1(KR)/DIAG*(1. + ALPHA + BETA
     1  - GAMMA)) - (2. + ALPHA - BETA + GAMMA))
   19 SUM = SUM + PART(KR)
      C = C+ T
      INT = INT - 1
      IF (INT) 20,20,17
   20 AMO(I) = 4.*PI*2.54E-09*SUM
      GO TO 51
   23 THKP = D(I)/(SQRTF(4.*RM*RM + D2))
      RTHKP = SQRTF(THKP)
      GAM = (1. - RTHKP)/(1. + RTHKP)
      Q = GAM/2. + 2.*(GAM/2.)**5. + 15.*(GAM/2.)**9.
      Q2 = Q*Q
   24 IF(D(I) - 2.*RM) 25,25,26
   25 Q4 = Q2*Q2
      Q6 = Q4*Q2
      Q8 = Q4*Q4
      EPSIL = 3.*Q4 - 4.*Q6 + 9.*Q8 - 12.*Q8*Q2
      AMO(I) =  T2*16.*PI*PI*RM*SQRTF(Q2*Q)*(1. + EPSIL)*2.54E-09
      GO TO 51
   26 AMO(I) =  T2*16.*PI*PI*RM*SQRTF(Q2*Q)*2.54E-09
   51 CONTINUE
```

```
      PUNCH 106,AL,(AMO(I),I = 1,NC1)
      IF(MSTOP) 1,1,9
    9 CALL EXIT
  101 FORMAT (4F10.4,3I5)
  102 FORMAT(8F10.3)
  106 FORMAT (4E20.5)
  110 FORMAT(I5)
      END
       5.5       1.0      .060     .0025   15   20   -1
      1.25      2.50      3.75      5.00      6.25      7.50      8.75      10.0
     11.25     12.50     13.75     15.00     16.25     17.50

      9.87      .182      .072       .01   12   19   -1
       4.5       5.0       5.5       6.0       6.5       7.5       8.0       8.5
       9.0       9.5       10.

       5.5        3.       .02     .0025   12   50    1
       3.1       3.4       6.8      10.2      13.6       17.      20.4      23.8
      27.2      30.6      34.0
ZZZZ

       201.18324E-06       119.69406E-06       746.06754E-07       503.70990E-07
       365.99023E-07       301.52756E-07       227.01090E-07       173.99551E-07
       135.47636E-07       106.99359E-07       856.02251E-08       693.10618E-08
       567.39769E-08       469.22088E-08       391.67899E-08

       407.89430E-06       117.42819E-06       106.78768E-06       974.70262E-07
       892.83772E-07       820.79063E-07       701.65849E-07       652.84269E-07
       610.28737E-07       573.44829E-07       541.86240E-07       538.99671E-07

       951.23176E-06       427.84745E-06       391.36904E-06       164.21534E-06
       804.81540E-07       438.19754E-07       259.19523E-07       163.85138E-07
       109.28593E-07       761.42385E-08       549.88622E-08       409.14988E-08
```

APPENDIX G

APPENDIX G

INTER-DISC CAPACITANCE

I Back-Turn Disc Winding

For solid insulation between discs:

$$C_D = \frac{1}{3} K \epsilon_0 2\pi R_m \frac{T_c}{D} 2.54 \times 10^{-2}$$

The dimensions are in inches and

$K$ = combined dielectric and fringing constant

$T_c$ = thickness of copper

$D$ = separation, copper to copper

$R_m$ = mean radius of coil

For a duct between the discs:

$$C_D = \frac{1}{3}\left(\frac{D_p}{2T_p} + \frac{1}{Duct}\right) K' \epsilon_0 2\pi R_m T_c 2.54 \times 10^{-2}$$

where: the dimensions are in inches

$D_p$ = dielectric of paper around wire

$T_p$ = thickness of paper around wire

Duct = duct width

$K^1$ = a fringing constant

II Drop-Down Disc Winding:

For a duct:

$$C_D = \frac{1}{4}\left[\frac{D_p}{2T_p} + \frac{1}{Duct}\right] K' \epsilon_0 2\pi R_m 2.54 \times 10^{-2}$$

Where the dimensions are in inches and:

$D_p$ = dielectric of paper

$T_p$ = thickness of paper around wire

Duct = duct width

$R_m$ = mean radius

$K^1$ = fringing constant

# APPENDIX H

# APPENDIX H

## FREQUENCY RESPONSE OF A PAIR OF DISCS

The voltage transfer function of the test circuit is:

$$\frac{V_{OUT}(s)}{V_{in}(s)} = \frac{\begin{bmatrix} 1 + 2s(C_T + C_D)(R + sL) - s^2 M C_g + 2s^2 M C_d \\ + s^2\left[C_D(C_T + C_g) + C_T(C_T + C_D)\right]\left[(R + sL)^2 - s^2 M^2\right] \end{bmatrix} R_L}{\begin{bmatrix} R_L \begin{bmatrix} 1 + s(C_g + C_T + 2C_D)(R + sL) + 2s^2 M C_D \\ + s^2\left[C_g(C_T + C_D) + C_T(C_T + 2C_D)\right]\left[(R + sL)^2 - (sM)^2\right] \end{bmatrix} \\ + \left[1 + sR_L C_L\right]\left[2(R + s(L + M)) + s(C_g + 2C_T)\right]\left[(R + sL)^2 - (sM)^2\right] \end{bmatrix}}$$

# APPENDIX H

## CALCULATION OF THE FREQUENCY RESPONSE OF A PAIR OF DISC COILS

```
ZZJOB 5                        4200024G                        HAMLIN
ZZFOR 52
*POBJP4
*LDISK
      SUBROUTINE DDISC
      DIMENSION XNUM(5) , DEN(5)
      COMMON XNUM,DEN,FQ,PHASE,XINCR,GAINO
      FQ = FQ + XINCR
      W = 2.*3.14159265*FQ
      W2 = W*W
      W3 = W2*W
      W4 = W3*W
      XNUMR = XNUM(1) - W2*XNUM(3) + W4*XNUM(5)
      XNUMI = W*XNUM(2) - W3*XNUM(4)
      DENR = DEN(1) - W2*DEN(3) + W4*DEN(5)
      DENI = W*DEN(2) - W3*DEN(4)
      GAINM = SQRTF((XNUMR*XNUMR+XNUMI*XNUMI)/(DENR*DENR+DENI*DENI))
      IF(DENR) 301,302,302
  302 TH1 = ATANF(DENI/DENR)
      GO TO 305
  301 TH1 =-ATANF(DENI/ABSF(DENR))
  305 IF(XNUMR) 303,304,304
  304 TH2 = ATANF(XNUMI/XNUMR)
      GO TO 306
  303 TH2 = - ATANF(XNUMI/ABSF(XNUMR))
  306 TH = TH1 - TH2
      PHASE = 180./3.14159265*TH
      IF(GAINM-4.0)311,311,310
  310 Y = GAINM/100. + 4.
      GO TO 312
```

```
  311 Y = GAINM
  312 X = LOGF(FQ)
      CALL PLOT(9,X,Y)
      IF(ABSF(GAINO - GAINM) - 0.05) 307,307,308
  308 PUNCH 100,FQ,GAINM,PHASE
  307 CONTINUE
      GAINO = GAINM
  100 FORMAT (F14.2,E15.5,F10.5)
      RETURN
      END
ZZFORX52
*POBJP4
C CALCULATION OF FREQUENCY RESPONSE OF DOUBLE DISC WITH CL,CG, AND RL
      DIMENSION XNUM(5) , DEN(5)
      COMMON XNUM,DEN,FQ,PHASE,XINCR,GAINO
  200 READ 101,R,RL,AL,AM,CD,CG,CL,CT
  402 READ 102,FQ,DELTA,FINAL,M
      PUNCH 101,R,RL,AL,AM,CD,CG,CL,CT
      PRINT 103
      PAUSE
      CALL PLOT (101,9.21034,18.42068,10.,2.30259,0.0,9.0,9.0,1.0)
      GAINO = 1.0
      PHASEO = -0.002
      CDT = CD + CT
      C2DGT = 2.0*CD + CG + CT
      CG2T = CG + 2.0*CT
      CC = CT*C2DGT + CG*CD
      ALM = AL + AM
      ALM2  = AL*AL + AM*AM
      XNUM(1) = RL
      XNUM(2) = 2.*R*RL*CDT
      XNUM(3) = RL*(2.*AL*CDT - AM*CG + 2.*AM*CD + R*R*CC)
      XNUM(4) = 2.*R*AL*CC*RL
      XNUM(5) = ALM2 *CC*RL
      DEN(1) = RL + 2.*R
      DEN(2) = R*RL*C2DGT + 2.*ALM + R*R*CG2T+ 2.*R*RL*CL
     0DEN(3) = RL*AL*C2DGT +  2.*AM*CD*RL + R*R*RL*CC+ 2.*R*AL*CG2T
```

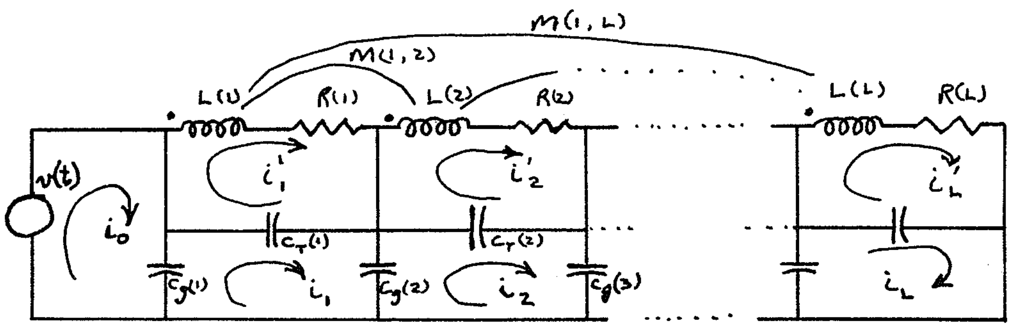
```
    1     + 2.*RL*CL*ALM + RL*CL*R*R*CG2T
      DEN(4) = 2.*R*AL*RL*CC + ALM2*CG2T + 2.*R*AL*RL*CL*CG2T
      DEN(5) = RL*ALM2*(CC + CL*CG2T)
    3 IF(FQ - FINAL) 4,13,13
  204 IF(M) 205,200,205
   13 PRINT 1101
      CALL PLOT(7)
      GO TO 204
    4 XINCR = DELTA
      IF(SENSE SWITCH 1) 1001,1000
 1001 XINCR = XINCR*10.
      GO TO 1000
 1000 HOLD = FQ
      CALL DDISC
      FREQ = FQ
   15 PROD1 = PHASEO*PHASE
      IF(PROD1)16,18,18
   18 GO TO 24
   16 FQ = HOLD
      STORE = PHASE
      IF(PHASE) 41,24,42
   41 PHASEO = ABSF(PHASE)
      GO TO 40
   42 PHASEO =   - PHASE
   40 XINCR =XINCR/10.
      IF(XINCR-0.25) 43,44,44
   43 FQ = FREQ
      PHASEO = STORE
      GO TO 3
   44 CONTINUE
      SIM = XINCR
   17 CALL DDISC
      HERE = FQ
   19 TOLS = .001
   20 IF(ABSF(PHASE)-TOLS)50,50,22
   22 TOLL = 88.0
```

```
   29 IF(ABSF(PHASE)-TOLL)30,50,50
   30 PROD2 = PHASEO*PHASE
      IF(PROD2)26,28,28
   28 XINCR = SIM
      GO TO 17
   26 FQ = HERE -SIM
      GO TO 40
   50 PHASEO = STORE
      GO TO 24
   24 FQ = FREQ
      GOTO 3
  101 FORMAT (8E10.2)
  102 FORMAT (3F10.0,I3)
  103 FORMAT (40H THIS PROGRAM REQUIRES PLOTTER AND PUNCH)
 1101 FORMAT(21H READ NEW SET OF DATA)
    2 CALL EXIT
      END
1.0       1000000.  320.00E-06280.00E-06180.00E-120.0100E-1320.000E-1232.000E-12
0.0       100000.   10000000. 1
ZZJOB                          2400042G                   HAMLIN
ZZDUP
*DELETDDISC
ZZZZ
```

APPENDIX I

APPENDIX I

ANALYSIS OF THE EQUIVALENT CIRCUIT




Mutual-Inductances between each coil

and every other coil.

The first loop equation :

$$\frac{d\,(v(t))}{d\,t} = \frac{i_0}{C_g(1)} - \frac{i_1}{C_g(1)} \qquad (\mathrm{I}-1)$$

$$i_0 = C_g(1)\,\frac{d\,v(t)}{d\,t} + i_1 \qquad (\mathrm{I}-2)$$

The first lower loop equation:

$$0 = \left(\frac{1}{C_g(1)} + \frac{1}{C_g(2)} + \frac{1}{C_T(1)}\right) i_1 - \frac{1}{C_g(2)}\,i_2 - \frac{1}{C_g(1)}\,i_0 - \frac{1}{C_T(1)}\,i_1' \qquad (\mathrm{I}-3)$$

Substitute I-2 into I-3 and write the remaining lower loop

equations.

$$\frac{dv(t)}{dt} = \left(\frac{1}{C_T(1)} + \frac{1}{C_g(2)}\right) i_1 - \frac{i_2}{C_g(2)} - \frac{i_1'}{C_T(1)}$$

$$0 = -\frac{i_1}{C_g(2)} + \left(\frac{1}{C_T(1)} + \frac{1}{C_g(2)} + \frac{1}{C_g(3)}\right) i_2 - \frac{i_3}{C_g(3)} - \frac{i_2'}{C_T(2)}$$

$$0 = -\frac{i_2}{C_g(3)} + \left(\frac{1}{C_T(2)} + \frac{1}{C_g(3)} + \frac{1}{C_g(4)}\right) i_3 - \frac{i_4}{C_g(4)} - \frac{i_3'}{C_T(3)}$$

$$\vdots$$

$$0 = -\frac{i_{L-1}}{C_g(L)} + \left(\frac{1}{C_T(L)} + \frac{1}{C_g(L)}\right) i_L - \frac{1}{C_T(L)} i_L'$$

These produce the matrix equation:

$$\begin{bmatrix} \frac{dv(t)}{dt} \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{C_T(1)} + \frac{1}{C_g(2)}\right) & -\frac{1}{C_g(2)} & 0 & \cdots & & 0 \\ -\frac{1}{C_g(2)} & \left(\frac{1}{C_T(2)} + \frac{1}{C_g(2)} + \frac{1}{C_g(3)}\right) & -\frac{1}{C_g(3)} & \cdots & & 0 \\ 0 & -\frac{1}{C_g(3)} & \ddots & & & 0 \\ 0 & 0 & & \ddots & & \\ & \vdots & & & & 0 \\ & \vdots & & & \ddots & -\frac{1}{C_g(L)} \\ 0 & \cdots & 0 & & -\frac{1}{C_g(L)} & \left(\frac{1}{C_T(L)} + \frac{1}{C_g(L)}\right) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ \vdots \\ i_{L-1} \\ i_L \end{bmatrix}$$

$$
- \begin{bmatrix} \frac{1}{C_T(1)} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{C_T(2)} & 0 & & \\ 0 & 0 & \frac{1}{C_T(3)} & & \\ & & & \ddots & 0 \\ & & & 0 & \frac{1}{C_T(L)} \end{bmatrix} \begin{bmatrix} i'_1 \\ i'_2 \\ i'_3 \\ \vdots \\ i'_L \end{bmatrix} \qquad (I-4)
$$

Equation (I-4) in matrix form is:

$$
\left[\dot{v}(t)\right] = \left[\frac{1}{CGT}\right]\left[Q\right] - \left[\frac{1}{CT}\right]\left[Q'\right] \qquad (I-5)
$$

The equations for the upper loops are:

$$
\begin{aligned}
0 &= R(1)\,\dot{i}'_1 + L(1)\,\ddot{i}'_1 + \frac{i'_1}{C_T(1)} + M(1,2)\,\ddot{i}'_2 + M(1,3)\,\ddot{i}'_3 \\
&\quad + M(1,4)\,\ddot{i}'_4 + \cdots + M(1,L)\,\ddot{i}'_L - \frac{i_L}{C_T(1)} \\
0 &= M(1,2)\,\ddot{i}'_1 + R(2)\,\dot{i}'_2 + L(2)\,\ddot{i}'_2 + \frac{i'}{C_T(2)} + M(2,3)\,\ddot{i}'_3 \\
&\quad + M(2,4)\,\ddot{i}'_4 + \cdots + M(2,L)\,\ddot{i}'_L - \frac{i_2}{C_T(2)} \\
0 &= \cdots \\
&\ \ \vdots \\
0 &= M(1,L)\,\ddot{i}'_1 + M(2,L)\,\ddot{i}'_2 + \cdots + M(L-1,L)\,\ddot{i}'_{L-1} + R(L)\,\dot{i}'_L \\
&\quad + L(L)\,\ddot{i}'_L + \frac{i'_L}{C_T(L)} - \frac{i_L}{C_T(L)}
\end{aligned}
$$

These produce a matrix equation:

$$
0 = \begin{bmatrix} L(1) & M(1,2) & M(1,3) & \cdots & M(1,L) \\ M(1,2) & L(2) & M(2,3) & \cdots & \vdots \\ M(1,3) & M(2,3) & L(3) & \ddots & \vdots \\ \vdots & & & \ddots & \vdots \\ \vdots & & & L(L-1) & M(L-1,L) \\ M(1,L) & \cdots & \cdots & M(L-1,L) & L(L) \end{bmatrix} \begin{bmatrix} \ddot{i}'_1 \\ \ddot{i}'_2 \\ \ddot{i}'_3 \\ \vdots \\ \vdots \\ \ddot{i}'_L \end{bmatrix}
$$

$$
+ \begin{bmatrix} R(1) & 0 & 0 & 0 & \cdots & 0 \\ 0 & R(2) & 0 & 0 & & 0 \\ 0 & 0 & R(3) & 0 & & \\ \vdots & & & \ddots & & \\ \vdots & & & & \ddots & \\ 0 & \cdots & & & \cdots & R(L) \end{bmatrix} \begin{bmatrix} \dot{i}'_1 \\ \dot{i}'_2 \\ \dot{i}'_3 \\ \vdots \\ \vdots \\ \dot{i}'_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C_T(1)} & 0 & 0 & & 0 \\ 0 & \frac{1}{C_T(2)} & 0 & & \vdots \\ 0 & 0 & \frac{1}{C_T(3)} & \cdots & \vdots \\ \vdots & & & \ddots & \\ \vdots & & & & \\ 0 & \cdots & & \cdots & \frac{1}{C_T(L)} \end{bmatrix} \begin{bmatrix} i'_1 \\ i'_2 \\ i'_3 \\ \vdots \\ \vdots \\ i'_L \end{bmatrix}
$$

$$
- \begin{bmatrix} \frac{1}{C_T(1)} & 0 & 0 & & 0 \\ 0 & \frac{1}{C_T(2)} & 0 & & 0 \\ 0 & 0 & \frac{1}{C_T(3)} & 0 & \\ \vdots & & & \ddots & \\ \vdots & & & & \ddots \\ 0 & \cdots & & \cdots & \frac{1}{C_T(L)} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ \vdots \\ i_L \end{bmatrix} \qquad \text{(I - 6)}
$$

$$[LM][\ddot{\mathcal{Q}}'] + [R][\dot{\mathcal{Q}}'] + \left[\frac{1}{CT}\right][\mathcal{Q}'] - \left[\frac{1}{CT}\right][\mathcal{Q}] \qquad (I-7)$$

If the currents in the lower loops are solved; the voltages at each node can be found by differencing the loop charge and dividing by the ground capacitance. Equations (I-5) and (I-7) are first solved for the loop currents. Equation (I-8) is derived from equation (I-5).

$$[\mathcal{Q}'] = \left[\frac{1}{CT}\right]^{-1}[\dot{v}(t)] + \left[\frac{1}{CT}\right]^{-1}\left[\frac{1}{CGT}\right][\mathcal{Q}] \qquad (I-8)$$

When equation (I-8) is substituted into equation (I-7), equation (I-9) is developed.

$$-[LM]\left[\frac{1}{CT}\right]^{-1}[\dddot{v}(t)] - [LM]\left[\frac{1}{CT}\right]^{-1}\left[\frac{1}{CGT}\right][\ddot{\mathcal{Q}}]$$

$$-[R]\left[\frac{1}{CT}\right]^{-1}[\ddot{v}(t)] + [R]\left[\frac{1}{CT}\right]^{-1}\left[\frac{1}{CGT}\right][\dot{\mathcal{Q}}]$$

$$-[\dot{v}(t)] + \left[\frac{1}{CGT}\right][\mathcal{Q}] - \left[\frac{1}{CT}\right][\mathcal{Q}] \qquad (I-9)$$

An integration of equation (I-9) is performed and the current is replaced with its differential, the charge.

$$[\mathcal{Q}] = [\dot{\Phi}]$$

$$\begin{aligned}
&[v(t)] + [R]\left[\frac{1}{CT}\right]^{-1}[\dot{v}(t)] + [L\,M]\left[\frac{1}{CT}\right]^{-1}[\ddot{v}(t)] \\
&= [L\,M]\left[\frac{1}{CT}\right]^{-1}\left[\frac{1}{CGT}\right][\dot{\mathcal{Q}}] + [R]\left[\frac{1}{CT}\right]^{-1}\left[\frac{1}{CGT}\right][\mathcal{Q}] \\
&\quad + \left[\left[\frac{1}{CGT}\right] - \left[\frac{1}{CT}\right]\right][\Phi]
\end{aligned}$$

solving for $[\ddot{\Phi}]$ where $\left[\frac{1}{CG}\right] = \left[\frac{1}{CGT}\right] - \left[\frac{1}{CT}\right]$

$$\begin{aligned}
[\ddot{\Phi}] = &- \left[\frac{1}{CGT}\right]^{-1}\left[\frac{1}{CT}\right][L\,M]^{-1}[R]\left[\frac{1}{CT}\right]^{-1}\left[\frac{1}{CGT}\right][\dot{\Phi}] \\
&+ \left[\frac{1}{CGT}\right]^{-1}\left[\frac{1}{CT}\right][L\,M]^{-1}[v(t)] \\
&- \left[\frac{1}{CGT}\right]^{-1}\left[\frac{1}{CT}\right][L\,M]^{-1}\left[\frac{1}{CG}\right][\Phi] \\
&+ \left[\frac{1}{CGT}\right]^{-1}[\ddot{v}(t)] \\
&+ \left[\frac{1}{CGT}\right]^{-1}\left[\frac{1}{CT}\right][L\,M]^{-1}[R]\left[\frac{1}{CT}\right][\dot{v}(t)] \qquad \text{(I-10)}
\end{aligned}$$

This equation is now in the form of a second order N-dimensional differential equation that can be solved using the Runge-Kutta Technique.

APPENDIX J

# APPENDIX J

## MATRIX MANIPULATION TO OBTAIN COEFFICIENTS TO DIFFERENTIAL EQUATION

```
ZZJOB 5                        2400042G          HAMLIN
ZZDUP
*DELETMINV
ZZZZ
ZZJOB
ZZFORX52
*PSTSN4
*POBJP4
C  PROGRAM MATRIX MANIPULATION TO OBTAIN COEFFICIENTS TO DIFF. EQUATION
C  READ IN CG(I),CT(I),R(I),ALM(I,J)
C   THE DIMENSION STATEMENT HAS TO BE ADJUSTED ACCORDING TO SIZE  OF MATRIX
C    0DIMENSION CG(   ),CT(   ),R(   ),CGT(   ,   ),EPS(   ,   ),
C    1   SA1CGT(   ,   ),ALM(   ,   ),GAMMA(   ,   ),ALPHA(   ,   )
C  DATA OUTPUT IS PUNCHED AS IT IS PRODUCED
     0DIMENSION CG(14),CT(14),R(14),CGT(14,14),EPS(14,14),
     1   SA1CGT(14,14),ALM(14,14),GAMMA(14,14),ALPHA(14,14)
    1 READ 101,NSECT,MSTOP
    2 READ 103,(I,CG(I),I = 1,NSECT)
    3 READ 103,(I,CT(I),I = 1,NSECT)
    4 READ 104,(I,R(I),I = 1,NSECT)
      READ 106,((I,J,ALM(I,J),J = 1,NSECT),I = 1,NSECT)
      PUNCH 103,(I,CG(I),I = 1,NSECT)
C   SET UP THE MATRIX 1/CGT AND SAVE
      DO 49 J = 1,NSECT
      DO 49 I = 1,NSECT
   49 CGT(I,J) = 0.0
      CGT(1,1) = 1./CG(2) + 1./CT(1)
      CGT(NSECT,NSECT) = 1./CG(NSECT) + 1./CT(NSECT)
      NSECT1 = NSECT - 1
      DO 60 I = 2,NSECT1
   60 CGT(I,I) = 1./CG(I) + 1./CG(I + 1) + 1./CT(I)
```

```
      DO 61 I = 1,NSECT1
      CGT(I+1,I) = -1./CG(I+1)
   61 CGT(I,I+1) = -1./CG(I+1)
C THE MATRIX 1/CGT HAS BEEN SET UP
      DO 69 J = 1,NSECT
      DO 69 I = 1,NSECT
   69 SA1CGT(I,J) = CGT(I,J)
C   INVERT THE MATRIX ALM AND CGT
      CALL MINV(CGT,NSECT,14,D)
   80 PUNCH 103,(I,CGT(I,1),I = 1,NSECT)
      CALL MINV(ALM,NSECT,14,D)
C   MULTIPLY 1/CT TIMES ALMINV AND STORE IN ALPHA
C  USE STORAGE AREA ALPHA TEMPORARILY
      DO 70 M1 = 1,NSECT
      DO 70 K1 = 1,NSECT
   70 ALPHA(K1,M1) = ALM(K1,M1)/CT(K1)
C   THE MATRIX 1/CT*ALMINV) IS NOW IN ALPHA TEMPORARILY
C  THE PRODUCT OF CGTINV*ALPHA PRODUCES MATRIX GAMMA
C   WE ARE NOW FINISHED WITH STORAGE AREA ALM
      DO 71 M3 = 1,NSECT
      DO 71 K3 = 1,NSECT
      GAMMA(K3,M3) = 0.0
C  EQUATE SA1CGT WITH ALM
      ALM(K3,M3) = SA1CGT(K3,M3)
      DO 71 M2 = 1,NSECT
   71 GAMMA(K3,M3) = GAMMA(K3,M3) + CGT(K3,M2)*ALPHA(M2,M3)
   81 PUNCH 103,(I,GAMMA(I,1),I = 1,NSECT)
C GENERATE 1/CG FROM 1/CGT - 1/CT AND STORE IN ALM
C  MULTIPLY GAMMA X 1/CG  TO OBTAIN ALPHA AND GAMMA  X  RCT TO OBTAIN EPS
      DO 90 K8 = 1,NSECT
   90 ALM(K8,K8) = ALM(K8,K8) - 1./CT(K8)
      DO 75 M4 = 1,NSECT
      DO 75 K4 = 1,NSECT
      EPS(K4,M4) = GAMMA(K4,M4)*R(M4)*CT(M4)
      ALPHA(K4,M4) = 0.0
      DO 75 M5 = 1,NSECT
```

```
   75 ALPHA(K4,M4) = ALPHA(K4,M4) + GAMMA(K4,M5)*ALM(M5,M4)
   83 PUNCH 103,(I,EPS(I,1),I = 1,NSECT)
   85 PUNCH 106,((I1,J1,ALPHA(I1,J1),J1 = 1,NSECT),I1 = 1,NSECT)
C  COMPUTE BETA BY MULTIPLY EPS X 1/CGT(SA1CGT)
C  USE STORAGE AREA GAMMA FOR BETA
      DO 76 M6 = 1,NSECT
      DO 76 K6 = 1,NSECT
      GAMMA(K6,M6) = 0.0
      DO 76 K5 = 1,NSECT
   76 GAMMA(K6,M6) = GAMMA(K6,M6) + EPS(K6,K5)*SA1CGT(K5,M6)
   87 PUNCH 106,((I2,J2,GAMMA(I2,J2),J2= 1,NSECT),I2 = 1,NSECT)
C  THE MATRICES ARE NOW CALCULATED
C THE INITIAL VALUES OF PSI ARE CGTINV(I,1) TIMES
C   THE DERIVATIVE OF THE INPUT VOLTAGE AT ZERO
  101 FORMAT (2I3)
  103 FORMAT (4(I6,E14.5))
  104 FORMAT (4(I6,F14.5))
  106 FORMAT (4(2I3,E14.5))
      IF(MSTOP) 89,89,1
   89 CALL EXIT
      END
```

```
C          SUBROUTINE MINV
C          PURPOSE  INVERT A MATRIX
C         USAGE   CALL MINV(A,N,IDIM,D)
C         DESCRIPTION OF PARAMETERS
C            A  INPUT MATRIX DESTROYED IN COMPUTATION AND REPLACED BY
C             RESULTANT INVERSE
C            N  ORDER OF MATRIX A
C               IDIM  MAGNITUDE OF DIMENSION IN CALLING PROG.
C            D  RESULTANT DETERMINANT
C   MUST BE A GENERAL MATRIX  NO SUBROUTINES AND SUBPROGRAMS REQUIRED
C   METHOD  STANDARD GAUSS JORDAN METHOD. THE DETERMINANT IS CALCULATED
C         A DET. OF ZERO INDICATES THAT THE MATRIX IS SINGULAR
      SUBROUTINE MINV(A,N,IDIM,D)
      DIMENSION A(1),L(40),M(40)
C     DOUBLE PRECISION A,D,BIGA,HOLD
C IF DOUBLE PRECISION IS DESIRED LEAVE OUT THE C FROM PREVIOUS STATEMENT
C         ABS IN STATEMENT 10 MUST BE CHANGED TO DABS.
C
C         SEARCH FOR LARGEST ELEMENT
C
      D=1.
      NK=-IDIM
      DO 80 K=1,N
      NK=NK+IDIM
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=IDIM*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( ABS(BIGA)- ABS(A(IJ)))15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
```

```
   20 CONTINUE
C        PRODUCT OF PIVOTS
      D=D*BIGA
      IF(D)22,150,22
C        INTERCHANGE ROWS
   22 J=L(K)
      IF(J-K)35,35,25
   25 KI=K-IDIM
       DO 30 I=1,N
      KI=KI+IDIM
       HOLD=-A(KI)
       JI=KI-K+J
       A(KI)=A(JI)
   30  A(JI)=HOLD
C        INTERCHANGE COLUMNS
   35 I=M(K)
      IF(I-K)45,45,38
   38 JP=IDIM*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI)=HOLD
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS CONTAINED IN
C        BIGA)
   45 DO 55 I=1,N
      IF(I-K)50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
C        REDUCE MATRIX
      DO 65 I=1,N
      IK=NK+I
      IJ=I-IDIM
```

```
      DO 65 J=1,N
      IJ=IJ+IDIM
      IF(I-K)60,65,60
   60 IF(J-K)62,65,62
   62 KJ=IJ-I+K
      A(IJ)=A(IK)*A(KJ)+A(IJ)
   65 CONTINUE
C        DIVIDE ROW BY PIVOT
      KJ=K-IDIM
      DO 75 J=1,N
      KJ=KJ+IDIM
      IF(J-K)70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
C        REPLACE  PIVOT BY RECIPROCAL
      A(KK)=1./BIGA
   80 CONTINUE
C        FINAL ROW AND COLUMN INTERCHANGE
      K=N
  100 K=(K-1)
      IF(K)150,150,105
  105 I=L(K)
      IF(I-K)120,120,108
  108 JQ=IDIM*(K-1)
      JR=IDIM*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI)=HOLD
  120 J=M(K)
      IF(J-K)100,100,129
  129 KI=K-IDIM
      DO 130 I=1,N
      KI=KI+IDIM
```

```
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI)=HOLD
      GO TO 100
  150 RETURN
      END
```

## DATA AND RESULTS

### INPUT

```
 3  1
    1  1.000000E-12       2  1.000000E-12       3  1.000000E-12
    1  1.000000E-12       2  1.000000E-12       3  1.000000E-12
    1            0.1      2            0.1      3            0.1
 1  1 1000.0000E-06    1  2 1000.0000E-06    1  3 1000.0000E-06    2  1 1000.0000E-06
 2  2 1000.0000E-06    2  3 1000.0000E-06    3  1 1000.0000E-06    3  2 1000.0000E-06
 3  3 1000.0000E-06
```

### OUTPUT

```
    1 100.00000E-14       2 100.00000E-14       3 100.00000E-14
    1 624.99997E-15       2 249.99998E-15       3 124.99998E-15
    1 624.62499E+00       2 249.25000E+00       3 124.12500E+00
    1 624.62499E-13       2 249.25000E-13       3 124.12500E-13
 1  1 624.00000E+12    1  2-623.99999E+12    1  3 000.00000E-99    2  1 249.00001E+12
 2  2-249.00000E+12    2  3 000.00000E-99    3  1 124.00001E+12    3  2-124.00000E+12
 3  3 000.00000E-99
 1  1 124.86250E+00    1  2-623.37499E-01    1  3 625.00000E-04    2  1 498.25001E-01
 2  2-248.75000E-01    2  3 249.99998E-04    3  1 248.12501E-01    3  2-123.87500E-01
 3  3 124.99998E-04
```

### INPUT

```
 5  1
    1  1.000000E-12       2  1.000000E-12       3  1.000000E-12       4  1.000000E-12
    5  1.000000E-12
    1  1.000000E-12       2  1.000000E-12       3  1.000000E-12       4  1.000000E-12
    5  1.000000E-12
    1            0.1      2            0.1      3            0.1      4            0.1
    5            0.1
 1  1 1000.0000E-06    1  2 1000.0000E-12    1  3 1000.0000E-12    1  4 1000.0000E-12
 1  5 1000.0000E-12    2  1 1000.0000E-12    2  2 1000.0000E-06    2  3 1000.0000E-12
 2  4 1000.0000E-12    2  5 1000.0000E-12    3  1 1000.0000E-12    3  2 1000.0000E-12
 3  3 1000.0000E-06    3  4 1000.0000E-12    3  5 1000.0000E-12    4  1 1000.0000E-12
 4  2 1000.0000E-12    4  3 1000.0000E-12    4  4 1000.0000E-06    4  5 1000.0000E-12
 5  1 1000.0000E-12    5  2 1000.0000E-12    5  3 1000.0000E-12    5  4 1000.0000E-12
 5  5 1000.0000E-06
```

OUTPUT

```
    1 100.00000E-14     2 100.00000E-14     3 100.00000E-14     4 100.00000E-14
    5 100.00000E-14
    1 618.18177E-15     2 236.36360E-15     3 909.09076E-16     4 363.63628E-16
    5 181.81813E-16
    1 618.18141E+00     2 236.36285E+00     3 909.08170E-01     4 363.62667E-01
    5 181.80833E-01
    1 618.18141E-13     2 236.36285E-13     3 909.08170E-14     4 363.62667E-14
    5 181.80833E-14
1   1 381.81855E+12  1  2-236.36385E+12  1  3-909.09195E+11  1  4-363.63670E+11
1   5-181.81833E+11  2  1-236.36388E+12  2  2 527.27326E+12  2  3-181.81836E+12
2   4-727.27354E+11  2  5-363.63669E+11  3  1-909.09170E+11  3  2-181.81836E+12
3   3 545.45507E+12  3  4-181.81836E+12  3  5-909.09180E+11  4  1-363.63668E+11
4   2-727.27330E+11  4  3-181.81836E+12  4  4 527.27324E+12  4  5-236.36387E+12
5   1-181.81832E+11  5  2-363.63663E+11  5  3-909.09190E+11  5  4-236.36384E+12
5   5 381.81853E+12
1   1 100.00000E+00  1  2-998.00000E-07  1  3-102.50000E-06  1  4-100.20000E-06
1   5-100.10000E-06  2  1-103.00000E-06  2  2 100.00000E+00  2  3-101.30000E-06
2   4-102.40000E-06  2  5-100.50000E-06  3  1-100.00000E-06  3  2-102.00000E-06
3   3 100.00000E+00  3  4-103.90000E-06  3  5-102.00000E-06  4  1-100.10000E-06
4   2-100.00000E-06  4  3-102.00000E-06  4  4 100.00000E+00  4  5-103.00000E-06
5   1-999.00000E-07  5  2-100.00000E-06  5  3-103.00000E-06  5  4-990.00000E-07
5   5 999.99990E-01
```

RUNGE-KUTTA SOLUTION OF DIFFERENTIAL EQUATIONS

```
C     RUNGE-KUTTA SOLUTION OF SECOND ORDER N-DIMENSION
C     DIFFERENTIAL EQUATIONS
C     READ IN ,NSECT,DT,FT,
C     READ IN I/CGT,GAMMA,EPS,ALPHA,BETA
C    0 DIMENSION CGT(   ),GAMMA(   ),EPS(   ),ALPHA(   ,   ),BETA(   ,   ),
C    1 PI(   ),PQ(   ),DEL1I(   ),DEL1Q(   ),DEL2I(   ),DEL2Q(   ),DEL3I(   ),
C    2DEL3Q(   ),DEL4I(   ),DEL4Q(   ),DELPI(   ),DELPQ(   ),V(   ),CG(   ),
C    3DRIV1(   ),DRIV2(   )
     0DIMENSION CGT(14),GAMMA(14),EPS(14),ALPHA(14,14),BETA(14,14),
     1 PI(14),PQ(14),DEL1I(14),DEL1Q(14),DEL2I(14),DEL2Q(14),DEL3I(14),
     2DEL3Q(14),DEL4I(14),DEL4Q(14),DELPI(14),DELPQ(14),V(14),CG(14),
     3DRIV1(14),DRIV2(14)
1     WRITE (3,105)
      READ (1,101) NSECT,MSTOP,DT,FT,A,B,C
2     READ (1,102) (I,CG(I),I = 1,NSECT)
      READ (1,102) (I,CGT(I),I = 1,NSECT)
      READ (1,102) (I,GAMMA(I),I = 1,NSECT)
      READ (1,102) (I,EPS(I),I = 1,NSECT)
      READ (1,103) ((I,J,ALPHA(I,J),J = 1,NSECT),I = 1,NSECT)
      READ (1,103)  ((I,J,BETA(I,J),J = 1,NSECT),I = 1,NSECT)
C     DERIVATIVE OF INPUT VOLTAGE  A(C - B)*1.E+06)
      VII = A*(C-B)*1.E+06
C     INITIAL VALUES OF CURRENT  PI
      H = DT*1.E-06
      T = 0.0
   11 DO 12 I = 1,NSECT
      PQ(I) = 0.0
   10 PI(I) = CGT(I)*VII
      DRIV1(I) = A*(GAMMA(I) - B*1.E+06*EPS(I) + B*B*CGT(I)*1.E+12)
   12 DRIV2(I)=A*(-GAMMA(I) + C*EPS(I)*1.E+06 - C*C*CGT(I)*1.E+12)
```

```
6   BO = EXP(-B*T)
    CO = EXP(-C*T)
    V(1) = A*(BO - CO)
13  DO 14 I = 1,NSECT
    DEL1Q(I) = PI(I)
    DEL1I(I) = DRIV1(I)*BO + DRIV2(I)*CO
    DO 14 J = 1,NSECT
14  DEL1I(I) = DEL1I(I) - ALPHA(I,J)*PQ(J) - BETA(I,J)*PI(J)
    BO = EXP(-B*(T + DT/2.))
    CO = EXP(-C*(T + DT/2.))
15  DO 16 I = 1,NSECT
    DEL2Q(I) = PI(I) +DEL1I(I)*H/2.
    DEL2I(I) = DRIV1(I)*BO + DRIV2(I)*CO
    DO 16 J = 1,NSECT
160DEL2I(I) = DEL2I(I) - ALPHA(I,J)*(PQ(J) + DEL1Q(J)*H/2.) -
  1     BETA(I,J)*(PI(J) + DEL1I(J)*H/2.)
17  DO 18 I = 1,NSECT
    DEL3Q(I) = PI(I) + DEL2I(I)*H/2.
    DEL3I(I) = DRIV1(I)*BO + DRIV2(I)*CO
    DO 18 J = 1,NSECT
180DEL3I(I) = DEL3I(I) - ALPHA(I,J)*(PQ(J) + DEL2Q(J)*H/2.) -
  1     BETA(I,J)*(PI(J) + DEL1I(J)*H/2.)
    BO = EXP(-B*(T + DT))
    CO = EXP(-C*(T + DT))
19  DO 20 I = 1,NSECT
    DEL4I(I) = DRIV1(I)*BO + DRIV2(I)*CO
    DEL4Q(I) = PI(I) + DEL3I(I)*H
    DO 20 J = 1,NSECT
200DEL4I(I) = DEL4I(I) - ALPHA(I,J)*(PQ(J) + DEL3Q(J)*H) -
  1       BETA(I,J)*(PI(J) + DEL3I(J)*H)
    DO 40 I = 2,NSECT
40  V(I) = (PQ(I-1) - PQ(I))/CG(I)
28  WRITE (3,104) T,(I,V(I),I = 1,NSECT)
29  DO 30 I = 1,NSECT
    DELPI(I) = H*(DEL1I(I) + 2.*DEL2I(I) + 2.*DEL3I(I) + DEL4I(I))/6.
    DELPQ(I) = H*(DEL1Q(I) + 2.*DEL2Q(I) + 2.*DEL3Q(I) + DEL4Q(I))/6.
```

```
      PI(I) = PI(I) + DELPI(I)
   30 PQ(I) = PQ(I) + DELPQ(I)
      T = T + DT
  200 IF(T - FT) 6,6,31
101   FORMAT (2I5,5F10.5)
102   FORMAT (4(I6,E14.5))
103   FORMAT (4(2I3,E14.5))
104   FORMAT (1H0,F10.3,12(I2,F8.5))
105   FORMAT (1H1,17H  NEW SET OF DATA)
31    IF(MSTOP) 32,1,32
32    CALL EXIT
      END
```

# BIBLIOGRAPHY

BIBLIOGRAPHY


1. Abetti, P. A. "Bibliography on the Surge Performance of Transformers and Rotating Machinery," Transactions, American Institute of Electrical Engineers. Paper 58, December, 1958, pp. 1150-64.

2. Waldvogel, P. and Rouxel, R. "A New Method of Calculating the Electrical Stresses in a Winding Subjected to a Surge Voltage," Brown Boveri Review. Vol. 43, 1956, pp. 206-13.

3. Dent, B. M., Hartill, E. R. and Miles, J. G. "A Method of Analysis of Transformer Impulse Voltage Distribution Using a Digital Computer," The Proceedings of the Institute of Electrical Engineers. Vol. 105, Part A, No. 23, October, 1958. pp. 445-59.

4. Rudenberg, R. "Performance of Travelling Waves in Coils and Windings," Transactions, American Institute of Electrical Engineers. Vol. 59, 1940, pp. 1031-40, pp. 1257-62.

5. Norris, E. T. "The Lightning Strength of Power Transformers," Journal, Institute of Electrical Engineers. Vol. 95, Part II, 1948, pp. 389-406.

6. Lewis, T. J. "The Transient Behavior of Ladder Networks of the Type Representing Transformer and Machine Windings," Proceedings Institution of Electrical Engineers. Vol. 101, Part II, 1954, pp. 541-53.

7. Abetti, P. A. "Transformer Models for the Determination of Transient Voltages," Transactions, American Institute of Electrical Engineers. Vol. 72, Part III, June, 1953, pp. 468-80.

8. Waldvogel, P. and Rouxel, R., loc. cit.

9. Dent, B. M., Hartill, E. R. and Miles, J. G., loc. cit.

10. Rosa, E. B. and Grover, F. W. "Formulas and Tables For the Calculation of Mutual and Self Inductance," Bureau of Standards. Bull. 8, January, 1912, p. 136.

11. Crowhurst, N. H. "Winding Capacitance," Electronic Engineering. November, 1949, pp. 417-31.

12. Waldvogel, P. and Rouxel, R., loc. cit., p. 208.

13. Knowlton, A. E. "Standard Handbook for Electrical Engineers," McGraw-Hill Book Company, 1941, p. 67.

14. The British Electrical and Allied Industries Research Association, "Surge Phenomena," Seven Years' Research For the Central Electricity Board (1933-1940), pp. 154-6, 1941.

15. Rosa, E. B. and Grover, F. W., loc. cit., p. 13.

16. Rosa, E. B. and Grover, F. W., loc. cit., p. 11.

17. Dent, B. M., Hartill, E. R. and Miles, J. G., loc. cit., p. 447.

18. Miles, J. G. "Frequency Spectre of Standard Impulse Wave Shapes," The Metropolitan-Vickers Gazette. Vol. 25, September, 1954, pp. 367-69.

19. Veverka, A. and Hon, A. "Disc-Coil Winding on Extra-High Voltage Transformers," Electrotech. Obzor (Czechoslovakia), Vol. 54, No. 7, pp. 304-9 (1965). In Czech.

20. Harris, F.K. "Electrical Measurements," John Wiley and Sons, Inc., 1962, p. 219.