THEORY AND DESIGN OF ELECTROTHERMAL LOUDSPEAKERS

A Thesis
Presented to
the Faculty of Graduate Studies and Research
Department of Electrical Engineering
University of Manitoba

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Electrical Engineering

by
Thomas Stephen Paige
May 1971
TO MY DAUGHTER

HEIDI
ACKNOWLEDGEMENT

The author wishes to thank his thesis advisor Prof. M. A. K. Hamid for supervising this project, and Mr. H. Unger for fabricating the experimental loudspeakers.

Thanks are also extended to the National Research Council of Canada for the financial support (Grant A3326) and to the Faculty of Graduate Studies and Research for assistance in the form of a University of Manitoba Graduate Fellowship.
ABSTRACT

The theory and design of a new type of loudspeaker which employs an electrothermal effect to reproduce sound is discussed in detail. This effect is a result of electrical modulation utilizing thermionic emission from a tungsten electrode in a seeded gas flame. It is shown that theory derived from basic principles can be used effectively to predict the response of such loudspeakers. The general analysis for spherical, cylindrical and planar wave source configurations is obtained using the Green's function technique. These solutions are verified by frequency response measurements of the sound pressure levels of physical models employing air-propane flames. The flames are seeded by an atomization process to improve conductivity.

Construction of two experimental horn-type electrothermal loudspeakers and their respective evaluation by both acoustic and psycho-acoustic tests reveals the many difficulties which must be overcome before such devices become practical. Factors such as efficiency, noise and distortion are discussed and suggestions are made for future studies.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. THEORY</td>
<td>4</td>
</tr>
<tr>
<td>2.1 The Modified Wave Equation</td>
<td>4</td>
</tr>
<tr>
<td>2.1.1 The Balance of Mass - The Equation of</td>
<td></td>
</tr>
<tr>
<td>Continuity</td>
<td>5</td>
</tr>
<tr>
<td>2.1.2 The Balance of Momentum</td>
<td>6</td>
</tr>
<tr>
<td>2.1.3 The Balance of Energy</td>
<td>7</td>
</tr>
<tr>
<td>2.1.4 Equation of State</td>
<td>8</td>
</tr>
<tr>
<td>2.1.5 Combination of Derived Expressions</td>
<td>11</td>
</tr>
<tr>
<td>2.2 Mechanism of Energy Transfer</td>
<td>13</td>
</tr>
<tr>
<td>III. SOLUTION OF THE MODIFIED WAVE EQUATION</td>
<td>17</td>
</tr>
<tr>
<td>3.1 Simple Source Solution</td>
<td>19</td>
</tr>
<tr>
<td>3.2 Cylindrical Source Solution</td>
<td>22</td>
</tr>
<tr>
<td>3.3 Plane Wave Source Solution</td>
<td>25</td>
</tr>
<tr>
<td>IV. PHYSICAL MODELS OF SOURCES</td>
<td>28</td>
</tr>
<tr>
<td>4.1 The Simple (Isotropic) Source</td>
<td>28</td>
</tr>
<tr>
<td>4.2 The Cylindrical Source</td>
<td>31</td>
</tr>
<tr>
<td>4.3 The Plane Wave Source</td>
<td>32</td>
</tr>
<tr>
<td>4.4 Comparison of Physical Models - Foundations of Design</td>
<td>32</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>V. BASIC PHYSICAL REQUIREMENTS</td>
<td>35</td>
</tr>
<tr>
<td>5.1 The Ionized Gas Region</td>
<td>35</td>
</tr>
<tr>
<td>5.2 The Problem of Seeding</td>
<td>37</td>
</tr>
<tr>
<td>5.3 The Seeding Compound</td>
<td>38</td>
</tr>
<tr>
<td>5.4 The Electrodes</td>
<td>41</td>
</tr>
<tr>
<td>5.5 The Coupling Circuit</td>
<td>42</td>
</tr>
<tr>
<td>VI. PARTIALLY ENCLOSED FLAME LOUDSPEAKER DESIGN</td>
<td>44</td>
</tr>
<tr>
<td>6.1 Radiation of Sound from the Open End of the Tube</td>
<td>47</td>
</tr>
<tr>
<td>6.2 Requirements for Plane Wave Propagation</td>
<td>47</td>
</tr>
<tr>
<td>6.3 Acoustical Impedance Transformers</td>
<td>48</td>
</tr>
<tr>
<td>6.4 Design Procedure</td>
<td>50</td>
</tr>
<tr>
<td>6.5 Description of Loudspeakers Constructed</td>
<td>51</td>
</tr>
<tr>
<td>VII. MEASUREMENTS</td>
<td>57</td>
</tr>
<tr>
<td>7.1 Measurements to Verify Theory</td>
<td>57</td>
</tr>
<tr>
<td>7.2 Measurements of the Response of the Experimental Loudspeakers</td>
<td>61</td>
</tr>
<tr>
<td>7.3 Psychoacoustic Tests</td>
<td>67</td>
</tr>
<tr>
<td>VIII. ADDITIONAL CONSIDERATIONS</td>
<td>70</td>
</tr>
<tr>
<td>8.1 Distortion</td>
<td>70</td>
</tr>
<tr>
<td>8.2 Flame Noise</td>
<td>70</td>
</tr>
<tr>
<td>8.3 Efficiency and Durability</td>
<td>72</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>IX. CONCLUSIONS</td>
<td>74</td>
</tr>
<tr>
<td>APPENDIX A  MEASUREMENT OF $c_g$ AND TEMPERATURE OF ELECTROTHERMAL REGION</td>
<td>75</td>
</tr>
<tr>
<td>APPENDIX B  RESONANCE WITHIN THE TUBE</td>
<td>77</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>78</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Ionization Potentials of Alkali Metals</td>
<td>38</td>
</tr>
<tr>
<td>6.1 Parameters of Experimental Loudspeakers</td>
<td>52</td>
</tr>
<tr>
<td>7.1 Measurement Data</td>
<td>60</td>
</tr>
<tr>
<td>7.2 Measurement Data for Experimental Loudspeakers</td>
<td>61</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>2</td>
</tr>
<tr>
<td>3.1</td>
<td>22</td>
</tr>
<tr>
<td>3.2</td>
<td>26</td>
</tr>
<tr>
<td>4.1</td>
<td>30</td>
</tr>
<tr>
<td>4.2</td>
<td>31</td>
</tr>
<tr>
<td>4.3</td>
<td>32</td>
</tr>
<tr>
<td>5.1</td>
<td>39</td>
</tr>
<tr>
<td>5.2</td>
<td>40</td>
</tr>
<tr>
<td>5.3</td>
<td>42</td>
</tr>
<tr>
<td>6.1</td>
<td>45</td>
</tr>
<tr>
<td>6.2</td>
<td>48</td>
</tr>
<tr>
<td>6.3</td>
<td>53</td>
</tr>
<tr>
<td>6.4</td>
<td>54</td>
</tr>
<tr>
<td>6.5</td>
<td>55</td>
</tr>
<tr>
<td>6.6</td>
<td>56</td>
</tr>
<tr>
<td>7.1</td>
<td>58</td>
</tr>
<tr>
<td>7.2</td>
<td>59</td>
</tr>
<tr>
<td>7.3</td>
<td>62</td>
</tr>
<tr>
<td>7.4</td>
<td>63</td>
</tr>
<tr>
<td>7.5</td>
<td>65</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

Electro-acoustic transducers remain the weakest link in systems which amplify and reproduce sound. It seems reasonable to believe that audible sound waves which are normally detected in air - a gaseous medium - should ideally be generated by a gaseous process. Most conventional loudspeakers depend on some type of solid vibrating surface which suffers from mass effects particularly at higher audible frequencies.

In 1954, Klein [1] described a new type of loudspeaker, the "ionophone", which reproduced music without a membrane or diaphragm. The ionophone loudspeaker consisted of an audio modulated radio frequency corona discharge coupled to a horn. Later, in 1960 a speaker similar to that developed by Klein proved very useful in providing high sound pressure levels at high frequencies for animal studies [2].

During a study of modulated airstream loudspeakers, Arnold [3] verified that pyroacoustic amplification could increase output levels of such loudspeakers by as much as 20 dB. In 1965 he introduced his modulated combustion or "Dragon" loudspeaker.

In 1967, Babcock, Baker and Cattaneo [4] presented a summary of observed interactions between flames and sound which included methods of modulating flames to reproduce sound. It was discovered that a flame, seeded with a salt to increase the conductivity, could
emit high sound levels with high fidelity in response to an electrical signal imposed upon it with a suitable electrode arrangement (see Fig. 1.1). In 1969 Burchard [5] presented a theoretical analysis of this peculiar phenomenon which he termed the "electrothermal loudspeaker".

Despite the acclaimed advantages of gaseous loudspeakers, no practical devices other than for laboratory purposes have been developed. It is the author's belief that the electrothermal loudspeaker with its substantial resource of heat energy represents an "active" type of loudspeaker capable of both amplifying and reproducing signals from either electronic or fluidic systems. The full potential of this
device can be realized only when its operation is perfectly understood. This warrants further investigation into the electrothermal phenomenon.

In this thesis the theory governing electrothermal acoustical behaviour is reviewed and the desired form of the wave equation is derived in Chapter II. Using the Green's function technique, solutions are obtained in Chapter III for particular geometrical configurations. These solutions are later used in Chapter IV to predict the frequency response characteristics of physical models corresponding to the mathematical models of Chapter III. In Chapter V details of the basic requirements for electrothermal loudspeakers are discussed.

Subsequent chapters are devoted specifically to the design of electrothermal loudspeakers and to the evaluation of their performance by acoustical measurements. Subjective listening tests are conducted to provide a "human" element to supplement this evaluation process.

In Chapter VIII additional factors such as efficiency, distortion, durability and flame jet noise are discussed. It is then possible to conclude the thesis in terms of the practicality of the electrothermal loudspeaker.
CHAPTER II

THEORY

2.1 THE MODIFIED WAVE EQUATION

Sound generation, like other wave phenomena, is usually analyzed by deriving a wave equation to mathematically describe the physical process. Wave equations for sound propagation in air have been dealt with in great detail by many authors. These equations, however, must be modified for reasons that shall become apparent later.

The electrothermal loudspeaker is an example of radiation of sound from a region of violent fluid motion. The theory of sound radiation from such a region has been discussed by Morse and Ingard [6]. Inside this region the motions of the gas are great enough so that the linear equations of wave motion described above no longer hold. Furthermore, energy is fed into the medium in the form of heat energy which makes the wave equation non-homogeneous. It is still possible to neglect the damping effects of viscosity and thermal diffusion, however, as long as boundary conditions are not rigidly imposed. Later when such conditions are imposed, their effects will be considered.

The modified wave equation follows directly from the relationships expressing the balance of mass, momentum and energy together with the equation of state of the gas. Each of these will be considered separately.
2.1.1 The Balance of Mass – the Equation of Continuity

The equation of continuity for the mass density of a fluid is [6]

$$\frac{\partial \rho}{\partial t} = q - \rho \nabla \cdot \mathbf{u}$$

where

- \(\rho\) = mass density of the fluid
- \(\mathbf{u}\) = velocity of fluid flow
- \(q\) = rate of generation of mass density per unit volume of fluid
- \(t\) = time

It is convenient to introduce a perturbation technique such that

$$\rho = \rho_0 + \delta$$

where

- \(\rho_0\) = static or average density of the fluid
- \(\delta\) = perturbation in the density resulting from acoustic waves

In general, for acoustical processes

$$\delta \ll \rho_0$$

therefore

$$\frac{\partial \rho}{\partial t} = \frac{\partial \delta}{\partial t}$$

and

$$\rho \approx \rho_0$$

Also, in the electrothermal effect, no mass density is created within
the fluid so that

\[ q = 0 \]

The linearized version of the continuity equation may be written as

\[ \frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{u} = 0 \]  \hspace{1cm} (2.1.1)

2.1.2 The Balance of Momentum

The second important relationship may be derived by writing Newton's second law for a small volume of gas located in a homogeneous medium [7]. Let

\[ \mathbf{F} = \text{external force per unit volume acting to accelerate a small volume of gas.} \]

The total force per unit volume acting on the volume of gas is

\[ \mathbf{F} - \nabla p \]

where \( p \) is the sound pressure or incremental change in atmospheric pressure within the small volume. This force must be equal to the rate of change of momentum so that

\[ \mathbf{F} - \nabla p = \rho \frac{\partial \mathbf{u}}{\partial t} \]

Applying the perturbation technique the linearized version of this equation will be

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \nabla p = \mathbf{F} \]  \hspace{1cm} (2.1.2)
where $\mathbf{F}$ can be expressed as the rate of change of momentum per unit volume of the gas due to external effects.

2.1.3 The Balance of Energy

To consider quantitatively the effect of introducing energy into a volume of gas, it is necessary to introduce a thermodynamic property known as entropy. An entropy increase corresponds to a change in the amount of heat energy per unit volume, so that

$$\frac{\partial Q}{\partial t} = \rho T \frac{\partial S}{\partial t}$$

where $Q$ is the heat per unit volume, $S$ is the entropy per unit mass and $T$ is the absolute temperature.

Also

$$\frac{\partial Q}{\partial t} = \text{the rate at which heat is transferred to or from the region of fluid being considered.}$$

It is safe to say that there is negligible heat loss due to viscosity and thermal conduction in the audible frequency range [7]. Define

\[ H \equiv \text{the rate at which heat energy is added to the region per unit volume so that} \]

\[ H = \rho T \frac{\partial S}{\partial t} \]
Again using the perturbation method

\[ T = T_0 + \tau \]

\[ S = S_0 + s \]

thus

\[ \rho_0 T_0 \frac{\partial s}{\partial t} = H \quad (2.1.3) \]

2.1.4 Equation of State

To establish a wave equation, it is desirable to eliminate all but one variable, say for instance sound pressure \( p \), from equations (2.1.1), (2.1.2) and (2.1.3). This is possible if the mass density \( \rho \) and the specific entropy \( S \) are expressed in terms of the pressure \( P \) and temperature \( T \).

\[ \rho = f(P,T) \]

\[ S = f(P,T) \]

Such functional relationships between two or more thermophysical variables are known as state equations.

To facilitate this discussion, some basic thermodynamic relationships must be reviewed. Most important of all are the measurable properties of gases defined as follows:

The isothermal compressibility

\[ \kappa_T = \frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial P} \right)_T \quad (2.1.4) \]

where \( \kappa_T \) is the fractional rate of change of volume (or density) with pressure at constant temperature.
The coefficient of thermal expansion

\[ \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = \frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p \]  \hspace{1cm} (2.1.5)

where \( \beta \) is the fractional change in volume (or density) with temperature at constant pressure.

Also define \( \alpha \) as the rate of increase in pressure with temperature at constant volume, that is

\[ \alpha = \left( \frac{\partial P}{\partial T} \right)_V = \frac{\beta}{K_T} \]  \hspace{1cm} (2.1.6)

The heat capacity \( C_p \) per unit mass at constant pressure can be expressed as [8]

\[ C_p = T \left( \frac{\partial S}{\partial T} \right)_p = -T \left( \frac{\partial S}{\partial P} \right)_T \left( \frac{\partial P}{\partial T} \right)_S \]  \hspace{1cm} (2.1.7)

To obtain analytical results, it is also necessary to make the approximation that the gas to be considered is an ideal gas, that is, it satisfies the relationship [9]

\[ P V = R T \]

where \( R \) is the gas constant. For ideal gases \( R \) may be expressed as [9]

\[ R = C_p - C_v \]  \hspace{1cm} (2.1.8)

where \( C_v \) is the heat capacity per unit mass at constant volume.

For sound of frequencies below \( 10^9 \) Hz in gases, it is a good approximation [6] to assume that the compression is adiabatic; that is, the entropy content of the gas remains nearly constant during the
compression. For adiabatic processes in ideal gases, the following relationships hold [9]:

\[ P \gamma \text{ constant} \quad (2.1.9a) \]

and

\[ T \, P^{(1-\gamma)/\gamma} \text{ constant} \quad (2.1.9b) \]

where \( \gamma \) is the ratio of the specific heat at constant pressure to that at constant volume, that is

\[ \gamma = \frac{C_p}{C_v} \quad (2.1.10) \]

For ideal gases, (2.1.4), (2.1.5), (2.1.6) may be reduced to

\[ K_T \rightarrow \frac{1}{T} \quad (2.1.11a) \]

\[ \beta \rightarrow \frac{1}{T} \quad (2.1.11b) \]

\[ \alpha \rightarrow \frac{P}{T} \quad (2.1.11c) \]

In accordance with the perturbation technique, the following approximations will be applied to the previous relationships:

\[ T \approx T_o \]

\[ P \approx P_o \]

\[ \rho \approx \rho_o \]

where \( T_o, P_o \) and \( \rho_o \) are the average or static values of temperature, pressure and mass density in the gas.

It is now possible to write the equation of state relating the change in density \( \delta \) and the change in specific entropy \( s \) in terms of the acoustic pressure \( p \) and the change in temperature \( \tau \), that is
\[ \delta = \left( \frac{\partial \rho}{\partial P} \right)_T \frac{\partial s}{\partial t} + \left( \frac{\partial \rho}{\partial T} \right)_P \frac{\partial s}{\partial t} \]

Using (2.1.4) and (2.1.5) this reduces to
\[ \delta = \rho_0 \kappa_T \frac{\partial s}{\partial t} - \rho_0 \beta \frac{\partial s}{\partial t} \]

or for ideal gases (using 2.1.11)
\[ \delta = \frac{\rho_0}{P_o} (p - \alpha t) \]  \hspace{1cm} (2.1.12)

Similarly
\[ s = \left( \frac{\partial s}{\partial T} \right)_P \frac{\partial s}{\partial t} + \left( \frac{\partial s}{\partial P} \right)_T \frac{\partial s}{\partial t} \]

Using (2.1.7) this reduces to
\[ s = c_p \left( \frac{T}{T_o} - \frac{\gamma - 1}{\gamma} \frac{p}{P_o} \right) \]  \hspace{1cm} (2.1.13)

2.1.5 Combination of Derived Expressions

Differentiating (2.1.12) with respect to time, one obtains
\[ \frac{\partial \delta}{\partial t} = \frac{\rho_0}{P_o} \frac{\partial s}{\partial t} (p - \alpha t) \]  \hspace{1cm} (2.1.14)

Differentiating (2.1.13) with respect to time and using (2.1.11) leads to
\[ \frac{\partial s}{\partial t} = \frac{c_p}{T_o} \left( \frac{\partial T}{\partial t} - \frac{\gamma - 1}{\alpha \gamma} \frac{\partial p}{\partial t} \right) \]  \hspace{1cm} (2.1.15)
Substituting (2.1.15) into (2.1.3) leads to

\[ H = \rho_0 c_p \left( \frac{\partial t}{\partial t} - \frac{(\gamma - 1) \partial p}{\alpha \gamma \partial t} \right) \]

which is equivalent to

\[ \frac{\partial}{\partial t} (p - \alpha t) = \frac{1}{\gamma} \frac{\partial p}{\partial t} - \frac{\alpha H}{\rho_0 c_p} \quad (2.1.16) \]

Substituting (2.1.16) into (2.1.14) gives

\[ \frac{\partial \theta}{\partial t} = \frac{\rho_0}{F} \frac{\partial p}{\partial t} - \frac{\rho_0 \alpha}{F} \frac{H}{\rho_0 c_p} \quad (2.1.17) \]

Define

\[ c^2 = \frac{\gamma P_0}{\rho_0} \quad (2.1.18) \]

thus

\[ \frac{\partial \theta}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t} - \frac{\alpha \gamma}{\rho_0 c_p} \frac{H}{c^2} \quad (2.1.19) \]

Using (2.1.8), (2.1.10) and (2.1.11), (2.1.19) becomes

\[ \frac{\partial \theta}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t} - (\gamma - 1) \frac{H}{c^2} \quad (2.1.20) \]

Using (2.1.20) it is now possible to write the continuity equation (2.1.1) as

\[ \frac{1}{c^2} \frac{\partial p}{\partial t} + \rho_0 \nabla \cdot u = \frac{(\gamma - 1)}{c^2} H \quad (2.1.21) \]
Differentiating (2.1.21) with respect to time, one obtains

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} + \rho_o \nabla \cdot \frac{\partial u}{\partial t} = \frac{(\gamma - 1)}{c^2} \frac{\partial H}{\partial t}$$

(2.1.22)

Taking the divergence of (2.1.2) one obtains

$$\nabla \cdot \rho_o \frac{\partial u}{\partial t} + \nabla \cdot \nabla p = \nabla \cdot F$$

(2.1.23)

Combining (2.1.22) and (2.1.23) leads to

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{(\gamma - 1)}{c^2} \frac{\partial H}{\partial t} - \nabla \cdot F$$

(2.1.24)

Equation (2.1.24) is the desired modified wave equation where $c$ represents the speed of sound waves propagating in a gas. It is necessary to discuss the non-homogeneous or "driving" term in more detail. This is possible by examining more closely the mechanism of energy transfer in the electrothermal loudspeaker.

2.2 MECHANISM OF ENERGY TRANSFER

The region of gas within an electrothermal loudspeaker consists of a high temperature gas flame, seeded with a vapour of an alkali salt, upon which an electric field is imposed. Such an "alkali vapour plasma" is an example of a weakly ionized gas or plasma.

The basic principle of energy transfer is well known [6]. The charged particles draw energy from the external electric field, and
some of this energy is transferred to the neutral gas atoms, which can lead to sound generation and amplification in the neutral gas. In general, the electrons maintain a temperature much higher than the neutrals leading to a constant transfer of energy.

In the time between collisions, the electrons (or ions) are accelerated in the direction of the field and receive a certain amount of energy. The collisions tend to randomize this motion so that most of the energy gained goes into random motion. There remains, however, a small net drift velocity $v_e$ of electrons given by

$$v_e = \mu_e E$$

where $\mu_e$ is the mobility of the electrons. Similarly for the ions

$$v_i = \mu_i E$$

The average rate of energy absorbed by the electrons per unit volume is

$$H_e = e N_e \mu_e E^2$$

where

- $e = \text{electronic charge}$
- $N_e = \text{number of electrons per unit volume}$

If $N_e$ is constant in time and space, then we can define the conductivity

$$\sigma_e = e N_e \mu_e$$

therefore

$$H_e = \sigma_e E^2$$

Similar relations hold for the ions, however, assuming $N_i \approx N_e$, $\mu_i$ is
much less than \( \mu_e \) (ions have much higher mass than electrons). Thus we can neglect the energy transfer of the ions and make the general conclusion

\[
H = \sigma E^2
\]  

(2.2.1)

In a similar manner we can express the rate of momentum transfer from the electrons and the ions to the neutrals. At acoustic frequencies, these tend to cancel out, and the net effect can be neglected so that \( F = 0 \) in equation (2.1.24).

The electric field which governs the electrothermal effect is most conveniently applied by a pair of electrodes in the plasma region. This field must consist of two components; a d.c. component which acts as a bias to assure symmetrical operation as well as to improve conductivity, plus an alternating component which produces the variations necessary for sound generation. Therefore

\[
E = E_{dc} + E_{ac} \sin \omega t
\]

If the distance between the electrodes is \( l \), then the field can be written as

\[
E = \frac{V_{dc}}{l} + \frac{V_{ac}}{l} \sin \omega t
\]

where

- \( V_{dc} \) = applied bias voltage
- \( V_{ac} \) = magnitude of alternating voltage
- \( \omega \) = frequency of alternating voltage
Inserting this into (2.2.1) leads to

\[
H = \frac{\sigma}{\chi^2} \left( V_{dc} + V_{ac} \sin \omega t \right)^2
\]

\[
= \frac{\sigma}{\chi^2} \left[ V_{dc}^2 + 2 V_{dc} V_{ac} \sin \omega t + \frac{1}{2} V_{ac}^2 (1 - \cos 2\omega t) \right]
\]

(2.2.2)

Differentiating (2.2.2) with respect to time, one obtains

\[
\frac{\partial H}{\partial t} = \frac{\sigma}{\chi^2} \left( 2 V_{dc} V_{ac} \omega \cos \omega t + V_{ac}^2 \omega \cos \omega t \right)
\]

if \( V_{dc} \gg V_{ac} \) the above expression reduces to

\[
\frac{\partial H}{\partial t} \approx \frac{\sigma}{\chi^2} \left( 2 V_{dc} V_{ac} \omega \cos \omega t \right) \quad (2.2.3)
\]

This approximation is not always valid as will be discussed later. If \( V_{dc} \) is not much greater than \( V_{ac} \), harmonic generation will result.

Substituting (2.2.3) into (2.1.24) and recalling that \( F = 0 \), we finally obtain

\[
\frac{1}{c^2} \frac{\partial^2 \rho}{\partial t^2} - \nu^2 \rho = 2(\gamma-1) \frac{\sigma V_{dc} V_{ac} \omega}{\chi^2 c^2} \cos \omega t \quad (2.2.4)
\]
CHAPTER III

SOLUTION OF THE MODIFIED WAVE EQUATION

It has been shown in Chapter II that the sound pressure within a region of electrothermal activity is governed by the equation

\[ \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{2 (\gamma-1) \sigma V_{dc} V_{ac}}{c^2 \xi^2} \cos \omega t \]  

(3.1)

It is now desired to solve this equation for several geometrical configurations which will prove useful in understanding the characteristics of electrothermal loudspeakers.

As mentioned before, the electrothermal loudspeaker represents a situation where sound is generated by pressure perturbations in a small region of space. These perturbations are then transferred to the surrounding air which is essentially unbounded. This type of situation lends itself very well to solutions by Green's functions techniques.

First, it is necessary to reduce (3.1) to a form suitable for Green's function analysis. Let

\[ B = \frac{2 (\gamma-1) \sigma V_{dc} V_{ac}}{c^2 \xi^2} \]  

(3.2)

hence (3.1) becomes

\[ \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = B \cos \omega t \]  

(3.3)
This equation applies only for the small region in violent fluid motion and can be extended to cover all space by introducing the Green's function $G$ such that

$$\frac{1}{c^2} \frac{\partial^2 G}{\partial t^2} - \nabla^2 G = B \delta(r-r_0) \cos \omega t \quad (3.4)$$

where $G$ can be assumed to be a product of a time and a space variable, that is

$$G = g(r) T(t) \quad (3.5)$$

also

$$\delta(r-r_0) = \begin{cases} 
1 & \text{if } r = r_0 \\
0 & \text{if } r \neq r_0 
\end{cases}$$

where $r_0$ and $r$ denote the locations of the source and observation points, respectively. To evaluate the effect of the entire region, the solution $G$ must be integrated over all such points in the electro-thermal region. Substituting (3.5) into (3.4) one obtains

$$\frac{g}{c^2} \frac{\partial^2 T}{\partial t^2} - \nabla^2 T = \delta(r-r_0) B \cos \omega t \quad (3.6)$$

Suppose

$$T = \cos \omega t$$

This is an assumption which can later be verified by substituting the solution $G$ back into the original equation (3.4). (3.6) then reduces to
\[- \frac{\omega^2}{c^2} g \cos \omega t - \nabla^2 g \cos \omega t = \delta(r-r_o) B \cos \omega t \]

or

\[ \nabla^2 g + \frac{\omega^2}{c^2} g = - \delta(r-r_o) B \]

Define

\[ K = \frac{\omega}{c} \]

so that

\[ \nabla^2 g + K^2 g = - \delta(r-r_o) B \quad (3.7) \]

which is suitable for Green's function analysis.

3.1 SIMPLE SOURCE SOLUTION

The most important solution of (3.7) is that of the simple or isotropic source. This solution gives the sound pressure \( p \) at a distance \( r \) from the source region whose largest dimension is much smaller than the wavelength of sound being radiated.

The shape of a simple source is irrelevant and is therefore assumed to be spherical with a small radius \( R \) such that

\[ KR \ll 1 \]

Throughout this discussion we are primarily interested in the far field, that is, \( r \gg r_o \) since the listener is generally far from the loudspeaker.

For the simple source the Green's function solution applicable
to (3.7) is well known:
\[ g(r|r_o) = \frac{\varepsilon^j Kr}{4\pi r} e^{-j Kr_o} \quad (r >> r_o) \]

To find the sound pressure \( p \), \( g(r|r_o) \) must be multiplied by the source strength \( B \) and integrated over the source region \( V \).

\[ p = \iiint_V B \frac{\varepsilon^j Kr}{4\pi r} e^{-j Kr_o} \, dV_o \]

In spherical coordinates,
\[ dV_o = r_o^2 \sin \theta \, d\theta \, d\phi \, dr_o \]

where
- \( r_o \) varies from 0 to \( R \)
- \( \theta \) varies from 0 to \( \pi \)
- \( \phi \) varies from 0 to \( 2\pi \)

The magnitude of the sound pressure may then be written as
\[ |p| = \frac{B}{4\pi r} \iiint_0^R \int_0^\pi \int_0^{2\pi} r_o^2 \sin \theta \, e^{-j Kr_o} \, d\phi \, d\theta \, dr_o \]

\[ = \frac{B}{r} \int_0^R r_o^2 \, e^{-j Kr_o} \, dr_o \]

Let
\[ I = \int_0^R r_o^2 \, e^{-j Kr_o} \, dr_o \]
This integral can be evaluated in terms of exponential functions. Then, using the relationship

\[ e^{-jKR} = \cos KR - j \sin KR \]

it is possible to extract the real part of the solution which may be stated as

\[
\text{Re}(I) = \frac{R^2 \sin KR}{K} + \frac{2R \cos KR}{K^2} - \frac{2\sin KR}{K^3}
\]

Recall that we assumed \( KR \ll 1 \) so that

\[
\sin KR \approx KR \\
\cos KR \approx 1
\]

thus

\[
\text{Re}(I) = R^3
\]

and

\[
|p| = \frac{B}{r} R^3
\]

Substituting the expression for \( B \) from (3.2)

\[
|p| = \frac{2(\gamma-1) \sigma Vdc Vac R^3}{\varepsilon^2 c^2 g^2} \frac{\omega}{r} \tag{3.1.1}
\]

where the term \( \varepsilon g \) has been introduced to correspond to the speed of sound in the electrothermal gas; to differentiate from the speed of sound in air \( c_a \). Thus for \( KR \ll 1 \) and \( r \gg r_o \) \( |p| \) is directly proportional to \( \omega \) and inversely proportional to \( r \).
3.2 CYLINDRICAL SOURCE SOLUTION

Another case of interest is the radiation of sound from an infinite cylinder, that is, the length \( l \) of the cylinder is much greater than the wavelength of the sound being generated, or mathematically

\[ K l \gg 1 \]

Referring to Fig. 3.1, \( r_o \) represents the radial position of an infinite line source within a cylinder of radius \( R \), with cross sectional area \( A \).

**FIG. 3.1 MATHEMATICAL MODEL OF CYLINDRICAL SOURCE**
Of primary interest is the solution for the sound pressure field outside the cylinder \((r >> r_0)\), so that we may neglect the \(\theta\) dependence and focus our attention on the radial coordinate.

The Green's function applicable to this configuration is [6]

\[
g(r|r_0) = \frac{1}{4} H_o(Kr) J_o(Kr) \quad r > r_0
\]

To find the sound pressure we must integrate:

\[
p = \int \int \frac{1}{4} B H_o(Kr) J_o(Kr) \, dA_0
\]

In cylindrical coordinates

\[
dA_0 = r_0 \, d\theta \, dr_0
\]

thus

\[
p = \frac{1}{4} B H_o(Kr) \int_0^R \int_0^{2\pi} r_0 J_o(Kr) \, d\theta \, dr_0
\]

In the far field \((Kr >> 1)\) we can use the asymptotic approximation

\[
H_o(z) \to \sqrt{\frac{2}{\pi z}} e^{i (z-\pi/4)}
\]

The magnitude of the sound pressure will be

\[
|p| = \frac{B}{4} \left( \frac{2}{\pi K_a r_0} \right)^{1/2} \int_0^R \int_0^{2\pi} r_0 J_o(K_{ga} r_0) \, d\theta \, dr_0
\]

where we have made the distinction:

- \(K_a\) refers to propagation in air
- \(K_{ga}\) " " " " thermal gas
thus
\[ |p| = \frac{2\pi}{4} B \left( \frac{2}{\pi K a} r \right)^{1/2} R \int_0^R r_o J_o(Kr_o) \, dr_o \]

let
\[ I = \int_0^R r_o J_o(Kr_o) \, dr_o \]

if
\[ x = Kr_o \]

then
\[ I = \frac{1}{K^2} \int_0^{KR} x J_0(x) \, dx \quad (3.2.1) \]

Using the relation for Bessel functions
\[ \left( \frac{1}{z} \frac{d}{dz} \right)^K \{ z^\nu J_{\nu}(z) \} = z^{\nu-K} J_{\nu-K}(z) \quad (k = 0, 1, 2, \ldots) \]

(3.2.1) reduces to
\[ I = \frac{R}{K} J_1(KR) \]

so that
\[ |p| = \frac{2\pi}{4} B \left( \frac{2}{\pi K a} r \right)^{1/2} I \]
\[ = \frac{2\pi}{4} B \left( \frac{2}{\pi K a} r \right)^{1/2} \frac{R}{K} J_1(K R) \]
Substituting the expression for $B$ from (3.2) and expressing $K_a$ and $K_g$ in the form

$$K_a = \frac{\omega}{c_a}$$

$$K_g = \frac{\omega}{c_g}$$

the solution becomes

$$|p| = (\gamma - 1) \sigma \frac{V_{dc} V_{ac} R}{c g^2} \left(\frac{2\pi c_a}{\omega r}\right)^{1/2} J_1(\frac{\omega R}{c g})$$

(3.2.2)

For small values of $KR$ ($KR << 1$)

$$J_1(KR) \approx \frac{KR}{2}$$

so that

$$|p| \approx (\gamma - 1) \sigma \frac{V_{dc} V_{ac} R^2}{c g^2} \left(\frac{\pi c_a}{2}\right)^{1/2} \left(\frac{\omega}{r}\right)^{1/2}$$

(3.2.3)

Equation (3.2.3) is identical to the solution developed by Burchard [5] using an independent solution method. It must be noted that (3.2.3) is valid only for frequencies such that

$$KR << 1$$

For higher frequencies the more complex expression (3.2.2) must be used.

### 3.3 PLANE WAVE SOURCE SOLUTION

It is desirable to obtain a one dimensional solution to equation (3.1). Such a solution is useful when considering radiation
from a region of electrothermal activity confined by a cylindrical container open at one or both ends.

Consider an infinite cylindrical tube containing a gas in electrothermal activity in the region \{-L \leq x \leq L\} (see Fig. 3.2). The tube is infinite in length so that we need not consider reflections of sound waves back to the region.

![Fig. 3.2 Mathematical Model of Plane Wave Source](image)

According to Beranek [7] we can assume plane wave propagation in the tube provided

\[ \frac{\lambda}{d} > 2\pi \quad \text{or} \quad K_a < 1 \]

The one dimensional Green's function \( g(x|\xi) \) applicable to (3.7) is [6]

\[
g(x|\xi) = \frac{j}{2K} e^{jk|x-\xi|}
\]

where

\[
|x-\xi| = \begin{cases} 
  x-\xi & x > \xi \\
  -(x-\xi) & x < \xi
\end{cases}
\]
The sound pressure in the region $|x| > L$ is therefore given by

$$p = \int_{-L}^{L} B \frac{j}{2K} e^{jK|x-x_o|} dx_o$$

Consider only the sound pressure wave travelling in the positive $x$ direction ($x > L$), so that

$$p = \frac{jB}{2K} e^{jKx} \int_{-L}^{L} e^{-jKx_o} dx_o$$

The magnitude of the sound pressure will be

$$|p| = \frac{jB}{2K} \int_{-L}^{L} e^{-jKx_o} dx_o$$

$$= \frac{jB}{2K} \left( e^{-jKL} - e^{jKL} \right)$$

From trigonometry

$$\sin x = \frac{1}{2j} (e^{jx} - e^{-jx})$$

so that

$$|p| = \frac{B}{K^2} \sin KL$$

Substituting for $B$ from (3.2) we obtain

$$|p| = \frac{2 (\gamma - 1) \sigma V_{dc} V_{ac} \omega \sin KL}{c^2 g^2 K^2}$$
Referring to Fig. 3.2 we note that

\[ L = \frac{l}{2} \]

Also

\[ K = \frac{\omega}{c} \]

The total solution for \( p \) becomes

\[ p = \frac{(\gamma-1) \sigma V_{dc} V_{ac}}{c \ell} \left\{ \frac{\sin \frac{\omega l}{2c \ell}}{\frac{\omega l}{2c \ell}} \right\} e^{j(K_a x - \omega t)} \]

This represents a wave propagating in the positive \( x \) direction with amplitude

\[ |p| = \frac{(\gamma-1) \sigma V_{dc} V_{ac}}{c \ell} \left\{ \frac{\sin \frac{\omega l}{2c \ell}}{\frac{\omega l}{2c \ell}} \right\} \quad (3.3.1) \]

A similar wave will propagate in the negative \( x \) direction with the same amplitude.
CHAPTER IV

PHYSICAL MODELS OF SOURCES

In Chapter III theoretical relations were developed which applied to various geometrical configurations of electrothermal acoustic radiation. These particular relationships were developed specifically to aid in the design of practical electrothermal loudspeakers. At this point it is necessary to draw analogies between the theoretical cases described in Chapter III and physical systems which are described approximately by analytical models of these geometrical configurations.

It must be noted that the solutions of Chapter III are valid only if there is a perfect acoustical impedance match between the gas and the air. The hot gas and air impedances will not differ significantly in this discussion and the resultant mismatch amounts to an error of less than 1 dB [5]. It will also be necessary to assume that the gas velocity is much less than the speed of sound, which certainly applies to the following cases.

4.1 THE SIMPLE (ISOTROPIC) SOURCE

A simple source may be physically any electrothermal region radiating sound into unbounded space. To be classified as such, the wavelength of sound to be radiated must be much greater than the largest dimension of the source. The shape of the source is irrelevant and is assumed for analytical purposes to be spherical. Such a situation can
be approximated physically by a pair of electrodes separated by a very small distance (1 cm) and located in the hot region [10] of an open gas flame as illustrated in Fig. 4.1.

**FIG. 4.1 PHYSICAL MODEL OF SIMPLE SOURCE**
4.2 THE CYLINDRICAL SOURCE

The theory for the infinite cylindrical source developed in Chapter III is of interest because of the fact that two electrodes placed widely apart (5 - 10 cm) in a gas flame constitute a region of electrothermal activity having radiation properties similar to an infinite cylinder when the wavelength of sound is less than the length of the flame. Such a situation is illustrated in Fig. 4.2.

FIG. 4.2 PHYSICAL MODEL OF CYLINDRICAL SOURCE
4.3 THE PLANE WAVE SOURCE

The one dimensional case of section 3.3 in physical terms, requires the enclosing of the flame in a cylinder open at both ends. A more practical alternative is to enclose the flame in a tube open at one end and apply sound absorbing material at the other end to prevent reflections of acoustical waves propagating in the negative direction as illustrated in Fig. 4.3.

![Diagram of an enclosed flame physical model]

FIG. 4.3 ENCLOSED FLAME PHYSICAL MODEL

4.4 COMPARISON OF PHYSICAL MODELS - FOUNDATIONS OF DESIGN

The physical models described above represent three independent foundations for design and development of practical electrothermal loudspeakers. To proceed further in this direction it is necessary to compare the merits of each foundation.

The sound intensity from a simple source is generally very small
and drops off rapidly with frequency (see eq. 3.1.1). Despite these disadvantages, the simple source serves two functions:

1. The simple source is the best situation for research aimed at investigating the true nature of electrothermal acoustic radiation and thus in the verification of the modified wave equation (2.1.24).

2. The simple source serves as a building block for more complex sources such as arrays or planes.

The cylindrical source, because of its larger dimensions, can produce greater sound intensities which increase with frequency at the rate of 3 dB per octave (see eq. 3.2.3). Thus an open flame may serve effectively as a high frequency or "tweeter" loudspeaker in the range of 5 - 20 KHz and also as an ultrasonic transducer beyond 20 KHz. One quality of this loudspeaker that sets it aside from most conventional tweeters is the fact that the directivity pattern will be essentially non-directional in a plane perpendicular to the axis of the flame. The cylindrical source will also be used in the verification of the modified wave equation.

The principal advantage of the one-dimensional case is the fact that the flame is partially enclosed and therefore sheltered to a certain extent from outside disturbances such as air currents or drafts. Also, in this case, the acoustic energy may be directed into a region of space in which it is most desirable.

It will become apparent later that the main problem in electro-
thermal loudspeakers is disturbances in the atmosphere, particularly slight drafts which tend to upset the electrical discharge in the electrothermal region. For this reason alone it is evident that partial enclosure of the flame is a necessity. Henceforth, further discussion of the design of electrothermal loudspeakers will be restricted to partially enclosed flames. Open flames will be used only for verification of theoretical relationships.
CHAPTER V

BASIC PHYSICAL REQUIREMENTS

In physical terms, an electrothermal loudspeaker consists of a region of ionized gas upon which is imposed an electrical signal introduced into the region by a pair of electrodes. The electrical signal consists of a bias voltage and a modulated voltage supplied from any amplified audio program source through a coupling circuit. The first stage in the design procedure is to analyze, in detail, each of these basic requirements.

5.1 THE IONIZED GAS REGION

To respond to electrical signals, the gas region must possess a workable conductivity. Thus, the main problem is the establishment of a stable discharge within the gas region to render it conductive. It is extremely important that the electrothermal gas region is not completely isolated from the atmosphere by any solid container or membrane, in order to preserve its truly gaseous nature. This means that the ionized gas region must be essentially at atmospheric pressure and not in a vacuum or at low pressure.

One method of establishing a stable discharge is that used by Burchard [5] and discussed in detail by Frost [11]. This method requires the use of a very high temperature (3000° K) gas flame seeded with an alkali metal salt, resulting in a workable conductivity. This
conductivity is the result of thermal ionization of the seeding compound in the electrothermal region. Although this method is very useful for experimental studies and measurements, it is not the most desirable method for practical loudspeakers; the main disadvantage being the fact that such high temperatures are obtained only by use of oxy-acetylene torches. These flames require accurate regulation of the combustion gases as well as numerous safety precautions. This method is also undesirable as stated in the discussion which will follow, wherein the gas is partially contained to alter its sound reproduction characteristics. High temperature flames require containers capable of withstanding these high temperatures without heat dissipation, which might result in the lowering of the working temperature and loss of conductivity.

An alternative method is to use a lower temperature air-propane flame. Air-propane torches producing a temperature of the order of 1000° K are a common market item and are extremely safe and easy to operate. At lower temperatures, however, one can no longer depend on thermal excitation of the seeding compound to yield the required conductivity. Workable conductivities have been obtained by gas discharges in air-propane flames and, although the nature of the discharge is not fully understood, a rational assumption can be made. Temperatures of the order of 1000° K can cause thermionic emission in metallic electrodes. The emitted electrons can be attracted to an electrode at high positive potential. The acceleration of such
electrons causes further ionization of the gas molecules between the electrodes. It is still necessary to introduce a seeding compound to stabilize the discharge. This is apparently due to the cancellation of the space charge in the region of the cathode as a result of the ionization of the seeding compound [12]. The voltage - current characteristics of such a discharge have an on-off type of response. When the voltage between the electrodes is below a certain critical value, no current will flow. As the voltage is raised to a critical point, the current increases abruptly and will continue to flow until the voltage is reduced below a second critical cut-off level. Care must be taken that the bias voltage is high enough to maintain the discharge to avoid "cut-off" distortion in the loudspeaker.

5.2 THE PROBLEM OF SEEDING

By far, the greatest time spent in the development of this project was devoted to providing a manner in which the alkali metal salt could be transported steadily to the electrothermal region. This problem has received considerable interest in the study of flame photometry [13, 14, 15] where samples of various elements and compounds are analyzed in a high temperature gas flame. Fells and Harker [16] have weighed the advantages of various seeding methods. Upon reviewing this literature and making many tests, the best method of seeding was concluded to be the spraying or atomization of an aqueous solution of the seeding compound. The seeding compound is dissolved in water and aspi-
rated as a mist by use of a compressed air spray technique. The mist is then directed to the burner and mixed with the air entering the burner ports. The final design of the aspirator unit is illustrated in Figs. 5.1 and 5.2. The presence of water vapour in the fuel gases results in a slight lowering of the operating temperature. Also, alkali metal salt solutions are highly corrosive agents. For this reason the aspirator unit must be constructed of corrosive resistant materials such as copper, brass or plastic.

5.3 THE SEEDING COMPOUND

The seeding compound must be a salt of a metal which is easily vapourized and has a relatively low ionization potential. The alkali metals are ideal for this purpose. A table of the alkali metals and their respective ionization potentials is given in Table 5.1 [14].

<table>
<thead>
<tr>
<th>Metal</th>
<th>Ionization Potential (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium</td>
<td>5.37</td>
</tr>
<tr>
<td>Sodium</td>
<td>5.12</td>
</tr>
<tr>
<td>Potassium</td>
<td>4.32</td>
</tr>
<tr>
<td>Rubidium</td>
<td>4.16</td>
</tr>
<tr>
<td>Caesium</td>
<td>3.87</td>
</tr>
</tbody>
</table>

TABLE 5.1 IONIZATION POTENTIALS OF ALKALI METALS

It is obvious from Table 5.1 that salts of caesium are the most
COMPRESSED AIR AT 2 psi

ATOMIZER (PLASTIC)

COPPER VESSEL

COPPER PLUG TO BURNER

MIST

SALT SOLUTION OF ALKALI METAL IN H₂O

"ACTUAL SIZE"

FIG. 5.1 THE ASPIRATOR UNIT
FIG. 5.2 PHOTOGRAPHS OF ASPIRATOR UNIT
effective seeding compounds investigated. In the experiments, caesium chloride was used in solution strengths of 1 - 5%. Higher concentrations did not prove any more effective and left deposits which disturbed the operation of the burner. Unfortunately, caesium chloride is a rather costly compound. In practical devices, potassium salts should be used since they are much more economical and almost as effective as the caesium salts in reducing the space charge at the cathode.

5.4 THE ELECTRODES

As explained previously, the chief mechanism of conductivity is thermionic emission from the metallic cathode. Such emission is governed by the well-known Richardson equation:

\[ J = A T^2 \exp \left( - \frac{\phi}{kT} \right) \]

where

- \( J \) = current density of electron emission
- \( T \) = temperature (°K)
- \( \phi \) = work function of cathode material
- \( k \) = Boltzmann constant
- \( A \) = an empirical constant usually determined experimentally

To obtain high current densities, it is essential that the work function \( \phi \) be very low. Work functions for various metals have been tabulated in the literature [17]. As well as having a low work function,
the cathode must have a high melting point in order to withstand the temperatures of the flame. It was found that either pure tungsten or thoriated tungsten electrodes (0.06 inches in diameter) were excellent for this purpose.

5.5 THE COUPLING CIRCUIT

The bias voltage and the alternating audio voltage must be applied to the electrodes in such a manner so that each may be controlled separately (isolated sources). The following circuit was found useful for this purpose (see Fig. 5.3).

![Schematic Diagram of the Coupling Circuit]

FIG. 5.3 SCHEMATIC DIAGRAM OF THE COUPLING CIRCUIT
The program source must be amplified by an audio power amplifier and coupled to the electrodes by a step-up transformer, to match the output impedance of the power amplifier with the impedance of the flame.
CHAPTER VI

PARTIALLY ENCLOSED FLAME LOUDSPEAKER DESIGN

In this section, the discussion will be limited to the sound reproducing characteristics of flames enclosed in a solid cylindrical tube, open at one end. It will be assumed that the waves will propagate in the tube in the positive x direction only (see Fig. 4.3), with plane wave fronts. This reduces the analytical model to the one-dimensional case described mathematically in equation (3.3.1).

Because of the wide range of sound pressures encountered in practice, it will be convenient to express the magnitude of the sound pressure p in terms of the sound pressure level in decibels defined as

$$\text{SPL} = 20 \log_{10} \frac{P}{P_{\text{ref}}} \text{ dB}$$

where

$$P_{\text{ref}} = 0.0002 \text{ microbar} \ (2 \times 10^{-5} \text{ newton/m}^2)$$

In Fig. 6.1 a plot has been constructed of SPL vs. $\frac{\omega L}{2c g}$ according to equation (3.3.1). This plot (Fig. 6.1) is extremely important since it represents the ideal characteristics of the loudspeakers to be designed as well as the relations between the various design parameters.

Referring to Fig. 6.1 the following definitions must be made:

$$\text{SPL}_o = 20 \log_{10} \frac{P_o}{P_{\text{ref}}}$$
FIG. 6.1 CHARACTERISTICS OF IDEAL ELECTROTHERMAL LOUDSPEAKER
where (from 3.3.1)

\[ p_o = \frac{(\gamma-1) \sigma V_{dc} V_{ac}}{c g \lambda} \]

Also define \( f_{cu} \) where

\[ f_{cu} = \text{upper cut-off frequency of the frequency corresponding to the first value of} \ \frac{\omega L}{2cg} \ \text{when} \ \text{SPL} \to -\infty \]

From a general view of Fig. 6.1 the following conclusions can be drawn concerning the characteristics of the ideal loudspeaker:

1. There is a region of flat response SPL\(_o\) for \( \frac{\omega L}{2cg} < \pi \).
2. There is a rapid drop off to the first minimum at \( \frac{\omega L}{2cg} = \pi \).
3. For \( \frac{\omega L}{2cg} > \pi \) there are a series of minima and maxima of diminishing SPL.

Flat response is usually accepted as the best criteria for loudspeaker design so that the first design equation can now be derived. For flat response the loudspeaker should be operated in the region of Fig. 6.1 described mathematically by the inequality

\[ \frac{\omega L}{2cg} < \pi \]

or since \( \omega = 2\pi f \)

\[ \frac{f_{cu}}{c g} < 1 \]  \( (6.1) \)

Up till now, the discussion has been restricted to the propaga-
tion of sound waves within the cylindrical tube. In order to be detected by listeners at a distance from the loudspeaker, the sound must be radiated into the air. This introduces several new problems which will now be considered.

6.1 RADIATION OF SOUND FROM THE OPEN END OF THE TUBE

The problem of the radiation of sound from an unflanged circular pipe has been discussed by Levine and Schwingr [18]. Beranek [7] has simplified this discussion by plotting the acoustic impedance of the air load at the end of the pipe for various values of Ka where a is the radius of the tube. From his plot one can conclude that for Ka > 1, the radiation impedance is approximately equal to the characteristic impedance of air. This matching of impedances is desirable in order to insure maximum radiation of sound without reflections at the surface where the end of the pipe opens to the atmosphere. This determines the second important relation, that is

\[ Ka > 1 \]  

(6.1.1)

6.2 REQUIREMENTS FOR PLANE WAVE PROPAGATION

Until now, it has been assumed that the sound waves propagate with plane wave fronts in the tube. For this to be true, Beranek [7] has stated that the ratio of the wavelength of sound to the diameter of the tube must be greater than about 6. This is essentially the same as saying that

\[ Ka < 1 \]  

(6.2.1)
When $Ka > 1$ resonances will occur within the tube. This three-dimensional problem is discussed briefly in Appendix B.

Note that equations (6.1.1) and (6.2.1) contradict one another. To overcome this difficulty it is necessary to introduce some type of acoustical impedance transformer at the end of the tube to modify the restriction imposed in (6.1.1).

6.3 ACOUSTICAL IMPEDANCE TRANSFORMERS

It is possible to transform the characteristic impedance of a small diameter tube to the characteristic impedance of air by using a horn as an acoustical transformer. It is not the purpose of this thesis to give a detailed analysis of radiation of sound from horn loudspeakers. Much literature has been devoted to this topic. It will be sufficient here to say that the horns used in the design were exponential horns, that is, the cross-sectional area expanded exponentially with $x$ as illustrated in Fig. 6.2.

![Diagram of exponential horn]

**FIG. 6.2 HORN WITH EXPONENTIAL CROSS-SECTION**
The differential equation for sound propagation in an exponential horn and the general solution to this equation are respectively [7]

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial p}{\partial x} - \frac{c^2}{\partial x^2} \frac{\partial^2 p}{\partial x^2} = 0$$

$$p = p_+ e^{-mx/2} e^{-j\sqrt{4k^2 - m^2/2}} e^{j\omega t}$$  \hspace{1cm} (6.3.1)

where $m$ is the flare constant which is a measure of how rapidly the cross section expands with $x$ and $p_+$ is the sound pressure at the throat of the horn.

There are two important factors to be drawn from (6.3.1):

1. The sound pressure is attenuated by a factor $e^{-mx/2}$ or the SPL is lowered by $20 \log_{10} e^{-mx/2}$.

2. There exists a lower cut-off frequency below which the impedance of the horn is purely reactive and no sound is radiated. This frequency is given by [7]

$$f_{c\ell} = \frac{mc}{4\pi}$$  \hspace{1cm} (6.3.2)

With the addition of the horn, (6.1.1) can be replaced by

$$KA > 1$$  \hspace{1cm} (6.3.3)

where $A$ is the radius of the mouth of the horn. The horn need not be circular and, in fact, square or rectangular horns are more easily constructed. In these cases $A$ will represent approximately one-half of the dimension of the mouth of the horn.
6.4 DESIGN PROCEDURE

With the design equations available it is now possible to design and construct loudspeakers by using the procedure below. This procedure should be used only as a guideline in loudspeaker construction. As in the design of musical instruments, the craftsman is at liberty to make modifications resulting in best performance.

1. Decide on the upper and lower cut-off frequencies of the loudspeaker to be constructed \( (f_{cu} \text{ and } f_{cl}) \).

2. Using equation (6.2.1) determine the maximum diameter of the tube.

3. Using equation (6.3.3) decide on the mouth diameter for the horn.

4. Using equation (6.3.2) determine the flare constant \( m \) for the horn.

5. Using equation (6.1) determine the distance required between the electrodes assuming that \( c_g \) is fixed. \( c_g \) is a function of the temperature of the electrothermal gas and must be known beforehand (see Appendix A). If the electrodes are too close together, undesirable arc discharge may result.

The design must always be compatible with the burner and flame used. This may set additional restrictions on the diameter of the cylinder enclosing the flame. In such cases, compromises must be made.
There are also limitations regarding bandwidth of flat response which cannot be stated easily in mathematical equations and are best observed in the measurements of the next chapter. As a general rule speakers designed for lower frequency reproduction will have undesirable resonances at higher frequencies.

6.5 DESCRIPTION OF LOUDSPEAKERS CONSTRUCTED

To best illustrate the problems and compromises required in the design of electrothermal horn loudspeakers, two such loudspeakers were constructed. The first loudspeaker was designed using the procedure suggested in Sec. 6.4 and was intended primarily to serve as a "tweeter" or high frequency loudspeaker. The second loudspeaker was constructed purposely to show the bandwidth limitations when lower frequencies are to be radiated.

The relevant design specifications and parameters for the experimental loudspeakers are tabulated in Table 6.1. The actual construction of the loudspeakers is illustrated in Figures 6.3, 6.4, 6.5 and 6.6.
<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Loudspeaker No. 1</th>
<th>Loudspeaker No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{cu}$</td>
<td>12 KHz</td>
<td>12 KHz</td>
</tr>
<tr>
<td>$f_{cl}$</td>
<td>600 Hz</td>
<td>200 Hz</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>5 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>$a$</td>
<td>0.5 cm</td>
<td>2.5 cm</td>
</tr>
<tr>
<td>$A$</td>
<td>7.5 cm</td>
<td>15 cm</td>
</tr>
<tr>
<td>$m$</td>
<td>0.22 cm$^{-1}$</td>
<td>0.06 cm$^{-1}$</td>
</tr>
<tr>
<td>$c_e$ (see Appendix A)</td>
<td>580 m/sec</td>
<td>580 m/sec</td>
</tr>
</tbody>
</table>

**TABLE 6.1** PARAMETERS OF EXPERIMENTAL LOUDSPEAKERS
FIG. 6.3 DIAGRAM OF LOUDSPEAKER NO. 1
FIG. 6.4 PHOTOGRAPH OF LOUDSPEAKER NO. 1
FIG. 6.5 DIAGRAM OF LOUDSPEAKER NO. 2
FIG. 6.6 PHOTOGRAPH OF LOUDSPEAKER NO. 2
Acoustical measurements of sound pressure levels were made for two purposes:

1. To verify the basic theory of electrothermal acoustical radiation.

2. To determine the true response of the experimental loudspeaker and thus the validity of the design equations.

Acoustical measurements were made in a large reverberant room using a Brüel and Kjaer precision sound level meter type 2203 in conjunction with an octave filter set type 1613. The microphone was located close to the source in each case so that only direct sound waves from the source were measured and excluding waves reflected from walls and other objects in the room. Furthermore, care was taken to prevent the formation of standing waves in the room by introducing irregularly shaped objects near flat walls. Lastly, sound levels were always generated above the noise level of the flame jet and corrections to SPL because of this noise were made using a correction table [7] when applicable. The measurement set-up is illustrated in Figures 7.1 and 7.2.

7.1 MEASUREMENTS TO VERIFY THEORY

Measurements were made using the practical simple and cylin-
FIG. 7.1 BLOCK DIAGRAM OF MEASUREMENT SET-UP
FIG. 7.2 PHOTOGRAPH OF MEASUREMENT SET-UP
drical sources described in Chapter IV in order to verify the validity of equations (3.1.1) and (3.2.3) respectively. Relevant recorded information is tabulated in Table 7.1.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Simple Source</th>
<th>Cylindrical Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>1 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>r</td>
<td>30 cm</td>
<td>30 cm</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>500 V</td>
<td>500 V</td>
</tr>
<tr>
<td>$V_{ac}$</td>
<td>100 Vrms</td>
<td>100 Vrms</td>
</tr>
<tr>
<td>I</td>
<td>50 ma</td>
<td>20 ma</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.013 mho/m</td>
<td>0.025 mho/m</td>
</tr>
<tr>
<td>R</td>
<td>0.5 cm</td>
<td>0.5 cm</td>
</tr>
</tbody>
</table>

TABLE 7.1 MEASUREMENT DATA

In each case $\sigma$ was deduced from the relation

$$\sigma = \frac{l}{ZA}$$

where

- $l$ = distance between electrodes
- $Z$ = flame impedance (electrical)
- $A$ = cross-sectional area of flame $\approx \pi R^2$

also $\gamma = 1.4$ for gases with diatomic molecules (which is approximately true for the gas being considered).

In Figures 7.3 and 7.4 the results of these measurements have
been plotted along with the theoretical plots deduced from equations (3.1.1) and (3.2.3) using the data of Table 7.1. Note that there is considerable scattering of data which can be attributed mainly to disturbances of the flame due to air currents and drafts in the room. The data does however show the general trend expected in the theory.

7.2 MEASUREMENTS OF THE RESPONSE OF THE EXPERIMENTAL LOUDSPEAKERS

Frequency response measurements of SPL were made in order to verify the characteristics of the electrothermal loudspeakers designed in Sec. 6.5. Using the measurement set-up of Fig. 7.1, the following information was recorded in Table 7.2:

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Loudspeaker No. 1</th>
<th>Loudspeaker No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{dc}$</td>
<td>600 V</td>
<td>700 V</td>
</tr>
<tr>
<td>$V_{ac}$</td>
<td>100 Vrms</td>
<td>200 Vrms</td>
</tr>
<tr>
<td>$L$</td>
<td>5 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>$I$</td>
<td>5 ma</td>
<td>25 ma</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.001 mho/m</td>
<td>0.001 mho/m</td>
</tr>
</tbody>
</table>

TABLE 7.2 MEASUREMENT DATA FOR EXPERIMENTAL LOUDSPEAKERS

The determination of $\sigma$ in this case is rather complicated because the electric field is no longer linear due to the nature of the electrodes. The values given above are rough approximations.

The microphone was located at the mouth of the horn of each
FIG. 7.3 FREQUENCY RESPONSE OF SIMPLE SOURCE
FIG. 7.4 FREQUENCY RESPONSE OF CYLINDRICAL SOURCE
In Figures 7.5 and 7.6 the results of these measurements are plotted along with the expected theoretical responses calculated by substituting the data of Table 7.2 into equation (3.3.1) and taking into consideration the reduction of SPL in the horn (see Sec. 6.3).

Referring to Figures 7.5 and 7.6, the following conclusions can be made:

1. The measured response of the first loudspeaker agrees well with the theoretical response.
2. In the measured response of the second loudspeaker there are three major inconsistencies;
   (a) There is a resonance in the low frequency range which is likely due to constructive interference of acoustic waves reflected back into the electrothermal region from the mouth of the horn.
   (b) Beyond 7000 Hz there are a series of resonant peaks. This is a result of the tube having a relatively large diameter thus violating equation (6.2.1) (see Appendix B).
   (c) The region of flat response has an SPL which is 10 dB lower than expected. This is most likely due to losses caused by viscosity and thermal diffusion as a result of enclosing the flame in the copper tube.

The characteristics of the second loudspeaker show that it might
FIG. 7.5 FREQUENCY RESPONSE OF LOUDSPEAKER NO. 1
FIG. 7.6 FREQUENCY RESPONSE OF LOUDSPEAKER NO. 2
possibly be useful as a public address loudspeaker for frequencies in the vocal range (0.5 to 5 KHz).

7.3 PSYCHOACOUSTIC TESTS

Since loudspeakers are designed primarily to be listened to by human ears it is obvious that no evaluation of the experimental results is complete unless the loudspeaker has been exposed to human subjects for analysis. There is a psychological quality of people which causes different persons to have different opinions about the same loudspeaker. It is important, therefore, to use a variety of people in such tests and look only for trends in their remarks which may then be used as data in psychoacoustic tests.

Seven subjects, both male and female, and of various ages were selected for the psychoacoustic tests. Each subject listened to a variety of types of music played through the experimental loudspeaker No. 2 and was asked to fill in the questionnaire which is given below.

ELECTROTHERMAL LOUDSPEAKER

SUBJECTIVE LISTENING TEST

Subject: Age: Occupation:

1. What is your first impression of the loudspeaker?

2. Describe, in your own words, the quality of the sound that you hear.

3. Additional comments (if any).
The results of this test are summarized below:

1. First Impressions

Words used to express the first impressions of the listeners included;

(a) different
(b) unique
(c) not practical
(d) unconventional
(e) amazing

2. Quality of Sound

(a) All subjects noticed the absence of the very low frequencies usually heard in conventional sound systems.
(b) Most of the subjects noticed that the noise level (flame jet) was disturbing.
(c) Most of the subjects agreed that the loudspeaker reproduced human voices very well.
(d) Some of the subjects noticed that high frequencies such as sounds of bells were reproduced very well.

3. Additional Comments

Only two listeners gave additional comments which were;

(a) "This speaker should eventually eliminate the present type of loudspeaker."
(b) "It sounds like a 1 1/2" speaker being slightly overdriven."

The reader is left to draw his own conclusions regarding the results of the psychoacoustic tests.
CHAPTER VIII

ADDITIONAL CONSIDERATIONS

8.1 DISTORTION

The most serious distortion in the experimental loudspeakers is harmonic distortion which was estimated to be in excess of 25%. The reason for this distortion is expressed mathematically in equation (2.2.2). Ideally the condition $V_{dc} >> V_{ac}$ is necessary to avoid harmonic distortion but it was found in practice that when this requirement was fulfilled the sound from the speaker was barely audible over the flame jet noise. The exponential horn itself will introduce additional harmonic distortion [7].

Another type of distortion, peculiar to electrothermal loudspeakers, can be termed cut-off distortion. This results from disturbances in the thermionic discharge and causes annoying fluctuations in sound level.

8.2 FLAME NOISE

Detailed studies on noise from open flames have been done by Smith and Kilham [19]. Flame noise is a function of burner geometry, turbulence, burning velocity as well as other related variables. Noise spectra were determined experimentally for the flames used in the loudspeakers and are presented in Fig. 8.1. From these plots it can be
FIG. 8.1 SPECTRA OF FLAME NOISE
concluded that flame noise is a serious problem in electrothermal loudspeaker design.

8.3 EFFICIENCY AND DURABILITY

The efficiency of an electrothermal loudspeaker can be defined as

\[
\eta = \frac{P_{\text{acoustic}}}{P_{\text{electrical}}} \times 100\%
\]

where

\[
P_{\text{electrical}} = \text{electrical power input}
\]

\[
P_{\text{acoustic}} = \text{acoustic power output} = IS
\]

where \(I\) is the sound intensity in watts/m\(^2\) and \(S\) is the surface area of the acoustic power radiator. The intensity can be expressed as

\[
I = \frac{P^2}{\rho_0 c} \text{ watts/m}^2
\]

Therefore by knowing the electrical power input and the sound pressure level at the mouth of the radiator, the efficiency may be calculated. Calculations for the designed loudspeakers show that the efficiency is very low (less than 0.1%).

The electrothermal loudspeakers designed required considerable maintenance. The electrodes were oxidized slowly and had to be replaced after several hours of operation. The flame consumed the contents of a 14 oz. propane cylinder in approximately 10 hours of oper-
ation and in a similar period of time it consumed approximately 80 cu. ft. of compressed air, and 10 grams of caesium chloride.
CHAPTER IX

CONCLUSIONS

The theory and design of electrothermal loudspeakers have been dealt with in many aspects. It has been shown that theory derived from basic physical, electrical and thermodynamic principles may be used effectively to predict the characteristics of electrothermal loudspeakers, and therefore, to aid in the design of such devices.

As previously stated, the heat energy of the flame represents an enormous resource of potential acoustic energy if modulated efficiently. Unfortunately, no pyroacoustic amplification was detected in the experiments and as a result the loudspeakers do not approach the degree of efficiency and feasibility of conventional electro-mechanical loudspeakers. This does not mean that amplification is an impossibility and future studies leading to amplification in electrothermal regions would certainly awaken interest in this type of loudspeaker.

In conclusion, it is hoped that this work has touched on many subjects which may encourage future development towards an "ideal" loudspeaker.
The following method was devised to measure $c_g$, the speed of sound in the electrothermal region. The flame is enclosed in a rigid metallic cylindrical pipe as shown in Fig. A.1.

The bottom plate of the cylinder is a heavy metallic material which readily reflects sound waves. Thus when sound waves are introduced into the cylinder by thermoelectric means, a resonance will exist whenever

$$\frac{\lambda}{2} = L \quad (A.1)$$

The frequency of resonance can be detected by varying the frequency of a sinusoidal generator and observing the maximum SPL on an SPL meter.
For the flame used in the experiments of Chapter VII, the following data was recorded:

Resonant frequency = 2300 Hz, \( L = 12.5 \) cm

thus from (A.1) it follows that

\[ \lambda = 25 \text{ cm} \]

Now

\[ c_g = \lambda f = 580 \text{ m/sec} \]

The speed of sound is a function of temperature given by [7]

\[ c_g = 331.4 \sqrt{\frac{T}{273}} \]

Thus

\[ T = 840^\circ K \]

It must be noted that \( c_g \) and \( T \) are not uniform throughout the electrothermal region and the above results are the average or "effective" values.
APPENDIX B

RESONANCE WITHIN THE TUBE

For a tube of radius \( a \), resonances will occur whenever

\[
J_1(Ka) = 0 \quad (B.1)
\]

As a result, there will be maxima in sound pressure on the axis of the cylinder whenever \( Ka = \beta_n \) where \( \beta_n \) are the eigenvalues or roots of (B.1). Furthermore there will be minima on the axis whenever \( Ka = \alpha_n \) where \( \alpha_n \) are the roots of the equation \( J_0(Ka) = 0 \).
BIBLIOGRAPHY


