THE UNIVERSITY OF MANITOBA

QUALITY OF RESPONSE FOR SYSTEMS INVOLVING TIME DELAY

by

DAVID FESENG LIANG

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ABSTRACT

This thesis deals with the quality of response of linear control systems involving time delay.

An extensive study, carried out to find the correlation of quality of step response with frequency response and with open loop root location is described. This study reveals that the phase margin fails to be a measure of relative stability and quality of response for some systems involving time delay; instead, gain margin may be used to serve the purpose.

The estimates of the quality of response for a large class of systems, based upon the principal pair of closed loop complex roots, have been found to be good. These estimates are especially good when an open loop pole is located at the origin.

The effects of delay approximands on the quality of response and the stability limit are investigated.

A design technique based upon gain margin is developed to design systems involving time delay. It is particularly useful for the design of linear control systems involving time delay to which the long-established design technique is simply not applicable.

ACKNOWLEDGEMENT

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CHAPTER I

GENERAL INTRODUCTION

1.1 INTRODUCTION

In feedback control systems, especially in the field of process control, one often encounters transportation delay; such delay may arise as a result of the time consumed while a system component travels for a finite time from one point in space to another.

In recent years, systems involving time delay have received considerable attention, but many of the studies have been devoted to the mathematical discussion of the differential-difference equation (3)#. The characteristic equations which describe systems with delay are transcendental in nature; the solutions are usually difficult to obtain and time consuming in computation. In fact, a whole book devoted to these equations has been written by Bellman and Cooke(2).

However, a designer is often more interested in obtaining a quick estimate of system response characteristics by an analysis that does not require the actual solution of the differential-difference equations.

Wright (26) has dealt with the effect of time delay simulation methods upon the stability. Elgerd and

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Numbers in parentheses refer to items in the Bibliography.

Stephens(12) have dealt with the effect of closed loop transfer function pole and zero locations on the transient response of linear control systems without delay.

In this thesis, the commonly used methods of root locus analysis and frequency analysis have been applied to various types of linear control systems involving time delay. The analysis and experimental results presented in Chapters II and III permit an estimate of the quality of response of some delay systems to be made once the parameter values are established, and in some cases, it enables one to realize practical system components when some of the desired performance characteristics and system parameter values are known.

In Chapter IV, the quality of response and the stability limit of various systems involving time delay using different types of delay approximands are compared with those of the actual delay.

The results presented in Chapters II and III are used in Chapter V to develop a design technique for realizing linear control systems using practical components which will satisfy the desired performance criteria, and to indicate the manner or method by which the systems must be adjusted or compensated to produce the desired performance characteristics.

1.2 MATHEMATICAL TIME DELAY APPROXIMATION

In the study of time delay control systems many different approximations have been developed. One of the most commonly used methods involves the polynomial approximation technique in which the Laplace Transform of the transportation delay is approximated by the rational polynomial fraction

$$e^{-sT} = \frac{b_0 s^n + b_1 s^{n-1} + \ldots\ldots + b_n}{s^n + a_1 s^{n-1} + \ldots\ldots\ldots + a_n} \qquad 1.1$$

(A) PADE APPROXIMAND

Expansion of the transfer function in a Maclaurin series produces

$$e^{-sT} = 1 - Ts + \frac{(Ts)^2}{2!} - \frac{(Ts)^3}{3!} + \frac{(Ts)^4}{4!} \cdots \qquad 1.2$$

Pade (19) approximated this well-known power series by defining two polynomials $N_{u,v}(Ts)$ and $D_{u,v}(Ts)$ such that

$$e^{-sT} = \lim_{\substack{u \to \infty \\ v \to \infty}} \frac{N_{u,v}(Ts)}{D_{u,v}(Ts)} \qquad 1.3$$

where $N_{u,v}(Ts) = 1 - \frac{u(Ts)}{(u+v)1!} + \frac{u(u-1)(Ts)^2}{(u+v)(u+v-1)2!}$

$$-\cdots + \frac{(-1)^u u(u-1)\cdots 2\cdot 1 (Ts)^u}{(u+v)(u+v-1)\cdots(v+1)u!} \qquad 1.4$$

and $D_{u,v}(Ts) = 1 - \frac{v(Ts)}{(u+v)1!} + \frac{v(v-1)(Ts)^2}{(u+v)(u+v-1)2!}$

$$-\cdots + \frac{v(v-1)\cdots 2\cdot 1(Ts)^v}{(u+v)(u+v-1)\cdots(u+1)v!} \qquad 1.5$$

Normal types of Pade approximations are obtained by setting u=v=n, where n is the order of approximation. The first three of these are

$$P_1(s) = \frac{2 - Ts}{2 + Ts} \tag{1.6}$$

$$P_2(s) = \frac{12 - 6Ts + T^2s^2}{12 + 6Ts + T^2s^2} \tag{1.7}$$

and

$$P_3(s) = \frac{120 - 60Ts + 12T^2s^2 - T^3s^3}{120 + 60Ts + 12T^2s^2 + T^3s^3} \tag{1.8}$$

The power series obtained from the division of the above fraction does not deviate from Eq.1.2 for a number of terms which depends on the value of n. In general, for higher values of n and smaller values of ωT, a better approximation is obtained.

(B) CASCADED LAG APPROXIMATION

The transfer function of transportation delay may also be expressed as

$$e^{-sT} = \lim_{n\to\infty} \frac{1}{(1+\frac{T}{n}s)^n} \tag{1.9}$$

and approximated by limiting n to some finite value. In particular, if n is taken to be one and two, then the first order cascaded lag is

$$C_1(Ts) = \frac{1}{(1 + Ts)} \tag{1.10}$$

and the second order cascaded lag is

$$C_2(Ts) = \frac{1}{(1+\frac{Ts}{2})^2} \qquad 1.11$$

The degree of accuracy of this approximation is again dependent upon the values of n and Ts.

For a given frequency ω, Pade one and second order cascaded lag approximands contribute the same amount of phase lag, viz:

$$\theta(\omega) = -2\tan^{-1}\frac{\omega T}{2} \qquad 1.12$$

At this frequency, the n-th order cascaded lag causes a gain attenuation of $1/(1+(\omega T/n)^2)^{n/2}$ but with any order of Pade approximand there is no gain attenuation at any frequency. As a result, the Pade approximands are advantageous over cascaded lags.

1.3 METHOD OF DYNAMIC SIMULATION

(A) ANALOG SIMULATION

Pade delay circuits

Analog circuits for Pade delay are obtained in Reference (7,14) and also suggested in (15). In general, the approximands to e^{-sT} may be written as

$$e^{-sT} = \frac{ABC....N - ABC....(N-1)s + ABC....(N-2)s^2 - ...As^{n-1} + s^n}{ABC....N + ABC....(N-1)s + ABC....(N-2)s^2 + ...As^{n-1} + s^n} \qquad 1.13$$

where A,B,C,...N are gain coefficients.

The simulation diagrams for the first ,second and n-th order Pade approximands are shown in Figures 1.1, 1.2 and 1.3, respectively.

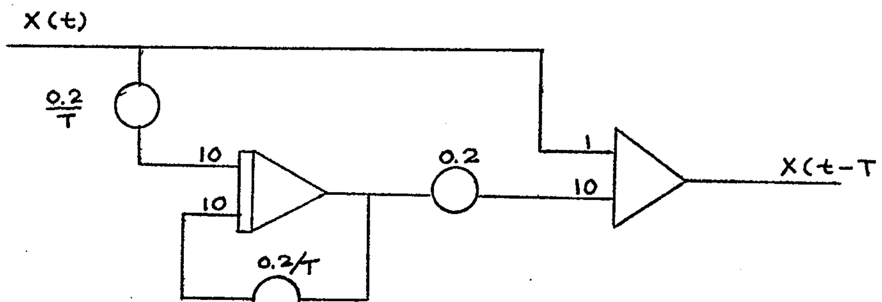


FIGURE 1.1 Diagram of first order Pade Approximand.

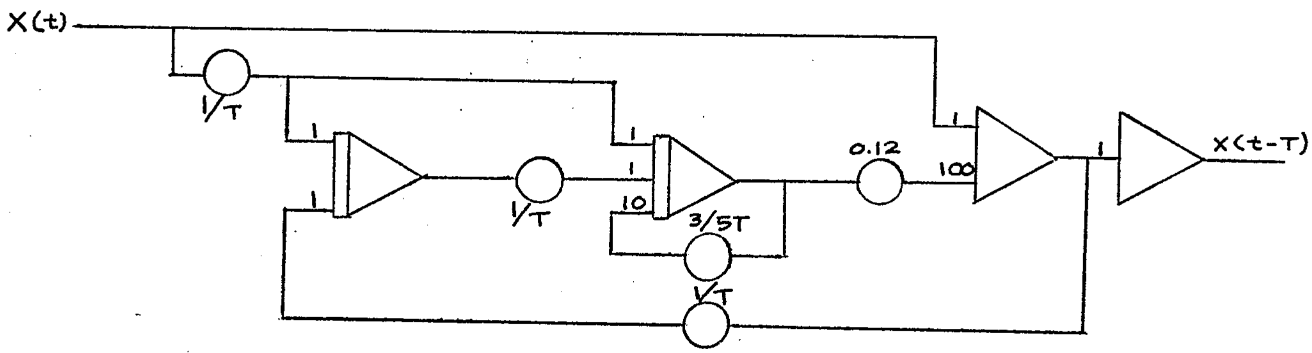


FIGURE 1.2 Diagram of second order Pade Approximand.

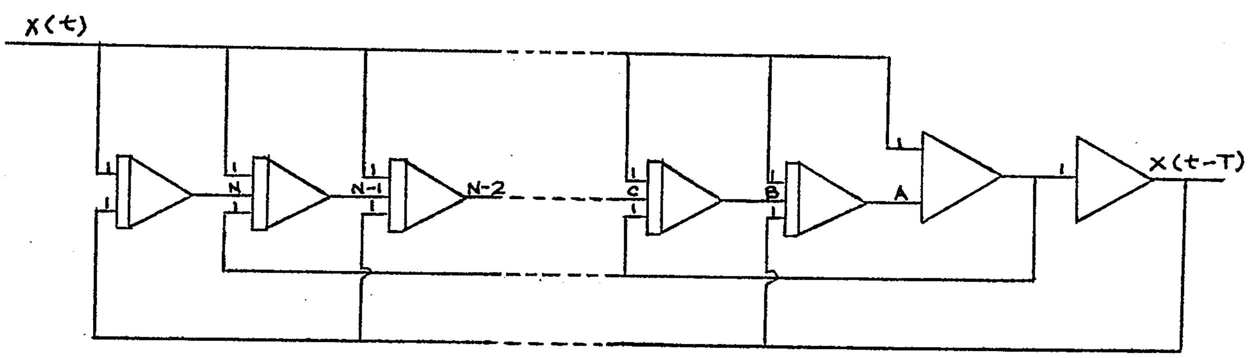


FIGURE 1.3 Diagram of n-th. order Pade Approximand.
(See reference 15)

Cascaded lag circuits

In general, the transfer function of the approximand is

$$C_n(s) = \frac{1}{\left(1 + \frac{sT}{n}\right)^n} \qquad 1.14$$

for which the computer circuit is shown in Figure 1.4.

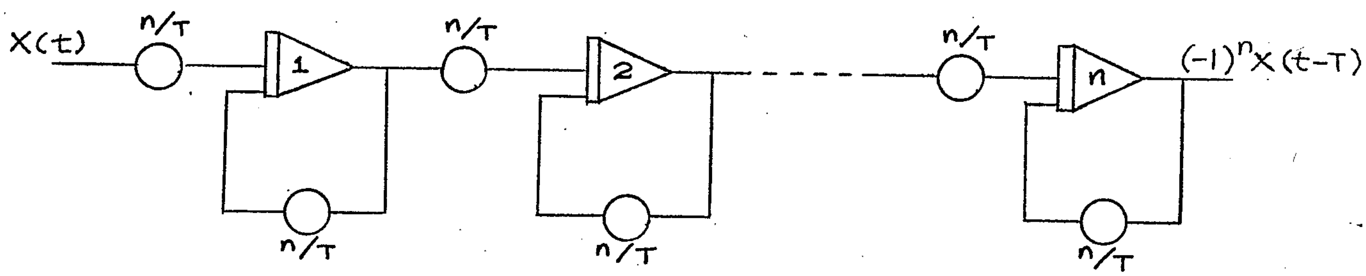


FIGURE 1.4 Diagram of nth. order cascaded lag Approximand.

(B) DIGITAL SIMULATION

The advancement of digital computing techniques has made various types of transfer functions including that of delay directly available in the continuous system modelling programme, i.e. C.S.M.P.(22).

Though the cost of digital progamming is probably higher than analog or hybrid simulation, simulation of time delay control systems using C.S.M.P. is rather simple, and the dynamic responses can be more accurately obtained and recorded. Therefore, the digital simulation method is used throughout most of the study in this thesis, especially when a high degree of accuracy is required.

Consider the rational transfer function

$$W(s) = \frac{N(s)}{D(s)} = \frac{b_o s^n + b_1 s^{n-1} + \ldots + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n} = \frac{Y(s)}{R(s)} \qquad 1.15$$

In order to eliminate the derivatives in the control force r(t), the transformation technique outlined in Athans and Falb (1) may be used. First define a state vector $\underline{x(t)}$ with components $x_1(t), x_2(t), \ldots x_n(t)$, such that

$$\begin{aligned}
x_1 &= y - b_o r \\
x_2 &= \dot{x}_1 - h_1 r \\
x_3 &= \dot{x}_2 - h_2 r \\
&\cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \\
x_n &= \dot{x}_{n-1} - h_{n-1} r
\end{aligned} \qquad 1.16$$

where

$$\begin{aligned}
h_1 &= (b_1 - a_1 b_o) \\
h_2 &= (b_2 - a_2 b_o) - a_1 h_1 \\
h_3 &= (b_3 - a_3 b_o) - a_2 h_1 - a_1 h_2 \\
&\cdot \\
&\cdot \\
&\cdot \\
h_n &= (b_n - a_n b_o) - a_{n-1} h_1 - a_{n-2} h_2 \\
&\quad - \ldots - a_2 h_{n-2} - a_1 h_{n-1}
\end{aligned} \qquad 1.17$$

The state equations become

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -a_n & -a_{n-1} & -a_{n-2} & \cdot & \cdot & \cdot & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \cdot \\ \cdot \\ \cdot \\ h_n \end{bmatrix} r \qquad 1.18
$$

and the system output is given by

$$y = x_1 + b_0 r$$

With these state equations, any type of delay approximand can be simulated without differentiation. A continuous system modelling programme for simulating a second or third order system is given in Appendix I for various types of delay approximands.

1.4 NORMALIZATION

In this thesis, many types of linear control systems involving time delay were investigated. To make the analysis applicable to any system independent of the speed of response and dependent on a minimum number of parameters, the well-known Laplace Transformation relation

$$L[f(\tfrac{t}{a})] = aL[f(t)] \qquad 1.19$$

was used to normalize time and thereby reduce the number of parameters by one ,by suitable choice of a. If the radial distance from the origin to the root of the system characteristic equation in the s plane is reduced by a factor 1/a, the corresponding speed of response is increased by a factor a.

Consider a general second order system involving time delay; the transfer function is represented by

$$\frac{C(S)}{R(S)} = \frac{K}{S(S+A)} e^{-S\tau} \qquad 1.20$$

Eq. 1.20 may be written as

$$\frac{C(S)}{R(S)} = \frac{K/A^2}{\frac{S}{A}(1+\frac{S}{A})} e^{-S\tau} \qquad 1.21$$

which with the following substitutions

$$\frac{S}{A} = s$$

$$\frac{K}{A^2} = k \qquad 1.22$$

and $A\tau = T$

becomes

$$\frac{c(s)}{r(s)} = \frac{k\ e^{-sT}}{s(s+1)} \qquad 1.23$$

Thus the equation with three running parameters A, τ and K has been reduced to a new equation with T and k as the running parameters.

Therefore a whole set of second order systems satisfying the relation $a\tau=T$ may be assessed by consulting the performance characteristics of Eq. 1.23 having T as its delay time and K/A^2 as its forward gain, the response time being increased by a factor of A, and the root locations reduced by a factor of 1/A.

Similarly, systems such as

$$\frac{Ke^{-S}}{(S+A)}, \quad \frac{Ke^{-S\tau}}{(S^2+2A\omega_n S+\omega_n^2)} \quad \text{and} \quad \frac{Ke^{-S\tau}}{S(S^2+2A\omega_n S+\omega_n^2)}$$

may be normalized to give

$$\frac{K/A}{(s+1)}\,e^{-(\tau A)s}, \quad \frac{(K/\omega_n^2)\,e^{-(\tau\omega_n)s}}{(s^2+2As+1)} \quad \text{and} \quad \frac{(K/\omega_n^3)\,e^{-(\tau\omega n)s}}{s(s^2+2As+1)}$$

respectively, with response time again being increased in the ratio of 1 to A.

CHAPTER II

FREQUENCY ANALYSIS

II.1 INTRODUCTION

Frequency analysis consists of a number of analytical and graphical procedures based on the sinusoidal steady state response of a system. Amongst other things it provides information about the relative stability of the system.

In 1932, Nyquist (18) developed his frequency domain stability criterion; it and its subsequent generalization (9) continue to be among the most useful techniques in the investigation of the system stability.

In particular, it has been found that the frequency analysis method is particularly useful in the determination of stability characteristics of systems involving time delay. The delay operator e^{-sT} has no pole or zero in the finite s-plane, and so only contributes a phase shift proportional to the frequency to the response function of the system.

Consider the single loop feedback control system shown below, where G(s) and H(s) are the transfer functions of the forward and feedback paths, respectively.

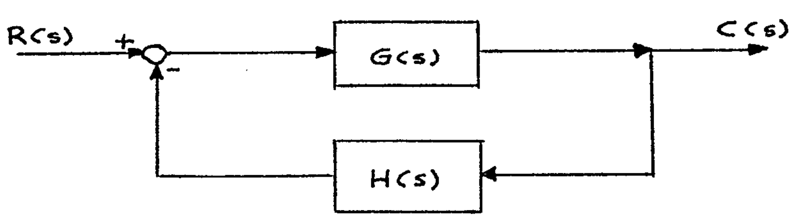


Figure 2.1 Block diagram of a single loop feedback control system.

The Nyquist stability Criterion states that a feedback control system is stable if, and only if, the contour $G(j\omega)H(j\omega)$ does not encircle the (-1,0) point in the $G(j\omega)H(j\omega)$ plane, where $G(s)H(s)$ has no pole in the right-hand s plane.

The phase crossover frequency, ω_m, is defined as the frequency for which the phase angle of $G(j\omega)H(j\omega)$ is 180^0.

The gain margin, G_m, is mathematically defined as

$$G_m = \left| \frac{1}{G(j\omega_m)\ H(j\omega_m)} \right| \qquad 2.1$$

The gain crossover frequency, ω_c, is the frequency at which the gain magnitude is unity.

The phase margin, Φ_m, is defined as

$$\Phi_m = 180^0 - \arg\left[G(j\omega_c)H(j\omega_c)\right]_{\text{degree}} \qquad 2.2$$

These quantities are illustrated in Figure 2.2.

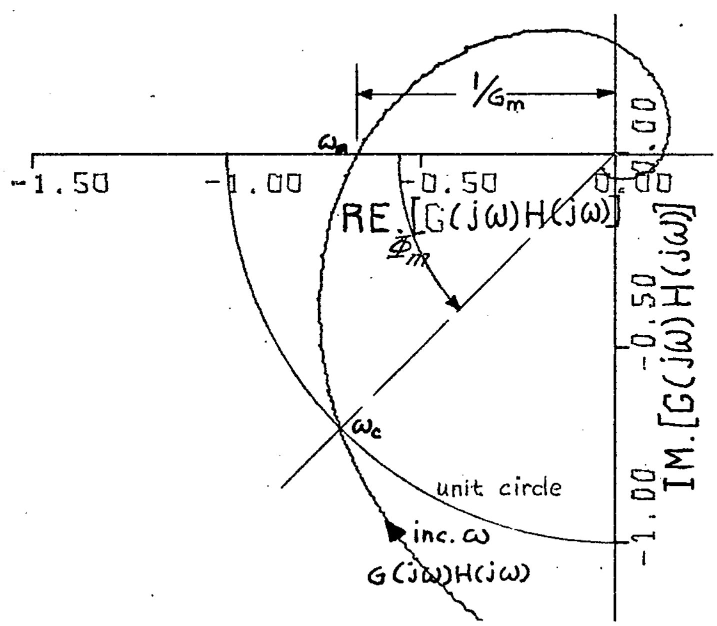


FIGURE 2.2 Nyquist diagram for $G(j\omega)H(j\omega)$.

The basic part of the study in this chapter is the examination of the adequacy of the phase margin and the gain margin in predicting the quality of response of linear control systems involving time delay.

II.2 EVALUATION OF PHASE MARGIN AND GAIN MARGIN

Gain Margin

To obtain the gain margin, G_m, it is necessary to locate ω_m, where

$$\arg\left[G(j\omega_m)\ H(j\omega_m)\right] = 180^\circ \qquad 2.3$$

For each given form of transfer function $G(j\omega)H(j\omega)$, ω_m is independent of the gain constant K.

Phase Margin

To obtain the phase margin, Φ_m, it is necessary to locate ω_c.

The negative of the open loop transfer function of any second order system without numerator dynamics involving time delay may be written as:

$$G_1(s)H(s) = \frac{K_1}{(As^2 + Bs + C)} \left(\begin{array}{l}\text{delay or delay}\\ \text{approximand trans-}\\ \text{fer function}\end{array}\right) \qquad 2.4$$

Now the Laplace transformation representing the actual delay e^{-sT} and the Pade delay approximands only introduce phase lag without altering the magnitude of the gain.

Since $|G_1(j\omega_{C_1})H(j\omega_{C_1})| = 1$, 2.5

$$K_1^2 = (-A\omega_{C_1}^2 + C)^2 + B^2\omega_{C_1}^2 , \quad 2.6$$

or $$A^2\omega_{C_1}^4 + (B^2 - 2AC)\omega_{C_1}^2 + (C^2 - K_1^2) = 0 \quad 2.7$$

and so $$\omega_{C_1} = \left[\frac{-(B^2-2AC) \pm [(B^2-2AC)^2 - 4A^2(C^2-K_1^2)]^{\frac{1}{2}}}{2A^2} \right]^{\frac{1}{2}} \quad 2.8$$

For $B^2-2AC>0$, $|K_1|$ must be greater than $|C|$ for ω_{C_1} to be real, and the positive sign preceding the interior radical must be used. When $(B^2-2AC)<0$ and $|K_1|<C$ both positive and negative signs preceding the interior radical may be used, in which case two different values of ω_{C_1} may be obtained; consequently, two values for the phase margin result. Moreover, when $B^2-2AC>0$ and $|K_1|<|C|$, no phase margin can be defined.

The phase margins of any second order system with either an actual delay, or Pade 1, or Pade 2 approximands, in the feedback loop are, respectively

$$\Phi_{m_{\text{actual delay}}} = 180^0 - [\tan^{-1}(\frac{B\omega_C}{C - A\omega_C^2}) + \omega_C T]^0 \quad 2.9$$

$$\Phi_{m_{\text{Pade 2}}} = 180^0 - [\tan^{-1}(\frac{B\omega_C}{C - A\omega_C^2}) + 2\tan^{-1}(\frac{6T\omega_C}{12-T^2\omega_C^2})]^0 \quad 2.10$$

and

$$\Phi_{m_{\text{Pade 1}}} = 180^0 - [\tan^{-1}(\frac{B\omega_C}{C - A\omega_C^2}) + 2\tan^{-1}(\frac{\omega_C T}{2})]^0 \qquad 2.11$$

The cascaded lag delay approximands, on the other hand, not only introduce phase lag but also change the magnitude of gain.

If the loop gain of a second order system with double cascaded lag delay approximand is increased by a factor of $(1+\frac{\omega_C{}^2 T^2}{4})$ from that of the Pade 1 delay approximand, the phase margin equal to that of the Pade 1 delay control system is obtained.

Consider now the third order system with a pole at the origin,

$$G_2(s)H(s) = \frac{K_2}{s(As^2+Bs+C)} \begin{pmatrix} \text{delay or delay} \\ \text{approximand transfer} \\ \text{function} \end{pmatrix} \qquad 2.12$$

The crossover frequency, ω_{C_2}, is a solution of the equation

$$K_2{}^2 = \omega_{C_2}{}^2 \; [(C - A\omega_{C_2}{}^2)^2 + B^2\omega_{C_2}{}^2] \qquad 2.13$$

which with the substitution $X = \omega_{C_2}{}^2$ is reduced to

$$K_2{}^2 = X[(C - AX)^2 + B^2 X)] \qquad 2.14$$

This is a cubic in X which, in practice, is time consuming to solve.

Let ω_{c_1} and ω_{c_2} be, respectively, the gain crossover frequencies for $G_1(s)H(s)$ and $G_2(s)H(s)$ as defined earlier. Now,

$$\arg\left[G_2(j\omega)H(j\omega)\right] = \arg\left[G_1(j\omega)H(j\omega)\right] - 90^0 \qquad 2.15$$

and, furthermore,

$$\omega_{c_2} = \omega_{c_1} \qquad 2.16$$

if, and only if $\quad K_2 = K_1\omega_{c_1} \qquad 2.17$

In order to determine the phase margin, Φ_{m_2}, for a given K_2, instead of solving for ω_{c_2} in Eq.2 14, the values of ω_{c_1} and K_1 satisfying Eq. 2.8 may be found, and then ω_{c_2} is found by searching for the particular values of ω_{c_1} and K_1 satisfying the relation $\omega_{c_1} K_1 = K_2$. The value of Φ_{m_2} is 90^0 less than the phase margin of $G_1(s)H(s)$ with ω_{c_1} as crossover frequency.

Similarly, in order to find K_2 for a given Φ_{m_2} one may search for ω_{c_1} and K_1 for which $\Phi_{m_1} = \Phi_{m_2} - 90^0$, and $K_2 = \omega_{c_1} K_1$.

Since the phase crossover frequency ω_m is independent of K for each particular feedback control system, whereas ω_c generally varies with K, the special technique described above for evaluating gain margin is comparatively easier to apply.

II.3 PHASE MARGIN AND TRANSIENT RESPONSE

For a second order closed loop linear control system, if the negative loop gain is

$$G(s)H(s) = \frac{\omega_o^2}{s(s^2 + 2\xi\omega_o)} \qquad 2.18$$

then the closed loop roots are

$$s_{1,2} = -\xi\omega_o \pm j\omega_o \sqrt{1 - \xi^2} \qquad 2.19$$

Dorf (10) has shown that

$$\Phi_m = \tan^{-1}[2\xi\{\frac{1}{(4\xi^4 + 1) - 2\xi^2}\}^{\frac{1}{2}}] \qquad 2.20$$

and that the relation between ξ and Φ_m may be approximated by the linear equation

$$\xi = 0.01\Phi_m^0 \qquad 2.21$$

with a maximum percentage error of 14% for $\xi < 0.7$.

Now, for a second order system with a pair of complex roots, the overshoot and normalized peak time may be written as

$$OS = e^{-\pi\xi/\sqrt{1-\xi^2}} \qquad 2.22$$

$$\omega_o T_p = \frac{\pi}{\sqrt{1-\xi^2}} \qquad 2.23$$

where ω_o is the radial distance from the closed loop root to the origin.

Thus with this linear approximation, ξ may be replaced by the phase margin to determine the transient performance of a system having a pair of closed loop complex roots. This relationship between the phase margin and quality of response is quite useful in the design of linear control systems (8,10,17,21,23,24).

A numerical and experimental study was carried out to investigate whether this characteristic could be further generalized to any second order feedback control system involving time delay.

For this purpose, consider a second order control system involving time delay and a pole at the origin with the transfer functions

$$G(s) = \frac{k}{s(s+1)} \tag{2.24}$$

$$H(s) = e^{-sT} \tag{2.25}$$

For given values of delay time, the overshoot, peak time and phase margin for various values of gain constant are obtained, the results are shown in Figure 2.3.

The same response parameters were experimentally obtained for two other systems involving time delay; the results are shown in Figure 2.4 for the system with one negative real pole

$$G(s)\,H(s) = \frac{k}{(s+1)}\,e^{-sT} \tag{2.26}$$

and in Figure 2.5 for the system with a double real pole

$$G(s)\,H(s) = \frac{k}{(s^2 + 2\,s + 1)}\,e^{-sT} \tag{2.27}$$

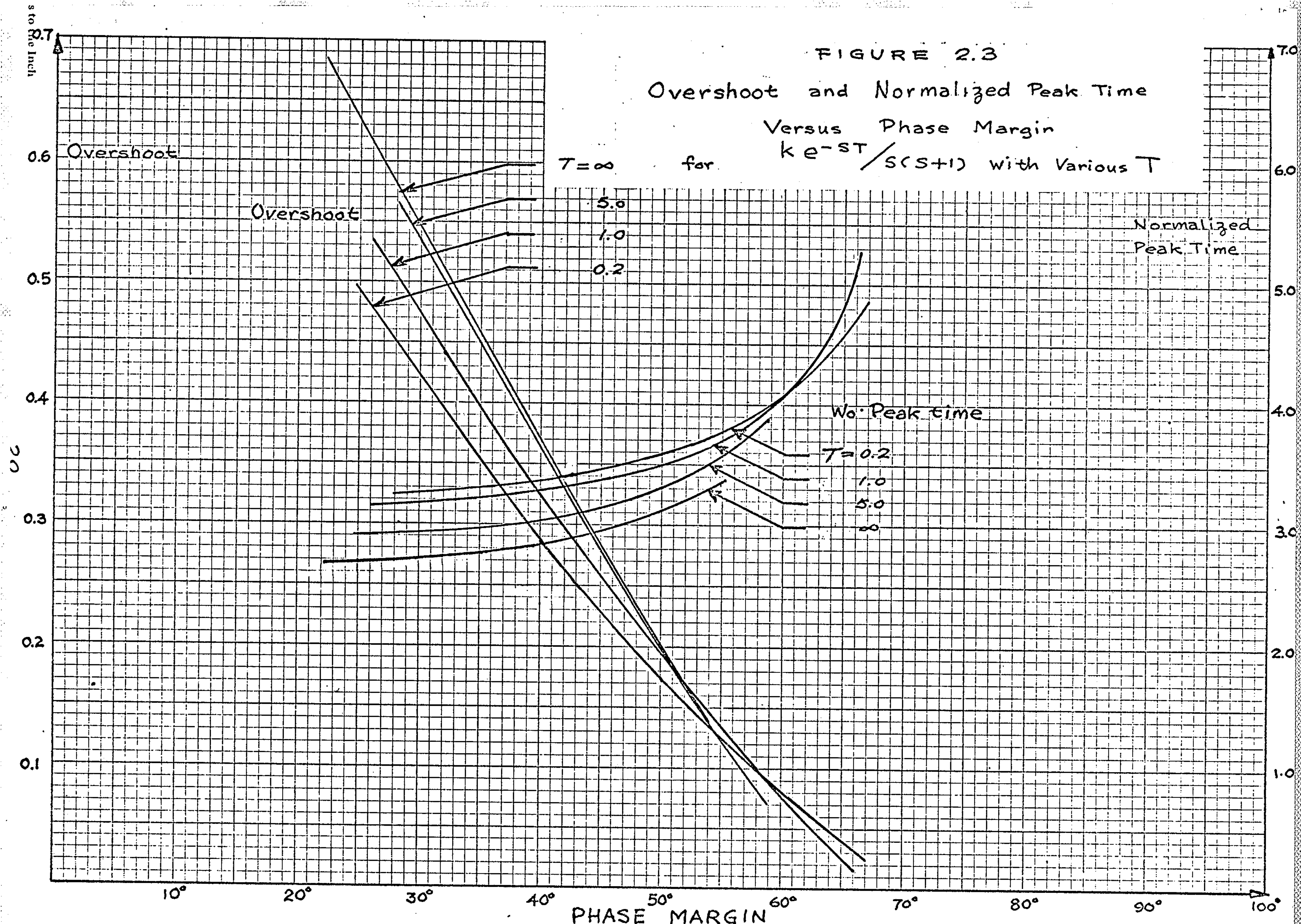


FIGURE 2.3
Overshoot and Normalized Peak Time
Versus Phase Margin
for $ke^{-sT}/s(s+1)$ with Various T
Overshoot
Overshoot
T=∞
5.0
1.0
0.2
Normalized Peak Time
Wo·Peak time
T=0.2
1.0
5.0
∞
0.7
0.6
0.5
0.4
0.3
0.2
0.1
7.0
6.0
5.0
4.0
3.0
2.0
1.0
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
PHASE MARGIN

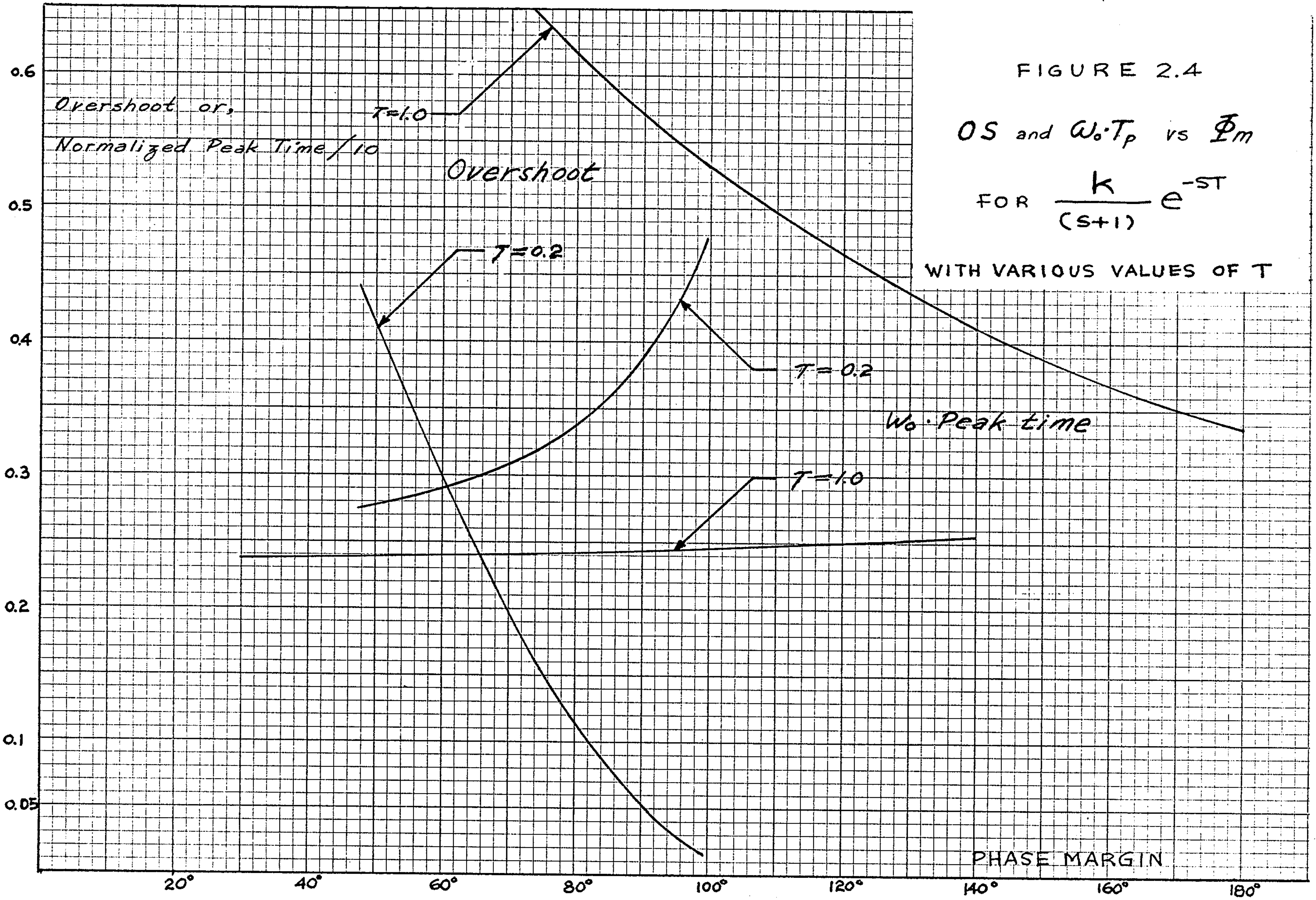


FIGURE 2.4

OS and $\omega_o \cdot T_p$ vs Φ_m

FOR $\frac{K}{(S+1)} e^{-ST}$

WITH VARIOUS VALUES OF T

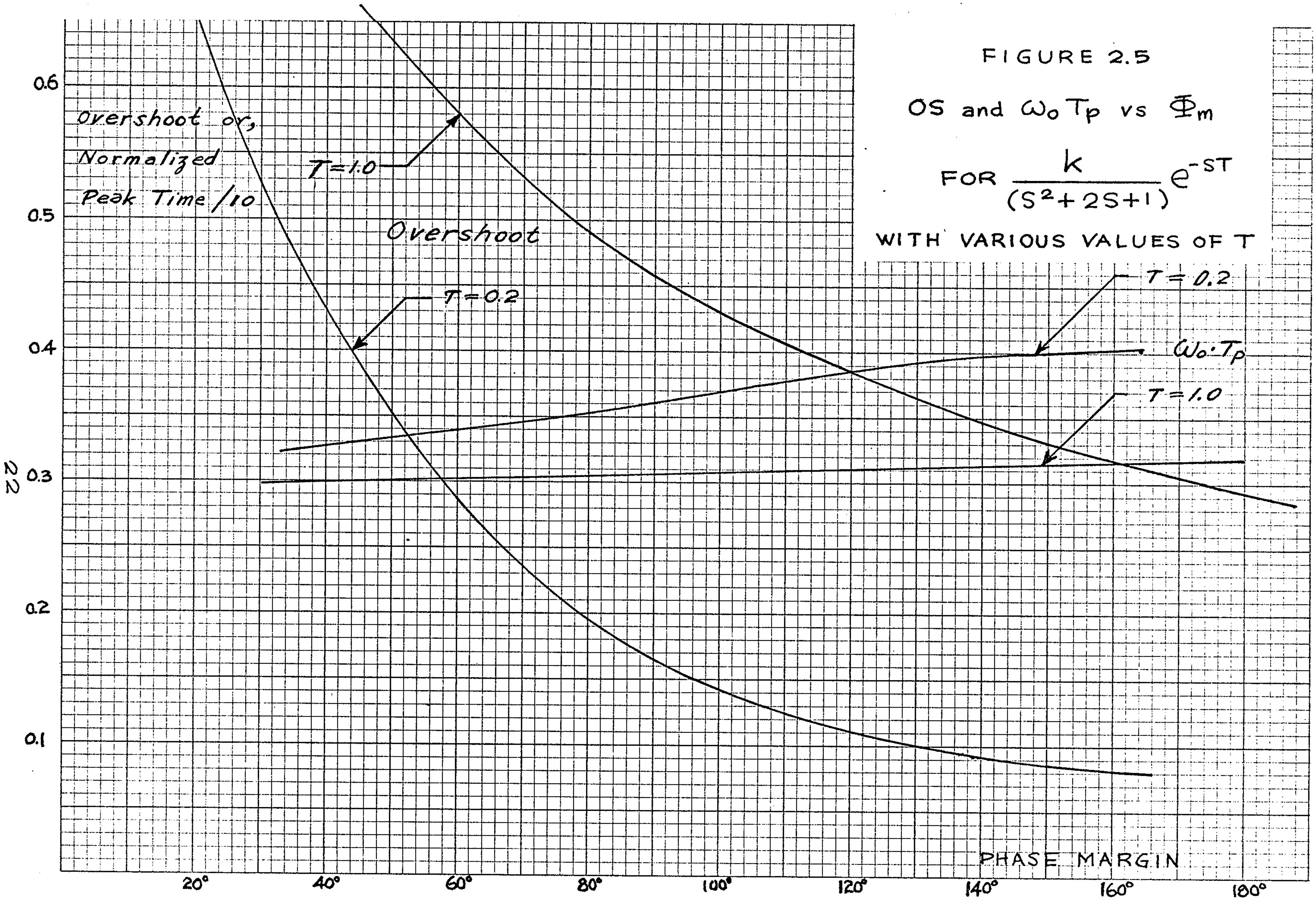


FIGURE 2.5

OS and $\omega_o T_p$ vs Φ_m

FOR $\frac{k}{(S^2+2S+1)} e^{-ST}$

WITH VARIOUS VALUES OF T

Now, consider the following two types of control systems

$$G_1(s)\ H(s) = \frac{k_1}{(s^2 + 2\ s + 1)}\ e^{-s} \qquad 2.28$$

$$G_2(s)\ H(s) = \frac{k_2}{s(s + 1)}\ e^{-s} \qquad 2.29$$

The frequencies for $G_1(s)H(s)$ and $G_2(s)H(s)$ with the phase margins of 45° are 0.894 and 0.403, respectively, for the loop gains of k_1=1.805 and k_2=0.434 and the gain margins of 1.405 and 2.66, respectively. From the experimental data shown in Figures 2.5 and 2.3 the overshoots for these two particular systems are 0.665 and 0.26, respectively. In contrast, the estimate based upon Eq. 2.21 and 2.22 is 0.21 for both systems.

The plot of ξ# versus Φ_m for $ke^{-sT}/(s+1)$ with T equal to 0.2, 1.0 and 5.0 is shown in Figure 2.6. In contrast to Eq.2.21, the damping constant ξ cannot be correlated with phase margin to any degree of accuracy.

In the study of Figures 2.3 through 2.5, for a system such as $ke^{-sT}/s(s+1)$ the phase margin seems to correlate with the percentage overshoot of unit step response to a certain degree of accuracy, i.e. independent of T, but

#In this thesis, unless otherwise specified, the damping constant ξ refers to the damping constant of the principal pair of complex closed loop roots.

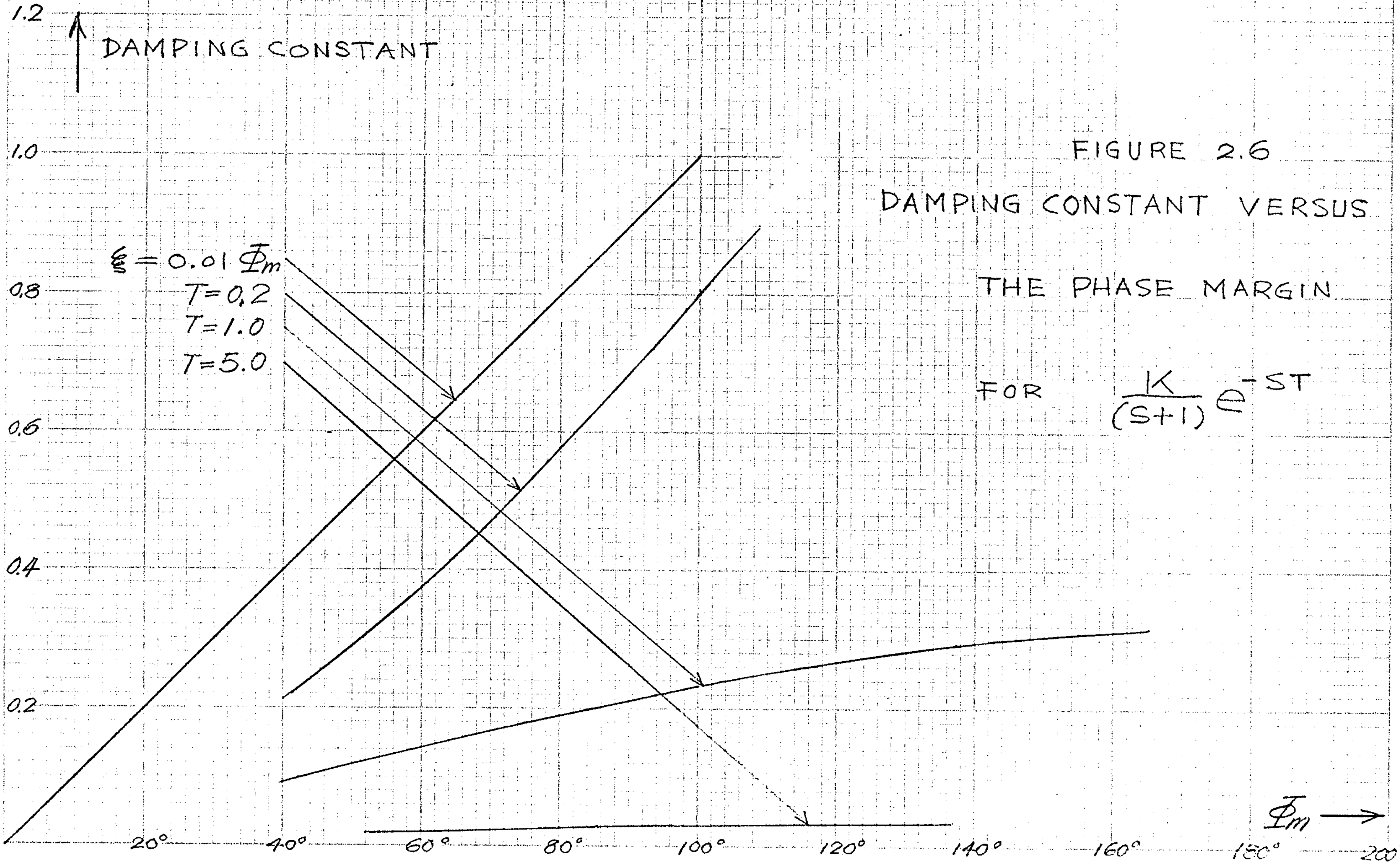


FIGURE 2.6

DAMPING CONSTANT VERSUS THE PHASE MARGIN FOR $\frac{K}{(S+1)} e^{-ST}$

from the above example and the presented data it is obvious that the phase margin fails to correlate with the step response for systems such as $ke^{-sT}/(s+1)$ and $ke^{-sT}/(s+1)^2$.

The Nyquist plots of $G_1(j\omega)H(j\omega)$ and $G_2(j\omega)H(j\omega)$ as defined in Eq.2.28 and 2.29 are presented in Figure 2.7, whereas, systems without delay, but with the same Φ_m, are presented in Figure 2.8.

FIGURE 2.7
Nyquist Plots for Systems Involving Delay

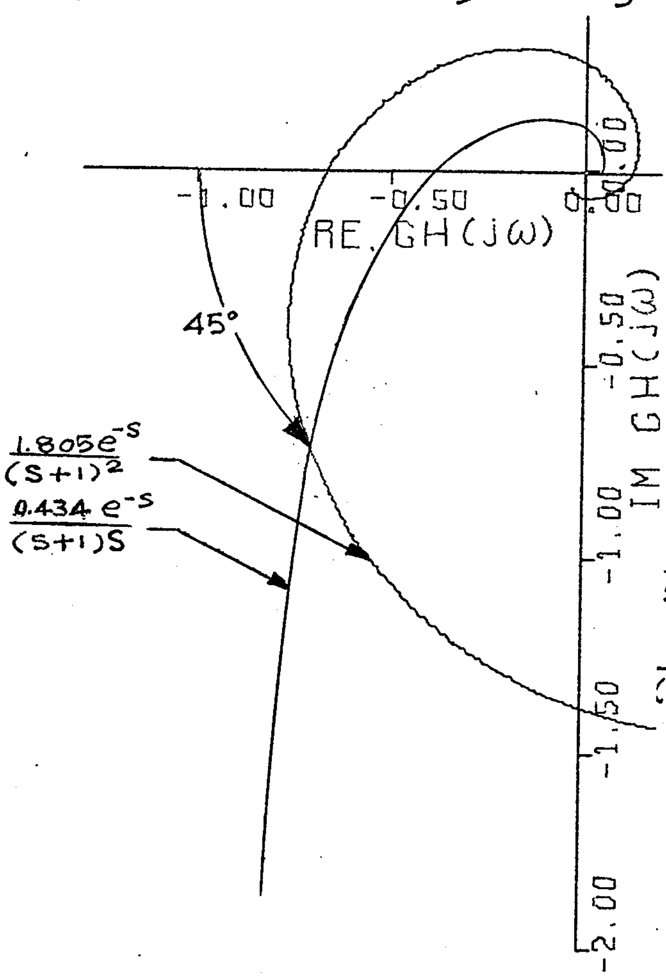
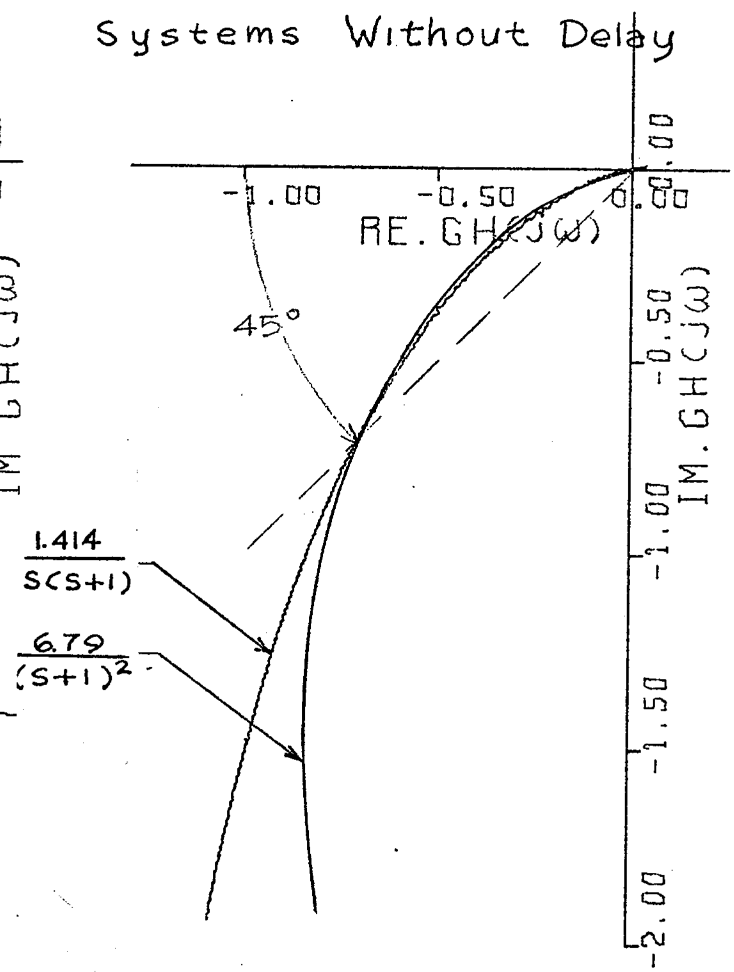


FIGURE 2.8
Nyquist Plots for Systems Without Delay



The Nyquist plots of the systems without delay are altogether different from those with delay in the manner they approach the real axis. This is due to the fact that the actual delay reduces the phase angle by an amount proportional to the product of the frequency ω and delay time T without any reduction in gain magnitude. Comparing Figure 2.7 to Figure 2.8, it appears that the transient characteristics of delay systems are less dependent on the phase margin than on the gain margin.

Moreover, in Section II.2, it has been shown that for any second order delay system of the form

$$G(s)\ H(s) = \frac{Ke^{-sT}}{(As^2 + B\ s + C)} \qquad 2.30$$

no phase margin can be defined when $|K| < C$ and $B^2 - 2AC > 0$.

As a result, for any control system involving time delay, if the frequency response curve as seen in the Nyquist plot passes through the negative real axis with a small change in magnitude, but accompanied by a sharp reduction in the phase angle, the phase margin can not be used to estimate accurately the quality of response of the system.

Thus, it is necessary to find another index upon which to base the design of linear control systems involving time delay.

II.4 GAIN MARGIN AND TRANSIENT RESPONSE

In this section, the correlation between transient response characteristics and gain margin is investigated.

Several types of systems have been investigated and the results are shown in Figures 2.9 through 2.11.

Figure 2.9: First order system

$$G(S)\ H(S) = Ke^{-S}/(S + A) \qquad 2.31$$

Figure 2.10: Second order system (excluding complex poles):

$$G(S)\ H(S) = Ke^{-ST}/(S + A)\ (S + B),\ T = 1.0 \qquad 2.32$$

(If $|B|>|A|$, $-A$ and $-B$ are the location of the "principal" and "secondary" poles, respectively.)

Figure 2.11: Third order system (excluding complex poles)

$$G(S)\ H(S) = \frac{Ke^{-ST}}{S(S + B)\ (S + C)},\ T = 1.0 \qquad 2.33$$

(If $|B|<|C|$, then B,C, are the "secondary" and "tertiary" poles, respectively.)

Figure 2.12 shows the Nyquist plot of three systems $ke^{-S}/(s+A)$ with A=0., 0.2 and 1.0. The forward loop gains for each of these systems were chosen to give a gain margin of 0.5, and the phase margins were calculated to be 44.8°, 56.06° and 121.5°, respectively —— a wide variation of phase margins for a given value of gain margin.

In the study of Figures 2.9 through 2.11, approximately

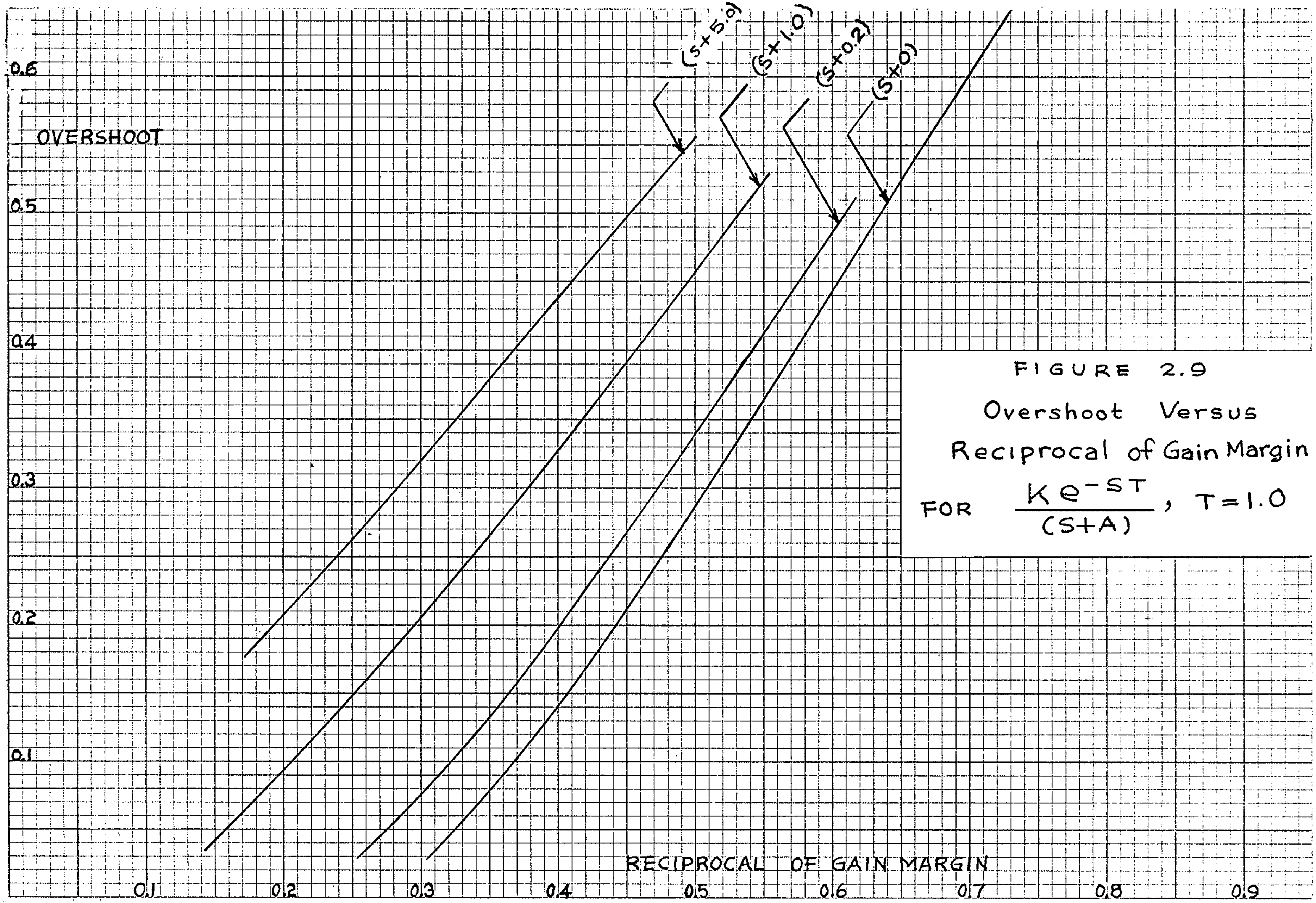


FIGURE 2.9
Overshoot Versus
Reciprocal of Gain Margin
FOR $\dfrac{Ke^{-ST}}{(S+A)}$, T=1.0

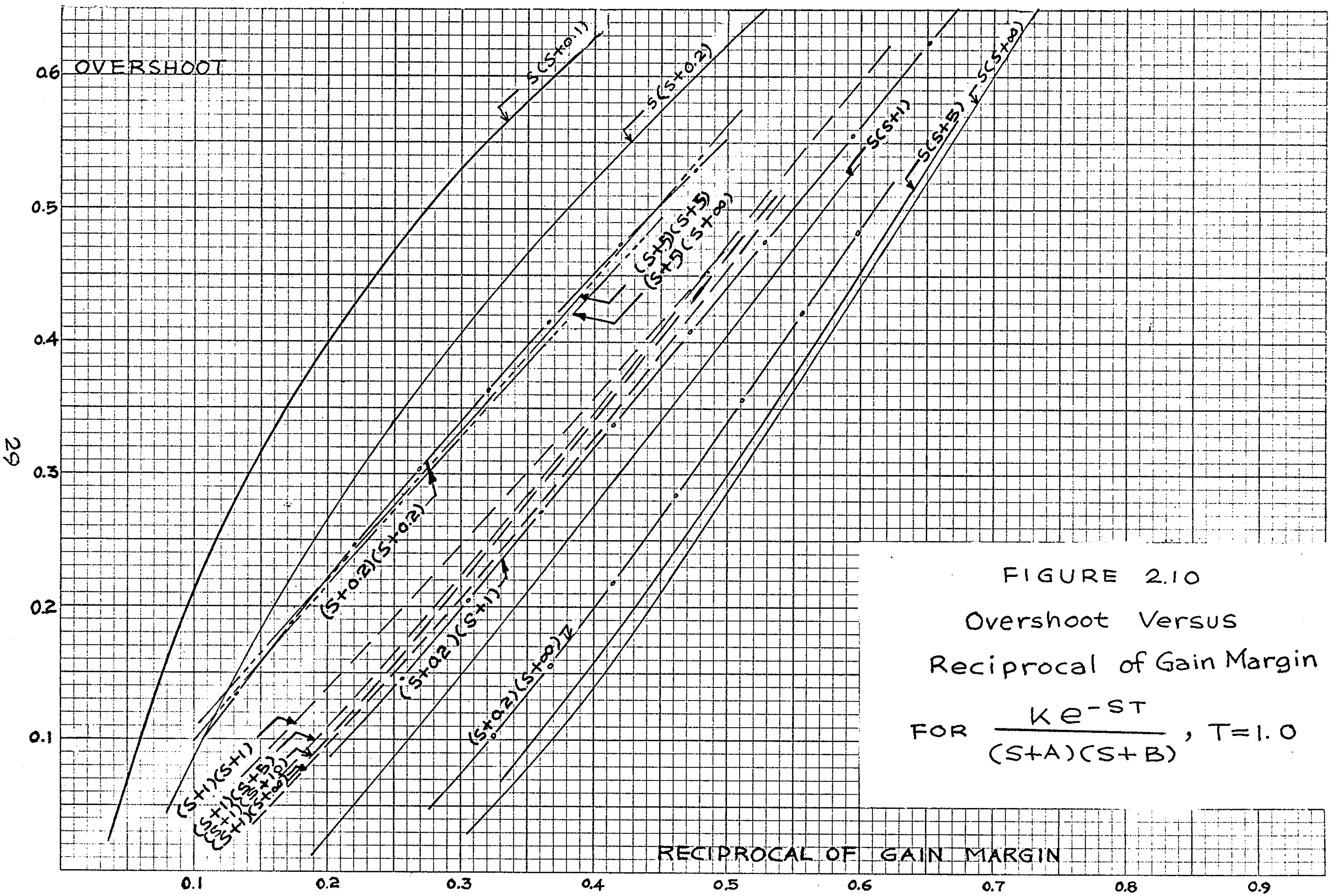


FIGURE 2.10
Overshoot Versus
Reciprocal of Gain Margin
FOR $\frac{Ke^{-ST}}{(S+A)(S+B)}$, T=1.0

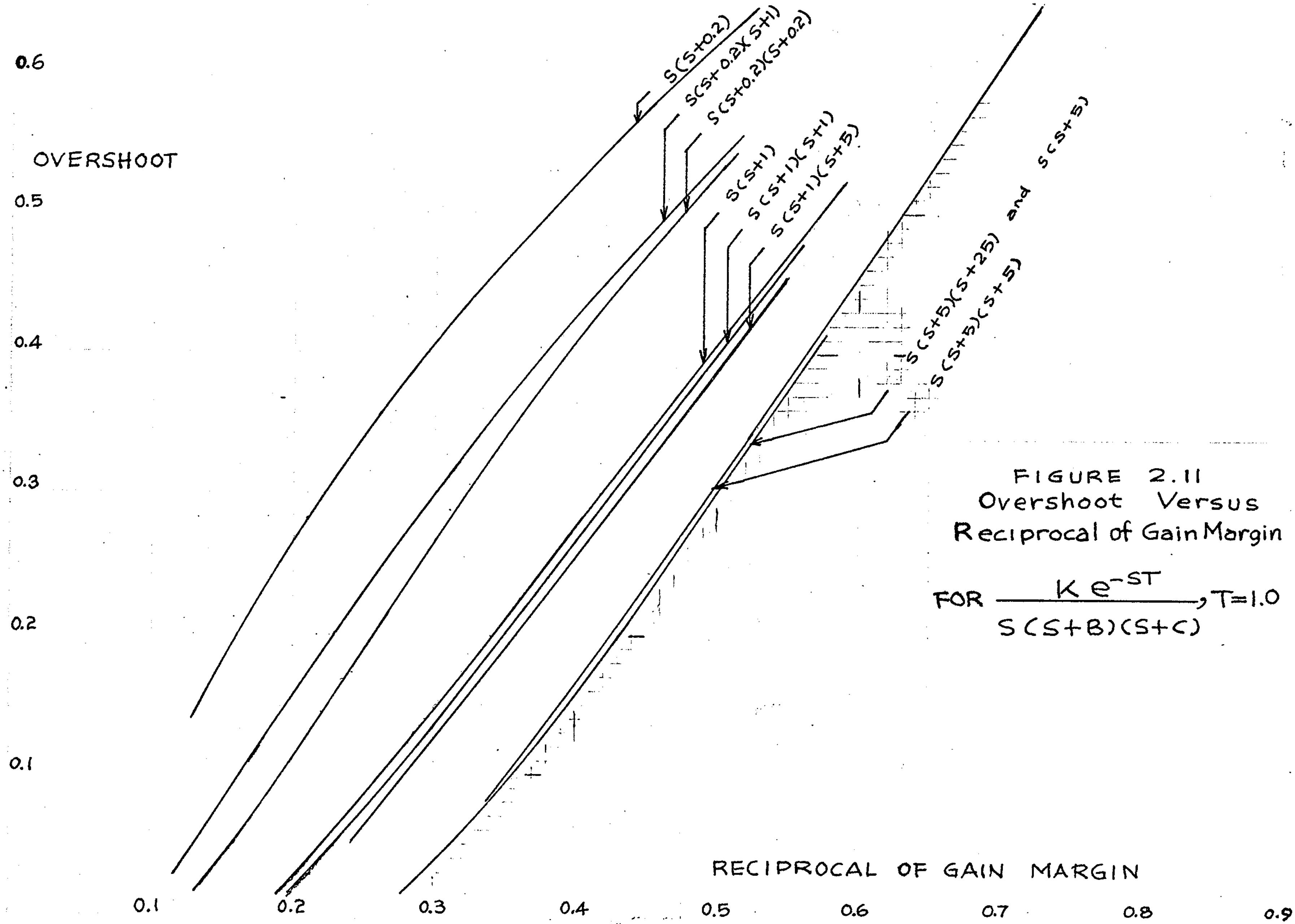


FIGURE 2.11
Overshoot Versus
Reciprocal of Gain Margin

FOR $\dfrac{K e^{-sT}}{S(S+B)(S+C)}$, $T=1.0$

FIGURE 2.12

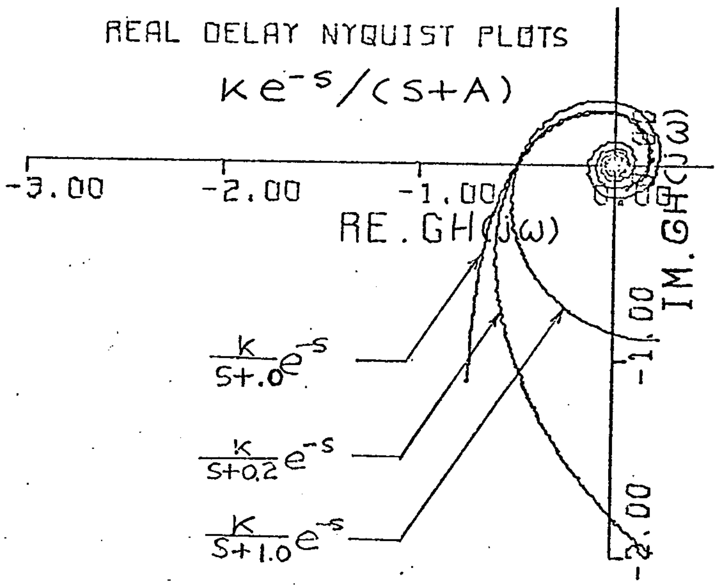
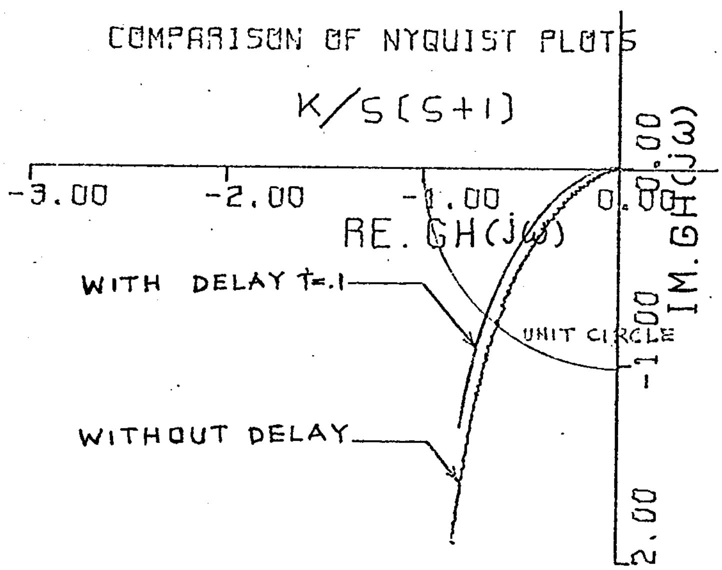


FIGURE 2.13



linear relationships between the overshoots and the reciprocal of gain margins are observed, the parameters for which depend upon both the location of the open loop poles and the values of the delay time.

On the basis of the presented graphs, a few general statements may be made concerning the correlation of overshoot and gain margin and the location of the open loop poles.

1. For a first order system (as shown in Figure 2.9), with any specific gain margin the closer the location of the open loop pole to the origin, the smaller the overshoot, i.e., for a given value of G_m, the overshoots are ordered as follows

$$\left(\frac{K_1 e^{-S}}{S + 5}\right)_{OS} > \left(\frac{K_2 e^{-S}}{(S + 1)}\right)_{OS} > \left(\frac{K_3 e^{-S}}{(S + 0.2)}\right)_{OS} > \left(\frac{K_4 e^{-S}}{(S + 0.0)}\right)_{OS} \qquad 2.34$$

2. For a second order system (excluding complex poles) as shown in Figure 2.10, with any specific gain margin the smaller the distance between the secondary and principal open loop poles, the larger the increase in overshoot over that of the first order system having only the principal open loop pole.

With any specific amount of increase in the distance between the secondary and principal open loop poles, the closer the principal pole is to the origin, the larger is the increase in the overshoot, i.e. for a given value of G_m, the overshoots are ordered as follows

$$\left(\frac{K_1e^{-S}}{S(S+0.1)}\right)_{OS} > \left(\frac{K_2e^{-S}}{S(S+0.2)}\right)_{OS} > \left(\frac{K_3e^{-S}}{S(S+1)}\right)_{OS} > \left(\frac{K_4e^{-S}}{S(S+5)}\right)_{OS}$$

$$> \left(\frac{K_\infty e^{-S}}{S(S+\infty)}\right)_{OS} = \left(\frac{Ke^{-S\#}}{S}\right)_{OS}, \qquad 2.35$$

$$\left(\frac{K_1e^{-S}}{(S+0.2)(S+0.2)}\right)_{OS} > \left(\frac{K_2e^{-S}}{(S+0.2)(S+1)}\right)_{OS} > \left(\frac{K_\infty e^{-S}}{(S+0.2)(S+\infty)}\right)_{OS}$$

$$= \left(\frac{Ke^{-S}}{(S+0.2)}\right)_{OS}, \qquad 2.36$$

$$\left(\frac{K_1e^{-S}}{(S+1)(S+1)}\right)_{OS} > \left(\frac{K_2e^{-S}}{(S+1)(S+5)}\right)_{OS} > \left(\frac{K_\infty e^{-S}}{(S+1)(S+\infty)}\right)_{OS} = \left(\frac{Ke^{-S}}{(S+1)}\right)_{OS}, \qquad 2.37$$

and

$$\left(\frac{K_1e^{-S}}{(S+5)(S+5)}\right)_{OS} > \left(\frac{K_\infty e^{-S}}{(S+5)(S+\infty)}\right)_{OS} = \left(\frac{Ke^{-S}}{(S+5)}\right)_{OS} \qquad 2.38$$

3. Third order systems (excluding complex poles) as shown in Figure 2.11 can have either a greater or lesser overshoot, when the third open loop pole is added to the system.

#The reader may refer to Section III.3, System II.

The correlation of the gain margin, overshoot and the location of open loop poles and zeros can be of great help to the designer. A design method based upon the open loop pole-zero location and the gain margin is developed in Chapter V, and a few practical examples are considered.

For any second order system without delay, the frequency response curve does not even pass through the negative real axis, whereas for systems such as $ke^{-sT}/s(s+1)$ with $T \leq 0.5$, the smaller T is the steeper is the line relating the overshoot and gain margin.

Therefore, the gain margin fails to be of any practical use for systems whose frequency response curve passes through the negative real axis with sharp reduction in both the phase angles and gain magnitudes. Such frequency response curve often occur in systems without delay.

Figure 2.13 shows the Nyquist plot of two particular systems with negative loop gains of $ke^{-0.1s}/s(s+1)$ and $ke^{-0.0s}/s(s+1)$, respectively; the inclusion of actual delay does not significantly alter the frequency response curve in the manner it approaches or crosses the negative real axis, and the step response of these systems are very well related to the phase margin , but obviously poorly related to the gain margin.

It is also found that the step responses of a set of systems such as $ke^{-sT}/(s^2+2As+1)$ for $A<0.6$ are also very poorly related to the gain margin.

As a result, the gain margin also has its limitation. The material presented here is intended to be of help to the designer to decide whether gain margin or phase margin may be used as an index for design.

II.5 CONCLUSION

Phase margin has been widely used as a measure of relative stability and as a principal parameter in the design of control systems without delay, but it fails to be a useful index for the prediction of the transient performance of systems whose frequency response curves pass through the negative real axis with sharp reduction in phase angle but little change in gain magnitude. In general, this may be attributed to the intrinsic nature of the time delay, e^{-sT}.

For these systems, gain margin may be utilized to provide information concerning the overshoot of the step response once the location of the open loop poles and zeros is known. This relationship may be of great help to the designer who designs in terms of the open loop poles and zeros.

One significant advantage of this approach is the simplicity of finding the gain margin and the location of the open loop poles and zeros.

CHAPTER III

ROOT LOCUS ANALYSIS

III.1 INTRODUCTION

The root locus method was conceived by W.R.Evans (13). This method enables one to determine the roots of the characteristic equation once the transfer functions of the forward and feedback paths are known.

From the information concerning the location of the closed loop poles, a complete and very accurate transient and steady state solution can be obtained. This chapter presents the digital computational technique for obtaining the root locus and gain locus of any system with rational or irrational transfer functions.

Transient response and root locus of various normalized delay control systems are investigated and the results presented in the form of the graphs are intended to be of help to the designer in the qualitative estimate of transient performance.

III.2 DETERMINATION OF ROOT LOCUS AND GAIN LOCUS

Consider a feedback control system with time delay in the feedback path. The closed loop transfer function is

$$W(s) = \frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1 + G(s)H(s)} \tag{3.1}$$

where

$$G(s) = K\,\frac{N(s)}{D(s)} \tag{3.2}$$

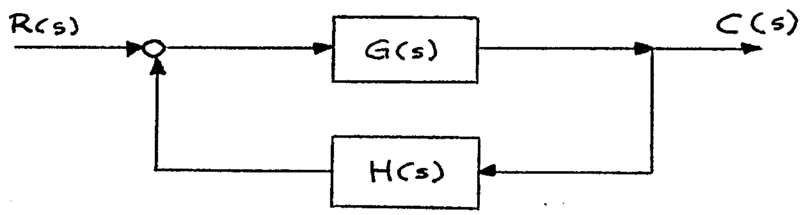


FIGURE 3.1 Block diagram of a feedback control system.

For the rational case which arises for various types of delay approximands, H(s) may be written as

$$H(s) = \frac{N_H(s)}{D_H(s)} \qquad 3.3$$

whereas for the actual delay

$$H(s) = e^{-sT} \qquad 3.4$$

The poles of W(s) must satisfy the characteristic equation

$$1 + G(s)\,H(s) = 0 \qquad 3.5$$

Thus it is necessary that

$$|G(s)\,H(s)| = 1 \qquad 3.6$$

$$\angle G(s)H(s) = 180^0 \pm n360^0 \qquad 3.7$$

where $n = 0, 1, 2, \ldots$

A Fortran IV programme was written (see Appendix II) to determine the root locus and the corresponding gain curves for any rational or irrational transfer function. This method is similar to the method of root locus determination of linear systems with actual delays as developed by Huang and Li (16), and is based upon the principle of finding the intersection points of the root loci with the imaginary axis as it is shifted along the real axis.

This method is based on the equation

$$|G(s)H(s)| \underline{\angle G(s)H(s)} = 1 \underline{\angle 180^{0} \pm n360^{0}} \qquad 3.8$$

If the complex variable $s = x + y\,j$, then

$$G(x,y) = \frac{KN\ (x,y)}{D(x,y)}$$

$$= \frac{K\ \mathrm{Re}\ N(x,y) + K\ \mathrm{Im}\ N(x,y)j}{\mathrm{Re}\ D(x,y) + \mathrm{Im}\ D(x,y)j} \qquad 3.9$$

and

$$\angle G(x,y) = \tan^{-1}\frac{\mathrm{Im}\ N(x,y)}{\mathrm{Re}\ N(x,y)} - \tan^{-1}\frac{\mathrm{Im}\ D(x,y)}{\mathrm{Re}\ D(x,y)} \qquad 3.10$$

Similarly,

$$\angle H(x,y) = \tan^{-1}\frac{\mathrm{Im}\ N_H\ (x,y)}{\mathrm{Re}\ N_H\ (x,y)} - \tan^{-1}\frac{\mathrm{Im}\ D_H\ (x,y)}{\mathrm{Re}\ D_H\ (x,y)} \qquad 3.11$$

Any point on the locus must satisfy the angle condition

$$\angle G(x,y) + \angle H(x,y) = -180^{0} + n360^{0} \qquad 3.12$$

at which,

$$K = -\frac{\mathrm{Re}\ D(x,y)\ \mathrm{Re}\ D_H(x,y) - \mathrm{Im}\ D(x,y)\ \mathrm{Im}\ D_H(x,y)}{\mathrm{Re}\ N(x,y)\ \mathrm{Re}\ N_H(x,y) - \mathrm{Im}\ N(x,y)\ \mathrm{Im}\ N_H(x,y)}$$

$$\text{or} \quad = -\frac{\text{Re } D(x,y)\ \text{Im } D_H(x,y) + \text{Re } D_H(x,y)\ \text{Im } D(x,y)}{\text{Re } N(x,y)\ \text{Im } N_H(x,y) + \text{Re } N_H(x,y)\ \text{Im } N(x,y)} \qquad 3.13$$

However, if only the absolute value of K is required, the following may be used in place of Eq. 3.13

$$|K| = \left|\frac{D(x,y)\ D_H(x,y)}{N(x,y)\ N_H(x,y)}\right| \qquad 3.14$$

Therefore, corresponding to any point on the root locus, the gain constant K can be evaluated.

Eq. 3.12 means that the algebraic sum of the phase angles of the vectors from all the finite poles and zeros of G(s)H(s) to any point on the locus must be -180^0 regardless of the region of the s plane under consideration.

For irrational cases such as the linear control system with actual delay in the feedback path

$$\begin{aligned} H(x,y) &= e^{-(x+yj)T} \\ &= e^{-xT}[\cos yT - j \sin yT] \end{aligned} \qquad 3.15$$

Therefore, Eq. 3.7 is reduced to

$$\angle G(x,y) = 180^0 \pm n360^0 + yT \qquad 3.16$$

for which

$$K = \frac{-\text{Re } D(x,y)\ e^{xT}}{\text{Re } N(x,y) \cos yT + \text{Im } N(x,y) \sin yT}$$

$$\text{or} \quad = \frac{-\text{Im } D(x,y)\ e^{xT}}{\text{Im } N(x,y) \cos yT - \text{Re } N(x,y) \sin yT} \qquad 3.17$$

or alternatively,

$$K = \left|-\frac{D(x,y)}{N(x,y)}\right| \qquad 3.18$$

Eq. 3.16 means that the algebraic sum of the phase angles of the vectors from all the finite poles and zeros of G(s) to a point on the locus is independent of the abscissa, x, and proportional to the ordinate, y. Thus it follows that the root locus of a system involving time delay has an infinite number of root locus branches.

The stability limit may be found without any difficulty by determining the value of y, with x set to zero, which satisfies Eq.3.12 or 3.16.

This method was extensively used in the work of this thesis to determine the root locus of various systems. It takes advantage of the continuity property of $\angle G(x,y)$ and $\angle H(x,y)$ (except at the open loop poles or zeros). Moreover, the phase angle of e^{-sT} is independent of the value of x and linearly proportional to the product of y and T. Consequently, with the aid of a spirule and a chart showing the linear relationship between the phase angle of e^{-sT} and yT, one may proceed to determine the root and gain locus# of any linear control system involving time delay without any difficulty.

III.3 ROOT LOCUS AND TRANSIENT RESPONSE

Consider a general second order system without delay with the following closed loop transfer function

#Gain locus refers to the plot of the logarithm values of gain constant versus the abscissa of the corresponding root locus.

$$W(s) = \frac{C(s)}{R(s)} = \frac{K}{(s^2 + 2\xi\omega_o s + \omega_o^2)} \qquad 3.19$$

The step response of this system is of the form

$$c(t) = E\left[1 - \frac{1}{\sqrt{1-\xi^2}} \; e^{-\xi\omega_o t} \sin(\omega_o \sqrt{1-\xi^2}t + \theta)\right] \qquad 3.20$$

The peak time, overshoot and 2% settling time are given by

$$T_p = \frac{\pi}{\omega_o \sqrt{1-\xi^2}} = \frac{\pi}{y} \qquad 3.21$$

$$OS = e^{-\xi\pi/\sqrt{1-\xi^2}} = e^{-\pi x/y} \qquad 3.22$$

and

$$T_s = \frac{4}{\xi\omega_o} = \frac{4}{x} \qquad 3.23$$

The quality of response, therefore, as measured in terms of the peak time, overshoot and settling time is only dependent on the coordinates of the closed loop complex pole.

In general, (17) if the closed loop transfer function of a stable system is of the form

$$\frac{C(s)}{R(s)} = \frac{\sum_{k=0}^{m} a_k s^{m-k}}{(s - p_1)^{m_1} (s - p_2)^{m_2} \dots (s - p_r)^{m_r}} \qquad 3.24$$

then the steady state unit step response is

$$c_{s.s} = \frac{a_m}{\prod_{j=1}^{r} (-P_j)^{mj}} \qquad 3.25$$

If all the poles of the transfer function in Eq.3.24 are simple, then

$$c(t) = c_{s.s} + \sum_{j=1}^{n} k_j e^{p_j t}, \; t > o \qquad 3.26$$

If there are α pairs of complex conjugate poles and $\beta=r-2\alpha$ real poles, then

$$c(t) = c_{s.s} + \sum_{j=1}^{\alpha} K_{Cj}\, e^{P_j t} + \sum_{j=1}^{\alpha} K_{Cj}^{*}\, e^{P_j^{*} t} + \sum_{j=1}^{r-2\alpha} K_{Rj}\, e^{-\lambda j t} \qquad 3.27$$

The transient response of these systems is mathematically defined in (17), but to determine the peak time, overshoot and settling time, one must evaluate K_{Cj} and K_{Rj} in addition to the information concerning the location of the closed loop poles and zeros. To perform these calculations is in most cases rather time consuming and probably deters the designer from using this technique in the study of the system performance characteristics.

In some cases of higher order linear control systems without delay, the effects of the pole-zero location on unit step response are given in the literature (5, 25, 12). This list of references is by no means exhaustive, but indicates the type of information one may obtain once the closed loop pole-zero configuration is established.

Since the linear control system involving time delay has an infinite number of root locus branches, merely determining them and the transient behavior of the system would consume a considerable amount of time, not to mention the realization of a practical compensator to satisfy the desired performance characteristics.

As a result, certain approximations are investigated for a number of systems.

SYSTEM I

Consider the set of first order linear systems for which the negative loop gain is

$$G'(S)\,H'(S) = \frac{K}{(S+A)}\,e^{-\tau S} \qquad 3.28$$

It was shown in Section I.4 that such set of systems may be suitably normalized to $ke^{-sT}/(s+1)$, where $T=A\tau$, $s=S/A$ and $k=K/A$ with the s plane shrunk by a factor of 1/A, and the response time stretched by the same factor.

The root locus and gain locus determination technique outlined in Section III.2 was applied to determine the principal root and gain locus for

$$G(s)H(s) = \frac{k}{(s+1)}\,e^{-sT} \qquad 3.29$$

with various values of T. The results are shown in Figure 3.2, and the root loci for such systems with T=0., 1.0 and 5.0 are shown in Figure 3.3.

Experimental step response results using the C.S.M.P. simulation method have been determined and are presented in Figure 3.4.

For a theoretical percentage overshoot of 15%, it was experimentally found that the theoretical# estimates are

#In this thesis, unless otherwise specified, the term theoretical estimate refers to theoretical results considering only the effect of a pair of principal complex closed loop roots.

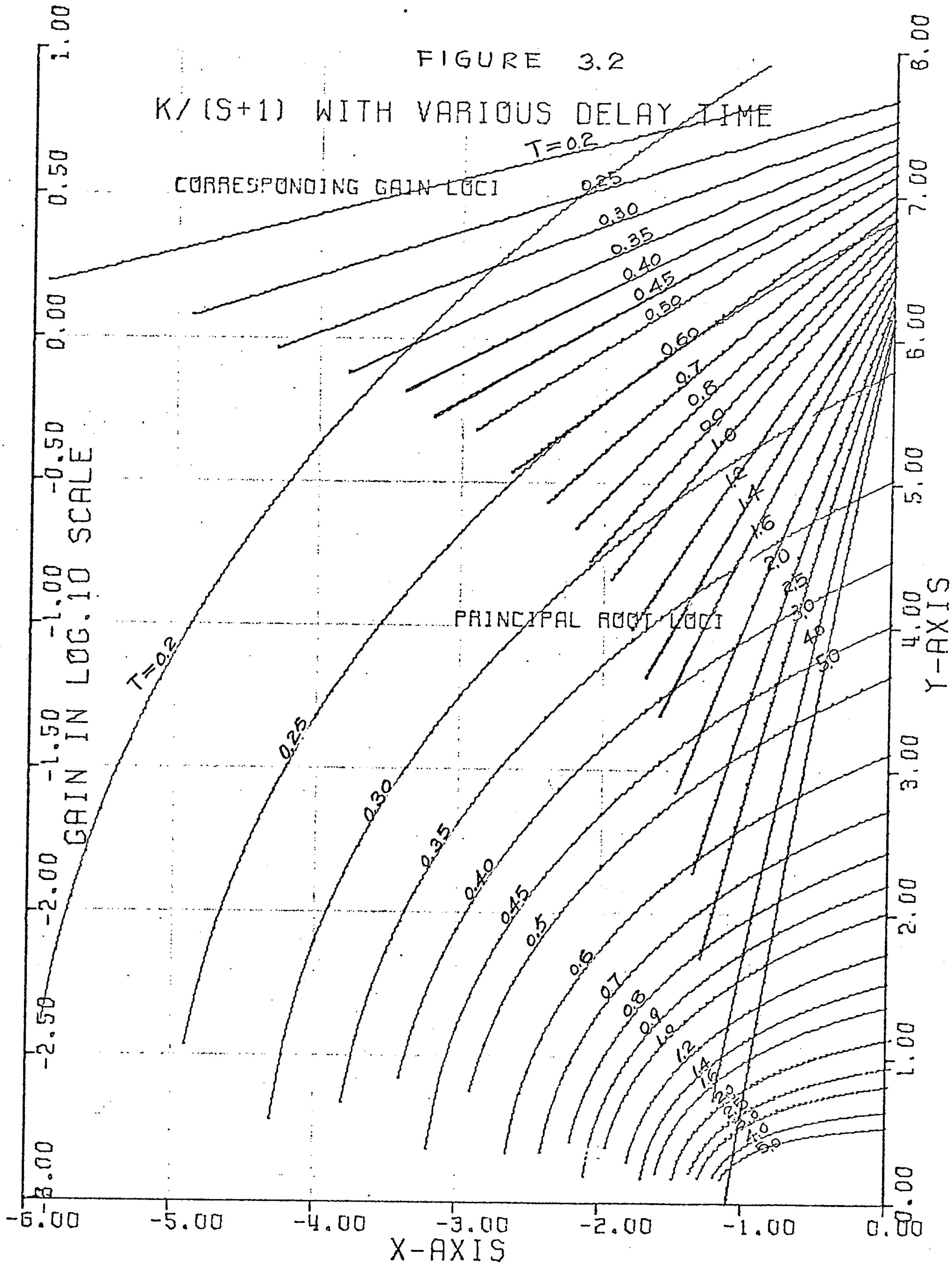


FIGURE 3.2
K/(S+1) WITH VARIOUS DELAY TIME
CORRESPONDING GAIN LOCI
T=0.2
0.25
0.30
0.35
0.40
0.45
0.50
0.60
0.7
0.8
0.9
1.0
1.2
1.4
1.6
2.0
2.5
3.0
4.0
5.0
PRINCIPAL ROOT LOCI
T=0.2
0.25
0.30
0.35
0.40
0.45
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
2.0
2.5
3.0
4.0
5.0
GAIN IN LOG.10 SCALE
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
Y-AXIS
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
X-AXIS
-6.00
-5.00
-4.00
-3.00
-2.00
-1.00
0.00

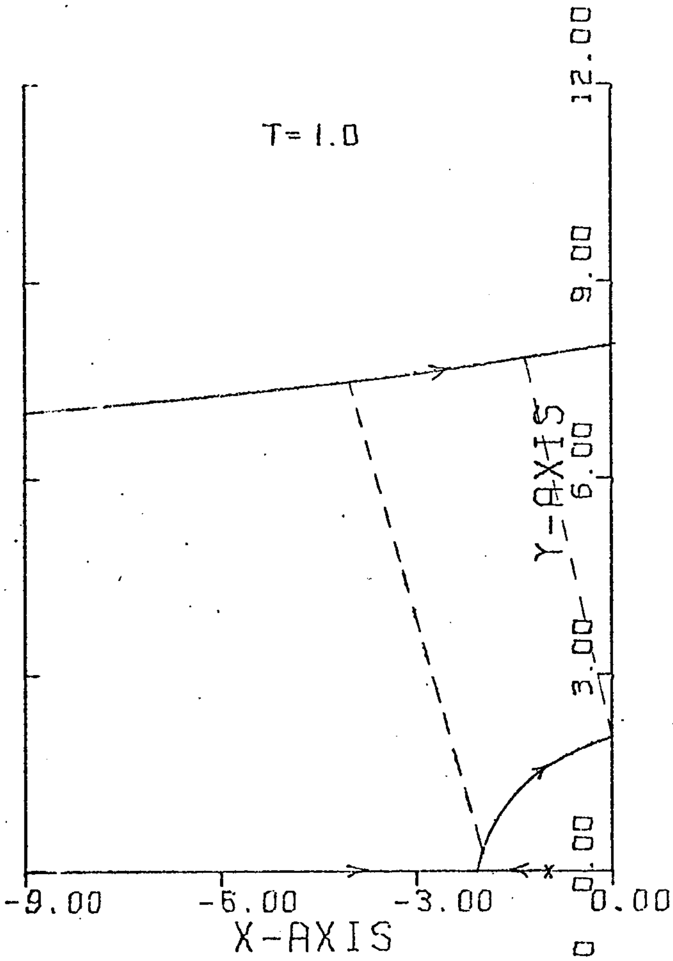
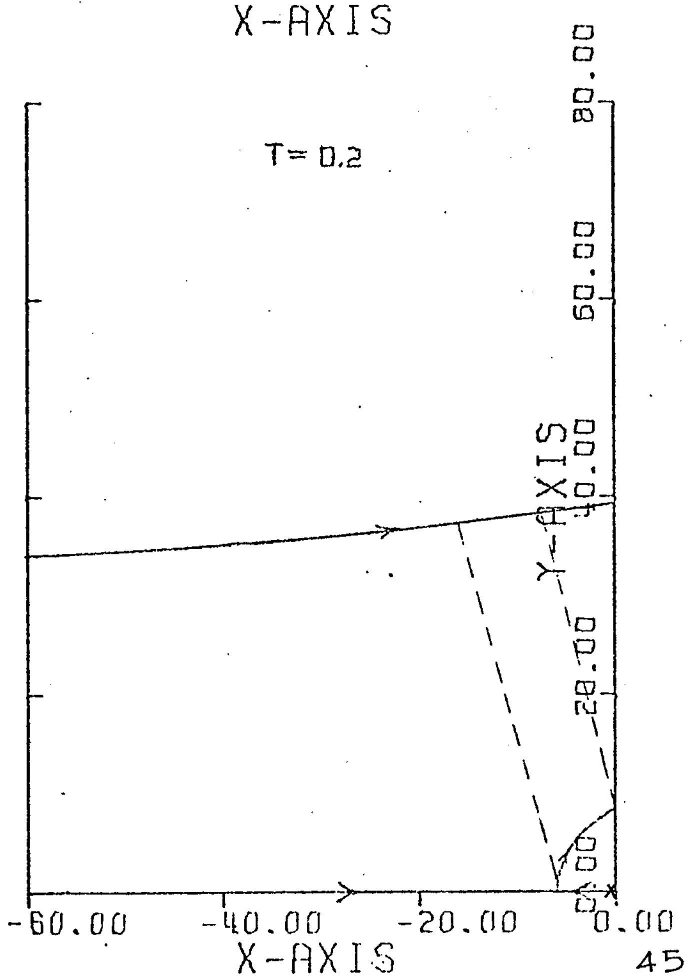


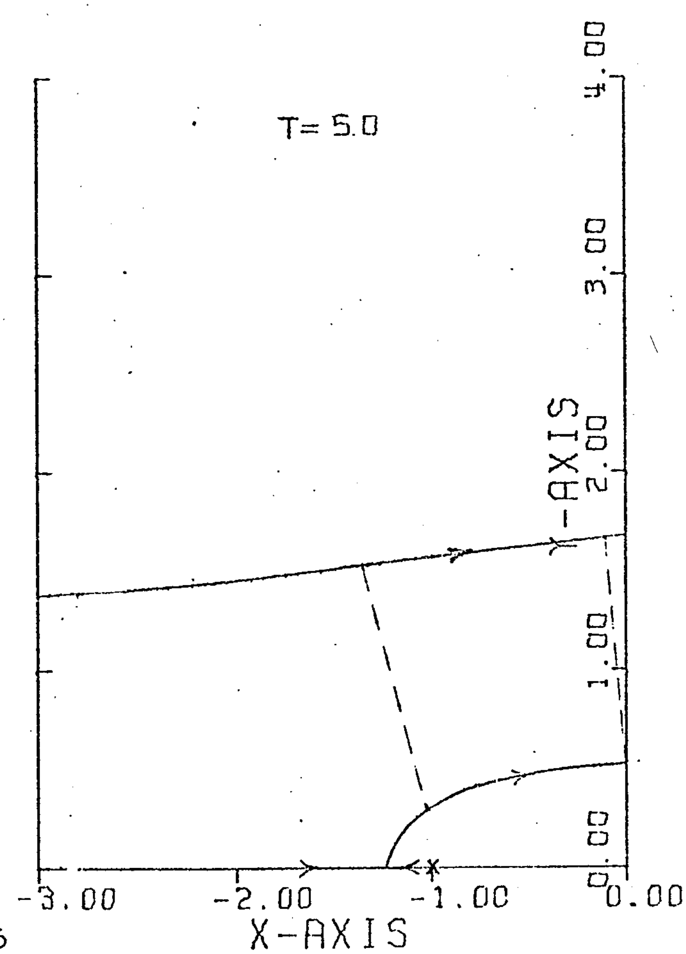
FIGURE 3.3

ROOT LOCUS OF $\frac{k e^{-ST}}{(S+1)}$

FOR VARIOUS TIME DELAYS

x — location of open loop pole.





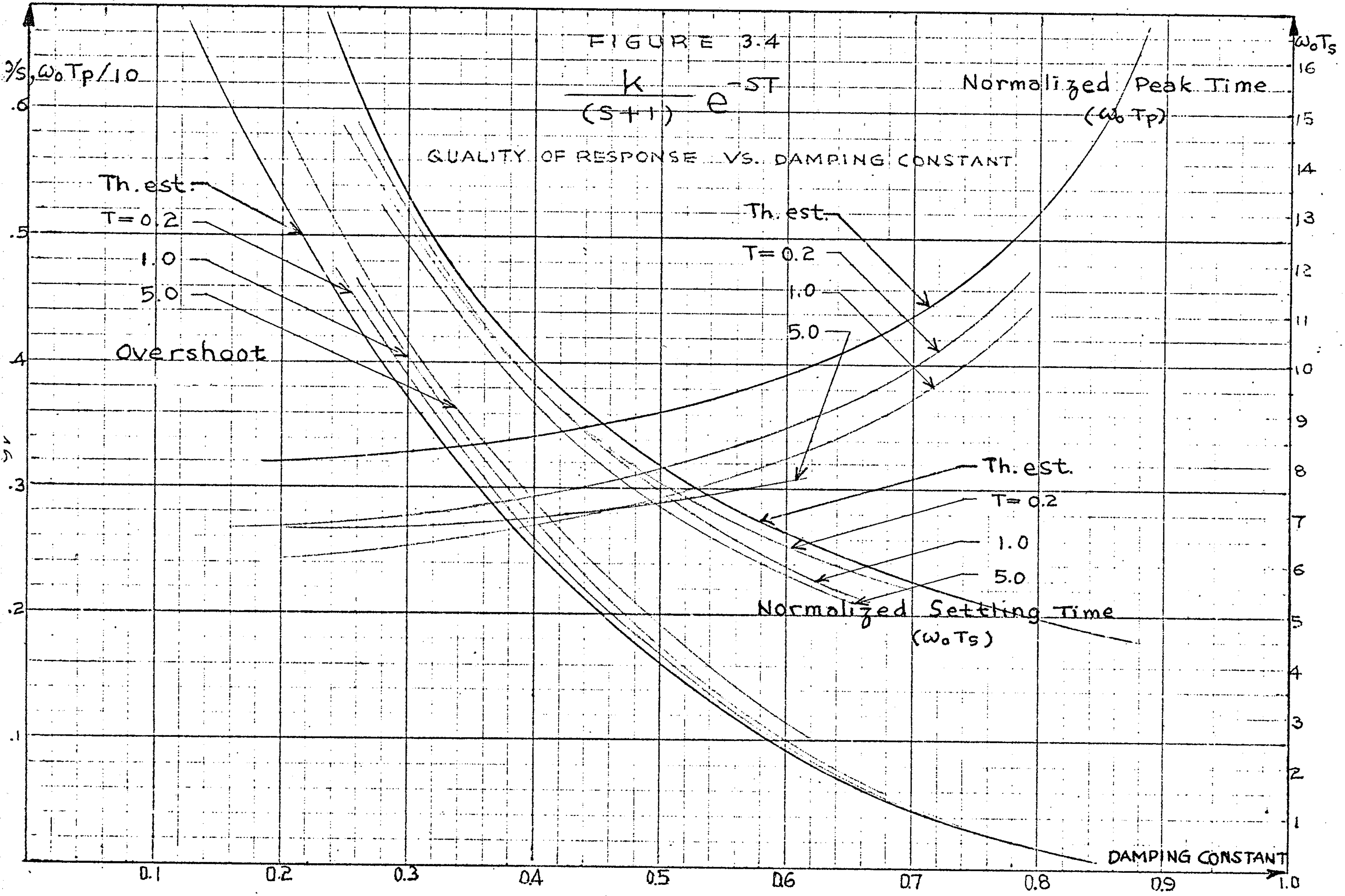


FIGURE 3.4
$\frac{k}{(S+1)} e^{-ST}$
QUALITY OF RESPONSE VS. DAMPING CONSTANT
%S, $\omega_0 T_p/10$
$\omega_0 T_s$
Normalized Peak Time
($\omega_0 T_p$)
Th. est.
T = 0.2
1.0
5.0
Overshoot
Th. est.
T = 0.2
1.0
5.0
Normalized Settling Time
($\omega_0 T_s$)
DAMPING CONSTANT
.6
.5
.4
.3
.2
.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

4.1%, 8.7% and 17.5% lower than the experimental percentage overshoots of $ke^{-sT}/(s+1)$ with $T = 0.2$, 1.0 and 5.0, respectively.

It can be observed in Figure 3.4, that as the delay time is increased, the discrepancy between the simulated percentage overshoot and the theoretical estimate increases.

In view of this observation, a further examination of Figure 3.3 was made. Here, it is apparent that for decreasing T values, the pair of secondary complex roots are increasingly farther away from the corresponding principal roots. Consequently, for smaller T values, the theoretical result, considering only the effect of the pair of principal complex roots, is expected to be a better estimate than those for larger T values.

The experimental results for normalized peak time and settling time are also presented in Figure 3.4; it is easily noted that the experimental results are lower than the theoretical estimates, and that the deviation increases for increasing delay time.

SYSTEM II

Consider the set of second order systems with a pole at the origin for which the negative loop gain is of the form

$$G'(S)\ H'(S) = \frac{K}{S(S + A)}\ e^{-\tau S} \qquad 3.30$$

Such a set of systems may be reduced to the normalized system

$$G(s)\ H(s) = \frac{k}{s(s + 1)}\ e^{-sT} \qquad 3.31$$

where

$$k = K/A^2 ,$$
$$s = S/A \qquad 3.32$$

and

$$T = A\tau.$$

The principal root loci for such a set of normalized systems are shown in Figure 3.5 and the root loci for T=0.2, 1.0 and 5.0 are shown in Figure 3.6.

For these systems, the theoretical estimates are surprisingly good especially with T = 0.2 and 1.0. In order to provide a better picture of the discrepancy between the experimental results and theoretical estimates, Table 3.1 is given.

TABLE 3.1

DELAY TIME T	GAIN CONSTANT K	PEAK TIME		2% SETTLING TIME		OVERSHOOT	
		Theoret. estimat.	Experim. results	Theoret. estimat.	Experim. results	Theoret. estimat.	Experim. results
0.2	0.3652	8.09108	8.0775	8.6957	9.255	.0242	.0241
	0.6442	4.6797	4.6800	9.3023	8.787	.1337	.1336
	1.0232	3.4084	3.4050	10.2564	9.825	.2647	.2645
	1.5085	2.7069	2.7037	11.7648	11.400	.3984	.3983
1.0	0.3442	6.8967	6.8300	14.286	14.365	.1450	.1454
	0.5222	5.2013	5.1200	20.000	19.618	.3534	.3560
	0.6788	4.5333	4.4400	28.572	27.780	.5301	.5350
5.0	0.9090	22.186	21.010	33.333	31.27	.0701	.0717
	0.1100	18.3934	17.263	40.000	39.74	.1589	.1626
	0.1326	16.1538	14.960	50.001	49.00	.2746	.2842
	0.1593	14.6257	13.400	66.670	67.40	.4158	.4326

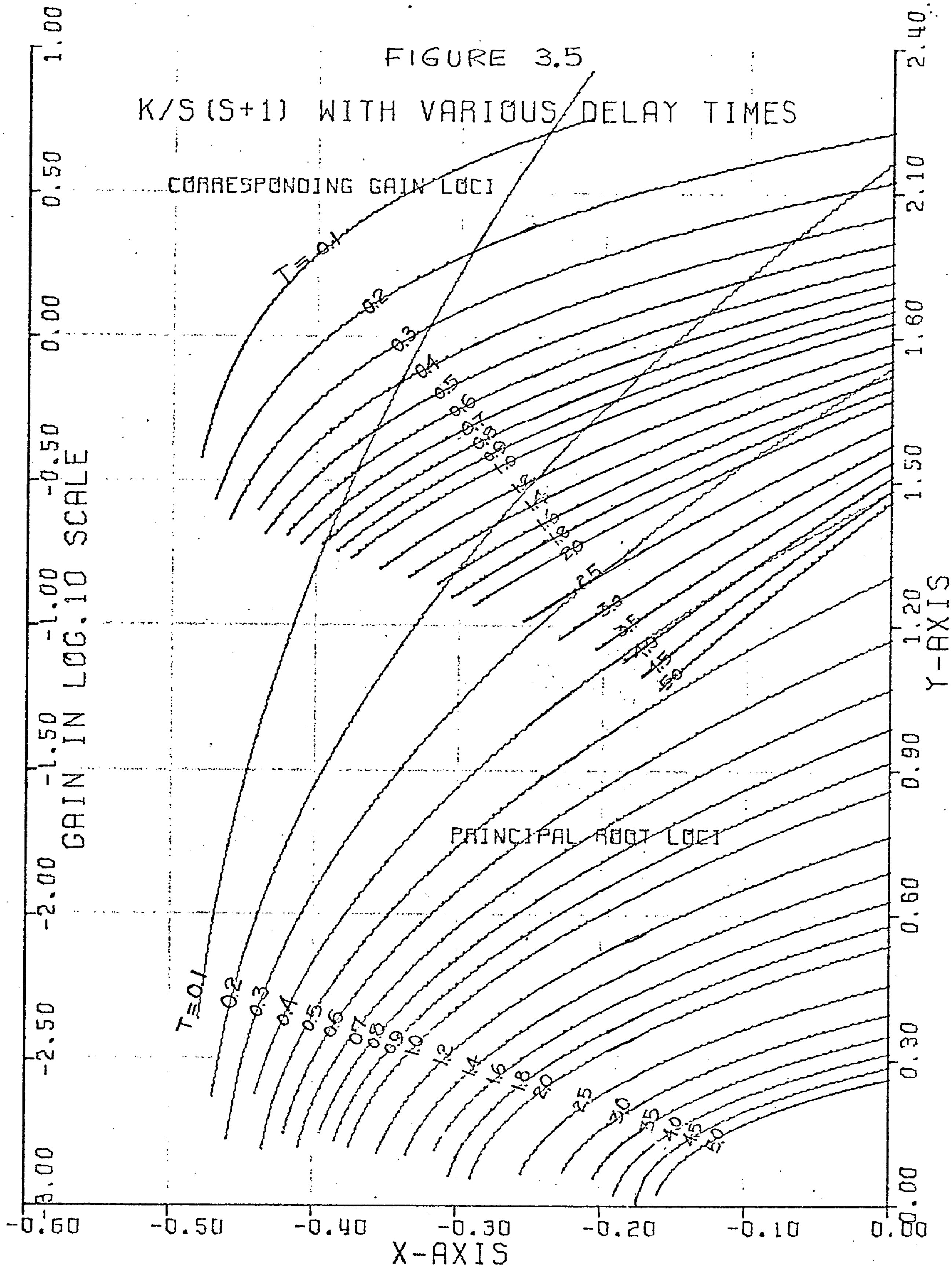


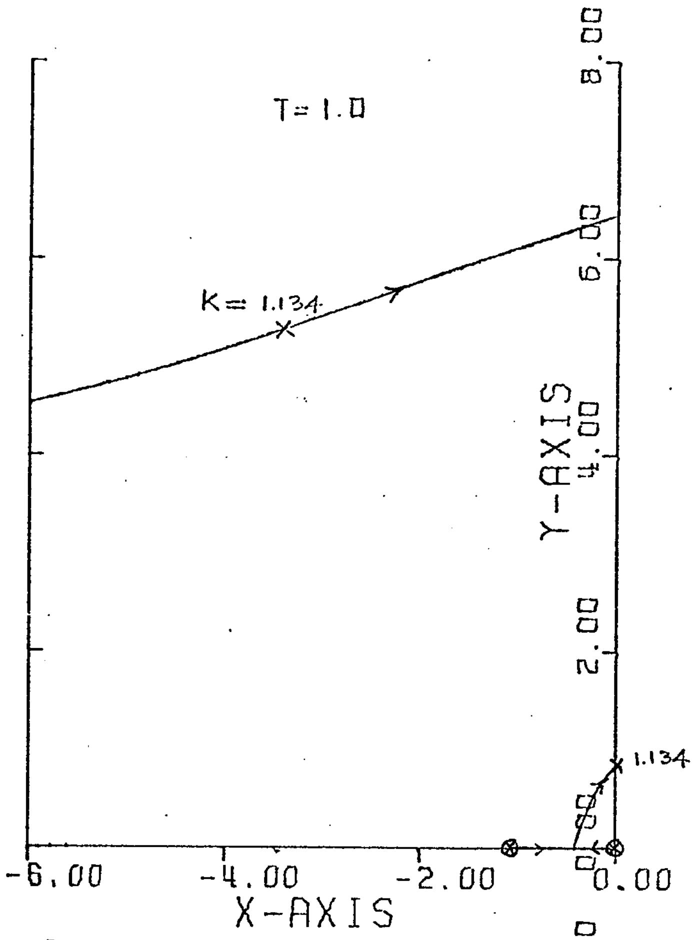
FIGURE 3.5
K/S(S+1) WITH VARIOUS DELAY TIMES
CORRESPONDING GAIN LOCI
PRINCIPAL ROOT LOCI
T=0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0
2.5
3.0
3.5
4.0
4.5
5.0
GAIN IN LOG.10 SCALE
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
Y-AXIS
2.40
2.10
1.80
1.50
1.20
0.90
0.60
0.30
0.00
X-AXIS
-0.60
-0.50
-0.40
-0.30
-0.20
-0.10
0.00

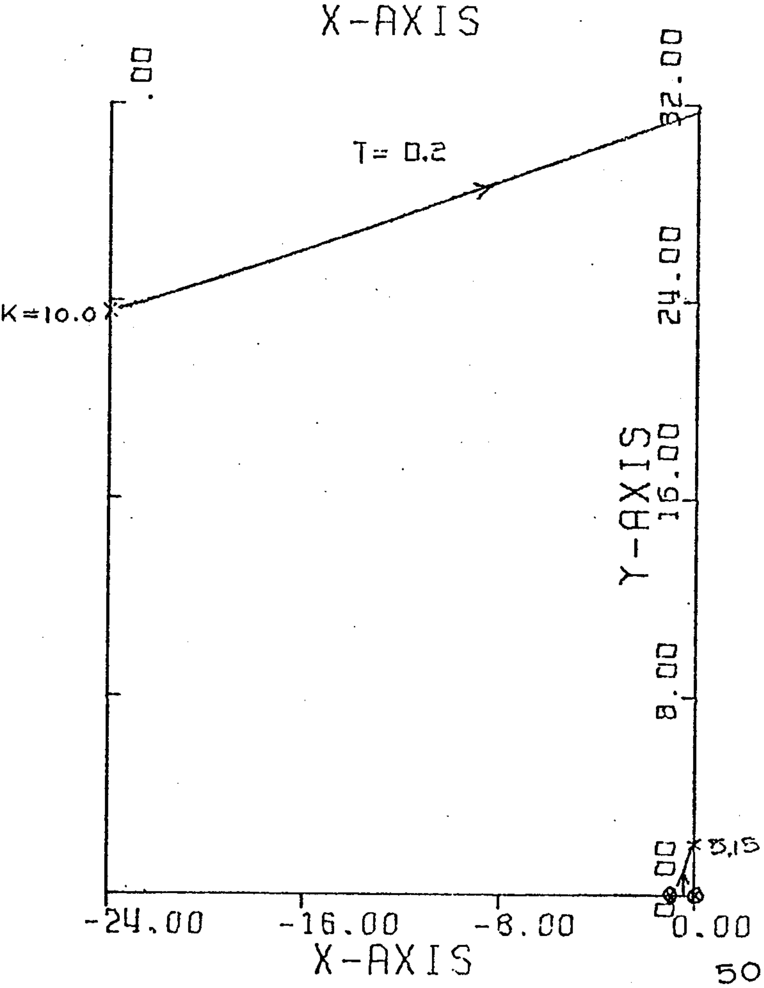
FIGURE 3.6

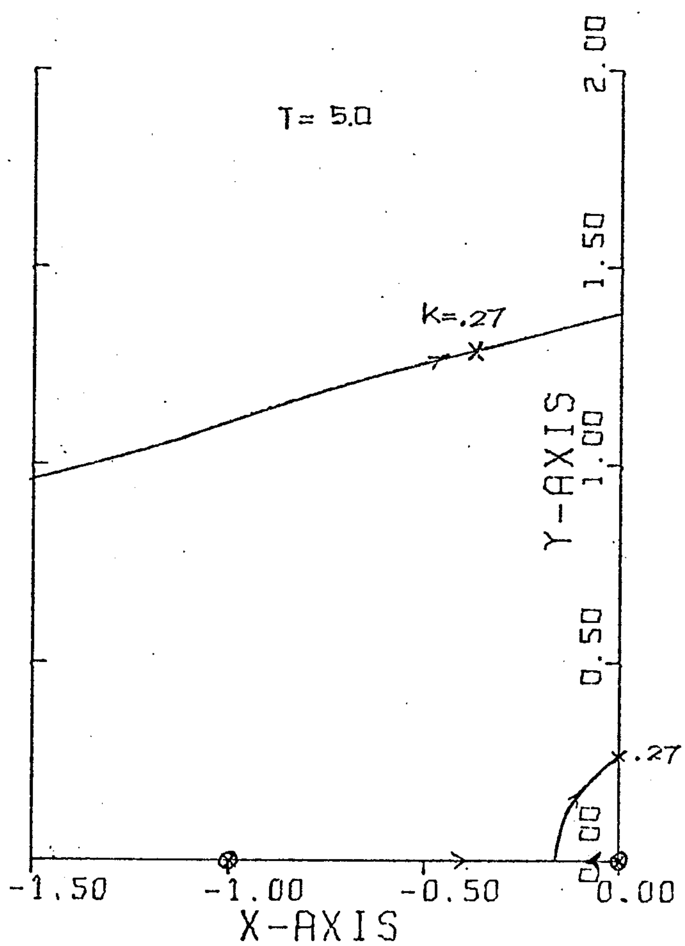
ROOT LOCUS OF $\frac{k}{S(S+1)} e^{-ST}$

FOR VARIOUS DELAY TIMES

⊗ – Location of open loop poles.







Even with a delay time of 5.0, for a theoretical percentage overshoot of 15%, for example, the estimate for percentage overshoot is only 2.5% lower than the simulation, whereas, the estimates for $\omega_o T_p$ and $\omega_o T_s$ are, respectively, 5.5% and 3.3% higher. Again, if Figures 3.3 and 3.6 are considered, the secondary root loci of system II are considerably farther away from their principal root loci compared to those of system I. Therefore, it is expected that the theoretical estimates of system II would be better than those of system I.

It is further observed that for such a set of systems, the estimate for overshoot is lower than the experimental result, whereas, the estimates for normalized peak time and settling time are higher. The discrepancy between the estimates and the experimental results increases for increasing T, but it is important to note that the discrepancies approach limits as $T \rightarrow \infty$. For example, the limits of these discrepancies are, respectively, 3.8%, 11.0% and 9.0% for OS, $\omega_o T_p$ and $\omega_o T_s$, with an estimated system overshoot of 15%.

Since $ke^{-Ts}/s(s+1)$ may be denormalized as $Ke^{-\tau S}/S(S+A)$, from the above observation, it is expected that for any given τ, the disparity between the estimate and experimental result increases for increasing A.

Consider the system such as $ke^{-sT}/s(s+1)$. The phase crossover frequency ω_m must satisfy the following equation

$$\omega_m T + \tan^{-1} \omega_m = \frac{\pi}{2} \qquad 3.33$$

if $T>>1$, then $\omega_m << 1$.

But for $|s|<<1$, the system such as $ke^{-sT}/s(s+1)$ is approximately equal to ke^{-sT}/s.

Consequently, for large values of T, $ke^{-sT}/s(s+1)$ may be expected to approach ke^{-sT}/s.

Moreover, $Ke^{-\tau S}/S$ may be normalized to ke^{-s}/s. Its principal root and gain loci are shown in Figure 3.7 and the quality of its step response is shown in Figure 3.8 with T approaching infinity. It is noteworthy that the error in the theoretical estimate for the set of systems $Ke^{-\tau S}/S$ is independent of the delay time τ.

On the other hand, for any system such as $Ke^{-\tau S}/S(S+A)$, the quality of response may be estimated by only considering the effect of a pair of principal complex roots. The maximum percentage errors between the estimates and experimental results are only 3.8%, 11.% and 9.0% for overshoot, normalized peak time and normalized 2% settling time, respectively, viz. the same as those of $Ke^{-\tau S}/S$.

A further examination was made to investigate whether the theoretical estimate can be improved by considering the effects of both pairs of secondary and principal complex

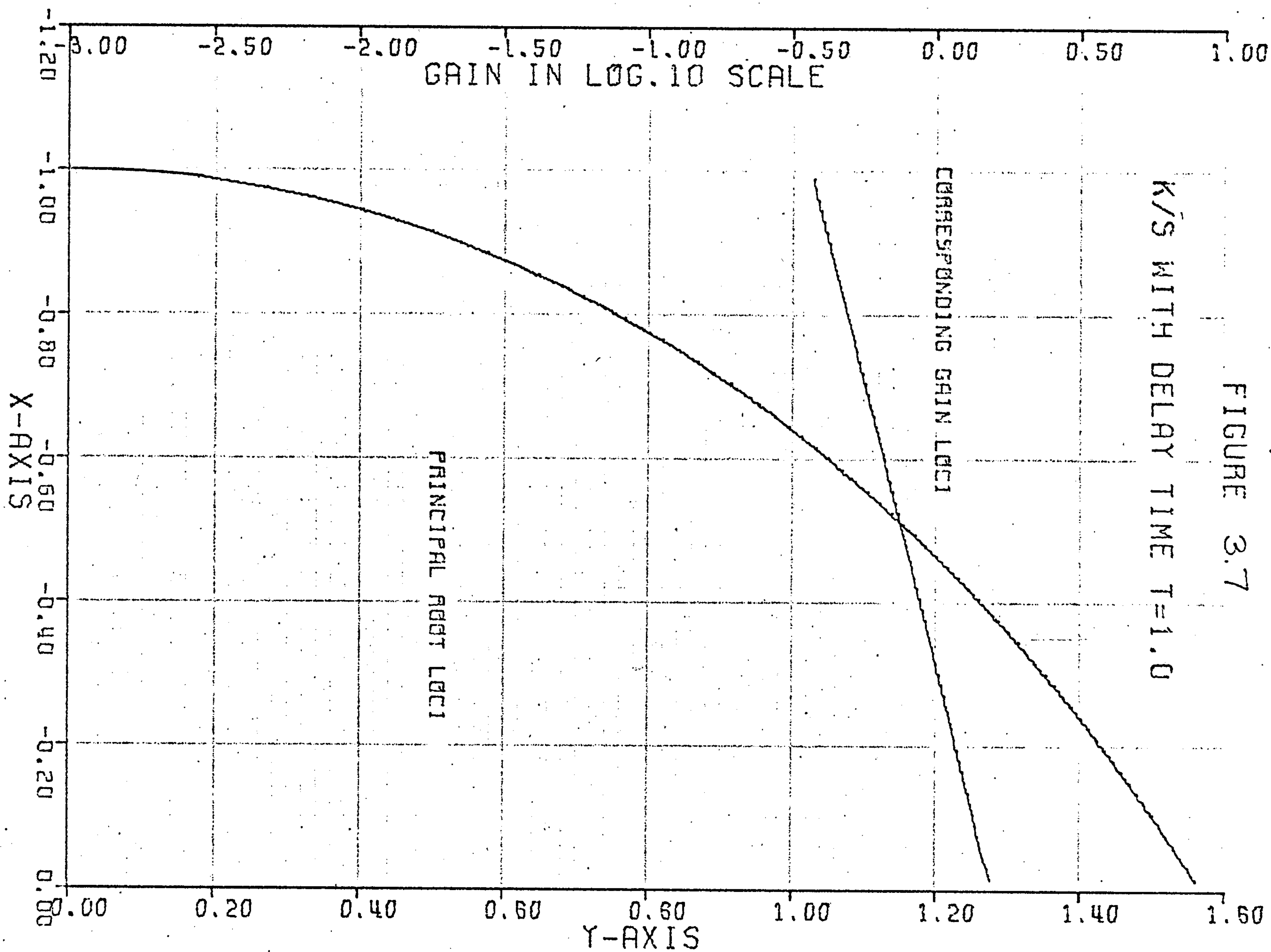


FIGURE 3.7

K/S WITH DELAY TIME T=1.0

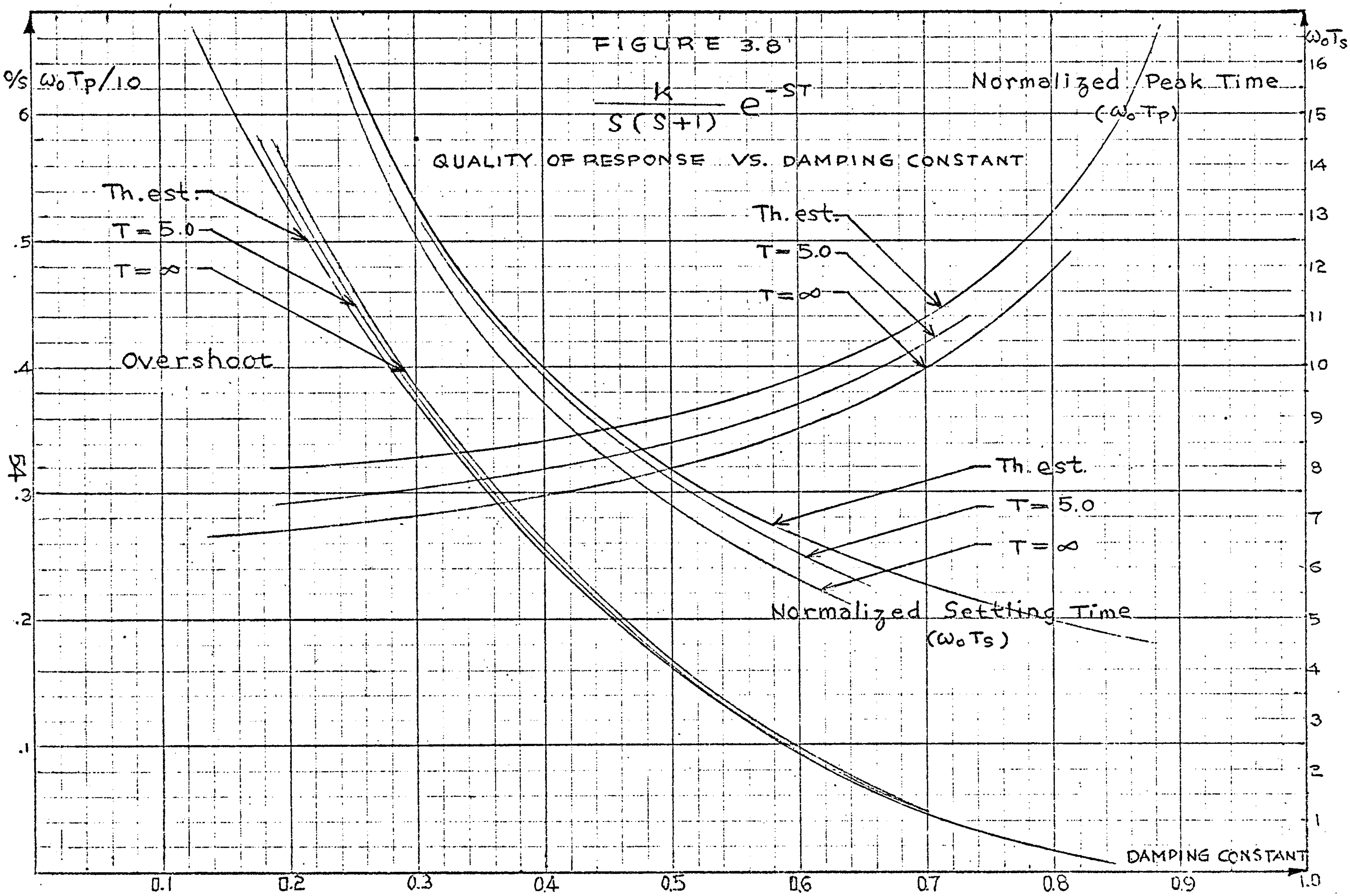


FIGURE 3.8
$\frac{k}{S(S+1)} e^{-ST}$
QUALITY OF RESPONSE VS. DAMPING CONSTANT
%/s $\omega_0 T_p/10$
$\omega_0 T_s$
Normalized Peak Time
$(\omega_0 T_p)$
Normalized Settling Time
$(\omega_0 T_s)$
Overshoot
Th. est.
T = 5.0
T = ∞
DAMPING CONSTANT
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
.1
.2
.3
.4
.5
6
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

54

roots.

As an example, a linear delay control system $G(S)H(S)=Ke^{-S}/S$ was investigated.

For K=1.183 the principal and secondary closed loop roots are located at (-0.2±1.432j) and (-1.9±7.64j), respectively.

The experimental results for a system with the pair of complex roots at (-0.2±1.432j) are as follows

$$T_p = 2.19387$$
$$T_s = 20.0004$$

and $$OS = 0.64483$$

Similarly, experimental data for a system with two pairs of complex roots are as follows

$$T_p = 2.2556$$
$$T_s = 18.2$$

and $$OS = 0.6644$$

The step response for the actual closed loop delay system produces

$$T_p = 1.850$$
$$T_s = 19.5$$

and $$OS = 0.6830$$

In general, when the effects of both pairs of the principal and secondary complex roots are considered in the investigation of the sets of systems such as $ke^{-sT}/s(s+1)$ and $ke^{-sT}/(s+1)$, the estimate of percentage overshoot is improved but the estimate of normalized peak time deteriorates.

Therefore, for these sets of systems the theoretical estimates considering only the principal pair of complex roots are not only simpler to obtain but also are better than the estimates considering both pairs of the principal and secondary complex roots.

System III

Consider the set of second order systems involving time delay with two negative real poles

$$G'(S)\,H'(S) = \frac{K}{(S + A)\ (S + B)}\ e^{-\tau S} \qquad 3.34$$

Such a set of systems may be rewritten as

$$\frac{K/A^2}{(s + 1)\ (s + \frac{B}{A})}\ e^{-(\tau A)s} \qquad \text{or} \qquad \frac{K/B^2}{(s + 1)\ (s + \frac{A}{B})}\ e^{-(\tau B)s}$$

with the substitution of S=As and S=Bs, respectively.

Therefore, such a set of systems may be assessed by referring to the system

$$G(s)H(s) = \frac{k}{(s + 1)\ (s + A)}\ e^{-Ts} \qquad 3.35$$

The principal root loci with various values of A are shown in Figures 3.9 through 3.11 for T = 0.2, 1.0 and 5.0 , respectively. And the plots of the quality of response

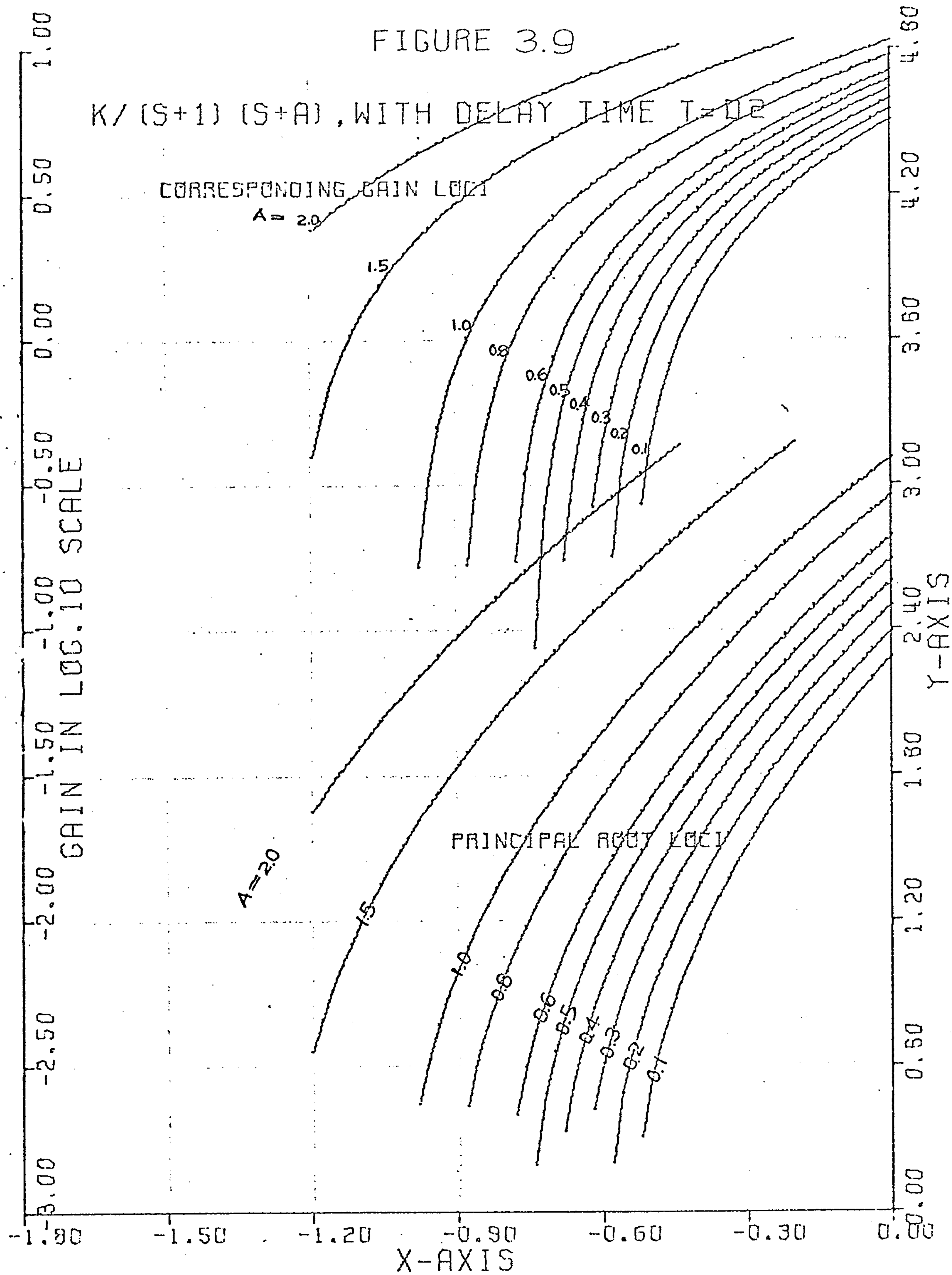


FIGURE 3.9

K/(S+1)(S+A), WITH DELAY TIME T=0.2

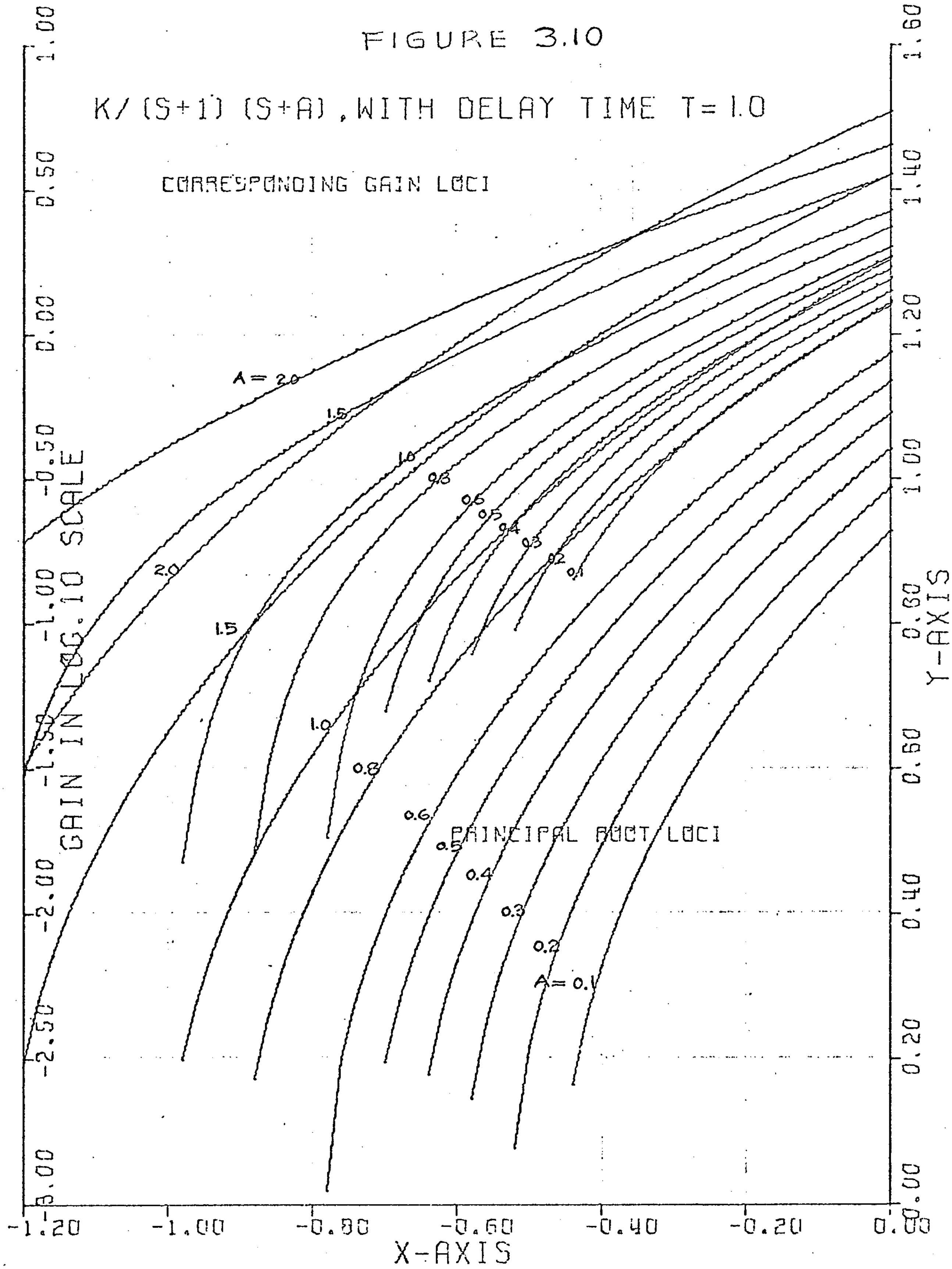


FIGURE 3.10
K/(S+1)(S+A), WITH DELAY TIME T=1.0
CORRESPONDING GAIN LOCI
PRINCIPAL ROOT LOCI
A=2.0
1.5
1.0
0.8
0.6
0.5
0.4
0.3
0.2
0.1
A=0.1
GAIN IN LOG.10 SCALE
Y-AXIS
X-AXIS
-1.20
-1.00
-0.80
-0.60
-0.40
-0.20
0.00
1.00
0.50
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
1.60
1.40
1.20
0.80
0.60
0.40
0.20

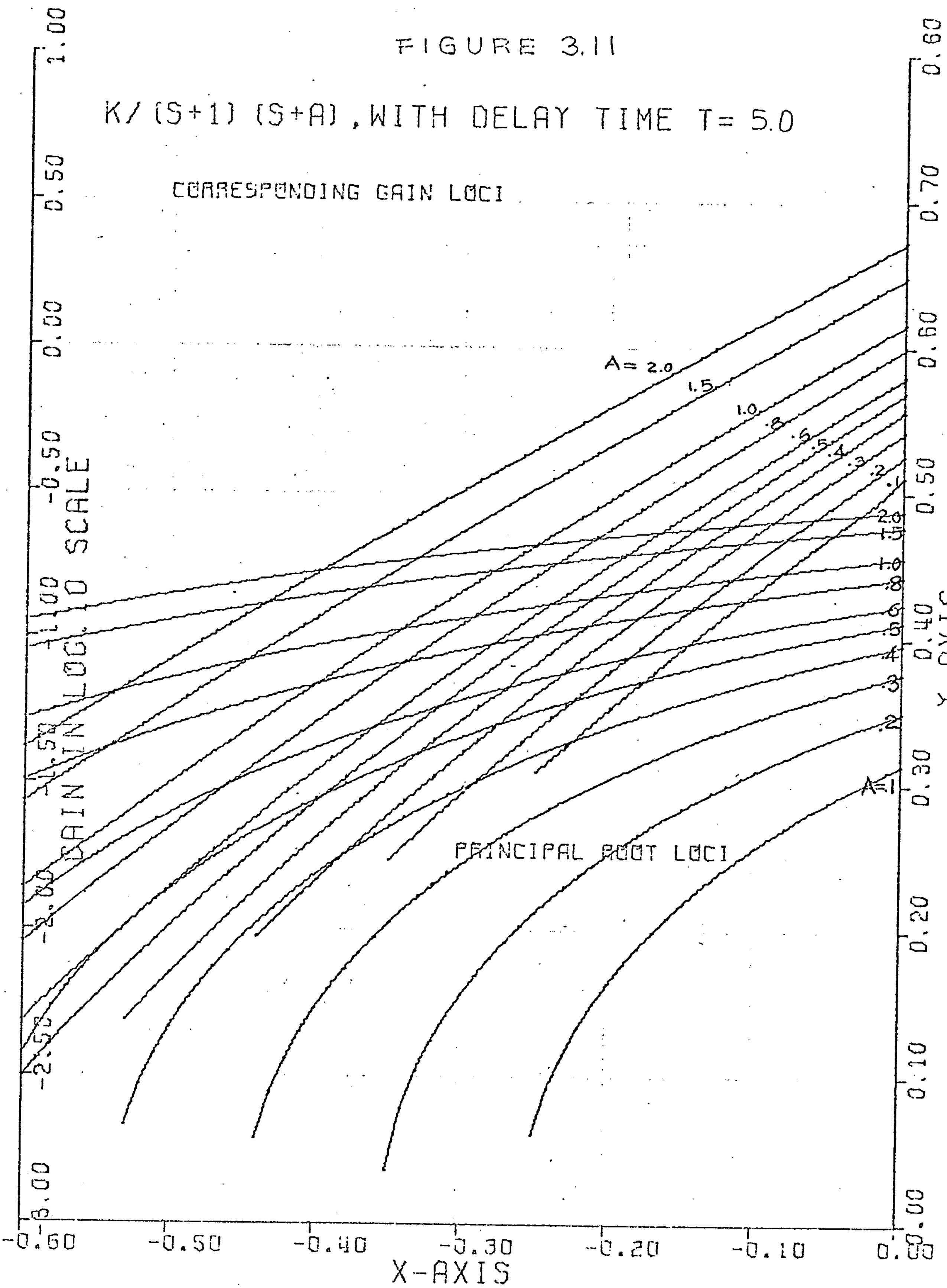


FIGURE 3.11
K/ (S+1) (S+A) ,WITH DELAY TIME T= 5.0
CORRESPONDING GAIN LOCI
PRINCIPAL ROOT LOCI
A= 2.0
1.5
1.0
.8
.6
.5
.4
.3
.2
.1
A=1
GAIN IN LOG.10 SCALE
Y-AXIS
X-AXIS
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.60
-0.50
-0.40
-0.30
-0.20
-0.10
0.00

versus the damping constant of the pair of principal closed loop complex roots for A = 0.2, 1.0 and 5.0 with various values of T are shown in Figures 3.12 through 3.14, respectively.

It is also observed that for such a set of systems, the estimate for percentage overshoot is lower than the experimental result, whereas, the theoretical estimates for the normalized peak time and settling time are higher.

The correlation between the error of the estimate and the location of the open loop poles is also investigated, and the results are shown in Figure 3.15.

From Figure 3.15, it is easily noted, for systems with the same delay time (T=1.0) and the same complex root damping constant, that as the open loop poles get farther away from the origin, the overshoot tends to increase, and the normalized peak time tends to decrease; but as the open loop poles get far enough from the origin, the curves tend to cross one another or even overlap. In any case, the theoretical estimate does provide the designer with reasonable information concerning the quality of response of the system.

System IV

The set of systems of the form

$$G'(S)\ H'(S) = \frac{Ke^{-\tau S}}{(S^2 + 2A\omega_n S + \omega_n^2)} \qquad \text{with } A < 1 \qquad 3.36$$

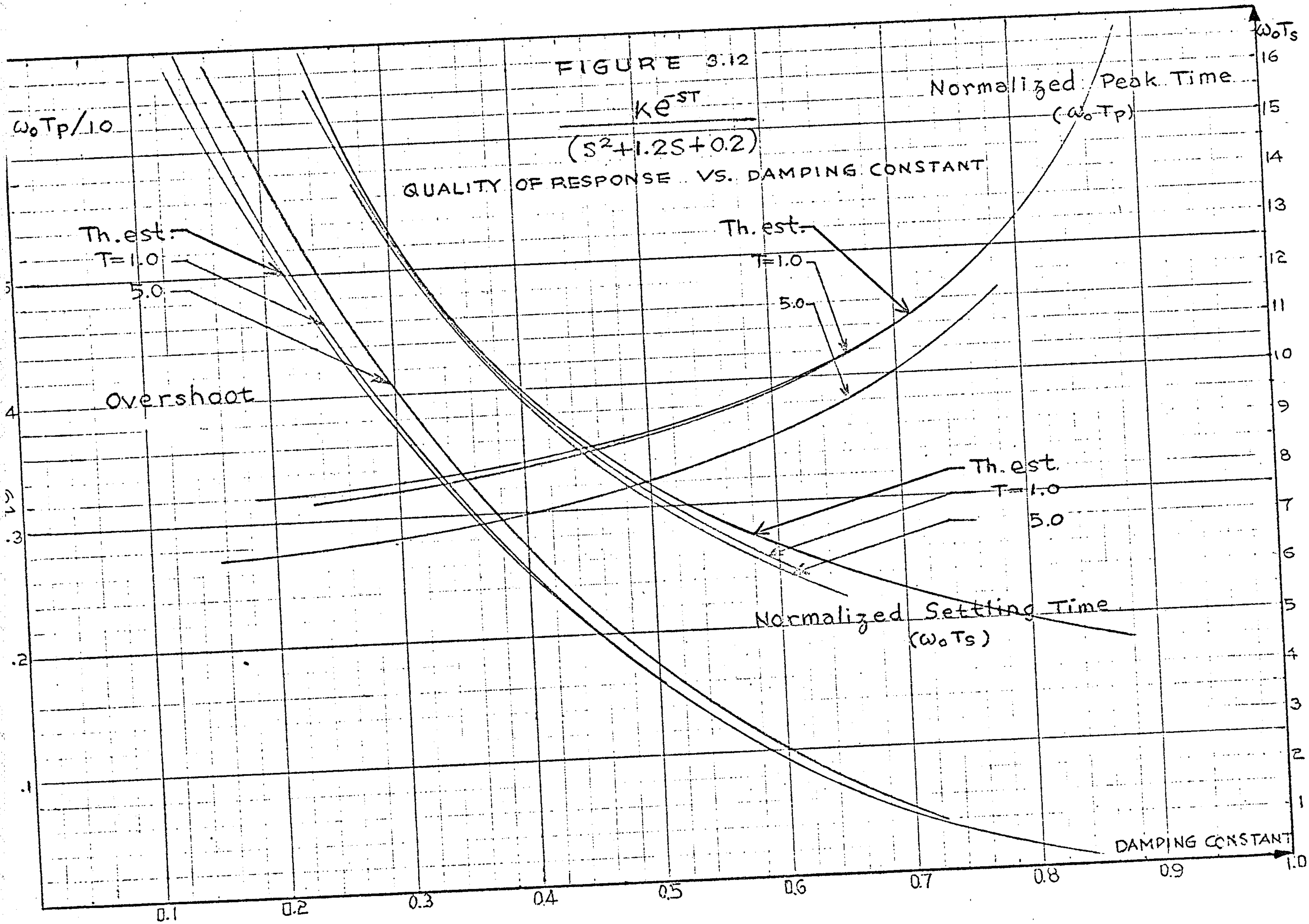


FIGURE 3.12
$\frac{Ke^{-ST}}{(S^2+1.2S+0.2)}$
QUALITY OF RESPONSE VS. DAMPING CONSTANT
$\omega_o T_p/10$
$\omega_o T_s$
Normalized Peak Time
$(\omega_o T_p)$
Normalized Settling Time
$(\omega_o T_s)$
Overshoot
Th. est.
T=1.0
5.0
DAMPING CONSTANT
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
.1
.2
.3
4
5
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

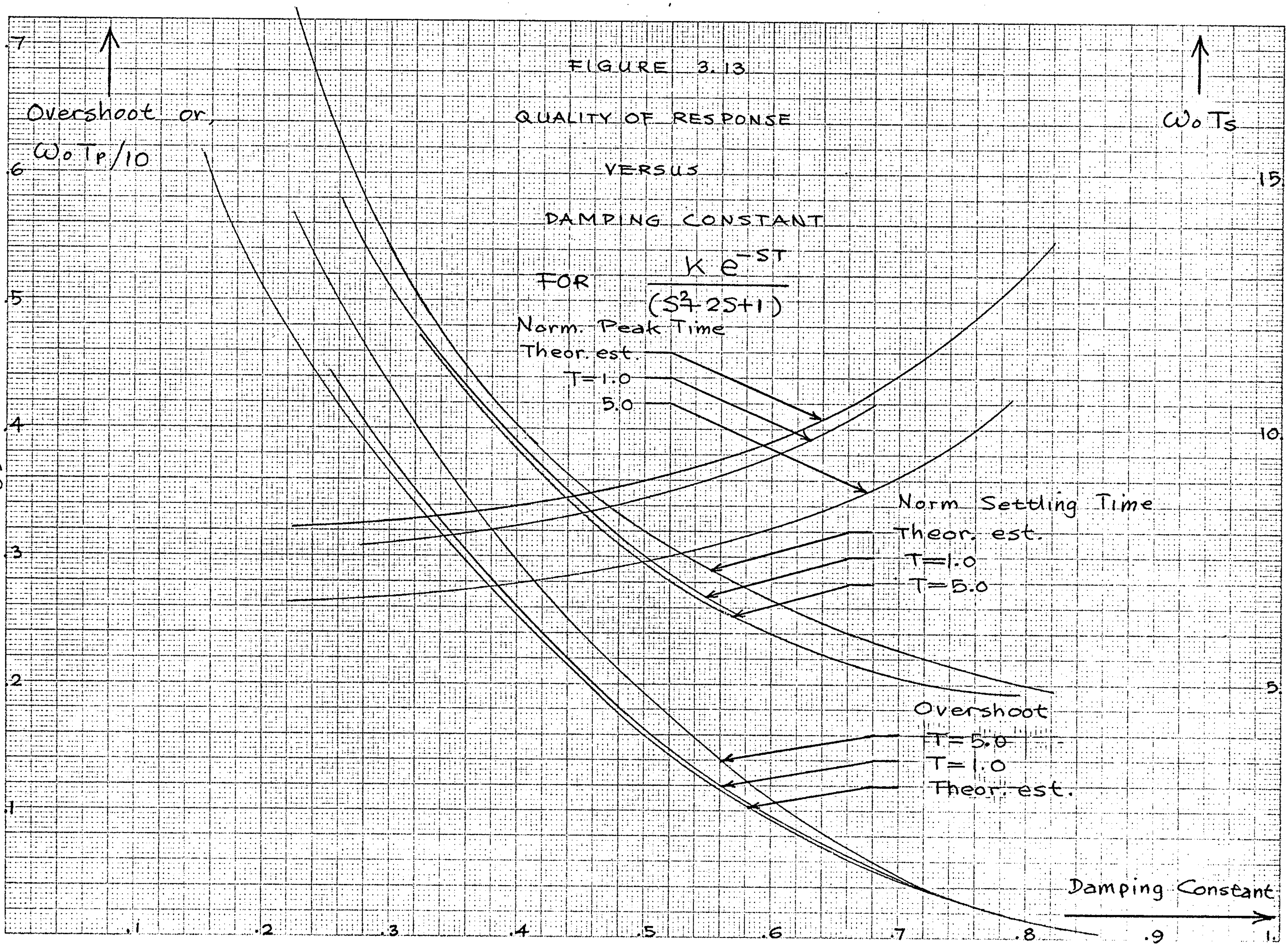


FIGURE 3.13

QUALITY OF RESPONSE VERSUS DAMPING CONSTANT

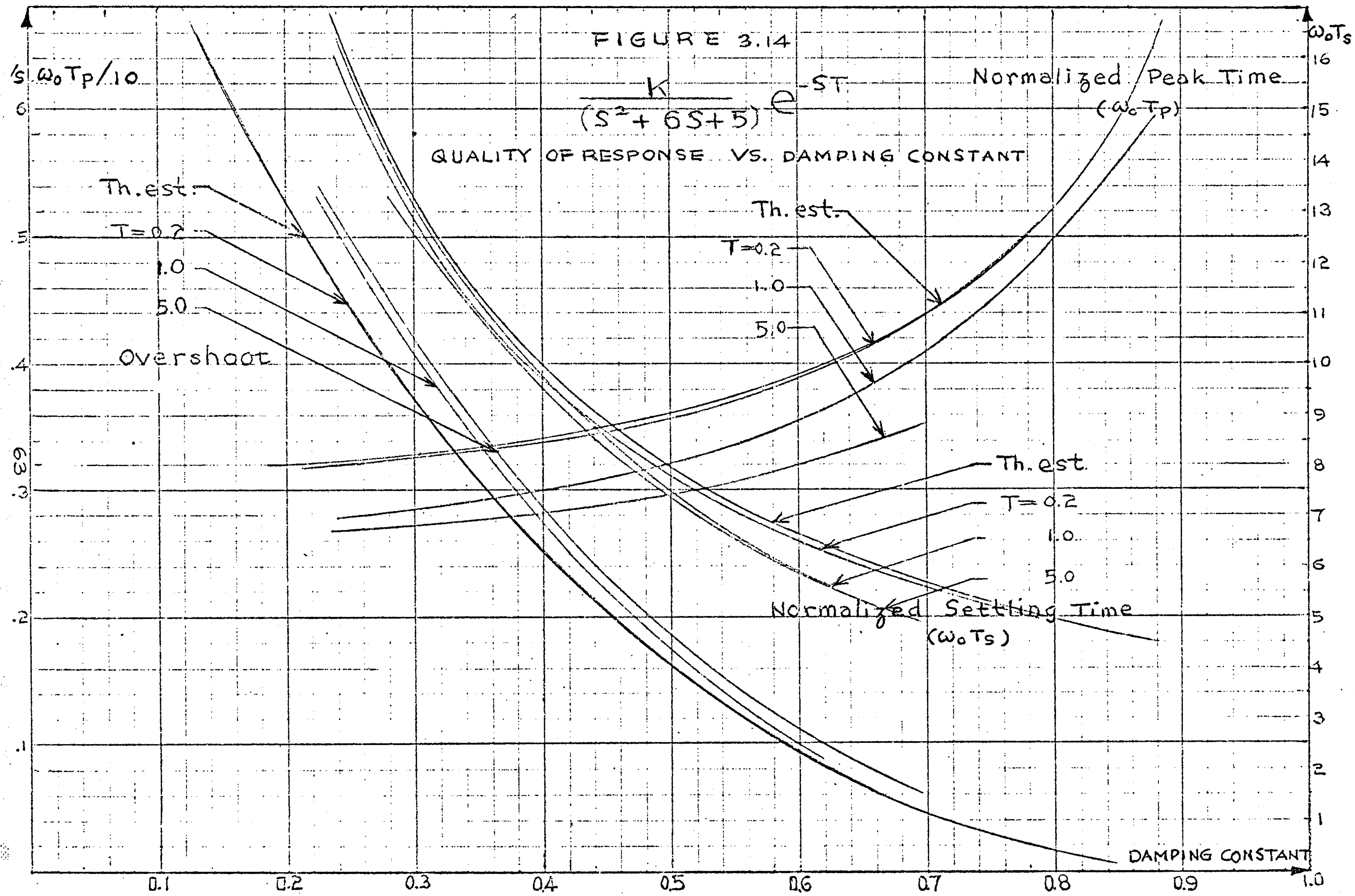


FIGURE 3.14
$\frac{K}{(S^2+6S+5)} e^{-ST}$
QUALITY OF RESPONSE VS. DAMPING CONSTANT
.5 $\omega_o T_p/10$
Normalized Peak Time
($\omega_o T_p$)
$\omega_o T_s$
Th. est.
T=0.2
1.0
5.0
Overshoot
Th. est.
T=0.2
1.0
5.0
Th. est.
T=0.2
1.0
5.0
Normalized Settling Time
($\omega_o T_s$)
DAMPING CONSTANT
.6
.5
.4
.3
.2
.1
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
63

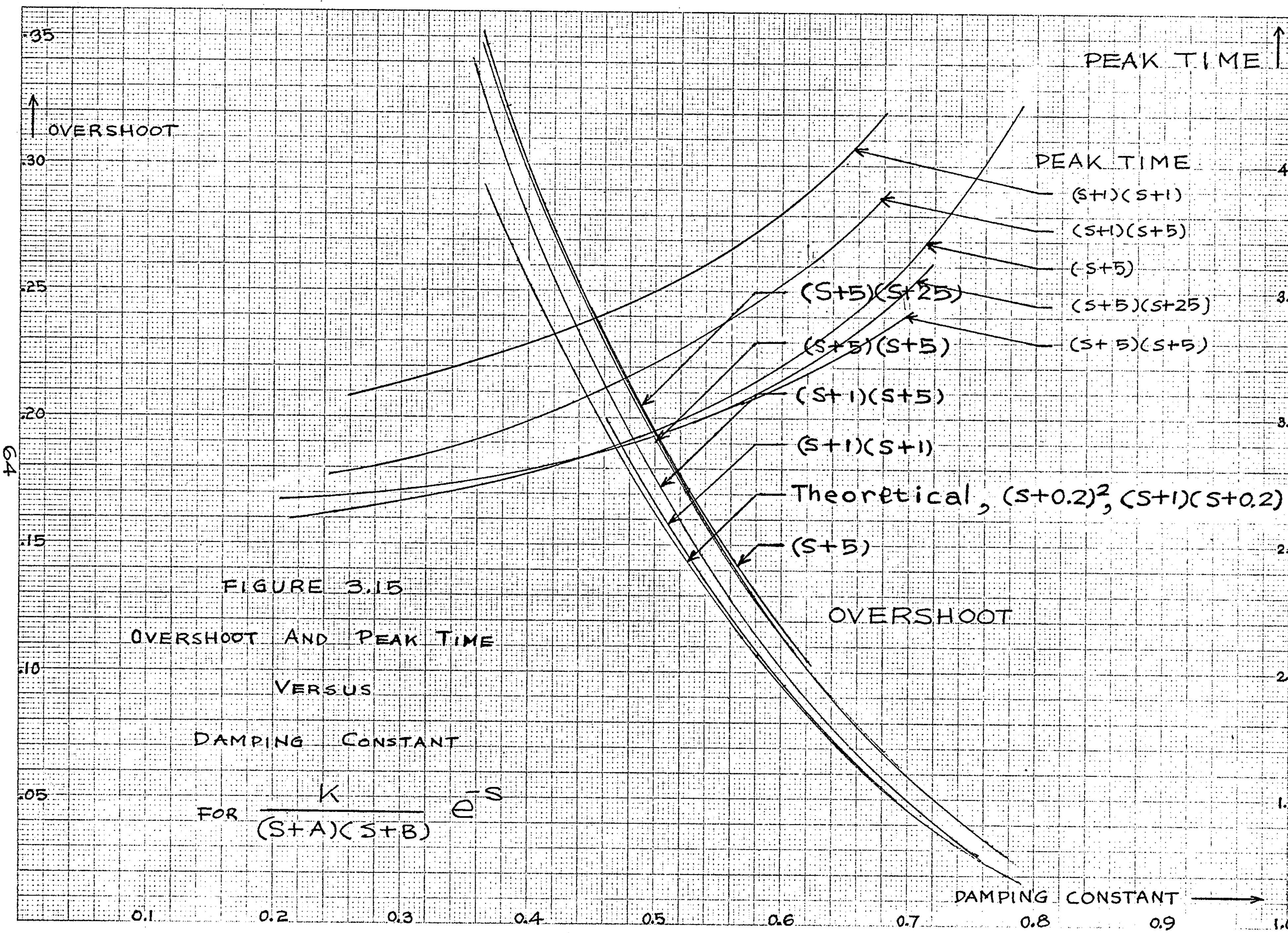


FIGURE 3.15

OVERSHOOT AND PEAK TIME VERSUS DAMPING CONSTANT

FOR $\frac{K}{(S+A)(S+B)} e^{-S}$

is investigated in the normalized form

$$G(s)\,H(s) = \frac{k}{(s^2 + 2A\,s + 1)}\;e^{-sT} \tag{3.37}$$

The graph showing the relation between the quality of response and the damping constant ξ for $ke^{-sT}/(s^2+1.2s+1)$ with T=0.2,1.0 and 5.0 is shown in Figure 3.16, whereas, the overshoot for systems with delay time T=5.0 but various values of A is shown in Figure 3.17.

In general, for smaller values of A and larger values of T, the theoretical estimates of the quality of response are very poor. In particular, for T = 5.0 and A =0.2, the pair of secondary complex roots is so close to the principal complex roots that the theoretical estimates considering only the effect of the pair of principal roots become very misleading. A section of the root loci of such systems with T=1.0 and 5.0 are shown in Figures 3.18 and 3.19, respectively.

In view of this, special attention has to be paid to the estimates for this set of systems.

Since for T = 0.2 and 1.0, the theoretical estimates are still reasonably good, the principal root and gain loci are shown in Figures 3.20 and 3.21 as a reference for design.

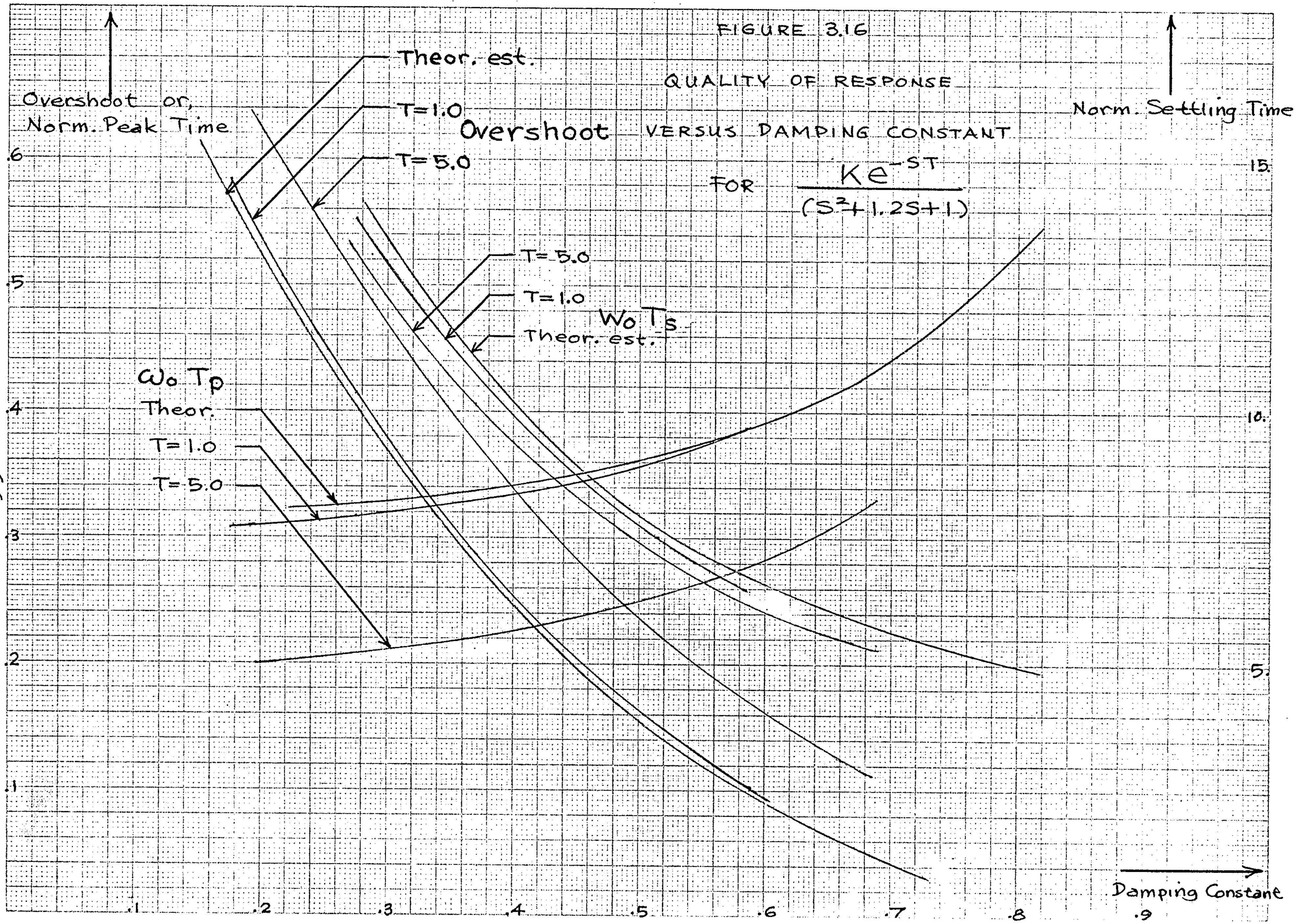


FIGURE 3.16
QUALITY OF RESPONSE
Overshoot VERSUS DAMPING CONSTANT
FOR $\frac{Ke^{-ST}}{(S^2+1.2S+1)}$
Overshoot or, Norm. Peak Time
Norm. Settling Time
Theor. est.
T=1.0
T=5.0
T=5.0
T=1.0
Wo Ts
Theor. est.
ωo Tp
Theor.
T=1.0
T=5.0
.6
.5
.4
.3
.2
.1
15.
10.
5.
.1
.2
.3
.4
.5
.6
.7
.8
.9
Damping Constant

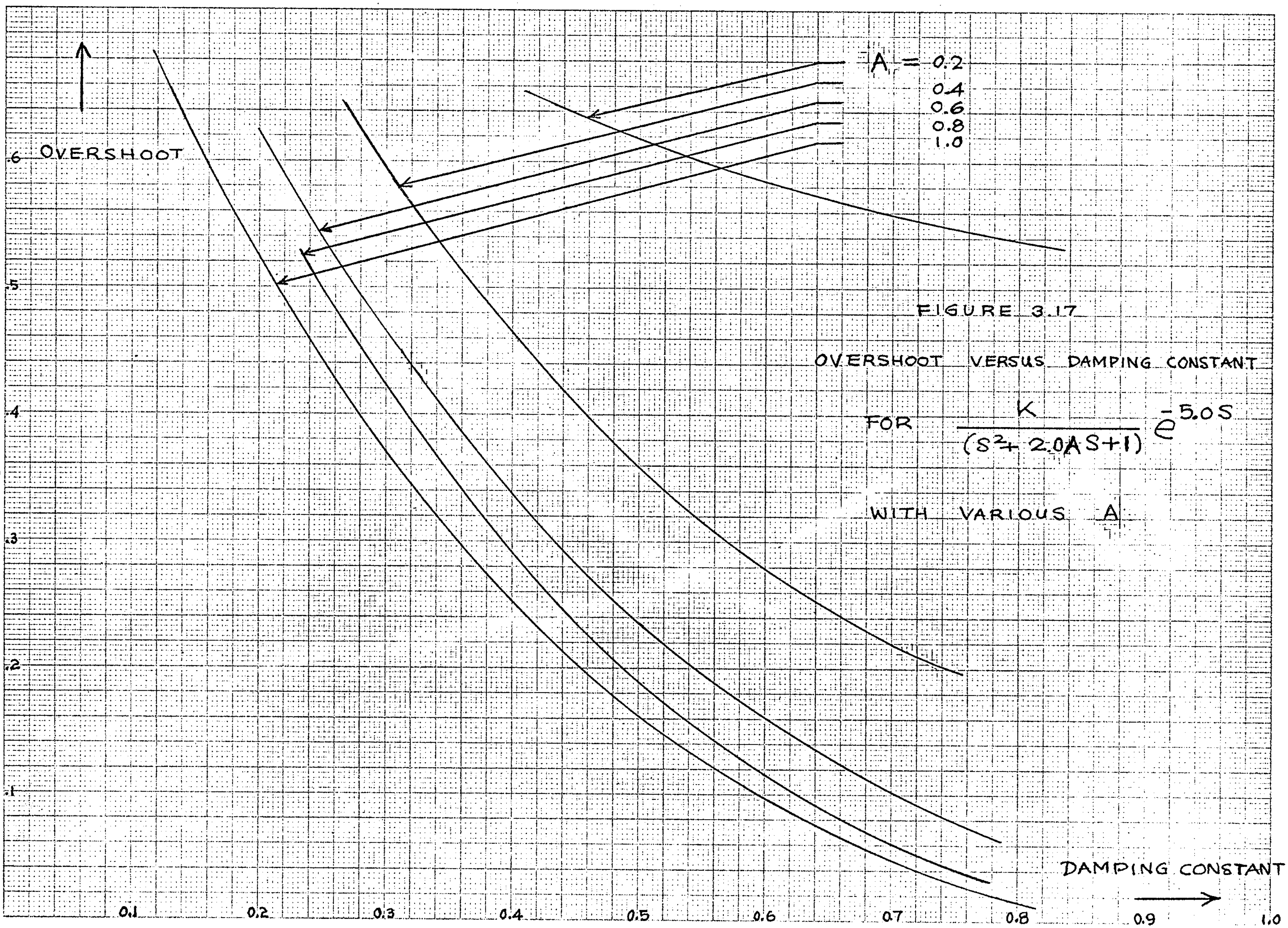


FIGURE 3.17

OVERSHOOT VERSUS DAMPING CONSTANT

FOR $\frac{K}{(S^2 + 2.0AS + 1)} e^{-5.0S}$

WITH VARIOUS A

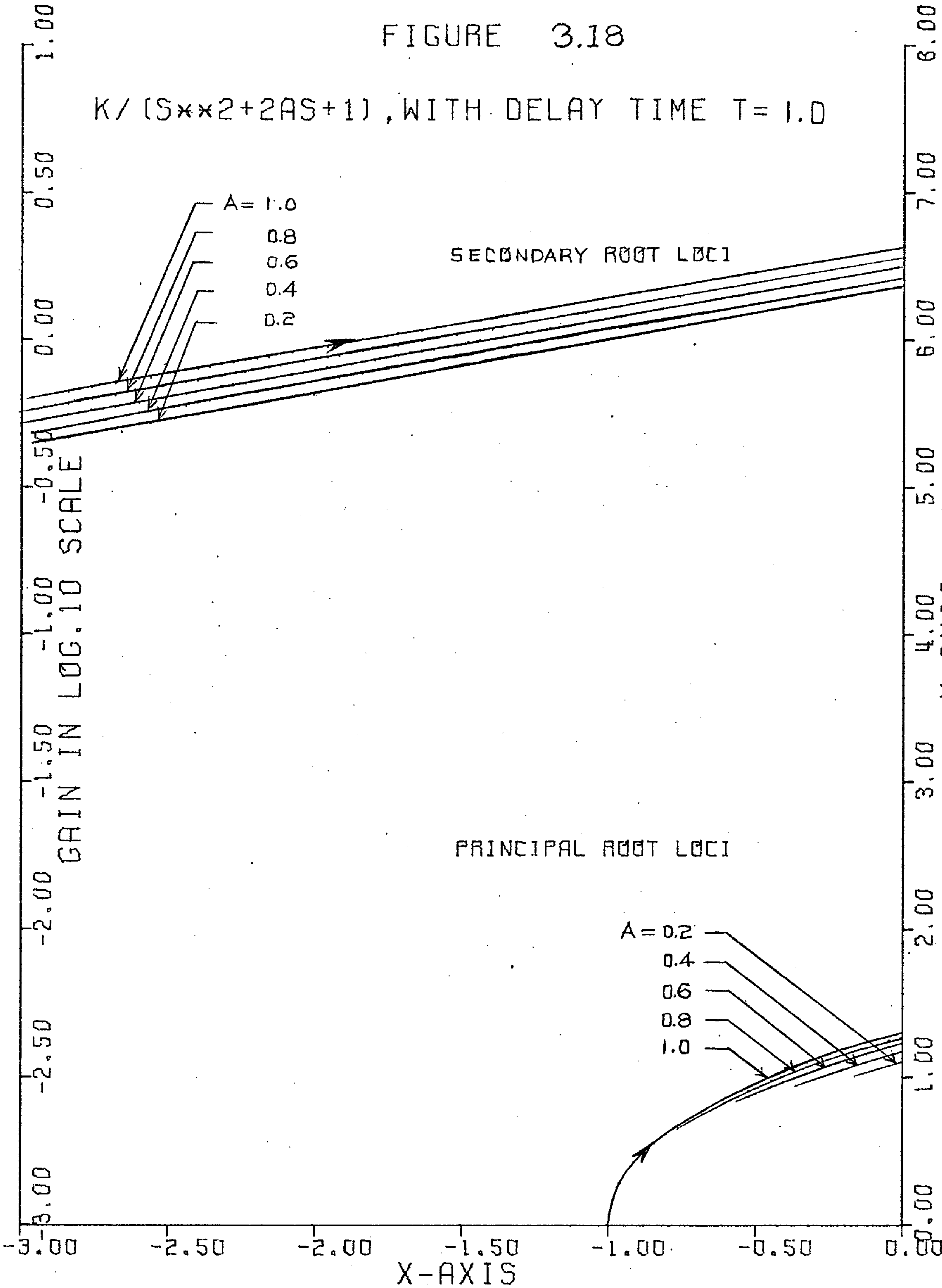


FIGURE 3.18
K/ [S**2+2AS+1] ,WITH DELAY TIME T= 1.0
A= 1.0
0.8
0.6
0.4
0.2
SECONDARY ROOT LOCI
PRINCIPAL ROOT LOCI
A = 0.2
0.4
0.6
0.8
1.0
GAIN IN LOG.10 SCALE
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
Y-AXIS
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
-3.00
-2.50
-2.00
-1.50
-1.00
-0.50
0.00
X-AXIS

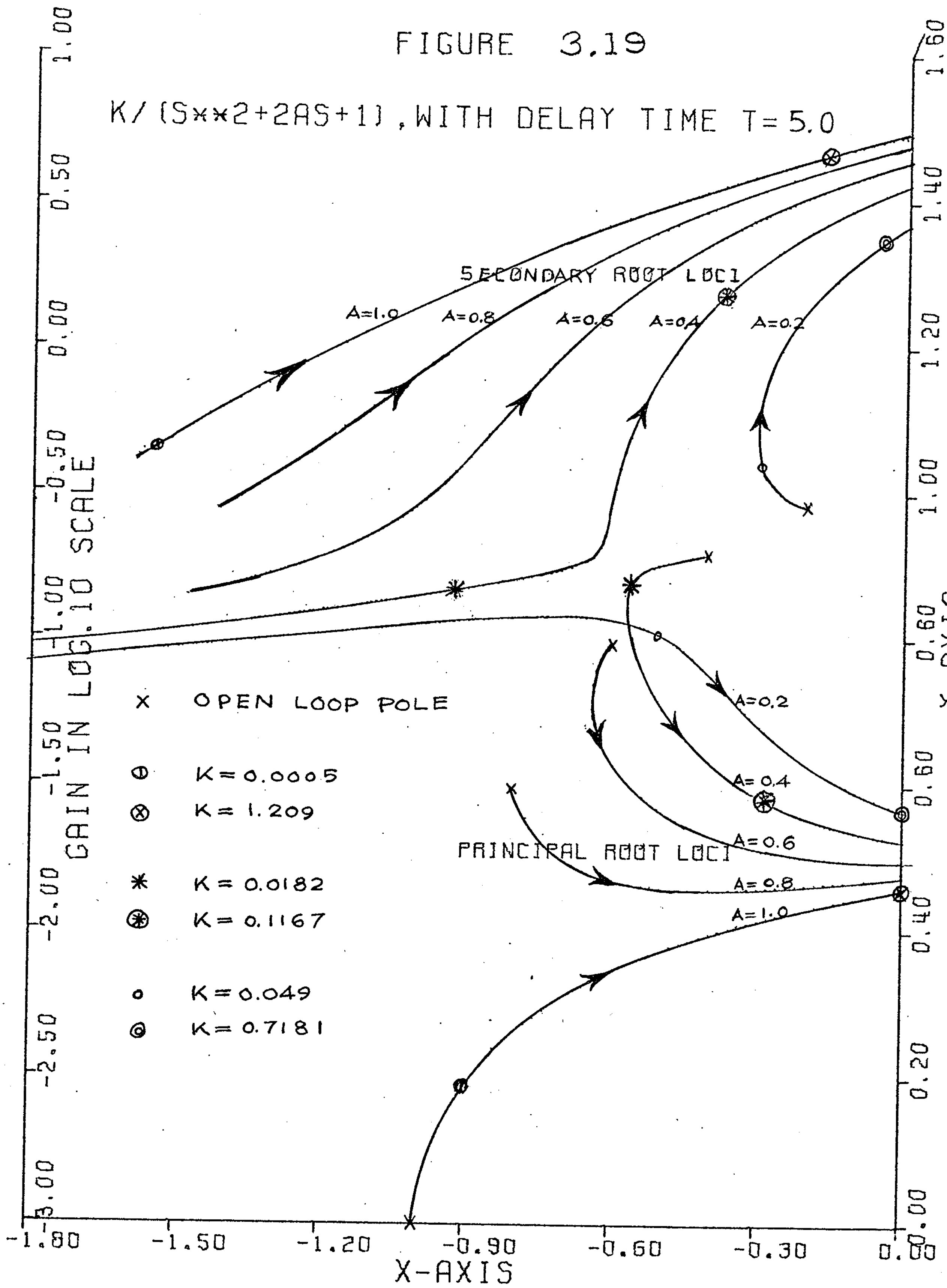
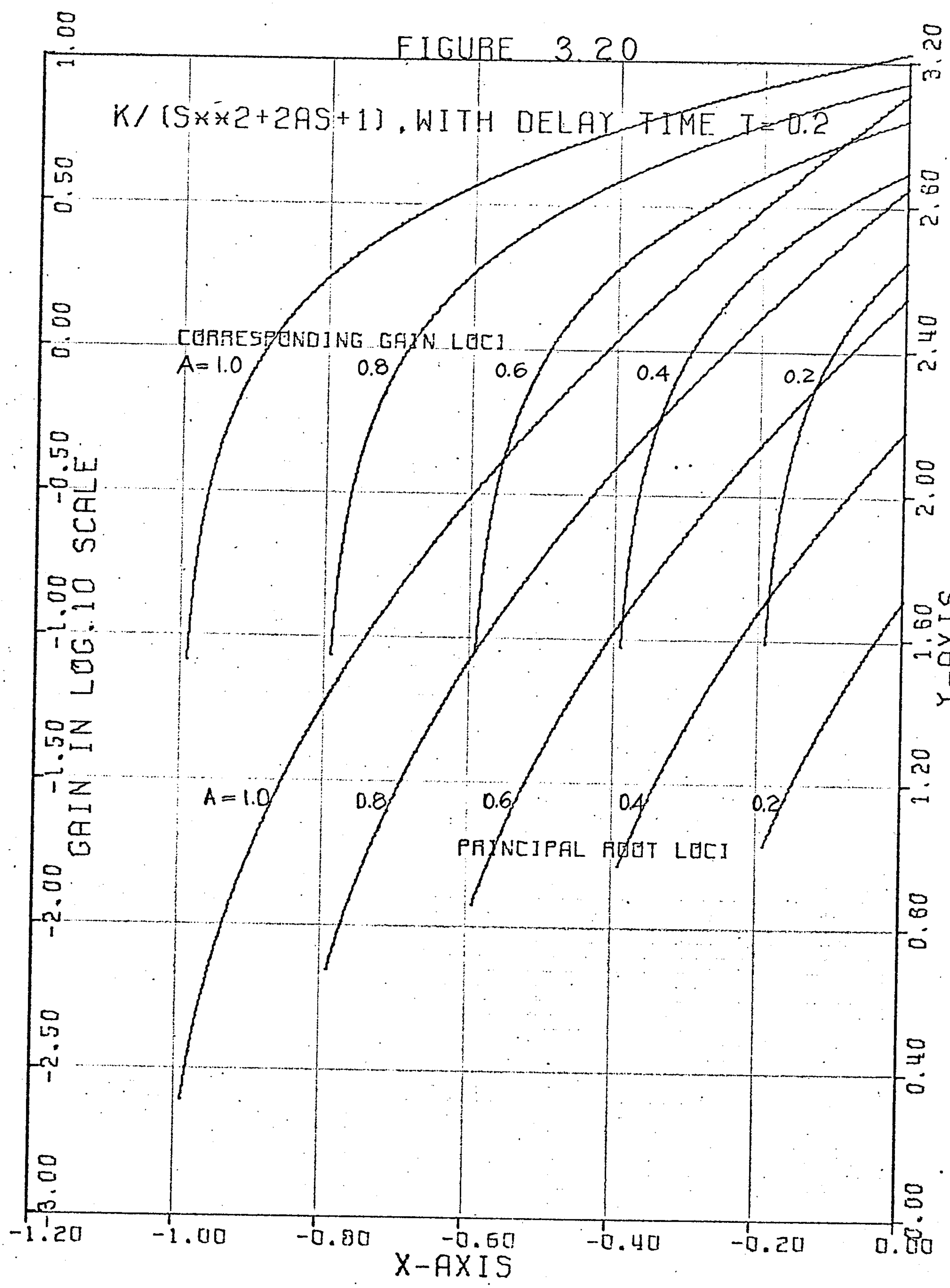


FIGURE 3.19

K/(S**2+2AS+1), WITH DELAY TIME T=5.0

FIGURE 3.20

K/(S**2+2AS+1),WITH DELAY TIME T=0.2



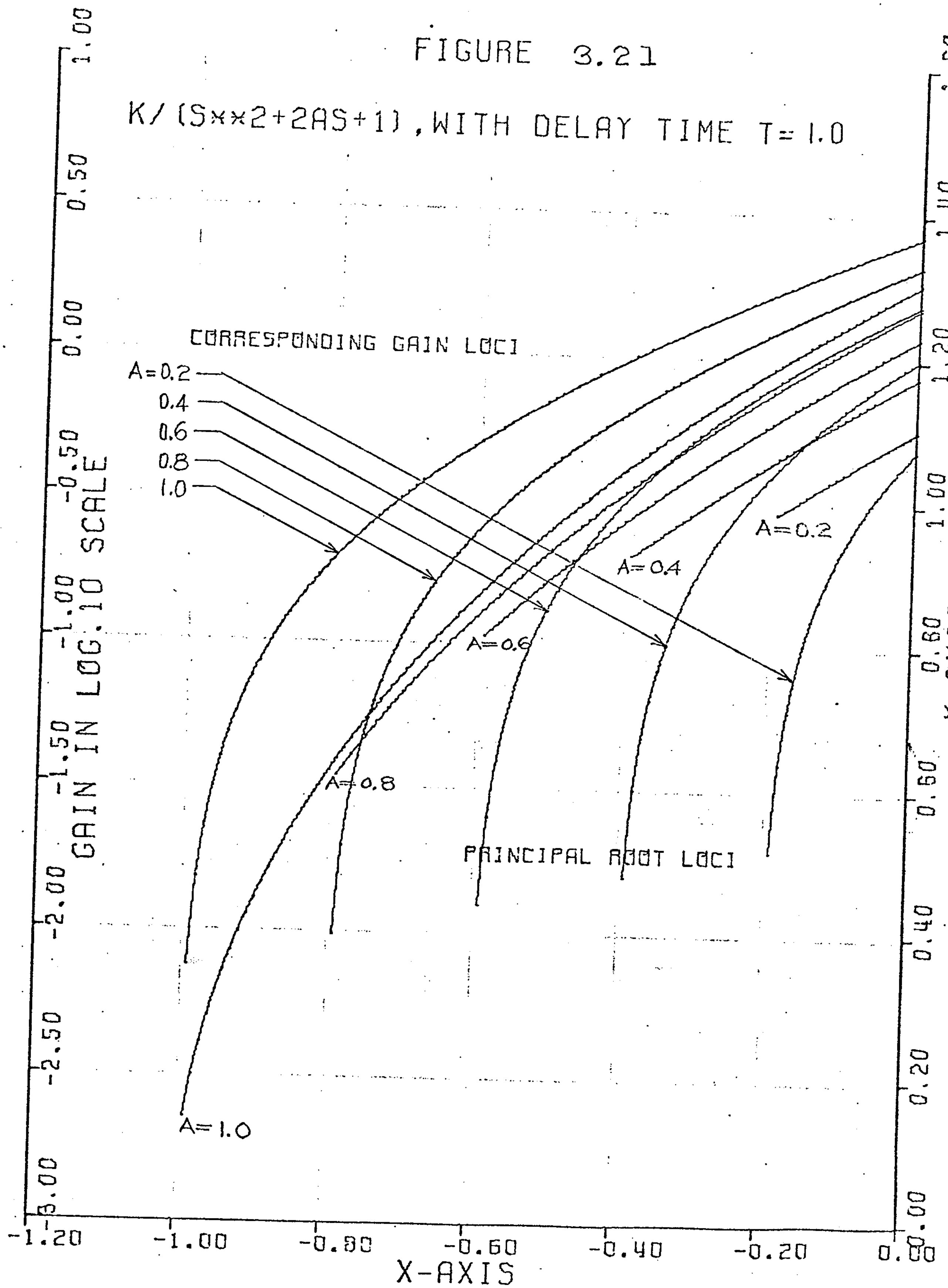


FIGURE 3.21
K/(S**2+2AS+1),WITH DELAY TIME T=1.0
CORRESPONDING GAIN LOCI
A=0.2
0.4
0.6
0.8
1.0
A=0.2
A=0.4
A=0.6
A=0.8
A=1.0
PRINCIPAL ROOT LOCI
GAIN IN LOG.10 SCALE
Y-AXIS
X-AXIS
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
-1.20
-1.00
-0.80
-0.60
-0.40
-0.20
0.00

System V

The set of systems of the form

$$G'(S)\ H'(S) = \frac{Ke^{-\tau S}}{S(S + A)\ (S + B)} \qquad 3.38$$

is also investigated in the normalized form

$$G(s)\ H(s) = \frac{k}{s(s + 1)\ (s + A)}\ e^{-sT} \qquad 3.39$$

The principal root loci and gain loci for such a set of systems are shown in Figures 3.22 through 3.24.

The plot of the quality of response versus the damping constant ξ for systems with T =5.0 and various values of A is shown in Figure 3.25.

Even for the system such as $ke^{-5s}/s(s+1)(s+5)$, the differences between the estimated and simulated results are negligibly small.

A comparison of the quality of response for

$$\frac{k}{s(s + A)}\ e^{-sT} \quad \text{and} \quad \frac{k}{s(s + A)^2}\ e^{-sT}$$

with T = 1.0 is shown in Figure 3.26.

Therefore, for this set of systems, the theoretical estimate provides very reasonable information about the quality of response of the systems.

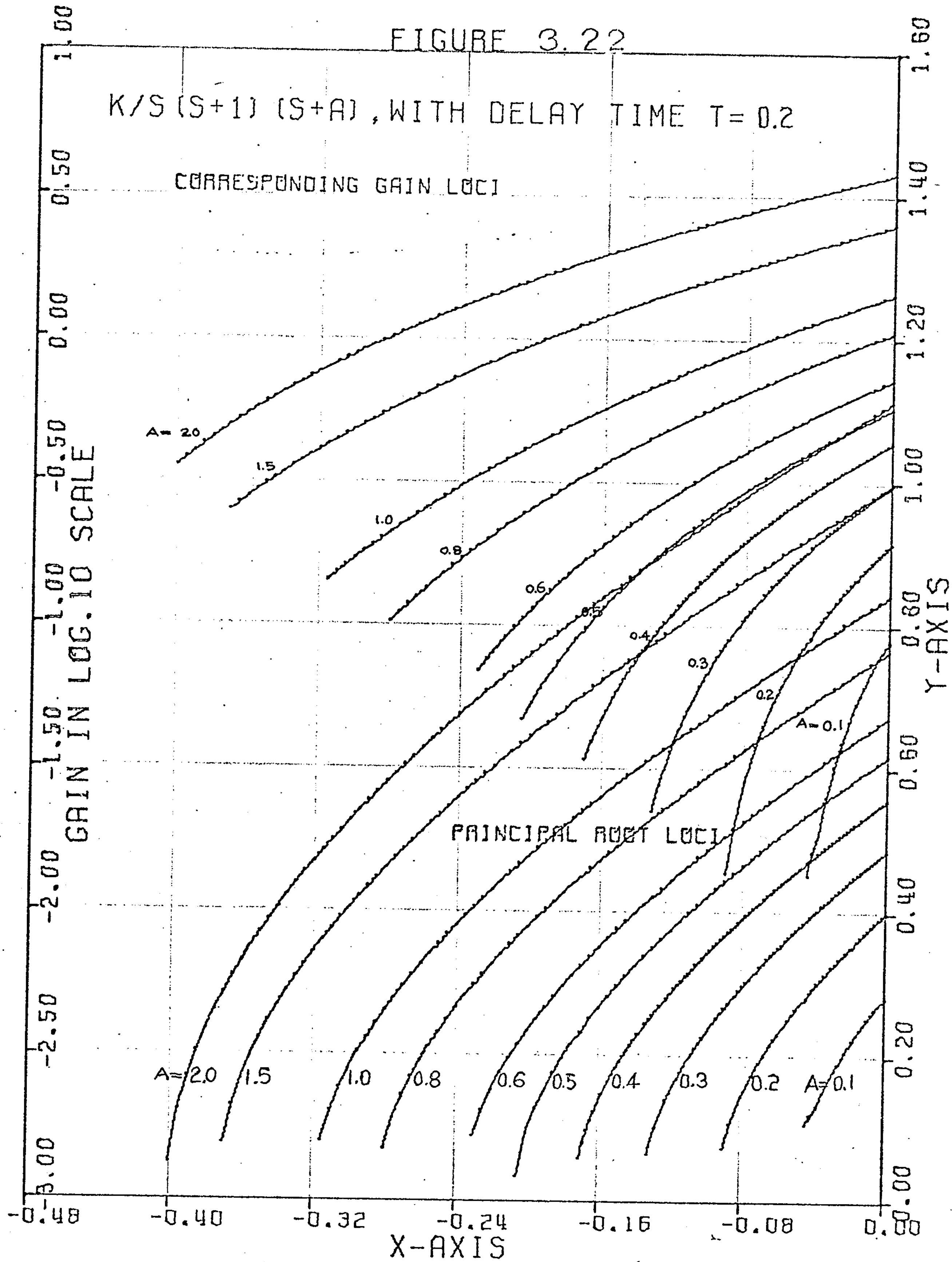


FIGURE 3.22
K/S (S+1) (S+A), WITH DELAY TIME T= 0.2
CORRESPONDING GAIN LOCI
A= 2.0
1.5
1.0
0.8
0.6
0.5
0.4
0.3
0.2
A= 0.1
PRINCIPAL ROOT LOCI
A= 2.0
1.5
1.0
0.8
0.6
0.5
0.4
0.3
0.2
A= 0.1
GAIN IN LOG.10 SCALE
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
Y-AXIS
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
X-AXIS
-0.48
-0.40
-0.32
-0.24
-0.16
-0.08
0.00

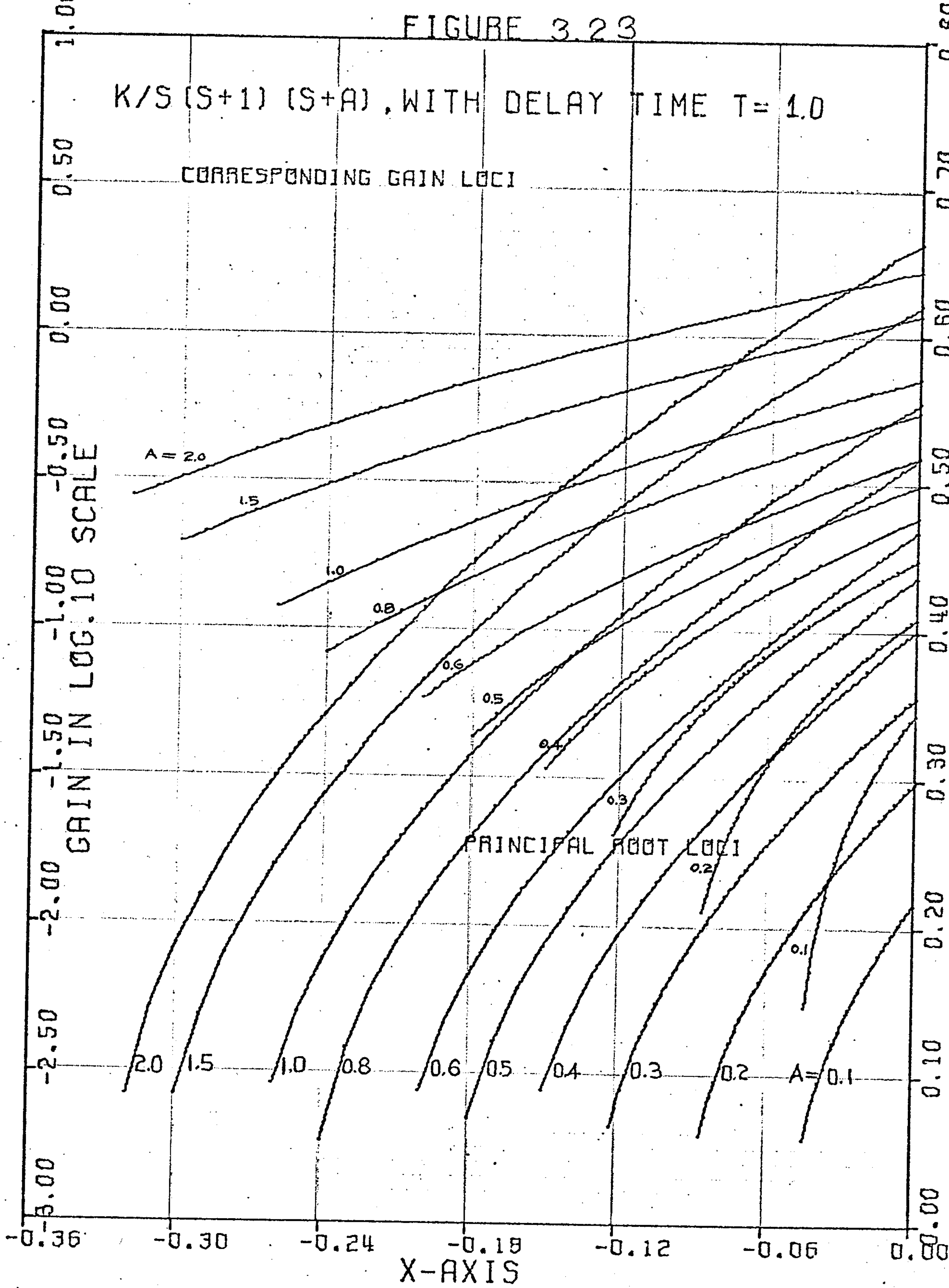
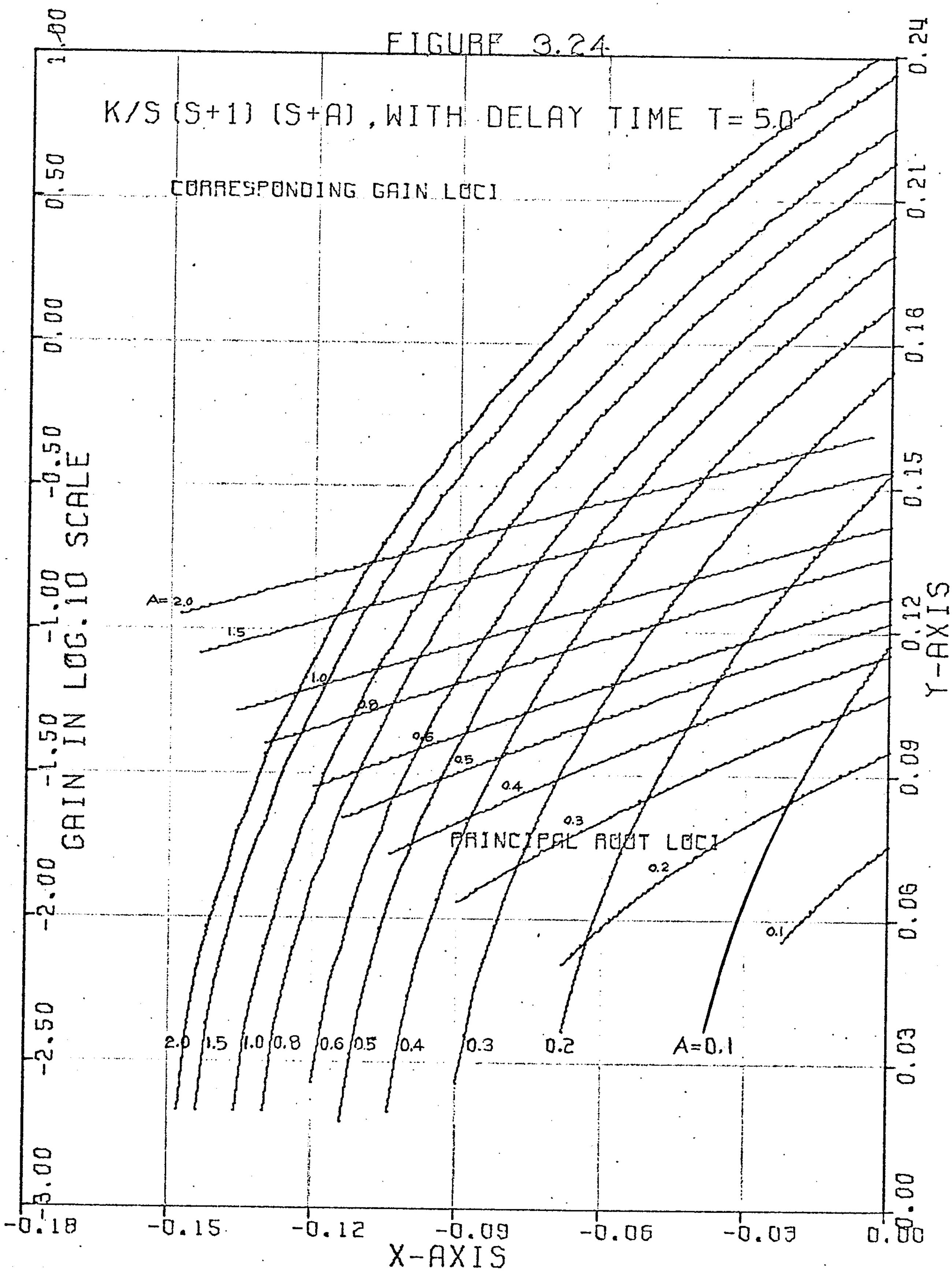


FIGURE 3.23
K/S (S+1) (S+A), WITH DELAY TIME T= 1.0
CORRESPONDING GAIN LOCI
PRINCIPAL ROOT LOCI
GAIN IN LOG.10 SCALE
X-AXIS
Y-AXIS
A= 2.0
1.5
1.0
0.8
0.6
0.5
0.4
0.3
0.2
A= 0.1

FIGURE 3.24


K/S (S+1) (S+A), WITH DELAY TIME T=5.0
CORRESPONDING GAIN LOCI
PRINCIPAL ROOT LOCI
A=2.0
1.5
1.0
0.8
0.6
0.5
0.4
0.3
0.2
0.1
A=0.1
GAIN IN LOG.10 SCALE
X-AXIS
Y-AXIS

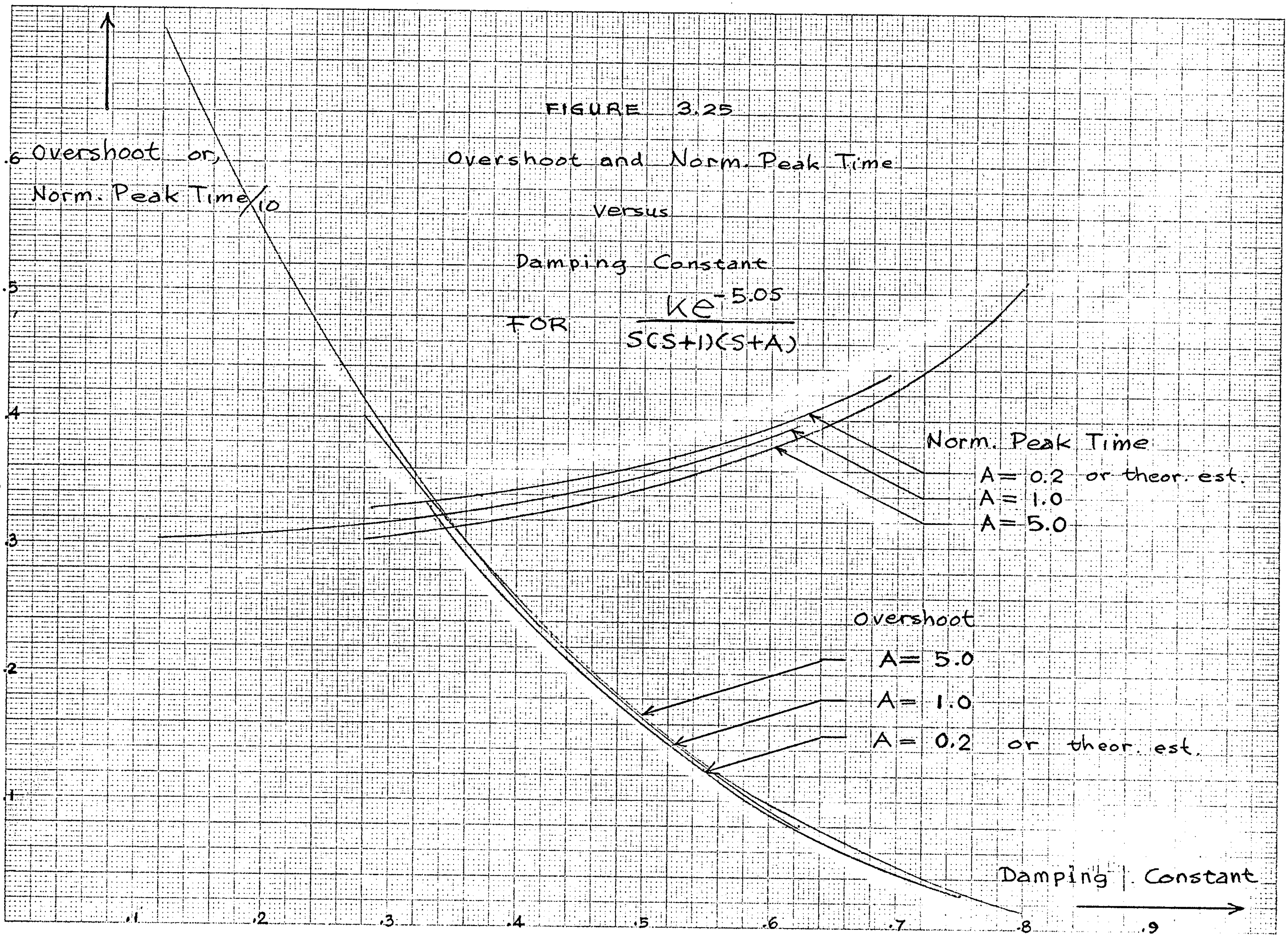


FIGURE 3.25
Overshoot and Norm. Peak Time
Versus
Damping Constant
FOR $\frac{Ke^{-5.05}}{S(S+1)(S+A)}$
Overshoot or
Norm. Peak Time/10
Norm. Peak Time
A = 0.2 or theor. est.
A = 1.0
A = 5.0
Overshoot
A = 5.0
A = 1.0
A = 0.2 or theor. est.
Damping Constant
.1
.2
.3
.4
.5
.6
.7
.8
.9
.6
.5
.4
.3
.2
.1

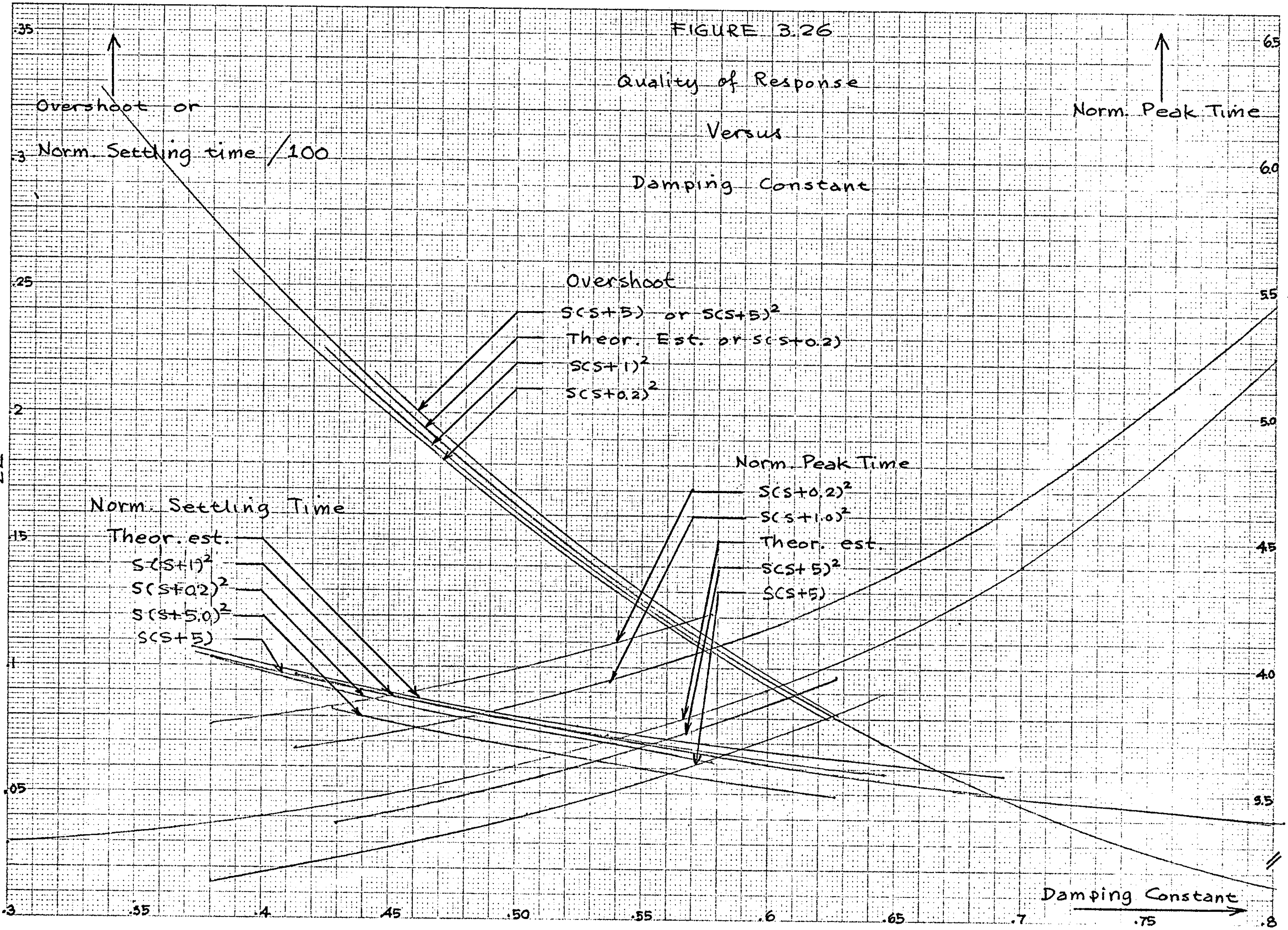


FIGURE 3.26

Quality of Response Versus Damping Constant

System VI

The set of systems of the form

$$G'(S)\ H'(S) = \frac{Ke^{-S\tau}}{S(S^2 + 2A\omega_n S + \omega_n^2)}\ , \text{ with } A < 1 \qquad 3.40$$

is also investigated in the normalized form

$$G(s)\ H(s) = \frac{ke^{-sT}}{s(s^2 + 2A\ s + 1)} \qquad 3.41$$

The graph showing the relationship between the quality of response and the damping constant ξ for various A values and T=5 is shown in Figure 3.27, whereas, the graph for systems with A = 0.6 and various delay times is given in Figure 3.28.

In contrast to system IV, for large T values, the error of the theoretical estimate is rather small, but for small T and K values the error is quite significant. This is probably due to the existence of the dominant negative real pole for smaller T and K values. To support this hypothesis, the root loci for this system with A = 0.6 and T = 0.2, 1.0 and 5.0 are presented in Figure 3.29. It is apparent that for T = 0.2 and smaller K values, the transient response of the system is dominated by the negative real root.

Therefore, for system VI, as long as $T=\omega_n\tau$ is larger than 1.0, the theoretical estimate is applicable. The principal root loci for T=1.0 and T=5.0 are shown in Figures 3.30 and 3.31, respectively.

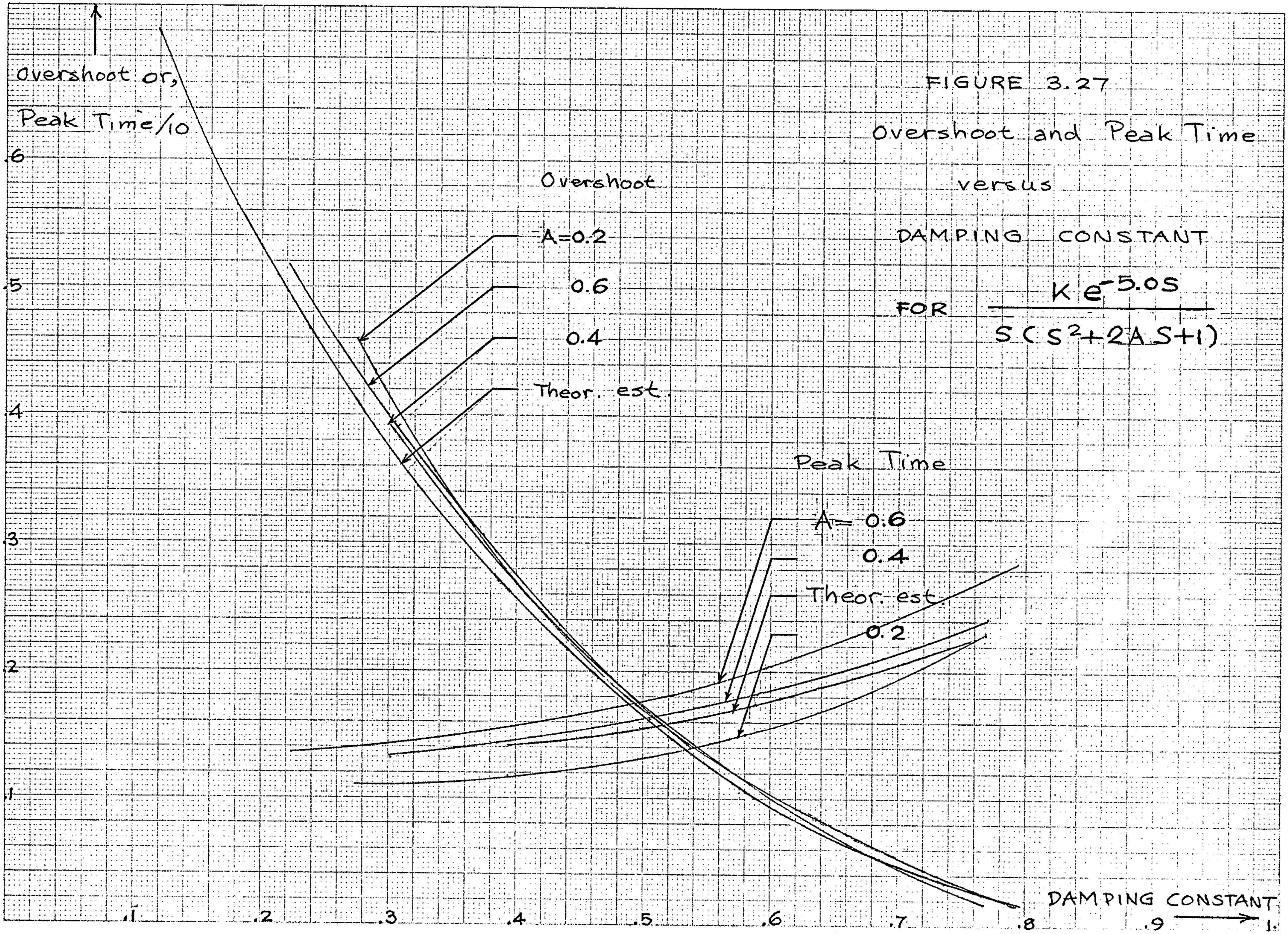


FIGURE 3.27

Overshoot and Peak Time versus DAMPING CONSTANT FOR $\frac{K e^{-5.0S}}{S(S^2+2AS+1)}$

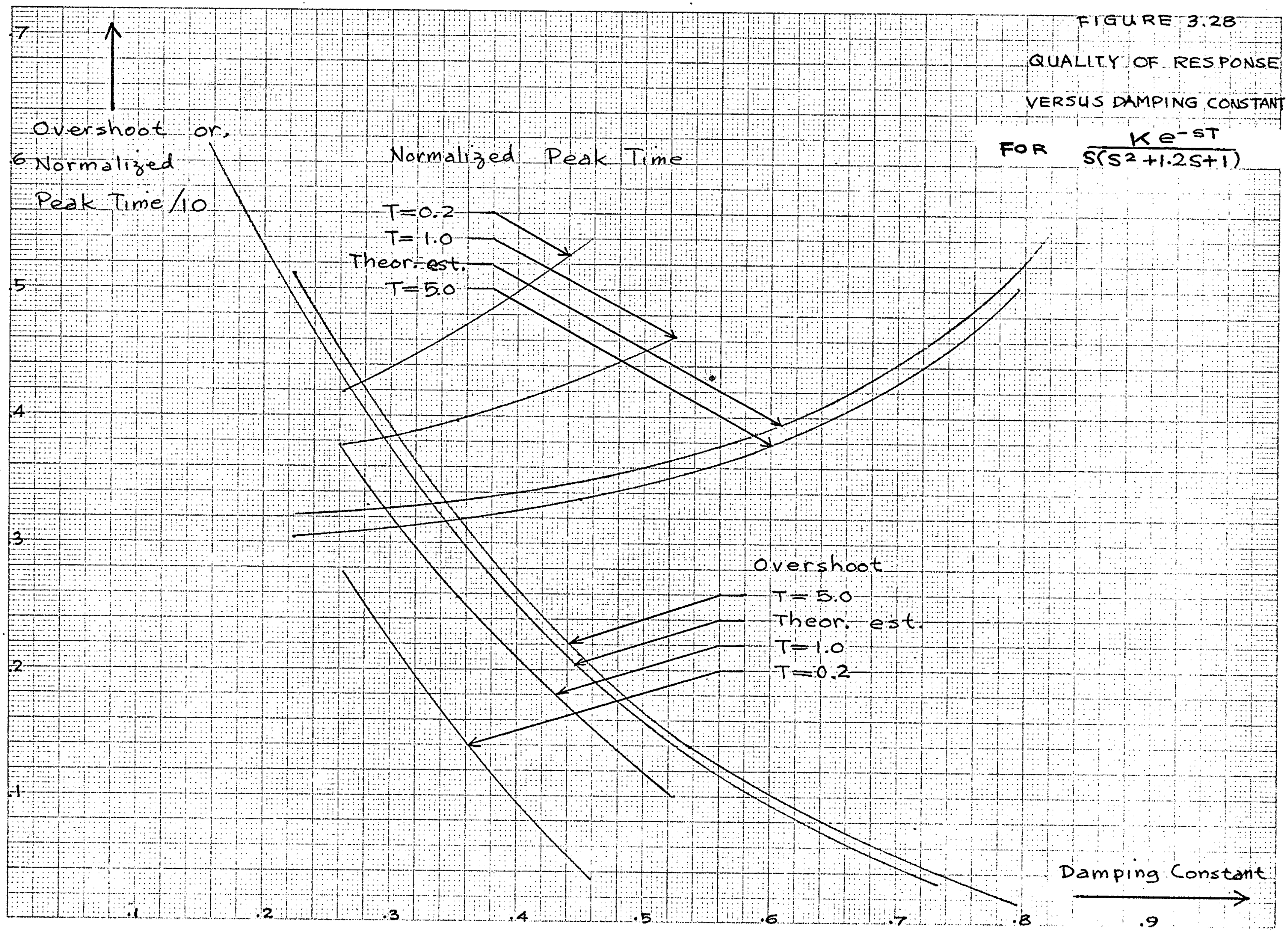


FIGURE 3.28
QUALITY OF RESPONSE
VERSUS DAMPING CONSTANT
FOR $\frac{Ke^{-sT}}{S(S^2+1.2S+1)}$
Overshoot or,
Normalized
Peak Time/10
Normalized Peak Time
T=0.2
T=1.0
Theor. est.
T=5.0
Overshoot
T=5.0
Theor. est.
T=1.0
T=0.2
Damping Constant
.1
.2
.3
.4
.5
.6
.7
.8
.9
.7
6
.5
.4
.3
.2
.1

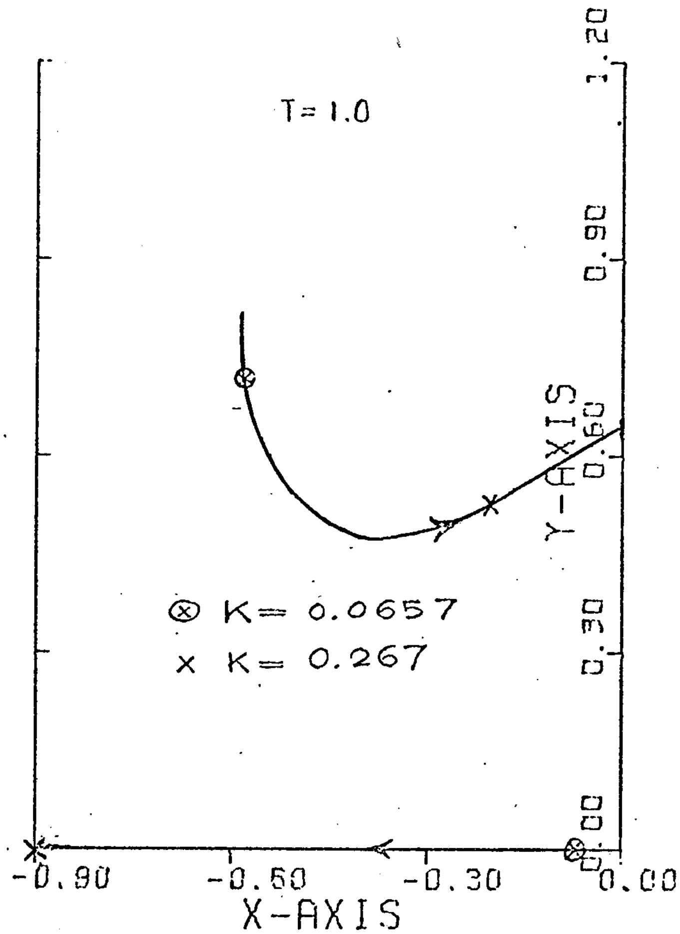


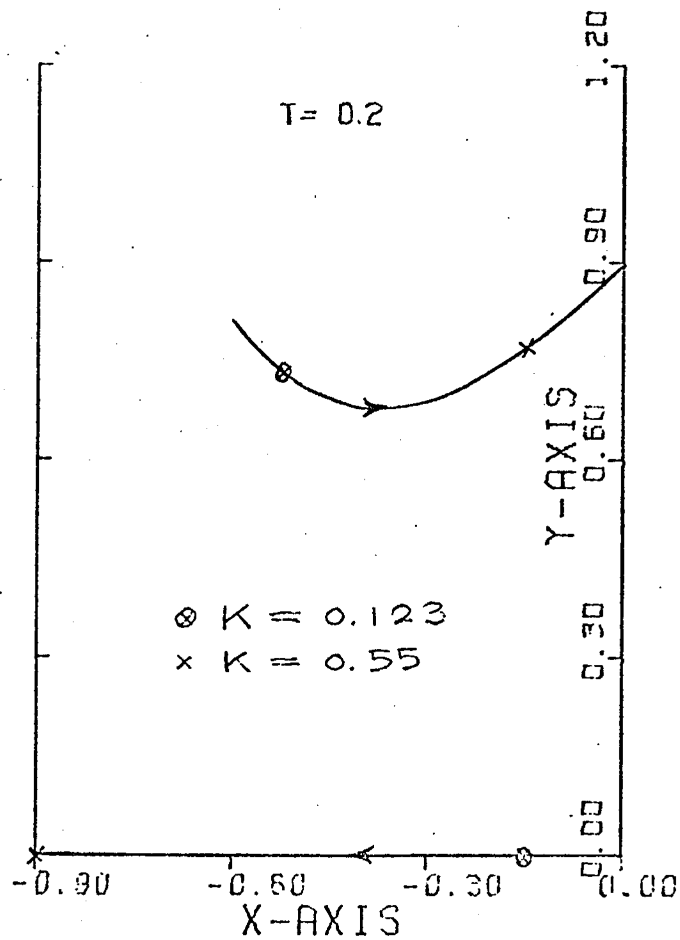
FIGURE 3.29

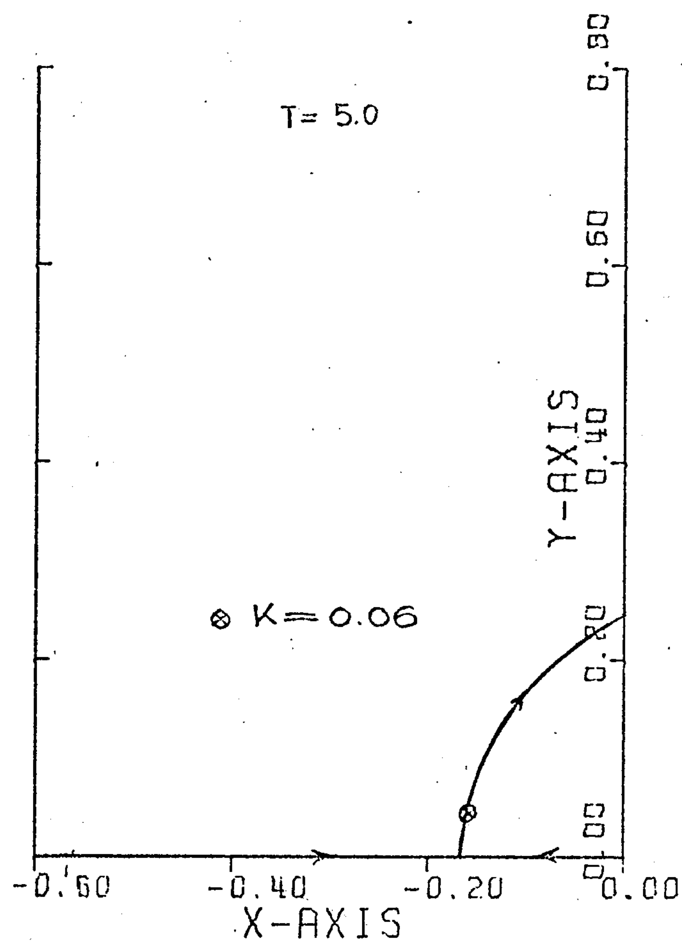
ROOT LOCUS OF

$Ke^{-sT}/s(s^2+2As+1)$

FØR VARIOUS DELAY TIMES

A = 0.6





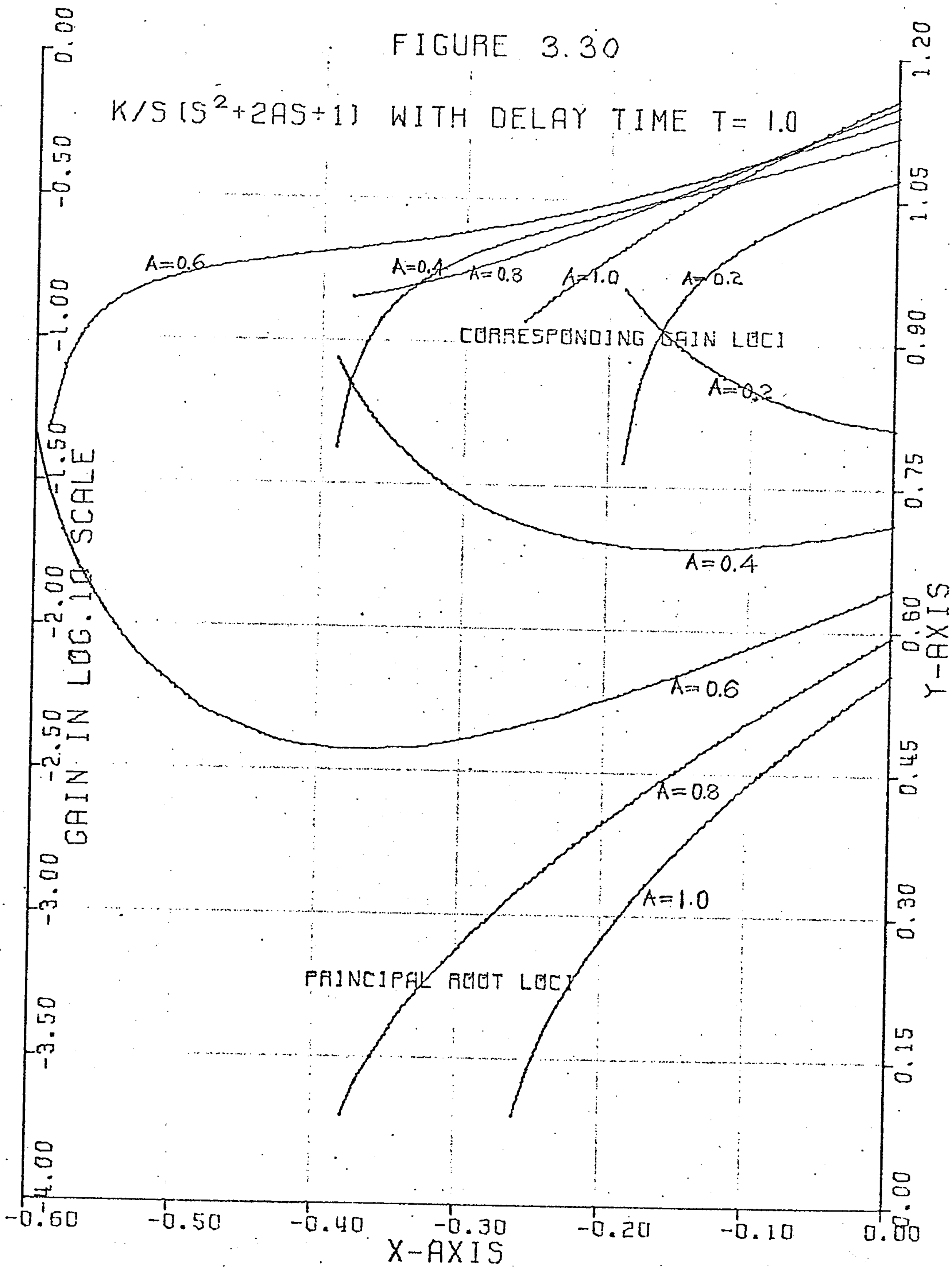


FIGURE 3.30
K/S(S²+2AS+1) WITH DELAY TIME T= 1.0
A=0.6
A=0.4
A=0.8
A=1.0
A=0.2
CORRESPONDING GAIN LOCI
A=0.2
A=0.4
A=0.6
A=0.8
A=1.0
PRINCIPAL ROOT LOCI
GAIN IN LOG.10 SCALE
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
-3.50
-4.00
Y-AXIS
1.20
1.05
0.90
0.75
0.60
0.45
0.30
0.15
0.00
X-AXIS
-0.60
-0.50
-0.40
-0.30
-0.20
-0.10
0.00

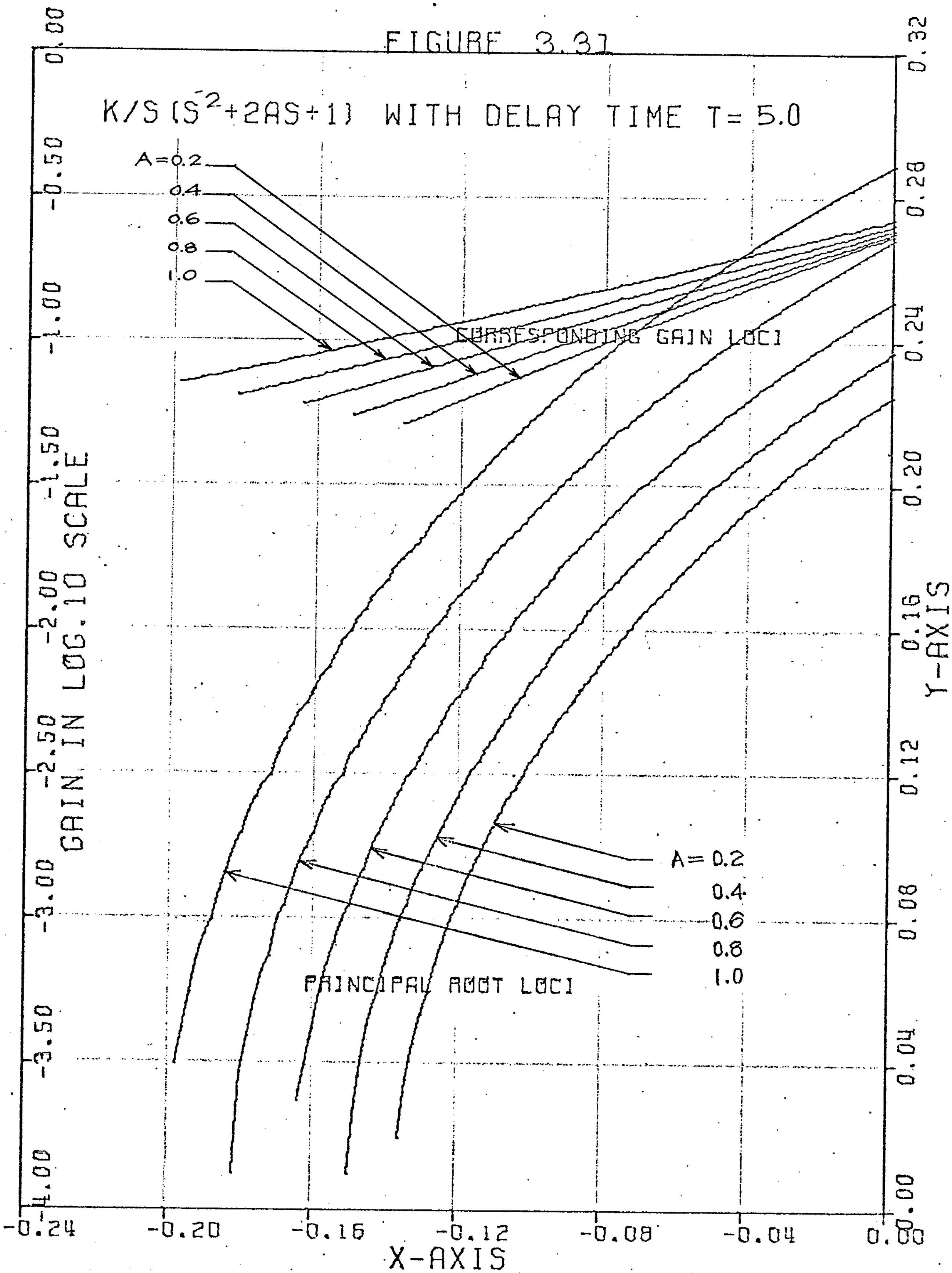

K/S(S^2+2AS+1) WITH DELAY TIME T= 5.0
A=0.2
0.4
0.6
0.8
1.0
CORRESPONDING GAIN LOCI
PRINCIPAL ROOT LOCI
A= 0.2
0.4
0.6
0.8
1.0
GAIN IN LOG.10 SCALE
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-3.00
-3.50
-4.00
Y-AXIS
0.32
0.28
0.24
0.20
0.16
0.12
0.08
0.04
0.00
X-AXIS
-0.24
-0.20
-0.16
-0.12
-0.08
-0.04
0.00

FIGURE 3.31

III.4 SOME EXAMPLES OF SYSTEM IDENTIFICATION AND ESTIMATION USING THE PRESENTED GRAPHS

In this chapter it is shown that the dynamic characteristics of some delay systems can be very well estimated by considering only the effect of a pair of complex roots. Moreover, the theoretical estimates of the quality of response as measured in terms of T_p, T_s and OS are only dependent upon the coordinates of principal root loci.

In this section, some very simple examples of system identification and estimation are presented. It is important to keep in mind that although the specification concerning T_p, T_s and OS enables us to determine the coordinate position of the desired system, more than one system may pass through any coordinate position. Therefore, more than one system may satisfy the specification.

Example 1

Given a feedback control system with forward loop transfer function G(S)=2/(S+3) and feedback path consisting of a transportation lag of 0.2 seconds, determine the quality of step response of this system. It is given that

$$G'(S)\ H'(S) = \frac{2}{S+3}\, e^{-0.2S} \qquad 3.42$$

With the substitution S = 3s,

$$G(s)\,H(s) = \frac{2/3}{(s+1)}\,e^{-0.6s} \tag{3.43}$$

Thus $\quad k = \frac{2}{3} = 0.667 \quad$ and $\quad T = A\tau = 0.6$.

For a gain constant of 0.667 ($\log K = \bar{1}.8241$) and $T = 0.6$, by referring to the gain loci shown in Figure 3.2, it is found that

$$x = -1.9$$

for which the ordinate of the corresponding root locus is

$$y = 1.85$$

Since

$$S = 3s,$$

$$X = -1.9 \times 3 = -5.7$$

and

$$Y = 1.85 \times 3 = 5.55$$

and so

$$T_p = \frac{\pi}{Y} = 0.566,$$

$$T_s = 0.701$$

and

$$OS = 0.04$$

The theoretical estimate for this normalized system has been found to be reasonably accurate. Therefore, the quality of response is determined with reasonable accuracy without the necessity of analog or digital simulation.

The following experimental results were obtained in digital simulation

$$T_p = 0.488,$$

$$T_s = 0.685$$

and

$$OS = 0.0448$$

Example 2

A linear feedback control system is known to have a transportation lag of 0.2 second, and a magnetic amplifier in the forward loop.

It is experimentally found that OS = 0.04 and T_p = 0.566. Identify this feedback control system.

The transfer function of the forward path is assumed to be of the form

$$G(S) = \frac{K}{S + A}$$

From the given data,

$$Y = \frac{\pi}{T_p} = \frac{\pi}{0.566} = 5.55$$

and

$$X = -\frac{Y}{\pi} \ln (25) = -5.70$$

Since

$$x = \frac{X}{A}, \quad y = \frac{Y}{A}, \text{ and } T = A\tau$$

then

$$\frac{y}{x} = \frac{Y}{X}.$$

Therefore, a trial and error method may be applied to find A such that the principal root locus passes through x =X/A with the slope of Y/X.

Figure 3.2 shows that with A=3.0, the principal root locus passes through $x=\frac{-5.7}{3}$ =-1.9 and Y/X=5.55/5.70=0.974 with k =0.66.

Therefore A =3.0 and K =3x0.66 = 1.98.

Example 3

Given the required system dynamic performance characteristics

$$OS = 0.04$$

and that $T_p = 0.566$

it is desired to design a delay control system with magnetic amplifier G(S)=K/(S+3) in the forward loop.

Since $Y = \frac{\pi}{T_p} = 5.55$

and $X = -\frac{Y}{\pi} \ln \left(\frac{1}{0.04}\right) = -5.7$

then $x = -\frac{5.7}{3} = -1.9$

and $y = \frac{5.55}{3} = 1.85$

From Figure 3.2, T is found to be 0.6 and k =0.66; therefore, K=1.98 and τ =0.2.

III.5 CONCLUSION

In this chapter it has been shown that the theoretical estimates are surprisingly good for some systems, especially when an open loop pole is located at the origin, whereas, for systems involving open loop complex roots, special care has to be taken in the theoretical estimation.

Since all the investigations were carried out in terms of the normalized systems, the graphs presented above allow the quality of response of whole classes of systems to be theoretically estimated.

Even though the investigations are not exhaustive, it would seem that a promising design method could be developed which would consume little time.

CHAPTER IV

STABILITY AND TRANSIENT RESPONSE OF SYSTEMS WITH VARIOUS METHODS OF DELAY SIMULATION.

IV.1 Introduction

In the study of many process control systems it is often necessary to generate a delay function or to simulate a delay on analog or digital computers.

The most commonly used method of delay simulation for analog computers employs Pade or cascaded lag delay approximands. These approximands invariably introduce some degree of phase or magnitude error which in turn affect the transient response of the simulated systems.

The purpose of the simulation is usually to investigate the stability limits and the dynamic characteristics of the systems. The dynamic effects on transient response due to the delay approximands are dependent upon the method of delay simulation being employed. Some delay approximands might introduce intolerable effects or even instability, which could possibly defeat the purpose of the simulation.

In this chapter, Pade one, Pade two and second order cascaded lag approximands are applied to simulate delay in otherwise linear lumped control systems. The dynamic characteristics and the stability limits of these simulated systems are compared with those of the systems with actual delay in digital computer simulation.

IV.2 COMPARISON OF ROOT LOCUS AND STABILITY LIMITS FOR SYSTEMS WITH VARIOUS DELAY APPROXIMANDS

The root loci of systems I through VI in their normalized forms with various delay approximands have been calculated.

The principal root loci of $ke^{-sT}/s(s+1)$ and $ke^{-sT}/(s+1)$ with various delay approximands and actual delay having delay times of 0.2, 1.0 and 5.0 are shown in Figures 4.1 and 4.2, respectively.

A few general statements based upon the presented graphs may be postulated

1. The disparity between the principal root loci of any system with various delay approximands and with actual delay increases for increasing delay time, e.g. $ke^{-sT}/(s+1)$.

2. Since $Ke^{-S}/(S+T)$ may be normalized to $ke^{-sT}/(s+1)$ with the substitution $S= sT$, the root locus of $Ke^{-S}/(S+T)$ is similar in shape to that of $ke^{-sT}/(s+1)$, only the coordinate scales and the gain calibration are altered by a factor T.

Hence, the comparison of the principal root loci of $Ke^{-S}/(S+T)$ with actual delay and with various delay approximands for various T, is the same as for the principal

FIGURE 4.1

COMPARISON OF ROOT LOCUS

FOR K/(S+1)

WITH VARIOUS TYPES OF

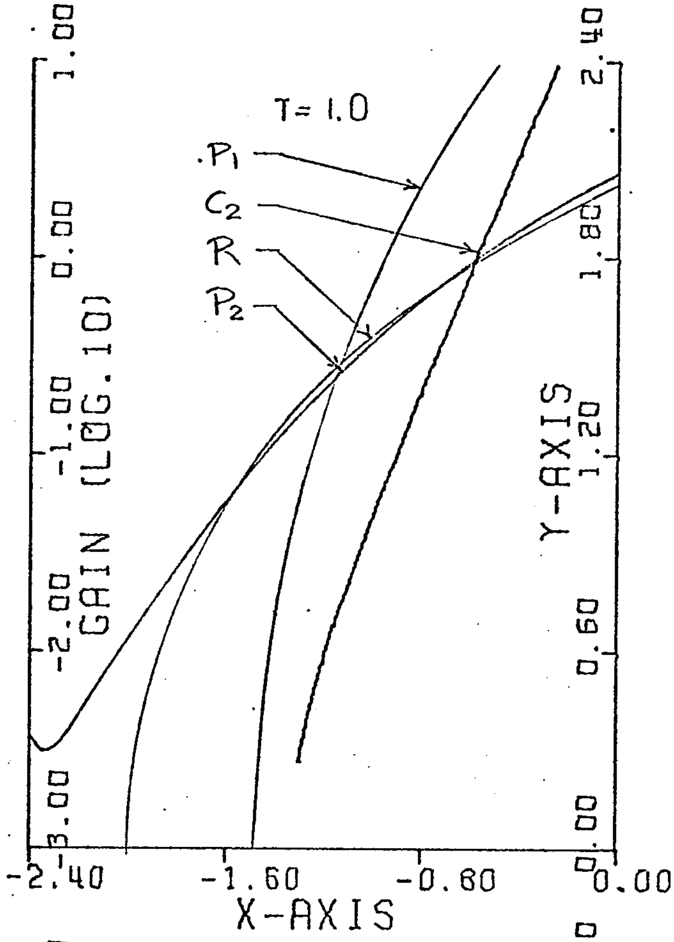
DELAY APPROXIMAND

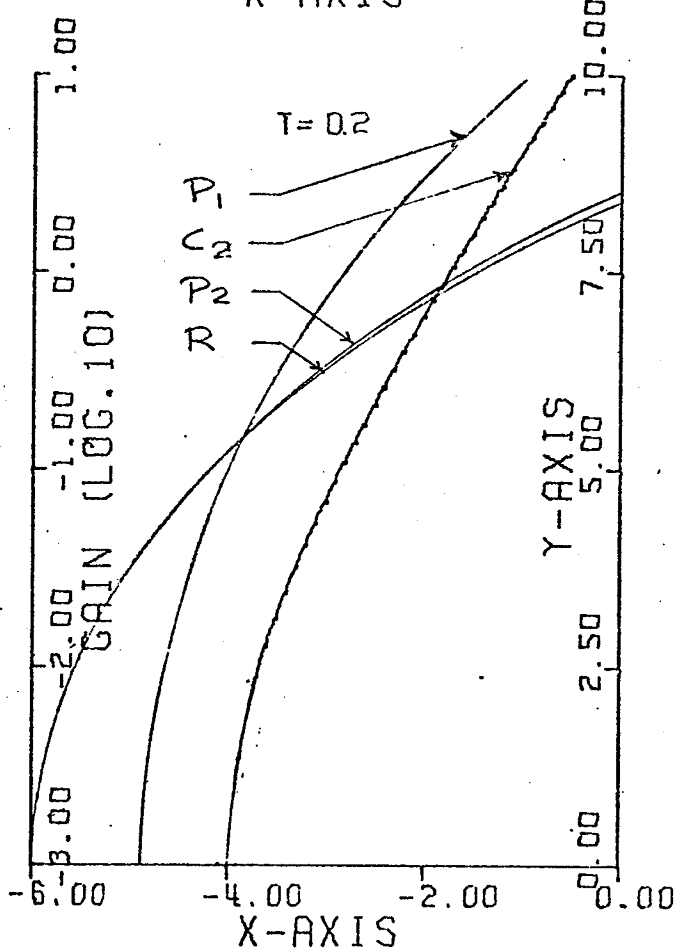
R — Actual delay

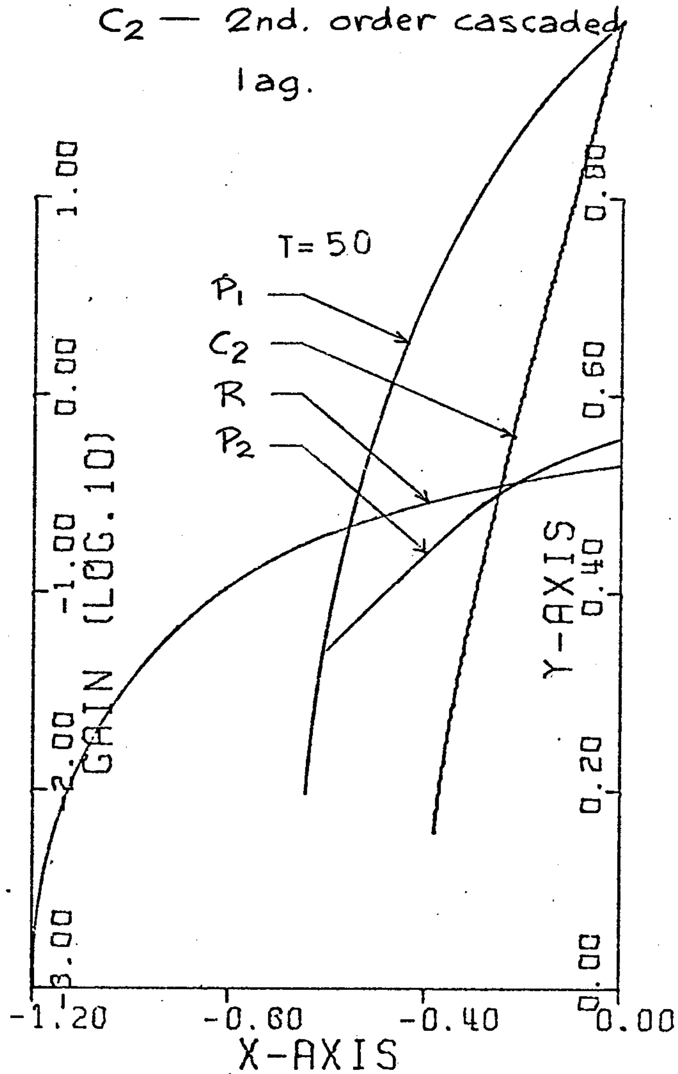
P_2 — Pade 2 Approximand

P_1 — Pade 1 Approximand

C_2 — 2nd. order cascaded lag.







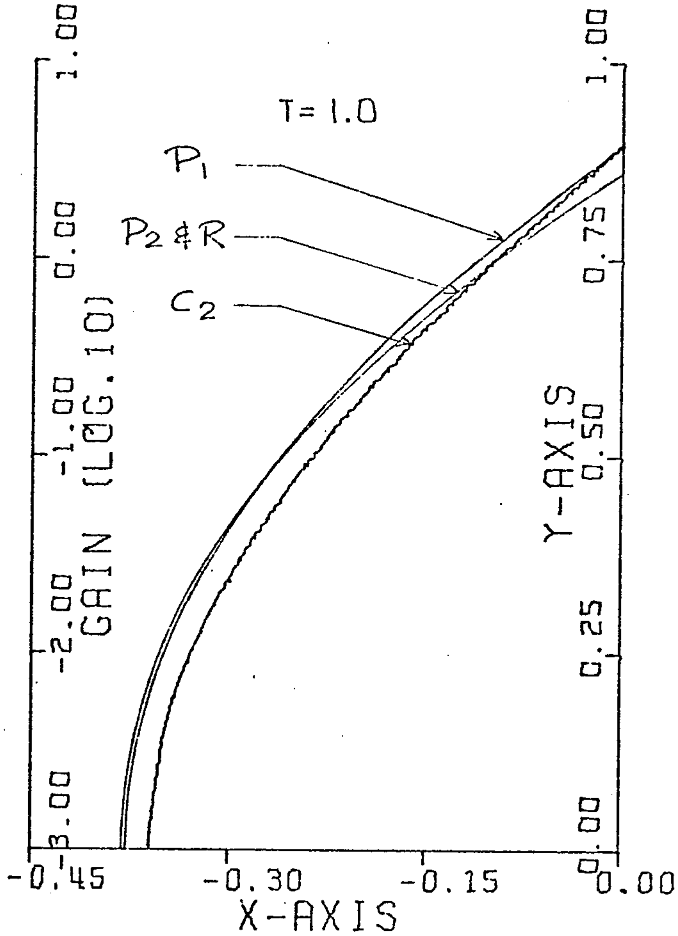


FIGURE 4.2

COMPARISON OF ROOT LOCUS

FOR K/(S+1)S

WITH VARIOUS TYPES OF

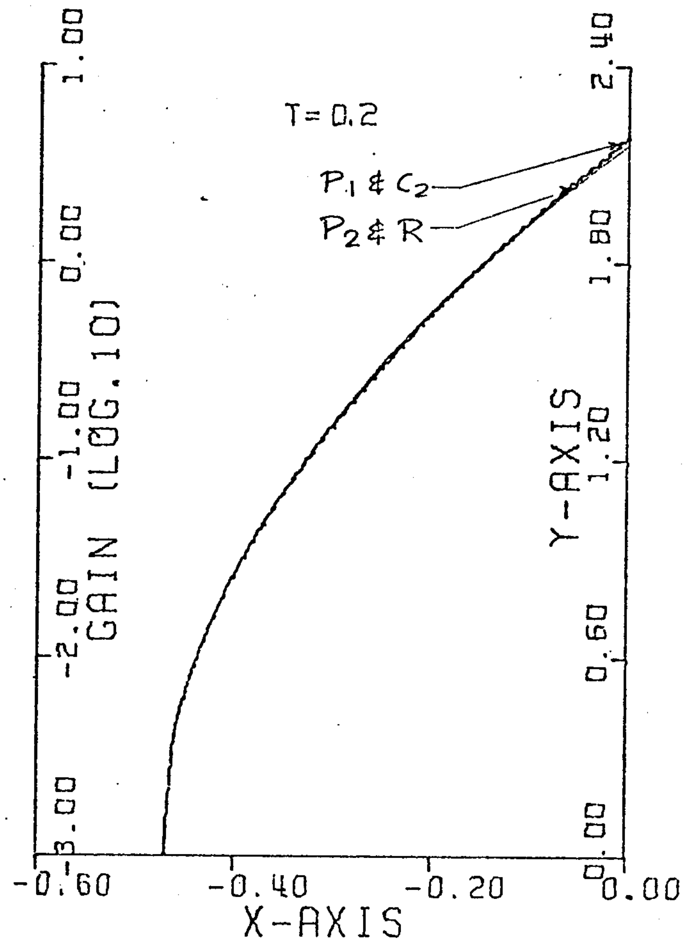
DELAY APPROXIMAND

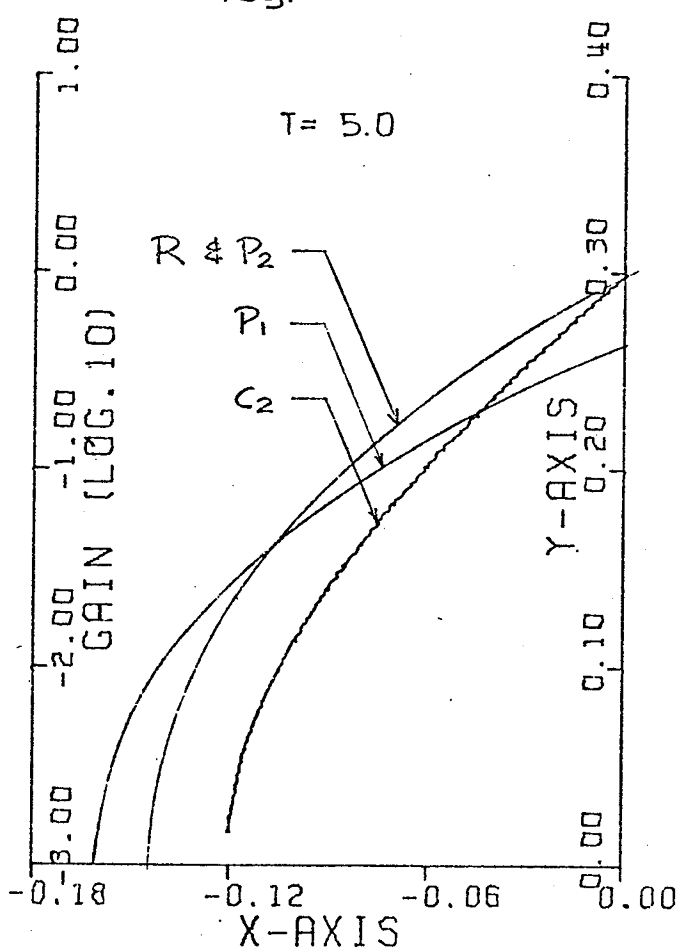
R — Actual delay

P_2 — Pade 2 Approx.

P_1 — Pade 1 Approx.

C_2 — 2nd. order Cascaded lag.





root loci of $ke^{-sT}/(s+1)$. Consequently, it may be concluded from Figures 4.1 and 4.2 that the disparity between the principal root loci with various delay approximands and actual delay increases with the distance from the origin to the principal open loop pole (or poles).

3. The principal root loci of systems with the Pade one approximand intersect the imaginary axis at exactly the same points as those of the second order cascaded lag approximands as would be expected. (The phase angles for $s=jy$ are identical for these two cases.)

4. The principal root loci of systems with actual delay intersect the imaginary axis with smaller ordinate than those systems with any order of Pade or cascaded lag delay approximands.

In order to examine the validity of the above generalizations, the following equation is examined

$$\angle H(o,y) + \angle G(o,y) = 180^0 \pm n\cdot 360^0 \qquad 4.1$$

$\angle G(0,y)$ is the same as the phase angle of $G(j\omega)$, with y substituted for ω, and $\angle H(0,y)$ the phase angle of $H(j\omega)$. Therefore, to find the value of y at the stability limit is to find the value of ω such that the phase angle of $G(j\omega)H(j\omega)$ is -180^o. In other words, it is the value of ω

such that

$$-\angle G(j\omega) = 180^0 + \angle H(j\omega) \qquad 4.2$$

To illustrate this more fully, in Figure 4.3 the phase angles of five different linear systems are compared to $180^\circ + \angle H(j\omega)$, where $\angle H(j\omega)$ are the phase angles of the actual delay or delay approximands such as Pade two, Pade one and second order cascaded lag.

For systems of this sort $-\angle G(j\omega)$ is a monotonically increasing function. Any order of Pade or cascaded lag approximands would introduce errors in phase angle such that the phase angles due to the actual delay unit are either equal to or smaller than the phase angles of the approximands. As a result, for these delay approximands, it is necessary to have a larger value of ω in order to satisfy Eq. 4.2. This implies that the root loci for systems with actual delay intersect the imaginary axis with the smallest ordinate for systems of the types considered.

Moreover, the deviation of the phase angle for the Pade or cascaded lag approximands from that of the actual delay decreases for higher order approximands. Since the gain constant is generally a monotonically decreasing function of ω, this means that the stability limits for systems with delay approximands approach those for the actual delay as the order of the approximands increases, and can never be

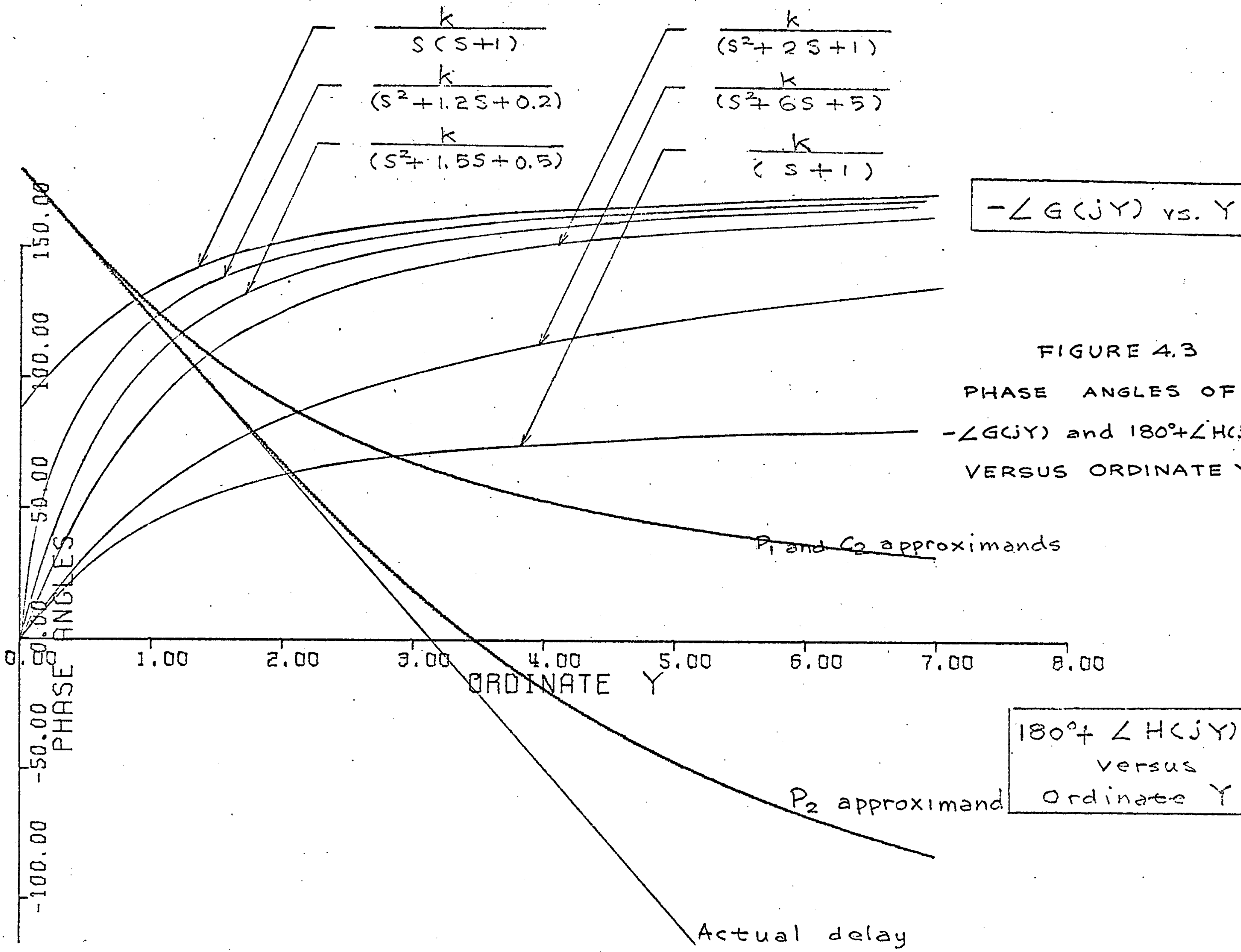


FIGURE 4.3

PHASE ANGLES OF $-\angle G(jY)$ and $180^\circ + \angle H(jY)$ VERSUS ORDINATE Y smaller than the respective stability limit of the actual system.

Therefore, it is expected that the error in the principal root loci would increase for increasing delay time T and also for increasing distance of the open loop poles from the origin.

IV.3 COMPARISON OF TRANSIENT RESPONSE FOR SYSTEMS WITH VARIOUS DELAY APPROXIMANDS

In Section IV.2 it has been shown that the error in the principal loci for systems with various delay approximands increases with increasing delay time and with increasing distance of the open loop pole from the origin. Consequently, the errors in the transient response would be expected to follow similar patterns.

The graphs showing the relation between overshoot and gain constant for systems with various delay approximands and the actual delay, for various values of delay time, are shown in Figures 4.4 through 4.6. It should be noted that the difference in overshoot for $ke^{-5s}/(s+1)(s+5)$ with the P_2 approximand and actual delay is as high as 20%, which is even higher than the theoretical estimate considering only the effect of a pair of complex roots for actual delay system. In contrast to the general statement by Wright (Dec.

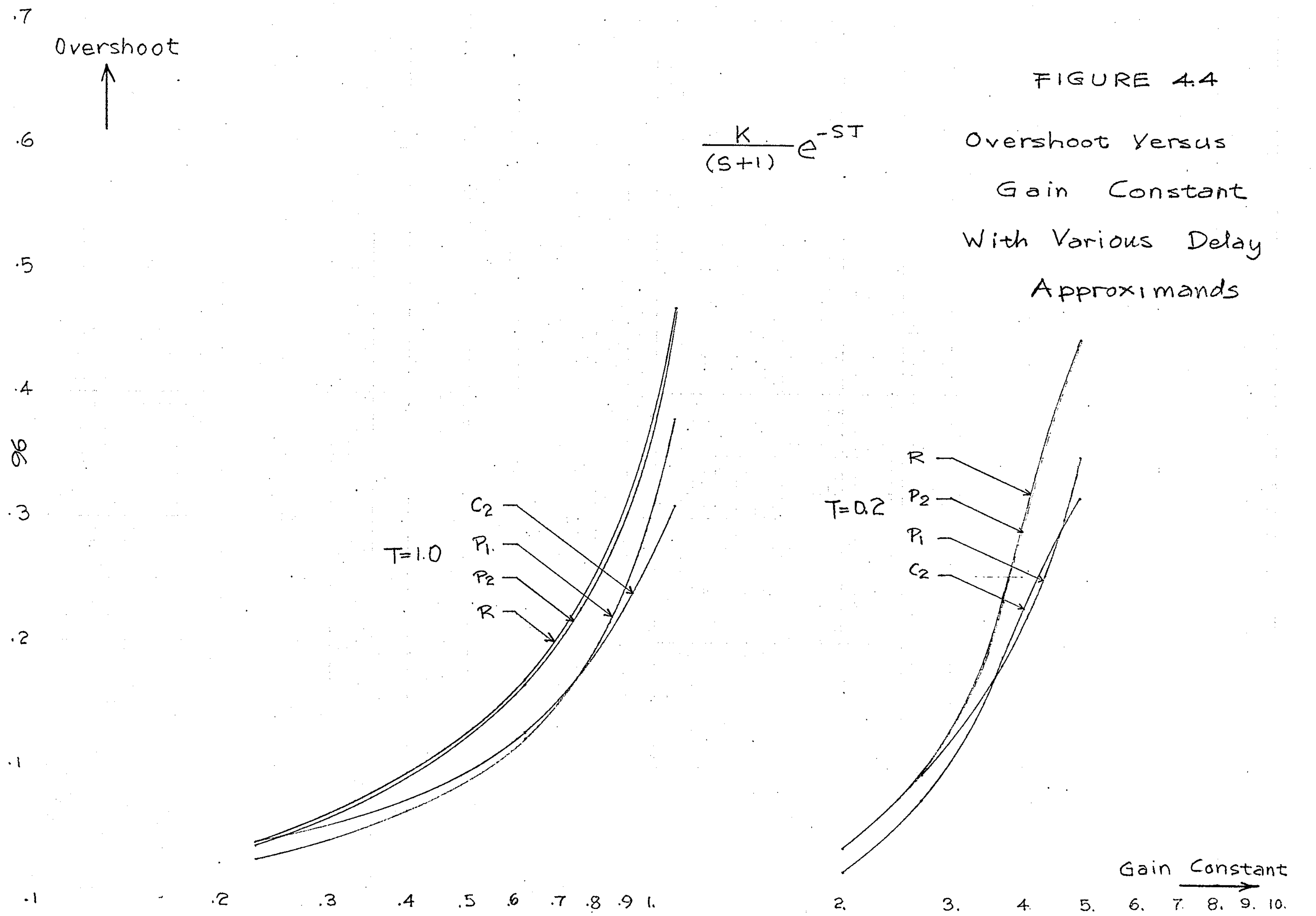


FIGURE 4.4

Overshoot Versus Gain Constant With Various Delay Approximands

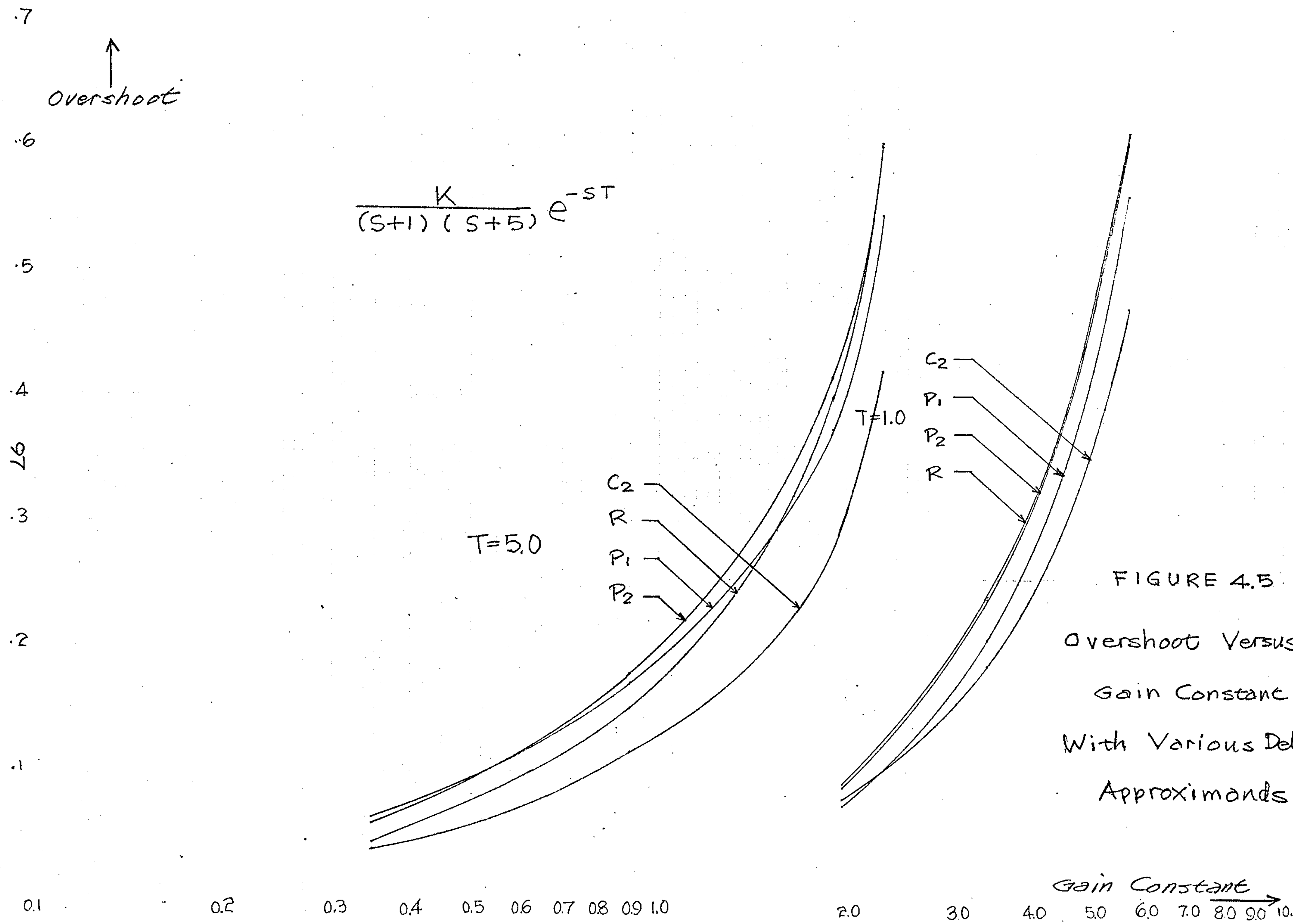


FIGURE 4.5

Overshoot Versus Gain Constant With Various Delay Approximands

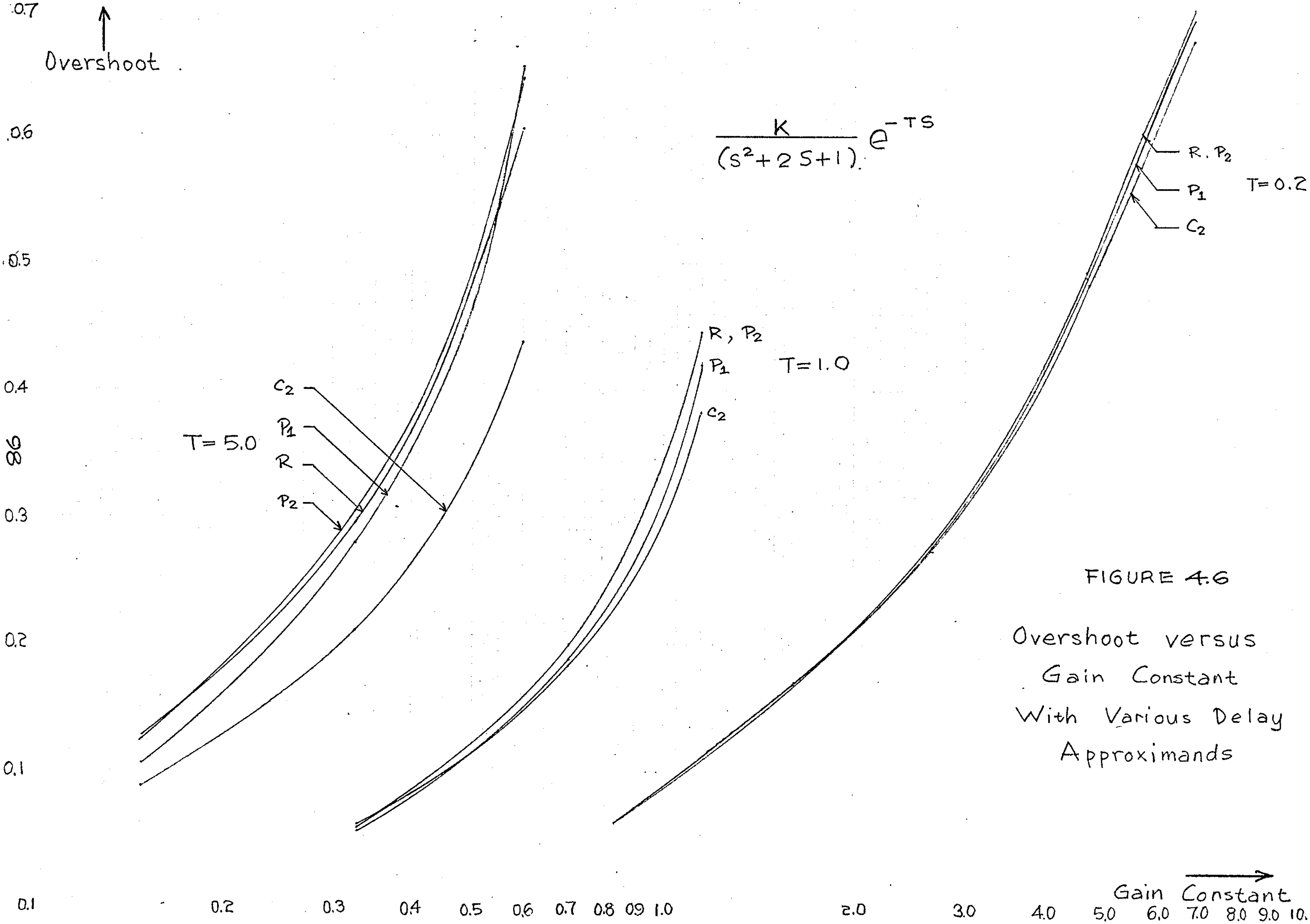


FIGURE 4.6

Overshoot versus Gain Constant With Various Delay Approximands

1965, pp.120 and 126), no generalization regarding the effect of delay approximand on the quality of response seem possible except for systems operating close to the stability limits. Such systems with delay approximands tend to have smaller overshoot and peak time, but larger settling time than those of the systems with actual delay. This generalization may be substantiated by considering the general trend of the principal root loci for higher values of gain constant.

VI.4 THE THEORETICAL ESTIMATION FOR SYSTEMS WITH VARIOUS DELAY APPROXIMANDS AND ACTUAL DELAY

The principal root loci of various control systems with delay approximands have been obtained and applied to estimate the system response.

From Figures 4.7 and 4.8, it is easily noted that the theoretical estimate for systems with delay approximands is not as good as those for the systems with actual delay.

For example, the theoretical estimate for $ke^{-sT}/(s+1)$ with a Pade one delay approximand to simulate T=1.0 is very poor compared to that of the actual delay. This implies that the theoretical estimates considering only the effect of the pair of principal complex roots for systems with delay approximands are not as good as those of the systems with

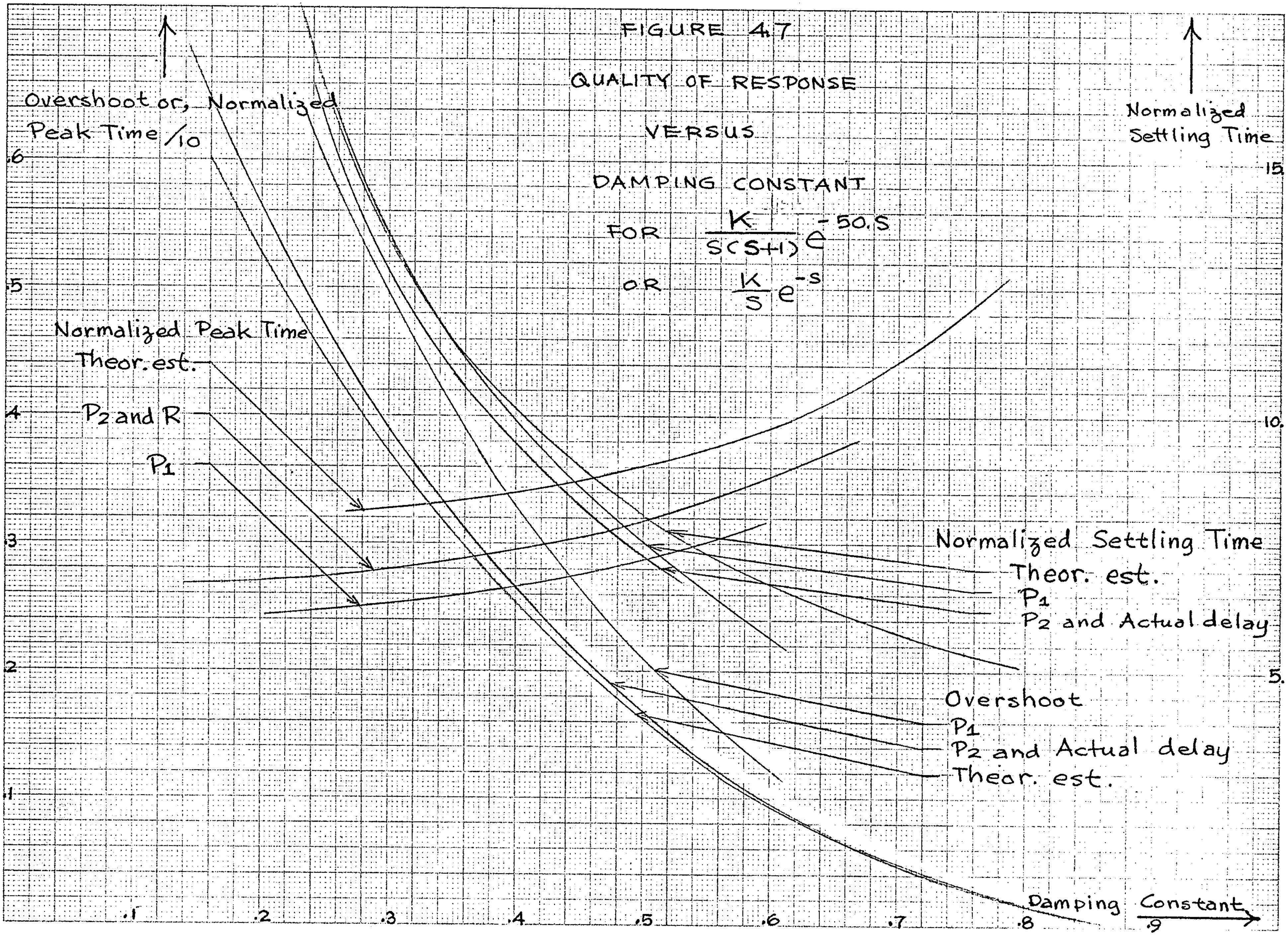


FIGURE 4.7
QUALITY OF RESPONSE
VERSUS
DAMPING CONSTANT
FOR $\frac{K}{S(S+1)}e^{-50.S}$
OR $\frac{K}{S}e^{-S}$
Overshoot or, Normalized Peak Time /10
Normalized Settling Time
Normalized Peak Time
Theor. est.
P2 and R
P1
Normalized Settling Time
Theor. est.
P1
P2 and Actual delay
Overshoot
P1
P2 and Actual delay
Theor. est.
Damping Constant
.1
.2
.3
.4
.5
.6
.7
.8
.9
.6
.5
.4
.3
.2
.1
15.
10.
5.

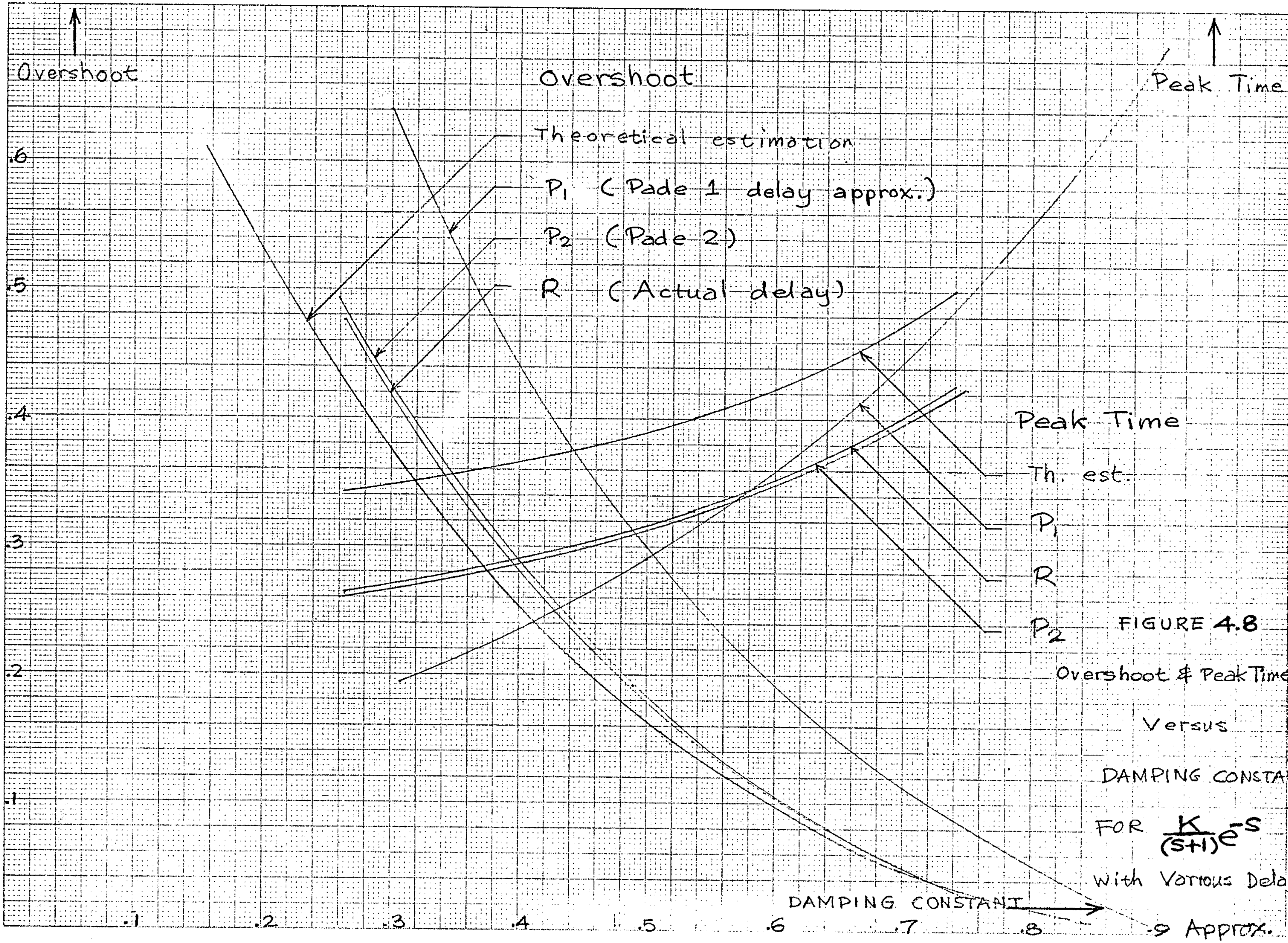


FIGURE 4.8

Overshoot & Peak Time Versus DAMPING CONSTANT FOR $\frac{K}{(s+1)}e^{-s}$ With Various Delay Approx.

actual delay. This generalization makes the root locus analysis neglecting higher order branches more desirable in systems with actual delay.

CHAPTER V

DESIGN OF LINEAR CONTROL SYSTEMS INVOLVING TIME DELAY

In Chapter IV, it has been shown that for some systems the theoretical estimates of the qualities of response are rather accurate, and in Chapter III that the gain margin may be used in place of the phase margin as a measure of relative stability.

For some systems it may be possible to choose the system parameters in order to provide the desired performance characteristics. However, for other systems, it is not enough merely to adjust the system parameters. Rather, the systems themselves must be changed. This chapter is mainly concerned with the design of linear control systems involving time delay using series compensation.

The two most commonly used methods of series compensation are considered: first, through the addition of a phase lead compensator and second, the addition of a phase lag compensator.

V.1 PHASE LEAD COMPENSATOR

The phase lead network is utilized to provide a phase lead angle, but it is accompanied by an increase in gain magnitude, which in turn shifts the gain crossover frequency to a higher value. For linear control systems without delay, the increase in phase lead angle is more than enough to compensate for the phase lag due to the increase in crossover frequency. As a result, an improved system may be obtained.

But for systems involving time delay, due to the intrinsic characteristics of the actual delay, e^{-sT}, the increase in phase lag angle due to the increase in crossover frequency is much larger than that of the system without delay. In general, except when the crossover frequency is sufficiently small, the addition of phase lead compensation can not improve system performance, and so this type of network may not be used.

V.2 PHASE LAG COMPENSATOR

The phase lag compensator is not used to provide a phase lag angle but rather to attenuate the crossover frequency.

For systems involving time delay, the phase lag compensator is particularly useful due to the intrinsic nature of the delay unit since it can lead to a considerable increase in both gain margin and phase margin. However, overdoing this results in sluggish response.

Consider the sets of normalized systems, $ke^{-sT}/s(s+1)$, $ke^{-sT}/(s+1)$ and $ke^{-sT}/s(s+1)(s+A)$. When T is sufficiently small the design method for systems without delay may be well applied. What "sufficiently small" means differs from one system to another depending on how much the delay unit affects the frequency response near the crossover frequency. However, the material presented in Chapter III may be helpful to the designer in this regard.

An investigation was carried out to determine a design method for two sets of systems involving delay: one with an open loop pole at the origin, and the other without such a pole.

V.3 CATEGORY I, SINGLE POLE AT S=0

For the sets of systems $Ke^{-\tau S}/S(S+A)$ and $Ke^{-\tau S}/S(S+A)(S+B)$ (B>A), when τA is larger than one, the system performance characteristics are quite similar to those of ke^{-sT}/s. Therefore the design method for ke^{-sT}/s is considered here.

With the argument presented above, consider then the lag compensator with transfer function

$$G_c(s) = \frac{(1 + \frac{s}{A'})}{(1 + \frac{Rs}{A'})} .$$

Figure 5.1 shows the effects of A' on the qualities of response for the compensated delay systems with the same gain constant and two different values of phase lag ratio, R.

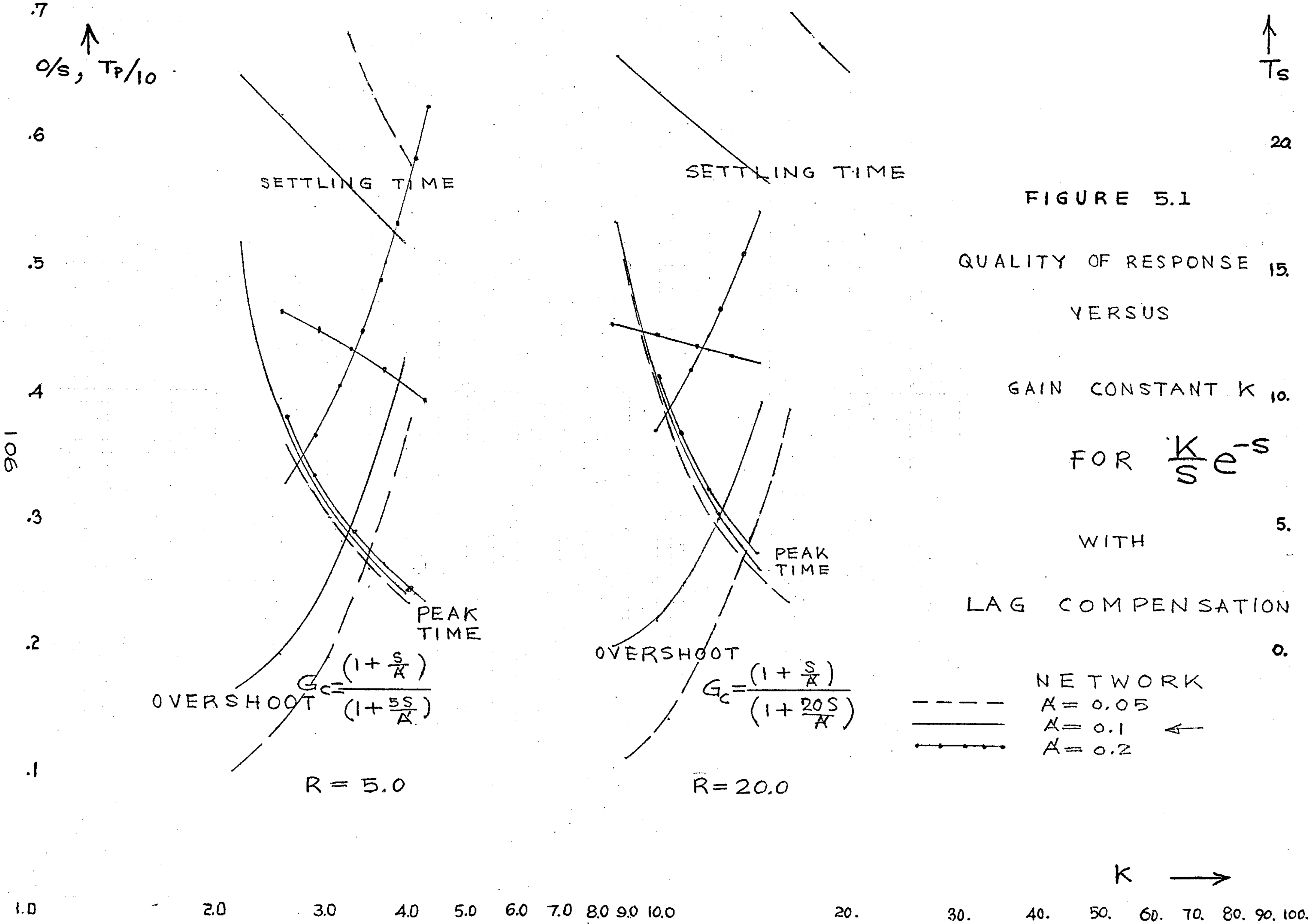


FIGURE 5.1

QUALITY OF RESPONSE VERSUS GAIN CONSTANT K FOR $\frac{K}{S}e^{-S}$ WITH LAG COMPENSATION

It is apparent that one must compromise among the conflicting demands on the system response characteristics. One such compromise is the "optimal choice" of $A'=0.1$. A smaller value of A' would result in a more desirable peak time and overshoot but an undesirable increase in settling time due to the fact that the step response does not oscillate around the steady state value. A larger value of A' would result in a more desirable (i.e. smaller) settling time but also a sharp increase in percentage overshoot.

The addition of the compensator

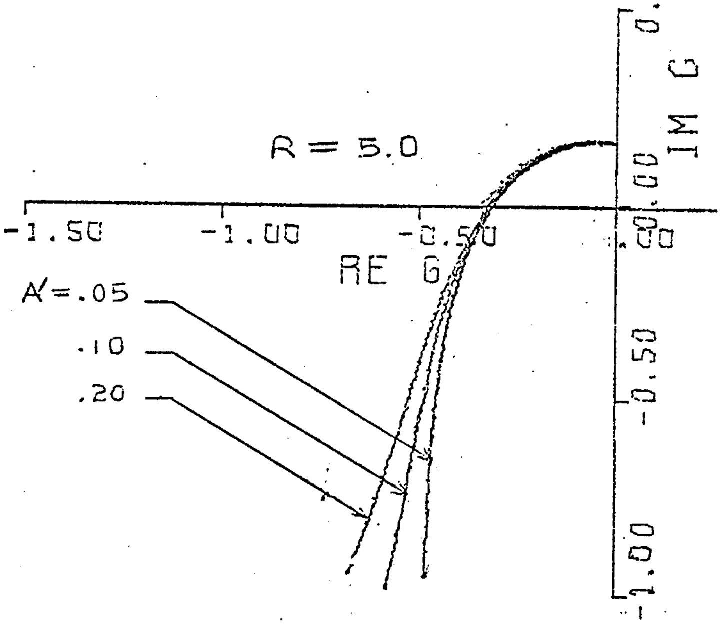
$$G_c(s) = \frac{(1 + \frac{s}{A'})}{(1 + \frac{Rs}{A'})}$$

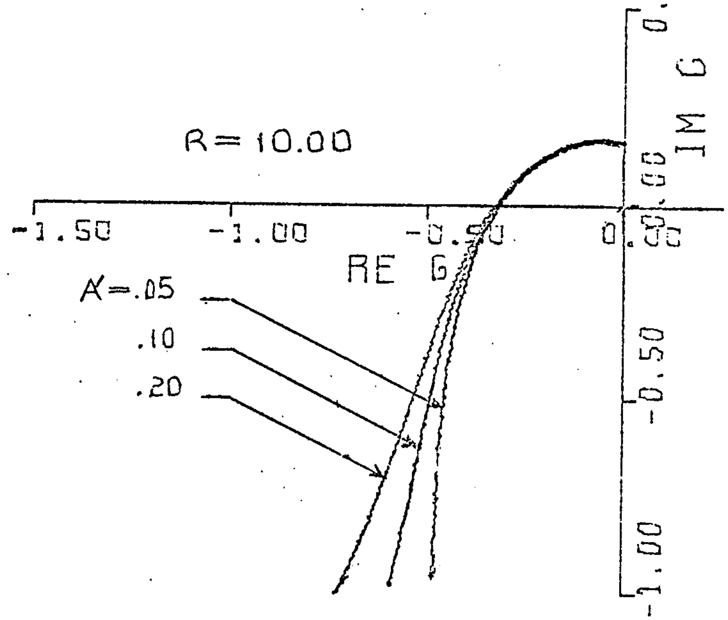
increases the gain margin by a factor approximately equal to the lag ratio R.

In Figure 5.2, the Nyquist plots of the uncompensated systems and compensated systems are compared for various values of A' and lag ratio R. Figure 5.3 shows the relation of the percentage overshoot, peak time, and the reciprocal of gain margin for

$$G(s)\ G_c(s)\ H(s) = \frac{k(1 + \frac{s}{0.1})e^{-s}}{s\ \ (1 + \frac{Rs}{0.1})}$$

Figure 5.3 indicates that for any specific value of percentage overshoot, a set of systems with various values of R and reciprocal of gain margin may satisfy the





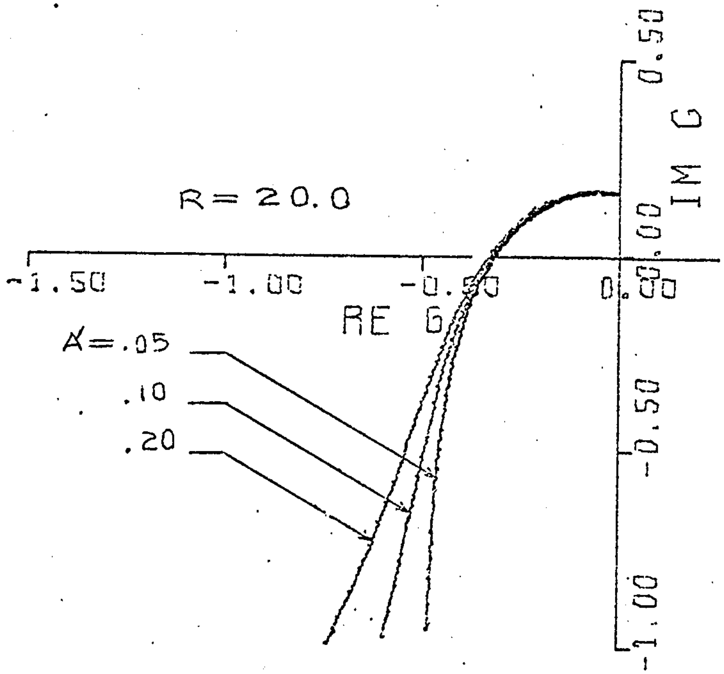


FIGURE 5.2

COMPARISON OF NYQUIST PLOTS

$$\frac{0.5\, R\, (1+S/A')\, e^{-S}}{S\, (1+ R\, S/A')}$$

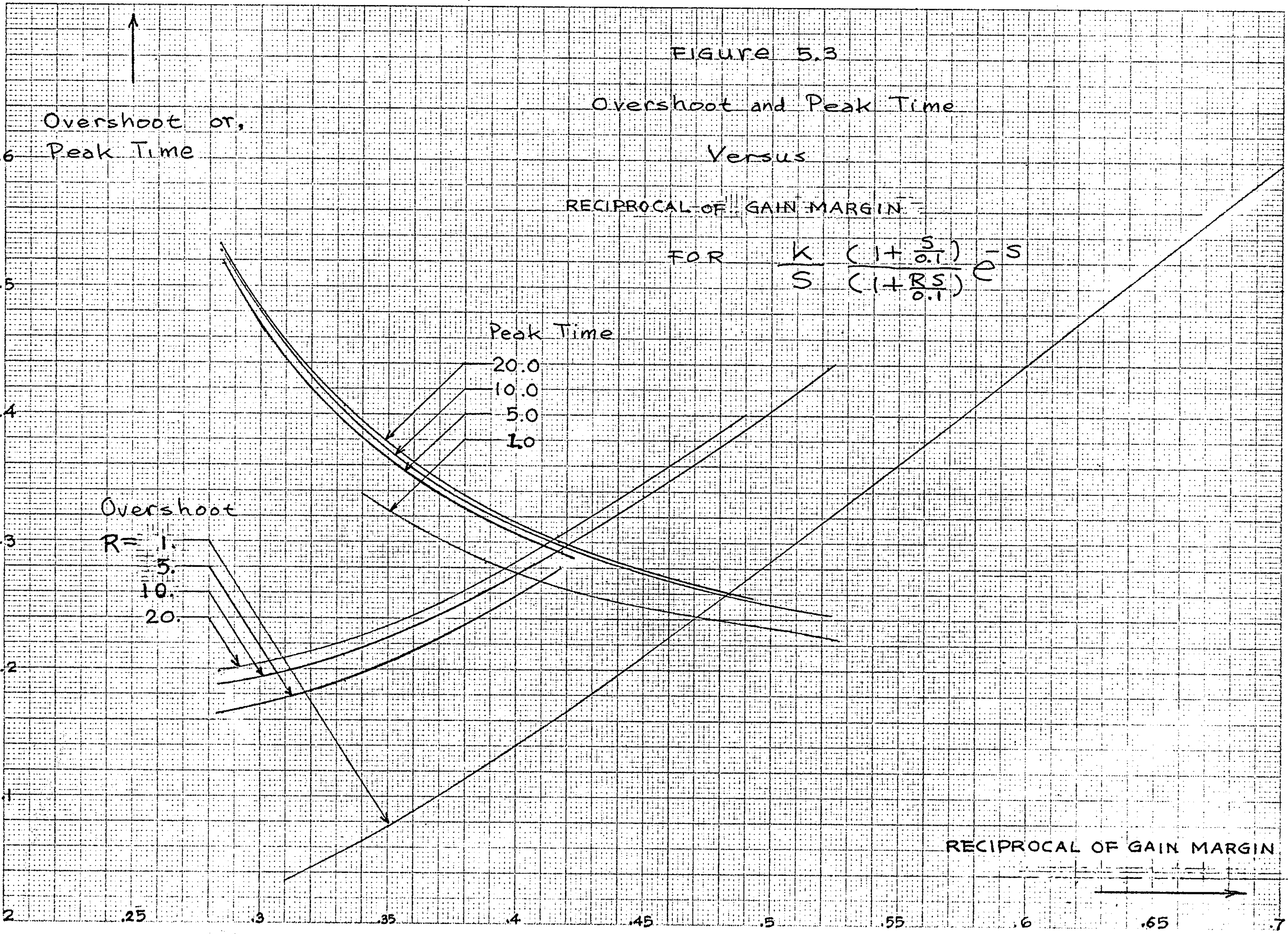


FIGURE 5.3
Overshoot and Peak Time
Versus
RECIPROCAL OF GAIN MARGIN
FOR $\frac{K}{S}\frac{(1+\frac{S}{0.1})}{(1+\frac{RS}{0.1})}e^{-S}$
Overshoot or, Peak Time
Peak Time
20.0
10.0
5.0
1.0
Overshoot
R= 1.
5.
10.
20.
RECIPROCAL OF GAIN MARGIN
.6
.5
.4
.3
.2
.1
.2
.25
.3
.35
.4
.45
.5
.55
.6
.65
.7

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specification, but due to the internal interaction of the system, gain margin is directly related to the values of R, and hence a trial and error method is necessary in selecting suitable R and $1/G_m$. It is found that $1/G_m$ may be set to 0.3 as the first trial value.

Example 1

Consider a single loop feedback control system with transfer functions,

$$G(S) = \frac{11.838}{S}$$

and

$$H(S) = e^{-0.1S}$$

The uncompensated system has the following response characteristics

$$OS = 0.6837$$

$$T_p = 0.1845$$

and

$$T_s = 1.9530$$

It is apparent that the above system is rather oscillatory.

Suppose it is necessary to design a compensator such that the percentage overshoot will not exceed 15% without increasing the ramp error.

The gain magnitude of the uncompensated system at phase crossover frequency , $\omega_m = 15.708$ is

$$|G(j\omega_m)| = 11.838/15.708 = 0.755$$

Assume then that $1/G_m$ is 0.30, then R must be approximately 2.5. The interpolation of Figure 5.3 indicates that for an overshoot of 15% the gain margin of the compensated system should be approximately 1/0.27. And hence,

$$R = \frac{0.755}{0.27} = 2.79$$

Therefore, the compensator could assume the form of

$$G_c(S) = \left(\frac{1 + S}{1 + 2.79S}\right)$$

Experimentally, the compensated system has the following response characteristics

$$OS = 0.1281$$

$$T_p = 0.53812$$

and

$$T_s = 2.135$$

Here, the design specifcation is not perfectly met. This is due to the fact that when R is smaller than 5.0, the interpolation of Figure 5.3 yields inaccurate results relating the transient performance and the reciprocal of gain margin.

Example 2

Given an uncompensated single loop feedback control system with transfer functions,

$$G(S) = \frac{50}{S\left(1 + \frac{S}{200}\right)^2}$$

and

$$H(S) = e^{-0.1S}$$

design a compensator such that the percentage overshoot will not exceed 20% without increasing the ramp error.

Since, approximately, $\frac{\pi}{2} + 2\frac{\omega_m}{200} + 0.1\omega_m = \pi$

or $\omega_m = 14.3$

then for the uncompensated system

$$|G(j\omega_m)| = \frac{50}{14.3} = 3.5$$

Assume that $1/G_m$ is 0.30, then R is roughly equal to 12.0. With reference to Figure 5.3 for R=12.0, the gain margin of 1/0.27 produces an overshoot of less than 20%.

Therefore,

$$R = \frac{3.5}{0.27} = 12.9$$

and so

$$G(S)G_c(S) = \frac{50}{S(\frac{S}{200}+1)^2}\,\frac{(1+S)}{(1+12.9S)}$$

Response characteristics determined by simulation for the compensated system are as follows

$$OS = 0.2096,$$

$$T_p = 0.59437$$

and $$T_s = 2.378$$

Since for this system T is sufficiently small, this design technique may be compared with the well-established design technique based upon the phase margin and the optimum choice of crossover frequency, i.e. the crossover frequency is set at the point of minimum phase lag to produce maximum phase margin.

For $$G_c(S) = \frac{(1 + \frac{S}{\omega_1})}{(1 + \frac{S}{\omega_2})} \qquad (\omega_1 > \omega_2)$$

suppose ω_c lies between ω_1 and 200 then, approximately,

$$\Phi = -\frac{\pi}{2} - (\frac{\pi}{2} - \frac{\omega_2}{\omega}) + (\frac{\pi}{2} - \frac{\omega_1}{\omega}) - \frac{2\omega}{200} - 0.1\omega$$

Here, ω_2 is likely to be much smaller than ω_{max}.

Differentiation with respect to ω leads to the maximum phase margin. Now approximately,

$$\frac{d\Phi}{d\omega} = \frac{\omega_1 - \omega_2}{\omega^2} - 0.11$$

If now R>>10, say, ω_2 may be neglected and hence,

$$\omega_c = \omega_{max} = \sqrt{\frac{\omega_1}{0.11}}$$

In order to get a phase margin of $\pi/4$

$$\Phi_c = -3\pi/4 = -\pi + \frac{\pi}{2} - \frac{\omega_1\sqrt{0.11}}{\sqrt{\omega_1}} - \frac{\sqrt{\omega_1}}{100\sqrt{0.11}} - 0.1\frac{\sqrt{\omega_1}}{\sqrt{0.11}}$$

thus $\omega_1 = 1.40$

and $\omega_c = 3.56$

and so $R = \frac{50}{3.56} = 14.05$

Hence, the compensator is of the form

$$G_c(S) = \frac{(1 + \frac{S}{1.40})}{(1 + \frac{14.05\,S}{1.40})}$$

Note that for R=14.05, the approximation $\omega_2 << \omega_1$ is justified.

Experimentally, this compensated system gave the following performance characteristics

$$OS = 0.2855$$
$$T_p = 0.6575$$

Comparing the two different compensated systems, the method developed in this thesis produces a compensator with smaller phase lag ratio and more desirable performance characteristics.

Moreover, the established technique produces a compensated system which is far from satisfactory.

V.4 CATEGORY II, NO POLE AT S=0

Since the sets of systems

$$\frac{K}{S + A} e^{-S\tau}, \quad \frac{Ke^{-S\tau}}{(S + A)(S + B)} \quad (B > A)$$

and

$$\frac{Ke^{-S\tau}}{(S + A)(S + B)(S + C)} \quad (C > B > A)$$

are quite similar to the sets of normalized systems

$$\frac{k}{(s + 1)} e^{-sT}$$

a design method was developed for $ke^{-sT}/(s+1)$ using the phase lag compensation network

$$G_c(s) = \frac{(1 + \frac{s}{A'})}{(1 + \frac{Rs}{A'})}$$

In Figures 5.4 through 5.6, the Nyquist plots of the compensated systems for various values of A' and phase lag ratio R are given.

The effects of A' on the qualities of step response of the compensated systems with the same gain constant and two different values of phase lag ratio for T=0.2, 1.0 and 5.0 are shown in Figures 5.7 through 5.9, respectively.

For a delay time of 0.2, with reference to the presented data and the comparison of Nyquist plots, it may be noted that for the compensated system with fixed gain constant and phase lag ratio, the overshoot of the step response increases with increasing A', but when A' is smaller than 0.9 , the step response of the compensated system does not oscillate around the steady state value, and hence causes the sharp increase in settling time.

Moreover, the choice of A=1.0 simply shifts the pole of the uncompensated system to a location closer to the origin, and this is known to provide a smaller overshoot, peak time and settling time —— all desirable transient characteristics.

Since the gain margin of such a compensated system is increased by a factor approximately equal to R, therefore,

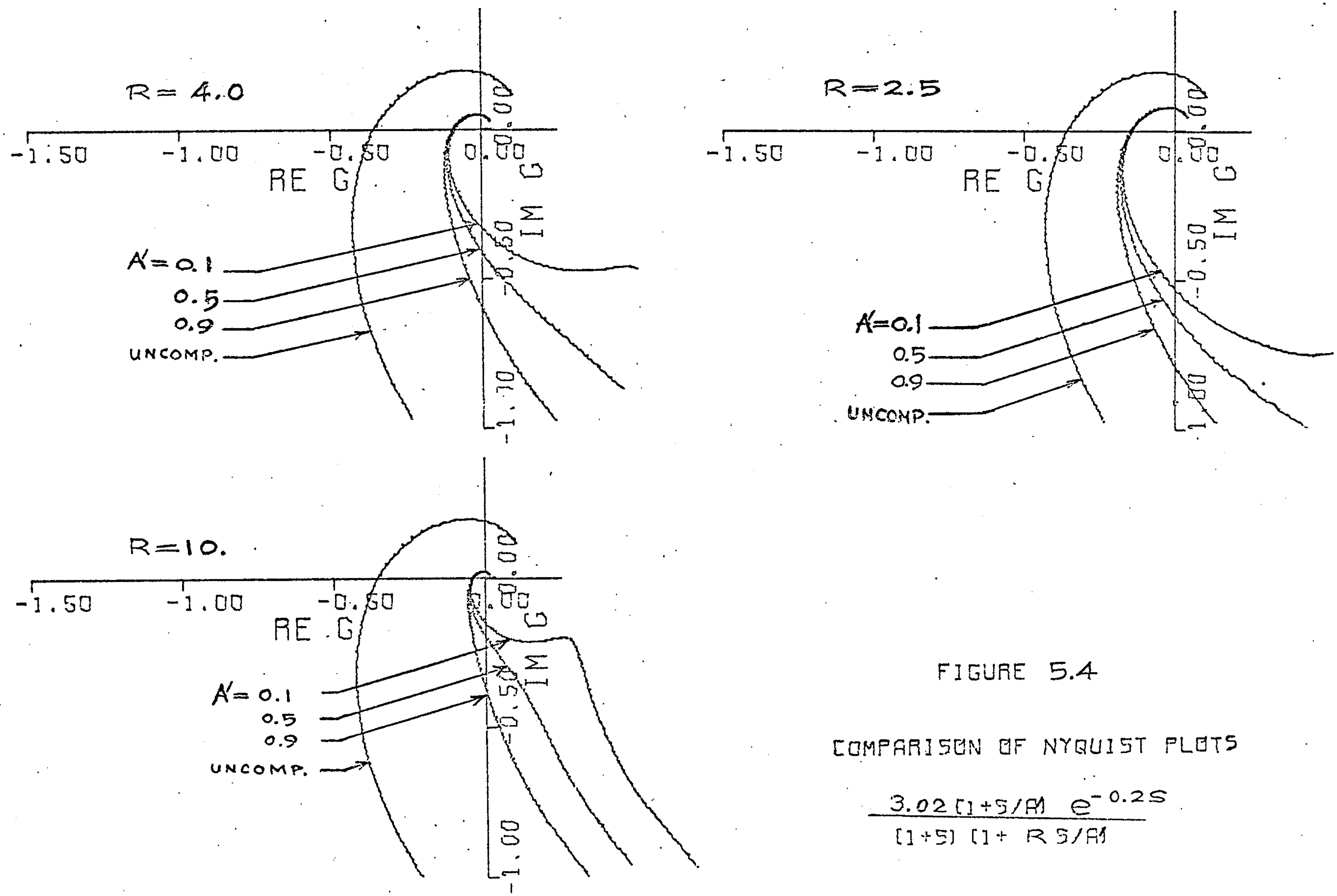


FIGURE 5.4

COMPARISON OF NYQUIST PLOTS

$$\frac{3.02\,[1+S/A]\;e^{-0.2S}}{[1+S]\;[1+RS/A]}$$

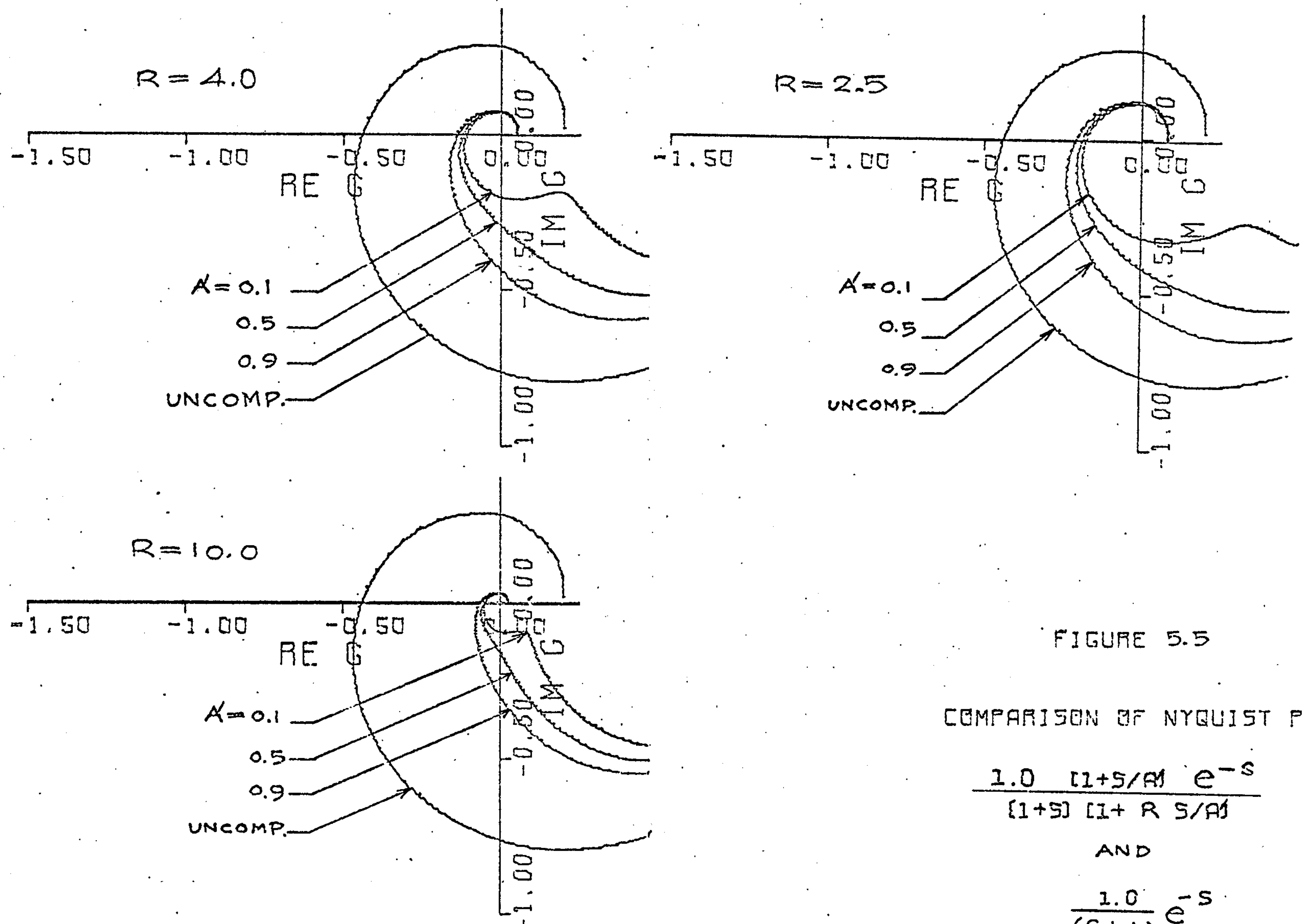


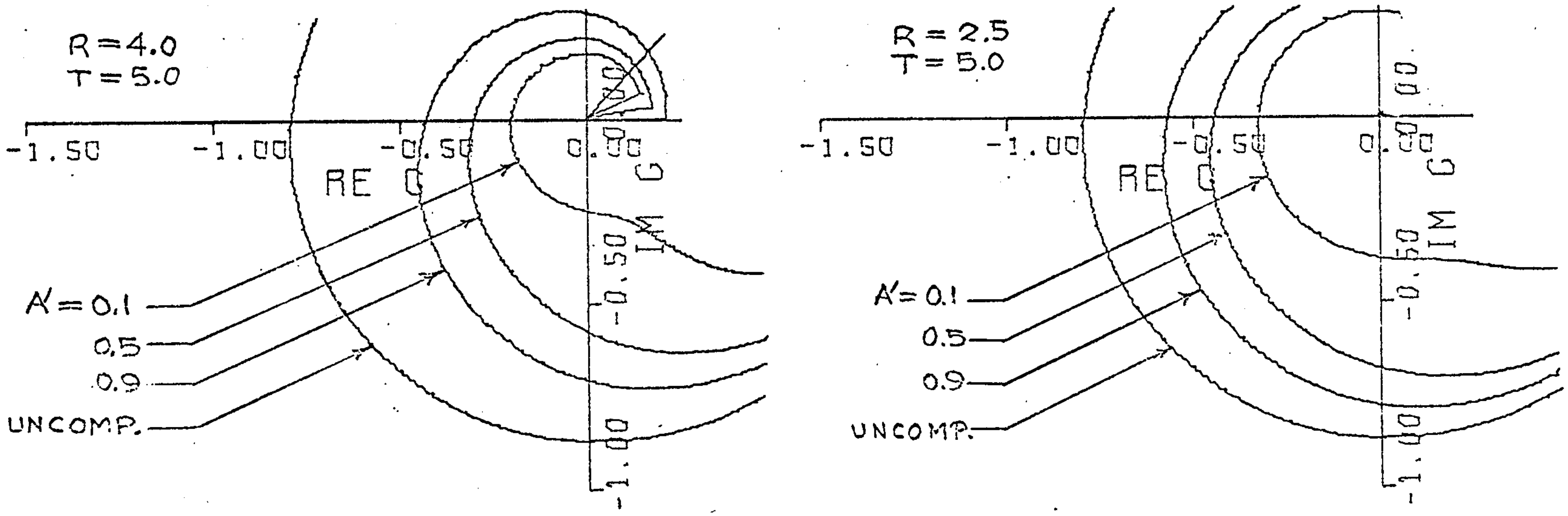
FIGURE 5.5

COMPARISON OF NYQUIST PLOTS

$$\frac{1.0\ [1+S/A]\ e^{-S}}{[1+S]\ [1+R\ S/A]}$$

AND

$$\frac{1.0}{(S+1)}e^{-S}$$



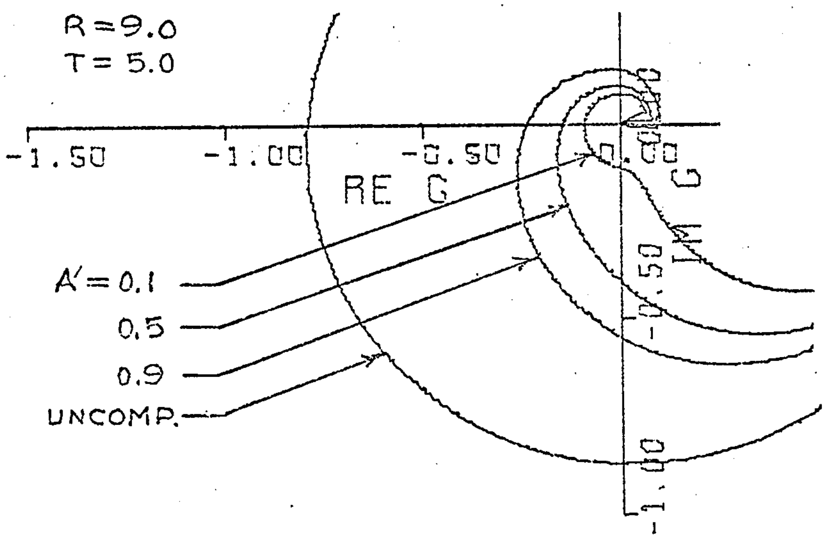


FIGURE 5.6

COMPARISON OF NYQUIST PLOTS

$$\frac{3.02\,(1+S/A)\,e^{-TS}}{(1+S)(1+RS/A)}$$

AND

$$\frac{3.02}{(S+1)}e^{-TS}$$

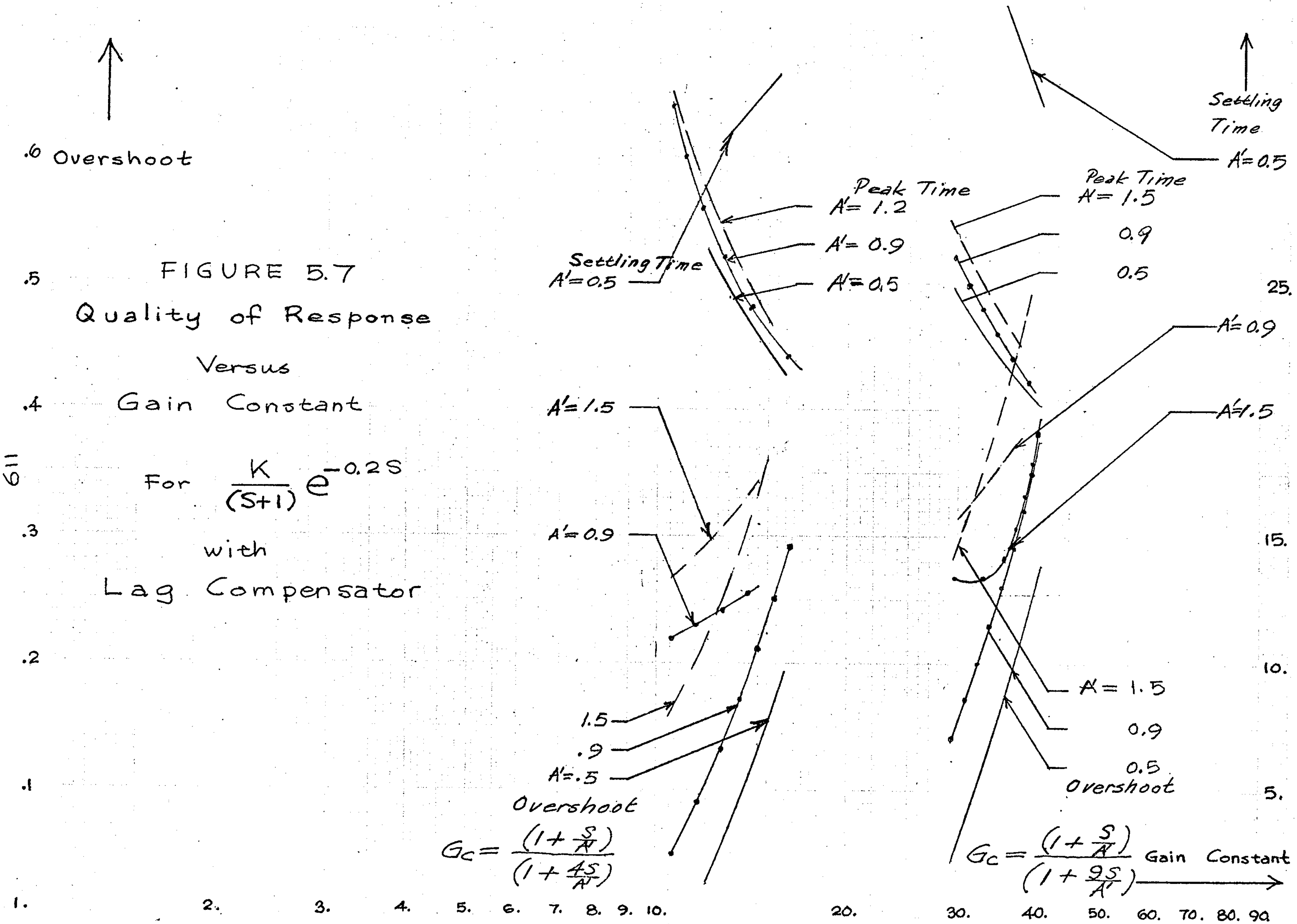


FIGURE 5.7

Quality of Response

Versus

Gain Constant

For $\frac{K}{(S+1)} e^{-0.2S}$

with

Lag Compensator

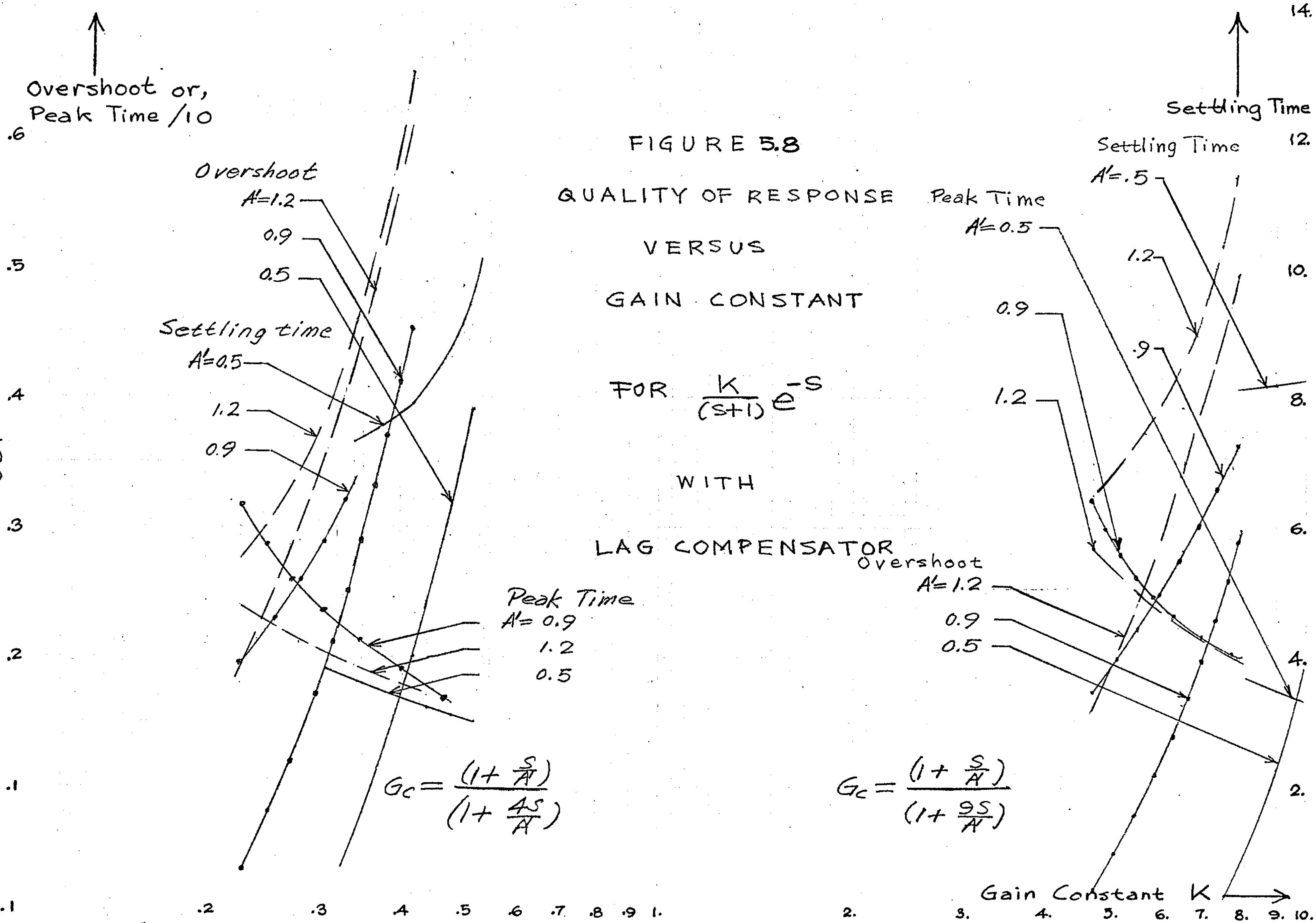


FIGURE 5.8

QUALITY OF RESPONSE VERSUS GAIN CONSTANT FOR $\frac{K}{(S+1)} e^{-S}$ WITH LAG COMPENSATOR

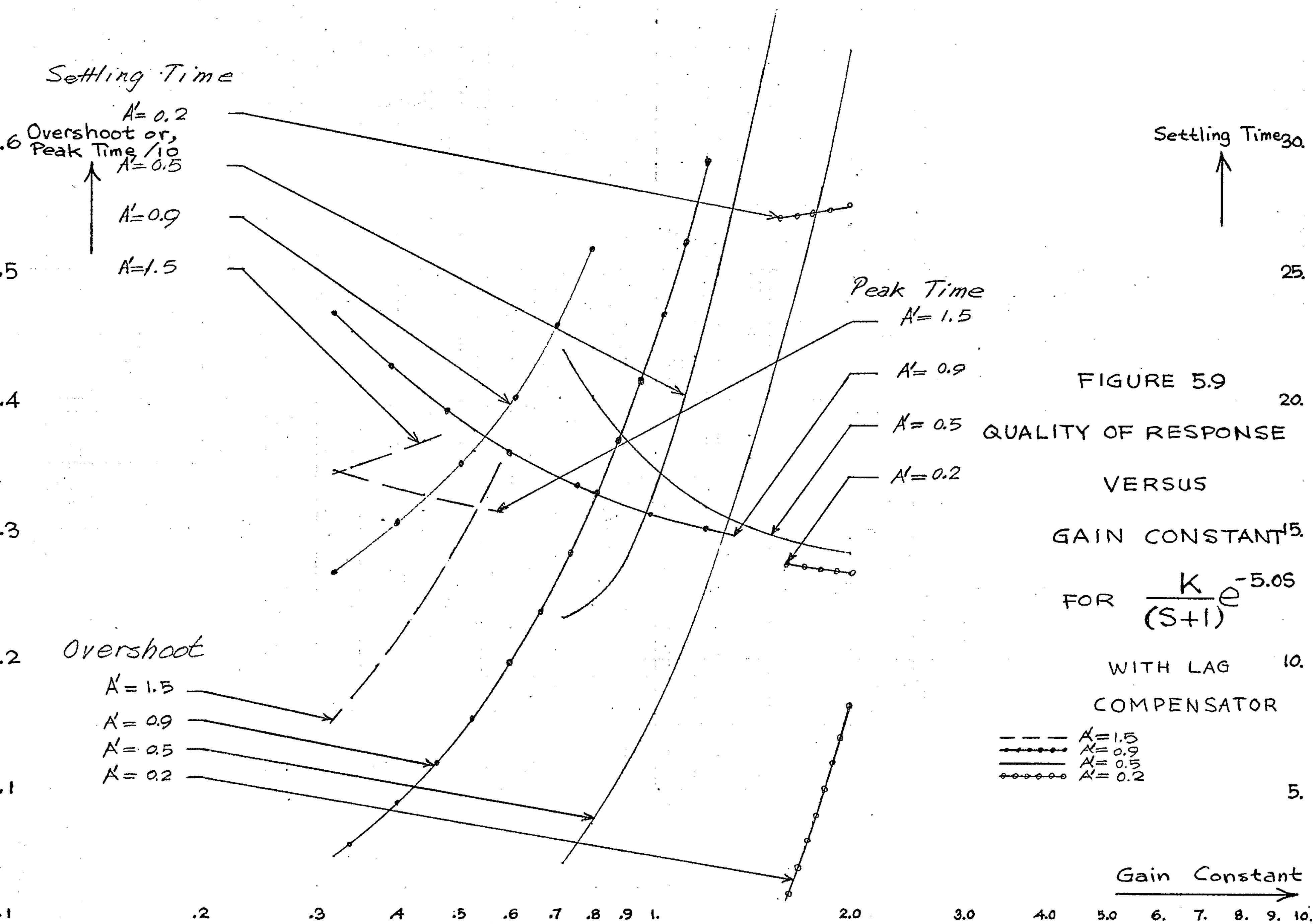


FIGURE 5.9

QUALITY OF RESPONSE VERSUS GAIN CONSTANT FOR $\frac{K}{(S+1)}e^{-5.0S}$ WITH LAG COMPENSATOR with reference to Figures 2.9 and 2.10, the designer can easily choose R once the design specifications are provided.

Likewise, for a delay time of 1.0, from the experimental data and the comparison of Nyquist plots, one is inclined to set A' to unity and shift the principal open loop pole closer to the origin in order to adjust the system to satisfy the design specification.

However, for a delay time of 5.0, A' may be set to 0.5 for more desirable performance. When A' is set to unity, the characteristics of this system are not particularly "optimal". The overshoot is considerably higher than that of the system with A' =0.5. However, the overshoot may be reduced by increasing the phase lag ratio. Also, when A' is set to unity, the transient characteristics of the compensated system can be more easily estimated. Consequently, A' is also set to 1.0 for the lag compensator of this system.

Example 3

Given a feedback system with a negative real pole at -1.0, a gain constant of 1.0, and a feedback path having a transportation lag of 1.0, design the series compensation network such that the overshoot will not exceed 16% without increasing the steady state error.

Since $$\omega_m = 2.03$$

so $$|G(j\omega_m)| = \frac{1}{2.26}$$

Assume that $1/G_m = 0.3$. Therefore R is approximately 1.5, and the compensated system is approximately $1e^{-s}/(1+1.5s)$. With reference to Figure 2.9 such a system is expected to have a gain margin of 1/0.27 for an overshoot of 16%. Thus it is necessary to have a phase lag ratio of approximately

$$R = \frac{1/2.26}{0.27} = 1.64$$

and a compensation network given by

$$G_c(S) = \frac{(1 + S)}{(1 + 1.64S)}$$

The digital simulation of this compensated system has the following qualities of response

$$OS = 0.1614,$$
$$T_p = 1.9000$$

and

$$T_s = 4.55$$

Example 4

Given a feedback control system, for which the negative loop gain is

$$G(S)\,H(S) = \frac{10e^{-S}}{(1 + S)\,(1 + \frac{S}{3})}$$

design a compensated system such that the overshoot will not exceed 20% without increasing the steady state error.

Since $$-\tan^{-1}\omega_m - \tan^{-1}\frac{\omega_m}{3} - \omega_m = -\pi$$

$$\omega_m = 1.625$$

The gain magnitude at this frequency is

$$|G(j1.625)| = 4.60$$

For a gain margin of 1/0.3, the phase lag ratio will be quite high. The interpolation of Figure 2.9 indicates that a desirable gain margin for the compensated system is roughly 1/0.23.

Therefore

$$R = \frac{4.60}{0.23} = 20$$

and hence

$$G_c(S) = \left(\frac{1 + S}{1 + 20S}\right)$$

The simulated system has the following qualities of response

$$OS = 0.1894$$

$$T_p = 7.380$$

and

$$T_s = 16.85$$

Example 5

If the gain of Example 4 is reduced to 5.0, design a compensated system to satisfy the above specification.

Since

$$|G(j1.625)| = 2.30$$

therefore,

$$R = 10.0$$

and hence

$$G_c(S) = \left(\frac{1 + S}{1 + 10S}\right)$$

The simulated step response characteristics are as follows

$$OS = 0.1680$$

$$T_p = 3.625$$

and $$T_s = 8.33$$

Here, the design technique based upon gain margin has been applied to four different types of systems, these compensated systems produce desirable performance characteristics. And even with the loop gain reduced by half, the phase lag ratio can be suitably adjusted to satisfy the design specification.

Example 6

Given a single loop feedback control system with negative loop gain (10,p.241)

$$G(S)\,H(S) = \frac{31.5e^{-S}}{(S+1)\,(30S+1)\,(\frac{S^2}{9}+\frac{S}{3}+1)}$$

design a compensated system such that the overshoot will not exceed 20% without increasing the steady state error. Since

$$\omega_m = 0.79$$

$$|G(j0.79)| = 1.121$$

Now, the uncompensated system has negative real roots at -0.0333 and -1.0. Therefore, if the complex roots are ignored, this system is similar to $ke^{-sT}/s(s+1)$. In order to have an overshoot smaller than 20% it is necessary to set the gain margin to approximately 1/0.34. Therefore

$$R = \frac{1.12}{0.34} = 3.3$$

and hence

$$G_c(S) = \frac{(1+30S)}{(1+99S)}$$

The digital simulation of the compensated system has the following qualities of response

$$OS = 0.1894$$

$$T_p = 7.3800$$

and

$$T_s = 16.85$$

Dorf has attempted to improve the system performance characteristics by simply adjusting the system to produce commonly acknowledged design criterion based upon the desirable values of phase margin. For this particular system he reduced the gain to a value of 17.7. This was found to give the following response characteristics

$$OS = 0.6078$$

$$T_p = 5.180$$

and

$$T_s = 36.1$$

It is noted that such a design technique leads to a system far from satisfactory.

V.6 CONCLUSION

In this chapter, it has been shown that the gain margin coupled with the "optimum" choice of the phase lag ratio may be used to design a compensation network for a linear control system involving delay.

The obvious advantage of this method is the simplicity in determining the lag compensation network and the applicability of this design technique to large sets of systems. It is particularly useful in the design of delay systems without a pole at the origin where the established design technique (based upon phase margin (10,p.241)) results in very unsatisfactory system performance characteristics.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

In this study, it has been found that gain margin instead of phase margin should be used in the analysis of transient performance of systems whose frequency response curves pass through the negative real axis with rapidly changing phase angles but little change in gain magnitudes.

The estimates of the quality of response for a large class of systems based on the principal pair of closed loop complex roots have been found to be good. These estimates are especially good when one open loop pole is located at the origin.

The differences between the transient responses of a system with actual delay and with various delay approximands increase for increasing delay time and also for increasing distance from the origin to the principal open loop pole.

In general, if the system has an open loop pole at the origin, the Pade 2 approximand may be applied to represent delay without introducing significant error. No generalizations of Wrights' observation, however, concerning the effect of delay approximand on the quality of response seem possible except for systems operating close to the stability limits.

A series compensation design technique based upon the theoretical and experimental results has been developed for linear control systems involving time delay of the form

1. $\frac{Ke^{-S\tau}}{S(S + A)}$ or $\frac{Ke^{-S\tau}}{S(S + A)\ (S + B)\dots(S + N)}$ $(A \leq B \leq C \leq \dots \leq N)$

for which $A\tau > 0.1$.

2. $\frac{Ke^{-S\tau}}{(S^2 + 2A\omega_n\ S + \omega_n^2)}$, for which $A > 0.6$.

3. $\frac{K}{(S + A)}\ e^{-S\tau}$ or $\frac{Ke^{-S\tau}}{(S + A)\ (S + B)\dots(S + N)}$ $(A \leq B \leq C \leq \dots \leq N)$

for any values of A, τ and B.

4. $\frac{K}{S(S^2 + 2A\omega_n\ S + \omega_n^2)}$ for any A and ω_n.

It is found that the series compensation method using a phase lead compensator will not improve system performance characteristics for most linear control systems involving time delay.

The sets of systems $\frac{Ke^{-S\tau}}{S(S + A)}$,

$\frac{Ke^{-S\tau}}{S(S + A)(S + B)\dots(S + N)}$ $(A \leq B \leq \dots \leq N)$ and $\frac{Ke^{-S\tau}}{S(S^2 + 2A\omega_n S + \omega_n^2)}$

may be normalized in the form of $ke^{-s}/s(s + a)$,

$\frac{ke^{-s}}{s(s + a)\ (s + b)\dots(s + n)}$ and $\frac{ke^{-s}}{s(s^2 + 2a\beta_n\ s + \beta_n^2)}$,

respectively.

For such sets of normalized systems for which $A\tau > 0.1$, a phase lag compensator such as

$$G_c(s) = \left(\frac{1 + \frac{s}{0.1}}{1 + \frac{Rs}{0.1}}\right)$$

may be inserted to produce the improved performance characteristics.

In order to satisfy the system performance characteristics specified in terms of the percentage overshoot, the lag ratio R has to be selected. Since the overshoot is dependent on both R and G_m, a trial and error method must be used to decide upon the value of R.

To delineate the procedures more fully, the following steps are suggested:

1. Normalize the system.
2. Calculate ω_m for the uncompensated system.
3. Calculate the gain magnitude of the uncompensated system at ω_m.
4. Choose $G_m = 1/0.30$ as the first trial value.
5. The first trial value of R is equal to $\dfrac{|G(j\omega_m)|_{uncomp.}}{0.30}$.
6. With reference to Figure 5.3 and specification on percentage overshoot, a more exact value of G_m may be chosen.
7. Based upon this new G_m, a more exact R may be determined.
8. Denormalize the system.

The sets of systems such as

$$\frac{K}{(S + A)} e^{-S\tau} \quad \text{and} \quad \frac{Ke^{-S\tau}}{(S + A)(S + B)\ldots(S + N)} \; (A \le B \le C \ldots \le N)$$

may be normalized in the form of

$$\frac{k}{(s + 1)} e^{-sT} \quad \text{and} \quad \frac{ke^{-sT}}{(s + 1)(s + b)\ldots(s + n)} \; (1 \le b \le c \ldots \le n)$$

For such sets of normalized systems, a phase lag compensator such as

$$G_c(s) = \frac{(1 + s)}{(1 + Rs)}$$

may be applied to improve the system performance characteristics.

The choice of R may follow the procedures presented above.

Other methods of compensation such as lag-lead compensation or feedback compensation have not been investigated. It would be desirable if more complete design techniques could be developed for systems involving delay.

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APPENDIX I

Digital Simulation using C.S.M.P.

All the systems examined in Chapters IV and V were solved by simulating the systems on the digital computer using the continuous system modelling programme (CSMP).

The continuous system modelling programme of delay function is a memory function consisting of sampling, storing and reading of input and output variables. Figure (A-1) gives the programme used for simulating

$$G_c(s)G(s)H(s) = \frac{31.5\ e^{-0.01s}}{(0.01s+1)(0.3s+1)[(\frac{0.01s}{3})^2+(\frac{0.01s}{3})+1]}\frac{(0.3s+1)}{(s+1)}$$

with a step input

$$R = \frac{32.5}{31.5}$$

Figure (A-2) gives the transient response of the system for a duration of 0.3 seconds.

Figure (A-3) gives the programme used for simulating

$$G_c(s)G(s)H(s) = \frac{31.5}{(0.01s+1)(0.3s+1)[(\frac{0.01s}{3})^2+(\frac{0.01s}{3})+1]}\cdot\frac{(0.3s+1)}{(s+1)}$$

$$\cdot\frac{[12 - 0.06s + (0.01s)^2]}{[12 + 0.06s + (0.01s)^2]}$$

```
****CONTINUOUS SYSTEM MODELING PROGRAM****

   ***PROBLEM INPUT STATEMENTS***

  PARAM T=0.01
  DYNAMIC
* SIMULATION OF DELAY CONTROL SYSTEM USING C.S.M.P.
*
* FORWARD LOOP TRANSFER FUNCTION IS OF THE FORM
* K(0.3S+1)/(0.01S+1)(0.3S+1)(S+1)((0.01/3)**2+(0.01/3)+1)
  Y1=REALPL(0.0,0.01,X)
  Y2=REALPL(0.0,0.3,Y1)
  Y3=CMPXPL(0.0,0.0,0.5,300.0,Y2)
  Y4=LEDLAG(0.3,1.0,Y3)
  C=31.5*Y4*9.0*10000.
* SIMULATION OF ACTUAL DELAY WITH T=0.01, Z=DELAY(N,P,C)
* WHERE P=DELAY TIME, N=NO. OF SAMPLING POINTS IN THE
* INTERVAL P
  Z=DELAY(40,0.01,C)
* UNIT STEP INPUT OF 32.5/31.5
  R=32.5/31.5
  X=R-Z
* PRINT PLOT THE OUTPUT RESPONSE C
  PRTPLT C
* FINTIME -- IS THE LENGTH OF TIME DESIRED IN THE SIMULATION
* SIMULATION OF DELAY TIME WITH T=0.01, Z=DELAY(N,P,C)
  TIMER DELT=0.00020, FINTIM=0.30, PRDEL=0.022,OUTDEL=0.006
* METHOD OF INTERGRATION DESIRED
  METHOD RKSFX
  END
  STOP

OUTPUT VARIABLE SEQUENCE
ZZ0013 Y4      C        Z        R       X        ZZ0003 Y1     ZZ0006 Y2
ZZ0010 ZZ0007 Y3       ZZ0014

 OUTPUTS     INPUTS     PARAMS   INTEGS + MEM BLKS   FORTRAN   DATA CDS
 18(500)   45(1400)    5(400)     5+  1=  6(300)    15(600)      15

        ENDJOB

          PROBLEM DURATION 0.0          TO   3.0000E-01

VARIABLE       MINIMUM          TIME         MAXIMUM          TIME
 C             0.0             0.0          1.1894E 00    7.3800E-02
```

FIGURE A-1

MINIMUM 0.0 C VERSUS TIME MAXIMUM 1.1894E 00

TIME	C
0.0	0.0
6.0000E-03	9.2170E-03
1.2000E-02	7.9901E-02
1.8000E-02	2.1658E-01
2.4000E-02	3.8133E-01
3.0000E-02	5.5115E-01
3.6000E-02	7.1400E-01
4.2000E-02	8.5931E-01
4.8000E-02	9.7970E-01
5.4000E-02	1.0720E 00
6.0000E-02	1.1360E 00
6.6000E-02	1.1737E 00
7.2000E-02	1.1887E 00
7.8000E-02	1.1854E 00
8.4000E-02	1.1685E 00
9.0000E-02	1.1428E 00
9.6000E-02	1.1123E 00
1.0200E-01	1.0806E 00
1.0800E-01	1.0505E 00
1.1400E-01	1.0238E 00
1.2000E-01	1.0019E 00
1.2600E-01	9.8525E-01
1.3200E-01	9.7381E-01
1.3800E-01	9.6722E-01
1.4400E-01	9.6478E-01
1.5000E-01	9.6566E-01
1.5600E-01	9.6898E-01
1.6200E-01	9.7389E-01
1.6800E-01	9.7960E-01
1.7400E-01	9.8549E-01
1.8000E-01	9.9104E-01
1.8600E-01	9.9592E-01
1.9200E-01	9.9991E-01
1.9800E-01	1.0029E 00
2.0400E-01	1.0050E 00
2.1000E-01	1.0061E 00
2.1600E-01	1.0065E 00
2.2200E-01	1.0063E 00
2.2800E-01	1.0056E 00
2.3400E-01	1.0047E 00
2.4000E-01	1.0036E 00
2.4600E-01	1.0025E 00
2.5200E-01	1.0015E 00
2.5800E-01	1.0006E 00
2.6400E-01	9.9990E-01
2.7000E-01	9.9936E-01
2.7600E-01	9.9900E-01
2.8200E-01	9.9880E-01
2.8800E-01	9.9874E-01
2.9400E-01	9.9880E-01
3.0000E-01	9.9893E-01

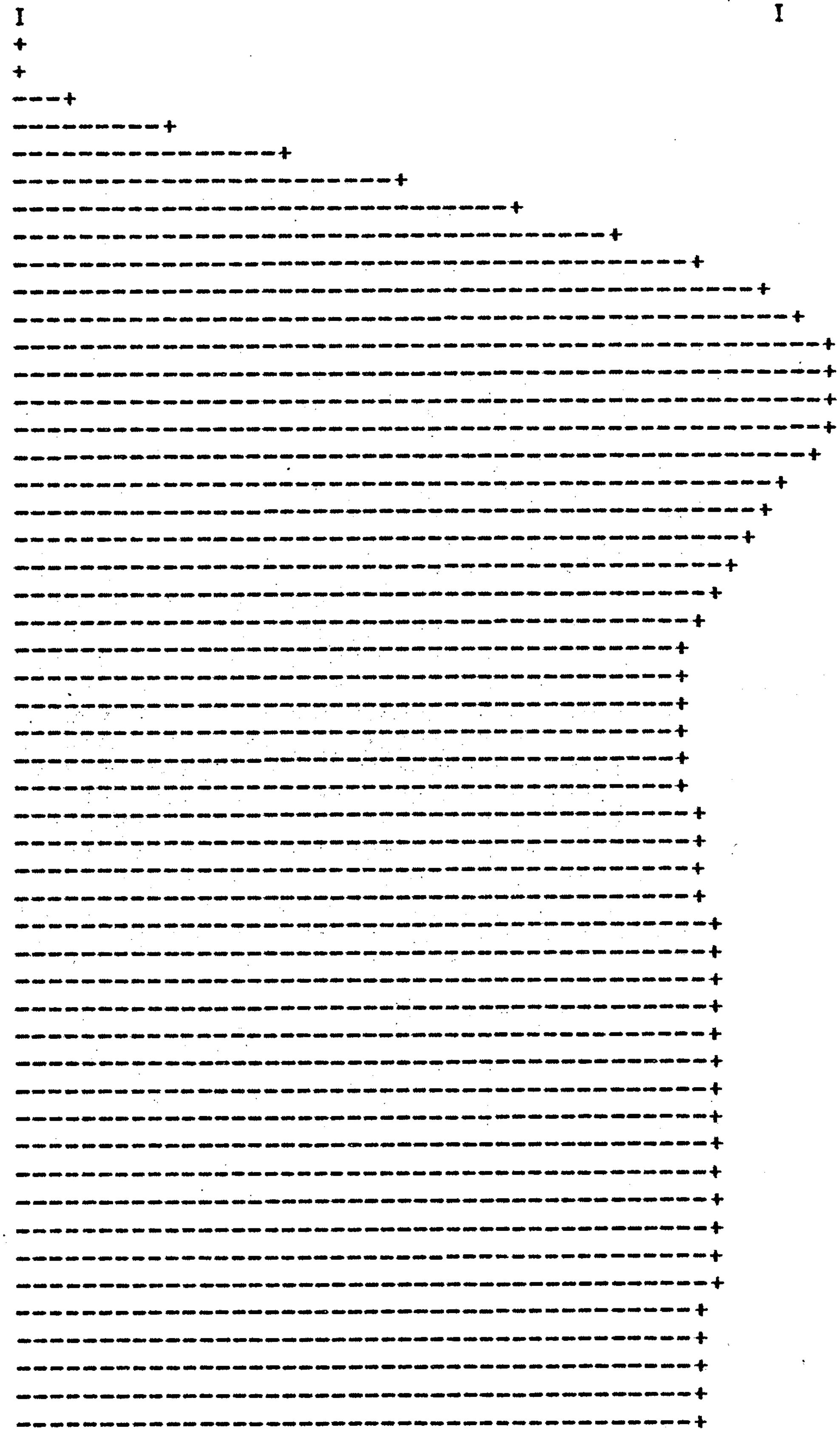

FIGURE A-2

```
***PROBLEM INPUT STATEMENTS***

  PARAM T=0.01
  DYNAMIC
* SIMULATION OF DELAY CONTROL SYSTEM USING C.S.M.P.
*
* FORWARD LOOP TRANSFER FUNCTION IS OF THE FORM
* K(0.3S+1)/(0.01S+1)(0.3S+1)(S+1)((0.01/3)**2+(0.01/3)+1)
  Y1=REALPL(0.0,0.01,X)
  Y2=REALPL(0.0,0.3,Y1)
  Y3=CMPXPL(0.0,0.0,0.5,300.0,Y2)
  Y4=LEDLAG(0.3,1.0,Y3)
  C=31.5*Y4*9.0*10000.
*      SIMULATION OF PADE 2 DELAY APPROXIMAND,WHERE THE POLYNOMIAL
*      FRACTION HAS BEEN TRANSFORMED TO ELIMINATE THE DERIVATIVES
       W1=Z2-12.0*C/T
       Z1=INTGRL(0.0,W1)
       W2=12.0*C/T-2.0*Z1/T
       Z2=REALPL(0.0,0.16667*T,W2)
       Z=Z1+C
* UNIT STEP INPUT OF 32.5/31.5
  R=32.5/31.5
  X=R-Z
* PRINT PLOT THE OUTPUT RESPONSE C
  PRTPLT C
* FINTIME -- IS THE LENGTH OF TIME DESIRED IN THE SIMULATION
* SIMULATION OF DELAY TIME WITH T=0.01, Z=DELAY(N,P,C)
  TIMER DELT=0.00020, FINTIM=0.30, PRDEL=0.022,OUTDEL=0.006
* METHOD OF INTERGRATION DESIRED
  METHOD RKSFX
  END
  STOP

OUTPUT VARIABLE SEQUENCE
ZZ0013 Y4      C        Z        R        X        ZZ0003 Y1       ZZ0006 Y2
ZZ0010 ZZ0007 Y3        ZZ0014 W1         Z1       ZZ0019 W2       ZZ0022 Z2

 OUTPUTS     INPUTS     PARAMS   INTEGS + MEM BLKS    FORTRAN   DATA CDS
 24(500)   60(1400)    5(400)     7+    0=  7(300)    21(600)      15

          ENDJOB
```

FIGURE A-3

APPENDIX II

DIGITAL COMPUTER PROGRAMME FOR DETERMINING THE ROOT AND GAIN LOCUS

Figure (B-1) gives the computer programme used for determining the root and gain loci of fourth order linear control systems (without numerator dynamics) with the Pade 2 delay approximand in the feedback path.

This programme may be modified to determine the root locus of any linear control systems with actual delay or any order of rational approximand in the negative loop gain.

For the case of actual delay, the computing technique is much simplified. The calculation of phase angles AN,AD,DAN and DAD is deleted; instead, DQ is replaced by

DQ = (-TD - YT) 180.0/3.1416

and CABS (K1)/CABS (K2) is replaced by e^{-xT}.

For any type of rational function, AN and AD may be modified to evaluate the phase angles from any point in the region due to its poles and zeros.

DEFINITION OF INPUT VARIABLES

X1, XL ------ The lower and upper limits of the abscissa of the region investigated.

Y1, YL ------ The lower and upper limits of the ordinate of the region investigated.

XINC, YING ------ The step sizes of the x and y axis increments.

T ------ The delay time.

DEFINITION OF OUTPUT VARIABLES

X, Y --- The coordinates of the root locus.

K --- The gain constant corresponding to the root locus (X,Y).

```
      DIMENSION T1(40),X1(40),XINC(40),XL(40),Y1(40),YINC(40),YL(40)
      DIMENSION DD1(30),DD2(30),DD3(30),DD4(30)
      REAL K
      COMPLEX TN1,TD1,TD2,TR
      COMPLEX K1,K2 ,K3
      READ(5,1) N
    1 FORMAT(I5)
       READ(5,2) (T1(I),DD1(I),DD2(I),DD3(I),DD4(I),X1(I),XL(I),XINC(I),
     1Y1(I),YL(I),YINC(I),I=1,N)
    2 FORMAT(7F5.2,3F10.3,F7.3)
C---- THIS IS A STANDARD PROGRAMME FOR DETERMINING THE ROOT LOCUS AND
C---- GAIN LOCUS OF ANY LINEAR SYSTEM WITHOUT NUMERATOR DYNAMICS IN THE
C---- FORWARD LOOP AND WITH ACTUAL DELAY OR ANY POLYNOMIAL DELAY
C---- APPROXIMAND IN THE FEEDBACK PATH.
      W1=1.5708
      W2=1.0
      DO 10 I=1,N
      X=X1(I)
      T=T1(I)
C---- D1,D2,D3,D4 ARE THE COEFFICIENTS OF THE DENOMINATOR OF THE
C---- FORWARD LOOP LINEAR TRANSFER FUNCTION:
C---- K/(D1*S**3+D2*S**2+D3*S+D4)
      D1=DD1(I)
      D2=DD2(I)
      D3=DD3(I)
      D4=DD4(I)
   22 Y=Y1(I)
   24 YD=Y+YINC(I)
      DX=YINC(I)
C---- A1,A2,A3 ARE THE COEFFICIENTS OF THE DELAY POLYNOMIAL APPROXIMAND
   23 A1=1.00
      A2=6.0/T
      A3=12.0/(T*T)
      ANR=A3-A2*X+A1*(X*X-Y*Y)
C---- CALCULATE THE PHASE ANGLES FROM ANY POINT IN THE REGION
C---- TO THE POLES AND ZEROS OF THE LOOP GAIN TRANSFER FUNCTION
      ANC=(2.0*X*A1-A2)*Y
      IF (ABS(ANR).LT.0.001) GO TO 41
      IF (ANR.GT.0.00) AN=ATAN(ANC/ANR)
      IF (ANR.LT.0.00) AN=ATAN(ANC/ANR)+3.1416
      GO TO 42
   41 AN=SIGN(W1,ANC)*SIGN(W2,ANR)
   42 ADR=A3+A2*X+A1*(X*X-Y*Y)
      ADC=(2.0*X*A1+A2)*Y
      IF (ABS(ADR).LT.0.001) GO TO 43
      IF (ADR.GT.0.00) AD=ATAN(ADC/ADR)
      IF (ADR.LT.0.00) AD=ATAN(ADC/ADR)+3.1416
      GO TO 44
   43 AD=SIGN(W1,ADC)*SIGN(W2,ADR)
   44 DANR=A3-A2*X+A1*(X*X-YD*YD)
      DANC=(2.*X*A1-A2)*YD
      IF (ABS(DANR).LT.0.001) GO TO 45
      IF (DANR.GT.0.00) DAN=ATAN(DANC/DANR)
      IF (DANR.LT.0.00) DAN=ATAN(DANC/DANR)+3.1416
      GO TO 46
   45 DAN=SIGN(W1,DANC)*SIGN(W2,DANR)
   46 DADR=A3+A2*X+A1*(X*X-YD*YD)
      DADC=(2.*X*A1+A2)*YD
```

FIGURE B-1

```
      IF (ABS(DADR).LT.0.001) GO TO 47
      IF (DADR.LT.0.00) DAD=ATAN(DADC/DADR)+3.1416
      IF (DADR.GT.0.00) DAD=ATAN(DADC/DADR)
      GO TO 48
   47 DAD=SIGN(W1,DADC)*SIGN(W2,DADR)
   48 TDR=D4+D3*X+D2*(X*X-Y*Y)+D1*(X*X*X-3.0*X*Y*Y)
      TDC=D3*Y+D2*2.0*X*Y+D1*(3.0*X*X*Y-Y*Y*Y)
      IF (ABS(TDR).LT..001) GO TO 49
      IF (TDR.GT.0.0) TD=ATAN(TDC/TDR)
      IF (TDR.LT.0.0) TD=ATAN(TDC/TDR)+3.1416
      GO TO 50
   49 TD=SIGN(W1,TDC)*SIGN(W2,TDR)
   50 DTDR=D4+D3*X+D2*(X*X-YD*YD)+D1*(X*X*X-3.0*X*YD*YD)
      DTDC=D3*YD+D2*2.0*X*YD+D1*(3.0*X*X*YD-YD*YD*YD)
      IF (ABS(DTDR).LT..001) GO TO 51
      IF (DTDR.GT.0.0) DTD=ATAN(DTDC/DTDR)
      IF (DTDR.LT.0.0) DTD=ATAN(DTDC/DTDR)+3.1416
      GO TO 52
   51 DTD=SIGN(W1,DTDC)*SIGN(W2,DTDR)
C---- SUMMATION OF ALL THE PHASE ANGLES
   52 DQ=(AN-AD-TD)*180.0/3.1416
      P=AN-AD-TD
      PP=DAN-DAD-DTD
      IF (ABS(P).GT.6.2832)   DQ=(P-SIGN(W2,(P))*6.2832)*180./3.1416
      DDQ=(PP)*180./3.1416
      IF (ABS(PP).GT.6.2832) DDQ=(PP-SIGN(W2,(PP))*6.2832)*180./3.1416
      IF ((DQ ) .GT.0.0) GO TO 5
      Z=(DQ)+180.0
      ZZ=(DDQ)+180.
      GO TO 6
    5 Z=(DQ)-180.0
      ZZ=(DDQ)-180.
C---- IS THE ANGLE CRITERIA SATISFIED TO THE REQUIRED DEGREE OF ACCURACY
    6 IF (ABS(Z).LT..002) GO TO 12
      IF (ABS(ZZ).LT..002)  GO TO 13
      IF (ABS(ZZ).LT.0.01)  GO TO 13
      IF ((ABS(YINC(I)-DX) .GT.0.001).AND.((ZZ/Z) .GT.0.000))  GO TO 34
      IF ((ABS(ZZ)+ABS(Z))  .GT.190.0)  GO TO 14
      IF(ZZ/Z) 11,13,14
   11 DX=-0.2*DX
      IF (ABS(DX).LT..00002)  GO TO 14
   34 Y=YD
      YD=Y+DX
      GO TO 23
   12 Y=Y
      GO TO 15
   13 Y=YD
C---- CALCULATE THE GAIN FOR THE CORRESPONDING ROOT LOCUS
   15 K1=CMPLX((2.0*X*A1-A2)*Y, A1*(X*X-Y*Y)-A2*X+A3)
      K2=CMPLX((2.0*X*A1+A2)*Y, A1*(X*X-Y*Y)+A2*X+A3)
      K3=CMPLX(D1*(X*X*X-3.*X*Y*Y)+D2*(X*X-Y*Y)+D3*X+D4, D1*(3.0*X*X*Y-
     1Y*Y*Y)+D2*2.0*X*Y+D3*Y)
      K=CABS(K3)  *CABS(K1)/CABS(K2)
      WRITE(6,8) D4,D2,D3,X,Y,K,T
    8 FORMAT(F3.1,' S**2+',F3.1,' *S+',F3.1,4F10.4)
   14 Y=Y+YINC(I)
      IF ( Y.LE.YL(I)) GO TO 24
   29 X=X+XINC(I)
```

FIGURE B-1 Cont'd.

```
      IF (X.LE.XL(I)) GO TO 22
   10 CONTINUE
      CALL EXIT
      END
```

FIGURE B-1 Cont'd.