THE UNIVERSITY OF MANITOBA

A COMPARATIVE STUDY OF THE MATHEMATICAL SKILLS OF CANADIAN, AND BRITISH PRE-UNIVERSITY STUDENTS

Being a Thesis Submitted to the Faculty of

Graduate Studies in Partial Fulfilment

of the Requirements for the Degree

of Master of Education

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by

William Swan

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ABSTRACT

The investigation was conducted in schools throughout Canada, England and Scotland during May and June, 1968.

PURPOSE OF THE INVESTIGATION

The purpose of the investigation was to obtain a comparative measure of Canadian mathematics achievement at the pre-university level.

THE INVESTIGATION

An investigator-devised test was used and test samples were selected from high schools in predominantly urban centres. Comparisons were made among the mean scores obtained within each of the following student groups in Canada and the other two countries:

1. All students in each test sample.
2. The top twenty students from each sample.
3. Students who had stated their intention to study mathematics beyond the high school level.

In addition, comparisons were made among the following groups of Canadian students who were following a pre-university course in which mathematics was compulsory:

1. Those wishing to discontinue the study of mathematics.
2. Those wishing to take additional mathematics in place of their existing program.

3. Those wishing to continue with their existing mathematics program.

ANALYSIS OF THE RESULTS

The significance of the difference of means for each of the comparisons was established by computing a critical ratio which then was referred to a table of normal distribution for evaluation.

The test used in the investigation had an estimated reliability, using the Kuder-Richardson formula 20, ranging from +0.859 for Canada to +0.904 for England.

Using this test as a criterion, the analysis of the results showed:

1. The English and Scottish test sample scores were significantly higher than the Canadian score.
2. The English and Scottish test scores for those students intending to study mathematics beyond the high school level, were significantly higher than the Canadian scores for the corresponding group.
3. The test scores of the top twenty English students was higher than either the Scottish or Canadian scores for the corresponding groups. Because of the small number of students involved this result was not tested for significance.
4. The test mean of Canadian students in compulsory mathematics programs was significantly higher for those students who wished to take additional mathematics than for those who preferred to continue with their existing program.

Similarly, the test mean of the students who wished to continue with their existing programs was significantly higher than for those students who would prefer to terminate mathematics and study another subject.

CONCLUSIONS

In each country the mean score of students intending to study mathematics beyond the high school level was greater than the mean of their total test sample. This indicates that students intending to continue their mathematics studies beyond the high school level could be given a more demanding program.

Evidence also suggested that the test mean of Canadian students in compulsory mathematics program was related to their attitude toward the subject.

In summary, the investigator suggests that the higher mathematics achievement of the English and Scottish students can be related to the following factors:

1. A stricter selective process for students entering high school in the English and Scottish systems.
2. Less extensive mathematics programs in those Canadian provinces in which mathematics is a required pre-university subject.
3. Greater specialization in the English and Scottish school systems. In these systems students are also given more freedom of choice in selecting their courses of study.

4. More time allocated to private study in English and Scottish school systems and correspondingly less time allocated to formal academic study.

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THE INVESTIGATION AND DEFINITIONS OF TERMS USED

In recent years, there has been a growing awareness in many countries of the importance of comparative education. This awareness has led to much international discussion of the topic, which, in turn, has led to innovations and changes in content and in teaching methods. The changes which have already taken place and the continual drive to improve objectives, ideals, content, and methods of education have given rise to a growing need for reliable information with which to compare the products of school systems, both on a national and international basis.

To provide such information, high school students of twelve of the world's major nations participated in a comparative study of mathematics achievement in 1964. This study, the first of its kind in the mathematics field, was conducted by the International Project for the Evaluation of Educational Achievement (IEA).

The outcome of the study provided a basis for analysis and comparison among the participating countries while enabling each to examine its system more critically.

However, Canada did not take part in this study and, as far as is known, there is no standard by which the mathematical achievement of Canadian students can be evaluated on an international level.

I. THE INVESTIGATION

Purpose of the Investigation

The primary purpose of the present investigation was to obtain a comparative measure of the mathematics achievement of Canadian students at the pre-university level. The following comparisons were made among student groups in Canada and in England and Scotland, two of the countries which had taken part in the international project:

1. The total samples from each of the three countries.
2. The top twenty students from each sample.
3. The students from each sample who had signified their intention to study mathematics beyond the high school level.

Other comparisons were made to evaluate the achievement of Canadian students following compulsory mathematics programs.

Background to the Investigation

Until early 1950, comparative education consisted, in the main, of written accounts of the educational systems of each country. Following this, many qualitative analytic studies took place in a number of countries. These studies resulted in descriptive reports such as the world survey of UNESCO and the publications of the International Bureau of Education and OECD (Organization for Economic Cooperation and Development). Although the need for cross-national data on comparative mathematical achievement had been recognized from time to time, there had been little significant development in the field until 1961. At that time B.S. Bloom of the University of Chicago presented a plan for a major international study in mathematics.

The plan received financial support from the United States Office of Education and was launched under the title of the International Study of Achievement in Mathematics.[1] The participating countries were England, Scotland, Australia, Belgium, Federal Republic of Germany, Finland, France, Israel, Japan, The Netherlands, Sweden, and the United States.[2] The plan included a comparative study of mathematics achievement among the pre-university populations in the participating countries. As mentioned previously, Canada did not take part in this international study.

The Need for Measuring Mathematical Achievement in Canada

Many countries, including Canada, are at present in the process of extending and improving their scientific and technological education. Because these programs have their roots in mathematics, there is a need to assess the adequacy of our national school system in this field. This could be done by conducting a study within the Canadian educational system with comparisons limited intra-nationally. However, Kneller[3] contends that comparative education is not worthy of the name unless it deals with the science of comparing educational systems on an international level. Hence, a comparative study on a cross-national basis seemed preferable for the present investigation.

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[1] Torsten Husen (ed.), International Study of Achievement in Mathematics (New York: John Wiley and Sons, 1967), p. 27.

[2] Ibid., p. 17.

[3] Padogogik Der Volker: Eine Einfuhrung in Die Vergleichende Erziehungswissenschaft, p. 503. Cited by George F. Kneller, Encyclopedia of Educational Research (3rd Edition, December, 1960), p. 317.

Such a study would have more value and greater adaptability if comparisons were made with some of the countries which had taken part in the international study.

II. DEFINITIONS OF TERMS USED

Pre-university Students

The national groups of pre-university students were defined as follows:

1. Canada: students in their final high school year who wrote a mathematics examination in June 1968 provided they had reached, at least, grade XII.

2. Scotland: students in the fifth year of high school who wrote the Scottish Certificate of Education, Higher Grade paper in mathematics in June, 1968.

3. England: students in the sixth form who wrote the General Certificate of Education, Advanced Level examination, in mathematics, in June, 1968.

Comparative Education

G.F. Kneller[4] views comparative education as an attempt to study education in different countries in the light of historical development, and pertinent educational theories and practices, and in consideration of social, cultural, and economic growth of these countries. This definition indicates the desirability of cross-national

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[4] George F. Kneller, Encyclopaedia of Educational Research, (3rd edition, December, 1960), p. 316.

studies in preference to studies limited to particular societies or cultures.

III. LIMITATIONS OF THE INVESTIGATION

Inevitably, a test intended for international participation, the administration of which cannot be controlled by the investigator, must have limitations. The most significant of these in the present investigation were:

1. The sample was not selected at random. Schools were selected from predominantly urban centres from as large a geographic area as possible in each country. The results of the test, therefore, could not reliably be generalized to the national populations.

2. Only five provinces elected to take part and participating students were, therefore, not an accurate cross-section of the Canadian pre-university year. However, because all Scottish high school students follow the same mathematics program, and English high school students are restricted to one of four programs, there was less chance of heterogeneity in these two groups.

3. The test was not necessarily a measure of the curriculum objectives of each country. However, it did possess a high reliability factor for each of the countries and, by comparison with the IEA test, it constituted a reasonable measure of the talents and skills considered most valuable in the pre-university mathematics program.

4. The investigator had no control over the administration of the test. However, there was no reason to doubt that this had been done efficiently and fairly by the principals and teachers concerned.

5. Much of the present thesis is already obsolete due to the

essential contemporaneity of comparative studies. No sooner have statistics been tabulated than they become out of date.[5] Already many of the Canadian school systems have introduced new mathematics programs which are soon to replace the existing traditional programs. The Scottish Department of Education introduced an alternate mathematics program in 1968 which will be followed by a new form of final examination in 1970. However, many of these changes result from other comparative studies and this factor of obsolescence is not entirely undesirable.

IV. ORGANIZATION OF REMAINDER OF THE THESIS

Chapter II is a review of current literature emphasizing the revival, functional aims, and changing methods of comparative education.

Chapter III outlines the educational systems in Canada, England and Scotland.

The investigation, its planning and administration are dealt with in Chapter IV. A comparison of the investigator-devised test with the IEA test shows the similarities of the two summary contents. Test validity is established by classifying the test questions in accordance with a taxonomy of educational objectives used in the IEA test construction.

Chapter V is a statistical analysis of the results. Some of the factors which may have affected the outcome are considered. The significance of the test results are established and test reliability

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[5]Thomas Woody, Future in Comparative Education. Proceedings of Second Annual Conference on Comparative Education (New York University, 1955), pp. 38-46.

factors calculated for each of the three countries.

The final chapter summarizes the outcome of the analysis. Conclusions are drawn relating to the mathematics achievement of Canada compared with the other two countries. Finally, implications of the study are considered and suggestions made for further research.

CHAPTER II

CURRENT LITERATURE ON COMPARATIVE EDUCATION

Origin and Revival of Comparative Education

Comparative education is commonly dated from Marc Antonie Jullien de Paris, French publicist and "father of comparative education." His ideas for establishing an international commission on education and a network of teacher educational institutes across Europe were first published in a short treatise in 1817.[6] Bereday[7] maintains that since then comparative education has resulted in some brilliant qualitative writing, but also, unfortunately, in some lifeless recounting of pedagogical facts without any attempt to analyse and compare. He claims that the present revival of comparative education is the result of the international situation, which has led to an unprecedented interest in pedagogy on the part of social scientists. However, the creation of UNESCO has probably been the greatest single factor in the present increasing interest in foreign educational systems.[8] Despite this revival, Husen and Postelthwaite,[9]

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[6] Stewart Fraser, Julien's Plan for Comparative Education 1816-1817 (Teacher College Columbia University, 1964).

[7] George Bereday, Comparative Methods in Education (Holt Rinehart and Winston, 1964), p. 9.

[8] George F. Kneller, "Comparative Education," Encyclopedia of Educational Research (3rd edition), p. 320.

[9] Torsten Husen, op. cit., p. 26.

as recently as 1967, stated that attempts to measure objectively the actual achievements of educational systems, and to explore empirically the factors related to such achievements, were, at that time, relatively few at the international level.

Aims of Comparative Education

Educators in the world's major countries are faced with the identical problem of making their programs world-minded in order to prepare students for the realities of the twentieth century. Sulwyn Lewis[10] viewed this as a formidable task claiming that the first need is for the removal of misunderstanding about others, their ideas and history. Brickmann[11] defines the aims of comparative education as the enhancement of human ideals and practices, the enlargement of cultural horizons, the reduction of international tensions, an enrichment of education considered as a philosophy, science, and cultural process. More precisely, he defines the functional aims as:

1. providing a framework, a set of techniques, as a basis of interpretation, and a source of hypothesis and conclusions about education.

2. furnishing reliable information about educational systems, ideas, problems and activities.

The establishment of international agencies such as UNESCO and OECD has helped achieve these aims and has intensified the exchange

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[10] Sulwyn Lewis, Towards International Cooperation (Pergammon Press, 1966).

[11] William Brickmann, Comparative Education in Theory and Practice Ed. Sociol. 116-75. George F. Kneller, "Comparative Education," Encyclopedia of Educational Research (3rd edition), p. 316.

and accumulation of valuable data relating to different patterns of educational organization, curricula, and teaching methods.[12]

Methods of Comparative Education

Comparative education, until recently, has tended to avoid evaluation, and has had to rely largely on descriptive material.[13] However, this tendency is changing and the present attitude of educators is that, given the goals of an education system, they are able to assess the outcome. King and Anderson[14] consider empirical and statistical techniques inseparable from national and international surveys. Even if this were not so, King[15] asserted, the increasing proportion of national investments in education and the resultant demand for efficiency and productivity call for less guesswork and more precision in comparative studies. The more quantifiable the data, the easier, and more reliable the analysis. This is not to say that comparative education consists only of collecting, sifting and analysing data. Such studies as the IEA project have relied on a balance between qualitative and quantitative methods, between theoretical and empirical methods. Whatever method is chosen, stated Sulwyn Lewis,[16] the comparative educator must still locate the pertinent issues,

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[12] Torsten Husen, op. cit., p. 13.

[13] Ibid., p. 13.

[14] Ibid., p. 26.

[15] Edmund King, Comparative Education. Vol. III, No. 3, 1967.

[16] Sulwyn Lewis, op. cit.

bearing in mind that what is important in one system may be of little significance in another.

IEA Study

The IEA was the first large scale empirical cross-national study of educational outcomes.[17] It has yielded not only extremely interesting research findings but also valuable administrative and methodological experience. Husen[18] considers that such studies can enable educationalists and ultimately those responsible for educational planning and policy-making, to benefit from the educational experiences of other countries. These studies enable educationalists to examine their own system in a more critical light. Cresswell,[19] however, criticizes the IEA test claiming that it is neither a reasonable measure of what the school children in the United States are learning now, nor a good instrument to gauge their mathematical skill. He contended that neither creativity nor originality can be identified through tests comprising multiple choice, machine scored, type questions. He went on to say that the low scores obtained by the United States was not just because its students had forgotten mathematics, but because its schools had been developed, and total curriculum planned with other objectives in mind. However, the next phase in the IEA project will involve the assessment of achievement in several subjects at the same

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[17] Torsten Husen, op. cit., p. 35.

[18] Ibid., p. 14.

[19] John L. Cresswell, "Are American Children Behind in Mathematics?" Parents Magazine (March, 1968), p. 58.

time.[20] Seven groups of subject matter experts in physics, biology, chemistry, English as a foreign language, French as a mother tongue, and as a foreign language, and civics/social studies have already prepared working papers giving a preliminary analysis of what to test in these subjects.

Present Status and Problems of Comparative Education

At present, education is the largest single investment, including defence, made by most governments; Canada is no exception here. Critical decisions are continually being made in the educational field which demand judgement based on the knowledge of the situation. One of the tasks of comparative educators is to provide information whereby these decisions can be formulated and their outcome evaluated. As Kneller[21] stated, there is need for more studies of comparative education, particularly in nations which hope to extend the values of their own culture. Even if the local educator is not interested in spreading his own ideas, the exchange of ideas from abroad will help him to examine his own system more clearly. Husen[22] pointed out empirical approaches in comparative education, and hence research which could tackle problems dealing with the "productivity" or "effectiveness" of the various school systems, have suffered from the lack of cross-nationally valid criteria of evaluation. Most of the

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[20]Torsten Husen, op. cit., p. 15.

[21]Kneller, op. cit., pp. 321-322.

[22]Torsten Husen, op. cit., p. 20.

present literature on comparative education is what Ulich[23] termed "an accumulation of direct rather than comparative information about educational trends in other countries." However, this situation is now changing with the establishment of the Center for Comparative Education at the University of Chicago, and the formation of the Council of the International Project for the Evaluation of Educational Achievement. Even with this increased interest, another problem arises in that literature and research of comparative studies quickly become obsolete. No sooner have facts and statistics been tabulated than they become out of date. As Woody[24] stated, one has to run fast to keep abreast of the rapid educational progress taking place in the world today, even before the creative task of making progress can begin.

Summary

In summary, although the last few years have seen a welcome increase in attempts to analyse educational problems through simultaneous comparisons, there were still, in 1967, very few complete studies at the international level. The present revival of comparative education must be maintained through national and international surveys using empirical and statistical techniques. Unless educators recognize the need for and the benefits to be derived from comparative education; unless they are prepared to allow their own system to stand and be

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[23] Robert Ulich, "Some Observations Concerning the Study of Comparative Education," Proceedings of the First Annual Conference on Comparative Education (New York University, 1954).

[24] Thomas Woody, op. cit., pp. 38-46.

measured against others; unless they are prepared to criticize and be criticized, then there is little hope of their own system rising beyond narrow provincialism.

CHAPTER III

DIFFERENCES AMONG THE EDUCATIONAL SYSTEMS

The school structure is one of the most important variables accounting for differences in the achievements of the pupils.[25] The differences in systems are, therefore, of greatest relevance, because a measure of comparative achievement would be of little use unless some account were taken of the cross-national differences in the respective educational systems.

Accordingly, information has been gathered regarding the important features of the school systems of the participating countries. This information includes the types of secondary schools, some of their characteristics, the method of student selection for these schools, and the percentage of students to be found at each stage.

Particular attention has been paid to the role of mathematics in the curricula. The program in some cases is designed for students intending to study mathematics beyond the high school level, in others, it is a prerequisite for university entrance. This leads to major differences in the program content which must be considered in any cross-national study.

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[25]Torsten Husen, op. cit., p. 221.

I. CANADIAN EDUCATION

In Canada, school attendance is compulsory up to the age of fifteen or sixteen, depending on the province in which the student resides.[26] Children commence the first grade of elementary school at about age six, although some enter kindergarten or regular classes before that age.[27]

Section 93 of the British North America Act assigns to the provinces exclusive jurisdiction over education.[28] Consequently, there are ten provincial school systems in Canada. Although they have much in common, some have unique features. Newfoundland has a denominational organization under one Department of Education. Quebec has two branches under the same department: one consists of the French and Roman Catholic sector, the other consists of the English and protestant sector. The federal government has responsibility for the North West Territories and all Indian and Eskimo education.[29]

High School Courses

In practice, there are three broad courses of instruction offered to students at the high school level. One course is designated for students who will attend university. In this program

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[26] Organization and Administration of Public Schools in Canada. Dominion Bureau of Statistics (third edition, 1966).

[27] Ibid., p. 11.

[28] Ibid., p. 13.

[29] Ibid., p. 12.

studies are planned to meet the entrance requirements of the local university. A second program for students preparing for specific technical positions includes a limited formal academic program, which is suited to their interests and abilities, and is offered by vocational high schools. The third program is intended for those students who do not wish to progress beyond high school, or to take specialized vocational training. This course, normally referred to as the general course, provides a broad general academic education which is less intensive than the university entrance program.

High School Pre-university Year

In most cases the matriculation course in its graduation year consists of a maximum of five subjects: English, mathematics, history or geography, one language, and one science. Table I provides a comparison of a number of factors relating to the pre-university grades in Canadian high schools. The data relate to the 1967-68 school year and was provided by the provincial Departments of Education for inclusion in this study. In general, students enter university at the end of grade twelve except in Ontario where they enter after grade thirteen, and in Newfoundland, where students write the university preparatory examination in grade eleven. In Quebec and Nova Scotia students may enter the first year of a four-year university course after grade eleven, or the second year of a four-year university course after grade twelve. The percentage of students reaching the final grade ranges from a high of 87 per cent in Alberta to a low of 32 per cent in Ontario. The Department of Education is the examining authority in each province, except in Ontario where individual schools

TABLE I

PRE-UNIVERSITY YEAR IN CANADIAN HIGH SCHOOLS
1967-68 SCHOOL YEAR

Province	Pre-university Grade	Percentage Reaching Pre-university Year	Percentage of Pre-university Grade Writing a Final Examination in Mathematics	Examination Authority
British Columbia	12**	75	70	Department of Education
Alberta	12	87	40	High School and University Matriculation Examination Board
Manitoba	12	44	compulsory	Department of Education
Saskatchewan	12	61	78	Department of Education
Ontario	13	32	60	Local Schools
Quebec	11*	***	90	Department of Education
New Brunswick	12**	35	85	Department of Education
Nova Scotia	11*	60	66	Atlantic Provinces Examining Board
Prince Edward Island	12	45	compulsory	Atlantic Provinces Examining Board
Newfoundland	11	40	compulsory	Department of Education

*Students may enter the first year of a four-year university course after grade XI, or the second year of a four-year course after grade XII.

**Some schools offer an additional year equivalent to the first year of university, Grade XIII.

***Data not available.

set their own examinations.[30]

Mathematics Program

With the exception of the North West Territories which has adopted the British Columbia program of studies, each province has its own mathematics syllabus. An outline of the common topics in set theory, arithmetic, algebra, geometry, trigonometry, and calculus contained in the Program of Studies of each province for the 1967-68 school year is presented in Table II. Ontario, where mathematics is not a required subject for matriculation, has the most extensive program, covering such advanced topics as groups, rings and fields, matrices and calculus. Alberta and British Columbia also include some of the more advanced topics in algebra and calculus and, as in Ontario, the courses are designed for those students intending to study mathematics at university. There are few major differences between the subject outlines in the other provinces although the Newfoundland program is understandably limited since it terminates in grade eleven.

II. THE ENGLISH EDUCATION SYSTEM

The Education Act of 1944 placed the English public school system under the charge of the Minister of Education whose responsibilities were defined as "the control and direction of the national policy for providing a varied and comprehensive educational service

[30] Ontario Department of Education. Letter, A.W. Banks, Registrar, December, 1968.

TABLE II

PROGRAM OF STUDIES IN MATHEMATICS IN CANADIAN HIGH SCHOOLS

Subject Matter	B.C.	Alta.	Man.	Sask.	Ont.	Que.	N.B.	N.S.	P.E.I.	Nfld.
Arithmetic										
Number systems	X	X	X							
Complex numbers	X	X	X	X	X	X				
Sets	X	X			X		X			
Algebra										
Polynomials	X	X	X	X	X	X	X	X	X	X
Systems of equations	X	X	X	X	X	X	X	X	X	X
Ratio, proportion, variation	X	X	X	X	X	X	X	X	X	X
Series and sequencies	X	X	X	X	X	X	X			X
Binomial theorem	X	X	X	X	X	X	X			
Permutations and combinations			X	X	X	X				
Probability		X		X	X					
Determinants	X	X	X	X	X					
Matrices	X	X		X	X					
Vectors		X			X	X				
Groups, rings and fields		X			X					
Mapping functions					X					
Geometry										
Plane	X	X	X	X	X	X	X	X	X	X
Analytic	X	X	X	X	X		X	X	X	
Trigonometry										
Logarithms	X	X	X	X	X	X	X	X	X	X

(continued)

Table II (continued)

Subject Matter	B.C.	Alta.	Man.	Sask.	Ont.	Que.	N.B.	N.S	P.E.I.	Nfld.
Exponential functions	X		X		X		X			
Trigonometric ratios	X	X	X	X	X	X	X	X	X	X
Expansion of multiple angles	X	X		X	X	X		X	X	
Identities	X	X	X	X	X	X	X	X	X	
Calculus										
Derivatives	X	X			X					
Integrals		X			X					

in every area."[31] At present, school attendance is compulsory from five to fifteen years of age, although the upper limit is to be extended to sixteen in the school year of 1970-71.[32] Under this system, students write the eleven-plus examination between the ages of eleven and twelve. On the basis of their performance on this examination, students are allocated to a particular type of secondary school. The externally imposed eleven-plus examination is diminishing in use and significance. It is gradually being replaced by locally prepared examinations.[33]

Secondary Schools

Essentially, there are four kinds of secondary schools in England: grammar schools, secondary modern schools, comprehensive schools, and secondary technical schools. The majority of candidates for university graduate from the grammar system. The grammar schools have long been the classical school in England and normally only the upper 20 per cent in the eleven-plus examination are accepted by them.[34] About 91 per cent of this group complete five or more years of secondary education. Those continuing beyond the fifth year usually study only three subjects at an advanced level during their sixth and seventh years.

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[31] Education Act (England), 1944, Section 1.

[32] George A. Beauchamp and Kathryn E. Beauchamp, Comparative Analysis of Curriculum Systems (The Kagg Press, 1966), p. 11.

[33] Ibid., p. 20.

[34] G.S. Osborne, Scottish and English Schools (Longmans Green and Company Ltd., 1966), p. 221.

Examination Authorities

The terminal examinations in the grammar schools are conducted by external examination boards. Members of these boards are not directly concerned with the operation of the schools. The major examining authorities are the Cambridge Local Examinations Council, the Northern Universities Joint Matriculation Board, the London University Examination Board, and the Oxford and Cambridge Joint Board.

Grammar School Certification

The grammar school programs in each subject are directed toward the General Certificate of Education, referred to as the G.C.E. These examinations are written by students at two levels, the Advanced level, and the Ordinary level. The Ordinary level examinations are normally written at the age of sixteen and a student may write eight, or even more subjects, at this level. However, the Ministry of Education state that only 20 per cent of any age group may be expected to attempt the Ordinary level examinations in four or more subjects.[35] The Advanced level examinations are written at the end of a two-year course following the Ordinary level. About 12 per cent of each age group reaches this level.[36] Most universities in England require at least two Advanced level, along with three Ordinary level, credits in any subjects to meet their "Certificate of Eligibility" requirement. However, since university entrance, in most cases, is highly

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[35] *Ibid.*, p. 217.

[36] Torsten Husen, *op. cit.*, p. 28.

competitive, a student normally requires three Advanced, along with three Ordinary credits to gain admission.

Non-university Courses

Comprehensive and technical schools offer programs leading toward university entrance but their programs are primarily directed toward the Certificate of Education, referred to as the C.S.E. According to the Secondary School Examination Council of England, the next 20 per cent of children below the G.C.E. group may be expected to achieve four or more credits in the C.S.E. examinations. A further 20 per cent can be expected to attempt the examination in individual subjects only. It was the Council's intention that a "Very Good" pass in the C.S.E. would be equivalent to a pass in the same subject at the G.C.E. Ordinary level,[37] which is well below university entrance standard. In the secondary modern schools most students follow a non-certificate course, although at the age of thirteen they are given an opportunity to transfer to grammar schools provided they achieve the required academic standing.

The Mathematics Programs

The four major examination boards mentioned on page 23 all provide a syllabus in pure mathematics, although each is ostensibly the same. The syllabus for the London University G.C.E. Advanced Level Examination in Pure Mathematics is shown in Appendix A. Although

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[37] George A. Beauchamp and Kathryn E. Beauchamp, *op. cit.*, p. 22.

this includes the basic algebra, geometry, and trigonometry, taught in the traditional mathematics programs in most Canadian provinces these topics are treated more extensively and to a greater depth than in the Canadian programs. Trigonometry, for example, includes periodic properties of trigonometric functions. Pure geometry is applied to the determination of angles made by planes and straight lines with one another. Calculus not only covers simple differentiation and integration, but the differentiation of inverse functions and simple examples of integration by substitution and integration by parts.

III. THE SCOTTISH EDUCATION SYSTEM

Scottish education is administered centrally by the Scottish Education Department in Edinburgh and locally by education authorities, usually in a county burgh. The Education (Scotland) Act of 1962 requires the parents of every child from five to fifteen years of age "to provide the efficient education for him suitable to his age, ability, and aptitude, either by causing him to attend a public school regularly or by another means." As in England, the school leaving age is to be raised from fifteen to sixteen in 1970.[38]

The Transfer

The step from primary to secondary education, known as the "transfer," takes place at age twelve, one year later than in England.

[38]Leslie S. Hunter, The Scottish Education System (Pergammon Press, 1968), p. 107.

At present each local authority arranges its own "transfer" scheme.[39] Consequently, there are thirty-five separate transfer schemes in existence. In all cases the type of secondary education to which students are transferred is determined by the outcome of the tests and assessments used in these schemes.

Types of Courses Available

Pupils are allocated to secondary courses of two main types. These courses are certificate courses leading to examinations for the award of the Scottish Certificate of Education, or non-certificate courses. A recommendation by the Scottish Education Department, in 1961, stated that 35 per cent of each age group should proceed to certificate course.[40] The 1962 Act states that a parent is not entitled to select a course of education for his child unless, in the opinion of the education authority, the child stands a reasonable chance of profiting from it.[41]

Secondary Schools

Perhaps the most satisfactory classification of secondary schools in Scotland at the present time, is in terms of courses and combinations of courses provided. A classification used by the Scottish Education Department is shown in Table III.[42] The schools

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[39] Education (Scotland) Act 1962, Section 30.

[40] Scottish Education Department, Transfer from Primary to Secondary Education, Cmmd, 1538 Edinburgh, H.M.S.O. Para 86.

[41] Education (Scotland) Act 1962, Section 2a.

[42] Leslie S. Hunter, op. cit., p. 87.

TABLE III

CLASSIFICATION OF SECONDARY SCHOOLS IN SCOTLAND

A. SELECTIVE SCHOOLS

1. Offering non-certificate courses only

2. Offering certificate courses only

B. COMPREHENSIVE SCHOOLS

Offering non-certificate courses, and complete Ordinary and Higher grade certificate courses

C. PART-SELECTIVE, PART-COMPREHENSIVE

1. Offering non-certificate courses, and years one and two of both Ordinary and Higher grade certificate courses

2. Offering non-certificate courses, and years one and two of Higher grade and years one to four of Ordinary grade certificate courses

3. Offering non-certificate courses, and years one to four of Ordinary grade certificate courses (no Higher grade)

in Category A1, are commonly called "junior secondary" schools and are equivalent to secondary modern schools in England. The schools in A2, are the "senior secondary" schools and correspond to grammar schools in England. Category B are "all through" comprehensive schools and offer both non-certificate, and complete Ordinary and Higher grade certificate courses, similar to the comprehensive schools in England. Category C schools are comprehensive but only provide Ordinary grade certificate courses. The present intention of the Scottish Education Department is that all secondary schools should eventually provide both certificate and non-certificate courses; in 1965 almost 65 per cent of secondary students attended Category C schools.[43]

Subject Selection

In most cases students select their own programs of studies based on their ability and aptitude but the ultimate responsibility for the content of any student program rests with the headmaster. Students may take five or more subjects at the Higher grade but past records show that only about 10 per cent of the students attempting Higher grade examinations obtain this number of passes.[44] Higher grade examinations correspond to Advanced level examinations in England or grade 12 examinations in Canada. If a student fails a Higher grade examination he may be awarded a credit at the Ordinary level provided

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[43] Ibid., p. 88.

[44] G.S. Osborne, op. cit., p. 218.

he attains a required standard. A similar system in England allows a student who fails an Advanced level examination to be awarded an Ordinary level certificate in that subject.[45]

University Entrance

Entrance to Scottish universities is governed by a standard minimum prescribed by the Scottish Universities Board at St. Andrews. The present minimum requirements are four credits at the Higher grade of the S.C.E., or three credits at this grade provided two of these are at Credit standard (60 per cent), or one is Very Good (70 per cent). As in England, the Board's certificate does not guarantee acceptance by any university, since admission is determined by individual faculty requirements, and by the number of places available. The shortage of places has led to competition in which the gap between the Entrance Requirements of the Board and actual entry conditions has widened considerably.[46]

The Mathematics Program

A copy of the Scottish Certificate of Education Board, "Syllabus - Mathematics on the Higher Grade," is contained in Appendix B. This syllabus covers most of the topics taught in the Canadian high schools, referred to in Table II. The geometry section includes three-dimensional figures, and the theorem of the three perpendiculars,

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[45] General Certificate of Education, 1968, Oxford Local Examinations. Regulations and Syllabus, p. 9.

[46] Leslie S. Hunter, *op. cit.*, p. 218.

which is beyond the level reached in any of the Canadian programs. On the other hand, it does not include such topics as matrices, vectors, and transformations now being taught in Alberta and Ontario. Although this syllabus meets the university entrance requirements, it is not necessarily the student's final high school year as there is an opportunity to continue studies toward a Sixth Year Certificate. Pure mathematics is then offered in the Elementary Analysis course, which includes algebra, with some set theory, calculus, and trigonometry, and in the geometry course, coordinate geometry of the circle and conic sections, vectors and matrices.[47]

IV. DIFFERENCES IN THE SCHOOL SYSTEMS

Time Spent on Mathematics

The mean number of minutes spent weekly on mathematics by each of the sample populations is given below.

Canada	343.39 minutes
England	295.39 minutes
Scotland	306.48 minutes

The Canadian time was heavily weighted by Ontario students who comprised almost one-third of the test sample. The Ontario mathematics program consisted of a total instructional time of approximately nine hours weekly.

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[47] Scottish Certificate of Education Examination Board, Conditions and Arrangements 1969.

Factors Affecting Achievement

Some of the factors to be considered when comparing student achievement in the three countries are summarized below.

1. Children enter elementary school at the age of five in England and Scotland, and normally one year later in Canada.

2. A highly selective system operates in England and Scotland. Only a small percentage of English and Scottish students reach the final high school year compared with Canada.

3. It is customary for Canadian students to study five subjects in the final year. Courses in the British system are more specialized, with a normal load of four subjects in Scotland and three in England.

4. Subject selection in England and Scotland is the prerogative of the student. In most Canadian systems students follow a set program to meet the university entrance requirements.

5. University entrance examinations are controlled by national authorities in England and Scotland, whereas in Canada, they are controlled by provincial Departments of Education.

6. University entrance in England and Scotland is not guaranteed by meeting the standard minimum requirements.

CHAPTER IV

THE TEST

The test items were drawn from the large body of content common to the three national programs. The investigator attempted to provide an extensive and representative sampling of the topic content from each country. The test, as well as having curricular validity for each country, had to meet an authoritative specification of the objectives of mathematics achievements. The objectives selected belonged to the cognitive domain which included those objectives dealing with the recall or recognition of learned material and the development of intellectual abilities.

The Test Groups

In selecting a test population from students in their final high school year, two important questions had to be answered if a realistic comparison were to be made between the selected groups. Firstly, it was necessary to know what percentage of students reach the pre-university year in each country, and secondly, what percentage of this group study mathematics. The comparative figures, along with the number of years' schooling completed by each group are listed in Table IV. Whereas over 50 per cent of Canadian students reach the pre-university year, a much smaller percentage reaches the same stage in the other two countries. The table also shows that a higher percentage of students study mathematics in their final high school

TABLE IV

PERCENTAGE OF STUDENTS REACHING PRE-UNIVERSITY YEAR
AND PERCENTAGE STUDYING MATHEMATICS
AT THAT LEVEL

Country	Percentage Reaching This Stage	Percentage Studying Mathematics	Number of Years Schooling	Number of Years High School
Canada	53	79	12	6
England	12	63	13	7
Scotland	18	73.8	12	5

year in Canada than in England or Scotland.

Population Sampling

Individual random sampling would have been impracticable because of the size of the eligible student population. On the other hand, the schools could have been used as the primary sample unit and the selected schools asked to take a sub-sample of their eligible student population. The sample reliability, in this case, would have depended almost entirely on the cooperation of the schools involved. The refusal of any school to participate, or its failure to take a random sample before testing, would have destroyed the randomness of the selection. The investigator ultimately decided to select 20 school systems from ten Canadian provinces along with 25 schools from Scotland, and the same number from England, all from predominantly urban areas. A total of 1,980 test papers were sent to the addressees listed in Appendix C.

Mean Ages of Participating Students

The mean ages of students taking part in the test were as follows:

Canada	17.56 years
England	17.48 years
Scotland	16.93 years

These figures do not provide the exact mean ages of the students. The questionnaire asked for student's ages in 'years,' hence the actual mean age of each test population was probably in the region of 0.5 years greater than the figures quoted above.

The Test

Test preparation. The thesis test was devised by the investigator and consisted of multiple choice type questions. The most important advantage of this type of test is that it makes possible much more extensive and representative sampling of the content topics than would otherwise be possible within reasonable time limitations.[48] A second advantage is that objective scores are easily obtained. To make the test more acceptable to school principals a time limit of one hour was imposed. With this condition in mind a test comprising 50 multiple-choice questions was planned. The aim was to make a survey of mathematical cognitive achievement without using a predetermined standard of what this achievement should be. Thus, no student was expected to answer every question on the test and a wide spectrum of achievement was anticipated.

Test items. Having taken into account the major high school mathematics programs in use in the three countries during the 1967-68 school year (see Chapter III), the investigator set out to design a test which, as far as possible, would include the topics of traditional mathematics common to all three countries. These topics, itemized under the broad headings of Arithmetic, Algebra, Elements of Analysis, and Geometry, are shown in Table V. The subject items are generally the same as those used to outline the program of mathematics studies in Canadian high schools in Table II. It should be noted at

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[48] Torsten Husen, *op. cit.*, p. 35.

TABLE V

LIST OF TOPICS FOR THESIS TEST

Arithmetic
General treatment of number systems
Natural numbers
Integers
Real numbers
Complex numbers
Algebra
Polynomials
Equations and inequalities
Systems of equations
Determinants
Probability
Elements of Analysis
Polynomial functions
Rational functions
Circular functions
Inverse circular functions
Logarithmic and exponential functions
Series
Geometry
Euclidean geometry
Analytical geometry
Trigonometry

this point that any categorization of mathematical topics is somewhat subjective and disagreements in allocation of items do occur.[49] The test questions are listed in Appendix D.

Test administration. Test arrangements were made by school principals in the participating schools. The investigator had requested that the test be written either just before or just after completion of the final examinations in the 1967-68 school year.

Test returns. Test returns were not as good as had been anticipated. Only 574 fully completed papers were returned. The returns are summarized in Table VI. Scotland had the largest percentage of returns, although this was not surprising under the circumstances. In Scotland the S.C.E. examinations finish a few weeks before the end of the school year but students are compelled to attend school during this period to complete the attendance regulations required for certification. Some of the headmasters indicated that they were pleased to administer the examination during what would otherwise be a "slack period." However, in Canada and England many schools stated that they were unable to give up time to write the test before the final examinations, and as students would not be returning to school afterwards, they were unable to participate.

Objectives and Validity

In the IEA project, test items were prepared by a committee of mathematicians and statisticians selected from the twelve

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[49] Ibid., p. 106.

TABLE VI

PERCENTAGE OF COMPLETED TESTS RETURNED
BY THE THREE COUNTRIES

Canada		England		Scotland	
Number Sent Out	Percentage Returned	Number Sent Out	Percentage Returned	Number Sent Out	Percentage Returned
980	22.9	500	22	500	47.8
(from five provinces)		(from seven schools)		(from fifteen schools)	

participating countries. The committee proposals form an authoritative statement of the principal objectives of mathematics instruction at the high school level. Therefore, they were used as a standard to indicate the extent to which the investigator-devised test measured the desired objectives of pre-university mathematics teaching.

Comparison Between IEA and Thesis Tests

Topical outlines of the IEA and thesis tests are presented for comparison in Table VII. The IEA student groups were defined as follows:[50]

1. Level 3a: Students in their final high school year studying mathematics as an integral part of their course for their future training, or as part of their pre-university studies. These are referred to as math specialists.

2. Level 3b: Students in their final high school year studying mathematics as a complementary part of their studies. These are referred to as nonmath students.

The thesis test was intended to cover both these groups, since in the Canadian provinces in which pre-university mathematics is compulsory, all students follow the same course whether or not mathematics is a prerequisite for their future studies. Perhaps the most notable difference between the two tests is the omission of calculus from the thesis. Although Scotland and England allocate a considerable part of their mathematics program to differential and integral calculus, seven of Canada's ten provinces do not offer this subject in their

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[50] *Ibid.*, p. 28 and 238.

TABLE VII

SUMMARY OF CONTENT OF IEA AND THESIS TEST

Subject Matter	Number of Test Items		
	IEA		Thesis
	Level 3a	Level 3b	
Arithmetic	3	12	6
Algebra	20	18	20
Euclidean Geometry	5	13	2
Analytical Geometry	8	5	6
Sets*	4	4	0
Trigonometry	3	3	4
Analysis	8	1	9
Calculus*	9	0	0
Probability	1	1	3
Logic*	8	1	0

*The thesis test was not designed to cover these topics.

school programs (see Table II). Accordingly the test was limited to pre-calculus topics in order to avoid unfairly weighting the content against Canada. This led to comments from a number of Scottish schools that, while a most important part of their program had been omitted from the test, several minor topics not normally covered in the Scottish schools higher grade syllabus had been included. However, this minimal disadvantage to Scotland can in no way be compared with the very great disadvantage which would have been incurred by Canada had calculus been included. Another major difference was that the thesis test was confined to traditional topics whereas approximately 15 per cent of the IEA test covered modern topics. These items were omitted from the thesis test as very few of the pre-university students in June 1968 would have completed a full course of this type; also, during the introductory stages of the modern mathematics courses, there were fewer common topics between the various school programs. Therefore, the experiment was designed to test traditional mathematics achievement only and was limited to students who had followed this type of course. The shortage of questions on Euclidean geometry in the thesis test arose from the disparity between the Canadian and British programs in this area. Most school systems in Canada finish Euclidean geometry in Grade XI whereas, in Britain, the course is continued well beyond this level and includes in some cases, such items as Ceva's theorem, Menelaus' theorem, the nine-point circle, and the Euler line, along with some topics in solid geometry. The IEA organizers were presumably faced with the same problem since five of the level 3b questions were also used in the 15 to 16 year-old age group; in fact, six of the level

3b questions were also used with the 13-year-old age group.[51] As the thesis test was limited to one hour it would have been unrealistic to include questions intended for a much younger age group. Nevertheless Table VII shows, as a pre-calculus test in traditional mathematics, the thesis test summary content parallels the pattern of the IEA test, with an emphasis on arithmetic, algebra, analytic geometry and analysis.

Validity. The validity of a test is often defined as the degree to which it measures what it purports to measure.[52] Reliability is a necessary but not sufficient condition for validity. Bloom and his co-workers[53] have produced a taxonomy of educational objectives in which they group cognitive objectives into the six major classifications of knowledge, comprehension, application, analysis, synthesis, and evaluation. More recently the IEA test construction committee and mathematics consultants agreed upon a short list of objectives which they believed would be accepted by most teachers of mathematics regardless of their nationality. These are:

A. Knowledge and information: recall of definitions, notation, concepts.

B. Techniques and skills: solutions.

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[51] Ibid., p. 105.

[52] Robert L. Ebel, Measuring Educational Achievement (Prentice-Hall Educational Series, 1965), p. 468.

[53] Julian C. Stanley, Measurement in Today's Schools (1964), p. 173.

C. Translations of data into symbols or schema, or vice versa.

D. Comprehension: capacity to analyse problems, to follow reasoning.

E. Inventiveness: reasoning creatively in mathematics.[54]

These behaviours, like Bloom's, are confined to the cognitive domain. The IEA committee classified behaviours A and B as "lower" mental processes and C, D, and E as "higher" mental processes. They recognized that the boundary between lower and higher processes was by no means sharp. In accordance with this taxonomy of educational objectives Table VIII presents a two-axis chart of the thesis content. The chart covers the 50 multiple-choice questions. In almost every case these questions have more than one objective. The totals indicate that the test emphasis is weighted towards the "lower" mental processes, diminishing from a high of 50 for A to a low of three for E.

The Questionnaire

The questionnaire, which was completed by all students taking part in the study, is found in Appendix E. The information provided by the students enabled comparisons to be made of the relative achievement between sub-groups of the test populations. It also enabled comparisons to be made of the mean ages of the test groups, and of the mean instructional time spent on mathematics, in the three countries.

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[54]Torsten Husen, op. cit., p. 81.

TABLE VIII

TWO AXIS CHART OF THESIS TEST OBJECTIVES

Course Content	Objectives					Total
	A[1]	B[2]	C[3]	D[4]	E[5]	
Arithmetic	6	5	4	3	0	18
Algebra	20	20	13	8	0	61
Euclidean Geometry	2	1	0	1	0	4
Analytic Geometry	6	4	2	2	0	14
Analysis	9	9	3	6	3	30
Trigonometry	4	2	2	0	0	8
Probability	3	3	0	1	0	7
Total	50	44	24	21	3	142

[1]Knowledge and information

[2]Techniques and skills

[3]Translation of data into symbols

[4]Comprehension

[5]Inventiveness

The Statistical Technique

The thesis test, Appendix D, was used as the basis of comparison. The bulk of computations were made on the Southern Alberta Institute of Technology IBM Computer 1800. The program produced the mean and standard deviation of the test sample of each country along with the following information:

1. Mean and standard deviation of the scores for students intending to study mathematics beyond the high school level.

2. Mean and standard deviation of the top 20 scores for each country.

3. Mean age of participating students from each country.

4. Mean weekly mathematics instructional time for each country.

In addition, the mean and standard deviation of the scores for the following groups of Canadian students, enrolled in compulsory mathematics programs, were computed:

1. Those wishing to drop mathematics and select another subject.

2. Those wishing to take additional mathematics.

3. Those who would prefer to continue with their present course.

Distribution of test scores. The Kolmogorov-Smirnov test was used to see whether the distribution of the thesis test scores, for each country, was a good fit to a normal distribution. O'Toole[55]

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[55]A.L. O'Toole, Elementary Practical Statistics (Allendoerfer Undergraduate Series), p. 181.

states that this test is a simple and powerful test of the goodness of fit of any theoretical distribution to an observed frequency distribution.

Comparison of test means. To discover whether two groups differ sufficiently in performance to enable one to state with confidence that there is a difference between the means of the populations from which the samples were drawn, we need to know the Standard Error of difference between the two sample means.[56] This general procedure was used throughout the analysis to compare test means. In each case the formula for the Standard Error of the difference between two uncorrelated or independent means, suggested by Garret[57] was used.

$$\sigma_D = \sigma_{(M_1 - M_2)} = \sqrt{\sigma^2_{M_1} - \sigma^2_{M_2}}$$

in which:

σ_{M_1} = The SE of the mean of the first sample

σ_{M_2} = The SE of the mean of the second sample

σ_D = The SE of the difference between the two sample means.

Tests of significance. A critical ratio was found by dividing

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[56]Henry E. Garrett, *Statistics in Psychology and Education* (David McKay Co., Inc., 1965), p. 213.

[57]*Ibid.*, p. 214.

the difference between the sample means by its standard error of difference ($CR = D/\sigma_D$). This operation reduced the obtained difference to a sigma score whose significance could then be evaluated from a table of Normal Probability Curve Areas.[58] The application of this method depended on the test scores having a normal distribution.

The statistical analysis follows in Chapter V.

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[58] *Ibid.*, p. 446.

CHAPTER V

STATISTICAL ANALYSIS OF THE DATA

The present chapter contains an analysis of the test results and presents some of the factors which may have affected the outcome of these results.

Test Reliability

The Kuder-Richardson formula KR20 was used to estimate the reliability of the test for each of the national samples. Formula KR20 is essentially a function of (a) the number of items, (b) the means and the standard deviations of the test scores, and (c) the standard deviations of the item difficulties.[59] Travers defined reliability by means of KR type formula, as the extent to which all items on a test can be considered to contribute to the measurement of a simple common variable.[60] Ferguson noted that if the items on a test have high inter-correlation with each other and are measures of the same attribute, then the reliability coefficient will be high.[61] To facilitate calculation of the reliability coefficients, item difficulties for the three populations were computed and are given in Table IX.

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[59] George A. Ferguson, Statistical Analysis in Psychology and Education (McGraw-Hill, 1966), p. 379.

[60] Robert M. Travers, An Introduction To Educational Research (The Macmillan Co., 1966), p. 187.

[61] George A. Ferguson, op. cit., p. 380.

TABLE IX

THESIS TEST - ITEM DIFFICULTY (p)

Question No.	Canada	England	Scotland
1	.39	.65	.39
2	.23	.51	.28
3	.24	.66	.48
4	.57	.86	.61
5	.51	.91	.79
6	.33	.90	.54
7	.66	.90	.81
8	.48	.78	.87
9	.51	.43	.34
10	.38	.61	.39
11	.82	.85	.78
12	.42	.88	.62
13	.36	.50	.41
14	.34	.19	.12
15	.48	.40	.42
16	.81	.98	.90
17	.62	.64	.56
18	.60	.83	.81
19	.42	.69	.43
20	.44	.69	.46
21	.49	.88	.71
22	.25	.50	.17
23	.09	.54	.22
24	.62	.80	.25
25	.70	.67	.69

(continued)

Table IX (continued)

Question No.	Canada	England	Scotland
26	.72	.80	.78
27	.74	.82	.86
28	.67	.92	.81
29	.76	1.00	.83
30	.46	.79	.35
31	.63	.86	.73
32	.79	.93	.82
33	.26	.55	.33
34	.36	.61	.46
35	.48	.74	.51
36	.45	.70	.53
37	.35	.65	.52
38	.42	.62	.39
39	.26	.59	.41
40	.49	.81	.54
41	.41	.69	.40
42	.19	.37	.19
43	.57	.84	.62
44	.39	.60	.62
45	.46	.62	.35
46	.27	.68	.29
47	.32	.79	.61
48	.49	.48	.23
49	.15	.40	.15
50	.05	.19	.09

"p" is the proportion of examinees passing each item.

The index difficulty "p", in the table, is the proportion of examinees passing each item. The item variance for each test item, which is the product of the item difficulty and the fraction of students failing each item, was then calculated. These results were used in the KR20 formula to establish estimates of reliability of the total test for each of the three countries.

The results are shown in Table X. The reliability factor for Canada was 0.859, for England was 0.904, and for Scotland was 0.902. This indicates that the test provides an estimate that has a high reliability for each of the countries, and the scores can be used for analyses involving individuals as well as those involving group comparisons.

TABLE X

RELIABILITY OF TEST

Country	$\sum p.q.$ (Item Variance)	Reliability Factor
Canada	10.10	0.859
England	8.69	0.904
Scotland	8.83	0.902

Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test[62] was used to measure the goodness of fit between the normal frequency distribution and the test scores obtained for each country. The results are shown in Table XI along

[62] A.L. O'Toole, op. cit., p. 181.

with the relevant significance levels. In each case the largest absolute value of the maximum difference was less than the critical value for $\alpha = 0.20$. Therefore, the normal distribution was a good fit for each of the observed samples.

TABLE XI

KOLMOGOROV-SMIRNOV TEST RESULTS

Country	Maximum Difference	Sample Population	Critical Values of Maximum Difference		
			Significance Level* .20	.05	.01
Canada	.042	225	.071	.090	.186
England	.106	110	.113	.130	.155
Scotland	.047	239	.070	.080	.106

*Reject null hypothesis if maximum difference greater than critical value[63]

Test Scores

The obtained test scores along with the standard deviations, for each country are contained in Table XII.

The Test Comparisons

Hypothesis 1: There is no significant difference in the test scores of the Canadian and English students.

[63] *Ibid.*, p. 183.

TABLE XII

SUMMARY OF RESULTS

Country	Total Test Population		Students Who Intend Studying Mathematics Beyond High School Level	
	Mean	Standard Deviation	Mean	Standard Deviation
Canada	23.12	7.99	24.99	7.73
England	34.91	8.67	37.10	8.26
Scotland	25.49	8.59	29.31	7.96

The results of the analysis are as follows:

Standard Error of Difference σ_D	Obtained Difference Between Means D	Critical Ratio D/σ_D
0.984	11.79	11.97

The critical ratio of 11.97 is much larger than 2.58 so the difference between the two means is significant at the .01 level. The null hypothesis is, therefore, rejected.

Hypothesis 2: There is no significant difference in the test scores of the Canadian and Scottish students.

The results of the analysis are as follows:

Standard Error of Difference σ_D	Obtained Difference Between Means D	Critical Ratio D/σ_D
0.770	2.37	3.08

The critical ratio of 3.08 is larger than 2.58 so the difference between the two means is significant at the .01 level. The null hypothesis is, therefore, rejected.

Hypothesis 3: There is no significant difference in the test scores of students in Canada and students in England who intend studying mathematics beyond the high school level.

The results of the analysis are as follows:

Standard Error of Difference σ_D	Obtained Difference Between Means D	Critical Ratio D/σ_D
1.11	12.11	10.9

The critical ratio of 10.9 is much larger than 2.58 so the difference between the two means is significant at the .01 level. The null hypothesis is, therefore, rejected.

Hypothesis 4: There is no significant difference in the test scores of students in Canada, and students in Scotland who intend studying mathematics beyond the high school level.

The results of the analysis are as follows:

Standard Error of Difference σ_D	Obtained Difference Between Means D	Critical Ratio D/σ_D
0.959	4.32	4.50

The critical ratio of 4.50 is larger than 2.58 so the difference between the two means is significant at the .01 level. The null hypothesis is, therefore, rejected.

Canadian Students in
Compulsory Mathematics Programs

In many Canadian provinces mathematics, at the pre-university level, is a compulsory subject. Students enrolled in compulsory mathematics programs were asked, if given the choice, which of the following alternatives they would select:

1. Continue with their present mathematics program.

2. Take additional mathematics.

3. Drop mathematics and select another subject.

The scores, standard deviations, and number of students in each group are contained in Table XIII.

TABLE XIII

TEST RESULTS OF CANADIAN STUDENTS IN COMPULSORY MATHEMATICS PROGRAMS

Group	Mean Score	Standard Deviation	Number of Students
Continue present program	19.67	6.81	61
Take additional mathematics	25.62	7.99	27
Drop mathematics	14.42	5.97	7

The significance of the obtained differences in the test scores of these three groups is tested in hypothesis 5 and 6.

Hypothesis 5. There is no significant difference in mathematics achievement between Canadian students who elected to continue their present mathematics program and those who would prefer to take additional mathematics.

The results of the analysis are as follows:

Standard Error of Difference σ_D	Obtained Difference Between Means D	Critical Ratio D/σ_D
1.76	5.95	3.38

The critical ratio of 3.38 is larger than 2.58 so the difference between the two means is significant at the .01 level. The null hypothesis is, therefore, rejected.

Hypothesis 6: There is no significant difference in mathematics achievement between Canadian students who elected to continue their present mathematics program and those who would prefer to drop mathematics and take another subject.

The results of the analysis are as follows:

Standard Error of Difference σ_D	Obtained Difference Between Means D	Critical Ratio D/σ_D
2.42	5.25	2.17

The critical ratio of 2.17 is larger than 1.96 so the difference between the means is significant at the .05 level. The null hypothesis is, therefore, rejected at the .05 level.

Top Twenty Scores from Each Country

The mean and standard deviation of the top twenty scores from each country are given in Table XIV. Although these scores involve a much larger percentage of the English test sample than either the Scottish or Canadian, the English mean is still much higher than the other two.

Synopsis of Results

1. The test reliability, as estimated by the Kuder-Richardson

TABLE XIV

TOP TWENTY SCORES FROM EACH COUNTRY

Country	Mean Score	Standard Deviation	Number of Students	Percentage of Test Population
Canada	38.10	2.42	20	8.9
England	44.15	2.32	20	18.2
Scotland	42.25	2.71	20	8.4

20 formula, was highest for England and lowest for Canada. Nevertheless, the Canadian reliability factor of .859 was still substantially high.

2. The Kolmogorov-Smirnov test indicated that the normal distribution was a very good fit for each of the national test samples.

3. Both the English and Scottish total sample population test means were significantly higher than the Canadian mean.

4. Both the English and Scottish test means for students intending to study mathematics beyond the high school level, were significantly higher than the test mean for the corresponding Canadian student group.

5. The test mean of Canadian students in compulsory mathematics programs who would prefer to take additional mathematics, was significantly higher than the mean of those who wished to continue their present program. Similarly, the test mean of the students who wished to continue with their present program was significantly higher than the mean of those students who would rather drop mathematics and take

another subject.

5. The mean of the top twenty English scores was greater than either the Scottish or Canadian mean for the same group.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND IMPLICATIONS OF THE STUDY

The present chapter summarizes the findings arising out of the analysis of the data. Conclusions are drawn relating to the differences in achievement between the three countries. Finally, implications of the study are considered and suggestions made for further research.

I. SUMMARY OF FINDINGS

1. Mathematics achievement of the following groups were compared.

   a. Total test samples.

   b. Students intending to study mathematics beyond the high school level.

   c. The top twenty students in each sample.

The English and Scottish students scored significantly higher than the Canadian students. In every case the null hypothesis was rejected at the .01 level.

2. Canadian students in compulsory mathematics programs who preferred to take additional mathematics scored significantly higher than those students who were satisfied with their existing program. Similarly, students who were satisfied with their existing program scored significantly higher than students who preferred to drop mathematics and take another subject.

3. In all three countries, the mean score of students intending to study mathematics beyond the high school level was higher than the mean score of the total sample.

II. CONCLUSIONS

The following conclusions are based on the synopsis of findings. Differences in the education systems of the three countries are also considered.

1. The higher mathematics achievement of English and Scottish students may, in part, be attributed to the following factors.

   a. Stricter selective process for students entering high school programs in England and Scotland.

   b. More specialized courses of study in the English and Scottish pre-university programs.

   c. Canadian students in many cases being compelled to take mathematics as a matriculation subject. In England and Scotland students select their own courses of study.

   d. Less extensive mathematics program in those Canadian provinces where mathematics is compulsory. Students who wish to study mathematics as part of their pre-university preparation are probably unable to achieve their maximum potential because of the restricted program necessary to enable the "average" student to pass the course. The test analysis indicated that the mean scores of students intending to study mathematics beyond the high school level was invariably higher than the total test sample mean scores.

2. Alberta, British Columbia, and Ontario, where the mathematics courses are designed for students intending to study mathematics at university have a more extensive curriculum content than the other provinces.

III. IMPLICATIONS

The findings of the study supported the need for further research in comparative education. Student scores achieved in mathematics tests are not the only measures of academic achievement. Many of the factors affecting the outcome of the test must be examined. Some benefits may be derived from further research along the following lines:

1. An examination of the objectives of the mathematics program in each province so that a measure may be made of the extent to which these objectives have been achieved.

2. A comparative study of achievement of university mathematics majors so that a considered judgement may be made of relative national mathematical skills. This would match only the brightest students from each country.

3. A comparative study of inter-provincial mathematics achievement to test the weaknesses and strengths of the individual programs.

4. An intra-provincial achievement test to examine the desirability of maintaining the present compulsory mathematics programs which are aimed at almost 60 per cent of the student

population.

5. Study the implications of having fewer academic sessions and more time devoted to private study in the pre-university year.

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APPENDICES

APPENDIX A

UNIVERSITY OF LONDON EXAMINATION BOARD

GENERAL CERTIFICATE OF EDUCATION

MATHEMATICS ADVANCED LEVEL

Paper I

The theory of the quadratic function and of quadratic equations. Simple algebraic functions. The theory of indices and logarithms.

Complex numbers; sum, product and quotient of complex numbers in algebraic form, Argand diagram.

Permutations and combinations. Elementary examples in the use of induction. The binomial theorem for a positive integral index. The use of the expansion $(1 + x)^n$, where n is fractional or negative; simple approximations. Determination of a linear law from experimental data.

Rectangular cartesian co-ordinates, including parameters, applied to the straight line, circle and parabola. Easy problems on loci.

Circular measure; small angles, including the use of limits such as $\sin \theta/\theta$ as θ tends to zero. The formulae for $\sin (A \pm B)$. Periodic properties of trigonometric functions. Sine and cosine formulae for a triangle with elementary applications. General solution of equations such as $\cos \theta = \cos \alpha$. Expression of $a\cos \theta + b\sin \theta$ in the form $r\cos (\theta - \alpha)$. DeMoivre's theorem for a positive integral index.

The differentiation of simple algebraic, trigonometric (excluding inverse functions), exponential and logarithmic functions. Differentiation of a sum, product, quotient and simple cases of a function of a function and of implicit functions. Applications to gradients, maxima and minima. Simple curve tracing.

Definite and indefinite integration of simple functions. Applications to area and volume.

Paper II

The convergence of geometric series. Summation of simple finite algebraic series. The relations between the roots of an algebraic equation and its co-efficients. The exponential and logarithmic series.

Identities, including partial fractions; the remainder theorem.

Handling of simple inequalities.

Further applications of deMoivre's theorem: the cube roots of ± 1.

Rectangular cartesian co-ordinates, including parameters, applied to the ellipse and hyperbola.

The determination of the angles made by planes and straight lines with one another. The mensuration and simpler properties of common solids including the tetrahedron, cone and sphere; any appropriate methods may be used. Cartesian co-ordinates in three dimensions; direction cosines, equations of a plane and of a straight line.

Differentiation of $\sin^{-1}x$ and $\tan^{-1}x$. Simple examples of integration by substitution and integration by parts. Applications of the calculus to rates of change, tangents and normals and centroids.

Further sketching of graphs including those of algebraic functions such as $(ax^2+bx+c)/(px^2+qx+r)$ and of simple transcendental functions.

APPENDIX B

SCOTTISH CERTIFICATE OF EDUCATION

MATHEMATICS HIGHER GRADE

Syllabus

Arithmetic

A knowledge of the arithmetic of the Ordinary grade syllabus will be assumed.

Geometry

Angles. Angle properties of parallel straight lines. Angle properties of triangles and of polygons.

The triangle. Isosceles and equilateral triangles. Similarity and congruence; minimum conditions. Inequalities concerning the sides and angles; shortest distance from a point to a straight line.

The quadrilateral. Various types and the properties associated with their angles, sides and diagonals.

The circle. Chord and perpendicular from centre. Length of chord and its distance from the centre. Common chord. Angles at the centre and at the circumference; angle in a semi-circle. Cyclic quadrilateral. Tangents, Common tangents to two circles. Tangent-chord-angle property. Touching circles. Rectangle properties of chords, secants, secant and tangent.

Three-dimensional figures. Angle between two planes; perpendicular and parallel planes; angle between a straight line and a plane; Theorem of the Three Perpendiculars.

Symmetry about a point and about a line.

Locus. Simple exercises in two and three dimensions.

Collinearity. Concurrency; perpendicular bisectors of sides, bisectors of angles, medians, altitudes of a triangle; centres of the circumscribed, inscribed and escribed circles, centroid, and orthocentre of a triangle.

Mensuration. Square; rectangle; parallelogram; triangle; trapezium; polygon by offsets. Pythagorean property of the right-angled triangle. Circle. Cube; rectangular solid; prism; pyramid; cylinder; cone; sphere.

Ratio. Intercepts made on transversals by three or more parallel straight lines; the particular case of equal intercepts. Division of two sides of a triangle by a straight line parallel to the third side; the particular case of bisection. Internal and external bisectors of an angle of a triangle. Similar figures, including ratio of areas.

Constructions: bisection of an angle and of a straight line; perpendicular to a straight line; angles of 30°, 45°, 60°, 90°; triangles and quadrilaterals, including scale drawing; division of a straight line into a number of parts which are equal or which are in a given ratio; circle through three points; tangents to a circle; inscribed and escribed circles of a triangle; triangle equal in area to a given quadrilateral.

(Proofs of theorems will not be asked for in the examination.)

Algebra

Generalized arithmetic. Formulae: construction, evaluation, change of subject.

Directed numbers.

Addition, subtraction, multiplication, and division.

Brackets. Factors: terms with common factor, expressions of the form $ax + ay + bx + by$, trinomials, difference of squares; reduction to these types by grouping.

Fractions.

Equations: simple; simultaneous linear in two and in three variables; quadratic; simultaneous equations in two variables--one equation quadratic and the other linear. Literal equations of the above types. Problems.

Elimination.

Remainder theorem, including application to factorization of polynomials. Solution of equations of higher degree than the second. Formation of equations with given roots.

Theory of quadratic equations; general solution; nature of roots; sum and product of roots.

Indices: proof of laws for $a^m \times a^n$, $a^m \div a^n$, $(a^m)^n$, $(ab)^n$ for positive integral indices; meaning of zero index; meanings of fractional and negative indices, derivation in numerical cases only; use of laws for any indices.

Logarithms: laws of logarithms including rule for changing base.

Surds: conjugate surds--application to rationalisation; irrational equations.

(The square root of an expression of the form $a + \sqrt{b}$ is not expected.)

Ratio and proportion.

Variation: direct, inverse, joint.

Arithmetic and geometric progressions: general term; means; sum to n terms; sum to infinity of geometric progression.

Compound interest.

Trigonometry

Definitions of sine, cosine, tangent, cosecant, secant, and cotangent for any angle. The fundamental relations.

Ratios of 0°, 30°, 45°, 60°, 90°.

Ratios of $(180^\circ + A)$, etc., in terms of the ratios for the angle A.

Formulae for $\sin(A + B)$, $\cos(A + B)$ (proofs will not be asked for in the examination). Formulae for $\tan(A + B)$, $\sin 2A$, $\cos 2A$, $\tan 2A$. Expression of the sum or difference of two sins or two cosines as a product, and conversely.

Equations. (General solutions are not required).

Elimination.

Triangle formulae:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R;$$

$$a^2 = b^2 + c^2 - 2bc\cos A;$$

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

with similar formulae for $\cos\frac{A}{2}$ and $\tan\frac{A}{2}$;

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\tan\frac{B+C}{2};$$

$$\triangle = \frac{1}{2}bc\sin A;$$

$$\triangle = \sqrt{s(s-a)(s-b)(s-c)}.$$

Solution of triangles, including the ambiguous case.

Radian measure: length of arc; area of sector and of segment.

Problems; identities.

Analytical Geometry

Introductory ideas: direction, sign.

Distance between two points.

Point dividing a straight line in a given ratio.

Gradient of the straight line joining two points.

Equation of a straight line in the forms:

$$\frac{y - y_1}{x - x_1} = m; \quad \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}; \quad y = mx + c.$$

The equation $ax + by + c = 0$ represents a straight line.

Parallel and perpendicular straight lines.

Equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$.

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle.

Intersection of a straight line and a circle.

Equation of the tangent at the point (x_1, y_1) on the circle $x^2 + y^2 = r^2$.

(Questions may be set which involve finding the equation of the locus of a point moving under given conditions.)

Graphs

Graphs of functions such as simple polynomials, $\frac{m}{x}$, $\frac{x}{x^2 + 1}$, $\log x$, 2^x, and trigonometric functions.

Sign of linear and quadratic functions.

Solution of equations; increasing and decreasing functions; turning points and turning values.

Sketch graphs. (Questions which involve asymptotes will not be set).

Derivation from experimental data of a linear law and of a law of the form $y = ax^n$.

Calculus

Derivative of a function: finding from first principles derivatives of functions of the form ax^n, for small positive and negative integral values of n.

Derivatives of sums and differences of these functions.

Rule (without proof) for differentiating ax^n for all values of n. Rule for differentiating a function of a function.

Applications of the derivative: the equation of the tangent to a curve at a point on it; the significance of positive, zero, and negative values of the derivative; maxima and minima; rates.

Integration as the inverse of differentiation. (Integration of x^{-1} is not expected.)

Definite integral: integral as an area; volumes of revolution.

APPENDIX C

SCHOOLS INVITED TO PARTICIPATE IN INVESTIGATION

A. Canadian School Districts

1. Alberta:

    (a) Edmonton School Board,

    10010 - 107A Avenue, Edmonton

    (b) Calgary School Board,

    Education Centre,

    515 Macleod Trail, Calgary

2. British Columbia:

    Board of School Trustees,

    Greater Victoria, Victoria.

3. Manitoba:

    (a) St. James School Division,

    2000 Portage Avenue, Winnipeg

    (b) Winnipeg School Division No. 1,

    1577 Wall Street, Winnipeg

4. New Brunswick:

    (a) Board of School Trustees,

    Fredericton

    (b) Board of School Trustees,

    School District No. 16, Moncton

5. Newfoundland:

    Anglican School Board,

    Bishop's College, Pennywell Road, St. Johns

6. Nova Scotia:

Board of School Commissioners,

Victoria Road, Dartmouth

7. Ontario:

(a) Board of Education,

North York

(b) Board of Education,

London

(c) Board of Education,

Brantford

(d) Board of Education,

Township of Toronto,

90 Dundas Street West, Toronto

(e) Board of Education

2135 Gill Street, Fort William

8. Prince Edward Island:

Board of School Trustees,

175 Spring Road, Charlottetown

9. Quebec:

(a) Protestant School Board of Greater Montreal,

600 Fielding Avenue, Montreal

(b) MacDonald Central School Board,

P.O. Box 222, Montreal

10. Saskatchewan:

(a) Board of Education,

Regina Public Schools and Regina Collegiate Institute,

1235 College Avenue, Regina

(b) Saskatoon Collegiate Institute Board,
211 - 4th Avenue, Saskatoon

11. One School Board which requested not to be identified in the report.

B. English Schools

1. Aylesbury Grammar School,
Walton Road, Aylesbury, Bucks.
2. Alleyn's School,
London, S.E. 22
3. Barton Peveril Mixed Grammar School,
Eastleigh, Hants.
4. Bishopshalt Grammar School,
Hillingdon, Middx.
5. Blackpool Grammar School,
High Furlong, Blackpool, Lancs.
6. Boston Grammar School,
Boston, Lincs.
7. Bristol Grammar School,
Bristol 8, Somerset
8. Brune Park County High School,
Gosport, Hants.
9. Dulwich College,
London, S.E. 21
10. Eltham College,
London, S.E. 9

11. Farnborough Grammar School,

Farnborough, Hants.

12. Havant County Grammar School,

Havant, Hants.

13. King's School,

Worcester, Worcs.

14. King Edward VII School,

Lytham, Lancs.

15. Latymer Upper School,

Hammersmith, London

16. Liverpool College,

Liverpool 18, Lancs.

17. Mathew Murray School,

Brown Lane, Leeds 12, Yorks.

18. Royal Grammar School,

Worcester, Worcs.

19. St. Albans Grammar School for Boys,

St. Albans, Herts.

20. St. Edmunds School,

Canterbury, Kent

21. Tadcaster Grammar School,

Tadcaster, Yorks.

22. Taunton School,

Taunton, Somerset

23. Tauntons School,

Highfield, Southampton, Hants.

24. Tiverton Grammar School,
    Tiverton, Devon.

25. Westminster School,
    London, S.W. 1.

C. Scottish Schools

1. Aberdeen Gordons School,
   Aberdeen, Aberdeenshire

2. Airdrie Academy,
   Airdrie, Lanarkshire

3. Arbroath High School,
   Arbroath, Angus

4. Ayr Academy,
   Ayr, Ayrshire

5. Banff Academy,
   Banff, Banffshire

6. Bathgate Academy,
   Bathgate, West Lothian

7. Dundee High School,
   Dundee, Angus

8. Dollar Academy,
   Dollar, Clackmannanshire

9. Elgin Academy,
   Elgin, Morayshire

10. Ellon Academy,
    Ellon, Aberdeenshire

11. Falkirk High School,
    Falkirk, Stirlingshire

12. George Watsons High School,
Edinburgh, Midlothian

13. Gordons Academy,
Huntly, Aberdeenshire

14. Harris Academy,
Dundee, Angus

15. Keith Academy,
Keith, Banffshire

16. Larbert High School,
Larbert, Stirlingshire

17. Leith Academy,
Leith, Midlothian

18. Montrose Academy,
Montrose, Angus

19. Morgan Academy,
Dundee, Angus

20. Perth Academy,
Perth, Perthshire

21. Powis High School,
Aberdeen, Aberdeenshire

22. Robert Gordons College,
Aberdeen, Aberdeenshire

23. Royal High School,
Edinburgh, Midlothian

24. Waid Academy,
Anstruther, Fifeshire

25. Wishaw High School,
Wishaw, Lanarkshire.

APPENDIX D

HIGH SCHOOL PRE-CALCULUS MATHEMATICS TEST

Time Allowed - One hour

In each case encircle the lettering indicating the selected answer.

1. If the digit 1 is placed after a two digit number whose tens' digit is x and whose units' digit is t, the new number is:

   (a) $10x + t + 1$ (b) $100x + 10t + 10$
   (c) $x + t + 1$ (d) $100x + 10t + 1$
   (e) None of these answers.

2. The largest number by which the expression $n(n^2-1)$ is divisible for all possible integral values of n is:

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 6

3. In the binary system which has a base 2; the first six positive integers are 1, 10, 11, 100, 101, 111. The numeral 11110 in the binary system would then be written in the decimal system as:

   (a) 30 (b) 32 (c) 64 (d) 10,000 (e) 28

4. The difference of the roots of the equation $X^2-4x-1 = 0$ is:

   (a) 3 (b) 0 (c) $2\sqrt{5}$ (d) $\sqrt{5}$ (e) 8

5. In the expression $\frac{x+1}{x-1}$ each x is replaced by $\frac{x-1}{x+1}$, the value of the resulting expression when $x = 1$ is:

   (a) 1 (b) 2 (c) 0 (d) -2 (e) -1

6. The value of $x^2 - 2x - 15$ can never be less than:

   (a) -16 (b) 1 (c) -1 (d) 0 (e) 14

7. For what angles between 0° and 360° is $\text{Cos } x^\circ = 1/2$:

   (a) 60° (b) 30° (c) 30° and 210° (d) 60° and 300° (e) 45°

8. $x^2 + y^2 - 3x + 8y + 2 = 0$ is the equation of a circle with its centre at:

   (a) $(-\frac{3}{2}, 4)$ (b) $(-3,8)$ (c) $(3, 8)$ (d) $(1, 1)$ (3) $\frac{3}{2}, -4)$

9. For what number X is $\sqrt{1-X}$ a real number:

(a) $X > 1$ (b) $X < 1$ (c) $X = -1$ (d) $X \leq 1$ (e) For all X's.

10. How many permutations of 4 objects taken 2 at a time are there:

(a) 24 (b) 16 (c) 12 (d) 6 (e) 8

11. The quantity factorial N may be symbolized N! Which of the following statements is false:

(a) $8! = 8 \times 7!$ (b) $2! = 3! - 4$ (c) $10! \div 9! = 10$

(d) $4! = 12$ (e) $2! - 1 = 1$

12. $(A^{-1} + B^{-1})^{-1}$ is equal to:

(a) $\frac{AB}{A+B}$ (b) $A + B$ (c) AB (d) $\frac{A+B}{AB}$ (e) None of these.

13. The largest of the following integers which divides each of the numbers of the sequence $2^5 - 2$, $3^5 - 3$, $4^5 - 4$, ----------, $n^5 - n$ is:

(a) 2 (b) 30 (c) 20 (d) 1 (e) 10

14. The number 1.14141414 ----------------------can be written as a common fraction. When reduced to lowest terms the difference between the denominator and numerator is:

(a) 3 (b) 14 (c) 8 (d) 22 (e) None of these.

15. The equation $x^2 + 4y^2 - 6x + 16y + 21 = 0$ is the equation of:

(a) a circle (b) a Hyperbola (c) a Parabola (d) Two straight lines (e) an ellipse

16. If $y^{-1} - 1$ is multiplied by y the product is:

(a) $1-y$ (b) $\frac{1}{y-1}$ (c) $\frac{1}{1-y}$ (d) $y-1$ (e) None of these.

17. One of the factors of $x^3 + 8$ is:

(a) $x^2 + 2$ (b) $x + 1$ (c) $x^2 - 2x + 4$ (d) $x^2 - 4$

(d) None of these.

18. If $\log_7 2 = a$ and $\log_7 3 = b$ then $\log_7 6$ is:

(a) ab (b) $\frac{a}{b}$ (c) a-b (d) 6 (e) a + b

19. Which one of the following statements is false for the equation:

$ix^2 - x + 2i = 0$ where $i = -1$

(a) The roots are imaginary (b) The discriminant is 9

(c) The roots may be found by using the quadratic formula

(d) The sum of the roots is 2 (e) The equation has two roots

20. The sum to infinity of $\frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \frac{1}{6^4} + \text{-----}$ is:

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{1}{36}$ (e) None of these

21. In which of the following equations is y neither directly nor inversely proportional to x:

(a) $x + 2y = 0$ (b) $xy = 4$ (c) $3x + y = 10$ (d) $\frac{y}{x} = \sqrt{5}$

(e) $x = 2y$

22. The number of points common to the graphs $x^2 + y^2 = 16$ and $y^2 = 9$ are:

(a) 2 (b) 4 (c) None (d) 1 (e) 3

23. If n! symbolizes factorial n then the series:

$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ ----------------

is the power series for:

(a) Cos x (b) Tan x (c) $\log_e x$ (d) e^x (e) Sin x

24. The conjugate of $a - ib$ is:

(a) $a + b$ (b) $a + ib$ (c) $a - ib$ (d) $a - b$

(e) None of these

25. $\text{Log}_{10}\ (10^{-9})$ equals:

(a) $\frac{1}{10^9}$ (b) 9 (c) -9 (d) 3 (e) 10^9

26. The points (6,12) and 0,-6) are connected by a straight line. Another point on the line is:

(a) (1,-5) (b) (1,1) (c) (0,0) (d) (12,6) (e) (2,0)

27. If 30 is divided into three parts proportional to 2, 4, and 6, the smallest part is:

(a) 10 (b) 15 (c) 2 (d) 1 (e) 5

28. If $\frac{x}{y} = \frac{3}{4}$ which of the following is true:

(a) $\frac{x-y}{y} = 4$ (b) $\frac{y}{y+x} = 4$ (c) $\frac{x+y}{y} = 7$

(d) $\frac{x}{y} = \frac{3}{8}$ (d) $x = \frac{4y}{3}$

29. Which of these statements is false:

(a) Sin A Cosec A = 1 (b) $\sin^2 A = 1 - \cos^2 A$

(c) $\tan A = \frac{1}{\cot A}$ (d) Sin A Cos A = 1 (e) Sec A Cos A = 1

30. $\frac{1}{i}$, where $i = \sqrt{-1}$ may be written:

(a) $1 + i$ (b) i^2 (c) $0 - 1i$ (d) 1 (e) -1

31. In how many different orders can the four letters A, B, C, D, be written, no letter being used more than once in each arrangement:

(a) 23 (b) 24 (c) 16 (d) 64 (e) None of these

32. The real factors of $x^2 + 4$ are:

(a) $(x + 2)(x - 2)$ (b) $(x + 2)(x + 2)$ (c) $(x - 2)(x - 2)$

(d) $(x^2 + x + 2)(x^2 - x + 2)$ (e) Non existent

33. When the sum of the first five terms of an arithmetic progression is one quarter the sum of the first ten terms; the ratio of the common difference to the first term is:

(a) 1:1 (b) 2:1 (c) 1:2 (d) 4:1 (e) 1:4

34. The values of x in the equation $\log_{10}(x^2 - 15x) = 3$ are:

(a) 5, -3 (b) 3, 1 (c) -25, 40 (d) 1, -3 (e) None of these

35. If, in the formula $X = \frac{an}{Y + ny}$

n increases, while a, Y, and y remain constant, then X:

(a) Decreases (b) Remains constant (c) Alternately increases and decreases (d) Increases (e) None of these

36. The geometric figure bound by $y = 3x + 2$, $y = 2 - 3x$, and $y = -2$ is:

(a) Quadrilateral (b) Right triangle (c) Isosceles triangle

(d) a Rhombus (e) An Equilateral triangle

37. A 25 foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the base of the building. If the top of the ladder slips 4 feet the bottom will slip:

(a) 8 ft (b) 4 ft (c) 6 ft (d) 7 ft (e) None of these

38. If the expression $\begin{vmatrix} a & c \\ d & b \end{vmatrix}$ has the value $ab - cd$ then the equation

$$\begin{vmatrix} 2x & -1 \\ x & x \end{vmatrix} = -3$$

(a) Has no roots (b) Has two real roots (c) Has two imaginary roots (d) Has many roots (e) Has exactly one root

39. A total of 21 handshakes is exchanged at a party. Assuming that each person present shakes hands with all the others then the number of people present is:

(a) 21 (b) 8 (c) 11 (d) 7 (e) None of these

40. If $4^{x+2} = 34 - 4^x$ then x equals:

(a) 2 (b) 4 (c) 1 (d) 6 (e) 0.5

41. The value of the expression $1 - \dfrac{1}{1 + \dfrac{x}{1-x}}$ when simplified is:

(a) 1 (b) x if $x \neq 1$ (c) $1-x$ (d) x if $x \neq -1$ (e) $x - 1$

42. The coefficient of x^6 in the expansion of

$$(1+x)\,(1+x^2)\,(1+x^3)\,(1+x^4)\,(1+x^5)\,(1+x^6)$$

is:

(a) 1 (b) 36 (c) 6 (d) 5 (e) None of these

43. The graphs of $x^2 + y = 10$ and $x + y = 10$ meet in two points. These points are:

(a) (0,1) and (1,0) (b) (0,0) and (0,0) (c) (0,10) and (1,9)

(d) (2,1) and (10,2) (e) (1,0) and (1,1)

44. The value of $\log_5 \dfrac{125 \times 625}{25}$ is:

(a) 5 (b) 6 (c) 10 (d) 3125 (e) None of these

45. Of the following statements the one that includes all values of x which will satisfy $2x - 3 < 7 - x$ is:

(a) $x < 0$ (b) $x > \frac{10}{3}$ (c) $x = 0$ (d) $x < \frac{10}{3}$

(e) $x < 6$

46. Where $i = \sqrt{-1}$, which of the following statements is false:

(a) $1 = 1$ (Cos 0° + i Sin 0°)

(b) $i = 1$ (Cos 90° + i Sin 90°)

(c) $-1 = 1$ (Cos 180° + i Sin 180°)

(d) $-i = i^3$ (e) $1 = 1$ (Cos 0° + i Sin 90°)

47. The locus of the centres of all circles of a fixed radius R passing through a common point is:

(a) A straight line (b) A circle (c) Two circles

(d) Two straight lines (e) None of these

48. $x^2 - 11y^2 = 25$ is the equation of

(a) an Ellipse (b) a Parabola (c) a Circle

(d) a Hyperbola (e) Two straight lines

49. The roots of the equation $px^2 + qx + r = 0$ will be reciprocal if:

(a) $p = r$ (b) $p = q$ (c) $q^2 = 4pr$ (d) $p = qr$

(e) $q = pr$

50. If $(x + \frac{1}{x})^2 = 3$ then $x^3 + \frac{1}{x^3}$ equals:

(a) 1 (b) 2 (c) 0 (d) 4 (e) None of these

APPENDIX E

HIGH SCHOOL PRE-CALCULUS

MATHEMATICS TEST

QUESTIONNAIRE

Participating students please complete the following questionnaire:

1. Age _____ yrs.
2. Do you plan to attend university sometime in the future?
   Yes ___, No ___
3. Length of school lesson periods __________ minutes.
4. How many periods of pure mathematics do you have weekly? _____
5. Is mathematics a compulsory subject in your school system?
   Yes ___, No ___
6. If the answer to question 5 is "Yes" answer the following question.

   If you were given the choice, which of the following courses would you choose?:

   (a) Discontinue mathematics and select another subject.

   (b) Take additional mathematics.

   (c) Continue with the present mathematics course.

   (a) ___, (b) ___, (c) ___.
7. Do you intend studying mathematics beyond the high school level?
   Yes ___, No ___.