THE DYNAMIC BEHAVIOR OF TRANSFORMER CORES

A Thesis

Presented to the Faculty of Graduate Studies and Research

University of Manitoba

In partial fulfillment of the requirements for the degree

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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by

John Douglas Poustie

May , 1970

ABSTRACT

This investigation was concerned initially with comparing the dynamic, or time varying, behaviour of two very different transformer cores. The various electrical characteristics of a core-winding assembly sometimes used as a "current transformer", and known as a bushing type C.T., were compared with the responses of a modified core and coil assembly taken from a power distribution transformer.

The relevant measurement techniques were thoroughly analyzed and are shown in the Appendix.

It was demonstrated that binomial equations; in one case a quintic, in the other a septic; are a fairly good representation for the average of the flux density vs magnetic field intensity, or B-H, characteristics. For certain situations, the average of the B-H curve may be represented by three segment straight line approximations.

The availability of a reliable two term equation is potentially very useful in harmonic or ferroresonant studies. Recent work has indicated that, for the investigation of the latter, loss representation is not of prime importance to the transformer model.

Using the binomial equations to model the core characteristics, the relationship between applied RMS voltage and resulting RMS current was developed.

A second prime concern of this investigation was the attempt to demonstrate that, for a series circuit containing linear resistance and capacitance as well as non-linear inductance, the analytical prediction of various aspects of ferroresonant behaviour was relatively straightforward

if the above binomial approximations are used, and that the agreement with test results was quite reasonable. Certain generalizations were made in regard to the "incremental describing function" for an odd powered polynomial relationship between current and flux linkages.

Finally, the excitation of the second and third subharmonics was accomplished and recorded, although no mathematical analysis was attempted in this area.

ACKNOWLEDGEMENT

The author wishes to thank Dr. G. W. Swift for suggesting the topic, arranging financial assistance through the National Research Council of Canada, and for his helpful suggestions during work on this thesis.

In addition, the help obtained from E. A. Dillon of Pioneer Electric Co. to obtain suitable core and coil samples is much appreciated.

John Poustie, B.Sc.(E.E.)

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INTRODUCTION

The first real objective of this thesis, which is an offshoot of the Doctoral Dissertation presented to Illinois Institute of Technology by G. W. Swift, was to obtain accurate flux density vs magnetic field intensity curves. These curves, also called B-H curves, were obtained by operational simplifier techniques and allow several electrical characteristics to be deduced regarding the sample, if frequency, flux density and time of switching are the experimental variables. Of most interest was the binomial equation, $i = C_1 \ell + C_n \ell^n$ which was proposed as the arithmetic average of the ℓ/i relationship, which is directly proportional to the B/H relationship.

In the previously mentioned Ph.D. thesis, the equation:

$$i = \ell + 4\ell^5$$

was presented as the per unit binomial best fitting the core and coils of a 5 kva distribution transformer manufactured by the Commonwealth Edison Co. of Chicago. Although the details of the core's construction were not known to the present author, it was of interest to compare the same results to the two cores of this thesis, for which constructional details were known.

It was also of interest to derive the theoretical "magnetization curve", that is, the I(RMS) vs E(RMS) relationship, using the binomial i/ℓ equation as the starting point, and then to compare the results with the actual magnetization test on the same core. Since agreement was not too close, presumably due to the inadequate modelling of iron losses, no

attempt was made to obtain the i/ℓ curve from the actual magnetization test results. The latter procedure would have been very useful, since to date, transformer manufacturers are not obliged to furnish B-H curves, but normally provide the magnetization data obtained from the acceptance tests.

While examining the ferroresonant response of a series R-L-C circuit, the author was able to measure the second and third subharmonics, and also became interested in the methods available for predicting ferroresonance analytically, and Chapter Seven represents an attempt to reconcile a mathematical prediction based on an approximate model to the measurements made on the circuit in the laboratory situation.

CHAPTER I

CONSTRUCTION OF CORE-COIL SAMPLES

The object of this Chapter is to compare physically the available test samples. The abbreviations C.T. and P.T. shall be used for "current transformers" and "power transformers" respectively.

Figure 1 gives physical details of the C.T. core, while in Figure 2, the P.T. is detailed.

Since the cores were available initially without windings, it was necessary to determine the number of turns to wind in order to accomplish further testing. In general, this is determined by applying the equation

$$E = \frac{4.44\, fN\, (B_m A_n)}{10^8}$$

where E = RMS voltage [usually called "rated" voltage],

f = frequency

N = number of turns (to be found)

B_m = peak flux density (to be chosen)

A_n = net steel cross sectional area.

If E is chosen, and since in this instance A_n is fixed, B_m is then chosen to restrict exciting current, as well as total core loss. Depending on steel type and treatment, and core construction, B_m may vary over a wide range.

For the C.T., E was chosen for convenience in testing to be 50 v RMS, and a figure of $B_m = 100$ kilolines/in^2 was thought to be

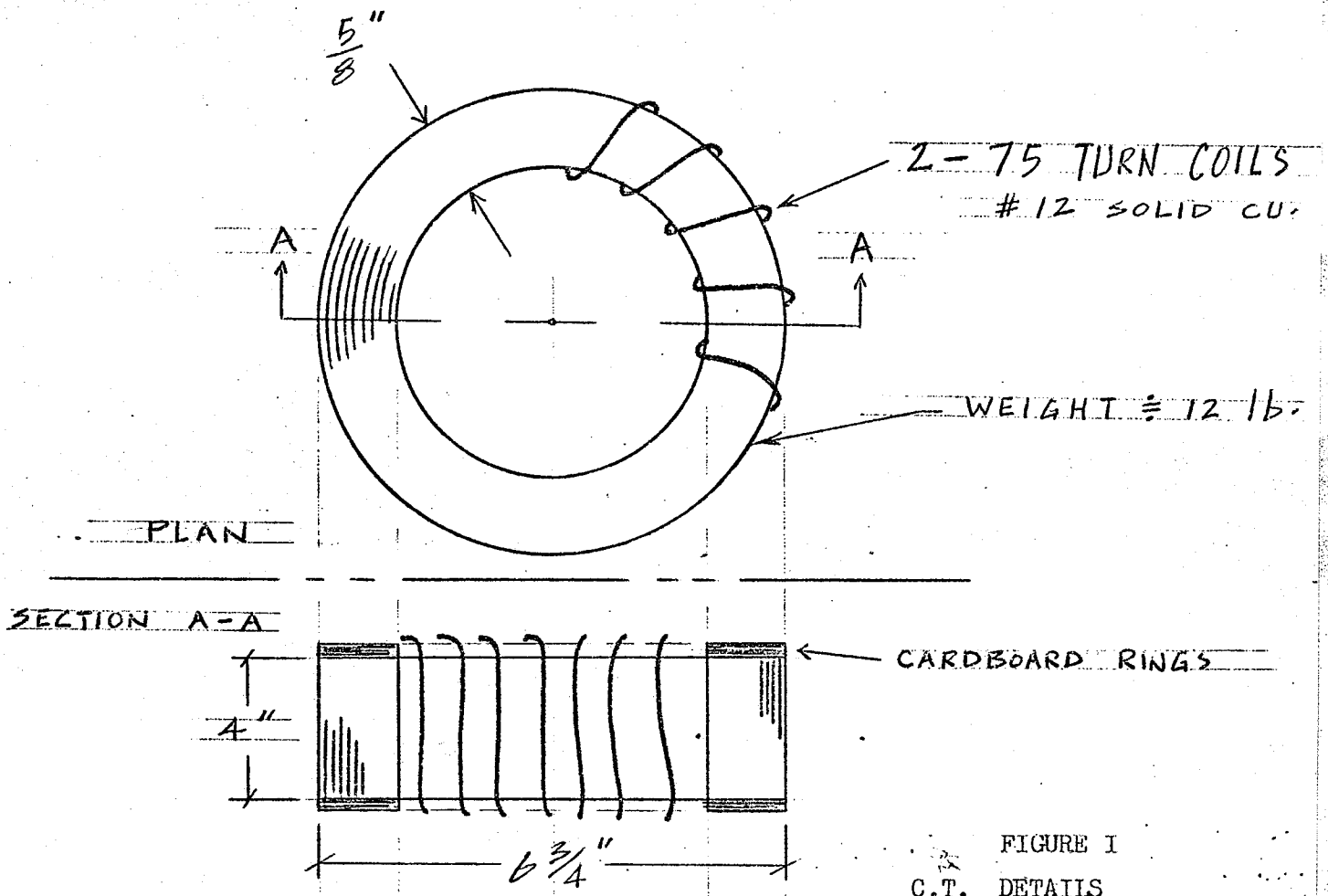

FIGURE 1
C.T. DETAILS

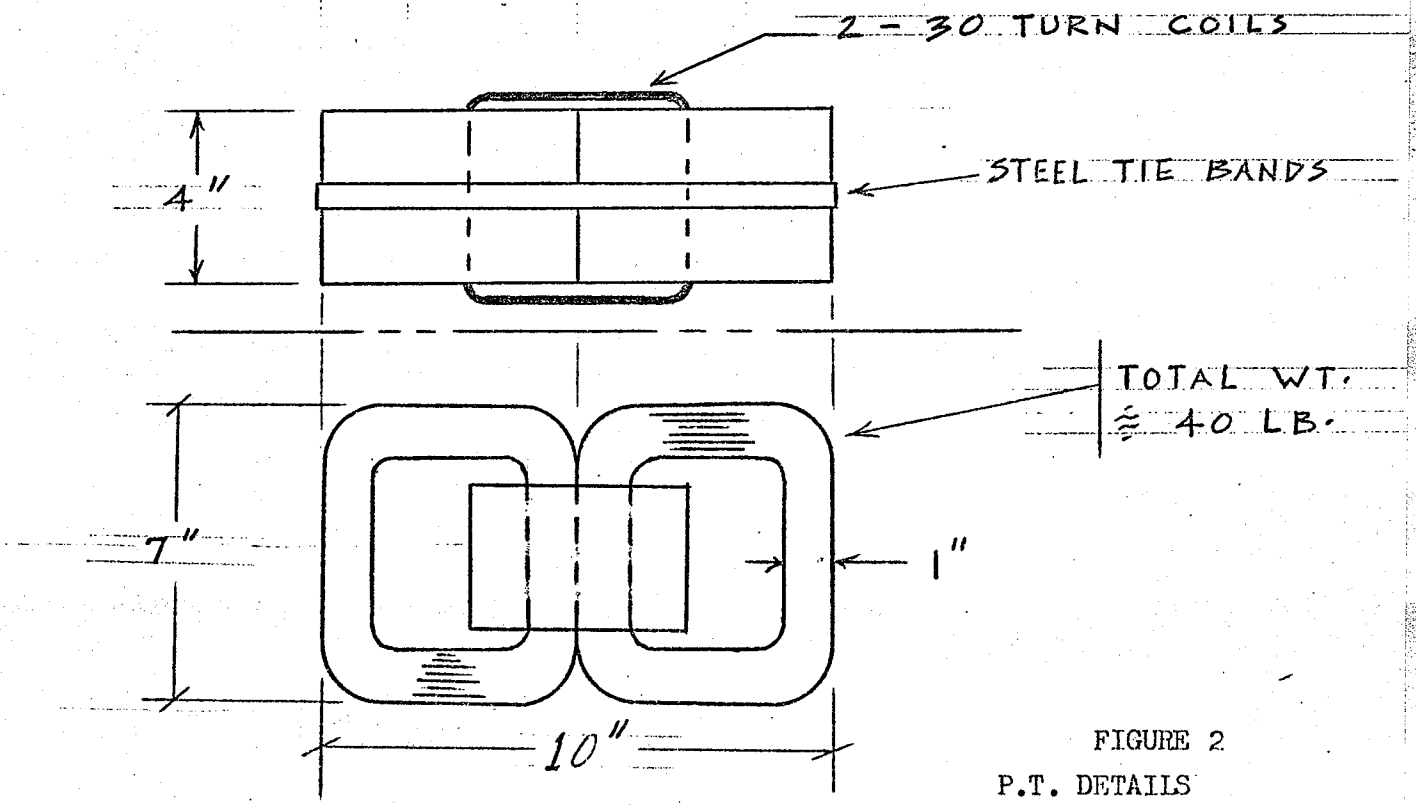

FIGURE 2
P.T. DETAILS

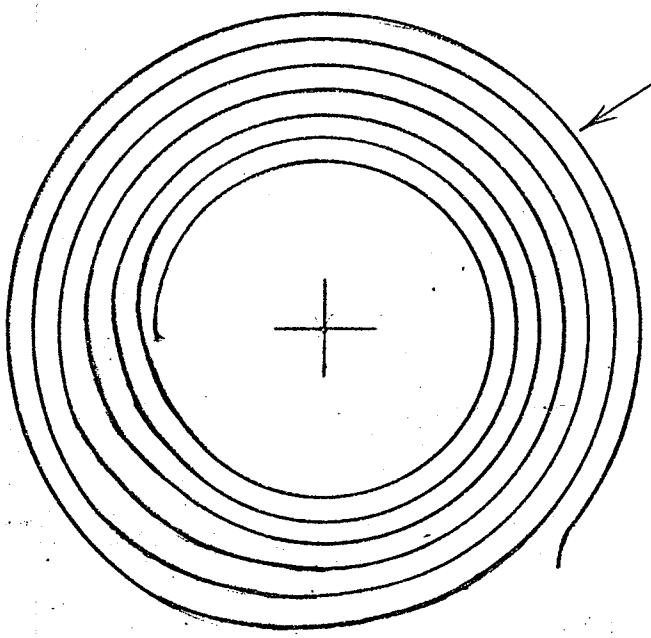

FIGURE 3
CT CORE CONSTRUCTION

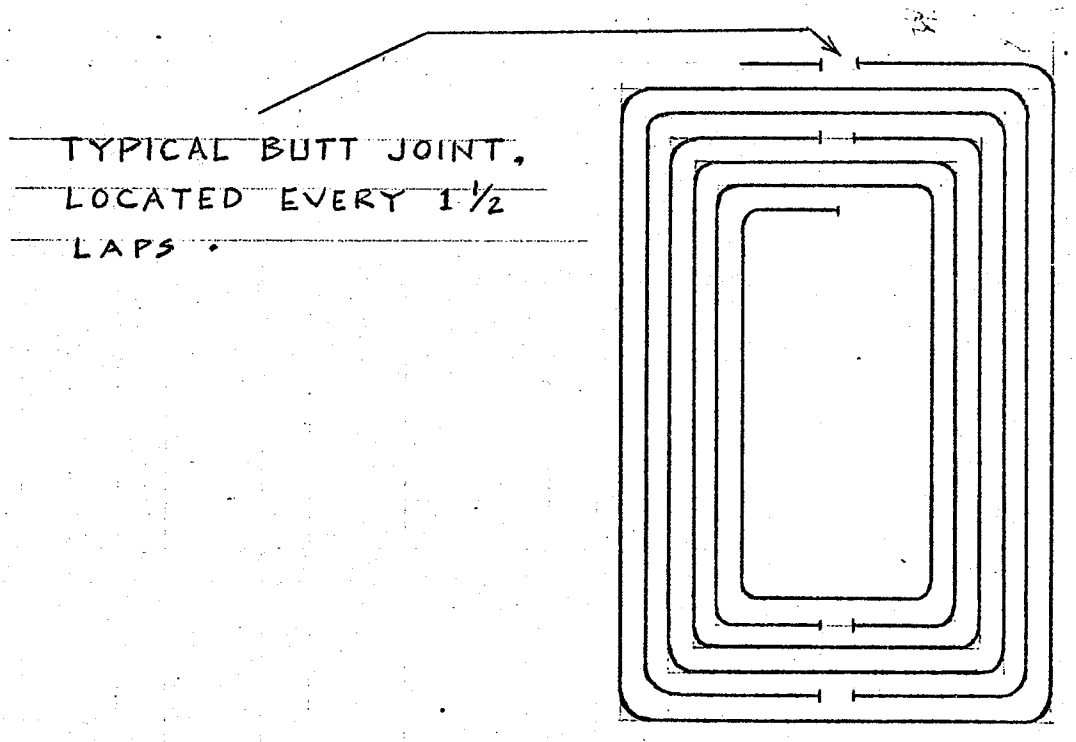

FIGURE 4
PT CORE CONSTRUCTION

satisfactory. This determined N = 75 turns. Two identical windings, of 75 turns and using #12 Heavy Formel solid copper were wound to allow the measurement of B-H curves, using an operational amplifier technique which will be explained later. The wire gauge was heavy enough to prevent winding overheating under all anticipated test conditions.

In the case of the P.T., E was also chosen to be 50 v RMS, while B_m = 83 kilolines/in^2 was used to restrict excitation current. Available test data on cores of similar construction indicated that excitation current would be much higher for the P.T. than for the C.T. core. Calculations indicated that 30 turns should be satisfactory, and two identical, 30 turn, #12 HF copper windings were wound as shown in Figure 2.

The material used in both cores was a modern, grain oriented transformer steel of approximately 11 mils thickness. The objective at this point was to emphasize differences in core cutting and assembly, rather than to discuss manufacturing specifications for the steel. To this end, Figures 3 and 4 illustrate core construction. It was felt that these samples represented extreme cases. The C.T. core could be said to approach the ideal, that is, fairly tightly wound on a machine, employing high grade steel, with no air gaps in the magnetic circuit, and no small radius bends in the steel. The P.T., on the other hand, which in this instance was salvaged, had air gaps (which tended to cause flux concentration) at 4 points on its periphery. It was assembled by hand onto the coil, and could have been mishandled, thus causing increased eddy current losses where interlayer insulation was scraped off, in addition to introducing metallic stresses which could not be relieved by "soaking" in a high temperature oven after the coil had been assembled on the core.

It is held by the author that the core of a large power transformer would lie in between the P.T. and the C.T. as far as the number and size of air gaps in the core, and relative amount of human handling goes.

This fact, then, might permit the application to large power transformers of some of the conclusions drawn in this thesis.

The author makes the presumption here of some prior knowledge of power transformer core assembly by his examiners.

CHAPTER II

RMS VOLTAGE AND CURRENT CHARACTERISTICS

The object of this Chapter is, at the same time as presenting comparative measurements of magnetization or open circuit data, including harmonic component determination, to examine the techniques used to obtain these characteristics.

Also, an attempt is made employing graphical analysis to show the influence of the shape of the B-H curve on the excitation current waveform.

Finally, an explanation for the shape of the actual magnetization curve is attempted, based on the B-H curve measurements to be presented in future chapters.

2.1 RMS responding meters.

Proper measurement of power frequency voltage and current was complicated by the following. While many types of meters, including Volt-Ohm Meters, Vacuum Tube Voltmeters, and moving iron instruments will read perfectly sinusoidal voltages accurately (assuming proper calibration), the source impedance may, in some instances, be sufficiently high to cause the terminal voltage to be distorted, expecially when currents flowing to the transformer on open circuit are not perfectly sinusoidal. Figure 5 depicts this circuit in a very general way, while Figure 6 shows what type of distortion could occur in the terminal voltage for an assumed current waveform. For an inductive source impedance, the harmonic distortion is intensified, since the current, the waveform of which is said to contain "harmonics", or higher frequency components, causes a relatively

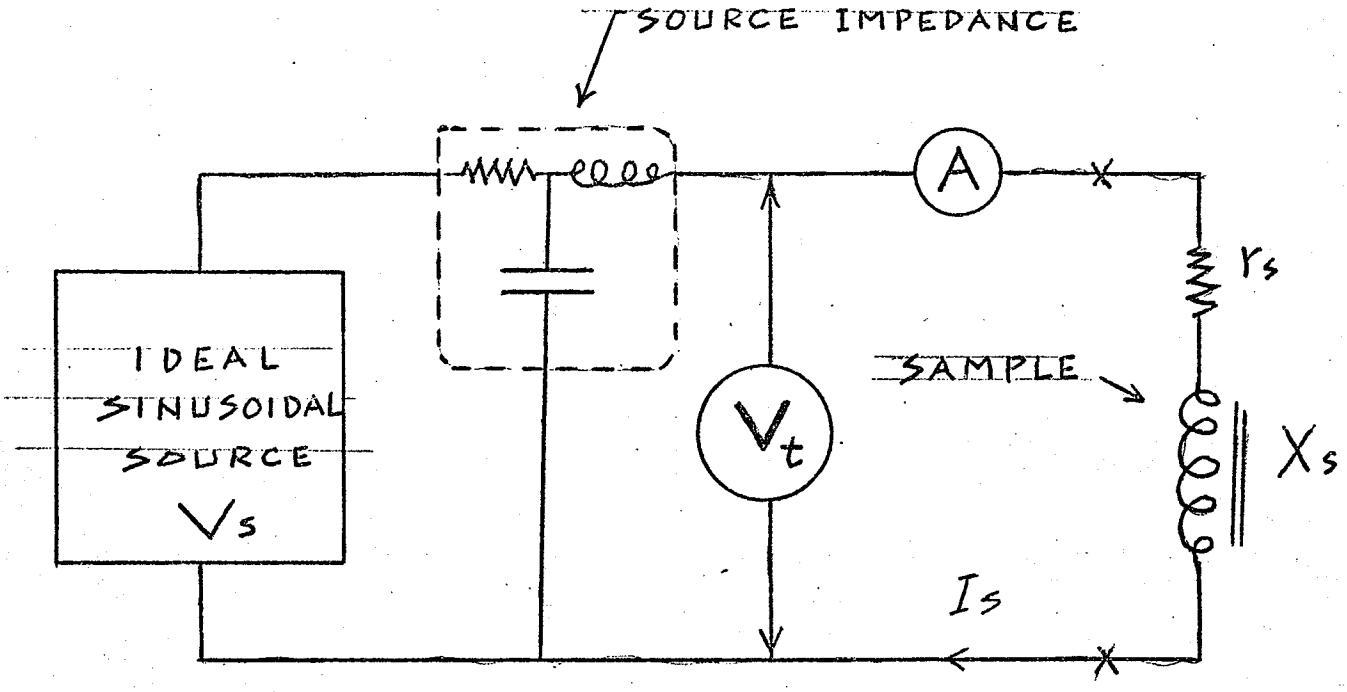

FIGURE 5

THE CIRCUIT USED TO MEASURE MAGNETIZATION CHARACTERISTICS

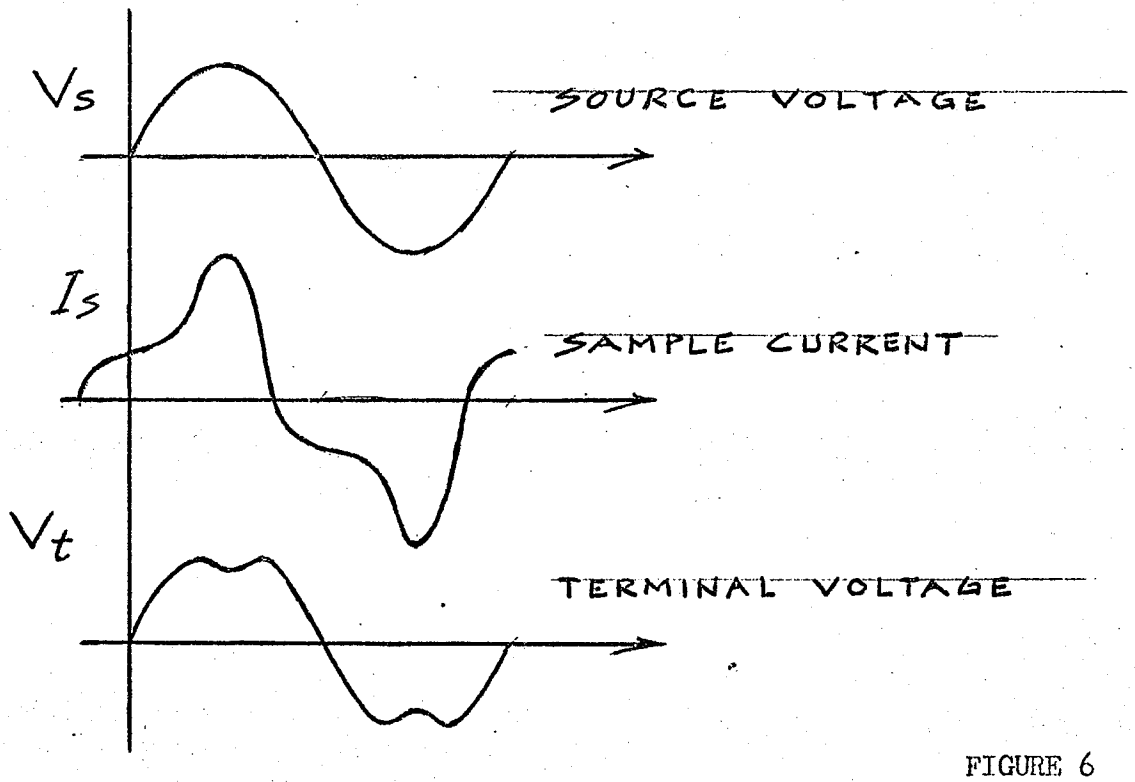

FIGURE 6

SHOWING POSSIBLE DISTORTION OF V_t & I_s

larger harmonic voltage drop across this same inductive source reactance. In transformer test installations, tooth ripple in the supply generator might cause unwanted (higher frequency) peaks in the test voltage.

It must be remembered that a VTVM, depending on the manufacturer, might respond either to the 0-peak or the peak-peak of a rectified sine wave. It is to be stressed that the VTVM is calibrated on a sinusoidal waveshape and will be in error for any distortion in the measured voltage. Similarly, a VOM will respond accurately to a sinusoidal voltage only; its movement being of the d'Arsonval type, fed from either a full wave or half wave rectifier unit.

In a meter having the moving iron type of construction, angular deflection is proportional to the square of the operating current and the rate of change of inductance with respect to angular deflection , so that calibration may be accomplished in terms of effective [RMS] voltage or current. According to Harris, "The waveform error of a modern instrument with short vanes is negligible for the third harmonic at power frequencies".

During the magnetization current measurements associated with this thesis, the author checked the applied voltage waveform with an oscilloscope to ensure that no distortion occurred. In this instance, the currents encountered were not larger than 3% of the source RMS current rating. Voltage magnitude was measured with a 75 volt range moving iron meter, while RMS currents were determined with a moving iron meter rated 1 amp RMS.

In a situation where source voltage distortion exists, the conversion of results back to an equivalent sine wave basis is exceedingly complicated. The interested reader is referred to Chapter II of reference # 3 in the bibliography .

2.2 Comparison of Magnetization Characteristics.

The characteristics are presented in Figure 7 A and 7 B . Certain similarities between the curves of 7 A deserve comments. Current increased with voltage, but not in a linear fashion, even for lower values of voltage. In both cases, as expected, $\Delta I/\Delta V$ becomes very large for larger voltages. On the other hand, the basic curve shapes are dissimilar, that is, the curves could not be superimposed in a per unit sense, even if different base quantities were chosen.

A transformer designer might point out that 83 kilolines per square inch was a poor choice for the design figure for the P.T., since the magnetization curve indicates that the core is into the saturation region at 50 volts RMS applied. The curve shows that 30 to 35 volts would have been a better choice for the "rating", now that the core is assembled. The C.T. design, however, worked out quite well for the assumptions made.

In Figure 8, tracings of the photographs of the actual current waveforms are shown. The large amount of distortion present in the C.T. current waveform is contrasted to the P.T. waveform at the chosen "rated" voltages.

It is to be stressed that the P.T. has, overall, a much more linear magnetization characteristic than does the C.T.

2.3 Harmonics in the Excitation Current.

It is of interest to analyze the excitation current waveform for both the C.T. and the P.T. The measurement technique used to accomplish this is analyzed in Appendix A. The results of these measurements are shown in Figure 9. The following calculations demonstrate the