ELASTIC AND INELASTIC SCATTERING OF PROTONS FROM ^{6}Li BETWEEN 25 and 45 MeV

by

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ABSTRACT

Using the University of Manitoba sector-focussed cyclotron a study has been made of the elastic and inelastic scattering of protons from ^{6}Li. Data were obtained at 25.9, 29.9, 35.0, 40.1 and 45.4 MeV. The 2.18 MeV (3^{+}, T = 0) state of ^{6}Li was found to be strongly excited, but the 3.56 MeV (0^{+}, T=1) state was quite weakly excited. To test the applicability of the optical model description for the scattering from such a light nucleus the elastic angular distributions have been analysed using the automatic search code SEEK. Available polarization angular distributions were included in the analysis. Good fits have been obtained for quite acceptable optical model parameters. Angular distributions for excitation of the 2.18 MeV level were measured at all five energies. Angular distributions for excitation of the 3.56 MeV level were measured at 25.9 and 45.4 MeV. An analysis in terms of a microscopic theory may give information about the spin-isospin dependent part of the effective interaction.

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CHAPTER 1

1-1 Introduction

The present thesis describes an experiment in which protons from the University of Manitoba sector focussed cyclotron were scattered by a ^{6}Li target. The incident protons had energies of 25.9, 29.9, 35.0, 40.1 and 45.4 MeV, and data were collected for elastic scattering and for inelastic scattering from the first and second excited states of ^{6}Li.

The elastic scattering data have been analysed using the optical model. An analysis of the inelastic scattering from the 3.564 MeV (0^{+}, T=1) level in ^{6}Li is planned for the near future according to a microscopic description of the interaction. It is suggested that the angular distributions corresponding to the 2.184 MeV (3^{+}, T=0) state in ^{6}Li would form an interesting subject for a copuled channels calculation.

It is the intention to examine in the first chapter in some detail the purpose of the experiment and the motives for a theoretical analysis of the data obtained, and to give an account of related investigations which have appeared in the literature. The main part of the discussion is contained in section 4, while the two preceeding sections are intended as a review of some of the basic material involved; section 2 contains an account of the fundamentals of the nuclear optical model, while section 3 deals with the present knowledge and understanding of the features of the ^{6}Li spectrum. The second chapter contains a description of the experimental arrangement and of the methods followed in

the reduction of the experimental data. In the third and final chapter the results obtained are presented and discussed. Appendix I contains the derivation of a few formulas used in the data reduction. Finally, tables of the differential cross-sections are given in appendix II.

1-2. The Optical Model

A comprehensive presentation of the optical model theory is outside the scope of this thesis and can be found for instance, in monographs written by Hodgson {1} and by Jones {2}. We shall limit ourselves to an account of the basic ideas together with a sketch of the historical development. Around 1950 the interaction of a neutron with a nucleus was thought to correspond to the following picture:

1) At very low energies (keV range), where the only channels open are elastic scattering and radiative capture, the dependence on energy of the differential and total cross-sections shows strongly peaked resonances. The scattering cross-section is the sum of terms corresponding to two different processes. The first (potential scattering) is a surface phenomenon in which the nucleus behaves as a hard sphere, while the second corresponds to the capture of the neutron to form a compound nucleus, which decays after a time of $\sim 10^{-14}$ seconds. The compound scattering is responsible for the resonances: the neutron can be absorbed only at those energies at which the wave function of the incident neutron at the nuclear surface satisfies the boundary conditions appropriate for the wave function describing a neutron inside the compound nucleus. At all other energies the nuclear surface is perfectly reflecting (resonance

scattering). The mathematical formulation of these ideas leads to the famous Breit-Wigner resonance formulas. {3}

2) At higher energies ($\gtrsim$8MeV) the number of channels available for the decay of the compound nucleus becomes so great that the probability of decay through the entrance channel is negligible. At the same time the separation between two adjoining states of the compound nucleus becomes less than their width. As a consequence the scattering is potential scattering. As the energy of the incident neutrons increases, the scattering cross-section remains constant while the reaction cross-section decreases regularly, being inversely proportional to the velocity.

In conclusion, at energies where the continuum theory is applicable, the total cross-section is expected to decrease smoothly with increasing incident energy. The same is expected to happen at lower energies for the "gross" variation of the cross-section, which is obtained by averaging over the resonances.

At the beginning of the fifties this picture was partially contradicted by experimental observations. On one hand, experiments like the one of Eisberg and Igo {4} (inelastic proton scattering at 32 MeV) gave strongly forward peaked angular distributions and relatively large values of the (p,p') cross-section while the compound nucleus theory predicts angular distributions symmetric about 90° and strongly favours the (p,n) process over the (p,p') process. On the other hand, evidence of phenomena that could not be explained in terms of the compound theory was given by the discovery of the giant resonances (Barschall {5}): plots of the total cross-section versus energy show broad peaks (of width ~1 MeV) whose posi-

tion and height vary regularly with the mass number of the target. This behaviour can be explained if one supposes that the nucleus, instead of being perfectly reflecting away from resonances, is partially absorbing. The potential scattering is then no more hard sphere scattering, the incident wave function penetrates into the nuclear well, and a resonance is produced when this can accomodate an integer number of half wavelengths.

From a mathematical point of view the partial absorption of the incident wave function can be obtained if one represents the nucleus by a complex potential well (optical potential). To illustrate this point, and to indicate how the cross-sections can be calculated, we shall consider the very simple case of $\ell=o$ neutrons and of a complex potential of the square well type:

$$V(\underline{r}) = -V-iW, \qquad r < R$$
$$= 0 \qquad\qquad r > R$$

Let us recall some well known formulas relative to the scattering of a plane wave by a central potential of range R. The asympotic solution of the Schrödinger equation, the differential cross-section for elastic scattering and the scattering cross-section are given by:

$$(1)\quad \psi(\underline{r}) = \frac{\sqrt{\pi}}{kr} \sum_{\ell=o}^{\infty} \sqrt{2\ell+1}\; i^{\ell+1} \left(e^{-i(kr-\frac{\ell\pi}{2})} - \eta_\ell e^{i(kr-\frac{\ell\pi}{2})} \right) Y_{\ell o}$$

$$(2)\quad \frac{d\sigma_{e\ell}}{d\Omega} = \frac{\pi}{k^2} \left| \sum_{\ell=o}^{\infty} \sqrt{2\ell+1}\,(1-\eta_\ell)\, Y_{\ell o} \right|^2$$

$$(3)\quad \sigma_{e\ell} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1)\ |1-n_\ell|^2 \ ;$$

where $n_\ell = e^{i2\delta_\ell}$ and the phase shifts δ_ℓ are obtained by imposing the continuity of the radial wave function and of its derivative at the boundary $r=R$.

In our case the radial solution is:

$$u = A\left(\sin \sqrt{\frac{2m}{\hbar^2}(E+V+iW)}\, r\right) \quad , \quad r < R$$

$$= B \sin\left(\sqrt{\frac{2mE}{\hbar^2}}\, r + \delta_0\right) \quad , \quad r > R$$

and the continuity condition:

$$\sqrt{\frac{2m}{\hbar^2}(E+V+iW)}\ \operatorname{cotg}\ \sqrt{\frac{2m}{\hbar^2}(E+V+iW)}\, R = \sqrt{\frac{2mE}{\hbar^2}}\ \operatorname{cotg}\ \left(\sqrt{\frac{2mE}{\hbar^2}}\, R+\delta_0\right)$$

gives a complex phase shift δ_0, so that $|\eta_0|<1$. As a consequence the amplitude of the outgoing wave (second term in the asympotic solution {1}) is less than the amplitude of the incoming wave, the difference being proportional to the number of particles absorbed per unit time. The same method gives the complex phase shifts in the case of any ℓ and different shapes of the potential, but, of course, the integration of the radial wave equation becomes in general far from trivial and numerical methods have to be employed.

Substitution of the phase shifts in the general formulas (2) and (3) gives the differential cross-section and the scattering cross-section while the reaction (absorption) and total cross-section are given by:

$$(4)\quad \sigma_{re} = \frac{\pi}{k^2} \sum_{\ell=o}^{\infty} (2\ell+1)\ (1-|\eta_\ell|^2)$$

$$(5)\quad \sigma_{tot} = \sigma_{el} + \sigma_{re} = \frac{\pi}{k^2} \sum_{\ell=o}^{\infty} (2\ell+1)\ 2\ \mathrm{Re}\ (1-\eta_\ell)$$

It should be noted that the optical model does not contradict the idea of the formation of a compound nucleus, but only the assumption of perfect reflection away from resonances. Actually at low energies the optical model and the compound nucleus theory are complementary, the former giving the general trend of the dependence of the total cross-section on the incident energy, the latter giving the details of this dependence.

At energies corresponding to the continuum, the optical model gives the cross-section for the formation of the compound nucleus, while its decay in one or another of the available channels, given the great number of these, can be treated from a statistical point of view (statistical theory). At these energies, however, the formation of the compound nucleus is not the only possible reaction mechanism, and, in particular, reactions involving charged particles, where emission from the compound nucleus is inhibited by the Coulomb barrier, follow generally the direct-reaction pattern.

In earlier studies of the optical model, like the one by Feshbach, Porter and Weisskoff {6}, a square well potential was used. It was found however, that a better agreement with experiment is obtained using a potential of the Woods-Saxon {7} type:

$$(6)\quad U(r) = \frac{-v-iW}{1+\exp\left|\frac{r-R}{a}\right|}$$

characterized by the four parameters, V. W. R, and a. V and W are the strengths of the real and imaginary part of the potential, R is the nuclear radius expressed in terms of the radius parameter r_0 and of the atomic number by $R = r_0 A^{1/3}$, and a is the diffuseness parameter. Generally better agreement with the experimental data is obtained when the values of r_0 and a for the real and imaginary parts are different.

An improvement is obtained considering, together with the volume absorption term (imaginary part of (6)), a surface absorption term, of the form $+4iW_D a_D \frac{d}{dr}\left(\frac{1}{1+\exp\left|\frac{r-r_D A^{1/3}}{a_D}\right|}\right)$ (derivative Wood-Saxon form). The reason for including this term is the following. The absorption process is due to the collisions of the incident neutrons with the nucleons of the target nucleus. Such collisions are opposed by the exclusion principle, particularly in the inside of the nucleus, where most shells are filled. So we expect the absorption to take place in preference at the surface of the nucleus, especially at lower energies, where the exclusion principle is more effective.

A further refinement of the optical potential is obtained with the inclusion of a spin-orbit term, which accounts for the polarization of the scattered particles. In analogy to the shell model potential, the spin-orbit term is chosen to be of the Thomas form. An absorption part in the spin-orbit term is sometimes included; however, its utility is doubtful, except at very high energies. Also, when

the optical model is applied to the scattering of charged particles, a Coulomb term V_c must be included in the potential.

Taking into account the additional terms discussed, we obtain what we shall call the "standard form" of the optical potential.

$$(7)\quad U(r) = V_c(r) - \frac{V}{1+\exp\left(\frac{r-r_o A^{1/3}}{a}\right)} - \frac{iW}{1+\exp\left(\frac{r-r_w A^{1/3}}{a_w}\right)}$$

$$+ 4iW_D a_D \frac{d}{dr}\left(\frac{1}{1+\exp\left(\frac{r-r_D A^{1/3}}{a_D}\right)}\right) + (Vs+iWs)\frac{4}{r}\frac{d}{dr}\left(\frac{1}{1+\exp\left(\frac{r-r_s A^{1/3}}{a_s}\right)}\right)(\underline{s}.\underline{\ell}.)$$

The potential (7) has been used extensively in the analysis of experimental data consisting of differential cross-section and polarization and total reaction cross-sections for a wide range of projectiles and target nuclei, at energies from a few MeV's to 150 MeV and over.

To fit, say, a differential cross-section angular distribution, the differential cross-sections are computed for all the angles considered using the potential (7) with a suitable set of starting parameters. Then the parameters are varied in order to minimize the quantity.

$$(8)\quad \chi^2 = \sum_{\substack{\text{all}\\ \text{angles}}} \left[\frac{\left(\frac{d\sigma}{d\Omega}\right)_{th} - \left(\frac{d\sigma}{d\Omega}\right)_{exp}}{\Delta\left(\frac{d\sigma}{d\Omega}\right)}\right]^2 ,$$

where $\left(\frac{d\sigma}{d\Omega}\right)_{th}$ and $\left(\frac{d\sigma}{d\Omega}\right)_{exp}$ are the theoretical and experimental values of the differential cross-section and $\Delta\left(\frac{d\sigma}{d\Omega}\right)$ is the experimental error.

The optical model potential (7) contains thirteen adjustable

parameters and usually some additional condition is imposed in order to reduce this number. For example, it is quite common to set either W=0 (only surface absorption) or W_D=0 (only volume absorption), and when both terms are considered it is usual to take r_W=r_D.

The parameters, which give the best fit, vary with type and energy of the projectile and with the atomic number of the target nucleus. Systematic optical model studies in progress in various laboratories have the purpose of finding the general trends in the variation of the best-fit parameters and the limits in the applicability of the model.

1-3. The Spectrum of ^{6}Li

The level scheme of the ^{6}Li nucleus is given in figure 1. According to the shell model the ^{6}Li nucleus consists of the complete 1s shell, containing two protons and two neutrons, plus one proton and one neutron in the 1p shell, i.e:, the ground configuration of ^{6}Li is $(1s)^4(1p)^2$.

With a purely central potential the ground configuration would be completely degenerate. Introduction of the interaction between the spin and orbital angular momenta of each particle and of two-body forces causes the degeneracy to be partially removed, the ground configuration splitting into levels corresponding to the different values of the total angular momentum J. The particles of the closed shell contribute zero angular momentum and positive parity, and may be ignored when computing the splitting of the ground configuration. Spin, parities, and relative energies of the lower levels of ^{6}Li are therefore those of a system of two particles, of quantum numbers n_1=n_2=1, l_1=l_2=1, s_1=s_2=½,

LEVEL DIAGRAM OF ^{6}Li

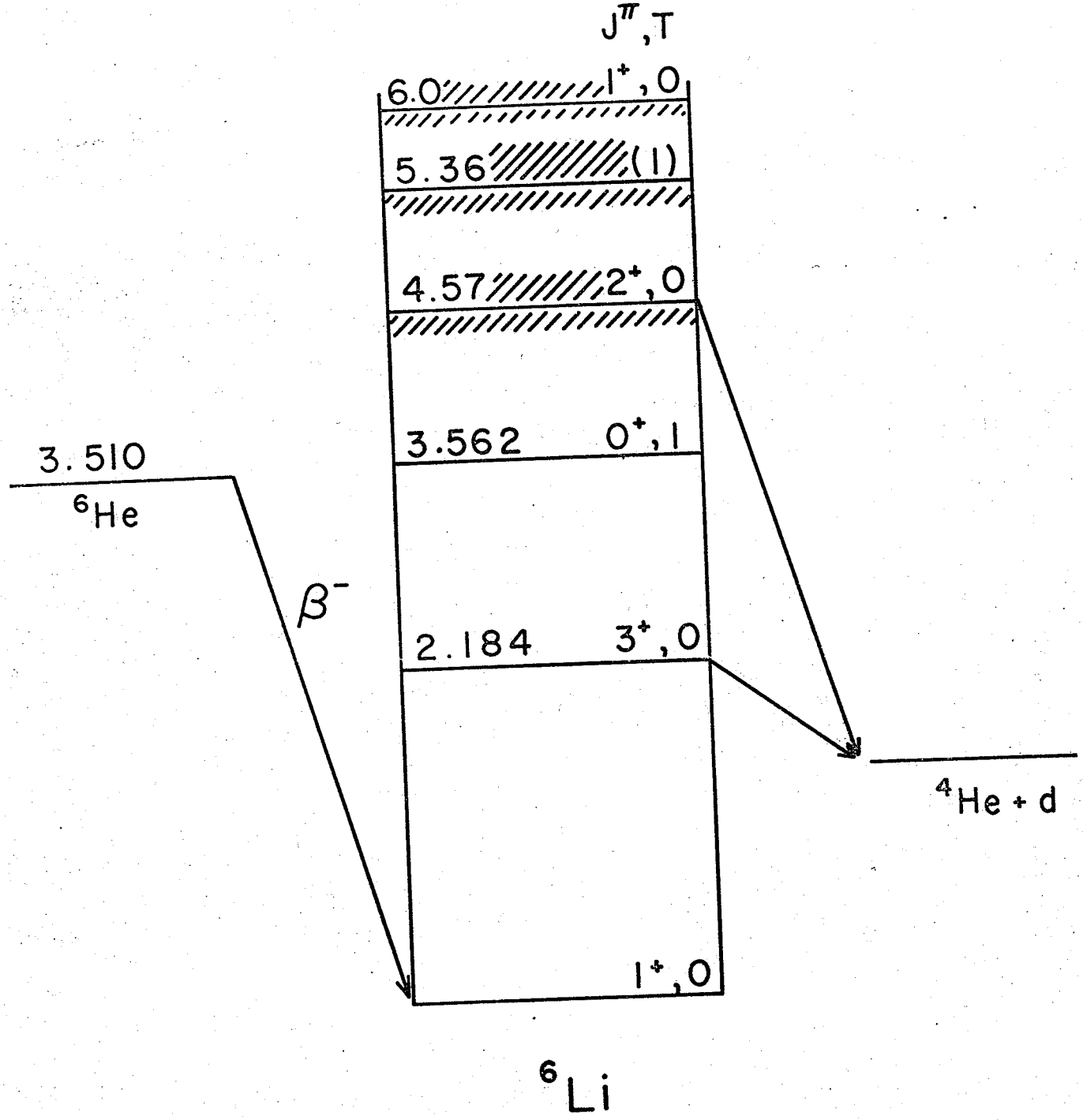

FIG. 1

$t_1=t_2=\frac{1}{2}$, $t_{z1}=\frac{1}{2}$, $t_{z2}=-\frac{1}{2}$, subject to the potential:

$$V_i=V_0 -a\, \underline{\ell}_i \cdot \underline{s}_i + V_{ij} ,$$

The parity of all levels of the ground configuration is given by $(-1)^{\ell_1 + \ell_2} = +1$; thus the states of negative parity whose existence has been suggested between 6 and 14 MeV belong to some higher configuration arising from the lifting of one p-nucleon to a higher shell.

The states of the ground configuration may be classified according to the value of the total isospin T. States with T=1 have the spin and space dependent part of the wave function antisymmetric and are analagous to those obtained when the two particles in the 1p-shell are both protons or both neutrons. The states with T=1 form isospin triplets together with the corresponding states of ^{6}Be and ^{6}He while the states with T=0 have the spin and space dependent part of the wave function symmetric. The values of the total angular momentum are obtained by coupling spins and orbital angular momenta of the two nucleons. The level scheme obtained depends on the coupling scheme adopted.

If one supposes that in the potential the spin-orbit term is much larger than the two-body interaction it is reasonable to use the j-j-coupling:

$$\underline{\ell}_1 + \underline{s}_1 = \underline{j}_1 \quad , \quad \underline{\ell}_2 + \underline{s}_2 \quad \underline{j}_2 \quad , \quad \underline{j}_1 + \underline{j}_1 = \underline{J} \quad (*)$$

(*) Here and in the following the symbols used are self explanatory and conform to the usual practice.

$$\psi_{JM}^{j_1 j_2} = \sum (\ell_1 m_{\ell_1} \tfrac{1}{2} m_{s_1} | j_1 m_{j_1})(\ell_1 m_{\ell_2} \tfrac{1}{2} m_{s_2} | j_2 m_{j_2}) \quad (*)$$

$$\cdot (j_1 m_{j_1} j_2 m_{j_2} | JM)\ \phi_{\ell_1 m_{\ell_1}}\ \phi_{\ell_2 m_{\ell_2}}\ X_{\frac{1}{2} m_{s_1}}\ X_{\frac{1}{2} m_{s_2}}$$

Considering the two-body interaction as a perturbation, then in zero order approximation states with the same values of j_1 and j_2 but different J and M are degenerate, and the levels of the $(p)^2$ configuration are obtained by letting the two nucleons occupy independently single particle states. The single particle levels of the p-shell are $p_{3/2}$ and $p_{1/2}$, the former being the lowest, so that, neglecting the two-body interaction completely, the states of $(p)^2$ configuration are given by $(p_{3/2})^2$, $p_{3/2}p_{1/2}$, $(p_{1/2})^2$, in order of increasing energy. Introducing the perturbation due to the two-body interaction the multiplet $(p_{3/2})^2$ splits according to the different values of J. These are determined in the following way. We can write:

$$\psi_{JM}^{3/2\ 3/2} = \sum_m (\tfrac{3}{2}\ M\ \tfrac{3}{2}\ M\text{-}m | JM) \psi_{3/2m}(1) \psi_{3/2M-m}(2)$$

$$= \pm \sum_m (\tfrac{3}{2}\ m\ \tfrac{3}{2}\ M\text{-}m | JM)\ \psi_{3/2m}(2)\ \psi_{3/2M-m}(1)$$

$$= \pm \sum_m (\tfrac{3}{2}\ M\text{-}m\ \tfrac{3}{2}\ m | JM)\ \psi_{3/2M-m}(2)\ \psi_{3/2m}(1)$$

where the sign on the right hand side is + for T=0 states, (which are symmetric with respect to the exchange of the space and spin coordinates

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(*) $(j_1, m_1, j_2, m_2 | j_3, m_3)$ denotes the Chebsch-Gordon coefficient for the coupling of the angular momenta $\underline{j}_1$ and $\underline{j}_2$ to $\underline{j}_3$

of the two nucleons) and is - for T=1 states. Using the following property of the Clebsch-Gordon coefficients:

$$(j_1 m_1 j_2 m_2 | JM) = (-1)^{j_1+j_2-J}(j_1 m_2 j_2 m_1 | JM)$$

we have:

$$\psi_{JM}^{3/2\ 3/2} = \pm(-1)^{3-J} \sum_m (\tfrac{3}{2}\ m\ \tfrac{3}{2}\ M-m | JM)\ \psi_{3/2\,M-m}(2)\ \psi_{3/2\,m}(1)$$

$$= \pm(-1)^{3-J}\ \psi_{JM}^{3/2\ 3/2}$$

Thus the values of J for the multiplet $(p_{3/2})^2$ are even if T=1 and odd if T=0, and spin and isospin of the first four states are(J=3,T=0) (J=1, T=0)(J=2, T=1),(J=0, T=1). This conclusion does not agree with the experimental results, which suggest T=0 for the J=2(4.57MeV) state.

The question of the applicability of the j-j model to the 1p shell nuclei has been examined in detail by Kurath {8}. This author, using the Hartree method and assuming j-j coupling, calculated the matrix elements of a two-body interaction of the form $V_{12}=P\exp\{-(\frac{r_{12}}{r_o})^2\}$ for the cases in which P is the Wigner, Majorana, Bartlett or Heisenberg operator. A comparison with the results obtained fifteen years earlier by Feenberg and Wigner {9} and Feenberg and Phillips {10} in L-S coupling shows that, while both models predict correctly the spins in some cases, none of them accounts satisfactorily for the spectra of all the 1p-nuclei. In particular, in the case of ^{6}Li, according to the j-j coupling model the order in increasing energy of the lower states is

(3,0) (0,1) (1,0) (2,1), while experimentally the ground state is (J=1, T=0), which is given correctly by the L-S model. The L-S coupling scheme is defined by:

$$\underline{\ell_1} + \underline{\ell_2} = \underline{L}$$

$$\underline{s_1} + \underline{s_2} = \underline{S} \qquad \underline{L} + \underline{S} = \underline{J}$$

$$\psi^{LS}_{JM} = \sum (\ell_1 m_{\ell_1} \ell_2 m_{\ell_2} | LM_L)(\tfrac{1}{2} m_{s_1} \tfrac{1}{2} m_{s_2} | SM_S)$$

$$\cdot (LM_L SM_S | JM)\, \psi_{\ell_1 m_{\ell_1}} X_{\frac{1}{2} m_{s_1}}\, \psi_{\ell_2 m_{\ell_2}} X_{\frac{1}{2} m_{s_2}}$$

For two p-nucleons the possible values of the quantum numbers L,S,J are given by:

$$L=0,1,2 \quad ; \quad S=0,1 \quad ; \quad J = |L-S|, \ldots, L+S-1, \; L+S$$

The value of the total isospin is determined by symmetry considerations. The space dependent part of the wave function has symmetry given by $\phi(1,2) = (-1)^L \phi(2,1)$ and the spin part is antisymmetric for singlet states (S=0) and symmetric for triplet states (S=1). The spin-space dependent wave function is then antisymmetric for even singlets and odd triplets, which will correspond to T=1, while we shall have T=0 for odd singlets and even triplets. Therefore the states of the $(p)^2$ configuration in L-S coupling are given by the following table:

L	S	J	T	$^{2T+1,2S+1}L_J$
0	0	0	1	$^1S_0$
0	1	1	0	$^3S_1$

con't.

L	S	J	T	$^{2T+1,2S+1}L_J$
1	0	1	0	$^{11}P_1$
1	1	0	1	$^{33}P_o$
1	1	1	1	$^{33}P_1$
1	1	2	1	$^{33}P_2$
2	0	2	1	$^{31}D_2$
2	1	1	0	$^{13}D_1$
2	1	2	0	$^{13}D_2$
2	1	3	0	$^{13}D_3$

In the case of pure L-S coupling and of a central potential the levels corresponding to the same values of L and S are degenerate. Feenberg and Phillips {9} give the energies of the various multiplets in terms of direct and exchange integrals of the Hartree theory:

$$\lambda = \int\psi(1)\phi(2)V_{12}\ \psi(1)\phi(2)d\tau_1 d\tau_2,$$

$$\kappa = \int\psi(1)\phi(2)V_{12}\phi(1)\psi(2)d\tau_1 d\tau_2,$$

where ψ and ϕ are two single particle wave functions. The indices 1 and 2 refer to the coordinates of the two nucleons and V_{12} is the interaction potential. The resulting order of the levels depends on the exchange character of the interaction and on the wave functions.

The intermediate coupling theory of the 1p-shell nuclei is due to Inglis {11,12} and Kurath {13}. These authors describe the system of the n nucleons of the $(1p)^n$ configuration by a Hamiltonian of the form:

$$(9)\quad H = \sum_i T_i + \sum_{i>j} V_{ij}(r_{ij}) O_{ij} + \sum_i a \underline{\ell}_i \cdot \underline{s}_i ,$$

where T_i is the kinetic energy operator for the i^{th} nucleon, $\underline{l}_i$ and $\underline{s}_i$ are the orbital and spin angular momenta, $V_{ij}(r_{ij})$ is a central two-body interaction and O_{ij} an exchange operator between nucleons i and j. The radial dependence of V_{ij} is assumed to be gaussian, the single particle wave functions are taken of the harmonic oscillator type, and the exchange operator O_{ij} is chosen to be the following linear combination of the space-exchange (Majorana) operator P and of the spin-exchange (Bartlett) operator Q:

$$O_{ij} = 0.8\ P + 0.2\ Q.$$

Having specified the interaction, the ratio of the direct to the exchange integral λ/κ depends only on the strength of the nuclear force and on the nuclear radius. Inglis and Kurath estimate $\frac{\lambda}{\kappa} = 6$ to be a reasonable value for the lp-shell. In the L-S coupling limit, which is obtained by setting a=0 in eq.(9), the order of the levels is now determined and their spacing depends only on κ, which is left as a free parameter in order to compare more easily the prediction of the theory to the experimental results. For the $(lp)^2$ configuration the order of the ^{2S+1}L multiplets is ^3S, ^3D, ^1S, ^1D, ^3P, ^1P.

The transition to intermediate coupling is described in terms of the parameter $\frac{a}{\kappa}$, which measures the relative strength of the spin-orbit coupling and of the two-body interaction. When $a \ll \kappa$ the description of the system in terms of the L-S model is still valid, but the coupling term $\sum_i a \underline{\ell}_i \cdot \underline{s}_i = A\ \underline{L} \cdot \underline{S}$, acting as a perturbation, removes the

degeneracy among levels of the same multiplet ^{2S+1}L. At the opposite limit, when a>> K, the spin orbit term dominates the interaction, and the (j-j) model is applicable. Numerical solutions, obtained by Inglis and by Kurath for the various mass numbers corresponding to the lp-shell, give position and spacing of the levels as a function of the parameter $\frac{a}{K}$. The value of this parameter which best describes each nuclide is then determined by comparison with the observed spectra.

In the case of ^{6}Li this value is given by $\frac{a}{K} \approx 1.3$, which is much lower than the values obtained for heavier nuclei of the lp-shell (≈ 5), and corresponds to essentially L-S coupling, the spin orbit interation representing only a perturbation which causes the splitting of the multiplets ^{2S+1}L.

In the preceeding we have discussed the spectrum of ^{6}Li from the point of view of the shell model. Another possible approach is represented by a description in terms of an α-cluster deuteron-cluster model {14}. A general review of the cluster model of nuclei is given by Wildermuth and McClure {15}.

According to the cluster model the ^{6}Li wave functions are written as:

$$\psi_{ijk} = A\phi_i(\alpha)\phi_j(d)\ X_k(\alpha\text{-}d)$$

where $\phi_i(d)$ is the internal wave function for the α-cluster, which depends on the space, spin and isospin coordinates of the four nucleons, $\phi_j(d)$ is the analagous wave function for the d-cluster, $X_k(\alpha\text{-}d)$ corresponds to the relative motion of the two clusters and A is the antisymmetrization operator.