A STUDY OF THE ULTIMATE STRENGTH OF A REINFORCED CONCRETE SQUARE SLAB SUPPORTED ON EDGE BEAMS CONTAINING A LARGE SQUARE OPENING AT ONE CORNER

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by

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NOTATION

α = ratio of positive steel to minimum steel used in slab

β = ratio of negative steel to minimum steel used in slab

γ = ratio of loaded lengths in negative to positive region = l_n/l_p

δ = deflection

ε_u = the maximum concrete strain

ε_y = steel strain at yield

θ = rotation

λ = load factor

A,B = constants

A_{cage} = concrete area enclosed by a reinforcing cage

A_s = area of tension reinforcement

C = compression in concrete

c = distance from extreme compression fiber to neutral axis at ultimate strength

= circumference of a reinforcing cage

D = internal dissipation of energy

d = effective depth

E = work done by external loads

F_{Y_L} = yielding force of one longitudinal bar

F_{Y_T} = yielding force of one stirrup

f_c = compressive stress in concrete

f'_c = ultimate compressive strength of concrete

f_s = tensile stress of reinforcement

f_y = yield strength of reinforcement

j = ratio of distance (jd) between resultants of compressive and tensile stresses to effective depth

K = constant

k = ratio of distance (kd) between extreme fiber and neutral axis to effective depth

k_1, k_2, k_3 = constants related to concrete stress distribution

L = length of free span

l = loaded length = $l_n + l_p$

l_n = loaded length of a strip in the negative moment region

l_p = loaded length of a strip in the positive moment region

M = bending moment

M_n = negative moment at support

M_p = maximum positive moment in span

M_T = torsional strength of reinforcing cage

M_u = ultimate bending moment

m = resisting moment of slab per linear foot due to minimum reinforcement used = 243 ft-lb/ft

m_x = bending moment per unit length acting on face x

m_y = bending moment per unit length action on face y

m_{xy} = twisting moment per unit length

n = modular ratio

= number of longitudinal bars at section considered

P_F = actual collapse load of the slab

P_U = collapse load according to upper bound solution

P_W	=	working load
$\bar{p}$	=	reinforcement ratio
p_b	=	balanced reinforcement ratio
R	=	reduction factor
R_L	=	equivalent yielding force of longitudinal bars per unit length
R_T	=	equivalent yielding force of a stirrup per unit length
R_{min}	=	minimum equivalent yielding force of steel per unit length
r	=	ratio of maximum tensile strength to maximum crushing strength
s	=	spacing of stirrups
T	=	tension in reinforcement
t	=	slab thickness
V	=	shear force
v	=	shearing stress
w	=	uniformly distributed load
Y	=	factor related to span length L
▬▬▬	=	strip
—·—·—·—	=	line of stress discontinuity
-+-+-+	=	contraflexure line
◄——	=	load dispersion
○	=	plastic bending moment of beams
●	=	plastic twisting moment of beams
〰〰	=	sagging cracks
- - - - -	=	hogging cracks

CHAPTER I

INTRODUCTION

1.1 Background

Since Ingerslev's work was first published in the early 1920's, yield-line theory for thin plates had drawn considerable attention to many researchers. One of them was Johansen[1] who extended and advanced Ingerslev's basic concept to a more comprehensive yield-line theory. Wood[2] had investigated many types of reinforced concrete floor slabs and discussed Johansen's works in detail. Jones[3] presented the application of yield-line theory to reinforced concrete slabs for practising engineers, as well as advanced students in engineering.

An alternative method for design of slabs is the strip method. The method was recently introduced by Hillerborg[4] and was discussed by Wood[5]. Armer [6] tested a number of slabs designed by the strip method.

Both methods, yield-line theory and the strip method, are based on the plastic theory and yield ultimate strength of reinforced concrete slabs. As distinct from the strip method, which is a design method, yield-line theory is an analysis procedure and based on the strength of steel in slabs.

Until recently, most reinforced concrete slabs have been designed according to the elastic theory. However, there are many shortcomings of the method. First, it cannot show how a slab behaves under an overload or a load at collapse. Second, the usual method is restricted to the design of slabs of regular shape only. Normally, the design is carried out as guided by codes of practice.

One type of a slab which faces designers is the floor slab with a large opening for a stairwell. The design cannot be carried out simply by the working stress method. Such slabs require analysis by elastic plate theory with the use of modern computers. The plastic theory is of some importance in such a case.

1.2 Object of Research

The purpose of this research was to show that the design of a reinforced concrete floor slab containing a large opening by Hillerborg's [4] simple strip method was consistent with experimental observation. A one-quarter scale model was employed as the study model. It was also hoped that some indication of the amount of reinforcement that could be saved in this type of design, as compared with design by the elastic theory (working stress design), could be obtained. In order to see how reliable the model functioned under a test load, deflections and the cracking pattern of the model were observed.

Finally, the type of failure and the collapse load were compared with those predicted by yield-line theory, taking into account the contribution of the membrane forces.

CHAPTER II

THEORY

2.1 Limit Analysis

Limit analysis is the plastic method for solving the collapse load of any structure assuming the formation of plastic hinges in the structure, and checking to see that the yielding criterion is satisfied throughout the structure. A structure is assumed to fail when the redistribution of moments takes place and one or more plastic hinges occur at a constant applied load.

In the early stage of development, the principles of limit analysis were based on perfectly plastic materials. Steel, because of its ductile behaviour, was the material to be approximated in such a manner. In steel design, the term plastic design, which is synonymous to limit analysis or design, is now commonly used.

In spite of the fact that plain concrete possesses a limited inelastic characteristic, the rotational capacity of the plastic hinge in the under-reinforced member is fairly large. This behaviour makes it possible to apply limit analysis to reinforced concrete structures. Therefore, slabs which have flexural ductility because of small percentages of steel reinforcement, can be analysed according to the concept of plastic analysis.

Baker et al.[7] summarized the conditions for any structure at collapse to satisfy a requirement of a plastic moment redistribution.

(i) Mechanism condition: The bending moment in the plastic hinges must be equal to the full plastic moments of resistance, and sufficient hinges must exist to develop a mechanism;

(ii) Equilibrium condition: The bending moments must be in equilibrium with the applied load throughout the structure;

(iii) Plastic condition: The bending moment anywhere in a structure must not exceed the full plastic moment of resistance.

If all these conditions are satisfied, the load factor at collapse, λ_c, according to limit analysis, can be obtained. But, for the case of a plate problem, the true load factor at collapse cannot be found precisely (except for a few special cases, the plastic condition cannot be solved), since the solution normally lies in a limited or bounded region, which is in between upper and lower limits. Therefore, an indirect approach to solve the problem by the principles of limit analysis must be used. That is to find both upper and lower bound solutions.

In order to find an upper bound solution, whose load factor λ is greater than or equal to λ_c, the first two conditions above must be satisfied. Similarly, in order to find a lower bound solution, the last two conditions above must be satisfied.

Theoretically speaking, if the collapse load from an upper bound solution coincides with that from a lower bound solution, it must be the true collapse load of the structure. Very few cases of slabs have been solved successfully for coincident upper and lower bound load factors. Since the true collapse load cannot be calculated in most cases, at least the gap between those bounds should be reduced to as much as possible.

2.2 Upper Bound Solution

Johansen[1], who made use of the concept of limit analysis, introduced yield-line theory to calculate the collapse loads of reinforced concrete slabs. By assuming the failure of a slab to occur along postulated lines along which steel was yielding, the collapse mechanism was formed. Work equations (i.e. the internal dissipation of energy and work done by the external loads) were then established to fulfil the equilibrium conditions. By doing so, the collapse load obtained was an upper bound solution, according to limit analysis. The method gave an unsafe solution which may be greater than or equal to the true collapse load ($\lambda \geq \lambda_c$).

In order to establish the work equations, the information concerning the amount of steel must be known in advance. Therefore, yield-line theory is more suitable for a method of analysis; the design can be carried out by a trial-and-error process only.

2.3. Lower Bound Solution

Hillerborg[4] suggested an alternative method -- the so-called strip method -- for designing reinforced concrete slabs by the plastic theory. The method is based on the equilibrium equation of plates:

$$\frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} - \frac{2\partial^2 m_{xy}}{\partial x \partial y} = -w$$

where m_x = bending moment per unit length acting on face x

m_y = bending moment per unit length acting on face y

m_{xy} = twisting moment per unit length

The basis of this method is the assumption that the twist $m_{xy} = 0$, and the slab load is then assumed to be carried by the strips in either the x- or y- direction by bending only. The new form of the equilibrium equation is obtained as:

$$\frac{\partial^2 m_x}{\partial x^2} = -Kw$$

$$\frac{\partial^2 m_y}{\partial y^2} = -(1-K)w$$

where K = constant.

This implies that the slab load will be carried by the strip in the x-direction only if K = 1, and in the y-direction only if K = 0.

Instead of being represented by the mathematical notations, the method can be carried out as a design procedure, as intended. To do so, each strip in a slab is considered as a beam. In order to fulfil the equilibrium equation, the boundary conditions must be applied to the strip. Reactions, shears and bending moments in the strip are then obtained from equilibrium. Furthermore, steel can be chosen to provide a resistance in each strip to the bending mement obtained. Therefore, the last condition of the requirements of limit analysis is automatically satisfied.

This design procedure is certainly a lower bound solution for the collapse load of the slab according to the principles of limit analysis. Since the yield criterion in the slab is nowhere violated, the calculated load will be equal to or less than the true collapse load. Moreover, since each strip is designed according to the bending moments obtained, the reinforcement in the strip can be arranged accordingly. This implies some economy in the amount of steel used which can be seen from the following:

(i) negative reinforcement can be stopped off at the point of contraflexure;

(ii) variable arrangement of steel in the various strips throughout the slab can be used.

One point to be mentioned here is that the limit analysis deals with bending moment fields only. Shear is not taken into consideration. This appears to be no great problem in most cases of slabs, since the shear load in slabs is usually low. Only in special cases must shear in slabs be checked.

2.4 Membrane Stresses

According to the principles of limit analysis, the upper bound solutions (from yield-line analysis) should give a higher collapse load than the true one. In reality, the true collapse load obtained from tests is usually greater than that indicated by yield-line theory. A lack of accuracy in the assumption of limit analysis results in an underestimation of the carrying capacity of a structure. Two basic assumptions are stated as follows:

(i) The material is perfectly plastic;

(ii) Geometric changes which occur when a structure deforms have an insignificant effect on the equilibrium of the structure.

Normally, a ductile material like steel shows a strain-hardening effect when the load applied has passed the initial plastic zone. This is also true for the case of a reinforced concrete member, due to its steel reinforcement. Moreover, when a slab starts to deform and geometric interference takes place, in-plane stresses are likely to develop in the plane of the slab. These are known as membrane stresses. Both effects increase the carrying capacity of the slab. Ockleston[8] found the compressive membrane stress effect, or arching action existed in his full scale tests in Johannesburg, especially when large restraint was furnished by the surrounding slab structure.

Furthermore, there are two types of membrane stresses:

(i) Compressive Membrane Stress:

Prior to the crushing of the concrete, the edge beams or surrounding structure act like rigid rings working against the expansion of the finite thickness of a slab when the clamped slab tries to deflect. The deflection is normally less than half the slab thickness. Since concrete is strong in compression, parts of the slab must jam into each other due to the compressive force exerted by the edge restraint (see Figure 1). This results in increasing the carrying capacity of the slab through an arching action.

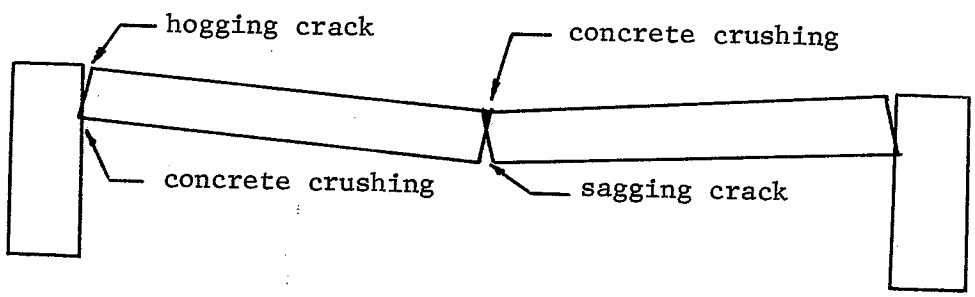

Figure 1. Membrane stresses in deformed slab.

(ii) Tensile Membrane Stress:

If the deflection of the slab is sufficiently large (usually greater than half the slab thickness), crushing of the concrete will occur in the region of the slab under which the compressive force is developed. The reinforcement in the tension zones of the slab will then act as a catenary system. This system is limited by the limit of plastic elongation of the reinforcement.

Again, an additional load can be carried by this action before the slab collapse. Therefore, slabs in general, develop large membrane stresses prior to failure.

The membrane stress effects have been mentioned in several previous works, but not too many researchers have carried out investigations to give a reliable estimate of the amount of enhancement of collapse load in such slabs. Wood[2] introduced the mathematical analysis for calculating the contribution of the membrane stress. It is termed an "enhancement factor" and equal to

$$1 + \frac{A^2}{4B}$$

where $A = \dfrac{\frac{t}{2d} - \frac{3r}{2}}{1 - \frac{3r}{4}}$

$B = \dfrac{\frac{3r}{4}}{1 - \frac{3r}{4}}$

$r = \dfrac{\text{maximum tensile strength}}{\text{maximum crushing strength}} = \dfrac{pf_y}{f'_c/0.85}$

t = slab thickness

d = effective depth

p = reinforcement ratio

f_y = yield strength of reinforcement

f'_c = ultimate compressive strength of concrete

Since the compressive membrane stress is developed in the slab at small deflections, the slab, which is a thin plate, may buckle at a load lower than the collapse load. It seems reasonable to use a reduction factor to take care of the instability of the slab. It was suggested by Wood that a factor of 0.5 should be used for $p < 0.004$. Therefore, the resulting enhancement factor becomes:

$$R\left(1 + \frac{A^2}{4B} \right)$$

where R = reduction factor.

CHAPTER III

DESCRIPTION AND DESIGN OF MODEL

3.1 Dimensions and Design Load for the Model

In practice, the span of most reinforced concrete floor slabs in office buildings or apartments ranges from 5 ft. to 25 ft. with thickness of 3 in. to 8 in., and the openings for stairwells normally have a rectangular shape, approximately 6 ft. x 10 ft. in size. For the sake of simplicity, the prototype was chosen so as to closely simulate an actual slab in common practice. It was an 8-in. two-way slab, 20 ft. square with a square opening of 8 ft. x 8 ft. at one corner. The slab was restrained on all four sides by 12 in. by 24 in. edge beams round the panel. Design working load was 200 psf. : with the dead load for the 8-in. slab equal to 100 psf. and live load taken as 100 psf.

For large-scale models (having a scale factor greater than about 1/5) it is generally possible to reproduce accurately all the geometric details of the prototype structure. Since the mechanical properties of the prototype and model materials are very similar, the behaviour of the structure can be observed through the elastic range and up to rupture. In the case of the quarter-scale model in this test, the laws of similitude were applied to the length scale only. Instead of converting the results obtained from the model tests to predict the behaviour of the full-scale structure, the slab model was built and tested as if it were a prototype structure itself. Therefore, all linear dimensions of the prototype were scaled down by a factor of 4. Dimensions and design load for the slab model (see Figure 2) were summarized as follows:

MODEL

Scale : one-quarter scale;

Dimensions : 5 ft. x 5 ft. reinforced concrete slab (clear span) of thickness 2 in., having a 2-ft. square opening at corner with 3-in. by 6-in. beams on all four sides of the slab;

Effective Area of the Slab : 21 sq.ft.;

Loading :

(i) Design working load:

Dead Load = 100 psf.

Live Load = 100 psf.

= 200 psf. = 4,200 lb.

(ii) Design ultimate load:

Dead Load = 1.5 x 100 = 150 psf.

Live Load = 1.8 x 100 = 180 psf.

w = 330 psf. = 6,930 lb.

(iii) Jack Load : The loading imposed was applied through the load setup on the slab at specified positions as shown in Figure 2. below.

Self wt.	=	25 psf.	=	525 lb.
Dead load from load setup	=	15 psf.	=	315 lb.
Loading imposed	=	290 psf.	=	6,090 lb.
w	=	330 psf.	=	6,930 lb.

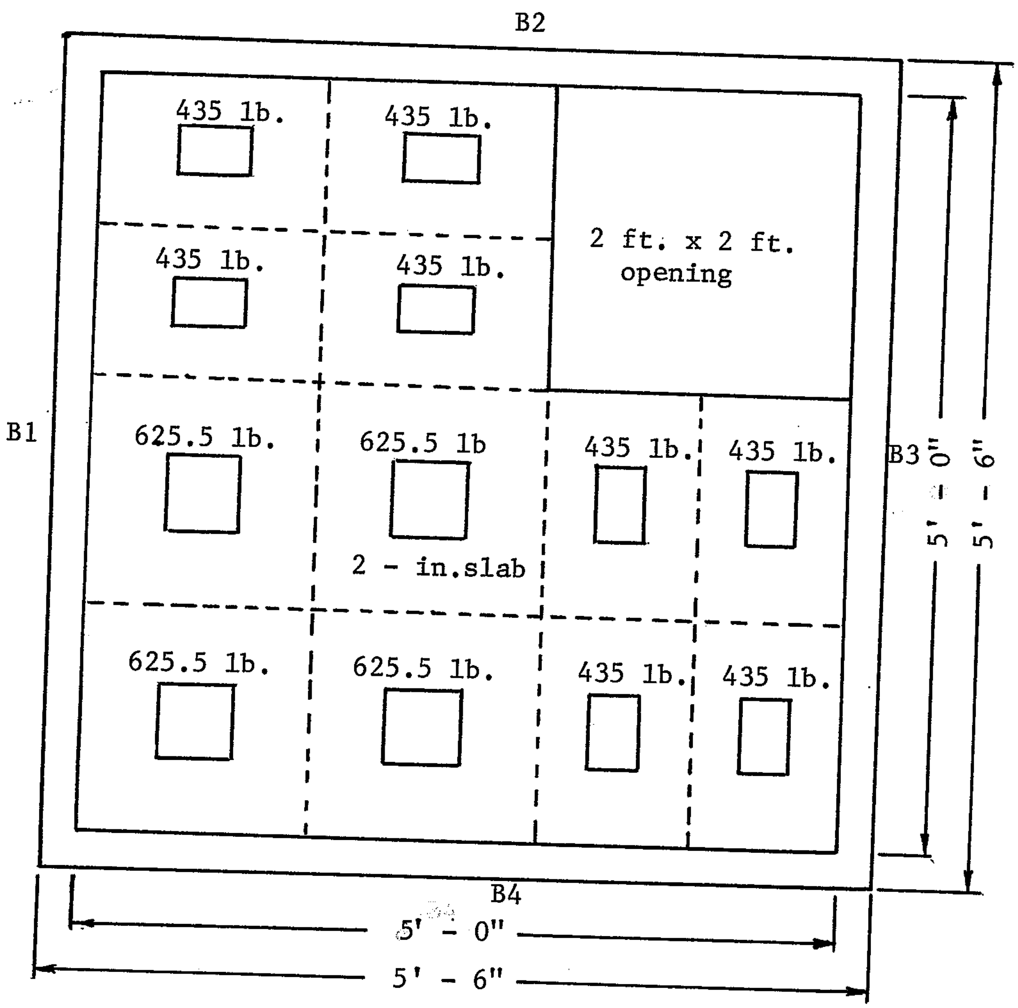

Figure 2. Slab Model.

3.2 Materials

A. Concrete

In order to obtain the strength of concrete used for designing the slab model it was necessary to perform trial mixes and compression tests. The cylinder specimens were made of 3-in. by 6-in. cylinder moulds filled in three layers of concrete. Each layer was tamped with a 3/8-in. diameter rod, approximately 24 inches long, using 25 strokes over the cross section of the mould. Due to the relationship between the maximum size of the coarse aggregate and the diameter of a cylinder mould, the diameter of a cylinder specimen, according to ASTM Standards, must be at least three times the nominal maximum size of the coarse aggregate used in the concrete. Therefore, the 3-in. by 6-in. cylinder moulds were used in accordance with the maximum size of coarse aggregate used in the test, which was 3/8-in. One major advantage in using this small mould is that since much less concrete is needed per specimen, more specimens can be made than with a 6-in. by 12-in. mould, therefore, a more accurate and more reliable result of an average compressive strength of the concrete can be obtained.

a) Aggregate

Fine sand was used as fine aggregate in mixing the concrete for the model. The sand particles were angular, The moisture content was 3.1 %, the specific gravity was approximately 2.65 and the finess modulus of the sand was 2.59. This was obtained from sieve analyses; the average result is shown in Table I: