# NOISE MEASURING TECHNIQUES
# AT ULTRA-HIGH FREQUENCIES

by

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## ABSTRACT

A method whereby the noise figure and the excess noise ratio of a device may be measured using succeeding-stage noise compensation is given. The procedure developed, measures the noise behavior of the first stage of a system and corrects for the noise contributions of the following stages.

Experimental results are presented for the noise figure and the excess noise ratio at UHF for a junction FET device. The existing noise models are also verified.

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# CHAPTER I

# NOISE CHARACTERIZATION

## One-Port Noise-Equivalent Representations

For the noise analysis of one-port networks various characterizations of noise exist. Representations commonly used are: noise temperature, noise conductance, noise resistance, equivalent saturated diode current, and excess noise ratio.

Any noise that has a power spectrum containing all frequency components in equal proportion is referred to as "white noise".

For any white-noise source a noise temperature, $T_e$, can be defined which is a measure of the available power of that white-noise source:

$$T_e = N_A/kB, \qquad \text{1.1}$$

where $N_A$ = available noise power of noise source measured in a bandwidth B.

As an example, consider the case when a temperature-limited diode is used as a white-noise source. For UHF purposes, the noise diode is terminated by the characteristic admittance of the system. The equivalent noise model of the diode noise source is shown in Figure 1.1.

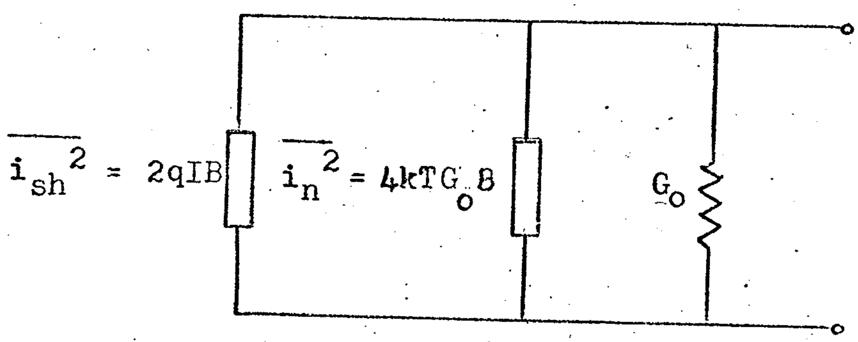



Figure 1.1 Noise-equivalent circuit of a diode noise generator

The diode generates shot noise which is proportional to the plate current, I. The terminating conductance generates thermal noise porportional to the physical temperature T of the conductor.

Short circuiting the generator's terminals, the mean-squared noise current becomes

$$\overline{i_{TOT}^2} = 2qIB + 4kTG_oB \quad \ldots\ldots\ldots\ldots\ldots \quad 1.2$$

This noise current can be expressed as a thermal noise source of conductance $G_o$ with a physical temperature $T_e$. Rearranging equation 1.2,

$$\overline{i_{TOT}^2} = 4kT_eG_oB \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1.3$$

where $$T_e = qI/2kG_o + T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1.4$$

Equivalently, the available noise power from the generator is

$$N_A = 2qIBG_o/4G_o^2 + 4kTG_o/4G_o$$

$$= qIB/2G_o + kTB \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.5$$

which can be rewritten,

$$N_A = kT_eB \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.6$$

where $T_e$ is given in Equation 1.4.

This example shows that the diode noise generator can be described by a thermal conductance $G_o$ at a physical temperature $T_e$, without affecting the terminal properties of the generator.

White noise across any two terminals of a network, measured in a bandwidth B, can be described by a noise current generator, paralleling the terminals.[2] The mean-squared value of the generator is

$$\overline{i_n^2} = 4kTg_nB \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.7$$

where $g_n$ is the equivalent noise conductance across the terminals, and T is the physical temperature of the network.

The fictitious noise conductance $g_n$ gives a measure of the noise across the two terminals. It does not necessarily represent an actual conductance in the network.

The dual to the equivalent noise conductance is the equivalent noise resistance. White noise may be described by its equivalent noise resistance $R_n$ referred across any pair of terminals. This fictitious resistance generates an equivalent mean-squared voltage paralleling the two terminals:

$$\overline{v_n^2} = 4kTR_nB \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.8$$

where T is the physical noise of the system.

Noise may also be represented by an equivalent mean-squared current generator

$$\overline{i_n^2} = 2qI_{EQ}B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1.9$$

in parallel with the terminals of a network, where $I_{EQ}$ is the equivalent saturated (temperature-limited) diode current of the network. In this case, the noise of the network is characterized by a generator having full shot noise, even though the true origin of the noise is not necessarily known.

The noise ratio, NR, of a two-terminal network is defined as

$$NR = N_A/ kT_oB \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1.10$$

where $N_A$ is the available output noise power measured in the bandwidth B, and $kT_oB$ is the available thermal noise power of the network's passive termination at standard temperature (290°K).

Noise ratio can also be expressed according to Equation 1.6 by

$$NR = T_e/T_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1.11$$

where $T_e$ is the equaivalent noise temperature of the network, and $T_o$ is the standard noise temperature.

An excess noise ratio, ENR, can also be defined:

$$ENR = (T_e - T_o)/T_o = T_x/T_o \quad \ldots\ldots\ldots\ldots \quad 1.12$$

where $T_x$ is the excess noise temperature.

Excess noise ratio is a measure of the noise produced by a one port network above the thermal noise contributed by its driving-point admittance.

Two-Port Noise-Equivalent Representations

Spot noise figure is a measure of the noise performance of two port networks. A definition of spot noise figure is the ratio of the noise power output of an actual two-port and its noise-free equivalent. Spot noise figure, at a specified frequency, has been defined by the IRE[3] as "the ratio of 1) the total noise power per unit bandwidth at a corresponding output frequency available at the output port when the noise temperature of the input termination is standard (290°K) to that of 1) engendered at the input frequency by the input termination. The standard noise temperature 290°K approximates the actual noise temperature of most terminations."

The important aspects of the previous definition should be pointed out. As defined, the spot noise figure is a point function of frequency. It is independent of output termination. The definition also states that the spot noise figure assumes that the input termination is at a physical temperature of 290°K.

When several networks are cascaded, each network contributes to the overall spot noise figure of the system according to [1]

$$F_{syst} = 1 + (F_1 - 1) + (F_2 - 1)/G_{A1} + (F_3 - 1)/G_{A1}G_{A2} + \cdots \qquad 1.13$$

where $G_{A1}$, $G_{A2}$, ... are available power gains of the individual stages, and $F_1$, $F_2$, ... are the spot noise figures of the individual stages. This expression holds for networks whether they amplify or attenuate.

The spot noise figure of an attenuating network at standard temperature is

$$F = 1/G_A \quad \text{.......................} \quad 1.14$$

where $G_A$ is the available gain of the network.

Equation 1.13 demonstrates that if the first stage of the cascade has a high gain, and the second stage has a relatively low spot noise figure, the system spot noise figure becomes

$$F_{syst} \cong F_1 \quad \text{.....................} \quad 1.15$$

Thus, design of the first stage is an important consideration for optimizing the noise performance of a system. Equations 1.13 and 1.14 show that attenuation in the front end of any amplifying system must be avoided if a low spot noise figure is desired.

Often the noise properties of only the first stage in the system are wanted. If the equality in Equation 1.15 does not hold, the contribution to system spot noise figure due to the second-order terms in Equation 1.13 must be taken into account. These terms form the expression for what is called the background spot noise figure, $F_{bkgd}$, of the system:

$$F_{bkgd} = F_{syst} - F_1 \quad \text{.............} \quad 1.16$$

In most amplifier systems, the signal occupies a finite bandwidth, and the spot noise figure may vary with frequency over this bandwidth. This consideration has prompted an average noise figure to be defined, which takes into account the total noise power in the channel. The IRE defines average noise figure as[3] "the ratio of 1) the total noise power delivered into the output termination by the transducer when the noise temperature of the input termination is standard (290°K) at all frequencies to 2) that portion of 1) engendered by the input termination. For heterodyne systems, 2) includes only that portion of the noise from the input termination which appears in the output via the principle-frequency transformation of the system, and does not include spurious contributions such as those from an image-frequency transformation".

Spot noise figure can then be written as a power ratio:

$$F = (GkT_oB + N_n)/GkT_oB \quad \ldots\ldots\ldots\ldots\ldots \quad 1.17$$

where $G = \dfrac{\text{output power delivered into load}}{\text{power available at input}}$

$kT_oB$ = available noise power from input termination

$N_n$ = output noise power contributed by transducer

Average noise figure and spot noise figure can be related by[3]

$$\bar{F} = \int F(f)G(f)\,df \Big/ \int G(f)\,df \quad \ldots\ldots\ldots\ldots \quad 1.18$$

where $\bar{F}$ = average noise figure

$F(f)$ = spot noise figure

$G(f)$ = transducer gain

If F(f) and G(f) are assumed constant over the measurement bandwidth, the average noise figure equals the spot noise figure. These conditions can be achieved by making the measurement bandwidth very narrow compared to the bandwidth of F(f) and G(f). With this in mind, it will be assumed that noise figure measurements designate spot noise figure, even though in actuality an average noise figure is being measured.

## CHAPTER II

## NOISE IN JUNCTION FETS

The measurement techniques developed in Chapter three were tested experimentally by examining the noise parameters of a junction FET (JFET). A brief outline of the JFET noise model will be given herein.

### Two-Port Representation of JFET Properties

In order to describe its behavior, the JFET must be considered as a linear two port having small signal voltages and currents as variables.

The admittance or y-parameters provide a useful characterization of two-ports at UHF. Figure 2.1 shows a linear network receiving a signal $I_s$ from a generator with output admittance $Y_s$, and delivering it to a load $Y_L$.

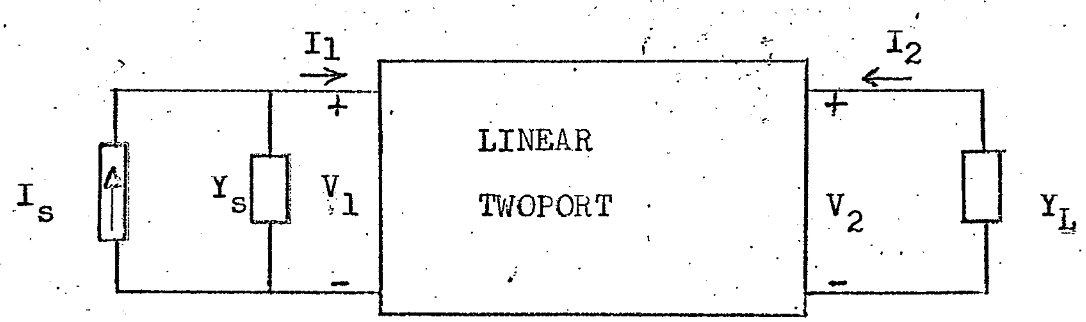



Figure 2.1 Two-port admittance representation of a network

The relationship between input and output variables is given by

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.1$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.2$$

The coefficients, or y parameters, are in general dependent on biasing and operating frequency.

The JFET, with its internal noise sources, can be considered as a linear noisy two-port network. The noise behavior within a narrow bandwidth B can be represented by two external noise generators connected to the network terminals, leaving the linear two-port noise free.[4] In general, the two noise generators are partially correlated with one another. One convenient way of describing JFET noise behavior is by lumping all noise contributions from within the device at the terminals as shown in Figure 2.2.[5]

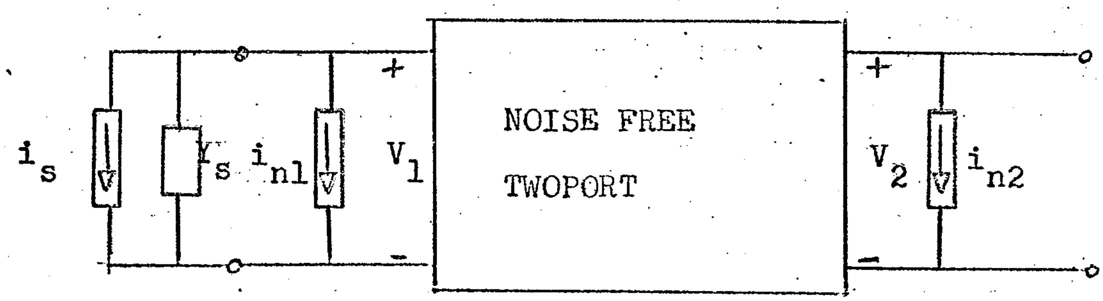



Figure 2.2 Representation of a noisy two-port connected to a signal source

The signal source with internal admittance is

$$Y_s = G_s + jB_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.3$$

connected to the input of the two port. The thermal noise of the signal generator is represented by the noise current generator $i_s$ of mean-squared value

$$\overline{i_s^2} = 4kT_oG_sB \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2.4$$

where $T_o$ is standard temperature (290°K). This noise generator is assumed to be uncorrelated with any noise generator of the noisy two-port.

Figure 2.2 can now be used for the determination of noise figure. The total noise power delivered to the output termination of a two port is proportional to the short-circuit mean-squared noise current of the output terminals. The noise power delivered to the output termination from any noise source in the network is proportional to the contribution made by that source to the output short-circuit current. The noise figure of the network in Figure 2.2, referring to Equation 1.17 then becomes[6]

$$F = 1 + \frac{1}{\overline{i_s^2}} \overline{\left| -i_{n1} + \frac{i_{n2}}{y_{21}} (Y_s + y_{11}) \right|^2} \quad \ldots \quad 2.5$$

where $y_{11}$ and $y_{21}$ are admittance parameters of the two port.

The noise current $i_{n1}$ can be separated into two components, $i_{n1}'$ and $i_{n1}''$, where the noise current $i_{n1}'$ is perfectly correlated with $i_{n2}$, and $i_{n1}''$ is completely uncorrelated with $i_{n2}$. Therefore:

$$i_{n1} = i_{n1}' + i_{n1}'' \qquad \text{or}$$

$$\overline{i_{n1}^2} = \overline{i_{n1}'^2} + \overline{i_{n1}''^2} \quad \ldots\ldots\ldots\ldots\ldots \quad 2.6$$

A correlation admittance is defined, where

$$y_{cor} = g_{cor} + jb_{cor} = y_{21}i_{n1}'/i_{n2} \quad .. \quad 2.7$$

The correlated current generator $i_{n1}''$ is expressed in terms of an equivalent noise conductance $g_n$ with the property

$$\overline{i_{n1}''^2} = 4kT_o g_n B \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.8$$

The output current generator $i_{n2}$, when transformed to the input terminals as an equivalent noise resistance $R_n$, has the value

$$\overline{i_{n2}^2} = 4kT_o R_n \left| y_{21} \right|^2 \quad \ldots\ldots\ldots\ldots \quad 2.9$$

From Equations 2.4 to 2.9, the noise figure of the network in Figure 2.2 can be re-written in the following form:

$$F = 1 + \frac{g_n}{G_s} + \frac{R_n}{G_s} \left| Y_s + y_{11} + y_{cor} \right|^2$$

$$= 1 + \frac{g_n}{G_s} + \frac{R_n}{G_s} \left[ (G_s + g_{11} + g_{cor})^2 + (B_s + b_{11} + b_{cor})^2 \right] \ldots\ldots\ldots\ldots \quad 2.10$$

The noise figure of the noisy two port can then be expressed as a function of four noise parameters: $g_n$, $R_n$, $g_{cor}$, and $b_{cor}$. In general, these quantities are a function of frequency and bias and do not depend on external circuitry.

JFET Small Signal High Frequency Model

Figure 2.3 shows an equivalent small signal model of an intrinsic JFET operated in the region beyond drain pinch-off[7,8].

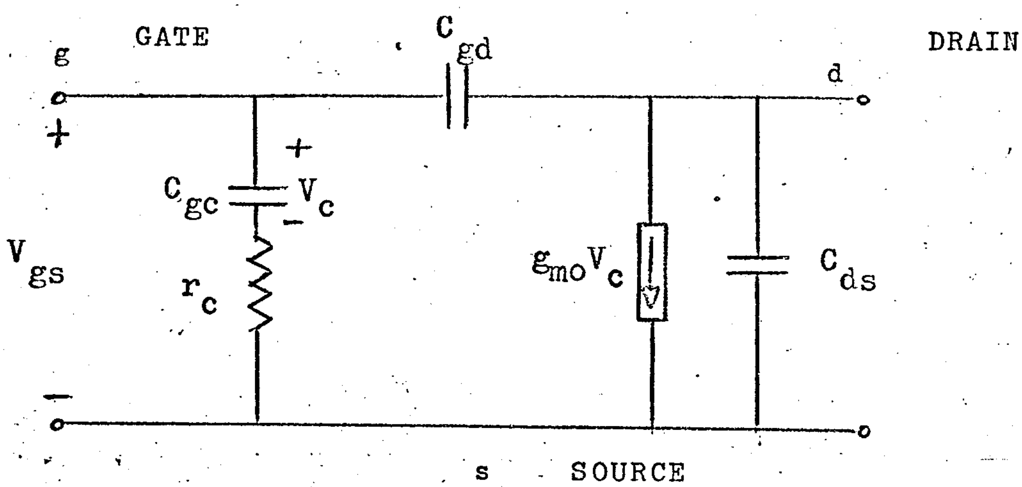



Figure 2.3 Small-signal model of intrinsic JFET

The series capacitor-resistor combination from gate to source is a lumped-element approximation of the distributed network formed between the gate and the active channel. $C_{gc}$ is the capacitance of the space charge diode of the reverse-biased gate-to-channel diode. The resistance $r_c$ is the effective resistance of the channel.

The intrinsic transconductance generator $g_{mo}V_c$ is controlled by the voltage across $C_{gc}$. This transconductance remains essentially constant with frequency; the high frequency performance being a function of the time constant $r_cC_{gc}$.

The capacitances $C_{gd}$ and $C_{ds}$ represent the intrinsic capacitances of the gate-drain and drain-source terminals, respectively.

Figure 2.4 shows a model of a physical UHF-JFET operating in the pinch-off region[9]. This model is useful well into the UHF region, until the lumped-element approximations are no longer valid and the lead inductance must be taken into account.

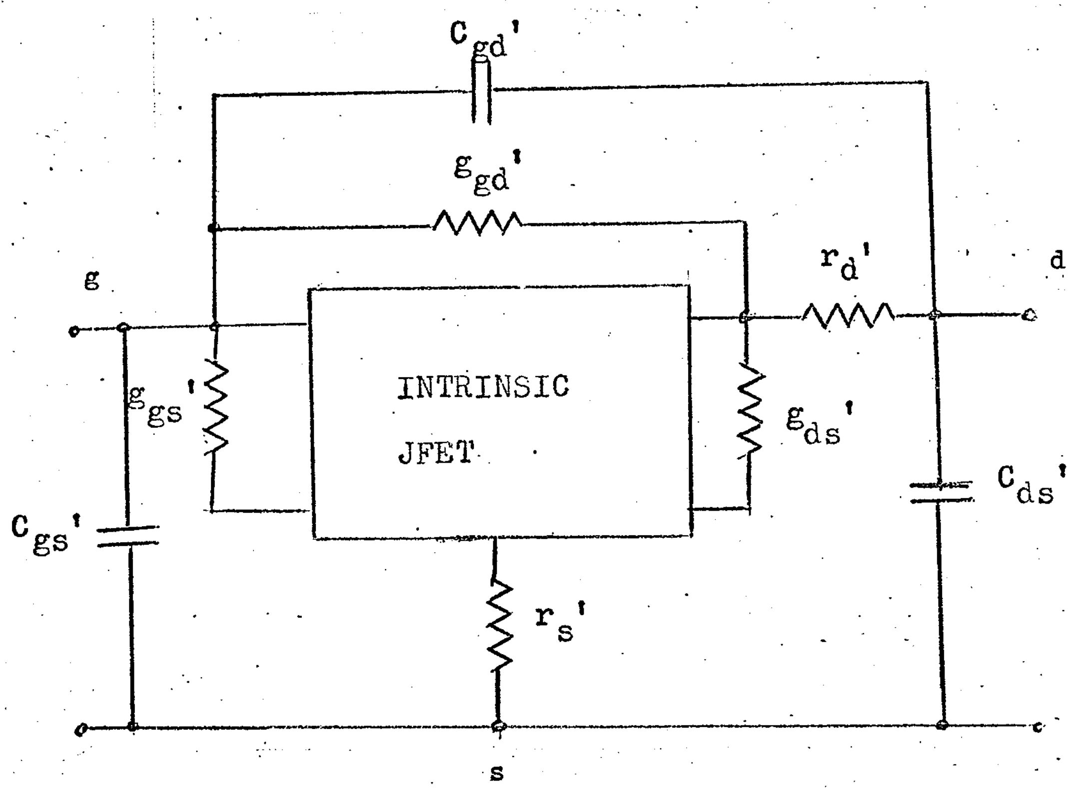



Figure 2.4 Practical JFET small-signal equivalent circuit for UHF

The conductance $g_{ds}'$, which is very small for a good device, represents the dynamic output conductance of the JFET.

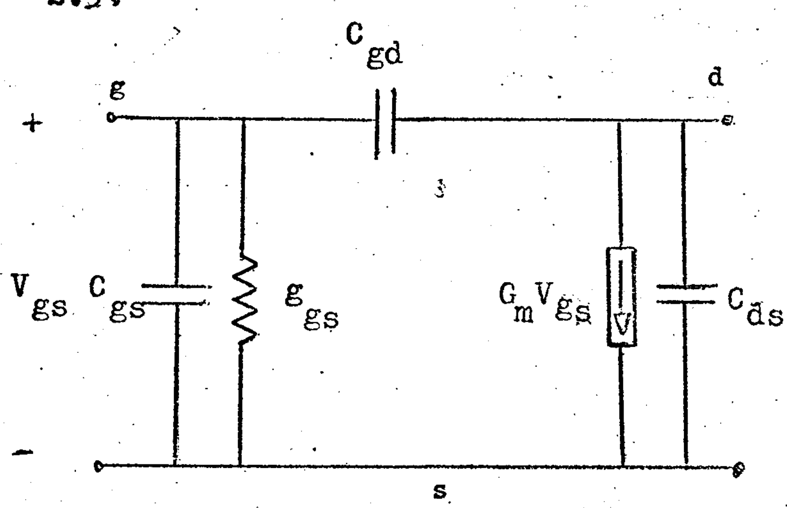
The conductances $g_{gs}'$ and $g_{gd}'$ represent the reverse-biased diodes between the gate and the drain, and also between the gate and the source.

Since the gate contacts of the JFET do not cover the entire channel, the series resistances $r_s'$ and $r_d'$ at the channel ends must be taken into account. The capacitances $C_{gd}'$, $C_{gs}'$, and $C_{ds}'$ include the effects of the case and inter-lead capacitances between the terminals of the device.

<u>JFET Noise Model at UHF</u>

At frequencies above the audio spectrum, thermal noise is the dominant noise in the JFET. Shot noise appears as a second-order effect due to a reverse leakage current in the gate-channel diode. Since this current is very small, the shot noise is usually negligible.

The equivalent circuit for the intrinsic JFET operating in the pinch-off region, and shown in Figure 2.3, can be transformed into the representation shown in Figure 2.5.




$$g_{gs} = \frac{r_c \omega^2 C_{gc}}{1 + (\omega/\omega_o)^2}$$

$$C_{gs} = C_{gc} \frac{\omega}{1 + (\omega/\omega_o)^2}$$

$$G_m = g_{mo} / (1 + j\frac{\omega}{\omega_o})$$

$$\omega_o = 1 / r_c C_{gc}$$

Figure 2.5 A small-signal model of the intrinsic JFET

The noise model of the intrinsic JFET contains two noise sources, both of thermal origin. The dominant one is the source representing the noise of the channel conductance. This noise is represented by a current generator $i_d$, paralleling the drain-source terminals,[10] where

$$\overline{i_d^{\,2}} = 4kT_o a g_{mo} B \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.11$$

The constant (a) is a function of the biasing parameters of the device. In the pinch-off region, $.6 < a < .7$ . The transconductance $g_{mo}$ is the low frequency value. This channel noise generator is independent of frequency.

At high frequencies, the input conductance, $g_{gs}$, must be taken into account in the noise model. The input conductance exhibts almost full thermal noise and its effect may be represented by a current generator

$$\overline{i_g^{\,2}} = 4kT_o g_{gs} B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2.12$$

paralleling the gate-source terminals[11]. In fact, the input conductance gives rise to exactly full thermal noise if the gate-source junction is biased at 0 volts.[8]

These two noise generators, $i_g$ and $i_d$, can now be added to the small signal model resulting in a noise model as shown in Figure 2.6.

The resistance $r_c$, from the equivalent model in Figure 2.3, is physically part of the channel. Thus the noise sources, $i_d$ and $i_g$, can be expected to be partially correlated since noise fluctuations in the channel are coupled both to the gate and the drain. The result of this correlation is an ultimate decrease in the noise figure of the device. For the JFET, correlation decreases

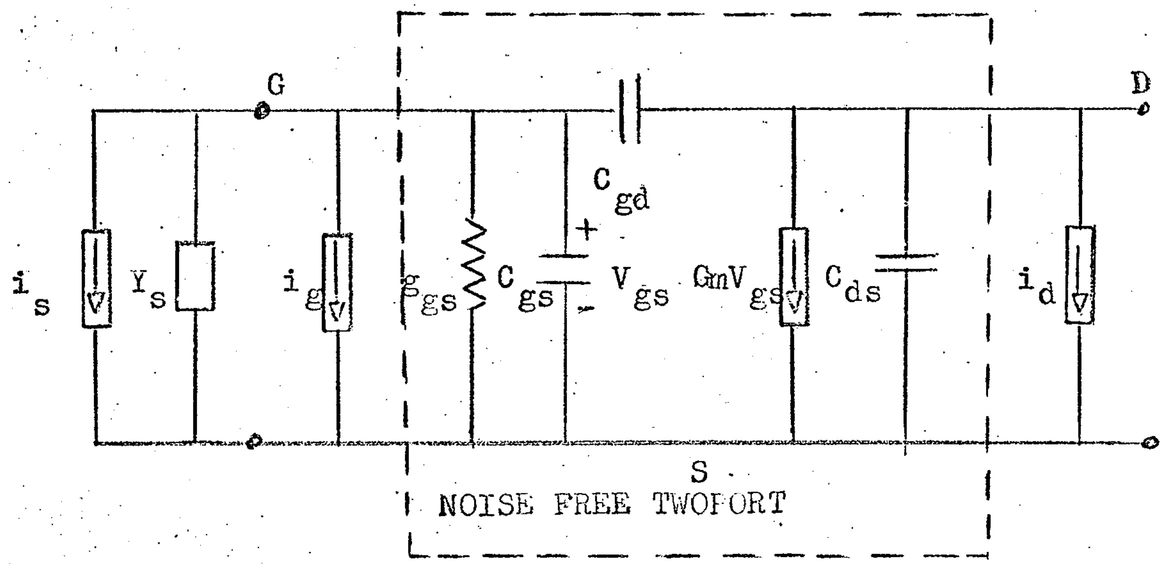



Figure 2.6 Intrinsic noise model of JFET in common-source configuration

noise figure by a factor of less than twenty percent[12] for all frequencies when the device behaves as the amplifier. Thus, it is a reasonable approximation to neglect the effect of correlation.

If correlation is neglected, a very simple expression results for the noise figure of the JFET. From Equations 2.10 to 2.12,

$$F = 1 + \frac{g_{gs}}{g_s} + \frac{R_{ns}}{g_s} \left| Y_s + y_{11s} \right|^2 \quad \ldots\ldots\ldots \quad 2.13$$

where $$R_{ns} = ag_{mo} / \left| y_{21s} \right|^2 \quad \ldots\ldots\ldots \quad 2.14$$

and $y_{11s}$ and $y_{21s}$ are admittance parameters of the common-source configuration.

In addition to the noise sources in the intrinsic JFET, thermal noise sources exist due to the parasitic resistances and conductances of the physical JFET. Lossy parasitic elements tend to increase the noise figure of a practical device. Parasitic susceptances, because they are loss-free, will not change the minimum obtainable noise figure at any one frequency.

Usually, parasitic elements have little effect on noise figure.[10] Also the effects of correlation and lossy parasitic elements tend to cancel the effect of one another. Correlation decreases noise figure and the parasitics increase it. Therefore Equation 2.13 can be expected to give reasonably accurate results for noise figure over a wide range of frequencies for the physical JFET in the common-source mode.

For JFET use at UHF, common-gate operation is preferred for presently available devices.[13] Common-gate operation allows reasonable power gains to be obtained in amplifier circuits. This configuration is unconditionally stable at all frequencies.

From the model of Figure 2.7, the following noise figure expression can be deduced[6]

$$F = 1 + \frac{1}{i_s^2} \overline{\left| -i_g + \frac{i_d}{y_{21g}} (Y_s + y_{11g} + y_{21g}) \right|^2} \quad .. \quad 2.15$$

where $y_{11g}$, $y_{21g}$ are common-gate admittance parameters.

Neglecting correlation and parasitics, the noise figure for the JFET becomes

$$F \cong 1 + \frac{g_{gs}}{G_s} + \frac{R_{ng}}{G_s} \left| Y_s + y_{11g} + y_{21g} \right|^2 \quad ...... \quad 2.16$$

where $R_{ng} = a g_{mo} / \left| y_{21g} \right|^2$ ........................ 2.17

referring to the arguments for the common-source mode.

It should be noted that

$$y_{11g} + y_{21g} = y_{11s} + y_{12s} = g_{gs} + j\ C_{gs} \quad ... \quad 2.18$$

Comparing Equation 2.16 with Equation 2.13, it follows that there is little difference in noise figures of these two configurations (common-gate and common-source).

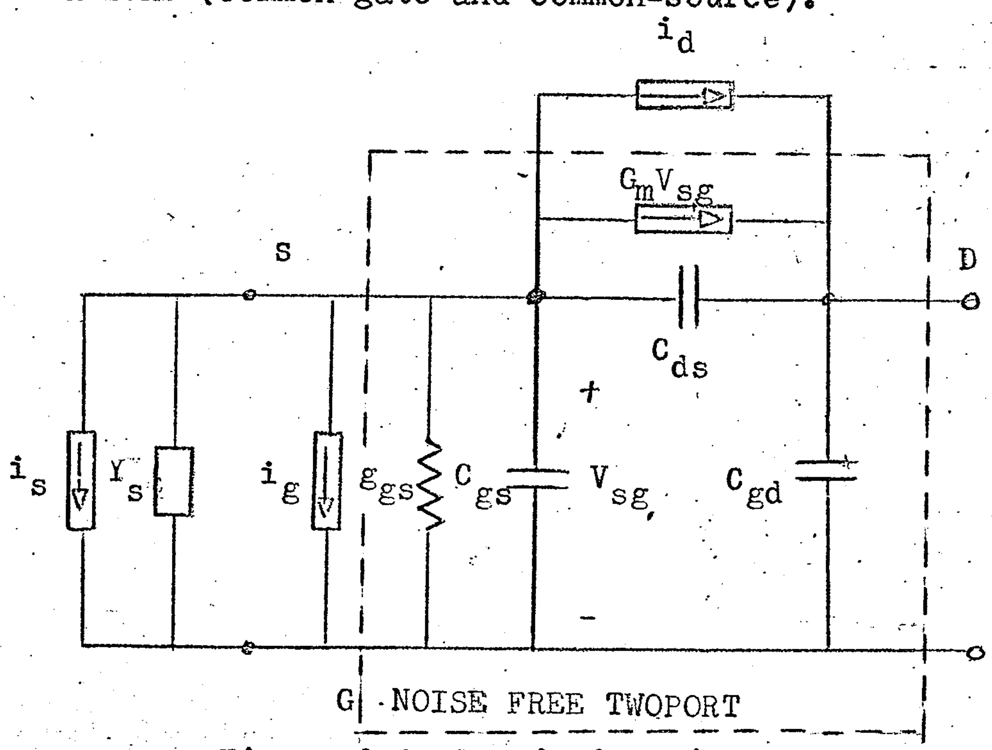



Figure 2.7 Intrinsic noise model of JFET in common-gate configuration (derived from Figure 2.6)