FAULT ISOLATION IN LINEAR ELECTRONIC CIRCUITS
BY UTILIZING THE FREQUENCY RESPONSE AND
THE MOEBIUS TRANSFORMATION

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ABSTRACT

FAULT ISOLATION IN LINEAR ELECTRONIC CIRCUITS BY UTILIZING THE FREQUENCY RESPONSE AND THE MOEBIUS TRANSFORMATION

A method of isolating (diagnosing) a single fault in a conventional linear electronic circuit, where only the input and output terminals are available for test measurements, is described.

A faulty circuit element is identified by comparing the measured magnitude and phase response of the faulty circuit, at a selected set of test frequencies, with a precomputed set of curves corresponding to all possible single fault conditions. These curves are shown to be arcs of circles, where the straight line is considered to be the special case of a circle with an infinite radius. Although the state equation approach is used in calculating the n-port parameters required for the precomputed curves, other methods such as topological formulas, ECAP(Electronic Circuit Analysis Program, supplied by IBM), or laboratory measurements could also be utilized.

The success of the Moebius transformation method, in identifying faults, was demonstrated by laboratory measurements and digital computer simulations. The results of fault diagnosis in several test circuits are presented.

This new approach to fault diagnosis provides a visual (graphical) identication of the faulty component and a precise evaluation of the faulty component value.

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INTRODUCTION

Motivation for the development of new fault isolation techniques

Prior to the introduction of a wide variety of complex electronic equipment, a technician could acquire adequate experience with the circuitry of the equipment in order to repair it effectively. However, with todays rapid change in circuitry, it is difficult for technicians to gain this experience and other methods must be introduced as an aid to the technician.

Another difficulty experienced in the fault diagnosis of a circuit is due to high packaging density of modern equipment. The procedure of probing the signal path has become difficult even in the discrete component circuitry. In an integrated circuit, however, once the device has failed, it is impossible to probe the signal path unless test points have been provided. Test points are generally undesirable since they cause a deterioration in the high frequency performance and, consequently, are usually not provided. Although an integrated circuit cannot be repaired, the manufacturer may want to identify the fault in order to modify the manufacturing process.

Statement of the problem and inherent assumptions

The problem that will be considered is the one of identifying a single fault in a linear, lumped, time-invariant network with a known topology and a set of known nominal component values. Identification of a fault requires that the faulty element and the value of the element be determined. If test points are required for unique fault identification, only a minimum number will be permitted; if possible, only the normal input and output terminals of the network will be used for test purposes.

The solution of the problem is based on the following assumptions:

(i) The fault is not intermittant, or the fault persists throughout the test measurement period.

(ii) Only those catastrophic failures, having a non-zero response at some terminal, can be identified uniquely, except in the case where a zero response at all terminals is unique to one fault.

(iii) Mutual inductance and the transformer turns ratio cannot be isolated as faults, since they violate the single element fault assumption. However, circuits, with mutual inductance and/or transformers with turns ratios, can be diagnosed if these parameters are excluded as possible faults.

If the value of the faulty parameter is not required, assumption (i) above can be relaxed and restated as:

(iv) The fault persists long enough to permit one significant test measurement.

A violation of assumption (ii), above, will be observable during the initial analysis of the circuit, and test points could be inserted into the circuit to satisfy the assumption.

Review of previous work

Seshu and Waxman[6] have given a solution to the problem by recognizing that the transfer function magnitude will change at some frequency if a circuit element changes. In terms of the Bode plot, this will be observable by a change in the location of the break frequencies, or the gain constant, or both. Thus by precomputing the gain between the break frequencies for all possible fault conditions, a fault dictionary of gain signatures is obtained. The magnitude response of the faulty circuit is then measured and compared to the list in the fault dictionary, and the faulty element is identified.

Since the number of transfer functions that must be evaluated for a reasonably complete fault dictionary is large, the symbolic transfer function is derived by topological formulas, and then, the fault signatures are calculated simply by substituting the appropriate values. This technique is necessary to shorten the computation time of the digital computer.

The method of Seshu and Waxman was investigated by Maenpaa, Stehman, and Stahl [2] and apparently proved to be quite successful. However, some of the problems encountered were: (i) the computation time and storage space required in the digital computer becomes quite large as the network size increases; (ii) no method is provided for interpolating between fault values, if a measured fault happens to fall between values listed in the dictionary.

Neu [4] has developed a method whereby a quantity $k(x_i)$ remains invariant from one n-port parameter to another if x_i is the circuit element that has changed. If x_i is another circuit element (i.e. one which could not produce the observed changes in the n-port parameters), $k(x_i)$ will be different for different n-port parameters. The quantity $k(x_i)$ is defined as:

$$k(x_i) = \frac{\frac{\partial T(x_i)}{\partial x_i}}{\Delta T}$$

where $k(x_i)$ = the "constant" of Neu's method

x_i = circuit element i

T = the n-port parameter

ΔT = the amount T has changed from its nominal value

To isolate a fault, $\frac{\partial T(x_i)}{\partial x_i}$ must be computed for all circuit elements and for two or more n-port parameters. The change in the corresponding n-port parameters due to the fault is then measured and $k(x_i)$ is computed. That circuit element x_i, for which all the $k(x_i)$ are constant, is then the faulty component.

The amount Δx_i, that the faulty component x_i has changed from its nominal value, can then be calculated from the equation:

$$\Delta x_i = \frac{Q+x_i}{k(Q+x_i)-1}$$

where Q is either the Thevenin immittance or transmittance at x_i. Neu only discusses resistive networks, although the procedure could probably be extended to include capacitors and inductors by using phasor analysis. The evaluation and measurement of several n-port parameters is still required, however, and a technique based on a single n-port parameter would probably be preferable.

Moroz[3] has shown that the coefficient of each power of s, the Laplace transform variable, of a transfer or driving point function can be written as:

$$k = \frac{ax_i + b}{cx_i + d} \tag{1.1}$$

where: k is any power of s of the transfer function (or driving point function),

x_i is any network parameter other than mutual inductance or a transformer turns ratio, and

a, b, c, d are real constants independent of x_i.

The constants a, b, c, and d must be computed for each power of s, for each circuit component. If the symbolic transfer function is used, the constants can be obtained directly by substitution. If a numerical circuit analysis technique is used, a, b, c, and d can be calculated by solving equation (1.1) for four values of k corresponding to four values of x_i. Three solutions are adequate in obtaining the ratio of three of the constants to the fourth one. The array of all coefficients a, b, c, and d is known as the coefficient signature.

The coefficients of the transfer function of the faulty network must be computed from test measurements and a curve fitting procedure. The measured coefficients are then used in solving equation (1.1) for x_i. This produces as many solutions for each x_i as there are independent coefficients in the transfer function polynominal ratio. If x_i is the faulty component, which caused the nominal transfer function to change to the new value, the solutions for x_i will all be constant and equal to the new value of x_i. If x_i is not the faulty component, the solutions for x_i will vary. As was suggested by Moroz, the solutions of the faulty x_i are not exactly constant, due to errors in the measurement procedure and in the curve fitting method. The x_i, which exhibits the least variance in the solution, is then chosen as the faulty component.

Significance of the new technique

One of the problems associated with the fault dictionary isolation technique is that of interpolating between the listed entries(gain signatures) in the dictionary when a fault falls between those listed. Since each element could fail with any value between zero and infinity, it is impossible to list all possible faults, and interpolation between the discrete steps of listed faults will often be required. The new technique, which is developed in the next section, permits a direct calculation of the value of the faulty component; the gain signature need not be evaluated and, therefore, the interpolation problem associated with the fault signature method is avoided.

It is shown that the use of the Moebius transformation method substantially reduces the number of n-port parameters that must be evaluated. Consequently, it becomes practical to use numerical evaluation of the n-port parameters by the state equation technique, rather than by the topological approach, without an increase in computer computation time. Since the state equation method requires much less storage than the topological formula method, the new technique also eliminates the large storage problem of the fault dictionary diagnosis.

If the network which is being investigated has two or more ports, more than one n-port parameter describing its behaviour exists. A two-port, for example, could be described by:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where, in general, the hybrid parameters: h_{11}, h_{12}, h_{21}, h_{22}, all differ. Seshu and Waxman[6] use only one of the h-parameters, in the above case, to determine a faulty element. However, in some cases, the same frequency signature exists for several different faults. Although a fault dictionary for another of the h-parameters might identify the fault uniquely, the computation time, required to compute the h-parameter and compile the additional fault dictionary, would increase significantly. The state equation technique of circuit analysis, which is impractical to use for compiling a fault dictionary, generates all n-port parameters simultaneously. Consequently, the new technique of fault diagnosis, which is suitable for state equation circuit analysis, can be used to investigate all of the h-parameters with only a small amount of additional computation.

An outline of the diagnosis technique

It is shown that the locus of an n-port parameter of a linear electronic network, as an element varies from zero to infinity, is an arc of a circle. This holds for all components except mutual inductances and transformer turns ratios. Since a circle is uniquely determined by any three points on the circle, we must evaluate or measure the n-port parameter for three values of each circuit element at each test frequency. Although any method may be used to evaluate the n-port parameter (e.g. ECAP, topological formula, state equations or laboratory measurements), the state equation approach was chosen as the most suitable , for reasons discussed in part IV. After the n-port parameters have been evaluated for three values of each component, the coefficients of the Moebius transformation are used in calculating the loci of the n-port parameters. The loci of the n-port parameters, at the test frequencies, are then plotted on graphs with one test frequency plot per graph.

The magnitude and phase of the n-port parameter of the faulty circuit are measured at the test frequencies and plotted on the $T(x_i)$ graphs. The plots are examined and the component x_i, which corresponds to the n-port parameter arc on which the measured n-port parameter of the faulty circuit falls, is the faulty component. After the faulty element has thus been detected, the coefficients of the Moebius transformation are used to calculate the value of the faulty component.

II

APPLICATION OF THE MOEBIUS TRANSFORMATION TO N-PORT PARAMETERS

Some of the properties of the well known Moebius transformation[1,8] are developed in the following section. The development, which is not essential for an understanding of the fault diagnosis technique, is included for completeness and ease of reference. The results of the development are used in the fault isolation computer program which is included in Appendix A.

The Moebius transformation

The Moebius transformation, which is also known by the names, bilinear transformation, fractional linear transformation and linear substitution, is a transformation of the form:

$$w = \frac{Az+B}{Cz+D} \qquad AD-BC \neq 0 \tag{2.1}$$

where A,B,C,D are complex numbers and z is a complex variable. The condition $AD-BC \neq 0$ is necessary, as otherwise w will reduce to a constant. In general, the transformation depends on three essential constants, namely, the ratio of any three of the constants A, B, C, D to the fourth one. If we choose three w_i corresponding to three z_i, then

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} \tag{2.2}$$

The R.H.S. fraction is known as the cross ratio of four numbers z, z_1, z_2, z_3 and the L.H.S. is the cross ratio of w, w_1, w_2, w_3. Thus, if we know w and three w_i corresponding to three z_i we can solve for z. To do this we note that there is a one to one correspondence between equation (2.2) and the following equation:

$$Sw = Tz$$

where $S = \begin{bmatrix} w_2-w_3 & -w_1(w_2-w_3) \\ w_2-w_1 & -w_3(w_2-w_1) \end{bmatrix}$

and $T = \begin{bmatrix} z_2-z_3 & -z_1(z_2-z_3) \\ z_2-z_1 & -z_3(z_2-z_1) \end{bmatrix}$

then $z = T^{-1}Sw$

where $T^{-1} = \begin{bmatrix} -z_3(z_2-z_1) & z_1(z_2-z_3) \\ -(z_2-z_1) & (z_2-z_3) \end{bmatrix}$

Then by the one to one correspondence:

$$z = \frac{aw + b}{cw + d} \tag{2.3}$$

where $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = T^{-1}S$

Solving (2.1) for z and equating to (2.3) we get:

$$z = \frac{Dw - B}{-Cw + A} = \frac{aw + b}{cw + d}$$

If z is real, $z = \bar{z}$, where $\bar{z}$ is the complex conjugate of z;

that is $\frac{aw + b}{cw + d} = \overline{\left(\frac{aw + b}{cw + d}\right)}$

Cross multiplying and simplifying we get:

$$(a\bar{c}-c\bar{a})|w|^2 + (a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + (b\bar{d}-d\bar{b}) = 0 \tag{2.4}$$

Multiplying the numerator and denominator of the last term by $(\bar{a}c-\bar{c}a)$:

$$(a\bar{c}-c\bar{a})|w|^2 + (a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + \frac{\bar{a}c(b\bar{d}-d\bar{b}) - \bar{c}a(b\bar{d}-d\bar{b})}{(\bar{a}c-\bar{c}a)} = 0$$

Adding and subtracting $b\bar{b}c\bar{c}$ and $\bar{a}a\bar{d}d$ to the numerator of the last term and regrouping terms we get:

$$(a\bar{c}-c\bar{a})|w|^2 + (a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + \frac{(\bar{a}d-\bar{c}b)(a\bar{d}-c\bar{b})}{(\bar{a}c-\bar{c}a)} = \frac{(ad-bc)\overline{(ad-bc)}}{(\bar{a}c-\bar{c}a)}$$

Dividing by $a\bar{c}-c\bar{a}$, provided $a\bar{c}-c\bar{a} \neq 0$:

$$|w|^2 + \frac{(a\bar{d}-c\bar{b})w}{(a\bar{c}-c\bar{a})} + \frac{(\bar{a}d-\bar{c}b)\bar{w}}{(\bar{a}c-\bar{c}a)} + \frac{(\bar{a}d-\bar{c}b)(a\bar{d}-c\bar{b})}{(\bar{a}c-\bar{c}a)(a\bar{c}-c\bar{a})} = \frac{(ad-bc)\overline{(ad-bc)}}{(\bar{a}c-\bar{c}a)(a\bar{c}-c\bar{a})}$$

Therefore:

$$\left(w + \frac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}\right)\left(\bar{w} + \frac{(a\bar{d}-c\bar{b})}{(a\bar{c}-c\bar{a})}\right) = \frac{(ad-bc)\overline{(ad-bc)}}{(\bar{a}c-\bar{c}a)(a\bar{c}-c\bar{a})}$$

But since $(w+m)(\bar{w}+\bar{m}) = (w+m)\overline{(w+m)} = |w+m|^2$

we have:

$$\left|w + \frac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}\right|^2 = \left|\frac{ad-bc}{\bar{a}c-\bar{c}a}\right|^2$$

After taking the square root of both sides we note that we have the equation of a circle:

$$\left|w + \frac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}\right| = \left|\frac{ad-bc}{\bar{a}c-\bar{c}a}\right| \qquad (2.5)$$

with centre: $\dfrac{(\bar{a}d-\bar{c}b)}{(\bar{a}c-\bar{c}a)}$

and radius: $\dfrac{(ad-bc)}{(\bar{a}c-\bar{c}a)}$

However, if $\bar{a}c-\bar{c}a = 0$, (2.6)

equation (2.4) becomes:

$$(a\bar{d}-c\bar{b})w + (b\bar{c}-d\bar{a})\bar{w} + (b\bar{d}-d\bar{b}) = 0, \qquad (2.7)$$

which is a straight line in w.

The initial assumption that $ad-bc \neq 0$ implies:

(i) $\overline{ad} \neq \overline{bc}$

(ii) a and c are not both zero

(iii) $\bar{a}$ and $\bar{c}$ are not both zero

(iv) d and b are not both zero

(v) $\bar{d}$ and $\bar{b}$ are not both zero