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ULTIMATE STRENGTH ANALYSIS OF REINFORCED CONCRETE GRILLAGES

by

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ABSTRACT

This dissertation deals with the assessment of the ultimate strength of conventionally reinforced concrete grillages, as occur in buildings and bridges, under concentrated loads and with particular emphasis on plastic hinge theory for grillages.

Two identical, ¼-scale models, were built in the laboratory for two tests. The first test involved applying a concentrated load at the center of the grillage. This test was performed in two sections. The first part proceeded until yielding occurred, and the second part proceeded to destruction. The second test involved applying a pair of concentrated loads to the interior longitudinal of the second grillage. This test continued until the mode of loading changed.

In the first test the collapse load was 1.30 times the load predicted by the method of combined ultimate design and plastic hinge theory. In the second test the collapse load was 1.23 times the predicted load. Increased concrete and steel strengths partially explained the increase in the collapse loads. Membrane and arching action in the interior grillage beams was also used to explain additional increase in the collapse loads. The concept of a hanging network of steel reinforcement was used to explain the large deflections. The behaviour of the two grillages under static load to destruction was also compared and discussed.

The test results from this study indicate that the combined ultimate strength and plastic hinge theory for reinforced concrete grillages is valid and safe, within certain limitations.

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NOTATION

A_{cage}	cross-sectional area enclosed by reinforcing cage
A_{sc}	area of compression steel in beam
A_{st}	area of tensile steel in beam
b	width of beam
b'	width of reinforcing cage
C	circumference or perimeter of reinforcing cage
d	overall depth of beam
d'	depth of reinforcing cage
d_n	depth of neutral axis of beam
$d_1, d_2 \ldots d_3$	depth to steel in levels 1,2....3
f'_c	crushing strength of 6 x 12-in. concrete cylinders
f'_{pr}	crushing strength of concrete prism
f_y, σ_y	yield stress in steel
f_{st}	stress in tensile steel
f_{yc}	yield stress in compression steel
f_{yt}	yield stress in tensile steel
F_y	yielding strength in one steel bar
K	coefficient
M	bending moment in a beam
M_u	ultimate moment of a beam
M_{IP}	ultimate moment of an interior beam
M_{EP}	ultimate moment of an exterior beam
M_T	torsional moment in a beam
M_{ET}	torsional moment in an exterior beam
$M_{T_{st}}$	total torsional moment developed by yielding steel (and its associated compressive concrete)
n	integer

NOTATION CONTINUED

n_x.........number of helices in reinforcing cage

p ....pitch of transverse reinforcement

P ....concentrated load

P_u ....ultimate concentrated load

R ....yielding strength per unit width, in torsional reinforcement

u ....crushing strength of 6 x 6-in. concrete cubes

V ....shear stress

δ ....deflection

θ ....rotation

α ....empirical coefficient

β ....angle of helices in torsional reinforcement; empirical coefficient

ϕ ....diameter of steel bar

ε_c ....ultimate compressive strain in concrete

E..........modulus of elasticity

CHAPTER I

INTRODUCTION AND REVIEW

1.1 Introduction.

With the constant introduction of new and better construction materials, engineering research is, to a greater extent, playing an important part in developing new analytical theories and building techniques. The goal of any research is to develop a logical analytical system which ensures that a structure will have sufficient strength to resist a system of loads imposed upon it, and to transmit these loads safely to its foundations.

In the past the method of ensuring the strength of a structure was by means of linear elastic analysis, which limited the maximum elastic stresses to a desired fraction of the stress at which elastic behaviour terminated. The maximum allowable stress for mild steel was set at a percentage of the yield stress, whereas for concrete, a percentage of the cylinder or cube crushing strength determined the "allowable working stress". Thus for several decades the Theory of Elasticity governed engineering analysis.

In conjunction with the Theory of Elasticity, laboratory investigation helped to formulate empirical methods of design and analysis. These methods often required simplifying assumptions in order to develop a practical design method and, because of this, they were rather conservative and uneconomical.

The elastic method of analysis proved most efficient for simple determinate structures where the elastic strength and the real strength were related linearly. However, this linear relationship was not valid for indeterminate structures unless they were made of an elastic, brittle material such as glass. In indeterminate structures constructed of mild and intermediate grades of steel, or where this type of steel is used as reinforcement in concrete, the "plastic" behaviour of this material permits the redistribution of stress at high loads. The ability of a ductile material such as mild steel, with its reserved strength, to redistribute load from high stress regions to less highly stressed areas was not readily recognized in practice, except in such classical examples as steel rivet groupings and fillets.

In the 1920's and 1930's it was recognized that elastic analysis of indeterminate structures produced excess strengths and investigations began to find the true ultimate strengths of such structural systems. However, it was not until after the Second World War, when building materials were in short supply, particularly in Europe, that the popularity of ultimate strength methods of analysis gathered momentum.

The use of Ultimate Strength Design for reinforced concrete has finally been recognized as more consistent than Elastic Design. Ultimate Strength Design recognizes the actual behaviour of a reinforced concrete member at ultimate capacity, and it is the only method that predicts the strength of a section. A specified overload factor is applied to each service load to obtain the desired ultimate load capacity. As the overload factor is used to design for the actual ultimate strength, a uniformly consistent, more accurate, safety factor will be obtained for all units of a structure. Elastic Design, or Working Stress Design, uses allowable stresses with the straight-line theory and the actual factor of safety is obscured and may vary widely with different types of members. With Ultimate Strength

Design it is permissible to apply a lower load factor to dead load than to live load, which is logical and desirable, as the dead loads to which a member is to be subjected are usually known with greater precision than live loads. Different load factors for live and dead load lead to a more realistic safety factor. Another advantage of Ultimate Strength Design is that it generally will result in more economical structures with safety uniform at all points. It allows full utilization of higher strength steels and concretes, whereas Working Stress Design underestimates the contribution of higher strength steels. Under Ultimate Strength Design, strength is of prime importance and serviceability and appearance are secondary.

Even though Ultimate Strength Design is slowly replacing Working Stress Design for simple designs, North American building codes still prescribe an elastic analysis for all indeterminate concrete structures. For buildings of the usual type of construction, spans, and story heights, codes permit the use of approximate methods of analysis for the determination of elastic moments and shears within certain ranges of variation in span lengths and loads. When the external moments and forces in a structure have been determined by the theory of elastic frames, the design of sections can proceed using Ultimate Strength Design.

Engineers are increasingly becoming aware that the behaviour of individual members in any composite structure is that of the structure as a whole, rather than that of separate units. The importance of considering the composite behaviour of the complete structure under load in both elastic and ultimate analyses is evident. It is easy to see that this type of analysis is rather difficult and it is only comparitively recently that composite action in structures has been studied in detail.

In understanding the strength of composite reinforced concrete structures, the study of flexural grillages increases our knowledge of the subject as it plays an important part in both buildings and bridges. In the past, however, in the analysis of buildings the skeletal framework has been isolated from the structure as a whole and then calculations have been made on the framework alone. In bridge work, where the main and transverse beams more clearly define a grillage, composite action between beams and slab has quite frequently been considered. However, where composite action has been used in analysis of bridge decks the usual method has been the equivalent T-beam system, which is in effect the grillage approach.

The strength analysis of a plastic grillage is governed by the principles of limit analysis(a). A correct solution is found when coincident upper and lower bound solutions have been obtained. The analysis of grillages by the plastic hinge method is dependent on the existence of sufficient plastic rotational capacity at each hinge to enable the complete mechanism to form without premature failure. Sufficient plastic rotational capacity exists in hinges of under-reinforced concrete beams. However, for beams having high flexural steel ratios or when considering torsion hinges, extreme caution must be exercised. Also it is not permissible to include the brittle strength of a member in such an analysis unless it can be shown that the "brittle hinge" is the last to form(b). In cases where brittle "hinges" are included in the strength analysis, failure will be catastrophic. Due to the high load factor recommended in cases of this nature(c), it may be preferable to assume a plastic hinge strength of zero at the brittle hinges and calculate the smaller but non-catastrophic plastic failure strength.

1.2 Review.

The amount of experimental work done on the composite action of reinforced concrete grillages has been extremely small. The earliest work done in this field was by Nylander(d). He performed two tests on reinforced concrete grillages in 1945. Each of the grillages was comprised of five beams and was supported at four points. The grillages were tested by loading the secondary beam at two central points. The results indicated that a failure load greater than that predicted by limit analysis was obtained and that torsional rotations in the grillages were from three to six times those observed in torsional control tests.

Work related to this study was also done by Heyman(e,f), who described a method for the limit design of transversely loaded square grids. He made the usual assumptions of plastic theory as applied to steel structures. However, the fundamental assumption that full moment redistribution occurs may lead to error if applied to concrete grillage structures where torsion failures are possible. A.L.L. Baker(g), in considering the design of reinforced and prestressed concrete frames, considered it necessary to calculate rotations in plastic hinges and proposed a method of doing this.

Another series of tests on concrete grillage bridges, known to the writer, was performed by Reynolds(b). In his research Reynolds used nine small scale prestressed bridges, comprising a preliminary structure, six right bridges, and two skew bridges. Good agreement was found between analytical results and experiment, with the ultimate load of each of the bridges tested slightly in excess of the estimated ultimate load. In Reynolds' analysis, however, the strength of brittle torsion "hinges" was included. These hinges were fortunately the last to form and the calculated load was realized. For all of the grillage bridges there was a decrease in strength after the maximum load was reached. This decrease was most severe in one of the skew grillages tested and was to be expected as a skew structure carries more load through torsion than does a right grillage. Reynolds' tests also serve to show that in order to develop a plastic failure, adequate shear strength is required.

More recently a series of concrete grid frame and beam tests, similar to those conducted in this study, were performed by Klus and Wang(h). Using computer analysis and model tests, it was indicated that moment distribution in grid frames could be predicted within ± 15 percent using the displacement method analysis and including the effects of torsion. If torsion was neglected in the analysis, it was found large errors could occur for certain loading conditions.

1.3 Object of the Tests.

The purpose of this study was to examine the design of right-angled concrete grillages using ultimate strength analysis in combination with a plastic hinge theory and to compare the theoretical with the actual test results. Certain contingencies were accounted for; the main beams were made rigid enough so that plastic hinges would form in the secondary interior beams, and also enough shear reinforcement was placed in all beams so as to ensure adequate shear strength. The behaviour of the structure was also to be observed.

It was hoped that the results of this study would aid in the understanding of reinforced concrete beam and slab composite structures. In conjunction with research done by Lansdown(i) and other studies initiated at the University of Manitoba(j,k), a valid method of predicting the ultimate strength of composite structures could, in time, be achieved.

The study itself included the building and testing of two identical ¼-scale model grillages. The results of the two tests to destruction, under different loading conditions, were then analyzed.

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