EQUILIBRIUM OF WIND LOADED INFLATABLE STRUCTURES

A THESIS
PRESENTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR A MASTERS DEGREE
IN ENGINEERING

PRESENTED TO THE
UNIVERSITY OF MANITOBA

BY

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SEPTEMBER, 1970

ABSTRACT

The shape of inflated fabric cylinders subject to wind loading was studied. A small deflection analysis and a large deflection analysis with a simplified pressure distribution were carried out. The results of a model study were compared with a direct numerical integration.

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INTRODUCTION

In this study, a number of aspects of inflated structures have been examined. The equilibrium equations have been derived in the natural or intrinsic coordinates of the membrane, in cartesian coordinates, and also in polar coordinates. A small deflection analysis has been carried out and a method of solution for a certain pressure distribution has been developed. For large deflections, an approximate shape determination has been made possible using a simplified pressure distribution. A numerical method was developed as an extension of this method and the results of this calculation have been compared favorably with the results from the model that was built to find the shape of the membrane under various conditions. The vibration or flutter of this model at low internal pressures in an airstream was investigated to find the conditions at which it occurred. A literature survey has been done on recent work in inflatable structures and included as an appendix.

CHAPTER I

BASIC EQUATIONS

In this chapter, the equilibrium equations for the membrane will be derived in several coordinate systems. The forms of the pressure distribution that will be used are given at the end of this chapter.

It was assumed that the inflated structure is a cylinder and that all loading is perpendicular to the axis of this cylinder so that the problem can be treated in two dimensions. The 'skin' of the structure has been considered as a weightless membrane which has no flexural stiffness. It will be shown that for the pressure assumptions used, the membrane may be treated as being inextensional. This leaves the problem in the form that will be considered. The forces on an element are the tensile membrane force, the normal pressure, and the tangential loading.

Intrinsic Coordinates

The natural or 'intrinsic' coordinates for the membrane are arc length 's', and the rotation of the tangent 'θ' (see fig. 1). If the relation between these two is specified, then the shape is defined.

It is convenient to consider equilibrium in two particular directions: parallel to the tangent at the midpoint of the element, and perpendicular to that tangent. In future, for brevity, these directions will be referred to as the tangential and normal directions respectively. All loads and forces are to be taken as being per unit length in the direction of the axis of the cylinder unless specified otherwise.

Consider the tangential direction. Equating the sum of the forces in the tangential direction to zero gives:

$$T \cos\frac{\delta\theta}{2} - (T + \delta T)\cos\frac{\delta\theta}{2} - F_s\,\delta s = 0 \qquad (1)$$

where: T is the tensile load in the membrane at the left hand end of the element,

$\delta\theta$ is the arc through which the tangent to the curve rotates in the length,

δT is the change in tensile load in the length,

F_s is the tangential load per unit area of the membrane,

and δs is the arc length of the element.

This can be rewritten as:

$$\delta T \cos\frac{\delta\theta}{2} + F_s\,\delta s = 0 \qquad (2)$$

If δs is allowed to become infinitesimally small, then $\delta\theta$ will approach zero and $\cos\frac{\delta\theta}{2}$ will approach one. This gives:

$$\frac{\delta T}{\delta s} = -F_s$$

In the limit, this becomes

$$\frac{dT}{ds} = -F_s \qquad (3)$$

In the normal direction, the equilibrium equation is:

$$T \sin\frac{\delta\theta}{2} + (T + \delta T)\sin\frac{\delta\theta}{2} - P\,\delta s = 0 \qquad (4)$$

where all symbols are as before and P is the normal pressure difference across the membrane. For small angles, $\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}$ and then:

$$T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} + \frac{\delta T \delta\theta}{2} - P\,\delta s = 0 \tag{5}$$

Ignoring the third term which is the product of two differentials and is thus very small leaves:

$$T\,\delta\theta - P\,\delta s = 0$$

or:

$$\frac{\delta\theta}{\delta s} = \frac{P}{T}$$

In the limit, as δs approaches zero, $\frac{\delta\theta}{\delta s}$ approaches $\frac{d\theta}{ds}$ to give:

$$\frac{d\theta}{ds} = \frac{P}{T} \tag{6}$$

The intrinsic coordinates give the simplest equilibrium equations. They are suitable for studying the effect of the variables but they are not the most convenient for the purposes of analysis because it is not possible to locate a point in space without performing an integration to obtain its 'x' and 'y' coordinates. The equations of equilibrium will also be derived for cartesian and polar coordinates since these are usually easier to work with.

Cartesian Coordinates

In cartesian coordinates, the equilibrium of the membrane element will be considered along the 'x' and 'y' coordinate axes (see fig. 2). The 'x' direction will be considered first. The equilibrium equation is:

$$0 = T\cos\theta - P\sin\left(\theta + \frac{\delta\theta}{2}\right)\delta s - (T + \delta T)\cos\left(\theta + \frac{\delta\theta}{2}\right) - F_s\,\delta s \cos\left(\theta + \frac{\delta\theta}{2}\right) \tag{7}$$

where $\tan\theta = \frac{dy}{dx}$ and other symbols have the same meaning as before.
Expanding and rearranging some terms gives:

$$T\cos\theta - T\cos\theta\cos\delta\theta + T\sin\theta\sin\delta\theta - \delta T\cos\theta\cos\delta\theta +$$

$$+\delta T\sin\theta\sin\delta\theta - P\sin\theta\cos\tfrac{\delta\theta}{2}\delta s - P\cos\theta\sin\tfrac{\delta\theta}{2}\delta s -$$

$$- F_s\,\delta s\cos\theta\cos\tfrac{\delta\theta}{2} + F_s\,\delta s\sin\theta\sin\tfrac{\delta\theta}{2} = 0 \qquad (8)$$

If δs is made small, then $\delta\theta$ becomes small and then $\sin\delta\theta$ approaches $\delta\theta$ and $\cos\delta\theta$ approaches one. Making these substitutions in (8) and ignoring as vanishingly small all products of two small quantities gives:

$$T\sin\theta\,\delta\theta - \delta T\cos\theta - P\sin\theta\,\delta s - F_s\,\delta s\cos\theta = 0 \qquad (9)$$

Dividing by $\delta s\cos\theta$ gives:

$$T\tan\theta\frac{\delta\theta}{\delta s} - \frac{\delta T}{\delta s} - P\tan\theta - F_s = 0$$

In the limit as δs approaches zero, $\frac{\delta\theta}{\delta s}$ becomes $\frac{d\theta}{ds}$ and $\frac{\delta T}{\delta s}$ becomes $\frac{dT}{ds}$ giving:

$$T\tan\theta\frac{d\theta}{ds} - P\tan\theta - F_s - \frac{dT}{ds} = 0 \qquad (10)$$

Now:

$$\frac{d\theta}{ds} = \frac{d\theta}{dx}\Big/\frac{ds}{dx} \quad ; \quad \frac{dT}{ds} = \frac{dT}{dx}\Big/\frac{ds}{dx} \quad ;$$

$$\tan\theta = \frac{dy}{dx} \quad ; \quad \frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \quad ; \qquad (11)$$

$$\text{and} \quad \frac{d\theta}{dx} = \frac{d}{dx}\left(\tan^{-1}\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}\Big/\left(1+\left(\frac{dy}{dx}\right)^2\right)$$

Substituting (11) into (10) gives:

$$T\frac{dy}{dx}\frac{\frac{d^2y}{dx^2}}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}} - \frac{\frac{dT}{dx}}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2}} - P\frac{dy}{dx} - F_s = 0 \qquad (12)$$

Equating forces in the 'y' direction to zero gives:

$$T \sin\theta + P\,\delta s \cos\left(\theta + \frac{\delta\theta}{2}\right) - (T + \delta T)\sin(\theta + \delta\theta) - F_s\,\delta s \sin\left(\theta + \frac{\delta\theta}{2}\right) = 0 \qquad (13)$$

Using the same method of simplification as was used for the equilibrium in the 'x' direction gives the equations:

$$P - \frac{dT}{ds}\tan\theta - T\frac{d\theta}{ds} - F_s \tan\theta = 0 \qquad (14)$$

and:

$$T\frac{\dfrac{d^2y}{dx^2}}{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{3/2}} \qquad \frac{\dfrac{dT}{dx}\,\dfrac{dy}{dx}}{\dfrac{ds}{dx}} \qquad - P + F_s\frac{dy}{dx} \doteq 0 \qquad (15)$$

which correspond to (10) and (12).

Equations (12) and (15) cannot easily be dealt with because of the powers of derivatives that occur. From (3) it can be seen that if $F_s = 0$, then $\frac{dT}{ds} = 0$, i.e. the tension is constant.

Substituting this condition into either (12) or (15) gives:

$$T\frac{d^2y}{dx^2} \bigg/ \left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2} - P = 0 \qquad (16)$$

which is the differential equation for a membrane if shear forces on it can be neglected and only the pressure need be considered. This equation is still not readily solved to find the complete solution y(x) which describes the shape of the structure. It turns out to be useful to have the equilibrium equation expressed in polar coordinates for the small deflection analysis.

Polar Coordinates

In polar coordinates the variables representing the coordinates are taken as R and ψ. Consider first the radial equilibrium of the element shown in fig. 3. The equilibrium equation is:

$$P\,\delta s\ \sin\left(\varphi + \frac{\delta\varphi}{2}\right) - F_s\,\delta s\ \cos\left(\varphi + \frac{\delta\varphi}{2}\right) + T\cos\left(\varphi + \frac{\delta\psi}{2}\right) -$$
$$(T + \delta T)\cos\left(\varphi + \delta\varphi - \frac{\delta\psi}{2}\right) = 0 \qquad (17)$$

where φ and ψ are the angles shown in fig. 3 and the other variables are as defined earlier. Expand (17) to get:

$$P\,\delta s\ \sin\varphi\cos\frac{\delta\varphi}{2} + P\,\delta s\ \cos\varphi\ \sin\frac{\delta\varphi}{2} - F_s\ \delta s\cos\varphi$$
$$\cos\frac{\delta\varphi}{2} + F_s\,\delta s\ \sin\varphi\ \sin\frac{\delta\varphi}{2} + T\cos\varphi\ \cos\frac{\delta\psi}{2} - T\sin\varphi$$
$$\sin\frac{\delta\psi}{2} - (T + \delta T)\cos\varphi\ \cos\delta\varphi\ \cos\frac{\delta\psi}{2} + \sin\varphi\ \sin\delta\varphi$$
$$\cos\frac{\delta\psi}{2}(T + \delta T) - (T + \delta T)\cos\varphi\ \sin\delta\varphi\ \sin\frac{\delta\psi}{2} -$$
$$-(T + \delta T)\sin\varphi\ \cos\delta\varphi\ \sin\frac{\delta\psi}{2} = 0 \qquad (18)$$

If δs is small, then the angles $\delta\varphi$ and $\delta\psi$ will be small and the substitutions: $\sin\delta\varphi = \delta\varphi$; $\sin\delta\psi = \delta\psi$; $\cos\delta\varphi = 1$; and $\cos\delta\psi = 1$ are valid. Ignoring all terms with more than one small quantity and collecting similar terms leaves:

$$P\ \delta s\ \sin\varphi - F_s\ \delta s\ \cos\varphi - \delta T\ \cos\varphi + T\sin\varphi$$
$$(\delta\varphi - \delta\psi) = 0$$

Divide by $\delta\psi\cos\varphi$.

$$P\tan\varphi\ \frac{\delta s}{\delta\psi} - F_s\ \frac{\delta s}{\delta\psi} - \frac{\delta T}{\delta\psi} + T\tan\varphi\left(\frac{\delta\varphi}{\delta\psi} - 1\right) = 0 \qquad (19)$$

As $\delta\psi$ approaches zero, then the ratios: $\frac{\delta s}{\delta\psi}$; $\frac{\delta T}{\delta\psi}$; and $\frac{\delta\varphi}{\delta\psi}$ approach the derivatives: $\frac{ds}{d\psi}$; $\frac{dT}{d\psi}$; $\frac{d\varphi}{d\psi}$ respectively.

Now:

$$\tan\varphi = -\frac{r}{\frac{dr}{d\psi}} \qquad \text{so} \qquad \varphi = \tan^{-1}\left(\frac{-r}{\frac{dr}{d\psi}}\right)$$

$$\text{and} \qquad \frac{d\varphi}{d\psi} = \left(r\,\frac{d^2r}{d\psi^2} - \frac{dr}{d\psi}\right) \Big/ \left(r^2 + \left(\frac{dr}{d\psi}\right)^2\right) \qquad (20)$$

$$\text{also:} \quad \frac{ds}{d\psi} = \sqrt{r^2 + \left(\frac{dr}{d\psi}\right)^2}$$

Substituting (20) into (19) after taking the limit gives:

$$\frac{-Pr}{\frac{dr}{d\psi}}\left(r^2+\left(\frac{dr}{d\psi}\right)^2\right)^{1/2}-F_s\left(r^2+\left(\frac{dr}{d\psi}\right)^2\right)^{1/2}\frac{dT}{d\psi}+$$

$$+T\left(\frac{-r}{\frac{dr}{d\psi}}\right)\left(\frac{r\frac{d^2r}{d\psi^2}-\left(\frac{dr}{d\psi}\right)^2 r^2-\left(\frac{dr}{d\psi}\right)^2}{r^2+\left(\frac{dr}{d\psi}\right)^2}\right)=0 \qquad (21)$$

or: $$P\ r\left(r^2+\left(\frac{dr}{d\psi}\right)^2\right)^{3/2}+F_s\frac{dr}{d\psi}\left[r^2+\left(\frac{dr}{d\psi}\right)^2\right]+$$

$$\frac{dT}{d\psi}\frac{dr}{d\psi}\left[r^2+\left(\frac{dr}{d\psi}\right)^2\right]+T\ r\left(\frac{d^2r}{d\psi^2}-2\frac{dr}{d\psi}-r^2\right)=0 \qquad (22)$$

In the tangential direction, the equilibrium condition is:

$$T\sin\left(\varphi+\frac{\delta\psi}{2}\right)-(T+\delta T)\sin\left(\varphi+\delta\varphi-\frac{\delta\psi}{2}\right)-F_s\,\delta s\sin\left(\varphi+\frac{\delta\varphi}{2}\right)+$$

$$+P\,\delta s\cos\left(\varphi+\frac{\delta\varphi}{2}\right)=0 \qquad (23)$$

Expanding (23) gives:

$$T\sin\varphi\cos\frac{\delta\psi}{2}\ \ T\cos\varphi\sin\frac{\delta\psi}{2}-(T+\delta T)\sin\varphi\sin\delta\varphi\sin\frac{\delta\psi}{2}-$$
$$-(T+\delta T)\sin\varphi\cos\delta\varphi\cos\frac{\delta\psi}{2}-(T+\delta T)\cos\varphi\sin\delta\varphi$$
$$\cos\frac{\delta\psi}{2}+(T+\delta T)\cos\varphi\cos\delta\varphi\sin\frac{\delta\psi}{2}-F_s\,\delta s\sin\varphi\cos\frac{\delta\varphi}{2}-$$
$$-F_s\,\delta s\cos\varphi\sin\frac{\delta\varphi}{2}+P\,\delta s\cos\varphi\cos\delta\varphi-P\,\delta s\sin\varphi$$
$$\sin\delta\varphi=0 \qquad (24)$$

If $\delta\psi$ is small, then δs and $\delta\varphi$ will be small and in the limit: $\sin\delta\psi=\delta\psi$; $\sin\delta\varphi=\delta\varphi$; $\cos\delta\varphi=\cos\delta\psi\simeq 1$. If these are substituted and second order terms are neglected, then the equation:

$$T\cos\varphi\,\delta\psi-\delta T\sin\varphi-T\cos\varphi\,\delta\varphi-F_s\,\delta s\sin\varphi-P\,\delta s\cos\varphi\simeq 0$$

results. Dividing by $-\delta\psi\cos\varphi$ gives:

$$T\left(\frac{\delta\varphi}{\delta\psi}-1\right)+\frac{\delta T}{\delta\psi}\tan\varphi+F_s\frac{\delta s}{\delta\psi}\tan\varphi+P\frac{\delta s}{\delta\psi}\simeq 0 \qquad (25)$$

Letting $\delta\psi$ approach zero and substituting for the derivatives as before gives:

$$T\left(\frac{r\dfrac{d^2r}{d\psi^2}-2\left(\dfrac{dr}{d\psi}\right)^2-r^2}{r^2+\left(\dfrac{dr}{d\psi}\right)^2}\right)-\frac{dT}{d\psi}\,\frac{r}{\dfrac{dr}{d\psi}}-\left(r^2+\left(\frac{dr}{d\psi}\right)^2\right)^{1/2}$$

$$\left(-P\quad\frac{F_s\ r}{\dfrac{dr}{d\psi}}\right)=0 \tag{26}$$

which can be rearranged to:

$$T\ \frac{dr}{d\psi}\left(r\ \frac{d^2r}{d\psi^2}-2\left(\frac{dr}{d\psi}\right)^2-r^2\right)-r\ \frac{dT}{d\psi}\left(r^2+\frac{dr}{d\psi}^2\right)-\left(P\ \frac{dr}{d\psi}-\right.$$
$$\left.-\ rF_s\right)\left(r^2+\frac{dr}{d\psi}^2\right)^{3/2}=0 \tag{27}$$

If the tangential loading can be neglected, and only a normal pressure remains, then (22) and (27) can be simplified to:

$$P\left(r^2+\left(\frac{dr}{d\psi}\right)^2\right)^{3/2}+T\left(r\ \frac{d^2r}{d\psi^2}-2\left(\frac{dr}{d\psi}\right)^2-r^2\right)=0 \tag{28}$$

if $r \neq 0$.

The Pressure Loading

The equilibrium equations have now been derived. It is necessary to know the pressure distribution in order to solve for the shape. It is theoretically possible to find the pressure distribution around an arbitrary shape by mapping the shape onto another shape for which the flow is known and determining the separation point by means of boundary layer theory, but it turns out to be prohibitively difficult. Instead of calculating the pressure distribution, it has been assumed that the pressure distribution is known and that it can be expressed as a Fourier series. Three pressure distributions are shown in fig. 17. Two were experimentally determined by Roshko, the other was found from a model described in chapter 4.

It has been assumed that the shear force is negligible. This means that the tension is constant by equation (3). Since the tension is constant, the elongation can be calculated beforehand and the arc length corrected. When this has been done, the elongation need not be considered afterwards so the problem can be treated as an inextensible one.

In order to solve the problem, some simplification is still necessary because it is not possible to find a closed solution to equations (16) or (28) when a Fourier series is substituted for the pressure P. Two different simplifying assumptions have been used. In the first method, it has been assumed that the pressure can be represented by a Fourier series of which the first term which is a constant is the largest and the other terms are much smaller. In this case, the membrane takes on a nearly circular shape and the small perturbation can be calculated after the differential equation has been modified. This is the small deflection analysis of chapter 2.

A second approach is to approximate the pressure by a number of regions of constant pressure with sharp jumps between them. In a region where the pressure is constant, the curvature is constant and so the shape is circular. In chapter 3 a pressure distribution consisting of two of these regions has been used.

CHAPTER II

SMALL DEFLECTION ANALYSIS

In this chapter, a method of solution for the shape of the membrane is given for small deflections from the no-wind shape. This restriction makes it possible to simplify the differential equation so that it can be solved.

The no-wind condition will be examined first. It has been stated that the tension T is a constant for the cases that will be considered. If there is no wind, then the pressure P is also a constant and equation (16) can be written as:

$$\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2} \Big/ \frac{d^2y}{dx^2} = \text{constant.} \tag{29}$$

Since the left hand side of (29) is the radius of curvature, the curve is a circular arc. If the arc length is chosen to be $\pi/2$ times the base, then the membrane will be a semicircle.

Let the semicircle be disturbed by a wind such that the change in pressure difference across the membrane $\Delta P(\psi)$ is small enough compared with the total pressure difference P.

In polar coordinates, the no-wind shape can be described by $r = a$, a constant if the origin is chosen correctly. The disturbed shape can then be described by $r = a + \varepsilon$ where ε is the change in radius and is a function of ψ. Let ΔP be such that ε will be small. Then $\left(\frac{d\varepsilon}{d\psi}\right)^2$ will be small compared with r^2 and then equation (28) can be approximated by:

$$P\ r^3 + T\left(r\ \frac{d^2r}{d\psi^2} - r^2 \right) = 0 \qquad (30)$$

The substitutions: $r = a+\varepsilon$; $P = P_o + \Delta P$ may be made. Since ε is small, $r \neq 0$ and so it may be divided out.

$$(P_o + \Delta P)\ (a + \varepsilon)^2 + T\left(\frac{d^2\varepsilon}{d\psi^2} - (a\ \varepsilon)\right) = 0 \qquad (31)$$

Dividing by T and rearranging

$$(a+\varepsilon)\ \left(\frac{P_o}{T}\ (a + \varepsilon) - 1\right)\ \frac{\Delta P}{T}\ (a+\varepsilon)^2 + \frac{d^2\varepsilon}{d\psi^2} = 0 \qquad (32)$$

Consider fig. 5. The tension T in the membrane can be found by a summation of forces in the vertical direction. If P_o and ΔP are so chosen that ΔP has an average value of zero, then it can be seen that a small perturbation ΔP will not alter the tension substantially. In (32) put $\Delta P = 0$ and $\varepsilon = 0$ to find the initial tension.

$$T = a\ P_o \qquad (33)$$

Putting this value into (32) gives

$$(a+\varepsilon)\left(\frac{P_o}{a\ P_o}\ (a + \varepsilon) - 1\right) + \frac{\Delta P}{a\ P_o}\ (a+\varepsilon)^2 + \frac{d^2\varepsilon}{d\psi^2} = 0$$

which gives:

$$\varepsilon + \frac{\Delta P}{a\ P_o}\ (a^2 + 2\ a\varepsilon + \varepsilon^2) + \frac{d^2\varepsilon}{d\psi^2} = 0$$

Ignoring ε^2 leaves:

$$\frac{d^2\varepsilon}{d\psi^2} + \varepsilon\left(1 + \frac{2\ \Delta P}{P_o}\right) = -\frac{a\ \Delta P}{P_o} \qquad (34)$$

In the case of potential flow, the pressure distribution due to the air flow over a circular cylinder is of the form: $c + d\ \cos\ 2\psi$. Since the cos 2ψ term is also a major part of the actual pressure distribution, this is the form of pressure distribution that will be considered for this analysis. Putting $P = -d\ \cos\ 2\psi$ since the constant 'c' can be included in the larger constant term P_o gives:

$$\frac{d^2\varepsilon}{d\psi^2} + (1 - 2d \cos 2\psi)\varepsilon = ad \cos 2\psi \qquad (35)$$

First Solution of The Reduced Equation

Putting equation (35) into the standard form of the Mathieu equation gives:

$$\frac{d^2y}{dx^2} + (a - 2q \cos 2x)\, y = r(x) \qquad (36)$$

where:

$y = \varepsilon$; $x = \psi$; $q = d$; $a = 1$; and $r(x) = ad \cos 2\psi$.

Since $a = 1$, this is an unstable case of the Mathieu equation according to fig. 4. Take the general solution of equation (36) to be of form:

$$y = A\, y_1(x) + B\, y_2(x) + y_p(x)$$

where $y_1(x)$ and $y_2(x)$ are independant solutions of the reduced differential equation and $y_p(x)$ is a particular integral. Take y_1 to be of form:

$$y_1 = e^{\mu x}\, \phi\, (x, \sigma) \qquad (37)$$

and let:

$$a = 1 + q f_1 + q^2 f_2 + q^3 f_3 + \dots \qquad (38)$$

$$\mu = q g_1 + q^2 g_2 + q^3 g_3 + \dots \qquad (39)$$

$$\phi = \sin (x - \sigma) + q h_1 + q^2 h_2 + q^3 h_3 + \dots \qquad (40)$$

where f_i, g_i, and h_i are functions of σ and h_i is also a function of x. Put equations (37), (38), (39), and (40) into the homogeneous differential equation corresponding to equation (36). Select the coefficients f, g, and h so that there is no term in $\cos (x - \sigma)$ in ϕ and so that the term in $\sin (x - \sigma)$ has the coefficient one. When these operations have been performed and the terms have been rearranged to a convenient form, the three series above become:

$$a = 1 - q\cos 2\sigma + \frac{1}{4}q^2\left(-1 + \tfrac{1}{2}\cos 4\sigma\right) + \frac{1}{64}q^3\cos 2\sigma +$$

$$+ \frac{q^4}{16}\left(\frac{1}{3} - \frac{11}{32}\cos 4\sigma\right) - \frac{11}{32}q^5\left(\frac{1}{9}\cos 2\sigma - \frac{13}{128}\cos 6\sigma\right) +$$

$$+ \frac{q^6}{8192}\left(-\frac{893}{27} + \frac{9181}{216}\cos 4\sigma - \frac{35}{4}\cos 8\sigma\right) - \ldots. \qquad (41)$$

$$\mu = -\tfrac{1}{2}q\sin 2\sigma \quad \frac{3q^3}{128}\sin 2\sigma - \frac{3q^4}{1024}\sin 4\sigma -$$

$$- \frac{q^5}{4096}\left(\frac{137}{9}\sin 2\sigma - \frac{9}{2}\sin 6\sigma\right) + \frac{{}^{6}}{16384}\left(\frac{337}{27}\sin 4\sigma - \frac{15}{4}\sin 8\sigma\right) + \ldots. \qquad (42)$$

$$\phi = \sin(x - \sigma) + S_3\sin(3x - \sigma) + S_5\sin(5x - \sigma) + S_7\sin(7x - \sigma) + \ldots. +$$

$$+ C_3\cos(3x - \sigma) + C_5\cos(5x - \sigma) + C_7\cos(7x - \sigma) + \ldots. \qquad (43)$$

$$S_3 = -\frac{q}{8} + \frac{q^2}{64}\cos 2\sigma - \frac{q^3}{512}\left(-\frac{14}{3} + 5\cos 4\sigma\right) + \frac{q^4}{4096}\left(-\frac{74}{9}\cos 2\sigma + 7\cos 6\sigma\right) + \ldots.$$

$$S_5 = \frac{q^2}{192} - \frac{q^3}{1152} + \frac{q^4}{4096}\left(-\frac{155}{54} + \frac{82}{27}\cos 4\sigma\right) - \ldots.$$

$$S_7 = -\frac{q^3}{9216} + \frac{q^4}{49152}\cos 2\sigma - \ldots$$

$$S_9 = \frac{q^4}{737280} - \ldots \qquad (44)$$

$$C_3 = \frac{3q^2}{64}\sin 2\sigma - \frac{3q^3}{512}\sin 4\sigma \quad \frac{q^4}{4096}\left(\frac{274}{9}\sin 2\sigma + 9\sin 6\sigma\right) \ldots$$

$$C_5 = \frac{-7q^3}{2304}\sin 2\sigma \quad \frac{11q^4}{27648}\sin 4\sigma \quad \ldots$$

$$C_7 = \frac{35q^4}{442368}\sin 2\sigma - \ldots$$

The parameter σ can be found from equation (41). If an approximation is sufficient, then the terms in $\cos 6\sigma$ and $\cos 8\sigma$ can be ignored, $\cos 4\sigma$ can be expanded in terms of $\cos 2\sigma$ leaving a quadratic equation in $\cos 2\sigma$:

$$\frac{q^2}{4}\left(1 - \frac{11}{64}q^2\right)\cos^2 2\sigma - q\left(1 - \frac{1}{64}q^2\right)\cos 2\sigma + \frac{65}{1536}$$

$$q^4 - \frac{3}{8}q^2 + 1 - a = 0 \qquad (45)$$

If greater accuracy is required, an iterative scheme can be used to include the terms in $\cos 6\sigma$ and $\cos 8\sigma$.

Second Solution of the Reduced Equation

The second solution of the homogeneous equation can be found by putting $-\sigma$ for σ; 'a' is an even function of σ and μ is an odd function of σ. This means that μ will change sign but have the same absolute value while 'a' is unaffected. The coefficients of the series for ϕ must be recalculated, however, since they are not all even or odd.

The Particular Integral

The particular solution can be found by the method of variation of parameters. McLachlan gives the particular integral as:

$$Y_p = -\frac{1}{C^2}\left(y_1(x)\int^x y_2(u)\, r(u)\, du - y_2(x)\int^x y_1(u)\, r(u)\, du\right) \qquad (46)$$

where $r(x)$ is the right hand side of the differential equation (43), y_1, and y_2 are two independent solutions of the homogeneous equation, and $-c^2 = y_2 y_1' - y_1 y_2'$ which is invarient.

The complete solution of the differential equation (43) is now:

$$y = A\, y_1(x) + B\, y_2(x) + y_p(x)$$

where A and B are arbitrary constants which must be selected to force the boundary conditions. The conditions in this case are the requirements that both ends are restrained and so the deflection at each end is zero.

It should be noted that x is not the independent variable, η is.

Returning to the original variables the shape can now be written as:

$$\varepsilon(\theta) = A\, y_1(\theta) + B\, y_2(\theta) + y_p(\theta)$$

CHAPTER III

SIMPLIFIED PRESSURE DISTRIBUTION

In the previous chapter, the shape of the membrane was examined for the case where the pressure distribution is the sum of a large constant value and a small variable component of any type. In this chapter, the case of large deflections and pressure variations is considered. The simplification in this chapter is the requirement that the pressure distribution be made up of regions of constant pressure with jumps between them. Specifically, pressure distributions similar to the one shown in fig. 7 which give rise to shapes like fig. 6 will be considered.

In order to specify the pressure distribution everywhere in fig. 7, three parameters must be known: P_1, $k = P_1/P_2$, and either β or α. These specify the magnitude of the pressure distribution, the relative size of the two constant pressure regions, and the location of the change in pressure. P_1, P_2 and β are shown on fig. 7 as the two pressures that act on the membrane, and the fraction of arc length from the leading edge to the jump in pressure. The last value α is the angle of the normal to the membrane at the jump in pressure measured as shown in fig. 6 which can be used instead of β.

Equation (16) can be rearranged to:

$$\frac{\frac{d^2y}{dx^2}}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}} = \frac{P}{T} \qquad (47)$$

The left hand side is the curvature of the membrane and the right hand side is a constant in a region where the pressure is constant. The curvature is thus constant in the region being considered and the radius of curvature is T/P. It can be seen that the differential equation will still be satisfied if P is altered as long as the ratio T/P is constant. For a given shape, if the pressure everywhere is doubled, and the tension is also doubled, then the differential equation is still satisfied. It can be seen that the magnitude of the pressure is not significant since the tension is a dependent variable. This leaves only two parameters that must be specified to fix the pressure: one to give the relative magnitude of the two pressures acting, and one to locate the position of the jump.

In order to be able to describe the shape fully, the centers of the arcs must be located. The variables required to do this are: α, d, and θ_1. With α and d the line of centers BD can be found. With the angle θ_1, the center C_1 can be located on this line and then the first arc AB may be drawn. The second center C_2 may be found readily since two points on the arc B and C are known and the center is known to lie on BD. Once the center has been found, the second arc BC can be drawn.

Since it is not possible to find α, d, and θ_1 by an explicit formula in a closed form solution, the problem has been solved by calculating the values k, α, and d for a number of values of α and θ_1. The problem has been normalized by letting the base length be a constant 10 units. The results of these calculations have been plotted in figs. 9 to 15. It is only necessary to use two or three of these figures in many cases but more have been included for flexibility and so the interaction of the variables may be seen. If for example, k and α are given, then d and θ_1 must be found. Fig. 10 is used to find θ_1, and then either fig. 9 or fig. 11 can be used to find d. If k and β are given, then α, θ_1, and d must all be found. This can be done by using figs. 12 and 13 to get d and θ_1 or fig. 13 to get θ_1 and then fig. 14 to get d.

Fig. 8 shows how the shape may be drawn for the particular case $\alpha = 60^\circ$ (1.047 radians), k = 0.5. From fig. 10, $\theta_1 = 1.9$ radians (or 109°), and from fig. 11, d = 6.45. Fig. 8b shows a family of curves with $\beta = 0.5$ which approximate the inflation shapes.

The method of calculation will now be described. The variables that must be related are:

θ_1, θ_2 the angles subtended by arcs AB and BC,

r_1, r_2 the radii or curvature of the two arcs,

α the angle that the line of centers makes with the horizontal.

β the fraction of arc length at which the pressure changes,

ℓ the arc length of the membrane,

s the length of the base of the structure,

d the distance AD in fig. 6.

The base length 's' is given by:

$$s = r_1 \cos(\pi - \theta_1 - \alpha) + (r_1 - r_2)\cos\alpha + r_2 \cos(\theta_2 - \alpha) \qquad (48)$$

The arc length is:

$$\ell = r_1\theta_1 + r_2\theta_2 \tag{49}$$

The fact that both ends of the membrane are fixed to the base at the same elevation is expressed by:

$$r_1 \sin(\pi - \alpha - \theta_1) + r_2 \sin(\theta_2 - \alpha) - (r_1 - r_2)\sin\alpha = 0 \tag{50}$$

An added restriction is the fact that the membrane is to be a semi-circle if both pressures are equal so:

$$\ell = \frac{\pi s}{2} \tag{51}$$

Rearranging (50) gives:

$$-r_1 = r_2\left[\sin(\theta_2 - \alpha) + \sin\alpha\right] / \left[\sin(\pi - \alpha - \theta_1) - \sin\alpha\right] \tag{52}$$

Substituting (51) and (52) into (49) gives:

$$\frac{\pi s}{2} = r_2\theta_2 - r_2\theta_2\left[\sin(\theta_2 - \alpha) + \sin\alpha\right] / \left[\sin(\pi - \alpha - \theta_1) - \sin\alpha\right] \tag{53}$$

Substituting (52) into (48) gives:

$$s = -r_2\left(\frac{\sin(\theta 2 - \alpha) + \sin\alpha}{\sin(\pi - \alpha - \theta_1) - \sin\alpha}\right)(\cos(\pi - \alpha - \theta_1) + \cos)) + r_2\left(\cos(\theta_2 - \alpha) - \cos\alpha\right) \tag{54}$$

Multiply (53) by $2/\pi$, equating the resulting expression for s to the right hand side of (54) and multiplying both sides by $\sin(\pi - \alpha - \theta_1) - \sin\alpha$ gives:

$$\begin{aligned} - \theta_1\left[\sin(\theta_2 - \alpha) \quad \sin\alpha\right] + \theta_2\left[\sin(\pi - \alpha - \theta_1) - \sin\alpha\right] = \\ - \frac{\pi}{2}\left[\cos(\pi - \alpha - \theta_1) + \cos\alpha\right]\left[\sin(\theta_2 - \alpha) + \sin\alpha\right] + \\ + \frac{\pi}{2}\left[\cos(\theta_2 - \alpha) - \cos\alpha\right]\left[\sin(\pi - \alpha - \theta_1) - \sin\alpha\right] \end{aligned} \tag{55}$$

The right hand side can be multiplied out and some terms cancel to give:

$$\theta_1 \sin(\theta_2 - \alpha) + \sin\,)- \theta_2 (\sin(\pi - \alpha - \theta_1) - \sin\alpha) =$$

$$\frac{\pi}{2} \cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2 - \sin\theta_1 - \sin\theta_2 \tag{56}$$

If θ_1 and α are selected, θ_2 can be found by iteration using the residual of equation (56).

The other parameters may be found as follows: Let r_1 take on any arbitrary value such as one. From (52) r_2 can be found. Using equation (43) allows 's' to be calculated. This will not be the normalized value but now r_1, r_2, and s can be scaled to make s the correct value. From fig. 6 by the law of sines,

$$d = r_1 \sin\theta_1 / \sin\alpha$$

which allows 'd' to be calculated. The pressure ratio $k = P_1/P_2$ is the reciprocal of the ratio r_1/r_2, and $\beta = r_1\theta_1/\ell$ where ℓ is the arc length from (51).

A listing of the program used to calculate the points for the graphs is included in appendix A.

CHAPTER IV

EXPERIMENTAL RESULTS AND CONCLUSIONS

In order to gain some appreciation of the magnitude of the deflections and of the behaviour of inflated structures, a small model was set up as shown in fig. 20. The wind tunnel was 30 inches in diameter and the model had a 24 by $5\frac{3}{4}$ inch base. The length of the plastic film that was used as a membrane was adjusted so that the model was a semi-circular cylinder when inflated with no wind blowing past it.

Figs. 21 to 24 show the model in various stages of inflation in an airstream. The air velocity was not uniform across the tunnel so that accurate measurements were not possible but the inflation process can be seen quite well in figs. 21 and 22. The dark lines of the model are a one inch square grid and each line is slightly more inflated than the one below it. The exact shape at any stage of inflation will probably not be identical to these since there will be disturbing forces in the membrane due to the fact that the model was not actually a true cylinder but these figures indicate what sort of shapes the membrane takes. Due to the low pressures required to inflate the membrane, it was not possible to give the inflation pressure more accurately than to say it was of the order of a tenth of an inch of water and the air velocity was about 40 feet per second. The last picture was taken with a much higher inflation pressure.

For a number of conditions, the shape of the membrane was found by using a frame with sliding pins that could be adjusted to just contact the plastic. When all the pins had been adjusted, the frame was moved and the coordinates at each point were recorded. With these coordinates, the curves in fig. 16 were drawn. An approximate pressure distribution was measured for the high velocity case and has been plotted in fig. 17. For comparison, the pressure distribution around a circular cylinder at roughly the same Reynolds number and also at the Reynolds number that would be encountered with a full size building have been included from Roshko.

An undesirable phenomenon that was encountered during the experiment was the vibration of the membrane at certain pressures of inflation and certain velocities. The vibration occurred when the membrane had the shapes shown in fig. 19 d and e. The range of pressures at which vibration was found was measured for a number of wind velocities and plotted in fig. 18. There are three regions in this figure: an intermittent forced vibration at low velocities and relatively high pressures which changed its amplitude considerably, large vibration of the membrane that occurred with only one of the three types of plastic, and a similar vibration that occurred for two types of plastic. The type of plastic has an influence on the damping since for the third type of plastic, it was not possible to find a resonant condition. Because of the limitations of the equipment it was not considered advisable to proceed with the investigation of this vibration since it was felt that it was necessary to gain more information from experiment before a theoretical study could be begun. It seems likely that the defects of the apparatus may have had a great influence on the vibration.

Two approximations to the shape of an inflated cylinder under wind loading have been described. Each of these requires that the pressure distribution around the cylinder be known and places a restriction on the pressure. The two step pressure distribution can be generalized to any reasonable number of steps. In this case, it is best to work with a different approach. The membrane can be divided into any number of regions. Each of these is assumed to have a constant pressure acting on it and so is a circular arc. Calculations are begun at one end by assuming a tension and an initial slope and adding the regions of the membrane one by one so that the slope is continuous and without jumps. If the tension and initial slope have been assumed correctly, then the final region will end at the correct point. If this is not the case, then the tension must be corrected to give the correct base length and the initial angle must be corrected to rotate the entire shape so that the base comes to lie at the correct angle. This method does not allow a study to be made of the behaviour of the shape the way that has been done in the chapter on the simplified pressure distribution, but it is a general method that will give the shape for any pressure distribution.

This method has been developed into a program included in appendis A and was applied to the problem of finding the shape that the membrane takes at the high speed flow case shown in fig. 16. This case was chosen since it was the only one for which the pressure distribution was measured. The computed shape is shown as the heavy line in fig. 16. The discrepancy between measured and calculated shapes is a horizontal shift of about one eighth of an inch. This could be due to some errors in the pressure distribution but is probably due to an error in the measurement of the shape of the lower ends of the curve. The lower ends of the curve were the least accurate because of the flexibility of the pins used for measuring the shape. The actual end point was found for only one end and the base length was used to find the other. An error of an eighth of an inch could have been made in the determination of the horizontal position of these ends.

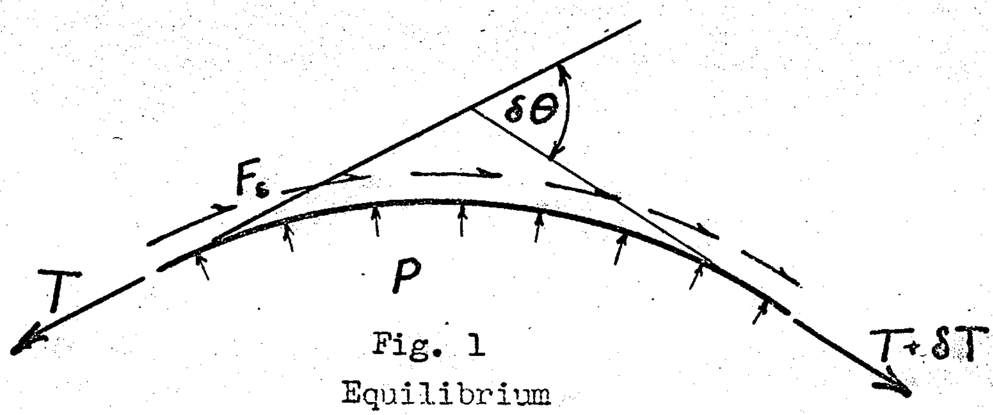


Fig. 1
Equilibrium
Intrinsic Coordinates

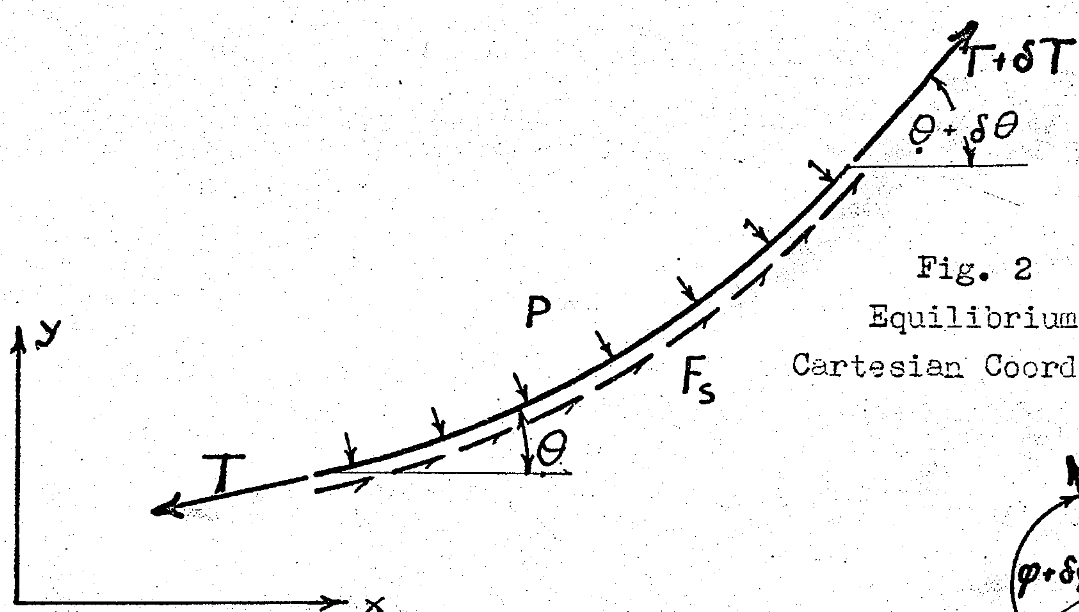


Fig. 2
Equilibrium
Cartesian Coordinates

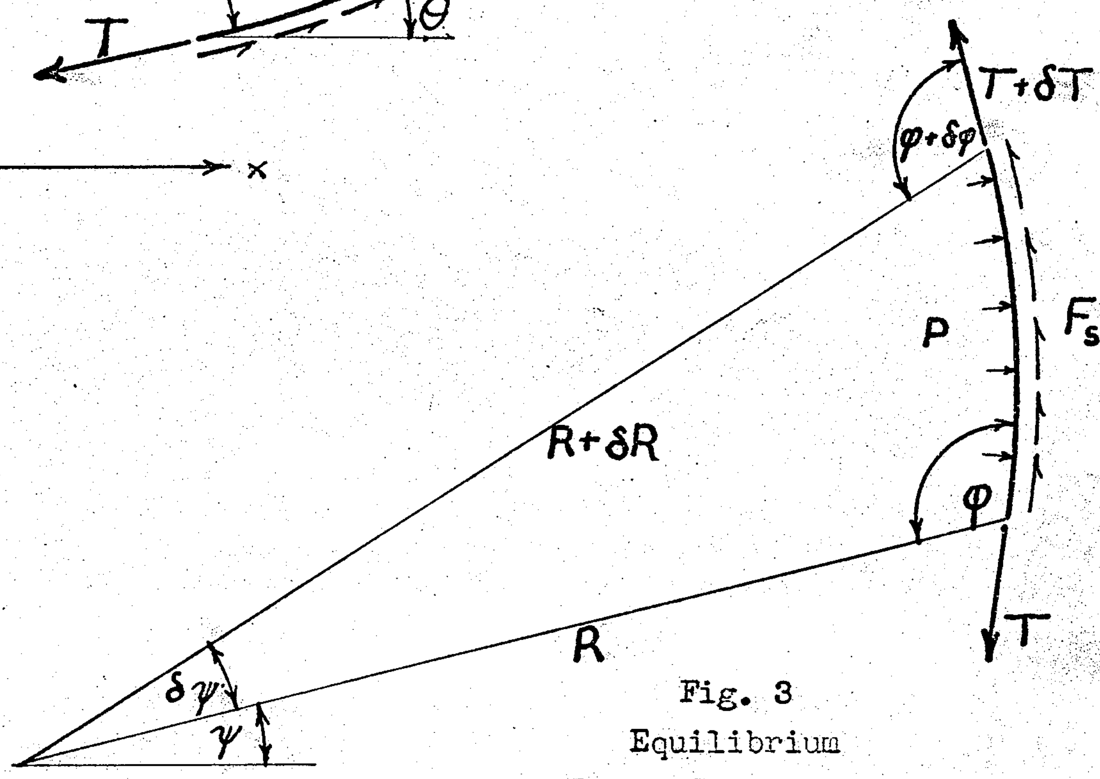


Fig. 3
Equilibrium
Polar Coordinates

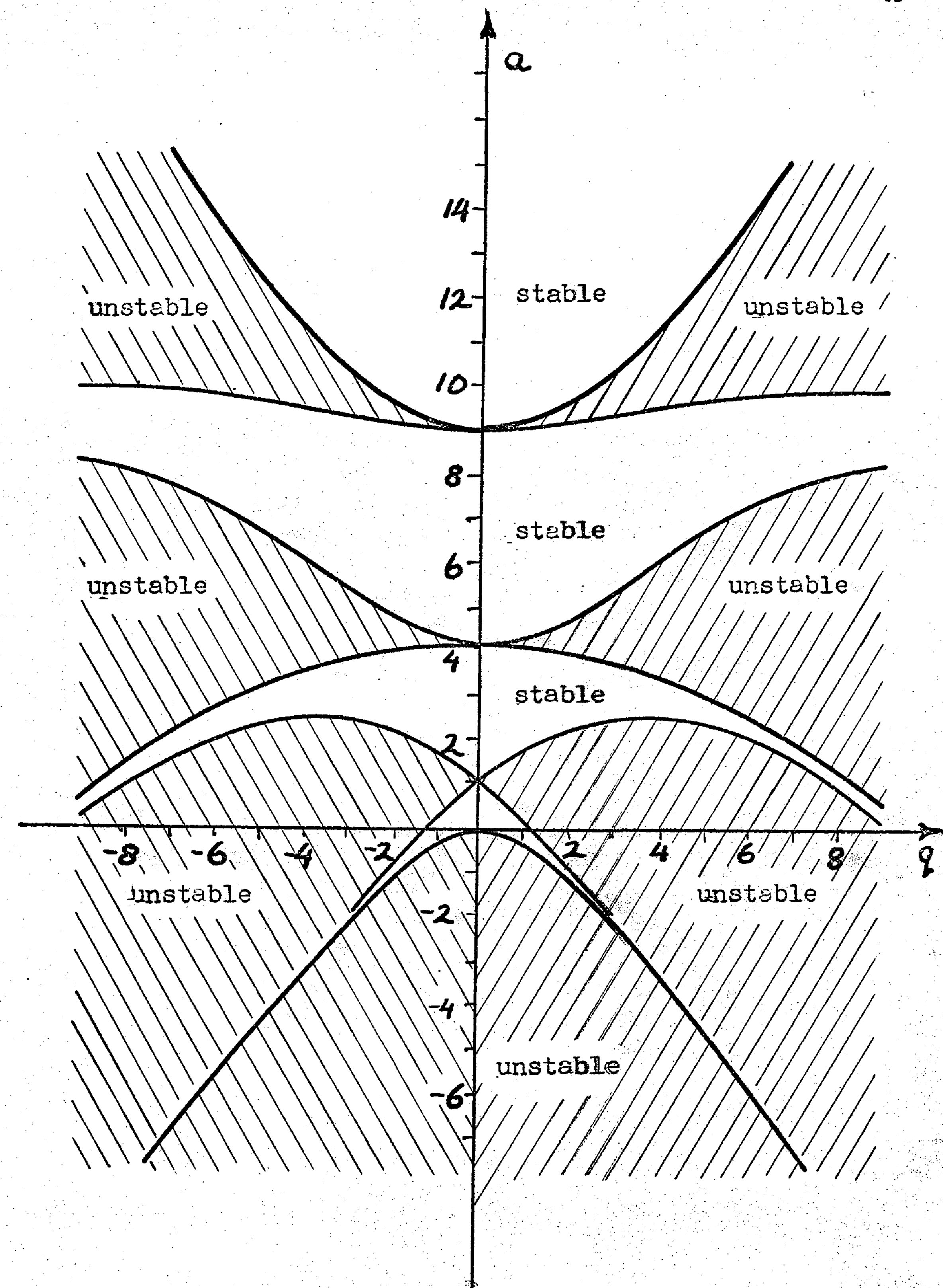
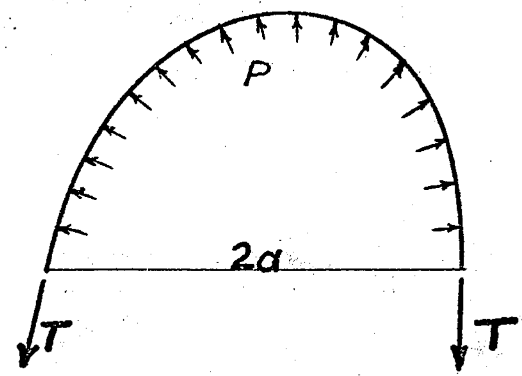


Fig. 4 Stability diagram for the Mathieu equation from McLachlan fig. 8B p 41

Fig. 5
Perturbation
Solution



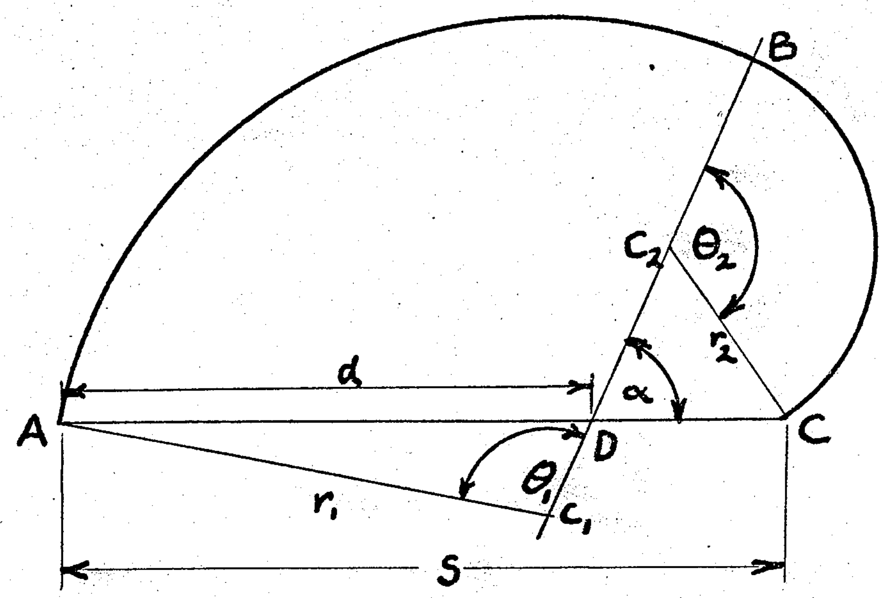


Fig. 6 Shape for Simplified Pressure Distribution

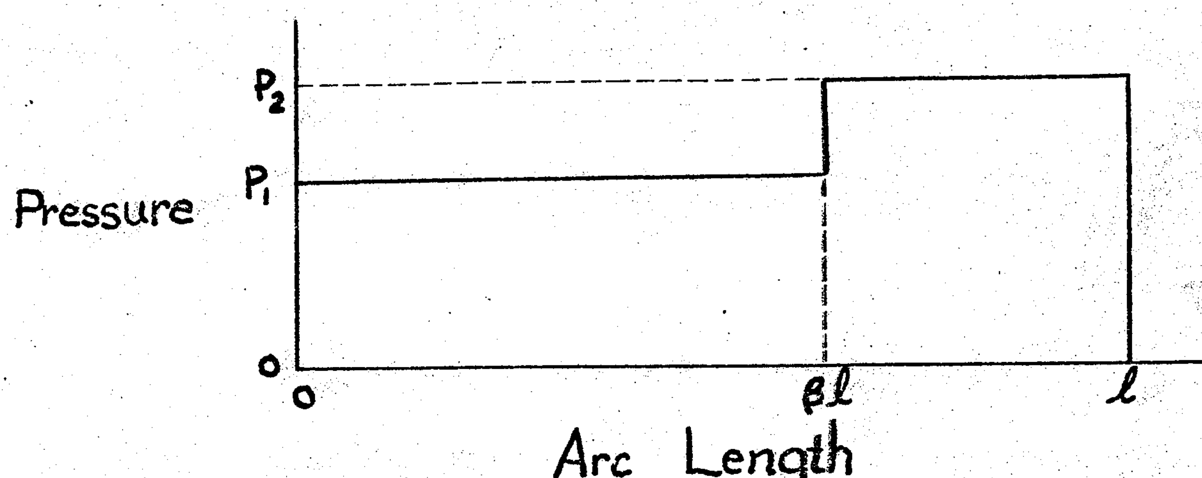


Fig. 7 Simplified Pressure Distribution

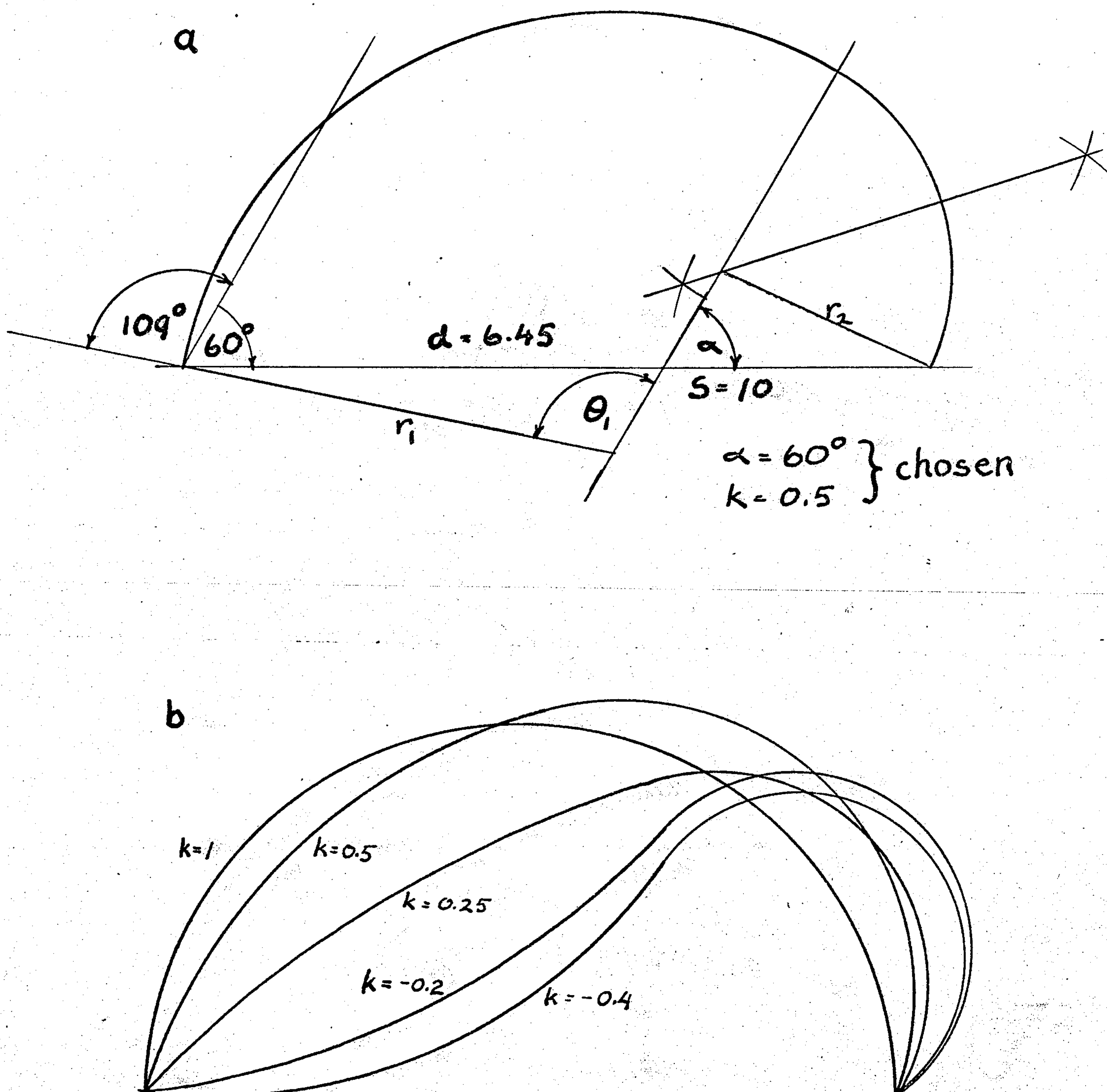


Fig. 8 Construction of simplified shape from graphs. a) method of construction. b) series of shapes with β = 0.5 and varying k.

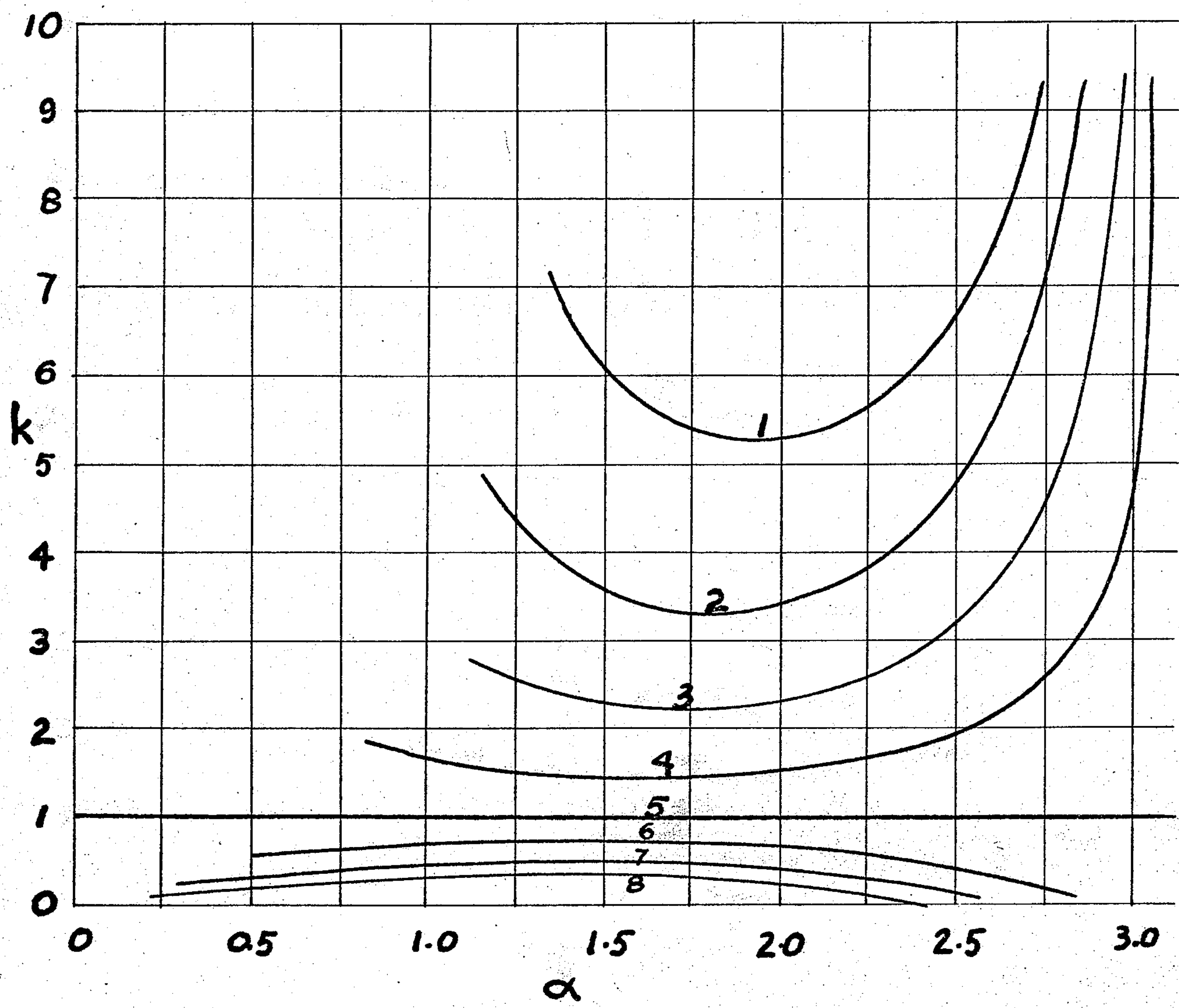


Fig. 9 k versus α, lines of constant d

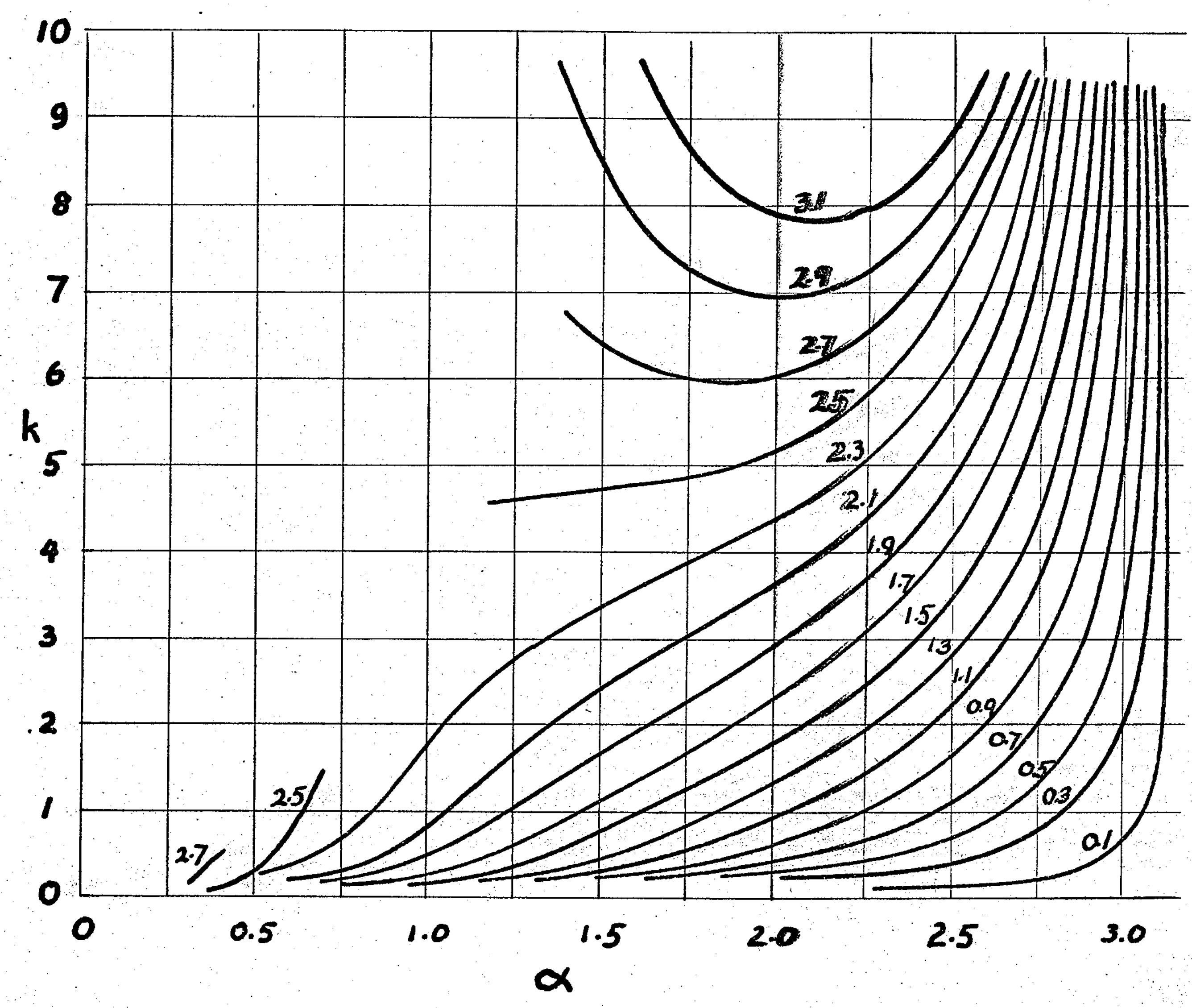


Fig. 10 k versus α, lines of constant θ_1

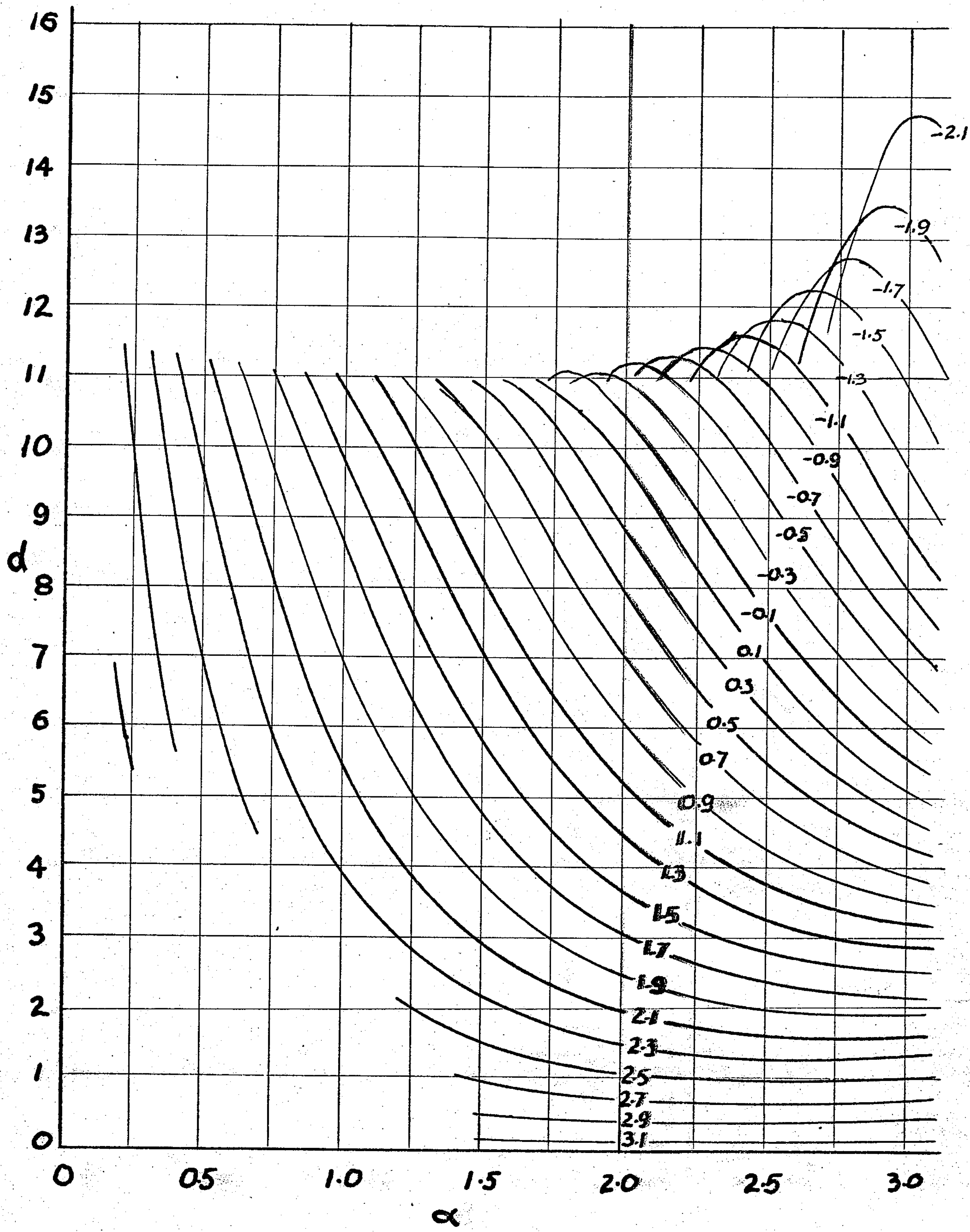


Fig. 11 d versus α, lines of constant θ_1

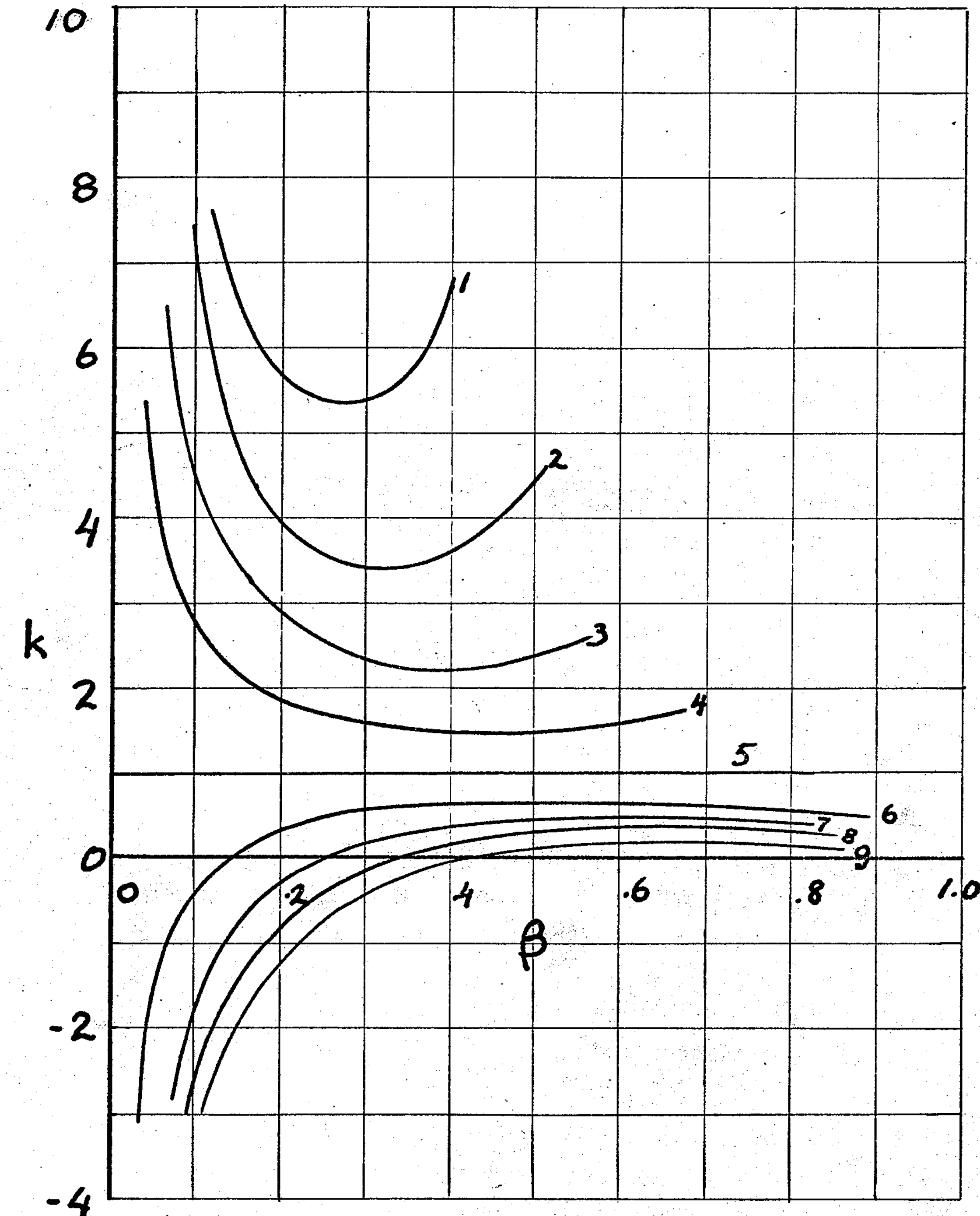


Fig. 12 k versus β, lines of constant d

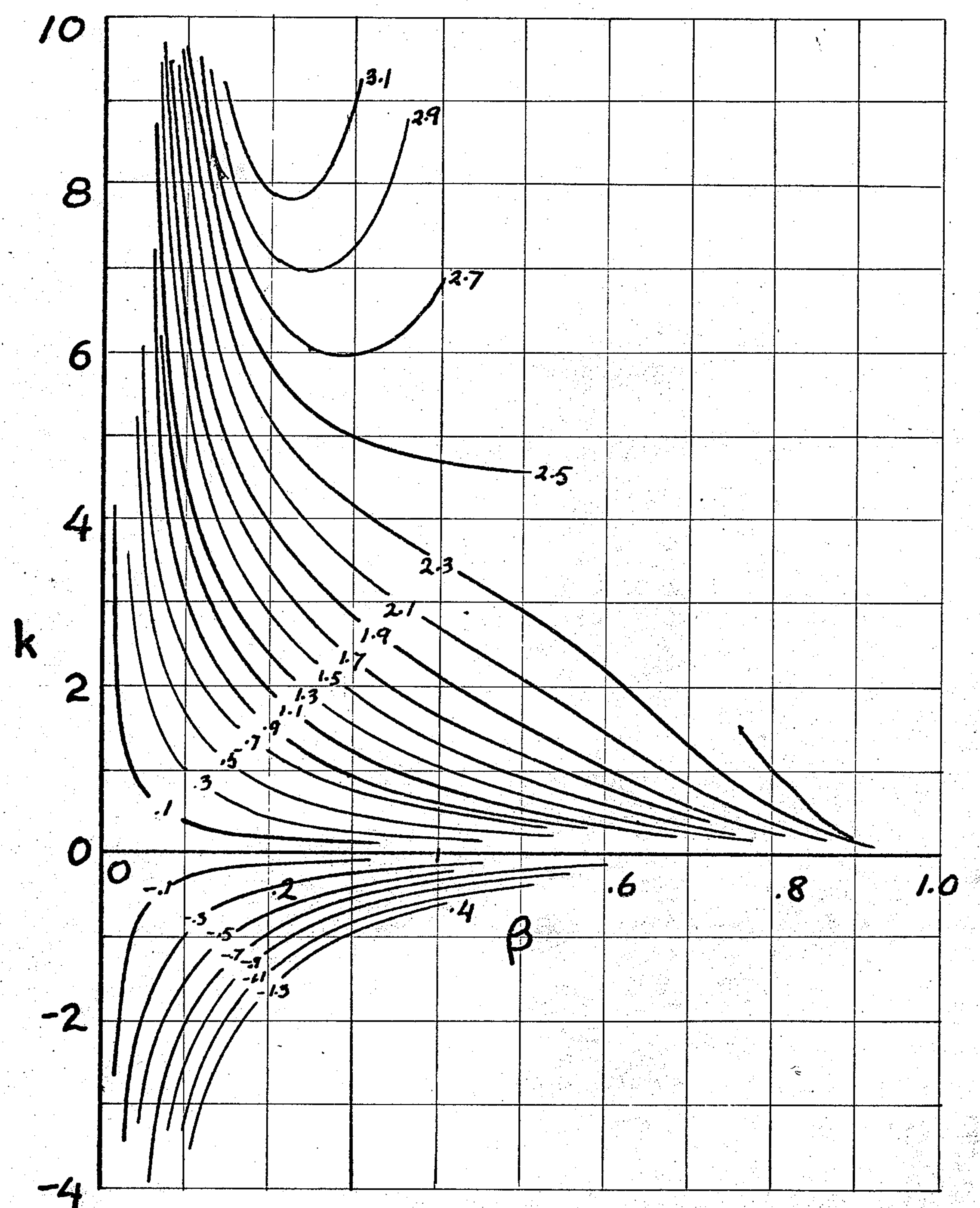


Fig. 13 k versus β, lines of constant θ_1

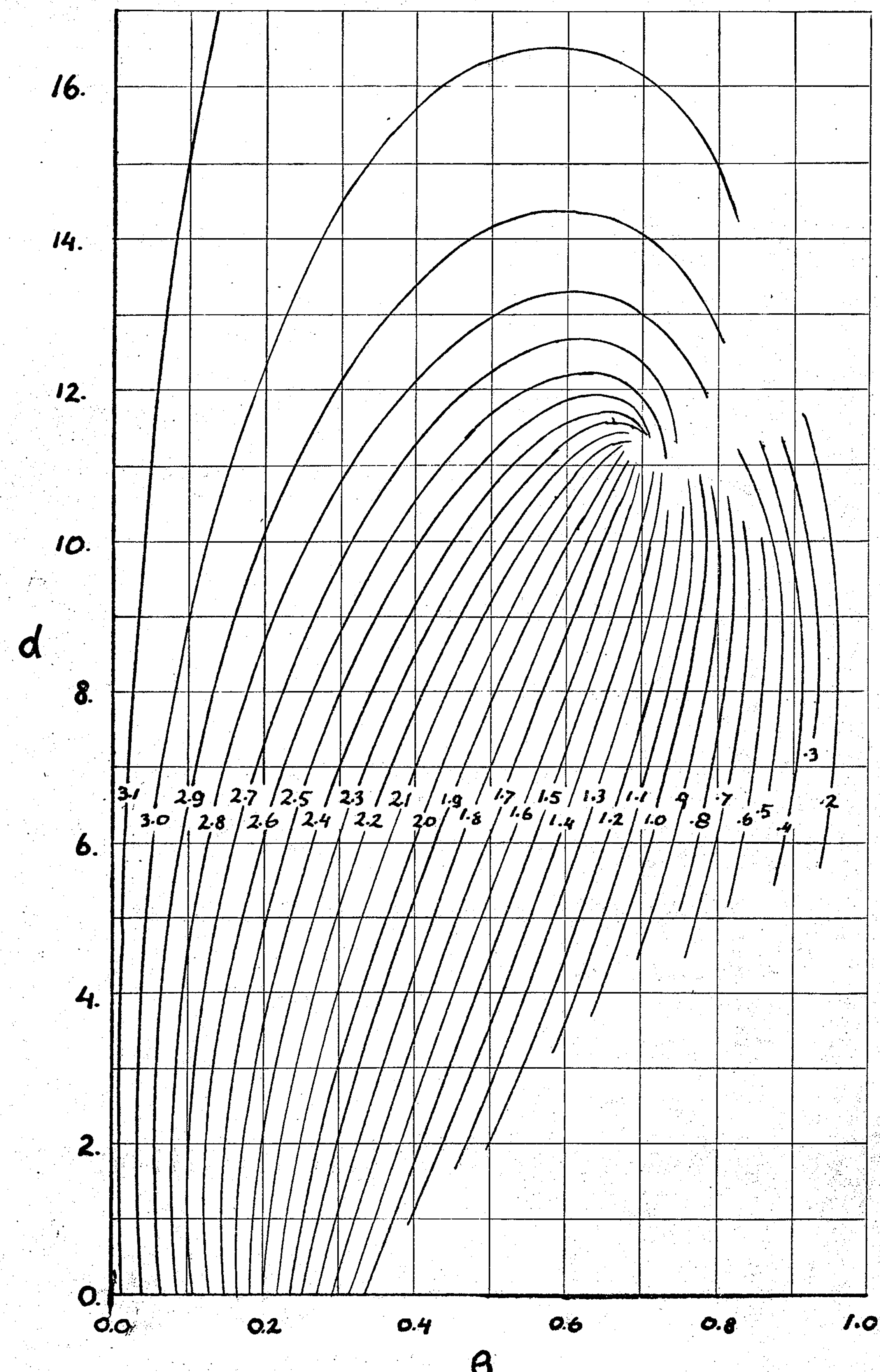


Fig. 14 d versus β, lines of constant α

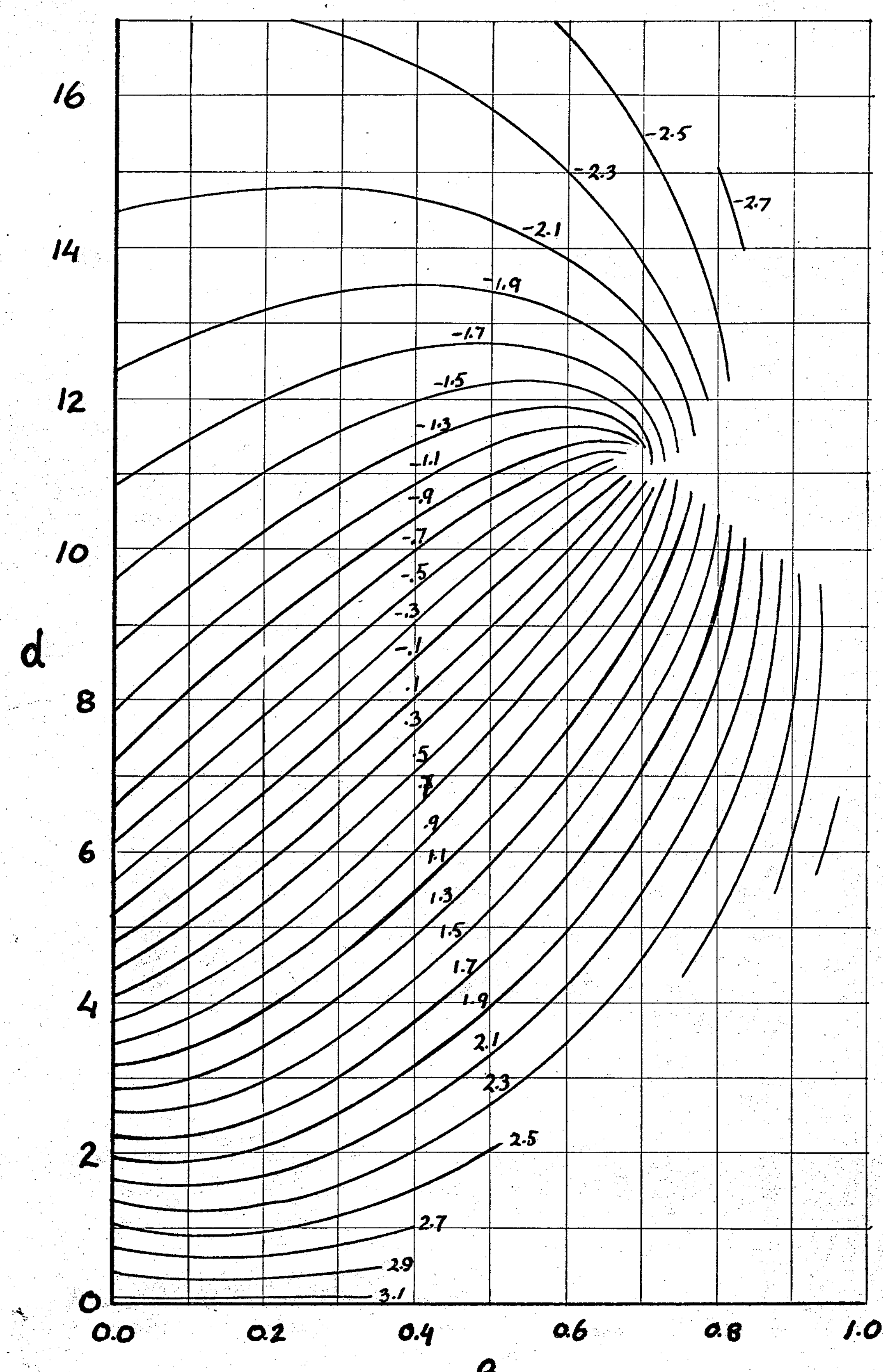


Fig. 15 d versus β, lines of constant Θ_1

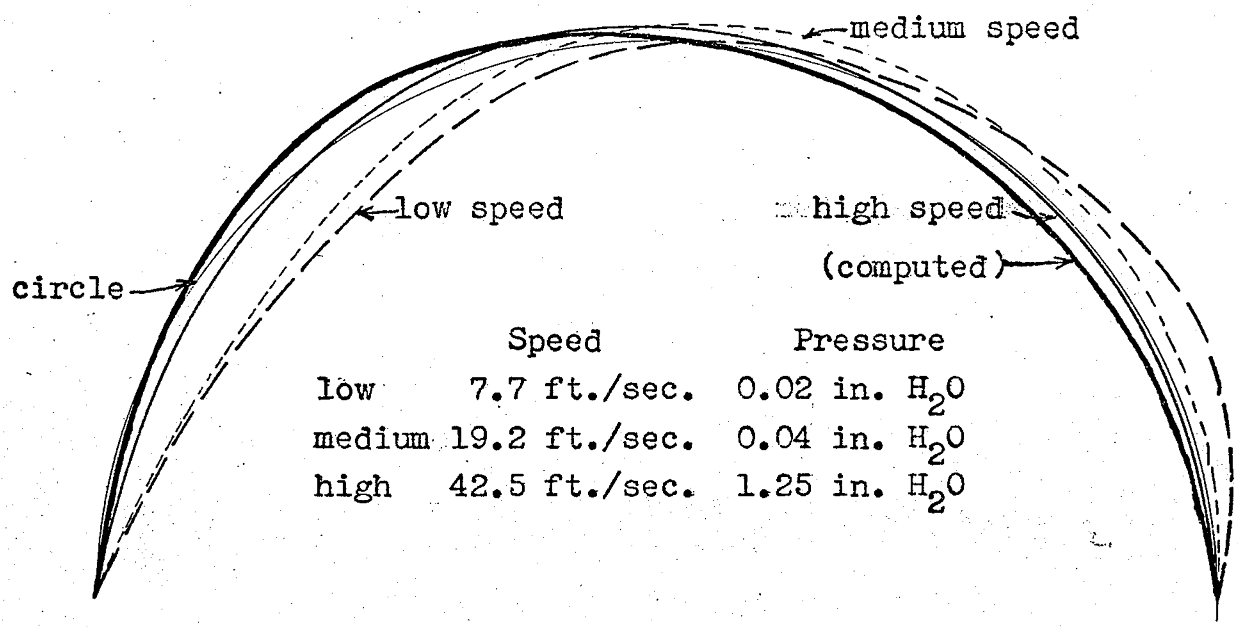


Fig. 16 Shapes Found from Model

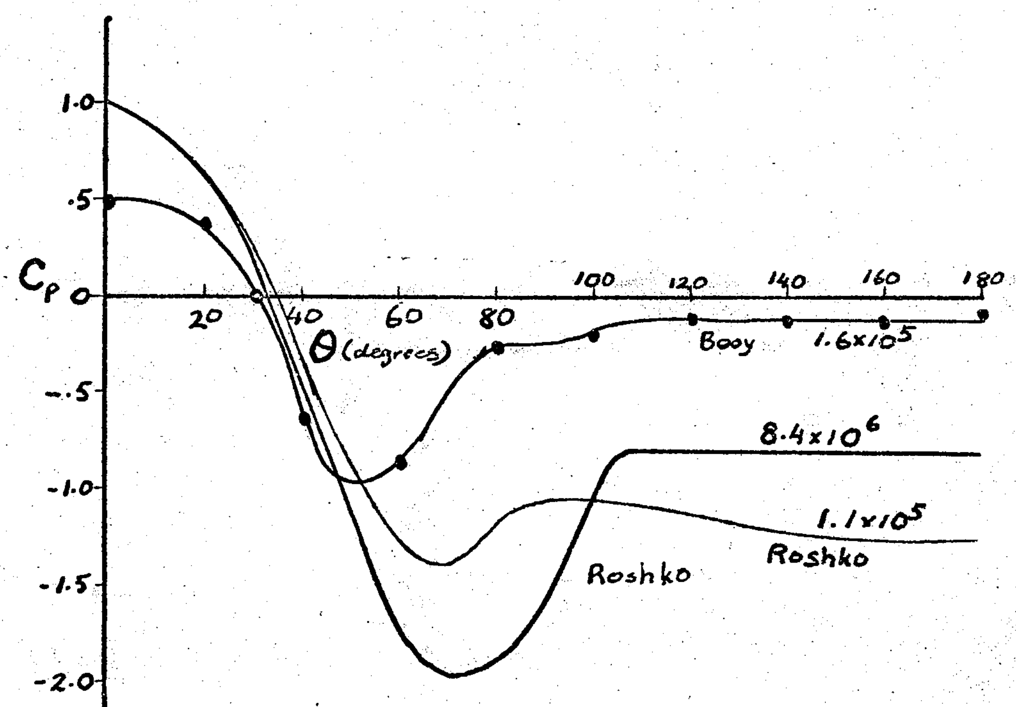


Fig. 17 Pressure distribution around model and two published pressure distributions for circular cylinders

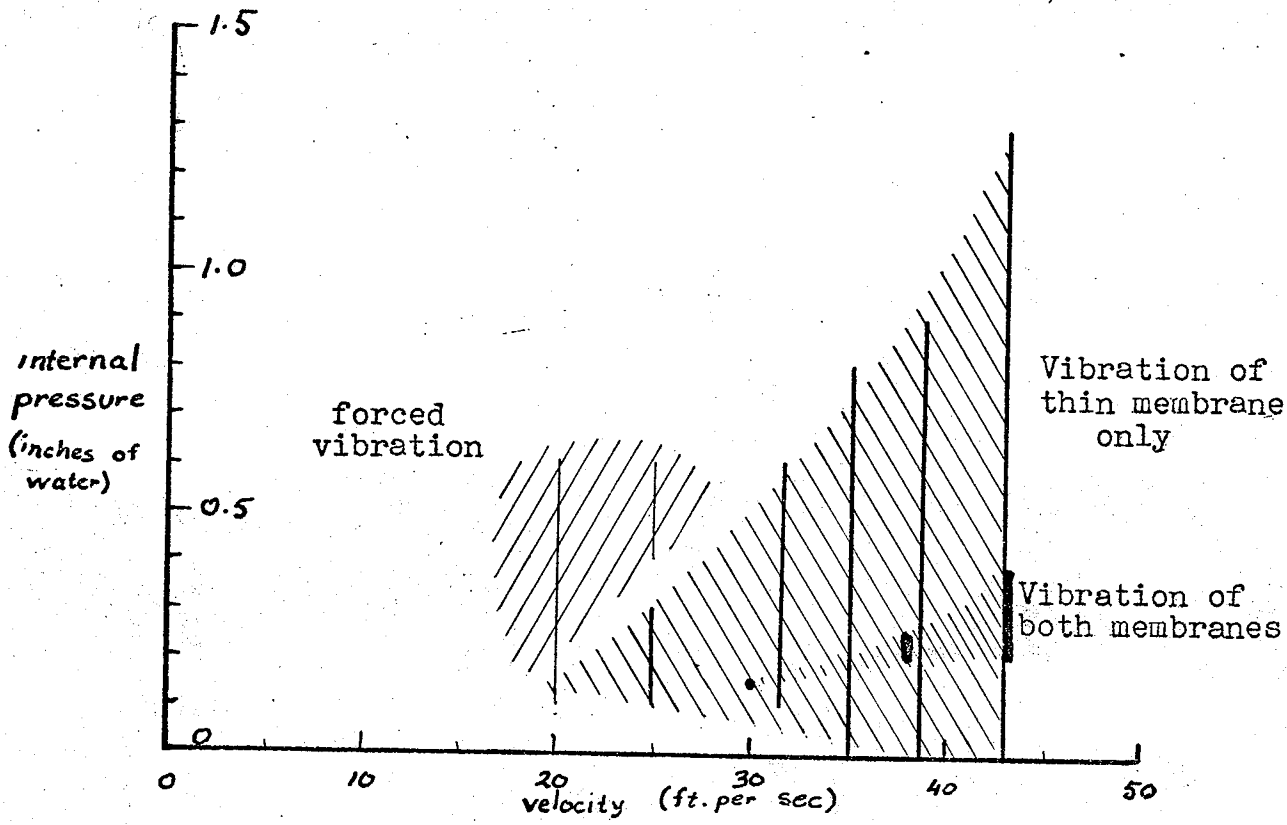


Fig. 18 Vibration Range Graph

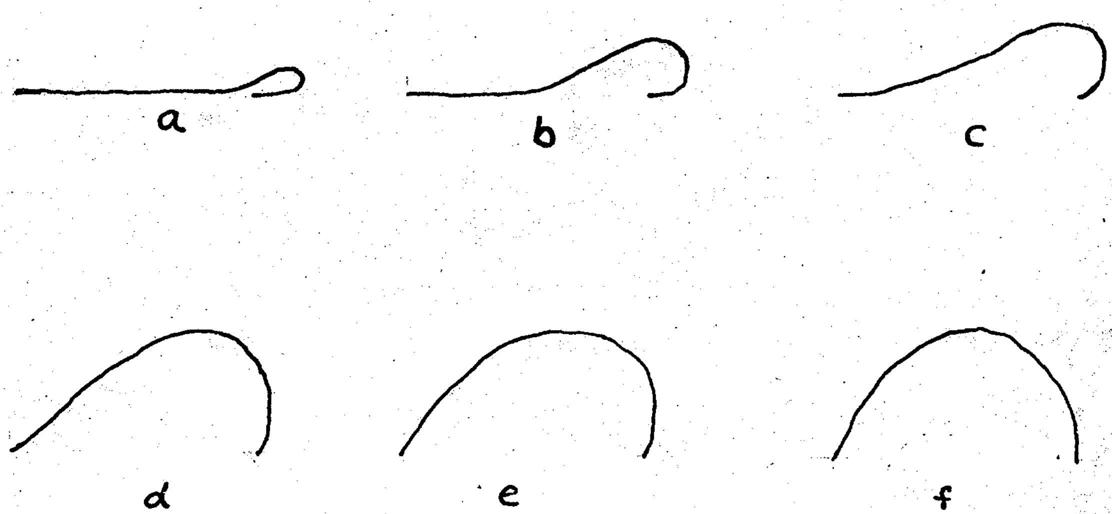


Fig. 19 Inflation of a Membrane with a Wind Blowing

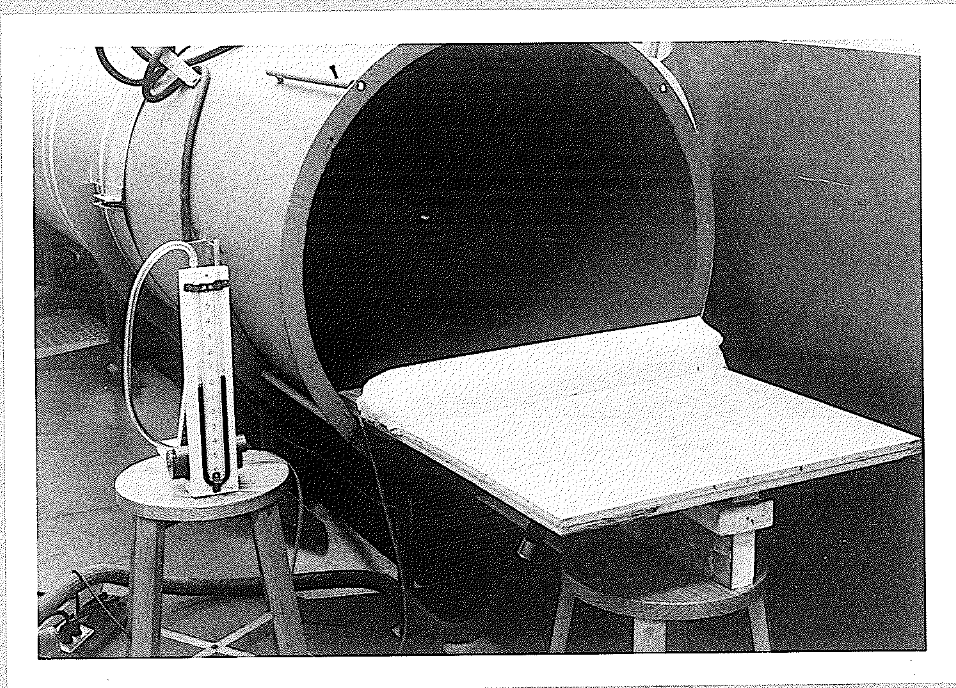


Fig. 20 Model used

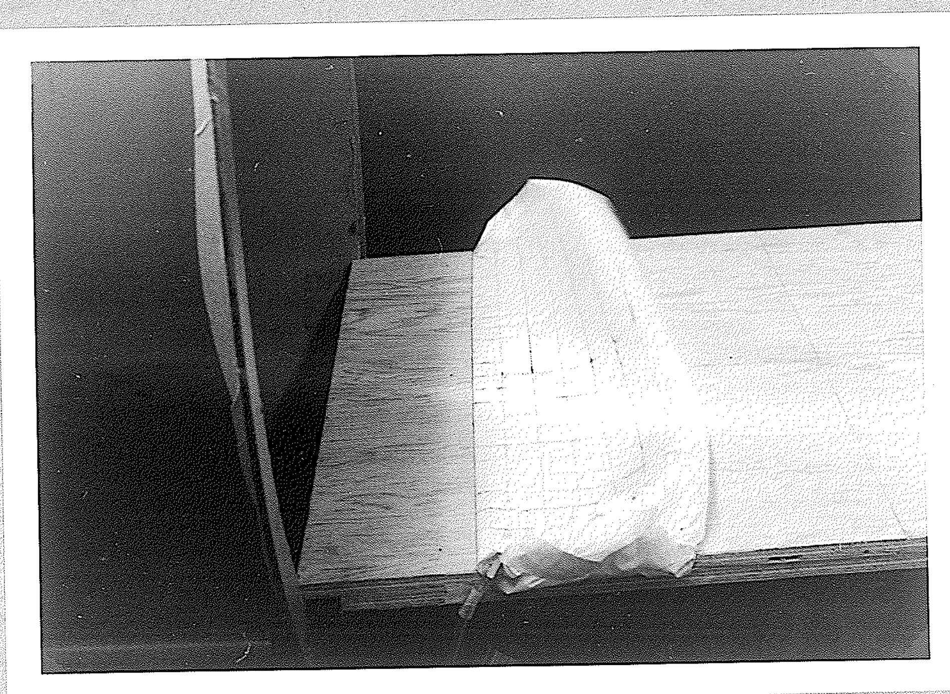


Fig. 21 Inflation of membrane with wind blowing
stage 1 beginning to fill

Fig. 22 Inflation of membrane with wind blowing
stage 2 almost full

Fig. 23 Inflation of membrane with wind blowing
stage 3 full but not yet pressurized

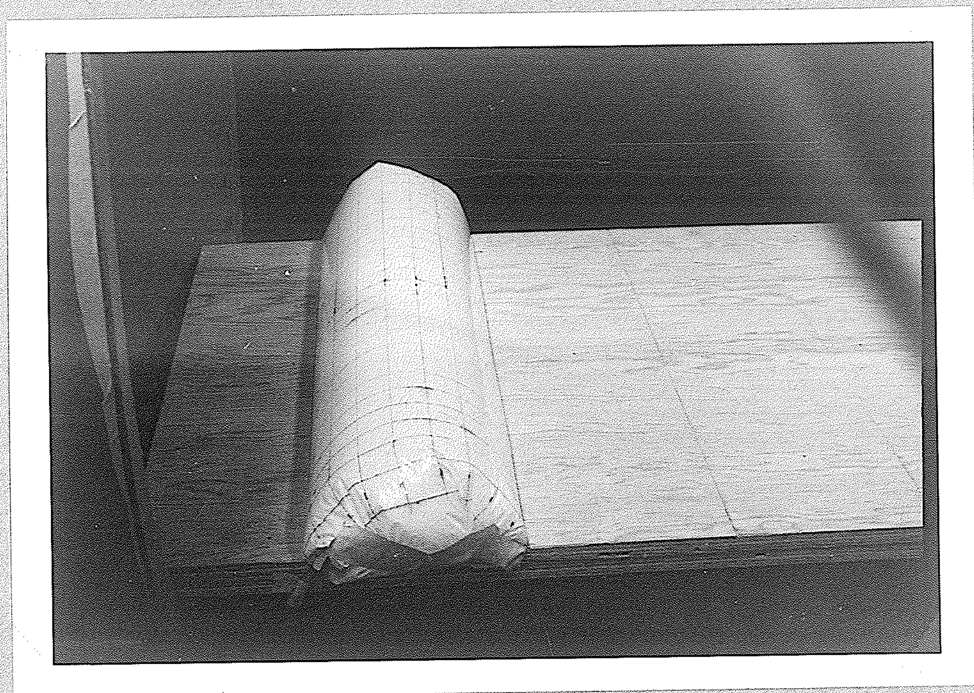


Fig. 24 Inflation of membrane with wind blowing
stage 4 pressurized

Appendix A Computer Programs

Program for simplified pressure analysis

```
      DIMENSION FAILT(900),FAILA(900)
      LOGICAL EN
      NCARD = 1
1     READ(5,102,END =7)RR1,INCR,SIGN
      WRITE(6,101)
      NLINE = 1
      K = 1
      NLOW = 1
      IF(SIGN.LT.0.) NLOW = 15
      DO 15 I = 1,31,2
      DO 15 J = NLOW,31,INCR
      R1 = RR1
      THETA1 = I / 10. * SIGN
      ALPHA = J / 10.
C     FIND TWO VALUES OF THE RESIDUAL TO START THE ITERATION FOR THETA2
      THIGH = 2.5
      IF(SIGN.LT.0.) THIGH = I / 10. + 1.0
      RLOW = 0.
11    RHIGH = RESID(THETA1,THIGH,ALPHA,SIGN)
      IF(RHIGH.GT.0.) GO TO 10
      RLOW = RHIGH
      TLOW = THIGH
      THIGH = THIGH + 0.5
      IF(THIGH .GT.6.)GO TO 16
      GO TO 11
10    IF(RLOW .LT.0.) GO TO 14
      TLOW = THIGH - 0.5
13    RLOW = RESID(THETA1,TLOW,ALPHA,SIGN)
      IF(RLOW.LT.0.) GO TO 14
      IF(SIGN.LT.0..AND.TLOW.LT.0.5-THETA1) GO TO 16
      TLOW = TLOW - 0.5
      IF(TLOW + 0.1)16,13,13
14    ITER = 0
C     START OF THE ITERATION LOOP
2     TNEXT = TLOW -(THIGH - TLOW) / (RHIGH - RLOW) * RLOW
      ITER = ITER + 1
      IF(ITER.LT.20) GO TO 8
C     ITERATION NUMBER EXCEEDS 20
      GO TO 6
8     RNEXT = RESID(THETA1,TNEXT,ALPHA,SIGN)
C     TEST FOR CONVERGENCE
      IF(ABS(RNEXT).LT..00001)GO TO 6
C     PUT TLOW  = TNEXT IF RNEXT IS NEGATIVE.
      IF(RNEXT.LT.0..AND.ABS(RNEXT).LT.ABS(RLOW))GO TO 4
C     SET THIGH  = TNEXT IF RNEXT IS POSITIVE.
      IF(RNEXT.GT.0..AND.ABS(RNEXT).LT.ABS(RHIGH))GO TO 5
C     IF NONE OF THE ABOVE TESTS IS SATISFIED, AND BOTH RHIGH AND RLOW
C        ARE OF THE CORRECT SIGN, USE THE AVERAGE OF THE TWO VALUES
C        OF THIGH AND TLOW.
      IF(RLOW.LT.0..AND.RHIGH.GT.0.) GO TO 12
16    FAILT(K) = THETA1
      FAILA(K) = ALPHA
      K = K + 1
      GO TO 15
C     THE CORRECTIONS TO THIGH AND TLOW FOLLOW
4     TLOW = TNEXT
      RLOW = RNEXT
      GO TO 2
```

```
5     THIGH = TNEXT
      RHIGH = RNEXT
      GO TO 2
12    TNEXT = THIGH + TLOW / 2.
      RNEXT = RESID(THETA1,TNEXT,ALPHA,SIGN)
      IF(RNEXT) 4,6,5
C     THE PROCESS HAS CONVERGED, FIND THE SHAPE NOW.
6     THETA2 = TNEXT
      COSA1 = COS(THETA1 + ALPHA)
      SINA = SIN(ALPHA)
      COSA = COS(ALPHA)
      SINA1 = SIN(ALPHA + THETA1)
      R2 = R1 *(SINA - SINA1) / (SIN(THETA2-ALPHA) + SINA)
      BASE = -R1 * COSA1 + (R1 - R2) * COSA + R2 * COS(THETA2 - ALPHA)
      R1 = R1 * 10. / BASE
      R2 = R2 * 10. / BASE
      BASE = 10.
      ARC = R1 * THETA1 + R2 * THETA2
      D = R1 * SIN(THETA1) / SINA
      X1 = -R1 * COSA1
      X2 = X1 + (R1-R2) * COSA
      Y1 = -R1 * SINA1
      Y2 = Y1 + (R1-R2) * SINA
      BETA = R1 * THETA1 / ARC
      R2R1 = R2 / R1
      EN =.FALSE.
      IF(J.EQ.31) EN =.TRUE.
     0IF(NCARD.LT.495)WRITE(7,106)THETA1,ALPHA,THETA2,R2,D,BETA,R2R1,EN,
     1NCARD
      NCARD = NCARD + 1
     0WRITE(6,104)R1,THETA1,ALPHA,THETA2,R2,ARC,BASE        ,D,BETA,R2R1,
     1X1,Y1,X2,Y2
      NLINE = NLINE + 1
      IF(NLINE.LE.28) GO TO 15
      NLINE = 1
      WRITE(6,101)
15    CONTINUE
      K = K - 1
      WRITE(6,105)
      DO 17 I = 1,K,4
17   0WRITE(6,103) FAILT(I),FAILA(I),FAILT(I+1),FAILA(I+1),FAILT(I+2),FA
     1ILA(I+2),FAILT(I+3),FAILA(I+3)
      GO TO 1
7     WRITE(6,107)NCARD
      CALL EXIT
101  0FORMAT('1 SHAPE OF MEMBRANE WITH SIMPLIFIED PRESSURE LOADING'/
     1       '0    R1    THETA 1   ALPHA   THETA 2    R2       L        S
     2  D         BETA  K=R2/R1    X1        Y1        X2       Y2 ')
102   FORMAT(F5.0,I5,F5.0)
103   FORMAT(' ',4(2F8.2,6X))
104   FORMAT('0',15F8.3)
105  0FORMAT('1  THE FOLLOWING CASES DID NOT WORK'/'0  THETA 1    ALPHA
     1     THETA 1    ALPHA        THETA 1    ALPHA         THETA 1    ALPHA')
106   FORMAT(8F9.5,L2,I6)
107   FORMAT('0',10X,'NCARDS',I6)
      END
      FUNCTION RESID(THETA1,THETA2,ALPHA,SIGN)
     0B1 = SIN(THETA2)* COS(ALPHA) - SIN(ALPHA) * COS(THETA2) + SIN(ALPH
     1A)
     0B2 = SIN(THETA1) * COS(ALPHA) + COS(THETA1) * SIN(ALPHA) - SIN(ALP
     1HA)
     0B3 = COS(THETA2) * SIN(THETA1) + COS(THETA1) * SIN(THETA2)- SIN(TH
     1ETA1) - SIN(THETA2)
      RESID =(THETA1 * B1 - THETA2 * B2 + 1.5707963 * B3) * SIGN
      RETURN
      END
```

<u>Program for generalized numerical solution of the shape</u>

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PIN(40),PCOEF(20),X(181),Y(181),DR(180),DT(180)
      COMMON NTERMS,NARCS,PCOEF
      COS(X) = DCOS(X)
      SIN(X) = DSIN(X)
      ABS(X) = DABS(X)
      ATAN(X) = DATAN(X)
      SQRT(X) = DSQRT(X)
      CALL TRAPS(1,1,1,0,1,1)
1     WRITE(6,109)
      READ(5,101,END=10) PINT,NPRES,NTERMS,NARCS,BASE,ARC
      READ(5,102)(PIN(I),I=1,NPRES)
      WRITE(6,114) PINT,BASE,ARC
      WRITE(6,115)(PIN(I),I=1,NPRES)
      NT = NTERMS + 1
      CALL FORCOS(PIN,NPRES,NT     ,PCOEF)
      WRITE(6,116)(PCOEF(I),I=1,NT)
      TOL = 0.0001 * BASE
      PINT = PINT - PCOEF(1)
      DO 2 I = 2,NT
2     PCOEF(I-1) = PCOEF(I)
      T = PINT * BASE / 2.0
      AOB = ARC / BASE
      ITER = 0
      N1 = NARCS + 1
      DL = ARC / NARCS
      X(1) = -BASE / 2.
      Y(1) = 0.
      ANGLE = 1.5707963
3     THETA = ANGLE
      ITER = ITER + 1
      DO 4 I = 1,NARCS
      PRES = PINT - Q(I)
      IF(PRES.LT.1.0D-10) GO TO 11
      R = T / PRES
      DTHETA = DL / R
      CX = X(I) + R * SIN(THETA)
      CY = Y(I) - R * COS(THETA)
      THETA = THETA - DTHETA
      X(I+1) = CX - R * SIN(THETA)
      Y(I+1) = CY + R * COS (THETA)
      GO TO 4
11    X(I+1) = X(I) + DL * COS(THETA)
      Y(I+1) = Y(I) + DL * SIN(THETA)
4     CONTINUE
      CHORD = SQRT((X(N1)-X(1))**2 + (Y(N1)-Y(1))**2)
      ERR = CHORD - BASE
      IF(ABS(ERR) - TOL) 6,6,5
5     T = T * BASE / CHORD
      ANGLE = ANGLE - ATAN(Y(N1)/(X(N1)-X(1)))
      IF(ITER-15)3,9,9
6     IF(ABS(Y(N1)).LT.TOL) GO TO 7
      ANGLE = ANGLE - ATAN(Y(N1)/(X(N1)-X(1)))
      IF(ITER.LT.17) GO TO 3
7     DO 8 I = 1,NARCS
      THETA = I * 3.1415927 / NARCS
      DX = X(I+1) + BASE / 2. * COS(THETA)
      DY = Y(I+1) - BASE / 2. * SIN(THETA)
      DT(I) = DX * SIN(THETA) + DY * COS(THETA)
```

```
 8      DR(I) =-DX * COS(THETA) + DY * SIN(THETA)
        WRITE(6,110) PINT,BASE,ARC,AOB,ITER,T
        WRITE(6,111)(X(I),Y(I),I=1,N1)
        WRITE(6,112)0.0,0.0,(DR(I),DT(I),I=1,NARCS)
ON*   OTHER COMPILERS MAY NOT ALLOW EXPRESSIONS IN OUTPUT LISTS
ON*   OTHER COMPILERS MAY NOT ALLOW EXPRESSIONS IN OUTPUT LISTS
        GO TO 1
 9      WRITE(6,110) PINT,BASE,ARC,AOB,ITER,T
        WRITE(6,111)(X(I),Y(I),I=1,N1)
        GO TO 1
 10     CALL EXIT
 101    FORMAT(F10.5,3I10,2F10.5)
 102    FORMAT(8F10.5)
 109    FORMAT('1')
 110    FORMAT(' PINT ',G13.6,' BASE ',G13.6,' ARC ',G13.6,' AOB ',G13.6,
       1' ITER ',I3,' TENSION ',G13.6)
 111    FORMAT('0',4(6X,'X',12X,'Y',10X)//(4(' ',2G13.6,3X)))
 112    FORMAT('0'4(4X,'RADIAL',5X,'TANGENTIAL',5X)//(4(' ',2G13.6,3X)))
 113    FORMAT('0',6G21.13)
 114    FORMAT('0 PINT ',G13.6,'  BASE ',G13.6,'  ARC LENGTH ',G13.6)
 115    FORMAT('0 PRESSURE AS SUPPLIED:'/(' ',8G13.6))
 116   0FORMAT('0 THE COEFFICIENTS OF THE FOURIER PRESSURE SERIES ARE:'/
       1 (' ',8G13.6))
        END

        FUNCTION Q(I)
        IMPLICIT REAL*8(A-H,O-Z)
        COMMON NTERMS,NARCS,PCOEF(20)
        COS(X) = DCOS(X)
        Q = 0.
        X = (I-0.5) * 3.1415927 / NARCS
        DO 2 J = 1,NTERMS
 2      Q = Q +PCOEF(J)* COS(X * J)
        RETURN
        END

        SUBROUTINE FORCOS (P,N,NTERMS,COEF)
        IMPLICIT REAL*8(A-H,O-Z)
C----
C---- THIS SUBROUTINE CALCULATES THE FOURIER COEFFICIENTS FOR A FUNCTION
C---- DEFINED ON THE INTERVAL  '0 ,PI'  THE FUNCTION IS P WITH 'N'
C---- POINTS, THE FIRST AT 0 AND THE LAST AT PI.  THE COEFFICIENTS ARE
C---- STORED IN COEF AND THERE ARE NTERMS OF THEM.      NOTE THAT THE
C---- EXPANSION IS A HALF RANGE EXPANSION IN COSINE TERMS ONLY.
C---- INTEGRATION IS DONE BY MEANS OF SIMPSON'S '1/3' AND '3/8' RULES
C----
        DIMENSION P(N),COEF(NTERMS)
        COS(X) = DCOS(X)
C---- ONCE THROUGH THE LOOP THAT STARTS HERE FOR EACH COEFFICIENT.
        DO 1 I = 1,NTERMS
        COEF(I) = 0.
        WEIGHT = 1.
C---- THIS LOOP DOES THE INTEGRATION
        DO 2 J = 1 , N
        X = (J-1) * 3.1415927 * (I-1) / (N-1)
        COEF(I) = COEF(I) + P(J) * COS(X) * WEIGHT
C---- SELECT THE CORRECT WEIGHTING FACTOR FOR THE NEXT TERM OF THE SUM.
        IF(WEIGHT.LE.2) GO TO 3
        IF((N-J.GT.4.OR.N-J.EQ.3).AND.WEIGHT.EQ.4.) GO TO 4
        IF(N-J.GT.3) GO TO 5
```

```
      IF(N-J.GT.1) GO TO 6
      IF(WEIGHT .EQ.4) GO TO 7
      WEIGHT = 1.125
      GO TO 2
3     WEIGHT = 4
      GO TO 2
4     WEIGHT = 2.
      GO TO 2
5     WEIGHT = 2.125
      GO TO 2
6     WEIGHT = 3.375
      GO TO 2
7     WEIGHT = 1.
2     CONTINUE
      COEF(I) = COEF(I) *2. / 3. / (N-1)
      IF(I.EQ.1) COEF(I) = COEF(I) / 2.
1     CONTINUE
      RETURN
      END
```

APPENDIX B

HISTORICAL BACKGROUND AND LITERATURE SURVEY

In recent years, temporary and portable buildings have been investigated in some detail. One type of building that is very well suited to this application is the air supported building. This has become a feasible structure with the use of modern materials. The requirements for a material are difficult to meet: airtightness, flexibility, good durability, resistance to creep, and the ability to retain these properties at whatever temperatures the structure is used.

Many materials satisfy some of these requirements, but few are satisfactory. Fabrics, especially the natural ones are not sufficiently airtight. Rubber can be used to make a fabric airtight but it deteriorates quickly. Metals and foils are airtight but cannot withstand the repeated flexing that is encountered with normal wind loading. Synthetics are best suited for these structures. The fabrics available can be coated with a durable layer of plastic or neoprene which makes them sufficiently airtight. These fabrics are also lighter than the natural fabrics which allows lower inflation pressures and thus simplifies the problems of entry and exit.

In most cases, inflatable structures are not suitable for use as permanent structures. They are limited to single story construction, they require inflation equipment to keep them up, and they give little resistance to heat loss so that they are expensive to heat. They are, however, suited for temporary applications where their light weight, portability, and low initial cost can offset the additional costs of heating, and the need for inflation. Since setup is very easy, they are often used for buildings that must be moved.

Inflated buildings can be grouped into three types depending on the location of the pressurized air. In the first type, the air is contained in small tubes at up to 100 psi. These tubes become rigid and act as a frame which supports the covering material which resembles a tent. In the second, the structure has a double wall, the air is contained between these two walls which are held together by regularly spaced cords. In the third type, the entire interior of the structure is pressurized. This is the type that will be discussed here. It is the simplest form of air supported structure and requires the lowest pressure to support it. The pressure must be contained at the entrance by the use of some form of airlock such as a revolving door, or simply two doors with suitable closers.

The second and third types of buildings are more interesting than the first in connection with this study so they will be examined in some detail. In the second type, the air pressure must be great enough to support the weight of both layers of fabric as well as resisting any external loads. The entire wall may be considered as a beam, in which the fabric can take only tension and the air will only provide compression. Failure will occur when the bending moment is great enough to make the tensile force in one membrane zero.

The air in the third type of structure can support it if the internal pressure is greater than or equal to the weight of the fabric. If an external load is applied, then the pressure must be increased by the magnitude of the load. If, however, the load is applied only to a small area, and it is not objectional to have a large deformation, then the membrane may be allowed to distribute the load and the pressure can be reduced.

The idea of using air pressure to support a structure was apparently introduced by Dr. Fredrick William Lanchester, an English engineer and researcher. In his patents of 1917, he proposed the use of a large shallow inflated dome for a field hospital. He planned and prepared a model, but the structure was never built.

In 1942, Herbert H. Stevens designed a factory that was to have an air supported roof. He proposed to make a shallow inverted saucer shaped roof about 1200 feet in diameter and 60 feet high. The roof material was to be 18 gauge steel with a layer of an insulating board and a tar and felt protecting cover and was to be supported by an internal pressure of 0.06 psi. This factory was not built either.

The first structure that was built was made and was built by Walter Bird who started to work on prototypes around 1946. He went into commercial production around 1956, but by that time had built a number of structures. Since that time, many buildings have been built by a number of firms. Some of these will be mentioned to indicate the uses that have been made of inflatable structures.

The first building to be erected was a radome built for the US government in 1949. It is necessary to protect radar equipment from the weather in far northern sites but the protective covering should interfere as little as possible with the radar. Fabrics are good for this application and air is a suitable method of supporting the fabric that has a minimum of interference with the electromagnetic waves.

A swimming pool cover and a tennis court cover (100,160) are fairly typical of the use made of many inflatable structures. These two domes are used at a school to make it possible to use these facilities all year round at a much lower cost than with a conventional building.

These structures have been used to advantage for the sheltering of construction sites. In the East Kootenay area of B.C. a large dome has been used to shelter crews assembling a large dragline in winter which was possible in spite of low temperatures and snow. In England, an inflated dome has been used to shelter the installation of transformers from rain to avoid corrosion. In order to reduce the size of the bubble, long cones were fitted to it and cables were passed through the vertices for hoisting so that the crane did not have to be inside the structure.

Another application for inflated structures is a small dam. In 1956, N. M. Imbertson devised this type of dam and since 1957 Firestone Co. has been producing the Imbertson Fabridam. It is only used for low dams since the stresses rise as the water height rises. The dams have been made 15 to 20 feet high and higher dams are anticipated. These dams are supported by filling with water although air could be used as well. In case of sudden freeze up, they can be heated to protect them from damage.

Descriptive Literature

As mentioned in the introduction, many uses have been found for air supported structures. In the first part of this survey, references to these applications will be listed. Most of these articles are not technical but descriptive. They serve to show the uses to which these structures may be and have been put.

Pearson (91) has given a history of inflatable structures and has summed up what has been done so far. Stevens (118) described a proposal for an air supported roof for a factory which was never built but is one of the first plans of its kind. Inflated frame buildings are described in three anticles (150, 152, 153). These buildings are now commercially available with up to 60 foot spans and in units 120 feet long. Inflated buildings and structures from here on are taken as refering to buildings in which the entire interior is pressurized.

A French company has been producing inflated buildings for sports activities and other applications that are described in a French article (157). Some buildings and also some other applications of inflated structures such as furniture, storage tanks for liquids, etc. have been described and illustrated in two articles (145, 151). Two articles in Engineering (146, 148) show an application for air supported buildings and discuss some of the factors that must be taken into account when a building of this type is being designed. For example, the decision of what height to select is determined by the needs of the application, but it is affected by the fact that for relatively high structures the wind has more effect while for low structures, the stresses will be much higher if a given overpressure is applied. Roland (101) shows a number of uses for inflatable structures in an article. Otto and Stomeyer (89) give a good discussion of inflatable buildings and include a number of applications. Otto (87) has a thorough discussion of inflated buildings and other structures in a large chapter of a book. Allison (1) describes air supported buildings and also double wall structures that have the pressurizing air only between the two walls and buildings that can stand by themselves but are inflated during severe wind conditions to stabilize them. Tesch (122) describes the uses of inflated houses and buildings that have been used in Sweden. He discusses their advantages and disadvantages and describes the construction details. Rawlings (96) discusses the use of metal plates for the membranes of inflated structures for storage tanks and roofs. Williams (133) describes the basics of air supported buildings and discusses their use in agricultural applications.

Some particular buildings have been described specifically in various articles. The possibility of using them for food storage has been considered (141). Two buildings that have been used for sports activities in a school in Litchfield, Conn. have been described in (160) and by Robertson (100). Two good descriptions of the Atomic Energy Pavilion used throughout South America are given by Otto (88) and Allison (2). A radio and television pavilion in Europe has been illustrated in (142). A large auditorium and display center (144) that was used to promote air travel has a 100 foot diameter and five foot thick walls. This building was not pressurized internally, only the walls were inflated. A California municipality used an aluminum coated vinyl-nylon roof to cover its water reservour (162) which is supported by air pressure.

A number of inflatable structures have been used to protect construction sites. A coal mining site in British Columbia was sheltered by a large dome so that construction could proceed through the winter on a large dragline that was being assembled (147). A similar building was used in Scotland to cover a factory extension (149) and in Holstein (161) to cover the site where houses were being built. A large stockpile of potash was stored inside an inflatable warehouse in St. Paul (159). For this type of application, there is no need to keep the roof inflated except during the loading and unloading operations. Jump (53) describes the use of an inflated tent to shelter a transformer while it is being erected in Great Britain. This tent speeds the erection considerably since the weather is not a factor to reckon with (158). An army field hospital (155) with tubular air cells has been developed. A gas turbine is used to support and to power the building according to Sterrett (117). Monego (80) and Mayer (76) describe the work done by military and NASA workers in the field of expandable structures.

Technical Literature

A considerable number of theoretical papers have been written on the subject of membranes and inflatable structures in recent years. A survey of these was done as part of this project. Habip (41) has reviewed a number of papers in this field that are not widely distributed or indexed and so they have been included in this survey for completeness. He notes that another survey has been done recently on expandable structures in space by Evans (29).

Leonard (61, 62) has studied the behaviour of inflated shells of revolution, and Forster and Schlusser (35, 36) have found the stresses and displacements of a spherical dome subjected to wind loads. The load that was used was taken from a building code.

Parisi (90) describes the use of inflatable cushions for crash protection in automobiles, and Kemmerer et al. (55) analyse this use in more detail.

The vibration and other dynamic deflections of membranes with various loadings and many shapes has been examined by a number of researchers. The vibration of a flat membrane has been studied by many of these. Weidman (131) studied the flutter of panels in a supersonic flow, Johns (49) has examined the flutter and divergence of panels for the subsonic and supersonic cases. Olcer (86) gives a general solution to the equation of a vibrating membrane, Sharp (110) gives a transform solution, and Leitner (67) discusses the vibration of a circular membrane as do Boyd (18) and Li et al. (68). Huffington discusses the vibration of membrane foils (47) and Mason (75) analyses the L-membrane problem. There are more papers on the vibration of flat membranes but only a sampling has been given since they do not concern the results in this thesis. The object in including them was to indicate that work has been done in this field.

The vibration of shells has been studied by using the membrane theory of shells. Ross (102) has found the membrane frequencies for a spherical shell, Hwang (48) has examined the more general axisymmetric shell, McIvor and Popelar (79) examined the dynamic stability of a shallow cylindrical shell and Ariaratnam and Sankar (8) have studied the dynamic snap through of shallow arches. Armenakas (7) discusses the accuracy of some dynamic shell theories.

Another shape that has been studied is the toroid. Leipins (66) has analysed them using a linearized theory.

The dynamics of inflatable structures has been examined by Leonard (63) and by Burggraf (19). The equations of motion of inflatable plates with moderately large deflections have been given by Bernstein (15).

Most of the above works were analytical in their approach. Some experimental results are included in the following papers. Stroud (120) has done static and vibration tests on inflatable plates. Jouriles (50) has done some torsion tests on fabric cylinders. Synthetic fabric cylinders have been tested by Loudenslager and Weth (70), and Slivka has done further tests on fabric structures (113). A number of tests on inflatable metal fabric structures have been summarized (156). These include bending, torsion, and combined loading on cylinders and plates. The decay of vibrations in an inflatable airfoil has been reported by Folks and Cross (34); Nickola (84) applied the Moire method to the measurement of the deflection of thin membranes.

The fabrics used for these structures have been studied both experimentally and analytically. Assaf (9) has discussed the types of fabrics being used. Williams and Goodman (134) investigated the materials to be used for a toroidal space structure. Shearing strains in fabrics were analysed by Costakos and Topping (23). Hoffman and Topping (44) analysed the interlaminar shearing stresses. Leonard (65) studied the shear stiffness of fabrics. Houmard (45) studied the effect of internal pressure on the torsional stiffness of fabric cylinders. Jouriles (52) measured the strains in fabric strips cyclically loaded and Topping (127) discussed the biaxial stress problem in fabrics. Cross and Madder (24) reviewed both theoretical and test data on fabrics. Davidson (25) and Mader (72) have both analysed the biaxial problem theoretically and Mader included some experimental work. Popper (94) has given rupture criteria for some biaxial loading conditions. Zendre and Deaton (140) tested the stiffness of stressed fabrics, Baker and Mikolajewski (12) considered the anisotropy of inflatable wing fabrics under biaxial stress. Backer et al. (11) have investigated the relationship between the structural geometry of a fabric and its properties.

The elementary mechanics and equations of equilibrium for inflatable structures have been investigated by various authors under a number of conditions. Wright (136) has analysed various configurations with rather complex shapes but using simple loadings. Trostel (87) has summarized membrane theory including an approximation for wind and snow loads for a simplified load on a spherical dome. Auge (10) has found the necessary and sufficient equilibrium conditions in tensor form for membranes. Simmonds (112) has given general equations of equilibrium and some solutions for uniform centrifugal loadings. Fang (30) has developed a nonlinear theory of elastically orthotropic inflatable shells and has applied it to the axisymmetrical deformation of shells of revolution. Hiderman and Bukhin (16) developed a small deflection theory for a membrane shell constructed of netting with a filler that has no rigidity and extensible fibers. Leonard (64) has given a general discussion of inflatable structures including material properties, buckling and collapse loads, deflections, vibration frequencies, and a linear theory of inflatable plates.

The study of large deformations in membrane shells and inflated bodies has been done by a number of workers. Solodilov (114) has studied the large deflections of a circular membrane attached to a deformable ring at its edge. Sherbourne and Lennox (111) have studied the large deflections of annular membranes, while Wilson and Slock (135) have worked on the deformation of a pressurized strip. Basuli (14) has studied the deflection of a certain cylindrical panel. Ross has calculated the large deflections of an inflated cylindrical tent (103). Rossettos (105) has analysed the large deflections of shallow spherical shells assymptotically. The large deflections of a membrane shell loaded by liquid pressure have been studied (154). The deformations of a locally loaded shallow spherical membrane have been found by Piechocki (92); and Marketos (74) has analysed the wrinkling of a spherical membrane. Goldberg (39) has analysed the more general problem of the large deformations of a rotationally symmetric membrane by linearizing the large deflections while Flugge and Chou (33) have examined the large deformations of a torus using a material that obeys Hooke's law. Grigor'ev (40) has examined the equilibrium of a membrane shell of revolution at large deflections. Moskowitz (81) has analysed the large deformations of fabric structures.

Schlecter (107) and Stein and Hedgepeth (116) have examined the conditions of wrinkling and have analysed membranes that are wrinkled.

Because many of the problems that are found do not have closed form solutions and many of the results are in the form of infinite series or else are approximations, numerical methods have been applied to the solution of membrane problems. Hart-Smith and Crisp (42) give a numerical solution for the large elastic deformations of thin rubber membranes. Archer (6) uses a finite difference methodto integrate Reissner's equations for the non-linear equations of shells of revolution. Halnins and Lestingi (54) use a type of numerical solution to analyse the elastic shell of revolution. He deals with the bending of the shells. Oden and Kubitza (85) use a finite element method to analyse the deformation of inflatable structures that are made of fabrics with a number of different constitutive relations. Andreeva (4) has solved the large deformation of a corrugated membrane numerically.

The stresses in membranes have been dealt with by Ames (3) for the axisymmetrical case and by Forster and Schlusser (35) for wind loads on a sphere. Ilias (28) has studied the problem of finding the stresses when the displacement boundary conditions are given.

Yen (137) superimposed the deflections from a small perturbation on the large deflections obtained by highly pressurizing a sphere. Wilde (132) examined the problem of designing an unstrained shape so that the specified final shape is obtained.

The characteristics of a membrane which has one or more holes have been studied by some workers. Vyas (95) used iteration and conformal mapping to analyse the effect of cutouts; Piechcki (93) examined the case where the hole is filled with a solid plug which would simulate the effect of doorways in the structure; and Cherepanov (20) examined the buckling of a membrane with holes under tension.

A number of reports deals with the deflections of membranes where one of the principal factors is the material's properties. MacFadden (71) describes the design and tests of a polypropylene balloon. Hart-Smith (43) inflated a flat rubber or mylar film to get a paraboloid. Foster (37) calculates the deformations of axisymmetric membranes made of Neo-Hookean materials, and L. Lomen (69) studied the instability due to thinning of the material walls of a sphere when inflated.

Some of the work done on shells can be applied to membranes since the membrane theory of shells is often used. Van Dyke (129) has studied a shell that is similar to the shapes used for some air supported strictures. Narayanaswami (83) discussed the nonlinear membrane problem of shells of revolution, and Reissner (97) worked on the finite inextensional deformations of shallow elastic shells with particular emphasis on the boundary conditions.

Rossettos (104) has examined a case for deep membranes of revolution which have nonlinear behaviour near the edges. Sujata (121) has discussed some features of framework stabilized inflatable structures. Schindler (108) has studied foldability. Donnel (27) has analysed the application of concentrated loads on inflatable structures by the use of a catenary curtain. Foster has studied the inflation of a plane circular membrane (38). Zagieboylo (139) has studied the effect of shock waves on inflatable structures and the effect of the structure on the wave inside it.

Some work has been done on particular applications. Waslton (130) discussed the design and analysis of tires as reinforced membranes. Kuby (58) applied a nonlinear membrane theory to pressure vessels in order to analyse the end caps of a cylindrical vessel without using the conventional discontinuity stresses. Deaton and Zendre (26) compared the stiffness of air filled structures to that of similar ones filled with a foam. Bird (17) discussed several aspects of the use of inflated domes for radomes. He summarized the results of up to two years of use on three radomes that were built. Anwar (5) analyses the use of air filled and water filled fabric dams. He examines the static case as well as the dynamic cases where there is water flowing over the dam.

The inflation of a toroidal membrane has been analysed by Sanders and Liepins (106), Murthy and Kiusalaas (82), and by Kydoniefs and Spencer

(59). Reissner (99) has studied the bending of toroidal membranes but see the review of his paper in Applied Mechanics Reviews vol. 17. review 3763 about errors in the paper. Schuerch and Schindler (109) have examined the problem of folding in connection with toroidal shells. Fernandez-Sintes (32) and Fernandez-Sintes and Nachbar (31) have worked on the related problem of finding the deformations in a highly elastic thin membrane subjected to uniform ring loads.

Another particular application that has received some attention is the inflated beam. Barton and Topping (13) have studied beams. Comer and Levy (21) have studied cylindrical cantilever beams. Kovalevsky and Rish (56) have studied fabric cylinders under various loading conditions. Reissner and Weinitschke (98) have examined finite pure bending of circular cylinders. Jouriles and Johnson (51) have tested fabric cylinders. Corneliussen and Shiels (22) have studied the stability of cantilever beams. Topping (128) has calculated the shear deflections and buckling characteristics of inflated members including the effect of the beam edges. Topping (123) has also examined the resistance of partially wrinkled cylinders to bending and shear loads, prismatic inflatable members, and the wrinkling and collapse of fabric cylinders in bending. The last of these is also considered by Topping and Chuperkoff (124). The applicability of deep wrinkler theory in assessing maximum bending moments of cylindrical beams has been investigated (143). Eliptical beams have been analysed by Mader (73); tapered cylinders by Kovalevsky and Rish (57); and uniformly loaded wings have been studied by Topping et al. (126).

An extension of inflatable beams is the inflatable panel. Inflatable panels consist of two layers of fabric separated by air under pressure

pressure and held together by drop cords at regular intervals. Stein (115) has examined inflatable panels. Houtz (46) has studied the buckling of panels in bending. Yurick and Marketos (138) have calculated the strains in a panel due to the inflation pressure. McComb (77) has developed a linear theory of elastic behaviour of plates and Stoffmacher (119) has given the nonlinear equilibrium equations and boundary conditions for plates with nonlinear strain-displacement relations. Topping and Marketos (125) have studied the effect of hysteresis and creep on the bending strength of panels. McComb and Leonard (78) have studied the effects of using slanted drop cords between the two membranes in order to increase the shear rigidity. Kyser (60) has studied the large deformations of a plate made up of an array of contiguous tubes.

REFERENCES

The first references are those referred to in the body of the thesis.

McLachlan, N. W. Theory and Application of Mathieu Functions, Oxford: Clarendon Press, 1947.

Roshko, A. "Flow Past a Circular Cylinder at High Reynolds Number," Journal of Fluid Mechanics, Vol. 10, no. 3 (May, 1961), pp. 345-356.

The references that follow have been referred to by number in Appendix B Historical Background and Literature Survey.

1. Allison, David. "Those Ballooning Air Buildings," Architectural Forum, July 1959, p. 134.

2. Allison, David. "Great Balloon for Peaceful Atoms," Architectural Forum, Nov. 1960, pp 142-145 ≁ .

3. Ames, W. F. "Stresses in Cylindrically Symmetric Membranes Reinforced with Extensible Cords", Franklin Institute Journal, Vol. 272 n. 3 Sept. 1961, pp 185-190.

4. Andreeva, L. E. "Numerical Solution of Large Deflection in Corrugated Membrane," Inzhenernyi Zhurnal, Mekhanika Tverdogo Tela, May-June 1967, pp. 83-89.

5. Anwar, H. O. "Inflatable Dams", ASCE Hydraulics Div., Vol. 93 n. Hy3, (1967), pp. 99-119.

6. Archer, R. R. "On the Numerical Solution of Non-linear Equations for Shells of Revolution", Journal of Mathematics and Physics, Vol. 41 n. 3, pp. 165-178.

7. Armenakas, A. E. On the Accuracy of Some Dynamic Shell Theories, ASCE Journal (Engineering Mechanics), Oct. 1967, pp. 95-110.

8. Ariaratnam, S. T. and Sankar, T. S. Dynamic Snap-Through of Shallow Arches with Stochastic Loads. AIAA journal Vol. 6 n 5 (May 1968), pp. 798-802.

9. Assaf, J. E. Military Applications of Coated Fabrics, American Dyestuff Reporter, Vol. 55 (May 9, 1966), pp. 376-381.

10. Auge, J. Applications of External Differentiation to the Theory of Membranes, International Association for Shell Structures, Bulletin n. 14 (1962) 18 pp.

11. Backer, S. et al. Textile Research Journal, Vol. 18, p. 650 (1948); ibid, Vol. 21, pp. 453, 635, 703 (1951).

12. Baker, A. and Mikolajewski, E., ASTIA, AD-159604 (1957).

13. Barton, J. A. and Topping, A. D., GER-10503 (1961).

14. Basuli, S., Large Deflection of a Certain Cylindrical Shell Panel, Journal of Applied Mechanics, Vol. 84 n 4 (Dec. 1962).

15. Bernstein, E. L. To Appear University of Florida (1964).

16. Biderman, V. L. and Bukhin, B. L. Equilibrium Equation for a Membrane Shell Constructed of Netting, Inzhenernyi Zhurnal Mekhanika Tverdogo Tela no. 1 (1966), pp. 81-89.

17. Bird, W. W., Large Air Supported Radomes for Sattelite Communications Ground Station Proceedings OSU-RTD Symposium On Electromagnetic Windows, 2-4 June, 1964, Vol. 5, 12 pp. (AD-605393).

18. Boyd, D. E., Dynamic Deformations of Circular Membranes, ASCE Journal (Engineering Mechanics), June 1966, pp. 1-16.

19. Burgraf, O. R., Some Dynamics Problems on Inflation of Bubbles in Space, NASA CR-203 52 pp. (1965).

20. Cherepanov, G. P., On Buckling Under Tension of a Membrane Containing Holes, Journal of Applied Mathematics and Mechanics, Vol. 27 n 2, pp. 405-420.

21. Comer, R. L. and Levy, S., AIAA Journal, vol 1 (1963), pp. 1652-

22. Corneliussen, A. H. and Shield, R. T. Finite Deformation of Elastic Membranes with Application to the Stability of an Inflated and Extended Tube, Archive for Rational Mechanics and Analysis, vol. 7 n 4, pp. 293-304 (1961).

23. Costakos, N. C. and Topping, A. D., GER-6594 (1955).

24. Cross, W. B. and Mader,W. M., GER-9133 (1958).

25. Davidson, D. A., ASD-TDR-63-485 (1963).

26. Deaton, J. W. and Zender, G. W., Stiffness Properties of Foam Filled Fabric Structures, NASA TN D-3267, 38 pp., Mar. 1966.

27. Donnell, L. H., Concentrated Loads on Inflated Structures, AIAA Journal, vol. 1 n 8, pp 1823-1828 (1963).

28. Elias, Z. M., Cartesian Formulation of Membrane Theory, ASCE Journal (Engineering Mechanics), April 1967, pp. 99-114.

29. Evans, G. R., Expandable Structures in Space: an Annotated Bibliography, SB-63-36, Lockheed Aircraft Corp., Sunnyvale, Calif., NASA accession no. N63-22729.

30. Fang, B., On a Theory of Inflatable Shells PhD Dissertation, University of Minnesota (1962).

31. Fernandez-Sintes, J. and Nachbar,W., Rotationally Symmetric Problems of Highly Inflatable Thin Membranes Subject to Ring Loads, SUNDAER no. 209, Dept. of Aeronautics and Astronautics, Stanford University (1964).

32. Fernandez-Sintes, J. Rotationally symmetric Problems of Highly Elastic Inflated Thin Membranes Subjected to Ring Loads Ph D Dissertation, Stanford University, 1965.

33. Flugge, W., Chou, S. C., Large Deformation Theory of Shells of Revolution, Journal of Applied Mechanics, vol 34 n 1, pp. 56-58, March 1967.

34. Foulks, E. D. and Cross, W. B., G E R-9156 (1959).

35. Forster, W. and Schlusser, K. H. The Membrane State of Stress of Wind Loaded Sperical Shells, Bauingenieur, vol. 42 n 1, pp. 21-29 (Jan. 1967).

36. Forster, W. and Schlusser,K.H., The Membrane State of Displacements of a Sperical Dome Subject to Wind Loads, Stavebnicky Casopis v 15 n 6, pp. 309-329 (1967).

37. Foster, H. O., Very Large Deformations of Axially Symmetrical Membranes Made of Neo-Hookean Materials, International Journal of Engineering Science, vol. 5 n 1, Jan. 1967, pp. 95-117.

38. Foster, H. O., Inflation of a Plane Circular Membrane, Journal of Engineering for Industry, vol 89 (Aug 1967), pp. 403-407.

39. Goldberg, M. A., A Linearized Large Deformation Theory for Rotationally Symmetric Membranes, ASME Transactions Ser. E vol 32, June 1965, pp. 444-445 or RAC-2236 Republic Avaiation Corp., Farmingdale, N.Y. (21 pp.).

40. Grigor'ev, A. S., Equilibrium of Membrane Shell of Revolution at Large Deformations Journal of Applies Mathematics and Mechanics, Vol 25 n 6 (1961), pp. 1618-1628.

41. Habip, L. M., A Review: Recent Work on Inflatable Structures, International Journal of Mechanical Sciences, vol. 7 n 2, pp. 149-152 (Feb. 1965).

43. Hart-Smith, L. J., Analysis of Inflatable Paraboloids, Mechanical and Chem. Eng. Trans. -- Australia Instn. Engrs., vol mc3 n 2 (Nov. 1967).

42. Hart-Smith, L. J. and Crisp. J. D. C. Large Elastic Deformations of Thin Rubber Membranes, International Journal of Engineering Science, vol 5 n 1 (Jan. 1967), pp. 1-24.

44. Hoffman, T. L. and Topping, A. D., GER-9938 (1960).

45. Houmard, J. E., GER-9931 (1960).

46. Houtz, N. E., GER-9167 (1959).

47. Huffington, N. J. (Jr.), Vibration of Membrane Foils, Martin Co. Research Report RR-74 39 pp. March 1966.

48. Hwang, C., Extensional Vibrations of Axisymmetric Shells, AIAA Journal, vol 3 n 1, pp. 23-26, Jan. 1965.

49. Johns, D. J., Some Panel Aero-elastic Instabilities, AGARD-474, Sept. 1963, 58 pp.

50. Jouriles, N., GER-6468 (1955).

51. Jouriles, N. and Johnson, J. E., GER-7126 (1955).

52. Jouriles, N., GER-7229 (1956).

53. Jump, L., Use of Inflatable Air Houses for Erection of Transformers on Site A E I Engineering, May-June 1966.

54. Kalnins, A. and Lestingi, J. F., On Non-Linear Analysis of Elastic Shells of Revolution, Journal of Applied Mechanics, vol. 34 n 1, pp. 59-64, March 1967.

55. Kemmerer, R. M., Chute, R., Hass, D.P., and Slack, W. K., Automatic Inflatable Occupant Restraint System, SAE paper 680033, Jan. 8-12, 1968, 19 pp.

56. Kovalevsky, L. and Rish, F. L., On the Structural Behaviour of Inflated Fabric Cylinders Under Various Loading Conditions in AFSC Aerospace Expandable Structures Conference Transactions AD-432006, May 1964, pp. 674-701.

57. Kovalevsky, L. and Rish, F. L., Structural Behaviour of Tapered Inflated Fabric Cylinders Under Various Loading Conditions in AFSC Aerospace Expandable Structures Conference Transactions, May 1965, pp. 659-685, AD-631406.

58. Kuby, G. H., Application of Non-Linear Membrane Theory to Pressure Vessels, Ph D Dissertation New York University, 1963.

59. Kydoniefs, A. D. and Spencer, A. J. M. Finite Inflation of Elastic Toroidal Membrane of Circular Cross Section, International Journal of Engineering Science, vol 5 n 4 (April 1967), pp. 367-391.

60. Kyser, A. C., A Contribution to the Theory of Pressure Stabilized Structures, NASA TN D-1919, 38 pp., May 1963.

61. Leonard, J. W., Behaviour of Pressure - Stabilized Inflatable Shells of Revolution, Ph D Dissertation, University of Illinois, 1966.

62. Leonard, J. W., Inflatable Shells (a three part article), ASCE Proceedings (Mechanics Division), April 1967, pp. 207-227; Dec. 1967, pp. 67-85; Oct. 1968, pp. 1231-1248.

63. Leonard, J. W., Non-Linear Dynamic Analysis of Structures, Final Report, NASA CR-97573 (1968), 33 pp.

64. Leonard, R. W., Brooks, G. W., McCombe, H. G. (Jr.)., Structural Considerations of Inflatable Re-Entry Vehicles, NASA TN-D-457 (1960), 23 pp. and Proceedings Recovery of Space Vehicles Symposium, Los Angeles, Calif. (1960), p. 62.

65. Leonard, R. W., J. Aero. Sci. vol 29, pp. 349- (1962); comment by A. D. Topping and author's reply, ibid, vol. 29, p. 757 (1962).

66. Leipins, A. A., Free Vibration of the Pre-Stressed Toroidal Membrane, NASA CR-58532, 59 pp., and AIAA Journal, Oct. 1965, vol 3 n 10, pp. 1924-

67. Leitner, A., Vibration in a Circular Membrane, American Journal of Physics, vol 35, pp. 1029-1031 (Nov. 1967).

68. Li, W. H. et al., Symmetric Vibration of Thick Circular Membrane, Journal of Applied Mechanics, vol 34, pp. 1020-1022 (Dec. 1967).

69. Lomen, D. O., Finite Deformation of Elastic Membranes with Application to the Stability of an Inflated Sphere Ph D Dissertation Iowa State University of Science and Technology 1964.

70. Loudenslager, O. W. and Weth, L. H., GER-845 (1946).

71. MacFadden, J. A., Polypropylene Balloon Study, Schjeldahl (G.T.) Co. Northfiels Minn., Oct 29 1964, 80 pp., AD-608098.

72. Mader, W. M., GER-9664 (1961).

73. Mader, W. M., GER-10505 (1962).

74. Marketos, J. D., GER 9341 (1959).

75. Mason, J. C., Chebyshev Polynomial Approximations for the L-Membrane Eigenvalue Problem, SIAM Journal on Applied Mathematics, vol 15 n 1, pp. 172-186 (Jan. 1967).

76. Mayer, N. J., NASA Research on Expandable Structures in AFSC Aerospace Expandable Structures Conference Transactions, May 1964, AD-432006, pp. 285-318.

77. McComb, H. G. (Jr.), NASA TN D-930 (1961).

78. McComb, H. G. (Jr.) and Leonard, R. W., J. Aero Sci., vol 29, pp. 476- (1962).

79. McIver, I. K. and Popelar, C. H., Dynamic Stability of a Shallow Cylindrical Shell, ASCE Proceedings (Engineering Mechanics), June 1967, pp. 109-128.

80. Monego, C. J., Air Supported Tents for Military Use in AFSC Aerospace Expandable Structures Conference Transactions, May 1964, AD-432006, pp. 400-431.

81. Moskowitz, G. D., Large Lateral Deflections of Textile Fabric Structures, Ph D Dissertation, Princeton Univ., (1963).

82. Murthy, M. V. V. and Kiusalaas, J., Toroidal Type Shells Free of Bending under Uniform Normal Pressure, Franklin Institute Journal, vol 282 (Oct. 1966), pp. 232-241.

83. Narayanaswami, O. S., Non-Linear Membrane Problem of Shells of Revolution, Ph D Dissertation Case Institute of Technology (1965).

84. Nickola, W. E., Dynamic Response of Thin Membranes by Moire Method, Experimental Mechanics, vol 6 n 12 (Dec. 1966), pp. 593-601.

85. Oden, J. T. and Kubitza, W. K., Numerical Analysis of Non-Linear Pneumatic Structures, NASA CR-84721.

86. Olcer, N. Y., General Solution of Equation of Vibrating Membrane Journal of Sound and Vibration, Nov. 1967, vol. 6 n 3, pp. 365-374.

87. Otto, F. (ed.), Tensile Structures, Volume 1, Pneumatic Structures, Cambridge, Mass., MIT Press, 1967.

88. Otto, F., Contribution a l'Architecture Pneumatique l'Architecture d'Aujourd'hui, vol 33, pp. 86-88 (June 1962).

89. Otto, F. and Stromeyer, P., Pneumatic Structures, AIA Journal, vol. 37 (April 1962), pp. 101-111.

90. Parisi, A. J., Latest Tests Prove Value of Instant Air Cushion, Product Engineering, Feb. 12, 1968.

91. Pearson, M., Pneumatic Structures, Architectural Review, vol. 142 (Oct. 1967), pp. 311-314 +.

92. Piechocki, W., Finite Deflections of a Locally Loaded Shallow Spherical Membrane Mechanika Teoretyczna i Stosowana, Vol. 2 n 2 (1964), pp. 45-58.

93. Piechocki, W., Finite Deflection of Spherical Membrane with a Central Hole. Acad. Polonaise Des Sciences - Bul. - Serie des Sciences Techniques, vol 11 n 12 (1963), pp. 757-762.

94. Popper, P. G., ASD-TDR-62-613 and ASD-TDR-62-457 (1962).

95. Ravindra, K. V., Cutouts in Membrane Shells, Ph D Dissertation Stanford University, 1966.

96. Rawlings, B., Inflated Ductile Metal Structures, Architectural Science Review, vol 10 n 2 (June 1967), p. 44.

97. Reissner, E. Note on Finite Inextensional Deformations of Shallow Elastic Shells, J. Math. Phys. vol 40 (1961), pp. 253-259.

98. Reissner, E. and Weinitschke, H. J., Finite Pure Bending of a Circular Cylindrical Tube, Quarterly of Applied Mathematics, vol 20 n 4 (Jan. 1963), pp. 305-319.

99. Reissner, E., On Bending of Pressurized Toroidal Membranes, J. Math. Phys., vol 42 n 1 (March 1963), pp. 38-46.

100. Robertson, N., Air Structures for School Sports, Case Study of Educational Facilities 9, Educational Facilities Laboratories, New York, 1964.

101. Roland, C., Inflatable Structures, Architectural Design, vol 36 n 7 (July 1966), pp. 340-360.

102. Ross, E. W. (Jr.), Membrane Frequencies for Spherical Shell Vibrations, AIAA Journal, May 1968, pp. 803-808, vol 6 n 5.

103. Ross, E. W. (Jr.), Aarge Deflections of an Inflated Cylindrical Tent, Journal of Applied Mechanics, December 1969, pp. 845-851.

104. Rossettos, J. N., Non-Linear Membrane Solutions for Symmetrically Loaded Deep Membranes of Revolution, NASA TN D-3297.

105. Rossettos, J. N., An Assymptotic Analysis for Large Deflections of Pressurized Shallow Spherical Membrane Shells, AIAA Journal, vol. 4 n 6 pp 1121-1123 (June 1966).

106. Sanders, J. L. (Jr.) and Leipins, A. A., Toroidal Membrane under Internal Pressure, AIAA Journal, vol 1 n 9 pp. 2105-2110 (Sept. 1963).

107. Schlechter, M., Buckling of Elastic Membrane on a Surface of Revolution, Quarterly of Applied Mathematics, vol 23 n 2, pp. 121-132 (July 1965).

108. Schindler, G. M., NASA TN D-1918 (1963).

109. Schuerch, H. U. and Schindler, G. M., A Contribution to the Theory of Folding Deformations in Expendable Structures with a Particular Application to Toroidal Shells NASA TN D-1690 28 pp. Dec. 1962; and AIAA Journal vol 1 n 4, pp. 873-878.

110. Sharp, G. R., Finite Transform Solution of Vibrating Annular Membrane, Journal of Sound and Vibration, July 1967, vol. 6 n 1, pp. 117-128.

111. Sherbourne, A. N. and Lennox, W. C., Elastic Large Deflections of Annual Membranes, ASCE Proceedings (Engineering Mechanics), vol. 92 n EM2, pp. 75-99.

112. Simmonds, J. G., The General Equations of Equilibrium of Rotationally Symmetric Membranes and Some Solutions for Uniform Centrifugal Loadings, NASA TN D-816 (1961).

113. Slivka,L.P., GER-6109 (1954).

114. Solodilov, Y. I., Large Deformations of a Circular Membrane Attached Along the Edge to a Deformable Ring, Soviet Engineering Journal, vol 5 n 2, pp. 316-322.

115. Stein, A. J., GER-8860 (1958).

116. Stein, M. and Hedgepeth, J. M., Analysis of Partly Wrinkled Membranes, NASA TN D-813, 32 pp., July 1961.

117. Sterrett, E., Turbine Engine Powers Emergency Ward Diesel and Gas Engine Progress, May 1965, vol 31 n 5, pp. 46-48.

118. Stevens, H. H. (Jr.), Air Supported Roofs for Factories Architectural Record, vol 92 (Dec. 1942), pp. 45-46.

119. Stoffmacher, G., Rept. SM-42817, Douglas Aircr. Co., Santa Monica, Calif. (1963).

120. Stroud, W. J., NASA TN D-931 (1961).

121. Sujata, H. L., Trans. Aerosp. Exp. Str. Conf., Dayton, Ohio (1963), p. 535.

122. Tesch, B., Inflatable Structures, Plastvarlden, vol 17 n 7-8, 9 Aug. 1967, pp. 72-74, 76, Sept. 1967, pp. 78-80 ≠.

123. Topping, A. D., GER-6400 (1954), GER-9876 (1960), GER-10282, (1961).

124. Topping, A.D. and Chuparkoff T, GER-7172 (1955).

125. Topping, A.D. and Marketos, J. D., GER-9098 (1958).

126. Topping, A.D. et al., GER-9245 (1961).

127. Topping, A. D., Aerospace Engineering, vol 20, pp. 18- (April 1961).

128. Topping, A. D., Shear Deflections and Buckling Characteristics of Inflated Members, Journal of Aircraft, vol 6 n 5, Sept. 1964, pp. 289-292.

129. Van Dyke, P., Tension Shell as a Membrane, AIAA Journal, vol. 4, pp. 2239-2241, Dec. 1966.

130. Walston, W. H. (Jr.), Design and Analysis of Inflated, Reinforced Membranes, Ph D Dissertation, University of Delaware, 1964.

131. Weidman, D. J., Experimental Flutter Results For Corrugation Stiffened and Unstiffened Panels, NASA TN D-3301, March 1966, 32 pp.

132. Wilde, P., Some Problems of Theory of Membranes Formed By Inextensible Cords, Archiwum Mechaniki Stosowanej, vol. 18 n 4 (1966), pp. 463-477.

133. Williams, I. L., Air Structures for Agriculture, Agricultural Engineering, vol 47 n 4 (April 1966), pp. 212-213 and ASAE report no. E920.

134. Williams, J. G. and Goodman, G. P., Structural and Materials Investigation of 1/8 Scale Model Space Structure of Toroidal Configuration and Filamentary Construction NASA TN D-2652, Feb. 1965, 4 Opp.

135. Wilson, P. E. and Slick, E. M., Finite Deformation of a Pressurized Membrane Strip, Journal of Applied Mechanics, vol. 32 n 2 (June 1965), pp. 451-453.

136. Wright, F. M., Elementary Mechanics of Flexible Membranes, SM-45750 (1963) 64 pp., Douglas Aircraft Co., Santa Monica, Calif.

137. Yen, C. L., A Study of Inflated Thin Shells, Ph. D Dissertation, Stanford University, 1966.

138. Yurick, R. A. and Marketos, J. D., GER-9526 (1959).

139. Zagieboylo, W., Shock Wave Interaction With Air-Supported Structures, Experimental Mechanics, Dec. 1967, pp. 532-536.

140. Zendre, G. and Deaton, J., ASTIA AD-261366 (1961).

141. Air Houses for Food Storage, Rubber and Plastics Age, Feb. 1964.

142. Air Supported Pavilion, Domus, n 455, pp. 22-27 (Oct. 1967).

143. Applicability of Deep Wrinkle Membrane Theory in Assessing Maximum Sending Moment of Inflated Fabric Cylinders and Packing Effects, (STAR 68:27925).

144. Astrosphere, Compressed Air, vol 73 n 1 (Jan. 1968), pp. 10-11.

145. Blow-Up Structures, Design n 233 (May 1968), pp. 67-71.

146. Carried On Air, Engineering, vol 187 (1959), p. 610.

147. Coal Mining Site Has Space-Age Look, Winnipeg Free Press, March 28, 1969, p. 23.

148. Design and Construction of Air Houses, Engineering, vol. 195, Jan. 18, 1963, p. 78.

149. Flexible Buildings Supported by Air Pressure, The Surveyor and Municipal Engineer, vol 123 n 3741, Feb. 15, 1964, pp. 63-64.

150. Inflatable Frame Buildings, Applied Plastics, vol 7 n 5, (May 1964), p. 23.

151. Inflatables -- Structures Built On Air, Product Engineering, vol 37 n 8 (April 11, 1966), pp. 77-79.

152. Portable Nylon Shelters With Inflatable Frame Engineering, vol 197 (1964), p. 395.

153. Walter Kidde Inflatable Frame Building, Engineering, vol. 197 (1964), pp. 738-739.

154. Solodilov, Y. I., Large Deformations of A Membrane Spherical Shell Loaded By Liquid Pressure, Inzhenernyi Zhurnal Mekhanika Tverdogo Tela, no 4 (July-Aug. 1966), pp. 151-153.

155. Rubber Covered Dacron Inflatable Field Hospitals, Rubber Age, vol 97 n 1 (April 1965), p. 81.

156. Analytical and Experimental Investigations of Coated Metal Fabric Expandable Structures for Aerospace Applications, ASD-TDR-63-542 (1963).

157. Structure Gonflable Nylfrance, l'Architecture d'Aujourd'hui, vol. 35,p. lxxiii, April 1964.

158. Tent That Speeds Erection On Site, Engineering vol. 199, (June 1965), p. 828.

159. To Use This Warehouse, Workmen Will Blow It Up, Chemical Engineering, vol 72 n 13 (June 21, 1965), p. 82.

160. Two Air Supported Buildings For Athletics, Architectural Record, vol 135 n 3 (March 1964) pp. 209-210.

161. Warm Air Tent For Building Construction, Engineer vol 211 (1961), p. 473.

162. Water Storage, Civil Engineering, vol. 32 (Sept. 1962), p.56.