AUTHOR .... Silberfarb, Samuel

TITLE .... On the direct product of a division and a simple matric algebra with certain notes on primitive idempotent elements.

Thesis .... M.A. 1926

On the Direct Product of a Division and a Simple Matric Algebra with Certain Notes on Primitive Idempotent Elements.

By

Samuel Silberfarb, B. A.

On the Direct Product of a Division and a Simple Matric Algebra with Certain Notes on Primitive Idempotent Elements.

Introduction

This paper establishes certain theorems in connection with an algebra A which is expressible as the direct product of a division algebra D and a simple matric algebra M. It is not assumed that D and M are sub-algebras of A. If D is of order δ and M of order n^2, then the order of $A=D \times M$ is δn^2. We let b and e represent the moduli of D and M respectively. It follows that the modulus of A is be. In agreement with the usual notation we write $e=\sum e_{ii}$, $(i=1,\ldots, n)$, where e_{ij} $(i,j=1,\ldots, n)$ represent the basal units of M.

The proofs of the paper are mainly dependent upon the material of sections 50 and 51 in Chapter VI of Dickson's "Algebras and Their Arithmetics". For the sake of reference, Algebras and Their Arithmetics will be abbreviated by A & A.

Theorem I

If dm=0, where d is any element of D, and m any element of M, then either d=0 or m=0.

For, if $d \neq 0$, then it possesses the inverse d^{-1}.

(1) Hence bm=0, where b is the modulus of D.

We may write $m=\alpha_{11}e_{11}+\alpha_{12}e_{12}+\alpha_{13}e_{13}++++\alpha_{rs}e_{rs}++++\alpha_{nn}e_{nn}$.

From (1), we obtain

(2) $\alpha_{11}be_{11}+\alpha_{12}be_{12}+\alpha_{13}be_{13}++++\alpha_{rs}be_{rs}+++\alpha_{nn}be_{nn}=0$.

If $m \neq 0$, there must be some $\alpha_{rs} \neq 0$.

Thus multiplying each member of (2) on the left by e_{rr} and on the right by e_{ss} we get $\alpha_{rs}be_{rs}=0$.

(3) Whence, $be_{rs}=0$.

Multiplying (3) on the right by e_{sr}, it follows that

$be_{rr}=0$, where $(r=1, \ldots, n)$.

Summing, we obtain $be_{11}+be_{22}+++ be_{nn}=0$.