THE UNIVERSITY OF MANITOBA

OFFSET PARABOLOID ANTENNAS FOR MULTIPLE BEAM APPLICATIONS

BY

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPATRTMENT OF ELECTRICAL ENGINEERING

WINNIPEG, MANITOBA

September 1979

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ABSTRACT

The performance of offset reflector antennas with offset feeds is investigated. Mathematical expressions for the reflector pattern calculations by both the vector current and the scalar aperture methods are developed and are used to compute the co-polar and the cross-polar radiations. A computer program is prepared which utilizes a two dimensional integration to compute the required patterns for both methods. It is found that the scalar aperture method succeeds in predicting the co-polar pattern quite accurately. However, this method always predicts lower cross polarization level than those predicted by the vector current method. The computation time required by both methods for their pattern calculations is found to be comparable. It is also found that the gain of the reflector is not affected by small displacement in the feed. Shifting the feed along the reflector axis of symmetry is recommended since it has a negligible effect on the co-polar radiation and improves the cross-polar one.

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Dr. L. Shafai for his continuous interest, helpful advice, cheerful and able guidance in all phases of this work.

The financial assistance provided by the Communication Research Centre and the department of Electrical Engineering of the University of Manitoba are gratefully acknowledged.

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Chapter I

INTRODUCTION

Recent developments in satellite and microwave communication systems as well as radio astronomical research have generated a great deal of interest in reflector type antennas. The reflector antennas are used with systems for communication or radar purposes in which a large value of gain, low side lobe level and low cross polarization are necessary.

At UHF frequencies and above, where pencil beams become practical, reflector antennas may be compared with other systems. The end fire or yagi antenna produces relatively high gain but suffers from a narrow frequency band. The phased arrays can be used to produce gain equal to those of reflector antennas, however, they are not universally useful, since they have complicated structures and are costly to produce. On the other hand, the reflector systems have simple geometries and are usually inexpensive to fabricate. They can also be serviced mechanically with far less sophistication than phased arrays. As a single monolithic unit on a mechanical scanning assembly, a reflector is relatively inexpensive to scan. A phased array, on the other hand, requires a phase shifter for each element or at least for each subarray. Therefore, the reflecting systems are the appropriate choice for most point to point communication systems unless the system requires certain particular characteristics, such as a wide and continuous scanning, where the phased array is a better choice [1].

In reflecting systems, the reflectors are mainly used to modify the radiation from a radiating element or feed. The reflectors that are of most interest in the antenna field are all derived from conic sections; Their cross section can take the form of any of the five conic sections circle, ellipse, parabola, hyperbola and the straight line. The reflecting surfaces are normally generated by translation of the curves or by rotation around the focal axis to generate a surface of revolution. Such reflectors, provided that their surfaces are smooth, are truely wide band devices, capable in principle of operation from radio to optical frequencies.

In single reflector systems, the parabola is the most commonly used conic section, with a primary source located at the focus and directed into the reflector area. Since the reflector surface is fixed, the focal system must therefore be designed to give the desired radiation pattern. Also in most practical antennas the reflector aperture angle, from the focus, is large and for a proper illumination its feed must have a wide angle radiation pattern. Because of its simplicity it has been studied exhaustively, but the description of all its properties is beyond the scope of this thesis. It is divided into two basic configurations, symmetric and offset. The symmetric configuration has been more popular and has been studied in more detail. With multiple off-axis feeds, it is also capable of producing multiple off-axis beams. However, this is at a cost of gain reduction.

The multiple reflector systems are also commonly used for their high magnification and short focal length. The Cassegrain antenna is the most common one among the multi-reflector systems. It consists of a main parabolic reflector and a secondary hyperbolic sub-reflector as shown in figure (1). The performance of the cassegrain system is affected by the blocking of the main aperture by the sub-reflector, the diffraction from the supporting structures and reflector edges and the spillover loss from the feed.

The use of reflectors as a microwave antenna started during the second world war with the invention of radar. The necessity to design useful systems stimulated a concentrated effort to develop a coherent and unified theory of reflector antennas founded on a solid base of physical

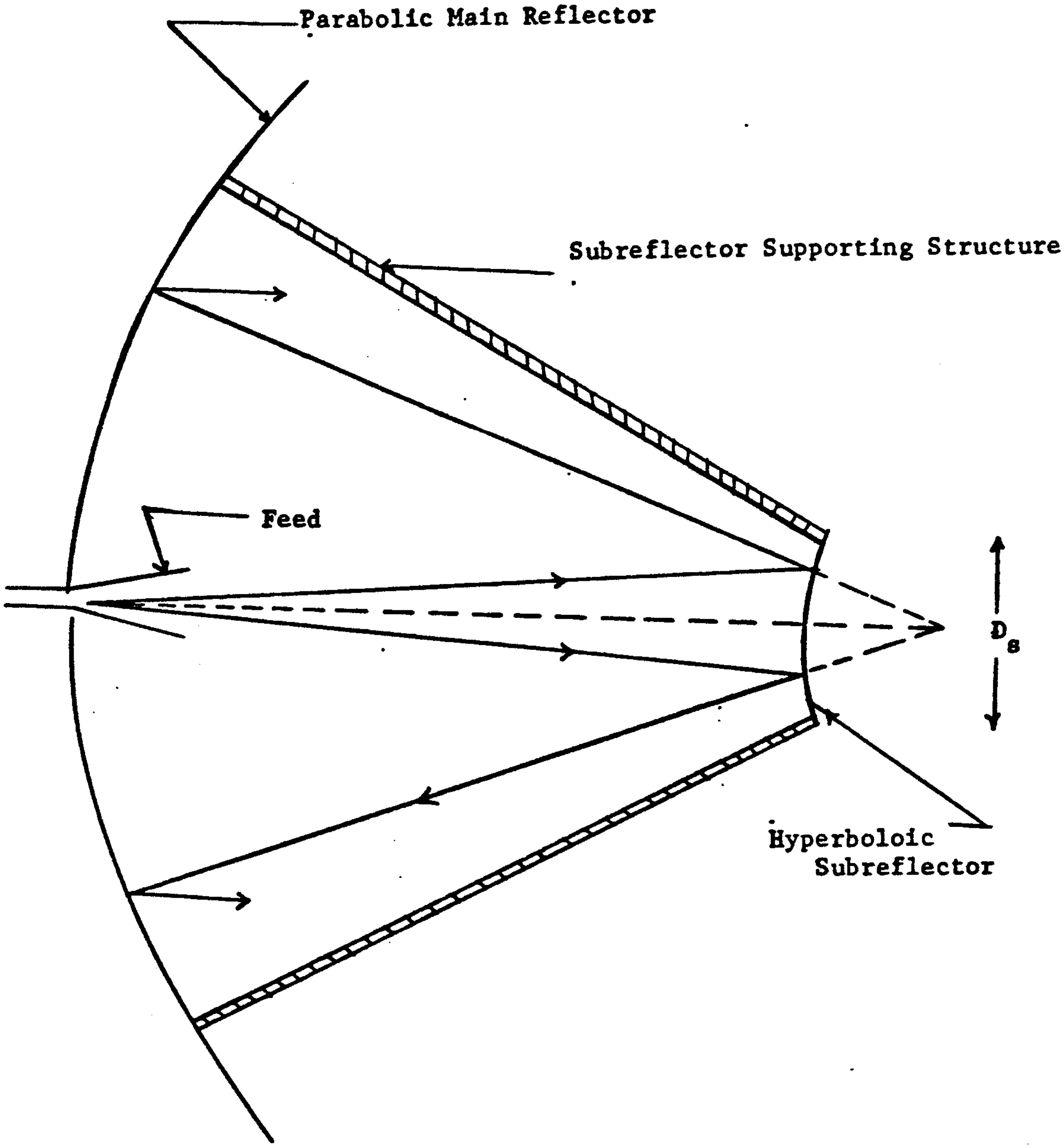

Figure 1. Classical Cassegrain Antenna .

electromagnetics theory. One of the first works published dealing with the polarization characteristics of reflector antennas was that of Cutler [2] who showed qualitatively that the ideal feed should radiate a spherical wave. He also examined the polarization characteristics of the then popular dipole feed and showed that it would give rise to cross polarization components in the reflected field.

The development of commmunication satellites and the arising need to track space probes during the sixties, demanded an increase in the aperture efficiency and a reduction in the feed spillover. The achievment of these goals was important since it would maximize the gain/temperature ratio (G/T) and reduce the cost. The reduction of the cost is essential since the cost of a single large reflector antenna is proportional to its diameter raised to the power 2.78 [3].

Galindo [4] showed that with a two reflector system it is possible to achieve arbitrary phase and amplitude distribution over the main aperture. Williams [5] described a modified cassegrain system, to which this approach was elegantly applied, leading to an improvement of about 2.5 % in the aperture efficiency.

By (1960) the horn had replaced the dipole as a reflector feed. But the characteristics of the horn's dominant mode were far from ideal for the purpose. This is mainly because its principle E and H plane radiation patterns are quite different. Potter [6] solved this problem in the case of the TE_{11} mode excited conical horn. He showed that a correct combination of TE_{11} and TM_{11} modes (generated by a step discontinuity in the diameter near the throat of the horn) at the horn aperture would lead to a radiation pattern having almost identical patterns in the E and the H planes.

Ruze [7] has discussed the beam shift and the degradation of a paraboloidal reflector performance with an offset feed. He also presented the beam characteristics of a tapered circular symmetric illumination. His calculations were for small offsets in the feed with respect to the wave length which made it possible to apply the scalar aperture method. On the other hand, Imbrial et al [8] computed the radiation pattern of a parabolic reflector with large lateral feed displacments of a symmetric paraboloid. They utilized both the vector current method and the scalar aperture theory and compared their computed results with experiments. In particular, they showed that the gain computed by the scalar aperture method tends to deviate significantly from the experimental ones as the feed displacement increases. The vector current method, on the other hand, gives satisfactory results.

In recent years, the use of offset paraboloid reflectors has found wide application, especially in satellite communication, because of certain advantages they offer over their axi-symmetric full paraboloidal counterpart. Their use in the past has tended to be restricted to applications where electrical performance specifications have been severe. This is due to the fact that the offset geometry is more difficult to deal with and more costly to fabricate. However, from a theoretical point of view, their study is relatively more accurate. In axi-symmetric paraboloids, the blocking of the aperture by the feed and its supporting structures results in the reduction of the antenna gain, a general degradation in the suppresion of side lobes and across polarized radiation which generally limits the performance of the antenna. Aperture blockage also makes the theoretical investigation of the axi-symmetric paraboloid more complicated than that of the offset reflector. Furthermore, the computation of the aperture blockage can only be carried out approximately, and the computed performance of the reflector with an aperture blockage seldom agrees accurately with experimental data [9].

The offset system has the advantage of reducing the mutual reaction between the reflector and the primary feed. It also makes it possible to use a larger focal length to diameter ratio while maintaining an acceptable structural rigidity. As a result, offset reflector primary feeds employ a larger radiating aperture, which in certain cases can improve the shaping of the primary pattern and give a better suppression of the cross polarized radiation emanating from the feed itself.

In many applications the structual pecularities of the offset reflector can be used to good advantage. For example, in the design of space craft antennas an offset configuration can often be accommodated more satisfactorily than an axi-symmetric design.

The initial work on the analysis of the offset reflector geometry was carried out by Cook et al [10] who were concerned with the analyses of a dual reflector open cassegrain system. The cross polarized radiation for linearly polarized excitation and the beam displacement for circularly polarized excitations have been investigated by Chu and Turrin [11]. They have given numerical calculations for the dependence of the radiation pattern upon the offset angle θ_0 , as well as, the half angle of illumination θ_c . They also provided detailed graphical data and a clear insight into the beam squinting properties of the circularly polarized prime focus fed offset reflectors. These depolarizing and squinting phenomena are often undesirable in high performance antennas, especially when frequency re-use is required. In many cases these effects constitute a major limitation in the application of an offset reflector, regardless of its other desirable properties.

Rudge [12] described two theoretical models for the prediction of the far field radiation from offset reflector antennas. The models can accommodate small offsets in the primary feed location, with respect to the reflector geometric focus and can thus be usefully employed in the study or design of multiple beam antennas. Experimental verification of the models has indicated that over a moderate range of angles about the antenna boresight, reliable predictions of the significant copolarized and cross polarized radiation can be obtained by the use of these methods. He has also shown that the maximum levels of reflector generated cross polarized radiation are comparatively insensitive to small offsets in the primary feed locations from the geometric focus.

A misunderstanding of the polarization properities of offset reflectors due to the belief that poor polarization is a property of the offset reflector, had put in the past restrictions on their application. Jacobsen [13] has shown that the reflector does not contribute to the cross polarization in the far field. Neither is it a necessary consequence of the asymetry of the configuration. He has shown that the cross polarization of the far field is due to the cross polarization of the primary feed after collimation by the reflector. Therefore the cross polarization can be decreased by improving the feed.

A new class of improved primary feed called the matched feed has recently been introduced [14]. The design of this feed type can be best explained by studying the focal plane field of offset reflectors. In the reception mode, a linearly polarized and uniform plane wave incident along the axis of a parabolic metallic surface induces a surface current which in turn gives rise to a particular field distribution in the focal plane region of the reflector. By reciprocity, any primary feed that can synthesize the entire focal plane field distribution would, upon transmission, produce a linearly polarized and uniform plane wave field distribution in the projected aperture with 100% aperture efficiency. This is why the concept is known as the matched feed concept, since it identifies the focal fields as those corresponding to an optimum feed aperture field distribution , in the sense of achieving maximum aperture efficiency [15]. This concept has been utilized, with considerable success, in the development of high efficiency feeds for both axi-symmetric [16] as well as offset systems [17], [18].

Bem [17] studied the electric field distribution in the focal region of an offset paraboloid. He determined the current distribution on the reflecting surface from the boundary conditions when a uniform plane wave is incident on the reflector. He treated each surface element as an elementary dipole producing an elementary field at a point near the focus. The total field at the focus is obtained by integration over the whole reflecting surface. He carried out the integration for a long focal length.
The focal field in the plane of symmetry was found to be;

$$E_x = K' \left[\frac{J_1(u)}{u} + j \frac{d \sin\theta}{4f} \frac{J_2(u)}{u} \cos\phi \right], \qquad (1)$$

$$E_y = K' \left[-j \frac{d \sin\theta_0}{4f} \frac{J_2(u)}{u} \sin\phi \right] \qquad (2)$$

where $u = \frac{k\,r}{4f} (1 + \cos\theta_0)$ and

$J_n(u)$ is the Bessel function of order n .

The solution for the wave polarization in the plane of asymmetry is similar to the above expressions, but is obtained by interchanging x and y and replacing ϕ with $(2\pi - \phi)$. That is ;

$$E_x = K' \left[-j \frac{d \sin\theta_0}{4f} \frac{J_2(u)}{u} \sin\phi \right], \qquad (3)$$

$$E_y = K' \left[\frac{J_1(u)}{u} + j \frac{d \sin\theta_0}{4 f} \frac{J_2(u)}{u} \cos\phi \right] \qquad (4)$$

An inspection of the above two cases shows that the cross-polar component is an asymmetrical function of ϕ with a magnitude which decreases with the decrease of the offset angle . The focal plane field is shown in figure (2) as a superposition of two configurations of the symmetric and the antisymmetric field distributions of two polarizations. For the primary feed to provide a match to these incoming fields, its aperture must exhibit similar linear polarization properties. It is a fact that conventional high performance axisymmetric feeds provide an excellent match to only the co-polar component, which leads to poor cross-polar radiation characteristics of the antenna. Rudge et al [14] explained how the symmetric component can be well matched by making use of higher order asymmetrical wave guide modes.

The matched feed used in this thesis is the same one designed by Aboul-Atta and Shafai [15]. They have shown that the improved class of primary feed devices must be at least bimodal in the sense of radiating a fundamental TE_{11} mode plus a compensating TE_{21} mode. The necessary condition is that these two modes be in phase quadrature at the aperture of the device. Since the compensating mode has a skew-symmetric field distribution in the aperture then upon radiation, its generated field becomes in phase with the

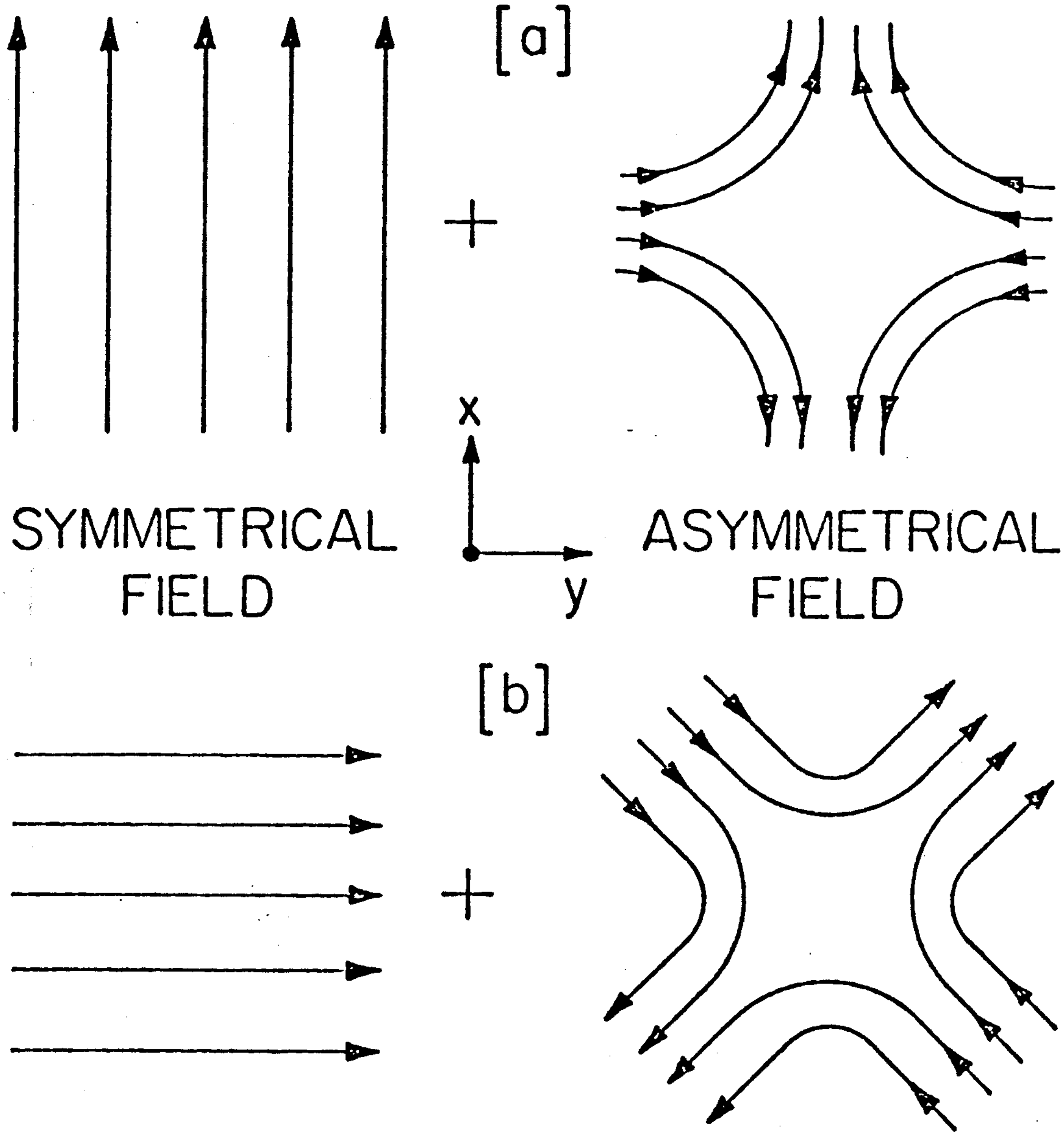

Fig. 2 Symmetric and asymmetric components of the offset reflector focal-plane field.
[a] Polarization in the plane of symmetry.
[b] Polarization in the plane of asymmetry.

fundamental radiation. Hence the total radiation field from a matched feed device at a distance R, may be expressed by [15]

$$E_T = [\ E^{(1)} + E^{(2)}\ ] \ \frac{\exp(-jkR)}{R} \qquad (5)$$

where $E^{(1)}$ and $E^{(2)}$ are real quantities representing the angular variation (θ, ϕ) of the respective radiation of the fundamental and the compensating modes.

$E^{(1)}(\theta)$ and $H^{(1)}(\theta)$ describe the E and the H plane radiation pattern functions of the fundamental mode and for TE_{11} circular waveguide mode are given by

$$E^{(1)}(\theta) = (1 + \cos\theta)\ \frac{J_1(u)}{u}\ , \qquad (6)$$

$$H^{(1)}(\theta) = (1 + \cos\theta)\ \frac{J_1'(u)}{1 - \left(\frac{u}{p_{11}}\right)^2} \qquad (7)$$

where $J_n(u)$ is the Bessel function of order n ,

p_{mn} is the n^{th} root of $J_m(u)$,

$$u = \frac{d'}{\lambda} \sin\theta$$

with d′ is the diameter of the primary feed aperture.

For the compensating TE_{21} mode the functions take the form

$$E^{(2)}(\theta) = \alpha\,(1 + \cos\theta)\,\frac{2\,J_2(u)}{u}, \qquad (8)$$

$$H^{(2)}(\theta) = \alpha\,(1 + \cos\theta)\,\frac{J_2'(u)}{\left(1 - \left(\frac{u}{p_{21}}\right)^2\right)} \qquad (9)$$

where α is the ratio between the TE_{11} and the TE_{21} modes. Equation (5) represents the new primary feed radiation that is to be used in the calculation of the performance of the antenna.

It should be noticed that the relative amplitude of the compensating mode is dependent upon the parameters of the reflector. These parameters are the offset angle θ_0 and f/d ratio of the reflector.

Aboul-Atta and Shafai [15] gave two examples to show the advantages of this type of feed and the effect of the reflector parameters on its performance. The first example was a reflector with 50^0 offset angle and f/d=0.4773 (aperture angle=45^0 , D=100λ). The original -17 dB cross-polar radiation of the reflector without TE_{11} mode, was reduced to less than -36 db after adding a TE_{21} mode with 0.3 mixing ratio. However, the overall antenna efficiency was dropped from 63.12 % to 62.18 % . This was due to the fact that the above aperture angle was no longer the optimum angle after adding the TE_{21} mode which modifies the feed pattern. The efficiency was then increased to 65.597 % by increasing the aperture angle from 45^0 to 50^0.

The second example showed the radiation pattern of a reflector with an offset angle of 30^0 and f/d=0.833. The required mixing ratio α was 0.07 which was much smaller than that of the previous case. It was found that the cross-polar radiation dropped to around -50 db of the co-polar field. In addition the antenna efficiency was found to be much larger.

From the above two examples, they have recomended that the geometry of the reflector be selected to have small offset angles and large f/d ratios. This geometry will yield a high antenna efficiency and will require a smaller mixing ratio . The small mixing ratios require smaller obstacles in the waveguide to generate the compensating mode, which improves its frequency performance and simplifies its design.

It should be noted that this method for the design of the feed is more accurate than that given by Bem [17]. Bem's expression is based on a series expansion and to increase its accuracy more terms have to be taken into account, that is to say more modes have to be generated which will complicate the design of the feed and will narrow its frequency bandwidth.

This thesis considers the offset reflector antennas for multiple beam applications, mainly for Canadian satellite use. Assuming a geostationary orbit, the communication needs of major Canadian cities may be accomodated by using only several narrow beams. From a central geostationary orbit, all Canadian cities can be seen by off axis beams clustered within $\pm 4^0$. We are therefore interested in the performance of offset systems capable of generating off axis beams within $\pm 4^0$, and with adequate polarization isolation. To allow for a possible off centre satellite location, beam shifts as large as $\pm 5^0$ will be considered.

In chapter two, the geometry of offset reflectors is defined in detail, and the different definitions of cross polarization are discussed. In chapter three, mathematical expressions necessary for reflector pattern calculations are developed for both the current distribution and the aperture field methods. In chapter four, the radiation patterns of offset reflectors with offset feeds are investigated by both the vector current and the scalar aperture methods. The computed patterns by the vector current method and a comparison of the two methods is presented in chapter five.

Chapter II

IMPORTANT DEFINITIONS ASSOCIATED WITH OFFSET SYSTEMS

2.1 INTRODUCTION

There is an ambiguity associated with the word offset as it is currently used in antenna engineering. One meaning refers to the reflector, an offset paraboloid is taken to be one which is not symmetrical about its axis of revolution. The portion of the reflector surface lying on one side of that axis is discarded altogether. Since the feed must still be located on or very close to the axis, this arrangement removes the feed from the region of highest aperture field intensity and reduces or may even eliminates blockage. Of course the axis of the feed is tilted so that its cone of illumination will be on that part of the reflector surface that remains. Otherwise, the spillover will be excessive. The other meaning associated with offset is one in which the phase centre of the feed is displaced laterally from the optical axis in order to squint the beam.

As there is an ambiguity associated with the word offset, there is also a lack of precise and unambiguous definition of cross polarization. The whole subject of cross polarization in reflector antennas had been somewhat confused for many years and had given rise to certain misunderstanding and controversies. Ludwig points out in his paper [19] this surprising deficiency and attempts to correct it. He discusses three different definitions that are commonly used. He then shows where they are applicable and how they differ.

In this chapter the two meanings associated with the word offset are clarfied and the offset reflector geometry is discussed in detail. The reflector's various parameters are defined and equations describing its surface for both spherical and rectangular coordinates are given. Also different definitions of cross polarization are discussed. Ludwig's third definition is used in this thesis for the predicted field components. This definition corresponds directly to the components measured using standard antenna range techniques.

2.2 OFFSET REFLECTOR GEOMETRY

An offset paraboloid is taken to be one which is not symmetrical about its axis of revolution. The geometry of an offset reflector is shown in figure (3). The basic parameters of the reflector are f, θ_0 and θ_c where f is the focal length of the parent paraboloid, θ_0 is the offset angle and θ_c, the half angle of illumination, is the half angle subtended at the focus by any point at the reflector rim.

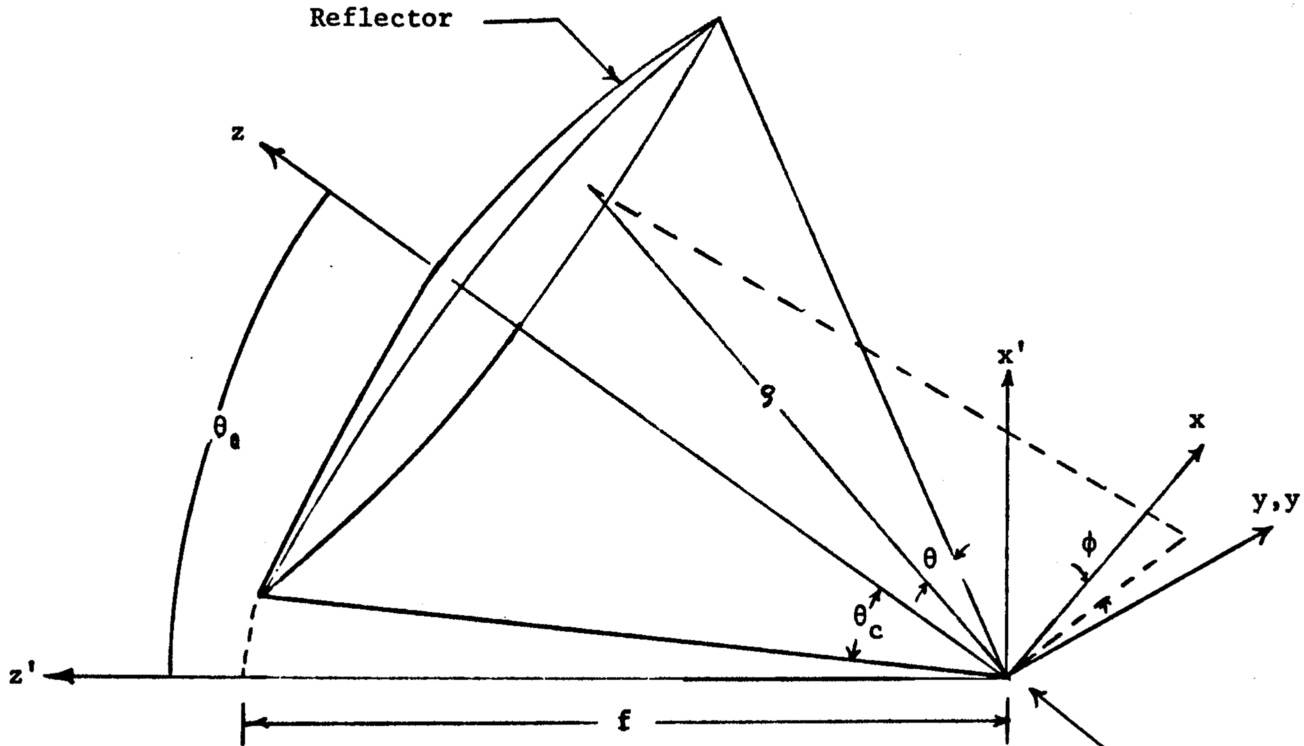

Figure 3. Offset Reflector Coordinate System.

Ingerson and Wong [20] defined an offset axis as the line through the focus that bisects the angle subtended at the focal point by the parabolic arc of the reflector in its plane of symmetry. It follows that the offset focal plane is the plane through the focus and perpendicular to the offset axis. With these definitions, the focal region characteristics of offset reflectors are quite analogus to those of symmetrical reflectors.

The offset reflector may be defined in two coordinate systems; x', y', z' which defines the parent paraboloid and the rotated system $x, y, z / \rho, \theta, \phi$ which is more convenient in describing the radiation pattern of a feed located at the reflector focus. The transformation from one system to another and the equation of the reflector in various coordinate systems are given in Appendix (A).

The physical contour of the reflector is elliptical but its projection onto the xy plane produces a true circle.
The parameters that are of interest in this work are :
The diameter d of the projected aperture

$$d = \frac{4 f \sin\theta_c}{\cos\theta_0 + \cos\theta_c} \qquad (10)$$

The differential surface element ds

$$ds = \frac{dx' \; dy'}{\cos \theta'/2} = \frac{r^2 \; \sin\theta \; d\,\theta \, d\phi}{\cos \theta'/2} \qquad (11)$$

And the distance r from the reflector focus to a point on a parabolic surface

$$r = \frac{2\ f}{1 + \cos\theta'} \qquad (12)$$

or

$$r = \frac{2\ f}{1 + \cos\theta \cos\theta_0 - \sin\theta \sin\theta_0 \cos\phi} \; . \qquad (13)$$

For any surface represented by a radius vector, the unit normal is given by [17]

$$\bar{n} = \frac{-\ \bar{r}_\theta \times \bar{r}_\phi}{|\bar{r}_\theta \times \bar{r}_\phi|} \qquad (14)$$

where

$$\bar{r}_\theta = \frac{d\bar{r}}{d\,\theta} \qquad \text{and} \qquad \bar{r}_\phi = \frac{d\bar{r}}{d\,\phi} \; .$$

In the rectangular coordinate system this may be given by

$$\bar{n} = n_x \; \bar{x} + n_y \; \bar{y} + n_z \; \bar{z} \qquad (15)$$

where

$$n_x = -\left(\frac{r}{4f}\right)^{\frac{1}{2}} (\sin\theta \cos\phi - \sin\theta_0) ,$$

$$n_y = -\left(\frac{r}{4f}\right)^{\frac{1}{2}} \sin\theta \sin\phi , \quad (16)$$

$$n_z = -\left(\frac{r}{4f}\right)^{\frac{1}{2}} (\cos\theta + \cos\theta_0) ,$$

$\bar{x},\bar{y},\bar{z}$ are the unit vectors of the x,y,z coordinate system .

The diameter of the parent paraboloid D is given by

$$D = 4 f \tan\left(\frac{\theta_0 + \theta_c}{2}\right) \quad (17)$$

while the distance X_0 from the axis of the parent paraboloid to the centre of the projected aperture is

$$X_0 = \frac{2 f \sin\theta_0}{\cos\theta_0 + \cos\theta_c} . \quad (18)$$

2.3 THE DEFINITION OF CROSS POLARIZATION

The possible use of orthogonal polarization to provide two communication channels for each frequency to save frequency bandwidth has led to interest in the polarization purity of antenna patterns. However, there is no universaly accepted definition of the cross polarization at present. According to IEEE standard definitions [20], the co-polarization and the cross-polarization is defined by comparing the source with a reference source. The co-polar field of the given source is then taken to be the component of the field which is parallel to the field of the reference source and the cross-polar field is the orthogonal component. This means that the co-polar field of a given source is defined by

$$\overline{E}_{co} = \frac{\overline{E} \cdot \overline{E}_{ref}}{E^2_{ref}} \overline{E}_{ref} \qquad (19)$$

where $\overline{E}$ is the electric vector field of a given source and $\overline{E}_{ref}$ is the electric vector field of the reference source.

Ludwig discussed three different definitions of the cross polarization in his 1973 paper [19]. He designated them as first, second, and third definitions according to the reference field being: 1) a plane wave, 2) the radiated E-field from a short electric dipole, 3) the E-field radiated by a Huygens source.