UNIVERSITY OF MANITOBA

Acquisition and Generalization of Prearithmetic Skills In Severely Retarded Men

by

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A Thesis

Submitted to the Faculty of Graduate Studies

In Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

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Abstract

Clearly specified behavioral definitions and testing and training procedures have been lacking in the field of teaching prearithmetic skills to the retarded. In the present study, formal testing and training procedures were used to assess and teach six behaviorally defined prearithmetic skills (i.e., reciting, counting moveable ordered items, counting fixed ordered items, counting fixed unordered items, size selection, and size identification), for numbers 1-5. A multiple baseline design across skills with replications across six severely handicapped students demonstrated the effectiveness of the training procedure which used modelling, guidance, fading, and a brief quiz. All students showed improvement on all skills and showed generalization across settings and items. Generalization across numbers (i.e., to numbers 6-10) was shown by the one student tested for this. With slight modifications in the behavioral definitions, the training procedure was also shown to be effective with one of the students who was nonvocal. This suggests that the definitions and procedures described could be useful to individuals interested in teaching prearithmetic skills to vocal or nonvocal retarded persons.

Acquisition and Generalization of Prearithmetic Skills in Moderately Retarded Men

In general, prearithmetic skills can be viewed as number-related behaviors which typically precede the acquisition of the basic arithmetic procedures of addition, subtraction, multiplication, and division. Behaviors such as reciting the number names in order, counting items, and identifying quantities are all examples of prearithmetic skills. Such skills are important prerequisites not only for acquiring further arithmetic skills, but also for such daily activities as telling time, handling money, accurate job performance, comparative shopping, requesting, following instructions, describing events, etc.

From the point of view of someone wishing to teach such skills to retarded individuals, there has been a noticeable lack of applied research in this field. Several studies have reported the teaching of prearithmetic skills (eg., Bijou, Birnbauer, Kidder, and Tague, 1966; Humphrey, 1966; Ross, 1970); however, they failed to adequately define the behaviors or describe the procedures used. Spradlin, Cotter, Stevens, and Freidman (1974) defined five prearithmetic behaviors relatively well, although Resnick, Wang, and Kaplan (1973) have reported probably the most extensive, well defined inventory to date. Unfortunately, neither Spradlin _et al_. nor Resnick _et al_. reported a training procedure to accompany their inventories.

Only two studies (Murrell, Hardy, and Martin, 1974; Swartz, 1974) were encountered with well defined behaviors and fully described procedures. Murrell *et al*. applied a fading procedure, in a multiple baseline fashion across numbers, to teach an institutionalized eight year old autistic boy to count out a required number of felt discs. The instructions for the task (eg., "Danny, what number is this?"), arrangement of stimuli (eg., markings on a piece of paper, presented numeral, position of tester's hand, etc.), and topography of the response (eg., orange discs were placed inside the outlined circle) were all completely described. Immediately following this training, three tests were conducted using different items (i.e., bingo chips, wooden blocks, and puzzle pieces) to test for generalization across items. Similar tests were conducted eight months later in the boy's home using the felt discs, bingo chips, and wooden blocks. Ninety per cent accuracy was reported over all tests.

Swartz (1974) described a training program which used prompts, praise, and tokens to teach a severely retarded boy to point to and count a maximum of five plastic tokens. He also specified the instructions, stimulus arrangements, and response topographies which formed the target behaviors.

It should be noted that the report of generalization by Murrell *et al*. is the only study encountered which reported an assessment of such generalization. For a more extensive, critical review of the literature on teaching prearithmetic skills, see Appendix A.

The purpose of the current research was to develop and implement testing and training procedures that would be effective in teaching a number of prearithmetic skills to mentally retarded young adults. In addition, it was designed to evaluate the generalization of these skills across settings and across items used. Furthermore, slight modifications in the definitions and procedures were introduced in order to adapt the training package to one nonvocal individual as well.

Method

Students

Six retarded young adult males served as students. All were residents of Valley View Centre, a provincial institution for the retarded with a resident population of about 750. The men ranged in age from 18 to 33 years (mean = 24.3 years) and had lived in the institution

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Insert Table 1 about here

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from 11 to 19 years (mean = 14.7 years). Four were classified as severely retarded, one as profoundly retarded, and one as moderately retarded (see Table 1). All men were selected on the basis of their being well behaved in individual sessions, minimally vocal (i.e., would imitate words or phrases), and deficient in the area of prearithmetic skills. The one exception was a nonvocal individual, Ken, who was included to probe the generality of the training procedure.

Table 1

Student Characteristics

Name	Age	Length of Stay	I.Q.	Level of Retardation
Allan	18	11	N.A.[a]	profound
David	24	12	30	severe
Frank	25	19	35	severe
Gerald	22	16	39	severe
Ken[b]	33	14	N.A.[a]	moderate
Randy	24	16	30	severe

[a]Formal test results were not available.

[b]Ken was a nonvocal individual.

Staff members were informed briefly of the purpose of the training procedure and selection requirements and were asked to suggest any individual for the research. Recommended individuals were pretested by testing a sample of several objectives and numbers (described in detail below) and providing brief instruction following errors. Individuals who were uncooperative, would not imitate, or were already proficient at the skills, were not accepted into the study.

Behavioral Objectives (Target Behaviors)

Six objectives were chosen for training which corresponded to those identified by Resnick *et al*. (1973). These objectives were described more completely than in Resnick *et al*., and are presented below:

Recite. When given a verbal instruction (eg., "Count to four"), the student was required to recite the numerals in the correct order from one to the specified number and then stop. Counting incorrectly, stopping short of the required number, or counting beyond it, were considered as errors.

Count moveable ordered items. When presented with a set of moveable items in an ordered linear array and a verbal instruction (eg., "Count these blocks by moving them onto the paper."), the student was required to move the items onto the paper, move each item once and only once, and count each as he did so.[1] Insufficient movement, incorrect counting, or a lack of correspondence between movement and counting, were considered as errors.

When finished, the student was required to identify the quantity of items when asked by the tester (eg., "How many blocks are there?"). Although recorded separately, both parts of the response (i.e., counting and quantity identification) were required to be correct before a reinforcer was given.

Count fixed ordered items. When presented with a set of fixed (although in fact the items were moveable, the student was not allowed to move them) items in an ordered linear array and a verbal instruction (eg., "Count these blocks by just touching them."), the student was required to touch or clearly point to the items once and only once, counting each as he did so. Excessive movement (i.e., more than one cm) of an item, incorrect counting, or lack of correspondence between touching or pointing and counting were considered as errors. Students were then required to identify the quantity as described above.

Counting fixed unordered items. Instructions and requirements were identical to the above objective except that the items were scattered at random (i.e., dropped from just above the table) onto the paper. Items landing off the paper were moved 1 cm onto it and all items had to be at least 1 cm apart. The tester made these alterations prior to giving the instruction.

Size selection. When presented with a set of moveable objects located off the paper and a verbal instruction (eg., "Show me four blocks."), the student clearly indicated a subset of the appropriate size by moving the required number of items onto the paper. No counting response or correspondence was required for this objective.

Size identification. When presented with three distinct sets of fixed items and a verbal instruction (eg., "Which group has four blocks?"), the student would point to or touch the group having the appropriate number of items. If the student counted the blocks first, the experimenter then repeated the instruction (eg., "So which group has four blocks?") and the next motor response was taken as the student's response. Again, no counting or correspondence was required for this objective.

The nonvocal objectives used with Ken were essentially the same as those identified above. Ken was given an abacus-like "counting frame" on which were located ten numbered blocks (approximately 2.5 cm by 2.5 cm by 2.5 cm) that could be moved from one side to the other. Where vocal students were required to name a number (eg., quantity identification), Ken was required to point to it. Where a vocal student was required to count the items on the table, Ken was required to touch each item and move one corresponding block (for a complete description of the nonvocal objectives, see Appendix B).

Training and Generalization Settings and Items

Settings in which tests were carried out were identified as either training, generalization, or novel. The training setting was a small secluded room (approximately 3 m by 3 m) which contained a table and several chairs. All training sessions were conducted in this setting. The generalization setting was a larger sitting room (approximately 10 m by 7 m) which contained some couches and soft chairs as well as a large table and chairs. Many posters, notices, and graphs covered the walls of this room. The novel setting was a term given to any other setting in which tests were conducted. It was at different times the bathroom, bedroom, office area, etc. and could be changed within a testing session.

Items which were used during the tests for demonstrating the skills were identified as either training, generalization, or novel. The training items were small, unpainted wooden blocks (approximately 2 cm by 2 cm by 2 cm). All training sessions were conducted using the training items. The generalization items were either blue plastic building blocks (approximately 1 cm by 1.5 cm by 3 cm) or .5 cm thick wooden discs (approximately 2 cm in diameter). When the generalization items were used, they were not mixed together; however, the tester would change from one type of item to the next several times during a testing session. The novel items were any other items used in testing. Typically, these were such things as pencils, cups, paper clips, balls, etc.

Experimental Design

Basically, a multiple baseline design across behaviors, with replication across students, was used to demonstrate the effectiveness of the training procedure.

Prior to any training, the following tests were completed: at least three tests of all objectives for numbers 1-5 (one student, David, received regular testing on numbers 6-10 as well) in the training setting with training items; one test in the training setting with generalization items; one test in the generalization setting with training items; one test in the generalization setting with generalization items; and one test in the novel setting with novel items. Following this series of tests, the training procedure was introduced with one objective only for numbers 1-5. Once the criteria for considering that an objective was learned were met, the student would receive one test in the training setting with training items. The training procedure was then introduced on the next objective in a multiple baseline fashion. After the third objective was trained, students received one test of each type described above. All students received a series of at least five tests (one of each type) following completion of training on the last objective.

Testing

Picture/color naming task. In order to supplement the rate of reinforcement near the beginning of the study, a simple picture/color naming task was introduced concurrent with the testing procedure. Several Peabody picture and color cards which the individuals could identify were presented at various times throughout the session. The tester supplemented the reinforcers earned by the student for correct performance of the various objectives with additional reinforcers. These were earned for correct identification of the picture or colors (the nonvocal individual was asked to point to a requested picture or color when presented with two of them) such that the student obtained a total of about 20 correct answers during the test. This procedure was not used for tests in the novel setting.

Tokens. During the testing and training sessions in the training and generalization settings, a pegboard and pegs were placed near the student and to one side with the pegs in the board along the edge farthest from the student. As the pegs were earned the student moved the pegs to the edge nearest him. All holes except those that were in use were covered with masking tape. When all of the pegs were earned, or at the end of the session, an "exchange" period was provided so that the student could exchange the pegs for a backup reinforcer. At this time, all of the pegs were returned to their original position and the student then selected from an assortment of candies and pieces of fresh fruit or vegetables.

In the case of tests conducted in the novel setting the pegboard and tokens were not used. Students received praise for correct responses and edibles at a rate that closely approximated the rate at which they would have received them if the tokens were used (i.e., 2-3 edibles during each test).

Procedure. Prior to each trial, the tester first determined if a picture/color naming task should be used in order to supplement the rate of reinforcement. If so, one or two such trials were conducted. If not, the tester arranged the testing items as necessary for the trial (eg., placed items in a row, arranged items in groups, piled items, etc.) and waited 3-5 seconds for the student's attention to the task. If attention was not achieved, the tester prompted the student to attend. The tester then gave the specific instruction appropriate for the objective being tested. If the student responded correctly, he was praised, and instructed to take a peg from one side of the pegboard and place it on the other. Instruction or guidance was provided for completion of this if necessary. If the student made an error, the tester simply looked down, marked the data sheet, and then went on with another trial. No errors were corrected and responses were recorded as either correct or incorrect. The particular objective and number tested on any given trial was determined by the tester. Typically, one to three numbers on a particular objective were tested and then another objective selected. The remaining numbers were tested later in the session. This procedure was repeated until all objectives and numbers were tested, at which time the test was terminated.

Training

Procedure. The training procedure consisted of two major sections: "acquisition" and "quiz". The instructor started the acquisition portion of the session by arranging any required items and directing the student's attention to the task. The objective that was being trained was then modelled correctly for the student. The modelling was preceded with a descriptive form of the instruction for that objective (eg., if counting fixed ordered items was demonstrated, the instructor would first say, "I am going to count these blocks and just touch them as I count them.", and then proceed to do so). Students needed only to attend to the demonstration and did not have to imitate or accompany the teacher at this point.

Following two or three such demonstrations, the instructor used whatever prompts, instructions, or guidance necessary to have the student successfully emit the objective. The specific nature of the extra prompts was established by the instructor to suit the individual student but usually consisted of pointing or gesturing and instructing the student to imitate or how specifically to handle the items. Errors at this point were minimal and success, regardless of additional prompts, was praised.

All requests for the student's performance of an objective were preceded by the specified instructions for that objective. Throughout the acquisition portion of the training procedure, the instructor frequently changed the number of items used between 1 and 5.

Once the student performed the objectives several times with the extra prompts, the instructor began fading the degree and number of prompts until the student could perform the objective correctly with only the instruction for that objective. If errors occurred during the fading process, the instructor would correct the error and increase the extra prompts in order to achieve successful performance. The specific fading steps were established by the instructor to suit the individual students but were not formalized as part of a written procedure. Such techniques as speaking more softly and providing fewer or less specific gestures or instructions frequently were effective.

If a student performed the objective correctly three consecutive times using different numbers of items each time, he was given a brief quiz. The quiz was used to test the performance of the objective on all numbers 1 through 5 when alternated with other response requirements. The quiz consisted of ten trials which alternated between a previously trained (P) objective and the current training (C) objective and started with a P objective. Any P objective could be required, and in cases where none existed (where the C objective was reciting) the picture/color naming task was used. Correct responses on the quiz were reinforced with praise and tokens. If errors occurred, the student was stopped immediately and a formal correction was given.

With the formal correction the instructor said "No. Watch." and then gave a descriptive form of the verbal instruction and demonstrated the objective clearly. The instruction was repeated and the student tried again. If the objective was then performed correctly, the student was reinforced as above. If he again made an error, the instructor simply looked down and recorded the data. The instructor then moved on to the next trial of the quiz.

In order to proceed to a new objective, the student had to achieve at least four correct (without correction) and no errors on both the C and P objectives during the quiz. Objectives were not changed within training sessions. If the quiz was "passed" the training session was stopped and any required testing was completed before the instructor proceeded to the next objective. If the quiz was not passed, the acquisition section was reintroduced. If the quiz was not passed twice more, the nature of the errors was evaluated and program changes made as needed. No program changes were required; only minor changes in the prompting and fading procedures were made.

<u>Variable interval reinforcement</u>. During the acquisition portion of the training session all correct responses received feedback and praise

(eg., "Hey, that's right. Good!"). In addition, in order to standardize the rate of reinforcement, the student also earned tokens for correct responses (prompted or not) on a variable interval schedule with a mean value of 60 seconds. The instructor was cued as to when reinforcement was "set up" and as to when the session should end by a cassette tape recorder via an earphone. Near the end of the study, the earphone malfunctioned and was not replaced since the audible signals did not appear to disrupt the performance of the students.

Tokens were exchanged for backup reinforcers as soon as all of the tokens were earned, or at the end of the session, regardless of the number earned. During the exchange period, the tape recorder continued to run, but the instructor ignored it unless the end of the session was indicated. If the end of the session was indicated while a quiz was in progress, the quiz was completed before terminating the session.

Problem behaviors. Any attempt to pick up or otherwise handle the session materials was immediately corrected (eg., the instructor said something like, "Wait! Don't pick it up, just point and touch it.") and occasionally the instructor modelled the correct way of responding or provided other kinds of prompts. Other mild or moderately disruptive behaviors (eg., reaching for a data sheet, peeking under a table, initiating a touching response prior to instruction, etc.) were handled in the same way. In the event that a disruptive behavior such as those described above occurred, a trial was terminated and repeated. The tester did not attempt to continue a trial which was disrupted. In the event of severe disruptions or lack of cooperation with required test activities, the tester warned the student only once then terminated the session after the next occurrence and, in a calm unresponsive manner, informed the student that he had misbehaved and that he would be returned to the dayroom area of the cottage. Termination of sessions occurred infrequently, and data from those few sessions which were terminated were not included in the data reported.

Results

Interobserver Reliability

Part of the training received by the instructor/tester from the author consisted of both individuals recording session data at the same time. Following this, the author provided feedback to the instructor/tester on her performance. Further interobserver measures were not taken due to the high degree of reliability obtained (i.e., approximately 95%) at this time. The behavior described in this paper proved to be defined sufficiently well such that reliability was not a problem.

Treatment Effects

Individual performance. Treatment effects were measured as changes in test performance. The mean test scores before and after training are presented in Table 2. It can be seen that all individuals showed higher mean test scores after receiving the training for all nine response measures. Only David, for numbers 6-10, showed no improvement (i.e., identifying the

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Insert Table 2 about here

Table 2

Mean Pre- and Posttraining Test Scores

Behaviors	Students															
	Allan		David(1-5)		David(6-10)		Frank		Gerald		Randy		Ken		Mean	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Rec	2.9	4.3	4.9	--	4.4	--	1.3	4.9	1.4	--	2.3	3.9	3.5	4.2	2.5	4.3
CMO	1.1	4.4	2.9	4.8	0.3	2.6	1.0	3.1	1.2	3.8	1.8	4.1	1.0	5.0	1.5	4.2
QMO	1.0	1.5	2.9	4.9	0.8	2.6	3.2	3.3	1.3	3.5	2.5	4.2	3.8	5.0	2.5	3.7
CFO	2.1	4.5	3.7	4.8	0.9	1.5	1.1	4.6	1.4	4.5	3.0	4.5	1.5	4.8	2.1	4.6
QFO	1.0	1.8	3.1	4.7	1.3	1.5	3.9	4.6	1.7	4.0	3.1	4.4	4.2	4.8	2.8	4.1
CFU	1.4	4.3	3.5	4.5	0.3	1.0	1.9	4.3	1.5	4.8	3.3	4.0	1.9	4.3	2.2	4.4
QFU	1.0	1.6	3.0	4.4	0.8	0.8	3.9	4.6	1.4	4.5	3.2	4.4	3.9	4.9	2.7	4.1
SS	1.9	3.7	3.9	4.3	1.3	2.4	3.3	3.5	0.7	3.3	2.7	4.3	2.4	3.3	2.5	3.8
SI	2.8	4.2	2.2	3.0	2.1	1.8	4.6	4.9	2.4	4.2	3.4	4.8	4.0	4.4	3.2	4.3

Note: Maximum score = 5.0

[a] Behaviors tested were reciting (Rec), counting moveable ordered items (CMO), identifying the quantity of moveable ordered items (QMO), counting fixed ordered items (CFO), identifying the quantity of fixed ordered items (QFO), counting fixed unordered items (CFU), identifying the quantity of fixed unordered items (QFU), size selection (SS), and size identification (SI). See text for complete definitions.

[b] The mean score excluded those for which no training occurred (i.e., David and Gerald on reciting numbers 1-5, and David on all behaviors for numbers 6-10).

quantity of fixed unordered items) or a reversal in this trend (i.e., size identification). None of these behaviors however, received training.

Gains on the test scores for the various behaviors were clearly greater for some individuals than for others. For example, Frank showed noticeable gains on reciting, counting moveable ordered, fixed ordered, and fixed unordered items (see Figure 1), whereas Gerald refused to recite with no items present until later in the study but did show improvement in all other behaviors (see Figure 2). Ken, the nonvocal individual, showed acquisition of reciting as well as all the quantity identification requirements during the baseline period and maintained these skills throughout the rest of the study (see Figure 3). Similar individualized improvements were noted for the other three students (see Figures A, B, and C in Appendix C).

Insert Figures 1, 2, and 3 about here

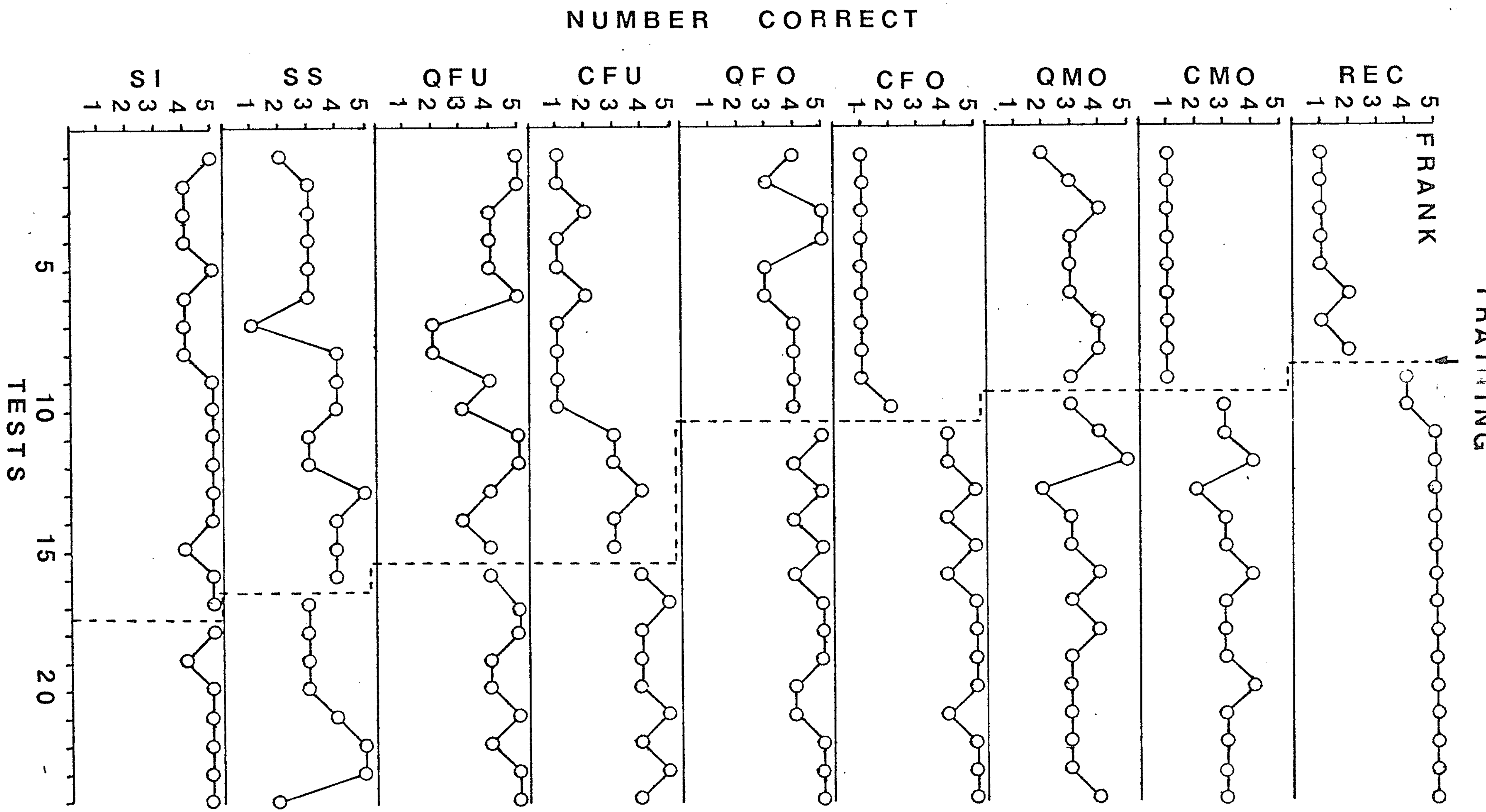
For some individuals and behaviors only minimal improvement occurred (see Table 2). Inspection of Figures 1, 2, and 3 reveals that this was frequently due to high (i.e., above 3.5) baseline scores (eg., Frank on identifying the quantity of fixed ordered and unordered items as well as on size selection; Ken, on reciting, identifying the quantity of moveable and fixed ordered items and fixed unordered items, and with size identification). This was also true of David (i.e., see Figure C in Appendix C).

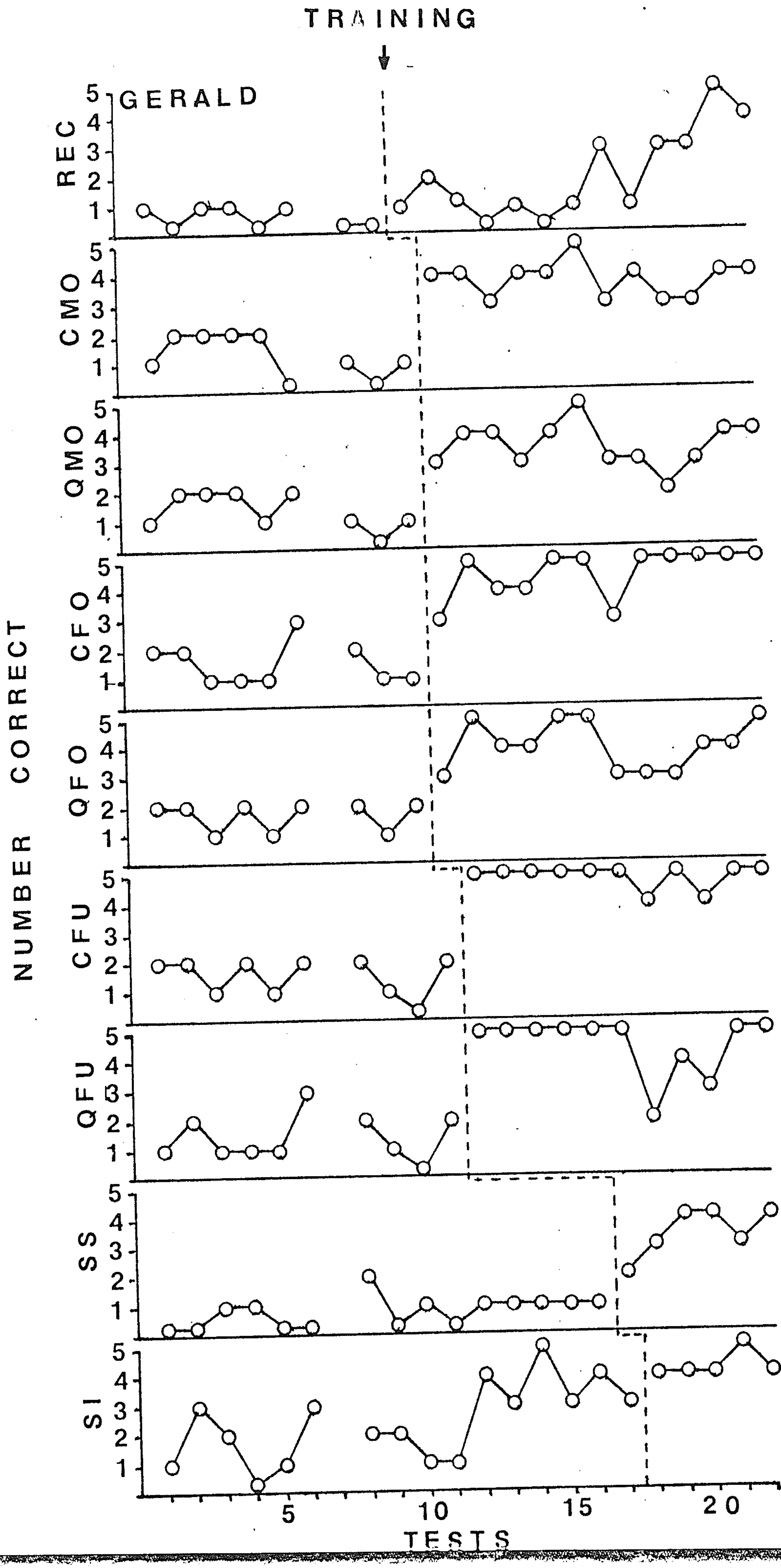
Figure Caption

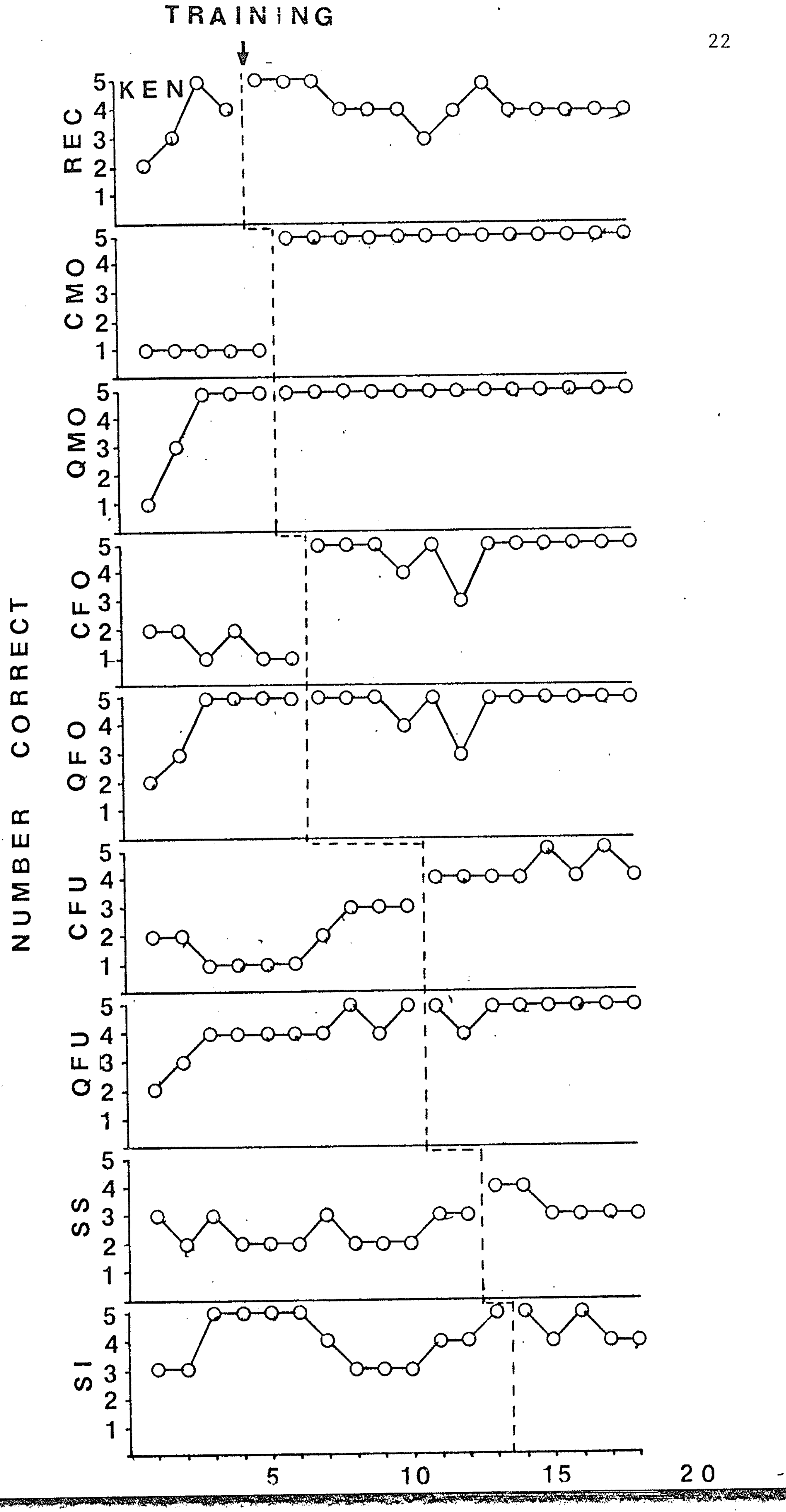
Figure 1. Number of correct (maximum = 5) on pre- and posttraining tests for Frank on reciting (REC), counting moveable ordered (CMO), fixed ordered (CFO), and fixed unordered (CFU) items, identifying the quantity of moveable ordered (QMO), fixed ordered (QFO), and fixed unordered (QFU) items, size selection (SS), and size identification (SI).

Figure 2. Number of correct (maximum = 5) on pre- and posttraining tests for Gerald on reciting (REC), counting moveable ordered (CMO), fixed ordered (CFO), and fixed unordered (CFU) items, identifying the quantity of moveable ordered (QMO), fixed ordered (QFO), and fixed unordered (QFU) items, size selection (SS), and size identification (SI).

Figure 3. Number of correct (maximum = 5) on pre- and posttraining tests for Ken on reciting (REC), counting moveable ordered (CMO), fixed ordered (CFO), and fixed unordered (CFU) items, identifying the quantity of moveable ordered (QMO), fixed ordered (QFO), and fixed unordered (QFU) items, size selection (SS), and size identification (SI).


NUMBER CORRECT
SI
SS
QFU
CFU
QFO
CFO
QMO
CMO
REC
1 2 3 4 5
FRANK
TRAINING
TESTS
5
10
15
20


TRAINING
GERALD
NUMBER CORRECT
REC
CMO
QMO
CFO
QFO
CFU
QFU
SS
SI
1 2 3 4 5
5 10 15 20
TESTS


TRAINING
KEN
REC
CMO
QMO
CFO
QFO
CFU
QFU
SS
SI
NUMBER CORRECT
1 2 3 4 5
5 10 15 20

In some cases, there was lack of improvement despite a baseline which was sufficiently low to allow improvement to occur. For example, Frank showed little improvement (i.e., less than 1.0) on identifying the quantity of moveable ordered items and on size selection, while Ken also showed only a small improvement on size selection. Similar small improvements are also seen with Allan (on identifying quantities of all counted items), David (on size identification), and Randy (on counting fixed unordered items).

Group performance. Figure 4 shows the mean test performance on each objective on pre- and posttraining tests averaged across all men. It can be seen that all objectives showed improvement following training. The smallest

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Insert Figure 4 about here

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improvement (i.e., 1.1) occurred on the objective of size identification while the largest improvement (i.e., 2.7) occurred on counting moveable ordered items. Such trends were representative of the performance of the individual men as well.

Generalization Across Settings, Items, and Responses

The students received training on only one behavior at a time, in the training setting, using training items. Improvement in other settings, with other items, or on other behaviors, are all examples of generalization. Table 3 shows the mean test scores averaged over all the pre- and posttraining tests for all behaviors for each student. This generalization data is shown

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Insert Table 3 about here

Figure Caption

Figure 4. Mean pre- and posttraining test performance on the various behavioral objectives averaged across all students.

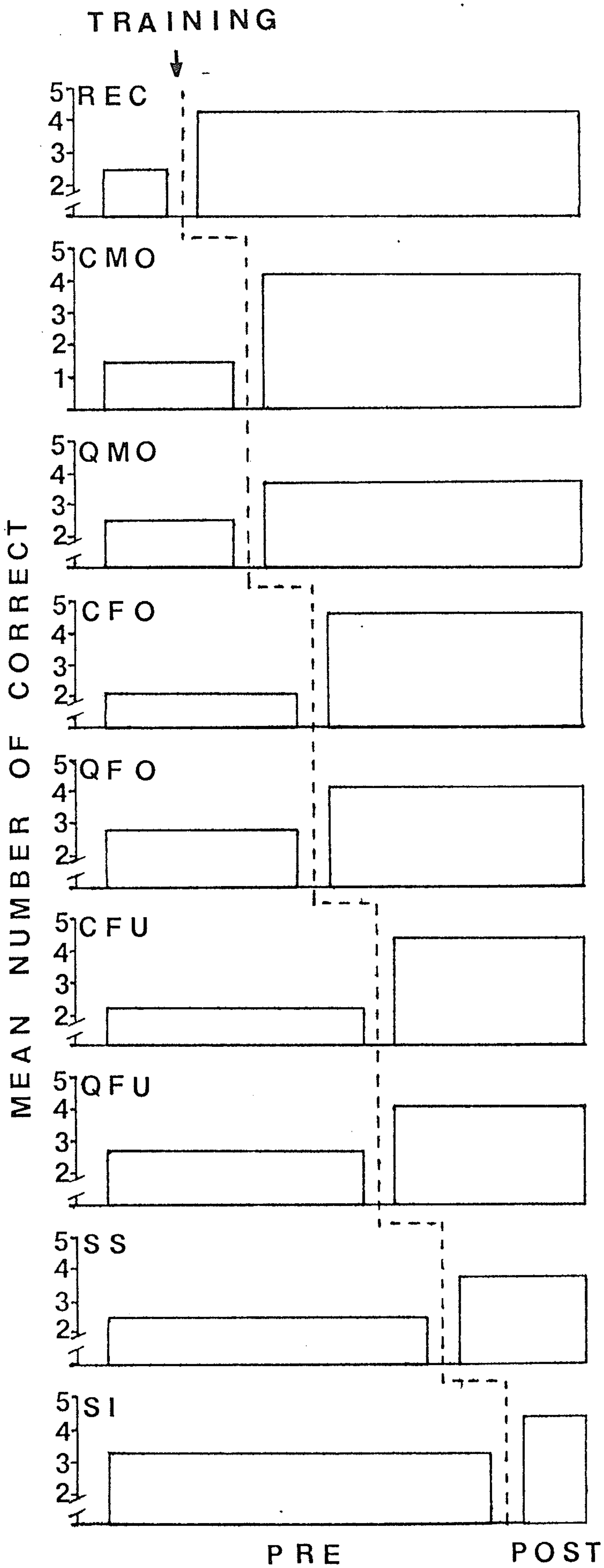
TRAINING
MEAN NUMBER OF CORRECT
REC
CMO
QMO
CFO
QFO
CFU
QFU
SS
SI
PRE
POST

Table 3

Pre- and posttraining test scores averaged across all behaviors for each subject for tests in training (T), generalization (G), and novel (N) settings, and with training (t), generalization (g), and novel (n) items.

	Pre					Post				
Students	T	G	Nn	g	t	T	G	Nn	g	t
Allan	1.8	1.8	1.3	1.8	1.3	3.7	3.4	3.6	3.2	3.4
David	3.2	3.0	3.3	2.9	3.2	4.4	4.4	4.2	4.6	4.5
Frank	2.8	2.6	2.4	2.8	2.7	4.3	4.1	4.3	4.1	4.3
Gerald	1.5	1.6	0.9	1.5	1.5	3.8	4.2	4.6	4.2	3.8
Randy	2.8	2.6	3.1	2.8	2.9	4.5	4.1	3.9	4.3	4.4
Ken	2.7	3.3	---[a]	3.3	2.7	4.7	4.5	4.2	4.6	4.6

Note: Maximum score = 5.0

[a]Due to an oversight, Ken did not receive an Nn pretraining test.

for each behavior for Frank, Gerald, and Ken in Figures 5, 6, and 7 respectively as a set of bar graphs before and after training. Generalization

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Insert Figures 5, 6, and 7 about here

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across settings and items is virtually complete and no clear systematic trends were evident.

Response generalization can be noticed only by inspecting the individual figures. Frank (see Figure 1) did show an increase in test scores on the behavior of counting fixed unordered items following test 10. This increase corresponded with the increase noted in counting fixed ordered items following training on this latter behavior between test 10 and test 11. Gerald (see Figure 2) also showed an increase in test scores on size identification following test 11 which corresponded with an increase on counting and identifying the quantity of fixed unordered items, following training on these behaviors between tests 11 and 12. No such increase could be noted with Ken (see Figure 3). Allan showed one such increase, Randy showed three, and David showed none (see Appendix C).

<u>Generalization Across Numbers</u>

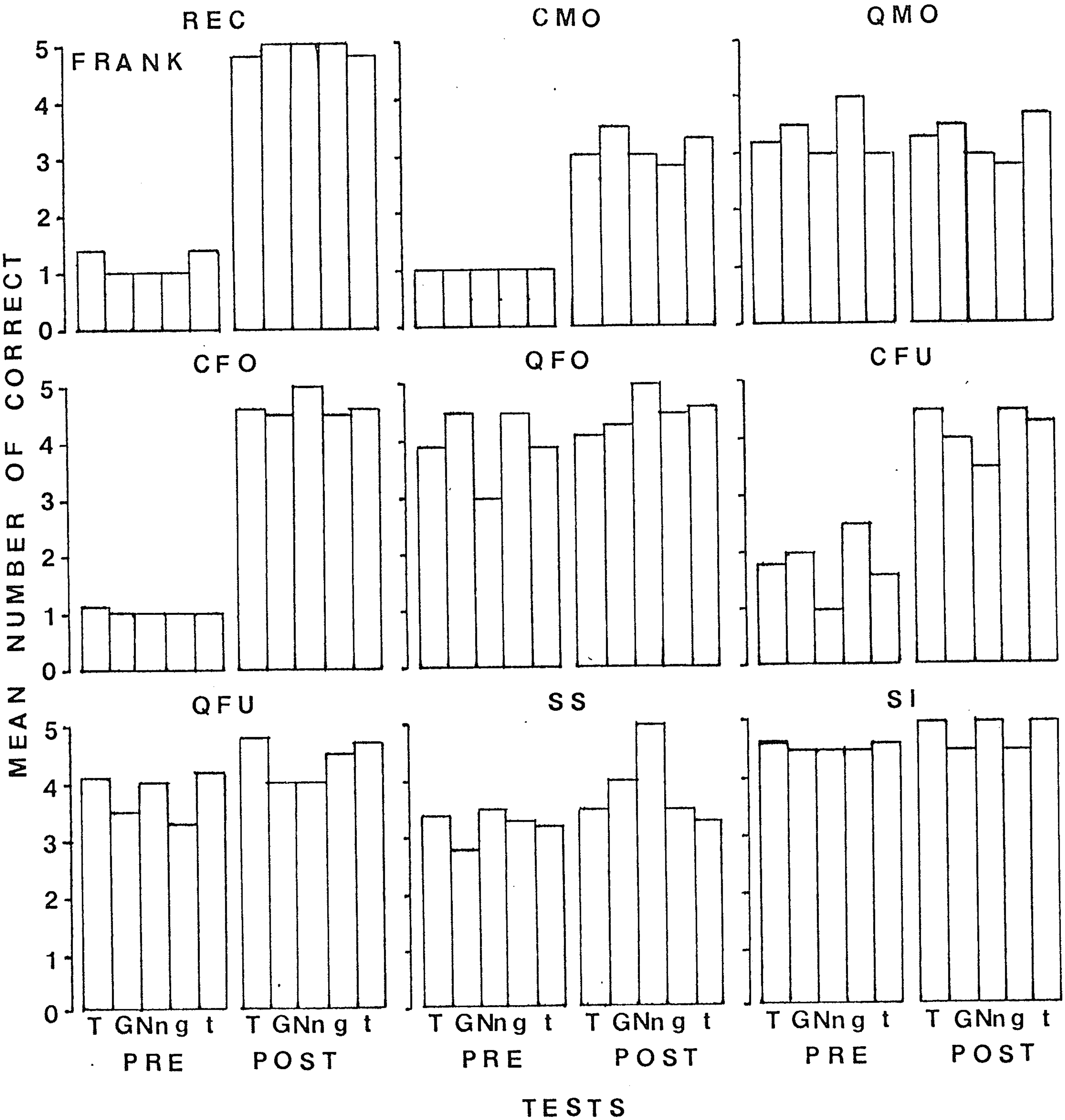
Table 2 shows that for numbers 6-10, six of the eight behavioral measures showed improvement in test scores following training on the corresponding behaviors for numbers 1-5. Upon inspection of Randy's data in Appendix C, it can be seen that for four of the behaviors the effect gradually disappeared over four to six sessions leaving little or no difference between pre- and posttraining test scores. No other individuals had sufficient numbers of probe tests to reveal any trend in the data, and are therefore not presented here.

Figure Caption

Figure 5. Mean pre- and posttraining performance by Frank on the various behavioral objectives in the training (T), generalization (G), and novel (N) settings and with training (t), generalization (g), and novel (n) items.

Figure 6. Mean pre- and posttraining performance by Gerald on the various behavioral objectives in the training (T), generalization (G), and novel (N) settings and with training (t), generalization (g), and novel (n) items.

Figure 7. Mean pre- and posttraining performance by Ken on the various behavioral objectives in the training (T), generalization (G), and novel (N) settings and with training (t), generalization (g), and novel (n) items.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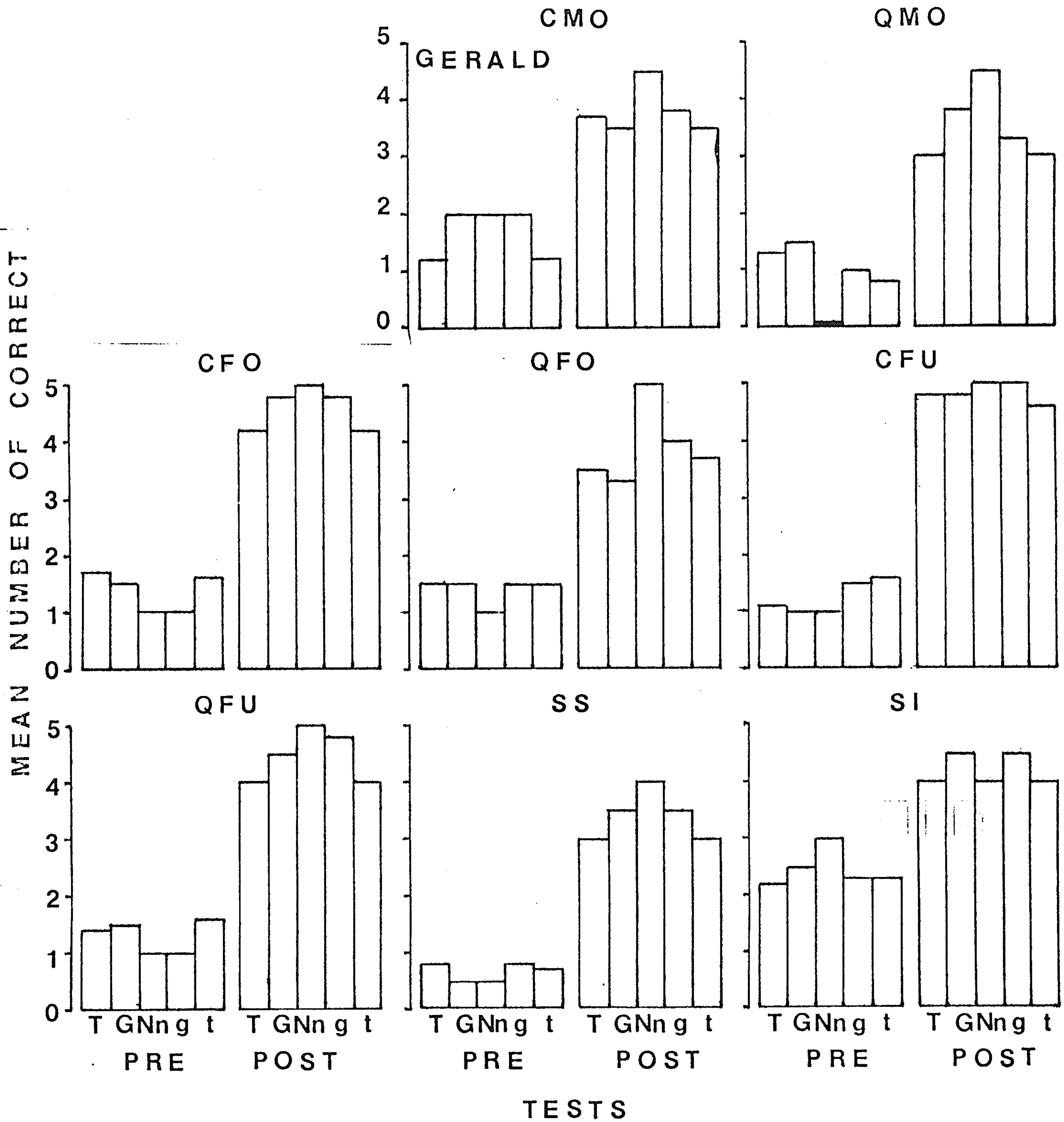

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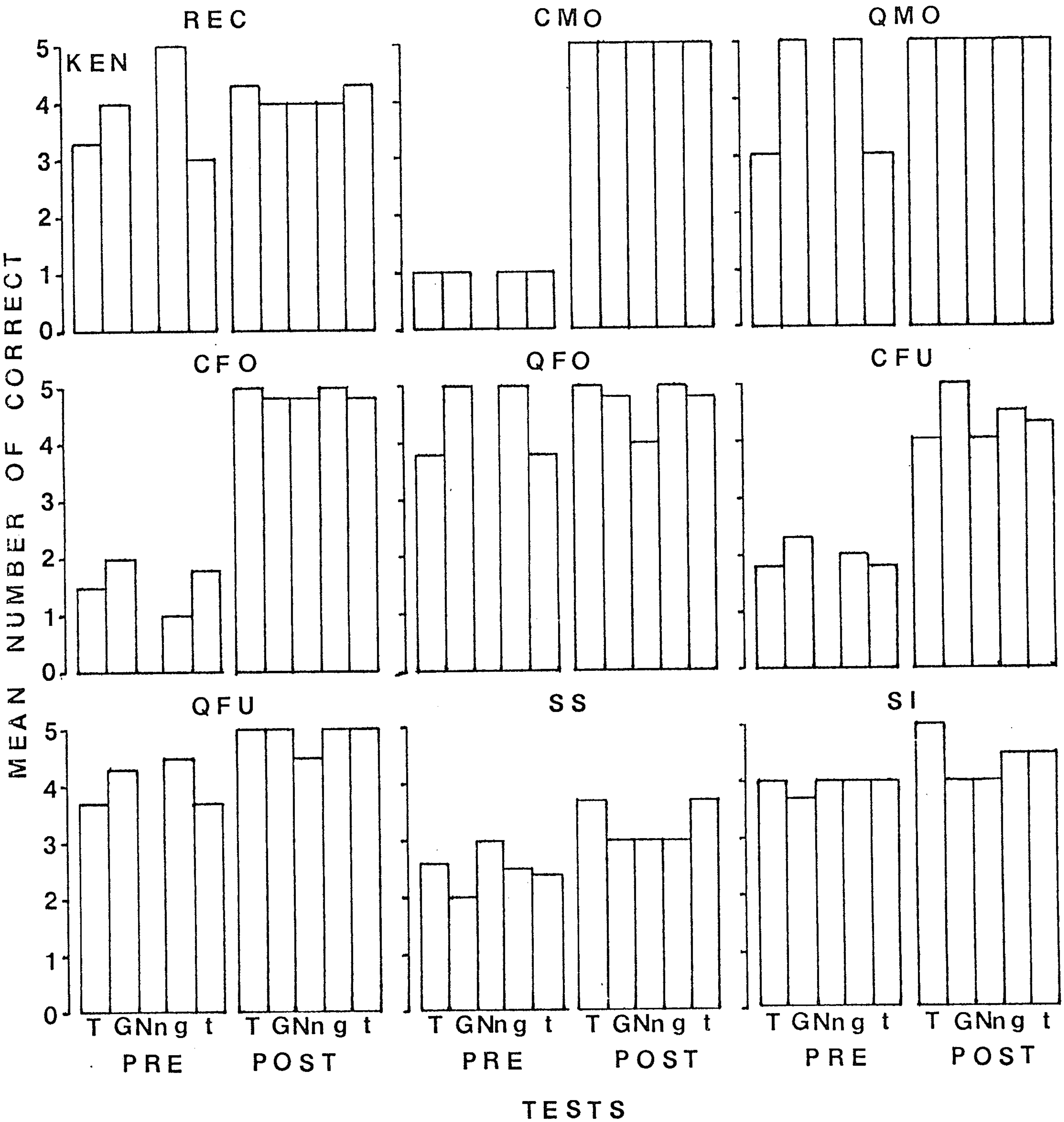
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Discussion

The area of teaching prearithmetic skills to the retarded has received little attention from applied researchers. With few exceptions (eg., Murrell *et al*., 1974; Swartz, 1974) reports of research in this area have failed to define behaviors specifically, describe procedures completely, demonstrate the effectiveness of the procedures experimentally, or show generalization of the skills.

In the present study, six behavioral objectives based upon those described by Resnick *et al*. (1973) were clearly defined. A formalized testing procedure was developed and used to assess the effectiveness of a specific training procedure which was applied systematically in a multiple baseline fashion across the six behaviors, and replicated with six retarded men. All of the men showed improvements on all behavioral measures following training when they received from 6 to 28 (mean = 15) one-half hour training sessions. In addition, all of the men showed generalization of this improvement across settings and items.

Although all the men showed increases in test performance from pre- to posttraining tests, this increase was occasionally small. Such small increases were often the result of pre-existing skills (eg., Frank and David), acquisition during baseline (eg., Ken and Randy), or individual idiosyncrasies (eg., Gerald simply refused to recite in the absence of items and Allan would consistently reply "two" when asked to identify the quantity of items).

In many cases, training resulted in performance that was consistently less than perfect. This was probably due to the criterion chosen for considering that a particular objective was "learned". The criterion would allow the individual to pass the quiz even if a prompt was required on a trial with a current training objective (see Method section). Because of this, students frequently were moved on to the next objective while their performance was only about 80% correct (i.e., 4 out of 5 correct). Inspection of the data from the training session in which quizzes were passed revealed that the performance on the quiz usually matched performance on the tests immediately following the training period. This suggests that if the quiz criterion were made more stringent (eg., 5 out of 5 correct on the current objective), performance on the tests following training might be better.

Of particular interest is the extent of generalization obtained. The current study showed complete generalization with the behaviors occurring in the presence of other settings and objects. This is of practical importance since it would allow training to occur in quiet secluded areas with convenient training items and not require further intervention strategies (eg., Stokes and Baer, 1977) in order to have the behaviors appear in functional situations.

Improvement in the performance of one behavior as a result of training on another (i.e., response generalization) occurred unpredictably throughout the data. The present study was not designed to identify the variables responsible for such generalization. One could speculate however that certain components (eg., touching one item at a time) of some of the components were similar and that training on one objective may have strengthened components of others. Such generalization could be an advantage from a practical point of view although experimentally this may lead to questionable demonstrations of a procedure's effectiveness by obscuring or confounding the treatment effects.

One further type of generalization deserves comment. If the number sequence is known and a generalized skill is learned (eg., counting moveable ordered items), then it might be expected that the skill would be displayed with other numbers not exposed to the training procedure. The occurrence of such generalization across numbers would be highly desirable since providing instruction with all numbers that may be necessary for an individual to know is simply impractical. Such generalization across numbers appears to have occurred with David on virtually all behaviors which received training and which showed improvement (see Figure B in Appendix C). Unfortunately, only two measures (i.e., counting and identifying the quantity of moveable ordered items) showed maintenance of these effects over time. Why some behaviors would show maintenance and others not, is not clear.

It is also noteworthy that with some slight changes in the behavioral definitions the procedure was able to successfully accommodate a nonvocal student. Ken's acquisition of the objectives either as a result of the training procedure or simply the series of baseline tests is encouraging. The use of such nonvocal communication systems with the retarded as Bliss (eg., Connolly, Note 1, 1979; Connolly and Silk, Note 2, Elder and Bergman, 1978; Vanderhiegen, Brown, MacKenzie, Reiman, and Scheibel, 1975), Non-SLIP (Carrier and Peak, 1975), and more recently, Pictogram-Ideogram Communication or PIC (Leonhart, Maharaj, Kleckner, and Peleshytyk, Note 3; Maharaj, Note 4), etc. has provided new opportunities for otherwise non-communicative individuals. The current study used a counting frame with numbered blocks on it as a communication device. Procedurally, this involved only minor changes in the behavioral definitions used and only slight variations in the specific nature of the components of the training procedure. It would not appear difficult to make further modifications which would allow for the teaching of such prearithmetic skills as are described in this study to individuals using such standard nonvocal communication systems as Bliss, PIC, or Non-SLIP.

Left unresolved by the current study are several important practical questions:

1. Which are the critical components of the training package?

2. Can changes in the training procedure or quiz criterion improve performance on the tests even further?

3. How does this training package compare with others designed to teach similar behaviors (or no package at all)?

4. How can the procedures be adapted to small group or classroom situations?

5. How can the procedures be adapted to nonvocal individuals using more standard methods of nonvocal communication?

6. Can procedures be developed to promote the use of these skills during the daily lives of these men?

These questions, as most, can only be answered with further systematic investigation. (For further discussion of the results see Appendix D).

References

Bijou, S.W., Birnbauer, J.S., Kidder, J.D., and Tague, C. Programmed instruction as an approach to teaching of reading, writing and arithmetic to retarded children. The Psychological Record, 1966, 16, 505-522.

Humphrey, J.H. An exploratory study of active games in learning number concepts in first grade boys and girls. Perceptual and Motor Skills, 1966, 23, 341-342.

Murrell, M., Hardy, M., and Martin, G.L. Danny learns to match digits with the number of objects. Special Education in Canada, 1974, 49, 20-22.

Resnick, L.B., Wang, M.C., and Kaplan, J. Task analysis in curriculum design: A hierarchically sequenced introductory mathematics curriculum. Journal of Applied Behavior Analysis, 1973, 6, 679-710.

Ross, D. Incidental learning of number concepts in small group games. American Journal of Mental Deficiency, 1970, 74, 718-725.

Spradlin, J.E., Cotter, V.W., Stevens, C., and Friedman, M. Performance of mentally retarded children on prearithmetic tasks. American Journal of Mental Deficiency, 1974, 78, 397-403.

Stokes, T.F., and Baer, D.M. An implicit technology of generalization. Journal of Applied Behavior Analysis, 1977, 10, 349-367.

Swartz, F. Effects of prompting, praise, and tokens on the development of counting behavior in a severely retarded boy. School Applications of Learning Theory, 1974, 7, 25-30.

Footnotes

[1]In order to assist in response definition, a sheet of 8½ inch by 11 inch (21.5 cm by 27.5 cm) paper was placed on the table top in the training and generalization settings.

Appendix A: Research on Teaching Prearithmetic Skills: An Evaluative Review from a Behavioral Perspective

What follows is a review paper to be published in the Journal of Special Education Technology.

Research on Teaching Prearithmetic Skills: An Evaluative Review from a Behavioral Perspective [1]

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Valley View Centre

[1]The author wishes to express his appreciation to Garry L. Martin, Joseph J. Pear, and John H. Whitely for their advice and guidance in the preparation of this manuscript. Reprints may be obtained from William B. Leonhart, Department of Psychology, Valley View Centre, Moose Jaw, Saskatchewan, Canada, S6H 4R2.

Running Head: Teaching Prearithmetic Skills

Abstract

This paper reviewed published literature which may be of use to both researchers and consumers of research interested in teaching prearithmetic skills to retarded and nonretarded children. The literature was then critically evaluated along the dimensions of behavioral, technological, analytic, generality, effective, applied and conceptually systematic as suggested by Baer, Wolf, and Risley (1968). It was concluded that published training procedures were weak with respect to provision of a comprehensive list of well defined behaviors, experimental analyses, and assessment of generalization. A behavior analysis approach for developing methods for teaching prearithmetic skills was advocated and some future research strategies suggested.

Research on Teaching Prearithmetic Skills: An Evaluative Review from a Behavioral Perspective

In general, prearithmetic skills can be viewed as number-related behaviors which typically precede the acquisition of the basic arithmetic procedures of addition, subtraction, multiplication and division. Behaviors such as reciting the number names in order, counting items, and identifying quantities are all examples of prearithmetic skills. Such skills are important prerequisites for acquiring further arithmetic skills as well as for effective daily functioning in activities such as telling time, handling money, accurate job performance, and so on.

In 1961, Burns noted that with respect to "the methods and materials of instruction (for teaching prearithmetic skills) - let it again be said that more research needs to be done in this area" (p. 59). Thirteen years later however, Spradlin, Cotter, Stevens, and Freidman (1974) observed that "there has been little research on the development of prearithmetic skills among retarded children. Moreover, training programs usually are designed for children who have already acquired counting skills" (p. 397). Taken together, this indicates a lack of research which clearly documents effective instructional procedures for teaching prearithmetic skills in general and with retarded children specifically.

Recent years have seen a greater emphasis placed upon teaching retarded children and adults. Individuals charged with this responsibility in the area of prearithmetic skills are faced with a difficult task of searching for the information that does exist and evaluating its usefulness.

This paper will review and critically evaluate the research in the area of prearithmetic skills with an emphasis upon research with retarded subjects published over the last 20 years. However, nonexperimental reports, textbooks, and research with nonretarded preschool children will be cited to illustrate points. In addition, in light of the extensive application of behavior anlaysis with retarded and preschool individuals and in the area of education in general, this paper will review literature from a behavioristic or behavior analysis position. Dimensions for such an evaluation were suggested by Baer, Wolf, and Risley in 1968 (i.e., behavioral, technological, analytic, generality, effective, applied, and conceptually systematic), and research will be evaluated along these dimensions.

Behavioral

"Applied research is eminently pragmatic; it asks how it is possible to get an individual to do something effectively" (Baer et al., 1968, p. 93). Behavioral research then, is concerned with identifying specific observable behaviors (vocal and nonvocal) which are emitted by an individual. In addition, Baer et al. also suggested that if human observers were used, then some measure of their reliability should be reported (usually as interobserver reliability).

Thus, prearithmetic skills should be described clearly and completely in terms of observable behaviors to the extent that two independent observers could agree on their occurrence. This interobserver agreement should be calculated and reported. In addition, since the occurrence of a particular response often depends upon the specific stimuli presented at the time (eg., a row of small wooden blocks) and some question or instruction (eg., "How many blocks are there?") it is important to include also such antecedent events in a description of the behavior. If one is to reliably identify an instance of the response, then a description of only its topography is insufficient.

Textbooks. Two early texts (Kennedy-Fraser, 1932; Thorndike, 1923) emphasized the importance of describing antecedent conditions (see Table 1).

Insert Table 1 about here

Kennedy-Fraser suggested an interesting testing and recording system for prearithmetic skills which identified the nature of an instruction given by a tester and a required response by a student (eg., verbal, written, presentation of a set of items, etc.). Although the system described the general nature of the antecedent stimuli and required responses, it did not describe specifically the content of the verbal instructions, extra prompts or instructions, placement of sets, etc., and leaves a good deal to be inferred by the reader.

Thorndike placed even greater emphasis upon clear definitions of arithmetic and prearithmetic skills as well as a functional analysis of such behavior in terms of antecedent stimuli, responses, and consequences.

Since then, many texts have appeared dealing in whole or in part with teaching prearithmetic and arithmetic skills to young children (eg., Clark and Eads, 1954; Copeland, 1970; Hartung, Van Engen and Mahoney, 1961;

Table 1

Acceptability of texts as judged on the dimensions of behavioral (beh), technological (tec), analytic (ana), generalization (gen), effective (eff), applied (app), and conceptually systematic (con).

Author(s)	Dimension						
	beh	tec	ana	gen	eff	app	con
Baumgartner (1965)	no	no	no	no	no	yes	no
Brady et al. (1971)	no	no	no	no	no	yes	no
Clark and Eads (1954)	no	no	no	no	no	yes	no
Copeland (1970)	no	no	no	no	no	yes	yes
Hartung et al. (1961)	no	no	no	no	no	yes	no
Hylton (1977)	yes	yes	no	no	no	yes	no
Ingram (1953)	no	no	no	no	no	yes	no
Kennedy-Fraser (1932)	no	no	no	no	no	yes	no
Kirk and Johnson (1951)	no	no	no	no	no	yes	no
Molloy (1963)	no	no	no	no	no	yes	no
Morton (1953)	no	no	no	no	no	yes	no
Peterson (1973)	no	no	no	no	no	yes	no
Stan (1969)	no	no	no	no	no	yes	no
Swensen (1973)	no	no	no	no	no	yes	no
Stephens (1977)	yes	yes	no	no	no	yes	no
Thorndike (1923)	yes	no	no	no	no	yes	yes

Morton, 1953; Stan, 1969; Swensen, 1969) or the retarded (eg., Baumgartner, 1965; Bradley, Hundziak, Patterson, 1971; Burns, 1961; Ingram, 1953; Kirk and Johnson, 1951; Molloy, 1963; Peterson, 1973). All have offered numerous suggestions as to important component skills and variables which may affect performance, but have not described specific behaviors, instructions, or stimulus arrangements.

Of noticeable exception to the above is a text by Stephens (1977) which presents very clear behavioral definitions such as "When presented with randomly arranged numeral cards (1-10), the student will put them in natural order". (p. 388). In addition, Stephens identified the specific instructions given by the teacher. Similarly, Hylton (Note 1) described a collection of prearithmetic activities extremely well. For example, one behavioral objective was described as follows: "Child will count 5 similar objects from left to right when they are placed in a low container before him and he is told, 'Count the objects'" (p. 13). She also went on to describe the specific placement of the objects, instructions used, and consequences for correct and incorrect responses.

Research reports. Reports of research or inventories for assessing the skills described above frequently failed to make complete descriptions of the relevant behaviors (see Table 2). For example, Humphrey (1966) used an "activity game" as a vehicle for teaching eight number concepts to first grade boys and girls. Neither the activities of the game nor the

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Insert Table 2 about here

Table 2

Acceptability of research reports as judged on the dimensions of behavioral (beh), technological (tec), analytic (ana), generalization (gen), effective (eff), applied (app), and conceptually systematic (con).

Author(s)	Dimension						
	beh	tec	ana	gen	eff	app	con
Beckworth and Restle (1966)	no	no	no	no	no	no	yes
Bijou et al. (1966)	no	no	no	no	yes	yes	yes
Brainerd (1973)	no	yes	no	no	no	no	yes
Brainerd (1974)	no	yes	no	no	no	no	yes
Brainerd and Fraser (1975)	no	yes	no	no	no	no	yes
Burns (1961)	no	no	no	no	no	no	no
Dunkley (1972)	no	no	no	no	no	yes	no
Humphrey (1966)	no	no	no	no	no	yes	no
Jacobson et al. (1973)	no	no	no	yes	yes	yes	no
Litrownik et al. (1976)	no	no	no	no	no	no	yes
Murrell et al. (1974)	yes	yes	yes	yes	yes	yes	yes
Potter and Levy (1968)	no	no	no	no	no	no	yes
Resnick et al. (1973)	yes	no	no	no	no	yes	no
Ross (1970)	no	no	no	no	no	yes	no
Spradlin et al. (1974)	no	no	no	no	no	yes	no
Swartz (1974)	yes	yes	yes	no	yes	yes	yes

assessment procedures were described. Ross (1970) also used game situations to teach prearithmetic skills to educable mentally retarded children. However, the behaviors were described only as rote and rational counting, one to one correspondence, understanding specific quantity terms, and concepts of numbers from one to five. No further description was given.

Similarily, Bijou, Birnbauer, Kidder and Tague (1966) developed individualized training programs for twenty-seven retarded children in reading, writing, and arithmetic. Children were taught to match dots or objects with numerals, count aloud, put numerals in their proper order, as well as some simple addition and subtraction. Unfortunately, the behaviors and methods of assessment were described no further. Likewise, Dunkley (1972) suggested an inventory of numbers three to nine which required the subjects to select buttons from a heap, mark number symbols, recognize number symbols, count by rote, and demonstrate the concept of ordinal numbers, but did not specify precisely the behaviors, instructions, or stimulus arrangements.

Spradlin et al. (1974) described five types of prearithmetic tasks. They were (a) rote counting to a specific numeral; (b) counting a set of objects which was either fixed or movable and ordered or unordered; (c) counting out a specified subset of objects from a larger unordered set; (d) using numerals which included writing numerals and matching numerals with a spoken numeral name or another numeral; and (e) set/numeral matching, which included matching a numeral with a given set of objects and matching a set of objects with either a given numeral or another set. Although the above descriptions by Spradlin et al. are considerably better than those mentioned previously, they still do not specify the instructions used or precisely what defined a matching or counting response.

Several articles have described the behaviors to be assessed or taught quite well. Perhaps the most successful attempt to develop an inventory of prearithmetic (and simple arithmetic) skills in terms of overt behavior was reported by Resnick, Wang and Kaplan (1973). They identified 39 steps or objectives which ranged from rote counting to simple addition and subtraction equation completions (eg., $\underline{x}$ + _ = $\underline{y}$; where $\underline{x}$ and $\underline{y}$ would be specified values and the subject would fill in the blank). They not only compiled a comprehensive (though not exhaustive) list of prearithmetic and basic arithmetic skills, but also identified the smaller units of these behaviors in terms of specific stimuli and response components. In addition, all the objectives were arranged in sequential or parallel order as to which should be taught first, second, third, and so on. This order was emperically verified using kindergarten children as subjects (Wang, Resnick and Boozer, 1971) and it was suggested that the sequence presented was approximately the normal developmental sequence. More recently however, Swartz (1974) and Dixon (1977) suggested that this order may be slightly different with retarded children.

Murrell, Hardy and Martin (1974) described a fading procedure to teach an eight year old autistic boy to count out a required number of felt discs. The instructions for the task (eg., "Danny, what number is this?"), arrangement of stimuli (eg., markings on a piece of paper, presented numeral, position of tester's hand, etc.), and topography of the response (eg., orange discs were placed inside the outlined circle) were all completely described.

Swartz (1974) described a training program which used prompts, praise, and tokens to teach a severely retarded boy to point to and count a maximum of five plastic tokens. Swartz also specified the instructions, stimulus arrangements, and response topographies which formed the target behaviors. Furthermore, Swartz was the only researcher to report interobserver reliability measures.

The problem then is that numerous sources offer suggestions about what, how, and when to teach or test various prearithmetic skills, but do not provide descriptions of observable behavior. If they do, they do not specify under what conditions these behaviors should occur. Such information is necessary if one is to identify target behaviors and structure situations which effectively bring about the occurrence of these behaviors. Also, although often the behaviors are easily observed, more effort should be made to assess the reliability of the observers who record such behavior.

Technological

A technological report is one in which "the techniques making up a particular behavioral application are completely identified and described" (Baer et al., 1968, p. 95). This requires that training methods be described to the extent that, from a reading of their description, a study could be replicated by another trained researcher. A report of a procedure for teaching prearithmetic behavior should describe completely all testing and training procedures including (as mentioned earlier) instructions, stimulus arrangements, and behaviors. In addition, a technological study would describe the subjects, apparatus, settings, programmed consequences of the target behavior, the background conditions (eg., a classroom token system, other related training, practice of behaviors, etc.).

Of the texts cited in the preceding section, none gave complete descriptions. Although one may not expect a text to describe specific procedures, it should provide useful information about teaching strategies and identifying conditions under which the strategies described will be effective (eg., in individual sessions, after previous verbal training, with concrete material, etc.). Such conditions were rarely discussed in the texts reviewed.

Research reports of training procedures did somewhat better than texts in describing their techniques, although many still fell short of being adequate (eg., Humphrey, 1966; Ross, 1970). For example, Ross suggested that the use of small group games produced "incidental learning" of number concepts in a group of educable mentally retarded children which exceeded the games made by a control group in a special education classroom. While Ross did report that the amount of time spent in game situations was equal to the amount of time spent on number concept training in the classroom, there was no description of the classroom methods or content, and insufficient information was given concerning the experimental or assessment procedures. It is unlikely that this research could be replicated on the basis of this procedural description. The same is true of the classroom situation for training prearithmetic skills described by Bijou et al. (1966).

Only Swartz (1970) and Murrell et al. (1974) reported their training procedures adequately. Swartz described procedures for evaluation, prompting, reinforcement and correction. Murrell et al. described similar features of their training procedure including several fading steps, generalization test, and a token system.

While their exists many sources of general advice about teaching prearithmetic skills, there is only a limited amount of published information which clearly specifies how to go about such a task. Applied behavior analysis has a history of precise description of methodology and as such, has much to offer in this respect. Regardless of the instructional approach chosen, it seems critical that researchers attend to and carefully report precisely how the behaviors were taught.

Analytic

To be analytic, a report should give "a believable demonstration of the events that can be responsible for the occurrence or nonoccurrence of a behavior" (Baer et al. 1968, p. 94). Baer et al. also made it clear that they favoured the use of within subject comparisons and the reporting of data from individual subjects in order to achieve this believability. A procedure for teaching prearithmetic skills, therefore, should demonstrate that it was the training procedure which produced any resulting acquisition of behavior (i.e., through systematic manipulation of the training procedure).

Again, the literature is lacking in good analytic research. Both Humphrey (1966) and Ross (1970) used group comparisons and did not systematically vary their training procedures, and in neither study were any individual data presented.

Murrell et al. (1974) did present individual data and used a design which approximated a multiple baseline across behaviors (eg., see Hersen and Barlow, 1976; Martin and Pear, 1978). They taught a subject to count out a required number of felt discs. Their training procedure was applied sequentially to numbers one through ten in a multiple baseline fashion; however, only one baseline test was done for all numbers prior to training. As training progressed from smaller to larger numbers, regular tests were conducted on previously learned numbers but no tests (except the initial baseline) were conducted on numbers not yet trained. Evaluation of the subject's performance on all ten numbers throughout the training, rather than only on previously learned numbers, would have produced a more complete multiple baseline design.

Swartz (1974) also reported individual data and used a ABAB reversal design to demonstrate the effectiveness of his training procedure. During the A conditions, no prompting or reinforcement was used and the subject was simply instructed to emit the appropriate touching or touching and counting response. Prompts, praise, and token reinforcement were then added during the B conditions. The training procedure was shown to produce and maintain appropriate pointing and counting.

Although Swartz achieved a reversal, it may not always be possible or desirable to use such a design. For this reason, and since numerous target behaviors can be identified (eg., Resnick et al. 1973) a multiple baseline design across behaviors seems ideal for conducting analytic research in this area if the potential problems of rapid acquisition and response generalization can be controlled.

Generality

"A behavioral change may be said to have generality if it proves durable over time, if it appears in a wide variety of possible environments, or if it spreads to a wide variety of related behaviors" (Baer et al., 1968, p. 96).

This dimension is especially relevant to the study of prearithmetic skills. These skills should be demonstrated with respect to a variety of items (eg., identify quantities of buttons, pencils, chairs, etc.), in several locations (eg., in classroom, at home, in a hallway, etc.), and the behavior should be durable over time (eg., a child might demonstrate a behavior at the beginning of a school year which was taught in the previous school year). Thus, any report of a procedure for training prearithmetic skills should demonstrate that generalization occurs across items and settings, and should show that the behavior is maintained following the termination of the training program.

Of the studies which taught prearithmetic skills, only two reported any measure of generalization. Jacobson, Bernal, Greeson, and Rich (1973) reported the assessment of generalization across a thirteen month time period. They used a concept development test (their own) to show that gains made by retarded children thirteen months earlier (following a successful training program) were maintained over time. Murrell et al. (1974) reported an excellent assessment of successful generalization across items, locations, and time. Following a procedure which taught a young autistic child to count out the correct number of felt discs in an institutional setting, three tests were conducted using different items (i.e., bingo chips, wooden blocks, and puzzle pieces). Similar tests were conducted eight months later in the subject's home using the felt discs, bingo chips, and wooden blocks. Ninety percent accuracy was reported on all tests.

Generalization as described above is a critical issue in teaching prearithmetic behaviors and it is somewhat surprising that assessment of generalization has been reported so infrequently. It would seem that tests of generalization across items and settings could easily be added to research designs and would require only a minimum of extra effort. Similarly, it would not be difficult to use a multiple baseline design (across responses) when teaching several prearithmetic skills to assess generalization across responses. Durability of the acquired behavior over time requires little effort to assess but requires a delay in reporting research. Reporting these various types of generalization seems well worth the extra effort and time required to acquire such data.

Effective

When Baer et al. (1968) discussed the effectiveness of a behavioral application, they suggested that "the theoretical importance of a variable is not usually an issue. Its practical importance, specifically its power in altering behavior enough to be socially important, is the essential criteria" (p. 96). Thus, a procedure for teaching prearithmetic skills should produce enough behavior change to be useful for the student. For example, the student may acquire behavior which is required for further training (eg., money handling skills, telling time, simple addition and subtraction), or which allows him to behave more effectively in his environment (eg., following instructions or making requests which involve numerically conceptual behavior).

Baer et al. also suggested that it is the people who must deal with the subject (eg., parents, teachers, peers, the subject himself, etc.) who are best qualified to judge the effectiveness of a behavioral intervention. Thus, a report of a training procedure should include some discussion (if only brief and incidental) of the potential impact any behavior change has had upon the functioning of that individual in his environment.

Humphrey (1966), Ross (1970), and Swartz (1974) all reported procedures which successfully taught prearithmetic skills but did not discuss the effectiveness (as defined above) of their procedures.

Bijou et al. (1966) did not report any specific data for the subjects of their research, but did report that the children who learned several prearithmetic skills were later taught simple arithmetic skills. Eventually, training was introduced which taught telling time and discrimination of monetary values. Although not explicitly stated, such results seem to indicate that the initial procedure for training prearithmetic skills produced changes in the behavior of the subjects which could be described as effective since it led to the acquisition of other behaviors which seemed to be of practical importance to the subjects and to the individuals with whom the subjects dealt.

Murrell et al. (1974) also discussed the effectiveness of their training procedure. They pointed out that after their subject was trained to count out varying numbers of items upon request, that his special education teacher made favourable comments concerning his counting performance. Although this report was less specific than that of Bijou et al., it did discuss the effectiveness of the training procedure with reference to an individual (i.e., the special education teacher) who had to deal with the behavior under study.

Applied

Baer et al. (1968) suggested that applied behavioral research should be directed towards solving problems which are of interest to man and society, rather than problems of theoretical interest only. The implication for research on prearithmetic skills appears to be that it should be directed towards identifying and teaching new, useful behaviors to individuals deficient in such skills, or towards improving the effectiveness and efficiency of procedures used to teach such skills.

Several investigators have studied theoretical issues such as the development of ordination and cardination (Brainerd, 1973, 1974; Brainerd and Fraser, 1975) as well as such variables as object arrangement, similarity, number (Beckwith and Rustle, 1966; Potter and Levy, 1968), color, shape (Beckwith and Rustle, 1966), meaningfulness (Potter and Levy, 1968), rule verbalization, and observer gender (Litrownik, Franzini, Turner, 1976). While it may be argued that the stimuli studied are of importance to the teaching of prearithmetic skills, and thus applied, it is the opinion of this author that the research seemed more concerned with the relevance of the variables to theory.

Spradlin et al. (1974) pointed out that "training programs usually are designed for children who have already acquired counting skills" (p. 397). This appears to be especially true of behavioral research. Although research of this type is clearly applied, it is frequently concerned with increasing the overall rate, quantity, or quality of performance on simple arithmetic tasks, and not with the teaching of more elementary prearithmetic skills.

On the other hand, there has been a distinct lack of applied behavioral research concerned with the actual teaching of prearithmetic skills. Humphrey (1966) and Ross (1970) both used indirect strategies (game-like situations) for improving prearithmetic skills of nonretarded and retarded children respectively. Only Bijou et al. (1966), Swartz (1974) and Murrell et al. (1974) reported training procedures which directly taught prearithmetic skills to individuals who were deficient in such skills and for whom such skills were relevant to their daily lives.

Conceptually Systematic

"[A field of study] will probably advance best if the published description of its procedures are not only precisely technological, but also strive for relevance to principle ... This can have the effect of making a body of technology into a discipline rather than a collection of tricks." (Baer et al., 1968, p. 96). There is no contradiction in suggesting that an applied (i.e., not primarily concerned with theory) analysis of behavior should be conceptually systematic. An applied behavior analysis is not concerned with proving that principles of behavior exist or testing theories about behavior. It is concerned with solving problems of behavior, identifying the principles responsible for the solution, and relating those principles to a larger body of scientific and technological knowledge.

An applied behavior analysis of prearithmetic skills then, should experimentally analyze the behavior to develop methods for teaching it, and relate these analyses and methods to the basic principles and procedures involved. Both Bijou et al. (1966) and Swartz (1974) related their methods to such principles and procedures. However, an excellent example of a conceptually systematic study is that of Murrell et al. (1974). They identified (and demonstrated by way of figures) several distinct fading steps through which they gradually changed the stimuli controlling a counting response (placing a specified number of felt discs in a required location) from a circle drawn on a piece of paper to the outstretched hand of the experimenter. The procedure was clearly identified and discussed as an example of a fading procedure.

An applied behavior analysis of prearithmetic skills which is conceptually systematic, serves two important functions. First, it serves to make "a body of technology into a discipline rather than a collection of tricks [which are] difficult to expand systematically and when they [are] extensive, difficult to learn and teach" (Baer et al., 1968, p. 96). Second, it serves to emphasize the <u>behavior</u> of the student and the analysis of this behavior.

GENERAL CONCLUSIONS AND FUTURE RESEARCH STRATEGIES

This paper reviewed available literature which might be of use to both researchers and consumers of research interested in teaching prearithmetic skills. This literature was evaluated along the dimensions suggested by Baer et al. (1968) for evaluating applied behavioral analyses and specific conclusions regarding these dimensions can be found in the relevant sections of this paper. Some more general conclusions and comments will now be presented.

Currently Available Literature

Although there is an abundance of research articles and textbooks which investigate the development of number concepts, compare various populations, and offer much advice about general strategies to teach prearithmetic skills, there are very few reports of procedures for teaching these skills. If an individual were interested in developing an effective and efficient procedure to teach such behavior, there is little available data-based literature to aid in that endeavor.

Utility of a Behavioral Analysis

It was suggested several times in this paper that an applied behavior analysis of prearithmetic behaviors has much to offer in the development of procedures for teaching such behavior. In what ways might it be useful? Of primary importance is the emphasis given by an applied behavior analysis to the clear and complete description of the behavior of interest in observable terms and to the specification of conditions under which the behavior is to occur.

Rather than describing the development of prearithmetic skills or comparing various populations with respect to these skills, an applied behavior analysis would develop and compare procedures for teaching these behaviors. It would stress the importance of describing these procedures, showing that they are responsible for the observed changes in behavior, and relating these procedures to already well developed principles and procedures.

An applied behavior analysis would also be concerned with producing behavior change that would be of importance to the subjects and to those with whom they had to deal. It would attempt to teach behaviors which would meet with natural reinforcers and thus allow the subjects to interact more effectively with their environment. Towards these ends, it would emphasize not only the assessment of changes in the behaviors under investigation in the experimental situation, but also the assessment of changes in other related behaviors, situations, and at times following the termination of the teaching program.

Future Research Strategies

Accepting that reports of good programs for teaching prearithmetic skills are lacking and that an applied behavior analysis can help to develop such programs, then research might be expanded in the following ways:

1. A comprehensive inventory of numerically conceptual behavior should be established and arranged in an order suitable for teaching (the bulk of this work seems to have been done already by Resnick et al., 1973, although their inventory could be expanded further).

2. A relatively standard evaluation format should be developed for testing the above inventory that would specify setting, instructions, stimulus arrangements, items used, and consequences for correct and incorrect

behavior. Several forms of this test might be developed for testing in various situations (eg., individual sessions, classroom, play area, etc.).

3. Effective training procedures should be developed, compared, and their components analyzed and their cost-effectiveness assessed. The effectiveness and efficiency of these procedures should be evaluated over many subjects, with different populations, and in group training situations. The most efficient procedures may be different in each of these cases.

4. Finally, researchers should begin devoting much more time and effort to assessing and programming for generalization of skills learned in the training situation to other stimuli, responses, and across time.

Reference Notes

1. Hylton, J.R. Quantative skills curriculum: number skills and currency skills for the trainable mentally retarded child. Portland, Oregon: Department of Special Education, Meiltnomah County Intermediate Education District, 1974.

REFERENCES

Baer, D.M., Wolf, M.M., and Risley, T.R. Some current dimensions of applied behavior analysis. Journal of Applied Behavior Analysis, 1968, 1, 91-97.

Baumgartner, B.B. Guiding the retarded child: An approach to a total educational program. New York: The John Day Company, 1965.

Beckwith, M., and Restle, F. Process of enumeration. Psychological Review, 1966, 73, 437-444.

Bijou, S.W., Birnbauer, J.S., Kidder, J.D., and Tague, C. Programmed instruction as an approach to teaching of reading, writing, and arithmetic to retarded children. The Psychological Record, 1966, 16, 505-522.

Bradley, M.A., Hundziak, M., and Patterson, R.M. Teaching moderately and severely retarded children: A diagnostic approach. Springfield, Illinois: Charles C. Thomas, 1971.

Brainerd, C.J. Mathematical and behavioral foundations of number. Journal of General Psychology, 1973, 88, 221-281.

Brainerd, C.J. Inducing ordinal and cardinal representations of the first five natural numbers. Journal of Experimental Child Psychology, 1974, 18, 520-534.

Brainerd, C.J., and Fraser, M. A further test of the ordinal theory of number development. Journal of Genetic Psychology, 1975, 127, 21-23.

Burns, P.C. Arithmetic fundamentals for the educable mentally retarded. American Journal of Mental Deficiency, 1961, 66, 57-61.

Clark, J.R. and Eads, L.K. Guiding arithmetic learning. New York: World Book Company, 1954.

Copeland, R.W. How children learn mathematics: Teaching implications of Piaget's research. New York, The MacMillan Company, 1970.

Dixon, M.H. Ranking some mathematics skills of preschoolers. Psychological Reports, 1977, 41, 223-226.

Dunkley, M.E. Mathematics and the disadvantaged child. Elementary School Journal, 1972, 73, 44-49.

Hartung, M.L., Ban Engen, H., and Mahoney, C. Teaching guide for our number workshop, #1. Toronto: W.J. Gage, Ltd., 1961.

Hersen, M., and Barlow, D.H. Single case experimental designs: Strategies for studying behavior change. New York: Pergamon Press, 1976.

Humphrey, J.H. An exploratory study of active games in learning number concepts in first grade boys and girls. Perceptual and Motor Skills, 1966, 23, 341-342.

Ingram, C.P. Education of the slow learning child. New York: The Ronald Press Company, 1953.

Jacobson, L.I., Bernal, G., Greeson, L.E., and Rich, J.J. Intellectual and conceptual acquisition in retarded children: A follow-up study. Bulletin of the Psychonomic Society, 1973, 1, 340-342.

Kennedy-Fraser, D. Education of the backward child. New York: D. Appleton and Company, 1932.

Kirk, S.A., and Johnson, G.O. Educating the retarded child. Cambridge, Massechusetts: Houghton Miffin Company, 1951.

Litrownik, A.J., Franzini, L.R., and Turner, Acquisition of concepts by TMR children as function of type of modelling, rule verbalization, and observer gender. American Journal of Mental Deficiency, 1976, 80, 620-628.

Martin, G.L. and Pear, J.J. Principles and tactics of behavior modification: What it is and how to do it. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1978.

Molloy, J.S. Trainable children curriculum and procedures. New York: The John Day Company, 1963.

Morton, R.L. Teaching children arithmetic. Morrison, New Jersey: Silver Burdett Company, 1953.

Murrell, M., Hardy, M., and Martin, G.L. Danny learns to match digits with the number of objects. Special Education in Canada, 1974, 49, 20-22.

Peterson, D. Functional mathematics for the mentally retarded. Columbus, Ohio: Charles E. Merrill Publishing Company, 1973.

Potter, M.C. and Levy, E. Spatial enumeration without counting. Child Development, 1968, 39, 265-272.

Resnick, L.B., Wang, M.C., and Kaplan, J. Task analysis in curriculum design: A hierarchically sequenced introductory mathematics curriculum. Journal of Applied Behavior Analysis, 1973, 6, 679-710.

Ross, D. Incidental learning of number concepts in small group games. American Journal of Mental Deficiency, 1970, 74, 718-725.

Skinner, B.F. About behaviorism. New York: Alfred A. Knopf, 1974.

Spradlin, J.E., Cotter, V.W., Stevens, C., and Friedman, M. Performance of mentally retarded children on prearithmetic tasks. American Journal of Mental Deficiency, 1974, 78, 397-403.

Stan, J.W. The teaching of mathematics in the elementary school, Scranton, Pennsylvania: International Textbook Company, 1969.

Stephens, T.M. Teaching skills to children with learning and behavior disorders. Columbus, Ohio: Charles E. Merrill Publishing Company, 1977.

Swartz, F. Effects of prompting, praise, and tokens on the development of counting behavior in a severely retarded boy. School Applications of Learning Theory, 1974, 7, 25-30.

Swensen, E.J. Teaching mathematics to children. (Second edition) New York: MacMillan Company, 1973.

Thorndike, E.L. The psychology of arithmetic. New York: The MacMillan Company, 1923.

Wang, M.C., Resnick, L.B., and Boozer, R.F. The sequence of development of some early mathematics behavior. Child Development, 1971, 42, 1767-1778.

Appendix B: Nonvocal Behavioral Objectives

The six nonvocal behavioral objectives used with Ken were:

A. Recite. When given a verbal instruction (eg., "Count to four") the student was required to move the response numerals on the counting frame, one at a time, from the right side of the frame to the left until the specified numeral is moved, and then stop. Moving more than one numeral at once, stopping (more than 5 seconds) short of the specified number, or continuing (moving the next numeral past the midway point for more than 5 seconds) past the specified numeral, was considered an error.

B. Count moveable ordered items. When presented with a set of moveable items in an ordered linear array and a verbal instruction (eg., "Count these blocks by moving them over the line") the student was required to move the items onto a 22" cm x 28" cm piece of paper once and only once, counting each (i.e., moving the corresponding response numeral) as he did so. Insufficient movement of blocks, simultaneous movement of more than one block or numeral, or lack of correspondence between movement of items and response numerals, was considered an error.

When finished, the student was required to identify (i.e., touch the correct response numeral) the quantity of items when asked by the tester (eg., "How many blocks are there?"). Although recorded separately, both parts of the response (i.e., counting and quantity identification) were required to be correct before reinforcement was given.

C. Count fixed ordered items. When presented with a set of fixed (although in fact the items were moveable, the student was not allowed to move them) items in an ordered linear array and a verbal instruction (eg., "Count these blocks by just touching them."), the student was required to touch or clearly point to the items once and only once, counting each as he did so. Excessive movement (more than 1 cm) of an item, incorrect counting, or lack of correspondence between touching or pointing and counting, was considered an error. Students were then required to identify the quantity as described above.

D. Count fixed unordered items. Instructions and requirements were identical to (C) above except that the items were scattered at random (dropped from just above the table) onto the paper. Items landing off the paper were moved 1 cm onto the paper and all items were placed at least 1 cm apart. The tester made these alterations prior to giving the instruction.

E. Size selection. When presented with a set of moveable objects located off the paper and a verbal instruction (eg., "Show me four blocks."), the student clearly indicated a subset of the appropriate size by moving the required number of items onto the paper. No counting or correspondence was required for this objective.

F. Size identification. When presented with three distinct sets of fixed items and a verbal instruction (eg., "Which group has four blocks?"), the student pointed to or touched the group having the appropriate number of items. If the student counted the blocks first, the tester then repeated the instruction (eg., "So which group has four blocks?"), and the next motor response was taken as the student's response. No counting or correspondence was required for this objective.

Appendix C: Extended Presentation of Results

Treatment Effects

Reciting. Four of the six students received training to criteria on reciting. Of these four, Frank showed the most improvement (i.e., from a mean pre-training test score of 1.3 to a mean posttraining test score of 4.9) while Ken showed only a small amount (less than 1.0) of improvement (i.e., from 3.5 to 4.2) as shown in Table 2 (see Results section). Following training, all four individuals except Randy showed a mean post-training test score above 4.0.

Counting moveable ordered items. All students received training on this objective and all showed noticeable (i.e., greater than 1.0) improvement following training. Table 2 shows that for all students, the mean test score improved from 1.5 prior to training, to 4.2 after. This mean score accurately reflects the improvement of each student.

Identifying the quantity of moveable ordered items. This objective was trained in combination with counting moveable ordered items. The degree of improvement on this objective was from 2.5 to 3.7 averaged over all students, less than for the previous objective. Much of this lack of improvement is accounted for by Allan and Frank who showed only small improvements of 0.5 and 0.1 respectively. Inspection of Figure A shows that

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Insert Figure A about here

Allan scored 1.0 on every baseline test, which was also true of identifying the quantities of fixed ordered and unordered items as well.

Figure Caption

Figure A. Number of correct (maximum = 5) on pre- and posttraining tests for Allan on reciting (REC), counting moveable ordered (CMO), fixed ordered (CFO), and fixed unordered (CFU) items, identifying the quantity of moveable ordered (QMO), fixed ordered (QFO), and fixed unordered (QFU) items, size selection (SS), and size identification (SI).

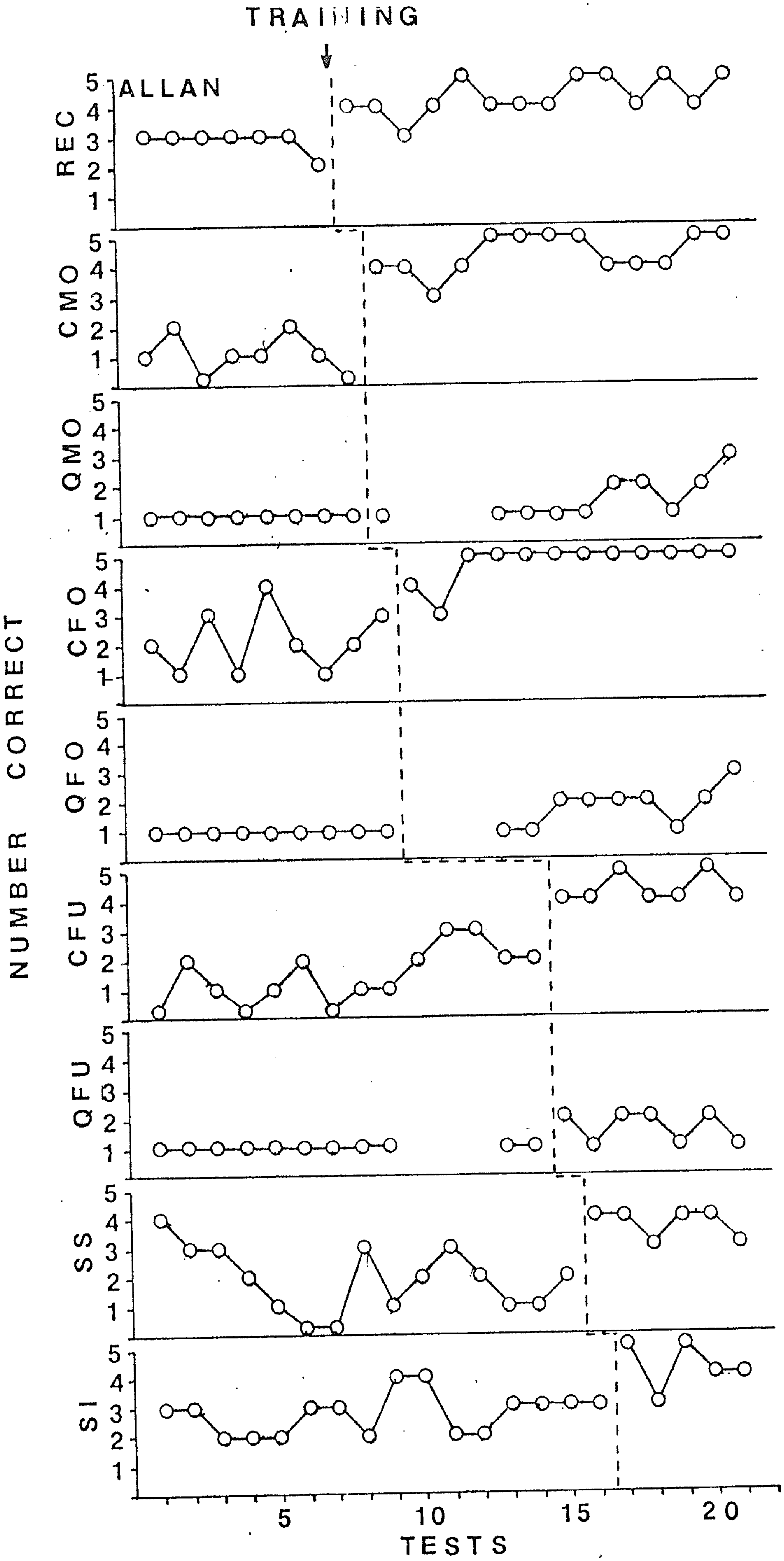
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Figure B. Number of correct (maximum = 5) on pre- and posttraining tests for Randy on reciting (REC), counting moveable ordered (CMO), fixed ordered (CFO), and fixed unordered (CFU) items, identifying the quantity of moveable ordered (QMO), fixed ordered (QFO), and fixed unordered (QFU) items, size selection (SS), and size identification (SI).

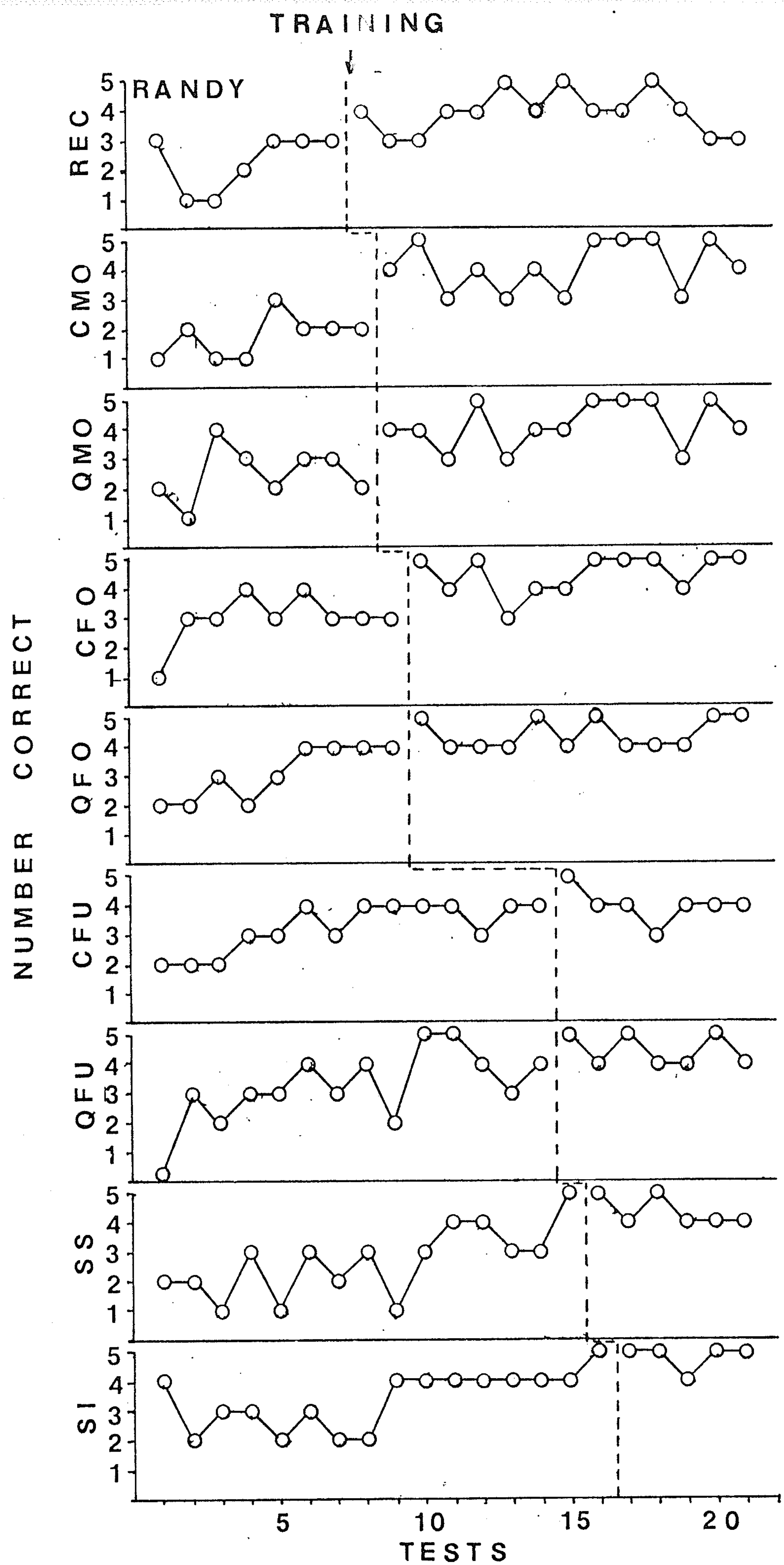

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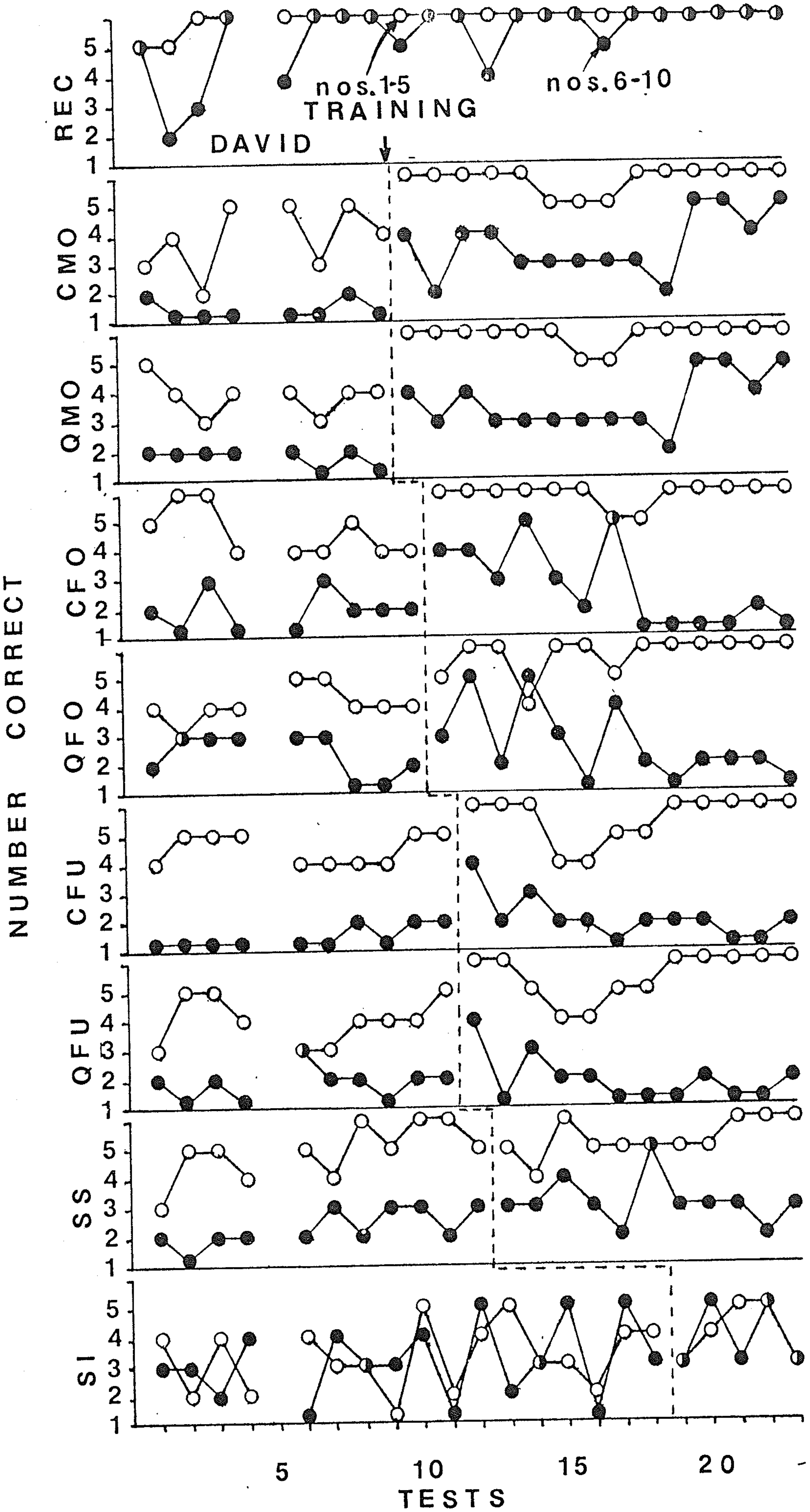
Figure C. Number of correct (maximum = 5) on pre- and posttraining tests for David on reciting (REC), counting moveable ordered (CMO), fixed ordered (CFO), and fixed unordered (CFU) items, identifying the quantity of moveable ordered (QMO), fixed ordered (QFO), and fixed unordered (QFU) items, size selection (SS), and size identification (SI).

Figure D. Mean pre- and posttraining performance by Allan on the various behavioral objectives in training (T), generalization (G), and novel (N) settings and with training (t), generalization (g), and novel (n) items.

Figure E. Mean pre- and posttraining performance by David on the various behavioral objectives in the training (T), generalization (G), and novel (N) settings and with training (t), generalization (g), and novel (n) items.

Figure F. Mean pre- and posttraining performance by Randy on the various behavioral objectives in the training (T), generalization (G), and novel (N) settings and with training (t), generalization (g), and novel (n) items.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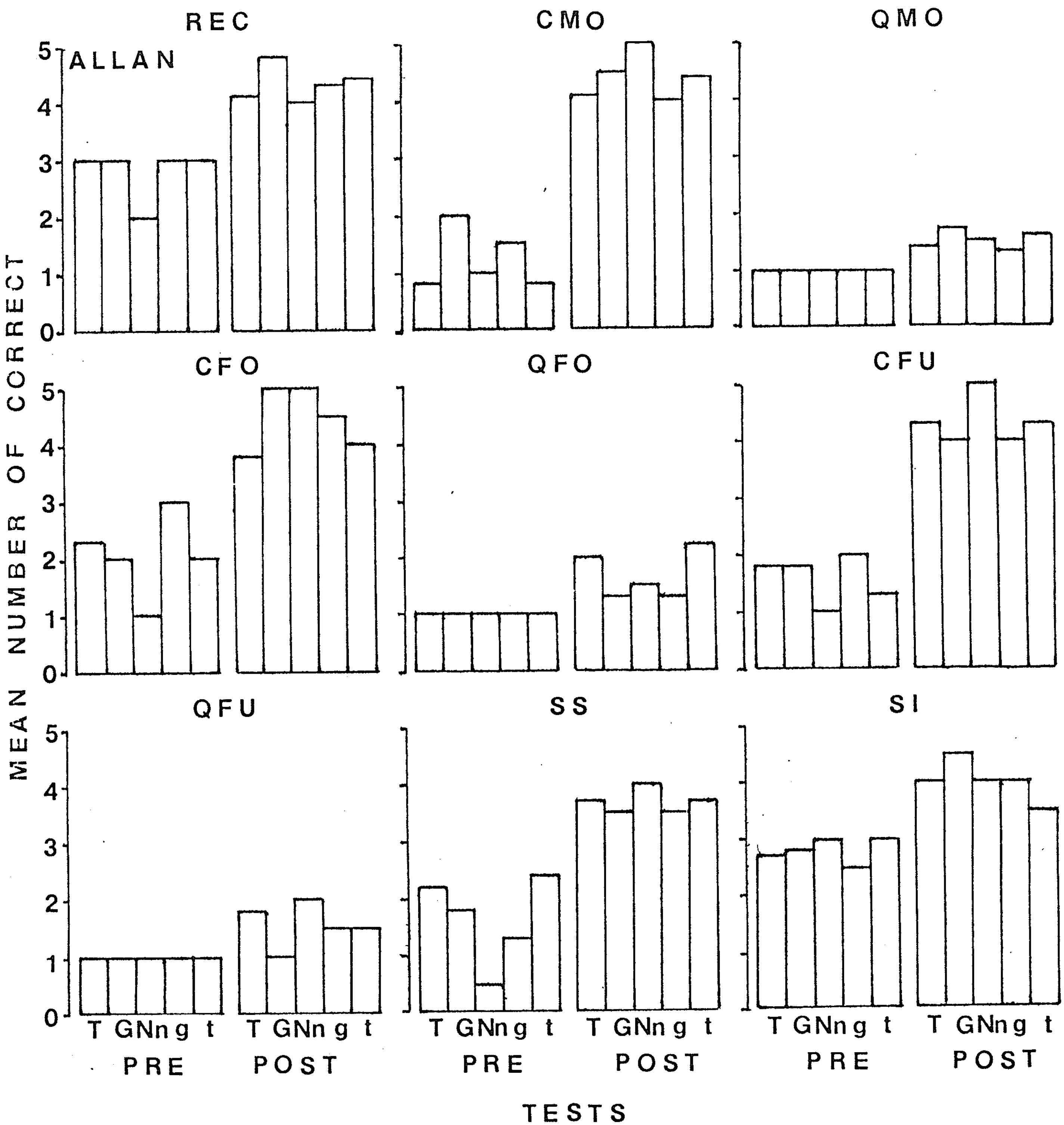

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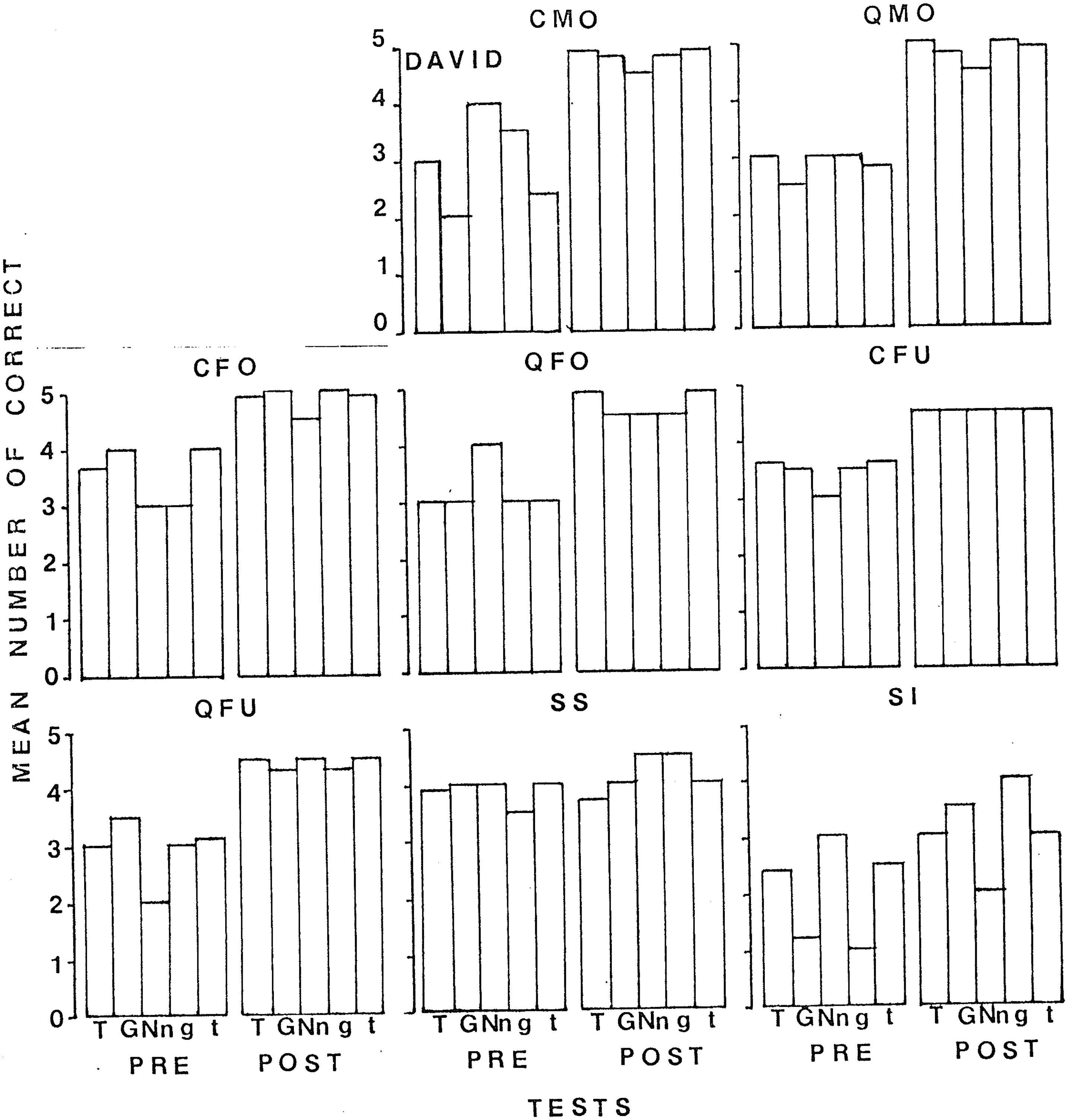
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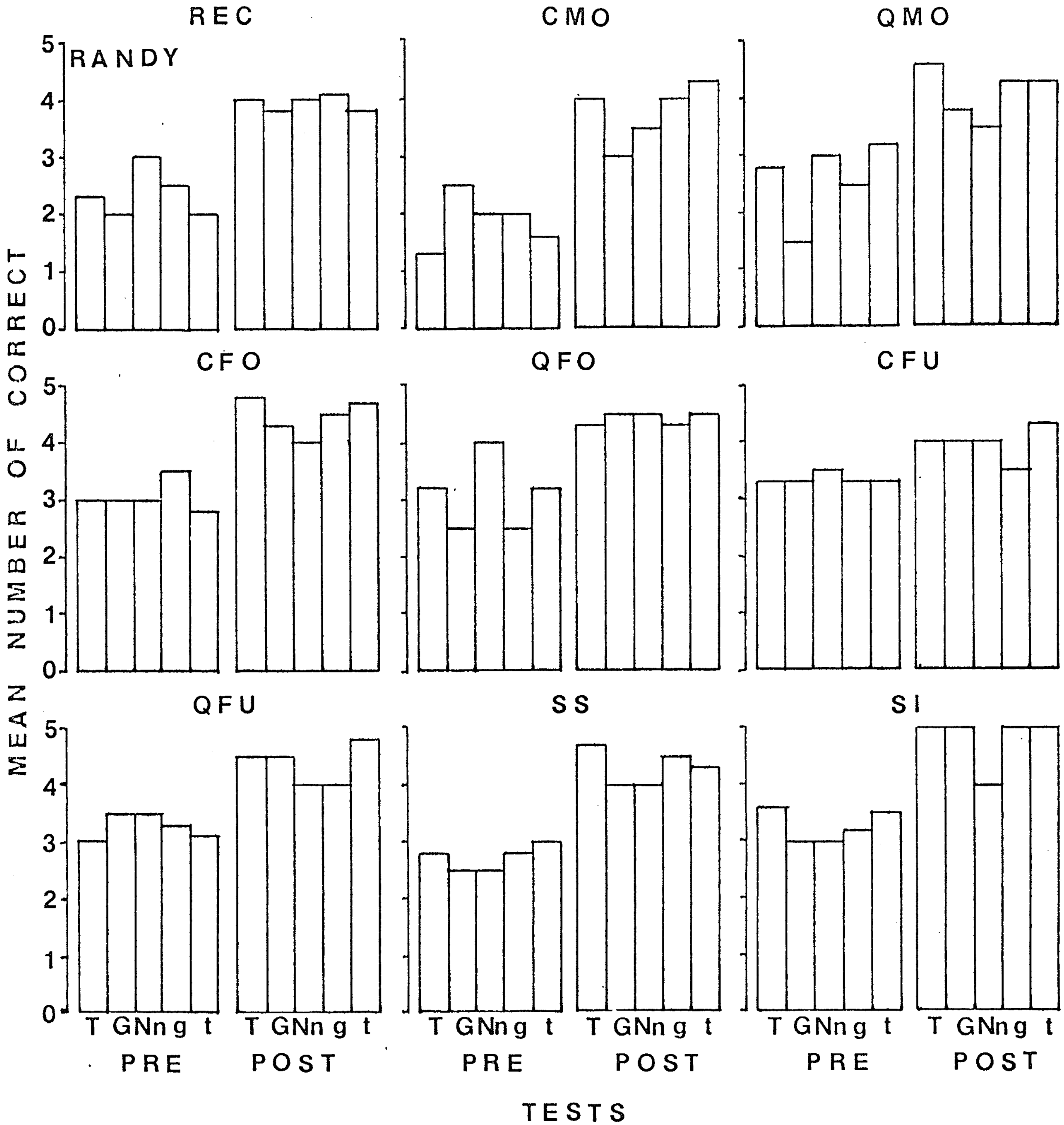
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<u>Counting fixed ordered items</u>. As with counting moveable ordered items, all students showed noticeable improvement on this objective. Averaged over all students, the mean pre-training test score was 2.1 and the posttraining test score was 4.6 which accurately represents the improvement shown by each student.

<u>Identifying the quantity of fixed ordered items</u>. Three of the students (i.e., Allan, Frank, and Ken) showed only small improvements on this objective. However, it should be noted that both Frank and Ken had high mean baseline scores (i.e., 3.9 and 4.2) which restricted the amount of improvement possible. Following training, the mean test score for all students was 4.1.

<u>Counting fixed unordered items</u>. Five of the students showed noticeable improvement on this objective. Only Randy showed a small degree of improvement (i.e., from 3.3 to 4.0) despite substantial room for such improvement. Although the training procedure produced only a small increase in performance, Figure B shows gradual improvement during the baseline period for this and several other behaviors (i.e., counting moveable ordered items, identifying the quantities of fixed ordered and fixed unordered items, as well as for size selection).

Insert Figure B about here

<u>Identifying the quantity of fixed unordered items</u>. Four of the students showed noticeable improvement on this objective. Again, despite a low mean baseline score of 1.0, Allan showed improvement only to 1.6 fol-

lowing training. A relatively high mean baseline score of 3.9 again limited the improvement possible for Frank who showed a small increase to 4.5. All other students showed increases which are represented by the mean scores of 2.2 and 4.4 before and after training respectively.

Size selection. Three of the students showed noticeable improvements on this objective. Frank, Ken, and David on the other hand all showed only small increases, although it should be noted that David's performance is perfect on the last three tests as Figure C shows.

Insert Figure C about here

Size identification. This skill required only that the students select one group out of three. Random selection should yield an average of 1.7 correct choices on each test. It is not too surprising then that the average mean pre-training score for this objective was 3.2. After training, it was 4.3, the smallest mean improvement of any of the objectives. Four of the men showed noticeable improvement. Only David and Frank did not; however, Frank's high mean baseline score of 4.5 limited such improvement.

Generalization

Across settings. The mean test scores for all tests conducted in the training, generalization, or novel settings were combined for each area, regardless of items used, and presented for each objective as bar graphs in Figures 5, 6, 7 (see Results section), and D, E, and F. Prior

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Insert Figures D, E, and F about here

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to training, both Frank and Gerald showed a tendency (i.e., in seven of the objectives) to perform slightly better on the tests in the training setting than in the novel setting. Gerald also showed a similar tendency to perform better in the generalization setting than in the novel setting. Otherwise, no systematic differences can be seen prior to training. Following training, Gerald's data showed a reversal to better performance in novel settings than in the training or generalization settings. Frank no longer showed differential performance and Randy appeared to do better in the training setting than in the novel setting. No other students showed any differential performance. Following inspection of Table 3, it can be seen that all the tendencies identified were small, with the largest being 0.8 (i.e., Gerald).

Across items. The mean test scores for all tests conducted with the training, generalization, or novel items were combined for each item used, and presented for each objective (see Figures 5, 6, 7, D, E, and F). Prior to training, only Gerald seemed to show differential performance with higher scores on training and generalization items than on novel items. Following training, a reversal of this trend can be seen with scores being higher for novel items than for generalization items and both of these higher than training items. Allan also tended to do best with the novel items and poorest with the generalization items; however, this latter trend was not as consistent as with the novel items. On the other hand, Randy tended to perform best with training items, more poorly with the generalization items, and poorest with the novel items. Again, these latter trends of Allan's and Randy's were weak and consistent only across five or six objectives. All other students showed no clear indication of differential performance with respect to items.

Across responses. Response generalization can be noticed only by inspecting the individual figures. Frank and Gerald showed some evidence of such generalization; whereas Ken did not (see Results section). David also showed no response generalization. Allan showed a slight improvement in counting fixed unordered items following test 9 which corresponded to the completion of training on fixed ordered items. Randy showed improvement on identifying the quantities of fixed ordered and unordered items, size selection, and size identification around test 9 or 10. These increases, however, may be part of a gradual upward trend in test scores during baseline.

Across numbers. Only David was tested on numbers 6-10 sufficiently often to detect any generalization. It seemed clear that there was a noticeable and maintained improvement in David's performance on counting and identifying moveable ordered items for numbers 6-10 as a result of training on numbers 1-5. A similar increase was seen for counting and identifying the quantities of fixed ordered items and fixed unordered items, however, this effect lasted only from three to six sessions and then performance returned to about its baseline level. Only a small increase occurred on size selection for numbers 6-10 following training on numbers 1-5 whereas on size identification, a slight decrease occurred.

Appendix D: Extended Discussion of Results

Visual inspection of the individual figures reveals the effectiveness of the training procedure somewhat better than Table 3. All students showed improvements in test scores on from four (i.e., Ken) to seven (i.e., Gerald) objectives which are characterized by changes immediately following the completion of training and posttraining data points which overlap little with those of the baseline conditions. In addition, the multiple baseline design demonstrated that it was the training procedure which was responsible for the observed improvements. Such within subject demonstrations have been lacking in the area of teaching prearithmetic skills in part as a result of an overall lack of research, with only Murrell, Hardy, and Martin (1974) using a partial multiple baseline design and Swartz (1974) using an ABAB reversal design. When a number of behaviors are to be taught, as in the present study, the multiple baseline design is easily incorporated as part of the overall instructional system.

Although improvements occurred frequently, they often were not large. Only about one-third of the improvements noted above could be considered large (i.e., greater than or equal to three), with neither David nor Randy showing any. Several of the men's baseline scores were too high to allow for a large effect even to occur. This was due either to the individual coming into the study with the skills already present to some extent (eg., Allan, on reciting and size identification; David, on all objectives except counting moveable ordered items and size identification; Frank, on identifying quantities, size selection, and size identification; Ken, on size identification; Randy, on counting fixed ordered items) or improvement during the baseline conditions (eg., David, on counting moveable ordered items; Frank, on counting fixed unordered items; Ken, on reciting and identifying the quantities of moveable and fixed ordered and fixed unordered items; Randy, on counting fixed unordered items, identifying the quantities of fixed ordered and fixed unordered items, and size selection). Either of these situations, along with the limit on possible progress to five correct, would prevent large improvements from occurring.

In some cases, even where further improvement in the posttraining test scores was possible, it did not occur. This was the case with Allan on identifying the quantity of moveable and fixed ordered and fixed unordered items, size selection, and size identification; David, on size identification; Frank, on counting and identification of quantity of moveable ordered items; Gerald, on counting and identification of the quantity of moveable ordered items and size selection; Ken, on size selection; and Randy, on counting fixed unordered items. With the exception of Allan who consistently replied "two" when asked to identify the quantity of items, it appears that the procedure was simply not successful in producing such behavior change. As noted in the Discussion section, this may be due to the criterion used for considering an objective to be learned.

All of the effects due to training noted above appeared to occur in all settings and with all items tested. Murrell _et_ _al_. (1974) also reported good generalization across items and settings, although their testing procedure contained a training component in it, whereas the current procedure did not. In addition, Murrell et al. only reported the training of one objective (for numbers 1-10) which closely resembled the size selection objective as described in this study.

Baer, Wolf, and Risley (1968) pointed out that generalization should not be taken for granted but that it should be planned and it does seem somewhat surprising that generalization was so extensive and complete. What trends there were towards differential performance in the various settings (eg., Gerald and Randy) following training were very small when averaged over all objectives (see Table 3 in the Results section). This was also apparent for each objective in the individual figures. Differences which do occur may have been due to such things as distractions in the generalization or novel settings which tended to hinder performance or, conversely, the effects of novelty which may have increased attention to the task.

Similarly, the generalization which occurred across items was virtually complete. Again, differences which did appear were small and not consistent across objectives or students. As with settings, generalization and novel items in some cases may have been distracting, or in others, may have increased attention. A detailed analysis of the effects of these settings and items is beyond the scope of this study.

None of the behavioral objectives were simply unitary actions. Each had several components which together made up the objectives. This was also pointed out by Resnick, Wang, and Kaplan (1973) in their analysis of the objectives in their inventory. For example, in counting fixed ordered items, an individual must touch an item not yet counted, say a number name, and repeat the sequence. With counting fixed unordered items, the sequence is the same, however, the location of the item touched is not a clear discriminative stimulus for which item should be touched next because of the random placement of items. One might assume then that instruction on counting fixed ordered items might result in improvements on counting fixed unordered items. This, in fact, appeared to happen with Frank. The fact that this did not noticeably happen with other students however, leads one to suspect that there may have been other important components of counting fixed unordered items which were not in common with counting fixed ordered items and, therefore, did not receive training during instruction on fixed ordered items.

Similarly, one could speculate about the common components between counting fixed unordered items and size selection to account for what appeared to be response generalization on size identification with Gerald or the gradual improvements on the identification of quantities of fixed unordered items, size selection, and size identification with Randy. None of the other students showed increases similar to Gerald's and the improvements by Randy appeared to be better described as an increasing trend during baseline.

It still may be that for some of the men some degree of response generalization did, in fact, occur. This particular study however, did not address the issue. It should be pointed out that such generalization would be welcome from an applied point of view (i.e., a reduction in overall training time), however if such generalization occurred on some objective in conjunction with training on that objective, the effects of a training procedure would be confounded and possibly obscured.

David was the only individual to receive extensive evaluation on numbers 6-10. This was due in part to his relatively good performance on numbers 1-5 initially (i.e., a larger assessment seemed desirable), and in part to the extra testing time that would be required if the other five men were tested (i.e., an assessment of numbers 6-10 would double the testing time required). Since it was felt that the baseline performance levels of the other men allowed enough room for improvement, they were not regularly tested on numbers 6-10. The occasional test that was done did not yield sufficient data for analysis and is not presented or commented on here.

David's data showed clear improvement on 6-10 as a result of training on numbers 1-5 on six of the eight objectives which received training. These results would indicate that David learned a basic skill which could be demonstrated with any number of items, as long as he could reliably recite the number sequence and maintain the one-to-one correspondence between the reciting and touching on moving responses. There was a problem with this latter requirement that prevented David from scoring higher still on counting objectives.

The puzzling aspect of David's data was the maintenance of the generalization for moveable ordered items and loss of it for fixed ordered and unordered items after approximately seven and three tests respectively. It may be that since David had a problem with one-to-one correspondence, the movement of the items in the moveable ordered situation made this requirement easier by providing extra visual and textural stimuli; whereas the fixed ordered arrangement reduced the intensity of these stimuli (i.e., items were only touched and the position changed only with respect to the pointing finger). The unordered arrangement further interfered with the visual stimuli since items spread out from the item just touched in many directions.

Performance on size selection and size identification showed no noticeable improvement on numbers 6-10 following training on numbers 1-5. This is understandable, however, since it was also true of numbers 1-5!

The performance of Ken, the nonvocal individual, was quite encouraging. It seems that individuals using such standard nonvocal communication systems as Bliss, Pictogram-Ideogram Communication (PIC), Non-SLIP, etc. might also be taught prearithmetic skills with only slight modifications in the behavioral definitions and training procedure.

Reference Notes

1. Connolly, E. A summary of three years of research using Bliss symbols to teach communication skills to institutionalized retarded adults. Paper presented at the meeting of the Association for Behavior Analysis, Detroit, May 1979.

2. Connolly, E., and Silk, D. Bliss symbols: training step program for nonverbal communication. Unpublished manuscript, 1976. (Available from the Manitoba School, Portage la Prairie, Manitoba).

3. Leonhart, W.B., Maharaj, S.C., Kleckner, J., and Peleshytyk, D. A comparison of initial recognition and rate of acquisition of Pictogram-Ideogram Communication (PIC) and Bliss symbols in institutionalized severely retarded adults. Paper presented at the meeting of the Canadian Association for the Mentally Retarded, Vancouver, October, 1979.

4. Maharaj, S.C. Pictogram-Ideogram Communication system. Unpublished manuscript, 1976. (Available from the Department of Speech Therapy, Valley View Centre, Moose Jaw, Saskatchewan, Canada).

References

Baer, D.M., Wolf, M.M., and Risley, T.R. Some current dimentions of applied behavior analysis. Journal of Applied Behavior Analysis, 1968, 1, 91-97.

Carrier, J.K. Application of a nonspeech language system with the severely language handicapped. In L.L. Lloyd (Ed.), Communication assessment and intervention strategies. Baltimore: University Park Press, 1976.

Elder, P.S., and Bergman, J.S. Visual symbol communication instruction with nonverbal, multiply-handicapped individuals. Mental Retardation, 1978, 16, 107-112.

Murrell, M., Hardy, M., and Martin, G.L. Danny learns to match digits with the number of objects. Special Education in Canada, 1974, 49, 20-22.

Resnick, L.B., Wang, M.C., and Kaplan, J. Task analysis in curriculum design: A hierarchically sequenced introductory mathematics curriculum. Journal of Applied Behavior Analysis, 1973, 6, 679-710.

Swartz, F. Effects of prompting, praise, and tokens on the development of counting behavior in a severely retarded boy. School Applications of Learning Theory, 1974, 7, 25-30.

Vanderhiegen, D., Brown, W.P., MacKenzie, P., Reinan, S., and Scheibel, C. Symbol communication for the mentally handicapped. Mental Retardation, 1975, 13, 34-37.