THE UNIVERSITY OF MANITOBA

A POTENTIAL FLOW REPRESENTATION OF THE TWO-DIMENSIONAL AUGMENTOR WING WITH FINITE JET THICKNESS

by

Vinh Quyen Tang

A THESIS

SUBMITTED TO THE UNIVERSITY OF MANITOBA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF MECHANICAL ENGINEERING

WINNIPEG, MANITOBA

March 1979

A POTENTIAL FLOW REPRESENTATION OF THE TWO-DIMENSIONAL AUGMENTOR WING WITH FINITE JET THICKNESS

BY

VINH QUYEN TANG

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

ABSTRACT

A method of potential flow solution for a simplified two dimensional augmentor wing with a thick uniform jet is presented. Also, a solution for the non-uniform jet is attempted.

In order to concentrate on the effect of the jet thickness and velocity profiles, the augmentor wing is simplified by assuming that the aerofoil is a flat plate and the augmentor has zero length. Thus, the thick jet starts at (and above) the trailing edge, inclined at an angle to the chord line.

For the uniform jet solution, the method replaces the aerofoil by a vortex distribution and uses source and vortex distributions at the jet origin and boundaries to represent the augmented jet. The source strength is related to the primary jet momentum coefficient. The problem is formulated by dividing the vortex distributions into line segments of linear and logarithmic strengths. The vortex strengths and the jet trajectory are determined by an iterative numerical method which requires the flow to be tangential to the aerofoil and jet boundaries, and the jet shape to be in equilibrium under the pressure loading.

When the jet has a thickness of only 0.5% of the chord, the solutions are in close agreement with linear theory and appropriate experiments. Solutions for a range of jet thickness (up to 9% of the chord) indicate that the lift coefficient and the jet trajectory are not affected very much by the jet thickness provided that the primary jet momentum coefficient is kept constant.

The solution for the non-uniform jet is formulated by approximating the jet by several uniform layers. Results are obtained for the special case of two equal thickness layers. It is found that, for a constant total primary jet momentum coefficient, a higher lift is developed when the lower part of the jet has a higher velocity than the upper part.

ACKNOWLEDGEMENTS

I would like to thank Dr. J. Tinkler for his invaluable help as a teacher and a research adviser. I greatly appreciate the long and late hours that Dr. Tinkler has spent generously to give me guidance, instruction, and assistance. Also, I would like to thank him for his concern and understanding of my personal difficulties.

I wish to thank Drs. D.J. Marsden, D.W. Trim and A. Trupp for their review of the thesis.

I wish to express my appreciation to the staff of the Mechanical Engineering Department, and my thanks to Mrs. Paulette Giardino for her typing of this manuscript and to Mr. P. Mitra and H. Weiss for their preparations of some of the drawings.

The grants from the Defence Research Board and National Research Council to support this work are acknowledged.

TABLE OF CONTENTS

LIST OF SYMBOLS

[A]	Coefficient matrix
A_1, A_2, A_3, A_4	Vortex segment end points
[B]	Column matrix
c	Chord length
C_1, C_2, C_3	Constants
C_J	Augmented jet momentum coefficient
$C_{J'}$	Primary jet momentum coefficient
C_L	Lift coefficient
C_M	Pitching moment coefficient (about the leading edge)
D	Function occurring in Equation 78
E	Function occurring in Equation 79
F	Function occurring in Equation 41
G	Slope of the jet center line
H	Function occurring in Equation 42
J	Augmented jet momentum
J'	Primary jet momentum
K	Coefficient in the expressions for singular vortex distributions
L	Lift
L	Function occurring in Equation 48
M	Function occurring in Equation 49
N	Total number of vortex segments
O	Origin of the x, y coordinates
O'	Origin of the ξ, η coordinates

P	Arbitrary point in the flow field
q	Source strength per unit length
R	Radius of curvature
S	Function occurring in Equation 68
T	Function occurring in Equation 69
u	x-component of the induced velocity
u'	ξ-component of the induced velocity
v	y-component of the induced velocity
v'	η-component of the induced velocity
U	Free stream velocity
U_u, U_e	Free stream velocities at upper and lower jet boundaries respectively
V	Augmented jet velocity
V_u, V_e	Augmented jet velocities at upper and lower jet boundaries respectively
W	Function occurring in Equation 64
x	Horizontal coordinate
y	Vertical coordinate
Z	Function occurring in Equation 65
α	Angle of attack
γ	Vortex strength per unit length
Γ	Circulation
δ	Jet thickness
η	Normal axis to the vortex segment line
θ	Angle measured from the source segment line to the line connecting the segment end point to control point ($0 < \theta < 2\Pi$)
Θ	Angle used in Equation 18

ξ	Tangential axis to the vortex segment line
ρ	Density
τ	Initial jet deflection angle
ϕ	Angle measured from the vortex segment line to the line connecting the segment end point to control point ($0 < \phi < 2\Pi$)
ψ	Tan ψ is the slope of the aerofoil and jet boundaries

LIST OF SUBSCRIPTS

a	average
A	of point A
B	of point B
c	of the chord
ct	center point of a segment
C	of point C
D	of point D
i	order of control points
i-j	(the induced velocity) at i^{th} control point due to j^{th} vortex element
j	order of vortex segments
k	constant
ℓ	linear
ℓ	of lower boundary
ℓd	linearly decreasing
ℓi	linearly increasing
N	number of vortex segments on the chord line and jet lower boundaries
N1	N + 1
N2	N + 2
N3	N + 3
NC	number of vortex segments on the chord line
NC1	NC + 1
NC2	NC + 2
NC3	NC + 3
oj	initial conditions of a vortex segment on the jet center line

P	of point P
P-s	(the induced velocity) at P due to a source distribution
P-γ	(the induced velocity) at P due to a vortex distribution
ps	peak strength
q	of the source distribution
u	of upper boundary
∞	at infinity (far upstream or downstream from the aerofoil)

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The development of high lift aerofoils is a step-by-step progress. Starting as early as 1920, methods of improving the aerofoil geometry to produce higher lift were studied. It was found, then, that the circulation around an aerofoil, which is directly proportional to the lift, could be improved by temporarily increasing the aerofoil camber and/or chord by means of mechanical flaps or slats. An example of a two dimensional aerofoil with a mechanical flap is presented in Fig. 1. In practice, there is a limit to the lift obtained by the modification of the aerofoil geometry because of the boundary layer separation which has an adverse effect on the circulation.

Means of controlling or retarding the flow separation were then tested. One method was the blowing of a high-speed jet sheet over the flap upper surface to suppress the separation. During the test of flow separation control in 1938, Hagedorn and Ruden (1) observed that the excess of the blown air originally used to control flow separation also resulted in a higher lift on the aerofoil.

Unfortunately the significance of this important observation was not recognized until more than a decade later, when the high-speed jet sheet was recognized as a solution to the high lift wings. In 1953, tests were done by Dimmock at the National Gas Turbine Establishment on the two dimensional elliptic aerofoil with a jet of air issuing from a slot near the trailing edge and at the lower surface of the aerofoil as seen in Fig. 2. The lift and pitching moment were measured by pressure plotting. The jet sheet behaved like an extended mechanical flap; thus, the name of the jet flap aerofoil was given.

The experimental results of the jet flap aerofoil by Dimmock and the empirical theory of Stratford were reported by M. Davidson in Ref. 2 showing that a very high lift coefficient could be obtained with the jet flap. Also another important result showed that the propulsive thrust was nearly equal to the momentum flux of the jet regardless of the jet deflection angle.

The first theoretical solution of a two dimensional jet flap was presented by Spence (3) in 1956. Spence's mathematical model of the jet-flap aerofoil consisted of a two dimensional flat plate aerofoil with an infinitesimally thin jet sheet issuing from the trailing edge. Ideal flow conditions were assumed, and the aerofoil and jet boundaries were replaced by vortex sheets whose distributed strengths were determined by satisfying the boundary conditions that the velocities on the aerofoil and jet boundaries must be tangential. An integro-differential equation for the vortex sheet strengths was formed. In order to obtain a less complex integro-differential equation, the jet sheet was assumed to be aligned with the aerofoil chord but the local slope of the jet streamline was that of the correct jet trajectory. Spence successfully solved this problem by approximating the distributed vortex strengths by the logarithmic functions, which satisfy the singularity behaviours of the velocity field at the leading and trailing edges, together with a Fourier series.

Spence's approximate solutions are applicable to a thin aerofoil at small angles of attack and small initial jet deflection angles, but the results are in good agreement with experimental findings even for an initial jet deflection angle of 50°. Spence's solutions have been regarded to be very reliable, however in recent years new methods of solutions for the jet flap were attempted.

Leamon and Plotkin (4) also used vortex sheets but allowed the boundary conditions to be satisfied on the real singular surfaces instead of the linearized ones. The distributed vortex strength on the chord and the jet center line trajectory were approximated by singular functions which satisfied the singular behaviours at the leading and trailing edges. The results deviate from the experimental and other theoretical results by 16 percent to 20 percent except for small jet momentum coefficients.

Sato (5) represented a circular cylinder with a jet flap with a number of discrete vortices located on the cylinder surface and along the jet. He then used conformal mapping to derive the flow about an elliptic aerofoil with a jet flap. Solutions of the effects of the jet flaps on an elliptic aerofoil section of 12.5% thickness chord ratio were found to be comparable to experimental results. Examples of calculations for other aerofoil sections were also presented.

At about the same time Herold (5) also presented a two dimensional, iterative solution for the jet flap. He also used the discrete vortex method but the thin aerofoil approximation was applied. His results agreed with those of Spence and experiment for low momentum coefficients, but in general, no marked improvement of the solutions by this method is noted.

Although no exact solution of the jet flap was found, efforts to improve the high lift system were never stopped in the laboratory. One of the practical difficulties of the jet flapped aerofoil was the control of the jet initial deflection angle. It was found that if the jet was ejected over a trailing edge flap (Fig. 3) instead of from the trailing edge, the jet would attach to the flap and leave the trailing edge at the same angle as the flap angle due to the Coanda effect. It was also recognized that this arrangement, known as the jet augmented flap, resulted in a remarkable improvement in lift. The first theoretical solution for the jet augmented flap was also presented by Spence (7) in 1958.

Almost a decade later, the jet-augmented flap arrangement was modified by adding a shroud to improve the thrust and lifting effectiveness. The latter arrangement is called an augmentor wing and is illustrated in Fig. 4. A jet issuing from a span-wise slot at the rear portion of an aerofoil emerges into a gap formed by the upper shroud and lower section of the flap, which directs the flow with a downward angle of deflection relative to the aerofoil chord. The flap is designed to allow mixing of the jet and the secondary induced air flow (Fig. 4) so that augmentation of momentum flux of the primary jet is obtained.

The arrangement of the augmentor wing contributes to high lift on two accounts: the presence of the jet induces an asymmetry in the main stream giving rise to a pressure lift on the aerofoil, and the reaction of the augmented jet momentum results in a contribution to lift (as well as a contribution to thrust). The augmented reaction lift is clearly an advantage over the jet flap arrangement.

Past augmentor wing investigations have been mainly laboratory experiments. In 1964, at the Fourth ICAS Conference in Paris, Whittley (8) presented a report on research progress which indicated the promise of the augmentor wing concept. Research work was then continued with tests on a large scale model in the NASA Ames 40 by 80 feet wind tunnel (9), and the results have shown a significant advantage of such a lifting system.

In 1969, Y.Y. Chan (10) contributed to the analytical solution of the augmentor wing by simplifying the model to that of a jet-augmented flap with the augmentor inlet suction represented by sinks at the hinge line on upper or lower surfaces. He showed that the lift coefficient could be improved by the augmentor wing arrangement. Later, Woolard in Ref. 11 tried to improve Chan's solution by redefining the sink strength.

Recently, Wilson et al (12) presented a new approach to the analysis of the augmentor wing, in which the restrictions of thin aerofoil and infinitesimally thin jet were avoided. The real two dimensional aerofoil and assumed-constant-thickness jet surfaces were used in the calculation of the potential flow field outside the jet. The solutions also allowed for the effects of the jet entrainment and the induced flow at the augmentor entrance by using source and sink distributions superimposed on the vortex sheets.

This thesis presents a theoretical model of the augmentor wing which, in the restrictive conditions of the ideal flow, represents a complete flow field around an augmentor wing including the jet flow. The object is to study the effects of the jet thickness and jet velocity or momentum distribution on the lift coefficient of the two dimensional augmentor wing.

CHAPTER II

MATHEMATICAL MODEL OF AUGMENTOR WING

II.1 Introduction

The velocity field induced by a distributed vortex is a very important concept in aerofoil theory as discussed, for example, in Chapter 12 of Reference 13. The flow about a two dimensional aerofoil can be represented by the flow resulting from the combination of a uniform stream with a distributed vortex, the actual strength distribution being determined by the shape of the aerofoil. The method provides a convenient means to determine not only the total lift but also the distribution of pressure on an aerofoil.

The problem of determining the aerodynamic coefficients of a given aerofoil profile is very difficult. However, as reported in Ref. 13, M. Munk introduced a method of approximation, known as the theory of thin aerofoil, which has proved to be very useful. The method replaces an aerofoil by its mean camber line which is assumed to deviate only slightly from the chord line.

Using the concept of vortex sheets and thin aerofoil theory, and based on Spence's solution for the jet flapped aerofoil, Chan (10) presented a theoretical model of the augmentor wing as shown in Fig. 5. The real augmentor wing was approximated by a flat plate aerofoil with a theoretical sink on the upper or lower surfaces of the aerofoil at the flap hinges. The sink was added to represent the suction flow at the flap hinge. The use of a sink, as in this case, or a distributed sink to represent the suction or entrainment is common. The strength of the sink was arbitrary, but in practice would be empirically determined. The jet thickness was assumed infinitesimal. The jet boundary conditions were satisfied on the linearized jet trajectory; i.e., on the extension of the chord line.

Woolard [11] interpreted Chan's suction coefficient as being based on the total mainstream flow into the augmentor and argued that it should have been based on the increase of flow (due to the jet entrainment), on the basis that no lift should be produced on a flat plate at zero angle of attack when the primary jet momentum is zero.

Another mathematical model of the two dimensional augmentor wing was presented by Wilson et al (12), based on the experimental model of the augmentor wing tested by Wang, Wright, and Mahal*. In Wilson's model, the aerofoil boundary was formed by connecting the front part of the real aerofoil boundary to the shroud and the flap chords by two planes, which were arbitrarily taken to represent the boundaries of the mixing zone of the primary jet and the secondary

---

*Wilson et al (12) incorrectly gave the reference as "Design Integration and Noise Studies for a Jet STOL Aircraft", Vol. IV, "Wind Tunnel Test Program", The Boeing Company, Commercial Airplane Group, Seattle, Washington, NASA CR-114286, May, 1972. Attempts to trace this report have been unsuccessful.

induced flow. The shroud and flap thickness were treated as negligible. The jet, whose thickness was constant and equal to the gap distance between the flap and the shroud, issued from the trailing edge. The aerofoil and the jet boundaries were divided into 200 segments of vortex and source-sink distributions. The segments were approximated by finite straight lines. The jet was truncated when it turned to within two degrees of the free stream. Then two end segments, having the same length as the adjacent segments, were added onto the jet.

II.2 Idealized Model of Augmentor Wing

The flow field about an augmentor wing is very complicated. For the practical purposes of engineering, the simplified or idealized models of the augmentor wing, such as those presented in the previous introduction, are often used in the analysis of the augmentor wing. These models, however, do not represent the complete augmentor wing aerodynamics.

In the real flow, the viscosity of air causes the development of boundary layers on the aerofoil and flap surfaces, and the entrainment along the jet. The boundary layers change the effective surfaces of the aerofoil and affect the drag and lift on the aerofoil. The entrainment increases the jet thickness and changes the jet momentum downstream. Another problem which complicates the augmentor wing aerodynamics is the mixing of the primary and induced flow inside the augmentor. The mixing process is very difficult to understand fully because of the turbulent nature of the flows and many parameters which affect the mixing, such as the augmentor configuration, the initial jet thickness, and the ratio between the initial jet thickness and augmentor thickness (11).

For the purposes of this analysis the flow is assumed incompressible, irrotational and inviscid, so that potential flow theory can be applied. In addition, the augmentor wing configuration is systematically simplified so that it is possible to solve the problem and to test its results at each stage, from the simplified case to the more complex one.

The simplified two dimensional augmentor wing model consists of a thin aerofoil with a downward deflected flap at the rear portion of the aerofoil and a parallel shroud just above the flap as shown in Fig. 6(a). The augmented jet is ejected between the shroud and the flap at the trailing edge in the direction of the flap chord line. Before studying the model in Fig. 6(a), a more simplified model where the flap and shroud chords are assumed to have zero-length will be examined. In the latter model, as shown in Fig. 6(b), the mixing of the jet and secondary induced flow are assumed to be completed in zero length. Furthermore, the functions of the shroud and flap in controlling the jet exit angle and their effects in turning the free stream flow at the trailing edge and shroud leading edge are assumed to be retained. Thus, the finite thickness jet will be considered to issue from the trailing edge at an angle to the aerofoil chord.

The model is simplified further by assuming that the aerofoil has no camber. The model is shown in Fig. 6(c) and consists of a flat plate at an angle of attack with a thick jet at the trailing edge.

II.3 Straight Uniform Jet

In Chapter 3 of Ref. 14, the technique of replacing two parallel vortices and a uniform flow by a source distribution is discussed. The technique is modified here to represent a jet by source and vortex distributions.

Here the two semi-infinite, parallel vortex distributions on two straight lines $y = \frac{\delta}{2}$ and $y = \frac{-\delta}{2}$, as shown in Fig. 7(a), are added to a source distribution on the y axis and between the two vortex distributions. It can be shown that if the source distribution has equal strength to the two vortex sheets, the resulting flow is a straight uniform jet flow of thickness δ in the region $x > 0$ and between the two vortex sheets (see Appendix A).

It is assumed, in this work, that the thick curved jet in a uniform flow can also be represented by a source distribution and vortex sheets of unknown strength distributions as shown in Fig. 8.

II.4 Mathematical Model of Augmentor Wing

It is desired now to use the concept of distributed vortices to construct a hypothetical model for the augmentor wing of Fig. 6(c).

The flat plate is replaced by a distributed vortex with unknown strength along its lines, and the jet is replaced by two vortex sheets at its boundaries and a source distribution at the trailing edge (Fig. 9). Their strengths are to be determined.

Thus, the resultant flow field is made up of the uniform flow, and the velocities induced by the distributed vortices and distributed sources. Because the velocity potential of the uniform flow, vortices and sources individually satisfy the Laplace equation, which is a linear equation, the potential of the resultant flow also satisfies the Laplace equation. The main boundary conditions are that the flow is tangential to the aerofoil surface and jet boundaries, and is undisturbed from the uniform stream (except in the jet) far from the aerofoil.

CHAPTER III

FORMULATION OF THE PROBLEM

III.1 Boundary Conditions

III.1.1 Kinematic boundary conditions

In using vortex sheets to represent the aerofoil and the jet, one of the physical conditions needed in determining the strength of the vortex distribution is that there will be no flow across the aerofoil and jet boundaries.

Generally, without knowing the type of vortex distribution, this condition means the vortex sheets should assume strength distributions that induce a velocity field such that the aerofoil and jet boundaries are streamlines. Mathematically, the velocities induced by the vortex sheets at any point on the boundary, when combined with the velocities induced by the source distribution and the uniform flow velocity should make a velocity tangential with the boundary at that point, or

$$\frac{U_\infty \sin \alpha + v_i}{U_\infty \cos \alpha + u_i} = \tan (\psi_i), \qquad (1)$$

where α is the angle of attack (Fig. 9) and $\tan (\psi_i)$ is the slope of the streamline at point "i". Also v_i and u_i are vertical and horizontal components of the induced velocity, and U_∞ is the main stream velocity far upstream.

III.1.2 Conditions at infinity

At downstream infinity the effects of the disturbances caused by the aerofoil and jet are negligible and the free stream velocity, U, is assumed to approach U_∞.

Concerning the jet trajectory, the physical condition requires the finite thickness jet to be aligned with the free stream at downstream infinity. Thus the jet velocity at infinity must be $(U_\infty + q_\infty)$ where $q_\infty\delta$ is the primary jet flow rate. The jet momentum coefficient measured at infinity is

$$C_{J_\infty} = \frac{J_\infty}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\rho(U_\infty + q_\infty)^2\delta}{\frac{1}{2}\rho U_\infty^2 c}, \qquad (2)$$

where J_∞ is the augmented jet momentum flux at infinity, δ and ρ are the jet thickness and density respectively, and c is the chord length, taken to be unity. The primary and augmented jet densities are assumed to be equal and constant. In this work C_{J_∞} is called the augmented jet momentum coefficient at infinity.