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CHARACTERISTICS OF LOOP YAGI ARRAYS WITH LOADED ELEMENTS AND THEIR APPLICATION TO SHORT BACKFIRE ANTENNAS

By

ALIREZA SHOAMANESH

A Thesis
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ABSTRACT

An analysis of wave propagation along an infinitely long array of single or two concentric loaded circular loops is presented. The travelling wave idea is applied to derive a relationship for the propagation constant along the structure. The analysis is based on a "circuit theory" method and gives the dispersion relation in terms of the mutual impedances between a reference and the other elements, in the array. In addition, Flouquet's theorem is used to account for the periodicity of the structure. It is found that capacitive loading increases the operating frequency range, for a given phase velocity of the travelling wave along the structure, and consequently, increases the bandwidth of the array. Capacitive loading of the inner array in a two concentric loop array, which operates in two passbands separated by a stop-band, increases both the separation of the pass-bands and the bandwidth of the second pass-band.

A study of the excitation problem in finite Yagi arrays shows that for arrays made of two concentric loops the choice of the outer loop as the feeder gives the most superior gain and input admittance characteristics.

As an example of multiple-loadings, the case of a doubly loaded array is considered. It is found that for a relatively smooth variation of the input admittance in a Yagi array, the exciter may be loaded resistively. Extensive investigation has also shown that to improve the gain bandwidth performance of a Yagi array, its directors and reflector must be loaded by capacitive and inductive loads, respectively. Similar result is also obtained for distributed impedance loadings.

Coaxial planar loops and their application to backfire antenna are also studied. First, a general method is described which enables one to study compact arrays and coaxial planar loop antennas with considerable ease. The technique is then used to investigate the radiation characteristics of coaxial planar loop arrays and backfire antennas constructed entirely with loops. The comparison between this type of backfire antenna and the conventional one with a solid reflector reveals that the optimum sizes of the reflector and the peripheral rim are approximately the same for both kinds of the reflectors. However, the optimum gain of the loop-reflector is found to be about 1dB less than that of the solid reflector aerial. In addition, it is found that as few as 6 loops are sufficient to construct the optimum size of the reflectors and almost the same number of loops are required to form the optimum peripheral rim. The new structure reduces the weight, windage and obstruction of view which accompany solid reflectors. In addition, the new antenna lends itself to exact analytical investigation.

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LIST OF SYMBOLS

Unless otherwise stated, the symbols most commonly used in this thesis have the following meaning:

LATIN ALPHABET:

a	radius of wire cross section
a_i	radius of wire cross section of the ith-element
A	a coefficient
b	radius of loop antenna
b_i	radius of the ith-element in a circular loop array
B	a coefficient
c	velocity of electromagnetic waves in free space
C	a coefficeint
C	lumped capacitance
C_{gi}	gap capacitance of the ith element
C'_{gi}	gap capacitance at $\phi = 0$ of the ith element
C''_{gi}	gap capacitance at $\phi = \pi$ of the ith element
d	spacing between two adjacent elements
d_F	spacing between exciter and disk (or loop) reflector
d_{ij}	distance between the ith and jth-element in an array
d_s	spacing of loop antenna from the ground plane

LATIN ALPHABET:

d_s	skin depth
dB	decibel
D	disk (or loop) reflector diameter
e	base of natural logarithms
E_θ, E_ϕ	θ and ϕ components of electric field
E_ϕ^a	ϕ component of applied electric field
E_ϕ^i	ϕ component of induced electric field
f	frequency
$F(\pi/2, \cdot)$	complete Elliptic integral of the first kind
G_d	directive gain in positive Z-direction
G_r	directive gain in negative Z-direction
$h_m^2(\cdot)$	second kind spherical Hankel function of order m
I	integral associated with complete Elliptic integral of the first kind
$I_i(\phi)$	total current of ith-element
I_i^n	coefficient of the nth element mode of ith-element
$\text{Im}(\cdot)$	imaginary part of a complex number
$J_m(\cdot)$	first kind Bessel function of order m
k	free space wave number
kb_i	size of the ith-loop in an array
kb_M	size of the largest loop in a loop reflector

LATIN ALPHABET:

K_{ij}^{n}, $K_{ij}^{'n}$, $\hat{K}_{ij}^{n}$	nth coefficient in Fourier series expansion of the kernel
ℓ	length of Yagi array
L	lumped inductance
$\ln(\cdot)$	natural logarithm
m_o	an integer number after which the impedance matrix can be considered a diagonal matrix
$m!$	factorial m
n	mode number
n_i	ratio of loop and wire radii, i.e., b_i/a_i
N	number of elements in an array of circular loops
P_D	dissipated power
P_r	radiated power
r, θ, ϕ	spherical coordinates
R	resistance
$R_{ij}(\cdot)$	distance between two current components normalized by radius b_i
$\mathrm{Re}(\cdot)$	real part of a complex number
$U_i(\cdot)$, $U_i^0(\cdot)$, $U_i^{\pi}(\cdot)$	a function equal to unity at the driving-point and zero elsewhere
v	phase velocity of electromagnetic waves
V_i, V_i^0, V_i^{π}	driving voltage of the ith-element
$W_{ij}(\cdot)$	kernel

y^n_{ij}	element of admittance matrix
Y_i	input admittance of ith-element
Z_i	internal impedance per unit length
Z_L, Z_{Li}, Z^0_{Li}, Z^π_{Li}	lumped impedance
Z^n_{ij}, Z'^n_{ij}, $\hat{Z}^n_{ij}$	element of impedance matrix

GREEK ALPHABET:

α_i	function
α_i	constant in tapered capacitive loading
β	phase constant of travelling wave
β	constant and $\ln(2\beta) = \gamma$
γ	Euler's constant
δ_{ij}	Kronecker delta function
$\delta\phi_i$	gap width of the driving-point of the ith-element
ε_0	permittivity of free space
ζ_i, ζ'_i	functions
η_0	characteristic impedance of free space
θ	spherical coordinate
$\bar{\theta}$	coefficient
μ_0	permeability of free space
σ	conductivity
ϕ	spherical coordinate
ϕ_{ij}	element of matrix
ψ	angle in surface current distribution
Ω	thickness parameter
ω	angular velocity

CHAPTER I

INTRODUCTION

The electromagnetic theory of circular loop antennas has received considerable attention in the past due to its advantages over dipole antennas, and its relatively simple geometry. The radiation pattern of a small loop antenna is similar to that of a dipole antenna and shows a figure eight shape in the vertical plane involving the loop axis. However, it tends to become directive in the axial direction as the loop circumference approaches one wavelength [1 - 2]. This fact suggests that a highly directive beam may be obtained if an array of coaxial loops of a proper dimension is utilized. A well suited application of the loop array is to employ loops as elements in a Yagi-Uda array, in which only the second element is excited. However, in contrast to the extensive studies available on dipole Yagi-Uda arrays, the literature on Yagi arrays of loop antenna is scarce. The goal of this thesis is to undertake a detailed investigation of these antennas.

1.1 Literature Review

The first general analysis of a circular loop as a transmitting antenna was carried out by Hallen [3]. He considered the loop antenna excited by a lumped generator, an idealized delta function generator, and obtained a formal solution for the current and the input admittance in the form of Fourier series. However, due to the occurrence of a singularity in the high order Fourier coefficients, he was unable to obtain numerical results except for loops small compared to the wavelength. Storer [4] reconsidered the problem but avoided the contribution from the higher order terms by approximating the series by an integral and evaluating it using the Caushy's principle value. He provided extensive tables and graphs of the input admittance and current distributions for loops up to a wavelength in circumference and a number of different wire cross sections. Storer also examined the validity of the constant current distribution on a small loop and was led to the conclusion that loops with circumference larger than 0.2 wavelength can not be considered small.

T.T. Wu re-examined the problem of evaluating the Fourier series. He pointed out that, assuming that the current flows along the center of the conductor, the expansion of Hallen and Storer does not converge everywhere on the antenna [5]. He examined the surface current distribution on a perfectly conducting loop and verified that the resulting Fourier series for $I(\phi)$ converges everywhere except at the driving point.

Later King et al. [6] computed the input admittance of a circular loop by taking a partial summation of the infinite series obtained by Wu. They suggested that a Fourier series solution with twenty terms is satisfactory for determining the admittance of thin $(\Omega = 2 \ln 2\pi \frac{b}{a} \geq 10)$ and $kb \leq 2.5$ loops in air and dissipative media. Here k, a, and b are the free space propagation constant and the wire and loop radii, respectively.

Inagaki et al. by assuming a finite gap at the driving point gave a theoretical basis for determining the required number of terms in the current and admittance computations [7]. They obtained an expression for the driving-point admittance in which the gap capacitance is a lumped representation of the effect of the gap width.

Iizuka et al. have analyzed an array of two identical parallel loop antennas. They have decomposed the voltage and current into symmetric and antisymmetric components. As a result, the simultaneous integral equations for the distribution of the current along the loops have been converted into a single integral equation similar to that of an isolated circular loop antenna which has already been studied [8]. This method, however, is not applicable to antennas with loops of different circumferences. The experimental work of Lindsay [9] is the first published study of relatively large circular loop Yagi arrays. Lindsay reported a gain difference of about 1.8dB in favour of loop arrays in comparison with dipole arrays of the same length. Later, Appel-Hansen reported similar experimental comparison [74] and [10]. He utilized a feeding system which consisted of a circular loop antenna as the exciter and a parasitic loop as reflector. For the director he selected three different configurations, an array of parasitic circular loops made of flat plate loops, a parasitic array of wire loops and a parasitic array of straight rods. Comparing the experimental gain of these directors, he found that for array lengths less than 2λ all three had similar gain performances, but for large arrays the gain of rod directors was somewhat higher. A similar problem was also examined by Takata and Sekiguchi [11]. Their numerical and experimental results show that for arrays lengths less than 2λ there is a gain difference of about 1 dB in favour of loop arrays over the arrays made of rod elements.

The first theoretical investigation of an array of coaxial circular loop antennas with arbitrary circumferences, has been carried out by Ito et al. [12] using Fourier series expansion, with emphasis on the existence of finite gaps at the driving points. The expression for the current distribution on each loop and the input admittance involve matrices, where the dimension of each matrix is related to the array size. Thus, in the investigation of loop arrays the required computer time and storage capacity increase with the array size and for very large arrays become excessive. To overcome these difficulties, the resonant property of loops was later applied by the author to develop an efficient method for the investigation of Yagi loop arrays [13] . For this method, the exact radiation fields and the current distributions of a finite and relatively large uniform array of circular loops were compared with that obtained using the dominant mode only. The discrepancies between the results of both radiation fields and current distributions were found to be negligible. Later, an approximate method was also utilized which facilitated the rapid and accurate analysis of large loop arrays [14]. In this method, a large loop array consisting of an arbitrary number of elements, was sub-divided into several small sub-arrays such that the first sub-array contained the active element. Neglecting all sub-arrays except the first two, the resulting two sub-arrays were solved simultaneously to yield loop currents. The results for the first sub-array were retained and were used in simultaneous solution of the second and third sub-arrays. The process was continued until all loop currents were obtained. The travelling wave approach has also been used to study coaxial loop arrays. Shen and Raffoul have applied this method to finite uniform Yagi arrays of equal loops to provide design parameters for a given bandwidth, directivity or array length [15]. Finally, the electrical properties of coaxial Yagi loop arrays were investigated by the author by applying the exact solution and the significance of each array parameter and their effects on antenna characteristics was determined [16]. Based on these works extensive design data for short and medium size arrays were obtained in [17].

In this thesis we extend the investigation of circular loop arrays to arrays with loaded elements. Wave propagation on an infinite structure is investigated and characteristics of finite arrays are studied. The work presented here is based on the Fourier series expansion method. Thus, a brief review of this theory for an array of coaxial circular loops with arbitrary circumferences is given in the next