THE UNIVERSITY OF MANITOBA

HIGH-Q ACTIVE RC BAND PASS FILTERS
USING TWO OPERATIONAL AMPLIFIERS

by

SIRUS SADUGHI

A Thesis
Submitted to The Faculty of Graduate Studies
In Partial Fulfillment of the Requirements for the Degree
of Master of Science

Department of Electrical Engineering

Winnipeg, Manitoba

February 1979

HIGH-Q ACTIVE RC BAND PASS FILTERS
USING TWO OPERATIONAL AMPLIFIERS

BY

SIRUS SADUGHI

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

ABSTRACT

Three new second order band pass filters are presented. Two of them are based on negative impedance converter techniques which have one and two operational amplifiers in their realizations. The single amplifier realization has better characteristics than a comparable Sallen and Key type, while the two-amplifier realization is superior to conventional RC-NIC filters from the sensitivity point of view. The third realization is a versatile multiple feedback filter which exhibits improved frequency response, reasonable Q-sensitivities, capability of independent tuning of ω_o, Q and R_{in}, and wide dynamic range. Experimental results are provided to support the viability of the proposed filters.

ACKNOWLEDGEMENTS

I wish to express my deep gratitude to Dr. H.K. Kim for his untiring supervision and encouragement. Thanks are due to Professor M. Yunik and Dr. J.S. Townsend for reviewing the manuscript as committee members, especially Professor M. Yunik, my coadvisor, for his helpful suggestions.

CONTENTS

CHAPTER I

INTRODUCTION

Active RC filters were first introduced in the thirties. Due to the bulky size of vacuum tubes, however, they were of little practical use. The situation changed dramatically in the advent of solid state components in the late forties. Research and development efforts in active filters in recent years have been influenced primarily by the progress and breakthroughs in linear microelectronic circuit technology. The reduction of size, weight, power consumption and an increase in electronic system reliability are the main factors for producing active filters in microelectronic form. Because of the inexpensive price of active elements (transistors and operational amplifiers), the addition of these elements in RC filters does not increase price significantly any longer. The application of active RC filters in industry is common in many areas such as consumer electronics, communication systems and precision instruments. The primary application of active RC filters is at relatively low frequencies where inductors are not suitable because of their bulky size and low quality. With presently available commercial operational amplifiers the useful frequency is roughly from dc to 500 kHz. Compared to the more conventional LC filters, active RC filters tend to be smaller in size and cheaper at lower frequencies.

There are various active RC realizations of filters using operational amplifiers. Because filters were specifically designed for different characteristics such as sensitivity, component count, stability, frequency response, dynamic range, gain, input and output impedance, etc., they are quite numerous. The purpose of this thesis is to investigate new types of band pass filters with better sensitivity and frequency response characteristics. Modern active network synthesis started by using negative impedance converters as a network element. These filter realizations proved to be highly sensitive to element changes. The other approaches were proposed later, using impedance inverters and converters, such as gyrators or Frequency Dependent Negative Resistors. Until state variable filters were introduced, many filters based on RC-amplifier coefficient matching technique were reported. They all failed to meet some of the specifications in practical filters and are therefore special purpose types. State variable filters proved to be the best but lacked frequency response. This thesis is an attempt to circumvent some of the shortcomings in realizing band pass filters discussed above. Chapter II reviews these methods of filter realizations, while Chapter III introduces several new configurations together with their characteristics and includes some experimental results.

CHAPTER II

GENERAL FILTER DESIGN

Active RC filter design begins with the approximation of the specified network transfer characteristic as a real rational transfer function in the complex frequency variable s, in the form of

$$T(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_o}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_o} \qquad n \leq m \qquad (2.1)$$

where m is the order of filter.

There are basically two approaches to filter design. In the direct realization approach, the filter transfer function given in (2.1) is synthesized in a manner similar to the passive filter synthesis using two ports. In the cascade realization approach the filter is synthesized as a tandem connection of second order filters. The transfer function T(s) in (2.1) can be decomposed into second order transfer functions $T_1(s)$, $T_2(s)$, ... , $T_k(s)$

$$T(s) = \prod_{j=1}^{k} T_j(s) \qquad (2.2)$$

where $k = \frac{m}{2}$ m even

$k = \frac{m+1}{2}$ m odd, then T_k is first order.

Each second order term has the form

$$T_j(s) = K_j \frac{s^2 + \alpha_{j1} s + \alpha_{jo}}{s^2 + \beta_{j1} s + \beta_{jo}} \qquad (2.3)$$

where $Q_j = \frac{\sqrt{\beta_{jo}}}{\beta_{j1}}$, $\omega_{oj} = \sqrt{\beta_{jo}}$.

The a_i and b_i for the network transfer function T(s) in (2.1) are real and hence the poles and zeros of T(s) are either real or occur in complex conjugate pairs. Since the poles of T(s) are in the left half plane, the βs are all positive. In other words, the denominator polynomial is strictly Hurwitz. On the other hand the αs may assume any real value including negative and zero depending on the location of the transmission zeros. The pairing of poles and zeros for the optimization of certain design criteria is itself an interesting problem but we shall assume that (2) and (3) are obtained. In general each block realizing T_i must be isolated from the ensuing block by means of a buffer amplifier. This implies an increase in the number of required amplifiers. However, easy access to inexpensive active elements, particularly operational amplifiers, justifies multi-amplifier design provided that improvement in performance is attainable. In general, low sensitivities, easy tunability, high-Q realizability are prominent features associated with multi-amplifier configurations.

2.1 Realization techniques

There are several methods for filter realizations. They include use of simulated inductance, frequency dependent negative resistor, RC-amplifier coefficient matching technique, generalized impedance converter, and state space method.

2.1.1 Simulated inductance or gyrator filters

In this type of filter, the transfer function is synthesized by a passive RLC network and then RC active netowrks simulating the inductors are embedded in place of inductors in the original passive network. The gain is always less than unity because of passivity of this kind of filter. The final design however is not simple. Since the output is taken from the passive network, a loading effect appears unless each section is perfectly matched. In general, two active elements are necessary for each inductance simulation. Gyrator filters are mainly used in moderate-Q high order filter realizations. The complexity of design is the same as in passive RLC designs and because of this similarity, the sensitivities are relatively low. The required number of components is high and specially in a cascade realization number of active elements increases. Several circuits are available for inductance simulation. They are basically impedance inverters which of course will have high frequency limitations due to the phase shift in the active elements. A typical of these circuits is Riordan gyrator [2] given in Fig. 2.1.

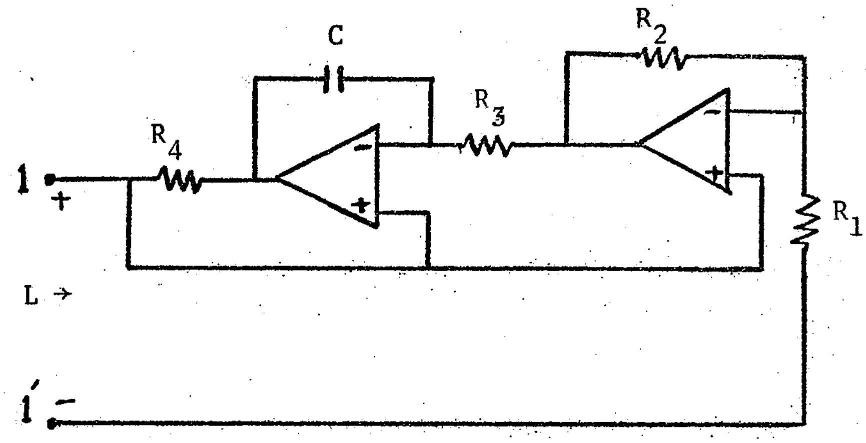


Fig. 2.1 Riordan gyrator

The input impedance at the terminals 11′, is approximately that of an inductance with a value of $L = \frac{R_1 R_3 R_4}{R_2} C$. The quality of this inductor is dependent on circuit parameters, however with $R_1 = R_2$ and $\omega R_3 C = 1$, it is possible to obtain high quality inductor at low frequencies, but due to phase shift in the operational amplifiers at high frequencies, subsequent instability may result.

2.1.2 Filters using Frequency Dependent Negative Resistors

This type of filter is obtained by transforming the network comprised of R, L, and C to another network with C, R, and a new element D. This new element is called Frequency Dependent Negative Resistor (FDNR) and has an input impedance equal to $\frac{1}{Ds^2}$. The transformation is obtained by dividing the impedances by s, and then a resistor transforms into a capacitor and an inductor transforms into a resistor and the capacitor into D.

A typical circuit realizing an FDNR is shown in Fig. 2.2.

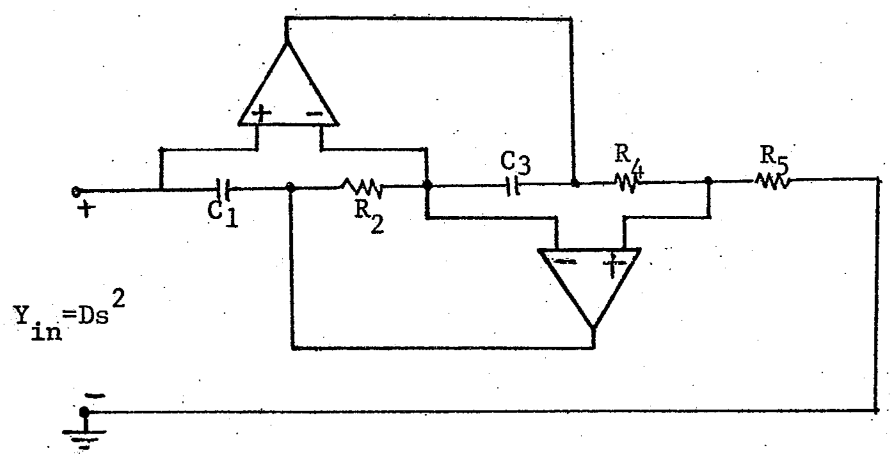


Fig. 2.2 Circuit realization of an FDNR

It can be shown that D in the above realization is given by

$D = \frac{R_2R_4C_1C_3}{R_5}$ provided that the operational amplifiers are ideal.

A filter using FDNRs works best when all the capacitors in the original network are grounded, which in turn results in grounded FDNRs. Therefore, this type of filter realization is suitable for high order low pass filters. Floating FDNRs are also realizable, but they are potentially unstable.

The complexity of design is the same as that of gyrator filters. Cascade realization is rarely used because there would be more passive and active elements. FDNR filters or in general positive impedance converter (PIC) filters exhibit the same characteristics as gyrator filters. Briefly, they have limited frequency response and a limited Q. Sensitivities are low, as well as the gain which is less than unity. In addition they use a large number of passive components.

2.1.3 RC-Amplifier realization

There are a number of circuits composed of R, C, and operational amplifiers, which realize general or special forms of transfer functions. The synthesis is based on coefficient matching and pole-zero cancellation is usually necessary to produce desired transfer function. Hence the number of components is high and for the minimal case, the number of capacitors used, is equal to the order of transfer function. The number of amplifiers in these kinds of filters is important and canonic types are defined on that basis. A typical filter in this category, using one OP-Amp, is the "Infinite gain realization" filter, which is shown in Fig. 2.3.

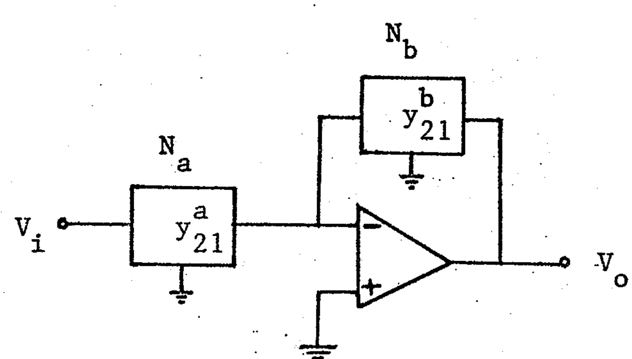


Fig. 2.3 Infinite gain realization of a transfer function

The voltage transfer function is given by

$$\frac{V_o}{V_i} = -\frac{y_{21}^a}{y_{21}^b} \tag{2.4}$$

where y_{21}^a and y_{21}^b are the short circuit transfer admittance of networks N_a and N_b. Figure 2.4 is the canonic realization of a second order band pass filter using the above configuration.

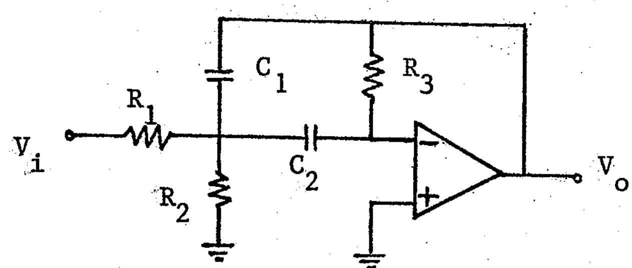


Fig. 2.4 Canonical form of infinite gain realization of a second order band pass filter

2.1.4 Generalized impedance converter method

In this type of realization, the transfer function is partitioned into RC realizable subnetworks connected by generalized impedance converter. Examples for RC-Gyrator and RC-NIC (negative impedance converter) realizations of second order low pass filters are given in Fig. 2.5.

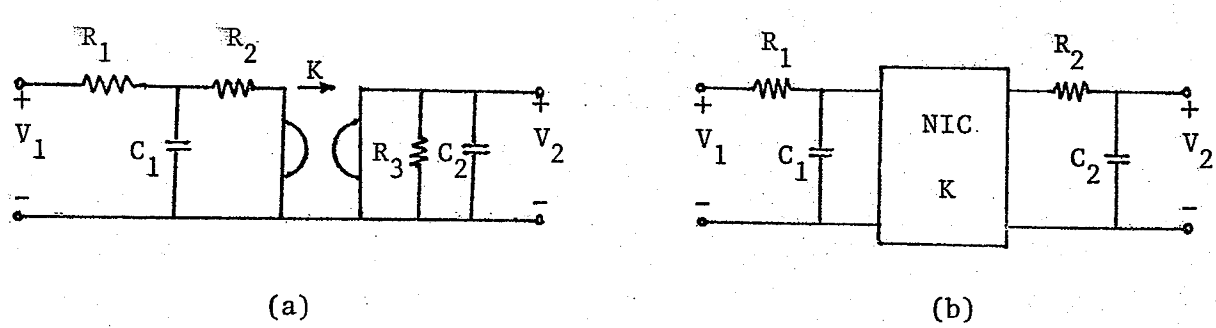


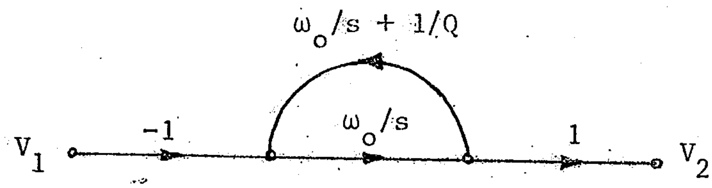
Fig. 2.5 RC-Gyrator (a) and RC-NIC (b) realizations of a second order low pass filter.

The technique employed in the realization is in fact RC:RL decomposition for RC-Gyrator type and RC:-RC decomposition for RC-NIC type. Both methods are cascade realizations and it should be mentioned that parallel realizations are also possible. The method of decomposition determines the sensitivity of transfer function to circuit parameters. There are optimal decompositions in both types which yield minimum sensitivities. [3] and [4].

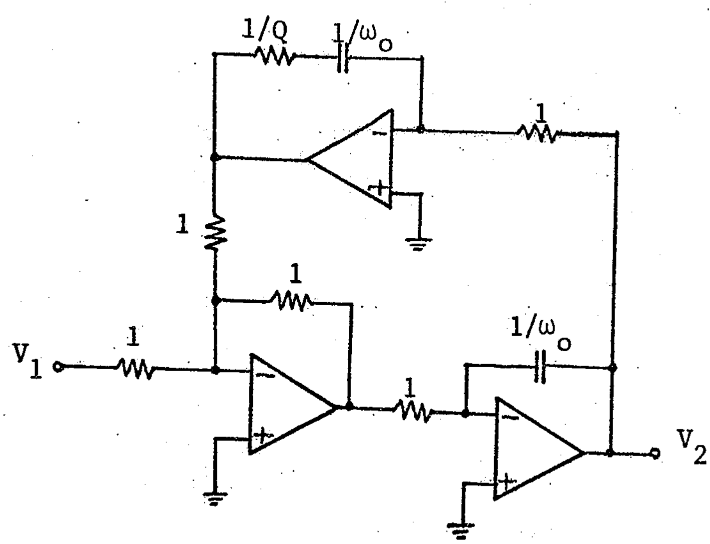
2.1.5 State variable method

This method of realization is based on analog simulation of the transfer function using integrators, invertors and summing amplifiers. This type of filter is suitable for high Q realizations and amenable for easy tuning and low sensitivity. A large number of active elements are required and the nonidealness of these amplifiers results in a moderate frequency response.

The design procedure starts with determining the signal flow graph of the transfer function. An example of the method is given in [5]. The resulting analog computer simulation can be constructed by means of integrators, invertors and summing amplifiers. Figure 2.6 shows the signal flow graph and analog computer realization for a second order band pass filter; $\frac{V_2}{V_1} = \frac{\omega_o s}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$.



(a)



(b)

Fig. 2.6 (a) A signal flow graph realizing the second order band pass filter

(b) An analog-computer realization of (a)

2.2 Single amplifier realization of second order band pass filters

All the previous methods of filter realization require at least two operational amplifiers except the RC-Amplifier coefficient matching technique which follows.

2.2.1 RC-Amplifier realization

Different single amplifier structures can be classified as negative feedback (NF) and positive feedback (PF) configurations [6], as shown in Figs. 2.7(a) and 2.7(b).

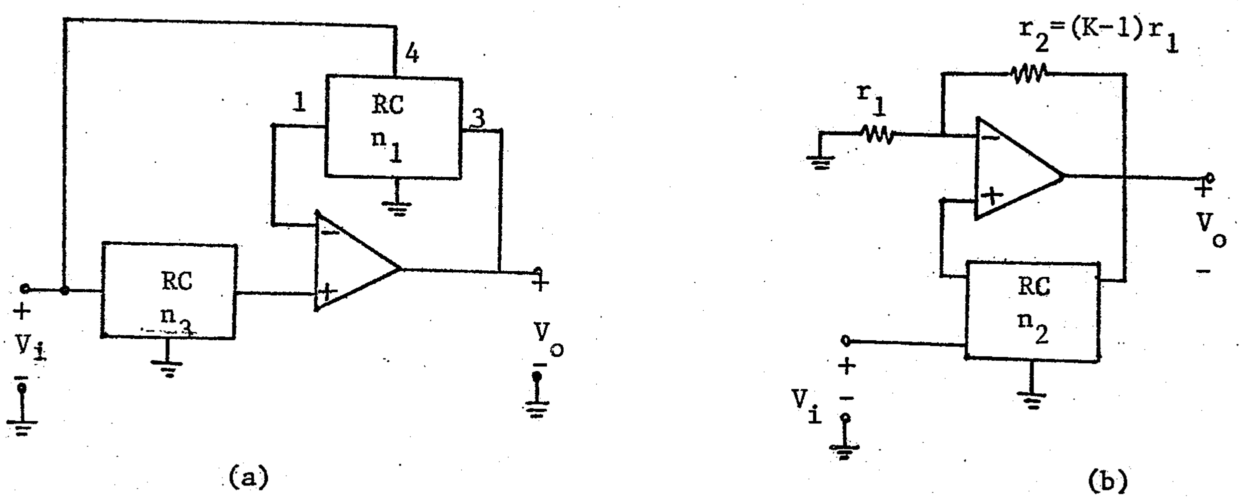


Fig. 2.7 NF(a) and PF(b) configurations for single amplifier transfer function realizations.

Positive feedback filters require a low gain amplifier and result in high Q-sensitivities, and wide bandwidth. On the other hand, negative feedback filters require high gain amplifier and yield low Q-sensitivities and narrower bandwidth. In both, NF or PF configurations, the type of RC network employed, determines the sensitivity properties of resulting filter.

Table 1 represents the characteristics of the various classes of filters.

Table 1

CLASS	PASSIVE Q-SENSITIVITIES	GAIN-SENSITIVITY PRODUCT	MAX. ELEMENT SPREAD
NF	Low	Q^2	Q^2
PF K > 1	Q	Q	Independent of Q
PF K = 1	Low	Q^2	Q^2

It has been shown that a better measure for active Q-sensitivities is the Gain Sensitivity Product GS^Q [7] defined as

$$GS_A^Q = AS_A^Q \tag{2.5}$$

where A is the amplifier gain and S_A^Q is the Q-sensitivity with respect to A. This measure becomes more meaningful when S_A^Q approaches zero figuratively due to extremely large value of A. This is the case for operational amplifier gain. Using the definition for sensitivity

$$\frac{\Delta Q}{Q} = S_A^Q \cdot \frac{\Delta A}{A} \quad . \tag{2.6}$$

If Q is a function of A, then it can be expanded in a Taylor series of the form

$$Q(\tfrac{1}{A}) = Q(0) + \frac{1}{A} \left. \frac{dQ}{d(\frac{1}{A})} \right|_{\frac{1}{A}=0} + \ldots\ldots \quad . \tag{2.7}$$

Truncating the series after the first two terms yields

$$Q(A) = Q_1 + \frac{Q_2}{A} \quad . \tag{2.8}$$

It is apparent that Q_1 and Q_2 are independent of A and depend only on the network. Calculating the Q-sensitivity with respect to A yields

$$S_A^Q = \frac{A}{Q} \cdot \frac{\partial Q}{\partial A} = \frac{A}{Q} \cdot \frac{-Q_2}{A^2} = \frac{-Q_2/Q}{A} \quad . \tag{2.9}$$

Thus $$\lim_{A\to\infty} S_A^Q = 0 \tag{2.10}$$

and

$$GS_A^Q = AS_A^Q = -\frac{Q_2}{Q} \tag{2.11}$$

which is nonzero.

GS^Q or in general GSP is a well-defined analytical expression irrespective of whether A is finite or infinite. Thus minimizing GSP optimizes the sensitivity of the network irrespective of the sensitivities alone.

G.S. Moschytz and P. Horn [7] concluded that the effect of finite operational amplifier gain bandwidth product on pole-ω_o and pole-Q can be expressed in terms of GSP. This interesting result predicts the high frequency limitations of NF and PF configurations. Examples of canonic single amplifier NF and PF realizations are infinite gain multiple feedback filter and Sallen and Key filter, respectively.

Figures 2.8(a) and 2.8(b) are the realizations of a second order band pass filter by these two methods.

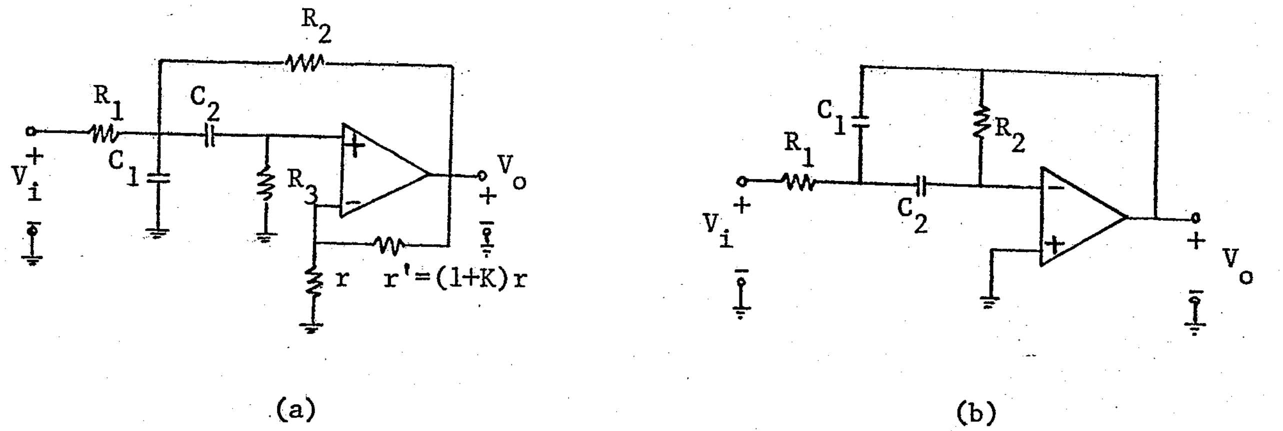


Fig. 2.8 (a) Sallen and Key; (b) Infinite gain multiple-feedback realizations of a second order band pass filter.

We shall investigate these two filters more in detail because they form an important base for the development in Chapter III

2.2.1 (a) Sallen and Key, second order band pass filter

The transfer function of this filter shown in 2.8(a) can be obtained as

$$T(S) = \frac{V_o}{V_i} = \frac{Ks/R_1C_1}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_2}+\frac{1}{R_3C_1}+\frac{1-K}{R_2C_1}\right)+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}} \quad . \tag{2.12}$$

The resonance frequency ω_o and quality factor Q are

$$\omega_o = \sqrt{\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}\,, \qquad Q = \frac{\sqrt{\dfrac{R_2C_1(R_1+R_2)}{R_1R_3C_2}}}{1+\dfrac{R_2}{R_1}+\dfrac{R_2}{R_3}\left(1+\dfrac{C_1}{C_2}\right)-K} \quad . \tag{2.13}$$

For equal-R, equal-C, we have

$$\omega_o = \frac{\sqrt{2}}{RC}\,, \qquad Q = \frac{\sqrt{2}}{4-K} \quad . \tag{2.14}$$

The ω_o and Q sensitivities are

$$S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = -\frac{1}{4},\quad S_{R_3}^{\omega_o} = S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = -\frac{1}{2},\quad S_K^{\omega_o} = 0 \tag{2.15}$$

$$S_{R_1}^{Q} = -\frac{1}{4}+\frac{1}{\sqrt{2}}Q,\quad S_{R_2}^{Q} = \frac{3}{4}-\frac{3}{\sqrt{2}}Q,\quad S_{R_3}^{Q} = -\frac{1}{2}+\sqrt{2}\,Q$$

$$S_{C_1}^{Q} = -S_{C_2}^{Q} = \frac{1}{2}-\frac{1}{\sqrt{2}}Q,\quad S_K^{Q} = 2\sqrt{2}\,Q-1 \tag{2.16}$$

$$GS^Q = KS_K^Q = 8\sqrt{2}\,Q - 8 + \frac{\sqrt{2}}{Q} \simeq 8\sqrt{2}\,Q \qquad (Q>>1)\ . \tag{2.17}$$

The effect of finite operational amplifier gain bandwidth product on the performance of this filter is given in [8]. The ω_o and Q variations are

$$\frac{\Delta\omega_o}{\omega_c} \simeq -4\sqrt{2}\,\frac{\omega_o}{GB}\ , \quad \frac{\Delta Q}{Q} \simeq 4\sqrt{2}\,\frac{\omega_o}{GB} \text{ provided that } Q{>>}1 \qquad (2.18)$$

where GB is the gain bandwidth product of the operational amplifier.

We can also obtain (2.18) by two explicit approximation formulas given in [9]

$$\delta \equiv \frac{\Delta\omega_o}{\omega_o} \simeq \frac{1}{2}\;\frac{\omega_o}{GB}\,\frac{1}{Q}\,(GS_K^Q)$$

$$\frac{\Delta Q}{Q} \simeq \frac{-\delta+4Q\delta\frac{\omega_o}{GB}}{1+\delta-4Q\delta\frac{\omega_o}{GB}} \qquad (2.19)$$

The gain sensitivity product for this filter is equal to approximately $8\sqrt{2}Q$ thus

$$\frac{\Delta\omega_o}{\omega_o} \simeq -\frac{1}{2}\,\frac{\omega_o}{GB}\cdot 8\sqrt{2} = -4\sqrt{2}\,\frac{\omega_o}{GB}$$

$$\frac{\Delta Q}{Q} \simeq -\delta = 4\sqrt{2}\,\frac{\omega_o}{GB} \qquad (2.20)$$

provided that $4Q\omega_o{<<}GB$.

If ω_o and Q have large values then according to (2.19), the Q-variation is not in the order of δ but much higher. For example, if $\omega_o = 10^4$ and $Q = 100$, then $\delta = 0.055$ and $\frac{\Delta Q}{Q} = .2$.

The ω_o and Q variations due to operational amplifier finite gain bandwidth product were shown to be dependent on GB and GSP. Thus for minimizing these variations we require lower gain sensitivity product and a low frequency operation. Some of the drawbacks of this filter are the low input impedance, high gain at the resonance frequency and high (passive and active) Q-sensitivities.
The input impedance and gain at resonance frequency are $-\frac{2}{3Q}R(\sqrt{2}+j)$ and $(2\sqrt{2}Q-1)$, respectively. Obviously, this filter is impractical for cascade realization of high-Q high order filters because of low input impedance and high gain.

2.2.1 (b) Infinite gain multiple feedback filter

The filter given in Fig. 2.8(b) has the following transfer voltage

$$T(s) = \frac{V_o}{V_i} = \frac{-\frac{1}{R_1C_1}s}{s^2+\frac{1}{R_2}(\frac{1}{C_1}+\frac{1}{C_2})s+\frac{1}{R_1R_2C_1C_2}} \quad . \tag{2.21}$$

The ω_o and Q are

$$\omega_o = \frac{1}{\sqrt{C_1C_2R_1R_2}} \quad , \quad Q = \frac{\sqrt{R_2/R_1}}{\sqrt{C_1/C_2}+\sqrt{C_2/C_1}} \quad . \tag{2.22}$$

For the case $C_1 = C_2 = C$ we have

$$\omega_o = \frac{1}{C\sqrt{R_1R_2}} \quad , \quad Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \quad . \tag{2.23}$$

The ω_o and Q sensitivities are

$$S_{R_1}^{\omega_o} = S_{R_2}^{\omega_o} = S_{C_1}^{\omega_o} = S_{C_2}^{\omega_o} = -\frac{1}{2}$$

$$S_{R_1}^{Q} = -S_{R_2}^{Q} = -\frac{1}{2}, \; S_{C_1}^{Q} = S_{C_2}^{Q} = 0 \quad . \tag{2.24}$$

The sensitivities with respect to passive elements are low. To find the active Q sensitivity we substitute the one pole roll-off model for operational amplifier with the gain of A. Figure 2.9 illustrates this model for the nonideal Op-Amp.

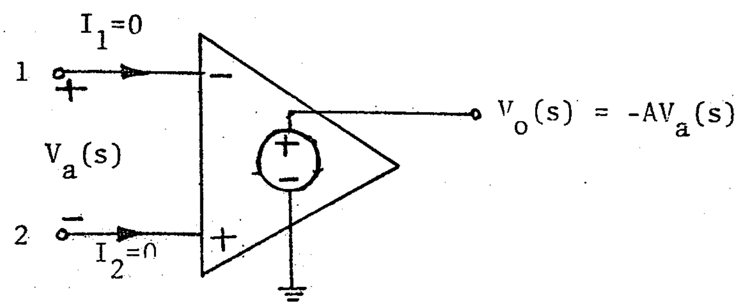


Fig. 2.9 One-pole roll off model for non-ideal operational amplifier

The gain of this amplifier is frequency dependent and equal to $A = \frac{GB}{s+\omega_a}$ where GB is the gain bandwidth product and ω_a is the 3dB down frequency of the operational amplifier. GB can also be written as $GB = A_o\omega_a$, where A_o is the dc open loop gain. Substituting the one pole roll-off model for the operational amplifier in the circuit of Fig. 2.8(b) yields

$$\frac{V_o}{V_i} = \frac{-R_2C_2s}{(1+\frac{1}{A})R_1R_2C_1C_2s^2 + [-R_2C_2+(1+\frac{1}{A})(R_2C_2+R_1C_1+R_1C_2)]s+1+\frac{1}{A}} \quad (2.25)$$

The Q and Q-sensitivity with respect to A are

$$Q_A = \frac{(1+\frac{1}{A})\sqrt{R_1R_2C_1C_2}}{-R_2C_2+(1+\frac{1}{A})(R_1C_1+R_2C_2+R_1C_2)} \quad (2.26)$$

if $C_2=C_1=C$, $R_2=4Q^2R_1$ then (2.26) reduces to

$$Q_A = \frac{Q(A+1)}{A+1+2Q^2} \quad (2.27)$$

with $S_A^{Q_A} = \frac{2Q^2A}{(A+1)(A+1+2Q^2)}$ (2.28)

if $A \to \infty \Rightarrow S_A^{Q_A} \to 0$.

But the GSP is finite as follows

$$GS_A^Q = AS_A^{Q_A} = \frac{2Q^2A^2}{(A+1)(A+1+2Q^2)} \tag{2.29}$$

and if $A \to \infty$ then

$$GS_{A\to\infty}^Q = 2Q^2 \ . \tag{2.30}$$

Using (2.19) yields

$$\delta = \frac{\Delta\omega_o}{\omega_o} \simeq -\frac{1}{2}\frac{\omega_o}{GB}\frac{1}{Q} \cdot 2Q^2 = -Q\frac{\omega_o}{GB}$$

and $$\frac{\Delta Q}{Q} \simeq \frac{Q\frac{\omega_o}{GB} - 4Q^2(\frac{\omega_o}{GB})^2}{1-Q\frac{\omega_o}{GB} + 4Q^2(\frac{\omega_o}{GB})^2} \ . \tag{2.31}$$

If ω_o and Q have moderate values then we can ignore the terms containing GB^2 in $\Delta Q/Q$ and in that case it simplifies to $Q\frac{\omega_o}{GB}$, thus

$$\frac{\Delta\omega_o}{\omega_o} \simeq -Q\frac{\omega_o}{GB} \ , \quad \frac{\Delta Q}{Q} \simeq Q\frac{\omega_o}{GB} \ . \tag{2.32}$$

The GSP for this filter is evidently much higher than that of Sallen and Key filter. Hence lower frequency response and higher sensitivity to operational amplifier parameters are expected. The input impedance and gain at resonance frequency are $R_1/2(1+j)$ and $2Q^2$ respectively. As in the case of the Sallen and Key filter, the low imput impedance and high gain make this filter impractical for high Q, higher order realizations.