THE ORDERLY DEVELOPMENT OF URBAN LAND
USING CONCEPTS OF ALGEBRAIC TOPOLOGY

A THESIS PRESENTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF CITY PLANNING

BY IAN McCREATH

DEPARTMENT OF CITY PLANNING
UNIVERSITY OF MANITOBA

FEBRUARY, 1979

THE ORDERLY DEVELOPMENT OF URBAN LAND USING CONCEPTS OF ALGEBRAIC TOPOLOGY

BY

IAN McCREATH

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF CITY PLANNING

Dedicated to my wife, Linda, who has
supported me in every way.

ABSTRACT

This thesis explores the nature of the theoretical foundations which would be required to establish a formal observational science of urban analysis which is closely related to the conventional urban planning process. The central hypothesis is that the various forms of structure defined by families of mathematical set can, conforming to the methods of algebraic topology, be used to correlate intuitive notions of the urban experience to experimental data derived from any urban neighborhood.

The development of this thesis assesses the value of formal science in relation to the deficiencies of current planning theory, examines the nature of the scale criteria which can provide the ability to impose the maximal degree of structure upon the individual experience of an observer under his particular circumstances of observation, and discusses the epistemological considerations that govern the comparison of data to ideas held by different observers. Consequently, a general evaluation of the anticipated requirements for a formal theory of urban observation is provided. Subsequently, based on the understanding of the observational basis of planning, it is possible to consider the approach to a formal design science closely related to conventional urban planning.

Particular mathematical structures considered in this study are the simplicial complex, partially ordered sets, and families of cover sets including compatibility classes, equivalence classes, and cliques. The study also considers the structural analysis of the simplicial complex

developed by R. H. Atkin, including the face ordering of simplices, q-connectivity, Q-analysis, the algebraic structure of patterns of choice, and the shomotopy concept.

Numerous planning concepts are discussed and related to mathematical structures, including land use zones, siting concepts, the mutual influence of activities in a neighborhood, compatibility of activities, convenience in a structure, and many others.

Particular forms of urban problems studied in some detail are the problem of social choice, the siting problem, measurement of urban influences in a neighborhood, and the land use zoning problem which has application to the determination of the largest sets of activities which may be located in each area of a specific neighborhood to provide the maximum freedom of choice among activities that is compatible with the preferences of the residents.

ACKNOWLEDGEMENTS

I wish to express my sincere appreciation to the numerous individuals who have helped me in this study. In particular Professor M. Carvalho, Professor Basil Rotoff, and Professor Venkataraman who served as my thesis advisers. In addition I wish to thank Professor Gould of Pennsylvania State University who gave me the opportunity to meet with a number of interested researchers in a conference on Q-analysis in 1978.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

I. The Objectives of the Study

1. Purpose

A significant improvement to the city planning process is possible, if changes are made to the currently accepted epistemology of planning. These changes will place the experimental process of urban planning on a similar basis to that which prevails in the conventional observational sciences such as physics. In current city planning it is frequently found that the particular configuration of objectives adopted in a plan bear little relation to the ends which may be implemented by feasible urban policy; and conversely, there are unintended side-effects to even the most well-conceived plan that are detrimental to many interests. Clearly, the theory and practice of city planning are not sufficiently well integrated to prevent this dichotomy; whereas the epistemology of experimental sciences contains an observational system that facilitates the effective description of their observations in terms of commonly held rational concepts about expected behavior.[1] As a result the problems

---

[1]Einstein noted, "The Belief in an external world independent of the percipient observer is the foundation of all science." Albert Einstein, "Maxwell's Influence on the Evolution of the Idea of Physical Reality," 1931. Only the idea of a rational standard of behavior is required to measure the deviation of actual behavior through description of differences only, rather than the totality of particulars in any situation. To catch the underlying rationale review both Gerald and Wigner's An Introduction to General System Thinking, (New York: Wiley, 1975), and also Sir Bertrand Russel's essay "The Relation of Sense Data to Physics," Contained in A Free Man's Worship, (London: Unwin Books, 1976), pp. 141-172.

that are recognized are situations structured by experimental means, and therefore the attention of the science is focused upon problems which are more immediately capable of solution by the science.[2] Whereas a city planner may believe that the solution to urban problems is immediate and obvious, an experimental science demonstrates which particular problems have these desirable characteristics.

The purpose of this study is to explore the theoretical option to use mathematical structure for the effective description of urban observations, and to discuss the solubility of planning problems in such terms.

In particular, this study employs the combinatorial and ordering properties of the simplicial complex to represent and analyze various aspects of urban land use phenomena. By introducing a particular concept of the orderly development of land use in an urban neighborhood, simplicial complexes representing the varying preferences of the residents will be compared to those representing the influences between activities actually present in the urban neighborhood; thereby facilitating decisions representative of the actual concerns of City Planners. Although the simplicial complex is a particular type of mathematical structure typical of algebraic topology, use is made only of the combinatorial and ordering properties of the complex in undertaking simple demonstrations for this study. However, there is reason to anticipate that further research in this area will find use for other properties of the simplicial complex.

---

[2]Problems come in two classes: soluble or insoluble. In a soluble problem the solution is implicit in its statement. The proper kind of problem statement requires a consensus among many observers. It may be that what confronts us in City Planning are not problems, but situations that cannot be resolved without some means of achieving consensus upon their essential structure.

2. Hypothesis

The central hypothesis of this study is that mathematical sets have the capability to represent some forms of data obtained by observation in an urban neighborhood, and the simplicial complexes constructed from these sets can represent significant aspects of the urban processes occurring in the neighborhood. Some urban problems defined in this structural urban analysis are soluble, and their solutions may supplement the intuition and judgement of the city planner in dealing with the urban problems that he perceives directly.

3. Components of the problem

In the broadest sense the problem that concerns this study is the rationalization of the theoretical basis of City Planning to make it more like conventional sciences. The main objective of this study is to outline a methodology for the proper conduct of all stages of planning using concepts of algebraic topology. This requires two partially incompatible objectives to be achieved: (1) The statement of a conceptual framework for the urban interpretation of mathematical structures to reveal the uses and the value of the approach; and (2) determination of means to implement the methodology by computation on a computer. The strategy of the study is to emphasize conceptual development at the expense of computational techniques.

A solution to the general problem can be obtained by resolving it into the following components:

a. Discuss the functional description of an urban neighborhood leading by mathematical representation satisfying criteria consistent with some intuitive preconditions for observation of urban phenomena.

b. Discuss the characteristics of the rational planning process using the functional type of description to provide complete, consistent, and independent modes of description compatible with the general experience of urban planners, with the epistemological constraints on the formation of explanations, and with the intuitive notions of planning.

c. Determine the general form of an urban problem that can employ functional explanations to solve problems in urban neighborhoods.

4. Particular Objectives of the Study

Since it is not possible to deal with this topic in complete generality, the discussion has been limited to the context of urban land use phenomena. Resolution of the hypothesis will satisfy the following objectives:

a. To determine how urban land use structures are to be represented by mathematical structures consistent with the physical constraints and the individual preferences that occur in an urban neighborhood.

b. To relate the planning process to mathematical processes that are suitable for the analysis of such representations, and to distinguish those forms determined by such analysis which are significant to the explanation of urban land use structure.

c. To determine the form of mathematical problem that represents the kind of problem which a planner wishes to solve in dealing with urban land use.

d. To discuss how such a problem can be solved and relate the solution of the problem to the orderly development of land uses.

5. Assumptions

The following assumptions are necessary to facilitate the development of this study.

a. Every individual has the capability to describe his own preference system, however incompletely, although the basic factors may be stated implicitly rather than explicitly.

b. Every individual's preference system expresses his assessment of his best interests at the time in question, and should be assumed (by definition) to be intrinsically rational. To judge a particular system as irrational is to expose one's ignorance of the factors that the subject used, perhaps implicitly, in his assessment.

c. An explanation is a model which sums up many particular individual preference systems relative to a common reference standard of behaviour, expressing the best that can be achieved without violating these individual preference systems. The idea of an absolute rational standard of behaviour is required only to provide a gauge against which actual behaviour can be compared permitting the apparent distortions to be assessed as the cost of individual irrationality.

6. Delimitations

The following delimitations are imposed upon the scope of the study in order to keep the problem within manageable limits.

a. Although this thesis considers only urban land use planning, it recognizes that city planning has far wider interests.

b. Consistent with the intent to explore this approach from the most general viewpoint, significant epistemological and mathematical concepts will be compared to planning concepts to provide the necessary basis for evaluating and understanding the application of mathematical techniques in city planning.

c. Consistent with the strategy to develop an outline of the methodology as a whole, particular techniques will be introduced in general terms, but only limited applications will be made sufficient to facilitate the understanding of concepts.

II. Description of the Problem

1. General

The central problem of planning is the problem of social choice first studied by Kenneth J. Arrow. This problem may be simply understood. Suppose a society consisting of two or more individuals is to construct a preference ordering over three or more alternatives belonging to the different preference systems of the individuals. What is wanted is a social welfare function to transform every possible pattern of preferences for the individuals into a single order of preference for the society. The permissible range of transformations which acceptably compose individual preference patterns into more global preference systems are limited by constraints imposed to insure fair, unbiassed treatment of the individual during construction of the social preference. Having defined a set of suitable constraints, Arrow rigorously proved the impossibility of forming a social welfare function.[3]

The impossibility of a welfare function is often interpreted as indicating the unsuitability of rational mathematical treatment of any human-oriented social science (that is, because suitable assumptions on the composition of human preferences cannot define a consistent welfare function, human preferences are uniquely indescribable by rational methods). On the contrary, descriptions of human preferences have the same shortcomings as any logical value system. For example, the

---

[3]Kenneth J. Arrow, Social Choice and Individual Values (New York: Wily, 1961). See also Morton D. Davis, Game Theory (New York: Basic Books, Inc., 1970) pp 191-192.

limitative theorems of logic (Church's Thesis, Goedel's Incompleteness Theorem, etc.) show that formal first order quantitative logic necessarily provides an incomplete description of a logical system (generally considered to be consistent) such as arithmetic. This development, while not welcome, has not occasioned any despair about the value of rational injury in logic, mathematics, or physics; instead a healthy awareness of the fundamental limitations to any human's ability to describe and to explain systems in general.

There is no reason why rational inquiry cannot search for the best possible descriptions of urban phenomena consistent with the limitations imposed by existing urban knowledge.

Friedmann has pointed out that the major consequence of Arrow's social welfare theorem is that it discredits for all time the suitability of abstract value systems for the determination of priority for concrete action in urban planning policy. The planner must replace abstract concepts such as the "public good" with a realistic appraisal of the conflicting preference systems of different communities of interest that are active with respect to any planning issue, and mediate these differences in accordance with the decision processes actually found to occur in urban areas.[4] In other words urban planning has the same concern for observation of individual species of objects in a system and their characteristic attributes as does physics.

Thus rational planning has the objective of finding the priorities which are the most realistic and rational for the different interests

---

[4]John Friedmann and Barclay Hudson, "Knowledge and Action: A Guide to Planning Theory" JAIP (January, 1974), p. 7

existing in any set of issues found in an urban situation. This objective is most readily achievable by reconstructing city planning theory as a formal observational science wherein rational concepts are described and constructed in terms confirmable by experiment. This will require a means of detecting and describing the significant communities of interest relative to an issue (resolution of social structure) and a means of mediating the conflicts in terms of the different interests of these communities using decision processes known to exist in urban areas (description of urban processes describing compatibility in terms of propagation of influences and trade-off tolerances).

Knowledge of those aspects of urban process and of individual preference systems enables the designer to consider different configurations of structures satisfying different preferences in different places while adequately separating conflicting aspects of the different interests. Clearly a knowledge of the structure of preferences that ranks them in terms of their generality will greatly facilitate the attempt by the designer to distribute benefits or exclude conflicts over the widest possible areas of an urban neighborhood.

2. Definition of Rational Planning

It is central to the development of this study that planning be regarded as a decision process, for it is through this view that mathematical analysis can be related to the act of decision-making in the city. Kaplan describes planning as the facilitation of decisions in the following sense:[5]

> "The application of behavioral science to policy is most self-conscious, deliberate, and explicit by way of planning, which may be defined as the enterprise of facilitating decisions and making them more realistic and rational. Decisions are facilitated as the choices are made more clear cut, and alternatives are more concretely and specifically demarcated. Decisions become more realistic as the values they involve are confronted with facts, and ideals are translated into concrete objectives. They become more rational as values are confronted with other values, and what Reichenbach calls 'entailed decisions' are taken into account."

Planning has evolved to meet particular needs that are not satisfied by the other decision processes, such as political dialogue, also occurring in the city. Therefore the requirement for planning is best understood in comparison to the act of normal decision-making.

Normal decision-making requires an act of self-deception concerning one's ability to make any decision at all. Suspension of doubt forecloses consideration of the feasibility and consequences of a decision outside except for a narrow range of circumstances.[6] This deception is fostered by abstract concepts such as "the public good" or "the general welfare" that implicitly assume an unfounded epistemological criteria, ie., there

---

[5] Abraham Kaplan, The Conduct of Inquiry (Scranton: Chandler Publishing Company, 1964), p. 403.

[6] Gibson Winters, Elements for a Social Ethic (New York: MacMillan, 1968). The meaning of suspended doubt is clear and immediately relevant to understanding the role of planning and its fundamental limitations.

always exists some simple and immediate decision leading to an equitable and unbiassed distribution of both the social benefits and burdens contained in a social system. While Arrow's Welfare Theorem exposes this fallacy, plans continue to be conceived having goals unrelated to practical means and often screening ulterior motives that accomplish the contrary ends by favouring some privileged group at the general expense.

Costs attributable to simple decision-making are usually known as externalities or opportunity costs. For example, isolated decision-making may mistakenly assume that a given solution provides great benefits to one area of the system, while overlooking even greater costs imposed upon some other area as a consequence; eg. polluting industry providing employment while degrading the environment. Extreme simplification may introduce bias by improperly reducing a number of alternatives to some favoured approach. Opportunity costs are incurred when an option that offers short-term benefits is selected despite great potential future costs. While it is true that there is no such thing as a free lunch, many decision-makers have found they need not be the ones who pay the bill. Emphasis upon a new concept of individual relative rationality can expose these areas of significant social costs.

3. Rationality in Planning

Despite the existence of cognitive limitations to any person's ability to describe and explain social preference systems, the approach taken by mathematical logic and physics would seem to demonstrate the importance of structural considerations for inquiry into limited areas of truth concerning urban process. Logic is normative and limitative in the sense that it can definitely identify among all patterns of behaviour those which are unacceptable modes of behaviour, even though it does not always facilitate the description of acceptable ones.

Planning has evolved in society to rectify the costs of simple-decision making, insofar as this is possible at all, by taking into account the greater interests of the whole society. If planning is to become a rational observational science, the ultimate source of knowledge must be the different preference systems that each individual is assumed to be capable of expressing (the grounds of description or of observation). The existence of some capability to make preferences definite is necessary, but no assumption as to their all-encompassing completeness is required. Knowledge gradually expands as empirical techniques are refined by new generations of researchers. However, the ultimate rationality of these individual expressions of self-interest must be assumed, whereas all standards of behaviour abstractly constructed from them must have their rationality demonstrated in an acceptable fashion. In any event, the rationality of an individual or a social group is a distinct characteristic of that group, expressing independently of any other such group or individual its own best interests. Rationality is assumed to be valid for a given group only under some range of circumstances consistent with the existence of the group as a distinct entity. Call this relative rationality and accept as part of the problem of a formal science of urban analysis the exploration of the conditions for existence of a distinct rationality as a property correlated to an identifiable social group.

Clearly a group will exist in the context of some issues or problems requiring a coherent organization of individuals having some common interests. Obviously the stability of a given group will determine the extent to which the interests of the group must be recognized in the mediation process, however, like any entity, groups are likely to have evolutionary stages in their development, and even very transitory social structures may have some significance in an urban area (signalling future development). This structure can be described in terms of mathematical sets.

Given any mathematical structure the significant question concerns the sub-structures it contains, together with the means of combining them to generate new structures. Two possibilities are open; either the composite structure will be a sub-structure again, or the composite structure is a new form having describable properties that contain the former structure as a special case. One should expect the corresponding behaviour to be exhibited by the group's characteristic rationality.

The required science should be able to identify the most general class of organization which may conceivably be considered urban. Various more or less arbitrary combinations of individual preferences systems can be combinationally generated, but an observational system must focus on those that exist (or could exist under some circumstances).

The acid test for the significance of a given group is that the rationality it expresses be relevant to some issue, and that the problem is sufficiently pressing for the individuals to reach some compromise of their own total interests in a greater common interest. It should not be surprising that individuals enter into such organizations in their own interests, and these may be distinct from those of other communities in which the individuals have not found it worthwhile to participate. In the broadest sense the rationality of individuals or groups simply expresses the quality of the conditions under which it will continue to exist in the social environment. The relevancy of rationality is immediately apparent.

In principle this system of abstraction should provide a "place" in its category system for different points of view with the relation of compatibility and incompatibility between each readily accessible. Mediation usually considers optimization of the common interests within a co-operative domain; hence, it should be viewed as a process of generating new forms of social organization. To say the process of mediation is rational assumes that some rules (insuring realism) are in practice. Conceiving these groups to be sets will facilitate the use of normal mathematical concepts of set, structure, order, and relation to determine the rational best interests of the whole social group.

Moreover, continuing levels of abstraction to larger aggregates of individuals will tend to describe what is most generally in the interest of the larger groups, enabling an efficient distribution of urban infrastructure to satisfy these interests using economical siting configurations.

Some of the difficulty inherent in the formalization of urban analysis can be side-stepped by first considering urban analysis as a natural science describing urban phenomena, then introducing the additional complications of the design orientation of planning. The component problems to establish urban analysis are the following:

a. Establish criteria for detecting the existence of significant social structures including;

    (1) An urban observational protocol describing the greatest detectable degree of structure to be found in the contents of any observational structure, or of the domain of view-points on which the observation is resolved.

    (2) A framework for the description of the signalling of compatibility and incompatibility of the different observational structures.

b. Establish criteria governing acceptable forms of explanations in urban planning which constructively establish newer more abstract social structures having their own rationality.

c. Establish a central organizing principle to describe some tendency of orderly development which describes decision process in a changing urban environment.

d. Develop techniques constituting a planning technology enabling these tendencies to be grasped by calculation and presented as a plan.

Ignorance of the exact parameters of the limitations to human observational and intellectual ability need not obstruct the study of the form which theories must take to account for the existence of these limitations. In general any scientific method reflects an assessment (possibly implicit) of the procedures required to construct concepts that respect these limitations. The absence of a scientific method (which reflects some degree of epistemological sophistication) usually invokes the requirement for an omniscient, and perhaps omnipotent, observer. The required forms must reflect an awareness of the social process of formal reasoning incorporated in the epistemic correlation between concepts and data. Rational logic in a science simply constrains the acceptable forms of reasoning by which concepts are constructed from observations to those which have some basis in experience. Rationality as a social value will emerge from a sense of the orderly development of urban areas as a consequence of the limited possibility of formally describing urban events.

The criteria incorporated in the definition of an urban observational protocol define a structure which in principle lists every possible mode of behaviour which may be considered urban-like in any way; but it does not assert that such modes of behaviour actually exist. Two forms of limitations are described by a protocol:

a. All perceptions from the environment that are definitely not urban-like (and therefore not of interest) are filtered out of consideration.

b. Observational criteria based on the currently available means of detecting urban behaviour distinguish between dynamic urban and describable urban phenomena.

Two distinct types of data will be observed in the urban environment:

a. Individual preferences describing those modes of organization which each person considers to be the best.

b. Actual data describing those modes of organization that actually exist in an urban neighborhood.

It will be found convenient to adopt an observational system using an abstract medium of description (sets). These data are then considered as being defined in a functional sub-space of the protocol describing the social relations and a physical backcloth sub-space of the observational domain consisting of the constraining relations. Obviously concepts will be constructed in the functional space, but their experimental confirmation requires their comparison to the physical space. The question of the use of a suitable medium of description is a fundamental consideration in the discussion of epistemic correlation, since concepts and perceptions are not in themselves directly comparable entities.

Having established the basis of an observational system, the question of a suitable framework of description for the definition of an urban process will be considered. Clearly the concept of plan describes a standard of behaviour which is to govern urban processes; hence planning as such can only be introduced in terms of the geometry of a descriptive framework for urban processes.

This interpretation of rational planning is most significant when one grapples with the requirements of a formal foundation for describing the planning process. This thesis contends that city planning can establish such a foundation independently of the grounds of any other science, and that significant advantages would accrue from doing so.[7]

Rationality applied to social choice must describe some central organizing principle incorporating the planner's intuitive notion of orderly development (include "good development" and exclude "bad development" to define an ordering of social structures). The best example is available in physical thermodynamics (entropy). The laws of thermodynamics organize the description of physical process as follows:

a. First Law (conservation of energy) defines the prerequisites that an organization must satisfy to be considered "physical-like."

b. The Second Law (tendency of processes) imposes a strong sense of the orderly development of physical process.

c. The Third Law essentially defines a zero of temperature.

d. The Zero (an addendum to the classical set) Law defines a necessary condition for an equivalence relation in terms of concept of temperature.

The First Law amounts to a condition on the process of communication between physical events which can also be imposed upon urban phenomena by considering the compatibility classes defined by urban signalling in the urban observational protocol (to be discussed in this thesis).

The Second Law is an invitation to examine the concept of strategic planning to see what tendencies can be defined that formalize urban development decisions under uncertainty. Decision procedures inherently

---

[7] W. Ross Ashby, Cybernetics (London: Chapman and Hall, Ltd., 1956) pp. 1-2. In his introduction Ashby recounts the idea and the advantages of independent foundations very explicitly.

require the assessment of the impact (costs) of alternative development policies, and the organizing principle must assess these clearly. It seems likely that Caratheodory's famous Accessibility Theorem re-expresses the problem of social choice in a form where the difficulties of Arrow's Welfare Theorem can be circumvented.[8] To do this requires formalization of the processes which occur along some path in a neighborhood (linking the different places where events occur) by rules which show the changing influences (acceptable at each stage of the process). It is interesting to note that the concept of a process is defined by Caratheodory's Theorem (which simply formalizes the Second Law) permitting the transition from static descriptions to non-constant (quasi-static) changes in described states. Inaccessible events in the neighborhood of any state are the necessary prerequisites to the Accessibility Theorem, and Arrow's Theorem implies these must exist.

The relevance of this to urban design is to state the obvious: preferences incompatible in the same place may become tolerable when removed to different places. The notion of place implies a geometrical system permitting different observers to agree as to what process occurs at some place despite the different perspective each has; and conversely, agree to what is different about the processes from their viewpoint.

Notions such as tolerance, influence, trade-off, signalling, physical backcloth, and functional space provide the framework for this organizing principle. Atkins' algebra of patterns provides a means of describing forces (possibly work), their impact (social costs), and the

---

[8]Mark W. Zemansky, Heat and Thermodynamics (New York: McGraw-Hill, 1957) p. 171-172.

different processes presented by patterns of choice. Much more development is required to firmly establish how the concept of orderly development can be formulated to describe the structurally stable institutions of society as those having the greatest significance.[9] The use of an algebra will facilitate assessment of costs and benefits. This will be done within the outline of planning as decision-facilitating that was promoted by Kaplan.[10]

4. Urban Structure

The fundamental requirement for a formal observational system in urban analysis is to identify naturally occurring phenomena that may be described in terms of mathematical structures. Without such criteria mathematical structure would remain an abstract concept unattached to any urban reality. The form of epistemic correlation to be adopted for this association must be selected with care. In view of the Arrow Impossibility Theorem one may anticipate that there will exist purely accidental forms of urban organization that cannot at this time be explained by mathematical structure; and conversely, there will exist concepts that are not associated to any concrete urban reality. However, adequate principles for orderly development will identify significant correspondences (those which are rational), employ criteria to effectively categorize urban phenomena into those which can be described by current

---

[9] Rene Thom, Structural Stability and Morphogenesis (New York: Benjamin, 1975), pp. 151-152.

[10] A. Kaplan, The Conduct of Inquiry, p. 403. His definition of planning was previously introduced on page 10.

means (making them worthy of immediate attention); and relegate those which are too "dynamic" to later generations of research.

Individuals deal routinely with questions of structure. Every problem which involves the allocation (distribution) of attention to different aspects of a situation by assignment of priority reflects a structural concern. The planner may distinguish various parts of the whole to which he attaches degrees of significance. For example, any community contains a large number of interest groups, different patterns of which are active in different issues. Each have their characteristic rationality, that is internally consistent; but as a whole society their interests are inconsistent with each other. Planning should identify areas of common interest to mediate a trade-off acceptable to all. This is clearly a structural consideration. As expressed by Friedmann, the importance of structural analysis in the problem of social choice is that, henceforth, it will be impossible for policy-makers to turn to some abstract ideal of "the public interest" for the priorities determining the allocations of social policy; rather these priorities will have to be sought where they were always found, in the various decision processes of the city.[11]

The central concepts of land use planning arise by abstraction from the idealization of the behavior of real property owners and closely parallel mathematical abstraction. The planner is familiar with the concept of fee simple ownership of land. By experience it has been found that conflicts between adjacent sovereign land-owners arise where

[11] John Friedmann and Barclay Hudson, "Knowledge and Action: A Guide to Planning Theory," JAIP (January 1974), p. 12. They express Arrow's Impossibility Theorem in terms of its significance to policy planners very concisely in this reference.

the interest of one leads to conflict with the absolute right of use by the other. With the density of land-owners found in a city it is obvious this simple concept is inadequate; therefore the absolute right to use land is structured into a bundle of distinct rights (development rights, eg.). Moreover, it would be inappropriate to assign all of these rights to particular individuals so new groups of individuals are distinguished and each is given, in its own right, the exercise of some property right(s) appropriate to it. The new groups are actually composite entities that are more than the aggregate of the participants (since they have in their own right particular rights not assigned to constituents). Unfortunately, these rights, their benefits, and their costs, are usually lumpy rather than distributed equally. Therefore an immediate conflict arises with respect to distribution because of the abstract nature of the larger community.

The planner must deal fairly and in an unbiased fashion with the distribution of benefits and burdens that have an inappropriate graininess. It is hardly surprising that distortions and stresses arise. Since the larger communities have some rights of their own, they may also have their own rationality which can be described and may also be in conflict. Hence, the dilemma facing the planner is the following: does he plan using the same principles when his own interests are in conflict? These "abstract" standards of behavior are real insofar as they concern the properties of a community which the composite group controls in its own right; hence, urban theory should be able to account for their effects upon policy.

The idea of structure is prevalent throughout city planning if we can but recognize it; and planners can also profit greatly from its use.

The mathematician will recognize the similarity of the discussion of property rights to the generation of new structures by generalization from existing ones. The immediate problem is to establish the conditions for existence of the abstractions (in their own right), the means of generating and combining them, and their satisfactory representations. The cost associated with distorted distributions of the rights of the whole to its constituents is more readily exposed from a structured viewpoint.

5. Mathematical Structure

A mathematical structure involves three inseparable ideas necessary to the explanation of urban development. These are order, relation, and structure. The fundamental unit of mathematical structure is the set. The structure specifies the distribution of different properties of the sets to particular subsets as described by the family of subsets found to be contained in the set. The presence or absence of different subsets in the family is determined by the relation which incorporates the rules of distribution (allocation). The order describing the precedence among subsets is determined by the way that they belong to each other specifying their development and their subservience to some greatest subset(s) of the family.

Topology is a science, as opposed to logic, in that it has a particular domain of interest which it attempts to describe. This domain encompasses the properties and the organizations derived from mathematical sets by the application of the general rules of logic. In a sense, which can be made very precise, topology is the science which explores the different ways in which any organization can be described. Interestingly, not every aspect of mathematical sets can be described; but those descriptions which are possible, by virtue of the ability of the set to encompass any logical properties, are also the best one can do in describing anything at all. The existence of any set implies the idea of a particular consensus as to the circumstances under which the structures are accepted as observable, although the existence theorems proven by the mathematician may be very remote from reality. It is the duty of the person who would employ these structures to show how the connection to reality may be made. Furthermore it is of interest that the mathematician habitually attempts to transform any problem into algebraic systems; be this a problem in geometry or some other abstract organization. It is only by means of algebraic systems that it is possible to actually compute results; and therefore the sense of representing urban structures by algebraic structures become comprehensible.

The epistemological importance of mathematical structure in urban analysis is to make the complexity of urban systems manageable by focusing attention upon the limits in which description of behavior is possible at all. By use of structured descriptions, points-of-view which reduce the burden on our limited cognitive capability are achieved. Mathematical structures cope with the description of unfamiliar, complex situations by providing:

a. A complete view broad enough to encompass all phenomena (preventing surprises).

b. A minimal view lumping together states unnecessarily discriminated.

c. An independent view that decomposes observed states into non-interacting qualities reducing the mental effort required.

6. The Planning Process

The distinct activities leading to preparation of a plan that solves some problem in an urban area are called the planning process. To meet the conditions of rationality heretofore discussed, the planning process must offer an explanation for behavior by representing urban neighborhood in terms of functional standards of behavior governing each piece of land in the neighborhood, where these are mutually compatible and consistent with the individual preferences of residents in each land area. Essentially this involves a transformation from individual preferences to new global standards of behavior, but the transformation must not overly distort the individual preferences; ie. it should be fair and unbiased in composing them while seeking their greatest common preferences to serve as standards.

The planning process must deal with the limited resolving capability of the instruments used to discriminate the structure of observations, and the limited cognitive ability of any planner to reduce these observations into manageable forms of description. To systematically compose these standards into configurations of objectives serving as a plan the stages of the planning process are assumed to be as outlined in Figure 1.

TABLE 1

THE STAGES OF THE PLANNING PROCESS

A. FUNCTIONAL DESCRIPTION OF URBAN SYSTEMS

PURPOSE	OBJECTIVES	ACTIVITIES
1. Preplanning (problem definition and analysis of factors). To use actual observations and experience incorporated by Delphi techniques into mathematical relations representing significant information about the urban area in a form suitable for further analysis.	1. To define the limit of significant observations as types of urban choice (the protocol of observation). 2. To determine the significant modes of interaction between choices (the signal relation). 3. To define quantitative signal levels of the relation. 4. To describe actual observations of choice existing in a neighborhood. 5. To describe the social relations as the preferences of individuals with respect to activities. 6. To describe the hierchial structure from signal relations. 7. To describe the backcloth precedence between individuals.	1. Determine activity and land protocol sets. 2. To determine signal sets. 3. Assign signal levels suitable for weighting signal relations. 4. Describe simplexes correlated with individual areas specifying preferences or existing structures. 5. Describe Delphi results as a weighted preference relation between activities. 6. Determine hierarchial cover sets and their relations.

TABLE 1 (CONT.)

THE STAGES OF THE PLANNING PROCESS

B. FUNCTIONAL EXPLANATION

PURPOSE	OBJECTIVES	ACTIVITIES
2. Planning (synthesis of alternatives). To systematically transform preferences relations into cover families using some relation depicting similarity between preference systems to determine alternate standards of behavior. The transformation is a "summing up" of individual experience by similarities to find the "greatest" common denominators" of that experience.	1. To resolve the maximal complete families of cover sets at different levels of consistency. 2. To take account of the orderliness of the standards of behavior at different signal levels. 3. To take account of the orderly nature of their ability to make actual behavior conform.	1. To discriminate standards of behavior using the face ordering, compatibility classes, equivalence classes and cliques. 2. To compare these classes at different signal levels. 3. To use different means of examining compatibility between preferences such as Q-analysis, or ordinary linear graphs to circumscribe different circumstances of observation.
3. Measurement (Evaluation of Alternatives). Comparison of actual structures to standards to determine the conformity with standards and assess the "costs of deviation."	1. To transform representations of existing structure into the same form as the standards. 2. To compare and assess similarities and differences between standards and actual structures.	a. To transform simplicial complexes into covers of the same form as the structure reflecting the effect of the backcloth relation on the pattern of influences. b. To use techniques such as Q-analysis to make differences between standard and actual structures explicit.

TABLE 1 (CONT.)

THE STAGES OF THE PLANNING PROCESS

C. IMPLEMENTATION

PURPOSE	OBJECTIVES	ACTIVITIES
4. Design. Determination of all alternatives consistent with all constraints.	To systematically generate the largest variety of land use activities to be sited in each area of the neighborhood without violating any standards.	Use a combinatorial design technique in conjunction with the algorithm for analysis of interconnected decision areas. (A.I.D.A.)
5. Regulation Determination of the alternate capacities reflected in various numerical indicators attached to sets of choice.	To determine the optimal assignment of capacities (eg. parking areas) to a structure.	Use linear programming.

7. The Plan

The plan is usually a document which attempts to impose order upon the changes occurring spontaneously in the urban environment. It does this by specifying a particular configuration of objectives together with action sequences leading to their attainment. The plan reduces uncertainty as to how its objectives are to be achieved by social action.

The study will elaborate the obvious capability of a mathematical order to express the elementary characteristics of a plan in terms of urban land use structures using combinatorial and ordering techniques.

Specifically the following characteristics of a plan will be dealt with mathematically:

a. The goal(s) of a plan are described by the concept of a limiting independent configuration of maximal elements of the ordering using the compatibility relations contained naturally in urban structures.

b. An objective is a maximal element of the urban structure under study.

c. The elements of action in the plan are the changes to the sets describing existing urban structure for which specific formal operations will be defined.

d. Development concerns action which changes the content of the sets themselves; growth concerns some change in the quantity of the elements of the sets without qualitatively changing their contents.

e. Orderly development describes any tendency of changes in the sets towards their maximal elements, such that each area is consistent with at least one standard.

f. Uncertainty describes the range of possible chains of sets representing possible sequences of development consistent with orderly development; or the uncertainty under a given standard of behavior as to which of its subsets will be found to occur.

g. Using the idea of standards of behavior, different strategies for development can be dealt with by mathematical explanation.

As the study elaborates the principle of orderly development in the following chapters, its close relation to the concept of statistical entropy used in communications theory or in physical thermodynamics will be evident. Consequently, the concepts of strategic planning will closely resemble some of the concepts used to describe physical process.

III. The Value of Structural Analysis in Planning

1. The Limitations of Planning

Although considerable debate is found in the planning literature concerning the shortfalls and limitations of rational analysis for the study of urban phenomena, and indeed any area concerned with human psychology, it is less common to find a careful examination of the limitations of contemporary planning in dealing with these phenomena. The mathematician can rightly claim that within the jurisdiction of their professional interest they have thoroughly explored the limits and nature of mathematical processes; in contrast the planner finds it difficult to formulate a definition of the city, let alone the interesting processes which occur therein. One of the major concerns which preceded this study was the feeling that urban planning could not define its own area of interest and its intrinsic limitations. In such conditions actions that subordinate the general interest to that of particular privileged groups can flourish undetected.

The current methods of city planning are essentially intuitive and eclectic in nature, and are perceived to have the following major deficiencies that are essentially attributable to inadequate investigation of the grounds of planning.

a. An inability to discriminate which problems are soluble. The existence of such criteria would conserve effort currently wasted on problems whose solution is socially desirable, but which are not soluble within existing technical means.

b. A lack of consensus on the significant aspects of urban process; and hence an inability to distinguish urban problems from those which merely occur in the city, and are the proper concern of other professions. Therefore planners often fail to grasp the social mandate which limits their scope of action.

c. The lack of consensus upon acceptable methods of analysis preventing the development of abstract concepts necessary to a general urban theory that would grasp the essentials of urban behavior.

d. Lack of interest in the philosophical investigation of many planning concepts leads to the continued presence of undetected fallacies in planning thought.

e. The lack of formal methods making planning difficult to teach. Present methods rely upon exposure and example.

f. Current intuitive methods of determining alternative courses of action are capable of resolving no more than a small portion of the total range of possible alternatives. Consensus upon solutions is difficult to achieve when so many unsurveyed possibilities exist as counter-examples.

g. Lack of standardization in the planning process and in plans makes direct experimental confirmation of theory difficult. Consequently experimental data, often obtained at great expense, cannot be generalized to other situations.

h. Lack of objectivity in the planning process makes it difficult to survey the steps by which the decision-maker arrived at a particular decision. Consequently decision-makers are not accountable to the public and many ulterior transactions escape the notice of those most greatly affected.

i. Current planning cannot take individual preference systems into account in deriving the global standards to be imposed on the neighborhood as a whole. Therefore such concepts as participative planning have no operative meaning, to the detriment of individual welfare.

The structural approach to planning can rectify these deficiencies.

2. The Potential of Mathematics in Planning

The problems of the planning process and the problems that are found in the planning process must be distinguished. Problems of the planning process are to be dealt with in the epistemology of the planning process. Particular problems in planning will be resolved by the use of mathematical processes which transform representations of individual preference systems into global standards of behavior useful for regulation and design.

The fundamental epistemological requirement to rationalize the planning process is to assume the rationality of individual behavior; notwithstanding that the nature of this rationality may not be well understood.[12] In this way the construction of a formal planning process is firmly grounded in individual behavior, as it is not in current planning theory. By constructing the global standards of behavior from individual systems of preference it is certain that the latter bear some

---

[12] Rene Thom, Structural Stability and Morphogenesis, pp. 151-152.

relation to actual behavior in contrast to many goals of current planning. In the present approach we are interested in the detection of structurally stable institutions of the land use in urban neighborhoods. Their functions must be explained by the models constructed from the various systems of preference introduced into the urban neighborhood by the residents. The fact that one may not understand the underlying psychology that gives rise to these preference systems will not prevent their description in a suitable form for inclusion in the planning process.

Some insight into the ability of the structural approach to provide new concepts to urban planning by introducing structural considerations may be obtained by examining the development of geometry. Two millennia ago geometry was a special case of the forms which were found in nature; ordinary space dominated geometry. In our era geometry dominates ordinary space in the sense that the forms which actually exist are a special case of geometry. Modern geometry provides a framework of concepts upon which objects can find their proper place with their interrelationships readily appreciated. Geometry is thus able to suggest new forms of relationship which have a possible place in reality, although they have not yet been experienced. Much of modern physics is a search for empirical confirmation of such forms suggested by structural considerations. The algebraicization of geometry played a strategic role in this development.[13]

Using a similar approach various urban institutions can be conceived as structurally stable forms of behavior because they are consistent to some degree which is sufficient to encompass the underlying preference

---

[13]W. Ross Ashby, An Introduction to Cybernetics (London: Meuthen and Co., 1968), pp. 1-2.

system of the participants in the institution. The use of structural concepts will introduce a quality of urban intuition in the capability to suggest the possible existence of new modes of organization under circumstances not yet experienced by the urban planner.

The planning process promoted in this study has the following general characteristics which redress the deficiencies of planning previously noted.

a. It provides a standard of behavior for the conduct of planning representative of a consensus of the participants.

b. It is concerned only with rational and describable urban phenomena and directs research into problems that can be solved by current means.

c. Its form of reasoning is objectively surveyable and effectively determines the outcome of the decision process.

d. It is systematic and grounded in the facts with its acceptable modes of analysis consistent with epistemological criteria.

e. It is usually self-conscious and constantly re-examines its basis of description for the validity of implicit assumptions and entailed decisions that it contains.

CHAPTER TWO

PREPLANNING: THE FUNCTIONAL DESCRIPTION OF URBAN NEIGHBORHOODS

I. The Functional Description of an Urban Neighborhood

1. The Concept of Functional Description

a. The Role of an Observational System in Planning

Preplanning is the initial phase of the planning process; concerned with the functional description of processes occurring in an urban neighborhood, and subject to the requirement that these descriptions be suitable for the functional explanation and the implementation phases of planning.

Modern system theory holds that any system of description is arbitrary, but recognizes that some systems are better than others for specific purposes. Preplanning must contain criteria that will determine the best possible functional description under any specific circumstances of observation. Such criteria ground the explanations of urban behavior in what can be described by available instruments and the prevailing group concepts. The conceptual basis for description will be determined by observational criteria which highlight particular aspects of observation as describable. Epistemologically it is insignificant that all aspects of behavior cannot be described by the criteria, but it is vital that criteria exist to permit some aspects to be described. Moreover, while it is necessary that the elements of the observational basis be describable, it is not necessary that they be capable of explanation. Explanations are to be constructed from the elements of the basis set.

More elaboration of the underlying concepts of an observational system can be found in Eugene Wigner's Introduction to General Systems Thinking,[1] R. H. Atkin's essay, "The Cohomology of Physics" in Quantum Theory and Beyond (in addition to his papers on Q-analysis), Sir Bertrand Russel's essay, "The Relation of Sense Data to Physics" found in A Free Man's Worship, and Herbert Simon's functional design concept contained in the Sciences of the Artificial.[2]

The concept of an observational system provides a device to impose an order on his raw perceptions, which is consistent with his own experience, and further must conform to the inescapable limits of observational and cognitive ability of any observer belonging to the larger scientific community. The order which is imposed by this system on the raw perception of events in an urban neighborhood is undeniably related to the objectives of planning.

Specifically the observational structure facilitates the preparation of a good plan that can minimize the costs incurred by normal decision-making in the following ways:

a. The observational structure provides a complete framework of description to simplify problems, derived from a shared protocol

---

[1]Eugene Wigner, An Introduction to General Systems Thinking, (New York: Wiley, 1976), pp. 87-94.

[2]The specific references to the works cited above are as follows: R. H. Atkin, "Cohomology in Physics," Quantum Theory and Beyond, (Cambridge: University Press, 1971), pp. 205-210; Sir Bertrand Russel, "The Relation of Sense Data to Physics," A Free Man's Worship, (London: Unwin Books, 1976) pp. 140-157; and Herbert Simon, The Sciences of the Artificial, (Cambridge: MIT Press, 1971) pp. 6-13.

of events (events which may possibly be observed), preventing any surprises from occurring during observations, and thereby reducing the effort to remember observations.

b. The framework provides the possibility of distinguishing between the preferences as to what should occur and the descriptions of what did occur or is occurring. Preferences and actual events are distinct types of data, and for each suitable concepts may be generalized using their ordering among themselves (as to consistency). Clearly significance should be attached to observed structures that are consistent with concepts derived from preferences.

c. The framework as a whole permits independent concepts and observational structures to find their own place, but the forms of relationship between them may be readily appraised to reduce transfer and opportunity costs.

d. Soluble problems may be distinguished from the insoluble situations.

b. Elements of the Epistemology of an Observational System

The essential features of any observational system evolve from the epistemological concepts discussed by Russel. Russel asserts that every individual lives in a private world containing all the different perceptions of the world accessible to the observer. The notion of "the place at which a perception is" must be construed as a place in the private perspective of an individual. In addition to the private spaces of every individual there is an overall space of perspectives in which every such space constitutes a single point. These private spaces must be ordered into a single space by means of testimony between individuals. Hence the concept of "the place from which a thing is observed" is also important. (Although no two spaces contain the same perceptions, it is clear that perceptions pertaining to the same object must have features in common to which definite representation is given by means of a protocol set.) The sets representing the common features of different individuals' experience obviously approximate the nebulous classes of experience found in the distinct spaces. Particular attention will be paid to the criteria making these sets definite.

The elements of a set describing a perception represent those aspects of experience called to attention by an observer made definite by the observational criteria. Although a perception is a form of experience, not every form of experience is a perception. Experience must be a cover set to generalize individual perception to the case where perception at different instances, different circumstances, or even those taken from the testimony of other observers may be considered. The concept of experience emphasizes the requirement to consider the social mechanisms that compose the experience of a whole group from that of its membership. While the experiences of different individuals are independent, when two or more are considered together they can no longer be arbitrary. The constraint is determined by the communications processes in the group and by its "pecking order."

The constraint is governed by the traditional assertion that physical factors cannot determine social relations; and conversely, social relations cannot determine the physical. The problem is to find an area of matching between the social and physical aspects to determine the choice at each instance of the neighborhood. Intrinsically, those qualities matching two independent aspects of the structure are called phase. If we are to view planning as the process of mediating conflict, then a knowledge of the representation of such social mechanisms is germane to the process of mediation. Modern mathematics studies the concept of connections which one may think of as the study of how a perspective of "the place at which a thing" is changes as the place from which it is viewed alters. This is the basis of the modern physical concept of field, but it would seem to be a concept equally applicable in city planning.

An urban field is generated by the mutual interaction of a configuration of groups or individuals, each of which have their own systems of preference, and as a whole accept some global standard of behavior. This global standard in turn acts to constrain individual behavior. If a system is to be a stable organization, then in some sense the global form must match the individual's preference. Stability assumes the consent of the governed to abide by standards imposed from above.

c. The Basic Requirements of an Observational System

To elaborate the mathematical concept, consider an inner environment defined by a space of functional social relations, and an outer environment describing different patterns of actual behavior. The images that these environments cast upon each other constitute their functional description. Where a suitable match between these images occurs, a stable barrier then exists to govern the transmission of signals between the inner and outer environment. The notion of phase is relevant to this barrier. Essentially the problem is to cast the physical patterns into a form comparable to the functional standards of behavior for every different set of circumstances in the neighborhood and to find the largest sets of choice compatible with all the constraints found in the whole neighborhood.

The simplicial complex can be used, much as a co-ordinate system is used, to map the observations which are the image of some particular elements of the set on which observations are resolved. The observations are assumed to be made in terms of definite sets, and particular observational criteria are required to establish a general consensus on what is to be observed. (This effectively limits the observational capability of the protocol and makes functional description heavily dependent upon observational data).

The effect of the phase property governing the changes in urban structures as it extends through the system or through time will be illustrated in terms of the siting problem. This chapter will specifically consider the following topics:

a. Use of Delphi techniques to determine the empirical input to preplanning;

b. Use of hard data for the description of the circumstances in a backcloth;

c. The introduction of standards of behavior; and

d. Interpretation of the mathematical representation of a neighborhood.

e. The siting problem as an example of a phase structure.

2. Global Structure and the Hierarchy of Cover Sets

The complex barrier that exists in an urban neighborhood can be intuitively decomposed into a set of signal relations, each of which leads to an analyzable set of simplicial complexes having an internal order discriminating standards of behavior for urban structure apparent at that signal level. An imposed external order represents the effect of the backcloth relations upon the structure as it extends through the system and through time.

While the signal images have among themselves a sense of being important which determines their order, the singleton sets of the backcloth on which they are resolved have no such necessary order. The composition of sets to reflect changes of viewpoint will be accomplished by using a calculus of simplicial complexes to form the union and intersection of whole relations; thereby studying the effect upon the available choice after a certain sequence of development. The imposed order represents social constraints in the backcloth. The general structure of a barrier will require elaboration of the concepts of phase, signal level, and connection before complete justice is given to the treatment of the complex signals in an urban neighborhood.

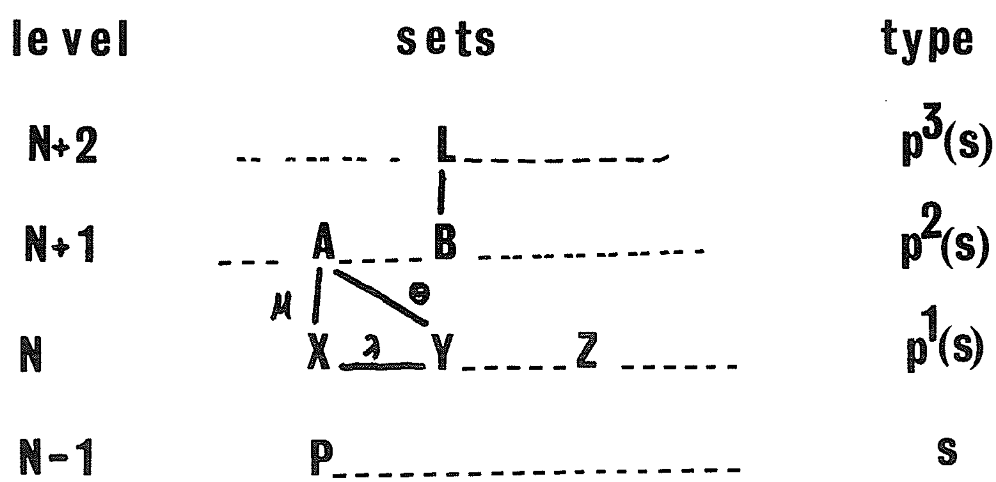
Obviously considerable, though straightforward, work must be done to make certain this calculus of complexes carries through the sense of orderliness from the lower level, and reflects the impact upon the extent to which activities can be sited in pieces of urban land. This of course is the implication behind Atkin's assertion that planning will study hierarchial arrangements of cover sets with the significance of a set at one level intuitively appreciated at the next higher or lower level.[3]

---

[3] R. H. Atkin, Mathematical Structure in Human Affairs, (Hieneman Educational Books, 1975), p. 118.

Figure 1

Hierarchial Cover Sets



HORIZONTAL (λ) AND VERTICAL (μ,θ) RELATIONS DESCRIBE THE SIGNALS IN THE ENVIRONMENT

3. The Nature of Urban Land Use Phenomena

Any science is concerned with certain phenomena and has the capability to recognize whether a given set of observations lay within its area of interest or not.[4] This is not to imply that an all-inclusive definition can specify, in every instance, the precise nature of subject phenomena; since the further clarification and extension of definition is a very active area of research in a science.[5] Nevertheless, it is usually possible to point out certain observations as being part of the central concern of a science.

Criteria, often implicitly assumed by researchers, are used to test specific circumstances for relevance to urban planning as they are encountered in observations. For example, to be considered a physical process, observations must conform to at least the laws of thermodynamics.

Notwithstanding that current planning cannot claim to understand urban behavior to this formal extent, it is possible to specify the form which such general criteria would take in a land use study.

There are three criteria generally used to establish the existence of significant observations in any area. These are public observability, repeatability, and persistence. Public observability is the requirement that a consensus of Land Use Planners recognize the phenomena as being essentially urban in nature; repeatability is one of the conditions necessary for such a consensus since it requires that the phenomena to happen often enough to be observed by a consensus of Planners.

---

[4] C. F. Von Weizsäcker, "The Unity of Physics," essay contained in Quantum Theory and Beyond, (Cambridge: University Press, 1971), p. 229.

[5] Ibid, p. 229.

Persistence requires that when it happens, it lasts long enough to register using the current empirical means of land use planning. Phenomena are usually recognized by their effect upon the environment and therefore functional descriptions are the norm. Since the most general characteristic of an urban function is the variety of choice, of services and of activities available to the resident of the city; it seems appropriate to recognize the significant characteristics of urban phenomena as being functionally identified with the variety of choices available to an urban resident.

This focus upon the choice available is in keeping with the spirit of the modern mathematical concept of information; discounting the importance of the actual nature of activities except that they must occur in urban land, but emphasizing the extent of choice available to residents on the land.

Recognition of urban phenomena can change over time as the tolerances determined by empirical means of observation are refined. The tolerance criteria structure observations into the categories - urban or non-urban - and place emphasis upon the distinct urban-like choices which are the most common and the most long-lasting relative to existing instrumentation. In Atkin's terms, they define the scale which is to be incorporated in a set of observations.[6] The set of observations considered to be urban, and also significant in any problem, is to be denoted the observational protocol. By means of the protocol it is possible to

[6]R. H. Atkin, "The Cohomology of Observations," essay contained in Quantum Theory and Beyond, p. 193.

study the general forms which functionally describe urban phenomena such as land using mathematical structure.

4. The Relation of Sets to Urban Phenomena

The mathematical set is a concept which denotes any collection of objects having any properties at all. It is the fundamental unit of mathematical structure, which is to be correlated initially with individual perceptions and with composite experiences.

The logical interpretation of sets allows their elements to represent equally well, either actual information or abstract properties. Sets providing a common medium for the representation of choice are a basis in terms of which both logical properties and actual information can be compared to establish an epistemic correlation between concepts and actual behavior.[7]

Sets and patterns of subsets can represent patterns of information according to the following definitions:

a. Definition

If S is a set of objects of any kind, then every conceivable property, which some of these objects may have and others not, can be fully characterized by specifying the subset of S whose elements have this property; therefore if two properties correspond in this sense to the same set, they are logically equivalent.

---

[7]Sir Bertrand Russel, "The Relation Between Sense Data and Physics," essay contained in a Free Man's Worship, (London: Unwin Books, 1976), pp. 140-144. Russell outlines here the requirements of an epistemology that rigorously organizes concepts in one's actual experience. The use of sets provides a medium for correlating one's preferences with what actually exists.

b. Definition

> Instead of correlating the subsets of S to properties of elements, we may equally well correlate them with all possible bodies of information concerning an otherwise undetermined element of S. Any such information asserts that this unknown element has a certain specified property.

Observe that in particular, logical disjunction of sets corresponds to their sum or union, conjunction to intersection, and negation to the complementation of a set. The empty set corresponds to absurd information, while subsets represent actual information.

Sets can represent equally well the hard data that may be obtained, for example by counting urban residents; or the preferences that are desired in an urban area. Dealing with the subset structure will permit one to order these preferences, arrange for them to dominate actual behavior in an urban neighborhood in a specific area, and thus correlate the patterns of choice with areas of urban land. The mutual relationships between land areas determine acceptable patterns of land use influences among the areas.

When patterns of choice are correlated with pieces of urban land, rational urban land use can be defined. Some concepts, essentially preferences, can be defined in theory but are found to describe no choices that actually exist; similarly some actual configurations of urban choice conform to no standards. An orderly approach to planning seeks to eliminate non-conforming choices, and reduce the conflict between different standards.

5. Detection of Relations in Urban Experience

In empirical planning the basic instrument is the ability of an individual to determine the structure of his perceptions and experience. This instrument is greatly facilitated by means of the Delphi techniques which have been developed in social sciences in recent years. Mathematical analysis makes available the ability to bring to explicit awareness the maximum amount of information that was implicitly known to the Delphi participants. Consequently the validity of the conclusions about phenomena depends upon the familiarity of those who initially define the problem for subsequent mathematical analysis.

The study must therefore touch briefly upon the ability of the Delphi methods to define significant sets and relations for analysis before discussing the presentation of Delphi information.

Definition

> A Delphi technique has the objective of facilitating the effort of an individual to impose the maximum degree of structure upon his individual experience, by providing a common protocol in terms of which he can describe his experience for comparison to that of other individuals, and which may then be transformed into suitable expressions of the group's experience.[8]

The Delphi is a structured methodology that limits the group's interest to possible observations using the observational protocol to focus upon some area of human experience. Individuals express their

---

[8] Thomas F. Saärinen, Environmental Planning: Perception and Behavior, (Atlanta: Houghton Mifflin and Company, 1976), p. 110. The definition extends Saarinen's discussion of structuring individual experience to the terms which seem to represent the function of the Delphi in preplanning.

experience in terms of relations between elements of the protocol, which are "summed up" into new sets expressing a consensus as to the total group experience.

It is difficult to see how the Delphi could be regarded otherwise than the means by which a consensus is derived from individual experience concerning the scale relations defining significant sets of observations that incorporate criteria such as those discussed in the last section. Some insight in this area can be obtained from the material discussed by Russel in the reference at footnote[4].

The Delphi is capable of providing the soft data describing individual preference systems which is necessary in addition to actual observations for solving land use problems. Thus, one of the most popular techniques in city planning is the goals-means evaluation matrix.[9] The technique attempts to establish the relation between goals and concrete means of attaining them.

6. The Study Design: Representation Using Delphi

In this section a study design is described which is representative of the outcome of a Delphi exercise undertaken to define an urban land use structure. The form of the product of such an exercise is illustrated to provide a basis to study further the mathematical analysis that is possible.

Note that the study design must contain both hard and soft data. Hard data for urban structures consists of sets determined by signal

---

[9]Morris Hill, "The Goals Achievement Matrix for Evaluating Alternative plans:, JAIP, (1968), pp. 19-29.

relations which define the choice associated with urban land, while soft data is concerned with structures that describe a resident's preferred limitation upon choices correlated with zones covering urban land.

The term backcloth has been used, and will be extensively used, to describe any structure representing the circumstances of observations. A backcloth can be understood as providing a basis which resolves the significant parts of observations (the streets) to focus on their particular differences (their associated choice) between them. Picturesquely, the backcloth acts like colored spectacles to vary the parts of a color pattern emphasized or melting into the background as the viewpoint is changed. This term is entirely relative to the context of its use: thus the study will speak of the backcloth of streets on which urban land use activities are discussed; but this framework as a whole is a backcloth for the discussion of urban standards shall govern activity in the whole neighborhood.

The analysis is focused on a specific area of Winnipeg, a portion of the Fort Rouge Community surrounding Osborne Street, that is shown on the map at Figure 2.

As shown in Figure 3, the map was reduced to a backcloth structure of streets defining what this study holds to be the significant areas of urban land on which activity occurs. In any study the extensive formal (legal) institution of land ownership ensures that there is no difficulty defining a backcloth of urban land areas. The relation of adjacency was used to define the relation between backcloth areas shown in Table 2. In fact this simple relation is necessary to introduce the idea of influences between urban land that ultimately gives rise to the

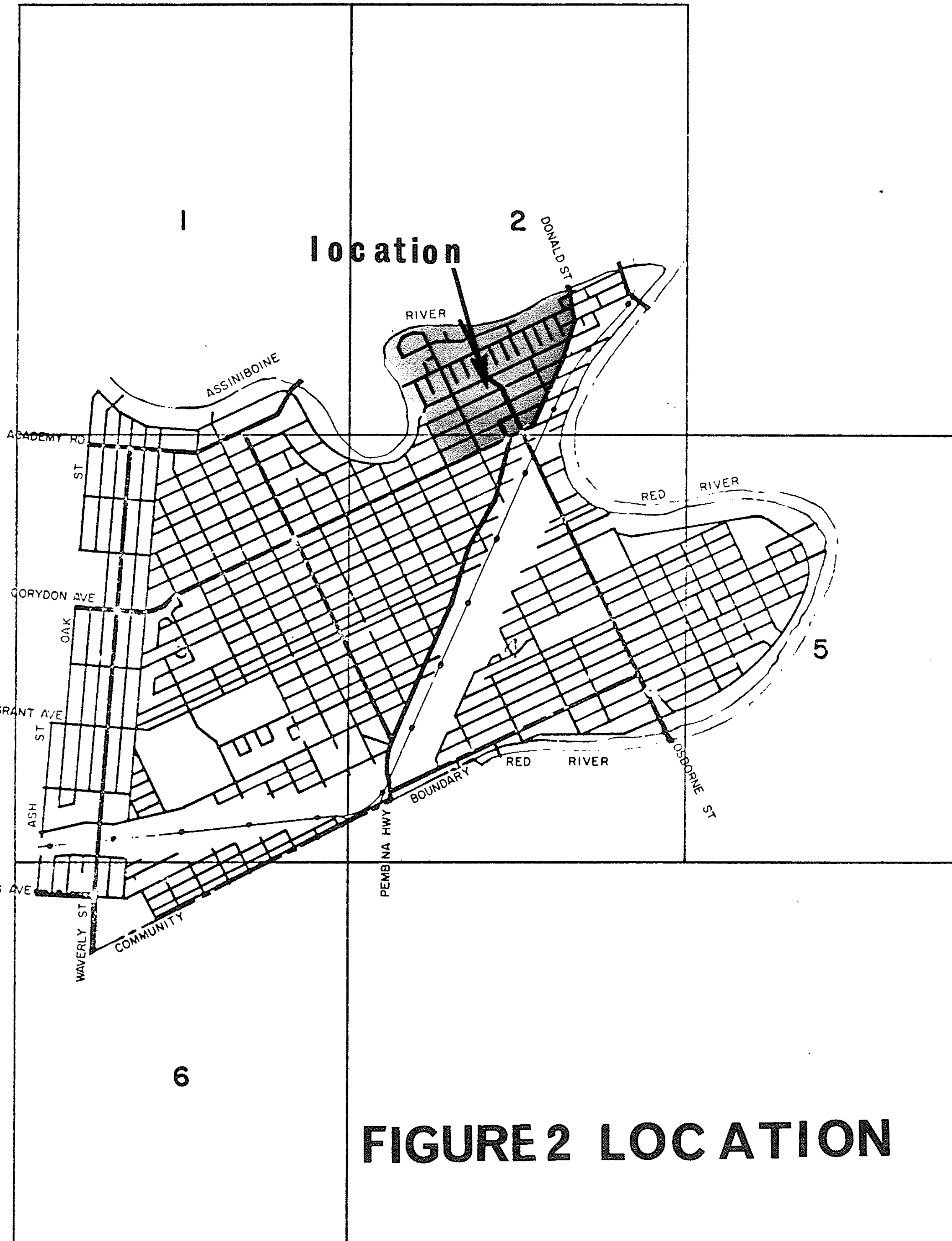


FIGURE 2 LOCATION

FORT ROUGE

KEY SHEET

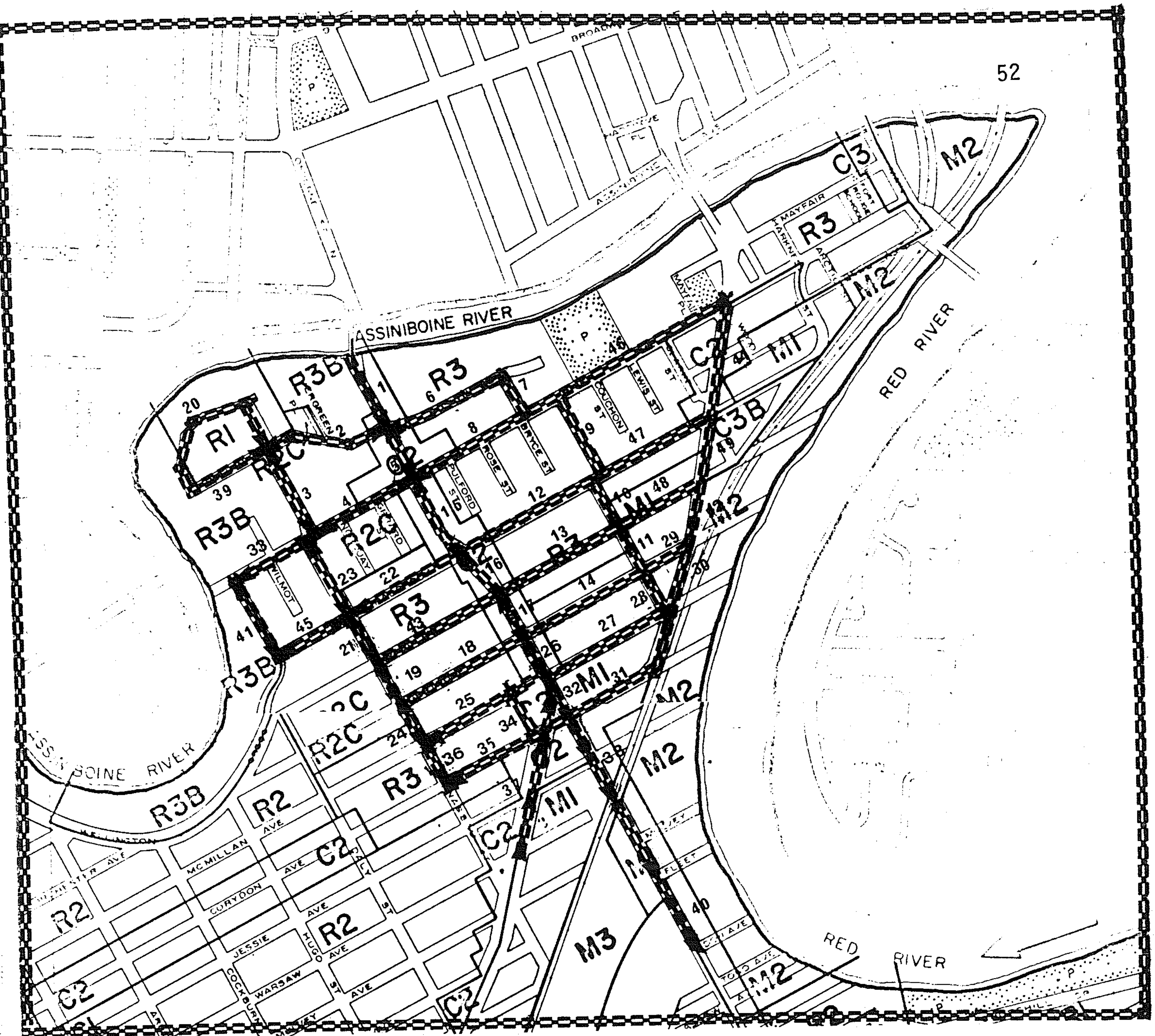


FIGURE 3

BACKCLOTH OF STREETS

Note: Because the analysis program was limited to a maximum of 50 vertices, the backcloth used in this study provides a very limited representation of the total relationship between the streets actually existing in the study area. This limitation can easily be removed by more efficient computer programming.

whole problem of conflicting land uses.[10] In this study the adjacency relation on the backcloth of streets represents accessibility, and determines the propagation of the influence of activities through the neighborhood. To provide a basis for defining levels of the accessibility signals, the idea of adjacency was extended to n-adjacency on the graph of the backcloth relation. The relation of n-adjacency incorporates the basis of the idea of signal levels, determining the possible extent of influence between backcloth areas, that propagate along the paths contained in the backcloth. Note that this in no way presupposes what influences shall be found to propagate, only that they shall be necessarily graded by their influence at the different levels. The signal levels of a backcloth can be efficiently computed by the methods noted in reference[11] using only the adjacency relation initially defined. The representation of a portion of the whole weighted relation shown in Table 3 should be typical of any backcloth signal level relation determined for a study design. Signal levels will always be represented by a weighted incidence matrix. The accessibility signal property, whether expressing transit time or distance between pairs of areas, is a fundamental feature of any study design.

The observational protocol is the set denoting urban land use activities that may occur in urban areas. The protocol for this simple

---

[10] Refer to the example of the problem of fee simple land ownership discussed in Chapter One for background to the problems of distribution.

[11] A. T. Berztiss, Data Structures: Theory and Practice, (New York: Academic Press, 1975), pp. 263-279. Also see M. Sakarovitch, "The k-Shortest Paths in a Graph," Transportation Research Quarterly 2, (1968), pp. 1-11. Both authors provide particularly useful algorithms that can be programmed to determine the weighted relation representing the signal relation given any backcloth relation.

urban functional space was defined, forestalling the need for any Delphi, by listing those activities actually occurring in the study area as contained in Henderson's City Directory for 1967 and 1976. The resulting protocol is listed in Table 4. Various sub-protocols reflecting a crude breakdown of activities into major urban functions are also distinguished in Table 4.

For completeness the illustration of a study design will introduce some concepts, necessarily determined in any study at this stage, which will not be explained until the discussion of the simplicial complex. The relation of the protocol to the backcloth of urban land is fundamentally important. The basic relation used is the incidence of activities from the protocol onto urban land areas. This concept is illustrated by Table 6 and Figure 5 contained in this chapter. To determine the patterns of influence present in each area as a result of the siting of activities for each accessibility level, multiply the incidence matrix of the complex $K_L(A)$ times the incidence matrix $K_L(L,R,n)$ for each level n. Table 5 shows an example of the simplex which would be found in such a composite complex.

While Table 5 is concerned with the effect of the siting of activities, the study is also concerned with the relations between activities themselves independent of what the backcloth relation actually produces. The determination of standards of behavior is a function of analysis to be carried on in the planning stage, but the preferences and theories about the desirable relations between activities to be used as a basis for planning must be determined in the preplanning design of the study.

Presumably, each individual will provide for each signal relation, a relation with an incidence matrix like that in Figure 4 below. Assuming ideas like co-operation or competition between activities, land use compatibility relations can be determined and later composed into standards like those shown in Table 4. Thus, the initial preference systems are a relation RcAXA between activities, but the standards are a relation R_2cFXA where F denotes a functional zone such as residential, commercial, or the like.

Figure 4

Example of a Preference Relation

Individual A   Signal Relation: Compatibility between activities based on desirable maximum separation.

Activity Set = (A_1,A_2,A_3,A_4,A_5,A_6)   Accessibility levels n = 0 to n = 5

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	2	5	0	4	1
A_2	2	0	3	0	1	3
A_3	5	3	0	2	4	5
A_4	0	0	2	0	1	0
A_5	4	1	4	1	0	1
A_6	1	3	5	0	1	0

Note - 0 obviously denotes activities that must be co-located

Note - the matrix is obviously symmetrical in nature.

There is a gap in what must actually be accomplished in a study design: ie. the discussion of the hierarchial arrangement of sets of data that reflect levels of concern in the study. For example, the planner is concerned with individuals, buildings, streets, neighborhoods, districts, communities and then whole cities each having its characteristic structures and signals that must somehow mesh as a whole. Atkin discusses the hierarchial arrangements at great length in the literature of Q-analysis.[12]

[12] R. H. Atkin, "A Approach to Structure in Architectural and Urban Design: Illustrative Examples," Environment and Planning B 2, (1975), pp. 21-57.

TABLE 2

ADJACENCY STRUCTURE IN THE BACKCLOTH*1

L1 = (L2, L5, L6)
L2 = (L1, L2, L3, L5, L6)
L3 = (L2, L3, L4, L23)
L4 = (L3, L4, L5, L8, L15, L23)
L5 = (L1, L2, L4, L5, L6, L8, L15)
L6 = (L1, L2, L5, L6, L7)
L7 = (L6, L7, L8, L9)
L8 = (L4, L5, L7, L8, L9, L15)
L9 = (L7, L8, L9, L12, L16)
L10 = (L9, L10, L11, L12, L13)
L11 = (L10, L11, L13, L14, L28, L29)
L12 = (L9, L10, L12, L15, L16, L22)
L13 = (L10, L11, L13, L16, L17, L22, L43)
L14 = (L11, L14, L17, L18, L26, L28, L29)
L15 = (L4, L5, L8, L12, L15, L16, L22)
L16 = (L12, L13, L15, L16, L17, L20, L22)
L17 = (L13, L14, L16, L17, L18, L20, L26)
L18 = (L14, L17, L18, L19, L24, L26)
L19 = (L18, L19, L20, L21, L24)
L20 = (L13, L16, L17, L19, L20, L21)
L21 = (L19, L20, L21, L22, L23)
L22 = (L12, L15, L16, L21, L22, L23)
L23 = (L3, L4, L21, L22, L23)
L24 = (L18, L19, L24, L25, L36)
L25 = (L24, L25, L26, L27, L32, L33, L34, L36)
L26 = (L14, L17, L18, L22, L23, L25, L26, L27)
L27 = (L25, L26, L27, L28, L30, L31, L32, L33)
L28 = (L11, L14, L27, L28, L29, L30)
L29 = (L11, L14, L28, L29, L30)

TABLE 2 (Con't)

L30 = (L11, L14, L27, L28, L29, L30, L31)
L31 = (L27, L28, L30, L31, L32, L33, L34, L35, L37, L38)
L32 = (L25, L26, L27, L28, L31, L32, L33, L35, L38)
L33 = (L3, L4, L23, L33, L41)
L34 = (L25, L33, L34, L35, L37)
L35 = (L21, L22, L25, L27, L28, L31, L32, L34, L35, L36, L37, L38)
L36 = (L24, L25, L35, L36)
L37 = (L31, L32, L34, L35, L37, L38)
L38 = (L31, L32, L35, L37, L38)
L39 = (L2, L3, L20, L39, L41)
L40 = (L38, L39)
L41 = (L33, L41, L45)
L42 = (L29, L30, L42, L48, L49)
L43 = (L13, L16, L17, L19, L21, L43)
L44 = (L44, L46, L47, L49)
L45 = (L21, L22, L23, L41, L45)
L46 = (L7, L8, L9, L44, L46)
L47 = (L9, L10, L44, L47, L49)
L48 = (L10, L11, L42, L48, L49)
L49 = (L42, L44, L47, L48, L49)

Note 1. The adjacency between designated areas of land (eg. L_1, L_2, etc.) is shown in structure form. $L_1 = (L_2, L_5, L_6)$ means that areas L_2, L_5, L_6 are each adjacent to L_1.

TABLE 3

EXTENSION OF SERVICE AREAS (n = 1 to n = 3)

The following table shows the extension of the service areas through the study area for the representative set of sites (Li, i = 1, 10 with the accessibility relation determined by the adjacencies of areas shown in Table 2. The notation Li = (...., $L^n j$...) indicates that Lj is first accessible to Li along some path at level n (accessible in n steps or less).

L1 = ($L^2 1$, $L^1 2$, $L^2 3$, $L^2 4$, $L^1 5$, $L^1 6$, $L^2 7$, $L^2 8$, $L^3 9$, $L^3 15$, $L^3 23$)

L2 = ($L^1 1$, $L^1 2$, $L^1 3$, $L^2 4$, $L^1 5$, $L^1 6$, $L^2 7$, $L^2 8$, $L^3 9$, $L^3 15$, $L^3 21$, $L^3 22$, $L^2 23$)

L3 = ($L^2 1$, $L^1 2$, $L^1 3$, $L^1 4$, $L^2 5$, $L^2 6$, $L^3 7$, $L^2 8$, $L^3 9$, $L^3 12$, $L^3 15$, $L^2 16$, $L^3 18$, $L^3 19$, $L^3 20$, $L^2 21$, $L^1 23$)

L4 = ($L^2 1$, $L^2 2$, $L^1 3$, $L^1 4$, $L^1 5$, $L^2 6$, $L^2 7$, $L^1 8$, $L^2 9$, $L^3 10$, $L^2 12$, $L^3 13$, $L^1 15$, $L^2 16$, $L^3 17$, $L^3 18$, $L^3 19$, $L^3 20$, $L^2 21$, $L^2 22$, $L^1 23$)

L5 = ($L^1 1$, $L^1 2$, $L^2 3$, $L^1 4$, $L^1 5$, $L^1 6$, $L^2 7$, $L^1 8$, $L^2 9$, $L^3 12$, $L^1 15$, $L^3 16$, $L^3 21$, $L^3 22$, $L^3 23$)

L6 = ($L^1 1$, $L^1 2$, $L^2 3$, $L^2 4$, $L^1 5$, $L^1 6$, $L^1 7$, $L^2 8$, $L^2 9$, $L^3 12$, $L^3 16$, $L^3 23$, $L^3 15$)

L7 = ($L^2 1$, $L^2 2$, $L^3 3$, $L^2 4$, $L^2 5$, $L^1 6$, $L^1 7$, $L^1 8$, $L^1 9$, $L^3 10$, $L^2 12$, $L^3 13$, $L^2 15$, $L^2 16$, $L^3 17$, $L^3 20$, $L^3 22$, $L^3 23$)

L8 = ($L^2 1$, $L^2 2$, $L^2 3$, $L^1 4$, $L^1 5$, $L^2 6$, $L^1 7$, $L^1 8$, $L^1 9$, $L^3 11$, $L^2 12$, $L^3 14$, $L^1 15$, $L^2 16$, $L^3 19$, $L^3 20$, $L^2 21$, $L^2 22$)

L9 = ($L^2 1$, $L^3 2$, $L^3 3$, $L^2 4$, $L^2 5$, $L^2 6$, $L^1 7$, $L^1 8$, $L^1 9$, $L^2 10$, $L^3 11$, $L^1 12$, $L^2 13$, $L^3 14$, $L^2 15$, $L^1 16$, $L^2 17$, $L^3 18$, $L^3 19$, $L^2 20$, $L^3 21$, $L^2 22$, $L^3 23$, $L^2 27$, $L^2 28$, $L^2 43$)

L10 = ($L^3 4$, $L^3 5$, $L^3 6$, $L^2 7$, $L^2 8$, $L^1 9$, $L^1 10$, $L^1 11$, $L^1 12$, $L^1 13$, $L^2 14$, $L^2 15$, $L^2 16$, $L^2 17$, $L^3 18$, $L^3 19$, $L^2 20$, $L^3 21$, $L^2 22$, $L^3 23$, $L^3 24$, $L^3 27$, $L^2 28$, $L^2 29$, $L^2 30$, $L^3 31$, $L^2 43$)

TABLE 4

COVER SETS FOR URBAN ACTIVITIES IN THE FORT ROUGE AREA AND ASSIGNMENT TO ZONE STANDARDS

1. Cover A1 - Residential Activities	Zone Assignment Structure*
a. High rise apartment	(R4, C3, C2)
b. Walk-up apartment	(R3, R4, C1, C2, C3, M1)
c. Multiple family dwelling	(R2, R3, R4, C1, C2, C3, M1)
d. Single family dwelling	(R1, R2, R3, R4, C1, C2, C3, M1)
e. Residence over commercial activity	(R3, C2)
f. Vacant lot (1) *(See note at end of table)	(ALL ZONES)
g. Vacant lot (2) *	(ALL ZONES)
2. Cover A2 - Local Commercial Activity	
a. Gas station	(C2, C3, M1, M2)
b. Grocery	(C1, C2, M1, M2)
c. Drugstore	(")
d. Restaurant/Lunch	(")
e. Laundromat/laundry service	(")
f. Delicatessen	(C3, C2, M1, M2)

Note: The table indicates the different zones with which the activity is considered to be consistent.

TABLE 4 (Con't)

3. Cover A3 - District Commercial Activity

		Zone Assignment Structure
a.	Supermarket	(C3, C2, M1, M2)
b.	Fruit dealer	(")
c.	Magazine/Tobacco	(")
d.	Florist	(")
e.	Dry Cleaner	(")
f.	Hotel	(C3, R3, R4, C2, M1, M2)
g.	Billiard parlor	(C3, C2, M1, M2)
h.	Shoe repair	(")
i.	Tailor/Seamstress	(C3, C2, M1, M2)
j.	Liquor Control/beer retail	(")
k.	Car wash	(C3, C2, M1, M2)
l.	Ice cream parlor	(")

4. Cover A4 - Regional Retail

a.	Fashion shop	(C3, C2, M1, M2)
b.	Furnishing/upholstery repair and retail	(")
c.	Retail distributors - various goods	(")
d.	Surgical/Optical supply	(")
e.	Pet store	(")
f.	Electronic/stereo/music store	(")
g.	Car Sales	(")

TABLE 4 (Con't)

Cover A4 - Regional Retail (Con't)

		Zone Assignment Structure
h.	Furriers	(C3, C2, M1, M2)
i.	Taxi office	(")
j.	Printing/stationery	(")
k.	Wine store	(")
l.	Textiles	(")
m.	Photo studio	(")
n.	Auto part retail	(")
o.	Art gallery/retail	(")
p.	Wig store/specialty	(")
q.	Special boutiques	(")
r.	Book store	(")
s.	Antique store	(")
t.	Musical instruments	(")
u.	Health food	(")
v.	Hobby store	(")

5. Cover A5 - Light Industrial Service and Manufacturing

a.	Equipment rentals	(M1, M2)
b.	Foundry	(M2)
c.	Pipe distributors	(M1, M2)
d.	Light manufacture and distribution	(")
e.	Plumbing/Heating	(M1, M2)

TABLE 4 (Con't)

Cover A5 - Light Industrial Service and Manufacturing - Con't

		Zone Assignment Structure
f.	Electrical supply/service	(C2, C3, M1, M2)
g.	Safety supply	(")
h.	Industrial supply and service	(M1, M2)
i.	Paint supply and service	(C2, C3, M1, M2)
j.	Contractors - general and special	(M2, M2)
k.	Engineering Consultants	(C2, C2, M1, M2)
l.	Building supply	(M1, M2)
m.	Welding	(")
n.	Architectural consultant	(C2, C3, M1, M2)

6. Cover A6 - Personal and Government Service

a.	Dentist	(R3, C2, C3)
b.	Doctor/Medical clinic	(")
c.	Barber	(C1, C2, C3, M1, M2)
d.	Hairdresser	(")
e.	Rehabilitation center - various	(R3, C2, C3)
f.	Manitoba hydro - facility	(R1, R2, R3, C1, C2, C3, M1, M2)
g.	Bank/Trust company	(C1, C2, C3, M1, M2)
h.	School	(R1, R2, R3)
j.	Interior design	(C2, C3, M1, M2)
k.	Firehall - engine company	(R1, R2, R3, C1, C2, C3, M1, M2)
l.	Real estate agent	(C2, C3, M1, M2)

TABLE 4 (Con't)

Cover A6 - Personal and Government Service - Con't

		Zone Assignment Structure
m.	Insurance agent	(C2, C3, M1, M2)
n.	Post office	(C1, C2, C3, M1, M2)
o.	Accounting/Advertising service	(C2, C3, M1, M2)
p.	Federal building	(")
q.	Consulate	(C2, C3)
r.	Hospital	(R2, C2, C3)
s.	Elderly housing	(R2, R3, R4, C1, C2, C3)
t.	Parking lot	(")
w.	Rental agents/Management services	(R1, R2, R3, C1, C2)

7. Cover A7 - Cultural/Organization/Institutions/Recreational

a.	Tennis club	(R1, R2, R3, C1, C2, C3)
b.	Associations - various	(R3, C1, C2, C3)
c.	Churches	(R1, R2, R3, R4, C1, C2, C3)
d.	Meeting halls	(R3, C1, C2, C3, R4)
e.	Union affairs	(C1, C2, C3, M1, M2)
f.	Ballet school	(R3, R4)
g.	Park	(R1, R2, R3, R4, C1, C2, C3)
h.	Recreation center	(R1, R2, R3, R4)
i.	Research institute	(C2, C3)
j.	Day school/Nursery	(R2, R3, R4, C1, C2)

TABLE 4 (Con't)

Cover A7 - Cultural/Organization/ Institutions
Recreational - Con't

	Zone Assignment Structure
k. Resource center/various	(R3, R4, C1, C2, C3)
l. Vacancy (8), (3), (4), (5), (6), (7), (9)	(All)

Note: Use of numbered vacancies was to distinguish areas having different numbers of vacant areas.

TABLE 5

EXTENSION OF ACTIVITY INFLUENCES

Suppose activity A1 was located in L1 and L6. Then at each accessibility level the influence of A is present in both L1 and L7. The simplex representing the influence of A1 would be defined by A1 = (L1, L7). Denote $A_i = (\ldots\ldots, L^n j, \ldots)$ as showing that Lj is first accessible to A_i along some path at level n. Then for the influence zones in Table 3 and activity A present as above, the simplex showing its accessibility for level 0, 1, 2, and 3 is as follows:

$$A1 = (L^0 1,\ L^1 2,\ L^2 3,\ L^2 4,\ L^1 5,\ L^0 6,\ L^1 7,\ L^1 8,\ L^1 9,\ L^3 10,\ L^2 12,\ L^3 13$$
$$L^2 15,\ L^2 16,\ L^3 17,\ L^3 20,\ L^3 22,\ L^3 23)$$

It is clear that any such simplex, at each accessibility level, is formed by the union of the simplexes representing the influence area for L_1 and L_7 at that level n. Overall, form the union for the weighted relation illustrated above by assigning the lowest accessibility index found in either L_1 or L_7 to the area related to A1. This relation is further discussed in section 2.d. following.

II. Structuring the Study Design

1. Introduction

a. The Problem

Each individual contributes to planning his own viewpoint of the urban function as perceived from the area in which he resides, but this viewpoint is also dependent on external conditions and the influences from other places in the neighborhood. The structural viewpoint strives to integrate all of these individual perspectives into a coherent whole in which the individual preferences, at the place where they are found, are nowhere violated by other influences in the neighborhood. Thus all the individual viewpoints are ordered in the various places where they have an influence: in other words what occurs at one place is determined by the neighborhood as a whole and every neighborhood influences some others by virtue of its content.[13]

These individual isolated viewpoints are ordered into their places by the backcloth patterns of communications between their areas. By taking account of the propagation of signals, the points where incompatible influences of the standards of different individuals occur can be identified and then systematically eliminated at their source. By analyzing the extension of influence through an area, one can determine which areas benefit from some siting decision, and which suffer by virtue of their location.

---

[13] Sir Bertrand Russel, "The Relation of Sense Data to Physics," pp. 152-155.

The use of individual social preferences requires the existence of a functional space of social relations that is independent of, but capable of comparison to, the backcloth. Although physical relations do not determine the social relations, the social relations cannot actually exist unless they match some existing patterns. The initial problem is to identify a structural representation for the data determined by Delphi techniques in which this mode of analysis occurs. The preplanning study design as structurally represented by the simplicial complex fulfills this requirement.

The rationale for functional description and explanation is provided by system theory. Simon gives a good account of the general nature of functional description/explanation.[14] Divide the neighborhood into an internal system that regulates its behavior through some relation to the external environment. The individual preferences with respect to a set of distinguishable signal relations acting through the backcloth describe the internal organization and constitute the barrier in functional descriptions of phenomena.

By enriching the complexity of the relations, which in various degrees represent the penetration of various signals through the barrier, very subtle aspects of the land use organization of an area can be revealed. The signals to be discriminated and their individual expressions are given by the preplanning Delphi exercise. Examples of signals are numerous; ie. accessibility, the retailer's domination of a market area for some specific product or service, access to recreational activities, noise or other nuisance effects of land use, and the like.

---

[14] Herbert Simon, The Sciences of the Artificial, (Cambridge: The MIT Press, 1971), pp. 6-10.

The simplexes of the simplicial complex correlate the effect of signals from the external environment upon individuals belonging to the domain of observation. Every simplex denoting an area is initially assumed to be independent, but the extension of the influences caused by the contents of any simplex propagating through the whole neighborhood can be studied by using signal levels attached to a backcloth ordering of the simplices. The resultant simplicial complexes showing the influences in each area can be ordered relative to each other and to standards. In the same way development of the neighborhood in time can be studied. In either case the effect of the whole neighborhood on each individual is immediately apparent and conflicts can be traced to their source and eliminated at each level. By correlating areas with influences on them or alternately the influence they produce on others, the simplicial complex provides a framework for functional description of urban phenomena.

The barrier concept emphasizes the importance of a limited number of signals in the system; to limit the complexity of the problem. The internal social organization of the neighborhood compiles individual preferences into standards of behavior according to their relation to the signal levels.

The study will focus first upon functional description of any signal relation using the orderliness in the complex to take account of the stable matching of the internal and external structure regulating signals.

Generally the concept of orderliness of development and of land use plan can be identified using only the simplicial complex.

2. The Urban Sense of Mathematical Relations

a. General

For the urban planner not familiar with the formal properties of a mathematical relation, a synopsis of the basic elements together with some excellent references is provided in Annex B. Henceforth the notation and operations defined in that Annex will be assumed in this study. It has been established that well-defined relations exist in the urban land and activities. For each signal relation considered significant each individual defines a different relation; but before considering the multiple relations involved in the barrier, the urban structures described by a single signal should be introduced.

The backcloth relation is taken to describe possible channels of communication, and signal level indicators must be defined on it for the effective use of the signal relation. For urban land, accessibility is one of the most significant; and various techniques can define the accessibility along a path between each pair of urban land areas. Distance or transit time are the most common measures, but others are surely possible for special purposes.

The signal level indicates the extent of the effort required to make some choice available to a resident of an area. By convention all choices above the zero level of direct incidence require some effort. Therefore a pattern of influences must be viewed as a possible choice requiring some action prior to its full realization as an actual choice, unlike the 0-level co-incidence of activities with the resident.

The following are basic types of urban choice situations that are conveniently represented by the mathematical concept of simplex. There are other types of basic urban structure reflecting a more coherent clustering of activities; but first consider the general application of the following which have sufficient unity to define an urban phenomenon. By imposing some limitation, either on the content of the choice set or upon its signal levels, standards are created. Since it is clear that a standard and an urban phenomenon, the former defined by Delphi and the latter by observation, are the same type of structure; they can be compared and the standard tested for its dominance of observed phenomena.

b. Types of Relations

Generally in this discussion $A = (A_i)$; $i = 1, N$ denotes a subset of the urban activity protocol containing N activities, while $L = (L_j)$; $j = 1, M$ denotes a subset of the urban land protocol containing M areas of land. The types of signal relations (denoted by R (S)) are all of the following type: $R(S_1) \subset LxL$, $R(S_2) \subset LxA$, or $R(S_3) \subset AxA$.

c. Type $R(S_1) \subset LxL$

A vivid and potent description of the function of a site in a neighborhood is provided by describing the areas it services (to which it provides access) in terms of the accessibility relation $R(S_1)$ describing the whole neighborhood. Thus, $L_i(n) = (\ldots L_j \ldots . L_m \ldots .)$ where $L_i \; R_n \; (S_1) \; L_j$ for n-adjacency level = n). Obviously the areas of land in the related set are serviced by the site because they are accessible along some path in n or less steps. The complex of simplices derived from the weighted accessibility relation R_n (see Table 3 for an example simplex) can be sliced to obtain this description and is used to describe the function of each site as its influence spreads through a neighborhood for each level of accessibility. Using this information one can study the suitability of sites to service a neighborhood as a whole for various activities. This simplex structure has application to zoning, market studies, or siting of public facilities. It is certainly one of the most basic and vivid structures directly interpretable by the structural analysis.

d. Type $R(S_2) \subset L \times A$

The influence of the whole neighborhood on a given area can be characterized as concisely as one would wish by a simplex. Using the convention that the presence of an activity at accessibility level n implies its presence continues at level n + 1, n + 2......N where N is the maximum level recognized. Thus, $L_i(N)$ denotes the influences on area L_i for all accessibility levels to N.

$L_i(N) = (\ldots, A_i(K), \ldots\ldots)$ $K \leq N$ meaning that each $A_i(K)$ cannot be accessed in less than K steps along some path (not specified), but once it enters the choice simplex it remains in thereafter. The reader will realize that this is a very gross characterization of choice since it highlights the closest instances of activity while ignoring the detailed structure such as instances of the same activity or the path used to access the activity. This structure is basic in monitoring the development of choice as a consequence of the extension of influence of a given site. (See Table 5.)

e. Type $R(S_3) \subset A \times A$

One can also recognize a functional relation between activities. This simplex is extremely important in defining standards since it is in terms of the relation between activities that non-spatial ideas like compatibility, convenience, competitiveness, or co-operativeness can define urban functions.

A simplex $A_i(N) = (\ldots A_j(n)\ )\ n \leq N$ defines the idea that all activities in the simplex are related (accessible to A_i in less than n steps). This could embody the idea that these activities do not compete with A_i in defining a function of the neighborhood, or that they co-operate with A_i to form a function of the system, or simply that they are compatible with A_i, or even that it is convenient for a user of A_i to have the other activities that close. Many of the significant ideas of social relations have the form $R(S_3) \subset A \times A$. (The preference relation Figure 4 for example.)

f. Standards Defined on Simplices

When using the mathematical binary relation to represent signals only related pairs of points are considered. Therefore the simplex represents the set of second elements of ordered pairs of the relation having a common first element. For actual Delphi techniques it is easiest to define the relation in terms of ordered pairs since individuals can easily relate this information, and then generate the possible ternary or higher order relations by suitable methods, if necessary. Therefore the simplex does not imply any necessary relation between elements of the set of second elements. Other structures that consider this internal relation are discussed later for particular problems.

The following are types of standards which can be introduced using the simplex structures defined:

a. Market or Service Area at Accessibility Level N:

By restricting the maximum accessibility parameter the largest set of areas serviced by a site can be constructed.

b. Influence Standard:

By fixing either the accessibility standard or the content of the simplex, (eg. $L_i = (A_i)$) the greatest degree of clustering, or the least degree of clustering permitted among urban activities at some accessibility level can be defined.

c. Functional Standard:

By fixing either accessibility or content of the functional relation between activities, a standard defining the greatest or least degree of clustering around some activity for reasons of convenience or compatibility can be defined. (eg. $A_i = (....A_j...)$

Thus, for example, clustering of activities that are equivalent to shopping complexes, manufacturing complexes, or recreational complexes can be distinguished from among the total influences of an area without regard to the specific area of the city where they happen to be found.

It should be clear that having identified the simplices that represent the function of individual areas or activities in terms of their perspective within the whole neighborhood, the next step is to define the totality of perspectives within any neighborhood as a collection of simplices.

In this regard it is worthwhile to introduce the formal properties of the simplicial complex interpreting them as urban concepts within the theoretical framework.

3. The Simplicial Complex

a. Introduction

The simplicial complex naturally represents the structure of perspectives of all elements of the backcloth within a whole neighborhood relative to some signal relation. It can, for example facilitate the comparison of an individual preference system to the other related individual preference systems that refer to the same relation and the same protocol. In this section the formal properties of the simplicial complex are briefly summarized. A more elaborate discussion of these properties is found in the literature of the Q-analysis listed in the bibliography.

b. The simplex

The simplex which was previously discussed has the idea of grouping together all of the ordered pairs of a binary relation having the same first element. It is formally defined for a relation $R \subset Y \times X$ in the following way:

Definition

If there exists at least one $Y_i \in Y$ such that a $(p + 1)$ - subset of X is R - related to it, call that $(p + 1)$ - subset of X a p- dimensional simplex or p- simplex. Denote it, $Y_i = (X_1, X_2 \ldots, X_{p+1})$ or as $Y_i = \sigma_p$, and call its name Y_i (possibly among many such names).

Any subset of this (p + 1) - subset of X is also R - related to Y_i and is therefore another simplex, say a q-dimensional face simplex of σ_p. This q-simplex is said to be a face of σ_p written $\sigma_q \leq \sigma_p$.

Thus each $Y_i \varepsilon R$ identifies a p - simplex for some p together with all of its faces. This collection of simplices defined by a relation is called a complex of simplices or simplicial complex K.

c. The Simplicial Complex

A simplicial complex is denoted by $K_Y(X;R)$ and the conjugate complex associated with it by the converse relation R^{-1} is denoted $K_X(Y;R^{-1})$. This notation may be remembered by the following device suggested by R. H. Atkin.[15]

$K_Y(X;R)$

Relation R	vertex or range set X
domain Y	incidence matrix (r_{ij})

$K_X(Y;R^{-1})$

Relation R^{-1}	vertex or range set Y
domain X	incidence matrix of R^T

It is usually assumed that the vertices of the complex are defined by the elements of the range set X of the relation YxX (ie. its protocol),

[15] R. H. Atkin, "An Approach to Structure in Architecture and Urban Design. Part 1: Introduction and Mathematical Theory." Environment and Planning B, (1974), p. 59.

while the names of simplices are the elements of the domain Y. This is reversed for the conjugate complex. The vertices define the 0-simplices or points of the complex.

The complex is naturally described by its incidence matrix having the elements of the range along the top and the elements of the domain along the side. Thus, any planning matrix is a simplicial complex. The simplices Y_i of the complexes defined in the incidence matrix of a relation form in the preceding section can be read off by noting in which columns j the X_j in row i are equal to 1. The incidence matrix of the conjugate complex of $K_y(X;R)$ denoted $K_x(Y;R^{-1})$ is found by taking the transpose of the incidence matrix, A^T, of the complex.

A complex can be given an orientation if for some reason it were necessary to introduce a positive sense in which the complex is to be transversed. Since the vertices, say X, are given the ordering of the natural numbers we say that the p-simplex

$$\sigma_p = (X_1, X_2, \ldots, X_{p+1})$$

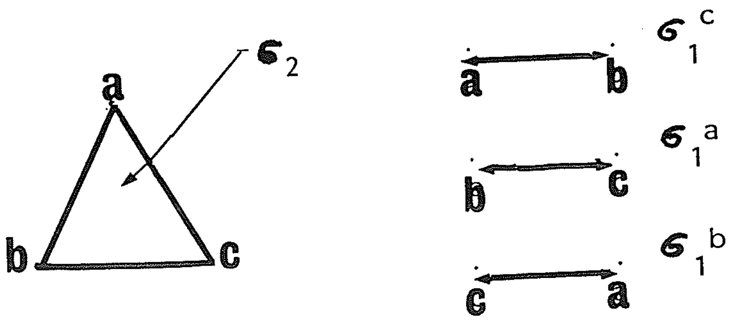
possesses a <u>positive orientation</u> if the sequence

$$(1, 2, \ldots, p+1)$$

is an <u>even permutation</u> of the same numbers with their natural ordering, and that it possesses a <u>negative</u> orientation when that permutation is <u>odd</u>. In the first place we denote the simplex by σ_p, or $+\sigma_p$, and in the second case by $-\sigma_p$. In this way every simplex σ_p, $p \geq 0$, possesses an orientation which is <u>naturally induced</u> and when this is done we say that the complex K possesses an orientation.

The dimension of a complex K is the greatest dimension of any simplex which belongs to it, ie. dim K = $\max_p (\ _p)$.[16]

Any subset $_n$ of a particular p-simplex is also a simplex of the complex K called the face of $_p$. This relation is also denoted $_n\ f\ _p$. To fix the idea of the face relation consider the following example of a two dimensional simplex with vertices a,b,c.[17]



$f\sigma_2 = (\sigma_1^a, \sigma_1^b, \sigma_1^c)$

For any simplex σ_n, $f\sigma_n = \bigcup_i \hat{\sigma}^i_{n-1}$ $\hat{\sigma}^i_{n-1} = (X_1, X_2 \dots \hat{X}_i \dots X_n)$ where $\hat{X}_i$ denotes the omission of vertex X_i from σ_n.

Since the empty set is a subset of every set it seems reasonable to include it as a face of every simplex in KY(X). When this is done we denote the empty set by the (-1) -simplex, $\sigma_{-1}{}^m$ and say that the complex K is thereby <u>augmented</u>; we write it as K^+ or as K_{-1}.

---

[16]Ibid, p. 59.

[17]Ibid, p. 59.

d. Face Ordering

Generally, the face ordering of a single simplex regarded as a complex is complete in the sense that it contains every one of its subsets. For a general complex, the ordering is only partial. When the ordering is partial some of the possible subsets of the range of the complex are missing, and also some faces belong to more than one simplex. This is the essence of the ability of the complex to be a framework for description. A science normally describes what exists by organizing a framework of observations that could possibly exist, and then using this framework to describe the states which are actually present or absent. Therefore the general concept of partial order should be discussed.

Ordering relations are particular types of relationships which may be used to impose a sense of precedence between different structures contained in the whole relation by the way in which they are or are not contained in each other. Different orderings can be classified by the reflexive, symmetric, and transitive properties. In this thesis the study will be primarily concerned with the following type of order.

A set A together with a specific ordering relation R defined in A is called an ordered set denoted by (A;R). Types of ordering relations used are designated by the following schemes.

$a < b$ means $a \leq b$ and $a \neq b$: read is strictly dominated by b.

$a \leq b$ means $b \geq a$: read b dominates a.

$b > a$ means $a < b$: read b strictly dominates a.

$a = b$ means $a \leq b$ and $b \leq a$: read a equals b.

$a//b$ means neither $a \leq b$ nor $b \leq a$: read a is not comparable to b.

$\neq$, $\not<$, $\not\leq$ are self explanatory.

These symbols denote the general concept of ordering between mathematical objects, of which the orderings of numbers is a particular example of mathematical objects that have such an ordering.

A particular order which is of concern in this study is the partial order. A partial order may be recognized by the following properties which its relation possesses.

P01 xRx for every x of S. (reflexive)

P02 if xRy and yRx, then necessarily $x = y$. (antisymmetry)

P03 if xRy and yRz, then xRz. (transitivity)

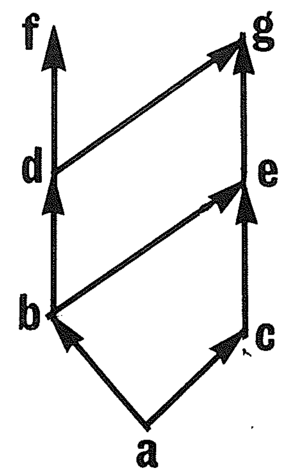
A simply ordered set has the following additional property: if x, y are elements of S, then either xRy or yRx. A simply ordered set is called a chain in the partially ordered set. In general a partial order may contain simply ordered chains, but not every subset is included in a chain since some pairs of elements are incomparable (x//y). In a complete order every pair of elements is comparable to each other, hence the trichotomy property holds. This property states that one of $x < y$, $x = y$, or $x > y$ must hold for every pair of elements.

The relation of strict predecessors (S,<) holds when $x \leq y$ but never $x = y$. This amounts to the assertion that transitivity does not hold. If the relation $x < y$ holds and in addition there exist no elements z such that $x < z < y$, then x is said to be the direct predecessor of y. One says that element y covers x in the chain.

The graph of an immediate predecessor relation is often of interest. It is called the Hasse diagram. The relation between a particular incidence matrix and its Hasse diagram is shown in the following example.

HASSE DIAGRAM AND INCIDENCE MATRIX

	A	B	C	D	E	F	G
A	0	1	1	0	0	0	0
B	0	0	0	1	1	0	0
C	0	0	0	0	1	0	0
D	0	0	0	0	0	1	1
E	0	0	0	0	0	0	1
F	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0



The incidence matrix of the strict predecessor relation can be derived from that of the partial order by noting that it must be an irreflexive and a transitive relation. Thus by eliminating entries along the diagonal of the incidence relation of the partial order, $(S,\leq)$ and by forming the composite of the relation with itself (multiplying the matrix by itself) and eliminating every entry in the incidence matrix which has a corresponding entry in the composite matrix, then one obtains the matrix of the strict predecessor relation.

In terms of the diagram and the incidence matrix given above, the concept of maximal or minimal elements can be introduced.

The maximal element in an ordered set is an element which is not dominated by any other element in the set. In other words if $a \varepsilon A$ is a maximal element, then $a \leq X$ implies that $a = x$. In the diagram above F, G are maximal elements, while A is minimal.

A chain is a particular order relation on subsets of the full set such that every set in the chain is simply ordered. In the diagram above a sequence such as (c,b,d,f) or (a,b,e,g) constitute chains within the partial order.

The above order relation can be extended to the subsets of the complex in the following way:

a. If $\sigma_q \leq \sigma_p$, then $\sigma_q \leq \sigma_p$: Every element belonging to σ_q also belongs to σ_p, but σ_p may contain elements not belonging to σ_q.

b. If $\sigma_q \leq \sigma_p$ and $\sigma_p \leq \sigma_q$, then $\sigma_p = \sigma_q$: Every element that belongs to σ_q also belongs to σ_p, and conversely.

c. If $\sigma_q \leq \sigma_p$ and $\sigma_p \neq \sigma_q$ then $\sigma_q < \sigma_p$: Every element belonging to σ_q also belongs to σ_p, but σ_p necessarily contains some elements not belonging to σ_q.

d. If neither $\sigma_p \leq \sigma_q$ nor $\sigma_q // \sigma_p$: Each simplex contains some elements not contained in the others.

The partial order carries into the complex representing the simplicial complex the following ideas usually contained in a land use plan.

a. The existing simplices represent the constraints imposed by observations describing current development; and therefore the observational ordering reflects the degree of development in different areas.

b. If the maximal elements of the order are standards, they represent possible objectives to be imposed limiting the choice available in each area.

c. The existence of more than one incomparable objective reflects the incompatibility of competing objectives, if the maximal elements are regarded as mutually exclusive.

d. The simply ordered chains in the partial order reflect partial paths of development that are consistent with at least one objective.

e. <u>The Face Operation</u>

It is clear that every simplex defined under the order relation is a simplex of the complex, therefore K(X;R) is closed under the order relation. This will be exploited to define the face operation on simplices of the complex. While the face operations will be mentioned at a later stage of the study, we introduce its definition here for completeness.

The face operator $f\sigma_p = \underset{i}{U}\hat{\sigma}^i_{p-1}, \hat{\sigma}^i_{p-1} = (X_1 X_2 \ldots . \hat{X}_i \ldots . X_{p+1})$ denotes union over all indices simplices with indices i. $\hat{X}_i$ denotes the omission of a particular vertex i to generate the (p - 1) - simplex σ^i_{p-1}.[18]

The co-face operator Δ is the dual operator to the face operator in that, given a p-simplex, it associates the p-simplex with all the (p+1) - simplices of which it is a p-face. It shall be defined later.[19]

The face and co-face operations reflect the action that any plan is concerned with; since a plan is concerned with actions consistent with objectives, these actions must be representable by operators like the above.

---

[18] R. H. Atkin, "An Approach to Structure in Architectural and Urban Design. Part 2: Algebraic Representation and Local Structure." <u>Environment and Planning B 1</u>, (1974), pp. 174-176.

[19] Ibid, pp. 176-179.

f. The Canonical Set of Simplices

The maximal simplices of the complex provide the most elementary notion of discrimination in a complex. Atkin shows that the set of maximal simplices, denoted $\overset{o}{Y}$, provide a canonical set defining every simplex of the complex.[20] They are also distinct in that they satisfy ones normal conception of discrimination of objects; that is, for all $Y_j \in K_Y(X;R)$, $Y_i \in \overset{o}{Y}$ only if $Y_i \, f \, Y_j$. Thus the elements of Y are either equal or distinct, and they completely characterize the choice in a simplex (as Atkins proves using the principle of inclusion-exclusion).[21] The maximal Y_i, $Y_j \in \overset{o}{Y}$ elements of the structure are discriminated against the background structure by virtue of this property: either $Y_i = Y_j$ or $Y_i // Y_j$. No other elements of the structure have this property. Thus locating the set Y amounts to the identification of the possible objectives in the complex consistent with current development.

g. Dilworth's Theorem

Having defined the face order as a partial ordering, the various combinatorial theorems proven about partial orderings on sets can be invoked. In particular, some significance can be attached to the subsets of the canonical set $\overset{o}{Y}$ which satisfy Dilworth's theorem.[22] Dilworth's theorem defines the minimal set of elements of $\overset{o}{Y}$ which are independent (incomparable to every other element of the set) and which together contain every element of the whole protocol. Needless to say

---

[20] Ibid, p. 176.

[21] Ibid. p. 176.

[22] Marshall Hall, Jr., Combinatorial Theory, (New York: Blaisdell Publishing Company, 1967), pp. 14-15 and 63-65.

Dilworth's theorem defines a basis for the structure as an independent set of simplices capable of describing every possible choice in the complex as the union of elements from the basis. The different goals are a configuration of objectives determined by a combinatorical procedure consistent with Dilworth's theorem.

4. The Interpretation of the Simplicial Complex

a. Introduction

The interpretation of urban structures using the simplex to describe particular perspectives of a neighborhood from different points of view has already been discussed. The ability of the simplicial complex to describe aspects of the whole neighborhood that are of interest to the planner will now be related. Using the minimal discrimination capability of the face ordering of the complex, the central ideas of a land use plan can be identified in relation to urban structures. While some of the discussions of this section are rather vague, they are intended to indicate the potential of the deeper applications of the theory.

b. Functions of the Complex: Representation of a Land Use Plan

(1) Framework for Description

The simplicial complex provides a means of representing clustering among activities contained in an urban protocol assuming different signals between the activities which are resolved over a backcloth set. An example of a protocol is given in Table 4 and also in Mathematical

Structure in Human Affairs.[23] The choices found in the complex for a given signal relation are correlated with definite urban areas. The idea of a cluster of activities being the result of signalling between activities leads to the idea that changes in choice flow through the structure of the complex, and this requires a means of discussing the dynamics of that flow. Evidence that some of these clusters arise from the particular compatibility or competition of urban activities makes the idea of an urban signal more comprehensible. As a framework, the protocol defines every logical possible combination of choices which can be resolved against the background and correlated to urban land by the signals. The signal relation is a subset of the power set P(S) of the protocol.[24] The missing sets must be explained as the action of the standards of behavior on the choice or as the action of the signal relation. The missing sets imply that the relative ordering of the choices will be partial. The choices assigned to urban land lead to a relative partial ordering of the sites reflecting their importance under the signal relation via their functional descriptions.

(2) The Relation of the Idea of a Land Use Plan to the Simplicial Complex

The ordering and combinatorial properties of a simplicial complex are sufficient to represent the idea of a land use plan which is to be imposed upon urban areas, although more complex structures are required to elaborate this idea into useful presentation of information. This

---

[23] R. H. Atkin, Mathematical Structure in Human Affairs, (London: Heineman Educational Books, 1975), pp. 118-120.

[24] The concept of a power set as a set and together with all of its subsets describes a complete relation.

shall be demonstrated by discussing the planning concepts that can be associated via Delphi techniques with the structures already introduced.

Simplices can describe urban perspectives in two distinct ways: a description of the choice actually available from a given viewpoint; and a description of the preferences that impose some limiting constraint (maximal or minimal) upon the choice that should be available at the viewpoint.

The partial order of the existing structures will reflect the current state of development, and provide a sense of precedence for areas relative to each other. The partial order of the standards imposed on each area reflect the extent to which the areas may develop. Hence, the simply ordered chains of a partial order represent possible paths of development. The current state of development has as many distinct paths leading to it as there are simply ordered chains in its families of subsets, while there are as many chains leading from it to some standard serving as an objective for the area. Unlike the abstract number system there is no unique successor or predecessor relation specifying a path of development as a simply ordered chain from a present state of development. From this one can see that there is considerable uncertainty in the course of development reflecting the real uncertainty a planner faces. Nevertheless, the simplicial complex provides the following elements of a plan:

a. Representation of the current state of development, and possibly specification of preferences as constraints;

b. Two operations, ie. the face and co-face operators, for determining preceding and succeeding development in an area.

c. The maximal elements representing the idea of an objective for the area.

d. A combinatorial means of determining a goal as a minimal set of objectives leading to solution of problems in an area.

The courses of development represent the different sequences of action that reach a goal, and the action elements of the plan are those changes in available choice determined by the operations on simplices of the complex. A goal reflects the orderly development of an urban area. Development may either be orderly in time or orderly in extension. If it is orderly in extension, then at every signal level examined it will be found that each area is consistent with at least one standard of behavior determined for that signal level. If it is orderly in time, then it will be found by comparison to standards of behavior applicable to each time interval that every area is consistent with at least one standard of behavior. Orderly development covered by the n-simplicial complexes for the n signal levels of the signal relation must be ordered into chains from level to level.

The standards of behavior are themselves orderable and their simply ordered chains can be made to depict the notions of convenience and compatibility that are imposed on the influences of an area as it extends its horizon through the neighborhood or through time. The order reveals the sense or the function of the simplexes in the neighborhood as a whole, and if one can systematically regulate the actual contents of the simplices the incompatiblities at any level may be detected and eliminated, as in the zoning problem to be developed in the following chapter.

Clearly, the significant use of the order is to match actual states of development to standards for future development eliminating non-conforming areas; and conversely, identifying for more careful study those standards which are rarely found to exist in urban area. The imposition of standards on an area creates a tendency to develop only lines consistent with the standard. Relative to any standard a definite idea of being more developed or less developed is defined. The concept of a force having both an intensity and a direction indicates any distortion of ideal patterns of development and should permit us to define costs explicitly.

(3) Operations, Forces and Orderly Development

Consistent with the idea that plans are concerned with changes to the development of urban areas, the simplicial complex contains the face and co-face operations to represent the possible changes. Obviously the changes in choice flowing through the structure must represent changes in the way that areas signal each other. In this section, mention is made of some of the conclusions drawn by Atkin during his more detailed study of the subject.

The co-face operation Δ acts on a simplex, say σ_p and associates with it all of the σ_{p+1}-simplices of which it is a face. It is convenient to consider the particular simplices of a complex as representing directions which may carry a numerical indicator. Thus $5\sigma_p^i$ might mean that the distinct choice σ_p^i is selected 5 times in the structure. This defines a numerical pattern of information that Atkin discusses thoroughly.[25]

---

[25]Note 15 refers to the paper which describes the algebra of patterns of choice that can be used to compute choice in a structure.

That the algebra permits choice to be computed by assigning numerical indicators to the elements of the algebra is important when the regulation of an existing structure is being considered. The co-face operation Δ symbolizes possible development choices. How these choices are constrained so as to remain orderly is of great interest to the planner.

The co-face operation involves the idea of a flow constraint on the choice that is determined by the whole structure on which it is defined. Thus given,

$(\bigcup_i \sigma_t^i, \pi^t) = \Sigma_i(\sigma_t^i, \pi^t)$

Thus $(f\sigma_{t+1}, \pi^t) = (\sigma_{t+1}, f^{-1}\pi^t)$

since by definition $f\sigma_{t+1} = \cup\sigma_t^i$

$(\sigma_{t+1}, f^{-1}\pi^t) = (\sigma_{t+1}, \Delta\pi^t)$

since $f^{-1}\pi^t = \Delta\pi^t$ by definition

$(\sigma_{t+1}, \Delta\pi^t) = \Sigma_i(\sigma_t^i, \pi^t)$

(σ_t^i, π^t) denotes the inner product between simplices at level t and the pattern at t.[26] f^{-1} denotes $_{t+1}$ which contain the $_t$ as a face. The notate π^t denotes the value assigned to p-dimensional simplices.

The constraint on choice inherent in the definition of $\Delta\pi^t$ is clear in that it can only be defined if σ_{t+1} exists with σ_t^i as a face. The values flow up through the structure as well and are contained by the flow constraints.[27]

The significance of these changes to patterns is discussed in some detail by Atkin. He notes that changes in patterns which are consistent

---

[26]See R. H. Atkin, Mathematical Structure in Human Affairs, p. 133. He discusses the flow constraints in the complex and the analogy to acceleration and velocity concepts.

[27]R. H. Atkin, Mathematical Structure in Human Affairs, p. 133. He discusses the flow constraints in the complex and the analogy to acceleration and velocity concepts.

with the structure are free to occur; ie. if these changes are consistent with a standard of behavior, the "forces" involved in a free change are virtual. Those changes not consistent with the structure are determined by other factors experienced as a forceful change by the residents. This is analogous to the relation between velocity and acceleration in physics.[28]

The idea of force as changes in a pattern of choice which are either free or abstracted is defined by Atkin.[29] These forces are not important in the current study, but their presence in the theory of simplicial complexes may help us to analyze the costs of irrationality in planning even as the concept of force measures the cost of effort in physics. The similarity of the formal properties of the face and co-face operations to the calculus of finite differences may point to a happy marriage between the theories of the two as a calculus for development in an urban area.

This possibility is inherent in the very idea of orderly development which, by imposing a set of standards on the choice in the area, creates a closed, ordered structure defining operators for development in the same way as they are defined in any algebra. The rule is that a particular pattern of choice plus a change to the choice must be consistent with at least one standard of behavior active in the area. In principle, one could define an addition table for every pair of elements describing

---

[28]Ibid, p.133, for further discussion of the flow of choice on a complex.

[29]Earl Glen Whitehead, Jr., Combinatorial Algorithms, (New York: Courant Institute, 1973), pp. 53-55.

choice that depicts whether a given development is allowed or not. The order, by establishing some patterns of choice as maximal, defines a tendency of development that is somewhat analogous to entropy in physics. Therefore, there is at least the possibility of a geometric/algebraic representation of significant ideas of urban development in the simplicial representation of urban structure.

c. The Representation of an Urban Area: An Example

Although the data presented in the beginning of this chapter describes a simplicial complex existing in an actual urban area, for the purposes of illustration a simpler example is provided in this section. Suppose the following complex was defined in some area. The possibility of a graphical representation of any relation is discussed in Appendix A while Atkin discusses the simplicial representation theorem that states that any complex dim K=N can be represented in an Euclidean space of at most 2N+1 dimensions.

The urban structure can be represented both algebraically and geometrically. Suppose the following complex was found to occur in some study area.

TABLE 6

INCIDENCE MATRIX OF A COMPLEX

Decision areas (streets) = (L1, L2, L3, L4, L5, L6)

Activities = (Drugstore, Florist, Bakery, Delicatessen, Grocery Store, Restaurant, Laundromat, Hobby Store)

	Drugs	Bakery	Florist	Deli	Grocery	Restaurant	Laundry	Hobby Store
L1	1	1	1	1	0	0	0	1
L2	0	0	1	1	1	0	0	0
L3	0	0	0	0	1	0	0	0
L4	0	0	0	0	0	1	1	0
L5	0	0	1	0	0	0	1	0
L6	0	0	0	1	0	1	0	0

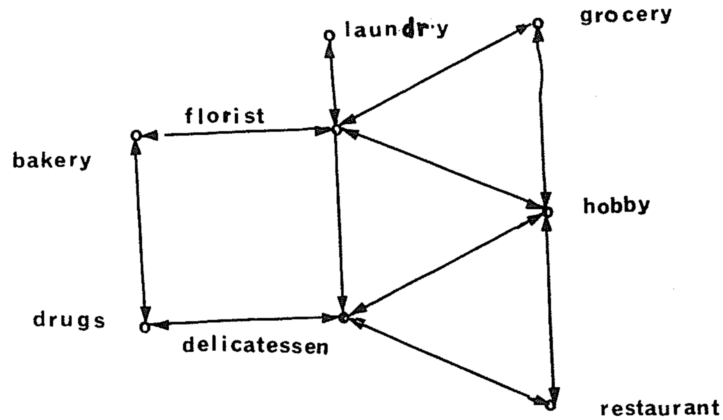
L1 is a σ_3, being defined as the pattern of choice (drugstore, bakery, florist, delicatessen). In terms of the relation to the concept of service areas which will be defined shortly, this complex defined to be the 0-level pattern of choice; that is every activity in the choice set may be selected directly by virtue of its presence in the area. The manner in which the incidence pattern of activities can be used to describe complex urban land use structures should be clear from this example.

The relation contained in the above complex can be represented graphically as in Figure 5.

FIGURE 5

GEOMETRIC REPRESENTATION OF A COMPLEX

$K_L(A;R)$



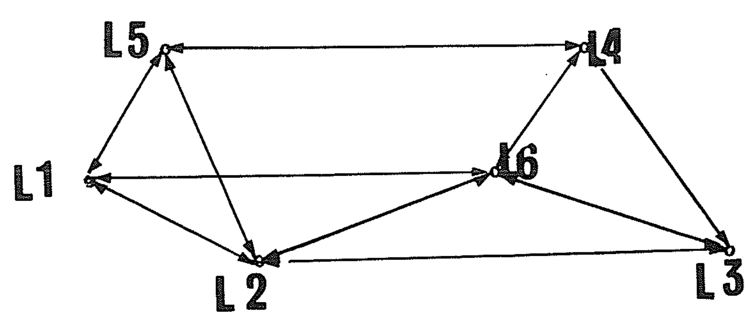
It should be noted that points are joined whenever the relation establishes that they are present in some area together. By taking the transpose of the above matrix, the inverse relation defining the streets which are related to each activity can be obtained, and this may also be given a graphical representation.

FIGURE 6

GEOMETRIC REPRESENTATION OF A CONJUGATE COMPLEX

$K_A(L;R^{-1})$

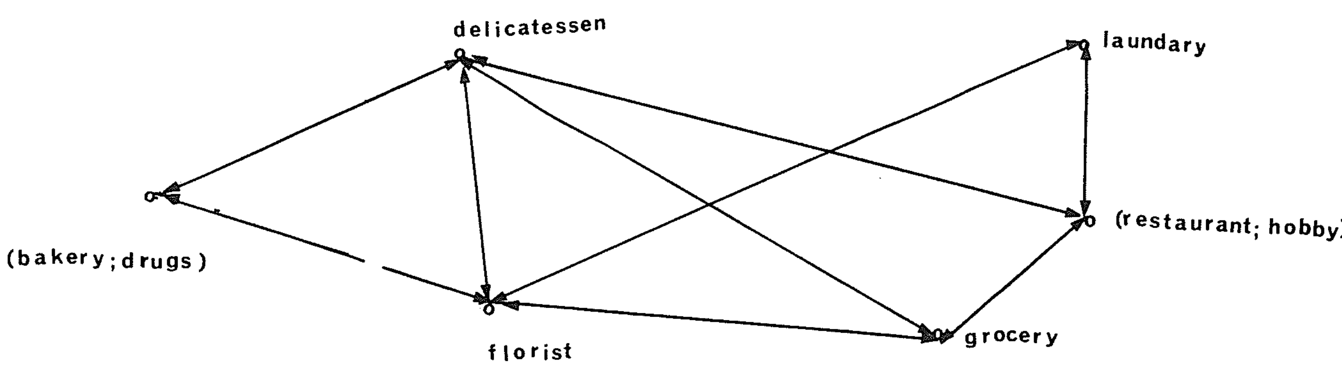
	L1	L2	L3	L4	L5	L6
Drugs	0	0	1	1	0	1
Bakery	0	0	0	1	1	0
Florist	0	0	0	1	0	1
Deli	0	1	1	0	0	0
Grocery	1	1	0	0	0	1
Restarant	1	1	0	0	1	0
Laundry	1	0	0	0	0	0
Hobby Store	1	0	0	0	0	0



The usefulness of the graphical relation is to permit the reader to visualize what is meant by compressing a functional space of relations into a physical space. Assume that every activity is initially described by a line graph wherein activities are joined only when those activities may occur together. This introduces the notion of compatibility relation between distinct types of urban activities. Since the compatibility relation permits some distinct sets to occur simultaneously, the initial linear graph can be compressed into a multiple graph defined on particular physical decision areas. The following example illustrates the compression possible in matching the functional space (Figure 6) to a physical space of decision areas.

FIGURE 7

COLLAPSE OF THE FUNCTIONAL RELATION



III. Definition of Significant Urban Structures in the Simplicial Complex

1. The Ordering of Sites

The partial ordering of urban structures is easily defined in terms of ideas like the following; the significance of which is obvious.

a. Consider the ordering of sites. The accessibility relation between sites has the form R c LXL where $L = (L_i)$ $i = 1, n$ are the sites of the neighborhood. In terms of this relation, for any pair of sites L_i, $L_j \varepsilon L$;

$L_i \leq L_j$ implies that every site L_k to which L_i provides access is also accessible from L_j, but L_j may provide access to some site not accessible from L_i. We say that L_i is dominated by L_j.

$L_i \leq L_j$ and $L_j \leq L_i$ implies $L_i = L_j$. The sites are equivalent in that each provides access to the same sites accessible from the other.

$L_i \leq L_j$ and $L_i \neq L_j$; L_i provides access to the same sites as are accessible from L_j, but L_j necessarily provides access to some not serviced by L_i. We say L_i is strictly dominated by L_j and denote it $L_i < L_j$.

$L_i // L_j$. Neither $L_i \leq L_j$ nor $L_j \leq L_i$. Each site services at least one site not serviced by the other. They are incomparable.

The canonical set L^0 describing the maximal elements discriminated in terms of the accessibility relation describes those sites which are the unique sites providing distinct service areas. In a very real sense, when one is looking for the best site for some activity in an area, it would be best to select among the sites in L^0 only.

But the combinatorial properties of the partial order can be used to solve an even more significant problem. This problem is, given a canonical set L^0, select and identify the minimum number of sites that must be serviced at some specified level of service to completely cover an area so that every street is within a certain distance of at least one source of service. The ordering of streets plays a fundamental role in the determination of solutions to this siting problem.

In addition to the ordering of sites among themselves for any specified service level, there is also a partial ordering of sites from accessibility level to accessibility level which provides a view of the orderliness of the service areas for each street as their horizons extend through the area. Naturally enough, this property is dependent on the backcloth relation of the streets reflecting their relative dominance under different conditions. It is through this ordering that the effects of signal levels on the propagation of phenomena through a neighborhood is perceived. It can be stated quite generally that the analysis of a given signal relation requires a knowledge, not only of the partial ordering of the simplices obtained by slicing any signal relation at some level, but also of the ordering of these signal levels relative to each other, to provide a view of their orderly development. It should be kept clearly in mind that the signal levels could equally well represent the widening of horizons through the background network of connections in the backcloth, or the changes to the area which occur over a period of time.

2. Introduction to the Siting Problem

The siting problem is a simple combinatorial covering problem. Accessibility is the necessary condition for the siting of urban activities on urban land. Although accessibility is not the only criteria upon which site selection is made, it may be argued that every suitable set of sites must first satisfy some accessibility requirements and then be evaluated by the application of further criteria to the set of sites which have the necessary accessibility. In this fashion accessibility criteria may be used to reduce the list of sites from which a selection is to be made for siting different types of urban activities. This, in itself, is of considerable advantage to the planner when evaluating a very large number of potential sites.

This problem is an example of sub-optimization in that the optimal sites for a single activity or group of activities ignores the constraints among the totality of activities. However, later discussion of the zoning problem will show that it is possible to determine the permissible configurations of activities in each area. If it should be found that none of the sites possible in the zoning problem correspond to these acceptable configurations, then an example of the cost of a conflict between individual and global rationality would have been found.

3. Application of the Problem

The siting problem is used to examine the service areas of different sites, ie. those sites which have strategic accessibility properties in order to service a study area. Using the set of areas which are distinct (maximal), the different possible minimal covering configurations of sites necessary to uniformly service the area is found. This may be for each accessibility level.

This simple application of the siting problem does not take into account any constraints upon the extent of overlap between the service areas of the sites; and hence ignores the possible impact of mutual competition. There is no requirement when examining the service areas to consider those which have no application to servicing the given activity. Therefore the geographic backcloth can be sliced to eliminate any areas which do not contain activities of interest.

The latter feature would be of significance in studying areas which serve as a suitable site in a market study. The demographic characteristics of each street could be noted, and if any street does not contain a significant market, it may be eliminated from further configurations. In this way, site configurations may be chosen to reflect the actual distribution of a market or the geographic backcloth.

An indication of the convenience of any accessibility structure for servicing a population may be obtained by using the siting problem. By generating the pattern from level to level one can determine at what level the structure starts to assume the characteristic of a uniform complex; ie. it takes on the structure of a simplex. The level is a significant structural property of the whole area determining the minimum level of accessibility to unify the whole structure; ie. if the activity distribution fails to place significant activities in the strategic sites, then more accessibility levels must be used to generate a uniform level of service. The level of uniform service can be interpreted as the accessibility level where every area is serviced by at least one instance of each specified choice.

The siting problem is of interest whenever the planner wishes to select sites so as to minimize investment in facilities in an area while retaining a uniform level of service to specified areas of the neighborhood, and also co-locating as many activities as possible on the main sites.

Final selection from among a large number of site configurations which cover the area may be based upon factors such as land cost, minimization of overlapping service areas, availability of sites, or compatibility between several different activities. Applications of this problem are the following:

a. For a particular map of streets upon which it is desired to site different neighborhood activities, the sites which are not distinct and dominant, may tentatively be assigned a residential function. (Since it is desirable that residences be serviced.)

b. If one desired to site playgrounds or bus-stops in which some definite maximum accessibility criteria is identified, one might select one of the suitable site configurations at the proper signal level.

c. Consolidate activities which may have some necessary interrelationship by selecting combinations of site configurations suitable to each activity and satisfying the mutual accessibility of activities.

d. Reduced competition between the service areas of given sites can be assured by selecting a configuration of sites with the smallest overlap in their service areas.

4. Significance of the Problem

The significance of the problem to the city planner is that the approach reduces the number of alternatives which must be examined by the planner in selecting sites having the necessary accessibility and neighborhood coverage.

The technique forces the examination of all possible alternatives prior to site selection. The planner is assured that each possible configuration is feasible by covering the entire area to some specified standard.

In an intuitive analysis there is no guarantee that all possible feasible configurations are considered in the analysis prior to the reduction of the choice set. In practice this means that a technique which is based upon experience cannot be guaranteed to bring into play all possible alternatives; and very likely alternatives with which the planner is not acquainted with by virtue of previous experience will not be considered. The potential of the mathematical approach to reveal alternatives implicit in the definition of pre-conditions of the problem, but not previously known, is a major asset. This systematic approach to the siting problem uses the geometry of a given area to objectively define possible solutions. Such objective solutions may be later surveyed when the consequences of the decision made on the basis of the analysis are available for comparison. In this way the design cycle is closed and feedback from actual experience may be used to improve the solution techniques. Usually conventional techniques based upon the experience of the analyst are not so easily amenable to inspection and correction by feedback data.

Currently a number of techniques exist for siting activities. Such techniques include the use of population maps to estimate the market population that might be tapped and permit the selection of a site somewhat near the center of mass of the population map. Usually potential sites determined in this way are then examined for their practical accessibility to the assumed market area. Other techniques depend upon site visits and an evaluation based upon the characteristics of the site and its neighborhood as assessed by the experience of the siting analyst.

In any of these techniques, there is no guarantee that the sites selected will together provide a minimum standard of service to an area, or that all possible sites have been examined. There is, on the other hand, no reason at all why these intuitive conventional techniques cannot be applied to the candidates which are determined from the mathematical analysis to further reduce the potential range of choice.

Conventional market surveys can not provide this type of information to facilitate planning. Hence, the siting problem is a novel approach to a type of problem having significance in urban planning.

5. Solution of the Siting Problem

A computer program using a backtrack routine to test every different combination of areas can determine the minimal combinations of simplices whose union is the whole range of the relation. Such an approach is combinatorial.

The back track routine is a system which systematically generates all possible combinations from a given set, testing each configuration to ensure it does not violate a stated condition, and outputting all configurations that completely satisfy the condition of pairwise incomparability. The covering problem is solved by testing for the smallest subsets of maximal areas which are pairwise incomparable.

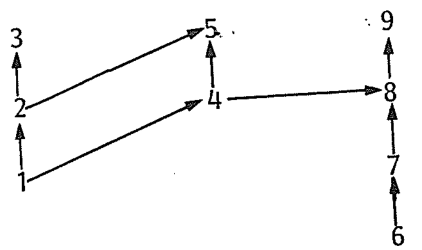
If $L_i//L_j$ and $L_i//L_k$, then for pairwise incomparability it is required that L_j be chosen such that $L_j//L_k$. This may be done by choosing any pair $L_i//L_j$ initially, forming their union, and eliminating from consideration any L_k of the maximal subsets which do not satisfy $L_k//(L_i \cup L_j)$. Choose successively the candidates from the smaller range of sets until no choice remains. At this point the sets selected in this way must be pairwise incomparable, although not necessarily the largest pairwise incomparable set.

The combinatorial approach may be the most efficient means of solving the siting problem. In fact, the example configurations in the demonstration following were derived by inspection because of the very significant reduction of initial candidates that occurred using only the maximal set L^0.

However, it is of interest to discuss the linear programming formulation of this problem because of the possible usefulness of linear programming for the solution of the more complex general siting program. This example illustrates how a partial ordering can be represented in a format suitable for the simplex algorithm used in linear programming which has some significance for the regulation of urban structure that involves linear programming.

FIGURE 8

A PARTIAL ORDER



3,5,9 are obviously maximal; 1, 6 are minimal.

A simplex tableau is defined as follows:

FIGURE 9

SIMPLEX TABLEAU FOR DILWORTH'S PROBLEM

top \ bottom	0	1	2	3	4	5	6	7	8	9
0	6	0	0	1	0	1	0	0	0	1
1	1	0	1	0	1	0	0	0	0	0
2	0	0	0	1	0	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	0	1	0	0	1	0
5	0	0	0	0	0	0	0	0	0	0
6	1	0	0	0	0	0	0	1	0	0
7	o	0	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	0

Row 0, contains a 1 in each column which represents the top of a chain. Column 0 contains a 1 in each row representing the bottom of a chain. The entry in cell (9, 0) makes the row 0 sum = column o sum = 9 (the number of elements in the order.)

Other elements are assigned such that

cell (i, j) = 1 if $i \leq j$ in the order and i is the immediate predecessor of j in some chain where $i \neq 0$ and $j \neq 0$

= o otherwise

cell (i, 0) = 1 if i is the top of a chain

= 0 otherwise

cell (0, j) = 1 if j is the bottom of a chain

= 0 otherwise.

For the definition of the objective function on this order proceed as follows:

define variable X,i,j for each position in the tableau

$$\max f = \sum_{i=0}^{9} \sum_{j=0}^{9} (C\ ij)\ (X\ ij)$$

subject to $Xij \geq 0$ for all i,j, (0,1,2,3,4,5,6,7,8,9)

$$\sum_{j=0}^{9} X0j = \sum_{j=0}^{9} Xj0 = 9$$

$$\sum_{j=0}^{9} X\alpha j = \sum_{j=0}^{9} Xj\alpha = 1 \text{ for } \alpha \varepsilon (0,1,2,3,4,5,6,7,8,9)$$

C00 = 1

C0j = 0

j ε {1, 9}

Cjo = 0

Cij = (0 if i j

(-N otherwise where N is a very large integer.

When this tableau is solved it will find the tops of disjoint chains which together are minimally sufficient to cover the whole area. The conditions of the problem make it necessary that a chain be regarded as a particular nested sequence of areas included in each distinct top element. Obviously the solution is not necessarily unique, but the minimal set of areas distinguished in this way will provide a particular solution to the problem.

6. Analysis of the Geographic Backcloth

Using the adjacency relation defined in Table 3 the signal structure from accessibility level 1 to level 7 was computed by successive multiplication of the incidence matrix for adjacency, and noting at what level pairs of areas first became connected. The resulting simplicial complex was then sliced from 1 to 7 to yield seven simplicial complexes describing the service areas of each possible site in the backcloth. Various means may be used to determine the partial ordering of the sites among themselves in the different complexes, but in the case of this study the Q-analysis compute program discussed in Chapter Three, Section 3.c and 3.d was used to establish the ordering.

The purpose of the analysis of the backcloth is to illustrate how the structures defined in this way clearly express the intuitive concept of a site servicing different areas, and how the ordering of the complex can be used to determine both the "best" sites and also the minimal number of such sites that must be used to completely service (cover) an area for some specified level of service.

Furthermore, the analysis also demonstrates that a site which is dominant at one level may or may not remain dominant at a higher level of accessibility; but no site which was not dominant at a lower level can be dominant at a higher level. The set of maximal areas at higher levels is always equal or less than those at lower levels; thus illustrating the existence of a partial ordering between the service areas from signal level to signal level.

Because the size of the simplicial complexes precludes their presentation, this analysis foreshadows the use of Q-analysis by employing the Q-structure vectors for the different complexes that were derived from the Q-analysis. These structure vectors reflect the extent to which the simplicial complex is fragmented, requiring more than one maximal simplex to cover it. It obviously reflects the degree of uniformity of the influences present in all of the areas in very compact form. Therefore Table 7 shows the global structure vectors at every level of accessibility from 1 to 7. It should be noted that in this small structure, the extension of influences as they interpenetrate very rapidly unifies the structure by level = 5. There are very few differences of influence in the complex. Hence, if everyone is willing to move up to 5 blocks for each choice, at least one instance of almost all activities can be found starting from almost every site. Although this may strike the reader as a very crude indication of service, it is remarkable in itself that analysis of mathematical structure can generate such indications. Much better reflections of the choice available to residents can be defined.

Table 8 shows the maximal sites in terms of the areas they service. Since the sets of dominant sites are relatively small, at least around levels three, four and five, the different minimal configurations of sites to service the areas were determined by inspection by testing the unions of maximal areas for completeness of coverage.

Table 9 gives an indication of the relative ordering of the maximal sites among themselves from level to level. This shows that a site dominant at one level can fail to be dominant at the next, but no non-dominant site at one level can become dominant at the next.

TABLE 7

STRUCTURE VECTORS

Accessibility Level (o)	Dimension (dim K)	STRUCTURE VECTOR
1	11	$(\overset{11}{1},1,2,2,5,9,1,7,3,3,1,9,4,3,\overset{0}{1})$
2	26	$(\overset{26}{1},1,1,2,3,4,4,3,6,6,6,7,6,7,7,6,7,$ $5,2,3,\overset{6}{1},..\overset{0}{1})$
3	36	$(\overset{36}{1}, \overset{35}{2}, ...\overset{30}{2}, \overset{25}{5}, 4,3,3,2,2,1,2,\overset{17}{1},$ $...\overset{0}{1})$
4	45	$(\overset{45}{1},1,1,1,2,3,\overset{39}{1}, ...\overset{0}{1})$
5	46	$(\overset{46}{1},1,1,1,2,2,\overset{40}{1}, ...\overset{0}{1})$
6	47	$(\overset{47}{1},1,1,1,1,2,\overset{41}{1}, ...\overset{0}{1})$

Analysis: Since only 47 areas were actually assigned on the complex, the maximum dimension possible is 47. The 0-level was not included since it would simply constitute a completely disconnected complex. Even though the component analysis is not included the reader can appreciate the effect of higher level adjacency in extending the service areas and rapidly welding an initially disconnected series of streets into a connected whole. Even at 0=4, the distinction between different maximal service areas is very small, smaller still at 0=5, or 0=6. The backcloth organization is a resource of an urban area to be exploited by proper management.

TABLE 8: MAXIMAL SIMPLICES OF THE BACKCLOTH

Accessibility Level (0)	Maximal Simplices (Dominant Sites)
1	L35,L32,L27,L31,L25,L20,L43,L19,L21 L22,L18,L17,L12,L13,L14,L15,L16,L17 L5,L6,L8,L9,L11,L45,L46,L36,L33 L23,L24,L7,L9,L10,L3,L44,L47,L48 L49,L39,L42,L40,L41,L30,L34,L26
2	L26,L27,L35,L25,L17,L16,L14,L15,L30 L34,L13,L31,L22,L39,L10,L11,L12,L29 L45,L23,L47,L48,L46,L42,L33,L49,L40 L41
3	L25,L26,L27,L29,L42,L16,L39,L40,L11 L45,L48,L47,L10,L33,L46,L41
4	L42,L25,L27,L45
5	L42,L45
6	L42,L45

Analysis: For example, one pairwise independent set at accessibility 0=3 was found to be (L26,L39,L45,L48). A good first choice for a site would be L45, followed by L42. This would lead to another following solution at accessibility 0=3, (L45,L42,L39,L48) as another possible solution at 0=3, part of which remained dominant at higher levels. The continued dominance of sites at higher levels is a desirable characteristic of a preferable solution.

For an accessibility level of four, the following solution was obtained by inspection: (L42,L45). This is the only solution at higher levels of accessibility to provide complete coverage. In essence, the relative sparsity of solutions is due to the influence of the arbitrary extension of the study area several blocks down Pembina Highway and down Osborn. Thus solutions are very sensitive to the geometry of the backcloth.

TABLE 9

PARTIAL REPRESENTATION OF THE INTER LEVEL BACKCLOTH ORDERING OF MAXIMAL ELEMENTS

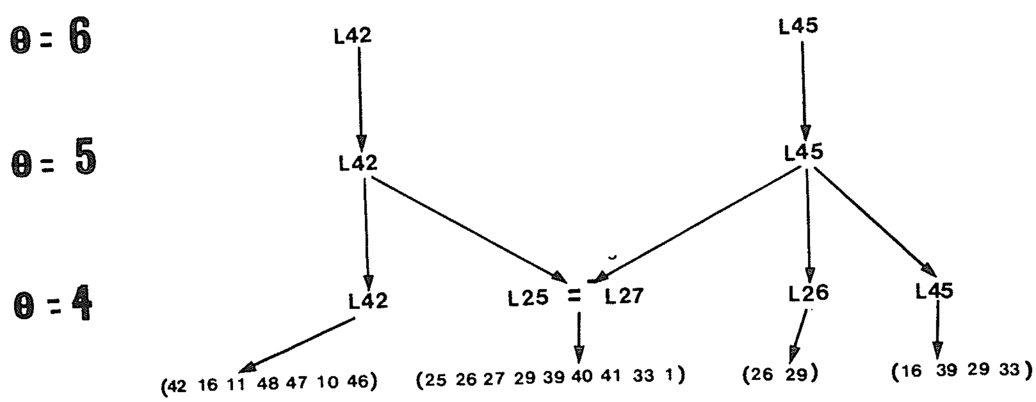


Table 9 tabulates some of the partial orderings between accessibility levels to show the ordering which exists between the maximal sites from level to level. Since any site at a specified level is transformed to the next level by forming its union with adjacent sites, more areas cannot be added to a non-dominant site than a dominant one. Therefore non-dominant sites cannot become dominant. However, it is possible that a previously dominant site will be included in another dominant site at the next level.

The nested chain of dominant sites extending from the lowest to the highest accessibility level defines a zoning structure. To rule out incompatibilities anywhere in the structure, it is important to know how the extension of a standard of behavior is ordered over the whole structure. Ideas like compatibility and convenience become very clear in this way.

The ordering can provide a decision procedure for selecting sites. Knowing that (L42,L45) remain dominant, one can select other patterns at lower desired levels of accessibility. Thus one knows that (L42,L45) which are sites at the lower level may attract additional loads or remain competitive at the higher levels because of their relative advantage over the other sites. For a commercial outlet L42, L45 should certainly be the first sites occupied using purely geometrical considerations.

7. Other Aspects of Urban Structure Using the Simplicial Complex

In addition to the purely geographic advantages of the backcloth functional aspects such as different levels of service in different areas can be captured quite simply on the complex. The following discussion defines these aspects as examples to clarify and stimulate further thought. Atkin provides some further examples in reference 30.

If one conceives of shopping complexes in general terms, then a relationship would be described between a largest and/or smallest set of activities having particular degrees of accessibility. The equivalence of modern shopping centers to natural forms in various shopping districts

---

[30] R. H. Atkin, V. Mancini, J. Johnson, "An Application of Algebraic Topology to Urban Structure," Urban Studies, (1971), pp. 221-242.

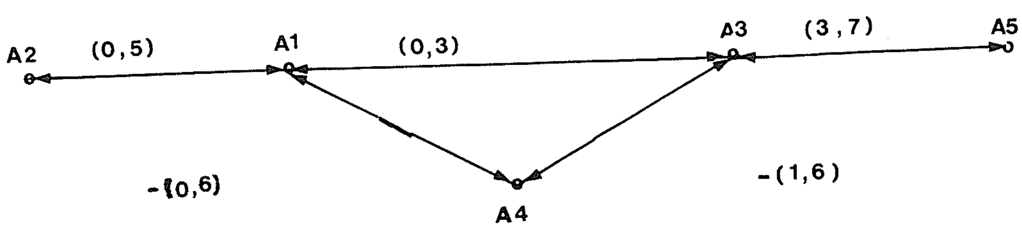
may be studied in this way. Since the difference between the maximal and minimal standards may provide a degree of freedom in the mutual influence of activities, shopping complexes can occur in a wide range of equivalent configurations. Hence one can study the relative advantages of naturally occurring shopping districts which may take many forms in a neighborhood.

The reason why this comparison is so general is that a preference should identify the least and greatest tolerances on the mutual accessibility parameters. Graphically, it is something like the following:

FIGURE 10

A FUNCTIONAL RELATION BETWEEN ACTIVITIES

Activity Set (A_1, A_2, A_3, A_4, A_5)



In such a graph the focus is on mutual relationships between pairs of activities. The absence of a bar would imply that the activities are indifferent. Designation (0,6) indicates that for compatibility the activities could be co-located, but should not be more than 6 units apart. Designation -(0,6) indicates incompatibility unless the activity is less than 0 (impossible) or greater than 6. Few would argue that such information is not available, but most would question how it is to be applied to structural analysis. The discussion of planning in the next chapter will examine methods to analyze preferences of this form.

CHAPTER THREE

ANALYSIS OF LAND USE STRUCTURE

I. Functional Explanation

1. Introduction

The study has now established that the simplicial complex is a suitable means of describing the functions of an urban neighborhood in terms of the elements of the observational protocol. This form of description facilitates the explanation of the behavior, using mathematical analysis. Functional descriptions organize the input data to a study under the control of the planner: the observational protocol determines the scope of the study to have a range of possible observations which is as complete as possible (thus preventing any surprises), while limiting the scope of data to that which has significant impact on neighborhood functions.

Planning and measurement are accomplished by using techniques obtained from the literature of Q-analysis, the standard mathematical literature, and the literature of computer science.[1] Together these techniques extract the maximum amount of useable information from the total information put into the study by the Delphi techniques. The usual objection that mathematical analysis cannot reveal any information not

[1] R. H. Atkin, Combinatorial Connectivities in Social Systems, (Basel: Birkhauser-Verlag, 1977). This book provides a summary of much of Atkin's work in Q-analysis.

originally present in the definition of the problem is valid; however, more organized, hence more useable information, is produced by the structural analysis. Although the study does not dwell upon the epistemological aspects of the functional explanation in the form presented, the interested reader can verify their acceptability by referring to the essay by Sir Bertrand Russel.[2]

Functional explanation is to be understood as a means of using the internal consistency inherent in the ordered structures to discriminate only the most significant states.[3] By use of the concept of covers for observation the complexity of describing the function of a system is greatly reduced; with luck the reduction achieved in this way will make an otherwise impractical combinatorial problem manageable; it is easier to grasp and manipulate cover sets (interpreted as standards of behavior) than the raw sets described by preplanning. Using the internal consistency of different observational states, cover sets that are similar to each other in some respect are aggregated. The consequence of aggregation is uncertainty when controlling or predicting the outcome of different circumstances. Predictions can be made only to the extent that observations are consistent with some standard covering them. Since an urban planner rarely controls all the variable factors in any urban neighborhood, some uncertainty is to be expected in any measurement process.

---

[2]Sir Bertrand Russel, "The Relation of Sense Data to Physics," contained in A Free Man's Worship.

[3]Gerald M. Weinberg, An Introduction to General System's Thinking, (New York: Wiley, 1975), p. 140. In this book Weinberg discusses the manner in which the metaphors of science bring the complexity of natural systems within our cognitive abilities.

Measurement is developed as a technique to test the consistency of the observed structures with the various standards of behavior. From this concept of measurement, the costs of irrationality may be assessed. Irrationality is held to exist whenever one standard is imposed upon an area to the exclusion of another. To properly assess the costs, Atkin's concept of force in an urban structure is required. Forceful changes to urban structure can be described in terms of the algebra of patterns in a complex.[4]

Structural analysis has the virtue of being a practical methodology that employs many standard algorithms from computer science. The way in which these algorithms generate new forms of structure is analagous to the way in which an instrument is capable of resolving the finer structures of an observational phenomenon. The idea of relative rationality frees us from the problem of attempting to find numerical or algebraic preference systems which completely order phenomena by substituting the generation of cover families which are as consistent and complete as possible. These distinct domains of preference will be pieced together into new wholes that are adapted to the particular circumstances of an urban neighborhood.

In the course of this chapter a state space concept will be introduced using the converse (to each other) notions of convenience and compatibility to bracket different observational phenomena into categories that provide a finely discriminating system of measurement for assessing the trade-off of different individual preference systems.

---

[4]R. H. Atkin, "Cohomology of Observations," essay contained in Quantum Theory and Beyond. Pp. 207-208 contain a discussion of the rationale for inserting an algebraic structure into the observations which are defined upon some observational backcloth.

Following a brief presentation of the theory of explanation in terms of mathematical structure planning and measurement concepts are presented under the following topics:

a. Discrimination by the face ordering of the complex.

b. Discrimination by the Q-analysis.

c. Illustration of the techniques and concepts using a state structure in an urban neighborhood.

d. The Clique Analysis.

e. The Zoning Problem (Application of the State Structure to Urban Zoning).

2. Families of Cover Sets

Planning and measurement together determine functional explanations of land use phenomena in an urban neighborhood. Atkin argues that the scale property which determine the membership of a set of observations represents the consensus of observers upon what is being observed.[5] Signals in the set of observations decompose the protocol into different families of subsets describing the experience of observers under different circumstances. The signal is represented by a family of cover sets which is consistent with the protocol.[6] The structural description is intrinsically hierarchial in nature so that the significance of any set is intuitively appreciated at a higher or lower hierarchial level.[7]

---

[5]Ibid, p. 193.

[6]Ibid, pp. 205-206.

[7]R. H. Atkin, Mathematical Structure in Human Affairs. (London: Heineman Educational Books, 1975) p. 118.

In the reference[8] Atkin shows how reasonable assumptions pertaining to the description of observations by a set lead to a cover of the observations that naturally define a simplicial complex. The following diagram shows how the protocol is normally resolved into a structure of observations on a backcloth using the simplicial complex. For the purposes of this chapter only certain conclusions from Atkin's discussion are of immediate interest.

FIGURE 11

THE STRUCTURE OF AN OBSERVATION

	$K_y(X_i R,n)$	Protocol of scales $S = (X_1, X_2 \ldots . X_n)$
resolution		
Backcloth set	$S_1 \to S_1=(Y_1, \ldots . Y_m)$	Incidence matrix of the signal relation (weighted by integars n)

A full barrier distinguishing N signal relation on a backcloth of M individuals in general requires MXN such structures.

It is clear that the protocol describes a complete framework for the description of observations in the given situation. The family of sets that is the power set of the protocol provides a cover in which every possible element of the family is also a possible mode of behavior. The structure of observations distinguished by the signal relation is a sub-family or sub-cover of the protocol. Since one is interested in

[8]R. H. Atkin, "Cohomology in Physics," pp. 207-208.

describing behavior in the simplest possible fashion, the maximal complete cover families determined by the signal relation are favored for the description of norms. They represent the largest sets of simplices that violate none of the restrictions of the signal. Every observed state is a sub-cover of at least one of them. (The greatest common denominator of the joint experience.)

Formally a covering is defined when X is a set at the N level whose distribution under a covering family of sets (N+1 level) is to be explained, then supposing A is an N+1 level set corresponding to X, the elements of A (subsets) are subsets of X and if $A = (A_1, A_2. A_3, A_n)$, $X = (X_1, X_2, \ldots. X_m)$ then

(i) $A_i c\ P(X)$ for i=1, ...n

(ii) $X = \underset{i}{U} A_i$

If in addition $A_i U A_j = \emptyset$ (the empty set), then A is a partition of X. The elements of A are quite distinct.

Each element of a cover family is interpreted as a particular standard of behavior. The essence of a standard of behavior is to classify and regulate: by classifying it identifies experiences which conform with some characteristic similarity; by regulating it rules out aspects of behavior that do not conform. A standard of behavior must be maximal. The emphasis of this chapter upon complete maximal cover families facilitates description of the most general consistent mode of behavior to ensure that the status of observed behavior can be unequivocally established. The use of the concept of standard implies some uncertainty in the prediction of actual outcomes under any given circumstances since any subset of the standard is acceptable as a mode of behavior. This manipulation of cover sets may make the combinatorial approach to planning feasible in a practical sense.

The simplification introduced by this approach must take account of two separate circumstances in the simplicial complex. The use of consistency relations such as the face order of the complex to aggregate the observational states into standards of behavior is the most direct means of reducing the degree of discrimination necessary in an observational scheme. Moreover, the backcloth used to resolve observations relates observation to singleton sets which have no intrinsic ordering. One must take account of the backcloth ordering must be taken into account to describe the different circumstances for observation and generate the different patterns of influence in the observational structure. The use of signal levels defined in terms of weighted relations on the backcloth, both physical and functional, will permit a matching of preference to the physcial circumstances. Thus, this matching may eventually lead to quantification of urban phenomena.

There is a particular theory of explanation that must be recognized for the use of structural analysis in urban planning. Theories are mapped onto standards of behavior to transform them into a more suitable form for direct comparison with structures representing direct observations. Using these mappings it is possible to understand how theory is used to classify behavior and how theory is to be confirmed by actual observation.

To provide empirical confirmation for a given theory a maximal cover family provides a bijection between the set of all possible signal relations and the set of all maximal cover families. We generate a maximal cover family from a theory relation since it is more convenient to compare sets to sets, but the standard contains the same information in a more useful form. The cover family shows the largest sets having some degree of internal consistency, a common denominator to their experience; and, hence, magnifies their significant differences. Now if one were to take the actual relation describing real behavior and generate the cover of it in the same way and if the covers were identical, the theory would be confirmed completely. Since one does not normally experience a relation as a whole, the complete cover family is not usually observed during actual observation; thus a cover family may contain a range of possible observations.

The considerations of this chapter deal with two problems:

a. How to generate complete cover families by transforming individual relations into covering standards that reflect significant kinds of similarity between urban experiences; and

b. How to generate and compare sets representing the influences of urban activities so that they are of the same form as the standards assumed.

Figure 12 shows the process that must be followed to achieve a functional explanation of existing urban structure. Figure 13 shows the form of relationship assumed between theory relations, the complete cover family, and actual structures being explained. This shows that explanation is intrinsically a comparison between levels of structure, reinforcing Atkin's assertion as to where the significance of a structure should be appreciated. (ie. The role of a structure or social group is appreciated at the next level of structure.)

FIGURE 12: RELATION BETWEEN COVER AND EXPLANATION

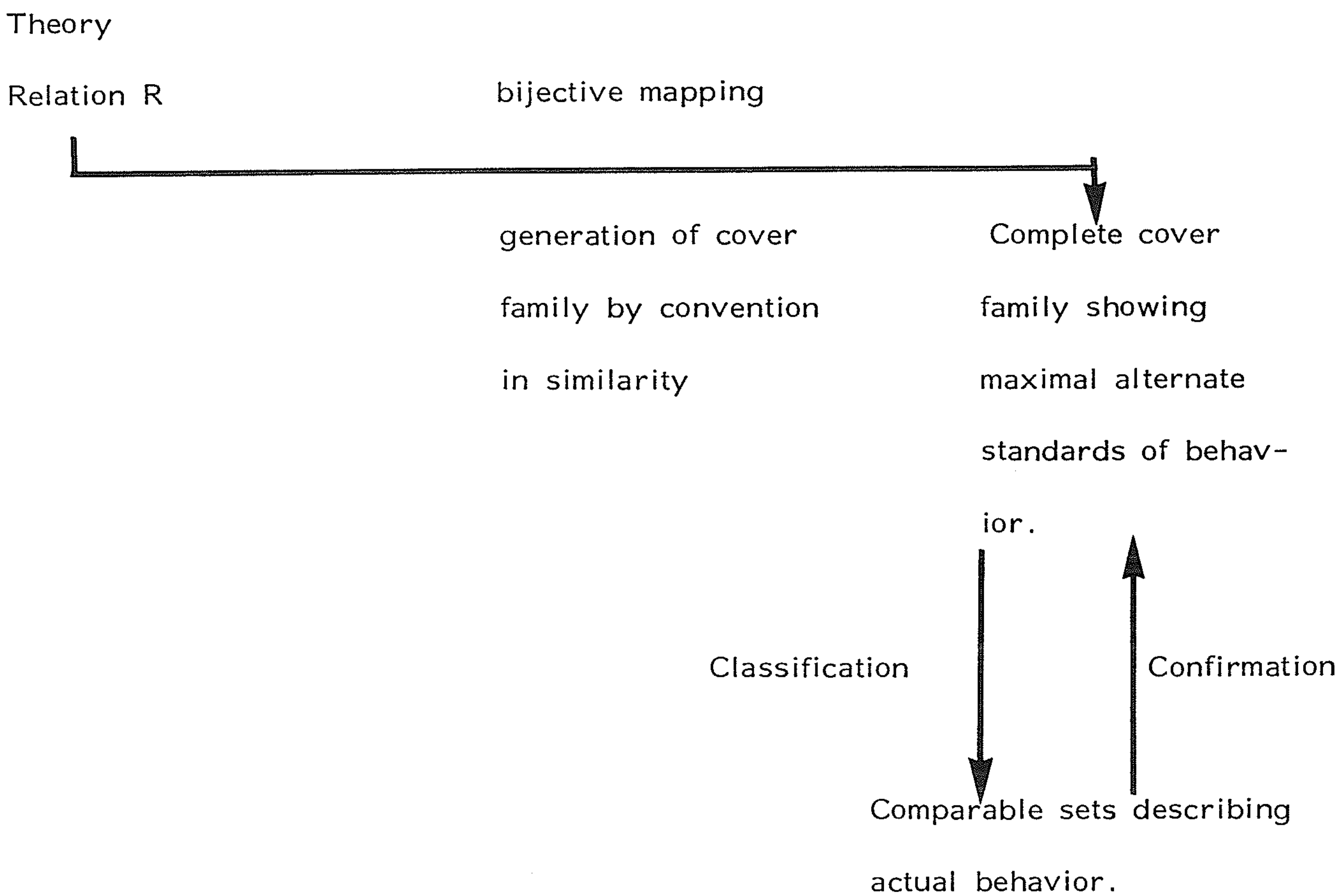


FIGURE 13

THE BASIS OF EXPLANATION IN STRUCTURAL ANALYSIS

Preplanning:

1. Urban Protocol → subset defining interest
2. System of individual relationships for each signal property of the urban protocol determined by Delphi → scale - hierarchy

It is a function of a particular problem to signal

Define backcloth
Generate signal complexes ordered by levels

The Problem

Uncertain structure of the network ordering signal properties and the backcloth social network

Planning →

Theory Defining Standards
For each distinct relation a possible hypothesis is constructed using some complete family of sets bijectively related to the relation. The different possible chains of properties can be compared by determining the fine structure of the cover sets relative to each other in some order.

Standards of Behavior
Uncertainty enters in this step

For each distinct pattern of cover sets consider it as a system of classification containing distinct standards of behavior.

Prediction with uncertainty of observation

Measurement to confirm standard of behavior

Actual Structure of observation

3. Discrimination

Discrimination denotes the existence of a significant difference between any pair of sets making them distinguishable when being compared; this difference is usually asserted in the form either structure A = structure B (they cannot be discriminated), or Structure A $\neq$ structure B (they can be discriminated), but $\neq$ or = may be replaced by any ordering relation. In fact both $(S, \leq)$ and $(S,=)$ will be used to discriminate different levels of structure with respect to both the simplex and the clique. There are two significantly different processes that lead to discrimination of the observational protocol with comparisons performed via the backcloth ordering.

The structure relative to which a comparison is made is called the modulus; ie. structure 1 is compared (modulo) structure 2. As a simple example of the structure of an observation (Figure 10), resolution of an observational protocol provides a comparison of a complete framework of possibly observable states (S) to a structure describing the circumstances of observation (viewpoints S^1). For any arbitrary signal relation the protocol may be resolved into an associated family of cover sets. This cover family has between its elements a partial order that permits discrimination to occur. Since the elements of S^1 are viewed as singletons, they impose no constraint on the discrimination operation. Magnification is a process which reveals the similarities and differences of the different viewpoints S^1 by constructing an equivalence relation from the partial ordering by discrimination. The Q-analysis is therefore one possible magnification process on the simplicial complex.[9]

---

[9]Ibid, p. 209.

When all the observable states on a scale S (ie. every subset of the class of observations) are manifest as singletons on a scale S^1, we shall say that S^1 is a resolution of S.[10] Resolution describes the conceptual limits to observation. For example, a simplicial complex in which every face of the complex is covered by a distinct element of the backcloth would be a resolution of the scale.

The significant levels of discrimination in structural analysis are the compatibility classes and the equivalence classes. The compatibility classes organize the cover sets describing observation into a partial order of distinct categories or modes of behavior, while the equivalence classes completely partition the elements of the structure. Since the overlapping sets characterize the structure of a partial order, it is natural to characterize equivalence classes as focused structures (analagous to physical magnification). Types of structure are distinguished by demanding different degrees of internal consistency (uniformity) among the elements.

In the methodology of Q-analysis, one has an effective instrument for extracting a significant amount of information from the Delphi processes. This information permits an examination of the transition from local to global forms of structure. The objective of the structural analysis is to systematically analyze the degree of uniformity on the underlying structure leading to the concept of a social standard of behavior.

The clique as a mathematical structure embodies the idea of a social standard of behavior in two respects.

---

[10] Ibid, p. 209.

a. Every constituent of the clique is compatible with every other constituent in the clique.

b. As a cover set, the clique implies complete uniformity of choice wherever it effectively dominates behavior.

Thus, the clique is the limit of resolution in an observational protocol relative to some signal relation. Whereas the protocol is a set describing every logically possible mode of behavior, the clique shows the largest complete sets of elements which are simultaneously observable. The methodology of Q-analysis systematically approaches this level of uniformity. Figure 14 shows the levels of discrimination contained in Atkin's methodology.

With reference to the literature of Q-analysis, Atkin enters into an extensive discussion of the concepts of local structure.[11] The social meaning of concepts such as q-adjacent chains, the shomotopy structure of the q-components, and the concept of q-hole is unclear unless understood that the local structure reveals the degree of uniformity between chains of q-connection that is imposed by social standards. The application of the concept of clique to urban zoning will be discussed in this chapter where it will provide a great deal of insight into the conflicts between standards of behavior.

---

[11] R. H. Atkin, Combinatorial Connectivities in Social Systems, pp. 64-68.

FIGURE 14

STRUCTURES FOR DISCRIMINATION OF SIGNAL RELATIONS

	Mathematical Technique	Structure Discriminated	Analogy	Urban Structure
1.	Generation of compatibility classes	Set of all simplices $Y_i \leq Y_j$ where $Y_j \in \hat{Y}$ The simplices of Y are representative structures magnified	Rough Focus	Influence neighborhoods service area of a site distinguish sites
2.	Generation of equivalence classes (Q-analysis)	q-components for levels $0 \leq q \leq \dim K$	Focus	Functionally distinct aspects of the urban neighborhood ie. shopping district single family residential
3.	adjacency between q-connected chains (pseudo-homotopy)	Compatibility covers of q-adjacent chains of the relation	Altering magnification to (new rough focus)	Uniformity structure of the distribution of choice in functional areas
4.	Q-shomotopy analysis of the q-loops	Mediation between standards of behavior in terms of their similarity	Approach to the limit of resolution (new focus)	Resolution of land use conflict
5.	Clique analysis	Clique cover family of the relation	New level of magnified structure	Urban zones or one-stop shopping districts

II. Planning and Measurement Using Global Analysis

1. Introduction

a. Objectives

In this section the magnification of urban structure using the face order and the Q-analysis of the simplicial complex will be considered. An urban structure is defined by the pattern of choice available to neighborhood residents by virtue of their location in the neighborhood and a particular signal relation. The simplex correlates a particular pattern of choice with a particular area of the neighborhood; thereby specifying the variety of choice available to a resident.

b. Computational Algorithms for Structural Analysis

One of the characteristics which Atkin demanded of any structural analysis suitable for city planning or architecture was that it be capable of implementation with a computer to simplify the processing of data concerning large and complex relationships that the designer must deal with.[12] In this section the study presents a brief resume of the algorithms necessary to implement, at least partially, the mathematical theory which has been hitherto discussed.

To write a basic Q-analysis program such as is contained in Annex C does not require great programming skill, although a suitable programming package to conveniently perform the necessary analysis under the control

---

[12] R. H. Atkin, "An Introduction to Structure in Architectural and Urban Design. 1. Introduction and Mathematical Theory." Environment and Planning B 1 (1974), p. 53.

of the planner in an interactive mode would obviously be much more complex.

One Fortran program was written to carry out the necessary analysis; but a second more efficient Fortran program was obtained from Pennsylvania State University. More effective algorithms than those pertaining to Annex C have been found, but these were not used in the present study. In most cases the reader can obtain the necessary information directly from the reference literature to prepare his own package. Annex C contains a complete Fortran program for the global analysis of a simplicial complex, but improvements to this program are discussed in the various sections.

2. Compatibility Classes of the Simplicial Complex

a. Interpretation of the Compatibility Class

The face ordering of the simplicial complex is one of its most obvious features. The use of this face ordering for the resolution of the preferable sites in the geographic backcloth has already been discussed. (Chapter Two) The face order has a potent capability to represent the significant functional characteristics of any site in an urban neighborhood.

The face order is a means of discriminating the significant elements of the cover by determining the elements of the canonical set $\mathring{Y}$. By definition the elements of $\mathring{Y}$ are defined by discrimination since a particular simplex Y_i of a complex $K_Y(S;R)$ belongs to $\mathring{Y}$ only if $Y_i \not\leq Y_j$, for all j in the complex.

Obviously the compatibility classes of $K_Y(X;R)$ are defined $C_j = (Y_i | Y_i \leq Y_j \text{ and } Y_j \varepsilon \mathring{Y})$.[13]

The face ordering of the cover family could be used to expose observations as subtle as the possible confusion between individuals and the roles which they occupy in a social organization, or as the conflict between authority and responsibility in the control structure of a group. Instances of confusion as to which functional categories govern specific areas are described by the face ordering of the different areas in a neighborhood. The authority-responsibility conflict of a social group will be put into an urban context by discussing the conflict between convenience and compatibility in an urban neighborhood (the phase space analysis of urban structure).

The compatibility class consists of all the greatest sets of individuals which are compatible with at least one other individual in the set. A compatibility relation is simply a relation which is reflexive and symmetric, but not necessarily transitive. An example of a social group having such classes will contain a number of languages. Individual A must be able to communicate with himself (thus the relation is reflexive); individual A must be able to communicate with any other with whom he shares a common language (thus symmetry). Communication occurs between pairs of individuals, but it is possible for two individuals to communicate with a third by an intermediary. These are chains of communication. The chains of communication are revealed by the different chains of nested sets in the partial order. The cover family determined by the compatibility classes is complete, but only partially ordered.

---

[13]R. H. Atkin, Combinatorial Connectivities in Social Systems, pp. 39-40.

b. Deriving the Shared-Face Matrix

The representation of a simplicial complex using the incidence matrix of the defining relation was fully discussed in the last chapter. Thus a simplicial complex describing the influence of a signal relation R on an observational protocol $X=(X_j)$, $j=1, m$ that is resolved relative to a backcloth of viewpoints $Y=(Y_i)$, $i=1, n$ is represented algebraically by a matrix $R=(r_{ij})$ where $r_{ij}=1$ if $Y_i R X_j$; $=0$ otherwise.

The shared face matrix S represents the intersection of the relation R with its conjugate R^{-1}. Annex B defines the intersection of the relations. In the incidence matrix, $S=RR^T$, where RR^T denotes the matrix multiplication of the incidence matrix R and its transpose R^T, and $S=(S_{ij})$, S_{ij} denotes the cardinality of the common face of Y_i and Y_j. For completeness the matrix of q-connectivities is $Q=S-1$ (1 is the identity matrix). The elements q_{ij} of Q denote the level of q-connection between the simplices of the complex. An example is discussed in section II.3.c.

c. Determination of the Order Relation of the Complex

Discrimination of the structure based upon the order relations in the complex can be quickly accomplished using the information contained in the matrix of q-connectivities. For each order relation in the complex, define a matrix of the same size as the matrix of q-connectivities. For each pair of simplices enter a "one" in the corresponding cell if the pair satisfies the given order relation and a "zero" otherwise. For a visual survey of ordered structures it is preferable that these relations are expressed in structure form (for each simplex list the set of other simplices that are in the given order relation).

Perform the following tests upon each pair of simplices in the complex to determine if the designated ordering relation is satisfied.

Let Q_{ij} denote the entry in the q-connectivities matrix corresponding to L_i and L_j.

If $Q_{ii} = Q_{jj} = Q_{ij}$, then $L_i = L_j$

If $Q_{ii} = Q_{ij} \leq Q_{jj}$, then $L_i \leq L_j$

If $Q_{ii} = Q_{ij} < Q_{jj}$, then $L_i < L_j$

If $Q_{jj} = Q_{ij} \leq Q_{ii}$, then $L_j \leq L_i$

If $Q_{jj} = Q_{ij} < Q_{ii}$, then $L_j < L_i$

If neither $Q_{ii} = Q_{jj}$ nor $Q_{jj} = Q_{ij}$ then $L_i // L_j$.

The ordering described in structure form is denoted as follows:

$L_i(=)$ = the set of simplices which are all equal to L_i

$L_i(\leq)$ = the set of simplices for which L_i is in the relation $\leq$

$L_i(<)$ = the set of simplices for which L_i is in the relation $<$

$L_i(//)$ = the set of simplices with which L_i is incomparable.

One is also interested in the simplices which are direct predecessors of each other. This relation is irreflexive and atransitive. Using the matrix derived in the first step of the order analysis for the relation $\leq$, first remove all diagonal elements from the matrix, multiply the resulting matrix by itself, and then compare the product to the original matrix on which the product was taken. If a "one" occurs in the product matrix and also in the original matrix, then remove this element from the matrix of $(S,\leq)$. The resulting matrix will be the a transitive and irreflexive incidence matrix of the direct predecessor relation.

To find the canonical set $\overset{\circ}{Y}$, if $K_Y(X;R)$ define the incidence relation between the Y_i such that $Y_i R Y_j$ for all Y_j such that $Y_i \leq Y_j$. The incidence matrix $O(\leq) = (O_{ij})$ is defined as follows:

$$O_{ij} = 1 \text{ if } Y_i \leq Y_j$$

$$= 0 \text{ otherwise.}$$

The maximal elements belonging to $\overset{\circ}{Y}$ can be recognized as those whose row entries are null in the order matrix O (expect for the diagonal and possibly any equal simplices).

d. The Phase Space Structure of an Urban Neighborhood

In physical thermodynamics, one of the most significant observations is the existence of states in a phase space which are inaccessible to the process from various starting points. Different physical processes are found to be equivalent to each other in the phase space representation. By employing the phase space representation of urban structure, different standards can be compared to each other and to representations of actual structure, and a very fine classification of modes of behavior in an urban neighborhood can be prepared.

Planners introduce the idea of orderly development in an urban area by decreeing that there exists some maximal degree of development which is tolerable to the residents of the area. This is done by imposing particular standards limiting the urban structure in an area to enforce gaps in the choice available to residents. The least one can expect when such maximal structures are imposed on a structure is that every urban area is consistent with at least one possible standard, if not more.

This idea of orderly development is quite analogous to the idea of an orderly transition between physical states in a physical process.

One can define an order upon the patterns of choice at each accessibility level using complexes such as $K_L(A,R^n)$. This ordering describes the variety of choice offered to the residents by the activities sited in specific areas. Dominant patterns that remain dominant from accessibility level to accessibility level indicate areas that are accessible to the greatest variety of choice as a consequence of the siting patterns of urban activity.

The ordering is readily defined at each accessibility level of a backcloth order relation, as it is for the signal levels of any preference relation. Each simplex $L_i(n)$ belonging to the set of areas in the backcloth denotes the maximum set of activities, each of which could be individually chosen by traversing some path of length $K \leq n$. One must carefully observe the convention that the simplex can guarantee choice of no more than one activity in any single act of choice along any specific path from the area L_i to the area on which the desired activity is sited. Only by considering cliques can the simultaneous relations of choice (cliques) be detected. However, the simplex relation does not rule out possible multiple choices.

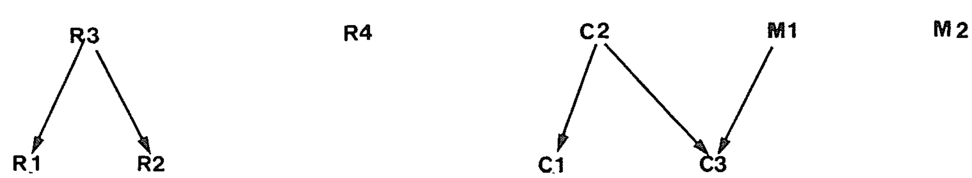
A standard imposed upon an urban area introduces gaps in the available choice. This will be illustrated using the assignment of activities to land use zones for the Fort Osborne area. These assignments were derived by direct application of the zoning criteria used by the Winnipeg Planning Department. It is hardly surprising that the zones should be partially ordered. The partial order is a direct consequence of the exclusion of some possible choices by particular zoning criteria. The designations have the usual meaning. (R denotes a class of residential land use, C commercial uses, and M is light industrial.)

The analysis to determine Figure 13 was performed during the computer analysis of the study area to be discussed in the following section.

The partial ordering into zones constitutes a set of mutually exclusive functional standards of behavior which cover the distribution of activities on urban land areas. The ultimate aim of the analysis is to reduce conflicts caused by the joint occurance of incomparable standards on overlapping areas of urban land.

The partial order in Figure 13 also illustrates the distinct chains of development that are consistent with the standards. An area which is currently zoned R1 under this order can develop into R3 and contain no nonconforming land uses. Similarly C3 can become either C2 or M1. The assignment of R1, R2, C1, or C3 zones to an area means there is an option for further development in the future. This option leaves an uncertainty about the ultimate idea of development that bears a strong relation to the concept of strategic development.

FIGURE 15: ORDERING OF ZONES IN THE STUDY AREA



An interesting application of the concept of partial order as a phase space is to use it for comparison of sites or standards to each other. The phase space structure arises from the intersection of different order relations describing the partial ordering of areas relative to each other, as represented by matrices of the type $O = (O_{ij})$ previously defined.

Consider a protocol which categorizes the individual activities according to some scheme. This scheme might be as simple as to take a protocol describing all of the influences in an urban neighborhood (ie. presence of playground within 2 blocks, etc.) and divide them into positive, negative, and indifferent categories. Those which are indifferent are ignored. Separately order the areas (S) within each category according to the following scheme.

a. Use the partial order described by $(S,\geq)$ for the "good influences."

b. Use the partial order described by $(S,\leq)$ for the "bad influences."

c. Set up an incidence matrix describing $O = (O_{ij})$ for each area, those other areas in the order relation.

d. Intersect the relations according to the scheme discussed in Annex B.

The information can be analyzed in two forms, one of which is similar to the q-analysis technique that will shortly be discussed. In the first case one can consider the elements of the protocol in the intersection of each pair of simplices. Particular significance is attached to those areas in the intersections for which i=j. The strucure in the intersection

L_i+L_i- tells one that area i has more positive aspects (is greater than) and less negative aspects (is less than) relative to the areas contained in its intersection set. However, the trade-off inherent in the other pairs are also significant, since they describe how, for example L_i+ compares to L_j-. Taking the relation itself to describe a new order, "is better than," treat the whole new order as a simplicial complex and obtain a comparison among the trade-offs that are present in the neighborhood. The result is a relation that gives very detailed information concerning the relative merits of each neighborhood in the area.

The use of the phase space for the classification of observed modes of behavior results in the imposition of a kind of "spectral analysis" in the urban neighborhood with the concept of signal level serving as the "energy level."

In urban planning particular significance is attached to changes of the structure either in time or as the service neighborhood of a site extends through the whole structure. These changes are orderly only if the structure at one level is contained in (or contains) the structure at the next higher (lower) level.

Thus the static classification of different simplices discussed above can be put in a dynamic context by examining the transition between different accessibility levels. The ordering tendencies providing a basis for the dynamics of the urban neighborhood are the ideas of compatibility and convenience.

Intuitively, compatibility implies the exclusion of some activities at some accessibility level as being incompatible influences on other activities in the set. However, compatibility is obviously sensitive to the levels of the signal relations such that as level n increases, the compatible sets will become larger. A compatible chain of development is one in which a simplex $L_i(n)$ of $K_L(A;R,n)$ is a subset of $L_i(n+1)$; ie. $L_i(n) \leq L_i(n+1)$ for $0 \leq n \leq \max N$ in the signal relation R. Obviously the effect of the compatibility relation is to disperse the elements of the protocol with the degree of dispersion increasing as n increases.

Convenience is a notion of a tendency based on the logical converse of the relation, $\leq$, ie. $\geq$. It tends to make the configurations of activities less dispersed (more compact) as n decreases. Therefore chains of transition of the form $L_i(n) \geq L_i(n+1)$ must be defined. Obviously the central tendency describing convenience is that more elements are contained in the sets of activities at lower levels, than those at higher levels.

The intersection of these two converse ordering relations obtained from individual preference relations can describe the activities at each accessibility level that are both compatible and convenient. One would expect that orderly development would focus on transitions between signal levels which are consistent with standards embodying ideas of compatibility and convenience at each signal level.

Hence, $C_{o(n)}$ describing a family of simplices that are compatibility standards at level n and $C_{v(n)}$ describing a family of convenience standards, compatible development describes a chain of observations $L_i(n) \leq L_i(n+1)$ where both $C_v^j(n) \leq L_i(n) \leq C_o^k(n)$ and $C_v^1(n+1) \leq L_i(n+1) \leq C_o^m(n+1)$, and $C_v^j(n) \geq C_v^1(n+1)$ and $C_o^k(n+1) \leq C_o^m(n+1)$.

Note that since only those $L_i(n) \leq C_v^j(n) \cap C_o^k(n)$ satisfy both convenience and compatibility for level n, one has the possibility of trade-offs inherent in ideas such that $L_i(n) \leq C_v^j(n) \cap C_o^n(n+1)$ indicating that to provide convenience at level n, the degree of compatibility appropriate to level n+1 has been accepted by the residents.

It is clear that these ideas permit a very sophisticated system of measurement to be employed in an urban neighborhood. This measurement can be quantitative as well as qualitative since there is no obstacle to examining a sufficiently large sample of neighborhoods for determination of the occupancy frequencies and transition frequencies of all logically possible states.

Although the phase space was conceived by examination of the measurement and planning ramifications of the partial order of the simplicial complex, it shall soon become clear that its significant application is to the clique structure of the neighborhood.

3. Equivalence Structures of an Urban Neighborhood

a. Introduction

One can understand the significance of Q-analysis with reference to the preceding discussion of the phase space structure. The intersection of two relations can be completely characterized by forming a new simplicial complex containing both the convenience and compatibility relations and then intersecting the resulting complex with itself. This generates a very detailed view of the comparison using the contents of the intersection sets of order relations, or one can focus upon the cardinality of the shared set (face) of two complexes. This latter view ignores the precise contents of shared-faces and considers a q-connection to be defined by equivalence between pairs of simplices having the same cardinality. The resulting structure covers the more detailed comparison of structures of the phase space.

It should be clear that there is no difficulty comparing simplices at this level, whether they represent standards or actual observation. The Q-analysis compares every simplex to every other simplex without regard to the precise contents of the shared face. The grading provides a very precise view of the covers of similarity that arise because, for example, a convenience and a compatibility structure overlap (intersect) without being completely identical.

b. The q-Connectivity and q-Components of Structure

The behavioral context of the Q-analysis has already been described. The basic order relation apparent in the simplicial complex is the face order, a partial order on sub-sets. This partial order is taken as indicative of an underlying compatibility relation as the basis of communication. Communication is possible in terms of a shared area of experience.[14] The common faces of the simplices of activities are apparently a disturbance in the structure causing an influence of some form to propagate throughout the structure. The systematic process generates q-components as the equivalence classes using the graded q-connectivity between the simplices providing a means of exploring patterns of communication based on the cardinality of a shared face.

[14] R. H. Atkin, Mathematical Structure in Human Affairs, pp. 26-33.

If two simplices σ_p^i, σ_s^k, share a common face which contains at least q+1 vertices, the pair of simplices are said to be q-connected to each other. Thus,[15] $\sigma_p^i \, N \sigma_s^k = \sigma_{q+1}^j$ (the shared face of the connection).

Given two simplices σ_p, σ_s in a complex $K_Y(X;R)$ they are said to be joined by a chain of q-connection if there exists a finite sequence of simplices $\sigma_{a_1}, \sigma_{a_2}, \ldots, \sigma_{a_h}$ such that (i) $\sigma_{a_1} \leq \sigma_p$ (ii) $\sigma_{a_h} \leq \sigma_s$ (iii) σ_{a_i} and $\sigma_{a_{i+1}}$ share a common face say σ_{b_i}, $i=1,\ldots,h-1$ and $b_i \geq q+1$ for all $1 \leq i \leq h-1$. The chain of connection is said to be of length h-1.

A q-component is an equivalence class for the relation of being q-connected at some level q of the grading.[16] It is defined to be a maximal chain of q-connection. In other terms it is a set of simplices, each simplex of which, is q-connected to at least one other simplex in the set. The simplices in the q-component constitute a cover set in the complete family of q-components and are in fact a partition since the q-connectivity is an equivalence relation. Associated with each q-component is the representative simplex of the component. Any activity which is present in the range of the relation and belongs to at least one simplex of the component also belongs to the representative simplex of the component. The representative simplex constitutes the union of every simplex in the component.

The representative simplex illustrates a simple standard of behavior in which the cover set prohibits behavior without regulating it. Every

---

[15] R. H. Atkin, Combinatorial Connectivities in Social Systems, p. 17.

[16] Ibid, pp. 18-22.

face of the representative simplex might be found somewhere in the component as the face of some simplex, but not necessarily. This represents the fact that the sequence of simplices is a sequence of successive choices, each of which is to be regarded as an alternative to the others, and which together yield some total choice. However, there are gaps in the possible chains of simplices precluding some choice of sequences entirely, while the total choice made possible by the combination of simplices cannot be realized in a single act of choice.

The representative simplex describes the pattern of choice in the sense that if a certain choice is not possible in the pattern it is not present in the representative of the component, but the fact that it is contained in the representative does not make it certain that it actually exists as a pattern in the component.

The q-component indicates the presence of a certain degree of similarity in the patterns of choice, a kind of clustering in urban structure. Consider the compatibility and convenience standards. Obviously the trade-off between C_o^i and C_v^j is represented in a q-connection. However, the q-connection considers three standards which have the same q-connectivity, but not the same common face to be equivalent; thus glossing over some of the finer details of their relationship. Q-analysis has the capability to rule out similarities of the ordering which do not exist in the structure.

c. The Q-analysis[17]

The Q-analysis is a procedure for systematically determining all the families of q-components of the complex for every level of q-connection from the 0-connected level up to the dimension of the complex.

In this section a manual procedure for performing the Q-analysis is described. In the next section a suitable computer program for larger data sets will be described.

The computation of the matrix of q-connectivities from the shared face matrix of the complex is the first step in the analysis. The shared face matrix X denotes a matrix whose entries in the cells m_{ij}, are integers describing the cardinality of the shared face between simplex i and simplex j. The matrix of q-connectivities is derived from the shared face matrix by subtracting 1 from each element. Thus

$M = (m_{ij})$ the shared face matrix,

$U = (1_{ij})$ the matrix whose only entry in each cell is 1,

$s = (s_{ij})$ the matrix which denotes the q-connection between simplices i and j,

$K = (K_{ij})$ the incidence matrix of a complex,

K^T is the transpose of the incidence matrix K,

$M = K\,K^T$

$S = M - U = K\,K^T - U.$

For the conjugate complex of a given complex the shared face matrix is derived using the relation

$S = K^T K - U.$

---

[17]Ibid, pp. 18-22.

Normal matrix multiplication is denoted in the relations above. Because the effect of the algorithm yields the matrix of q-connections which is a symmetrical relation, the matrix S is also symmetrical and can be represented using only the upper triangular form. The following example illustrates the derivation of the matrix of q-connectivities from a simple complex.

FIGURE 16: EXAMPLES OF AN INCIDENCE MATRIX FOR Q-ANALYSIS

	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}	L_{11}
Z_1	0	0	1	0	1	0	1	0	0	0	1
Z_2	0	0	1	0	1	1	1	1	0	0	0
Z_3	0	0	0	0	0	1	1	1	0	0	0
Z_4	0	1	0	0	1	0	1	1	0	0	0
Z_5	0	0	0	0	0	0	1	0	0	0	0
Z_6	1	0	0	0	0	0	0	0	1	0	1

The q-connection matrix of the above complex is shown in Figure 17.

FIGURE 17: MATRIX OF Q-CONNECTIVITIES

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
Z_1	3	2	0	1	0	0
Z_2	-	4	2	2	0	-1
Z_3	-	-	2	1	0	-1
Z_4	-	-	-	3	0	-1
Z_5	-	-	-	-	0	-1
Z_6	-	-	-	-	-	2

The following procedure may then be applied manually to the matrix of q-connectivities in order to perform the Q-analysis which is the following:

FIGURE 18: Q-ANALYSIS OF A COMPLEX

$q = 4$	(Z_2)	$Q_4 = 1$
$q = 3$	$(Z_1)\ (Z_2)\ (Z_4)$	$Q_3 = 3$
$q = 2$	$(Z_1,\ Z_2,\ Z_3,\ Z_4)\ (Z_6)$	$Q_2 = 2$
$q = 1$	$(Z_1,\ Z_2,\ Z_3,\ Z_4)\ (Z_6)$	$Q_1 = 2$
$q = 0$	$(Z_1\ Z_2\ Z_3\ Z_4\ Z_5\ Z_6)$	$Q_0 = 1$

The following procedure was used to determine the Q-analysis;

1. Select an entry of maximum degree; ie. $m_{12}=4$.
2. Follow up the column and across the row to determine any other entries equal or greater than 4.
3. For each such entry found in step two, the corresponding simplex is equivalent and should be entered in the set for that class.
4. For each such entry check each row and column to determine entries equivalent to these.
5. Repeat until no new entries are found.
6. Select another entry not included in the previous class and repeat the process to define a new set.
7. Repeat until no new entires can be found which are not in some equivalance class.
8. Select an entry of the next lower degree; ie. 3 and repeat until all equivalent sets have been identified for q=3.
9. Continue process until sets for q=0 have been identified.

d. The Q-finder Routine

The Q-finder routine can be visualized in the following manner. Regard the matrix of q-connectivities as a weighted relation. For each level of q-connection from 0 to the dimension of K, the appropriate incidence matrix of the relation may be found by slicing the weighted relation according to the rule,

$$\text{for some q-level } Q,\ M_{ij} = 1 \text{ if } q_{ij} \geq Q$$
$$= 0 \text{ otherwise.}$$

The resulting incidence matrix $M = (m_{ij})$ may be treated as the incidence matrix of a linear graph for all intents and purposes. Therefore any computer algorithm designed to determine the components of an undirected (symmetrical) linear graph can be used to find the q-components for each level of q-connection in the Q-analysis.

In the program presented in Figure 17 the q-finder routine is a sub-routine. The main program calls the component finder each time the q-connectivities matrix is sliced to a new level of q connection.

The steps in the process are the following:

1. Define a list T_K with dimension $\geq$ cardinality Y, the number of simplices in the complex.
2. Define the shared face matrix QFACE(1,J) containing the appropriate entry for each pair of simplices in the complex.
3. Define variable Q designating the level of the Q-connection being analyzed.
4. Define a program flag, either F=0 or F=1.
5. Initialize all components of T_K = 0, F=0, and variable K=1, Q=0, COUNT=1.

6. For each K check for diagonal values (ie. $J \geq K$) of QFACE (K,J) $\geq$ Q to determine the simplices which are to be assigned to the same component as K.

7. Assign the first diagonal element QFACE (K,K) in which QFACE $\geq$ Q to the kth element of T (T_k) by setting T_k=COUNT.

8. Check each entry 1 of QFACE (K,1) in the kth row and assign to T_1, for those 1 which are $\geq$ Q, the value of COUNT according to the following tests

if $T_1 = 0$	then T_1 = COUNT
if T_1 = COUNT	then T_1 = COUNT
if $T_1 \geq$ COUNT	then COUNT = T_1 and set all further T_j = COUNT to T_K and set F=1
if $T_1 \leq$ COUNT	COUNT = T_k Set all T_j = COUNT to T_1 and F=1.

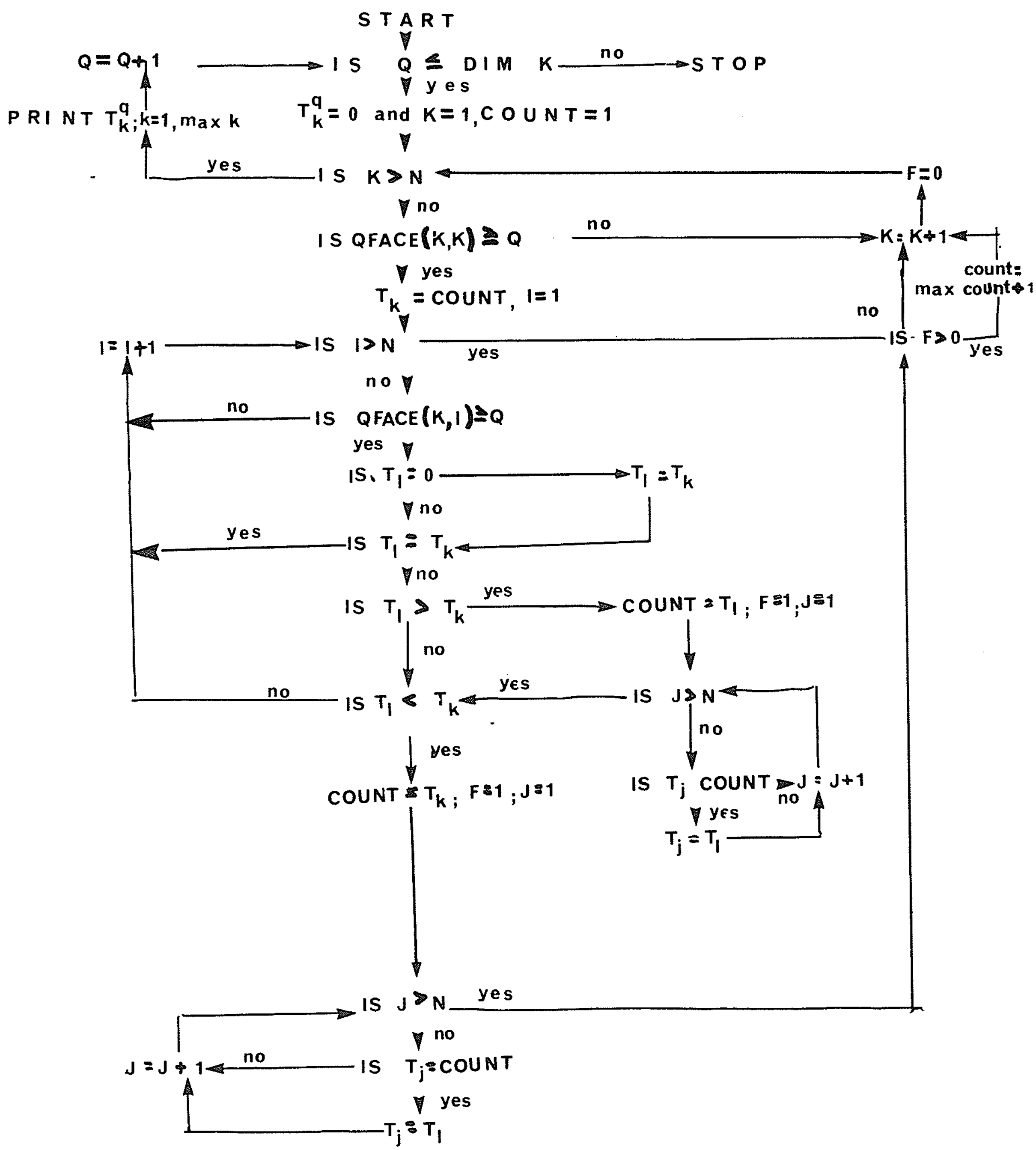
9. Test if F is greater than 0. If no, set COUNT to the maximum value of COUNT which has occurred in the current cycle plus one. If yes, proceed to the next value of K using current value of COUNT.

10. Iterate for all K and then output components.

11. Repeat for each $0 \leq Q \leq \dim K$.

FIGURE 19

FLOW DIAGRAM FOR THE Q-FINDER ROUTINE



Refer to the flow diagram at Figure 19 for the structure of the program.

The standard computer program described here performs adequately, but it is possible to do significantly better. In particular, when further analysis of the complex is desired, the K-tree formalism is an excellent representation of linear graphs which may be used to find both components and cycles of the complex very rapidly once it has been programmed. The K-tree formalism together with the component-finder algorithm and the cycle-finder algorithm that can be applied to the analysis of a complex is described in the reference.[18] It is recommended that this approach be used to write new programs for the Q-analysis in lieu of the program offered at the end of this study.

e. Some Structural Indications in the Q-Analysis

There are three important indicators derived from the Q-analysis initially.[19] These are the eccentricity of the simplices, denoted ecc (σ), the global structure vector denoted Q (the obstruction vector is derived from the structure vector), and the critical q-value, q_c. These indicators are extensively discussed and used in the various sources.

Eccentricity as the name suggests is an indicator of how well connected a particular simplex is to the global structure. For example, we would expect that a high rise apartment building in a single family

---

[18] A. T. Berztiss, Data Structures: Theory and Practice, (New York: Academic Press, 1975), pp. 146-153 and pp. 284-296. These sections describe the K-tree formalism which is an excellent algorithm for some parts of structural analysis.

[19] R. H. Atkin, Mathematical Structure in Human Affairs, pp. 21-22 and 107-116.

residential neighborhood would be rather eccentric. Such a situation would arise in the case where the incidence vector of the high rise apartment building over the streets of the neighborhood was compared to the incidence vector for other common types of housing. It follows that, for the dimension of the simplex describing high-rise incidence on areas, the dimension at which this vector was connected to other simplices would be very small. Atkin has suggested the measure ecc $(\sigma) = (\hat{q} - q)/(q + 1)$ to describe this condition. $\hat{q}$ denotes the dimension of the simplex, while q denotes the maximum value of q for which this simplex is connected to any other simplex in the complex.

q_c is the greatest value of q for which all simplices of the complex merge into one connected component. This is an indication of how well the whole structure is connected together. One can see intuitively that in a shopping example, it is desirable that the various simplices describing commercial activity be as well connected with each other as possible to provide a wide range of choice to the shopper in a small area. This gives us a sense of the compactness of a commercial area.

We have previously defined the structure vector as the number of equivalence classes at each q-level. This structure vector, when redefined as the obstruction vector by subtracting unity from each component of the vector which is less than the dimension of the complex, defines the obstruction vector.

$Q = (Q_{n-1}-1,\ Q_{n-2}-1, \ldots . Q_1-1,\ Q_o-1)$

Atkin has shown how this vector may be taken as an indication of the obstruction to the flow of patterns in the complex. Essentially each distinct component at some q-level indicates freedom within the component to make some choices of activity, but no simplex which is not contained in the component can be chosen freely at that q-level. We shall shortly discuss the concept of patterns upon the complex and the use of the obstruction vector.

In the preceding example of Q-analysis the global structure vector was:

$$Q = \overset{4}{(1}\ 3\ 2\ 2\ \overset{0}{1)}$$

The obstruction vector was $\hat{Q} = \overset{3}{(2}\ 1\ 1\ \overset{0}{0)}$

Eccentricities were as follows:

$\text{ecc}\ (Z_1) = \frac{3-2}{2+1} = \frac{1}{3}$	$\text{ecc}\ (Z_4) = \frac{1}{3}$
$\text{ecc}\ (Z_2) = \frac{2}{3}$	$\text{ecc}\ (Z_5) = \frac{0}{1} = 0$
$\text{ecc}\ (Z_3) = 0$	$\text{ecc}\ (Z_6) = 2$

4. Analysis of a Neighborhood Structure

The following analysis illustrates some of the ideas previously discussed using the study design set up in Chapter Two. Its significance is apparent in that it clearly demonstrates the existence of a partial ordering in the neighborhood structure, the existence of the idea of standard in the zoning imposed on the neighborhood, and the idea of measurement/classification of urban structure. The demonstrations are very crude, but the possibility of extension to more complex forms of structure is obvious.

Table 10 shows the Q-analysis of a joint structure of standards and land use. Measurement requires that the standards, defined by the assignment of activities to zones, be inserted into the simplicial complex $K_L(A;R)$ as dummy streets. From the resulting Q-analysis the degree of consistency of different streets and different zones can be read. The clearest conclusion evident in the Q-analysis is that there is a partial order resulting from the dominance of the standards or areas of land. Moreover, those areas of land that do not conform to any zoning standard can be read immediately from the Q-analysis. These are shown in Table 12.

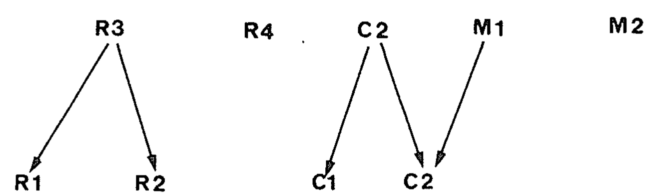
In the current analysis little use is made of the detailed fine structure contained in the equivalence classes. However, it will be noted that at different levels of q-connectivity various land areas are consistent with more than one structural standard. One possible interpretation is that at some earlier stage of development the land area was consistent with more than one standard leaving open a significant choice concerning the future path of development. When an area is developed so as to reduce the number of zones to which it is consistent an opportunity cost is incurred. It also illustrates the sense in which, prior to full development, zones can be said to float over the area. What the structure illustrates clearly is the normal and intuitive concept of development control in city planning. These concepts are significant and more fully discussed in the zoning problem.

The zoning structures in the Q-analysis dominate the resulting complex to a great extent. The zone which is the most tolerant is the C2 classification. However, it is apparent from the structure vector that the overall structure is really rather uniform throughout all Q-levels. Thus, the introduction of the standards does not produce as great a fragmentation of the structure as one would expect if truly discriminating standards were used. This result is generally indicative of the fact that the legal zoning tool is not capable of making the fine discriminations which one would desire of an urban design tool, although it is a step in the right direction.

Table 11 shows the consistency relation of development areas and zones. This information was extracted from the face ordering of the complex. Figure 20 shows the partial ordering of the standards among themselves as a result of the consistency relation of the standards. It is clear that they overlap to some extent.

FIGURE 20

HASSE DIAGRAM OF ZONES



The concept of zoning applied in this section shows that there exist common subsets to different zones. These common subsets are the basis of the possibility of mediating conflicting land use conforming to different standards. If more than one standard governs the same area, then the largest permitted choice of urban activity is the common subset of the governing standards.

In the comparison of actual observed structures, for example the simplices of a complex $K_L(A;R)$ to standards defining zones in the neighborhood, there are some standards wholly contained in others. There may occur orderings of the form $S_i \leq L_j \leq S_k$. This describes the concept of intensification of land use resulting in an area that is in transition from one zone to another.

The detailed classification of urban land areas in Table 12 was derived using the ordering of the structures among themselves in comparison to the ordering of the zones among themselves. It is a simplified example of the potentially rich and complex phase structure of a neighborhood. The following definitions were used to derive the ordering of Table 11.

Given streets L_i (i=1, 49) and standards S_j (j=1, 9).

Classification using zoning can differ according to whether the overall scheme of development is orderly (has a strict strategy governing the whole area) or natural. Areas subject to standards are either definitely governed by one or subject to a future option. The following cases can be recognized:

a. Orderly with an option: $S_i \leq L_i \leq S_m$ and also $S_i \leq L_i \leq S_n$ with $S_m//S_n$; or

b. Orderly and definite: $S_i \leq L_i \leq S_m$ (L_i is not dominated by any other standards)

c. Natural with an option: $L_i \leq S_m$, $L_i \leq S_n$, $S_m//S_n$, with $S_i \leq S_m$, $S_i \leq S_n$ but $L_i//S_i$ or

d. Natural and definite: $S_i \leq S_m$, $L_i \leq S_m$, but $S_i//L_i$.

Clearly a finely discriminating standard is required for orderly development, although natural standards can provide some degree of classification as shown in Table 12. The orderly pattern of development can occur where a development strategy is imposed upon the neighborhood by planners. The existence of options in orderly development indicates areas of strategic choice, but no such pattern need be found in the absence of a strategy.

The high proportion of transitional areas indicated in Table 12 shows that the zoning in the study area is not orderly. This could be caused by incomplete development of the areas, by inconsistent uses in the zones as a result of previous development, or by the fact that the area is in transition from one type of neighborhood to the other. Intuitively the area has such a variety of uses that it is undoubtedly undergoing a transition. The problem is caused by zoning standards that do not effectively discriminate the significant varieties of structure found in the area. Nevertheless, the capability of the tool and the suitability of the concept are borne out by this demonstration.

TABLE 10

Q-Analysis (O-Accessibility Level) Complex $K_L(A)$
Fort Osborne Study Area

Zones (R1, R2, R3, R4, C1, C2, C3, M1, M2)

Areas (L1 to L49)

Dimension of Complex = 87

```
                      87   79      75      71  32            25  23       12
Structure Vector = (1 ....2,3,3,3,3,2,2,2,1...2,2,2,1...2,1 ...3,3,1,2, 1
                                  0
                    2,1,2,2,1,1,1,)
```

Q-level	Components
87-80	(C2)
79	(C2,C3) (M1)
78-75	(C2,C3) (M2)(M1)
74-71	(M2,M1) (C2,C3)
71-33	(M2,M1,C2,C3)
32	(M2,M1,C2,C3,C1)
31-29	(M2,M1,C2,C3,C1) (R3)
28-27	(M2,R3,C1,C2,C3,M1)
26-23	(L15,M2,R3,C1,C2,C3,M1)
	(L15,M2,R3,C1,C2,C3,M1) (R4)
21	(L15,M2,R3,C1,C2,C3,M1,R4)
20-17	(L15,L26,M2,R2,R3,R4,C1,C2,C3,M1)
16-14	(L15,L26,M1,M2,R1,R2,R3,R4,C1,C2,C3)
13-12	(L15,L17,L26,M1,M2,R1,R2,R3,R4,C1,C2,C3)
11	(L9) (L15,L17,L22,L26,M2,M1,R1,R2,R3,R4)
10	(L16) (L8) (L15,L17,L22,L26,M1,M2,C1,C2, C3,R1,R2,R3,R4)

TABLE 10 (Con't)

Q-Level	Component
9	(L8,L15,L16,L17,L22,L26,M2,M1,R1,R2, R3,R4,C1,C2,C3)
8	(L3,L8,L15,L16,L17,L22,L26, ALL ZONES) (L14) (L3,L4,L6,L8,L13,L14,L15,L16, L17,L22,L26,L37, ALL ZONES)
6	(L3,L4,L6,L8,L13,L14,L15,L16,L17,L22, L26,L37,L47, ALL ZONES) (L18)
5	(L3,L4,L16,L13,L14,L15,L16,L17,L18, L22,L26,L33,L35,L37,L46,L47,L48, ALL ZONES)
4	(L3,L4,L5,L6,L8,L13,L14,L15,L16,L17, L18,L22,L26,L33,L35,L37,L43,L46,L47, L48, ALL ZONES) (L40)
3	(L3,L2,L4,L5,L6,L8,L13,L14,L15,L16, L17,L18,L19,L22,L23,L24,L26,L27,L28,L33 L35,L36,L37,L40,L43,L44,L45,L46,L47, L48, ALL ZONES) (L25)
2	(L2,L3,L4,L5,L6,L8,L9,L10,L12,L13,L14, L15,L16,L17,L18,L19,L22,L23,L24,L25, L26,L27,L28,L30,L33,L35,L36,L37,L38, L39,L40,L41,L43,L44,L45,L46,L47,L48, ALL ZONES)

TABLE 10 (Con't)

Q-level	Component
1	(L2,L3,L4,L5,L6,L8,L9,L10,L12,L13,L14, L15,L16,L17,L18,L19,L22,L23,L24,L25, L26,L27,L28,L30,L33,L35,L36,L37,L38, L39,L40,L41,L43,L44,L45,L46,L47,L48, ALL ZONES)
0	(ALL except L7,L21,L31,L42,L49 which are not in the complex)

TABLE 11: CONSISTENCY STRUCTURES

Consistency Structures $(S,\leq)$ and $(S,<)$ are both described in the following table.

$L1 = (L\bar{\bar{1}},\underline{L3},\underline{L6},\underline{L8},\underline{L13},\underline{L15},\underline{L17},\underline{L19},L2\bar{\bar{0}},\underline{L22},L26,\underline{L30},\underline{L33},\underline{L35},L37,\underline{L47},\underline{L48},\underline{R1},\underline{R2},R3,R4,C1,C2,C3,M1)$

$L2 = (L\bar{\bar{2}},\underline{L22},\underline{L33},\underline{L46},\underline{L47},R4,C2,C3)$

$L3 = (L\bar{\bar{3}},\underline{C2},\underline{C3})$

$L4 = (\underline{C2},L\bar{\bar{4}})$

$L5 = (\underline{L15},C2,L\bar{\bar{5}})$

$L6 = (\underline{C2},L\bar{\bar{6}})$

L7 = (not in complex)

L8 = (dominant maximal)

$L9 = (C2,\underline{C3},\underline{M1},L\bar{\bar{9}})$

$L10 = (\underline{L4},\underline{L5},L1\bar{\bar{0}},L15,\underline{L19},\underline{L46},C1,C2,\underline{M1})$

$L11 = (\underline{L17},L1\bar{\bar{1}},L50,C2,\underline{C3},\underline{M1})$

$L12 = (L2,L1\bar{\bar{2}},\underline{L4},\underline{L8},L18,\underline{L14},L22,\underline{L23},\underline{L24},L33,L43,\underline{L45},L46,L47,\underline{L48},R3,R4,C2,C3,M1)$

$L13 = (\underline{M1},L1\bar{\bar{3}})$

$L14 = (L1\bar{\bar{4}})$ (dominant maximal)

$L15 = (\underline{C2},L1\bar{\bar{5}})$

$L16 = (L1\bar{\bar{6}})$ (dominant maximal)

$L17 = (\underline{C2},L1\bar{\bar{7}})$

$L18 = (L1\bar{\bar{8}})$ (dominant maximal)

$L19 = (\underline{C1},C2,L1\bar{\bar{9}})$

$L20 = (L\bar{\bar{1}},\underline{L3},\underline{L6},\underline{L8},\underline{L13},\underline{L15},\underline{L17},\underline{L19},L2\bar{\bar{0}},\underline{L22},L26,\underline{L30},\underline{L33},\underline{L35},L37,\underline{L47},\underline{L48},R1,R2,R3,R4,C1,C2,C3,M1)$

Legend

$\bar{\bar{L}}_i$ = indicates equality of simplices

$\underline{L}_i$ = indicates the simplex is predecessor of the indicated simplex

TABLE 11 (Con't)

Consistency Structure (S,≤) and (S,<)

L21 = (not in complex)

L22 = ($\underline{\mathrm{C2}}$,L2$\bar{\bar{2}}$)

L23 = ($\underline{\mathrm{L22}}$,L2$\bar{\bar{3}}$,R3,C2,C3)

L24 = (L47,L2$\bar{\bar{4}}$,$\underline{\mathrm{R3}}$,R4,C2,$\underline{\mathrm{C3}}$,M1)

L25 = (L2$\bar{\bar{5}}$)(dominant maximal)

L26 = ($\underline{\mathrm{C2}}$,L2$\bar{\bar{6}}$)

L27 = (C2,$\underline{\mathrm{C3}}$,$\underline{\mathrm{M1}}$,L2$\bar{\bar{7}}$)

L28 = ($\underline{\mathrm{M2}}$,L2$\bar{\bar{8}}$)

L29 = ($\underline{\mathrm{M1}}$,$\underline{\mathrm{M2}}$,C2,$\underline{\mathrm{C3}}$,L2$\bar{\bar{9}}$)

L30 = ($\underline{\mathrm{L26}}$,L3$\bar{\bar{0}}$,$\underline{\mathrm{L37}}$,M1,$\underline{\mathrm{M2}}$,C2,C3)

L31 = (not in complex)

L32 = ($\underline{\mathrm{M1}}$,$\underline{\mathrm{M2}}$,C2,$\underline{\mathrm{C3}}$,L3$\bar{\bar{2}}$)

L33 = ($\underline{\mathrm{R3}}$,C2,$\underline{\mathrm{C3}}$,L3$\bar{\bar{3}}$)

L34 = (not in complex)

L35 = (C2,$\underline{\mathrm{C3}}$,L3$\bar{\bar{5}}$)

L36 = (C2,$\underline{\mathrm{C3}}$,L3$\bar{\bar{6}}$)

L37 = (C2,$\underline{\mathrm{C3}}$,$\underline{\mathrm{M1}}$,L3$\bar{\bar{7}}$)

L38 = ($\underline{\mathrm{M1}}$,$\underline{\mathrm{M2}}$,$\underline{\mathrm{C2}}$,$\underline{\mathrm{C3}}$,L3$\bar{\bar{8}}$)

L39 = ($\underline{\mathrm{R4}}$,L3$\bar{\bar{9}}$)

L40 = (L4$\bar{\bar{0}}$) (dominant maximal)

L41 = ($\underline{\mathrm{L2}}$,$\underline{\mathrm{L3}}$,L22,L4$\bar{\bar{1}}$,R4,C2,C3)

L42 = (not in complex)

L43 = $\underline{\mathrm{R3}}$,$\underline{\mathrm{R4}}$,C1,C2,$\underline{\mathrm{C3}}$,M1,L4$\bar{\bar{3}}$)

Consistency Structure $(S,\leq)$ and $(S,<)$

L44 = $(\mathrm{C2},\underline{\mathrm{C3}},\mathrm{L4}\bar{\bar{4}})$

L45 = $(\underline{\mathrm{L22}},\underline{\mathrm{L43}},\mathrm{L4}\bar{\bar{5}},\mathrm{R2},\mathrm{R3},\mathrm{C2},\mathrm{C3})$

L46 = $(\mathrm{L43},\underline{\mathrm{C2}},\mathrm{L4}\bar{\bar{6}})$

L47 = $(\underline{\mathrm{C2}},\mathrm{L4}\bar{\bar{7}})$

L48 = $(\underline{\mathrm{M2}},\mathrm{L4}\bar{\bar{8}})$

L49 = (not in complex)

M2 = $(\mathrm{M}\bar{\bar{2}})$ (dominant maximal)

R1 = $(\underline{\mathrm{R3}},\mathrm{R}\bar{\bar{1}})$

R2 = $(\underline{\mathrm{R3}},\mathrm{R}\bar{\bar{2}})$

R3 = $(\mathrm{R}\bar{\bar{3}})$ (dominant maximal)

R4 = $(\mathrm{R}\bar{\bar{4}})$ (dominant maximal)

C1 = $(\underline{\mathrm{C2}},\mathrm{C1})$

C2 = $(\mathrm{C}\bar{\bar{2}})$ (dominant maximal)

C3 = $(\mathrm{M1},\mathrm{C3},\underline{\mathrm{C2}})$

M1 = $(\mathrm{M}\bar{\bar{1}})$ (dominant maximal)

TABLE 12

CLASSIFICATION OF LAND AREAS

The classifications contained in this table were derived from Table 10 and Table 11 using the definitions for each type of ordering.

a. Non-Conforming Streets (streets not consistent with any zone standard) L8,L14,L16,L18,L25,L40.

b. Definite R1 (Orderly or natural classification system) L_i R_1, L_1, L20.

c. Definite R2 (Orderly or natural classification system) L_i R2 and L_i R1 none.

d. Definite R3

   a. (Orderly system) none (R_1 L_i R_3 or R_2 L_i R_3 incomparable to other zones),

   b. (Natural system) none (L_i R_3 and incomparable to other zones).

e. Definite R4 (Orderly or natural) L_i R4 and incomparable to other zones. L35.

f. Definite C1 (Orderly or natural) (L_i C_1 C_2 and incomparable to other zones) L15.

g. Definite C2

   a. Orderly (C1 L_i C2 or C3 L_i C2 and incomparable to other zones) none,

   b. Natural (L_i C2 and incomparable to other zones) L4, L5, L6, L15, L22, L26, L46, L47.

h. Definite C3 (Natural or orderly) L_i C3 C2 L_i C3 M1 and incomparable to other zones. L3, L9, L11, L24, L27, L44.

TABLE 12 (Con't)

j. Definite M1

a. Natural (L_i M1 and L_i C3) L13,

b. Orderly (C3 L_i M1 and L_i incomparable to other zones) none.

k. Definite M2 (Natural or Orderly) (L_i M2 and L_i incomparable to other zones) L28, L48.

The remainder of the areas involve future options of some form.

III. Planning and Measurement Using Local Structure

1. Introduction

Local analysis in contrast to global analysis examines the contrast between the structures discriminated using the simplex as a unit of structure and the structures using the clique as a unit of structure, together with the shomotopy analysis describing the transition between these two levels of structure. The local analysis examines closely the uniformity of different types of structures. The clique structure has the greatest degree of uniformity and therefore describes the concept of a standard of behavior in structural analysis. The use of the clique as the limiting form of structure in the local analysis lends clarity to the local analysis of structure. Algorithms to implement local analysis become much more complex. However, the real problem in local analysis is to ensure that the structure taken to describe actual observation is defined in the same way as the structure defining standards of behavior. The reader will note that the concepts of phase space, use of the orderings, even the concept of compatibility and equivalence classes; that were important in the global analysis remain important at the local level.

2. Analysis of Uniformity in Patterns of Choice in a Complex

A social standard of behavior imposes a degree of uniformity upon a group of individuals in the sense that it prohibits modes of behavior that are not consistent. Conventionally, one conceives of a standard or standards of behavior being associated with the central interest or role of some particular group. It is conventional to think of the preferences of each group as being internally consistent, although the totality of such groups will present incompatible standards of behavior. The social problem is to determine a configuration of standards that regulate each distinct group to provide the maximum freedom of behavior consistent with their own aims and that regulate the conflicts that occur between different incompatible standards.

The mathematical representation of social uniformity is the complete cover of cliques of a symmetrical binary relation.

A clique is a set of elements from the observational protocol generated relative to a signal relation such that every element in the set is compatible with every other element in the set. A complete clique cover is the set of all maximal distinct cliques contained in the relation. For completeness, cliques with 0, 1, or 2 elements must be formally recognized, unlike the normal social concept of cliques.

The clique cover represents the limits to the ability to discriminate in a given signal relation. Each clique is a complete relation that in effect defines a new observational protocol. Whereas the original protocol showed the totality of observations which might occur in a given signal relation, the clique shows the largest distinct complete set of observations. The uniformity of patterns of choice makes them suitable representations of the standards of behavior in a social preference system.

The clique cover defines a new structure having the same formal properties as a simplicial complex. For example, under the closed partial order there is a face operation, and one can discuss the q-connections of the cliques, etc.

The distinction between the patterns of choice offered by the simplex or clique structures identifies the problem of simultaneous choice in urban analysis. The simplices were defined as the set of pairs of elements having the same first element. There need be no necessary relation between the elements of the range in the simplex. The pattern of choice described by the simplex is interpreted as an offer which at most guarantees choice of a single element at one time.

Clearly, the discrimination of the cliques in the relation determines the possible largest sets of multiple or simultaneous choice in the structure. For example, every ternary relation must also be a binary relation, but not every ternary relation derived from a binary relation is necessarily significant to the structure. The cliques are a cover with this sense of simultaneity. If one wishes to iterate description of choice to higher levels, the clique covers provide new protocols with which one can return to the residents to solicit their opinions on the desirable multiplicity of choice for the urban structure.

Consider the set (a,b,c,d) together with relation R. If this set formed a simplex of the relation with respect to element "a" one would have at least (aRb,aRc,aRd). Additional elements of the relation such as (bRc,bRd) are not necessary although they may possibly exist. If, however, the structure forms a clique with respect to a, then (aRb,aRc, aRd,bRc,bRd,cRc) is necessary. The set of elements are pairwise related and every pair is present. In a clique every element satisfied the defining relation with every other element and hence represent simultaneous choice.

It is clear that not every face of a simplex forms a clique, but a clique must always be a face of a simplex. Given a simplex of some relation, the simplex may be analyzed into all of its subsets, but not every subset of the simplex will be a possible simultaneous choice. In particular, the 0-vertices of the relation may be regarded as at least 0-cliques.

The relation between the set of undirected (symmetrical) relations and the set of complete clique covers is a function (onto). This can be guaranteed by properly selecting the algorithm by which the clique covers are generated. Each element of the domain of the relation will belong to at least one cover set, and all of the maximal cliques belong to the cover family by definition. Therefore each undirected relation determines a cover family of cliques, and the set of all undirected relations should by definition determine all possible cover families of this form. To demonstrate that the relation is unique it is necessary to prove that there are no two distinct relations, say R1 and R2 on the same domain set A which have identical complete families, say CR1(A) = CR2(A). The hypothesis of distinct R1 and R2 means that there are at least two elements a_i, a_j belonging to A such that although $a_i R1 a_j$ is true, $a_i R2 a_j$ is not. From $a_i R1 a_j$ it follows that there is a pair (a_i, a_j) in some member of CR1(A). Since CR1(A) = CR1(A) it follows that there is a member of CR1(A) which is identical to that member of CR1(A) containing the subject pair. It therefore follows that R2 contains the pair (a_i, a_j). This implies that $a_i R2 a_j$ is a contradiction. Therefore the mapping of relations onto clique covers is unique where maximal cliques are used.

The whole simplicial complex defined by the signal relation is decomposed into a number of sub-complexes associated with the levels of the signal relation. There will be one complete clique cover for each signal level. The interpretation of the clique structure describes urban zoning in terms of the constraints upon choice for activities which become available to the resident of an area as his horizon expands with increasing signal levels.

The significant complex for zoning is $K_L(A,R^n,n)$ where L is the backcloth streets, A is the protocol set of urban activities, and R^n denotes the weighted relation describing the choice of activities offered by accessible service areas under the backcloth ordering for each accessibility level.

Assume further that the complex has been sliced for each n and reduced to a clique cover family for each level.

The concept of simultaneity in an urban sense must be introduced. Simultaneity means that a given set of activities can be selected at the same time. In this context, it is taken as a matter of convention that if the basic areas discriminated in the geographic backcloth are streets, all activities actually sited on a given street can be selected simultaneously. This introduces the concept of a one-stop shopping area. (ie. one can select every activity by parking once in an area.) The individual backcloth street is treated as a 0-accessibility level clique conventionally requiring no effort to choose its associated activities. The properties that this 0-clique has are the following:

a. Every activity in the street can be chosen by travelling at most some distance (length of the street) from any other activity.

b. Every activity can be selected by traversing at most some distance (the length once).

It is clear that the former property can be generalized to higher accessibility levels allowing us to set up efficient search patterns for determining an ideal shopping plan. The collapse which occurs in going from the idea of a 1-clique to a zero clique may be interpreted as the distinction between a potential decision and an actual decision in urban analysis.

Suppose one knew which cliques of activities at each accessibility level are assigned to each one-stop shopping area of a given neighborhood. Suppose also that one wished to visit some particular set of activities with the fewest stops. Obviously the areas that one must visit are included within the one-stop shopping areas of the lowest accessibility level clique that contains all of the choices. One would then examine the choice attached to the (n-1) - cliques that belong to the n-clique, searching for one which either contains the desired choice as a face, or the ones containing the largest faces whose union contains the desired choice. One would proceed in this way down to the lowest level which represented one's preferred walking distance. The resultant cliques would each describe one-step shopping areas where one can park in any of their areas to obtain a multiple choice. Obviously the nesting of cliques from level to level can specify a systematic decision procedure to determine the parking areas providing access to a desired choice of shopping activities.

It should be immediately apparent that while it is easy to determine the preferable multiple choices of activities from the preference relations, there is a problem in determining the assignment of these choices to specific areas.

Fortunately this problem is easily overcome. From the complex $K_L(L,R^n,n)$ generate the cover of cliques on the backcloth siting relation. Knowledge of this complete clique cover allows us to decompose the incidence matrix of the backcloth at each accessibility level into the direct sum of the incidence matrices of the cliques. Direct sum here means that addition of corresponding elements of the incidence matrices of the clique is performed modulo 2. (ie. 1 + 1 = 1, 0 + 0 = 0, 1 + 0 = 0 + 1 = 1.)

Each clique incidence matrix can be multiplied by the incidence of activities in areas. Thus one has a clique relation of the form $(L \times L) \times (L \times A) = (L \times A)$. In effect the pattern of choice for each one-stop shopping area is the union of the activities which are contained in each individual area belonging to the clique.

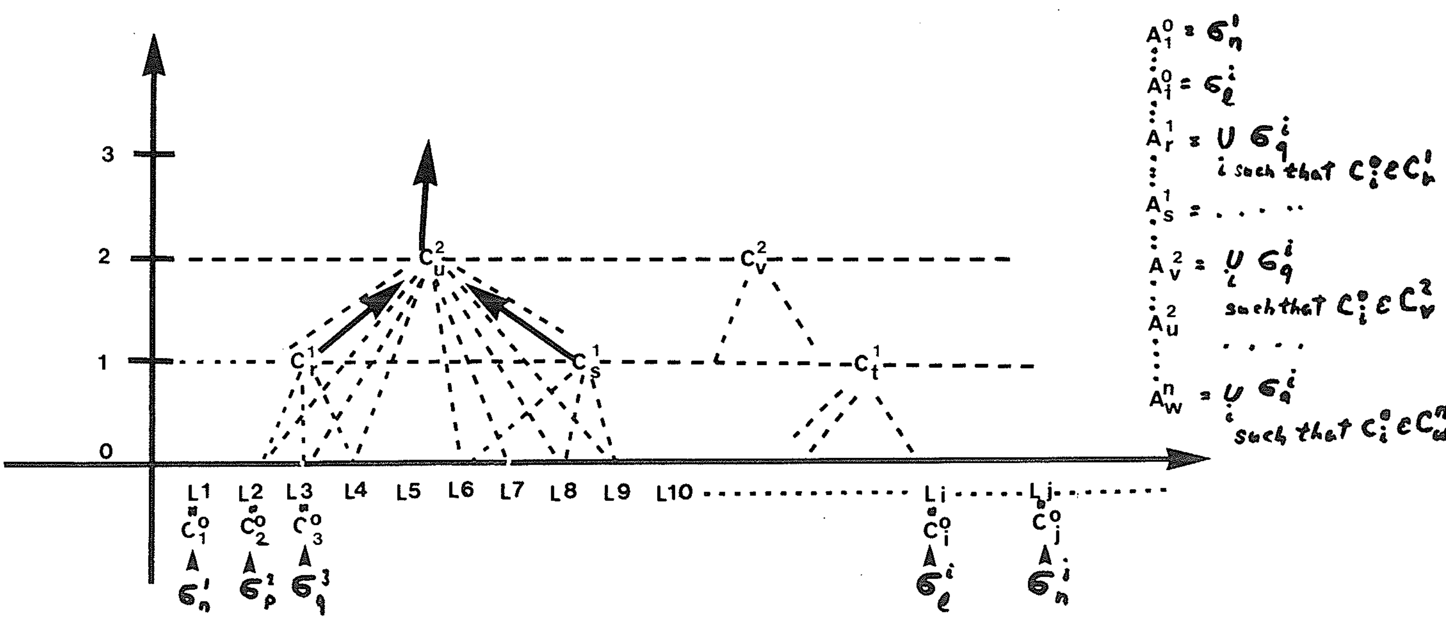
Clearly, the presence of an activity in any area of a clique defined at some accessibility level results in the presence of its influence in every other area belonging to that clique. In this way the cliques of activities directly defined by the backcloth order are determined in a form suitable for comparison to the cliques determined from preference relations. In other words, urban land use zones defined by residents' preferences can be compared to the actual patterns of influence resulting from the detailed siting of activities.

Let A_i^o denote the ith clique of activities actually sited on an area L_i. This clique is identical with the simplex σ_n^o associated with L_i defined by the incidence relation of activities on areas (at accessibility level = 0).

Let C_i^n denote the ith clique of areas at level n. This clique defines a set of areas L_i which are contained in the set C_i^n. There will exist for level n a complete family of such maximal sets of areas, ie. $C = (C_1^n \ldots . C_k^n \ldots . C_m^n)$. A_k^n denotes the set of multiple choice on clique C_k^n. $A_k^n = \bigcup_i \sigma_n^i$ for i such that $L_i \varepsilon C_k^n$.

A number of interpretations may be attached to this form of relationship between areas, and their cliques. Land use zones denote cliques of mutually compatible activities. Thus, the relationship provides a means to relate the pattern of activities directly incident in each area to the different interpenetrating zones, and to determine the standard of service provided by the whole structure to each area at each signal level. Since, in any analysis, one wishes to fix the zones acting on each individual area to provide both a minimal standard of service and a maximal pattern of influence that does not violate any zoning constraints, the utility of the clique decomposition is evident. The structural scheme is illustrated in the Figure 21 below.

FIGURE 21: CLIQUE ANALYSIS OF ZONES



The clique structure defined in this way reveals a property of urban structure, designated as phase, representing the way in which distinct patterns of choice arise by the propagation and interference of influence from their sources in the area. The actual pattern of choice evident in any area is the result of the interference of one or more cliques at that area. The phase quality of structure can be used to fit together the patterns which result in consistent patterns at different accessibility levels which satisfy a given standard of service assumed to be the minimum acceptable in the area. In order to fit the patterns together properly it is necessary to study the ordering of cliques at different accessibility levels and select a suitable chain from the partial order of cliques revealed to provide a consistent growth of influences from the source.

Recall that, in the phase space analysis, the idea of compatibility and convenience focused attention upon those chains of transitions from signal level to signal level which were direct successors of the elements at the next lower level. Clearly, all of the significant urban zoning standards that will be developed from individual preference relations must incorporate the idea of orderly development exemplified by the phase space analysis.

Each zone must be orderly throughout the whole structure embodying a particular trade-off between compatibility and convenience. For a large sample it will be possible to identify the natural frequency and stability of particular zones in the different urban neighborhoods. Mathematical zoning is a tool that will yield valuable long term development information about the behavior of urban neighborhoods.

These structural ideas are capable of embodying the idea of strategic planning. The simple fact that the cliques standards overlap resulting in ambiguity of classification means there is an inherent uncertainty as to the ultimate disposition of development in an area. Each time a developmental change occurs, which forecloses alternatives for future development by reducing the set of final development standards, an opportunity cost is incurred. Picturesquely, at an intermediate stage of development, the zones float. This uncertainty in the governing structure is desirable since a planner cannot regulate the occurrence of actual activities. By preserving options with respect to the future location of some types of activities, he can make decisions as required. This decision is made with the knowledge that there is always at least one chain of development open that preserves compatibility and convenience of land use. Where a number of chains are open, select the chain which leaves the greatest variety of ultimate development states to incur the least opportunity costs.

This defines the zoning problem. It is possible that by working with covers rather than the actual incidence of activities, modern computers will be able to handle the combinatorial complexity of the problem.

3. Clique Analysis

A suitable program for generating the complete family of cliques contained in an undirected graph is contained in Annex D. The reference describes not only the suitable algorithm, but provides a program written in Algol.[20]

Generally the clique-finder algorithm uses a method of programming known as branch and bound backtrack programming. The virtue of the particular algorithm presented in the reference is that it contains a method of testing each branch to identify those branches which cannot possibly lead to a clique at the earliest possible point. The algorithm automatically determines the complete set of maximal cliques which defines the necessary cover family of the complex.

4. Shomotopy Structure

The purpose of the decomposition of a backcloth relation into cliques is to identify areas where the cliques overlap within each accessibility level. These overlapping areas identify potential conflicts between zoning standards which are ignored by conventional land use zoning. The assumption that zones are distinct ignores the edge effects of the propagation of influences between zones, but the use of cliques can reveal the edges to facilitate the mediation of conflict.

The mediation of conflicts between zones is accomplished by using the largest common subset shared by two distinct zoning standards. Although the conflict areas receive influences from both incompatible

---

[20]Coen Bron & Josep Kerbosch, "Finding all the Cliques of an Undirected Graph," CACM (1973) pp. 575-577.

standards, they contain only the common subset of activities which are compatible with these influences; moreover, they contribute to each zone only influences compatible with both standards.

If the distinctly incompatible activities in each zone are concentrated in areas outside the overlap, the greatest overall variety of development is possible without incurring any conflict. Thus, the geometry of a neighborhood can be exploited by the clique analysis to achieve compatible development.

The clique structure may provide insight into the dynamic significance of similarity between adjacent q-connected chains of a component introduced by Atkin.[21] A clique is a maximal cycle which contains, say, n identical simplices. The cardinality of these simplices is p. Since they are identical, every p - 1 loop in the clique is p - 1 connected and p - 1 adjacent; hence, perfectly uniform. It is this uniformity of influences within the clique that constitutes the importance of the structure to urban analysis, but much further research is required to clarify the urban design problem along these lines.

---

[21] R. H. Atkin, Combinatorial Connectivities in Social Systems, pp. 57-68.

IV. Stages of Structural Analysis: Synopsis

The functional explanation of behavior in an urban area explains global behavior in terms of local behavior using the relations that functionally describe both preferences and actual behavior. A mathematical analysis program to implement this scheme of explanation contains the following steps:

1. Define in data files the incidence relations describing each individual preference system with respect to a given protocol for each signal relation.
2. Generate the influence service areas using the signal levels on the backcloth ordering relation yielding the different complexes $K_L(L,R^n,n)$ for each signal relation. Algorithms were discussed for determining the distance between each pair of backcloth areas.
3. Generate the pattern of choice associated with these different complexes. Each complex $K_L(A,R^n,n)$ for a given signal relation is the composition of $K_L(L,R^n,n) \times (K_L(A,R^o,0)$.
4. Use the Q-analysis program to define the compatibility classes and equivalence classes. From this information it is possible to study:
   a. The siting program which optimizes site selection for distinct activities without regard to the overall compatibility of influence.
   b. Analyze the preferability of different patterns of choice associated with distinct areas at each accessibility level by defining a phase space with a structure of trade-offs.

5. An efficient algorithm for determining the different q-loops within components was referred to. Using the same general scheme of backtrack branch-and-bound programming employed in the clique-finder algorithm, the identification of the q-shomotopy equivalence classes of the component, the q-holes, and the cliques should be possible. Normally this mode of analysis will be applied to standards.
6. The clique finder algorithm discussed can be used as a means of defining the actual consistency of observations in an urban area.

The implementation of structural analysis in urban design problems is the subject of the following chapter.

CHAPTER FOUR

APPLICATION OF STRUCTURAL ANALYSIS TO URBAN DESIGN

I. Introduction

The original goal of extending the structural analysis into urban design has been found to be overly ambitious within the scope of the current study. Nevertheless, an outline is proposed as a basis for future research. Structural analysis is consistent with the stages of the planning process outlined earlier.

Fundamentally, the design problem of urban planning is implicit in the structure of input data in the preplanning stage. Two essentially different types of input data were recognized: data defined on the observational backcloth; and data defined on the functional space of social preferences.

The problem is to match standards of behavior in the functional space to decision areas on the backcloth ensuring that no social constraints are violated, while also violating none of the constraints of the observational backcloth. Clearly, the incidence of certain social groups in a particular decision area will restrict the choice of standards of behavior for given decision areas. Tolerances in the preference relations will provide some freedom of choice to adjust standards of behavior for different influence patterns generated by the backcloth. The objective of analysis is to determine feasible configurations of standards of behavior that provide the most compact distributions of activities on the decision areas. The specification of patterns of land use to maximize the freedom of individual choice without violating any of the constraints of the social preference relations is the design problem of planning.

II. The Design Concept

The phase space of the clique cover discussed in the previous chapter is the basis for this design problem. The ideas of convenience and of compatibility are central to the implementation of the design process. Mediation must occur within the overlapping decision areas where conflict between distinct zones may exist.

Design uses a top-down approach. Select a set of zones at the highest accessibility level. The choice at successively lower levels is constrained by the choice at the higher level containing the particular clique on which a decision is to be made. The existence of a standard of behavior on a backcloth clique at any decision level automatically constrains the maximum activities that can be sited in any area contained in the clique. Convenience and compatibility will further reduce the possible choices at each level. This combinatorial problem can be solved by standard backtrack programming techniques.

The only real complication in the design process arises from the fact that the cliques in general overlap at each level. Therefore a means of mediating the standards in the conflict decision areas must be specified. The choice on any decision area at a given level is subject to a sideways order specifying the total independent configuration of zones at that level.

It is fortunate that a combinatorial design technique exists. It was developed by the Institute of Operations Research in Great Britain and is termed the Analysis of Interconnected Decision Areas (AIDA). The basic paper describing the algorithm is included as Annex F.

The first step is to identify the decision areas with the backcloth cliques at each accessibility level. Then the strategy graph of these areas will be developed. In this strategy graph, if two areas do not overlap, the choices made thereon are independent. As shown in the annex, a strategy graph describes the choices prohibited in connected areas as a result of specific decisions in any area. Where a bar exists between pairs of areas, the question of the degree of conflict between incompatible standards that will be accepted must be defined, remembering that the smaller the conflict accepted, the fewer activities will be permitted in the overlapping areas.

Therefore the detailed specification of bars between possible choices in each decision area will be derived from the following rules:

1. If there is no overlap between decision areas, no bar will exist between them;
2. Where a bar exists, the pairs of possible zones must be examined and those which are too incompatible must be rejected.

The possible choices on each decision area has been partially determined by the vertical order. Call the set of decision areas as follows:

$D^n = (D^n_1, \ldots\ldots\ldots\ldots D^n_m)$ D^n_k is the kth cliques of areas at the n-accessibility level.

The zones to be considered with each decision area are the clique standards previously defined as possibly consistent with the area.

Thus $C^n(i) = (C_1^i, \ldots\ldots C_p^i)$ where $C^n(i)$ denotes the choice of clique standards compatible with the ith decision area.

The AIDA algorithm discussed in the paper at Annex F can specify the consistent overall sets of choice, one for each decision area, which violate none of the constraints of the strategy graph represented by bars. The algorithm can generate all and only the feasible configurations which are the independent urban designs at that level.

One of the interesting aspects of this design problem is the possibility of a strategic approach to planning. This possibility becomes most clear when one orders the solutions on their respective decision areas. To some extent there will be a degree of overlap between the solutions acceptable in each area. This means that the decision to site a distinct type of facility may or may not rule out some of the solutions. By selecting sites which rule out the least possible solutions the planner can keep open the greatest number of options for future development open. Only when the potential solutions have been reduced as far as possible are the final limits of development reached.[1]

---

[1]J. K. Friend and J. N. Jessop, Strategic Choice and Local Government, (London: Tavistock, 1971). In this book the authors discuss the results of an operations research inquiry into local government in Great Britain. A sequential or strategic decision process is deemed necessary to planning under these conditions. The strategic planning process they outline is related structural analysis in the design stage.

III. Regulation

It should be clear that the solution of the structural problem only determines the potential choices. For the designer, the problem of matching the potential of the urban structure to the capacity of physical resources remains to be solved. For example, the provision of a great variety of commercial services is of no avail when these services are not provided with suitable circulation systems to facilitate customer parking and transportation of customers from the market area. The provision of recreational facilities to residents is of little use, if these services do not have the capacity to handle the demand.

This question is obviously related to the shifting dynamics of the patterns of choice in a neighborhood. The question of the capacity of a structure arises whenever one is concerned with the distinction between the consistency of a pattern of choice with the overall structure and the ability of the structure to accommodate the different quantitative patterns of choice that actually exist. Atkin deals with this question algebraically. The algebra of patterns of choice is a fundamental consideration in any theory of urban indicators.

The fundamental considerations of the algebra of patterns of choice is described in the reference paper.[2] The approach is based on the fact that one may establish a mapping of simplices onto numerical values; these mappings define co-simplices. Thus

$\sigma_i^P : \sigma_i^p \to J$ (J is the system of integers, for example)

---

[2] R. H. Atkin, "An Approach to Structure in Architectural and Urban Design. Pt. 2. Algebraic Representation and Local Structure," Environment and Planning B (1974), pp. 173-191.

The idea of a pattern of choice with a value is expressed by a graded set of co-simplices defined on the various p-dimensional simplices of the complex as follows:

π denotes the value of a pattern on a complex.

$= (\pi^n, \pi^{n-1}, \ldots\ldots, \pi^1, \pi^o)$ where n = dim K

$\pi^q = (\sigma_q^i$, all i) $\rightarrow$ number system.

With suitable restrictions these algebraic structures may be added and subtracted, multiplied by a scalar, and generally treated as any other algebraic structures.

Bearing in mind that these values symbolize the quantity of choice made in certain instances, one must carefully examine how the algebraic operations relate to the numerical quantity when the operations are consistent with a covering structure.

It is clear that the addition and subtraction of patterns can be visualized as adding and subtracting quantities of choice similar to a vector. Thus, $(5\sigma_p^i + 1\sigma_{p-2}^j) + (2\sigma_p^i + 3\sigma_{p-2}^j) = (7\sigma_p^i + 4\sigma_{p-2}^j)$.

For operations dealing with changes to patterns, the face and co-face operations are similar to the calculus of finite differences in the manner in which they carry numerical values through the structure. They are defined as linear operations on the changes in value rather than on the values themselves. Hence, as Atkin shows in the reference paper, there is a distinction between the change in the structure and its value. In other words, the formal operations reveal the total extent of change on the structure, but the distribution of this change to different patterns must be clearly distinguished.

If there are h_p p-simplices in the structure K, a basis for the co-simplices is the set of h_p mappings (σ^p, i = 1, ...h_p) where

$$\sigma_i^p\ (\sigma_p^j) = 0 \text{ if } i \neq j$$
$$= 1 \text{ if } i = j.$$

This may be used to define the value of the patterns by the inner product notation. Every co-simplex is the sum of the σ_s^p ($\sigma_i^p = \Sigma_i\ \sigma_i^p$).

One can thus carry the analogy to a vector space further by assuming that each simplex denotes a possible "direction" in the space. The symbolism introduces algebraic indeterminates "x_i" to correspond to the vertices σ_i^0 of the complex. The algebraic "x_i" are the basis set for an n-dimensional module V(J) over the integers J. In the reference papers the important aspects of the algebra of choice are worked out and the reader is advised to refer to them for the details. Rather than repeating this material here without drawing any fresh conclusions, some observations on the significance of the capacity in the complex to the design problem of urban planning will be made to indicate the approach and the use of this theory in city planning.

The quantity, which is indicated by the integers J, must be something which refers to a resource limitation of the urban area. Supply and demand of some resource determines this matching. In addition, the quality must be such that it can be mapped on the simplices of the complex which denote the choice available or the choice demanded. The quality must be common to all of the choices with which it is associated since it denotes an interaction between the choices.

Such possible qualities are: the requirement for parking of an urban activity, the services that are significant to different sets of activities, flows of money, rents, or similar aspects of urban dynamics. Clearly, the use of different indicators on the raw backcloth structure may point out the significance of a certain sub-complex of the neighborhood while slicing out other aspects completely. In terms of any such quality it is clear that patterns of choice can only be added when they interact with each other in this sense.

These patterns of choice introduce the possibility of an unbiased view of the choice in the neighborhood. Every logically possible choice set must be included initially, and if any are excluded, the reasons for exclusion must be explained. This will correct a tendency of urban planning to overlook some less obvious choices that the existing neighborhood does not favour.

The algebra of patterns of choice is intrinsically aspatial. To deal with each specific instance of choice in a given area make each occurrence of the same type of choice in a different area notationally distinct. The interaction between activities must be explicitly introduced using the signal levels through the medium of linear programming.

The possibility of linear programming arises from the fact that the geometrical representation of a simplicial convex is in terms of closed convex polyhedra.

ie; a 2-simplex $(v_1v_2v_3)$ can be represented by

$$1 = \theta_1 + \theta_2 + \theta_3 \quad \theta_i \leq 1 \quad P = \theta_1P_1 + \theta_2P_2 + \theta_3P_3$$

The indeterminate θ_i in this representation can be used to define permissible limits upon (tolerances of) the choice denoted to changes in the indicators attached to the choice. This is to say for each simplex there is a possible supply and demand relation which relates its capacities to every other instance of the same choice on the complex. It may be necessary to return to the distinct decision areas defined by the cliques at each signal level to determine the frequency of every distinct type of choice at each signal level, for these distinct choices to be related to the preferences of the residents. Moreover, the distinct occurrences of an activity must be inter-related in terms of their capacities to specify the choice they supply to the urban system.

In essence, the use of linear programming on the choice structure of the urban phase space gives a representation suitable for optimization techniques.

It is tempting to speculate that the algebra of choice together with the calculus of finite differences might provide a geometrical-algebraic language suitable for the expression of choice on an urban backcloth. This problem bears investigation by suitably qualified researchers in the future.

The design process proceeds in two phases: design to optimize the richness of variety of potential choice in an urban area; and regulation to adjust the physical resources of the urban neighborhood to satisfy the choices of the residents. The introduction of an algebra is natural to describe the dynamic behavior of the structure.

CHAPTER FIVE

CONCLUSIONS OF THE STUDY

I. Reflections Upon the Objectives of the Study

The genesis of this study is to be found in the conviction that the existence of many forms of unexamined values in city planning demands that a deep inquiry be made into the logical foundations of the discipline, and that this inquiry be conducted within the framework of conventional epistemology, as in other sciences. Only in this fashion can the limitations of the discipline, imposed by the social mandate for planning and by the capabilities of the current technology of planning, be explored within a framework of fundamental principles. Since one of the most potent techniques used in formal science is to describe the limits of what is feasible using the knowledge of what is impossible, the study places considerable emphasis upon the problem of social choice as the fundamental form of the problem that encompasses the aims and objectives of city planning.

Realistically, progress on the theoretical side of this thesis has been limited in this area. While the study has no difficulty in clearly relating the problem of social choice to the conventional aspects of planning theory, the key role of the problem of social choice in the proper development of a mathematical formalism for planning is not adequately clarified. Reasons were stated, however, for the belief that the central notion in this avenue of inquiry would be the concept of urban field, determining the different maximal configurations of standards of behavior that are consistent with, and appropriate to, the preferences of affected individuals. In later research it is proposed to study the conceptual organization of an urban field, thought to be a form of mathematical structure having properties at least analogous to the idea of a fiber bundle as used in physical gauge theory to geometrize the concept of field. The result, if practicable, would be a very concise formulation of the urban design problem in a very satisfactory fashion.

The positive accomplishments of this thesis are numerous, despite its many areas of weakness. Working from the conventional notions of urban planning theory, five stages were distinguished that are thought to encompass the required methodology for structural analysis in urban planning in the logical order of execution.

Working within this framework, the role of the modern delphi techniques in attempting to impose the maximum degree of structure upon the observational experience of the individual to facilitate comparisons between values themselves is readily appreciated. The application of sets to describe these structures greatly enhances the techniques. Moreover, the direction to be taken by subsequent analysis to transform raw data into conceptually more useable forms is made evident. Structural analysis enables the maximum amount of useable data to be drawn from the raw experience of the planner.

The thesis is thought to provide a convincing case for the observational basis of planning. Illustrations of the further analysis of raw structures by application of partial ordering for the siting problem and the preference structure (convenience and compatibility) providing a finely discriminating system of measurement would seem convincing.

The approach to the design problem is considerably less well established, but the case which is made (obstructed by the lack of clear examples) does provide plausibility to the case for interpreting design as a signal level-by-signal level process of matching land use zones (preference cliques) to one-stop shopping areas (backcloth cliques). Certainly, with some effort to program the technique, its practicality may be verified. The judgement of the thesis must be — plausible.

In summary, the origin of the study was in a very vaguely defined problem area - but it seems fair to conclude that the problem at least is considerably more well-clarified than was the case hitherto. As a bonus a number of novel and seemingly promising planning techniques have been proposed. The real conclusion of this study can best be summed up in the following hypotheses suggesting challenging areas of future research to extend our knowledge in this vital area.

II. Directions for Future Research

For the mathematician there is the challenging problem of reformulating the problem of social choice by extending its interpretations in a novel way, reminiscent of the gauge theories of physics. Since maxima of the partial ordering seem to most nearly correspond to structures which would be recognized (physically) as objects, the problem of social choice is nothing but a statement that two incompatible objects cannot occur in the same place at the same time. Intuitively, the designer recognizes the existence of a partially independent structure of functions in any system and attempts to satisfy the needs of incompatible functions in different places in the system at the same time. Of course, this assumes that the concept of place has meaning. Generally the logical notion of a connected chain of events (processes) is required for "place" to be defined. Changing places require that the ordering considered be at worst an acyclic order to have the necessary concept of proximity. The key to introduction of formal geometry in urban analysis is to recognize the related concepts of: accessibility governed by a strictly limited order between events, the possibility of orderly development indirectly connecting otherwise inaccessible stages of development, and the concept of standards of behavior describing the largest structures of compatible preferences which exist in the social structure. Obviously, Atkin's algebra of patterns has application to this question.

For the computer analyst there is the technical problem of implementing the techniques discussed in this study within a sufficiently sophisticated programming package to handle large volumes of data efficiently.

For the urban planner, the application of the siting problem, the measurement technique, and the zoning problem would be of great interest. In these terms, planning can become an empirical science.

Planning studies should attempt to correlate the existence of certain types of standards of behavior with external parameters denoting the socioeconomic status of a neighborhood. The purpose of standards is standardization. Structural analysis will become much more effective as a tool for design when it has been reduced to the handbook level of application.

The application of linear programming to the structures should be studied in greater detail. In this area some practical questions involving traffic studies, the structure of services in a neighborhood, and the commercial viability of shopping complexes can be answered.

The zoning problem should be studied in a greater detail. It should be possible to examine the distortion of individual rationality by the global standards. For example, using the solutions of the siting problem which are optimal for a given type of activity, to what extent is there a distortion of this rationality in comparison to the locations permitted by the zoning problem? Such distortions are a manifestation of the concept of urban field.

It would be interesting to know whether there is any productive outcome from the combination of the face algebra with the calculus of finite differences. The algebraic structure of the various urban neighborhoods should be assessed carefully. It may be possible that the phase space will reveal some previously unsuspected symmetries that are of great value to the urban designer.

Far from being an abstract study, the pursuit of these areas of research may eventually lead to very practical results for the urban planner and designer. Mathematical thought can serve the urban planner well, in the same fashion as it has served our understanding of sophisticated aspects of physical process. The fact that the vast majority of urban residents may not realize how they are organized, or appreciate the methods by which improvements to urban organization are developed, will not prevent their enjoyment of the benefits derived from it.

ANNEX A

GLOSSARY OF TERMS

The following glossary provides a partial listing and definition of terminology which is employed in the text of this thesis.

Acquaintance (relation of): An epistemological notion introduced by Sir Bertrand Russel to describe the correlation between formal concepts generalized from experience and particular aspects of observation (perceptions). The relation of acquaintance describes the actual perception of an observer under some specified external condition in terms of a common set of elementary observations.

Accomodation: A notion distinguishing the concept of behavior consistent with a standard of behavior from the possibility of the occurrence of a consistent form of behavior under specified circumstances. Quantitative limits imposed on the activity in a structure may reduce the feasible choice to some subset of the consistent forms of behavior these can overload the capacity of the structure without violating any of its logical constraints upon possible standards for choice.

Backcloth: A concept that there is an underlying relatively static structure which provides a basis for the dynamic forms of behavior that are actually observed. A backcloth serves as a consistent standard of behavior to be used as a frame of reference in analysis and may be applied equally well to the description of the functional activities and physical aspects of an urban system, i.e. the map of the area is a geographic backcloth, the authority of individuals in a group describes how to compare their points of view within a formal organization. The backcloth defines the scope of observation related by some signal property to the different viewpoints resolved as significant states for the study.

Backtrack programming: A basic combinatorical technique which has wide application in exhaustive search procedures. Efficient versions of the backtrack programming technique use integer programming with branch-and-bound techniques detect unprofitable search tracks quickly and expedite the search.

Basis: The term usually describes the minimal set of sub-structures which are capable of representing every other sub-structure possible as a pattern in a given structure. It also describes the qualities of a value which cause it to be assigned some weight in a judgement.

Capacity (of a structure): If, after assignment of some numerical indicators to a sub-structure to describe the extent of choice on a particular pattern for assignment of the available resources to be shared among all choices, then the capacity is a limit upon the total resource available within the structure, and is used to determine the acceptable patterns which can be accommodated.

Chain: A simply ordered sequence of subsets of a structure, i.e. one in which each subset is linked in some order to the next in sequence. Chains are generally used to describe the state-transition structure of behavior in a system.

Change: An alteration of the system which may be either a change in capacity indicators (growth) or a change in the actual nature of feasible choices (development). Change covers both the growth and development concepts in planning in the same way as vector quantities involve both change of direction and magnitude.

Choice (in a pattern): Since the planner is always working with standards of behavior, limiting without actually regulating the actual behavior of a system, the problem of choice arises. The decision is to select one particular subset from a range of subsets specified by a pattern to reduce uncertainty. In the zoning problem patterns of choice applied to decision areas are used to systematically reduce the freedom of choice until only one configuration is possible, determining the decision.

Classification (functional decomposition): For any relation there will exist a pattern of maximal sub-structures uniquely associated with the given relation. Although aspects of the pattern may be common to more than one such structure, every possiblity in the relation is covered by at least one standard structure.

Clique: A maximal complete sub-graph of a graph based upon some signal relation. Every element in the clique is related to every other element. The clique describes subset of a simplex denoting the possibility of simultaneous choice of all of its elements.

Combinatorial: A system of complete enumeration of all possible configurations which may be possible in a given situation (i.e. combinatorial decision problems).

Combination: A combination is any subset defined without regard to order of the constituents of a set (i.e. ABC and CBA are different permutations of a set, but are the same combination).

Completeness (of description): The logical requirement that every possible subset of a standard of behavior be viewed as a possible form of behavior in the system. Therefore it is the aim of a natural process of description to account for both the gaps (or forbidden choices) and also the relative frequency of occurrence of particular forms of behavior. Completeness is an idea which provides the possibility of describing dynamic behavior on a relatively static backcloth.

Component (as in q-component): A combination of sub-structures in which every substructure included in the component is related to at least one other substructure in the component.

<u>Conservation Law</u>: A constraint upon the total capacity of the structure which requires that available capacity be shared, such that every transaction must maintain a total balance between the capacity used and capacity available. Conservation is an accommodation rule which determines legal combinations of patterns from those which are illegal and requires symmetrical conditions to describe the transactions compatible with the system. The idea of a conservation law will describe the minimum requirements for a process to be considered urban-like.

<u>Constructability</u>: A requirement for orderliness that is applied to a sequence of transactions in a system. Distinct structures are formed combinatorically from primitive elements of the system in a sequence of steps which are each decideable by the rules of the system. In logical discussions some decideability criterion is used to distinguish the permissible forms of substructure, but in application to a scientific method, plausibility criteria are used instead to permit deductions from more doubtful, but reasonable, concepts and relations.

<u>Cover</u>: A fundamental consistency structure describing a standard of behavior. A cover is a family of subsets such that every subset structure representing possible forms of behavior is included in at least one subset of the cover. Not every subset of the cover is necessarily a permissible form of behavior.

<u>Consensus</u>: A basis of agreement upon the common properties of some concept or concepts. A consensus in general constitutes an ordering of viewpoints.

Cycle (loop): Is a simply ordered chain of structures in which the only structure that occurs more than once is the initial and final structure. Any constrained behavior must be cyclic.

Decision Area: A concept borrowed from Analysis of Interconnected Decision Areas to describe the situation where an element of the backcloth which interacts in some way with the other elements is to have a unique standard of behavior arbitrarily assigned to it that is compatible with the choices made simultaneously on the other decision areas.

Decision Problem: The condition confronted when one tries to assign a unique choice from the family of possible choices to an area of the backcloth. The choice sets are mutually exclusive alternatives and only one may apply to any decision area; but the choices made on all decision areas must, as a whole, be as independent as possible. The decision areas, by virtue of the residents living on them, may partially determine the choice by delimiting the set of standards acceptable to the residents; while the mutually incompatible functional standards also partially determine which configurations of choice are independent or interacting between the decision areas. An algorithm such as the Analysis of Interconnected Decision Areas is required to determine the feasible choice configurations on the study area.

Delphi Technique: Denotes any methodology which attempts to impose the maximum degree of structure upon the experience of the individual with respect to a particular protocol of observations by transforming it into a structure in which the experience of different individuals can be directly compared.

Describeability: The concept denoting those urban patterns of choice which can be represented by set-like structures. Patterns of behavior that cannot be reduced to this form are considered to be too dynamic for structural analysis. The ontological criteria to distinguish describeable urban behavior from the dynamic forms of behavior were discussed in detail.

Design: A process occurring in a number of stages, such as analysis, synthesis, and evaluation, in which data is collected, classified; then used to form potential solutions to a problem; and are ultimately evaluated to select the best possible answer to the problem.

Design Problem (zoning problem): Is a topological problem in which desirable functional relationships expressing compatibility and convenience are transformed into physical structures expressing siting patterns that are consistent with some constraining cover without excessively distorting the functional forms of relationship with which the analysis commenced.

Development: A structural change in which new forms of possible choice are incorporated in an existing structure.

Distinct: Two subsets are said to be distinct or discriminated when neither is a subset of the other (they each contain distinct elements). A partial order fails to completely order distinct subsets relative to each other and they are to be regarded as mutually exclusive alternatives.

Distribution: Measurement concerned with comparison of behavior to some standard essentially determines which of the alternate standard structures are consistent with actual behavior one at a time. In contrast, design attempts to study the distributions of possible standards across the whole structure and their inter-relationships as a whole.

Environmental Cluster: Is a term used to designate a mathematical structure representative of actually or potentially existing urban structure. It introduces the possibility of distinction between mathematical structures and urban structure to describe the interface between forms of mathematical structure representing possible forms of urban structure which is capable of mathematical representation. Use of this term is a reminder that not all urban phenomena are describeable in this way.

Epistemic Correlation: Sets can be used to establish an epistemic correlation between concepts and actual data since they form a common medium to both distinct types of entities. A correlation consists of a definite choice of some elements by a specific individual at a specific location in time and space, who is subject to known external conditions of observation.

Epistemology: The aim of epistemology for the purposes of this thesis is to describe how the concept of relationship may be used to transform a definite individual's perceptions into suitable categories of behavior forming a group's consensus on to the description of normal behavior in a definite area of interest. It is concerned with how one may have the means of describing experience in any case.

Equality: In principle, a statement that two structures are identical in every respect including their constituents and their ordering.

Equivalence: Is a much weakened form of the equality relation which demands that possibly distinct objects are similarly ordered with respect to some of their characteristic properties, and generally represent an agreement that for some purposes these objects can be treated as the same, without necessarily being the same in all aspects.

Explanation: This concept denotes a transformation from a situation which is poorly understood into new terms of description whose significance is more readily appreciated, and which also preserves some degree of homologous correspondence to the terms of the original situation. Explanation is usually limited to particular aspects of the situation.

Force: A change in a structure which has two aspects such that the change may be either in intensity (concerned with alteration of an indicator of choice (growth)), or in development (alteration of the occurrance of a given possibility in the structure). A force is usually defined upon some backcloth providing a basis for comparison, and can be used to reflect the costs due to the stress of non-rationality.

Formalization: Is a social process which occurs within a group in which both the concepts and the means of discussion of those concepts are reconstructed to make them decideable for a successively greater range of circumstances; i.e., a plan is a formalization in which the situation is simplified for discussion with explanations to reduce the costs of decision, and to specify the action sequences leading to objectives.

Functional Aspects: Denotes those patterns of the urban observational structure which provide significant services to the urban resident.

Framework: Is a particular structure serving as a backcloth that permits the existence of significant phenomena to be recognized, and the relations between these forms to be readily appreciated.

Fundamental Problem of Planning: The fundamental problem of planning is the process of transforming distinct local forms of experience or preferences of a system into global standards behavior which are compatible with the available physical resources, and will accommodate the maximum volume of local preferences.

Geometric Realization: Is a particular concrete representation of a complex using graphs or matrices to facilitate data processing.

Global Property: Is the most general set of properties which all members of a structure have in common.

Grading of a Structure: Is the ranking of components defined in a structure based upon increasing q-connectivity between the structures, or alternately according to a sequence of signal level indicators.

Graph: Is a form of mathematical structure which is a concrete representation of a mathematical relation using points to represent vertices of the relation and arcs to join the related vertices.

Grounds (of inquiry): Are the justifications which identify the most significant fundamental concepts distinguishing the objects of concern and their different forms of relationship, which cause them to be accepted by a consensus of the profession.

Growth: A change in an indicator attached to the existence of a particular form of phenomena which does not introduce new possible forms of choice into the structure.

Hierarchy: Is the recognition that every structure as a whole may enter into relationships with other whole structures at different levels of concern. The significance of a given level of a structure is more readily appreciated in relation to other structures at a higher level, i.e., in a hierarchy the function of a group is defined locally (at a lower level), but its role is appreciated globally.

Hole: A q-hole is a minimal cycle of q-connected structures required to realize the representative simplex of a given component. A hole is the basis of representation of the component and represents the best one can expect to do in consistently incorporating a given protocol into a constructible realization of that protocol at some level of q-connectivity in some component of the structure. A hole corresponds to the idea of a social object in the same way as Bertrand Russel defined a concept of "matter" out of sensory perceptions as a limit.

Independent: The distinct (maximal) subsets of a complete cover are mutually exclusive in that at most one may be chosen under particular circumstances, but in considering a sequence of decision it is desirable that every successive choice may be made independently of choices which have gone before. Independent choices can only approximately be realized by studying the phase relationships of the structure.

Intersection: Is a fundamental set operation which defines a new set in terms of the elements held in common between a number of sets.

Local Property: Is a property which is true under particular circumstances in some part of a structure but not necessarily true for the whole structure.

Magnification: This describes the process of discriminating successively finer structures in a observational protocol that approach the limits of observation where the protocol is fully resolved. In magnification the subsets of the backcloth are composed into new subsets by virtue of some similarity to generate the largest subsets that are internally consistent. These largest sets form a new cover family of the protocol that are distinct from each other in some respects while internally consistent in other respects.

Mapping (transformation): Is a function defined upon the elements of one structure as a domain which relates each element of the domain to at most one element of the object structure which the domain is mapped into.

Ontological Criteria: Criteria which delimit the interest of an observer to forms of observations which are publically observable, repeatable, and consistent with the observational capabilities of a science.

One-Stop Shopping Area (Clique): A particular type of urban structure which denotes a set of urban activities that can be chosen simultaneously relative to the signal relation that governs the definition of the cliques, i.e., the one stop shopping area is a set of activities that can be reached by parking in the area and walking no more than a specified distance to reach every activity in the clique in turn.

Obstruction: The condition described by the Q-analysis caused by missing combinations of choice in the simplicial structure disrupting the free flow of patterns of choice.

<u>Ordering</u>: A form of relation defined over a set of objects which establishes a system of precedences between different subsets that may be distinguished in the relation. Orderings are usually classified by the extent to which they possess the properties of being reflexive, symmetric, and transitive.

<u>Pattern</u>: A concept used in several senses in this thesis. A system of sub-sets distinguished by a structure may be regarded as a pattern of information in the sense that it is a preliminary announcement of the extent to which information may become available as the range of choice among the subsets is narrowed down by the application of data. A pattern may also be thought of as the system of numbers which are attached to the elements of structure to specify the data.

<u>Perception</u>: A particular part or aspect of the whole which is singled out by an act of attention at some instant. A perception distinguishes a part of the whole pattern as data which is actually observed.

<u>Phase</u>: An aspect of structural analysis which arises by virtue of the requirements, in actual decisions, to distinguish between the presence of the influence of an activity in an area and the actual presence of the activity. These have different outcomes for decision and the discrimination of phase relationships enables the planner to study the manner in which the parts of the system fit together as a new whole by systematically generating phase transitions consistent with the structure as it develops in time or extends through the whole structure.

Phase Space: In simplest terms the phase space is the totality of different combinations of observations that are logically possible for a given observational protocol. A variety of different analyses may be performed, using the ordering of observations on the phase space under different circumstances and for different signal properties, to determine which states of the phase space are accessible to the observer and which are not.

Protocol: The set of elements of observation which are determined by the group to have in common some property which places them in the urban domain of observation. The protocol represents a consensus of urban planners which defines the scope of urban phenomena to be recognized.

Q-connection: Simplices which are discriminated as being distinct under the face ordering of the simplicial complex may nevertheless be equivalent in terms of some common shared face which defines their q-connection.

Quasi-static: A concept of physical thermodynamics which defines the essential conditions to which a process must conform to in order to be formally describeable. Quasi-static may also describe the nature of action sequences in strategic planning.

Rational Process: Any discriminable events forming a decideable sequence in which the order of succession of the events in the process is connected by well-defined rules. Any process is rational in this sense.

Relation: A rule which assigns to each element of a set of objects (the domain) elements of another set (the range).

Regulation: A stage of the implementation process concerned with the matching of signal tolerances to the capacity of a structure specifying the physical resources of the area. Whereas the structure of an area may in principle support a certain variety of choice, there remains the question of the freedom of individual residents to actually use the choices given the physical resources available in the neighborhood. A given pattern of choice may be consistent with the structure of the neighborhood, but insufficient capacity may not accommodate the pattern physically.

Resolution: This concept describes the extent to which discrimination of the finer structure in a protocol is correlated with the discrimination of the finer structure of the observational context associated with the protocol. Although a great number of states may be potentially capable of resolution in the protocol, there is no guarantee that the apparatus of observation can observe all of them in any context. In the limit of resolution, the 2^{N-1} non-null subsets of any protocol would have observational significance in some context.

Signal Property: The different subsets actually observed that are consistent with the urban observational protocol are associated with some relation that is said to signal the influences between the urban activities. Where possible the signal relation should have an assigned level structure that enables the planner to distinguish the degree of tolerance to different signals in the environment.

Scale Property: Certain signals are common to all urban activities and therefore generate the observational protocol. The scale is therefore the particular signals which define the context in which one shall recognize the phenomena as being essentially urban.

Simultaneity: This concept is particularly associated with the question of when a particular collection of entities shall be considered as an entity in its own right, or in other words a new whole composed of smaller wholes having properties that are particular to the collection as a whole. For example, the collection of perceptions experienced by an individual at one instant is a whole by virtue of simultaneous occurrance, but the collection of a group of remembered perceptions common to an individual that may have occurred at different instances is a whole of quite a different type. Even more so is the collection of the experiences of a group of individuals. Structures may aggregate experiences in these different ways, but one significant question pertaining to the realization of these structures in a concrete way will be the question of the largest sets of choice which can be chosen in a single instance.

Simplex: The structure defined by a relation in which every element contained in the simplex has a common first element in the domain of the relation.

Simplicial Complex: A collection of simplices belonging to the same relation, usually taken to describe an actual observed process.

Standard of Behavior: A set of observations which is maximal under some relation so that it may be regarded as a constraint prohibiting some occurrence of particular choices in the structure. Every subset of a standard of behavior is held to be a possible occurrence, but the standard does not regulate either particular occurrences or mandate the occurrence of some particular choice.

Strategic: A planning concept associated with a sequential decision process where choice at some stage of decision delimits the range of future choices without specifying which choice shall be made in the future.

Tolerance: A concept applied to a standard which is discriminated in terms of an observational parameter such as distance, and which permits an individual to specify a range of this parameter that is acceptable to him for the given signal relation being studied. Tolerance relations provide the signal level structure of preferences which must be matched to the patterns of influence generated by different siting patterns of a backcloth. They are fundamental to the study of phase structure in a complex.

Urban Structure: The set of activities correlated by a signal relation to some observer at a specific location in time and space, who is subject to some specified conditions of observation.

Uncertainty: Three kinds of uncertainty are distinguished in a planning science, uncertainty of observation, uncertainty of value, and uncertainty of relationship.

ANNEX B

THE MATHEMATICAL RELATION

1. Representation of a Relation

The matematical binary relation representing the signalling between subsets of the protocol and the backcloth on which the protocol is resolved compares two finite sets to each other. Consider the set (Y_i) i = 1, m of elements of the backcloth called the domain of the relation and the set (X_j), j = 1, n of elements of the protocol, called the range. All possible comparisons of the elements of (Y_i) to the elements of (X_j) is represented by the cartesian set of ordered pairs (Y_i,X_j). Any binary relation R will be a subset of (Y_i,X_j). Thus $R c (Y_i,X_j)$ i = 1,m, j = 1,n, and (Y_i, X_j) belong to R if $Y_i R X_j$. The converse relation, R^{-1}, is naturally defined R^{-1} c (X_j, Y_i) with $(X_j, Y_i) \varepsilon R^{-1}$ whenever $(Y_i, X_j) \varepsilon R$.

A relation may be concretely represented by either an incidence matrix or a linear graph.

Consider a matrix $M = (M_{ij})$ such that $M_{ij} = 1$ if $(Y_i, X_j) \varepsilon R$
$= 0$ otherwise.

The incidence matrix of the converse relation R^{-1} is denoted by the transpose of $M(M^T)$.

The linear graph of a relation can be drawn simply by joining each pair of vertices of a graph that are in the relation.

For structural analysis it is necessary to consider the concept of weighted relation. The entries in the incidence matrix of a weighted relation are the integers in addition to 0 or 1. From any weighted relation a whole series of incidence matrices describing a complex can be obtained by slicing the matrix.

The $(\emptyset_{ij})$ are a set of slicing parameters for a weighted matrix N. Define the incidence matrix of a relation, M, that is covered by N in terms of the elements (n_{ij}) as follows:

$M_{ij} = 1$ if $n_{ij} \geq \emptyset(n_{ij})$ $\quad$ $\emptyset_k(n_{ij})$ denotes the slicing parameter

$= 0$ otherwise $\quad$ associated with matrix element n_{ij}.

The slicing procedure is important in sealing with weighted relations representing the signal levels of individual preference relations.

2. Intersection of Relations

In order to deal with the trade-off of preferences in a phase space representation, such as that discussed in the urban zoning problem on either a complex of simplices or a complex defined on cliques, the intersection of relations is necessary. The example of intersection as described here requires that two different relations be compared on a common backcloth. In the context of urban land this means that possible forms of relation between signal relations are considered only if they occur on the same areas of urban land.

The intersection of two signals, say R_1 and R_2, can define a new relation $R_{12} \subset (A1 \times A2) \times L$ where L is the common domain and A1 or A2 are the ranges of the two relations. R_{12} is defined if $x \varepsilon A1$ and $y \varepsilon A2$, then;

$((x,y,),L_i) \varepsilon R_{12}$ if and only if there exists an L_i such that $(L_i,x) \varepsilon R_1$ and $(L_i,y) \varepsilon R_2$.

The intersection of two complexes defined according to this rule can be determined by multiplying the incidence matrix of one complex times the incidence matrix of another complex. Refer to Mathematical Structure in Human Affairs, pages 121-124 for a more detailed discussion of the intersection of complexes.

3. Functions

A relation is the general case of the concepts of ordering (relation), function, and maximal cover sets. Since these concepts are necessary to the rationale of our study, they are briefly elaborated here.

A function (or mapping) is a special form of relation. It can be recognized in the incidence matrix as having at most one element in any matrix row. In a function at most one element of the range can correspond to any element of the domain. The element of the range set B corresponding to an element of A, called the image of A, is usually denoted $\mu(a) = b$. By virtue of the functional relationship $\mu(a) \neq \mu(a')$ implies $a \neq a'$ (but $\mu(a) = \mu(a')$ does not necessarily imply $a = a'$). The domain of the function must always be the whole of the domain set A. Therefore the domain must be restricted to the subset $(a) \varepsilon A$ such that $\mu(a) \varepsilon B$ for every $a \varepsilon A$. A mapping denotes the case when the whole of A is a function on B. A function is into when $\mu(A) \subset B$. If the function is onto, then every $b \varepsilon B$ is identified with a $\mu(a)$ for some $a \varepsilon A$. A mapping may also be one-one and into (injection). In this case $\mu(a) = \mu(a')$ implies $a = a'$, although some $b \varepsilon B$ may not be a $\mu(a)$ for some $a \varepsilon A$. When M is onto it is called a surjection. When a mapping is both injective and onto it is called a bijection.

A mapping settles the question of the distribution of elements of the range to elements of the domain in a very satisfactory fashion. It is a rule which says that for any choice from the domain a specific choice in the range must be made.

Given the set F = (Tom, Dick, Harry) the decision to award prizes of ($1, $2, $3, $4, $5) can be resolved by different mappings $M: F \to P$

1. An into map M, permits sharing of prizes and not all prizes are distributed

   M_1(Tom) = $1, M_1(Dick) = $1, M_1(Harry) = $5.

2. An injective map M_2 does not distribute all prizes but provides a unique one for each:

   M_2(Tom) = $1, M_2(Dick) = $2, M_2(Harry) = $4.

3. If the set of prize winners is extended a surjective map M_3 is possible:

   M_3(Tom) = M_3(June) = $1, M_3(Dick) = $2, M_3(Harry) = $3, M_3(Mary) = $4, M_3(Susan) = $5.

4. Supposing June drops out of the competition so that the competition set becomes restricted. A bijective map is possible.

   M_4(Tom) = $5, M_4(Dick) = $1, M_4(Harry) = $2, M_4(Susan) = $3, M_4(Mary) = $4.

   In this case no prizes are shared, no prizes escape, and no one fails to win a prize. The range of prizes is completely ordered by the domain of people.

4. Compatibility and Equivalence Classes

Suppose one wished to classify a group of people with respect to their punctuality, one would use a relation such as x is more punctual than y. As long as every person arrived at different times and every person usually at a meeting attended the meeting, one could order the set of individuals according to the order in which they arrived at a meeting. If there was a group of people absent, the order would be only partial since it is impossible to apply this definition of the ordering to those absent from the meeting. Moreover, if some people arrived together, then the order would also fail. (Two people arriving together are incomparable in terms of the order relation is more punctual than, although these elements each can be compared to other elements in terms of the relation). By making an abstraction to the effect that individuals arriving together are equivalent for the purposes of the relation and that those individuals absent from the meeting are all equivalent to each other, a distribution of individuals that is completely ordered is derived. If one chose to describe the usual behavior of individuals at a series of meetings such that the ordering relation is no longer completely transitive, then the individual could arrive at different points for different meetings as long as he was always more punctual than another. This would define a partial ordering of the sets with some individuals appearing in more than one of the classes.

The use of classes to govern distributions of a field of elements to be ordered transforms every possible relation of a given protocol bijectively onto every possible set consisting of all of the maximal sets under the ordering relation. The various sets of the set of standards govern the possible distributions of the elements representing behavior to be explained. This is the essence of the theory of explanation using mathematical structure to impose some rational sense upon complex behavior.

annex c

A FORTRAN PROGRAM FOR Q-ANALYSIS

ANNEX C

```
,*** PSU-WATFIV  PROGRAM TO DETERMINE Q-ANALYSIS
C     ***********************************************************************
C     *
C     * PROGRAM : TOPONOQ(-HOLE SEARCH)
C     *
C     *     THIS PROGRAM ANALYZES THE STRUCTURAL TOPOLOGY OF A DATA
C     *          SET UTILIZING THE METHODOLOGY OF DR. R. H. ATKIN
C     *
C     *     INPUT :
C     *
C     *     PARAMETER CARD -
C     *
C     *           COLS.   1 - 4     NUMBER OF ROWS IN INPUT MATRIX
C     *
C     *           COLS.   5 - 8     NUMBER OF COLS. IN INPUT MATRIX
C     *
C     *           COL.       12     PUNCH OPTION  -  PUNCH = 1
C     *
C     *           COL.       16     NUMBER OF FORMAT CARDS    (MAX. OF 5)
C     *
C     *           COL.       20     PRINTING OF INPUT MATRIX
C     *                                  PRINT WHEN COL. 20 = 1
C     *
C     *     TITLE CARD -
C     *
C     *           COLS.  1 - 80     TITLE  (MAX. OF 80 CHARACTERS)
C     *
C     *     FORMAT CARD(S) -
C     *
C     *           COLS.  1 - 80      FORMAT FOR INPUT MATRIX (20A4)
C     *
C     *     NAMES CARDS -
C     *
C     *           (1) LIST OF NAMES FOR ROW ITEMS (NROWS OF THEM).
C     *
C     *           (2) LIST OF NAMES FOR COLUMN ITEMS (NCOLS OF THEM).
C     *
C     *            - IN FORMAT (16(1X,A4))
C     *
C     *            - START EACH LIST ON A NEW CARD.
C     *
C     *
C     *     DATA CARDS -
C     *
C     *                 - READ ACCORDING INPUTED FORMAT
C     *                 - MAXIMUM SIZE (150,150)
C     *
C     *     USUAL JOB SPECIFICATIONS -
C     *
C     *                              -  T=25,P=2500,S=280
C     *
C     *      PROGRAM VERSION AS OF MARCH 30,1976. Written by
C     *      Stephen L. Morgan, Pennsylvania State University
C     *      Modified by I.D. McCreath, University of Manitoba,
C     *      to determine the face ordering of the Complex.
C     *
C     ***********************************************************************
C
C
```

```
      IMPLICIT INTEGER*2 (A-Z)
      DIMENSION  FMT(100),TITLE(20),INPUT(150,150), ORDER(150,150)
      INTEGER*4 FMT,TITLE,NAMES,CNAMES(150)
      COMMON MAT(150,150),NAMES(150)
C
C     INPUT - INPUT MATRIX OF ORIGINAL DATA
C     MAT   - DERIVED SIMILARITY MATRIX OF Q-CONNECTIVITES (ATKINS)
C
C
      CALL SETTAP('TN',16)
C          SETTAP IS A P.S.U. COMPUTER CENTER PRINTER FORMS SUBROUTINE
C
      DO 10 I=1,150
      DO 10 J=1,150
      INPUT(I,J)=99
      ORDER(I,J) = 0
   10 MAT(I,J) = -1
      READ(5,1000) NROWS,NCOLS,IPUNCH,NFMT,IPRINT
 1000 FORMAT(5I4)
C
      READ(5,1001) (TITLE(I),I=1,20)
      WRITE(6,2002) (TITLE(I),I=1,20)
 1001 FORMAT(20A4)
 2002 FORMAT('1',20A4)
C
      IF(NFMT.GE.1.AND.NFMT.LE.5) GO TO 17
      IF(NFMT.LT.1) WRITE(6,2000)
 2000 FORMAT('-','NUMBER OF FORMAT CARDS NOT SPECIFIED')
      IF(NFMT.GT.5) WRITE(6,2001)
 2001 FORMAT('-','NUMBER OF FORMAT CARDS EXCEEDS 1')
      GO TO 999
C
   17 NFMT=NFMT*20
      READ(5,1002) (FMT(I),I=1,NFMT)
 1002 FORMAT(20A4)
      NFMT=NFMT/20
      WRITE(6,2003)NROWS,NCOLS,IPUNCH,IPRINT,NFMT,(FMT(I),I=1,NFMT)
 2003 FORMAT('-','USER SPECIFICATIONS'/'0',I10,' - NUMBER OF ROWS',
     */' ',I10,' - NUMBER OF COLS.'/' ',I10,' - PUNCH OPTION ',
     *'- PUNCHED OUTPUT = 1'/' ',I10,' - PRINTING OF INPUT MATRIX, PRINT
     * = 1'/' ',I10,' - NUMBER OF FORMAT CARDS',
     */' ',10X,'   FORMAT FOR READING INPUT MATRIX :',5(/' ',20X,20A4))
C
      READ(5,1003) (NAMES(I),I=1,NROWS)
      READ(5,1003) (CNAMES(I),I=1,NCOLS)
      WRITE(6,2009) (NAMES(I),I=1,NROWS)
 1003 FORMAT(16(1X,A4))
      WRITE(6,2010) (CNAMES(I),I=1,NCOLS)
 2009 FORMAT(' ','ROW NAMES -'/(' ',T10,20(2X,A4)))
 2010 FORMAT(' ','COLUMN NAMES -'/(' ',T10,20(2X,A4)))
C
      DO 18 I=1,NROWS
   18 READ(5,FMT) (INPUT(I,J),J=1,NCOLS)
      IF(IPRINT.NE.1) GO TO 39
      WRITE(6,2004)
 2004 FORMAT('1','INPUT MATRIX')
      CALL MATOUT(INPUT,NROWS,NCOLS)
C
   39 MAXQ=0
C
```

```
C      MAXQ=0 DUMMY VALUE HERE, INITIALIZED TO CALL LATER IN SUBPROGRAM  00000122
C           MAXMAT                                                        00000123
C                                                                         00000124
       DO 20 I=1,NROWS                                                    00000125
       DO 20 L=I,NROWS                                                    00000126
       DO 20 J=1,NCOLS                                                    00000127
C                                                                         00000128
C      AT THIS POINT, WE ARE CREATING THE Q-CONNECTIVITIES MATRIX         00000129
C                                                                         00000130
C           TAKING EACH ROW IN TURN AND COMPARING IT WITH (1) ITSELF TO  00000131
C           CREATE THE DIAGONAL, AND (2) ALL SUBSEQUENT ROWS.            00000132
C                                                                         00000133
       IF(INPUT(I,J).NE.INPUT(L,J).OR.INPUT(I,J).EQ.0) GO TO 20          00000134
       MAT(I,L)=MAT(I,L)+1                                                00000135
       MAT(L,I)=MAT(I,L)                                                  00000136
    20 CONTINUE                                                           00000137
C    'INSERT CODE SEGMENT HERE'                                          00000138
C      WE ARE CALLING SUBPROGRAM MAXMAT WHICH WILL FIND THE MAXIMUM       00000139
C           ELEMENT IN THE Q-CONNECTIVITIES MATRIX (MAT). THIS IS MAXQ   00000140
C           INITIALIZED ABOVE.                                           00000141
C                                                                         00000142
       CALL MAXMAT(NROWS,MAXQ)                                            00000143
C                                                                         00000144
       WRITE(6,2005)                                                      00000145
  2005 FORMAT('1','SHARED FACE MATRIX - KROW(COLUMN)')                    00000146
       CALL MATOUT(MAT,NROWS,NROWS)                                       00000147
       IF(IPUNCH.NE.1) GO TO 22                                           00000148
       WRITE(7,3000)                                                      00000149
       WRITE(7,3002) ((MAT(I,J),J=1,NROWS),I=1,NROWS)                     00000150
  3000 FORMAT(' '/'SHARED FACE MATRIX - KROW(COLUMN)')                    00000151
  3002 FORMAT(20I4)                                                       00000152
    22 CONTINUE                                                           00000153
C                                                                         00000154
C      WE ARE CALLING SUBPROGRAM QFINDR WHICH FORMS THE MAIN              00000155
C           ANALYTICAL PART OF THE PROGRAM.                              00000156
C                                                                         00000157
       CALL QFINDR(NROWS,MAXQ)                                            00000158
C                                                                         00000159
C      HAVING COMPLETED THE ANALYSIS ON THE ORIGINAL Q-CONNECTIVITES     00000160
C           MATRIX, WE REPEAT THE ANALYSIS ON THE TRANSPOSED INPUT       00000161
C           MATRIX TO CREATE A SECOND Q-CONNECTIVITES MATRIX.            00000162
C      RATHER THAN ACTUALLY TRANSPOSING, WE COMPARE THE COLS. INSTEAD    00000163
C           OF THE ROWS OF THE INPUT MATRIX.                             00000164
C                                                                         00000165
       DO 25 I=1,NCOLS                                                    00000166
       NAMES(I)=CNAMES(I)                                                 00000167
       DO 25 J=1,NCOLS                                                    00000168
    25 MAT(I,J)=-1                                                        00000169
C                                                                         00000170
       DO 30 J=1,NROWS                                                    00000171
       DO 30 I=1,NCOLS                                                    00000172
       DO 30 L=I,NCOLS                                                    00000173
       IF(INPUT(J,I).NE.INPUT(J,L).OR.INPUT(J,I).EQ.0) GO TO 30          00000174
       MAT(I,L)=MAT(I,L)+1                                                00000175
       MAT(L,I)=MAT(I,L)                                                  00000176
    30 CONTINUE                                                           00000177
C                                                                         00000178
       MAXQ=0                                                             00000179
       CALL MAXMAT(NCOLS,MAXQ)                                            00000180
C                                                                         00000181
```

```
       THE FOLLOWING CODE SEGMENT WAS
       INSERTED BETWEEN CARD NUMBER
       138 and 139 IMMEDIATELY FOLLOWING
       THE STATEMENT " 20 CONTINUE"

       THIS CODE SEGMENT TESTS THE
       FACE ORDERING OF THE INPUT
       MATRIX USING THE INFORMATION
       CONTAINED IN THE MATRIX MAT(I,J)
       TO DETERMINE THE PARTIAL ORDERING
       WHICH IS OUTPUT IN STRUCTURE
       FORM USING THE MATRIX ORDER(I,J)

       DO 21 I = 1, NROWS
       DO 21 L = 1, NROWS
       IF( MAT(I,L).EQ.MAT(I,I)) ORDER(I,L)=1
       IF(MAT(I,L).EQ.MAT(L,L))ORDER(L,I)=1
       IF(ORDER(I,L).NE.0.AND.ORDER(I,L).
         EQ.ORDER(L,I))ORDER(I,L)=2
       IF(ORDER(I,L).EQ.2)ORDER(L,I)=2
    21 CONTINUE

       WRITE(6,2071)
  2071 FORMAT('1', 'PREDEDENCE STRUCTURE')
       CALL MATOUT(ORDER,NROWS,NROWS)
       WRITE(6,2027)
  2027 FORMAT('1', 'STRUCTURE OF DIRECT
         PREDACESSORS IN COMPLEX')

       DO 2028 ICL = 1, NROWS
       DO 2028 JCL = 1, NROWS
       DO 2028 LCL = 1, NROWS
       IF (ORDER(ICL,JCL).EQ.0)GO TO 2029
       IF (ICL.NE,JCL) GO TO 2030
       ORDER(ICL,JCL)=0
       GO TO 2029
  2030 MARK = ORDER(ICL,LCL)*ORDER(LCL,JCL)
       IF (MARK.NE.1) GO TO 2031
  2031 CONTINUE
  2029 CONTINUE
  2028 CONTINUE
       CALL MATOUT(ORDER,NROWS,NROWS)
```

```
      WRITE(6,2007)
 2007 FORMAT('1','SHARED FACE MATRIX - KCOLUMN(ROW)')
      CALL MATOUT(MAT,NCOLS,NCOLS)
      IF(IPUNCH.NE.1) GO TO 35
      WRITE(7,3001)
      WRITE(7,3002) ((MAT(I,J),J=1,NCOLS),I=1,NCOLS)
 3001 FORMAT(' '/'SHARED FACE MATRIX - KCOLUMN(ROW)')
   35 CONTINUE
C     CALL ECCEN(NCOLS)
      CALL QFINDR(NCOLS,MAXQ)
C
  999 STOP
      END
```

```
      SUBROUTINE MAXMAT(N,MAXQ)
      IMPLICIT INTEGER*2 (A-Z)
      INTEGER*4 NAMES
      COMMON MAT(150,150),NAMES(150)
      DO 10 I=1,N
      IF(MAXQ.LT.MAT(I,I)) MAXQ=MAT(I,I)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MAXLST(LIST,N,CURENT)
      IMPLICIT INTEGER*2 (A-Z)
      DIMENSION LIST(150)
      MAX=0
      DO 10 I=1,N
      IF(MAX.LT.LIST(I)) MAX=LIST(I)
   10 CONTINUE
      CURENT=MAX+1
      RETURN
      END
```

```
      SUBROUTINE QFINDR(N,Q)
C
C     THE SUBPROGRAM PARAMETERS ARE THE SAME AS IN THE MAIN
C          PROGRAM.  AT FIRST, Q HAS THE VALUE OF MAXQ, BUT THIS
C          WILL DECREASE BY ONE AT EACH STAGE OF THE ANALYSIS, UNTIL
C          Q = -1.
C
C
C
C     COMP   -  A VECTOR OR LIST IN WHICH THE NUMBERS OF THE COMPONENTS
C                  ARE STORED, TO WHICH EACH ELEMENT IS ASSIGNED
C                  AT EACH VALUE OF Q.
C
C     CURENT  -  THE VALUE OF THE COMPONENT NEXT TO BE ASSIGNED.
C
C     FLAG  -  EQUAL 0, WHEN NEXT VALUE OF CURENT=CURENT+1
C              EQUAL 1, WHEN NEXT VALUE OF CURENT<PRESENT VALUE
C
C     ISTPUT  -  THE STRUCTURE VECTOR, WHICH IS ALSO THE OBSTRUCTION
C                   VECTOR WHEN UNITY IS SUBTRACTED FROM EACH ELEMENT.
C
      IMPLICIT INTEGER*2 (A-Z)
      DIMENSION COMP(150),ISTPUT(150)
      COMMON MAT(150,150),NAMES(150)
      INTEGER*4 NAMES
C
      KOUNTR=1
   10 DO 20 I=1,N
   20 COMP(I)=0
   30 CURENT=1
```

```
      FLAG=0
C
      DO 40 I=1,N
      IF(MAT(I,I).LT.Q) GO TO 40
C
C
C     IF DIAGONAL CELL (MAT(I,I)) < Q THE ITEM CANNOT BE IN ANY
C          CLUSTER AT THIS POINT.
C     IF DIAGONAL CELL IS NOT < Q THE ITEM HAS TO BE ASSIGNED TO
C          SOME COMPONENT
C
C
      COMP(I)=CURENT
      DO 50 K=1,N
C
C     CHECKING ALL CELLS IN ALL ROWS WHOSE DIAGONAL CELL IS NOT
C          < Q, WE ARE ALSO RESOLVING ANY CONFLICTS IN ASSIGNMENT
C          VALUES DUE TO FOMERLY DESIGNATED CLUSTERS THEMSELVES
C          JOINING TOGETHER.
C
      IF(MAT(I,K).LT.Q) GO TO 50
      IF(COMP(K).EQ.0) COMP(K)=COMP(I)
      IF(COMP(K).EQ.COMP(I)) GO TO 50
      IF(COMP(K).GT.COMP(I)) GO TO 60
      GO TO 70
   60 CURENT=COMP(K)
      FLAG=1
      DO 80 J=1,N
      IF(COMP(J).EQ.CURENT) COMP(J)=COMP(I)
   80 CONTINUE
   70 CONTINUE
      IF(COMP(K).LT.COMP(I)) GO TO 90
      GO TO 50
   90 CURENT=COMP(I)
      FLAG=1
      DO 100 J=1,N
      IF(COMP(J).EQ.CURENT) COMP(J)=COMP(K)
  100 CONTINUE
   50 CONTINUE
      IF(FLAG.GT.0) GO TO 110
      CALL MAXLST(COMP,N,CURENT)
  110 FLAG=0
   40 CONTINUE
      KOUNT=0
      CALL OUTPUT(COMP,N,Q,KOUNT)
      ISTPUT(KOUNTR)=KOUNT
      KOUNTR=KOUNTR+1
      Q=Q-1
      IF(Q.LT.0) GO TO 120
      GO TO 10
  120 KOUNTR=KOUNTR-1
      KOUNTQ=KOUNTR-1
      WRITE(6,2000) KOUNTQ,(ISTPUT(I),I=1,KOUNTR)
 2000 FORMAT('3',T48,'FROM',T52,I2,T60,'TO .... 0'/' ',T10,'STRUCTURE VE
     *CTOR',T50,20I4,(/T50,20I4))
      DO 130 I=1,KOUNTR
  130 ISTRUT(I)=ISTPUT(I)-1
      WRITE(6,2020) KOUNTQ,(ISTPUT(I),I=1,KOUNTR)
 2020 FORMAT('-',T48,'FROM',T52,I2,T60,'TO .... 0'/' ',T10,'OBSTRUCTION
     *VECTOR',T50,20I4,(/T50,20I4))
```

```
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(LIST,N,Q,KOUNT)
      IMPLICIT INTEGER*2 (A-Z)
      DIMENSION LIST(150),OUTLST(150)
      INTEGER*4 NAMES
      COMMON MAT(150,150),NAMES(150)
      MAX=0
      CALL MAXLST(LIST,N,MAX)
      MAX=MAX-1
      ISTART=1
      IFLAG=0
    5 DO 10 I=1,N
      IF(LIST(I).EQ.ISTART) GO TO 20
   10 CONTINUE
      NEXT=ISTART+1
      DO 30 I=1,N
      IF(LIST(I).GE.NEXT) LIST(I)=LIST(I)-1
   30 CONTINUE
   20 ISTART=ISTART+1
      IF(ISTART.GT.MAX) GO TO 40
      GO TO 5
   40 CONTINUE
      WRITE(6,2000) Q
 2000 FORMAT('3','Q-VALUE  - ','(',I2,')')
      KOUNT=1
      KK=0
  110 DO 80 I=1,N
      IF(LIST(I).EQ.KOUNT) GO TO 90
      GO TO 80
   90 KK=KK+1
      OUTLST(KK)=I
   80 CONTINUE
```

```
188          IF(KK.EQ.0) GO TO 100
189          WRITE(6,2001) KOUNT,(NAMES(OUTLST(I)),I=1,KK)
190     2001 FORMAT('0',30X,'COMPONENT  - ','(',I2,')',T60,12(A5,' '),(/T66,11(
            *A5,' ')))
191          KK=0
192          KOUNT=KOUNT+1
193          GO TO 110
194      100 KOUNT=KOUNT-1
195          RETURN
196          END


197          SUBROUTINE MATOUT(ANYMAT,N,M)
       C
       C     -MATOUT- SUBROUTINE TO WRITE OUT ANY INTEGER MATRIX
       C     MATRIX TO BE PRINTED IS (ANYMAT)
       C     ANYMAT IS ASSUMED TO HAVE (N) ROWS, AND (M) COLUMNS
       C
       C                                                 STEPHEN L. MORGAN
       C                                                          (07/08/75)
       C
198          DIMENSION ANYMAT(150,150),PMT(7),AMT(5),FIELD(10),REPEAT(10),MM(16
            *),BMT(5),NBR(10),JLIST(150)
199          INTEGER*2 ANYMAT,N,M,IBIG,I,J,NN,MM,NUM,MA,MB,JLIST
200          INTEGER*4 AMT,BMT,PMT,REPEAT,FIELD
       C
201          DATA PMT(1),PMT(2),PMT(3),PMT(5),PMT(7)/'(1X,','I3,1','X,  ',
            *'I   ',')    '/,
            *AMT(1),AMT(3),AMT(5)/'(5X,','I   ','//)  '/,
            *FIELD/1H4,1H5,1H6,1H7,1H8,1H9,2H10,2H11,2H12,2H13/,
            *REPEAT/2H30,2H24,2H20,2H17,2H15,2H13,2H12,2H10,2H10,1H9/,
            *BMT(1),BMT(3),BMT(5)/'(   ','I   ',')   '/,
            *NBR/30,24,20,17,15,13,12,10,10,9/
       C
202          IBIG=-30000
203          DO 60 I=1,150
204       60 JLIST(I)=I
205          DO 30 I=1,N
206          DO 30 J=1,M
207       30 IF(IBIG.LT.ANYMAT(I,J)) IBIG=ANYMAT(I,J)
208          DO 40 I=1,10
209          IF(IBIG.LT.0) IBIG=-1*IBIG
210          IF(IBIG.LT.10**I) GO TO 50
```

```
   40 CONTINUE                                                          00000379
   50 FMT(6)=FIELD(I)                                                   00000380
      FMT(4)=REPEAT(I)                                                  00000381
      AMT(4)=FIELD(I)                                                   00000382
      AMT(2)=REPEAT(I)                                                  00000383
      BMT(4)=FIELD(I)                                                   00000384
      BMT(2)=REPEAT(I)                                                  00000385
      NUM=NBR(I)                                                        00000386
      MM(1)=0                                                           00000387
      DO 1 I=2,16                                                       00000388
      J=I-1                                                             00000389
    1 MM(I)=MM(J)+NUM                                                   00000390
      NN=(M-1)/NUM+1                                                    00000391
      IF(M.GT.195) GO TO 20                                             00000392
      DO 10 I=1,NN                                                      00000393
      IF(I.NE.1) WRITE(6,15)                                            00000394
   15 FORMAT('2','CONTINUED MATRIX'//)                                  00000395
      MA=MM(I)+1                                                        00000396
      J=I+1                                                             00000397
      MB=MM(J)                                                          00000398
      IF(MB.GT.M) MB=M                                                  00000399
      WRITE(6,AMT) (JLIST(J),J=MA,MB)                                   00000400
      DO 5 J=1,N                                                        00000401
    5 WRITE(6,FMT) J,(ANYMAT(J,K),K=MA,MB)                              00000402
   10 CONTINUE                                                          00000403
      RETURN                                                            00000404
   20 DO 25 I=1,N                                                       00000405
      WRITE(6,12) I                                                     00000406
   12 FORMAT('0','ROW',I4)                                              00000407
   25 WRITE(6,BMT) (ANYMAT(I,J),J=1,M)                                  00000408
      RETURN                                                            00000409
      END                                                               00000410

,COMPILE TIME=     0.59 SEC,EXECUTION TIME=     1.11 SEC, CORE USED-OBJECT=   15280 ,ARRAYS=   93100 ,TOTAL=  192512  BYTES
```

```
DATA FROM ATKIN'S "MATHEMATICAL STRUCTURE IN HUMAN AFFAIRS" - PG. 28

USER SPECIFICATIONS

        20 - NUMBER OF ROWS
        10 - NUMBER OF COLS.
         0 - PUNCH OPTION - PUNCHED OUTPUT = 1
         1 - PRINTING OF INPUT MATRIX, PRINT = 1
         1 - NUMBER OF FORMAT CARDS
             FORMAT FOR READING INPUT MATRIX :
                    (30I2)

ROW NAMES -
            P1    P2    P3    P4    P5    P6    P7    P8    P9    P10   P11   P12   P13   P14   P15   P16   P17   P18   P19   P20
COLUMN NAMES -
            A1    A2    A3    A4    A5    A6    A7    A8    A9    A10
```

INPUT MATRIX

	1	2	3	4	5	6	7	8	9	10
1	1	0	1	0	0	1	0	1	1	1
2	1	1	0	0	0	0	1	1	0	0
3	0	1	1	1	1	0	0	1	0	1
4	1	0	1	0	0	0	0	0	0	0
5	0	1	0	0	0	1	0	0	1	1
6	0	1	0	0	0	0	0	1	0	0
7	0	0	1	0	0	1	0	0	0	0
8	0	1	0	0	1	0	0	1	0	1
9	1	0	1	0	0	1	0	0	0	0
10	0	1	0	1	0	0	0	0	0	0
11	1	0	0	0	1	0	1	0	1	0
12	0	0	1	0	0	0	0	0	0	1
13	0	0	0	1	0	0	0	1	0	0
14	1	1	0	0	1	0	1	0	1	0
15	1	0	1	0	0	1	0	1	0	1
16	0	1	0	0	1	1	0	1	0	0
17	0	1	1	0	0	0	1	1	1	0
18	1	0	1	0	1	1	0	0	0	0
19	0	1	0	1	0	0	1	0	0	1
20	1	0	1	0	1	0	0	1	1	0

SHAPED FACE MATRIX - KROW(COLUMN)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	5	1	2	1	2	0	1	1	2	-1	1	1	0	1	4	1	2	2	0	3
2	1	3	1	0	0	1	-1	1	0	0	1	-1	0	2	1	1	2	0	1	1
3	2	1	5	0	1	1	0	3	0	1	0	1	1	1	2	2	2	1	2	2
4	1	0	0	1	-1	-1	0	-1	1	-1	0	0	-1	0	1	-1	0	1	-1	1
5	2	0	1	-1	3	0	0	1	0	0	0	0	-1	1	1	1	1	0	1	0
6	0	1	1	-1	0	1	-1	1	-1	0	-1	-1	0	0	0	1	1	-1	0	0
7	1	-1	0	0	0	-1	1	-1	1	-1	-1	0	-1	-1	1	0	0	1	-1	0
8	1	1	3	-1	1	1	-1	3	-1	0	0	0	0	1	1	2	1	0	1	1
9	2	0	0	1	0	-1	1	-1	2	-1	0	0	-1	0	2	0	0	2	-1	1
10	-1	0	1	-1	0	0	-1	0	-1	1	-1	-1	0	0	-1	0	0	-1	1	-1
11	1	1	0	0	0	-1	-1	0	0	-1	3	-1	-1	3	0	0	1	1	0	2
12	1	-1	1	0	0	-1	0	0	0	-1	-1	1	-1	-1	1	-1	0	0	0	0
13	0	0	1	-1	-1	0	-1	0	-1	0	-1	-1	1	-1	0	0	0	-1	0	0
14	1	2	1	0	1	0	-1	1	0	0	3	-1	-1	4	0	1	2	1	1	2
15	4	1	2	1	1	0	1	1	2	-1	0	1	0	0	4	1	1	2	0	2
16	1	1	2	-1	1	1	0	2	0	0	0	-1	0	1	1	3	1	1	0	1
17	2	2	2	0	1	1	0	1	0	0	1	0	0	2	1	1	4	0	1	2
18	2	0	1	1	0	-1	1	0	2	-1	1	0	-1	1	2	1	0	3	-1	2
19	0	1	2	-1	1	0	-1	1	-1	1	0	0	0	1	0	0	1	-1	3	-1
20	3	1	2	1	0	0	0	1	1	-1	2	0	0	2	2	1	2	2	-1	4

Q-VALUE - (5)

COMPONENT - (1) P1

COMPONENT - (2) P3

```
Q-VALUE  - ( 4)

          COMPONENT  - ( 1)          A1   A2   A3   A6   A8
          COMPONENT  - ( 2)          A5
          COMPONENT  - ( 3)          A7
          COMPONENT  - ( 4)          A9
          COMPONENT  - ( 5)          A10

Q-VALUE  - ( 3)

          COMPONENT  - ( 1)          A1   A2   A3   A5   A6   A7   A8   A9   A10
          COMPONENT  - ( 2)          A4

Q-VALUE  - ( 2)

          COMPONENT  - ( 1)          A1   A2   A3   A4   A5   A6   A7   A8   A9   A10
```

```
Q-VALUE  - ( 1)
                    COMPONENT  - ( 1)          P1   P2   P3   P4   P5   P6   P7   P8   P9   P10  P11  P12
                                                    P13  P14  P15  P16  P17  P18  P19  P20

Q-VALUE  - ( 0)
                    COMPONENT  - ( 1)          P1   P2   P3   P4   P5   P6   P7   P8   P9   P10  P11  P12
                                                    P13  P14  P15  P16  P17  P18  P19  P20

    STRUCTURE VECTOR              FROM 5      TO .... 0
                                       2    5    9    1    1    1

    OBSTRUCTION VECTOR            FROM 5      TO .... 0
                                       1    4    8    0    0    0
```

SHAPED FACE MATRIX - KCOLUMN(ROW)

	1	2	3	4	5	6	7	8	9	10
1	8	1	5	-1	3	3	2	3	3	1
2	1	9	1	2	3	1	3	5	2	3
3	5	1	9	0	2	4	0	4	2	3
4	-1	2	0	3	0	-1	0	1	-1	1
5	3	3	2	0	6	1	1	3	2	1
6	3	1	4	-1	1	6	-1	2	1	2
7	2	3	0	0	1	-1	4	1	2	0
8	3	5	4	1	3	2	1	9	2	3
9	3	2	2	-1	2	1	2	2	5	1
10	1	3	3	1	1	2	0	3	1	6

Q-VALUE - (9)

COMPONENT - (1) A2

COMPONENT - (2) A3

COMPONENT - (3) A8

Q-VALUE - (8)

COMPONENT - (1) A1

COMPONENT - (2) A2

COMPONENT - (3) A3

COMPONENT - (4) A8

```
Q-VALUE  - ( 7)

          COMPONENT  - ( 1)          A1
          COMPONENT  - ( 2)          A2
          COMPONENT  - ( 3)          A3
          COMPONENT  - ( 4)          A8

Q-VALUE  - ( 6)

          COMPONENT  - ( 1)          A1
          COMPONENT  - ( 2)          A2
          COMPONENT  - ( 3)          A3
          COMPONENT  - ( 4)          A5
          COMPONENT  - ( 5)          A6
          COMPONENT  - ( 6)          A8
          COMPONENT  - ( 7)          A10

Q-VALUE  - ( 5)

          COMPONENT  - ( 1)          A1     A3
          COMPONENT  - ( 2)          A2     A8
          COMPONENT  - ( 3)          A5
          COMPONENT  - ( 4)          A6
          COMPONENT  - ( 5)          A9
          COMPONENT  - ( 6)          A10
```

```
Q-VALUE  - ( 1)
                     COMPONENT  - ( 1)          A1   A2   A3   A4   A5   A6   A7   A8   A9   A10


Q-VALUE  - ( 0)
                     COMPONENT  - ( 1)          A1   A2   A3   A4   A5   A6   A7   A8   A9   A10


                              FROM 9     TO .... 0
      STRUCTURE VECTOR           3   4   4   7   6   5   2   1   1   1

                              FROM 9     TO .... 0
      OBSTRUCTION VECTOR         2   3   3   6   5   4   1   0   0   0


ACCOUNT: U6230          MAXIMUM TIME (SEC):   25  NET CPU (SEC):        2
DATE: 03/30/77 IDENT:   ACTUAL TIME, INCLUDING 2.0 SEC SYSTEM TIME:     5 @ $.11/SEC = $  0.55
USER: MORGAN S          LINES PRINTED:       572  CARDS PUNCHED:        0 @ $.35/100 = $  0.00
DESTINATION: AA         MAXIMUM RECORDS:    2500  TOTAL RECORDS:      572 @ $.07/100 = $  0.42
                        CARDS READ:          439                ***** TOTAL COST = $  0.97  JOB NAME PL 1 41639
```

```
Q-VALUE  - ( 4)
          COMPONENT  - ( 1)        A1  A2  A3  A6  A8
          COMPONENT  - ( 2)        A5
          COMPONENT  - ( 3)        A7
          COMPONENT  - ( 4)        A9
          COMPONENT  - ( 5)        A10

Q-VALUE  - ( 3)
          COMPONENT  - ( 1)        A1  A2  A3  A5  A6  A7  A8  A9  A10
          COMPONENT  - ( 2)        A4

Q-VALUE  - ( 2)
          COMPONENT  - ( 1)        A1  A2  A3  A4  A5  A6  A7  A8  A9  A10
```

Algorithm 457 annex d

Finding All Cliques of an Undirected Graph [H]

Coen Bron* and Joep Kerbosch† [Recd. 27 April 1971 and 23 August 1971]

* Department of Mathematics † Department of Industrial Engineering, Technological University Eindhoven, P.O. Box 513, Eindhoven, The Netherlands

Present address of C. Bron: Department of Electrical Engineering, Twente University of Technology, P.O. Box 217, Enschade, The Netherlands.

Key Words and Phrases: cliques, maximal complete subgraphs, lusters, backtracking algorithm, branch and bound technique, ecursion

CR Categories: 3.71, 5.32

Language: Algol

escription

Introduction. A maximal complete subgraph (clique) is a omplete subgraph that is not contained in any other complete ubgraph.

A recent paper [1] describes a number of techniques to find aximal complete subgraphs of a given undirected graph. In this aper, we present two backtracking algorithms, using a branch-nd-bound technique [4] to cut off branches that cannot lead to a ique.

The first version is a straightforward implementation of the asic algorithm. It is mainly presented to illustrate the method used. his version generates cliques in alphabetic (lexicographic) order.

The second version is derived from the first and generates iques in a rather unpredictable order in an attempt to minimize e number of branches to be traversed. This version tends to proice the larger cliques first and to generate sequentially cliques ving a large common intersection. The detailed algorithm for rsion 2 is presented here.

Description of the algorithm - Version 1. Three sets play an portant role in the algorithm. (1) The set *compsub* is the set be extended by a new point or shrunk by one point on traveling ong a branch of the backtracking tree. The points that are eligible extend *compsub*, i.e. that are connected to all points in *compsub*, e collected recursively in the remaining two sets. (2) The set *ndidates* is the set of all points that will in due time serve as an tension to the present configuration of *compsub*. (3) The set *t* is the set of all points that have at an earlier stage already ved as an extension of the present configuration of *compsub* and now explicitly excluded. The reason for maintaining this set *t* will soon be made clear.

The core of the algorithm consists of a recursively defined tension operator that will be applied to the three sets just deibed. It has the duty to generate all extensions of the given nfiguration of *compsub* that it can make with the given set of ndidates and that do not contain any of the points in *not*. To t it differently: all extensions of *compsub* containing any point *not* have already been generated. The basic mechanism now nsists of the following five steps:

p 1. Selection of a candidate.
p 2. Adding the selected candidate to *compsub*.
p 3. Creating new sets *candidates* and *not* from the old sets by removing all points not connected to the selected candidate (to remain consistent with the definition), keeping the old sets in tact.
Step 4. Calling the extension operator to operate on the sets just formed.
Step 5. Upon return, removal of the selected candidate from *compsub* and its addition to the old set *not*.

We will now motivate the extra labor involved in maintaining the sets *not*. A necessary condition for having created a clique is that the set *candidates* be empty; otherwise *compsub* could still be extended. This condition, however, is not sufficient, because if now *not* is nonempty, we know from the definition of *not* that the present configuration of *compsub* has already been contained in another configuration and is therefore not maximal. We may now state that *compsub* is a clique as soon as both *not* and *candidates* are empty.

If at some stage *not* contains a point connected to all points in *candidates*, we can predict that further extensions (further selection of candidates) will never lead to the removal (in Step 3) of that particular point from subsequent configurations of *not* and, therefore, not to a clique. This is the branch and bound method which enables us to detect in an early stage branches of the backtracking tree that do not lead to successful endpoints.

A few more remarks about the implementation of the algorithm seem in place. The set *compsub* behaves like a stack and can be maintained and updated in the form of a global array. The sets *candidates* and *not* are handed to the extensions operator as a parameter. The operator then declares a local array, in which the new sets are built up, that will be handed to the inner call. Both sets are stored in a single one-dimensional array with the following layout:

| *not* | *candidates*

index values: 1.....*ne*..............*ce*....

The following properties obviously hold:

1. $ne \leq ce$
2. $ne = ce$: empty (*candidates*)
3. $ne = 0$: empty (*not*)
4. $ce = 0$: empty (*not*) and empty (*candidates*) $=$ clique found

If the selected candidate is in array position $ne + 1$, then the second part of Step 5 is implemented as $ne := ne + 1$.

In version 1 we use element $ne + 1$ as selected candidate. This strategy never gives rise to internal shuffling, and thus all cliques are generated in a lexicographic ordering according to the initial ordering of the candidates (all points) in the outer call.

For an implementation of version 1 we refer to [3].

Description of the algorithm - Version 2. This version does not select the candidate in position $ne + 1$, but a well-chosen candidate from position, say s. In order to be able to complete Step 5 as simply as described above, elements s and $ne + 1$ will be interchanged as soon as selection has taken place. This interchange does not affect the set *candidates* since there is not implicit ordering. The selection does affect, however, the order in which the cliques are eventually generated.

Now what do we mean by "well chosen"? The object we have in mind is to minimize the number of repetitions of Steps 1-5 inside the extension operator. The repetitions terminate as soon as the bound condition is reached. We recall that this condition is formulated as: there exists a point in *not* connected to all points in *candidates*. We would like the existence of such a point to come about at the earliest possible stage.

Let us assume that with every point in *not* is associated a counter, counting the number of candidates that this point is not connected to (*number of disconnections*). Moving a selected candidate into *not* (this occurs after extension) decreases by one all counters of the points in *not* to which it is disconnected and introduces a new counter of its own. Note that no counter is ever

eferen...

A... J.G., and Minker, J. An analysis of some graph eoret... clu...ter techniques, *J. ACM 17* (1970), 571–588.

B... E. Unpublished report. U. of Toronto

Br... C., Kerbosch, J.A.G.M., and Schell, H.J. Finding ques ... an undirected graph. Tech. Rep. Technological U. of ndho...en, The Netherlands.

Li... John D.C., et al. An algorithm for the traveling sales-an pr...lem. *Oper. Res. 11* (1963), 972–989.

M...n, J.W. and Moser, L. On cliques in graphs. *Israel J. lath.* 3 (1965), 23–28.

M...gan, G.D., and Corneil, D.G. Corrections to Bier one's ...orithm for generating cliques. *J. ACM 19* (Apr. 972), 2...–247

lgorithm

```
rocedure output maximal complete subgraphs 2(connected, N);
 value N;  integer N;
 Boolean array connected;
ommer: The input graph is expected in the form of a symmetrical
 Boolean matrix connected. N is the number of nodes in the
 graph. The values of the diagonal elements should be true;
egin
 integer array ALL, compsub[1 : N];
 integer c;
 procedure extend version 2(old, ne, ce);
   value ne, ce;  integer ne, ce;
   integer array old;
 begin
   integer array new[1 : ce];
   integer nod, fixp;
   integer newne, newce, i, j, count, pos, p, s, sel, minnod;
   comment The latter set of integers is local in scope but need
      not be declared recursively;
   minnod := ce;  i := nod := 0;
DETERMINE EACH COUNTER VALUE AND LOOK FOR
MINIMUM:
   for i := i + 1 while i ≤ ce ∧ minnod ≠ 0 do
   begin
      p := old[i];  count := 0;   j := ne;
COUNT DISCONNECTIONS:
      for j := j + 1 while j ≤ ce ∧ count < minnod do
           if ¬ connected[p, old[j]] then
           begin
              count := count + 1;
SAVE POSITION OF POTENTIAL CANDIDATE:
              pos := j
           end;
TEST NEW MINIMUM:
      if count < minnod then
      begin
         fixp := p;  minnod := count;
         if i ≤ ne then s := pos
         else
         begin s := i;  PREINCR: nod := 1 end
      end NEW MINIMUM;
   end i;
   comment If fixed point initially chosen from candidates then
      number of disconnections will be preincreased by one;
BACKTRACKCYCLE:
   for nod := minnod + nod step −1 until 1 do
   begin
INTERCHANGE:
      p := old[s];  old[s] := old[ne + 1];
      sel := old[ne + 1] := p;
FILL NEW SET not:
      newne := i := 0;
      for i := i + 1 while i ≤ ne do
           if connected[sel, old[i]] then
           begin newne := newne + 1;  new[newne] := old[i] end;
FILL NEW SET cand:
      newce := newne;  i := ne + 1;
      for i := i + 1 while i ≤ ce do
           if connected[sel, old[i]] then
           begin newce := newce + 1;  new[newce] := old[i] end;
ADD TO compsub:
      c := c + 1;  compsub[c] := sel;
      if newce = 0 then
      begin
         integer loc;
         outstring(1, 'clique = ');
         for loc := 1 step 1 until c do
              outinteger(1, compsub[loc])
      end output of clique
      else
      if newne < newce then extend version 2(new, newne, newce);
REMOVE FROM compsub:
      c := c − 1;
ADD TO not:
      ne := ne + 1;
      if nod > 1 then
      begin
SELECT A CANDIDATE DISCONNECTED TO THE FIXED
POINT:
           s := ne;
LOOK: FOR CANDIDATE:
           s := s + 1;
           if connected[fixp, old[s]] then go to LOOK
      end selection
   end BACKTRACKCYCLE
 end extend version 2;
 for c := 1 step 1 until N do ALL[c] := c;
 c := 0;  extend version 2(ALL, 0, N)
end output maximal complete subgraphs 2;
```

Remark on Algorithm 323 [G6]
Generation of Permutations in Lexicographic Order [R.J. Ord-Smith, *Comm. ACM 11* (Feb. 1968), 117]

Mohit Kumar Roy [Recd. 15 May 1972]
Computer Centre, Jadavpur University, Calcutta 32, India

In presenting Algorithm 323, *BESTLEX*, for generating permutations in lexicographic order, the author has mentioned the number of transpositions. It may be remarked here that equal numbers of transpositions are required by both *BESTLEX* and the previously fastest algorithm, Algorithm 202 [1]. The exact number of transpositions (T_n) necessary to generate the complete set of $n!$ permutations is given by

$$T_n = n!\,(\psi_{n-1}) - (n+1)/2, \quad \text{if } n \text{ is odd, and}$$
$$T_n = n!\,(\psi_{n-2}) - n/2, \quad \text{if } n \text{ is even,}$$

$$\text{where } \psi_{2n} = 1 + \frac{1}{2!} + \frac{1}{4!} + \cdots + \frac{1}{(2n)!} \doteq 1.543 \text{ for } n \geq 3.$$

The above expressions do not include the few extra transpositions (equal to the integral part of $n/2$) required by *BESTLEX* to generate the initial arrangement from the final one, as this portion has

Fig. 1. Random graphs show the computing time per clique (in ms) versus dimension of the graph (in brackets: total number of cliques in the test sample).

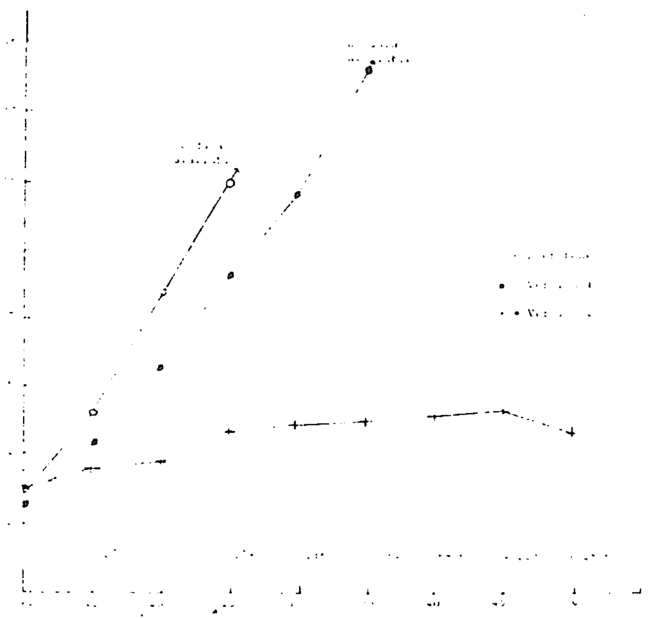
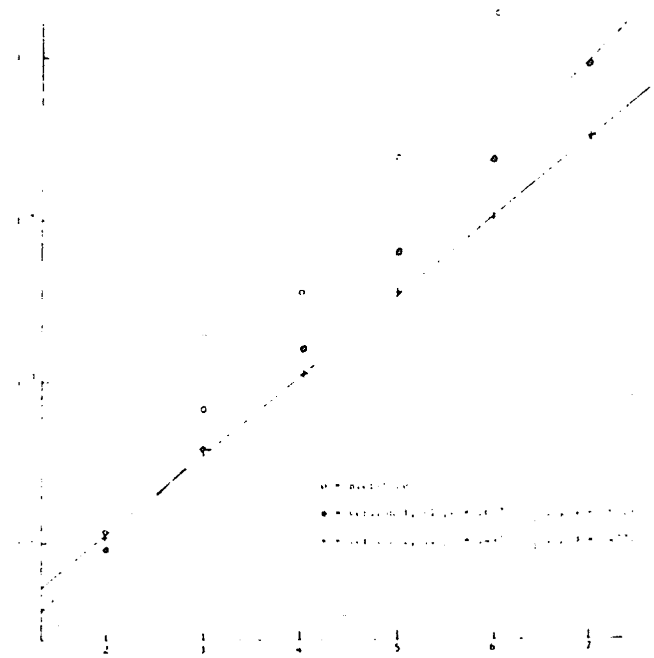


Fig. 2. Moon-Moser graphs show the computing time (in ms) versus k. Dimension of the graph = $3k$. Plotted on logarithmic scale.



decreased by more than one at any one instant. Whenever a counter goes to zero the bound condition has been reached.

Now let us fix one particular point in *not*. If we keep selecting candidates disconnected to this fixed point, the counter of the fixed point will be decreased by one at every repetition. No other counter can go down more rapidly. If, to begin with, the fixed point has the lowest counter, no other counter can reach zero sooner, as long as the counters for points newly added to *not* cannot be smaller. We see to this requirement upon entry into the extension operator, where the fixed point is taken either from *not* or from the original *candidates*, whichever point yields the lowest counter value after the first addition to *not*. From that moment on we only keep track of this one counter, decreasing it for every next selection, since we will only select disconnected points.

The Algol 60 implementation of this version is given below.

Discussion of comparative tests. Augustson and Minker [1] have evaluated a number of clique finding techniques and report an algorithm by Bierstone [2] as being the most efficient one.

In order to evaluate the performance of the new algorithms, we implemented the Bierstone algorithm[1] and ran the three algorithms on two rather different testcases under the Algol system for the EL-X8.

For our first testcase we considered random graphs ranging in dimension from 10 to 50 nodes. For each dimension we generated a collection of graphs where the percentage of edges took on the following values: 10, 30, 50, 70, 90, 95. The cpu time per clique for each dimension was averaged over such a collection. The results are graphically represented in Figure 1.

The detailed figures [3] showed the Bierstone algorithm to be of slight advantage in the case of small graphs containing a small number of relatively large cliques. The most striking feature, however, appears to be that the time/clique for version 2 is hardly dependent on the size of the graph.

[1] Bierstone's algorithm as reported in [1] contained an error. In our implementation the error was corrected. The error was independently found by Mulligan and Corneil at the University of Toronto, and reported in [6].

The difference between version 1 and "Bierstone" is not so striking and may be due to the particular Algol implementation. It should be borne in mind that the sets of nodes as they appear in the Bierstone algorithm were coded as one-word binary vectors, and that a sudden increase in processing time will take place when the input graph is too large for "one-word representation" of its subgraphs.

The second testcase was suggested by the referee and consisted of regular graphs of dimensions $3 \times k$. These graphs are constructed as the complement of k disjoint 3-cliques. Such graphs contain 3^k cliques and are proved by Moon and Moser [5] to contain the largest number of cliques per node.

In Figure 2 a logarithmic plot of computing time versus k is presented. We see that both version 1 and version 2 perform significantly better than Bierstone's algorithm. The processing time for version 1 is proportional to 4^k, and for version 2 it is proportional to $(3.14)^k$ where 3^k is the theoretical limit.

Another aspect to be taken into account when comparing algorithms is their storage requirements. The new algorithms presented in this paper will need at most $\frac{1}{2}M(M+3)$ storage locations to contain arrays of (small) integers where M is the size of largest connected component in the input graph. In practice this limit will only be approached if the input graph is an almost complete graph. The Bierstone algorithm requires a rather unpredictable amount of store, dependent on the number of cliques that will be generated. This number may be quite large, even for moderate dimensions, as the Moon-Moser graphs show.

Finally it should be pointed out that Bierstone's algorithm does not report isolated points as cliques, whereas the new algorithm does. Either algorithm can, however, be modified to produce results equivalent to the other. Suppression of 1-cliques in the new algorithm is the simplest adaption.

Acknowledgments. The authors are indebted to H.J. Schell for preparation of the test programs and collection of performance statistics. Acknowledgments are also due to the referees for their valuable suggestions.

MISCELLANEOUS

Analysis of Interconnected Decision Areas: an Algorithm for Project Development

THERE are many situations where the complete definition of a project depends on a choice from a variety of options. When these choices are technically interdependent, the task of exploration of the field of possibilities is a complex one. Since problems of this kind are widespread, an understanding of their mathematical nature might well lead to considerable benefit. This communication concerns only the mathematical problem and does not discuss practical implementation.

In the development of a project certain general considerations enter into the fulfilment of total requirements, but the choice is made by a stepwise procedure in which technical compatibilities are examined within small groups of possibilities. This 'natural' procedure may lead to considerable backtracking and waste of effort. Hence we wish to develop a more systematic approach. In what follows, the general considerations will be called 'decision-areas'; within each of these decision-areas there are mutually exclusive 'options'. Options with respect to different decision-areas may or may not be compatible. A feasible overall solution is one that chooses one option from each decision-area in such a way that overall compatibility obtains. Some mathematical description of these basic concepts follows.

We will introduce the idea of a 'strategy graph' consisting of N points, v_i, representing decision-areas, and line segments representing the technical interdependence of pairs of decision-areas.

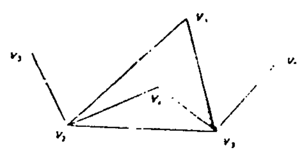
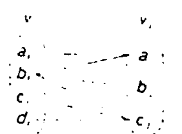


Fig. 1. Example of a strategy graph with six points

Each decision-area v_i consists of a set of options n_i. A binary matrix A_{ij} is introduced to delineate the compatibility of each of the n_i options in v_i with each of the n_j options in v_j for all decision-areas v_i and v_j which are adjacent in the strategy graph. An entry of A_{ij} is 1 if the two options are compatible, and 0 otherwise.



$$A_{ij} = \begin{array}{c} \\ a_i \\ b_i \\ c_i \\ d_i \end{array} \begin{array}{c} \begin{array}{ccc} a_j & b_j & c_j \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Fig. 2. Two adjacent decision-areas and their compatibility matrix

We will further define an 'α-combination' of options as a set of N options, one from each decision-area such that no two options are incompatible. The following problems can then be posed:

(1) How many α-combinations exist for a given strategy graph and given compatibility relationships?

(2) What are the α-combinations themselves?

(3) Given that there is a numerical 'cost' associated with each option, what is the total cost of each of the α-combinations?

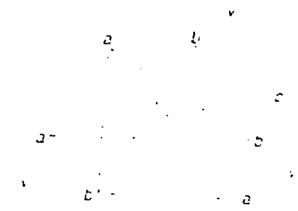
It can be shown that any strategy graph can be expressed in the form of a complete graph (in which every pair of points are adjacent) by the introduction of universal compatibility matrices between decision-areas not previously adjacent, of the form U_{ij} with all entries having the value 1. This will leave every α-combination unchanged, and does not alter their number.

We will now develop an algorithm for finding the t[o]... number, ν, of α-combinations. Considering one decisi[on] area, say v_1, with options $a_1, b_1, \ldots$, we write:

$$\nu = \nu(a_1) + \nu(b_1) + \ldots$$

where $\nu(a_1)$ is the number of α-combinations contain[ing] the option a_1, etc.

We first describe a method for handling the compl[ete] graph K_3 with three points, as illustrated in Fig. 3, developing an explicit formula for $\nu(a_1)$.



$$A_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_{23} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Fig. 3. Three mutually adjacent decision-areas and their compatibility matrices

For two matrices A and B of the same size $m \times$ ... the elementwise product is denoted $A \times B$. Defi[ne] $C_{23}(a_1)$ as the matrix in which the rows are associat[ed] with the first row of A_{12}, the columns with the first row [of] A_{13}, and the entries are obtained by multiplying t[he] associated row and column values. In the example:

$$C_{23}(a_1) = \begin{array}{c} \\ 1 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 1 & 0 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

For a given matrix M, let ΣM be the sum of its entrie[s]. Then it is evident that:

$$\nu(a_1) = \Sigma(A_{23} \times C_{23}(a_1))$$

For Fig. 3, we find:

$$\nu(a_1) = \Sigma\left(\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\right)$$
$$= \Sigma\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = 2$$

Clearly the two α-combinations in Fig. 3 containing a[$_1$] are $a_1b_2a_3$ and $a_1a_2b_3$. Similarly, $\nu(b_1) = 0$ in Fig. 3, an[d] hence by equation (1), $\nu = 2$.

By a straightforward generalization of this method, [a] given complete graph K_N can be reduced by one point [at] a time. Thus eventually K_3 is obtained. Hence the fir[st] of the three problems may be handled.

To speak of the last two problems, a device originally due to Kirchhoff[1] may be invoked. This involves labellin[g] the links which join each pair of adjacent options in a given strategy graph by assigning a variable to each lin[k] for purposes of identification. Then the method outlined here for the number of α-combinations will yield instead the α-combinations themselves. Knowing every α-combination, the cost of each is immediately determined by the costs of the options involved.

This work has arisen out of a study of building design undertaken as part of research work for the Buildin[g] Industry Communications Research Project.

FRANK HARARY

Research Center for Group Dynamics,
University of Michigan, Ann Arbor.

NEIL JESSOP
JOHN LUCKMAN
JOHN STRINGER

Institute for Operational Research,
3 Devonshire Street, London, W.1.

[1] Kirchhoff, G., *Poggendorf Annal.*, 72, 497 (1847).

BIBLIOGRAPHY

The following references are provided as a guide to the various areas which this study touched upon and were consulted during the study, although not necessarily cited in the text of the report.

I. Bibliography of the Methodology of Q-Analysis

(1) Atkin, R. H., "Abstract Physics," II Nuevo Cimento, 38/1, (1965) pp. 496-517.

(2) Atkin, R. H., "A Generalization of Special Relativity Theory," unpublished, 1966.

(3) Atkin, R. H., "Cohomology of Observations," in T. Bastin (ed.), Quantum Theory and Beyond (Cambridge: The University Press, 1971) pp. 191-211.

(4) Atkin, R. H., "From Cohomology in Physics to Q-Connectivity in Social Science," International Journal of Man-Machine Studies, 4, (1972) pp. 139-167.

(5) Atkin, R. H., Urban Structure (Colchester: Department of Mathematics, Urban Structure Research Report No. 1, 1972).

(6) Atkin, R. H., A Survey of Mathematical Theory (Colchester: Department of Mathematics, Urban Structure Research Report No. 2, 1972).

(7) Atkin, R. H., A Study Area in Southend-on-Sea, (Colchester: Department of Mathematics, Urban Structure Research Report No. 3, 1973).

(8) Atkin, R. H., A Community Study of the University of Essex, (Colchester: Department of Mathematics, Urban Structure Research Report No. 4, 1974).

(9) Atkin, R. H., Mathematical Structure in Human Affairs, (London: Heinemann Educational Books, 1974).

(10) Atkin, R. H., "An Approach to Structure in Architectural and Urban Design," Environment and Planning B, 1, (1974) pp. 51-67; 2 (1974) pp. 173-191; and 3, (1975) pp. 21-57.

(11) Atkin, R. H., Methodology of Q-Analysis: A Study of East Anglia-1, (Colchester: Department of Mathematics, Regional Research Project Report No. 5, 1975).

(12) Atkin, R. H. and I. Witten, "A Multi-dimensional Approach to Positional Chess," International Journal of Man-Machine Studies, Vol. 7, 1975, pp. 727-750.

(13) Atkin, R. H., Multidimensional Man, (Colchester: Department of Mathematics, 1977), mms. for publisher.

(14) Atkin, R. and J. Johnson, A Study of East Anglia III (Colchester: Department of Mathematics, Research Report 7, 1976).

(15) Atkin, R. H., Hartson, W. and I. Witten, "Fred CHAMP, Positional Chess Analyst," International Journal of Man-Machine Studies, Vol. 8, (1976) pp. 517-529.

(16) Atkin, R. H., "An Algebra of Patterns on a Complex I," International Journal of Man-Machine Studies, (1974) pp. 285-307.

(17) Atkin, R. H., Combinatorial Connectivities in Social Systems, (Basel: Birkhauser-Verlag, 1977).

(18) Johnson, J. H., Methodology of Q-Analysis: A Study of East Anglia-2, (Colchester: Regional Research Project Report No. 6, 1976).

(19) Atkin, R. H., "An Algebra of Patterns of a Complex II," International Journal of Man-Machine Studies 8 (1976), pp. 483-498.

(20) Johnson, J. H., "The Q-Analysis of Road Intersections," International Journal of Man-Machine Studies 8 (1976) pp. 531-548.

(21) Melville, Brian, "Notes on the Civil Uses of Mathematics," International Journal of Man-Machine Studies 8, (1976) pp. 501-515.

II. Basic References for Mathematical Structure and Logic

(22) Ayres, Frank, J., Theory and Problems of Modern Algebra, (New York: McGraw-Hill, Schaum's Outline Series, 1965).

(23) Beth, Everet W., The Foundations of Mathematics, (New York: Harper & Row, 1966).

(24) Delong, Howard, A Profile of Mathematical Logic, (Reading: Addison-Wesley, 1970).

(25) Hilton, P. J. and Wylie, S., Homology Theory, (Cambridge: University Press, 1967).

(26) Keese, John W., An Introduction to Algebraic Topology, (Belmont: Brooks-Cole Publishing Company, 1970).

(27) Kemeny, John G., Snell, J., and Thompson, Gerald, Introduction to Finite Mathematics, (Englewood Cliffs: Prentice Hall, 1965).

(28) Lipshutz, Seymour, Theory and Problems of General Topology, (New York: McGraw-Hill, Schaum's Outline, 1965).

(29) MacLane, Saunders and Birkhoff, Garret, Algebra, (London: MacMillan Company, 1967).

(30) Scroggs, James, Linear Algebra, (Belmont: Brooks-Cole, 1970).

(31) Vick, George R., Elementary Mathematical Structure, (Columbus, Ohio: Merril Publishing, 1970).

(32) Hohn, Franz E., Matrix Algebra, (New York: MacMillan, 1958).

III. Graph Theory

(33) Berge, Claude, The Theory of Graphs and Applications, (London: Meuthen and Co. Ltd., 1962).

(34) Berge, Claude, Graphs and Hypergraphs, (London: North Holland Publishing Company, 1973).

(35) Behzad, Mehdi, and Chartrand, Gary, Introduction to the Theory of Graphs, (Boston: Allyn and Bacon Inc., 1971).

(36) Johnson, David E. and Johnson, Hohny R., Graph Theory with Engineering Applications, (New York: The Ronald Press Company, 1972).

(37) Harary, Frank, Proof Techniques in Graph Theory, (New York: Academic Press, 1969).

(38) Harary, Frank, and Norman Robert Z. and Dorwin, Cartwright, Structural Models and Equations, (New York: Wiley and Sons, 1965).

(39) Higman, Bryan, Applied Group Theoretic and Matrix Methods, (New York: Dover, 1964).

(40) Maxwell, Lee M., Reed, Myril B., The Theory of Graphs: A Basis for Network Theory, (New York: Pergamon Press, 1971).

IV. Applied Combinatorial Mathematics and Computer Programming

(41) Beckenbach, Edwin, ed., Applied Combinatorial Mathematics, (New York: Wiley and Sons, 1964).

(42) Collins, P. A., Collins, R. J., Peterkin, C. R., Scuse, D. H., Staten, R. G., Structured Fortran with WATFIV-S and WATFOR-11S, (St. Pierre: Babbage Research Center, 1976).

(43) Hall, Marshall, Combinatorial Theory, (Toronto: Blaisdell Publishing, 1967).

(44) Horowitz, Ellis, and Sartaj, Sahni, Fundamentals of Data Structure, (Woodland Hills: Computer Science Press, Inc., 1976).

(45) Whitehead, Earl Glenn, Combinatorial Algorithms, (New York University: Courant Institute, 1973).

(46) Bron, C. and Kerbosch, J., "Algorithm 457: Finding all Cliques of an Undirected Graph," CACM 16, (1973), pp. 575-577.

(47) Ferber, K. and Jurgensen, H., "A Program for the Drawing of Lattices," In Computational Problems in Abstract Algebra, (J. Leech, ed.) (Oxford: Pergamen Press, 1970) pp. 83-87.

(48) Fox, B. L., "Calculating kth Shortest Paths," Infor 11, (1973) pp. 66-70.

(49) Floyd, R. W., "Algorithm 97: Shortest Path," CACM 5, (1962) p. 345.

(50) Gibbs, N. E., "A Cycle Generating Algorithm for Finite Undirected Graphs," JACM 16, (1969) pp. 564-568.

(51) Stine, H. S., Discrete Mathematical Structures and Applications, (Chicago: Scientific Research Institute, 1973).

(52) Wallace, Andrew H., Algebraic Topology: Homology and Cohomology, (New York: W. A. Benjamin Inc., 1970).

(53) Welch, J., "A Mechanical Analysis of the Cycle Structure of Finite Undirected Linear Graphs," JACM 13, pp. 205-210.

(54) Sakarovitch, M., "Shortest Chains in a Graph," Transportation Research 2, 1968, pp. 1-11.

(55) Harary, Jessop, Stringer, Luckman, "An Algorithm for Project Development," Nature London 206, (1964) p. 118.

V. Formal Methods in City Planning and Architecture

(56) Alexander, C., Notes on a Synthesis of Form, (Harvard: University Press, 1964).

(57) Berechman, Joseph, "Toward a Land Use Planning Model for New Towns," International Journal of Systems Science, Vol. 7, No. 2, (1976) pp. 225-239.

(58) Boyce, David E., "Toward a Framework for Defining and Applying Urban Indicators in Plan Making," Urban Affairs Quarterly, December, (1970) pp. 145-171.

(59) Cousins, J., "Topological Organization of Architectural Space," Architectural Design, (October 1970) pp. 145-171.

(60) Hill, Morris, "Goals-Achievement Matrix for Valuating Alternative Plans," J.A.I.P. (1968) pp. 19-29.

(61) Friend, J. K., Jessop, J. R., Strategic Choice and Local Government, (London: Tavistock, 1970).

(62) Levins, P. H., "The Use of Graphs to Decide the Optimal Layout of Buildings," Architects Journal, (Oct. 1964) pp. 809-815.

(63) Karsky, J. G., Structure of Transportation Networks, (University of Chicago, Department of Geography Research Paper No. 83) 1964.

(64) Kaplan, Abraham, Conduct of Inquiry, (Scranton, P. A.: Chandler, 1964).

(65) March, Lionel and Steadman, Phillip, The Geometry of Environment, (London: Meuthen, 1971).

(66) Mack, Ruth, Planning on Uncertainty, (New York: Wiley, 1971).

(67) Saarinen, Thomas F., Environmental Planning: Perception and Behavior, (Boston: Houghton-Mifflin Co., 1976).

VI. Methodology of Systems Modelling

(68) Ashby, W. Ross, An Introduction to Cybernetics, (London: Meuthen and Co. Ltd.; University Paperback, 1964).

(69) Bertanlanffy, Ludwig von, General System Theory, (New York: George Brazillier Ltd., 1968).

(70) Klir, George J., An Approach to General Systems Theory, (New York: Van Nostrand Reinhold Co., 1969).

(71) Klir, G. J. and Uyttenhive, J. J., "Computerized Methodology for Structure Modelling," Annals of Systems Research 5, (1976) pp. 29-66.

(72) Klir, G. J., "Identification of Generative Structures in Empirical Data," International Journal of General Systems 3, (1976), pp. 89-104.

(73) Klir, G. J., "On the Representation of Activity Arrays," International Journal of General Systems 2, (1975) pp. 145-168.

(74) Laslo, Ervin, The Systems View of the World, (New York: George Brazillier Ltd., 1972).

(75) Luce, Duncan and Raiffa, Howard, Games and Decisions, (New York: John Wiley & Sons, Inc., 1967.

(76) Simon, Herbert A., The Sciences of the Artificial, (Cambridge, MIT Press, 1969).

(77) Von Neumann, John and Morgenstern, Oscar, Theory of Games and Economic Behavior, (Princeton: Princeton University Press, 1953).

(78) Weinberg, Gerald M., An Introduction to General Systems Thinking, (New York: Wiley, 1975).

(79) Iberall, A. S., Toward a General Science of Viable Systems, (New York: McGraw-Hill Publishing Company Ltd., 1972).

VII. Philosophy of Science and Planning-Physical Analogies and Theoretical Physics

(80) Bateson, Gregory, Steps to an Ecology of Mind, (New York: Ballentine Books, 1972).

(81) Born, Max, Natural Philosophy of Cause and Chance, (New York: Dover Publications, 1964).

(82) Beth, Everet W., The Foundations of Mathematics, (New York: Harper and Row, Publishers, 1966).

(83) Delong, Howard, A Profile of Mathematical Logic, (Reading: Addison-Wesley Publishing Company Ltd., 1970).

(84) Eddington, Sir A. S., The Nature of the Physical World, (Ann Arbor: University of Michigan Press, 1958), (Ann Arbor Paperbacks 1958).

(85) Fraser, J. T., Of Time, Passion, and Knowledge, (New York: George Brazillier Ltd., 1975).

(86) Neurath, Otto, gen. ed., International Encyclopedia of Unified Science, Vol. I-II, (Chicago: University of Chicago Press, 1962) Vol. II no. 2; The Structure of Scientific Revolutions, by Thomas S. Kuhn.

(87) Russel, Bertrand, "The Relations of Sense Data to Physics," in Great Ideas of Modern Science, pp. 185-210. Edited by Robert W. Marks. (New York: Bantam Books Inc., Bantam Science and Mathematics Series, 1967).

(88) Von Weizsacker, C. F., "The Unity of Physics," in Quantum Theory and Beyond, pp. 229-262. Edited by Ted Bastin (Cambridge: Cambridge University Press, 1971).

(89) Jaynes, J. T., "Information and Statistical Mechanics," Phy Rev 106, pp. 620-630.

(90) Goldman, Stanford, "Mechanics of Individuality in Nature 1," Foundations of Physics 2, (1971) pp. 395-408.

(91) Goldman, Stanford, "The Mechanics of Individuality in Nature II, Barriers Cells, Phase Space," Foundations of Physics 3, 2, (1973) pp. 203-228.

(92) Colodny, Robert G., ed., The Nature and Function of Scientific Theories, (Pittsburgh: University of Pittsburgh Press, 1970).

(93) Pierce, J. R., Symbols, Signals and Noise, (New York: Harper & Row 1961).

(94) Thom, Rene, Structural Stability and Morphogenesis, (New York: Benjamin, 1973) Operations Research and Linear Programming.

(95) Gaven, Donald P. and Thompson, Gerald L., Programming and Probability Models in Operations Research, (Monterey, C. A.; Brooks-Cole, 1973).

(96) Hillier, Frederick and Lieberman, Gerald, Introduction to Operations Research, (San Francisco: Holden-Day, 1967).

(97) Loomba, Paul, Linear Programming, (New York: McGraw-Hill, 1964).

VIII. Nature of Planning and Urban Processes

(98) Caldwell, Lynton Keithe, Environment, (New York: Doubleday, 1970).

(99) Dunn, Edgar E., Economic and Social Development: A Process of Social Learning, (London: Johns Hopkins Press, 1971).

(100) Dyckman, J. W., "Planning and Decision Theory," J.A.I.P. XXVII, (November, 1961), pp. 335-345.

(101) Friedman, J. and Hudson, Barclay, "Knowledge and Action: A Guide to Planning Theory," J.A.I.P. (January, 1974) pp. 2-14.

(102) Friedman, J.,Retracking America, (New York: Doubleday, 1973).

(103) Thompson, W. I., At the Edge of History, (New York; Harper & Row, 1971).

(104) Mumford, Lewis, The City in History, (New York: Harcourt, Brace & World, Inc., 1961).

(105) Spengler, Oswald, The Decline of the West, Vol. I-II, (New York: Alfred A. Knopf, Inc., 1926).

(106) Holling and Goldberg, "Ecology and Planning," J.A.I.P. (July, 1971).

(107) Lynch, Kevin, The Image of the City, (Cambridge, Mass.: The MIT Press 1960).

(108) Long, N. E., "Local Community as an Ecology of Games," Journal of Sociology, LXIV, p. 252.

(109) Winters, Gibson, Elements for a Social Ethic, (New York: MacMillan 1968).

(110) Webber, M., Dyckman, J. W., Foley, D. L., Guttenberg, A. W., Wheaton, W. L., Wurster, C. B., Explorations into Urban Structure, (Philadelphia: University of Pennsylvania Press, 1964).