THE UNIVERSITY OF MANITOBA

NEW TECHNIQUES FOR LOAD-FLOW AND OUTAGE STUDIES OF VERY LARGE-SCALE POWER SYSTEMS INCLUDING MULTI-TERMINAL HVDC NETWORKS

BY

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE

STUDIES IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ELECTRICAL ENGINEERING

WINNIPEG, MANITOBA, CANADA

FEBRUARY 1979

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ABSTRACT

This dissertation presents new techniques for load-flow and outage studies of very large power systems.

A new technique for the load-flow calculations of integrated multi-terminal dc/ac systems is developed. This technique is fast, efficient and reliable and is therefore an improvement over known procedures. The representation of dc system is such that it leads to simple and efficient calculations and saving in storage requirement and yet it is so general that a multi-terminal dc network of any configuration and control strategies can be easily simulated.

A new method for outage studies is developed. This method is very fast and suited for single or multiple outages. It provides voltages and active and reactive power flows in the post-contingency state with acceptable accuracy.

For these new techniques the fast-decoupled method has been chosen because of its inherent superiority in terms of speed of calculation, storage requirement, reliability and simplicity in addition to noticing its wide-spread acceptance by power industry. Also, sparsity is fully exploited by using Zollenkopf's method and other sparsity techniques.

This thesis also describes two new diakoptical techniques for load-flow solution of very large scale power systems (2,000 or more buses) using the bus-admittance matrix and the fast decoupled methods. These new diakoptical techniques enlarge the scope of the load-flow and outage studies presented in this thesis by removing the restriction imposed by the core storage of computers on the size of a system that can be solved.

The proposed four new techniques have been tested on a number of power systems. Solution algorithms are presented in details and sample systems are solved to show the correctness and working of the proposed methods.

ACKNOWLEDGEMENTS

The author would like to express his appreciation and deep gratitude to Professors R.M. Mathur and R.W. Menzies, Joint Advisors, for their constant supervision and great efforts. These have made this research work possible. Their helpful assistance and guidance throughout the preparation of the thesis, and their financial support have been invaluable.

Special thanks are due to Professors A. Wexler and P.N. Shivakumar, and to Mr. R.E. Harrison of Teshmont Consultants Ltd., for their advice and suggestions.

Special acknowledgement goes to both the Electrical Power and Machines Department, Cairo University, and to the Department of Electrical Engineering, University of Manitoba for partial support, understanding, and co-operation during the course of this research.

The author is grateful to the University of Manitoba for providing a fellowship and to Manitoba Hydro for granting the Donald M. Stephens Memorial Fellowship.

Particular appreciation is due to Miss M. Wiebe for her tireless efforts in expertly typing the manuscript.

The author expresses his sincere appreciation to his mother and all the members of his family, for their patience, encouragement and love.

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CHAPTER I

INTRODUCTION

1.1 Background: (63 - 76)

There are a number of features of HVDC transmission that have been recognized for having the potential to reduce costs, improve the flexibility and reliability of HVDC and also improve the performance of the connected ac systems.

All existing HVDC systems transfer energy between two points with a single rectifier station supplying a single inverter station , the main differences between systems being the length and type of line between the stations, the size and number of poles and the nature of the connected ac systems. However, the interest in multi-terminal operation, and dc line tapping has increased recently and some systems are already planning to include such features.

Multi-terminal operation is an attractive alternative to several two-terminal HVDC links for the following reasons:

(a) The net installed capacity of converters is less than that required for several two-terminal HVDC links

(b) The losses in the transmission lines and in the converter stations are lower.

(c) The inherent overload capability of overhead lines can be used advantageously for a more flexible mode of operation.

The potential applications of multi-terminal HVDC systems would seem to be the following:

(a) ac network interconnections ;

(b) bulk power transmission ; and

(c) reinforcement of highly loaded ac networks.

The possible configurations of multi-terminal HVDC systems are series, mesh and radial or (tee) connections.

In the case of series connected stations, the direct current is common to all stations and is controlled by one station. All the other stations control the power by varying the inverter voltage by firing angle control or transformer tap changer control. Operation is essentially at constant current equal to full-rated current, or some optimum value depending upon the variations in load demand at each station. In comparison to parallel-connected stations, power reversal can be carried out by means of firing control in a very short time. The series-connected multi-terminal system is grounded at one location only, whereas both station neutrals in the point to point link are usually grounded. This actually subdivides the system into two independent subsystems and is a prerequisite for using two current controllers.

Series connection of converter stations admittedly has a number of characteristics that should make it attractive to system planners. These have to be weighed against such drawback as reduced flexibility to future extension and higher losses during partial load. It is therefore felt that the main application of series connected converter stations would be the tapping of bulk power d.c. transmission lines where the power tapped off in one or more places is only a fraction of the total link capacity.

The parallel connection of converter stations results in an interconnected system similar to those presently used with ac. Such a system clearly promises the highest degree of versatility. The common variable here is the system voltage, any number of substations can easily be added. Looking at the control aspect, transmission voltage would be governed by one station, all other stations control the power via their direct current. Parallel connection of converter stations would seem to be the most likely alternative to be chosen for a multi-terminal HVDC system. For power reversal in one station only, polarity reversal switches are necessary with switching performed at current zero in conjunction with the converter controls.

In most practical situations fast power reversal is not a necessity due to other system considerations and slow speed switching would be adequate. If required, fast power reversal can be obtained using suitable dc circuit breakers.

Two configuations can be used, radial and mesh connections. Their major differences are in breaker requirements and transmission line costs. For the radially connected system, if any line is to be taken out of service for maintenance, no breakers are required; converter control can be used to reduce line current to zero and disconnect switches can isolate the lines. Of course, secure telecommunication channels for interlocking and control setting are required during this procedure.

With the meshed system however, as a rule this is not possible. Here, a breaker capable of switching load currents is therefore required in any case. It can be operated without using telecommunication channels. As far as transmission line cost is concerned, at a first glance, the star connected system seems to be more attractive. However, both alternatives should be compared on an equal basis, which is transmission security.

Based on the current state of HVDC development and keeping in mind the requirements of some new systems being planned, one can, with reasonable confidence, say that multi-terminal HVDC transmission systems will form subsystems of power transmission network in the near future. The requirement, therefore, is that we must have digital computer programs which are flexible and which can efficiently handle a multi-terminal HVDC/ac system for load-flow and stability studies. Hence, one part of the research work described in this thesis is devoted to a detailed development of a technique for load-flow solution of integrated multi-terminal HVDC/ac systems. The technique is superior in a number of ways as compared to all known procedures.

A power system continuously experiences changes in its operating condition. These changes can either be due to load demand variations, planned rescheduling of power generation, disconnecting lines and transformers for maintenance or as a consequence of system faults. The effect of these disturbances is investigated both during system planning and operation. Transient and dynamic stability of power systems, considering that faults are experienced at different locations, is investigated to provide acceptable quality of service to the consumers. Quite often, a faulty element is automatically disconnected from the system by the protective devices. The system configuration, therefore, changes. Even before the dynamic and transient performance is investigated, it is advantageous to know whether (or not) the modified system would be stable from the steady state considerations alone. Also before lines and transformers are removed from the system for maintenance and repairs, it is essential to ensure that the modified system would be stable. In addition, line outage studies are a desirable part of a comprehensive system security monitoring process. The effect of load changes and generation rescheduling can be easily evaluated but the outage simulation of a line or transformer is more complex because these contingencies change the system configuration. One of the obvious solutions is the use of the well known load-flow techniques. The use of ac power flow solutions are too cumbersome and expensive for contingency analysis. A fast and approximate technique may be sufficient in most cases. Therefore, another phase of the research presented in this thesis is devoted to the development of a very fast, although approximate technique to provide the post contingency load-flow in the event of a single or multiple element outages. The technique requires one iteration starting from the load flow data of the base system and identification of outages.

The ever increasing size of present day power systems imposes great burdens on analytical methods now in use. This is the result of the large quantities of core storage and high computation times required for very large power systems. These circumstances not only apply to the load-flow problem but to all power system problems. In recent years, a number of papers have appeared involving piecewise solution of large scale electrical networks. Systems are torn up into isolated subdivisions. Each subdivision is handled independently for the partial solution of the problem. Then, the so called intersection model is constructed and solved for the full solution of the problem.

Known piecewise load-flow solution methods were first applied to Z and Y matrix routines. It has been demonstrated that tearing is an effective method for overcoming the size of problems that are encountered. Most of these techniques suffer from certain disadvantages as will be discussed later. Hence, the author finds it necessary to devise a new diakoptical technique for Y matrix load-flow solution of very large size networks that combines all the advantages and avoids disadvantages of already known procedures.

On the other hand, two piecewise algorithms using Newton-Raphson (N-R) load-flow method have been developed, the first by tearing the power system and the second by mathematically tearing the Jacobian matrix rather than the system itself. Also in these methods there are certain limitations and some drawbacks that should be encountered. Therefore, a part of the research work described in this thesis is devoted to a diakoptical fast-decoupled load-flow solution of very large scale networks which combines many of the advantages of the existing good methods for both power system load-flow solutions and numerical techniques. It also eliminates the drawbacks of other known techniques.

1.2 Contributions:

The research work described in this thesis makes at least four contributions:

(1) A new technique for load-flow calculations of integrated multi-terminal dc/ac systems.

(2) A very fast technique for post contingency load-flow in the event of a single or multiple element outages.

(3) An algorithm for piecewise load-flow solution of very large size power systems using the bus-admittance matrix.

(4) A new, exact, diakoptical fast decoupled load-flow solution of very large scale electrical networks.

The contributions are claimed to be distinct advancement in the current state of art.

1.3 Development of Chapters:

Chapter 2 is a brief review of known techniques for load-flow solution of ac power systems.

Chapter 3 details the development of a new load-flow technique for an integrated multi-terminal dc/ac system.

Chapter 4 presents known methods and the proposed technique for outage studies in power system planning and operation.

Chapter 5 presents a brief review of known piecewise load-flow solutions and describes development of two new diakoptical techniques for load-flow solution of very large size power systems.

Chapter 6 presents the major contributions and suggestions for future work.

CHAPTER II

LOAD-FLOW METHODS FOR AC SYSTEMS

2.1 Introduction (1),(2)

Load-flow calculations provide power flows and voltages for a specified power system subject to the regulating capability of generators, condensers and on-load tap changing transformers, as well as specified net interchange between individual operating systems. This information is essential for the continuous evaluation of the current performance of a power system and for analyzing the effectiveness of alternative plans for systems expansion to meet increased load demand. These analyses require the calculation of numerous load-flows for both normal and emergency operating conditions.

The load-flow problem consists of the calculation of power flows and voltages of a network for specified terminal or bus conditions. A single-phase representation is adequate since power systems are usually balanced.

Associated with each bus are four quantities: the real and reactive power, the voltage magnitude and the phase angle. Three types of buses are represented in a load-flow calculation: A PQ bus, at which the total injected power is specified. A PV bus, at which the total injected active power is specified and the voltage magnitude is maintained at a specified value by reactive power injection. A system slack (or swing) bus is selected to provide the additional real and reactive power to supply the transmission losses, since these are unknown until the final solution is obtained. At this bus the voltage magnitude and phase angle are specified.

The overall load-flow problem can be divided into the following subproblems:

1. The formulation of a suitable mathematical network model. The model must describe adequately the relationships between voltages and powers in the interconnected system.
2. A specification of the power and voltage constraints that must apply to the various buses of the network.
3. Numerical computation of the load-flow equations subject to the above constraints. These computations should give us, with sufficient accuracy, the values of all bus voltages.
4. When all bus voltages have thus been determined, we must, finally, compute the actual load flows in all transmission lines.

2.2 Power System Equations: (1,2)

Network Performance Equations

The equation describing the performance of the network of a power system in impedance form is

$$(E\ bus) = (Z\ bus)\ (I\ bus) \tag{2.1}$$

or in admittance form is

$$(I\ bus) = (Y\ bus)\ (E\ bus) \tag{2.2}$$

The elements of Y bus matrix are calculated as follows:

The diagonal element Y_{ii} is obtained as the algebraic sum of all admittances incident upon node i.

The off-diagonal elements $Y_{ij} = Y_{ji}$ are obtained as the negative of the admittance connecting nodes i and j.

Bus Mismatches and Solution Accuracy Criteria

The power at bus i is

$$P_i - jQ_i = E_i^* I_i = E_i^* \sum_{k=1}^{n} Y_{ik} E_k \qquad (2.3)$$

Since $\quad E_i = V_i e^{j\theta i}$, $E_i^* = V_i e^{-j\theta i}$ $\quad$ (*means conjugate)

and $\quad Y_{ik}$ = the (i,k)th element of the bus admittance matrix

$= G_{ik} + j B_{ik}$, the real and imaginary components of power at bus i are

$$P_i = V_i \sum_{k\varepsilon i} (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) V_k$$

$$Q_i = V_i \sum_{k\varepsilon i} (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) V_k$$

where $\theta_{ik} = \theta_i - \theta_k$

and $k\varepsilon i$ denotes a bus k (including $k = i$) directly connected to bus i.

Active and reactive power mismatches ΔP_i and ΔQ_i are

$$\Delta P_i = P_i \text{ (scheduled)} - P_i \qquad (2.4)$$

$$\Delta Q_i = Q_i \text{ (scheduled)} - Q_i \qquad (2.5)$$

The most common convergence criterion used in practice is

$\Delta P_i \leqslant C_p$ for all PQ and PV buses

$\Delta Q_i \leqslant C_q$ for all PQ buses

where C_p and C_q are tolerances chosen typically in the range

.01 to 10 MW/MVAR. Bus voltage-change tests are often used for load-flow algorithms in which mismatches are not readily available. Such tests are sensitive to the convergence rate of the solution process and are usually used as initial stopping criteria, after which the mismatches are computed and tested.

Line Flow Equations

After the iterative solution of bus voltages is completed, line flows can be calculated. The current at bus i in the line connecting bus i to k is

$$i_{ik} = (E_i - E_k)\, y_{ik} + E_i\, \bar{y}_{ik}/2$$

where y_{ik} = line admittance

$\bar{y}_{ik}$ = total line charging admittance

The power flow, real and reactive, is

$$P_{ik} - jQ_{ik} = E_i^* \, i_{ik}$$

$$= E_i^* (E_i - E_k)\, y_{ik} + E_i^* E_i\, \bar{y}_{ik}/2 \qquad (2.6)$$

where at bus i the real power flow from bus i to bus k is P_{ik} and the reactive power is Q_{ik}.

Similarly at bus k, the power flow from k to i is

$$P_{ki} - j\, Q_{ki} = E_k^* (E_k - E_i)\, y_{ik} + E_k^* E_k\, \bar{y}_{ik}/2 \qquad (2.7)$$

The power loss in line $i - k$ is the algebraic sum of the power flows determined from eqs. 2.6 and 2.7.

2.3 Solution Techniques of AC Load-Flow Problem

2.3.1 Gauss Iterative Method Using Y bus: (1)

The solution of a load-flow problem is initiated by assuming voltages for all buses except the slack bus. Then, currents are calculated for all buses except the slack bus "s" from the bus loading equation

$$I_i = (P_i - j\,Q_i)/\,E_i^* \qquad i = 1,2,\ldots,n \quad i \neq s \tag{2.8}$$

where n is the number of buses in the network. The performance of the network can be obtained from the equation I bus = Y bus E bus. Selecting the ground as the reference bus, a set of (n - 1) simultaneous equations can be written in the form

$$E_i = \frac{1}{Y_{ii}} \left(I_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} \; E_k\right) \qquad i = 1,2,\ldots,n \quad i \neq s \tag{2.9}$$

The bus currents calculated from eq. 2.8 , the slack bus voltage, and the estimated bus voltages are substituted into eq. 2.9 to obtain a new set of bus voltages. These new voltages are used in eq. 2.8 to recalculate bus currents for a subsequent solution of eq. 2.9. The process is continued until changes in all bus voltages are negligible. After the voltage solution has been obtained, the power at the slack bus and line flows are calculated.

2.3.2 Gauss Iterative Method Using Z bus: (1)

Selecting an intital set of bus voltages, bus currents are calculated from

$$I_i = (P_i - jQ_i) / E_i^* - y_i E_i \qquad i = 1,2, \ldots, n \quad i \neq s \tag{2.10}$$

where y_i is the total shunt admittance at bus i and the shunt connections are treated as current sources.

A new estimate of voltages is then obtained from the bus impedance network equation

$$E\ bus = Z\ bus\ I\ bus + E_R \tag{2.11}$$

where E_R is the vector whose elements are all equal to the voltage of the slack bus and the bus impedance matrix, formed by using the slack bus as reference, is of dimension (n - 1) x (n - 1). Eq. (2.11) can be expressed as follows

$$E_i^{m+1} = E_s + \sum_{\substack{k=1 \\ k \neq s}}^{n} Z_{ik} I_k^m \qquad i = 1,2,\ldots, n \quad i \neq s$$

where Z_{ik} = the (i,k) th element of the bus impedance matrix.

$$I_k^m = (P_k - jQ_k)/(E_k^m)^* - y_k E_k^m$$

and m is an iteration counter.

2.3.3 Gauss-Seidel Iterative Method Using Y bus: (1,3)

The bus voltage eq. 2.9 can also be solved by the Gauss-Seidel iterative method. In this method, the new calculated voltage E_i^{m+1} immediately replaces E_i^m and is used in the solution of the subsequent equations, 2.3.4 Gauss-Seidel Iterative Method Using Z bus (1,4,5)

The bus voltage eqs. 2.11 are solved one at a time in the order established by the bus coding. After each equation is solved to obtaine a new estimate of bus voltage, the corresponding bus current is recalculated. Then, the load-flow equations are given by

$$E_i^{m+1} = E_s + \sum_{\substack{k=1 \\ k\neq s}}^{i-1} Z_{ik} I_k^{m+1} + \sum_{\substack{k=i \\ k\neq s}}^{n} Z_{ik} I_k^{m} \qquad \begin{array}{l} i=1,2,\ldots,n \\ i\neq s \end{array} \tag{2.12}$$

where $I_k^{m+1} = (P_k - j\, Q_k)/(E_k^{m+1})^* - y_k\, E_k^{m+1}$

A similar approach by applying Gauss-Seidel to load-flow problems using bus impedance matrix, is found in [5]. In this method, unlike [4], ground is selected as reference bus and each load is reduced into a tie impedance to ground. The technique of reference [4] is simpler and converges in the same number of iterations as [5] for similar-sized systems.

2.3.5 Newton-Raphson Method: (6,7,8)

The generalized Newton-Raphson method is an iterative algorithm for solving a set of simultaneous nonlinear equations in an equal number of unknowns $F(X) = 0$. At a given iteration point, each function $f_i(X)$ is approximated by its tangent hyperplane. This linearized problem is constructed as the Jacobian matrix equation

$$F(X) = -J\ \Delta X \tag{2.13}$$

which is then solved for the correction ΔX . The square Jacobian matrix J is defined by $J_{ik} = \partial fi/\partial x_k$, and represents the slopes of the tangent hyperplanes. Matrix J is highly sparse in the load-flow applications and eq. 2.13 is solved directly and rapidly by using sparsity techniques.

The Newton method's convergence is sensitive to the behavior of the functions F(X) and hence to their formulation. The more linear they are, the more rapidly and reliably Newton's method converges. Nonsmoothness, i.e., humps in any function fi(X) in the region of interest can cause convergence delays, total failure, or misdirection to a nonuseful solution.

Since the chosen load-flow functions F(X) tend not to be too nonlinear and reasonably good initial estimates are available, these difficulties are encountered infrequently. In fact, applied to the vast majority of practical load-flow problems, Newton's method is very reliable and extremely fast in convergence.

The Newton load-flow formulations adopted to date use for F(X) the bus power or current mismatch expressions and designate the unknown bus voltages as the problem variables (X). Mathematically speaking, the complex load-flow equations are nonanalytic and cannot be differentiated in complex form. In order to apply Newton's method, the problem is separated into real equations and variables. Rectangular or polar coordinates may be used for the bus voltages.

The polar power-mismatch version is the most widely used of all formulations, whose Jacobian matrix eq. 2.13 can be written for convenience of presentation in the partitioned form

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \qquad (2.14)$$

Slack bus mismatches and voltage corrections are not included in 2.14 and likewise ΔQ_i and ΔV_i for each PV bus are absent. The submatrices H, N, M and L represent the negated partial derivatives of 2.4 and 2.5 with respect to the relevant θ's and V's, e.g., $H_{ik} = -\partial \Delta P_i/\partial\theta_k$. If buses i and k are not directly connected, their "mutual" terms in the J matrix are zero, and J is thus highly sparse, with positional but not numerical symmetry.

The polar power mismatch version converges to high accuracy, nearly always in 2 to 5 iterations, from a flat start ($V = 1$ per unit and $\theta = 0$) independent of a system size. The accepted formulation 2.14 can be improved by a minor modification which very often reduces the number of iterations by one and can avoid divergence in some extreme cases. Noting that the performance of Newton's method is closely associated with the degree of problem nonlinearity, the best left-hand defining functions are the most linear ones.

If eq. 2.5 is divided throughout by V_i, only one term (Q_i scheduled $/V_i$) on the right-hand side of this equation is not linear in V. Moreover, for practical values of Q_i scheduled and V_i, this non-linear term is numerically relatively small. It is therefore preferable to use a problem defining function $\Delta Q/V$ on the left-hand side of 2.14

in place of ΔQ. Dividing ΔP by V can also be helpful, but has a relatively small effect, since the active power component of the problem is not strongly coupled with voltage magnitudes.

A number of schemes are available for attempting to improve the performance of Newton's method. One of the simplest of these is to impose limits on the permissible sizes of the voltage corrections at each iteration, thereby helping to negotiate humps in the defining functions. With its quadratic convergence, Newton's method takes maximal advantage of good initial voltage estimates. Some programs perform one or two Gauss-Seidel iterations before the Newton process [6]. This is beneficial provided that the relatively weak Gauss-Seidel method does not diverge when faced with a difficult problem. A most rapid and reliable Newton program can be created by calculating good initial angular estimates using the dc load flow and also good voltage magnitude estimates by a similar technique [7].

The computing time per iteration of Newton's method rises, on an average, little more than linearly with the number of buses in the system. Since the number of iterations is size-invarient, the superiority of Newton's method increases rapidly speedwise over previous methods as the size of the system to be solved increases. For typical large systems, the computing time for one Newton iteration is roughly equivalent to seven Gauss-Seidel iterations [6].

2.3.6 Decoupled Methods (8,9)

The Decoupling Principle:

An inherent characteristic of any practical electric power transmission system operating in the steady-state is the strong interdependence between active powers and bus voltage angles, and between reactive powers and voltage magnitudes. Correspondingly, the coupling between these "P-θ" and "Q-V" components of the problem is relatively weak. Applied numerical methods are generally at their most efficient when they are able to take advantage of the physical properties of the system being solved. In the load-flow problem there has been a recent trend towards this objective by "decoupling" (solving separately) the "P-θ" and "Q-V" problems.

Decoupled Newton Method (10)

In 2.14 the elements to be neglected are those contained in submatrices [N] and [M]. Eq. 2.14 is then separated into two smaller matrix equations,viz., the P-θ and Q-V problems and are

$$\Delta P/V = A\,\Delta\theta \tag{2.15a}$$

$$\Delta Q/V = C\,\Delta V \tag{2.15b}$$

where A and C are negated Jacobian matrices. In this method dividing ΔQ by V is important, since it substantially reduces the nonlinearity of the Q - V problem.

Eqs. 2.15a and 2.15b can be constructed and solved simultaneously with each other at each iteration. First solve 2.15a for $\Delta\theta$ and use the updated θ in constructing and then solving 2.15b for ΔV. The first calculated values of θ are accurate to within a few degrees, even when starting from $\theta = 0$. The first solution of 2.15b then usually gives remarkably good values for V, within say, 0.3 per cent of the final solution. The decoupled method converges at least as reliably as the formal Newton version.

For convergence to practical accuracies, it usually takes a similar number of iterations. The computation time per iteration is 10 to 20 per cent less than for the formal Newton method.

Fast - Decoupled Method (9)

The first step in applying the decoupling principle is to neglect the coupling submatrices [N] and [M] in eq. 2.14 , giving two separated equations

$$[\Delta P] = [H] \quad [\Delta\theta] \tag{2.16}$$

$$[\Delta Q] = [L] \quad [\Delta V/V] \tag{2.17}$$

where $H_{km} = L_{km} = V_k V_m (G_{km} \sin \theta_{km} - B_{km} \cos \theta_{km})$ for $m \neq k$

$$H_{kk} = -B_{kk} V_k^2 - Q_k \quad \text{and} \quad L_{kk} = - B_{kk} V_k^2 + Q_k$$

In practical power systems the following assumptions are almost always valid:

$$\cos \theta_{km} \simeq 1 \quad , \quad G_{km} \sin \theta_{km} << B_{km} \quad , \quad Q_k << B_{kk} V_k^2$$

So that good approximations to 2.16 and 2.17 are:

$$[\Delta P] = [V.\ B'\ .\ V] \quad [\Delta\theta] \tag{2.18}$$

$$[\Delta Q] = [V.B''\ .\ V] \quad [\Delta V/V] \tag{2.19}$$

At this stage of the derivation, the elements of the matrices [B'] and [B''] are strictly elements of [-B] .

The decoupling process and final forms are now completed by:

(a) omitting from [B'] the representation of those network elements that predominantly affect MVAR flows, i.e. Shunt reactances and off-nominal in-phase transformer taps.

(b) omitting from [B"] the angle-shifting effects of phase shifters.

(c) taking the left-hand V terms in 2.18 and 2.19 on to the left-hand sides of the equations and in 2.18 removing the influence of MVAR flows on the calculation of [$\Delta\theta$] by setting all the right-hand V terms to 1 per unit. Note that the V terms on the left-hand side of 2.18 and 2.19 affect the behaviors of the defining functions and not the coupling.

(d) neglecting series resistances in calculating the elements of [B'], which then becomes the dc approximation load-flow matrix. This is of minor importance, but it is found experimentally to give slightly improved results.

With the above modifications the final fast-decoupled load-flow equations become

$$[\Delta P/V] \quad = \quad [B'] \; [\Delta\theta] \tag{2.20}$$

$$[\Delta Q/V] \quad = \quad [B''] \; [\Delta V] \tag{2.21}$$

Both [B'] and [B"] are real, sparse and have the structure of [H] and [L] respectively. Since they contain only network admittances they are constant and need to be factorized once only at the beginning of the study. [B"] is symmetrical and if phase shifters are absent or accounted for by alternative means [B'] is also symmetrical.

The immediate appeal of 2.20 and 2.21 is that very fast repeat solution for [$\Delta\theta$] and [ΔV] can be obtained using the constant factors of [B'] and [B"]. These solutions may be iterated with each other in some defined manner towards the exact solution, i.e. when [ΔP/V] and [ΔQ/V] are zero.

The method converges very reliably, usually in 2 to 5 iterations for practical accuracy on large systems. The method has the decoupled property of giving a very good approximate solution after the first one or two iterations. Provided that the [ΔP/V] and [ΔQ/V] functions are calculated efficiently, the speed per iteration is roughly 5 times that of the formal Newton method and two-thirds that of the Gauss-Seidel methods. The storage requirements of the fast decoupled method are about 40 % less than those of Newton's method.

The fast decoupled method offers a uniquely attractive combination of advantages over the established methods, including Newton's, in terms of speed, reliability, simplicity and storage, for conventional load-flow solutions. It is therefore chosen as the method for solving the ac system load flow equations in the thesis.

CHAPTER III

A NEW LOAD-FLOW SOLUTION TECHNIQUE OF INTEGRATED MULTI-TERMINAL DC/AC SYSTEMS

3.1 Introduction

The increasing interest in the operational feasibility and potential applications of multiterminal HVDC systems leaves a need for investigating load-flow methods suitable for including the dc system in an overall ac/dc load-flow solution. The size of ac system invariably greatly exceeds the size of even a most extensive multiterminal HVDC system, so the component of computing time used by the dc system is expected to be relatively short.

A dc system when viewed from the ac side of the converter appears either as a load, at a rectifier terminal, or as a source, at an inverter terminal, of active power. Reactive power is however, absorbed at both kinds of converter terminals. The amount of active and reactive power flow depends on a number of variables in the ac and dc systems.

Historically, as the requirement of inclusion of dc links in a power system load-flow program appeared, several papers described sequential solution methods.[11-16] In a sequential approach, the dc system load-flow solution including terminal constraints is formulated separately so that the terminal conditions can be imposed on the interconnection buses in an ac load-flow program. Each ac solution establishes the terminal ac bus voltages for the dc solution and then accepts the subsequent P (active power) and Q (reactive power) converter loading from the dc solution.

The general principle of the sequential solution method is to alternate between ac and dc load-flows until all the variables converge to the required accuracy. There are 3 basic types of convergence tests: [26]

(a) mismatch tolerances in the HVDC load-flow

(b) mismatch tolerances in the ac load-flow, and

(c) convergence of the interface quantities for which the tolerances are smaller than those in (a) and (b) by a factor of 10.

A number of alternative iterative sequences could be applied:

(i) converging the dc load-flow accurately before each ac iteration;

(ii) achieving a rough convergence of the dc load-flow and successively increasing the accuracy after each ac iteration;

(iii) alternating between accurately converged dc and ac solutions; and

(iv) alternating between single ac and dc solutions.

A typical number of dc iterations lies in the range 4-8, with scheme (i) requiring the highest and (iv) the lowest. Due to the short time required to perform a dc iteration as compared to an ac iteration, the differences between the different approaches are not crucial. Nevertheless, it is evident from the literature that the sequential solution is inherently inefficient [17-21], although it is attractive because it utilizes existing programs for ac systems with minimum modifications and by virtue of a separate dc system solution a greater flexibility in its modelling exists.

As the need for ac/dc system studies surged upwards, an earnest effort to devise the best procedure of computation with however little improvement over the sequential technique became paramount. Unified load-flow solutions [17-21] have been developed to take into account the interdependence between the ac and dc systems by solving simultaneously at each iteration the complete set of dc and ac systems equations within one load-flow program. A potent new technique [22] is developed by the author which incorporates multi-terminal HVDC systems within fast decoupled load-flow programs as an integral part of the ac iterative procedure. Inclusion of a conventional

2-terminal HVDC network forms only a special case and is solved readily by the new technique. In this technique, the representation of dc systems is such that it leads to simplifications in calculations and savings in storage requirements and yet it is so general that a multi-terminal dc system of any configuration and control characteristics can be easily simulated. This technique is fast, efficient and reliable and is therefore an improvement over known procedures.

The unified approach is more efficient and gives faster and more reliable convergence than the sequential approach, but requires complex programming.

3.2 Mathematical Formulation of HVDC Load-Flow Equations

3.2.1 Representation of HVDC Terminal

For efficient programming the dc link model should contain the minimum possible number of equations and variables. This would normally restrict the dc representation to plant components between, and including, the converter transformers in order to alter the tap-ratios without the need for recalculating the ac network admittance matrix.

Fig. 3.1 shows the basic converter model used in the analysis.

Series and/or parallel connection of the converters may be necessary to achieve the desired dc voltage or current. Fig. 3.2 illustrates a multiple bridge converter model. For this model, the transformer reactance is assumed to be the same for all transformers and transformer tap-ratios are also assumed to be equal.

The equivalent circuit of a single bridge HVDC converter model with its transformer represented by an equivalent π circuit is shown in Fig. 3.3. In this model, 'a' is the transformer turns-ratio, 'Y' is the transformer admittance, 'V' is the converter ac busbar voltage, 'Vd' is direct voltage and 'Id' is direct current.

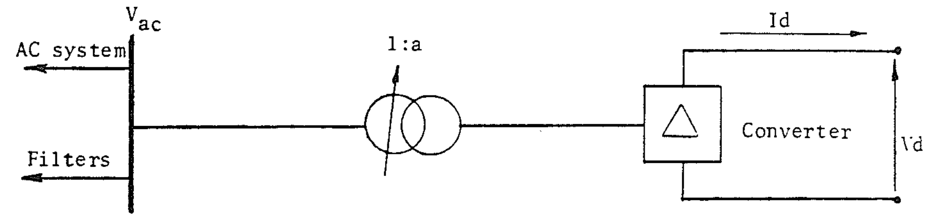


Fig. 3.1 Single-bridge converter model

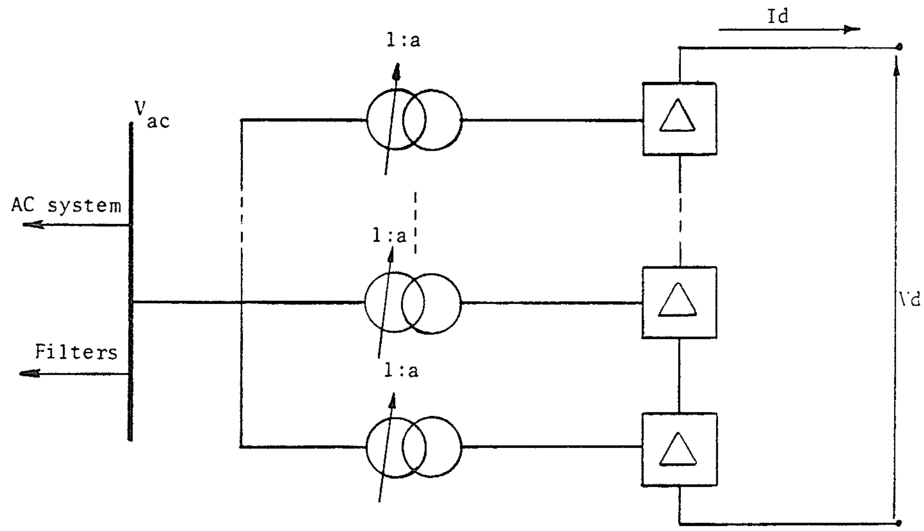


Fig. 3.2 Multiple-bridge converter model

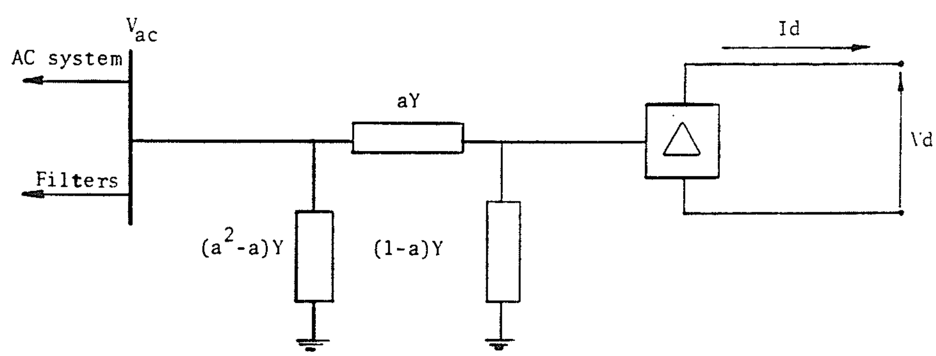


Fig. 3.3 Equivalent-circuit for single-bridge converter model

3.2.2 Choice of the Per-Unit System

In order to mesh the ac and dc equations directly in a load-flow program, the dc equations must be expressed in a per-unit system that is compatible with the ac per-unit system. Computational simplicity is achieved by using common power and voltage base parameters for both ac and dc systems. Both ac and dc per-unit systems are listed below.

AC Per-Unit System

$(\text{VA base})_{ac} = \text{MVA}$ (3-phase power)

$(\text{V base})_{ac} = E_L$ (Line to line) kV

Therefore $(\text{I base})_{ac} = (\text{MVA} \times 10^3) / \sqrt{3}\ E_L$ A

and $(\text{Z base})_{ac} = [(\text{V base})_{ac} / \sqrt{3}\ (\text{Ibase})_{ac}] \times 10^3$

$= E_L^2 / \text{MVA}$ Ω

DC Per-Unit System

$(\text{VA base})_{dc} = \text{MVA}$

$(\text{V base})_{dc} = E_L = (\text{V base})_{ac}$ kV

Therefore $(\text{I base})_{dc} = (\text{MVA} \times 10^3) / E_L$ A

and $(\text{Z base})_{dc} = (\text{V base})_{dc} / (\text{I base})_{dc}$

$= E_L^2 / \text{MVA}$ Ω

Hence $(\text{I base})_{dc} = \sqrt{3}\ (\text{I base})_{ac}$

and $(\text{Z base})_{dc} = (\text{Z base})_{ac}$

3.2.3 Converter Equations

The following basic assumptions are made in the derivation of the equations representing an ac/dc converter [23,24] :

1) All harmonics of voltage and current produced by a converter are filtered completely and the dc current is free of ripple.

2) The ac voltages and currents at the interface bus are balanced

and are sinusoidal waveforms of constant frequency.

3) The converter transformer has no resistance and requires no exciting current.

4) The valves of the converter have no forward voltage drop.

The equivalent circuit of Fig. 3.3 has been redrawn in Fig. 3.4 for a general single bridge converter terminal, for including the system variables and their reference directions. The reference of analysis is chosen to be the interface ac/dc bus as shown in Fig. 3.5. In these figures,

$I\underline{/-(\psi+\phi)}$ is the (r.m.s.) fundamental component of the transformer secondary current,

$E\underline{/-\psi}$ is the transformer secondary voltage,

ϕ is the power factor angle, and

θ is the control angle - either α (firing angle) for a rectifier terminal or δ (extinction angle) for an inverter terminal.

Applying Kirchoff's Current Law at node "m" of Fig. 3.4 we have

$$aY\,(V\underline{/0} - E\underline{/-\psi}) = \text{SIGN}\ I\underline{/-(\psi+\phi)} + (1-a)\,Y\,E\underline{/-\psi} \tag{3.1}$$

where SIGN = 1 for a rectifier terminal

and = -1 for an inverter terminal

Substituting Y by jB, where B is the transformer susceptance and $j = \sqrt{-1}$ eq. 3.1 in its complex form could be separated into the following two equations:

$$\text{SIGN}\ I\ \sin(\psi+\phi) = B\,E\cos\psi - a\,B\,V \tag{3.2}$$

$$0 = \text{SIGN}\ I\cos(\psi+\phi) + B\,E\sin\psi \tag{3.3}$$

I is related to Id by the expression

$$I = (\sqrt{6}/\pi)\ Id \tag{3.4}$$

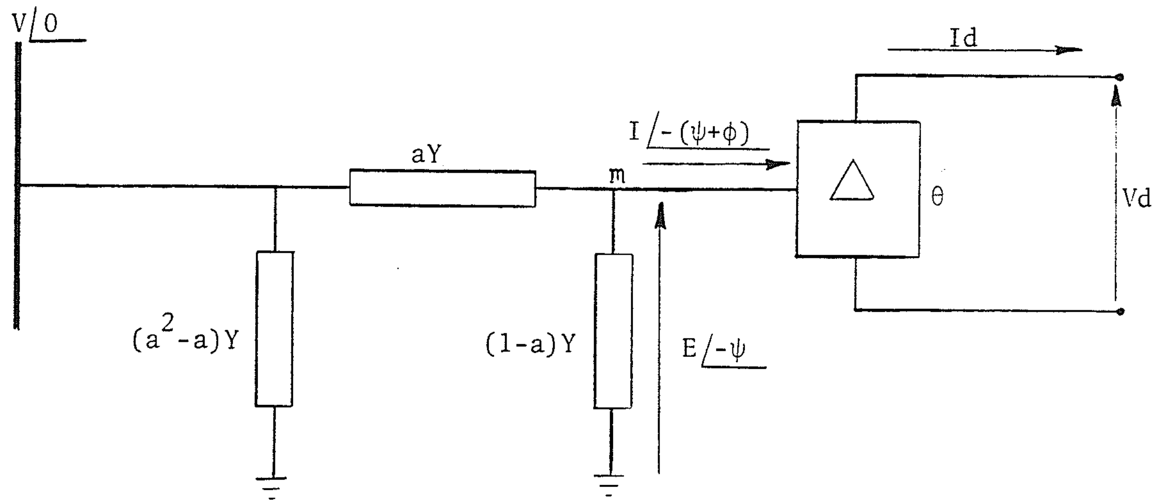


Fig. 3.4 Equivalent-circuit for an HVDC terminal

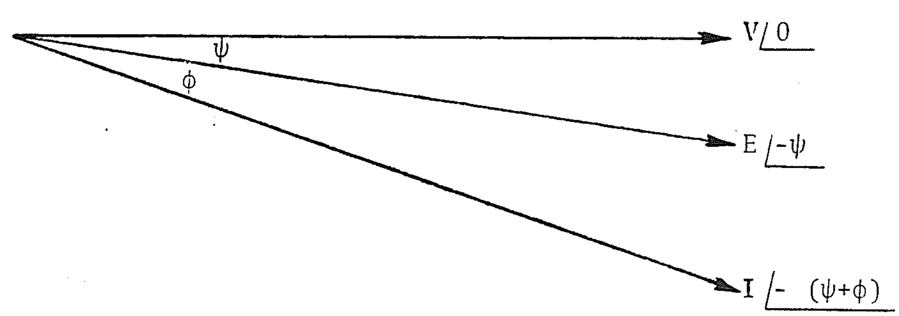


Fig. 3.5 Phasor diagram of a terminal

which is exact only if γ (overlap angle) = $0°$, but, which is true with a maximum error of 4.3% at $\gamma = 60°$ and only 1.1% for $\gamma \leqslant 30°$ (the normal operating range). The variation of (I/Id) with overlap angle is negligible for normal operating conditions [23]. The relation 3.4 is expressed in per-unit as

$$I = Kd\ Id \tag{3.5}$$

where $Kd = 3\sqrt{2} / \pi$

substituting from eq. 3.5 into eqs. 3.2 and 3.3 and writing in per-unit form, we get

$$SIGN\ Kd\ Id\ \sin(\psi+\phi) = B\ E\ \cos\psi - a\ B\ V \tag{3.6}$$

$$0 = SIGN\ Kd\ Id\ \cos(\psi+\phi) + B\ E\ \sin\psi \tag{3.7}$$

Neglecting losses, active ac power (Pac) equals dc power (Pd). For a single-bridge converter terminal

$$Pac = Pd \tag{3.8}$$

i.e. $\sqrt{3}\ I\ E\ \cos\phi = Id\ Vd$ (3.9)

Substituting from eq. 3.4 into eq. 3.9, we have

$$Vd = (3\sqrt{2}/\pi)\ E\ \cos\phi \tag{3.10}$$

In case of NB bridges, connected in series, per pole, for a terminal we have

$Pac = 2\ NB\ \sqrt{3}\ I\ E\ \cos\phi$,

$Pd = 2\ Id\ Vd$/pole, therefore Vd/pole is given by

$Vd = (3\sqrt{2}/\pi)\ NB\ E\ \cos\phi$

In general, the direct voltage per pole, of a terminal, is expressed in per-unit as

$$Vd = Ke\ E\ \cos\phi \tag{3.11}$$

where $Ke = (3\sqrt{2}/\pi)\ NB$

Also, the direct voltage of a single-bridge converter terminal is

$$Vd = (3\sqrt{2}/\pi)\ E \cos\theta - Id\ (3/\pi)\ Xc \tag{3.12}$$

where Xc is the bridge commutation reactance. In case of NB bridges per pole, the direct voltage per pole in per-unit is expressed by

$$Vd \doteq (3\sqrt{2}/\pi)\ NB\ E \cos\theta - Id\ (3/\pi)\ Xc(\text{equivalent})$$

where Xc (equivalent) is the equivalent (series-parallel combination of the individual bridges) commutation reactance of the terminal. Alternatively, the direct voltage per pole, of a terminal, is expressed in per-unit as

$$Vd = Ke\ E \cos\theta - Kc\ Xc(\text{equivalent})\quad Id \tag{3.13}$$

where $Kc = 3/\pi$

In summary, each terminal has the following converter equations:

$$\text{SIGN}\ Kd\ Id \sin(\psi+\phi) = B\ C \cos\psi - a\ B\ V$$

$$0 = \text{SIGN}\ Kd\ Id \cos(\psi+\phi) + B\ E \sin\psi$$

$$Vd = Ke\ E \cos\phi$$

$$Vd = Ke\ E \cos\theta - Kc\ Id\ Xc\ (\text{equivalent})$$

3.2.4 Network Equations

A linear formulation of network equations is used for the three possible network connections as follows:

(a) Mesh System

A single line diagram of n-terminal mesh connected system is shown in Fig. 3.6, for the case of n = 6.

The nodal currents - Id_1, Id_2 ..., Id_6 - are positive for rectifiers and negative for inverters.

The relation between the nodal dc currents and the dc bus voltages, in per-unit, could be written in the following matrix form

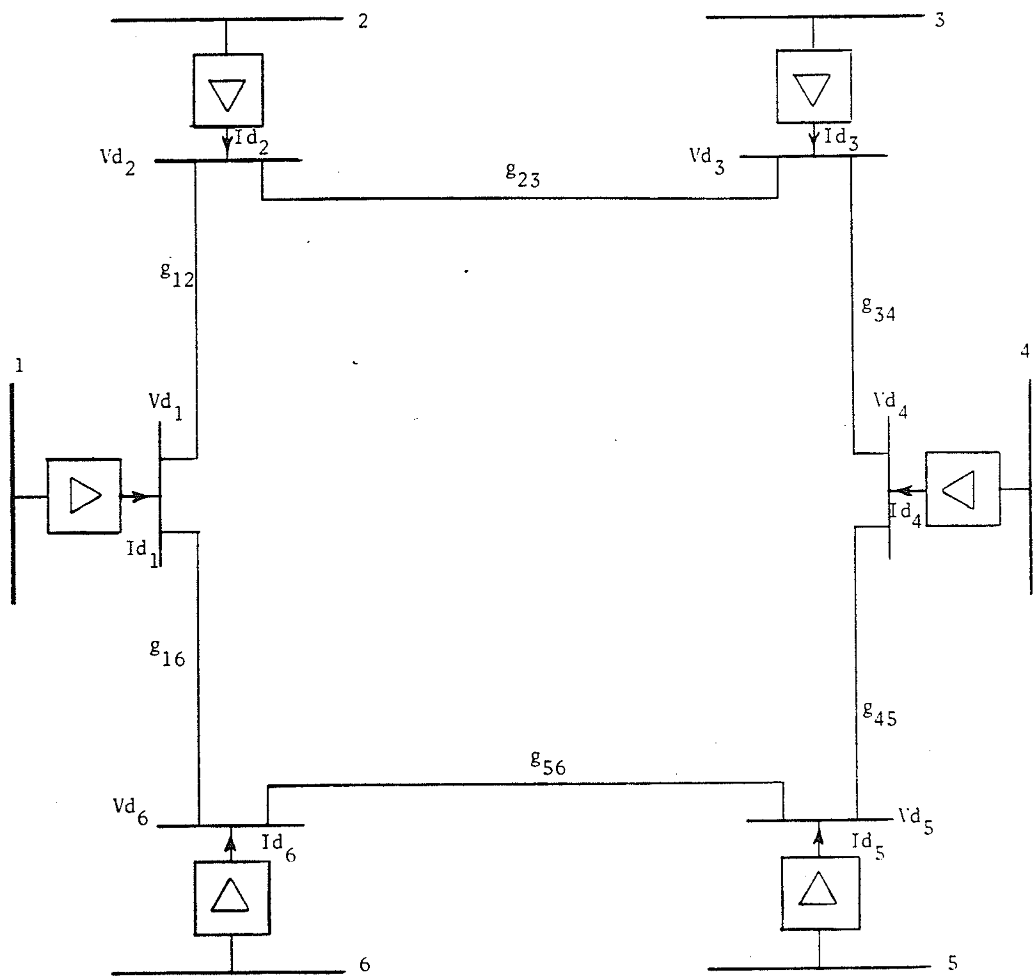


Fig. 3.6 6-Terminal mesh connected system

$$
\begin{bmatrix} Id_1 \\ Id_2 \\ Id_3 \\ Id_4 \\ Id_5 \\ Id_6 \end{bmatrix}
=
\begin{bmatrix}
g_{12} + g_{16} & -g_{12} & & & & -g_{16} \\
-g_{12} & g_{12} + g_{23} & -g_{23} & & & \\
& -g_{23} & g_{23} + g_{34} & -g_{34} & & \\
& & -g_{34} & g_{34} + g_{45} & -g_{45} & \\
& & & -g_{45} & g_{45} + g_{56} & -g_{56} \\
-g_{16} & & & & -g_{56} & g_{16} + g_{56}
\end{bmatrix}
\begin{bmatrix} Vd_1 \\ Vd_2 \\ Vd_3 \\ Vd_4 \\ Vd_5 \\ Vd_6 \end{bmatrix}
$$

Table 3.1 Network equations for the 6-terminal mesh system.

$$[Id] = [G][Vd] \tag{3.14}$$

where [Id] is the injected current vector,

[Vd] is the dc voltage vector, and

[G] is the conductance matrix, which is symmetrical and its elements are calculated as follows:

$$G_{ii} = \sum_{j\varepsilon i} g_{ij} \tag{3.15}$$

$G_{ij} = G_{ji} = -g_{ij}$ ($j \varepsilon i$ means node j directly connected to node i)

$i,j = 1,2,3\ldots.,n$ (n is the number of terminals)

The network equations of the system shown in Fig. 3.6 are written in matrix form in table 3.1.

(b) Radial System

A single line diagram of an n-terminal radial connected system with m-tap-buses is shown in Fig. 3.7. For this system n=6 and m=4 (T_1, T_2, T_3 and T_4).

The conductance matrix of the system including the tap-buses is calculated by

$$G_{ii} = \sum_{j\varepsilon i} g_{ij}$$

$$G_{ij} = G_{ji} = -g_{ij} \tag{3.16}$$

$i,j = 1,2,3,\ldots., n+m$

The network equations of the system of Fig. 3.7 are written in a matrix form in table 3.2.

The current injected at tap-buses T_1, T_2, T_3 and T_4 are zero ($Id_{T_1} = Id_{T_2} = Id_{T_3} = Id_{T_4} = 0$). Tap-buses should be assigned numbers larger than the number of terminals in the system, as shown in Fig. 3.7,

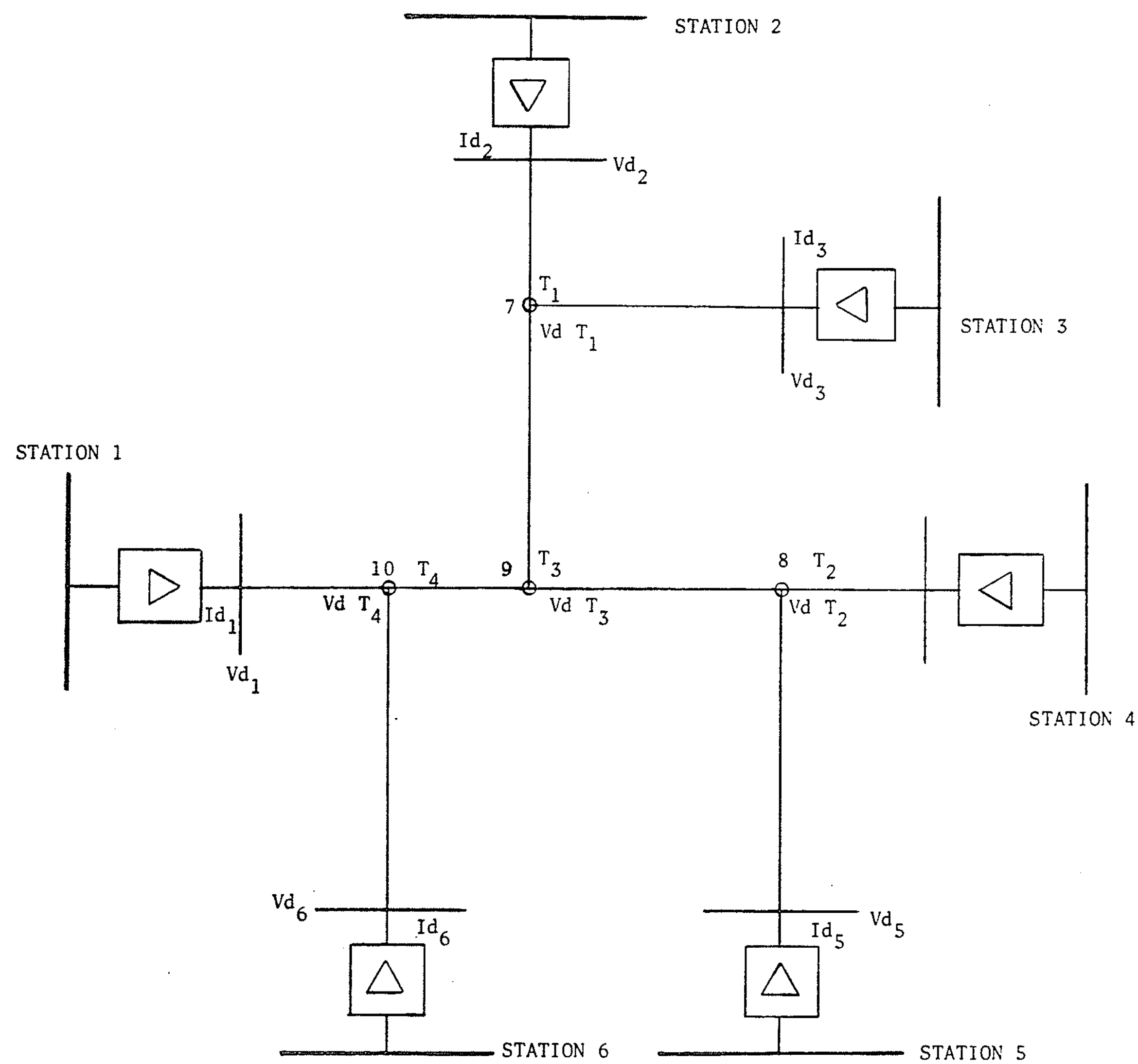


Fig. 3.7 6-Terminal radial system with 4-tap-buses

Table 3.2 Network equations for the 6-terminal radial system with 4-Tap buses

	Id_1		g_{110}									$-g_{110}$	Vd_1	
	Id_2			g_{27}					$-g_{27}$				Vd_2	
	Id_3				g_{37}				$-g_{37}$				Vd_3	
	Id_4					g_{48}				$-g_{48}$			Vd_4	
	Id_5	=					g_{58}			$-g_{58}$			Vd_5	
	Id_6							g_{610}				$-g_{610}$	Vd_6	
$Id\ T_1$	Id_7			$-g_{27}$	$-g_{37}$				G_{77}		$-g_{79}$		Vd_7	$Vd\ T_1$
$Id\ T_2$	Id_8					$-g_{48}$	$-g_{58}$			G_{88}	$-g_{89}$		Vd_8	$Vd\ T_2$
$Id\ T_3$	Id_9								$-g_{79}$	$-g_{89}$	G_{99}	$-g_{910}$	Vd_9	$Vd\ T_3$
$Id\ T_4$	Id_{10}		$-g_{110}$					$-g_{610}$			$-g_{910}$	G_{1010}	Vd_{10}	$Vd\ T_4$

TAP BUSES

$$G_{77} = g_{27} + g_{37} + g_{79}$$

$$G_{88} = g_{48} + g_{58} + g_{89}$$

$$G_{99} = g_{79} + g_{89} + g_{910}$$

$$G_{1010} = g_{110} + g_{610} + g_{910}$$

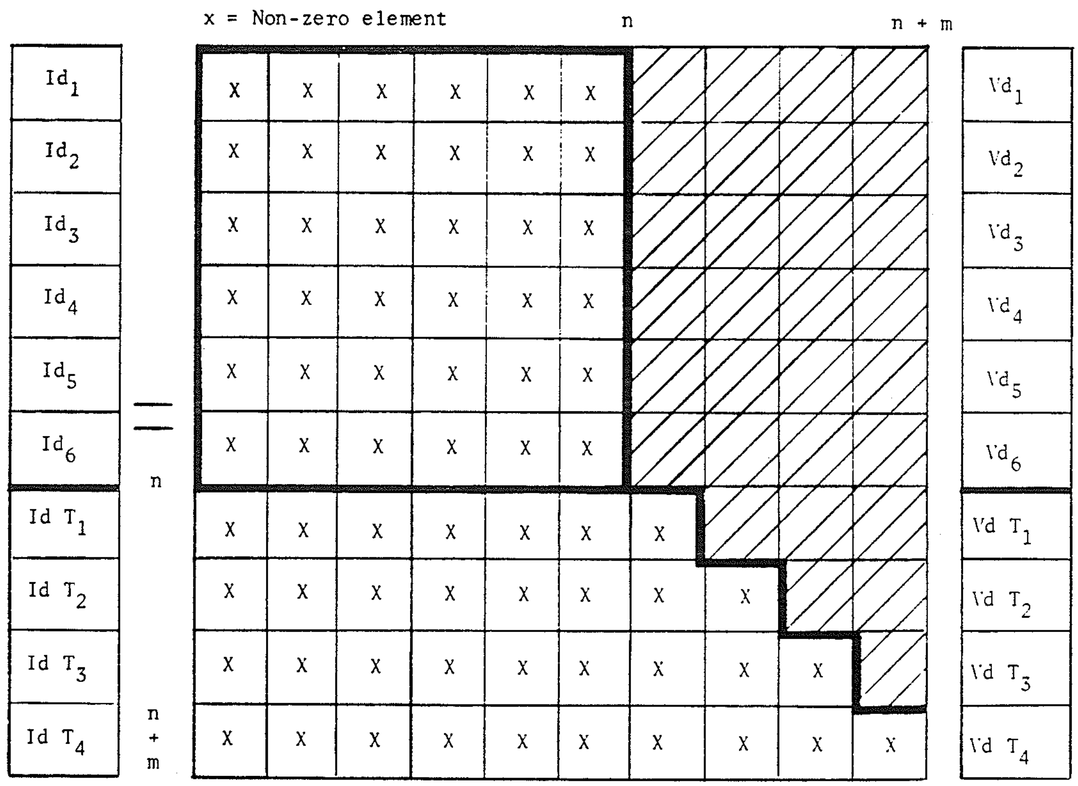


Fig. 3.8 Reduced conductance matrix for the radial connected system.

therefore they appear in the bottom rows and columns as shown in table 3.2. The tap-bus voltages (Vd_{T_1} , Vd_{T_2} , Vd_{T_3} and Vd_{T_4}) are irrelevant to the conversion process. They should be eliminated to reduce the number of equations to be solved, which leads to increased program efficiency.

Elimination of the tap-buses is achieved by a variant of Gauss column elimination, starting with the element $G_{\bar{n}-1,\bar{n}}$, where $\bar{n}$=n+m. This means that after m pivoting eliminations the hatched part in the matrix gets completely full of zeros as shown in Fig. 3.8. Due to the fact that the converter dc bus voltages are independent of the tap-bus voltages, only the upper left hand (nxn) matrix is relevant to the load-flow solution which is conveniently stored.

(c) Series System

In series connected system with (n-1) terminals on power control, the n^{th} terminal accommodates losses.

The common loop current is determined from

$$\Sigma\, Vd_i + Id\ R = 0 \tag{3.17}$$

3.2.5 Control Equations (16, 26, 73-75)

At a rectifier, the tap changer maintains the firing angle within typically $14° \leqslant \alpha \leqslant 16°$ for a minimum control angle of 5 - 7°. It ensures a margin of current control beyond $\alpha_{minimum}$ so that small fluctuations of ac voltage do not result in frequent changes in control allocations.

For a nominal firing angle of 15°, the margin is

$$\Delta Vd = Vdo\ (\cos 5° - \cos 15°) \simeq .03\ Vdo$$

= .03 x ideal no load voltage and is applicable for any value of Id or Xc. In practice tap-changer control is made according to

α - measurements. For the program, the dc bus voltage is formulated by allowing a 3% change in Vdo

i.e. $$Vd = .97 \ Ke\ E \cos \theta_{min.} - KcXc\ Id \tag{3.18}$$

Alternatively, allowing a 3% change in Vd

$$Vd = .97\ (Ke\ E \cos \theta_{min.} - KcXc\ Id) \tag{3.19}$$

results in a dc voltage larger, in theory, by 0.3%, assuming that KcXc Id is 10% of Vdo but is negligible, in practice, due to expected dead-band in the tap-changer control. The selection of either eq. 3.18 or 3.19 is arbitrary. Both equations provide a realistic solution to the voltage margin problem imposed by the multi-terminal application.

To determine the preferred inverter extinction angle, a strategy similar to that used for rectifiers can be used. That is, a voltage margin can be incorporated to serve the same purpose as in rectifiers on constant power (current) control. The same ΔVd can be introduced for the inverter as determined previously. For $\delta_{min.}$ typically $16° \leqslant \delta \leqslant 18°$, a preferred δ is about 21-22°.

At the voltage controlling station, the tap-changer keeps the voltage at the desired value Vd = Vd specified.

The tap-changer equation for a station on constant power (current) control is $Vd = .97\ (Ke\ E \cos \theta_{min.} - KcXc\ Id)$.

The system dc voltage is determined by one terminal with converters operating with either their minimum delay angle (α min.) if a rectifier, or their minimum extinction angle (δ min.) if an inverter. For this terminal, the control equations are:

$$Vd = Vd\ sp. \tag{3.20}$$

$$\cos \theta = \cos \theta_{min.} \tag{3.21}$$

Other terminals operate with constant power (current) control.

In reference [74] the principles and characteristics of 3 different control methods are shown: the current margin method, the voltage limiting method, and the operation to a current voltage characteristic.

The most suitable control methods are the current margin and voltage limiting methods. However, the author, and as is clear from many publications, supports the current margin method.

For both current margin and voltage limiting control methods, the dc voltage is determined by one terminal with converters operating with their minimum control angle. Other terminals are operating with current/power controls. Tap-changing control accommodates the desired dc voltage at the voltage controlling station and the preferred values of α and δ at stations on current (power) control.

Equations 3.20 to 3.23 represent control equations for the current margin method.

However, for voltage limiting control methods equations 3.20 to 3.22 apply and the tap-changer equation for terminals controlling current (power) is given by

$$\begin{aligned} Vd &= .95 \ Vd \ \text{limit} \\ &= .95 \ K_e \ E \cos \theta_{min} - .95 \ Kc \ Xc \ Id. \end{aligned}$$

For each terminal, two equations describe the converter controls, one from each group

$$\text{Group 1.} \quad Id = Id_{sp.} \qquad Pd = Pd_{sp.} \tag{3.22}$$

$$\text{Group 2.} \quad Vd = .97\ Ke\ E \cos \alpha_{min.} - .97\ Kc\ Xc\ Id \qquad Vd = .97\ Ke\ E \cos \delta_{min.} - .97\ Kc\ Xc\ Id \tag{3.23}$$

3.2.6 Converter Active and Reactive Power

The active and reactive power at the primary side of the converter transformer of a terminal are

$$Pdc = \sqrt{3}\ E\ I \cos \phi \tag{3.24}$$

$$Qdc = \sqrt{3}\ E\ I \sin \phi + (18/\pi^2)\ Xc\ Id^2 \tag{3.25}$$

Using the per-unit system, discussed before, Pdc and Qdc are given by

$$Pdc = E\ I \cos \phi \tag{3.24a}$$

$$Qdc = E\ I \sin \phi + (18/\pi^2)\ Xc\ Id^2 \tag{3.25a}$$

Substituting from eq. 3.5 into eqs. 3.24a and 3.25a we have

$$Pdc = Kd\ Id\ E \cos \phi \tag{3.26}$$

$$Qdc = Kd\ Id\ E \sin \phi + Kd^2\ Xc\ Id^2 \tag{3.27}$$

A dc converter is represented by a load which either absorbs active power (Pdc) and reactive power (Qdc) at a rectifier terminal or absorbs active power (-Pdc) and reactive power (Qdc) at an inverter terminal.

3.2.7 Filter Representation

Filters are capacitive at power frequency and located at the interface buses.

They can be either represented in the formulation of the bus admittance matrix as shunt elements, at the respective buses as shown in Fig. 3.9 or their effect can be simulated by subtracting the reactive power they supply ($Qc = V^2 Bf$, Bf is filter susceptance) from the reactive power demand of the dc terminal (Qd) to give a net reactive power loading (Qd - Qc), which is used to modify the injected reactive power at the interface (ac/dc) bus as shown in Fig. 3.10.

Both methods of filter representation are reliable.

3.2.8 Effect of Ground Current

Refer to Fig. 3.11.

It applies for a parallel connection only, since Vg = 0, in a series systems because there is only one ground point.

Normally, the grounding resistance at a terminal and the imbalance between pole-currents are small so that the voltage across the grounding resistance is negligible. Vg is given by

$$Vg = Rg\ (Idp - Idn) \tag{3.28}$$

where Rg is the grounding resistance and Idp and Idn are the positive and negative pole currents respectively.

If Vg has to be taken into consideration, the direct voltage Vd has to be modified to Vp by adding Vg as follows

$$Vp = Vd + Vg$$

Therefore, eqs. 3.11, 3.13, 3.14, 3.20 and 3.23 are modified to be

$$Vp = Ke\ E \cos \phi + Vg \tag{3.11}$$

$$Vp = Ke\ E \cos \theta - Kc\ Xc\ Id + Vg \tag{3.13}$$

$$[Id] = [G][Vp] \tag{3.14}$$

$$Vp = Vp \text{ specified} \tag{3.20}$$

$$Vp = .97\ (Ke\ E \cos \theta_{min.} - Kc\ Xc\ Id) + Vg \tag{3.23}$$

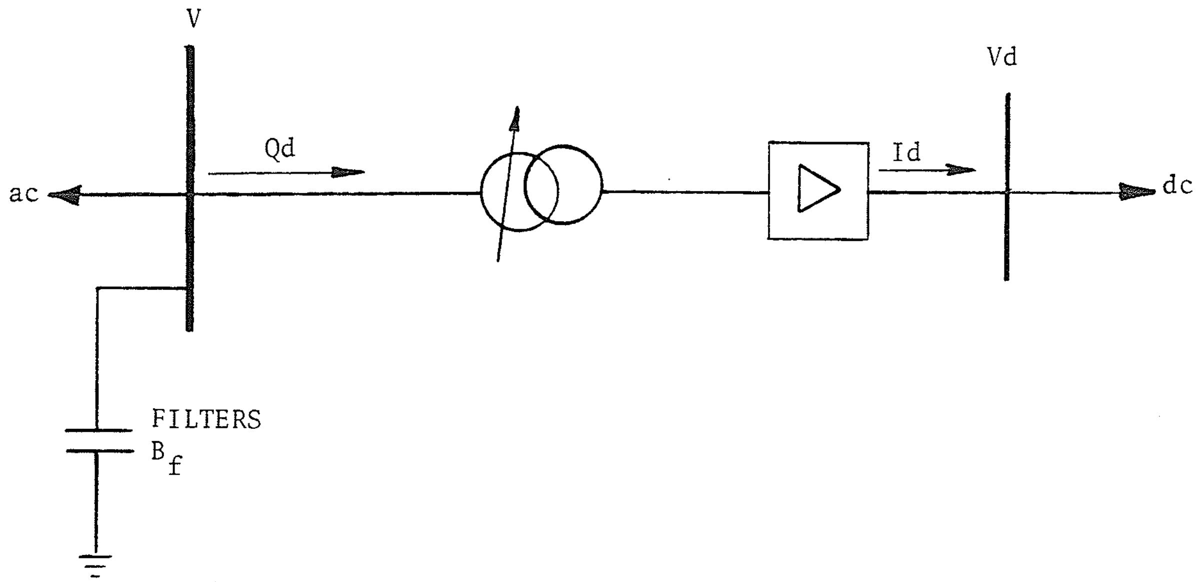


Fig. 3.9 Representation of filters on the ac side

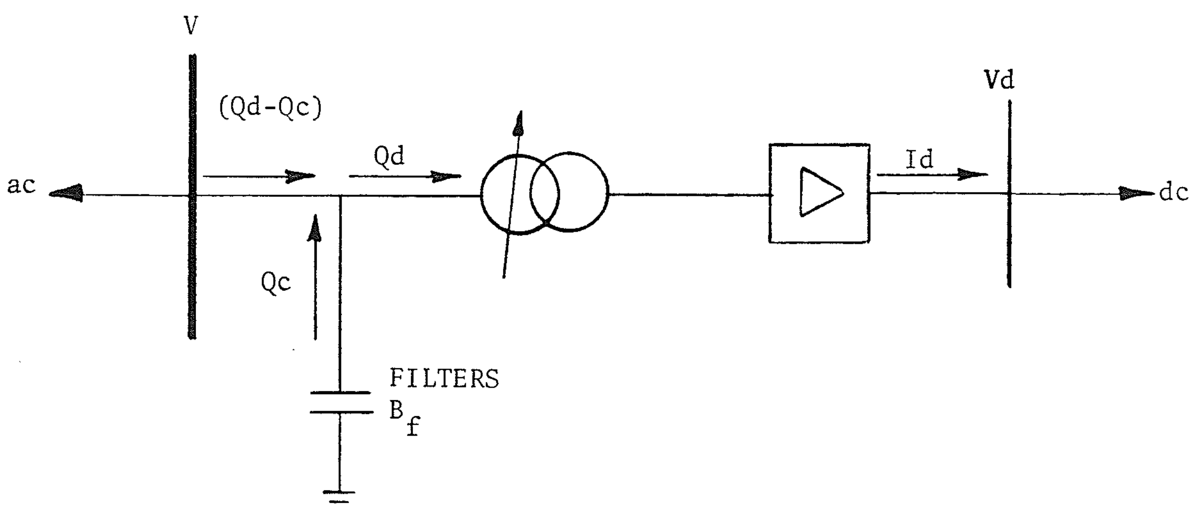


Fig. 3.10 Representation of filters on the dc side

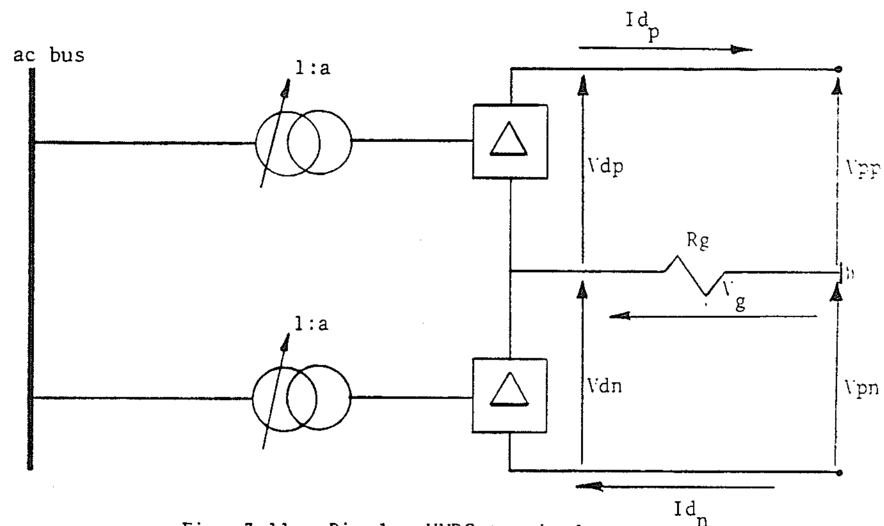


Fig. 3.11 Bipolar HVDC terminal

For zero ground current with non-negligible Rg,

$$Idp = Idn \tag{3.29}$$

3.3 HVDC Load-Flow Solution

3.3.1 Choice of Variables

The variables at each terminal are chosen to be -

E, ψ, a, ϕ , Vd, Id and cos θ

To eliminate trigonometrical nonlinearity and avoid overflows with infeasible operation modes, cos θ is used as a variable instead of θ.

3.3.2 Calculation of the Initial Conditions of the DC System

Two methods for the calculation of the initial conditions have been investigated. The salient features of which are as follows:

Method 1:

At each station, the ideal no-load direct voltage is calculated from

$$Vdo = Ke\ E \cos \theta_{min.} \tag{3.30}$$

The dc current is deduced from eq. 3.30 and the knowledge of the desired power interchange by

$$Id_1 = Pset/Vdo \tag{3.31}$$

A first estimate of the dc bus voltage is

$$Vd_1 = Vdo - Id_1\ Kc\ Xc \tag{3.32}$$

Better evaluation of the dc current and voltage are

$$Id_2 = Pset/Vd_1 \tag{3.33}$$

$$Vd_2 = Vdo - Id_2\ Kc\ Xc \tag{3.34}$$

Method 2:

(i) An initial estimate of the current can be calculated as

$$Idi = Pset\ i/Vd\ set \tag{3.35}$$

where Pset i = power interchange at station number i

i = 1,2,3,....

and Vdset = desired dc voltage at the voltage controlling station.

(ii) The dc bus voltages will be different from the set dc voltage at the voltage controlling station by the transmission line voltage drops which can be neglected for the purpose of calculating the initial values.

(iii) The control angles at the stations on power (current) control are close to $\alpha = 15°$ for rectifiers and $\delta = 22°$ for inverters assuming $\alpha_{min.} = 7°$ and $\delta_{min.} = 18°$

(iv) The initial value of the angle ϕ of 25° provides a reasonable initial reactive power level. Also the angle ψ has an initial value of zero.

(v) The tap-ratio a is chosen to be equal to 1(one) for all converter transformers.

The two methods have been found to be reliable, but the second method results in better initial values.

3.3.3 Method of Solution

Newton's method is used to solve the dc system equations in the form

$$R = A \ \Delta X \tag{3.36}$$

where R is the residual vector

ΔX is the change in the dc variables,

$$\Delta X = [\Delta Id \quad \Delta Vd \quad \Delta\cos\theta \quad \Delta E \quad \Delta\phi \quad \Delta\psi \quad \Delta a]^T$$

(T means transpose)

and matrix A is the dc Jacobian matrix $= -\dfrac{\partial R}{\partial X}$

The residual vector R is calculated as follows:

(a) Converter Equations

For each dc terminal we have the following four residuals:

$$R_1 = Vd - Ke\ E \cos\theta + Kc\ Xc\ Id \tag{3.37}$$

$$R_2 = Vd - Ke\ E \cos\phi \tag{3.38}$$

$$R_3 = SIGN\ Kd\ Id \cos(\phi+\psi) + B\ E \sin\psi \tag{3.39}$$

$$R_4 = SIGN\ Kd\ Id \sin(\phi+\psi) - B\ E \cos\psi + a\ B\ V \tag{3.40}$$

(b) Network Equations

$$Rg = G\ Vd - Id \tag{3.41}$$

for n-terminal dc system, Rg has n elements.

(c) Control Equations

Constant Voltage Control:

$$Rc = \cos\theta_{sp} - \cos\theta \tag{3.42}$$

$$Rt = Vd\ sp - Vd$$

Constant Current Control:

$$Rc = Id_{sp} - Id$$

$$Rt = .97\ Ke\ E \cos\theta_{min.} - .97\ Kc\ Xc\ Id - Vd \tag{3.43}$$

Constant Power Control:

$$Rc = Pdsp - Pd$$

$$Rt = .97\ Ke\ E \cos\theta_{min} - .97\ Kc\ Xc\ Id - Vd \tag{3.44}$$

The rows of eq. 3.36 are obtained as follows:

(a) Converter Equations

$$R_1 = Ke \cos\theta\ \Delta E - \Delta Vd - Kc\ Xc\ \Delta Id + Ke\ E\ \Delta \cos\theta$$

$$R_2 = Ke \cos\phi\ \Delta E - Ke\ E \sin\phi\ \Delta\phi - \Delta Vd$$

$$R_3 = -B \sin\psi\ \Delta E + SIGN\ Kd\ Id \sin(\psi+\phi)\ \Delta\phi$$
$$+ [SIGN\ Kd\ Id \sin(\psi+\phi) - B\ E \cos\psi]\ \Delta\psi - SIGN\ Kd \cos(\psi+\phi)\ \Delta Id$$

$$R_4 = B \cos\psi\ \Delta E - SIGN\ Kd\ Id \cos(\psi+\phi)\Delta\phi - [SIGN\ Kd\ Id$$
$$\cos(\psi+\phi) + B\ E \sin\psi]\ \Delta\psi - SIGN\ Kd \sin(\psi+\phi)\ \Delta Id - BV\ \Delta a$$

(b) Network Equations

$Rg = - G\ \Delta Vd + \Delta Id$

(c) Control Equations

$Rc = \Delta \cos\theta$

or $= \Delta\ Id$

or $= \Delta\ Pd = Id\ \ \Delta Vd + Vd\ \ \Delta Id$

$Rt = \Delta\ Vd$

or $= \Delta\ Vd + .97\ Kc\ Xc\ \ \Delta Id - .97\ Ke \cos\theta_{min.}\ \ \Delta E$

Fig. 3.12 illustrates the Jacobian matrix A for a 3-terminal (terminals m, k and n) HVDC system. Terminal m is on dc voltage control, terminal k is on dc current control and terminal n is on dc power control.

As shown in Fig. 3.12, some diagonal elements of the dc Jacobian matrix can be very small or zero. To avoid complications arising from these zero or near zero elements, for the computation of Δx, we should either use partial pivoting, in which case we have to use the full Jacobian matrix, or we can use the step-by-step solution [22] described below, for which we must store only the conductance matrix G which is very small and of dimensions n x n. For a 3-terminal dc system, the full Jacobian, Fig. 3.12, is a 21 x 21 matrix, whereas the conductance matrix is only 3 x 3. Therefore, in the step-by-step solution, storage requirement is only 1/49th. If sparsity techniques are used for storing the dc Jacobian matrix, the saving is less .

Step-by-Step Solution

Consider a 3-terminal (terminals m, k and n) HVDC system such that

m is on dc voltage control

k is on dc current control, and

n is on dc power control

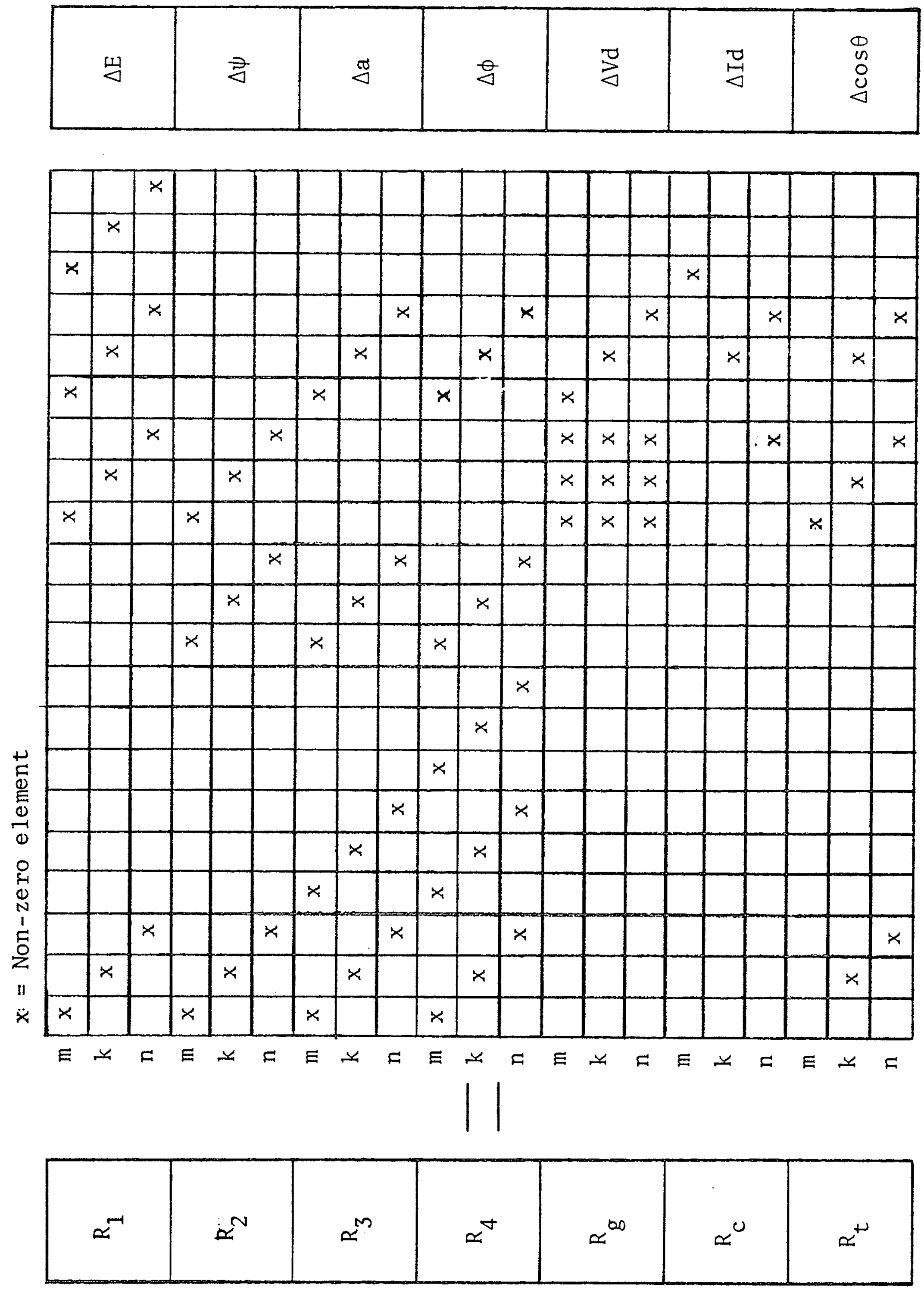


Fig. 3.12 DC Jacobian matrix for 3-terminal system

Calculation of ΔVd_m , ΔId_m , ΔVd_k , ΔId_k , ΔVd_n and ΔId_n:

Using control and network equations, we have

$$\Delta Vd_m = Rt_m \tag{3.45}$$

$$\Delta Id_k = Rc_k \tag{3.46}$$

$$\Delta Pd_n = Rc_n \tag{3.47}$$

or $\quad Vd_n \; \Delta Id_n + Id_n \; \Delta Vd_n = Rc_n$

i.e. $\Delta Id_n = Rc_n/Vd_n - (Pd_n/Vd_n^2)\, \Delta Vd_n$ (3.47)

Network equations give $-Rg = G\, \Delta Vd - \Delta Id$

i.e.,

$$\begin{bmatrix} -Rg_m \\ -Rg_k \\ -Rg_n \end{bmatrix} = \begin{bmatrix} g_{mm} & g_{mk} & g_{mn} \\ g_{km} & g_{kk} & g_{kn} \\ g_{nm} & g_{nk} & g_{nn} \end{bmatrix} \begin{bmatrix} \Delta Vd_m \\ \Delta Vd_k \\ \Delta Vd_n \end{bmatrix} - \begin{bmatrix} \Delta Id_m \\ \Delta Id_k \\ \Delta Id_n \end{bmatrix} \tag{3.48}$$

From Eqs. 3.45 to 3.48, the increments ΔId_m, ΔVd_k and ΔVd_n are computed as follows:

$$\begin{bmatrix} \Delta Id_m \\ \Delta Vd_k \\ \Delta Vd_n \end{bmatrix} = \begin{bmatrix} -1 & g_{mk} & g_{mn} \\ 0 & g_{kk} & g_{kn} \\ 0 & g_{nk} & (g_{nn}+Pd_n/Vd_n^2) \end{bmatrix}^{-1} \begin{bmatrix} -Rg_m - g_{mm} \; Rt_m \\ -Rg_k - g_{km} \; Rt_m + Rc_k \\ -Rg_n - g_{nm} Rt_m + Rc_n/Vd_n \end{bmatrix} \tag{3.49}$$

If terminal n is on constant current control, eq. 3.49 becomes

$$\begin{bmatrix} \Delta Id_m \\ \Delta Vd_k \\ \Delta Vd_n \end{bmatrix} = \begin{bmatrix} -1 & g_{mk} & g_{mn} \\ 0 & g_{kk} & g_{kn} \\ 0 & g_{nk} & g_{nn} \end{bmatrix}^{-1} \begin{bmatrix} -Rg_m - g_{mm} \; Rt_m \\ -Rg_k - g_{km} Rt_m + Rc_k \\ -Rg_n - g_{nm} Rt_m + Rc_n \end{bmatrix} \tag{3.49a}$$

Calculation of $\Delta \cos \theta$ and ΔE:

Constant Voltage Control

$$\Delta \cos \theta = Rc \tag{3.50}$$

From eq. 3.37 we have

$$-R_1 = Rt - Ke \quad E \quad Rc - Ke \cos \theta \, \Delta E + Kc \, Xc \quad \Delta Id$$

The only unknown in this equation is ΔE which is given by

$$\Delta E = (R_1 + Rt - Ke \, E \, Rc + Kc \, Xc \, \Delta Id) / \, Ke \cos \theta \tag{3.51}$$

Constant Current or Constant Power Control

From Eq. 3.43, we have

$$\Delta E = (-Rt + .97 \, Kc \, Xc \, \Delta Id + \Delta Vd) / \, .97 \, Ke \cos \theta_{min.} \tag{3.52}$$

Then from eq. 3.37 we have

$$\Delta \cos \theta = (R_1 + \Delta Vd - Ke \cos \theta \, \Delta E + Kc \, Xc \, \Delta Id) / Ke \, E \tag{3.53}$$

Calculation of $\Delta\phi$, $\Delta\psi$, and Δa:

The following steps are applied for all terminals. From eq. 3.38

$$\Delta\phi = (-R_2 - \Delta Vd + Ke \cos \phi \, \Delta E) / Ke \, E \sin \phi \tag{3.54}$$

From eq. 3.39

$$\Delta\psi = [R_3 + SIGN \, Kd \cos(\psi+\phi) \, \Delta Id + B \sin \psi \, \Delta E - SIGN \, Kd \, Id \sin(\psi+\phi) \Delta\phi \,] / \, (SIGN \, Kd \, Id \sin(\psi+\phi) - B \, E \cos \psi) \tag{3.55}$$

From Eq. 3.40

$$\Delta a = [-R - A_1 \, \Delta Id + A_2 \, \Delta E - A_3 \, \Delta\phi - \{ SIGN \, Kd \, Id \cos(\psi+\phi) + B \, E \sin \psi \} \, \Delta\psi] / B \, V \tag{3.56}$$

where $A_1 = SIGN \, Kd \sin(\psi+\phi)$

$A_2 = B \cos \psi$

and $A_3 = SIGN \, Kd \, Id \cos(\psi+\phi)$

At the end of each dc iteration, however, violations for the upper and lower limits of converter transformer taps "a" and firing or extinction angles "θ" are corrected.

Eqs. 3.45 through 3.56 provide the elements of ΔX vector which modify the values of dc variables and are used as initial values for the next iteration.

3.4 Load-Flow Solution of Integrated Multi-terminal DC/AC Systems

In the previous sections, the formulation and analysis of a model of multi-terminal HVDC system are discussed. A review of load-flow methods for ac system is presented in chapter (2). This section presents a new technique used to incorporate multiterminal HVDC System Jacobian equations within a fast-decoupled ac power system load-flow program.

3.4.1 Combined Jacobian Matrix and Equations of an Integrated AC/DC System

Interdependence exists between real and reactive power residuals of the ac system and the dc system variables, and also between the dc system residuals and the ac system variables. Thus, it is convenient to combine the ac system and the dc system Jacobian matrix equations, and remove the necessity to solve and interface between the two networks.

For the ac system alone, using the fast-decoupled method, recalling eqs. 2.20 and 2.21 and renaming [B'],[B"] by $[AJ_1]$ and $[AJ_4]$, respectively, we have

$$\left[\frac{\Delta P}{V} \right] = [AJ_1]\ [\Delta\theta]$$

$$\left[\frac{\Delta Q}{V} \right] = [AJ_4]\ [\Delta V]$$

Integration of ac and dc system equations yields

$$\begin{bmatrix} \frac{\Delta P}{V} \\ \frac{\Delta Q}{V} \\ R \end{bmatrix} = \begin{bmatrix} AJ_1 & C & PX \\ D & AJ_4 & QX \\ B & RV & A \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta X \end{bmatrix} \tag{3.57}$$

In eq. 3.57 the submatrices of the combined Jacobian matrix are derived in Appendix A. Submatrices AJ_1 , AJ_4, C and D have the same structure and values as in the ac system (C = 0 and D = 0).

Submatrices PX, QX, B and RV consist mainly of zero elements, (B = 0), except for the elements associated with the ac/dc buses.

Hence, the load-flow equations of an integrated ac/dc system are

$$[\frac{\Delta P}{V}] = [AJ_1][\Delta\theta] + [PX][\Delta X] \tag{3.58}$$

$$[\frac{\Delta Q}{V}] = [AJ_4][\Delta V] + [QX][\Delta X] \tag{3.59}$$

$$[\ R\ ] = [RV][\Delta V] + [A][\Delta X] \tag{3.60}$$

From eq. 3.58

$$[\frac{\Delta P}{V}] - [PX][\Delta X] = [AJ_1][\Delta\theta] \tag{3.61}$$

or

$$[\frac{\Delta P}{V}]_{INT} = [AJ_1][\Delta\theta]$$

where

$$[\frac{\Delta P}{V}]_{INT} = [\frac{\Delta P}{V}] - [PX][\Delta X]$$

$$= [\frac{\Delta P}{V}]_{ac} - \underbrace{[\frac{P}{V}]_{\text{dc terminals}} - [PX][\Delta X]}_{\text{only n elements}}$$

From eq. 3.59

$$[\frac{\Delta Q}{V}] - [QX][\Delta X] = [AJ_4][\Delta V]$$

or

$$[\frac{\Delta Q}{V}]_{INT} = [AJ_4][\Delta V] \tag{3.62}$$

where

$$[\frac{\Delta Q}{V}]_{INT} = [\frac{\Delta Q}{V}] - [QX][\Delta X]$$

$$= [\frac{\Delta Q}{V}]_{ac} - \underbrace{[\frac{Q}{V}]_{\text{dc terminals}} - [QX][\Delta X]}_{\text{only n elements}}$$

Substituting from eq. 3.62 into eq. 3.60 we get

$$[R] = [RV][AJ_4]^{-1}\{[\frac{\Delta Q}{V}] - [Qx][\Delta X]\} + [A][\Delta X]$$

$$= [RV][AJ_4]^{-1}[\frac{\Delta Q}{V}] - [RV][AJ_4]^{-1}[QX][\Delta X] + [A][\Delta X]$$

i.e. $$\{[R] - [RV][AJ_4]^{-1}[\frac{\Delta Q}{V}]\} = \{[A] - [RV][AJ_4]^{-1}[QX]\}[\Delta X] \tag{3.63}$$

Let $$[\Delta R]_{INT} = [RV][AJ_4]^{-1}[\frac{\Delta Q}{V}] \tag{3.64}$$

and $$[\Delta A]_{INT} = [RV][AJ_4]^{-1}[QX] \tag{3.65}$$

Therefore, eq. 3.63 becomes

$$\{[R] - [\Delta R]_{INT}\} = \{[A] - [\Delta A]_{INT}\}\ [\Delta X]$$

or $$[R]_{INT} = [A]_{INT}\ [\Delta X] \tag{3.66}$$

where $[R]_{INT} = [R] - [\Delta R]_{INT}$

and $[A]_{INT} = [A] - [\Delta A]_{INT}$

Eqs. 3.61, 3.62 and 3.66 are combined to give

$$\begin{bmatrix} (\frac{\Delta P}{V})_{INT} \\ (\frac{\Delta Q}{V})_{INT} \\ R_{INT} \end{bmatrix} = \begin{bmatrix} AJ_1 & & \\ & AJ_4 & \\ & & A_{INT} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta X \end{bmatrix} \tag{3.67}$$

In the above manipulations and for storage, sparsity techniques are fully exploited.

3.4.2 Effect of the Integration of an AC/DC System on the Load-Flow Equations of Each System in the Absence of the Other

In this section, comparison is made between load-flow equations of an ac system with and without dc terminals. Also between dc system load-flow equations with and without integrating both ac and dc systems.

DC System:

As stated before in eq. 3.36, Newton equations of dc system alone are

$$[R] = [A][\Delta X]$$

But for an integrated ac/dc system, load-flow equations of the dc system are given by eq. 3.66 as

$$[R]_{INT} = [A]_{INT}\ [\Delta X]$$

Comparing eqs. 3.66 and 3.36, one can notice that

(1) n elements of the residual vector [R] should be modified due to the integration of both ac and dc systems.

(2) n x 3n elements of the dc Jacobian matrix [A] should be modified due to the integration technique.

AC System:

It has been shown that fast-decoupled load-flow equations of an ac system in the absence of dc terminals are (eqs. 2.20 and 2.21)

$$[\frac{\Delta P}{V}] = [AJ_1]\ [\Delta\theta]$$

$$[\frac{\Delta Q}{V}] = [AJ_4]\ [\Delta V]$$

whereas the load-flow equations of an ac system in an integrated ac/dc system are given by (eqs. 3.61 and 3.62)

$$[\frac{\Delta P}{V}]_{INT} = [AJ_1][\Delta\theta]$$

$$[\frac{\Delta Q}{V}]_{INT} = [AJ_4][\Delta V]$$

Comparison between eqs. 3.61 and 2.20 reveals that

(1) n elements of the active power mismatches vector $[\frac{\Delta P}{V}]$ of the ac system should be modified due to the integration technique.

(2) $[AJ_1]$ is exactly the same for both cases.

Further by comparing eqs. 3.62 and 2.21, one observes that

(1) n elements of the reactive power mismatches vector $[\frac{\Delta Q}{V}]$ of the ac system are modified due to the integration of ac and dc systems, and

(2) Unlike [19] submatrix $[AJ_4]$ of an integrated ac/dc system remains exactly the same as in an ac system in the absence of a dc network.

3.4.3 Method of Solution

The following steps are taken: (Flowchart- Appendix E)

1. Calculate the initial values for both ac and dc system variables.
2. Form and factorize AJ_1 and AJ_4, using the Bi-factorization method.
3. Calculate $(\Delta P/V)_{ac}$,$(\Delta Q/V)_{ac}$ and R.
4. Calculate RV, QX and PX.
5. Calculate ΔX:

   (a) Elements of ΔX excluding Δa are calculated as in the normal case of dc system using the step-by-step method as outlined earlier.

   (b) Residual vector R_4 has to be modified due to integrating ac and dc systems as follows

$$R_{4_{INT}} = R_4 - \Delta R_{INT}$$

where

$$\Delta R_{INT} = RV \cdot AJ_4^{-1} \cdot (\frac{\Delta Q}{V})$$

$$= RV \cdot \Delta V_1$$

Zollenkopf's method is used to calculate ΔV_1 and then by multiplying it by RV using a sparsity technique we get ΔR_{INT} .

(c) A technique is developed using the same routines of Zollenkopf, which can be used in the multiplication of two sparse matrices, to calculate $\Delta A_{INT} = RV \cdot AJ_4^{-1} \cdot QX$. The only effect of ΔA_{INT} is to modify the value of A_1, A_2 and A_3 of eq. 3.56.

(d) Δa is computed from eq. 3.56 using $(R_4)_{INT}$ in place of R_4 and the modified values of A_1, A_2 and A_3. This completes the first dc iteration. Update X.

6. Modify n elements of $(\Delta P/V)_{ac}$ given by step 3 and corresponding to the ac/dc bus bars to get $(\Delta P/V)_{INT}$ and then use Zollenkopf's to solve for $\Delta\theta$ according to eq. 3.61, update θ.

7. Use the updated values of X and θ in forming $(\Delta Q/V)_{INT}$ then solve for ΔV according to eq. 3.62, update V.

This completes the first iteration of the integrated ac/dc system. The pattern for subsequent iterations is the same except that we start from step (3), since matrices AJ_1 and AJ_4 are constant and need to be factorized only once at the beginning of the solution. Convergence tests are used for the integrated system with the criteria

$$\max|\Delta P| \leqslant \varepsilon_p$$

$$\max|\Delta Q| \leqslant \varepsilon_q \quad \text{for all buses}$$

where $\max|\Delta P|$ and $\max|\Delta Q|$ are largest absolute elements of active power mismatches vector $[\Delta P]$ and reactive power mismatches vector $[\Delta Q]$, respectively. ε_p and ε_q are specified tolerances.

3.5 Application of the Proposed Technique to Sample Systems

In the absence of any existing systems the proposed load-flow solution technique was tested on hypothetical systems. These systems are based on the AEP - 14 bus system described in [15]. The procedure

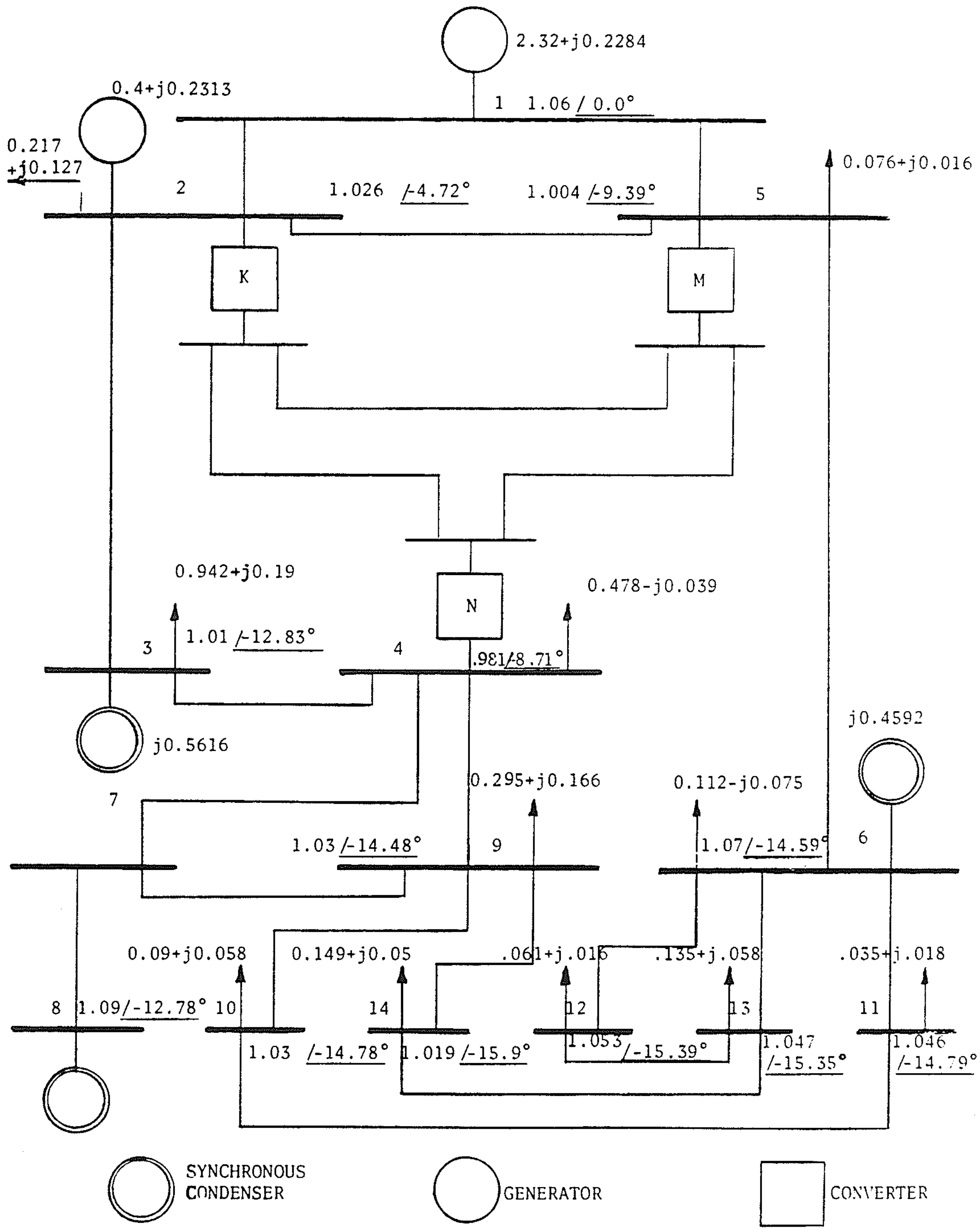


Fig. 3.13 Sample system, including loads, generation and bus-voltages.

Table 3.3

Data for the ac lines

LINE	BUSBAR		R p.u.*	X p.u.*	B p.u.*	TRANS-FORMER TAP %
	SENDING	RECEIVING				
1	1	2	.01938	.05917	.0528	
2	1	5	.05403	.22304	.0492	
3	2	3	.04699	.19797	.0438	
4	2	4	.05811	.17632	.0374	
5	2	5	.05695	.17388	.0340	
6	3	4	.06701	.17103	.0346	
7	4	5	.01335	.04211	.0128	
8	4	7	-	.20912	-	-2.2
9	4	9	-	.55618	-	-3.1
10	5	6	-	.25202	-	-6.8
11	6	11	.09498	.19890		
12	6	12	.12291	.25581		
13	6	13	.06615	.13027		
14	7	8	-	.17615		
15	7	9	-	.1101		
16	9	10	.03181	.08450		
17	9	14	.12711	.27038		
18	10	11	.08205	.19207		
19	12	13	.22092	.19988		
20	13	14	.17093	.34802		
21	9	9	-	-5.26		

*100 MVA base

R = resistance , X = reactance , B = line charging

Table 3.4

Data for 2 terminal dc system

	Converter 1	Converter 2
Trans. reactance, p.u.	0.10	0.07
Comm. reactance, p.u.	0.10	0.07
Filter susceptance, p.u.	0.4902	0.6301
Resist. of dc line, p.u.	0.00334	
Const. dc current, p.u.	0.456	----
Const. dc voltage, p.u.	----	1.284
Minimum angle, degrees	7°	18°

Table 3.5

(a) Data for 3 terminal mesh connected dc system

	Converter M	Converter K	Converter N
Transformer reactance, p.u.	0.10	0.07	0.04
Commutation reactance, p.u.	0.10	0.07	0.04
Constant current, p.u.	---	0.4362	0.916
Constant voltage, p.u.	1.286	---	---
Minimum angle, degrees	7°	7°	18°
Dc line resistances, p.u.	0.005	0.005	0.005
	R_{MK}	R_{KN}	R_{NM}

(b) Data for 3 terminal radial connected dc system

Same data given in (a) except

R_M = .005 p.u.

R_K = .005 p.u.

R_N = .005 p.u.

Table 3.6

Line flows for the integrated ac/dc system

Line	SB	EB	Real Power	Reactive Power	Line	SB	EB	Real Power	Reactive Power
1	1	2	1.5641	0.1218	12	6	12	0.0776	0.0338
1	2	1	-1.5215	-0.0493	12	12	6	-0.0768	-0.0322
2	1	5	0.7559	0.1066	13	6	13	0.1670	0.1046
2	5	1	-0.7276	-0.0421	13	13	6	-0.1648	-0.1002
3	2	3	0.7298	-0.0588	14	7	8	0.0000	-0.3246
3	3	2	-0.7060	0.1137	14	8	7	0.0000	0.3420
5	2	5	0.4146	-0.0117	15	7	9	0.2876	0.0502
5	5	2	-0.4053	0.0050	15	9	7	-0.2876	-0.0414
6	3	4	-0.2401	0.2579	16	9	10	0.0570	-0.0180
6	4	3	0.2489	-0.2698	16	10	9	-0.0569	0.0182
8	4	7	0.2877	-0.2433	17	9	14	0.0921	-0.0010
8	7	4	-0.2877	0.2742	17	14	9	-0.0911	0.0032
9	4	9	0.1615	-0.0789	18	10	11	-0.0312	-0.0759
9	9	4	-0.1615	0.0976	18	11	10	0.0317	0.0771
10	5	6	0.4367	-0.2375	19	12	13	0.0143	0.0163
10	6	5	-0.4367	0.2992	19	13	12	-0.0142	-0.0162
11	6	11	0.0657	0.0966	20	13	14	0.0551	0.0572
11	11	6	-0.0646	-0.0942	20	14	13	-0.0541	-0.0552

DC TERMINALS

FROM	TO	REAL POWER	REACTIVE POWER
2	K	0.5609	0.2241
4	N	-1.1720	0.6331
5	M	0.6147	0.2590

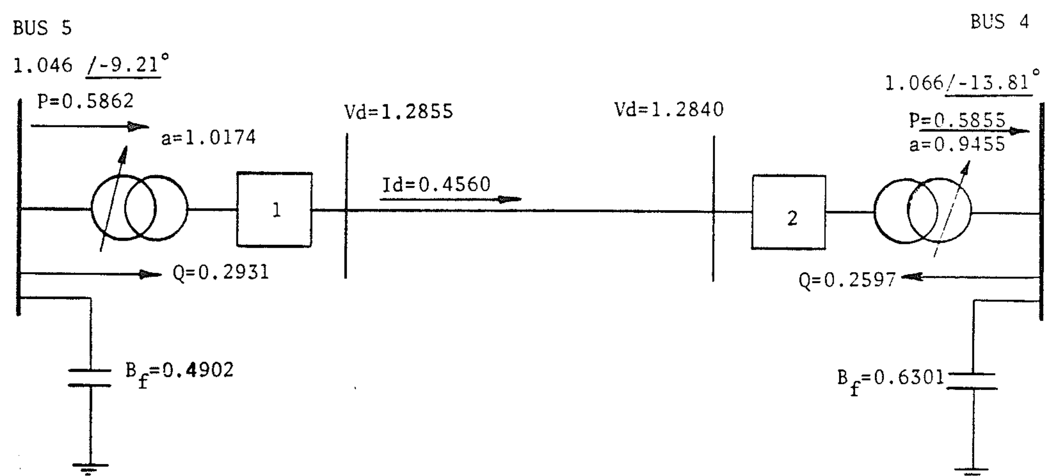


Fig. 3.14 Dc parameters of a 2-terminal system resulting from the ac-dc load-flow.

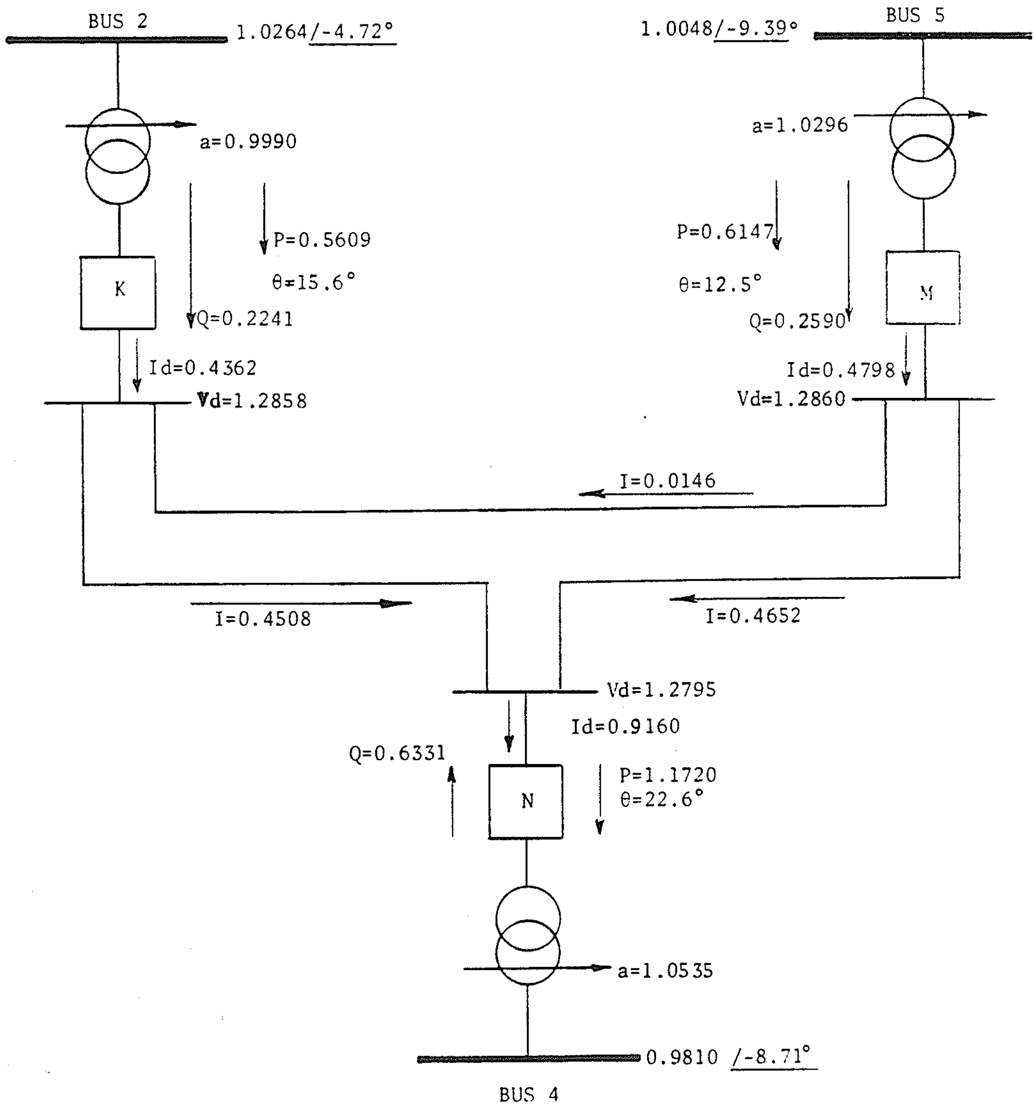


Fig. 3.15 Dc parameters of a 3-terminal mesh connected system resulting from the ac-dc load flow.

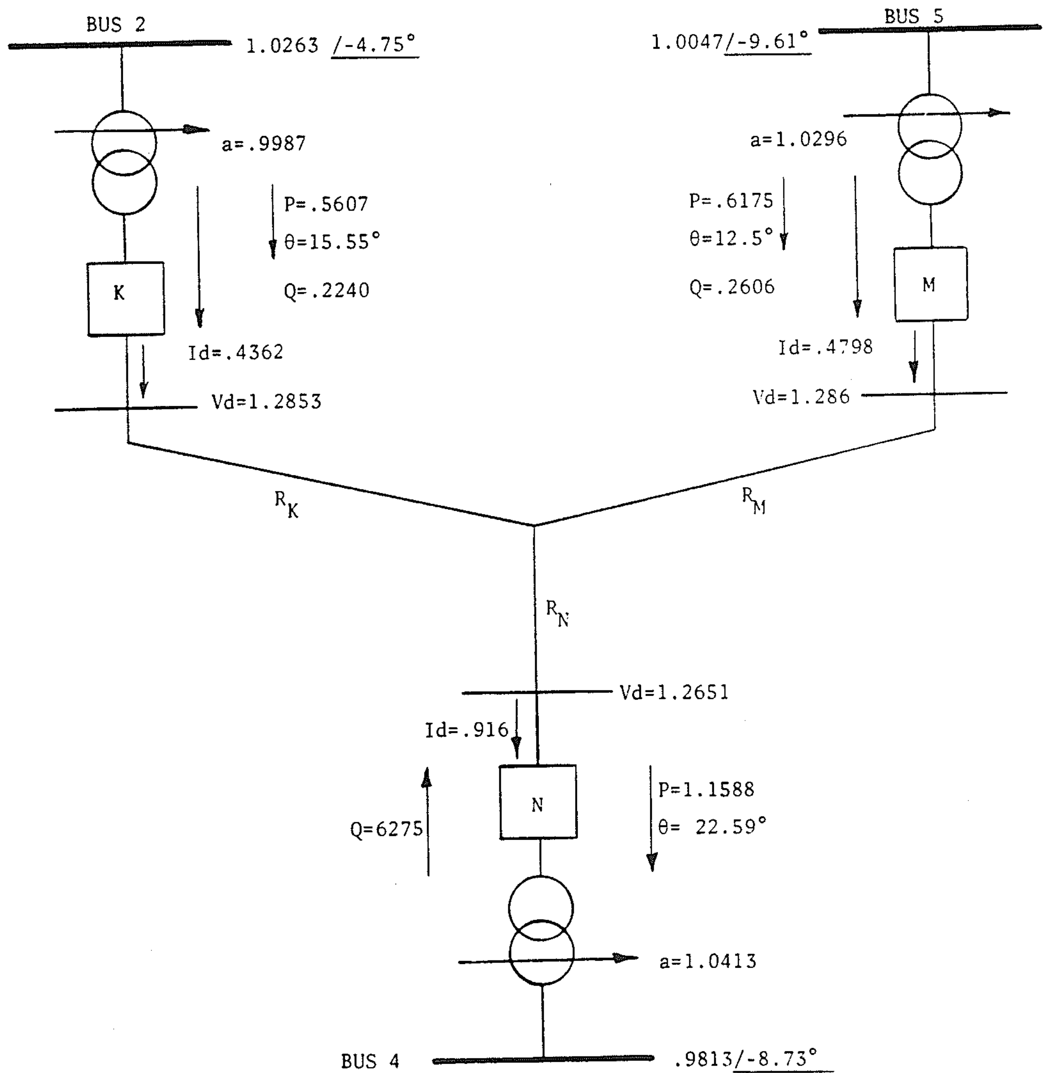


Fig. 3.16 Dc parameters of a 3-terminal radially connected system resulting from the ac-dc load-flow.

was tested on three systems. The first system had a two terminal dc obtained by replacing ac line 4-5, the second system had a three terminal dc mesh connection obtained by replacing lines 4-5 and 2-4 (Fig. 3.13), and the third system was the same as the second one, but with radial connection.

Data for the ac system are summarized in table 3.3. Data for dc systems are given in tables 3.4 and 3.5. The power setting on dc lines was made equal to the active power carried by replaced ac lines.

Each system required 4 iterations. Table 3.6 provides the details of the results obtained for the second system which has a 3-terminal mesh connected dc system.

Details of the resulting dc parameters for both 2-terminal and 3-terminal dc systems are given in Figs. 3.14, 3.15 and 3.16.

Accuracy of the Solution

The program has a default accuracy of .0005 per-unit for all ac and dc variables. Tolerances of less than 0.5 MW and 5 MVARS mismatches per bus are easily obtained. The dc residuals of zero values are also obtained.

Speed of the Solution

The proposed load-flow solution technique of integrated multiterminal dc/ac systems converges in 4-6 iterations irrespective of the number of terminals.

The number of iterations may be increased in the following cases:

(i) If the transmission voltage is modified due to a tap-limit violation, in this case at least one extra iteration is required.

(ii) If the calculated tap-setting is not practically feasible at more than one station, 2 additional iterations may be required.

(iii) If the transformer turns-ratios have to be corrected because we assume in the derivation of the dc equations that the transformer turns-ratios are continuously variable, whereas actually transformers have discrete tap-steps.

(iv) If a practical solution for the operating conditions is required, for example after each iteration a test has to be made to ensure that the voltage controlling station has the lowest loaded voltage limit. If the test result is not satisfactory, the program should assign voltage control to the station with the lowest loaded voltage limit. Each reallocation of voltage control results in an extra iteration before final convergence is achieved.

3.6 Conclusions

This chapter presents a fast technique for load-flow studies of integrated ac/dc systems which embodies the following features:

(1) It employs fast decoupled load-flow techniques. It is found that the reliability, computational speed and storage advantages offered by the basic fast-decoupled algorithm [9] are preserved as far as the ac network is concerned.

(2) The dc system is formulated in a most general way such that any multi-terminal system of any configuration and control characteristics are easily accomodated.

(3) The choice of the dc system variables and equations makes the calculation procedure very simple. A considerable saving is evident from the requirement of factorizing AJ_4 only once.

(4) The step-by-step solution for dc system is faster and requires less storage as compared to other known procedures.

(5) The overall efficiency of the proposed technique is greatly improved by using Zollenkopf's bi-factorization and other sparsity techniques.

(6) In summary, the technique is fast, efficient and reliable and is therefore a distinct improvement over known procedures.

CHAPTER IV

A NEW METHOD FOR OUTAGE STUDIES

4.1 Introcuction

The major objective of power transmission system planning and operation engineers is to have a sound system operating at its peak efficiency. However, a good system should be able to survive contingencies, leading to system elements outages, and settle down to a near optimal state without over-loading of and over-voltages at system elements. It is very important to have techniques which can determine post contingent network voltages and power flows in a most economical way. Generally, an experienced systems engineer, having identified an efficient system can guess at the crucial components which if removed from the system may lead to over-voltages or over-loadings. However, he also needs a tool to calculate the consequent system state and verify his guess. Since a number of cases must be examined, the economy and speed of calculations are of paramount importance. An absolute accuracy in calculations is of secondary importance as the results within engineering tolerances are generally sufficient. It is more so because all critical cases have to be analysed in greater details anyway.

The analysis of this chapter is restricted to the outages of ac elements of an integrated dc-ac system. Consistent with the above-mentioned requirements for contingency/outage analysis this chapter presents a very fast, although approximate, technique[39] to provide the post-contingency load-flow in the event of a single or multiple element outage . The technique requires one iteration starting from the load-flow data of the base system and identification of outages. On account of the nature of technique it is safe to assert that solutions consequent to changes in the generation or the loads can be accommodated with the same speed. In principle the proposed method is based on the use of fast decoupled load-flow technique and the concept of simulating a line or transformer outage by injecting power equal to that flowing into the outaged element at appropriate buses. This technique reverts the changed network configuration back to the original in a mathematical model. The usefulness of this technique has been illustrated very well by Sachdev and Ibrahim[35]. By combining the outage simulation technique with the fast decoupled load-flow technique a number of advantages are gained. As compared to [35] our method does not require calculation of power injection modification factors and reduces the sensitivity matrix elements calculation effort to half. Since our method needs only one iteration and uses bi-factorization and sparse matrix programming techniques it is, we believe, the fastest.

In this chapter the proposed method is outlined by considering the outage of one element. Later, the technique is generalized to take into account multiple outages.

In the following at first, some known techniques are discussed and later details of the proposed technique are presented.

4.2 Review of Known Techniques for Outage Studies

Contingency analysis methods that do not model the outage probabilities of system elements but rather simulate element outages, by actual removal, are considered to be deterministic methods. The use of ac power flow methods for outage analysis may be considered a deterministic approach, characterized by excellent accuracy but otherwise expensive computationally and excessively demanding of engineering time. Methods appropriate for contingency analysis have been developed and are summarized as follows

A. DC Power Flow Contingency Analysis:-[27]

The simplest, but perhaps most inaccurate method for analyzing the effects of not only single but also multiple contingencies, is a method based on the dc power flow equations. These equations, usually (N-1) in number, where N is the number of buses, model only the real power flow and ignore the reactive power flow. In many cases all line resistances are neglected and the line flows are assumed to be considerably smaller than the steady state stability limit of the line. The result of all these assumptions is that we obtain a linear model of the network to facilitate performing multiple-contingency outages using the principle of superposition.

To develop the dc power flow equations, we must start with the standard ac power flow equations

$$P_i = \text{Real}\,(E_i^* \sum_j Y_{ij}\, E_j) \tag{4.1}$$

$$Q_i = -\text{Im}(E_i^* \sum_j Y_{ij}\, E_j) \tag{4.2}$$

$$i=1,2,\ldots,N$$

Assuming the voltage magnitudes remain unchanged and equal to their base - case values, the N net injected reactive power equations can be ignored, leaving only the N real power equations. Considering bus 1 is the slack, we have

$$P_i = \text{Real}\ (E_i^* \sum_j Y_{ij}\ E_j)$$

$$= \text{Real}\ (V_i\ e^{-j\theta i} \sum_j\ Y_{ij}\ \ e^{j\delta ij}\ V_j\ \ e^{j\theta j})$$

$$= \text{Real}\ (\sum_j V_i\ V_j\ \ Y_{ij}\ \ e^{j(\ \delta ij\ +\ \theta j\ -\ \theta i)})$$

, with the line resistances neglected, $Y_{ij} = j\ B_{ij}$ and

$$P_i = \Sigma\ V_i\ V_j\ \ B_{ij}\ \ \sin(\theta_i - \theta_j) \qquad i=2,3,\ldots.,N$$

Under the assumption that

$$V_i\ V_j\ \ B_{ij}\ \ \sin\ (\theta_i - \theta_j) << V_i\ V_j\ \ B_{ij}\ ,$$

$(\theta_i - \theta_j)$ must be sufficiently small that the sine of the angle difference can be replaced by the angle difference, thus

$$P_i = \sum_j V_i\ V_j\ \ B_{ij}\ \ (\theta_i - \theta_j)$$

$$= - \sum_{j=1}^{i-1}\ V_i\ V_j\ \ B_{ij}\theta_j + \theta_i \sum_{j\neq 1}\ V_i\ V_j\ \ B_{ij}$$

$$- \sum_{j=i+1}^{N}\ V_i\ V_j\ \ B_{ij}\ \theta_j$$

Letting

$$K_{ii} = \sum_{j\neq i}\ V_i\ V_j\ \ B_{ij}$$

and $K_{ij} = -V_i\ V_j\ \ B_{ij}$

We get

$$P_i = \sum_{j=1}^{i-1} K_{ij}\ \theta_j + K_{ii}\ \theta_i + \sum_{j=i+1}^{N} K_{ij}\ \theta_j$$

$$i=2,3,\ldots,N$$

Or in matrix form

$$P = K\ \theta \tag{4.3}$$

Clearly, if for a fixed set of power injections P, a line or lines are removed, both the K matrix and the θ vector will change from their base-case values K° and θ^S by an amount ΔK and $\Delta\theta$ such that

$$\begin{aligned} P^\circ &= (K^\circ + \Delta K)(\theta^S + \Delta\theta) \\ &= K^\circ\ \theta^S + \Delta K\ \theta^S + K^\circ\ \Delta\theta + \Delta K\ \Delta\theta \end{aligned}$$

Neglecting $\Delta K\ \Delta\theta$, then

$$P^\circ \cong K^\circ\ \theta^S + \Delta K\ \theta^S + K^\circ\ \Delta\theta \tag{4.4}$$

But $P^\circ = K^\circ\ \theta^S$ (from 4.3), hence eq. 4.4 becomes

$$0 = K\ \theta^S + K^\circ\ \Delta\theta$$

Therefore

$$\Delta\theta = -(K^\circ)^{-1}\ \Delta K\ \theta^S \tag{4.5}$$

Eq. 4.5 provides the changes in the bus voltage angles due to network changes.

Since the change in the power flow in line ij is

$$P_{ij} = -K_{ij}\ (\Delta\theta_i - \Delta\theta_j) \tag{4.6}$$

Where $\Delta\theta_i$ and $\Delta\theta_j$ are the i th and j th components of $\Delta\theta$, the effects of line outages on line flows are readily available. It is important to emphasize that $(K°)^{-1}$ need only be computed once, namely at the beginning of the contingency studies. As long as the base case remains the same, $(K°)^{-1}$ is valid.

Using eq. 4.5 is a simple matter in both cases of single and double or higher order contingencies. Though this approach of outage study is the simplest, and fastest, its accuracy is low and it does not provide information regarding reactive power flow in the system elements.

B. Z-Matrix Method for Contingency Analysis (5, 27, 28, 29)

Z-matrix methods, as the name indicates, make use of the bus impedance matrix associated with both the base-case system (system having accurate load-flow solution) and the system modified by either line removals or line additions. The Z-matrix for a system can be obtained in several ways. It can be obtained directly by inverting the bus admittance matrix or it can be constructed sequentially by using available algorithms.

The fundamental approach to contingency analysis using the Z-matrix is to inject a fictitious current, into one of the buses associated with the element to be removed, of such value that the current flow through the element equals the base-case flow, all other bus currents being set equal to zero. In effect, this procedure creates throughout the system a current flow pattern that will change in the same manner as the current flow pattern in the ac power flow solution when the element in question is removed.

The requirement that the fictitious injected current creates the same current flow in the element to be removed as in the base-case is considered a constraint.

Implementing the ideas discussed, assume that we want to evaluate the effects of an outage of line km, given that the base-case system flows and voltages are available. We may include all MVA loads in the Z-matrix by first converting the MVA loads using

$$Z\ Load_i = V_i^2 \ / \ S_i^* \tag{4.7}$$

and applying any of the Z-matrix building algorithms. Knowing the Z-matrix, we may write the bus voltage equation

$$\begin{bmatrix} E_1 \\ E_2 \\ \cdots \\ \cdots \\ E_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots\cdots & Z_{1N} \\ Z_{21} & \cdots\cdots & Z_{2N} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ Z_{N1} & \cdots\cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ \cdots \\ I_N \end{bmatrix} \tag{4.8}$$

Since we do not yet know how much current to inject into bus k to create the flow I_{km} that equals the base-case flow I_{km}^s, it is logical to start by letting $I_k = 1.0$ p.u., thus from eq. 4.8, injecting a unit current into bus k gives

$$[\ E_1 \ \cdots\cdots \ E_N]^T = [Z_{1k} \ \cdots\cdots \ Z_{Nk}]^T$$

Therefore

$$I_{km} = (E_k - E_m) \ / \ Z\ line\ km$$

$$= (Z_{kk} - Z_{mk}) \ / \ Z\ line\ km \tag{4.9}$$

Obviously $I_{km} \neq I^{s}_{km}$ the first time; therefore we need to calculate an adjustment parameter, d, where

$$d \triangleq I^{s}_{km} / I_{km} \tag{4.10}$$

Multiplying d by the initial unit injection results in a new injection, $I_k = d$, and new current flow

$$I_{km} = (E_k - E_m) / Z \text{ line } km = d\,(Z_{kk} - Z_{mk})/Z \text{ line } km$$

$$= I^{s}_{km}$$

Due to the injection $I_k = d$, the current flows in all the other elements are

$$I_{ij} = (E_i - E_j) / Z \text{ line } ij \qquad \text{all } ij \neq km$$

$$= d\,(Z_{ik} - Z_{jk}) / Z \text{ line } ij \tag{4.12}$$

The next step is to remove line km and calculate the new current flow pattern $\tilde{I}_{ij}$ in the new system.

Once the new currents are available, current flow changes due to removing line km are

$$\Delta I_{ij} = \tilde{I}_{ij} - I_{ij} \quad \text{for all } ij \tag{4.13}$$

Of course, when $ij = km$, $\tilde{I}_{km} = 0$, therefore

$\Delta I_{km} = - I^{S}_{km}$, with this result the need for adjusting the fictitious current I_k to create a flow of I^{S}_{km} in line km is seen to be crucial.

Calculating the current flow pattern $\tilde{I}_{ij}$ in the modified network $\tilde{Z}$, in which line km has been removed, requires only that we inject current $I_k = d$, as before, into the modified network.

The voltages that result from this injection can be used to determine the needed flows:-

$$\begin{bmatrix} \tilde{E}_1 \\ \tilde{E}_2 \\ \\ \tilde{E}_N \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{11} & \cdots\cdots\cdots & \tilde{Z}_{1N} \\ \tilde{Z}_{21} & \cdots\cdots\cdots & \tilde{Z}_{2N} \\ & & \\ \tilde{Z}_{N1} & \cdots\cdots\cdots & \tilde{Z}_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} d\ \tilde{Z}_{1k} \\ d\ \tilde{Z}_{2k} \\ \\ d\ \tilde{Z}_{Nk} \end{bmatrix} \quad (4.14)$$

Therefore

$$\tilde{I}_{ij} = (\tilde{E}_i - \tilde{E}_j) \ / \ Z \text{ line } ij$$

$$= d\ (\tilde{Z}_{ik} - \tilde{Z}_{jk}) \ / \ Z \text{ line } ij \quad (4.15)$$

where $\tilde{I}_{km} = 0$

Further, substituting eqs. 4.15 and 4.12 into eq. 4.13 , we get

$$I_{ij} = \frac{d}{Z \text{ line } ij} \ [\ (\tilde{Z}_{ik} - \tilde{Z}_{jk}) - (Z_{ik} - Z_{jk})] \quad (4.16)$$

The modified network $\tilde{Z}$ must be known before eqs. 4.14 through 4.16 can be used. Its formulation can be found in [27].

If double contingencies are to be considered, the same basic procedure is used, except that two fictitious currents must be injected into the system, creating in the two lines to be removed a current flow equal to their base-case flow values. Further, the Z-matrix must be modified appropriately to reflect the removal of the two lines. Upon injecting the two bus currents into the original and modified systems, the desired changes in current flow can be obtained.

In general, the accuracy of the Z-matrix method is far better than the accuracy of the dc power flow method. In fact, the results are comparable to those obtained using an ac power flow.

C. Peterson, N.M. et al, (31)

[31] introduced an approach for the approximate iterative solution of linearized a.c. power flow equations, that is suited for the steady-state analysis of both the real and reactive effects of a series of contingent line outages.

Their technique is based on the direct solution of a sparse system of linear algebraic equations by ordered triangular factorization and sparse matrix programming techniques. Also it is based on two symmetric real matrices which remain constant for the complete solution cycle. Therefore each matrix is triangularized only once and the effect of a line outage is simulated without changing the matrices.

Starting with the equations of real and reactive power injections at node k,

$$P_k = V_k^2 \; G_{kk} + V_k \sum_{m \varepsilon \alpha_k} V_m \,(G_{km} \cos \theta_{km} + B_{km} \sin\theta_{km}) \tag{4.17a}$$

$$Q_k = -V_k^2 \; B_{kk} + V_k \sum_{m \varepsilon \alpha_k} V_m \,(G_{km} \sin \theta_{km} - B_{km} \cos\theta_{km}) \tag{4.17b}$$

α_k is the set of branches connected to node k. Noting that the B_{kk} term can be separated into its components as follows:

$$B_{kk} = \sum_{m\varepsilon\alpha_k} (-t_{km}\, B_{km} + BY_{km}) + B_{C_k}$$

where B_{km} is the transfer susceptance of branch km

t_{km} is the tap ratio when branch km is a transformer

(when it is a transmission line, $t_{km} = 1.0$)

BY_{km} is the charging susceptance of the k leg of the equivalent π of the line km

and B_{C_k} is the susceptance of the shunt capacitor or reactor at node k.

Also knowing that

$$\sin\theta = \theta + (\sin\theta - \theta)$$

$$\cos\theta = 1 + (\cos\theta - 1)$$

Substituting in 4.17a we get

$$V_k \sum_{m\varepsilon\alpha_k} B_{km}\,\theta_{km} = P_k - V_k^2\, G_{kk} - V_k \sum_{m\varepsilon\alpha_k} V_m\, G_{km}$$

$$-V_k \sum_{m\varepsilon\alpha_k} V_m\,\{G_{km}\,(\cos\theta_{km} - 1) + B_{km}\,(\sin\theta_{km} - \theta_{km})\} \quad (4.18)$$

Noting that $\theta_{km} = \theta_k - \theta_m$, the L.H.S. of eq. 4.18 can be arranged in the form of a system of N linear equations with (N) voltage phase angles as variables, i.e. it could be written in matrix notation as $[A][\theta]$, where $[\theta]$ is an (N) vector of node voltage phase angles and [A] is an (NxN) matrix with elements

a_{km} is defined as follows

$$a_{kk} = V_k \sum_{m \varepsilon \alpha_k} V_m \; B_{km} \tag{4.19a}$$

$$a_{km} = -V_k \; V_m \; B_{km} \tag{4.19b}$$

On the R.H.S. of eq. 4.18 the first and second terms of the Taylor's series expressions of the sine and cosine functions are substituted. The R.H.S. can then be written in terms of new symbols $[P_k^{'}]$ and $[P_k^{''}]$, which are defined as follows

$$P_k^{'} = P_k - V_k^2 \; G_{kk} - V_k \sum_{m \varepsilon \alpha_k} V_m \; G_{km} \tag{4.20a}$$

$$P_k^{''} = V_k \sum_{m \varepsilon \alpha_k} V_m \; (G_{km} \frac{\theta^2_{km}}{2} + B_{km} \frac{\theta^3_{km}}{6}) \tag{4.20b}$$

The complete system of equations for real power can then be written as

$$[A][\theta] = [\;\; [P^{'}] + [P^{''}] \;\;] \tag{4.21}$$

Similar analysis applied to the reactive power equations leads to the following results

$$[C][V] = [\;\; [Q^{'}] + [Q^{''}] \;\;] \tag{4.22}$$

where

$[V]$ is an $N^{'}$ vector of node voltage magnitudes,

$N^{'}$ is number of PQ nodes

and $[C]$ is an $(N^{'} \times N^{'})$ matrix with elements C_{km} defined as follows

$$C_{kk} = \sum_{m\varepsilon\alpha_k} t_{km} \; B_{km} \tag{4.23a}$$

$$C_{km} = -B_{km} \quad \text{for} \quad m \; \varepsilon \; \gamma_k \tag{4.23b}$$

where γ_k is the set of all PQ nodes connected to node k

$[Q']$ and $[Q'']$ are defined as follows:-

$$Q'_k = \frac{Q_k}{V_k} + V_k \left(\sum_{m\varepsilon\alpha_k} t_{km} \; BY_{km} + B_{C_k} \right) + \sum_{m\varepsilon\psi_k} V_m \; B_{km} \tag{4.24a}$$

$$Q''_k = -\sum_{m\varepsilon\alpha_k} V_m \left\{ G_{km}\left(\theta_{km} - \frac{\theta^3_{km}}{6}\right) + B_{km} \frac{\theta^2_{km}}{2} \right\} \tag{4.24b}$$

where ψ_k is the set of all PV nodes connected to node k.
The technique simulates the effect of a branch outage on the solution without changing the triangularized matrix to reflect the branch change. An outage of a line in branch km causes four changes in matrix [A] as follows

$$\Delta a_{kk} = \Delta a_{mm} = -\Delta a_{km} = -\Delta a_{mk}$$

The changes in [A] can be expressed in matrix notation as

$$[A'] = [A] + \Delta a_{km} \; [M_A][M_A]^T \tag{4.25}$$

where $[M_A]$ is an N vector that is all zeros except element k which is +1 and element m which is -1

The inverse of $[A']$ is given by [32]

$$[A']^{-1} = [A]^{-1} - C \; [Z_A][M_A]^T \; [A]^{-1} \tag{4.26}$$

where

$$C = \left(\frac{1}{\Delta a_{km}} + [M_A]^T [Z_A] \right)^{-1}$$

and

$$[Z_A] = [A]^{-1} [M_A] \tag{4.27}$$

For a branch outage eq. 4.21 can be written as

$$\left[[A] + \Delta a_{km} [M_A][M_A]^T \right] \left[[\theta] + [\Delta\theta] \right]$$

$$= \left[[P'] + [P''] \right]$$

where $[\Delta\theta]$ is the vector of the phase angle corrections to account for the line outage.

A similar expression can be derived from eq. 4.22 for the voltage correction $[\Delta V]$.

$$\left[[C] + \Delta C_{km} [M_C][M_C]^T \right] \left[[V] + [\Delta V] \right]$$
$$= \left[[Q'] + [Q''] \right]$$

Here ΔC_{km} is the change in element km of $[C]$ resulting from the line outage, $[M_C]$ is an N' vector that is all zeros except for element k which is +1 and element m which is -1.

$[\Delta\theta]$ and $[\Delta V]$ are then computed as follows:-

$$[\Delta\theta] = -\left(\frac{1}{\Delta a_{km}} + Z_{Ak} - Z_{Am} \right)^{-1} (\theta_k - \theta_m) [Z_A] \tag{4.28}$$

$$[\Delta V] = -\left(\frac{1}{\Delta C_{km}} + Z_{Ck} - Z_{Cm} \right)^{-1} (V_k - V_m) [Z_C] \tag{4.29}$$

where

$$[Z_C] = [C]^{-1} [M_C] \tag{4.30}$$

The scalars Z_{Ak} and Z_{Am} are elements of $[Z_A]$.

The scalars Z_{Ck} and Z_{Cm} are elements of $[Z_C]$.

θ_k and θ_m are elements of $[\theta]$, the solution vector of eq. 4.21 without the line outage, and V_k and V_m are elements of $[V]$, the solution vector of eq. 4.22 without the line outage. Thus, by solving eq. 4.21 as a linear system to obtain the solution $[\theta]$ the solution for $[\Delta\theta]$, the correction to account for a line outage, can be obtained by first solving for $[Z_A]$ by a repeat solution and then solving eq. 4.28. A parallel statement applies for the voltage solution in eq. 4.29.

Though this approach has good overall performance, it requires some computational processing of the basic state data before line outages can be simulated.

D. B. Stott and O. Alsac[9]

[9] has applied a technique similar to [31], based on the decoupled load flow solution. As stated before, the fast decoupled load-flow equations are given by

$$[\frac{\Delta P}{V}] = [B'][\Delta\theta] \tag{2.20}$$

$$[\frac{\Delta Q}{V}] = [B''][\Delta V] \tag{2.21}$$

All outages must of course be reflected correctly in the calculation of $[\frac{\Delta P}{V}]$ and $[\frac{\Delta Q}{V}]$, for the outage of a series branch two non-sparse vectors $[X']$ and $[X'']$ must be calculated, each requiring one repeat solution using the factors of $[B']$ and $[B'']$ respectively.

After each solution of eq. 2.20, $[\Delta\theta]$ is corrected by an amount

$$- C' \; [X'][M']^T \; [\Delta\theta] \tag{4.31}$$

where C' is a scalar given by

$$C' = (1/b + [M']^T \; [X'])^{-1},$$

$$[X'] = [B']^{-1} \; [M'] \; ,$$

$[M']$ = column vector which is null except for

$[M']_k = a$, $[M']_m = -1$,

b = line or nominal transformer series admittance ,

and a = off-nominal turns ratio referred to the bus corresponding to row m , for a transformer

= 1.0 for a line

Similarly, after each solution of eq. 2.21, $[\Delta V]$ is corrected by an amount

$$-C'' \; [X''] \; [M'']^T \; [\Delta V] \tag{4.32}$$

where

C'' is a scalar given by

$$= (1/b + [M'']^T \; [X''] \;)^{-1}$$

$$, \; [X''] = [B'']^{-1} \; [M'']$$

and $[M'']$ is formulated in a similar way like $[M']$.

It was found[9] that the (1θ, 1V) iteration scheme remains the best in this application and it was confirmed that using the base-case solution to start the outage calculations is on an average distinctly better than using the normal flat voltage start each time.

E. Sachdev and Ibrahim[35]

[35] has proposed that the outage of a line be simulated using the sensitivity matrix (which is the inverse of the Jacobian matrix) and the changes of power injected into the system at the buses connected by this line. Fig. 4.1 depicts the simulation of an outage of a line connecting load buses k and m. The basic and final states of the system buses k and m are shown in figs. 4.1a and 4.1b respectively. The voltages at these buses would not change, from that in the final system state, if this transmission line is reconnected and power equal to that flowing in the line is injected into these buses. Fig 4.1C, therefore, simulates the outage of this line without physically changing the system configuration. Power flow, in the line connecting bus k and m, in the simulated final state is given by

$$P'_{km} = V'_k \; V'_m \; [G_{km} \cos(\theta'_k - \theta'_m) + B_{km} \sin(\theta'_k - \theta'_m)]$$

$$-V'^2_k \; (G_{km} - G'_{km}) \tag{4.33a}$$

$$Q'_{km} = V'_k \; V'_m \; [G_{km} \sin(\theta'_k - \theta'_m) - B_{km} \cos(\theta'_k - \theta'_m)]$$

$$+V'^2_k \; (B_{km} - B'_{km}) \tag{4.33b}$$

$$P'_{mk} = V'_m V'_k \left[G_{km} \cos(\theta'_m - \theta'_k) + B_{km} \cos(\theta'_k - \theta'_m)\right]$$
$$-V'^2_m \; (G_{km} - G'_{km}) \qquad (4.33c)$$

$$Q'_{mk} = V_m V_k \left[G_{km} \sin(\theta'_m - \theta'_k) - B_{km} \sin(\theta'_m - \theta'_k)\right]$$
$$+V'^2_m \; (B_{km} - B'_{km}) \qquad (4.33d)$$

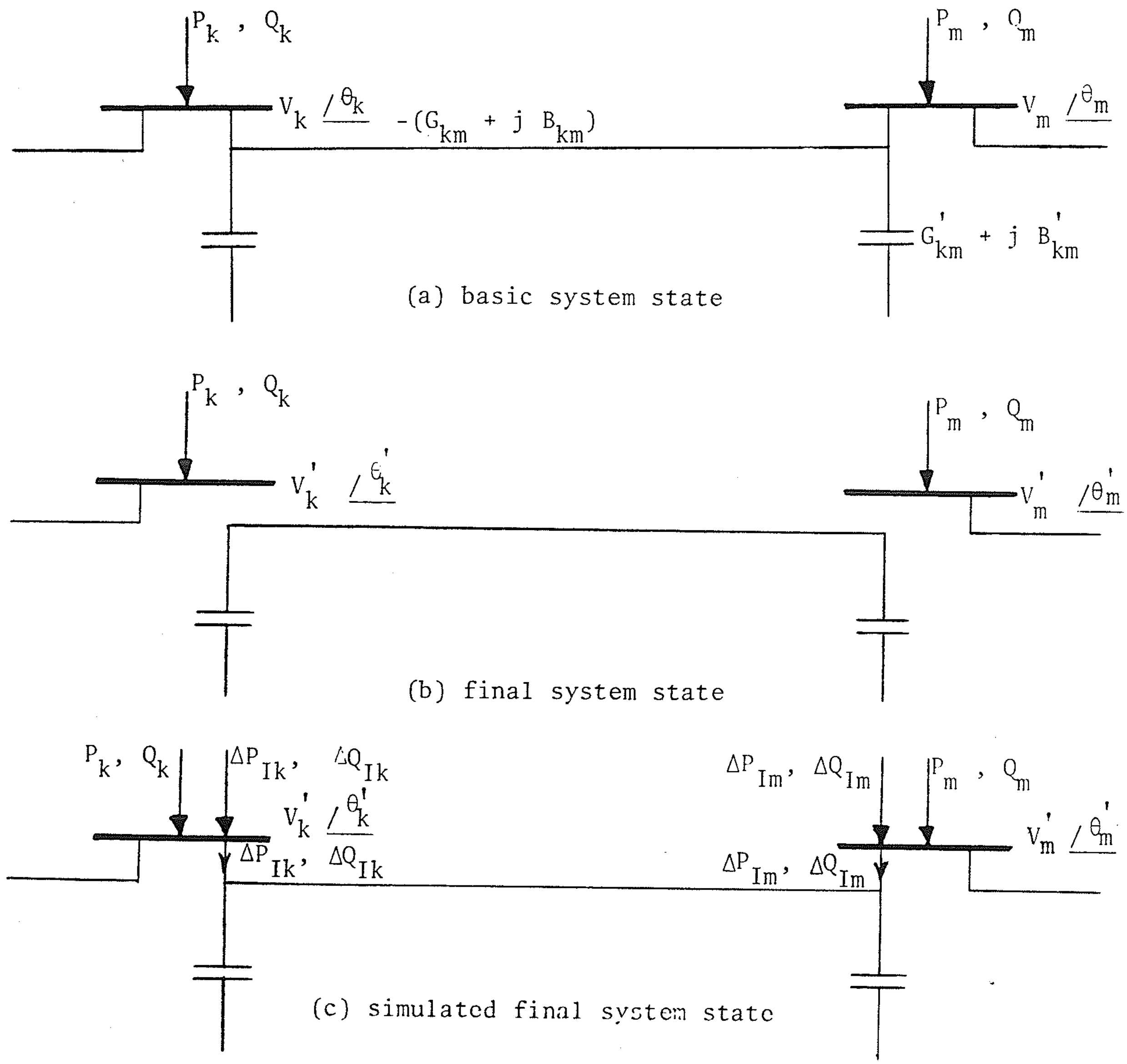


Fig. 4.1 The outage simulation of a line connecting the system buses k and m.

Where

$$G_{km} + j\,B_{km} = (k,m)\text{th element of nodal admittance matrix}$$

$$G'_{km} + j\,B'_{km} = \frac{1}{2}\ \text{(line charging admittance for line km)}$$

and
$$\begin{aligned} V'_m &= V_m + \Delta V_m \\ V'_k &= V_k + \Delta V_k \\ \theta'_m &= \theta_m + \Delta\theta_m \\ \theta'_k &= \theta_k + \Delta\theta_k \end{aligned} \tag{4.34}$$

As is evident from Fig. 4.1c, the change of power injected into the buses k and m should equal the line flow in the final system state,

$$\begin{aligned} \Delta P_{Ik} &= P'_{km} \\ \Delta P_{Im} &= P'_{mk} \\ \Delta Q_{Ik} &= Q'_{km} \\ \Delta Q_{Im} &= Q'_{mk} \end{aligned} \tag{4.35}$$

Since the outage of a line connecting two load buses k and m is being simulated by the changes of power injected into buses k and m only, eq. 4.36 is formulated using the sensitivity matrix as follows

$$\left[\begin{array}{cc|cc} C_{kk} & C_{km} & D_{kk} & D_{km} \\ C_{mk} & C_{mm} & D_{mk} & D_{mm} \\ \hline E_{kk} & E_{km} & F_{kk} & F_{km} \\ E_{mk} & E_{mm} & F_{mk} & F_{mm} \end{array}\right] \left[\begin{array}{c} \Delta P_{Ik} \\ \Delta P_{Im} \\ \hline \Delta Q_{Ik} \\ \Delta Q_{Im} \end{array}\right] = \left[\begin{array}{c} \Delta\theta_{k} \\ \Delta\theta_{m} \\ \hline \Delta V_{k} \\ \Delta V_{m} \end{array}\right] \qquad (4.36)$$

Eqs. 4.33 and 4.36 and the equality constraint of eqs. 4.34 and 4.35 represent a nonlinear relationship between the changes in real and reactive power injected into the buses k and m and the magnitudes and phase angles of the voltages at these buses.

Eqs. 4.33 and 4.36 can be solved using an iterative technique (more than one iteration is required), the values of ΔP_{Ik} , ΔP_{Im}, ΔQ_{Ik} and ΔQ_{Im} thus calculated are used to solve for the changes in the entire system state.

Assuming that the system loads and the real power output of various generating plants remain unchanged, all except the above four elements of the controlling vector would be zero, the elements of only four columns of the sensitivity matrix are therefore required to be computed to evaluate the state simulating the outage of a line connecting two load buses.

The changes of power injection required for simulating the outage of a line (km) connecting different types of buses are summarized in table (4.1), for each calculated change of power the elements of only one column of the sensitivity matrix are required to obtain the final system state.

Table (4.1)

TYPE OF BUSES CONNECTED BY THE LINE		POWER INJECTIONS REQUIRED TO BE CALCULATED
BUS k	BUS m	
Generator bus	Swing bus	ΔP_{Ik}
Load bus	Swing bus	ΔP_{Ik} , ΔQ_{Ik}
Generator bus	Generator bus	ΔP_{Ik} , ΔP_{Im}
Load bus	Generator bus	ΔP_{Ik} , ΔQ_{Ik} , ΔP_{Im}
Load bus	Load bus	ΔP_{Ik} , ΔQ_{Ik} , ΔP_{Im}, ΔQ_{Im}

The transformer outage is simulated in the same manner as the line outage in case the transformer is operating at nominal tap setting. If the transformer tap is connected to, say, bus k and off nominal tap setting , t_{km} , is being used, the voltage V_k' in eq. 4.33 is replaced by V_k'/t_{km} . Except for this change, the simulation procedure remains the same.

The system solution obtained by the proposed technique will be slightly different from that of the Newton-Raphson approach because the elements of the sensitivity matrix are calculated from the pre-contingency bus voltages. This error can be partly compensated by using power injection modification factors (PIMF). Since each transmission element has a unique effect on the system and this effect is likely to be similar at different loading conditions, a single value of the PIMF may be used for each transmission element. PIMF for a line connecting buses k and m can be determined by comparing the real power injections at buses k and m calculated by this technique and without the use of PIMF and those calculated from a load-flow by the Newton-Raphson technique. Therefore, the values of ΔP_{Ik} ,ΔP_{Im} ,ΔQ_{Ik} and ΔQ_{Im} calculated from eqs. 4.35 and 4.38 should be modified by using PIMF before calculating the changes in the entire system state.

Comparing with Stott method, the latter has a considerable storage advantage but this technique requires the upper and lower triangular factors of the complete Jacobian matrix.

4.3 New Method

In principle, a line or transformer outage causes a system configuration change. An equivalent mathematical model is made [35] which has the original configuration except that at the terminal buses of the outaged element additional active and reactive power, equal to that which would flow if the line was present with the modified system voltages, are injected, as shown in fig. 4.1. The maximum number of ΔP_I, ΔQ_I variables for each element outage is 4. Hence, if these are known, four columns of the sensitivity matrix are required for obtaining the system state.

4.3.1 Single Branch Outage: Mathematical Model

Consider the outage of a line connecting buses k and m in fig. 4.1. In the simulation, under final operating conditions, the real and reactive power flows in the line km at the two buses are given by eq. 4.33. Rewriting these, we have

$$P'_{km} = V'_k V'_m \; [G_{km} \cos \theta'_{km} + B_{km} \sin \theta'_{km}] - V'^2_k \; (G_{km} - G'_{km})$$

$$Q'_{km} = V'_k V'_m \; [G_{km} \sin \theta'_{km} - B_{km} \cos \theta'_{km}] + V'^2_k \; (B_{km} - B'_{km})$$

$$P'_{mk} = V'_m V'_k \; [G_{km} \cos \theta'_{mk} + B_{km} \sin \theta'_{mk}] - V'^2_m \; (G_{km} - G'_{km})$$

$$Q'_{mk} = V'_m V'_k \; [G_{km} \sin \theta'_{mk} - B_{km} \cos \theta'_{mk}] + V'^2_m (B_{km} - B'_{km})$$

where $\theta'_{km} = \theta'_k - \theta'_m = - \theta'_{mk}$

As shown in Appendix B, P'_{km} is given by

$$P'_{km} = P_{km} \text{ (basic state)} + \Delta P_{km} = P_{kmo} + \Delta P_{km} \qquad (4.37)$$

Similarly $P'_{mk} = P_{mko} + \Delta P_{mk}$

$Q'_{km} = Q_{kmo} + \Delta Q_{km}$

and $Q'_{mk} = Q_{mko} + \Delta Q_{mk}$

Changes in line powers (ΔP_{km} , ΔP_{mk}, ΔQ_{km} , ΔQ_{mk}) are expressed in terms of changes (corrections) in bus voltage magnitudes and angles (Appendix B)

$$[\Delta P]_{k,m} = [\sqrt{V_k \; V_m}]\,[B_3]_{k,m} \; [\Delta\theta]_{k,m} \qquad (4.38)$$

likewise

$$[\Delta Q]_{k,m} = [\sqrt{V_k \; V_m}]\,[B_4]_{k,m} \; [\Delta V]_{k,m} \qquad (4.39)$$

Therefore the changes of active and reactive power injected into the buses k and m to simulate an outage of the line km could be computed from eqs. 4.35, 4.37, 4.38 and 4.39. These changes are given as

$$\begin{bmatrix} \Delta P_{Ik} \\ \Delta P_{Im} \end{bmatrix} = \begin{bmatrix} P_{kmo} \\ P_{mko} \end{bmatrix} + \begin{bmatrix} \Delta P_{km} \\ \Delta P_{mk} \end{bmatrix}$$

$$= \begin{bmatrix} P_{kmo} \\ P_{mko} \end{bmatrix} + \begin{bmatrix} \sqrt{V_k V_m} & 0 \\ 0 & \sqrt{V_k V_m} \end{bmatrix} \begin{bmatrix} B3_{kk} & B3_{km} \\ B3_{mk} & B3_{mm} \end{bmatrix} \begin{bmatrix} \Delta\theta_k \\ \Delta\theta_m \end{bmatrix} \qquad (4.40)$$

$$\begin{bmatrix} \Delta Q_{Ik} \\ \Delta Q_{Im} \end{bmatrix} = \begin{bmatrix} Q_{kmo} \\ Q_{mko} \end{bmatrix} + \begin{bmatrix} \Delta Q_{km} \\ \Delta Q_{mk} \end{bmatrix}$$

$$= \begin{bmatrix} Q_{kmo} \\ Q_{mko} \end{bmatrix} + \begin{bmatrix} \sqrt{V_k V_m} & 0 \\ 0 & \sqrt{V_k V_m} \end{bmatrix} \begin{bmatrix} B4_{kk} & B4_{km} \\ B4_{mk} & B4_{mm} \end{bmatrix} \begin{bmatrix} \Delta V_k \\ \Delta V_m \end{bmatrix} \qquad (4.41)$$

Assuming that the loads and real power output of generating plants remain unchanged and since the outage of a line km connecting two load buses k and m is being simulated by the changes in the power (active and reactive) injected into buses k and m only based on the fast-decoupled load-flow method,

$$[\frac{\Delta P}{V}] = [B'][\Delta\theta] \text{ or } [\Delta\theta] = [S1][\frac{\Delta P}{V}] \qquad (4.42)$$

$$[\frac{\Delta Q}{V}] = [B''][\Delta V] \text{ or } [\Delta V] = [S2][\frac{\Delta Q}{V}] , \qquad (4.43)$$

only two columns of each sensitivity matrix [S1] and [S2] are required. For other combinations of Generator/Swing and Load buses as shown in table 4.1 lesser columns are required. A method of calculating a column of sensitivity matrix is shown in Appendix B.

Therefore the corrections ($\Delta\theta_k$, $\Delta\theta_m$, ΔV_k, ΔV_m) are related to the changes of active and reactive power injections (ΔP_{Ik}, ΔP_{Im}, ΔQ_{Ik}, ΔQ_{Im}) by

$$\begin{bmatrix} \Delta\theta_k \\ \Delta\theta_m \end{bmatrix} = \begin{bmatrix} S1_{kk} & S1_{km} \\ S1_{mk} & S1_{mm} \end{bmatrix} \begin{bmatrix} \Delta P_{Ik} / V_k \\ \Delta P_{Im} / V_m \end{bmatrix} \tag{4.44}$$

$$\begin{bmatrix} \Delta V_k \\ \Delta V_m \end{bmatrix} = \begin{bmatrix} S2_{kk} & S2_{km} \\ S2_{mk} & S2_{mm} \end{bmatrix} \begin{bmatrix} \Delta Q_{Ik} / V_k \\ \Delta Q_{Im} / V_m \end{bmatrix} \tag{4.45}$$

Substituting from eq. 4.44 into eq. 4.40, we get

$$\begin{bmatrix} \Delta P_{Ik} \\ \Delta P_{Im} \end{bmatrix} = \begin{bmatrix} P_{kmo} \\ P_{mko} \end{bmatrix} + \begin{bmatrix} \sqrt{V_k V_m} & 0 \\ 0 & \sqrt{V_k V_m} \end{bmatrix} \begin{bmatrix} B3_{kk} & B3_{km} \\ B3_{mk} & B3_{mm} \end{bmatrix} \begin{bmatrix} S1_{kk} & S1_{km} \\ S1_{mk} & S1_{mm} \end{bmatrix} \begin{bmatrix} \Delta P_{Ik} / V_k \\ \Delta P_{Im} / V_m \end{bmatrix} \tag{4.46}$$

But

$$\begin{bmatrix} \Delta P_{Ik} / V_k \\ \Delta P_{Im} / V_m \end{bmatrix} = \begin{bmatrix} \frac{1}{V_k} & 0 \\ 0 & \frac{1}{V_k} \end{bmatrix} \begin{bmatrix} \Delta P_{Ik} \\ \Delta P_{Im} \end{bmatrix} = [V]^{-1}_{k,m} \; [\Delta P_I]_{k,m}$$

Eq. 4.46 could be written in the form

$$[\Delta P_I]_{k,m} = [P_o]_{k,m} + [\sqrt{V_k V_m}]\,[B_3]_{k,m}\,[S1]_{k,m}\,[V]^{-1}_{k,m}\,[\Delta P_I]_{k,m}$$

Let $[SP] = [\sqrt{V_k V_m}][B_3]_{k,m}\,[S1]_{k,m}\,[V]^{-1}_{k,m}$

Since $[\sqrt{V_k V_m}]$ and $[V]^{-1}_{k,m}$ are diagonal matrices, therefore

$$[SP] = [B_3]_{k,m}[S1]_{k,m}\,[V]^{-1}\,[\sqrt{V_k V_m}]$$

The product $[V]^{-1}_{k,m}\,[\sqrt{V_k V_m}] = \begin{bmatrix} \frac{1}{V_k} & 0 \\ 0 & \frac{1}{V_k} \end{bmatrix} \begin{bmatrix} \sqrt{V_k V_m} & 0 \\ 0 & \sqrt{V_k V_m} \end{bmatrix}$

$$= \begin{bmatrix} \sqrt{V_m / V_k} & 0 \\ 0 & \sqrt{V_m / V_m} \end{bmatrix}$$ $= [I]$ = unity matrix, based on the

assumption that $\sqrt{V_k / V_m} \cong 1.0$, i.e. the square root of voltage magnitude of both ends of a line are approximately equal, hence [SP] is given as

$$[SP] = [B_3]_{k,m}\,[S1]_{k,m} \tag{4.47}$$

From eqs. 4.46 and 4.47 we have

$$[\Delta P_I]_{k,m} = [P_o]_{k,m} + [SP][\Delta P_I]_{k,m} \tag{4.48}$$

$$[\Delta P_I]_{k,m} - [SP][\Delta P_I]_{k,m} = [P_o]_{k,m}$$

$$[\,[I] - [SP]\,]\;[\Delta P_I]_{k,m} = [P_o]_{k,m}$$

Therefore ΔP_{Ik} and ΔP_{Im} are given by

$$[\Delta P_I]_{k,m} = [\;[I] - [SP]\;]^{-1}\,[P_o]_{k,m} \tag{4.49}$$

Similarly $[\Delta Q_I]_{k,m} = [\ [I] - [SQ]\ ]^{-1}\ [Q_o]_{k,m}$ (4.50)

where $[SQ] = [B_4]_{k,m}\ [S2]_{k,m}$ (4.51)

Therefore the changes of the injected active and reactive power (ΔP_{Ik} , ΔP_{Im} , ΔQ_{Ik} , ΔQ_{Im}) could be calculated in terms of at most two columns of each sensitivity matrix (S1, S2) , the series and charging admittances of the outaged line, and the basic load-flow (Po, Qo).

The calculated values of ΔP_{Ik} and ΔP_{Im} are used in eq. 4.42 to obtain the solution for $[\Delta\theta]$ of the whole system. Also, the calculated values of ΔQ_{Ik} and ΔQ_{Im} are used in eq. 4.50 to compute the solution for $[\Delta V]$ of the whole system. $[\Delta\theta]$ and $[\Delta V]$, the correction vectors to account for an outage of the line km, are used to calculate the post-contingency state, $[\theta']$ and $[V']$, for the whole system. Knowing the post-contingency system voltages, the post-contingency line flows can, therefore, be determined.

4.3.2 Generalization for Multiple Branch Outages (Flowchart-Appendix E)

Consider outage of lines 1, 2, ..., n connected between load buses (a, b), (c, d), ... , (y, z).

The changes in the injected power (ΔP_I, ΔQ_I) into buses (a, b), (c, d) ... , (y, z) are given by

$${}_{2n}[\ \Delta\ \text{INJECT}] = {}_{2n}[\text{POWER}_o] = {}_{2n}[\ \Delta\ \text{POWER}] \qquad (4.52)$$

where [Δ INJECT] are the changes in injected powers at the 2n buses (a, b, c, ..., z), $[\text{POWER}_o]$ are the precontingency power flows in the lines and [Δ POWER] are changes in these power flows due to outages.

Eq. 4.52 applies for both active and reactive powers.

The relations between [Δ POWER] and changes in the bus voltage magnitudes and angles are expressed in a general form by the following equations.

$$\begin{bmatrix} \Delta POWER_{ab} \\ \Delta POWER_{ba} \\ \Delta POWER_{cd} \\ \Delta POWER_{dc} \\ \vdots \\ \Delta POWER_{yz} \\ \Delta POWER_{zy} \end{bmatrix} = [\sqrt{VV}\,] \cdot \begin{bmatrix} \bar{B}_{ab} & & & \\ & \bar{B}_{cd} & & \\ & & \ddots & \\ & & & \bar{B}_{yz} \end{bmatrix} \begin{bmatrix} \Delta X_a \\ \Delta X_b \\ \Delta X_c \\ \Delta X_d \\ \vdots \\ \Delta X_y \\ \Delta X_z \end{bmatrix}$$

or $[\ \Delta\ POWER] \quad = \quad [\sqrt{VV}]\ [\bar{B}]\ [\Delta X]$ (4.53)

where for $\Delta POWER = \Delta P; \quad \Delta X = \Delta\theta$ and

$$\bar{B}_{a,b} = [B_3]_{a,b} = B_{ab}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = -B_{ab\,\text{line}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

B_{ab} = imaginary part of (a,b)th element of the nodal admittance matrix

= -series susceptance of line ab = $-B_{ab\,\text{line}}$

or for Δ POWER $= \Delta Q; \quad \Delta X = \Delta V$ and

$$\overline{B}_{a,b} = [B_4]_{a,b} = \begin{bmatrix} B_{ab} - 2B'_{ab} & -B_{ab} \\ -B_{ab} & B_{ab} - 2B'_{ab} \end{bmatrix}$$

$$= B_{ab} \begin{bmatrix} C & -1 \\ -1 & C \end{bmatrix} = -B_{ab}\,_{\text{line}} \begin{bmatrix} C & -1 \\ -1 & C \end{bmatrix}$$

where $C = \dfrac{B_{ab} - 2B'_{ab}}{B_{ab}} = 1 + \dfrac{\text{line charging susceptance}}{\text{line series susceptance}}$

Similarly with $\overline{B}_{c,d}$, ... , and $\overline{B}_{y,z}$

Note that $C = 1$ for calculating ΔP

and $[\sqrt{VV}\,]$ is a diagonal matrix given by

$$[\sqrt{VV}] = \underbrace{\begin{bmatrix} \sqrt{V_a V_b} & 0 & & & & & \\ 0 & \sqrt{V_a V_b} & & & & & \\ & & \sqrt{V_c V_d} & 0 & & & \\ & & 0 & \sqrt{V_c V_d} & & & \\ & & & & \ddots & & \\ & & & & & \sqrt{V_y V_z} & 0 \\ & & & & & 0 & \sqrt{V_y V_z} \end{bmatrix}}_{2n} \; 2n$$

ΔINJECT and ΔX are related by a small portion S of the whole sensitivity matrix [Fig. 4.2] as shown below

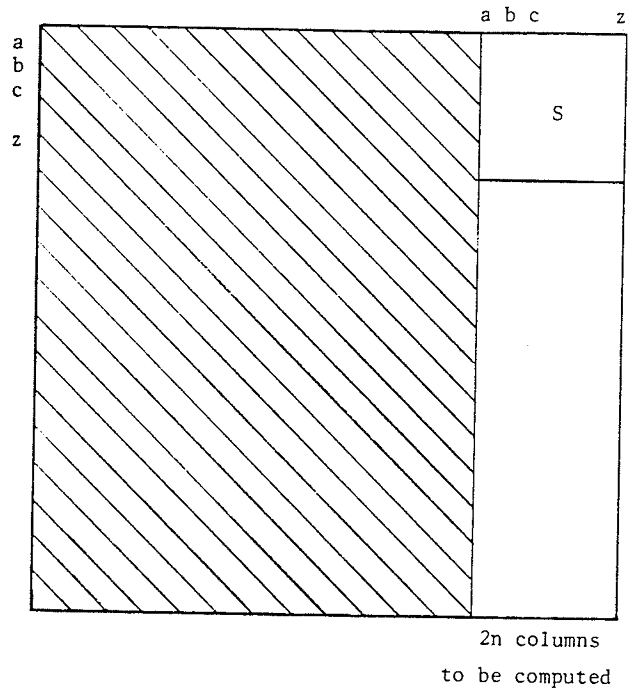
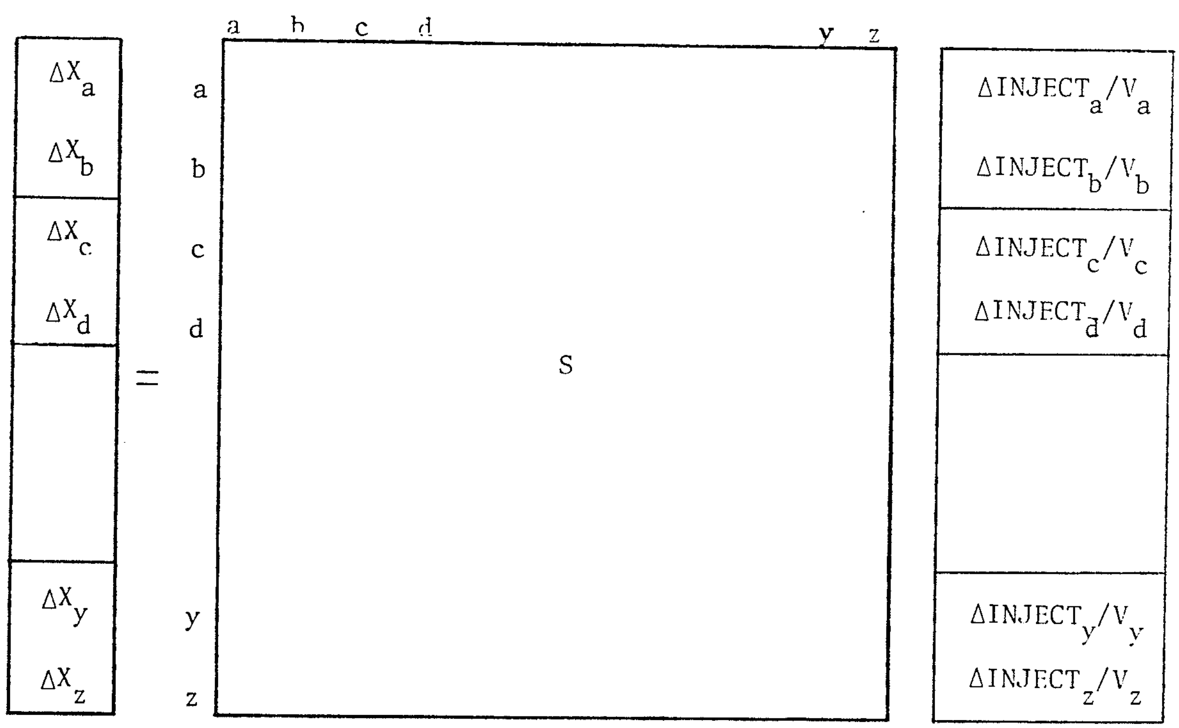


Fig. 4.2 Sensitivity Matrix



i.e. $[\Delta X] = [S] \left[\frac{\Delta INJECT}{V} \right]$ (4.54)

From eqs. 4.53 and 4.54

$$[\ \Delta\ POWER] = [\sqrt{VV}\ ]\ [\bar{B}]\,[S]\,[V]^{-1}\ [\ \Delta INJECT]$$
$$= [\bar{B}]\,[S]\ [\sqrt{VV}\ ]\,[V]^{-1}\ [\ \Delta INJECT]$$
$$= [BS]\ \ [\ \Delta INJECT] \qquad (4.55)$$

since $\sqrt{VV}$ and $[V]^{-1}$ are diagonal matrices,

$[\sqrt{VV}\ ]\,[V]^{-1} = [I]$ (unity matrix), since $\sqrt{V_i\ /\ V_j} \simeq 1$

and $[BS] = [\bar{B}]\,[S]$

Formulation of [BS] :

Matrix [BS] is formulated row by row

Row a = (c x row a of [S] -row b of [S]) x B_{ab}
Row b = (c x row b of [S] -row a of [S]) x B_{ab}
Row y = (c x row y of [S] -row z of [S]) x B_{yz}
Row z = (c x row z of [S] -row y of [S]) x B_{yz}

Therefore we do not need to store $[B_3]$, $[B_4]$, $[\sqrt{VV}]$ and $[V]^{-1}$. All that we need is [S]. Matrices $[B_3]$ and $[B_4]$ are multiplied by [S] {S may be either S1 or S2} implicity.

From eqs. 4.52 and 4.55 we have

$$[\Delta\text{INJECT}] = [\text{POWER}_o] + [\text{BS}][\Delta\text{INJECT}]$$

i.e. $$[\Delta\text{INJECT}] - [\text{BS}][\Delta\text{INJECT}] = [\text{POWER}_o]$$

$$[\,[\text{I}] - [\text{BS}]\,]\,[\Delta\text{INJECT}] = [\text{POWER}_o]$$

Let $[\text{BS}'] = [\text{I}] - [\text{BS}]$, then $[\Delta\text{INJECT}]$ is given as

$$[\Delta\text{INJECT}] = [\text{BS}']^{-1}\,[\text{POWER}_o] \tag{4.56}$$

From eq. 4.56 $[\Delta\text{INJECT}]$ is calculted since $[\text{POWER}_o]$ is known from the precontingency load-flow.

By knowing the 2n columns of the sensitivity matrix shown by the unhatched portion in Fig. 4.2 , $\Delta\theta$ and ΔV , the corrections due to lines outages, are calculated to update $[\theta']$ and $[V']$ of the whole system in the post contingency condition. Post contingency line flows are therefore determined.

In this technique we need just one iteration starting from the basic state to calculate the changes in the system state due to single branch or multiple branch outages.

4.4 Application of the Proposed Technique to a Sample System

Test Results

The aforementioned technique has been applied on AEP-14 bus sample system for single and double line outages.

For single line outages, lines 7 (high loading), 11 (medium loading) and 20 (light loading) are removed one at a time, whereas for double line contingencies, lines (16 and 17) and (3 and 20) are removed in pairs.

For comparison, an exact load-flow solution is also obtained in which the outage is simulated by physically removing the line.

Table 4.2 summarizes the results for active power flows in the case of single line outages. This table also shows the precontingency flows in the system given by the fast-decoupled solution.

For the contingency of line 7 it also provides results obtained by using Z-matrix [28,29] and Stott's [9] techniques (one iteration) to show the accuracy of the results on a comparative basis.

Table 4.3 shows a comparison between reactive power flows on lines obtained by full load-flow solution and that by the proposed technique due to the outages of line 7 (heavily loaded) and line 20 (lightly loaded).

Table 4.4 provides the results of active power flows for double line outages.

Table (4.2)

A comparison of the active power flow calculated by the exact and proposed techniques (single line outages)(MW)

LINE	NO OUTAGE LOAD-FLOW	OUTAGE OF LINE 20 EXACT LOAD-FLOW	OUTAGE OF LINE 20 PROPOSED TECHNIQUE	OUTAGE OF LINE 11 EXACT LOAD-FLOW	OUTAGE OF LINE 11 PROPOSED TECHNIQUE	OUTAGE OF LINE 7 EXACT LOAD-FLOW	OUTAGE OF LINE 7 PROPOSED TECHNIQUE	Z MATRIX METHOD	B.STOTT'S METHOD
1	154.07	156.34	153.93	156.47	153.90	175.66	166.62	174.51	175.02
2	79.89	76.28	79.42	76.15	80.19	61.79	64.03	58.19	58.15
3	83.08	72.83	83.29	72.96	83.38	107.82	101.82	87.83	88.43
4	38.64	56.17	47.92	56.31	51.92	63.69	56.78	87.78	88.68
5	46.51	41.37	46.05	41.23	49.49	17.10	23.78	12.61	12.69
6	14.10	23.66	19.64	23.55	17.86	8.66	7.20	9.14	8.37
7	64.01	63.47	66.02	64.65	60.80	.00	.00	.00	.00
8	24.97	29.52	26.47	30.45	26.18	12.91	15.47	18.30	17.87
9	14.06	16.61	14.86	17.11	14.50	7.24	8.70	10.22	10.04
10	49.50	42.36	46.19	40.91	38.98	69.30	64.86	60.32	61.28
11	10.46	11.39	13.72	.00	.00	22.53	18.11	17.16	16.79
12	8.37	6.56	6.66	8.77	6.66	9.89	9.50	9.37	9.27
13	19.47	13.21	12.95	20.94	16.38	25.68	23.31	23.19	22.75
14	.00	.00	.00	.00	.00	.00	.00	.00	.00
15	24.97	29.52	26.35	30.45	25.69	12.91	15.37	18.11	17.83
16	2.28	1.40	.87	12.58	10.53	9.23	5.45	4.06	3.60
17	7.25	15.22	13.85	5.47	8.56	.09	.55	3.23	3.45
18	6.72	7.60	9.81	3.51	2.39	18.26	14.27	13.06	12.50
19	2.18	.41	.40	2.58	.55	3.68	2.74	3.04	2.86
20	7.86	.00	.00	9.67	4.76	15.40	12.64	12.50	11.69

Table (4.3)

A comparison of reactive power flow calculated by the exact and proposed techniques. (MVARS)

LINE	NO OUTAGE LOAD FLOW	OUTAGE OF LINE 7		OUTAGE OF LINE 20	
		EXACT	PROPOSED	EXACT	PROPOSED
1	19.74	24.69	22.65	19.87	19.71
2	2.40	3.62	1.70	2.10	2.09
3	2.69	1.33	1.58	2.64	2.67
4	5.12	5.26	7.43	5.44	5.72
5	7.54	3.32	3.84	7.07	7.08
6	2.71	4.51	6.06	1.83	2.10
7	8.02	.00	.00	6.74	7.89
8	9.57	11.51	8.76	9.61	10.09
9	.0106	.47	.39	.025	.020
10	12.65	18.84	22.75	13.29	13.25
11	9.44	6.83	5.31	9.41	9.89
12	3.21	2.77	2.61	2.26	2.62
13	10.29	8.75	8.08	5.48	7.52
14	28.35	33.29	27.81	31.43	31.68
15	17.35	20.53	17.32	20.57	20.23
16	1.31	1.50	2.17	1.48	1.43
17	.08	1.97	2.59	5.00	3.04
18	7.12	4.37	3.56	7.29	7.33
19	1.43	.90	.68	.55	1.14
20	5.35	3.04	2.75	.00	.00

Table (4.4)

A comparison of the active power flow calculated by the exact and proposed techniques (double line outages) (MW)

LINE	DOUBLE OUTAGE OF LINES 3 and 20		DOUBLE OUTAGE OF LINES 16 and 17	
	EXACT LOAD FLOW	PROPOSED METHOD	EXACT LOAD FLOW	PROPOSED METHOD
1	142.51	149.33	155.47	154.07
2	107.55	93.60	77.46	76.82
3	000.00	00.00	71.92	79.10
4	72.32	69.46	52.90	40.30
5	84.97	77.44	42.15	45.37
6	94.14	78.67	24.92	18.23
7	119.92	85.43	49.62	54.08
8	23.90	24.99	18.87	24.97
9	13.38	14.07	10.63	14.06
10	51.62	49.62	59.51	53.78
11	20.65	18.64	12.78	10.46
12	6.56	8.30	9.95	8.37
13	13.21	16.23	25.58	19.47
14	00.00	00.00	00.00	00.00
15	23.90	24.96	18.87	24.97
16	7.45	5.09	00.00	00.00
17	15.23	10.54	00.00	00.00
18	16.47	12.90	9.09	6.80
19	4.10	2.10	3.73	2.18
20	00.00	.00	15.32	10.86

Solution Accuracy

The average and maximum errors for voltage magnitudes and angles are less than (1% and 2%) and (3% and 5%) respectively. Considering the purpose of contingency analysis it is felt that the accuracy is quite acceptable.

A comparison of a line flow given by the proposed technique and the exact solution has been made in terms of a percentage of the line rated capacity and not the line flow. This, to us, seems a logical choice, because the main purpose of the outage study, after all, is to determine and detect if there are any lines overloaded. Also, we should not keep on changing the base for a line under different loading conditions. An evaluation of results shows that for single line outage, the average error is less than 1% whereas the maximum error is less than 6%. The corresponding figures for double line outages are 2% and 10%.

If the actual loading of line is used as the basis, table 4.5 compares our errors with the errors of other accepted techniques, for the outage of line 7. As it is clear from this table only four lines (marked with *) have errors higher than those of Z-matrix and B. Stott's methods and the remaining sixteen lines have errors less than those obtained by these accepted two techniques. Clearly, a line which is predicted to carry 0.55 MW power as compared of .09 MW should not matter; because it is no way near its rating. However, if it is said that the error in this situation is 511.1% it is bound to cause concern. For this case, the errors with Z-matrix method and Stott's method are 3488.9% and 3733.3% respectively.

Table (4.5)

% Error based on the line flows for the outage of line 7

(Refer to the last 4 columns of Table 4.2)

LINE	PROPOSED TECHNIQUE	Z MATRIX METHOD	B. STOTT'S METHOD
1	5.15*	.655	.364
2	-3.625	5.83	5.89
3	6.27	18.54	17.98
4	10.85	-37.82	-39.24
5	-39.06*	26.26	25.79
6	16.86*	-5.54	3.35
7	0	0	0
8	-19.83	-41.75	-38.42
9	-20.16	-41.16	-38.67
10	6.41	12.96	11.57
11	19.62	23.83	25.48
12	3.94	5.26	6.27
13	9.23	9.70	11.41
14	0	0	0
15	-19.05	-40.28	-38.11
16	40.95	56.01	60.99
17	511.1	3488.9	3733.3
18	21.85	28.48	31.54
19	25.54*	17.39	22.28
20	17.92	21.75	24.09

*Cases where the error by the proposed technique is larger than these using the other two techniques.

Solution Speed

Since the post-contingency results are always obtained in one iteration starting from the pre-contingency load-flow the speed of solution is undoubtedly very high.

4.5 Conclusions

1. The attractive feature of modified power injection technique for the simulation of transmission system elements has been combined with fast-decoupled load-flow technique to provide a new very fast contingency evaluation method.

2. The method provides final load-flow results of acceptable accuracy.

3. The technique is generalized and is shown to be suitable for multiple contingencies.

4. The chapter provides details of other outage simulation methods for completeness of the subject.

CHAPTER V

PIECEWISE LOAD-FLOW SOLUTIONS OF VERY LARGE SIZE POWER SYSTEMS

5.1 Introduction (44,60)

Diakoptics (known variously as the method of tearing or by civil engineers as substructuring)[40] was conceived and developed by Kron.[41] The basic idea of diakoptics is to analyze a system by tearing it into its desirable component parts, and to solve each component part as if the others did not exist. The solutions of the component parts are then combined and modified to take the interconnections into consideration. The results are as exact as if the system had never been torn apart.

Kron applied his findings to the inversion of large matrices and stated that by tearing the system into "n" subdivisions, inverting each subdivision and reconstructing them, the total time could be cut by a factor of $2/n^2$.

The method of tearing is applicable to both electric and nonelectric, physical as well as economic (operations research), linear or nonlinear systems.

The original work by Kron was in the area of electric networks. A most interesting account of Kron's work is available in [42], this work was extended and enlarged by Happ who presents a full account of diakoptics and electrical network theory in his book [43]. In his large number of publications he has advanced the technique with particular reference to electrical power distribution systems.[44-48]

Brameller has made use of the theory of diakoptics on a number of problems [49, 50, 51].

In control system applications ideas similar to diakoptics have appeared under the general term "decomposition techniques". A very useful introduction to diakoptics is given in [52].

Some of the inherent advantages of diakoptics are

1. Because diakoptics allows a large system to be torn into n smaller sub-systems, a small computer can be used to solve a large system which otherwise cannot be handled.

2. A much smaller recomputational effort, and rebuilding effort of models has to be made as compared to conventional methods when :

   a) a change in any one subdivision occurs ;

   b) a change in the interconnections of the sub-divisions is to be considered ; and

   c) additional subdivisions are to be added.

There are certain inherent disadvantages in diakoptics also. Two major disadvantages of diakoptics, as stated by Happ [44], are that, first, several steps are required to obtain a solution, as compared to only a ZI operation in the conventional impedance matrix load-flow method, and second, that an intersubdivision matrix has to be formed.

Sections 5.2 and 5.3 provide a summary of the known decomposition and piecewise methods used for load-flow studies. In sections 5.4 and 5.5 two new load-flow techniques based on diakoptics and employing efficient load-flow techniques are presented.

5.2 Decomposition Techniques

The piecewise solution of a very large system is obtained by tearing the system into "n" parts (A,B,...,N), called subdivisions. There is no limitation on the lines of tear (L) except that no mutuals should be present between lines belonging to different subsystems. The total system equations can be written as

$$
\begin{bmatrix} E_A \\ E_B \\ E_C \\ E_N \\ V_L \end{bmatrix}
\begin{array}{c} A \\ B \\ C \\ N \\ L \end{array}
=
\begin{array}{c}
\begin{array}{ccccc} A & B & C & N & L \end{array} \\
\begin{bmatrix} Z_A & & & & \\ & Z_B & & & \\ & & Z_C & & \\ & & & Z_N & \\ & & & & Z_L \end{bmatrix}
\end{array}
\begin{bmatrix} I^{TA}+I^{T'A} \\ I^{TB}+I^{T'B} \\ I^{TC}+I^{T'C} \\ I^{TN}+I^{T'N} \\ I^{L} \end{bmatrix}
\tag{5.1}
$$

For each area: E is the voltage vector,

Z is the impedance matrix,

I^T is the external current source vector,

$I^{T'}$ is the current vector due to the interconnection of the subdivisions,

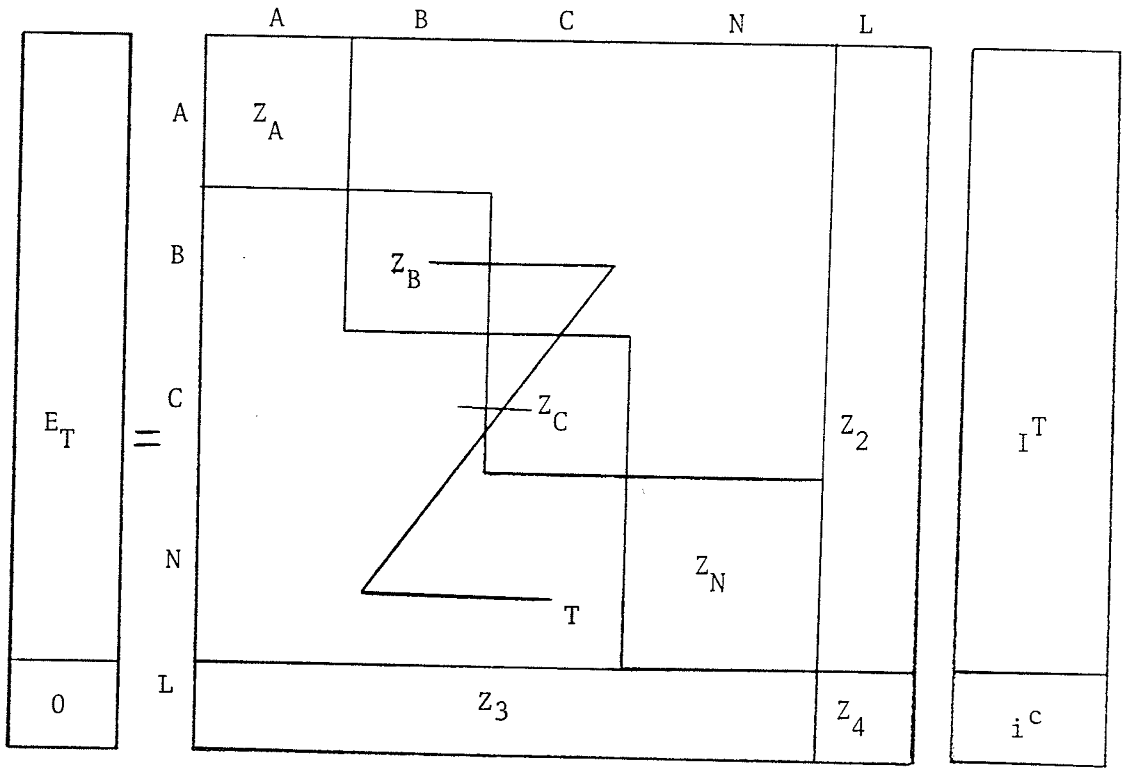
I^L and V_L are branch currents and voltages across the proposed removed branches, and

Z_L is a diagonal matrix of the removed lines impedances.

With the correct currents applied at the points of cut, the subdivisions can be individually solved by any conventional method, from which the solution of the total network is obtained. However, the values of injected currents ($I^{T'}$) are generally not known. Two piecewise methods are described in Ref. [48] for incorporating $I^{T'}$ in the solution procedure. The first method (called boundary iteration method) determines $I^{T'}$ by an iterative method and in the second method (called diakoptic method) $I^{T'}$ is determined in a noniterative or stepwise manner.

In a boundary iteration method, and in the linear case where I^{T} is known, the voltages across the cut branches (V_L) can be calculated from the initial estimate of the voltages of the subdivisions. Then the current I^{L} and $I^{T'}$ can be calculated. Knowing I^{T} and $I^{T'}$, the voltages in the subdivisions can be determined. With the new tie voltages available, corrections in $I^{T'}$ can be made and the procedure repeated. In the nonlinear case, such as load-flow, it leads to an outer loop to satisfy the boundary conditions.

In diakoptics, using the impedance matrix. eq. 5.2 is written. In this equation: Z_T consists of the impedance submatrices of the torn areas (A,B,C,...,N). Z_2, Z_3 and Z_4 are submatrices that reflect the interconnection of the subdivisions. They can be constructed from Z_T as explained in Ref. [48]. All voltages (E_T) are measured with respect to ground, all current (I^{T}) represent the injected bus-to-ground currents and all currents (i^{c}) represent the currents injected at the points of tear.



(5.2)

The solution of (5.2) can be obtained as follows

$$i^c = -Z_4^{-1} Z_3 \quad I^T \tag{5.3}$$

$$E_T = Z_T I^T + Z_2 \, i^c \tag{5.4}$$

When eliminating i^c from 5.4 by substituting 5.3 into 5.4 we obtain

$$E_T = (Z_T - Z_2 Z_4^{-1} Z_3) \; I^T \tag{5.5}$$

With current I^T known, a solution can be obtained by the two-step algorithm above or by the use of eq. 5.5. Note that the sparsity in eq. 5.2 is retained by the use of eqs. 5.3 and 5.4, but that it is not retained in eq. 5.5.

A six-step piecewise algorithm was derived in Ref. [48] which is completely equivalent to solving 5.3 and 5.4. Its major features are that it does not require the formation of the submatrices Z_2 and Z_3 and is more flexible.

The six-step piecewise algorithm is as follows:

1. Obtain solution of torn subdivisions excluding tie currents to other subdivisions

$(E_T^{(0)} = Z_T\ I^T)$

2. Compute voltages across torn subdivisons given intersubdivision (CUT) branch sign convention

$(e_c' = E_L^{(0)})$

3. Compute the currents injected at the points of tear

$(i_c = Z_4^{-1}\ e_c')$

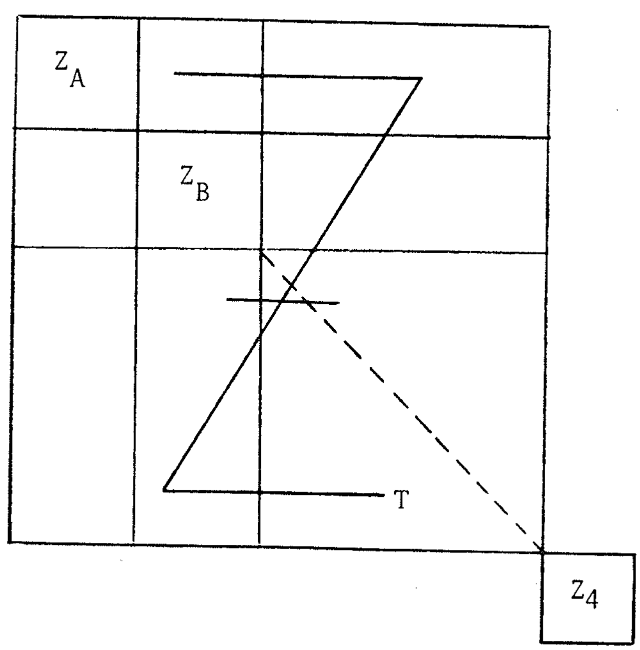
4. Convert i^c to injected tie currents I^{ties} by assigning signs (I^T)

5. Obtain voltage contributions in subdivisions due to tie currents I^T $(E_T^{(1)} = Z_T\ I^T)$

6. Total voltage solution is the sum of the voltages obtained in steps (1) and (5)

$(E_T = E_T^{(0)} + E_T^{(1)})$

The models used require only the individual subdivision models and Z_4 as shown in 5.6



(5.6)

The Z-matrices of the subdivisions, which explicitly represent the subdivision solutions, are not required to execute the algorithm above. Step (1) can be accomplished by elimination methods, triangular factorization methods, by nodal iterative procedures using admittances, or by other iterative procedures.

As a brief summary of piecewise techniques, the boundary iteration method requires an iterative procedure in the linear case, whereas the diakoptic method requires no iterations. In nonlinear cases,(48) such as network programs, the interconnection of the torn subdivisions is expected to increase the number of iterations required in the boundary iteration method, whereas this need not be the case in the diakoptic approach. The advantage of the boundary iteration method is its simplicity.

An observation was made from a theoretical stand-point that it appears that piecewise methods allow different iteration procedures for solving the load-flow or other similar non-linear applications.

Sasson [53] has classified decomposition techniques according to whether branches are cut or whether nodes are cut. He and Carre [55] have shown that the actual places chosen for cuts are of considerable importance in relation to the speed of convergence. They have recommended that partitioning should be made at places where the weakest couplings exist. In electrical system, the branches with highest reactances or smaller susceptances can be chosen for interconnection. This may be true in case of a boundary type iteration. But in diakoptics and as reported in [45,46] the introduction of tearing retains the identical convergence characteristics as the original untorn problem and is independent of the lines torn.

The classification - diakoptics versus boundary iteration - is better because the work in [45] could really be in both of the Sasson's categories. The diakoptic theory that the program in [45] is based upon decomposed networks by cutting through branches and nodes as outlined further in [44]. This is reflected in the program which allows any branch to have zero impedance, which signifies decomposing by cutting nodes.

5.3 Known Piecewise Load-Flow Methods

In load-flow methods such as bus impedance matrix and Newton-Raphson, the number of iterations required to yield an acceptable result appears to be independent of a system size. But the solution time and computer storage requirement vary with the number of buses. These obvious practical limitations have restricted the application of these methods for large size systems. The division of system into subsystems and handling each of them independently have overcome these barriers and now the methods may be used for any system size. The following is a brief review of the known piecewise load-flow methods.

A. Happ et al [45, 46] have applied a decomposition technique based on the theory of a diakoptic method, especially designed for use with their Z-matrix load flow method[5]. Their method was the first to appear in the literature in which the load-flow problem is solved by decomposition techniques to overcome the size limitation inherent in Z-matrix methods. It proposes that the system be decomposed into parts (A, B, C...) by cutting certain transmission lines. The impedance matrix of each part (Z_A, Z_B, Z_C...) is formed independently, then for each part form Z_2 submatrix ($Z2_A$, $Z2_B$, $Z2_C$, ...). Next, a Z_4 intersubdivision matrix is constructed to transmit effects between the parts, as shown in Fig. 5.1.

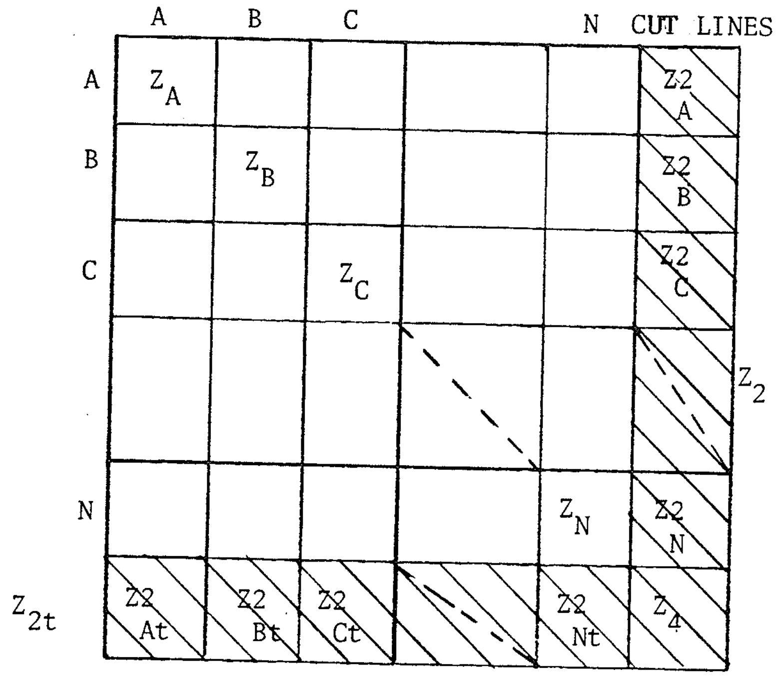


Fig. 5.1 Submatrices for piecewise Z-matrix load-flow method

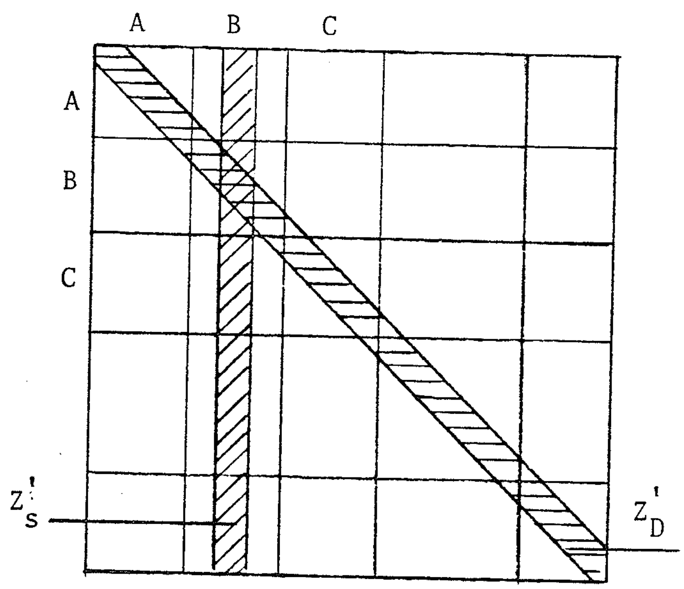


Fig. 5.2 Swing bus and voltage regulated buses vectors

During the iterative solution the voltages at each bus are given by $E_n = Z_{n1}I^1 + Z_{n2}I^2 + \ldots.$, with the cut line buses serving as sources of injection current. The magnitude of the injection current is determined from the difference in voltage at the two ends of the cut line and the apparent impedance across the tear.

In the voltage iteration for the entire system, each area in turn can then be treated independently using the Z-matrix only of the area being considered.

If an injection current adjustment is made to a bus n, a corresponding counteradjustment must be made to the swing bus injection current I^S to hold the swing bus voltage E_S at a fixed magnitude and angle,

$$E_S = Z_{S1}I^1 + Z_{S2}I^2 + Z_{Sn}\,(I^n + \Delta I^n) + \ldots.$$
$$+ Z_{SS}\,(I^S + \Delta I^S) + \ldots.$$

The swing bus axis Z_S' of the interconnected (untorn) system must therefore be computed. For voltage regulated buses a vector Z_D' is required [Fig. 5.2]. Reference should be made to the original publications for computational details [5, 45, 46]. The convergence characteristic of the torn Z-matrix load-flow is identical to the untorn.[46]

B. Sasson[53] has applied a decomposition technique to the nonlinear programming load-flow method [54] which is based on the minimization of a function made up of the sum of the squares of the load-flow equations.

In his method, branches are essentially cut twice, once on each of the two terminal nodes of the branch, and not on the branch itself. Only the cut node on the far side of the interconnection branch will act as a source. The near node will be a normal node of the section containing it.

Fig. 5.3 shows the decomposition technique that results in an overlapping of the subsystems. A decision is made to cut through the branch between nodes 1 and 2. Normally, subsystem A contains node 1 and subsystem B contains node 2. In this scheme, subsystem A contains node 1 and node 2, node 1 appearing as a normal node while node 2 appears as a constant voltage node. The reverse is true for subsystem B. When subsystem A is solved initially, node 2 remains constant at its initial value. When subsystem B is solved, node 1 remains at the value that it took when subsystem A was solved and node 2 takes on a new value. This value for node 2 is used as a constant when A is solved again.

It is noted that the interconnection branch is part of both subsystems. There is no need for any additional computation apart from the load-flow solution of each subsystem. The method thus proceeds iteratively from subsystem to subsystem until all conditions are simultaneously met.

In this decomposition method and as discussed before, the interconnection branch on which convergence characteristics depend, should be of smaller susceptance.

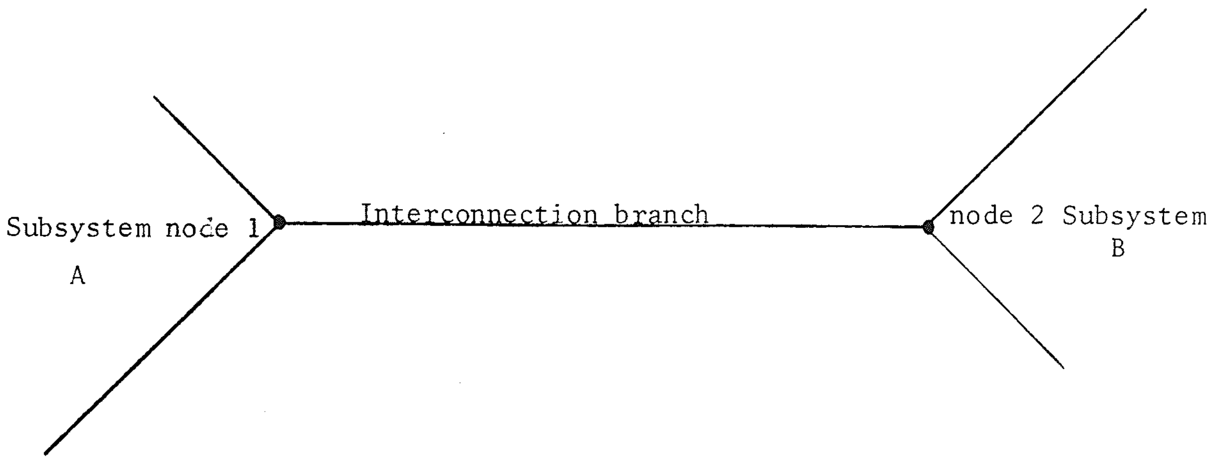


Fig. 5.3 Representation of a decomposition technique

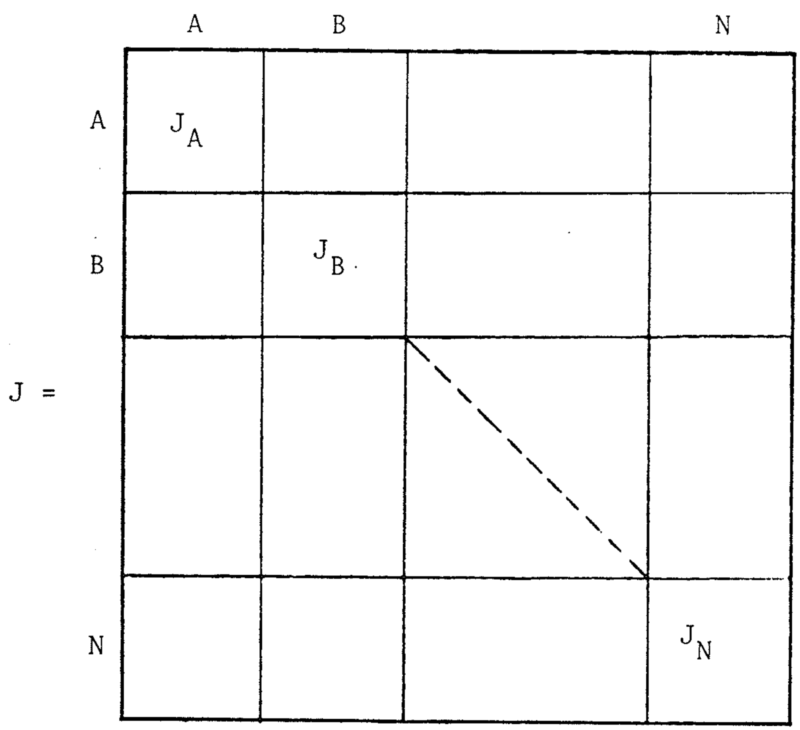


Fig. 5.4 Piecewise N-R Jacobians

C. Laughton[56] has applied decomposition techniques to load flow analysis using the nodal impedance matrix. In his paper, he reported that diakoptics is a special form of partitioning.

D. Happp and Young [47] have developed a piecewise algorithm using Newton-Raphson (NR) load-flow method. The NR load-flow as described in reference [6] is based on a Jacobian which is a function of admittances, voltage magnitudes and angles. Instead of requiring a large Jacobian for a system, smaller Jacobians can be used which represent the torn subdivisions similar to the area Z's in the piecewise solution of the impedance matrix load-flow. The Jacobians of the subdivisions form a block diagonal form (bdf) as shown in Fig. 5.4.

The iteration procedure is as follows:

(1) Since the initial voltages are known (either from a flat voltage start, or from a previous case), the initial tie flows ($I^{ties,\ old}$) between subdivisions may be computed by

$$I_{ij}^{tie} = (E_i - E_j)\, Y_{ij}^{tie} ,$$

where all quantities are complex numbers. This initial value of tie current is stored in a special list. The voltage E_i and E_j are the voltages that exist at this point in the solution at each end of the tie line admittance Y_{ij}^{tie}.

(2) The data for the first subdivision is loaded into memory and the Jacobian of the subdivision is formed for the first iteration. The buses to which tie-lines are connected have been tagged as special buses by input, and the total tie-line current entering such a bus can be found from the data in step (1). The power and reactive power delivered to the bus is computed

$$P_i + jQ_i = (E_i)(\Sigma\, I_{ij}^{tie})^*$$

the real and reactive power are treated as though the ties were replaced by an equivalent load during the formation of the Jacobian. The residual error in active power and reactive power is then determined in the usual manner for each bus of the subdivision. The required change in the voltage and angle of each bus is determined. A new set of bus-voltages and angles in the subdivision are now known (E_T).

(3) The admittance matrix for this subdivision is now formed and stored in the region of memory previously occupied by the Jacobian. The admittance matrix is next factorized and used together with the tie line currents to calculate the change in subdivision voltages due to the presence of the tie currents. The components are subtracted from the voltages determined in step (2)

$$E_T^{(0)} = E_T - Z_T \, I^{ties,old}$$

(4) The procedure described in steps (2) and (3) is repeated for each of the subdivisions. The factorized form of the admittance matrix for each subdivision is retained in peripheral storage for use in subsequent iterations.

(5) The intersubdivision impedance matrix (Z_4) - which has been formed by a separate subroutine - is now brought into core (into the region previously occupied by a factorized subdivision admittance matrix). The matrix (Z_4) is factorized and used with the voltages across the tie lines in order to form the tie-currents:

$$E_{L_{ij}}^{(0)} = E_{Ti}^{(0)} - E_{Tj}^{(0)}$$

$$i^c = (Z_4)^{-1} E_L^{(0)}$$

$$I^{ties,new} = i^c \text{ (except for signs)}$$

The factorized matrix $(Z_4)^{-1}$ is stored in peripheral memory for future iterations.

(6) Each of the subdivision factorized admittance matrices (Z_T) along with the subdivision bus voltages $(E_T^{(0)})$ is brought into memory, and new bus voltages are calculated

$$E_T = E_T^{(0)} + Z_T\, I^{\text{ties, new}}$$

This completes the first iteration. The pattern for subsequent iterations is the same except that it is not necessary to refactorize the subdivision matrices (Z_T) and the intersubdivision matrix (Z_4).

E. Roy [57] has developed a piecewise load-flow solution of large electrical power systems by nodal admittance matrix, applying the principle of superposition : "The voltage at any node of a linear electrical network due to a number of node currents is the algebraic sum of the voltages due to each nodal current acting alone at a time and others being ignored". This simple law of linear circuits is applied to solve large scale electrical networks when they are torn into subdivisions by cutting appropriate lines.

There are two types of currents:

(a) Externally injected currents at the nodes, which are the same as the node currents of the original untorn network [I].

(b) Injected currents due to tear or cut, this is the actual current in the lines of tear (i).

The voltage due to (a) is called apparent nodal voltage (E') and due to (b) is called correction voltage (e). The nodal voltages of the original network are given by

$$E = E' + e$$

Reference should be made to the original publication for computational details.

Roy's technique permits an average engineer to solve his large scale practical problems in pieces without having knowledge of terminology, topology and diakoptics. In his method there is the possibility of selecting a slack bus in each area (subdivision).

This method is similar to that of Brameller et al [52]. Both methods develop their solution models in nodal admittance matrix by using simple circuit laws. It is comparatively easy to write Brameller's matrices directly by inspection than to write equations involving these matrices implicitly. The explicit use of these simple matrices supplemented by the programming techniques of chapter seven makes Brameller's method more systematic and its implementation more straightforward.

In his paper, Roy, even if he was not aware of it, has shown an algebraic equivalent to the diakoptic method. The six-step procedure introduced by Happ resembles Roy's approach almost point by point. In general, Roy has done diakoptics from network equations and algebra, without the need of introducing new concepts and basing himself only on the superposition theorem for linear systems.

F. Kaustri and Potti [58] have developed an algorithm based on the diakoptics equation for block diagonalizing and solving the Jacobian matrix and hence the NR load-flow problem. Their method of partitioning is similar to the one suggested by Sasson.[53]

They applied diakoptics directly to the Jacobian matrix, rather than to the power system itself or its admittance matrix as was previously done, without approximations, with the result that the method has the same convergence property as the one piece solution. In their method, there is a limitation due to extra storage required for the additional right hand side vectors. This may become excessive if many tie lines are to be cut.

5.4 A New Diakoptic Technique For Load-Flow Solution of Very Large Power Systems Using Bus-Admittance Matrix

In this section an exact diakoptic technique [61] for load-flow studies based on bus admittance matrix is described in which full advantage of Zollenkopf's bi-factorization[25] and other sparsity techniques is taken. Since the technique is exact, it produces the same final result and retains the convergence property of the original untorn system. The method provides for a reference bus in each torn subdivision by selecting a number of temporary buses (TB). The voltages and angles of these temporary buses are not assigned, but instead are calculated in each iteration.

The selection of temporary reference buses avoids singularity and a need of converting loads to equivalent impedances, reduces the size of subdivision matrix by one and leads to rapid convergence of subdivision equations.

The proposed technique is simpler to understand and implement. For comparison, the example solved in Ref. [52], pp. (170), is resolved in Appendix (C), using the proposed technique.

5.4.1 Admittance Matrices of the Torn System

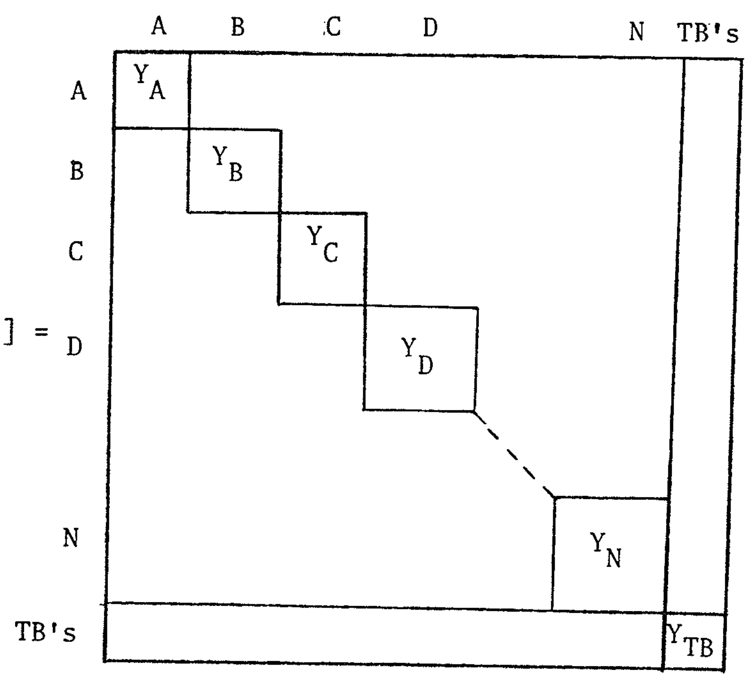
A given large network is torn into N subdivisions (A,B,C,...N) by cutting the appropriate lines. Subdivision is generally guided by the need to limit the size of each subdivision and is generally performed to separate identifiable power systems e.g. territories of utilities, provinces, etc. The only restriction on the lines, which should be cut for subdivision, is that no mutual coupling must exist between subdivisions.

In addition to a slack bus, selected for the untorn system, a number of temporary buses (TB) are selected to provide each subdivision with a reference bus (TB or slack). This procedure eliminates singularity and makes the bus admittance matrix well-conditioned. For computational simplicity, a temporary bus should not be connected to a line to be cut for subdivision. An N subdivision system, therefore has δ = (N-1) temporary buses.

The bus admittance matrix [Y] for each subdivision is formed in the usual manner, using its selected reference (slack or TB) bus.

An additional diagonal matrix $[Y_{TB}]$ of dimensions (δ,δ) is formed for the temporary buses (TB's) of all subdivisions. An element of $[Y_{TB}]$ represents the algebraic sum of the admittances of all lines incident on to a temporary bus. By combining the admittance matrices [Y] of all, N, subdivisions with $[Y_{TB}]$ a block diagonal admittance matrix [Y(bdf)], as shown in 5.7, of the torn system is obtained.

[Y(bdf)] =



(5.7)

5.4.2 Admittance Matrix of the Untorn System

The bus admittance matrix [Y] of the untorn system is represented as

$$[Y] = [Y_1] + [Y_2] \tag{5.8}$$

where $$[Y_1] = [Y(bdf)] + [C]\ [M]^{-1}[C]^t \tag{5.9}$$

[Y(bdf)] - is given in eq. 5.7

[C] - is a subset of the tie line - bus connection matrix of dimensions (α,ψ),

α - total number of buses in the network

ψ - number of lines cut for N subdivisions

It has +1 and -1 as the non-zero elements and is formed by inspection, by using the procedure outlined in Ref. [52].

$[M]^{-1}$ - is a diagonal matrix of dimension (ψ,ψ) where each element represents the admittance of a cut line.

and $$[Y_2] = [F][I][K]^t + [K][I][F]^t \tag{5.10}$$

$[Y_2]$ is a very sparse matrix, containing very few elements corresponding to the buses connected to the TB's in the system. The formulation and form of matrices [F] and [K] are given in Appendix (C) and [I] is a unity matrix.

The relationship of eq. 5.8 is better understood when one examines the example given in Appendix (C).

5.4.3 Solution procedure

The load-flow equations of an n-bus system with bus no. 1 as the slack bus are

$$\left(\frac{S_i^*}{V_i^*} - Y_{i1} V_1 \right) = \sum_{j=2}^{n} Y_{ij} V_j \qquad (5.11)$$

$$i = 2, 3, \ldots, n$$

or, in a matrix form,

$$[J] = [Y][V] \qquad (5.12)$$

where $[Y]$ is the bus admittance matrix

$[J]$ is the system bus injected current vector

and $[V]$ is the required bus voltage vector.

For the untorn system the exact voltage vector is calculated by

$$[J] = [Y][V]_{exact}$$

$$= [Y_1][V]_{exact} + [Y_2][V]_{exact} \, .$$

Hence,

$$[V]_{exact} = [\Delta Y]^{-1}[V] \qquad (5.13)$$

where $$[V] = [Y_1]^{-1}[J] \qquad (5.14)$$

and $$[\Delta Y] = [I] + [Y_1]^{-1}[Y_2] \qquad (5.15)$$

From the above equations, at first, $[V]$ is calculated, without actually inverting the matrix $[Y_1]$, and then $[\Delta Y]$ is calculated to give $[V]_{exact}$. For all these calculations sparsity techniques are utilized to the fullest extent.

5.4.4 Procedure for Obtaining $[Y_1]^{-1}$

$[Y_1]^{-1}$ is obtained by using Householder formula[59] as

$$[Y_1]^{-1} = [Y(bdf)]^{-1} - [Y(bdf)]^{-1}[C][[M] + [C]^t [Y(bdf)]^{-1} [C]]^{-1} [C]^t [Y(bdf)]^{-1} \qquad (5.16)$$

Defining $[Z_c]$ of dimensions (ψ,ψ) as the intersubdivision matrix by

$$[Z_c] = [M] + [C]^t\ [Y(bdf)]^{-1}\ [C] \tag{5.17}$$

equation 5.16 is rewritten as

$$[Y_1]^{-1} = [Y(bdf)]^{-1} - [Y(bdf)]^{-1}\ [C]\ [Z_c]^{-1}\ [C]^t\ [Y(bdf)]^{-1} \tag{5.18}$$

The component matrices of eq. 5.18 have already been defined. It should be noted that for eq. 5.18 no matrix inversion is required and sparsity techniques are used to a maximum extent.

The intersubdivision matrix $[Z_c]$, defined by eq. 5.17, is easily constructed by using elements of $[M]$ and very few elements from very few columns of matrix $[Y]^{-1}$ of each subdivision corresponding to its buses connected to the cut lines in a similar way as that of Ref. [52].

To compute the ith column of a matrix $[Y]^{-1}$, let a column vector $[ei]$ be defined as a zero vector except its ith element which is unity, therefore,

$$[Y][Y]^{-1}[ei] = [I][ei]$$

or

$$[Y][Zi] = [ei]$$

where $[Zi]$ is the ith column of $[Y]^{-1}$ and is calculated by solving the factorized matrix $[Y]$ and $[ei]$ by Zollenkopf bi-factorization direct solution subroutine.

5.4.5 Calculation of [V]

$[V]$ defined in eq. 5.14 is obtained by substituting for $[Y_1]^{-1}$ from eq. 5.18, i.e.

$$[V] = [Y(bdf)]^{-1}[J] - [Y(bdf)]^{-1}\ [C][Z_c]^{-1}\ [C]^t[Y(bdf)]^{-1}[J] \tag{5.19}$$

$$= [V_1] - [V_2] \tag{5.20}$$

where $[V_1] = [Y(bdf)]^{-1}[J]$, bus voltages in the subdivisions

and $[V_2] = [Y(bdf)]^{-1}[C][Z_c]^{-1}\ [C]^t\ [V_1]$ (5.21)

= correction in bus voltages due to tearing.

5.4.6 Steps for Solution

1. Form and factorize the bus admittance matrix $[Y_A]$ for the first subdivision by using Zollenkopf's bi-factorization and other sparsity techniques.

2. Using the subdivision bus injected current vector $[J_A]$, and $[Y_A]$ solve for the bus voltage vector $[V_{1A}]$ for the first subdivision.

$$[Y_A][V_{1A}] = [J_A]$$

Note that no explicit matrix inversion is needed.

3. Repeat steps (1) and (2) for each subdivision to obtain $[V_1]$ for the subdivided system.

4. For the determination of correction voltage vector $[V_2]$, due to tearing, proceed to calculate

(i) Cut line voltages $[E_L] = [C]^t[V_1]$

(ii) Cut line currents $[I_c] = [Z_c]^{-1}\ [E_L]$

(iii) Injected tie currents $[I_{tie}] = [C][I_c]$

and (iv) finally $[V_2] = [Y(bdf)]^{-1}[I_{tie}]$ (5.22)

From eq. 5.22 for each subdivision we can write

$$[Y_A][V_{2A}] = [I_{tie\ A}] \tag{5.23}$$

since $[Y_A]$ is already factorized in step (1), $[V_{2A}]$ is easily obtained without inversion. Similarly, the complete bus correction voltage vector $[V_2]$is formed.

5. The solution $[V] = [V_1] - [V_2]$ is therefore obtained.

6. The step (5) excludes temporary buses for which

$$[V_{TB}] = [Y_{TB}]^{-1} [J_{TB}]$$

Since, temporary buses are chosen such that they are not connected to lines to be cut for subdivision $[V_{2\ TB}] = 0$

$$\text{i.e. } [V_{TB}] = [V_{1\ TB}] \tag{5.24}$$

7. Obtain $[V]_{exact}$ from $[V]$ by using eqs. 5.13 and 5.15. Sparsity techniques are fully exploited for this procedure. Steps (1) to (7) complete the first iteration for obtaining the bus voltage vector $[V]_{exact}$ starting from the known bus current injection vector [J].

In the event [J] is not given and instead power (S) is given, eq. 5.11 is used and the iterative solution is continued until convergence is obtained satisfying a chosen tolerance criterion.

5.4.7 Results

The proposed technique has been applied to a number of power systems. The results obtained by applying the above described technique are exactly the same and show the same convergence when the untorn system is solved as a whole. In order to illustrate the working and showing the exactness of the proposed solution technique, a sample system of Ref. [52] is taken up in Appendix (C).

5.4.8 Conclusions

This section presents an exact diakoptic technique for the load-flow of very large power systems employing bus admittance matrix. The proposed technique has the following attractive features.

1. It makes a full use of bi-factorization and other sparsity techniques and therefore makes the solution computationally efficient.
2. The solution technique is exact and hence produces the same solution and has the same convergence property as the untorn system.
3. By virtue of the selection of a temporary bus as reference in each subdivision (excepting one which contains the slack bus) the singularity of bus admittance matrices is avoided and instead they are well-conditioned. The method can therefore be freely applied to low voltage power systems which have little or no shunt admittance in their network representations.
4. The method does not impose any restriction on the impedance of lines to be cut for subdivision.
5. The method is easier to understand and implement than some other known techniques.

5.5 A New, Diakoptic, Fast-Decoupled Load-Flow Method For Very Large Power Systems

A new diakoptic technique for load-flow solution using a bus-admittance matrix has been presented in section (5.4). This technique is exact, reliable and easier to understand and implement than any other known technique employing admittance (or impedance) matrices. It should be applied at some utilities who have an admittance (or impedance) matrix load-flow program.

This section presents another new diakoptic technique for load-flow problems [62] based on the fast-decoupled method.[9]

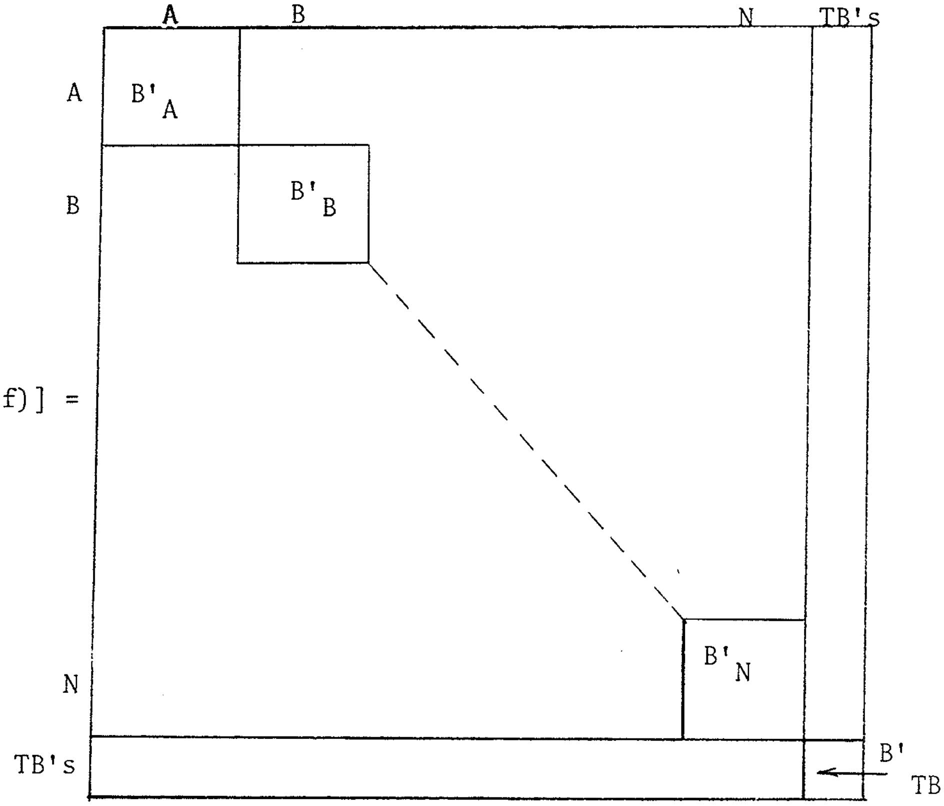
For this technique the author has chosen the fast-decoupled load-flow technique because of its inherent superiority in terms of speed of calculation, storage requirement, reliability and simplicity in addition to noticing its wide-spread acceptance by power industry. This load-flow method forms the basic block for the development of a new diakoptic technique which exploits sparsity and uses Zollenkopf's bi-factorization technique. The proposed technique is exact and therefore provides accurate solution and retains the same convergence characteristics as the untorn system if it were to be solved as a whole. As compared to many known methods the proposed technique does not impose tearing restrictions, make approximations and run the risk of computational breakdown due to singularity of the admittance matrix.

5.5.1 Formation of Subdivision Matrices

A given large network is torn into n subdivision (A, B, C, ...N) by cutting connecting lines with only one restriction that no mutual coupling should exist between lines of different subdivisions. Generally, an objective guide for tearing is to separate networks of different utilities, identifiable power pools, etc. A restriction[53] on the impedance of the lines to be cut is unnecessary for this technique. In addition to the slack bus of the original system which would lie in one subdivision, a temporary bus (TB) is selected for reference in each of the remaining subdivisions for avoiding singularity and ensuring that all subdivision matrices are well conditioned. Thus, $\delta = (n-1)$ temporary buses are required for the system. For simplicity no TB should be connected to the lines to be cut for subdividing the system. The voltage and angle of each TB is not specified but calculated in each iteration.

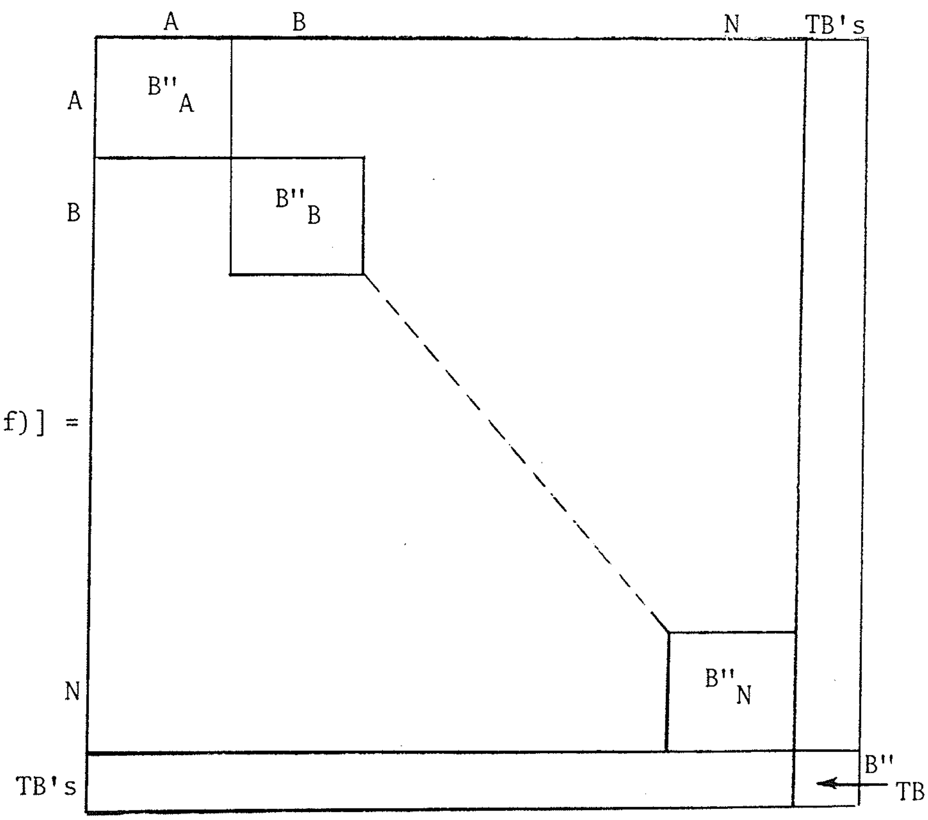
Submatrices [B'] and [B''], as required for Fast-decoupled Load-flow[9], are established for each subdivision, with respect to its reference - a slack bus or a TB. In addition, diagonal matrices $[B'_{TB}]$ and $[B''_{TB}]$ of dimensions (δ,δ) representing TB's of the system are obtained where a diagonal element of $[B'_{TB}]$ is the algebraic sum of (1/X) of all lines incident on the bus (TB) and that of $[B''_{TB}]$ is the algebraic sum of (-B) of the incident lines on the bus. Combinations of [B'] of all subdivisions with $[B'_{TB}]$ and [B''] of all subdivisions with $[B''_{TB}]$ give block-diagonal-form (bdf) matrices, [B'(bdf)] and [B''(bdf)], as shown in eqs. 5.25 and 5.26.

[B'(bdf)] =



(5.25)

[B''(bdf)] =



(5.26)

5.5.2 Matrices of the Untorn System

The matrices [B'] and [B"] of the untorn system are expressed as

$$[B'] = [B'_1] + [B'_2] \tag{5.27}$$

and
$$[B''] = [B''_1] + [B''_2] \tag{5.28}$$

where
$$[B'_1] = [B'(bdf)] + [C][M']^{-1}\,[C]^t \tag{5.29}$$

and
$$[B'_2] = [F'][I][K]^t + [K][I][F']^t \tag{5.30}$$

The matrices $[B''_1]$ and $[B''_2]$ are expressed similarly by replacing primed matrices by double primed e.g. $[M']^{-1}$ by $[M'']^{-1}$ etc.

In eq. 5.29: [B'(bdf)] is already known, [C] is a subset of bus connection matrix of dimensions (α,ψ), where α- number of buses in the system and ψ- number of lines cut for subdivision. It is constructed by inspection and has only +1 or -1 for non-zero elements. As in Ref. [52] the element C_{aj} (row a, column j) of [C] is given as

C_{aj} = +1 , if the line j is directed towards bus a

= -1, if the line j is directed away from bus a

= 0, if bus a does not include line j or is the slack bus.

For convenience, in programming, the connection matrix is condensed into a two column table as shown in Appendix (D).

$[M']^{-1}$ is a diagonal matrix of elements = $\frac{1}{X}$ of the cut lines.

$[M'']^{-1}$ is also a diagonal matrix of elements (-B) of the cut lines.

Both, $[M']^{-1}$ and $[M'']^{-1}$ are of (ψ,ψ) dimensions.

For eq. 5.30 matrices [F'] and [K] are defined in Appendix (D) and [I] is a unit matrix. The matrices $[B'_2]$ and $[B''_2]$ are very sparse. They contain elements corresponding only to buses connected to the TB's as can be seen in Appendix (D).

5.5.3 Bus Angle Change Vector

In fast-decoupled load-flow procedure[9]

$$\left[\frac{\Delta P}{V}\right] = [B'] \, [\Delta\theta]_{exact}$$

$$= [B'_1][\Delta\theta]_{exact} + [B'_2][\Delta\theta]_{exact}$$

by substituting from eq. 5.27

$$\text{or} \quad [\Delta\theta]_{exact} = [\Delta B']^{-1} \, [\Delta\theta] \tag{5.31}$$

$$\text{where} \quad [\Delta B'] = [I] + [B'_1]^{-1} \, [B'_2] \tag{5.32}$$

$$\text{and} \quad [\Delta\theta] = [B'_1]^{-1} \, \left[\frac{\Delta P}{V}\right] \tag{5.33}$$

5.5.4 Bus Voltage Change Vector

Since, $\left[\frac{\Delta Q}{V}\right] = [B''][\Delta V]_{exact}$

By substituting for [B''] from eq. 5.28 and rearranging we get

$$[\Delta V]_{exact} = [\Delta B'']^{-1} \, [\Delta V] \tag{5.34}$$

$$\text{where} \quad [\Delta B''] = [I] + [B''_1]^{-1} \, [B''_2] \tag{5.35}$$

$$\text{and} \quad [\Delta V] = [B''_1]^{-1} \, \left[\frac{\Delta Q}{V}\right] \tag{5.36}$$

In all above manipulations sparsity techniques are used advantageously and no explicit matrix inversions are performed.

5.5.5 Derivation of $[B_1']^{-1}$ and $[B_1'']^{-1}$

The Householder formula[59] gives

$$[B_1']^{-1} = [[B'(bdf)] + [C][M']^{-1} [C]^t]^{-1} \tag{5.37}$$

$$= [B'(bdf)]^{-1} - [B'(bdf)]^{-1}[C][[M'] + [C]^t$$

$$[B'(bdf)]^{-1}[C]]^{-1} [C]^t [B'(bdf)]^{-1}$$

Let [ISDM'] of dimensions (ψ,ψ) be defined as the intersubdivision matrix given by

$$[ISDM'] = [M'] + [C]^t [B'(bdf)]^{-1} [C] \tag{5.38}$$

then eq. 5.37 can be rewritten as

$$[B_1']^{-1} = [B'(bdf)]^{-1} - [B'(bdf)]^{-1} [C][ISDM']^{-1}[C]^t$$

$$[B'(bdf)]^{-1} \tag{5.39}$$

Similarly

$$[B_1'']^{-1} = [B''(bdf)]^{-1} - [B''(bdf)]^{-1} [C][ISDM'']^{-1} [C]^t$$

$$[B''(bdf)]^{-1} \tag{5.40}$$

where $$[ISDM''] = [M''] + [C]^t [B''(bdf)]^{-1} [C] \tag{5.41}$$

In the above, no matrix inversion is required, instead sparsity is fully exploited by using Zollenkopf's method and other sparsity techniques.

5.5.6 Formulation of Intersubdivision Matrices [ISDM'] and [ISDM'']

The procedure for both [ISDM'] and [ISDM''] is the same, excepting that for [ISDM''], [B'] and [M'] are replaced by [B''] and [M''] respectively. Rewriting eq. 5.38, we have

$$[ISDM'] = [M'] + [C]^t [B'(bdf)]^{-1} [C]$$

$[M']$ is the inverse of diagonal matrix $[M']^{-1}$ and hence is easily obtained.

An explicit inversion of $[B'(bdf)]$ is not needed because all we need is to calculate only a few elements from very few columns of the subdivision matrices $[B']^{-1}$ corresponding to the buses connected to the cut lines.

For calculating the ith column of a matrix $[B']^{-1}$ let $[e_i]$ be a zero vector excepting its unity ith element therefore,

$$[B'][B']^{-1}\ [e_i] = [I][e_i]$$

or $$[B'][S_i'] = [e_i]$$

where $[S_i']$ is the ith column of the matrix $[B']^{-1}$. It is calculated by solving the factorized Jacobian matrix $[B']$ and $[e_i]$ by Zollenkopf's Bi-factorization direct solution subroutine.

Main diagonal elements:

The diagonal element $ISDM'_{ii}$ of $[ISDM']$ corresponds to the cut line i between bus k of area A and bus m of area B.

It is given as

$$ISDM'_{ii} = M'_{ii} + S'_{kk} + S'_{mm}$$

where S'_{kk} is the kth element of $[S'_k]$ of area A

and S'_{mm} is the mth element of $[S'_m]$ of area B

when one end of a cut line is connected to a slack bus, its corresponding element is zero.

Off diagonal elements:

Consider element $ISDM'_{ij}$ corresponding to cut lines i and j when line i connected between buses k in area A and m in area B and line j is connected between buses l in area A and n in area B.

Then

$$ISDM'_{ij} = S'_{lk} \text{ (area A)} + S'_{nm} \text{ (area B)}$$

where S'_{lk} is the lth element of $[S'_k]$

and S'_{nm} is the nth element of $[S'_m]$

When both cut line i and j do not terminate in the same area, its corresponding element is zero.

When the assumed directions of lines i and j are dis-similar the sign of the calculated element is reversed.

5.5.7 Determination of $[\Delta\theta]$

Substituting for $[B_1']^{-1}$ for eq. 5.39 in eq. 5.33, we get,

$$[\Delta\theta] = [B'(bdf)]^{-1} [\tfrac{\Delta P}{V}] - [B'(bdf)]^{-1} [C][ISDM']^{-1} [C]^t [B'(bdf)]^{-1} [\tfrac{\Delta P}{V}]$$

$$= [\ \Delta\theta_1] - [B'(bdf)]^{-1} [C][ISDM']^{-1} [C]^t [\ \Delta\theta_1]$$

$$\triangleq [\ \Delta\theta_1] - [\ \Delta\theta_2] \tag{5.42}$$

where $[\ \Delta\theta_1]$ is the independently calculated increment in bus angles of the torn subdivision.

and $[\ \Delta\theta_2]$ is correction in the bus angles increment due to tear.

5.5.8 Determination of $[\Delta V]$

In a manner similar to above

$$[\Delta V] \triangleq [\ \Delta V_1] - [\ \Delta V_2] \tag{5.43}$$

$[\Delta V_1] = [B''(bdf)]^{-1} [\frac{\Delta Q}{V}]$, is the independently calculated increment in bus voltages of the torn subdivisions

and $[\Delta V_2] = [B''(bdf)]^{-1} [C] [ISDM'']^{-1} [C]^t [\Delta V_1]$ (5.44)

is correction in the bus voltage increment due to tear

5.5.9 Procedural Steps For Solution (Flowchart- Appendix E)

1. Calculate flows on the ties between subdivisions

$$I_i = (E_k - E_m) Y_{km} = (E_k - E_m) / Z_{km} \quad (5.45)$$

where $E_k (V_k \angle \theta_k)$ and $E_m (V_m \angle \theta_m)$ are voltage (complex) that exist at this point in the solution at the ends of tie line i and Y_{km} is the tie line admittance.

2. Form [B'] for the subdivision A, factorize it by using sparsity and Zollenkopf Bi-factorization method.

3. Form $[\frac{\Delta P}{V}]$ vector for subdivision A by considering tie lines as loads at the connecting buses. Net active power flow in the tie lines connected to bus k is given by

$$P_k = \mathrm{Re}\,(E_k \sum_{\text{tie lines}} I_i^*) \quad (5.46)$$

Solve the factorized matrix [B'] from step (2) and

$[\frac{\Delta P}{V}]$ to obtain $[\Delta\theta_1]$ from

$$[B'][\Delta\theta_1] = [\frac{\Delta P}{V}] \quad (5.47)$$

Note that no matrix inversion is needed and sparsity is fully exploited.

4. Repeat steps (2) and (3) for all n torn subdivisions and obtain $[\Delta\theta_1]$ for the whole system.

5. Compute [$\Delta\theta_2$].

   (a) Form $[C]^t[\Delta\theta_1] = [\Delta\theta_{C1}]$ - simple algebraic summation using the condensed notation for storing the connection matrix.

   (b) Obtain and factorize [ISDM'] and then obtain

   $[\Delta\theta_{C2}] = [ISDM']^{-1}[\Delta\theta_{C1}]$.

   (c) Form $[\Delta\theta_{C3}] = [C][\Delta\theta_{C2}]$

   (d) Compute $[\Delta\theta_2] = [B'(bdf)]^{-1}[\Delta\theta_{C3}]$ by rewriting the relation as

   $[B'(bdf)][\Delta\theta_2] = [\Delta\theta_{C3}]$

   or $[B'][\Delta\theta_2] = [\Delta\theta_{C3}]$ independently for each subdivision.

   Then, by using the factorized matrix from step (2) for each subdivision calculate its [$\Delta\theta_2$].

6. Solve for $[\Delta\theta] = [\Delta\theta_1] - [\Delta\theta_2]$

7. For temporary buses excluded from the previous step

   $$[\Delta\theta_1]_{TB} = [B']^{-1}_{TB}\,[\frac{\Delta P}{V}]_{TB}$$

   Since temporary buses are not connected to cut lines

   $$[\Delta\theta_2]_{TB} = 0$$

   or $$[\Delta\theta\ ]_{TB} = [\Delta\theta_1]_{TB} \qquad (5.48)$$

8. Modify $[\Delta\theta]$ to obtain $[\Delta\theta]_{exact}$ by using eq. 5.31.

9. Update bus angles $[\theta]$ by

   $$[\theta]_{new} = [\theta]_{old} + [\Delta\theta]_{exact}$$

10. Repeat step (2) but for [B''].

11. Repeat step (3) but for $[\frac{\Delta Q}{V}]$

    Net reactive power flow at bus k is given by

$$Q_k = \text{Im}\,(E_k \sum_{\text{tie lines}} I_i^*) \tag{5.49}$$

    For each subdivision obtain $[\Delta\theta_1]$ from

$$[B''][\Delta V_1] = [\frac{\Delta Q}{V}] \tag{5.50}$$

    following the procedure of step (3).

12. Repeat steps (10) and (11) to obtain $[\Delta V_1]$ for all subdivisions.
13. Repeat the procedure of step (5) to obtain $[\Delta V_2]$ by using appropriate matrices from eq. 5.44.
14. Obtain $[\Delta V] = [\Delta V_1] - [\Delta V_2]$.
15. Modify $[\Delta V]$ to $[\Delta V]_{exact}$ by using eq. 5.34.
16. Update bus voltage $[V]$ by using

$$[V]_{new} = [V]_{old} + [V]_{exact}$$

The above steps describe the first iteration.

In subsequent iterations we do not refactorize the matrices [B'(bdf)], [B"(bdf)], [ISDM'] and [ISDM"].

Also, we do not recalculate the submatrices used for obtaining $[\Delta\theta]_{exact}$ and $[\Delta V]_{exact}$ from $[\Delta\theta]$ and $[\Delta V]$ respectively.

Iterations are continued until convergence is attained according to a chosen tolerance criterion.

5.5.10 Results

The proposed diakoptical technique has been tested on a number of power systems for obtaining their load-flow solution. The results of the system when torn or when solved as a whole are exactly the same and for a given power mismatch require equal number of iterations.

A simple example is provided in Appendix (D) to verify the correctness of the proposed technique.

5.5.11 Conclusions

This section presents a new diakoptical technique for load-flow studies of a very large power system with the following attractive features:

1. It uses the fast-decoupled load-flow technique and combines it with diakoptics. In the process of solving, it uses bi-factorization and other sparsity techniques and therefore minimizes core storage and saves execution time as compared with other known techniques.
2. There is no theoretical limit on the size of the problem which can be solved by it.
3. No restrictions, such as impedances, on the lines to be cut are imposed.
4. The proposed technique is exact that is, the convergence property and the final solution of a system are not altered due to tearing.
5. An exact representation of the tie lines is provided for both matrices [B'] and [B''] and for active and reactive power mismatches vectors.
6. No elements of the individual subdivision impedance matrices are required in the calculation of the intersubdivision matrix which saves significant computation time.
7. The selection of temporary reference buses in subdivision eliminates any singularities the type of which can upset some known procedures when the technique is applied to low voltage networks which may not have equivalent shunt admittances.

5.6 Summary

This chapter is devoted to the study of load-flow of very large systems. At first a detailed review of the known techniques, namely diakoptics, decomposition and piecewise solution is provided (sec. 5.1 to 5.3). The later half of the chapter (sec. 5.4 and 5.5) has been devoted to the description of two new diakoptic techniques based on Bus-Admittance and Fast-Decoupled load-flow solution methods.

The new techniques have been tested on a number of power systems. For completeness one example for each of the two techniques is presented in appendices.

The new diakoptic techniques enlarge the scope of the load-flow studies presented in this thesis by removing the restriction on the size of the Power-Systems imposed by computers.

CHAPTER VI

MAJOR CONTRIBUTIONS

The following major contributions are made in this thesis:

1. A fast, efficient, reliable technique for load-flow solution of integrated dc/ac systems is developed. It accommodates all configurations and control characteristics of multi-terminal HVDC networks. It is unquestionably an improvement over all known techniques.

2. A very fast technique for outage studies is presented. This technique is useful for the economy of studying numerous cases and the selection of critical cases which can be studied by full load-flow solution. The new techniqe can be used for single or multiple outages. It provides the post-contingency load-flow solution - both active and reactive power flows and system voltages - with acceptable accuracy in just one iteration starting from the pre-contingency load-flow.

3. A diakoptical method for load-flow solution of very large power systems using bus-admittance matrix is developed. It provides the same solution and the same convergence characteristics as the untorn system. It can be applied to power systems with all voltage levels.

4. An exact diakoptical fast-decoupled load-flow technique for very large power systems is developed. It employs the best known ac load-flow method, diakoptics and sparsity techniques. It saves core storage and computational time and is therefore

superior to all other known techniques.

This technique does not impose restrictions on

- the size and the voltage level of a system
- the impedance of the cut lines.

It retains the same convergence property and the same final solution as the untorn system.

Suggestions for Future Work

1. Expansion and improvement of the load-flow program of integrated multi-terminal dc/ac systems to be included in dc/ac transient stability study programs.

2. Modification of the outage simulation program to investigate outage studies of integrated dc/ac power systems. The program should be able to study outages in either one of the ac or dc systems , or in both.

3. Expansion of the diakoptical load-flow programs to produce load-flow, outage simulation and stability studies of very large scale ac and/or integrated multi-terminal dc/ac power systems.

APPENDIX A

Submatrices of the Jacobian Matrix of an Integrated AC/DC System

Submatrix AJ_1

The real power at a busbar connected to a converter terminal is

$$P = P(ac) + P(dc)$$

where $P(dc) = Kd\ E\ Id \cos \phi$

The partial differentials for the Jacobian elements are

$$AJ_1 = \frac{\partial P}{\partial \theta}/V = \frac{\partial P(ac)}{\partial \theta}/V + \frac{\partial P(dc)}{\partial \theta}/V$$

$$\frac{\partial P(dc)}{\partial \theta}/V = 0 \quad \text{since} \quad P(dc) \text{ is not function of } \theta$$

$$\frac{\partial P}{\partial \theta}/V = \frac{\partial P(ac)}{\partial \theta}/V$$

Hence, submatrix AJ_1 is exactly the same as for the ac system in the absence of any dc terminals, i.e. constant and symmetrical in value and position. This means we have to factorize it only once before the iterative solution of the integrated ac/dc system.

Submatrix C

$$\frac{\partial P}{\partial V}/V = \frac{\partial P(ac)}{\partial V}/V + \frac{\partial P(dc)}{\partial V}/V$$

$\frac{\partial P(ac)}{\partial V}/V = 0$, by the decoupling principle.

$\frac{\partial P(dc)}{\partial V}/V = 0$, since P(dc) is not function of V.

Therefore submatrix C continues to be a null matrix as in an ac system. The choice of expression for P(dc) eliminates the confusion of having to reconcile that $\frac{\partial P(dc)}{\partial V}/V = 0$ as needed in the formulation adopted by Arrillaga [19].

Submatrix PX

Submatrix PX is defined as

$$PX = \frac{\partial P(dc)}{\partial X}/V$$

since $\quad \dfrac{\partial P(ac)}{\partial X} = 0$

PX turns out to be a very sparse matrix. Its non-zero elements are contained in n rows. Each row with only three non-zero elements. It can be subdivided into submatrices A_2 , B_2 , C_2 and a null matrix PX_0, as shown in Fig. A.1 where N1 is the size of AJ_1 and ND is the number of dc variables = 7xn.

The non-zero elements of A_2 , B_2 and C_2 are stored in vectors

$$A_2 = \frac{\partial P(dc)}{\partial E}/V = Kd\ Id \cos\phi /V$$

$$B_2 = \frac{\partial P(dc)}{\partial \phi}/V = -Kd\ Id\ E \sin\phi\ /V$$

$$C_2 = \frac{\partial P(dc)}{\partial Id}/V = Kd\ E \cos\phi\ /V$$

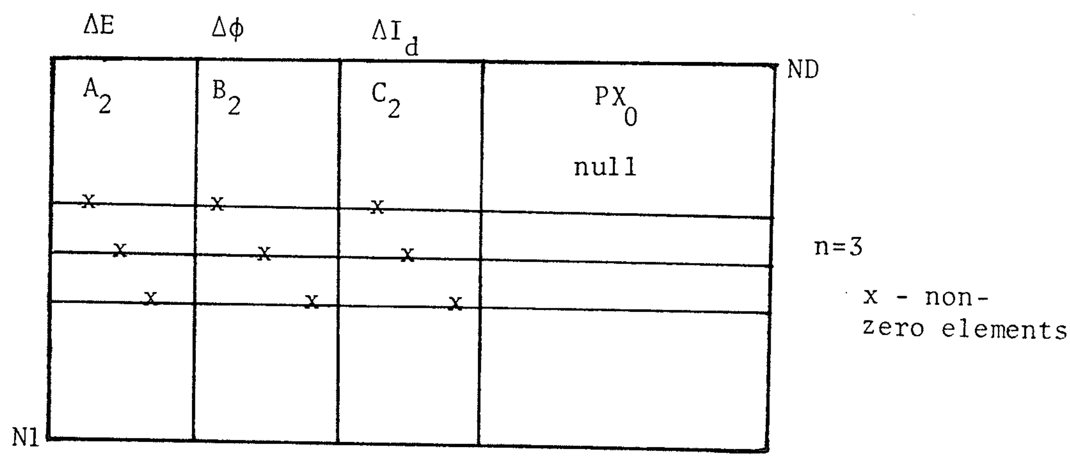


Fig. A.1

Submatrix D

The reactive power at a busbar connected to a converter terminal is

$$Q = Q(ac) + Q(dc)$$

where $Q(dc) = Kd\ E\ Id \sin\phi + Kd^2\ Xc\ Id^2$

$$\frac{\partial Q}{\partial \theta}/V = \frac{\partial Q(ac)}{\partial \theta}/V + \frac{\partial Q(dc)}{\partial \theta}/V$$

$\frac{\partial Q(ac)}{\partial \theta}/V = 0$, by the decoupling principle

$\frac{\partial Q(dc)}{\partial \theta}/V = 0$, since Q(dc) is not function of θ .

Therefore, submatrix D continues to be a null matrix as in an ac system.

Submatrix AJ_4

$$AJ_4 = \frac{\partial Q}{\partial V}/V = \frac{\partial Q(ac)}{\partial V}/V + \frac{\partial Q(dc)}{\partial V}/V$$

$\frac{\partial Q(dc)}{\partial V}/V = 0$, since Q(dc) is not function of V.

Hence, AJ_4 stays exactly the same as in an ac system without the dc terminals. Since the elements of AJ_4 do not change it has to be factorized only once before iteration starts and in this way it leads to saving in computer time as compared to Arrillaga [19] where on account of changes in the diagonal elements associated with the dc terminals, AJ_4 must be factorized in each iteration. This is a note-worthy simplification offered by the method. It is estimated that in our method 75% to 80% of the AJ_4 factorization time is saved.

Submatrix QX

Submatrix QX is defined as

$$QX = \frac{\partial Q(dc)}{\partial X}/V$$

since $\frac{\partial Q(ac)}{\partial X} = 0$

QX like PX is a very sparse matrix containing only 3n non-zero elements shown in Fig. A.2 where N4 is the size of AJ_4 . The elements of A_1, B_1 and C_1 are calculated as follows

$$A_1 = \frac{\partial Q(dc)}{\partial E}/V = Kd\ Id \sin\phi\ /V$$

$$B_1 = \frac{\partial Q(dc)}{\partial \phi}/V = Kd\ E\ Id \cos\phi\ /V$$

$$C_1 = \frac{Q(dc)}{\partial Id}/V = (Kd\ E \sin\phi + 2Kd^2\ Xc\ Id)\ /V$$

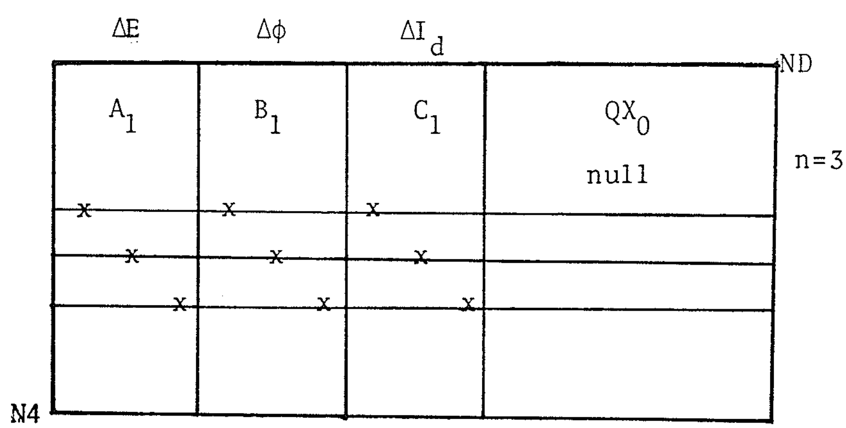


Fig. A.2

Submatrix B

DC system residuals are not function of θ , hence, submatrix B remains a null matrix.

Submatrix RV

Only residuals R4 of the dc system are function of V, therefore

$$RV = \frac{-\partial R4}{\partial V} = -aB$$

Hence, RV is a very sparse matrix with only n elements as shown in Fig. A.3. These elements are stored in a vector form.

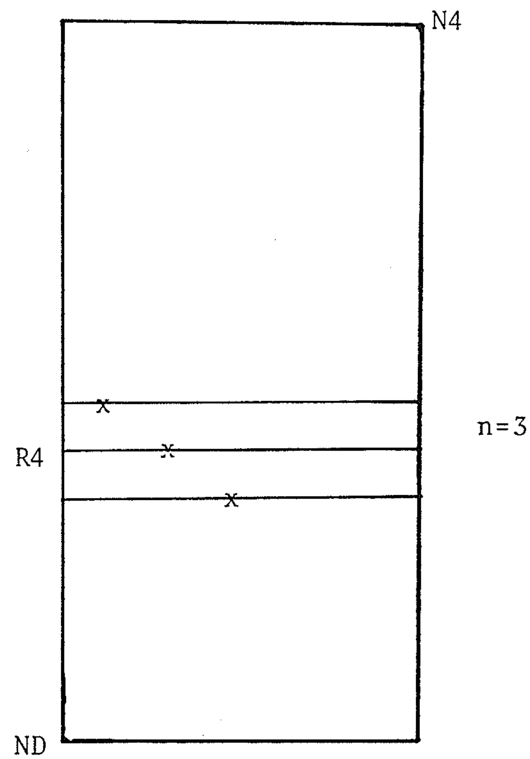


Fig. A.3

APPENDIX B

B.1 Expressions of Active and Reactive Power Flows in a Simulated Outaged Branch

With specific reference to Fig. 4.1 that depicts the simulation of an outage of a line connecting load buses k and m, eqs. 4.33 and 4.34, and knowing that

1. V_k, V_m; θ_k and θ_m are elements of the load flow solution without line outage (basic state).
2. ΔV_k, ΔV_m, $\Delta\theta_k$ and $\Delta\theta_m$ are corrections to account for the line outage, we have

$$P'_{km} = (V_k + \Delta V_k)(V_m + \Delta V_m)[G_{km}\cos(\theta_{km} + \Delta\theta_{km})$$

$$+ B_{km}\sin(\theta_{km} + \Delta\theta_{km})] - (V_k + \Delta V_k)^2(G_{km} - G'_{km}) \qquad \text{(B.1)}$$

where $\theta_{km} = \theta_k - \theta_m$, $\Delta\theta_{km} = \Delta\theta_k - \Delta\theta_m$

Assuming that

$$\Delta V_k \, \Delta V_m = 0$$

$$, \quad \overline{\Delta V_k}^{\,2} = 0$$

, $\Delta\theta_{km}$ is sufficiently small that

$$, \quad \sin(\Delta\theta_{km}) = \Delta\theta_{km}$$

$$, \quad \cos(\Delta\theta_{km}) = 1$$

$$, \quad \Delta V_k \, \Delta\theta_{km} = 0$$

and $\quad \Delta V_m \, \Delta\theta_{km} = 0$

Therefore P'_{km} is given by

$$P'_{km} = V_k V_m \,(G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) - V_k^2 (G_{km} - G'_{km})$$

$$+ \quad [V_m \,(G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) - 2\, V_k \,(G_{km} - G'_{km})]\, \Delta V_k$$

$$+ \quad V_k \,(G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km})\, \Delta V_m$$

$$+ \quad V_k V_m \,(-G_{km} \sin\theta_{km} + B_{km} \cos\theta_{km})(\Delta\theta_k - \Delta\theta_m)$$

i.e. $P'_{km} = P_{km}$ (basic state) + ΔP_{km} (due to line outage)

$$= P_{km}\ (\text{b.s.}) + \Delta P_{km} \tag{B.2}$$

Of course $\Delta P_{km} = 0$ at normal conditions and the basic power P_{km} flows in the line km.

By similar analysis we have

$$\begin{aligned} P'_{mk} &= P_{mk}(\text{b.s.}) + \Delta P_{mk} \\ Q'_{km} &= Q_{km}(\text{b.s.}) + \Delta Q_{km} \\ Q'_{mk} &= Q_{mk}(\text{b.s.}) + \Delta Q_{mk} \end{aligned} \tag{B.3}$$

Applying the equality constraints 4.35, we have

$$\begin{aligned} \Delta P_{Ik} &= P_{km}(\text{b.s.}) + \Delta P_{km} \\ \Delta P_{Im} &= P_{mk}(\text{b.s.}) + \Delta P_{mk} \\ \Delta Q_{Ik} &= Q_{km}(\text{b.s.}) + \Delta Q_{km} \\ \Delta Q_{Im} &= Q_{mk}(\text{b.s.}) + \Delta Q_{mk} \end{aligned} \tag{B.4}$$

B.2 Application of the Decoupling Principle for Calculating the Changes in Line Active and Reactive Power Flows (36-38)

The changes in the active and reactive power flows in an outaged line km could be expressed in terms of the corrections (ΔV_k, ΔV_m, $\Delta\theta_k$ and $\Delta\theta_m$) as follows.

The line flow equations as discussed before are

$$P_{km} = V_k V_m [G_{km} \cos(\theta_k-\theta_m) + B_{km} \sin(\theta_k-\theta_m)] - V_k^2 (G_{km} - G'_{km})$$

$$Q_{km} = V_k V_m [G_{km} \sin(\theta_k-\theta_m) - B_{km} \cos(\theta_k-\theta_m)] + V_k^2 (B_{km} - B'_{km})$$

where $Y_{km} = G_{km} + j B_{km}$ = (k,m)th element of nodal admittance matrix.

The changes in line power flows are expressed in terms of partial derivatives of line flows with respect to busbar voltage magnitude and angle as follows

$$\begin{bmatrix} \Delta P_{km} \\ \Delta Q_{km} \end{bmatrix} = \begin{bmatrix} J_{km-q} \end{bmatrix} \begin{bmatrix} \Delta\theta_q \\ \frac{\Delta Vq}{Vq} \end{bmatrix} \tag{B.5}$$

It should be noted that each row of J_{km-q} has only four non-zero elements, corresponding to $\Delta\theta_k$, $\Delta\theta_m$, ΔV_k and ΔV_m. The line flow Jacobian matrix J_{km-q} is composed of four submatrices

$$\begin{bmatrix} J_{km-q} \end{bmatrix} = \left[\begin{array}{c|c} R & U \\ \hline T & S \end{array}\right] \tag{B.6}$$

Applying the decoupling principle, i.e. neglecting the coupling matrices U and T we get

$$[\Delta P_{km}] = [R][\Delta\theta q] \tag{B.7}$$

$$[\Delta Q_{km}] = [S][\frac{\Delta Vq}{Vq}] \tag{B.8}$$

Elements of [R] and [S] can be derived from the partial derivatives of line flows with the assumptions

$$\cos(\theta_k - \theta_m) = 1.0$$

and $G_{km} \sin(\theta_k - \theta_m) << B_{km}$

Therefore, the changes in line power flows ($\Delta P_{km}, \Delta Q_{km}, \Delta P_{mk}$ and ΔQ_{mk}) in terms of the changes of buses voltages magnitude and phase angle (ΔV_k, ΔV_m, $\Delta\theta_k$ and $\Delta\theta_m$) are

$$\left[\begin{array}{c} \Delta P_{km} \\ \hline \Delta P_{mk} \end{array}\right] = \left[\begin{array}{c|c} V_k V_m B_{km} & -V_k V_m B_{km} \\ \hline -V_k V_m B_{km} & V_k V_m B_{km} \end{array}\right] \left[\begin{array}{c} \Delta\theta_k \\ \hline \Delta\theta_m \end{array}\right] \tag{B.9}$$

and

$$\begin{bmatrix} \Delta Q_{km} \\ \Delta Q_{mk} \end{bmatrix} = \begin{bmatrix} -V_k V_m B_{km} + 2V_k^2 (B_{km} - B'_{km}) & -V_k V_m B_{km} \\ -V_k V_m B_{km} & -V_k V_m B_{km} + 2V_m^2 (B_{km} - B'_{km}) \end{bmatrix} \begin{bmatrix} \frac{\Delta V_k}{V_k} \\ \frac{\Delta V_m}{V_m} \end{bmatrix}$$

$$\ldots\ldots (B.10)$$

Assuming $\sqrt{V_k/V_m} = 1.0$, i.e. the square root of voltage magnitude of both ends of a line are approximately equal, the above equations become

$$\begin{bmatrix} \Delta P_{km} \\ \\ \Delta P_{mk} \end{bmatrix} = [\sqrt{V_k V_m}] \begin{bmatrix} B_{km} & -B_{km} \\ \\ -B_{km} & B_{km} \end{bmatrix} \begin{bmatrix} V_k & 0 \\ \\ 0 & V_m \end{bmatrix} \begin{bmatrix} \Delta\theta_k \\ \\ \Delta\theta_m \end{bmatrix}$$

$$\ldots\ldots (B.11)$$

$$\begin{bmatrix} \Delta Q_{km} \\ \\ \Delta Q_{mk} \end{bmatrix} = [\sqrt{V_k V_m}] \begin{bmatrix} B_{km} - 2B'_{km} & -B_{km} \\ \\ -B_{km} & B_{km} - 2B'_{km} \end{bmatrix} \begin{bmatrix} V_k & 0 \\ \\ 0 & V_m \end{bmatrix} \begin{bmatrix} \frac{\Delta V_k}{V_k} \\ \\ \frac{\Delta V_m}{V_m} \end{bmatrix} \qquad \text{(B.12)}$$

Eq. B.10 could be rewritten in the form

$$[\Delta P \text{ line}]_{k,m} = [\sqrt{V_k V_m}]\ [B_3]_{k,m}\ [V]_{k,m}\ [\Delta\theta]_{k,m}$$

Also eq. B.11 could be rewritten in the form

$$[\Delta Q \text{ line}]_{k,m} = [\sqrt{V_k V_m}]\ [B_4]_{k,m}\ [V]_{k,m}\ [\frac{\Delta V}{V}]_{k,m}$$

Where

$[V]$ and $[\sqrt{V_k V_m}]$ are diagonal matrices

$[B_3]$ and $[B_4]$ are calculated as follows

$$B_{3_{km}} - m = B_{4_{km}} - m = -B_{km}$$

$$B_{3_{km}} - k = B_{km} \qquad \text{(B.13)}$$

$$B_{4_{km}} - k = B_{km} - 2\,B'_{km}$$

The decoupling process is completed by

- Omitting from $[B_3]$ the representation of these network elements that predominantly affect reactive power flow, e.g. shunt reactances.
- Omitting from $[B_4]$ the angle shifting effect of phase shifters.
- Setting all elements of $[V]$ to 1.0 per unit, then we have

$$[\Delta P \text{ line}]_{k,m} = [\sqrt{V_k V_m}]\ [B_3]_{k,m}\ [\Delta\theta]_{k,m} \qquad \text{(B.14)}$$

$$[\Delta Q \text{ line}]_{k,m} = [\sqrt{V_k V_m}]\ [B_4]_{k,m}\ [\Delta V]_{k,m} \qquad \text{(B.15)}$$

B.3 Computation of a Column of a Sensitivity Matrix

Simulation of a line outage by the proposed technique uses the elements of at most two columns of a sensitivity matrix. To compute the ith column of a sensitivity matrix

Let a column vector $[e_i]$ be defined such that its ith element is one and all others are zero, therefore

$$[J][S][e_i] = [I][e_i]$$

$$[J][S_i] = [e_i] \tag{B.16}$$

where $[S_i]$ is the ith column of the sensitivity matrix $[S]$, and is calculated by solving the factorized Jacobian matrix $[J]$ and e_i by Zollenkopf Bi-factorization method by calling the direct solution subroutine.

Our technique is based on the decoupled load-flow method equations

$$[\frac{\Delta P}{V}] = [B'][\Delta\theta]$$

$$[\frac{\Delta Q}{V}] = [B''][\Delta V]$$

The sensitivity matrices corresponding to the fast-decoupled matrices $[B']$ and $[B'']$ are given by

$$[S1] = [B']^{-1}$$

$$[S2] = [B'']^{-1} \tag{B.17}$$

APPENDIX C

Consider a sample system as shown in Fig. C.1. Bus R is the system slack (swing) bus. The system is torn into three subdivisions 1, 2 and 3 by cutting lines j, k and 1. Buses g and h are selected as temporary reference buses (TB's) in subdivisions 1 and 2 respectively. For computational simplicity, assume that all line admittances are one per unit.

The current injection vector [J] is given by

[J] =

a	b	c	d	e	f	g	h	t
6	-1	-6	-6	3	-2	0	9	

C.1 Proof of Equation (5.8):

[Y] matrix of the untorn system is given by C.1.

[Y] = untorn system (C.1)

	a	b	c	d	e	f	g	h
a	3	-1	-1				-1	
b	-1	3		-1			-1	
c	-1		3	-1				-1
d		-1	-1	4		-1		-1
e					2	-1		
f				-1	-1	3		
g	-1	-1					2	
h			-1	-1				2

In the following analysis and for storage, sparsity techniques are fully exploited.

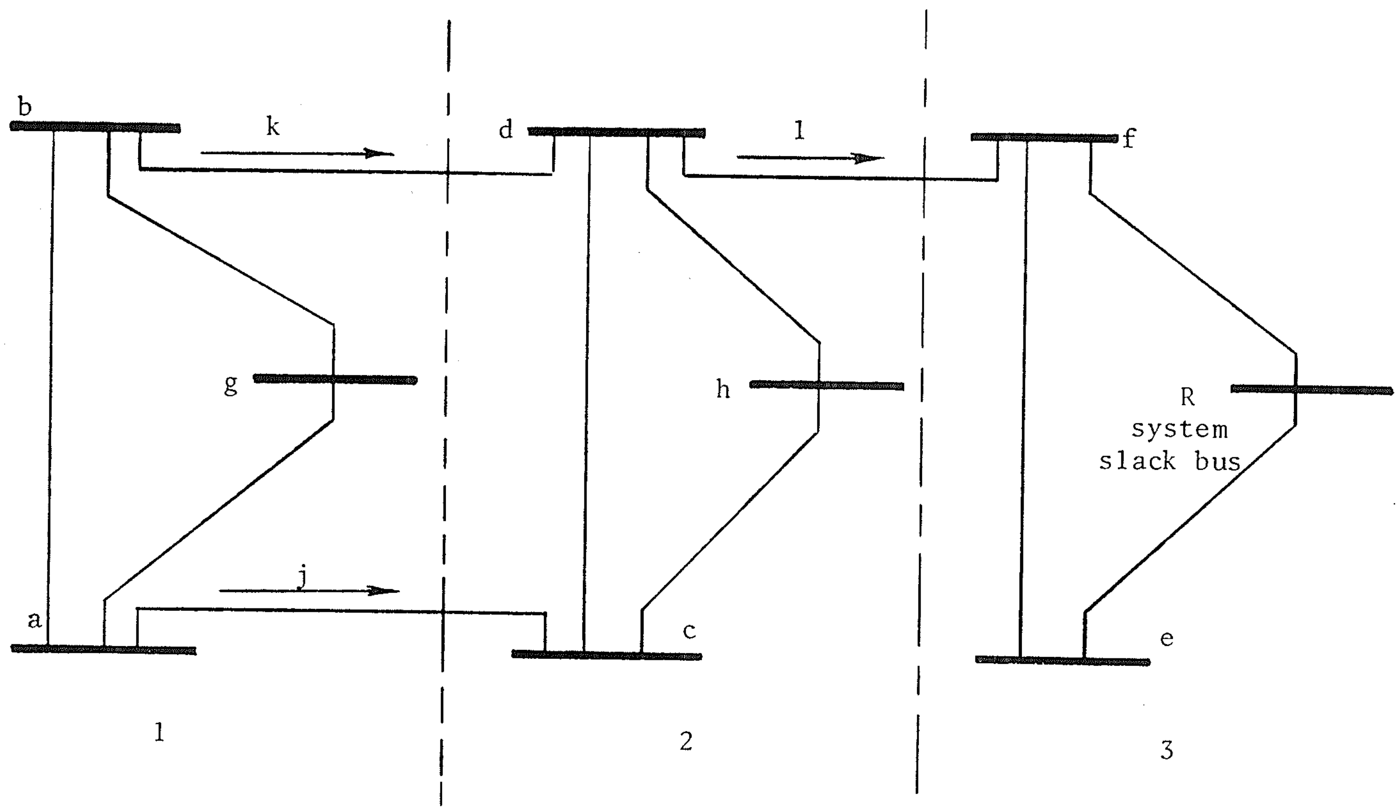


Fig. C.1 A sample system

[Y(bdf)] is given by C.2

[Y(bdf)] =

	a	b	c	d	e	f	g	h
a	2	-1						
b	-1	2						
c			2	-1				
d			-1	2				
e					2	-1		
f					-1	2		
g							2	
h								2

(C.2)

(TB's)

The connection matrix [C] and matrix [M] are given by

[C] =

	j	k	l
a	-1		
b		-1	
c	1		
d		1	-1
e			
f			1
g		ZEROS	
h			

[M] =

	j	k	l
j	1		
k		1	
l			1

(C.3)

Matrix [C] is stored in the condensed form as follows

	FROM	TO
j	a	c
k	b	d
l	d	f

The product $[C][M]^{-1}[C]^t$ is

	a	b	c	d	e	f	g	h
a	1		-1					
b		1		-1				
c	-1		1					
d		-1		2		-1		
e								
f				-1		1		
g								
h								

(C.4)

An element $F_{i,g}$ is given by (-admittance) of the line connecting bus i and TB g, [F] is given as:

[F] =

	g	h
a	-1	
b	-1	
c		-1
d		-1
e		
f		
g	ZEROS	
h		

(C.5)

Define a matrix [K] as follows

$$[K] = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array}\!\!\begin{array}{c} \begin{array}{cc} g & h \end{array} \\ \left[\begin{array}{cc} & \\ & \\ \multicolumn{2}{c}{\text{ZEROS}} \\ & \\ & \\ & \\ \hline 1 & \\ & 1 \end{array}\right] \end{array} \longleftarrow \text{Unity Matrix} \qquad \text{(C.6)}$$

The produce ($[F][I][K]^t + [K][I][F]^t$) is, therefore, given by

$$\begin{array}{c|cccccc|cc|} & a & b & c & d & e & f & g & h \\ \hline a & & & & & & & -1 & \\ b & & & & & & & -1 & \\ c & & & & & & & & -1 \\ d & & & & & & [F] & & -1 \\ e & & & & & & & & \\ f & & & & [F]^t & & & & \\ \hline g & -1 & -1 & & & & & & \\ h & & & -1 & -1 & & & & \\ \hline \end{array}$$

Let $[Y_2] = [F][I][K]^t + [K][I][F]^t$ (C.7)

From C.2, C.4 and C.7 , the summation

$[Y(bdf)] + [C]\ [M]^{-1}\ [C]^t + [Y_2]$ is given by

$$\begin{array}{c|cccccccc|}
 & a & b & c & d & e & f & g & h \\
\hline
a & 3 & -1 & -1 & & & & -1 & \\
b & -1 & 3 & & -1 & & & -1 & \\
c & -1 & & 3 & -1 & & & & -1 \\
d & & -1 & -1 & 4 & & -1 & & -1 \\
e & & & & & 2 & -1 & & \\
f & & & & -1 & -1 & 3 & & \\
g & -1 & -1 & & & & & 2 & \\
h & & & -1 & -1 & & & & 2 \\
\hline
\end{array} \qquad (C.8)$$

From C.1 and C.8

$[Y]$ untorn system $= [Y(bdf)] + [C][M]^{-1}\ [C]^t + [Y_2]$

or $[Y] = [Y_1] + [Y_2]$

where $[Y_1] = [Y(bdf)] + [C][M]^{-1}\ [C]^t$

C.2 System Solution

$[Y(bdf)]^{-1}$ is

$$
\frac{1}{3}\;
\begin{array}{c|cccccccc|}
 & a & b & c & d & e & f & g & h \\
\hline
a & 2 & 1 & & & & & & \\
b & 1 & 2 & & & & & & \\
c & & & 2 & 1 & & & & \\
d & & & 1 & 2 & & & & \\
e & & & & & 2 & 1 & & \\
f & & & & & 1 & 2 & & \\
g & & & & & & & \frac{3}{2} & \\
h & & & & & & & & \frac{3}{2} \\
\hline
\end{array}
$$

For this example $[Y(bdf)]^{-1}$ is calculated explicitly, but in the proposed technique no matrix inversion is required and sparsity is fully exploited by Zollenkopf's method. The intersubdivision matrix $[Z_c]$ is

$$
Z_c = \frac{1}{3}\;
\begin{array}{c|ccc|}
 & j & k & l \\
\hline
j & 7 & 2 & -1 \\
k & 2 & 7 & -2 \\
l & -1 & -2 & 7 \\
\hline
\end{array}
$$

$$[V_1] = \frac{1}{3}\begin{array}{cccccc|cc} a & b & c & d & e & f & g & h \\ \hline 11 & 4 & -18 & -18 & 4 & -1 & 0 & 13.5 \end{array}$$

$$[E_L] = \frac{1}{3}\begin{bmatrix} -29 \\ -22 \\ 17 \end{bmatrix}$$

$$[I_C] = \frac{1}{96}\begin{bmatrix} -330 \\ -168 \\ 138 \end{bmatrix}$$

$$[V_2] = \begin{array}{cccccc|cc} a & b & c & d & e & f & g & h \\ \hline 828 & 666 & -966 & -942 & 138 & 276 & 0 & 0 \end{array}\ \frac{1}{3x96}\ t$$

Therefore $[V] = [V_1] - [V_2]$, is given by

$$[V] = \frac{1}{3}\begin{array}{c|c} a & 2.3750 \\ b & -2.9375 \\ c & -7.9375 \\ d & -8.1876 \\ e & 2.5625 \\ f & -3.8750 \\ \hline g & 0 \\ h & 13.50 \end{array}$$

The product $Y_1^{-1}\, Y_2$ is computed using sparsity techniques and is

$$
\frac{1}{96}\;
\begin{array}{c c}
 & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} &
\left[\begin{array}{cccccc|cc}
 & & & & & & -62 & -28 \\
 & & & & & & -61 & -26 \\
 & & & & & & -29 & -28 \\
 & & \text{Z} & \text{E} & \text{R} & \text{O S} & -25 & -50 \\
 & & & & & & -5 & -10 \\
 & & & & & & -10 & -20 \\
\hline
-48 & -48 & & & & & \multicolumn{2}{c}{\text{ZEROS}} \\
 & & -48 & -48 & & & & \\
\end{array}\right]
\end{array}
$$

$[\Delta Y] = [I] + [Y_1]^{-1}\, [Y_2]$

$[\Delta Y]\, [V]_{exact} = [V]$, using sparsity techniques $[V]_{exact}$ is calculated and is given as

$$
[V]_{exact} = \begin{array}{c c}
\left[\begin{array}{c} 7 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 6 \\ 8 \end{array}\right] &
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array}
\end{array}
$$

APPENDIX D

Consider a sample system as shown in Fig. C.1. Bus R is the system slack (swing) bus. The system is torn into three subdivisions 1, 2 and 3 by cutting lines j, k and l. Buses g and h are selected as temporary reference buses (TB's) in subdivisions 1 and 2 respectively. For computational simplicity, assume that all line reactances are 0.2 per unit.

D.1 Proof of Equation (5.27)

[B'] matrix of the untorn system is given by D.1

[B'] = untorn system (D.1)

	a	b	c	d	e	f	g	h
a	15	-5	-5				-5	
b	-5	15		-5			-5	
c	-5		15	-5				-5
d		-5	-5	20		-5		-5
e					10	-5		
f				-5	-5	15		
g	-5	-5					10	
h			-5	-5				10

In the following analysis and for storage, sparsity techniques are fully exploited.

[B'(bdf)] is given by D.2,

[B'(bdf)] = (D.2)

	a	b	c	d	e	f	g	h
a	10	-5						
b	-5	10						
c			10	-5				
d			-5	10				
e					10	-5		
f					-5	10		
g							10	
h								10

(TB's)

The connection matrix [C] and matrix [M'] are given by D.3.

$$[C] = \begin{array}{c|ccc|} & j & k & l \\ \hline a & -1 & & \\ b & & -1 & \\ c & 1 & & \\ d & & 1 & -1 \\ e & & & \\ f & & & 1 \\ \hline g & & \text{ZEROS} & \\ h & & & \\ \hline \end{array} , \quad [M']^{-1} = \begin{array}{c|ccc|} & j & k & l \\ \hline j & 5 & & \\ k & & 5 & \\ l & & & 5 \\ \hline \end{array} \qquad (D.3)$$

Matrix [C] is stored in condensed form as follows

	FROM	TO
j	a	c
k	b	d
l	d	f

The product $[C][M']^{-1}[C]^t$ is

$$\begin{array}{c|cccccc|cc|} & a & b & c & d & e & f & g & h \\ \hline a & 5 & & -5 & & & & & \\ b & & 5 & & -5 & & & & \\ c & -5 & & 5 & & & & & \\ d & & -5 & & 10 & & -5 & & \\ e & & & & & & & & \\ f & & & & -5 & & 5 & & \\ \hline g & & & & & & & & \\ h & & & & & & & & \\ \hline \end{array} \qquad (D.4)$$

An element $F'_{i,g}$ is given by $-1/X$ of the line connecting buses i and g

$$[F'] = \begin{array}{c|cc|} & g & h \\ \hline a & -5 & \\ b & -5 & \\ c & & -5 \\ d & & -5 \\ e & & \\ f & & \\ \hline g & \text{ZEROS} & \\ h & & \\ \hline \end{array} \qquad (D.5)$$

Define a matrix [K] as follows

$$[K] = \begin{array}{c|cc|} & g & h \\ \hline a & & \\ b & & \\ c & & \\ d & \multicolumn{2}{c|}{\text{ZEROS}} \\ e & & \\ f & & \\ \hline g & 1 & \\ h & & 1 \\ \hline \end{array} \longleftarrow [I] \text{ unit matrix} \qquad \text{(D.6)}$$

The product $([F'][I][K]^t + [K][I][F']^t)$ is, therefore, given by

	a	b	c	d	e	f	g	h
a							-5	
b							-5	
c								-5
d						[F']		-5
e								
f				$[F']^t$				
g	-5	-5						
h			-5	-5				

(D.7)

Let $[B'_2] = [F'][I][K]^t + [K][I][F']^t$

From D.2, D.4 and D.7, the summation

$[B'(bdf)] + [C][M']^{-1}[C]^t + [B_2']$ is given by

	a	b	c	d	e	f	g	h
a	15	-5	-5				-5	
b	-5	15		-5			-5	
c	-5		15	-5				-5
d		-5	-5	20		-5		-5
e					10	-5		
f				-5	-5	15		
g	-5	-5					10	
h			-5	-5				10

(D.8)

From D.1 and D.8

$$[B'] \text{ untorn system } = [B'(bdf)] + [C][M']^{-1}[C]^t + [B'_2] \quad (D.9)$$

or $[B'] = [B_1'] + [B_2']$

where $[B_1'] = [B'(bdf)] + [C][M']^{-1}[C]^t$ (D.10)

D.2 System Solution

To understand the proposed technique, let us consider the calculation of one iteration using the [B'] of the untorn system and then by using the proposed diakoptical technique

For computational simplicity, assume $(\frac{\Delta P}{V})$ of the sytem is given by

$$[\frac{\Delta P}{V}] = \begin{array}{cccccccc} a & b & c & d & e & f & g & h \\ \end{array}$$

a	b	c	d	e	f	g	h
.06	-.01	-.06	-.06	.03	-.02	0	.09

t

We have $[\frac{\Delta P}{V}] = [B'][\Delta\theta]$, using [B'] as given in (D.1), the solution $[\Delta\theta]_{exact}$ is given by

$[\Delta\theta]_{exact} = 1/5$

a	b	c	d	e	f	g	h
.0700	.0500	.0400	.0300	.0200	.0100	.0600	.0800

t

(D.11)

Using the proposed diakoptical technique the following steps are taken:

(a) Calculation of $[\Delta\theta]$

$[\Delta\theta]$ vector is computed using eq. 5.42.

For this example $[B'(bdf)]^{-1}$ is calculated explicitly but in the proposed technique no matrix inversion is required and sparsity is fully exploited by Zollenkopf's method.

$$[B'(bdf)]^{-1} = \frac{1}{15} \begin{array}{c|cccccccc|} & a & b & c & d & e & f & g & h \\ \hline a & 2 & 1 & & & & & & \\ b & 1 & 2 & & & & & & \\ c & & & 2 & 1 & & & & \\ d & & & 1 & 2 & & & & \\ e & & & & & 2 & 1 & & \\ f & & & & & 1 & 2 & & \\ g & & & & & & & 3/2 & \\ h & & & & & & & & 3/2 \\ \hline \end{array} \quad \text{TB's}$$

$$[\Delta\theta_1] = \frac{1}{1500} \begin{array}{|c|c|c|c|c|c|c|c|} a & b & c & d & e & f & g & h \\ \hline 11 & 4 & -18 & -18 & 4 & -1 & 0 & 13.50 \\ \hline \end{array}^{\,t}$$

$$[\Delta\theta_{C1}] = [C]^t [\Delta\theta_1] = \frac{1}{1500} \begin{bmatrix} -29 \\ -22 \\ 17 \end{bmatrix}$$

Intersubdivision matrix is given by

$$[ISDM'] = 1/15 \quad \begin{array}{c|ccc|} & j & k & l \\ \hline j & 7 & 2 & -1 \\ k & 2 & 7 & -2 \\ l & -1 & -2 & 7 \\ \hline \end{array}$$

$$[\Delta\theta_{C2}] = 1/9600 \quad \begin{array}{c|c|} j & -330 \\ k & -168 \\ l & 138 \\ \hline \end{array}$$

$$[\Delta\theta_{C3}] = \begin{array}{|cccccccc|} a & b & c & d & e & f & g & h \\ \hline 330 & 168 & -330 & -306 & 0 & 138 & 0 & 0 \\ \hline \end{array}^{\,t} \quad \frac{1}{9600}$$

$$[\Delta\theta_2] = 1/(1500\text{x}96) \begin{array}{|cccccccc|} a & b & c & d & e & f & g & h \\ \hline 828 & 666 & -966 & -942 & 138 & 276 & 0 & 0 \\ \hline \end{array}^{\,t}$$

Therefore, $[\Delta\theta] = [\Delta\theta_1] - [\Delta\theta_2]$ is given by

$$[\Delta\theta] = 1/5 \begin{array}{|cccccccc|} a & b & c & d & e & f & g & h \\ \hline .0079 & -.0098 & -.0265 & -.0273 & -.085 & -.0129 & 0.0 & .0458 \\ \hline \end{array}^{\,t}$$

(D.12)

$[\Delta B']$ is computed and is given as

$[\Delta B']$ = (D.13)

	a	b	c	d	e	f	g	h
a	1						-.6458	-.2917
b		1					-.6354	-.2708
c			1				-.302	-.604
d				1			-.2604	-.5208
e					1		-.0521	-.1042
f						1	-.1042	-.2084
g	-.5	-.5					1	
h			-.5	-.5				1

$[\Delta\theta]_{exact}$ is calculated using sparsity techniques - (that store only the nonzero elements excepting ones on the diagonal, i.e. 16 elements) and $[\Delta\theta]$ given by D.12 according to the equation

$$[\Delta B'][\Delta\theta]_{exact} = [\Delta\theta]$$

$$[\Delta\theta]_{exact} = 1/5 \begin{array}{cccccccc} a & b & c & d & e & f & g & h \\ \end{array}$$
$$\boxed{.0700 \;\; .0500 \;\; .0400 \;\; .0300 \;\; .0200 \;\; .0100 \;\; .0600 \;\; .0800}^{\,t}$$

which is exactly the same as D.11.

APPENDIX E

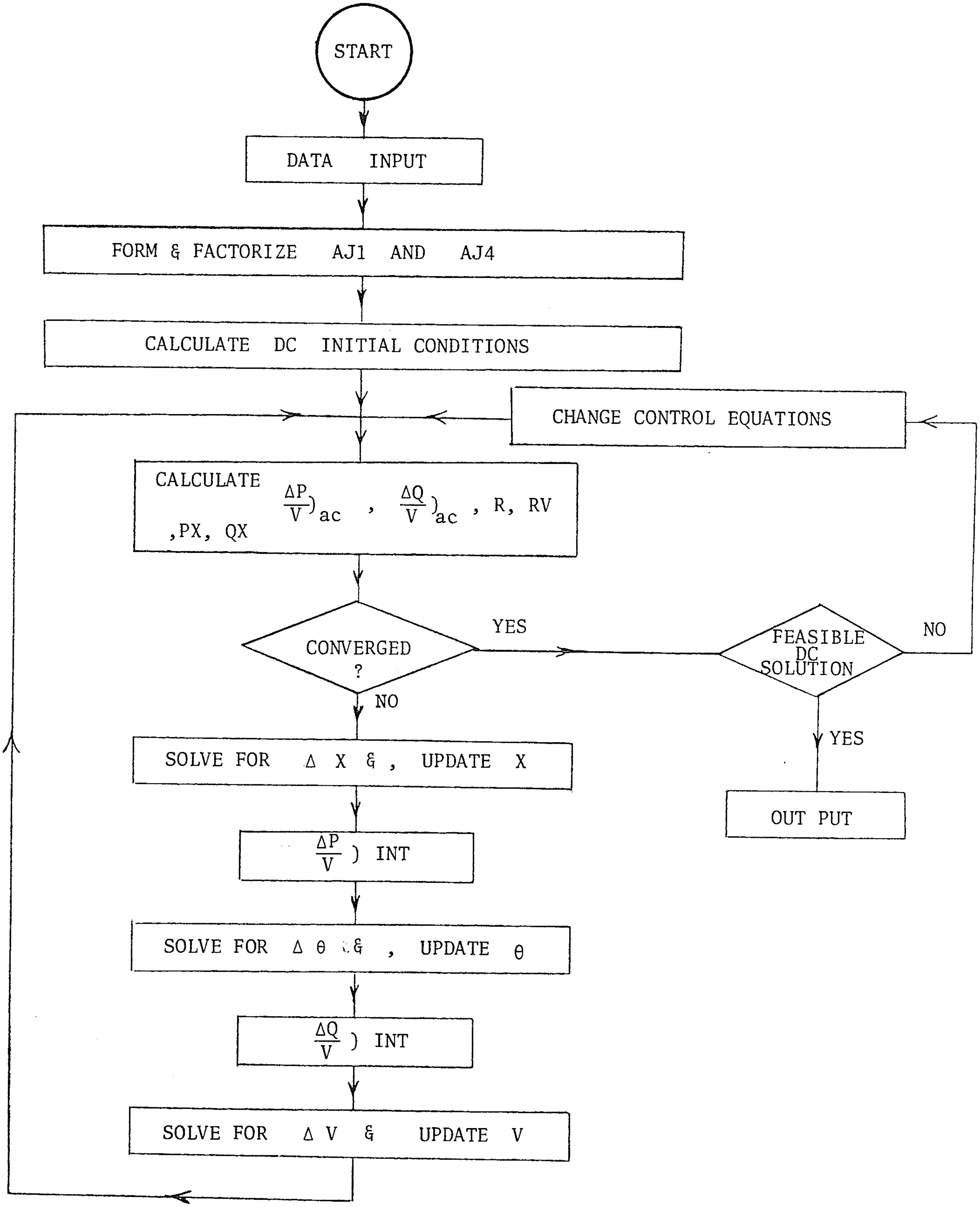


Fig. E.1 Simplified flow-chart of ac/dc load-flow

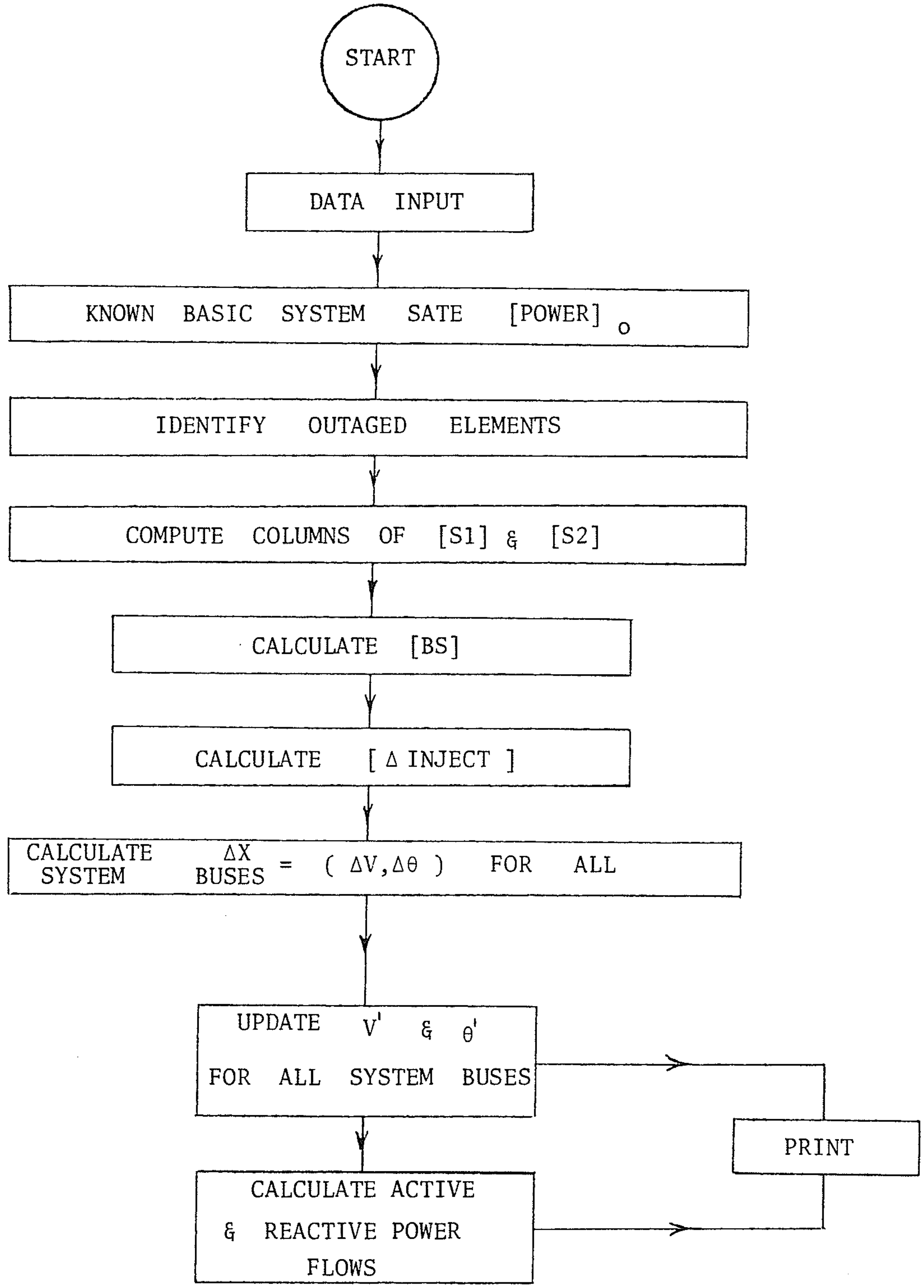
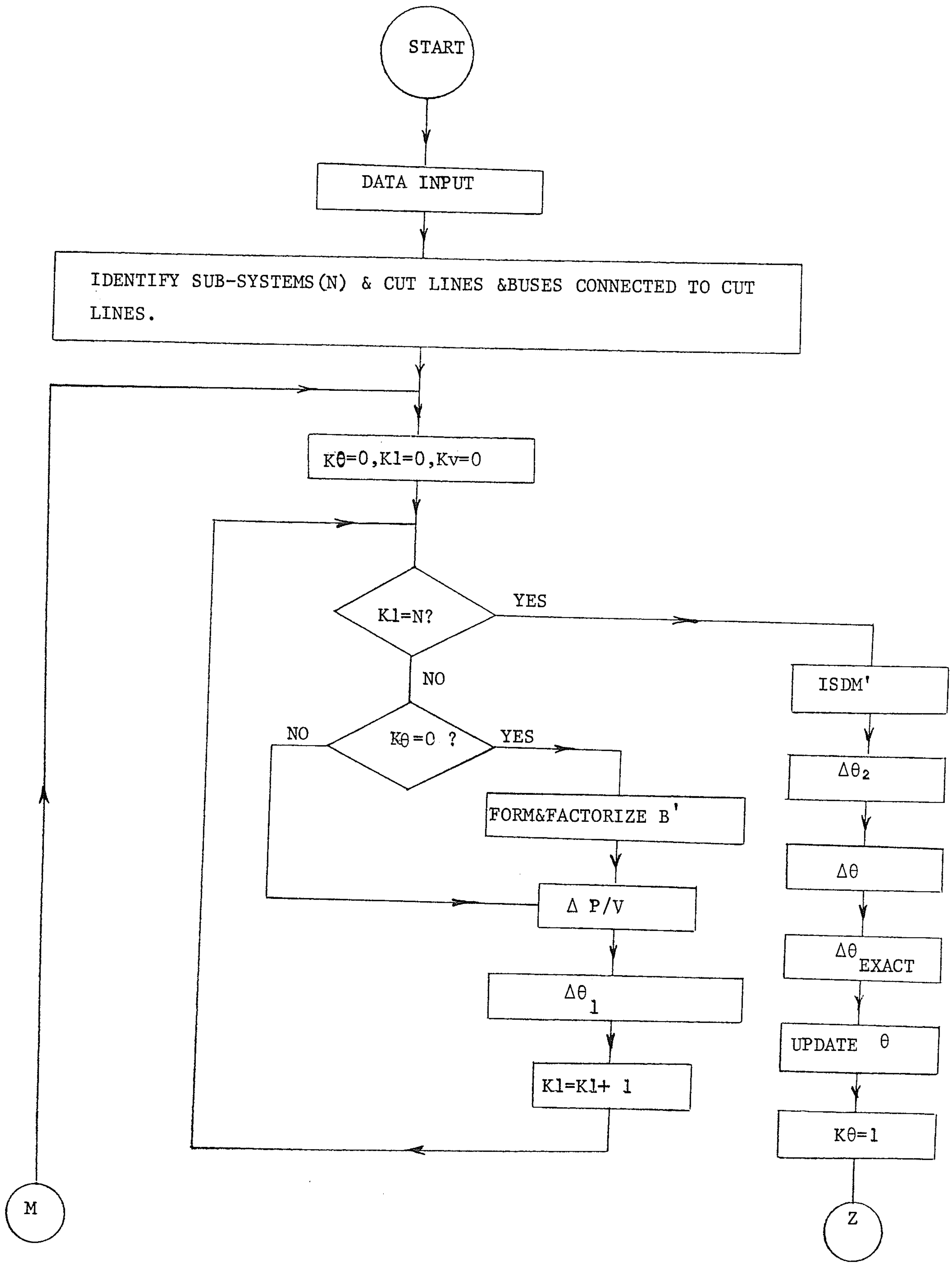


Fig. E.2 Simplified flow-chart of outage simulation.


START
DATA INPUT
IDENTIFY SUB-SYSTEMS (N) & CUT LINES &BUSES CONNECTED TO CUT LINES.
Kθ=0,Kl=0,Kv=0
Kl=N?
YES
NO
Kθ=0 ?
NO
YES
FORM&FACTORIZE B'
Δ P/V
Δθ 1
Kl=Kl+ 1
ISDM'
Δθ2
Δθ
Δθ EXACT
UPDATE θ
Kθ=1
M
Z

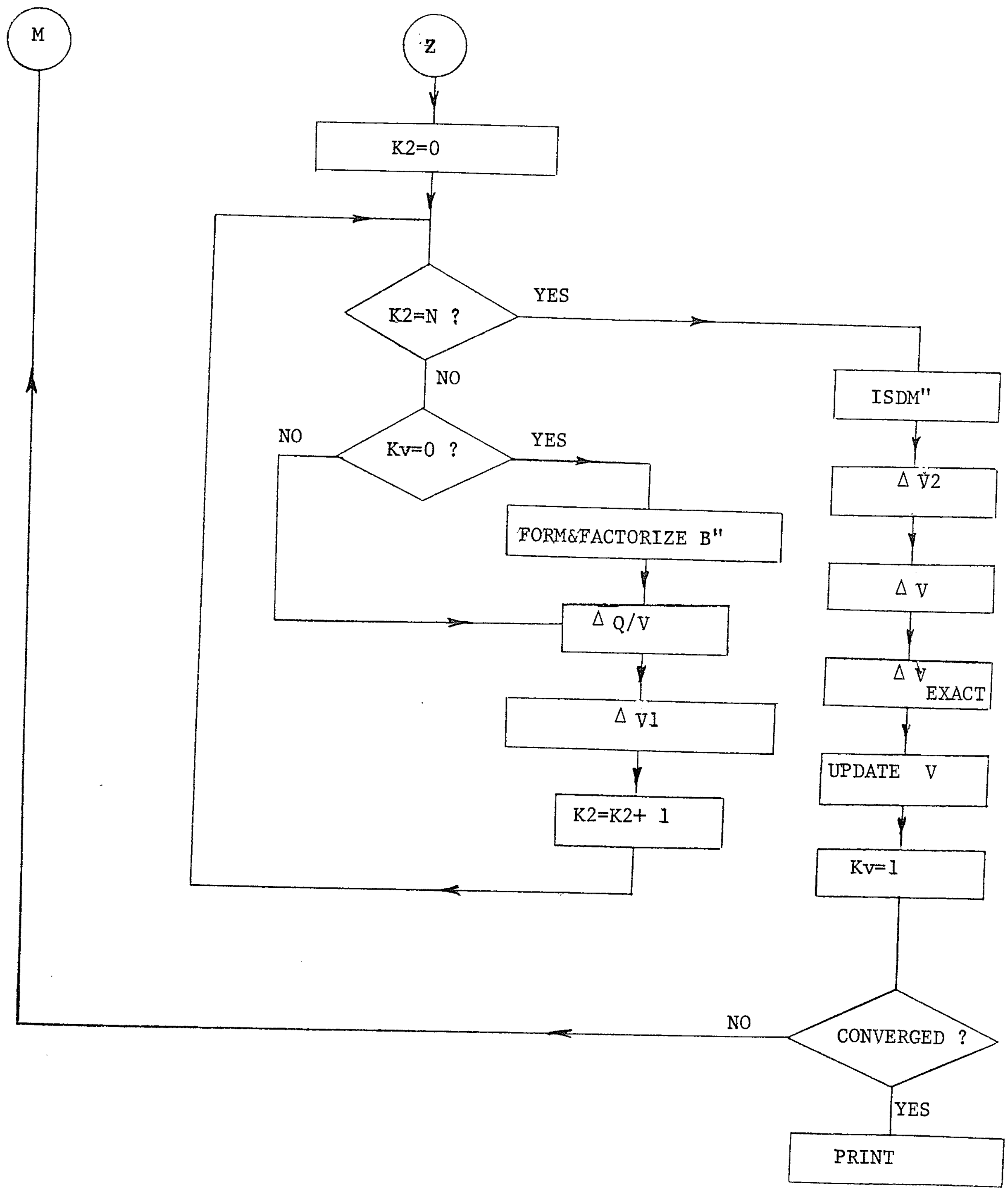


Fig. E.3 Simplified flowchart of diakoptical fast-decoupled load-flow.

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