THE UNIVERSITY OF MANITOBA

AN EVALUATION OF THE ACTUARIAL STRUCTURE OF THE CROP INSURANCE PROGRAM IN CANADA

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN

PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

by

Shih-Ping Sun

DEPARTMENT OF AGRICULTURAL ECONOMICS AND

FARM MANAGEMENT

October, 1976

"AN EVALUATION OF THE ACTUARIAL STRUCTURE OF THE CROP INSURANCE PROGRAM IN CANADA"

by

SHIH-PING SUN

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

ABSTRACT

AN EVALUATION OF THE ACTUARIAL STRUCTURE OF THE CROP INSURANCE PROGRAM IN CANADA

by

Shih-Ping Sun

Crop insurance has provided a systematic way of dealing with variable crop yields and unpredictable crop losses for those farmers who have purchased crop insurance. However, the insured farmer has to pay a premium for the entitlement of indemnity in the event of crop loss. The institution which underwrites crop insurance has to charge premiums in order to recoup the cost of operating the scheme. Although crop insurance programs in Canada and most other countries are subsidized by governments, these programs generally proceed with the goal that they can be carried out under the basis of self-sustaining growth over the long-run. Fulfillment of these provisions necessitates an accurate premium rate schedule, not only to ensure the financial viability of the crop insurance program but also to satisfy the demand for the program from the insureds' standpoint.

The basic purpose of this study, therefore, is to develop an actuarial structure for establishing a premium rate schedule for the operation of a crop insurance scheme. More specific objectives of this study are:

(1) to explore the nature of the demand for and the supply of crop insurance with particular reference to the need for an accurately estimated insurance premium; (2) to establish the theoretical base for setting a crop insurance premium as well as to evaluate the actuarial structures of current crop insurance programs by examining their statistical properties and problems, and their economic foundations and implications; (3) to recommend a model for estimating the insurance premium and to establish an experience rating system for the crop insurance scheme.

The crop insurance scheme is an actuarially based program. In principle, it is supposed to be self-sustaining, i.e., pure premiums are calculated in such a way that total premiums collected should equal total indemnities paid over a long-run period. Generally, the appropriate data required for estimating the insurance premium are the actual yield data collected on the individual farm. Instead, aggregate averages for yield data are used. It is further assumed that the crop yield data are normally distributed.

The results of this study indicate that the distribution of crop yields may be of any kind; a normal distribution may occur as a special case. What seems to be needed is a general estimation method which has the capability of taking into account any case that may arise in practice. Pearson distributions, which permit an estimate of the probability distribution without specifying the algebraic form of the distribution prior to the estimation process, are used in this study to meet the need. It has been found that the estimated Pearson distribution can represent the observed distribution of crop yields better than a normal distribution. This result implies that using an estimated Pearson distribution the premium rate for crop insurance could be more accurately determined than by using a normal distribution.

In most of the currently operating crop insurance programs, the premium rate is charged on an area basis. There are deficiences which may result from using a single class rate for insurance of farmers who actually have different risks in connection with their crop productions. An experience rating system is developed in this study to reallocate the indemnity costs of the program among the insured farmers according to individual farmers' performance. The experience rating system considers not only the frequencies of the indemnity claims but also the levels of actual realized yields on the individual farms. It offers a discount or charges a penalty as well as adjusts the insurance coverage. Furthermore, determination of the experience rating table is based upon proper revision of the distribution in the light of additional data about the farmer(s) in question.

A number of policy implications due to the results are noted. The most significant implication is that crop insurance increases the farmer's ability to withstand unfavorable economic outcomes resulting from severe reduction in crop yield, thus allowing the farmer to make better plans for his farm business without the threat of bankrupting the farm unit due to large loss. However, crop insurance enables the insured farmer to gain the maximum level of return for a given resource input only when the premium rate is equal to expected indemnity for the individual farmer. Another implication of the findings is useful for farm planning; that is, a decision for maximizing profit and minimizing risk must be based not only on expected yields and variance but also skewness and kurtosis of the yields as well. A production decision based only on the first two measures of the risk concerned is inadequate. Farmers cannot depend on the long-run average yield occurring each year, they must take account of the uncertainty they face not only in terms of variance, but also in terms of skewness and peakness in the yield distribution.

ACKNOWLEDGEMENTS

I am grateful to many organizations and individuals who have supported my graduate study and contributed to the completion of this thesis. In particular, I would like to express my appreciation to the following:

I am deeply grateful to my advisor, Dr. R. E. Capel, for his help in the organization and the writing of this thesis. Without his guidance, constant encouragement, and help with patience the completion of the study would not have been possible.

I am also grateful to Dr. M. H. Yeh for his guidance and encouragement especially when the study was under his supervision before he was on leave in 1972.

Special thanks are due to the thesis committee members, Dr. L. R. Rigaux and Dr. G. I. Paul, for their important suggestions towards improvement. I am deeply indebted to them. Thanks are also due to Dr. S. N. Kulshreshthra, University of Saskatchewan, for his careful reading of the thesis and valuable suggestions.

I also wish to think the Canada Department of Agriculture and the University of Manitoba for their financial support for my graduate study.

Finally, I would like to extend my sincere appreciation to my wife, Fei-wei, for her patience and understanding and my parents who sacrificed much to finance my education in Taiwan.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

RATIONALE OF THE STUDY

Agriculture faces a two-fold uncertainty; one arising from price-instability and the other arising out of yield uncertainty. Widespread crop losses generally create severe hardships not only for farmers but for the entire rural community and the whole economy. Yearly fluctuations in the yield of the major grain crops are the chief source of many uncertainties in farm planning. This affects investment practices, particularly decisions concerning land, machinery, fertilizer use, and selection of enterprise combinations. Uncertainty breeds inefficient use of resources. It prevents farmers from obtaining maximum product from given resources and inhibits production at minimum cost for a given level of output. This study therefore focuses attention on the consequences of yield uncertainty. Particularly, it centered on some quantitative aspects of crop insurance schemes.

I.1. Crop Insurance and the Canadian Program

Crop insurance is considered as a precautionary measure which can be used by farmers to stabilize their income against crop failure which is due to adverse weather, or to related physical crop conditions beyond their control. Basically, there are three categories of crop insurance; namely, all-risk crop insurance, area yield insurance, and weather crop insurance. The three categories can be distinguished by the methods used for calculating premiums and indemnities. In this study crop insurance refers to all-risk crop insurance.

For area yield insurance, premiums and indemnities are based upon the yield attained from an area. Indemnities are paid to any insured farmer in any year in which the average area yield for the year falls below a specified level (the level may be defined as some percentage of the long-run average yield for the area). Premium rates are based upon the estimated amount of indemnities payable under each of the indemnity options. The program executed under the Prairie Farm Assistance Act is quite similar to area yield insurance. "The legislation provides for a levy of one percent on all grain marketed through the Canadian Wheat Board to help cover indemnities paid to farmers under the program, ... The basis for payment is the average yield of wheat in a given block or township."[1]

For weather crop insurance, premiums and indemnities are based upon weather records of the locality in which

---

[1]Federal Task Force on Agriculture, Canadian Agriculture in Seventies (Ottawa: Queen's Printer, 1970), p. 390.

insurance is sold. Indemnities are paid to any insured farmer in any year in which the weather, in terms of some measurable criteria, is beyond certain limits of tolerance. Factors such as rainfall, temperature, evaporation, and hail are generally included in the formula which is used to determine when the indemnity should be paid.[2] Premium rates are based upon the estimated amount of indemnities payable under each of the indemnity options.

For all-risk crop insurance a base-yield is established for each farm--such as the average yield over 25 years for a well defined area.[3] Insurance is offered to cover some specified percentage of the base-yield. In Canada, as the Crop Insurance Act states: "the amount of the insurance to be effected on any crop in any area or any farm in any area, shall not exceed 80 percent of the average yield of the crop in the area or on the farm whichever is the greater."[4] If the actual yield in any year falls below the insured yield, an indemnity which is equal to the difference between

---

[2] H.G. Halcrow, "Actuarial Structures for Crop Insurance," Journal of Farm Economics, 31:422, 1949.

[3] Manitoba Crop Insurance Corporation, Tenth Annual Report (Winnipeg: The Corporation, 1969), p. 4.

[4] Economics Branch, Canada Department of Agriculture, Federal Agricultural Legislation, Canada 1969, Publication No. 69/19 (Ottawa: The Department, 1969), p. 24.

the insured yield and the actual yield is payable to the insured farmer. Premium rates are based on probabilities of crop yields falling below the insured levels.

Attempts to provide crop insurance as a systematic protection against the uncertainty of crop yields for farmers have been made in Canada since the early 1920's. A private company was encouraged to enter the crop insurance business in Western Canada during the 1920's but it withdrew from the field after a short lived and costly experience.[5] This failure induced several feasibility studies mainly done by governments, on the subject of all-risk crop insurance.[6] Every study recommended provision of crop insurance to all Canadian farmers. As a result, the Canadian Parliament passed the Crop Insurance Act on July 18, 1959. This Act enables the Minister of Agriculture to enter into agreements with the provinces, whereby the federal government contributes 50 percent of the administrative costs and 25 percent of the premium incurred in the operation of crop insurance schemes.[7] In addition,

---

[5]Federal Task Force on Agriculture, op. cit., p. 389.

[6]Manitoba Crop Insurance Committee, <u>Crop Insurance in Manitoba</u> (Winnipeg: Economic Survey Board, Province of Manitoba, 1940), and Royal Commission on Agriculture and Rural Life, Province of Saskatchewan, <u>Crop Insurance</u> (Regina: Queen's Printer, 1956).

[7]Economics Branch, Canada Department of Agriculture, op. cit., p. 24.

there was provision for federal loans to the provinces in years when indemnities exceeded the reserves available for paying claims. In 1964, an amendment to the Act provided that, as an alternative to loans, the federal government might reinsure part of the provincial risk involved in an approved crop insurance plan. Since 1960, eight provinces have provided crop insurance plans for their farmers; they are Manitoba (1960), Saskatchewan (1961), Prince Edward Island (1962), Alberta (1965), Ontario (1966), British Columbia (1967), Quebec (1968), and Nova Scotia (1969). There were 89 crop insurance plans available to farmers and 46,326 farmers had purchased crop insurance in 1971.[8] Crop insurance administrators continually strive to improve the terms and conditions of crop insurance contracts to meet the needs of farmers in various areas.

---

[8]A detailed list of the crop insurance plans available in Canada may be found in Canada Department of Agriculture, Annual Report of the Minister under the Crop Insurance Act, 1971/72 (Ottawa: The Department, 1972). The number of insured farmers reached 53,403 in 1970. According to the annual report, the main decreases in participation were in the provinces of Quebec, Alberta, and Saskatchewan. The Wheat Inventory Reduction Program instituted in 1970 and carried through to 1971 with the introduction of the Grassland Incentive Program may account for decreases in participation in Alberta and Saskatchewan where wheat is the main crop. Participation was down in Quebec because of administrative difficulties, in particular the delay in paying indemnities, which caused many farmers to misunderstand and reject the program. However, the farmers participating in 1971 only constitute about 18 to 24 percent of the potential users of the crop insurance program.

Crop insurance has provided a systematic way of dealing with variable crop yields and unpredictable crop losses for those farmers who have purchased crop insurance; it permits the insured farmer to protect himself against serious financial loss due to natural hazards over which he has little or no control. Furthermore, crop insurance reduces the impact of widespread crop losses on the rural community. To this extent, most business associated with agriculture would benefit from the program.

I.2. Problems and Objectives

Crop insurance is not a free good. The insured farmer has to pay a premium for the entitlement of indemnity in the event of crop loss. The institution which underwrites crop insurance has to charge premiums in order to recoup the cost of operating the scheme and to make profits. In other words, insurance premiums play a very important role in the market for crop insurance. An insurance premium is a "price" which, when given the other relevant demand and supply forces, may or may not equalize the quantity of insurance demanded and the quantity supplied. Although crop insurance programs in Canada and most other countries are subsidized by governments, these programs generally proceed with the expectation that they can be carried out under the basis of self-sustaining growth over a period of time.[9] Fulfillment of these provisions necessitates a sound statistical base for estimating the premium rate, not only to ensure the financial viability of the crop insurance program but also to satisfy the demand for the program from the insureds' standpoint.

I.2.1. Premium Determination and Present Premises

The crop insurance program is an actuarially based program. In principle, it is supposed to be self-sustaining. The premiums paid by farmers should bear a close relationship to the risk involved and the level of coverage selected. Pure premiums are calculated for each homogeneous group as well as for each crop in such a way that total premiums collected should equal total indemnities paid over a long-run period.[10] The foundation on which a sound crop insurance program rests is explained in the subsequent discussion - one of the basic issues is the determination of a pure premium rate schedule. The statistical base for determining

[9] Manitoba Crop Insurance Corporation, op. cit., p. 1, and The Crop Insurance Commission of Ontario, Third Annual Report, Fiscal Year Ended March 31, 1969 (Toronto: Ontario Department of Agriculture and Food, 1969), p. 4.

[10] Pure premium, unlike gross premium, excludes other costs incurred in the operation of a crop insurance program. It is a matter of policy, which is not examined in this study, whether the other costs should be paid out of gross premiums or be paid by governments. The latter is assumed but this assumption does not appear as a necessary condition in the analysis.

such a schedule is described below with the following notations being used in the statistical specification:

P = annual pure premium rate (bushels/acre),

c = insured yield (bushels/acre),

x = crop yield (bushels/acre), a random variable,

and $f(x)$ = probability density function of crop yields.

Theoretically, the annual pure premium rate per acre to be paid by farmers should be equal to the expected annual loss cost per acre, i.e., $E(\ c - x;\ x < c\ )$, to the insurance agency. For each particular acre of land, a loss cost occurs when the actual yield falls below the level of protection provided or insured yield, i.e., $x < c$. The formula for calculating the premium rate is

$$(1) \qquad P = E(\ c - x;\ x < c\ ) = \int_0^c (\ c - x\ )\ f(x)\ dx.$$

The level of insured yield is usually calculated as a percentage of the average yield specified by the insurance agency or by government policy decisions.[11] Equation (1) implies that the premium rate can be determined

---

[11] In 1966, the Crop Insurance Act was amended to allow more flexibility in the provisions of crop insurance plans. One of the amendments is to raise the limit of coverage from 60 percent of the average yield to 80 percent. Canada Department of Agriculture, Annual Report, op. cit., p. 2.

if the probability density function of crop yields, $f(x)$, is known. However, the actual probability distribution of crop yields is unknown. The crucial factor determining the pure premium rate is how to estimate the probability distribution of crop yields. Generally, the insurance contract is made with the individual farmer. Consequently, the appropriate data required for estimating the distribution of crop yields is the actual yield data collected on the individual farm. However, a series of yield data for each individual farm is not available. Instead, aggregate (district or area) averages for yield data are used. This assumes that different farms in a homogeneous region, usually classified in accordance with soil fertility, topography, and the history of crop production, have not only the same productivity but also the same yield variability as the average for crop yields for an area over a period of years. It is further assumed that the crop yield data are normally distributed. These simplifications are currently employed by most crop insurance programs for the purpose of determining the premium rate schedule.

I.2.2. Problems

It may be noted that the characteristics of a farm unit, the nature of its organization, and the quality of the farmer's management ability affect not only the level of expected yield but also the variability of crop yields. Farmers within a homogeneous area may not have the same probability distribution of crop yields. Further, the actual probability distribution of crop yields can be of any type; the normal distribution may occur only as a special case. Problems caused by the simplifying assumptions used in determining the premium rate are briefly discussed below.

(a) normal distribution: Suppose g(x) is the estimated normal distribution of crop yields, then P_n, where

$$(2) \qquad P_n = \int_0^c (c - x) \, g(x) \, dx$$

is the premium rate according to the estimated normal distribution. If P_n is greater than P, as defined earlier in equation (1), the farmers would find that the current premium rate is too costly for them to subscribe to crop insurance. Conversely, if the premium rate according to the normal distribution, i.e., P_n, is less than the premium rate according to the actual yield distribution, i.e., P, then the crop insurance authority will find that the program cannot balance the premiums received and the indemnities paid.

(b) heterogeneous distribution: On the other hand, the probability distributions of crop yields are unlikely to be the same for all members of a group of farmers in an area. The premium rate calculated using the same probability distribution of crop yields for the group may result in a greater or a lesser premium (for an individual farmer) than his expected indemnity. In the case of the premium being relatively higher than the expected indemnity, the farmer will naturally not subscribe to insurance for protection. Consequently, the crop insurance program will fail to attract a large majority of farmers, especially the low-risk farmers who make less indemnity claims. Only farmers with better than average expectations of collecting indemnity would be attracted to such a program. As a result, the administrators of the program would soon find that farmers who had frequently claimed indemnities would stay in the program while other farmers who had never claimed any indemnity would gradually drop out. The crop insurance program would have problems due to lack of participants and/or inability to become financially self-supporting.

(c) aggregate distribution: In determining premium rates, problems may also arise concerning the use of average crop yields for an area. For example, if half the farmers in the rating area consistently get below-average yields and the other half get above-average yields, then an insurance program using an aggregate distribution for calculating the insured yield will be relatively more attractive to those with lower yields, but not to others. Such a program is inherently destined to reach only half of the potential market. On the other hand, farmers are exposed to a far greater uncertainty than average yields for an area would imply. Therefore, if the variance in the area average is used as an estimate of the variance in yield on an individual farm, the variability in yields that will be experienced on each farm will be underestimated. If the variability of crop yield is underestimated, the premium rate will be underestimated. The administrators of the program will then find that the indemnities paid cannot balance the premiums received. The inability to become a financially self-sustaining program in the long-run is obviously implied.

From an administrator's point of view, it may be argued that a biased premium rate can be simply adjusted upward or downward according to actual experience. It could also be argued that there has been a lack of theoretical foundation for determining the size of adjustment which could result in frequent adjustments being made to the premium rate, suggesting perhaps that the insurance scheme was structurally faulty or chaotic. Farmers might lose confidence in the scheme and hesitate to subscribe to insurance for protection, and the administrator might have difficulty in managing the scheme.

From an economic point of view, an actuarially unsound crop insurance program may also have adverse effects on the pattern and efficiency of resource use, i.e., a pronounced change may take place in plans for the farm. The effects on resource use may depend upon the relationship between the farmer's expected indemnity and the required premium payment. In the case of the expected indemnity being higher than the premium payment, insured farmers may restrict resource use to the level necessary to qualify for insurance which he would otherwise handle differently. On the other hand, since the expected loss is larger than the premium, the cost of uncertainty inherent in the yield variability is substituted for a lesser amount of regular premium payment. Therefore, the farmer would have a net gain through subscribing to crop insurance. This may be viewed as a downward shift in the expected average cost curve. Crop insurance would then become a program which subsidized farmers operating on uneconomic units. It may also be criticized for inducing inefficiency of resource use. In the case of the expected indemnity being lower than the premium payment, the farmer is better off choosing to stay out of the insurance. Under such a premium indemnity schedule, the use of crop insurance for precaution against uncertainty becomes a real cost to the farm. The additional cost, equal to the difference between the expected indemnity and the premium payment, cause a rise in the average cost curve. Although the shift in the average cost curve does not affect the optimum input combinations for producing a given output, it will have effects on the profitability of producing that product. A different pattern of resource allocation compared to that used without crop insurance may have to be introduced. For example, farmers may shift his resources towards uninsured activities, e.g., livestock.

I.2.3. Objectives

In recent years, a number of research efforts have been devoted to the subject of crop insurance. However, those projects have approached crop insurance from either an institutional or a farmer's viewpoint. They have attempted to suggest changes in the crop insurance program to provide an adequate coverage or to introduce the best insurance strategies for farmers.[12] However, very little has been done on examining the actuarial structure of a crop insurance program toward establishing a sound insurance premium indemnity schedule. A study from this approach would assist the insurance

---

[12]Great Plains Agricultural Council, Crop Insurance in the Great Plains, Montana Agricultural Experiment Station, Bulletin 617 (Bozeman: Montana State University, 1967), and G.A. Smith, E.L. LaDue, and R.S. Smith, Crop Insurance Strategies in High Risk Production Areas, A.E. Res. 72-11 (Ithaca: Department of Agricultural Economics, Cornell University, 1972).

agency to achieve the goal of being self-sustaining over the long-run, as well as helping farmers and the economy as a whole to attain the most efficient use of resources. The present study builds on the preceding criticisms of a hypothetical actuarially unsound crop insurance program by addressing itself to the following questions:

> Is there a demand for crop insurance? What are the motivations or behavior reactions leading farmers to participate in a crop insurance program? Is there any economic advantage in favor of crop insurance? Under what conditions can the economic advantage be realized? What is the nature of the supply of crop insurance? Is the supply restricted by financial viability? What is the theoretical basis supporting crop insurance as a self-sustaining program?
>
> What is the theory suggesting that crop insurance premium determination should be based upon the actual distribution of crop yields of each individual farm? Do crop yields obey a specific distribution? Is it possible to estimate the actual distribution instead of assuming that crop yields are normally distributed?

Under the present situation where a series of years of crop yield data of each individual farm are not available, is there a consistent approach that may be adopted for adjusting the premium based upon an aggregate yield distribution (estimated from district or area average data) to the premium based upon the farmer's individual yield distribution?

I.3. Hypotheses

Three hypotheses emerging from the previous discussion form the subject matter of the present study. these are stated below:

First, the demand for and the supply of crop insurance do exist. Insurance premium plays an important role in the market mechanism of crop insurance affecting buyers' motivation to subscribe to insurance as well as supplier's financial viability of operating crop insurance. Although crop insurance programs in most countries are operated and subsidized by governments, an unbiased premium is still crucial in terms of efficient use of resources and benefits to the economy.

Second, an actuarially sound crop insurance program is one in which premium rate determination is based upon the probability distribution of crop yields. Since the true probability distribution of crop yields is unknown, the alternative is a better estimated distribution of crop yields. Under this condition, the crop insurance program administrators are able to assess the actual loss probability of the reduction in crop yield. Therefore, the premium rate can be more accurately estimated than by any other method.

Third, aggregate yield data (compiled from a district or a risk area), currently used for determining the insurance premium indemnity schedule, are not appropriate for estimating the crop yield distribution. Farmers in a homogeneous region do not necessary conform to the same crop yield distribution. More specifically, the influence of individual factors, such as the farm manager's ability and performance on the yield distribution cannot be neglected. Under the present situation, yield data collected from an individual farm are not adequate for estimating an individual yield distribution. An aggregate distribution which is estimated from the area average data has to be used in calculating the premium rate in the initial stage of an insurance program. However, subject to the qualifications mentioned earlier, an experience rating system, which adjusts the premium indemnity schedule actually effected on the individual insured farmer, should be established.

I.4. Assumptions

The present study is based upon the following assumptions:

(a) A farmer's production response involves the utility choice between the degree of uncertainty and the level of expected return.

(b) The crop yield distribution can be estimated statistically.

The implications of these assumptions are discussed below.

Crop production is subject to a variation in yield. Given an individual's psychological makeup and capital position, the optimum production plan depends not only upon the level of expected return but also upon the likely deviation from the expectation. The fear of suffering liquidation of the farm unit due to large losses induces an intermediate end of striving for security in the short-run planning perspective. The introduction of a production procedure with higher expected returns involves trading off the higher expected return against extra uncertainty. Therefore, individual farmers must choose among alternatives between the level of expected return and the degree of uncertainty, and the indifference curves slope upward as shown in Figure 1. Insurance changes the future uncertainty of low yields into risk. Variability phenomena can be incorporated into the cost. Therefore, farmers are able to use their limited resources more efficiently, whereas under uncertainty they would obtain either a less-than-maximum return or a greater-than-minimum cost for given resource inputs. The demand for crop insurance may be derived from a need to guarantee a minimum level of return from crop production and/or to increase the farmer's ability to withstand unfavorable economic conditions.

The second assumption implies that the variability in crop yields is one which has some uniformity of behavior, so that it is possible to measure and estimate the probability of loss in the future. Therefore, it becomes feasible to supply crop insurance. However, such uniformity of behavior can only be established when sufficient data have been collected concerning the uncertainty in question.

I.5. Organization of the Study

This study is organized as follows.

Chapter II focuses attention on the economic

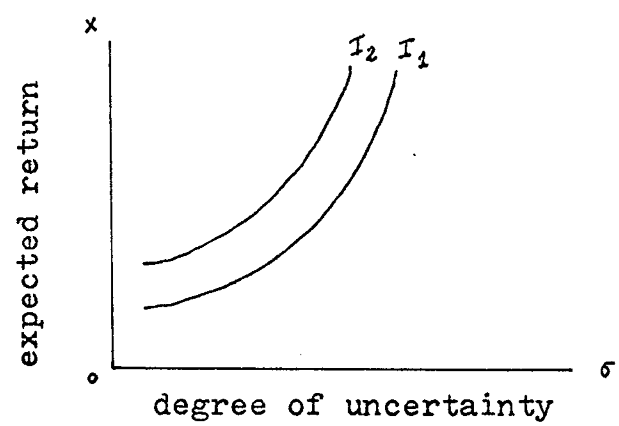


Figure 1. Indifference Curves Indicating Substitution between Expected Return and Uncertainty.

aspects of a crop insurance program which considers the premium rate to be the most important factor affecting both the supply of and the demand for the program. After discussing the risk and uncertainty phenomena in crop yields, the applicability of the theory of risk to a crop insurance program is examined. This chapter is further used to establish the actuarial requirement for a financially self-sustaining crop insurance program with a comment on the current government subsidy. It also establishes the motivations for farmers to participate in the program, i.e., why there is a demand for crop insurance. Finally, it elaborates on the necessity of having an accurate estimate of the insurance premium and the economic implications of a biased premium on resource allocation.

Chapter III examines the statistical base for a crop insurance program. First, it establishes the theory suggesting the use of crop yield distribution for crop insurance premium rate determination and explains the supposition of the study, i.e., crop yield distribution is not restricted to a specific type. This implies that for the purpose of examining an unknown distribution, a general estimation method is needed to take account of any case that may arise in practice. This chapter further provides a review of the existing statistical bases of various crop insurance programs. The statistical properties, problems, economic foundations, and implications of these statistical bases are examined. Finally, Pearson distributions which permit an estimate of the probabilty distribution without specifying the algebraic form of the distribution prior to the estimation process are introduced. The implications of using Pearson distributions in crop insurance premium rate determination are also discussed in this chapter.

In Chapter IV, Pearson distributions are applied to wheat yield data of 14 crop districts of the Province of Manitoba to show that crop yield distribution can be of any type. The estimated Pearson distributions are compared with the estimated normal distributions, and are tested statistically as well. Finally, the differences in premium rates based on the estimated Pearson distribution and the normal distribution are examined.

Chapter V stresses the individual approach in the development of crop insurance program. It deals with the influence of individual factors on the crop yield distribution of a particular farm unit. It also illustrates the theoretical background of establishing an experience rating system and the shortcomings of existing methods. An appropriate and consistent approach is suggested in the final section of this chapter.

Suggestions and implications of this study are stated in the final chapter.

Data collected exclusively from the Province of Manitoba are used in the following analyses. However, the methodology used and the implications derived from the analyses have general applicability to crop insurance programs in any country.

CHAPTER II

THE ECONOMIC FOUNDATION OF A CROP INSURANCE PROGRAM

Crop insurance is not a free good. Insured farmers have to pay a premium to be entitled to an indemnity in the event of crop loss. The institution which underwrites crop insurance has to charge a premium in order to recoup the cost of operating the scheme and to make economic profit. In other words, the insurance premium plays a very important role in the crop insurance market. Although crop insurance programs in Canada and most other countries are being operated and subsidized by governments, the inherent economic significance of the insurance premium remains unchanged. Crop insurance programs generally proceed with the expectation that they can attract a large number of farmers to participate in the program and be carried out under the basis of self-sustaining growth in the long-run. In order to fulfill these provisions, it necessitates a sound premium indemnity schedule for operating the crop insurance program. This chapter deals with the economic aspects of a crop insurance program by examining the theoretical foundation of the program and the implications of the insurance premium on the supply of and demand for the program. Finally, it discusses the effects of crop insurance on resource allocation with reference to the need for an accurately estimated insurance premium.

II.1. Supply of Crop Insurance

From the functional standpoint, insurance is a social device whereby the uncertainties of individuals are combined in a group; small periodic premium contributions by the insured provide a fund out of which those who suffer losses may be reimbursed. However, insurance is possible only when the variability of the event concerned is measurable, and when the loss event is one over which the insured has only limited control. Furthermore, an insurance scheme involves mutuality. In the long-run the cost of insurance must be paid out of the premium collected, so that policyholders are themselves financing the cost of their insurance. Crop insurance is an application of the theories of risk and insurance to provide farmers with a form of systematic protection against yield uncertainty. The applicability of those theories to a crop insurance program and their implications on the determination of an insurance premium are discussed below.

II.1.1. Risk, Uncertainty, and Crop Insurance

Following F. Knight,[13] risk and uncertainty

---

[13] F.H. Knight, Risk, Uncertainty and Profit (Boston: Houghton Mifflin Company, 1921), p.233.

are distinguished as two different phenomena. Risk refers to situations where parameters (such as mean, variance, and other higher order moments) of the probability distribution of outcomes are known (either through calculation *a priori* or from statistics of past experience). In other words, the variability of the outcomes can be measured empirically or quantitatively. Uncertainty, on the other hand, refers to those situations in which the parameters of the probability distribution of outcomes cannot be empirically or quantitatively determined.

Time involved in crop production precludes perfect forecasting of yield. Decision-making must take place in an environment of either uncertainty or risk. This arises because the crop farmer must formulate an expectation of the future yield as the basis for his production plan but there is no way by which the expectation can be verified quantitatively in the planning stage. In any given year actual realized yield may be higher or lower than the expected yield; actual yield will seldom be equal to expected. Moreover, the fact that the actual yield is higher or lower than the expected yield in one year has little or no effect on the likely outcome in the following year. Since no essential relationship can be established between subsequent yield observations, the year-to-year fluctuation in crop yields may therefore be considered as randomly distributed over time in the manner of a stochastic variable. Even if the distribution of crop yields is unknown *a priori*, it is possible to estimate the probability distribution empirically by gathering a number of observations. The law of large numbers further ensures that the reliance to be placed upon the estimated distribution is increased when the number of observations increases.[14]

The above discussion suggests that the yield outcome, x, in a single year, which cannot be predicted without error, is an uncertainty phenomenon. However, risk is present when the probability distribution of yields, f(x), is established over a period of years. When the probability distribution of the yield outcomes is known, it is possible to estimate the number of years in which the yield will fall in each yield interval. If crop loss is defined as occurring when the yield falls below a specified level, c, then, given the yield distribution, the annual average loss per acre, x_1, can be estimated as the mathematical expectation of the yield below that level, i.e.,

$$x_1 = E(\, c - x;\; x < c \,) \qquad (3)$$

By cumulating an amount equal to the annual average

[14] W. Feller, *An Introduction to Probability Theory and Its Applications*, Vol. I, 3rd Edition (New York: John Wiley, 1968), pp. 243-263.

loss each year, the farmer should be able to average out the financial disaster resulting from a severe reduction in yield.

However, farmer in most countries lack economic means and resources, and cannot make plans depending upon the mathematical expectation which may only be realized over an extended period. The possibility of bankruptcy of a farm unit due to uncertain crop loss in the earlier establishing stage still exists. One way to get around the problem is to transfer the risk to another party with a strong financial capability of absorbing the ultimate outcome. Insurance is based upon the idea of providing an institution to accept the transfer of risk. By contributing an amount equal to the annual average loss cost to the insurance company, it is possible for an individual farmer to distribute uncertain large crop losses over time.

The transfer of personal risk is one element of insurance. However, the insurance is completed only where many individual risks are combined. An insurance scheme involving two farmers would be little better than each individual taking care of his own risk personally. In other words, it is necessary for the insurance scheme to attract a large number of participants. Under such a scheme, uncertain losses of individuals are combined into a group; small regular premium contributions by the insured population provide a fund out of which those who suffer losses may be reimbursed. The incidence of loss is broad-based through insurance so that even the impacts of the heaviest loss upon an individual insured can be easily absorbed by the insured group as a whole. However, attention must be drawn to the function of insurance in connection with the distribution of losses over the insured group. That the insured in the long-run pay for all the losses is undoubtly true, but the distribution of losses is only an indirect result of the insurance.

The above discussion leads to the following conclusions. The variability of crop yields may be viewed as a risk phenomenon in the long-run, and therefore, crop insurance is possible. Furthermore, it is necessary to the insurance company that a sufficient number of the farmers subscribe to insurance. In order to recoup the cost of operating the program, the insurance agency has to estimate the indemnity claims and charge insurance premiums to the insured farmers. In other words, the supply of crop insurance is tied in with the financial viability of the program. The significance of the insurance premium to the insurance program is discussed below.

II.1.2. Crop Insurance Progran and Insurance Premium

The financial viability of the crop insurance program relies on an actuarially sound premium indemnity schedule rather than simply charging an actuarially sound amount of premium. The necessary condition to meet the actuarial requirement is that the premiums paid by a farmer using the crop insurance should be equal to the indemnities reimbursed to him over a long period of time and all farmers facing similar probabilities for similar indemnities would be assessed similar premiums. In other words, no farmer would be penalized or subsidized consistently through the use of insurance. If this condition cannot be met, adverse selectivity would be widespread; farmers with better than average expectation of collecting indemnity would be the ones most frequently attracted to the program and others would likely remain outside the program. As a result, the financial viability of the crop insurance program would be affected.

It may be argued that the insurance company can simply raise the premium rate. However, the problem is that when the premium rate is not properly set the development of the adverse selection could destroy the whole insurance business. For example, as the premium level rises, the people who buy insurance are those who are increasingly certain that they will need insurance. Thus as the premiums go up, the better risks no longer buy insurance, leaving a higher proportion of poor risks to share the costs of indemnity payments. But with only poor risks in the pool the premiums must rise again.

This further squeezes out the better risks and premium will have to go up again. As the process continues, the final result may be that insurance is not offered at any price.

Since crop insurance is an actuarially based program, there should be no difference whether it is operated by government or a profit oriented commercial company, at least in terms of operational efficiency and risk management.[15] However, current crop insurance plans are consistently being subsidized by governments which inhibits private commercial companies to compete in the crop insurance market. Subsidy to the crop insurance may be justified on the ground that the subsidy, which would bring about the widespread use of crop insurance, might be considerably less than the amount of money which has been spent on feed and seed loans, direct relief payment, disaster payments, etc. It has also been claimed that the income stabilizing effect generated by crop insurance may accrue to segments of the economy other than agriculture. Nevertheless, insurance involving subsidy, especially subsidy extending to the pure premium,

---

[15]If (1) scale economies are possible
(2) externalities to other sectors are present
(3) other government support programmes are substituted for farm income maintenance,

then, there are strong arguments for a government scheme versus a private sector plan, particularly when universal coverage is desirable. The arguments here are analogous to those in the literature on public goods in general.

may have important adverse effects on the efficiency of resource allocation.[16] For example, when wheat insurance is subsidized, it may encourage the growing of wheat on land that is submarginal for wheat production. Therefore, it is preferable that the crop insurance program is operated on a self-sustaining basis in the long-run.

The above discussion implies that an actuarially sound crop insurance program requires the premium rates to be adequate, reasonable, and equitable. It is evident that the premium rate structure of an insurance scheme should provide an income sufficient to meet claims and expenses, as well as a possible profit factor. On the other hand, insurers must establish reasonable rate levels even when the full force of competition is not operative in rate determination, inasmuch as excessive rates may cause farmers to assume or self-insure their risks whenever it is possible. Since the insurance contract is with the individual farmer, the supply of crop insurance should not merely be based upon aggregate gains or losses in any year or over a period of years. Instead, the analysis must distinguish between the losses which may be due to incorrect actuarial procedure, such as the rate structure failing to reflect the actual

[16]The relationship between crop insurance program and resource allocation will be examined in detail in Chapter II.3.

character of the risks of the particular policyholders, and to those losses which may occur from time to time because of adverse crop conditions.

II.2. Demand for Crop Insurance

The insurance premium has been examined as an important element in determining the financial viability of a crop insurance program and thus the supply of the program. It was also pointed out that, in order to operate on a safe and efficient base in terms of distribution of risks, it is necessary for the insurance program to attract a majority of farmers to participate and stay in the program. Farmers who participate in a crop insurance program are required to pay an insurance premium to the insurance agency in exchange for being entitled to receive indemnity in the event of crop loss. However, the motivation for farmers to subscribe to insurance requires further discussion. Crop insurance has been used by farmers in many countries as a precautionary measure against yield uncertainty by guaranteeing a minimum level of return from their investment in crop production. It may also generate higher expected incomes which might not be realized in a short period in the absence of crop insurance. This aspect of a crop insurance program is discussed below.

II.2.1. Crop Insurance as a Precautionary Measure

The variability of crop yields has traditionally been a serious problem in the agricultural industry. Severe crop losses have led to distress on the part of the farmer, and have often prevented the expansion of the farm firm or have caused outright bankruptcy. Moreover, bankruptcy is irreversible. Once the farm firm is liquidated, the manager cannot simply roll the wheel of chance once again to let the probabilities catch up. However, to cope with wide variations in crop yields, there are a number of other strategies besides crop insurance which can be employed by farmers to lessen the severity of yield reduction.

In order to withstand unfavorable economic outcomes in lean years, farmers may create and maintain a financial or credit reserve in good years. In addition to debt-payment rescheduling, farmers may concentrate their major machinery and household improvements in good crop years and curtail such expenditures in the poor years, with less impact on the farm operations and family living. This strategy is only possible for the older or better established farmer and it is referred to as self-insurance or latent insurance. But it is almost impossible for the beginning or low income farmer. On the other hand, maintaining financial reserves can be a relatively uneconomic way of dealing with production risk when a farmer has other efficient uses for liquid assets. Even if a farmer "pays the premium" himself there is a time element involved. He might not be able to accumulate enough "paid premiums" before a severe loss occurs.

Governments have also established alternatives to help farmers to meet losses from natural hazards, such as the reduction of land rent and taxes, emergency loans, and direct payments. These are no doubt of considerable benefit to farmers, however, their main shortcoming is that farmers cannot expect them as a right. Furthermore, their scopes largely depend on the policies and resources of governments. Even if concessions and relief in case of crop failure are so guaranteed by law, their permanency may not only be doubtful but also not strictly desirable; a large expenditure on a farm relief program represents an additional burden on government. Prospects of continuing relief are liable to stigmatize farmers as an economically dependent group and are likely to be questioned by the non-agricultural community.

Diversification may also be employed by farmers as a precaution against uncertainty where the immediate objective is not so much profit maximization as stability of income. The farmer may choose not to specialize in a single product over time, even where substitution ratios and price ratios would so dictate. Instead, plans may be laid for producing several products to minimize the likely deviations of the outcomes. The hope of the diversification is that if the return from one product is low, the return from another may be high. The criticism of diversification is that when it is adopted to meet uncertainty it almost always necessitates a sacrifice; it results either in less-than-maximum product from given resources, or conversely, does not allow mimimum cost for a given output. Furthermore, the extent to which diversification is practiced as a means of reducing production variability is unknown.

Insurance of growing crops is largely free from the above drawbacks. It gives reliable protection against the uncertainty of crop yields in return for a regular premium payment. Further, it enables the insured farmer to improve his credit. Bankers and other credit institutions are willing to provide larger and more adequate loans since their borrowers are expected to have more stable and reliable incomes and can offer more tangible security for such loans. In fact, the Manitoba Agricultural Credit Corporation has accepted subscription to crop insurance as a security for a guaranteed line of credit. Adequate financial assistance will provide farmers with greater possibility of making optimum production plans in farming business.

However, it is necessary here to emphasize that crop insurance cannot improve the basic income position of the farm on an uneconomic unit, it only provides the farmer with a means of reducing income variation and guaranteeing a minimum level of return from his investment in crop production.

Survival is an end in the short-run for some farmers to attain profit maximization in the long-run. Under this circumstance, crop insurance may be chosen as a precaution against potentially disastrous yield reduction which may liquidate the farm unit. On the other hand, crop insurance also increases the farmer's ability to withstand unfavorable economic outcomes resulting from severe reduction in crop yield. As a result, it may allow farmers to make better plans for their farm businesses without the threat of bankrupting the farm unit due to large loss. Therefore, for some farmers crop insurance may generate higher expected incomes which may not be realized in a short period in the absence of crop insurance. The motivation for farmers to subscribe to crop insurance in connection with this aspect is discussed below.

II.2.2. Crop Insurance and Farm Planning

Individuals frequently must, or can, choose among alternatives that differ, among other things, in the degree of uncertainty to which the individual will be subject.[17] In the case of farm planning, the

---

[17] M. Friedman and L.J. Savage, "The Utility Analysis of Choices Involving Risk," Journal of Political Economy, 56:279-304, 1948.

production response also implies choices between the level of expected return and the degree of uncertainty. Production with higher expected return may tend to increase the uncertainty of the expected return that may be realized in the future. The concept of degree of uncertainty is frequently referred to as the dispersion of expected returns. In the statistical context, dispersion may be measured by the variance, standard deviation, or range of expected (or realized) values. Uncertainty threatens the farmer with the possibility of liquidating the farm unit and therefore, for a given level of expected income, an increase in uncertainty reduces the farmer's utility. In other words, individual farmers must and do choose, on a utility basis, among production alternatives considering not only the level of the production return but also the degree of uncertainty to be incurred. The problem confronting the farmer at the production decision-making stage is how to allocate his resources between alternative production strategies; either a higher expected return with more uncertainty or a lower expected return with less uncertainty.

Since uncertainty reduces the individual farmer's utility, the introduction of a production procedure with higher expected return involves trading off higher expected output against extra uncertainty. Consequently, an indifference map can be drawn for each individual farmer and takes the form of I_0, and I_1 as shown in Figure 2.

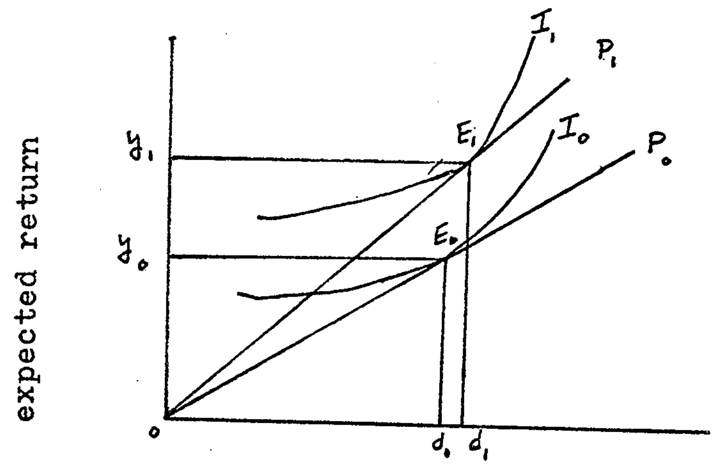


Figure 2. Impact of Crop Insurance Program on the Substitution between Expected Return and Uncertainty.

In the above figure, each indifference curve represents a locus of combinations of expected return and uncertainty between which the individual farmer has the same preference. Each curve slopes upward to the right as a result of the assumption that expected return is good, which will result in an increase in utility, and uncertainty is bad, which will detract some utility. It follows, from this assumption, that if his expected return is increased the individual farmer will be better off unless uncertainty is also increased to hold him at the same level of satisfaction as before. For the same reason, the indifference curves are to be interpreted as reflecting higher levels of satisfaction as one moves upward and and to the left; say from I_0 to I_1 in Figure 2. The curves are convex downward, because it is posited that the higher the expected return generated from production, the less does this higher expected return mean to the individual farmer and hence the smaller the increase in uncertainty he is willing to bear to increase his expected return further. In addition, it may be pointed out that each a family of indifference curves will differ in slope for different individuals depending on their attitudes toward uncertainty and even for the same individual depending on his income or financial position. The slope of the indifference curve represents the individual's marginal rate of substitution between expected return and uncertainty.

The individual farmer's choice among production alternatives is constrained by his production opportunity curve. This shows the combinations of expected return and uncertainty the farmer may actually choose from using the resources which he controls. The line op_0 in Figure 2 is the production opportunity curve. It slopes upward and to the right, implying that as the level of expected return increases, the possible loss or uncertainty becomes greater.[18] If the production

---

[18]The point has been emphasized in J.R. Hicks, Value and Capital (London: Oxford University Press, 1948), p. 200. To simplify the illustration a linear production opportunity curve is used. However, The following analysis will not be affected when a non-linear relationship is postulated.

opportunity curve considers only returns from investment within the farm unit, the curve will cross the origin, i.e., if the farmer produces nothing the prospective return is zero and the chance of loss, or uncertainty, is zero.

Equilibrium is therefore established when the indifference curve becomes tangent to the production opportunity curve indicating that maximum utility has been attained. Here the marginal rate of substitution between expected return and uncertainty on the farmer's preference map is equal to the marginal rate of substitution between expected return and uncertainty on his production opportunity curve. As shown in Figure 2, the optimum choice is denoted by an investment yielding an expected return of oy_0 and a dispersion of outcome of od_0. The equilibrium or stability condition can be interpreted this way: the farmer may refuse to go beyond E_0 because (1) increasingly greater amounts of prospective return are necessary (on the indifference curve) to make him willing to undertake greater uncertainty while (2) the most probable expected return (on the production opportunity curve) cannot increase at a rate great enough to compensate for the greater uncertainty. The farmer is, therefore, unwilling to select possibilities beyond E_0. Maximum utility is denoted since higher indifference curves could not be attained with the opportunities available while any combination on the opportunity curve above or below E_0 would lie on a lower indifference curve and represent a smaller total utility.

However, when the farmer considers subscribing to crop insurance in his production plan, the optimal decision will no longer remain at E_0 in Figure 2. Crop insurance has the effect of reducing the degree of uncertainty associated with crop production by guaranteeing a minimum level of return equal to the insured yield. The reduction in the degree of uncertainty is realized from the fact that when actual yield falls below insured yield the farmer will be reimbursed for the deviation. Since pure premium is calculated in such a way that the total premiums paid equal the indemnities received in the long-run, subscribing to crop insurance will not alter expected returns. Decreasing the degree of uncertainty for all levels of expected return results in a steeper production opportunity curve. The new production opportunity curve is tangent to a higher indifference curve. Instead of being in equilibrium at E_0, the farmer is now at E_1, which in Figure 2 is to the right and above E_0. Thus he is expecting more return and bearing more uncertainty if the uncertainty is measured under the old context. More important, a higher degree of satisfaction can be attained by the insured farmer.

However, the amount of expected income associated with the higher utility does not have to be associated with a higher degree of uncertainty as shown in Figure 2. It is quite possible to draw the indifference curve so that with crop insurance the insured farmer is in equilibrium at a combination with the same or less uncertainty than before; these possibilities are shown in Figure 3. The result derived from the analysis is an empirical matter rather than one of theory, i.e., depending on the nature of the indifference curves posited. It is no more than a case of the substitution effect and the income effect (or wealth effect). With crop insurance, the substitution effect leads to a higher degree of uncertainty being chosen when the ability to withstand

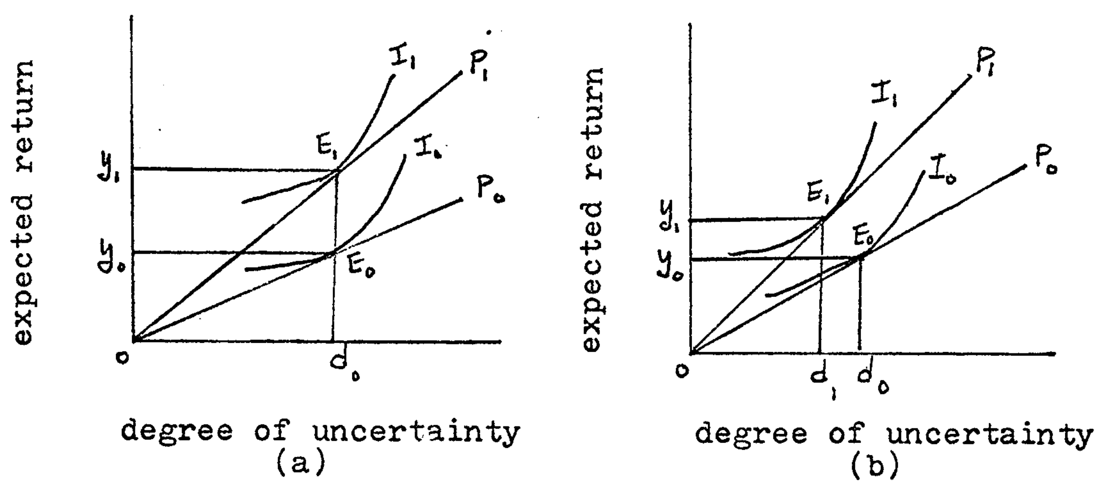


Figure 3. Possible Cases Resulting from Subscribing Crop Insurance (a) Higher Expected Return at the Same Degree of Uncertainty; (b) Higher Expected Return at a Lower Degree of Uncertainty

unfavorable economic conditions increases, but the income effect could go either way. So long as the insured farmer has been satisfied with the level of expected income, the income effect may work in a different direction compared to the substitution effect.

The above discussion suggests that the motivation for farmers to participate in the crop insurance program is more complex than that is implied by describing it as a precautionary measure against yield uncertainty. By examining the hypothetical individual's production opportunity curve and indifference map, the results indicate that crop insurance may also generate a higher level of satisfication, one which might not be realized in the absence of crop insurance. The higher level of satisfication is attainable by either increasing the individual farmer's ability to withstand yield uncertainty and/or increasing the level of expected return from farm production.

II.3. Crop Insurance and Resource Allocation

The use of crop insurance may have various effects on the pattern and efficiency of resource allocation depending on the relationship between the required insurance premium and the farmer's expected indemnity. Three distinct types of situations may be postulated and explained below.

Plans in crop production must be made at one point in time for a product which will be forthcoming at a future point in time. Farmers must therefore form expectations, taking account not only the most probable value of the future outcome but also of the likely deviation of the actual outcome from the expected outcome. A great majority of farmers in most countries, due to insufficient economic means and resources, are not able to absorb large losses. The fear of suffering liquidation of the farm unit inhibits the realization of profits over an extended period. In attempting to obtain a continuous stable income so as to survive in business in the short-run, a farmer may have to take precautions that result in a less-than-maximum product from given resources or, conversely, do not allow a minimum cost for a given output, even though substitution ratios and price ratios would so dictate. Instead, plans which imply inefficient resource use are laid for product(s) which result in a minimum variation in farm income over time.

Under a crop insurance scheme, the chance of an uncertain loss is transferred to the insurance agency. The insured farmer is free from the threat of farm liquidation due to a large uncertain loss. Furthermore, the individual farmer can substitute small regular premium payments for irregular losses; a fixed cost, the premium, will be substitute for an uncertain cost, the cost of yield variability. The level of fixed cost generally has no effect on the optimum decision in the short-run planning stage.[19] In the long-run, the premium payments are expected to be balanced with the indemnities received; the total average cost curve should remain the same as the one without considering insurance. The farmer will be able to gain the maximum level of income for a given resource outlay according to the equation of substitution ratios and to come closer to the point of optimum resource allocation in a more consistent manner than he could achieve without insurance. However, the ideal situation holds only when the premium payments are balanced by the indemnities for the individual farmer.

In the case of the expected indemnity being higher than the premium payment, the farmer may restrict resource use to the level necessary to qualify for insurance which he would otherwise handle differently. Furthermore, since the expected indemnity is higher than the premium payment, a lesser amount of regular premium payment the cost of uncertainty inherent in the yield variability is substituted for. Therefore, the

---

[19]The level of fixed cost has significance for short-run optimization in one special case, i.e., when there is a loss equal to the farmer's fixed cost, and then the optimum decision is to discontinue production. J.M. Henderson and R.E. Quandt, <u>Microeconomic Theory</u> (New York: McGraw-Hill Book Co., 1958), p. 57.

insured farmer has a net gain through subscribing to crop insurance. This may be condered as a downward shift in the expected average cost curve. If this becomes the reason for farmers to operate on uneconomic units, the crop insurance program will be criticized for inducing inefficient use of resources.

In the case of the expected indemnity being lower than the premium payment, the farmer is better off choosing not to purchase insurance. Under such a premium indemnity schedule, the use of crop insurance becomes a real cost to the farm. The additional cost, equal to the difference between the expected indemnity and the premium payment, causes a rise in the average cost curve. Although the shift in the average cost does not affect the optimum input combination for producing a given output, it will affect the profitability of producing that product. A different pattern of resource allocation compared to that used without crop insurance may be introduced.

The economic significance of crop insurance on resource allocation can also be discussed in terms of risk management. Men differ much in their ability to judge risks. Division of labour in estimating risks can mean that those who are especially trained to it will be the ones who will undertake it. Moreover, their ability will be further developed through experience and training gained on the job. The possibility of transferring the uncertainty of crop production to the insurance agency for a fixed premium frees the farmer from the paralyzing influence of uncertainty and enables him to make the best of use his resources in the direction of the business of farming. The gain to the society from crop insurance is obtained partly through reduction in the cost of carrying risks when they are borne by those who have the most ability to estimate risks and partly through the increase in efficiency of those who are abnormally sensitive to the influence of uncertainty.

A premium rate in the insurance circle is the price an insurer charges for each unit of a risk that is transferred to it. From the standpoint of the insured, a rate may be viewed as the price paid for a unit of protection. In other words, the premium rate is one of the most important factors affecting both the demand for and the supply of the crop insurance program. An accurately estimated premium rate is also important with respect to the efficient utilization and allocation of resources.

The problem of premium rate determination is complicated in all lines of insurance by the fact that most insurance rates apply to a period of time subsequent to the time they were calculated. Not only are the proportion of past losses, i.e., the ratio of losses to total exposure, reliably estimated, but adjustments must also be made to account for any changes in conditions that are likely to modify future losses as compared with those of the past. The approach taken in setting rates and premiums may use one of two fundamental philosophies, namely, class rating or individual risk rating. A class rate is obtained in principle by apportioning the anticipated losses for a given class of coverage over all of the exposure units. As mentioned before, there are deficiencies which may result from using a single class rate for insurance of farmers who actually have different risks in connection with their crop productions. It is recommended that an individual risk rating approach be used. However, this is not to say that the rate paid by a policyholder under the individual risk approach will have to be determined solely by the experience of the policyholder. It merely implies that the premium paid for transferring the risk is dependent directly upon the experience of a particular insured. The application of the individual risk method of premium rate determination frequently involves the procedure of setting up a basic initial rate and an experience rating system adjusting the basic initial rate to a more equitable level in accordance with the actual character of the insured.

The statistical base for determining an initial premium rate and its empirical application are presented in the following two chapters. This study will then recommend a procedure for establishing an experience rating system for a crop insurance program.

CHAPTER III

THE STATISTICAL BASE FOR A CROP INSURANCE PROGRAM

Crop insurance programs in Canada are still relatively new and considerable experimentation is going on. Different approaches are being used to determine the level at which the production of a crop should be guaranteed and the premium that needs to be charged for such a guarantee. However, the approaches being used can generally be classified into two categories. One is known as the "actual loss cost technique" which is based upon the simulated cost, i.e., a cost which would have been incurred in the past. This technique was first used by the Federal Crop Insurance Corporation in the United States during the nineteen thirties. The other is a more theoretical one which is based on the normal distribution. In this chapter, these two statistical bases are first reviewed and evaluated. Their statistical properties and problems, and their economic foundations and implications are examined. This chapter will then introduce the basic rationale of Pearson distributions, an estimation procedure, and the criterion for choosing a particular type of distribution. Finally, the use of Pearson distribution is explained and the reasons for preferring Pearson distributions will be presented.

III.1. Distribution of Crop Yields and Crop Insurance

Crop losses due to natural causes have some similarity to property losses or damages due to fire or other accidents. However, there are considerable differences between property and crop insurance in terms of the risk involved. Since the nature of the risk involved in the operation of a crop insurance scheme differs from that in other types of insurance, different approaches must be used to determine insurance premiums, i.e., distribution of outcomes vs. distribution of loss experience.

Insurable hazards, other than crop losses, tend to be randomly distributed over time and space, but more important, they have a lower individual probability of occurance than crop losses due to natural hazards. In other words, among the insured population, an indemnity claim is a rare event. In addition, insurance plans other than all-risk crop insurance only deal with individual risks. Thus, the insured hazards exhibit the statistical behavior of "extreme values",[20] a theory concerned only with the probability of losses, and not with the distribution

---

[20] D. Bostwick, "Probability Analysis of Yield Data, Including the Extreme Value Statistical Distribution," in Management Strategies in Great Plains Farming, H. W. Ottoson (ed.), Great Plains Council Publication No. 19 (Lincoln, Nebraska: University of Nebraska, 1961), p. 55-9.

of events which are not losses. To estimate such rare events, the appropriate base is the distribution of loss experience, especially when the rare events are several standard deviations away from the central mass of observations. It is more feasible, then, to prepare a probability schedule by taking a count of such rare events over time.

Crop insurance is not limited to coverage of rare events. It insures against crop loss from any and all natural causes, and this increases the probability that a loss claim will be made on each insured crop. It is obvious that having some crop loss from at least one natural cause is not a rare event. It is possible for the insurance agency to estimate the loss probabilities of each cause and add them up. However, it is not a simple addition and the task would be formidable. Since crop insurance covers crop losses from all causes, and since indemnity is calculated against the insured yield, the focus should be concentrated on determining an appropriate insured yield and the probability of a claim on assuring the specified yield. In addition, crop insurance may offer several insurance coverages and the insured yield may well extend into the central mass of yield observations approaching the mean. If the guaranteed yield were low, for example, 25 percent of the average yield, a crop loss would be a rare event. The statistical behavior would fall in the category of extreme values. But as the guaranteed yield is raised, the probability of a loss claim rises. Therefore, the statistical behavior shifts out of the category of extreme values into the category of central mass.[21] Furthermore, an indemnity claim is based on the difference between the average yield on the insured farm and the guaranteed yield. If the average yield on the farm is equal to the guaranteed yield, then half the acre yields may be expected to fall below the guaranteed yield and to have the potential of making a loss claim. Consequently, the distribution of outcomes becomes the appropriate base for determining the crop insurance premium indemnity schedule.

III.2. Actual Loss Cost

In the development of crop insurance programs, the crop insurance agencies had very little experience which could be used as the basis for establishing a dependable premium-indemnity schedule. However, by 1938, the United States Department of Agriculture had developed a technique for determining the insured yield and the annual premium rate for its crop insurance program. This technique used the actual yields of the

---

[21] The application of extreme value distribution requires that the population should be distributed in an exponential manner, D.L. Brakensiek and A.W. Zingg, Application of the Extreme Value Statistical Distribution to Annual Precipitation and Crop Yields, ARS 41-13 (Washington: U.S. Department of Agriculture, 1957), p. 2.

insurable crop over a number of years as the base for calculating the insurance coverages and the premium eates.[22]

III.2.1. Premium Rate Estimation

The procedure used to calculated the coverage level and the annual premium is describeed as follows. Information was collected with regard to the yield per acre for a number of years and an average annual yield was calculated. Then, an insurance protection or coverage level, as a percentage of the average yield, was computed for each year under consideration, and the annual loss cost per acre was determined for each year. The annual loss cost was defined as the amount by which the annual yield fell short of the imputed insured yield and the total loss cost is the sum of the annual loss costs over the whole period. Finally, the premium rate is calculated by dividing the total loss cost by the number of years considered.

Let x_i be the recorded yield for the i-th year and let c be the insured yield. The total loss cost over a

---

[22] P.K. Ray, Principles & Practices of Agricultural Insurance, (Calcutta: Bookland Private Limited, 1958), p.57. Examples of the use of this technique can be found in J.C. Gilson, "Instability in Agriculture and Crop Insurance," (paper presented at the Farm Conference Week, March, 1962, University of Manitoba, Winnipeg), and in Royal Commission on Agriculture and Rural Life, Province of Saskatchewan, 1956, op. cit., p. 114.

period of time is the summation of ($c - x_i$) for all i, such that $x_i < c$. Hence, the average annual loss cost or premium rate, P, is calculated by equation (4),

$$(4) \qquad P = \sum (c - x_i)/ N \qquad 0 \leq x_i < c,$$

where N is the total number of annual yield observations. If there are n years where the yield falls below the coverage level, then equation (4) can be rewritten as equation (5).

$$(5) \qquad P = (n/N) \cdot c - \sum x_i/N \qquad 0 \leq x_i < c.$$

In this equation, n/N is equivalent to the probability that the actual yield will fall below the insured yield and x_i/N is the average yield per acre when a crop loss does occur.

However, there have been various modifications to the technique when it is used practically in the field. For example, in Prince Edward Island, Nova Scotia, Quebec, and Ontario the preference is to provide each insured farmer with a production guarantee for a crop based on the average yield of the crop on the individual farm over the preceding 5 to 10 years. The premium is based upon the average loss cost of this protection for all farmers in the area or province. In Nova Scotia and Ontario the crop insurance plan for apples is designed to establish a premium as well as a coverage for each farmer on the basis of crop yield on his farm over the preceding six years. Where yield records of a farm are lacking, the record of the average yield for the area is used as the basis for determining the insurance coverage in the initial years of participation. In Quebec because past yield records were not available, the farmers were permitted to declare their average or normal yields, with the understanding that in the event of a claim the insurance agency would judge the appropriateness of the average yield declared by the farmers.[23]

III.1.2. Limitations

One of the limitations of the actual loss technique is that it may not provide an unbiased estimate of the premium rate. If there is a continuous distribution of crop yields, the premium rate is calculated using the formula, shown in equation (1) on page 8. Based on this, the actual loss cost technique should provide an estimator like equation (6).

$$(6) \qquad P_a = 1/N \cdot \Sigma(\, c - x_i \,)\, f_f(x_i) \qquad 0 < x_i < c\,,$$

where x_i is the yield variable, $f_f(x_i)$ is the frequency of yield at x_i bushels per acre, and N is the number of yield observations. The notation $f_f(x_i)/N$; i.e., the relative frequency of yield at x_i, is similar to $f(x)$;

[23]Canada Department of Agriculture, Annual Report, op. cit., p.3.

i.e., the density function of a continuous yield distribution. Whether P_a is an unbiased estimator of P depends upon how close the $f_f(x_i)$, for $0 < x_i < c$, is to the true distribution of crop yields. In other words, discrete characterization of continuous phenomenon may lead to error in the case of missing information.

Another limitation arising from the actual loss cost technique is that any statistic measure cannot yield total information. Although actual loss experience is the best set of data for estimating the theoretical loss cost, it may not be the best estimator. This is because the original data are subject to irregularity. In order to provide a systematic and consistent way of smoothing the origianl data, many theoretical distributions are established in the field of statistics to meet various needs.

III.3. Normal Distribution

In part because of inadequacies in the actual loss cost technique for insurance premium rate determination, an alternative procedure has been developed. It is called the normal distribution technique whereby the area under an estimated normal distribution is used to estimate the average annual loss cost or pure premium rate.[24]

[24] R.K. Ray, op. cit., p. 62, and R.R. Botts and J.N. Boles, "Use of Normal Curve Theory in Crop Insurance Ratemaking," Journal of Farm Economics, 40:733-740, 1958.

This technique has been used by the Manitoba Crop Insurance Corporation for determining risk area premium rate since 1959.[25]

III.3.1. Theoretical Basis

The normal distribution technique was originally tried by the United States Department of Agriculture for determining county premium rates for those counties where yields tend to be normally distributed over time around the county average. Since then, the technique has been widely used in various countries, mainly because most statistics tend to be normally distributed or their distributions converge to a normal distribution. In statistical theory, various laws of large numbers and different versions of the central limit theorem have been derived to justify using the normal distribution to approximate an unknown population distribution. With respect to crop yield data, the law of large numbers and the central limit theorem may be applied as follows. The recorded crop yield series frequently refers to the average yield either on a farm unit or for an area. The central limit theorm states that the sample means tend to be normally distributed when the sample size becomes large. The law of large numbers further ensures that the reliability increases as the sample size is increased.

[25] Manitoba Crop Insurance Corporation, "Method Used to Determine Theoretical Loss Cost per Acre" (Winnipeg: The Corporation, 1968). (Mimeographed.)

For a crop insurance program, the sample size becomes larger as more and more farmers join and stay in the program. Therefore, the use of a normal distribution for crop insurance premium rate determination seems statistically sound.

With regard to crop insurance, one may also look at crop yields in another way. Each planting may be seen as having the potential of harvesting a maximum crop unless or until natural hazards take their toll. This view assumes that under a specific (soil-climate) environment and a specified technology, there is a pure potential yield that would occur except for natural hazards and random factors. Because natural hazards do occur, the potential is seldom reached. It may be believed that in most instances the potential yield is considerably above the yields actually experienced by most farmers. On rare occasions when the season is perfect, no disease or insects appear, and all other circumstances are ideal, the actual yield experience may really approach the ultimate potential.

It should be noted that the potential yield may not be the physically ultimate yield but rather the economic potential yield. Today, farmers have available a wide variety of production technology from which to choose depending upon their production style and management strategies. It is traditional to believe that farmers use the strategy of maximum total net profit in which they push the application of inputs to the extent that it pays. However, due to errors in economic life, most if not all, yields actually attained fall below the potential yield. Tintner has distinguished two types of errors that occur in economic behavior.[26] Errors of the first kind result from the fact that an individual, who otherwise acts rationally and makes correct forecasts of all relevant future data, fails for some reason to make all adaptions of the economic factors which he controls in such a manner as to give his maximum utility or profit. Errors of the second kind are the effect of erroneous forecasts. The reasons for these errors in economic life are due to institutional obstacles, the influence of tradition, imperfect organization, negligence, and similar causes.

Since natural hazards and errors in economic behavior are numerous, they are more or less of random character. In other words, they are then much like the errors that occur in the natural sciences and which are treated in the so-called theory of errors.[27] Therefore, it would not be surprising if their effects on crop yield would follow the normal law of errors. As a result,

---

[26] Gerhard Tintner, The Variate Difference Method, Cowles Commission Monograph No. 5 (Bloomington, Indiana: Principia Press, Inc., 1940), pp. 2-5.

[27] Gerhard Tintner, "A Note on Economic Aspects of the Theory of Errors in Time Series," Quarterly Journal of Economics, 53:144, November, 1938.

it may be concluded that the potential yields may not obey a normal distribution in the statistical sense. Instead, yields may tend to be normally distributed because a normal distribution of adverse crop hazards and erroneous management strategies cut actual yields down to below their potential.

II.3.2. Premium Rate Estimation

As pointed out in Chapter I, the pure premium rate is equal to the expected annual loss cost to the insurance agency. The premium rate is calculated by the following formula:

$$(7) \qquad P_n = E\,(\,c - x\,;\,x < c\,) = \int_0^c (\,c - x\,)\,g(x)\,dx$$

If the continuous yield variable, x, is normally distributed with mean, u, and variance, σ^2, then the probability density function of the normal distribution takes the form of equation (8).

$$(8) \qquad g(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\tfrac{1}{2}\,\frac{(x-u)^2}{\sigma^2}\right\} \quad -\infty < x < \infty$$

Equation (7) can then be written as equation (9);

$$(9) \qquad P_n = \int_c^c (\,c - x\,)\,\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\tfrac{1}{2}\,\frac{(x-u)^2}{\sigma^2}\right\} dx$$

This expression for calculating the pure premium rate can be simplified in the following steps;[28] first, equation (9) can be written as

$$P_n = c\int_{-\infty}^{c} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\tfrac{1}{2}\,\frac{(x-u)^2}{\sigma^2}\right\} dx$$

$$- \int_{-\infty}^{c} x\,\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\tfrac{1}{2}\,\frac{(x-u)^2}{\sigma^2}\right\} dx$$

Let $$A = \int_{-\infty}^{c} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\tfrac{1}{2}\,\frac{(x-u)^2}{\sigma^2}\right\} dx$$

which represents the probability that actual yield per acre will fall below the insured yield c.

Then $$P_n = c\,A - \int_{-\infty}^{c} x\,\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\tfrac{1}{2}\,\frac{(x-u)^2}{\sigma^2}\right\} dx$$

Further, the second term can be simplified by the following transformations, i.e., let

$$z = \frac{x-u}{\sigma}, \quad x = \sigma z + u, \quad dx = \sigma\,dz, \text{ and } \quad K = \frac{C-u}{\sigma}$$

Then, $$\int_{-\infty}^{c} x\,\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\tfrac{1}{2}\,\frac{(x-u)^2}{\sigma^2}\right\} dx$$

$$= \int_{-\infty}^{K} (\sigma z + u)\,\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\tfrac{1}{2} z^2\right) \sigma\,dz$$

---

[28] The integration is supposed to be taken from 0 to c for the calculation of premium rates. However, in order to obtain a simplified formula, the integration will be taken from $-\infty$ to c. When the formula is used and the probability density at the negative side of the distribution does not vanish, then there is a different implication for the use of normal distribution.

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{K} z \exp\left(-\tfrac{1}{2} z^2\right) dz + \frac{u}{\sqrt{2\pi}} \int_{-\infty}^{K} \exp\left(-\tfrac{1}{2} z^2\right) dz$$

$$= \frac{\sigma}{\sqrt{2\pi}} \left(-\exp\left(-\tfrac{1}{2} z^2\right) \right)_{z=-\infty}^{z=K} + u\,A$$

$$= -\sigma d + u\,A$$

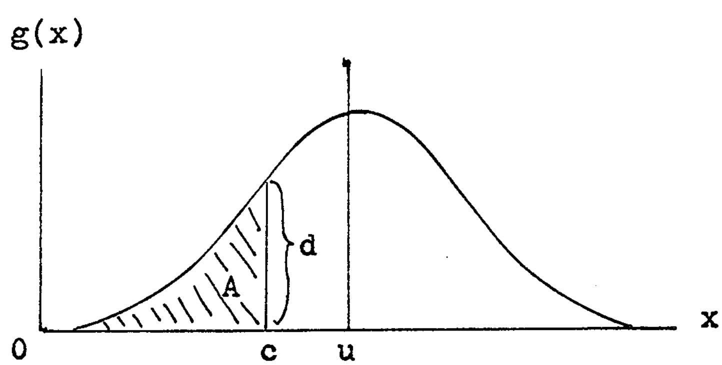
Therefore,

(10) $$P_n = c\,A - (-\sigma d + u\,A)$$

$$= \sigma d - (u - c)\,A$$

The algebraic notations d and A in equation (10) are further explained below.

Figure 4 shows a typical normal distribution with crop yield, x, in bushels per acre plotted on the horizontal axis, and probability density, g(x), plotted on the vertical axis. The perpendicular erected at u indicates the average yield u bushels per acre and the perpendicular erected at c indicates the insured yield c



Crop Yield (bushels per acre)

Figure 4. Theoretical Normal Distribution

bushels per acre. The shaded portion, A, represents the probability that actual yield per acre will fall below the insured yield level c, and d is the ordinate of the normal distribution at c.

The average yield, $\bar{x}$, and the standard deviation, $\hat{\sigma}$, calculated from any given set of data can be used as estimates of u and σ. The insured yield, c, can be computed as a certain percentage of the average yield, $\bar{x}$. With the table of ordinates of the standard normal distribution, the value of d can be obtained. The value of A can be obtained using the table of cumulative normal distribution.

III.3.3. Limitations of the Technique

That field crop yields conform to a normal distribution seems pausible. The distribution of field crop yields is determined by resources used, technology, weather, and some unexplained random variation. These facotrs will now be defined and discussed in more detail.

(a) Resources: Land, labour, capital, and other production resources influence yields. For example, yield expection and variability on a highly productive land may be quite different from those on a less productive land. However, the impact of relative differences in resources used can be ignored for purposes of crop insurance if areas can be stratified into homogeneous crop production areas or farms. Changes in the quality of resources can be considered as technological improvement which will be discussed below.

(b) Technology: This includes innovations in machnization, seed varieties, management strategy, and recommended practices resulting from improved farmer education and information dissemination. These effects produce a significant upward trend in yield per acre. But raising yield expectation also increases the crop value that may be lost. New technology has provided additional protection against crop loss from such hazards as disease, insect damage, weed infestation, and so on. Nevertheless, epidemics still occur. Moreover, technology can do little to prevent crop loss from freezing temperature, hail damage, and severe drought.

(c) Weather: This includes both direct weather influences (rainfall, temperature, etc.), and indirect influences (insect damage, plant disease, etc.). In a given year, the distribution of crop yields may be skewed either to the left or to the right due to the weather, as described below. Excellent weather conditions throughout the entire growing season, particularly during the germination, flowering, heading, and harvesting seasons, must prevail if high yields are to be obtained. Such crop yields do occur and phenomenally high yields are recorded. These not only raise average yields but also cause the distribution of crop yields to be skewed to the left. On the other hand, bad weather, for example, too much or too little rain or heat, during any one of the several critical periods is sufficient to reduce yield drastically, even though ideal weather is the rule during the preceding and succeeding parts of the growing season. Such bad crop years do occur, reducing average yields and tending to cause distribution of crop yields to be skewed to the right.

(d) Random Variation: This is caused by all other factors influencing yields. In the absence of precise knowledge, it may be assumed that random variation tend to make the distribution of crop yields a normal one.

In any given year, all of these factors are operative, and have a composite influence on crop yields. The actual distributions of crop yields may vary significantly from crop to crop, from soil to soil, and from climate to climate. As an illustration of this fact, thirty years time series data from the Delta Branch of the mississippi State Experiment Station indicate asymmetry and nonnormal peakness in field crop yields.[29]

It is clear that the probability of occurance of a below average yield, from a distribution skewed to the right would be greater than it would be from a normal distribution. The problem connected with using the normal distribution for insurance purposes may be explained further as follows.

The problem for premium calculation arises from

---

[29] R.H. Day, "Probability Distribution of Field Crop Yields," Journal of Farm Economics, 47:713-41, 1965.

difficulty in making a correct estimate of the total probability to the left of the insured crop level. It is useful to discuss this in terms of the shape of the distribution and the existence and position of the inflection point. A normal distribution is symmetrical and bell shaped and has an inflection point at one standard deviation on each side of the mean. A question arises as to whether there is an inflection point in the distribution of crop yields. On the left hand side of a normal distribution the slope increases at an increasing rate until it reaches its maximum or infinity at the inflection point. After the left side inflection point the slope decreases at an accelerating rate until it reaches zero at the maximum point. This characteristic implies that probabilities before the inflection point are much smaller than those after the inflection point. Figure 5 illustrates this property. The probability before the inflection point, A, represented by area C, is smaller than the probability after the inflection point, represented by area B. If the actual distribution of crop yields

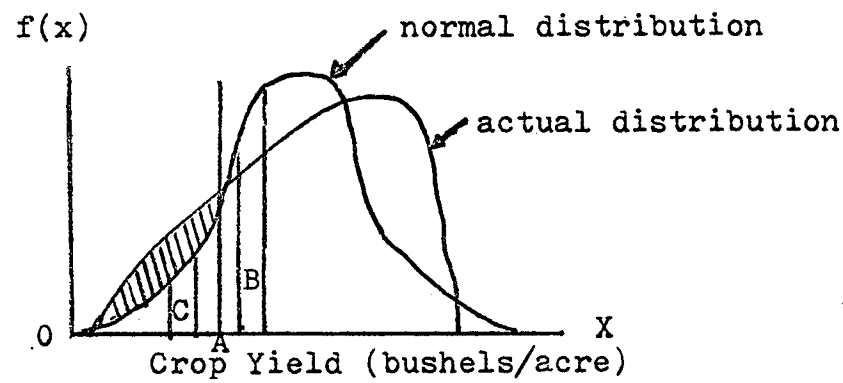


Figure 5. Example Showing Biased Estimation of Probability

does not have an inflection point, then a biased estimation of probability can be expected. The shaded area between the normal and the actual distribution in Figure 5 is an example showing underestimation of probability when a normal distribution is (incorrectly) assumed. Underestimation of probability implies underestimation of the premium rate for an insurance program.

Further, a normal distribution extends to infinity in either direction while the distribution of crop yields may theoretically be considered as a finite one. It is clear that crop yields have a finite lower limit on their range, namely, zero. It also seems clear that even under the most favorable circumstances they must possess a finite upper limit. In actual practice whether or not the distribution of crop yields is finite makes little difference, as long as the estimated distribution gives a good approximation to the yield distribution. The problem arising from the use of the normal distribution is that the tails of the normal distribution approach zero only after three standard deviations from the mean. If such an estimated distribution is used in premium ratemaking, the estimated premium rate will be biased.

The well known noraml distribution is a very simple way to represent a distribution. It has been widely used for the purpose of obtaining an estimate of the probability in the neighborhood of the mean.

For example, within two or three standard deviations from both directions of the mean, the probability enclosed by the normal distribution can give a satisfactory approximation to that enclosed by a non-normal distribution. However, the normal distribution may not satisfactorily describe some special events, especially in the actuarial field; such as the number of car accidents or the number of insurance claims in a unit of time, etc.. What seems to be needed is a general estimation method which has the capability of taking account of any case that may arise in practice. The following section introduces Pearson distributions as an alternative for estimating an empirical distribution.

III.4. Pearson Distributions

Karl Pearson has developed a system of distributions which includes a wide variety of forms. "It has been found that in many cases the Pearson distributions provide a remarkably good fit to observation."[30] It is believed that Pearson distributions can be used to estimate the actual distribution of crop yields for the purpose of crop insurance premium ratemaking.

III.4.1. The Initiation of Pearson Distributions

Pearson distributions were initiated from the most common characteristics of a unimodal distribution.

---

[30] M.G. Kendall and A. Stuart, The Advanced Theory of Statistics, Vol. I (London: Charles Griffin, 1958), p. 152.

Such a distribution generally starts at zero, rises to a maximum, and then falls sometimes at the same rate but more often at a different rate. In other words, the tangency of the probability density function, dy/dx, is equal to zero at the tails and the maximum of the distribution, where $y = f(x)$ is the probability density of the random variable x. Karl Pearson suggested a differential equation, which takes the form of equation (11), to express them.

$$dy/dx = y\ (\ x - A\ )\ /\ h(x) \tag{11}$$

where h(x) is a general function of the random variable x, and may take the ascending power of x as equation (12).[31]

$$h(x) = b_0 + b_1 x + b_2 x^2 \tag{12}$$

Equation (11) indicates that $dy/dx = 0$, such that $y = 0$, or $x = A$. The integrations of the differential equation, i.e., equation (11), constitute the various members of the Pearson distributions. The results of the integrations are presented in Table 1.[32]

The parameters of Pearson distributions can be estimated in terms of moments by the method of moments.[33]

---

[31]For practical purposes it will be sufficient that equation (12) includes powers of x up to only the square.

[32]A detailed derivation of the integrations can be found in W.P. Elderton and N.L. Johnson, System of Frequency Curves (London: Cambridge University Press, 1969).

[33]Ibid., pp. 12-34, and M.G. Kendall and A. Stuart, op. cit., Vol I, pp. 148-149.

Table 1

The Family of Pearson Distributions

No. of Type	Origin	Functional Form
Main Type		
I	mode	$y=y_0(1+x/a_1)^{m_1}(1-x/a_2)^{m_2}$
IV	mean + va/r	$y=y_0(1+x^2/a^2)^{-m}\ e^{-v\ \tan^{-1}(x/a)}$
VI	a before start of curve	$y=y_0(x-a)^{q_2}x^{-q_1}$
Transition Type		
Normal Distribution	mode (=mean)	$y=y_0\ e^{-x^2/2\sigma^2}$
II	mode (=mean)	$y=y_0(1-x^2/a^2)^m$
VII	mode (=mean)	$y=y_0(1+x^2/a^2)^{-m}$
III	mode	$y=y_0(1+x/a)^{ra}\ e^{-rx}$
V	start of curve	$y=y_0x^{-p}\ e^{-r/x}$
VIII	end of curve	$y=y_0(1+x/a)^{-m}$
IX	end of curve	$y=y_0(1+x/a)^m$
X	start of curve	$y=y_0\ e^{-x/\sigma}$
XI	b before start of curve	$y=y_0\ x^{-m}$
XII	mean	$y=y_0\left(\frac{(3+B_1)+B_1+x}{(3+B_1)+B_1-x}\right)^{(B_2/(3-B_1))}$

Source: W.P. Elderton and N.L. Johnson, <u>System of Frequency Curves</u> (London: Cambridge University Press, 1969)

The estimation procedures for the three main types of Pearson distributions are introduced below.

III.4.2. Estimation Procedure

The family of Pearson distributions contains three main types and ten transition types of distribution as shown in Table 1. In this section the characteristics of and estimation procedures for the three main types, which are frequently met in practical applications, will be reviewed and discussed. Because the identification of a Pearson distribution depends on equation (12), the following presentation will start from the characteristics of this equation.

(a) Type I Pearson Distribution

If the roots of equation (12), i.e., $h(x)=b_0+b_1x+b_2x^2$, are real and of opposite sign, then the integration of equation (11), gives the Type I Pearson distribution as follows;

$$(13) \qquad y = y_0 \left(1+x/a_1 \right)^{m_1} \left(1-x/a_2 \right)^{m_2} \qquad -a_1 \leqslant x \leqslant a_2$$

where $m_1/a_1 = m_2/a_2$ and with origin at the mode.

The Type I Pearson Distribution is a skewed distribution with a range from $-a_1$ to a_2. The origin of the distribution is located at the mode. If the parameters m_1 and m_2 are approximately equal, the distribution is nearly symmetrical. The Type I Pearson distribution may be bell-shaped. However, a variety of shapes may occur depending upon the magnitudes of m_1 and m_2. If both m_1 and m_2 exceed zero, the distribution has a unique mode and vanishes at both ends. If either m_1 or m_2 is negative, the ordinate at the corresponding terminal is infinite, and the distribution is J-shaped or its minor-inage. If both m_1 and m_2 are negative, the distribution is U-shaped, starting and ending with infinite ordinates and having an anti-mode instead of a mode as the usual origin. Figure 6 illustrates the corresponding cases that may occur for a Type I Pearson distribution.

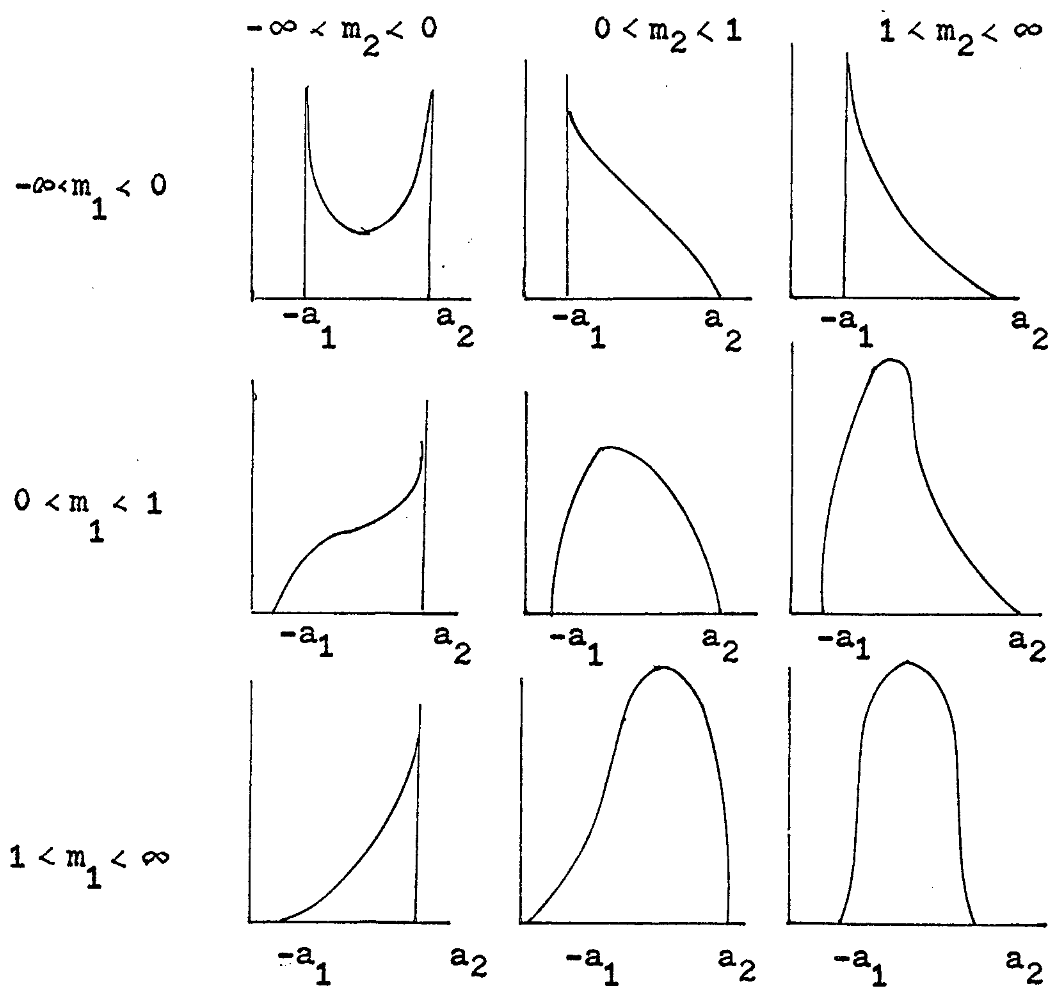


Figure 6. Forms of Type I Pearson Distribution

The parameters of a Type I Pearson distribution can be estimated in the following order,

(14) $$r = 6(B_2 - B_1 - 1)/(6 + 3B_1 - 2B_2)$$

(15) $$a_1 + a_2 = \tfrac{1}{2}\sqrt{u_2}\sqrt{\{B_1 (r+2)^2 + 16(r+1)\}}$$

The m's are given by

(16) $$\tfrac{1}{2}\left\{r-2 \pm r(r+2)\sqrt{\frac{B_1}{B_1(r+2)^2+16(r+1)}}\right\}$$

when u_2 is positive m_2 is the positive root.

(17) $$y_0 = \frac{1}{a_1+a_2} \frac{m_1^{m_1}\ m_2^{m_2}}{(m_1+m_2)^{m_1+m_2}} \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1)\Gamma(m_2+2)}$$

(18) $$\text{mode} = \text{mean} - \tfrac{1}{2}\frac{u_3}{u_2}\ \frac{r+2}{r-2}$$

In the case of J-shaped distribution, it is best to express the distribution in the form of

(19) $$y = y_0\, x^{m_1} (a_1 + a_2 - x)^{m_2} \qquad -a_1 \leq x \leq a_2$$

with the origin at the start of the distribution

and

(20) $$y_0 = \frac{1}{(a_1+a_2)^{m_1+m_2+1}} \cdot \frac{\Gamma(m_1+m_2+2)}{\Gamma(m_1+1)\ \ \Gamma(m_2+1)}$$

(b) Type IV Pearson Distribution

If the roots of equation (12), i.e., $h(x)=b_0+b_1x+b_2x^2$, are imaginary, the integration of equation (11) gives the

Type IV Pearson distribution as equation (21)

(21) $$y = y_0 \left(1+\frac{x^2}{a^2}\right)^{-m} e^{-v \tan^{-1}(x/a)} \qquad -\infty < x < \infty$$

with origin at the mean + va/r.

The distribution is skewed and has unlimited ranges in both directions. u_3 and v have opposite signs, i.e., when u_3 is positive v is negative. The parameters of the distribution can be estimated in the following order;

(22) $$r = 6(B_2-B_1-1)/\ 2(B_2-3B_1-6)$$

(23) $$m = \tfrac{1}{2}\ (r+2)$$

(24) $$v = \frac{r\ (\ r-2\ )\ \sqrt{B_1}}{\sqrt{\{16(r-1)\ -\ B_1(r-2)^2\}}}$$

(25) $$a = \sqrt{\frac{u_2}{16}}\ \sqrt{\{16\ (r-1)\ -\ B_1(r-2)^2\}}$$

(26)[34] $$y_0 = \frac{1}{a\ F(r,v)}$$

(27) $$\text{mode} = \text{mean} - \tfrac{1}{2}\ \frac{u_3(r-2)}{u_2(r+2)}$$

(c) Type VI Pearson distribution

If the roots of equation (12), i.e., $h(x)=b_0+b_1x+b_2x^2$, are real and of the same sign, then the integration of

---

[34] The calculation and explanation of the special function F(r,v) can be found in W.P. Elderton and N.L. Johnson, op. cit., pp.68-75.

equation (11) gives the Type VI Pearson distribution, or equation (28),

$$(28) \qquad y = y_0 \, x^{-q_1} \, (x-a)^{q_2}$$

where $a<x<\infty$ if a is positive and $-\infty<x<a$ if a is negative. This distribution is either skewed or bell-shaped, and when q_2 is negative, the distribution is J-shaped.

The parameters of a Type VI distribution can be estimated in the following order,

$$(29) \qquad r = 6(B_2-B_1-1)/(6+3B_1-2B_2)$$

$$(30) \qquad a = \tfrac{1}{2}\sqrt{u_2}\,\sqrt{\{B_1\,(r+2)^2 + 16(r+1)\}}$$

If u_3 is negative, then a is negative or a negative sign should be imposed. q_2 and $-q_1$ are given by

$$(31) \qquad \frac{r-2}{2} \pm \frac{r(r+2)}{2}\sqrt{\frac{B_1}{B_1(r+2)^2 + 16(r+1)}}$$

$$(32) \qquad y_0 = \frac{a^{q_1-q_2-1}\;\Gamma(q_1)}{\Gamma(q_1-q_2-1)\,\Gamma(q_2+1)}$$

$$(33) \qquad \text{mode} = \text{mean} - \tfrac{1}{2}\,\frac{u_3(r+2)}{u_2(r-2)}$$

It has been found that it is easier to work with the form of equation (34).

(34) $$y = y_0 \left(1 + x/a_1 \right)^{-q_1} \left(1 + x/a_2 \right)^{-q_2}$$

with the origin at the mean,

where

(35) $$y_0 = \frac{(q_2+1)^{q_2} (q_1-q_2-2)^{q_1-q_2} \Gamma(q_1)}{a(q_1-1)^{q_1} \Gamma(q_1-q_2-1) \Gamma(q_2+1)}$$

(36) $$a_1 = \frac{a (q_1-1)}{(q_1-1) - (q_2+1)}$$

(37) $$a_2 = \frac{a (q_2+1)}{(q_1-1) - (q_2+1)} .$$

III.4.3. Identification

The integrations of equation (11), which constitute the family of Pearson distributions, depend entirely upon the characteristics of the quadratic function of the random variable x; namely equation (12). Therefore, Pearson distributions can be identified by the coefficients of the general function and written as a criterion, k, as follows;

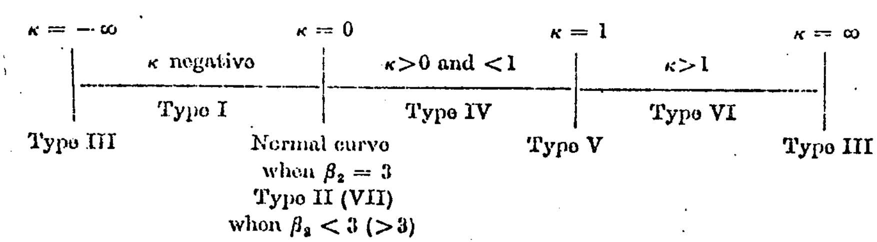
(38) $$k = b_1^2 / 4b_0b_2 .$$

For example, in a Type I Pearson distribution the roots of equation (12) are real and of opposite sign.

This condition is equivalent to the criterion, k, being less than zero. Similarly, in a Type IV Pearson distribution the criterion is between zero and one, and in a Type VI Pearson distribution the criterion is greater than one.

These three main types of distribution actually include all possible distributions. But in the limiting points, i.e., when one type of distribution changes into another in terms of k, there may be one or more algebraically simpler forms called transition distributions. For example, when k increases from a negative to a positive value, there is a point at which k is equal to zero. At this point the following distributions are equivalent: the Type II with $B_2 < 3$, the normal curve with $B_2 = 3$, and the Type VII with $B_2 > 3$.

The criterion, k, may be of any value from negative infinity to positive infinity. The following diagram shows the different types of distribution which include all the possible values of the criterion, k, and do not overlap.



Using the method of moments, the coefficients of equation (12), i.e., b_0, b_1, and b_2, can be written in terms of moments.[35] Substituting them into equation (38), the criterion, k, takes the form of equation (39);

$$(39) \qquad k = \frac{B_1 \, (B_2+3)^2}{4 \, (2B_2-3B_1-6) \, (4B_2-3B_1)} = f(u_2, u_3, u_4)$$

When a Pearson distribution is applied to estimate an unknown distribution, the criterion, k, can be used to identify the type of Pearson distribution to which the unknown distribution belongs. The estimation procedure involves interpreting the sample B_1 and B_2 coefficients as being the theoretical ones for purposes of calculating the criterion, k. As in all statistical estimation, the usual testing problem arises from the statistical unreliability of conclusions drawn form the sample B_1 and B_2 coefficients. For example, if the null hypothesis is that the unknown distribution was drawn from a Type I population, what would be the probability under this hypothesis of obtaining sample B_1 and B_2 coefficients that fall in Type IV and Type VI distributions? How far from the Type I distribution would the sample B_1 and B_2 coefficients have to fall in order for one to infer, at a given level of probability, that is did not in fact arise from the Type I population?

---

[35] Ibid., pp. 35-41.

Unfortunately, these quite legitimate questions cannot be answered because the requisite sampling theory has never been discovered.[36]

III.4.4. Implications

The problem of finding a model to describe insurance indemnity payments is familiar and important in the actuarial field. The essence of this problem rests on the knowledge of the probability distribution of the variable exposed to risk. Such a distribution is also needed in connection with the determination of the "proper" premium rate, the "proper" risk reserve, and the resulting reinsurance deemed necessary.

Theoretical distributions, such as binomial, Poisson, and normal distributions, do provide a simple way to represent a simple type of probability event. However, it frequently happens that the probabilities are not known a priori, and it is impossible to tell which theoretical distribution is the best to use in the analysis. What seems to be needed is an apparatus which has the capability of finding a distribution from the sample data and taking account of any case that may occur in practice. Previous discussions reveal that Pearson distributions fulfill this requirement and can be used.

---

[36] R.H. Day, op. cit., p. 732.

Using Pearson distributions for estimating the unknown distribution has the advantage that there is no need for an a priori assumption about the algebraic form of the distribution. The type of Pearson distribution which approximates the unknown population distribution can be identified in the estimation process by the criterion, k, i.e., a function of the skewness and kurtosis coefficients of the unknown distribution. By this approach it is possible to reduce the errors caused by incorrectly specifying the form of the unknown distribution. The statistical foundations of using skewness and kurtosis coefficients to identify a Pearson distribution is explained as follows.

In statistical theory, it has been noted that the characteristic function determines the moments when they exist, and that the characteristic function also determines the distribution function. It does not, however, follow that the moments completely determine the distribution, even when moments of all orders exist. Only under certain conditions will a set of moments determine a distribution uniquely, but, fortunately for statisticians, those conditions are obeyed by all the distributions arising in statistical practice. For all ordinary purposes, therefore, a knowledge of the moments, when they all exist, is equivalent to a knowledge of the distribution function: equivalent, that is, in the sense that it should be possible theoretically to exhibit all the properties of the distribution in terms of the moments. In particular, it may be expected that if two distributions have a certain number of moments in common they will bear some resemblance to each other. If, say, moments up to those of order n are identical then as n tends to infinity the distributions approach identity. Consequently, it may be expected that one should be able to identify an unknown distribution by calculating the sample moments. Although, the lower moments do not determine a distribution explicitly, they might be found to be identical to those of another distribution of known form. As concluded by Kendall and Stuart: "In practice, approximations of this kind often turn out to be remarkably good, even when only the first three or four moments are equated.[37]

Pearson distributions have been used in actuarial work for a long time. In 1936, Riebesell showed that the distribution of actual and expected loss ratios of fire insurance businesses should be calculated as a Type IV Pearson distribution and that the results differed strongly from the normal one.[38] In 1937, Wold referred to the successful

---

[37] M.G. Kendall and A. Stuart, op. cit., Vol. I. p.87.

[38] H.L. Seal, Stochastic Theory of A Risk Business (New York: John Wiley & Sons, 1969). p.5. In the second chapter of this book, Seal has reviewed various probability distributions that have been proposed or utilized in the insurance field.

fitting of a Pearson Type III distribution to Swedish rural fire insurance company claim ratios.[39] In 1938, Rossmann chose the Type I Pearson distribution for the loss ratios distribution experienced by a French hail insurance company.[40] Recently, Welten has applied Pearson distributions to the distribution of fire insurance claim ratios and found that a Type I Pearson distribution was appropriate.[41]

Pearson distributions typically estimated by the method of moments have been regarded from two rather different standpoints. The method has been criticized when the observed data are regarded as samples from a population and it is desired to find a mathematical representation of that population. In such cases the moments calculated from observations are only estimates of the population parameters and they do not, in general, lead to the most efficient estimates of the population parameters.[42] As a result, R.A. Fisher launched the method of maximum likelihood with an attack on Pearson's

---

[39] H.O.A. Wold, "A Technical Study on Reinsurance," C.R. XI Cong. Intern. Actu., Paris, 1:549-59, 1937.

[40] H.L. Seal, op. cit., p. 5.

[41] C. P. Welten, "Estimating of Stop Loss Premium in Fire Insurance," Astin Bulletin, 2:356-61, 1964.

[42] If the distribution is normal, the method of moments is efficient. M.G. Kendall and A. Stuart, op. cit., Vol. II, p. 65.

method of moments.[43] But, efficient, maximum likelihood methods have seldom been applied to the main types of Pearson distribution. The reason for this becomes immediately evident to anyone who attempts to do so. The likelihood equations are nonlinear and extremely oomplicated. Furthermore, computational experiments for maximum likelihood estimation of the Pearson distributions conducted by R.H. Day indicated no substantial changes in the empirical results.[44]

From an actuarial point of view, the objective frequently is to obtain an estimate of the distribution which can satisfactorily reflect its characteristics based on observation, as well as permitting accurate graduation and interpolation. There is no further inference to be made on the estimated parameters of the distribution, i.e., the coefficients of the distribution. Pearson distributions estimated by the method of mements are generally satisfactory.[45]

In the following chapters, Pearson distributions will be applied to wheat yield data of the 14 crop

---

[43] A detailed discussion of the efficiency of moments in determining the parameters of a Pearson distribution has been given by R.A. Fisher, "On the Mathematical Foundation of Theoretical Statistics," Philosophical Transactions of the Royal Society of London, (a), 222:309-368, 1922.

[44] R.H. Day, op. cit., p. 734.

[45] M.G. Kendall and A. Stuart, op. cit., Vol. I p. 154.

districts of Manitoba for estimating the distributions of wheat yields. The results will be used as examples showing the applicability of Pearson distributions for the purposes of crop insurance premium ratemaking.

CHAPTER IV

ESTIMATED DISTRIBUTION OF CROP YIELDS AND THE PURE PREMIUM RATE

In this chapter Pearson distributions are applied to wheat yield data of the 14 crop districts of Manitoba for estimating the distributions of wheat yields. The chi-square statistic is then used to test the fit of the estimated Pearson distribution to the observed data. Comparisons of the same statistics calculated from the estimated normal distribution are made in terms of goodness of fit. Finally, these two estimated distributions are used for pure premium ratemaking, and the results are compared.

IV.1. Source of Data

Fifty year (1921-70) wheat yield records for the 14 crop districts of Manitoba are used as the data bases for estimating the distributions of wheat yields. The data set is presented in Appendix I.

Most of the agricultural region is located in the southern part of the Province. In 1970, wheat harvested acreage was estimated to be 1.4 million acres,[46] while wheat production was 30.5 million bushels, and the value

---

[46] Department of Agriculture, Province of Manitoba, Manitoba Agriculture, 1971 Edition (Winnipeg, The Department, 1971), p. 7.

of the 1970 wheat crop was 44 million dollars. Figure 7 shows the location of crop districts. Wheat yields for each district are derived from Statistics Canada surveys which are integrated with the Manitoba crop reporting program.[47]

The delineation of crop districts is not identical with that of crop risk areas which are currently used by the Manitoba Crop Insurance Corporation.[48] This discrepancy means that the estimated premium rates are not comparable with current premium rates used by the Corporation.[49] Nevertheless, this limitation does not destroy the usefulness of this study which is intended to introduce the use of Pearson distributions for estimating distributions of yields for crop insurance premium ratemaking.

IV.2. Estimated Distribution of Wheat Yields

All the Pearson distributions are determined by the first four moments, except for some of the transition types which can be determined by fewer than four moments. For example a normal distribution can be determined by

---

[47] Ibid., p. 84.

[48] The allocation of the crop risk areas of the Province of Manitoba is shown in Figure 9.

[49] The current premium rate schedule is also not the full pure premium rate schedule; i.e., the Federal government share of 25 percent of the pure premium rate is not included.

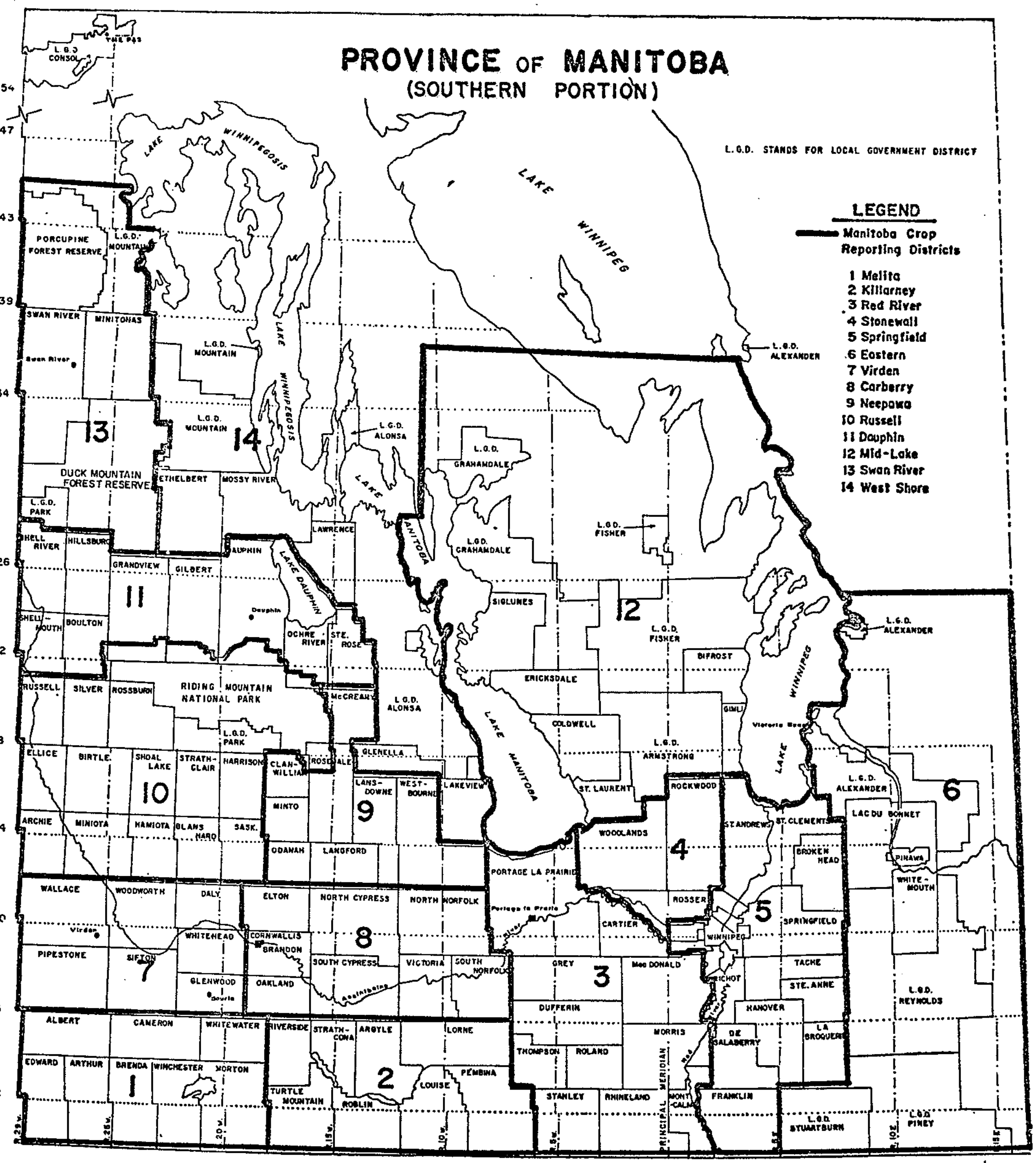


Figure 7. Manitoba Crop Reporting Districts (1970)

Source: Department of Agriculture, Province of Manitoba, Winnipeg.

the first two moments, i.e., the mean and the variance. In general, Pearson distributions can be estimated by going through the following steps;

(1) Determining the numerical values of the first four moments for the distribution to be estimated.

(2) Determining the numerical values of the B_1 and B_2 coefficients and the criterion, k.

(3) Substituting the sample moments and the B_1 and B_2 coefficients for their theoretical values in the functional relationship between them and the parameters of the distribution, e.g., equations (14) through (18), for the parameters of a Type I distribution.[50]

Applying these steps to the wheat yield data of the 14 Manitoba crop districts gives results as follows.

Results for the first two steps of the estimation procedure are shown in Table 2. The figures in column (2) of this table are the first moment, mean, or average of the wheat yields. The average wheat yield for all crop districts is about 20 bushels per acre. District 1 has the smallest average yield, 17.02 bushels per acre, and district 13 has the largest, 24.96 bushels per acre.

The variance and standard deviation of the yields

---

[50] A computer program for estimating three main types of Pearson distribution and the normal distribution has been prepared in Appendix I.

Table 2

Basic Statistics for Estimating Pearson Distributions

District (1)	mean (2) (bushels/acre)	u_2 (3)	u_3 (4)	u_4 (5)
1	17.02	42.10	-161.64	5189.75
2	19.54	35.85	-80.02	2907.75
3	19.78	24.17	5.24	1418.03
4	20.74	27.51	18.62	1874.73
5	20.86	23.12	-16.70	1234.68
6	17.84	20.97	14.86	1103.88
7	19.90	29.53	-93.21	2053.16
8	20.74	32.91	-90.57	2668.03
9	20.46	29.81	-91.18	2139.96
10	22.90	37.09	-141.49	3624.01
11	20.66	30.14	-67.42	2050.17
12	20.08	31.83	-37.48	1945.07
13	24.96	37.07	-87.74	2972.51
14	18.02	27.62	15.74	2199.43

Source: Calculated from wheat yield data (1921-70), Department of Agriculture, Province of Manitoba, Winnipeg.

Table 2 (continued)

District (6)	Standard Deviation (7) (bushels/acre)	B_1 (8)	B_2 (9)	k (10)
1	6.49	0.3502	2.9286	-0.2418
2	5.98	0.1390	2.2631	-0.0590
3	4.92	0.0020	2.4277	-0.0013
4	5.24	0.0167	2.4772	-0.0116
5	4.81	0.0226	2.3103	-0.0120
6	4.58	0.0240	2.5100	-0.0173
7	5.43	0.3375	2.3550	-0.1250
8	5.74	0.2301	2.4635	-0.1063
9	5.46	0.3140	2.4089	-0.1244
10	6.09	0.3924	2.6348	-0.1744
11	5.49	0.1660	2.2566	-0.0677
12	5.64	0.0436	1.9197	-0.0152
13	6.09	0.1511	2.1625	-0.0577
14	5.26	0.0118	2.8838	-0.0331

are shown in columns (3) and (7) respectively. The average standard deviation for all districts is about 5 bushels per acre. The lowest is 4.81 bushels per acre for crop district 5, and the highest is 6.49 bushels per acre for district 1.

From the means and variances normal distributions of yields were estimated for the 14 crop districts and are plotted with a crossed line in Figure 8. When a normal distribution is estimated it is assumed that the distribution is symmetric and of normal peakness, i.e., B_1 is zero and B_2 has a value of three.

Based upon the first four moments, the B_1 and B_2 coefficients are calculated and shown in columns (8) and (9) of Table 2 respectively. However, none of the B_1 coefficients is exactly equal to zero and neither is the B_2 coefficients equal to three. Also, as shown in column (4) of Table 2, there are 10 out of the 14 crop districts where the third moments, u_3, is negative. They are crop district 1, 2, 5, 7, 8, 9, 10, 11, 12, and 13. This suggests that the distributions for these ten districts should be considered to be skewed to the left. For the other four districts, where the u_3's are positive, the distributions should be considered to be skewed to the right.

Column (9) of Table 2 shows the B_2 coefficients. These are all less than three, and this implies that the distributions have less slope than the normal distribution.

In terms of statistical significance, there are several tests of normality which can be used, though they are not very satisfactory for reasons explained below. There are two tests due to E. Pearson.[51] One is a skewness test using the square root of the B_1 coefficient. This test may be used for a sample of as few as 11 observations. The other is a kurtosis test using the B_2 coefficients, though this test can be safely applied only to a minimum sample size of 200. However, the null hypothesis of the test of normality should consist of <u>two</u> components simultaneously; namely, the B_1 coefficient is equal to zero and the B_2 coefficient is equal to three, i.e.,

$$(40) \qquad H_0: \quad B_1 = 0 \text{ and } B_2 = 3$$

However, a joint probability distribution for this kind of test is not available yet. For these reasons, this study approaches the problem of whether the crop yields are normally distributed in a different way. In the following section, a chi-square test will be used to compare the estimated normal distribution with the estimated Pearson distribution in terms of the goodness of fit to the observed data. The following paragraphs explain further the estimation of a Pearson distribution of wheat yields.

---

[51] E.S. Pearson and H.O. Hartley, <u>Biometrika Tables for Statistician</u>, Vol. 1 (London: Cambridge University Press, 1962).

The criterion, k, which identifies the type of Pearson distribution should be used to represent the unknown distribution, and is calculated according to equation (39). It is shown in column (10) of Table 2. All the k's in column (10) are negative, suggesting that the Type I Pearson distribution should be used to represent the distribution of wheat yields for all of the 14 crop districts in Manitoba. Using equations (14) through (18) the parameters of the Type I Pearson distribution have been estimated and are shown in Table 3. These 14 estimated Type I Pearson distributions are plotted with a solid line in Figure 8. As can be seen from Figure 8 there are six crop districts where the distribution of wheat yields is bell-shaped; i.e., districts 1, 3, 4, 5, 6, and 14. There are seven where the distribution is shaped like a cocked hat; i.e., districts 2, 8, 9, 10, 11, 12, and 13, and there is one district where the distribution is J-shaped; i.e., district 7. The shape of the distribution can also be recognized by the magnitudes of the parameters m_1 and m_2. A bell-shaped Type I Pearson distribution occurs when m_1 and m_2 are both greater than one, as in crop district 1 where m_1 is equal to 4.72 and m_2 is equal to 1.22. A cocked-hat-shaped Type I Pearson distribution occurs when either of m_1 and m_2 is between zero and one and the other is greater than one, as in crop district 2 where m_1 is equal to 1.20

Table 3

Parameters of Estimated Type I Pearson Distributions for Wheat Yields, Province of Manitoba

Crop District	a_1	a_2	m_1	m_2	y_0	Origin
1	34.36	8.86	4.72	1.22	0.061	20.23
2	20.24	6.09	1.20	0.36	0.059	23.51
3	13.59	14.99	2.59	2.85	0.074	19.59
4	13.57	18.08	2.57	3.43	0.070	20.18
5	14.30	10.06	1.96	1.38	0.073	21.65
6	11.79	16.65	2.68	3.79	0.080	17.27
7	25.96	-3.93	0.77	-0.12	0.094	5.22
8	21.90	5.61	1.75	0.45	0.065	24.62
9	22.74	0.69	1.06	0.03	0.080	27.59
10	26.16	3.08	1.71	0.20	0.068	28.81
11	19.29	4.23	1.06	0.23	0.066	25.23
12	20.17	0.46	0.29	0.01	0.061	28.67
13	21.90	2.68	0.76	0.09	0.061	31.70
14	28.43	41.59	16.22	23.74	0.075	17.71

Notes: i. A computer program for calculating these parameters was prepared and is shown in Appendix I.

ii. A Type I Pearson distribution takes the form

$$y = y_0 \left(1 + \frac{x}{a_1} \right)^{m_1} \left(1 - \frac{x}{a_2} \right)^{m_2} \qquad -a_1 \leqslant x \leqslant a$$

with origin at the mode.

Source: Based on wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.

and m_2 is equal to 0.36. A J-shaped Type I Pearson distribution occurs when one of the parameters m_1 or m_2 is negative, and the other positive, as in district 7 where m_1 is equal to 0.77 and m_2 is equal to -0.11.

It is clear that crop yields have a finite lower limit in their range, namely, zero. It also seems clear that they have a finite upper limit, even under the most favorable circumstances. A Type I Pearson distribution with a limited range seems to be the best one in the family of Pearson distributions for representing the distribution of crop yields. Furthermore, it seems they are more likely to have a bowl, cocked-hat, or bell-shaped distribution than one which is J or U-shaped. However, a J-shaped one did occur as the estimated distribution for district 7. This result may be due to sampling error. Samples drawn from a highly skewed and highly peaked population might appear to be from a J-shaped distribution.

Frequency histograms of the yields of the 14 districts are also plotted in Figure 8. In order to compare the fit of the estimated distribution to the histograms a third graph is plotted for each district. However, these graphs only reveal the degree of fit to a certain extent. Therefore, a more precise statistical test, the chi-square test, will be applied to the problem in the following section.

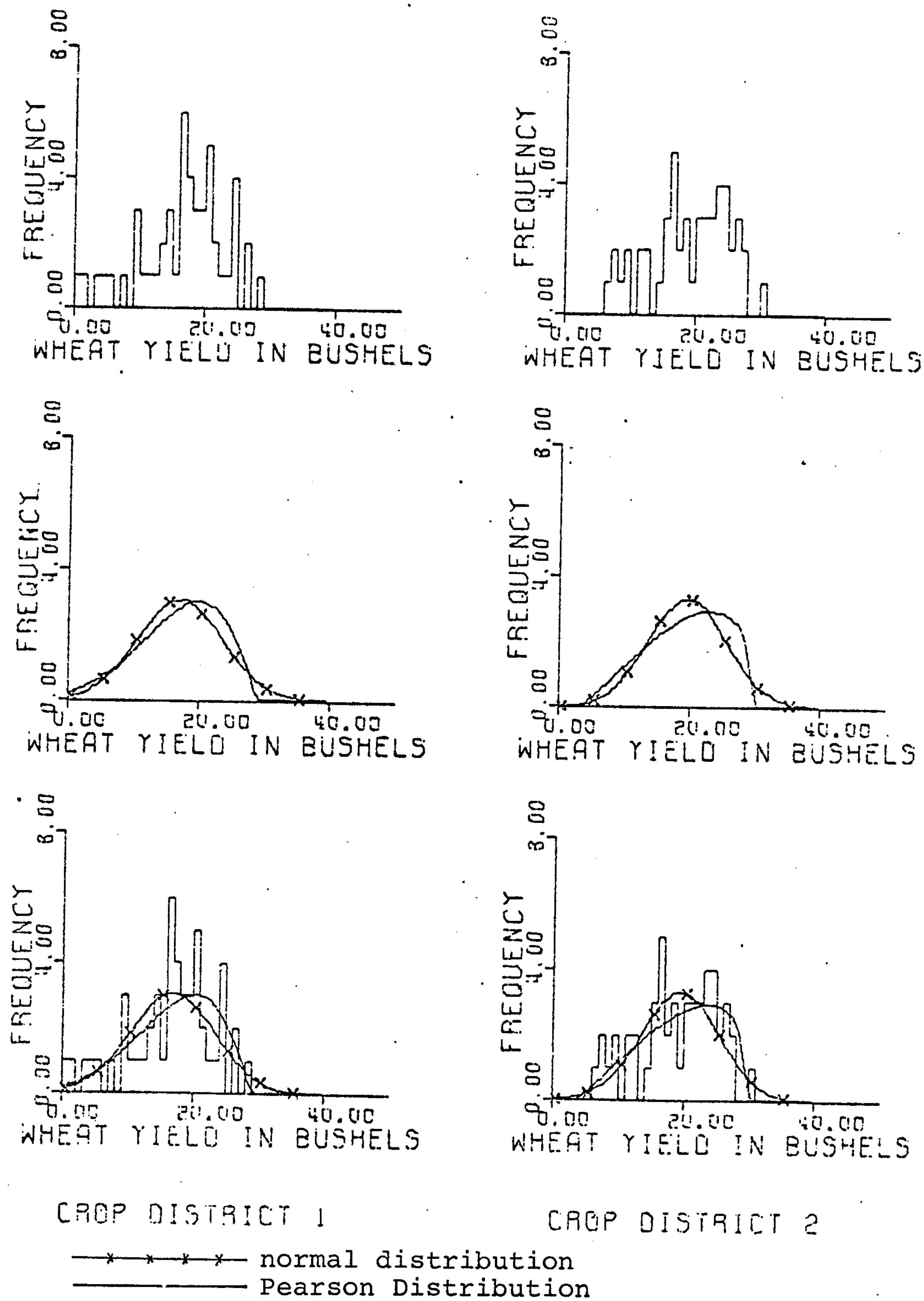
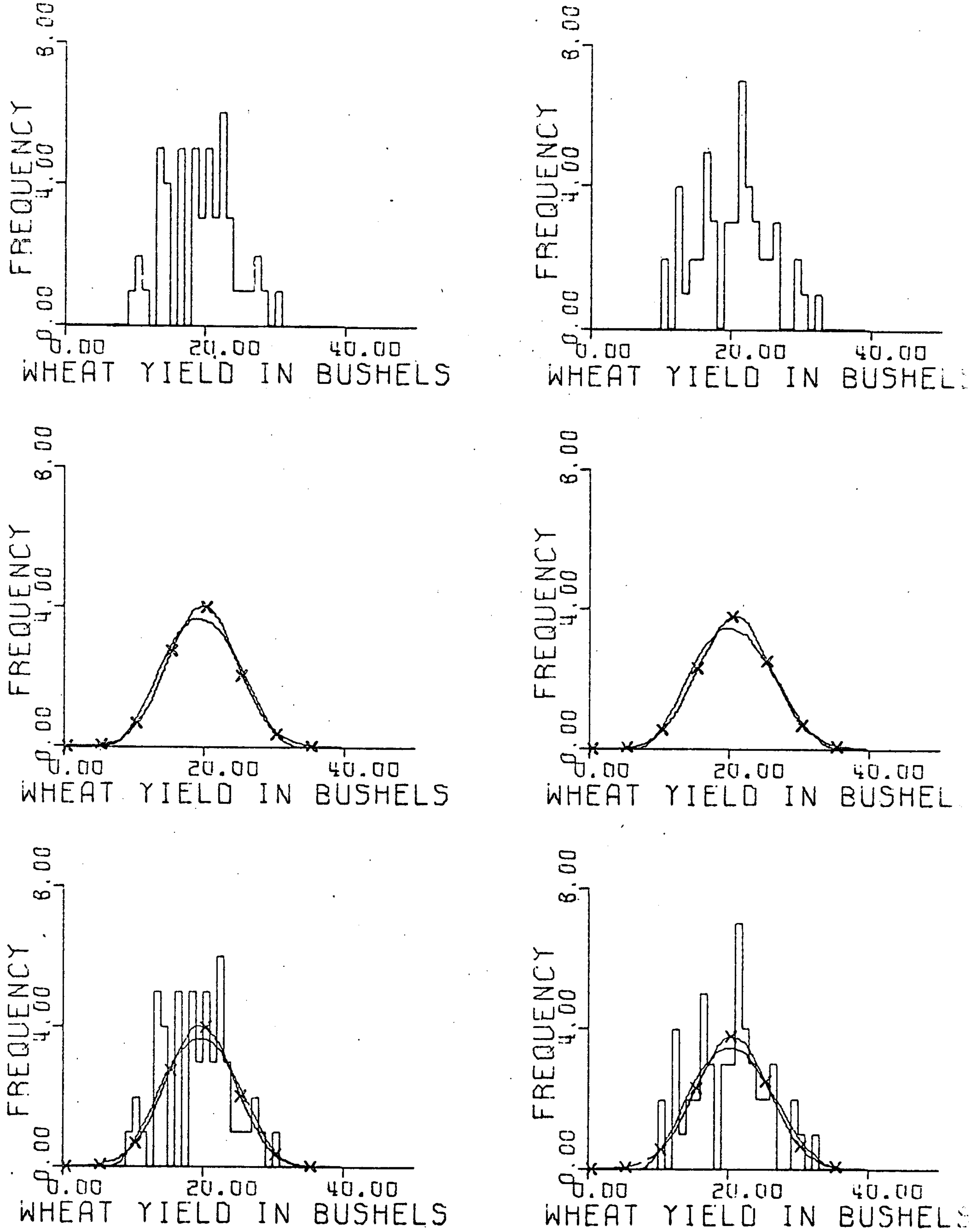


Figure 8. Histograms and Estimated Distributions of Wheat Yields, by Crop District, Manitoba

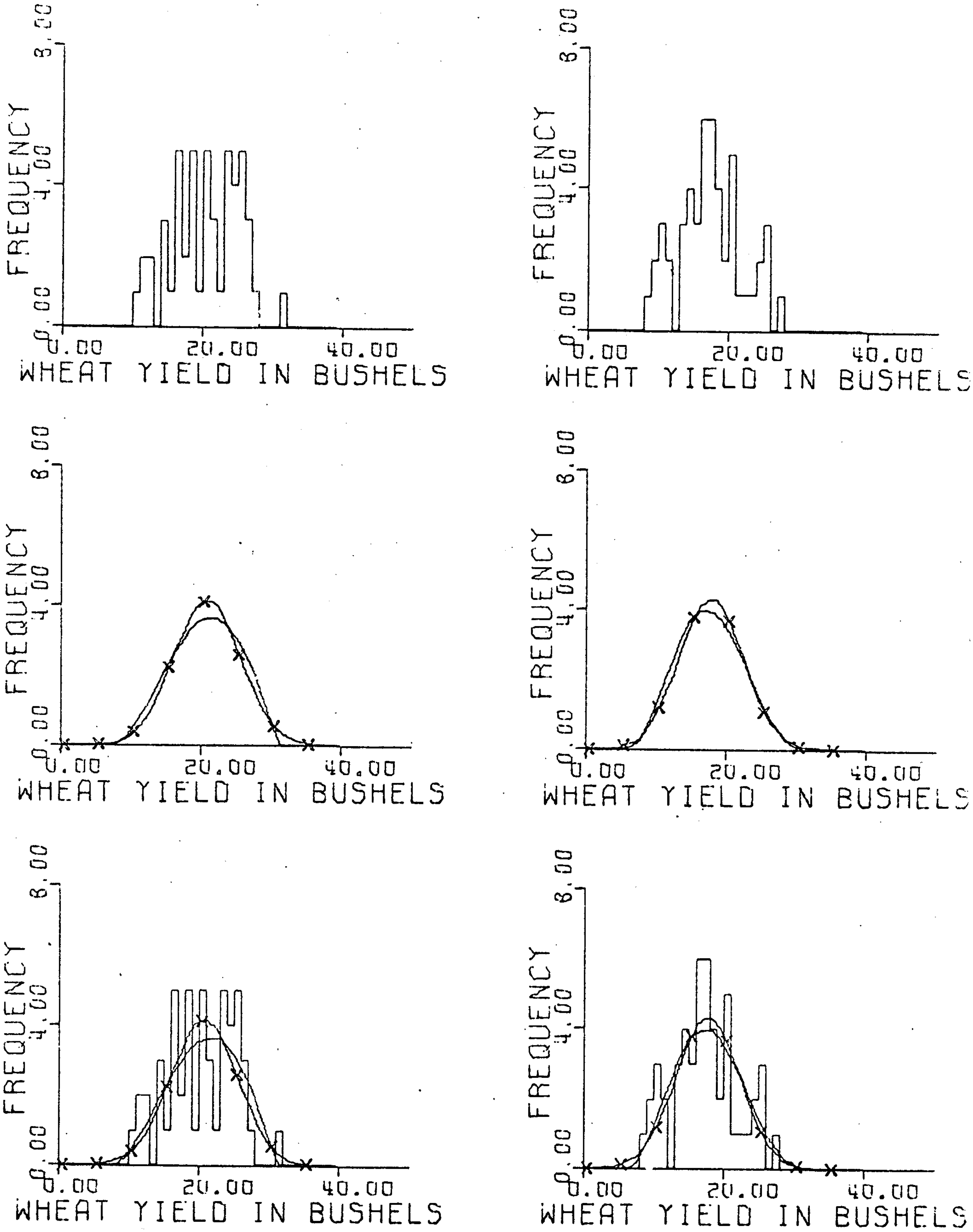
Source: Based on wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.



CROP DISTRICT 3

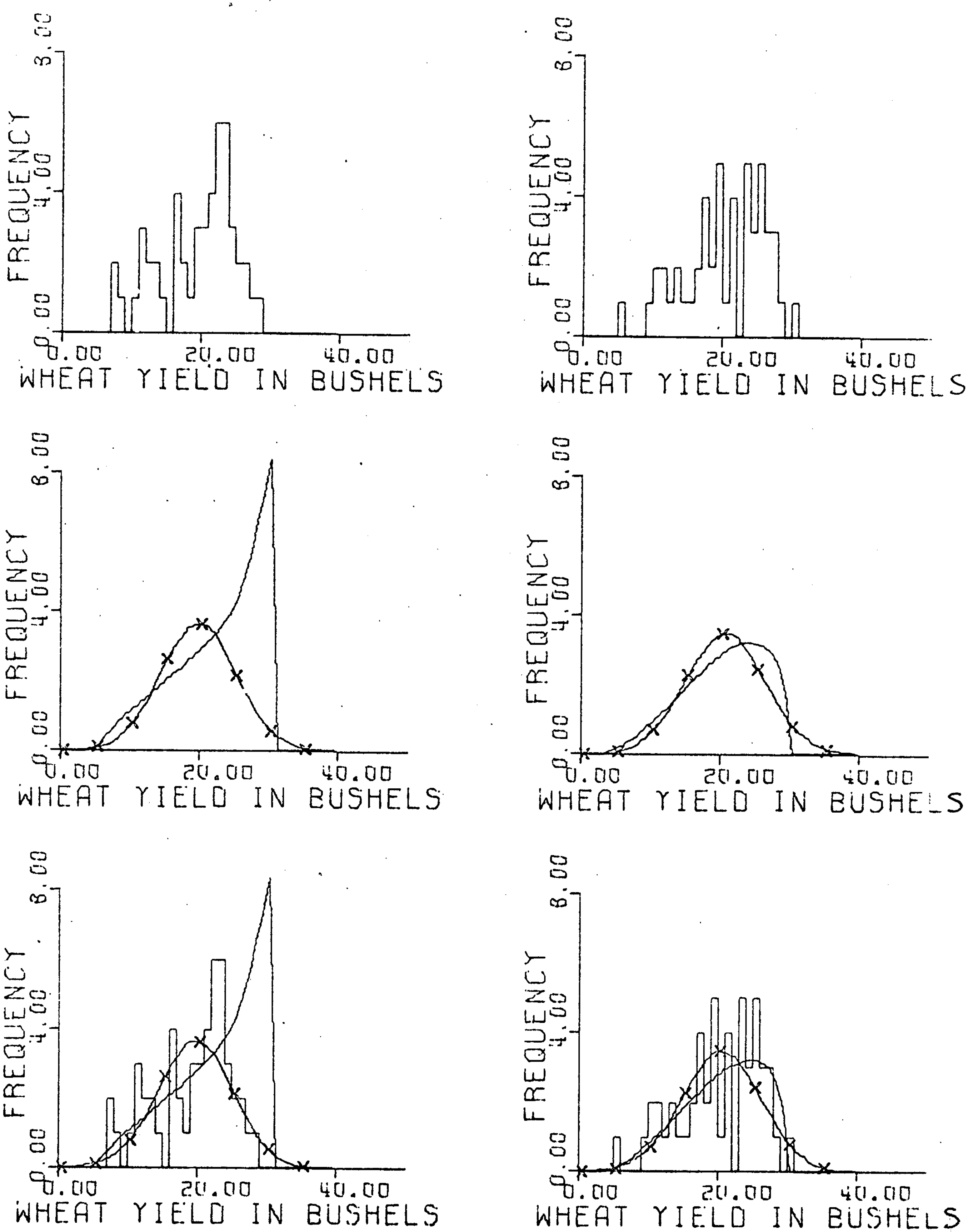
CROP DISTRICT 4

Figure 8 (continued)



CROP DISTRICT 5 CROP DISTRICT 6

Figure 8 (continued)



CROP DISTRICT 7

CROP DISTRICT 8

Figure 8 (continued)

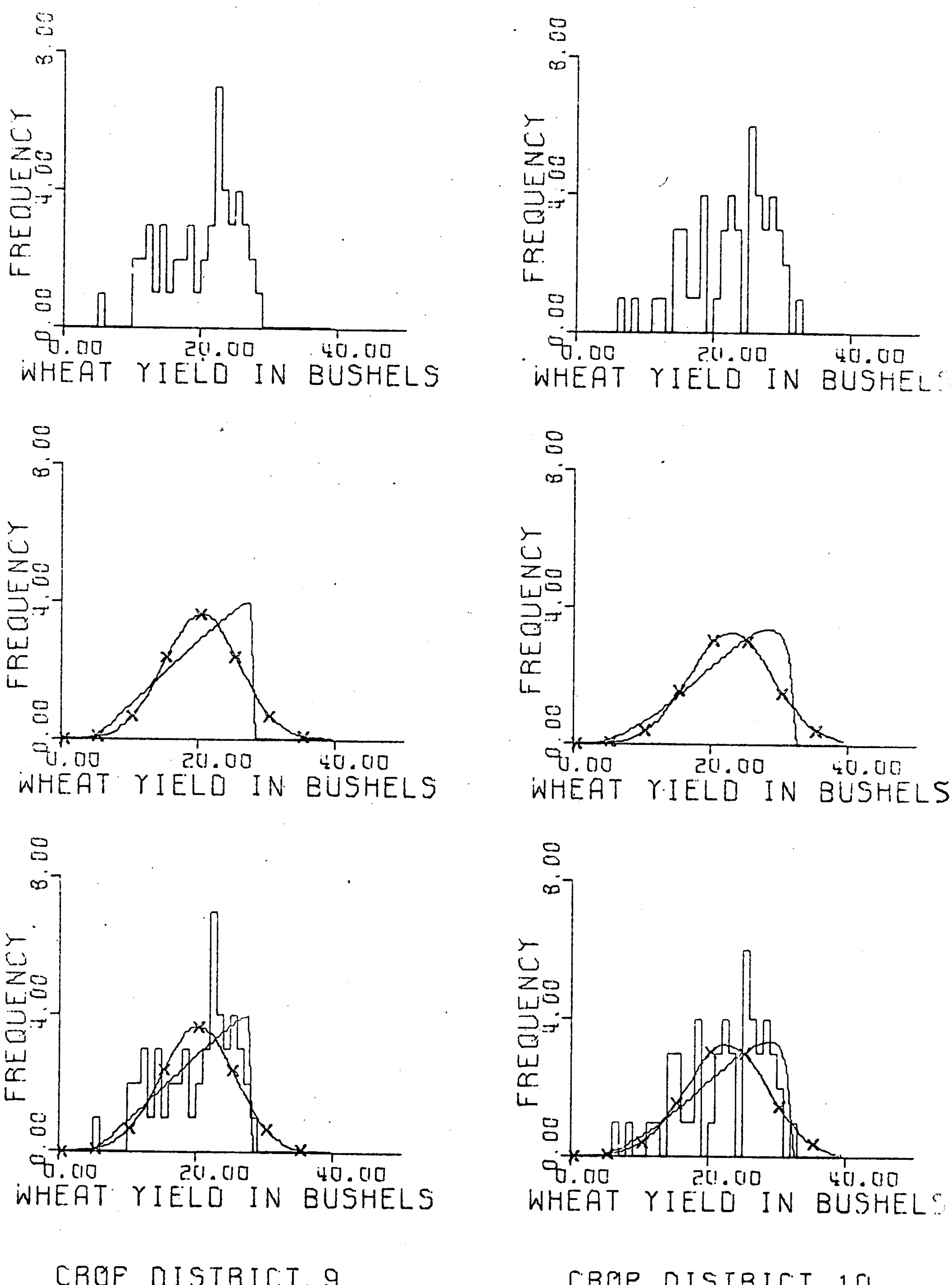
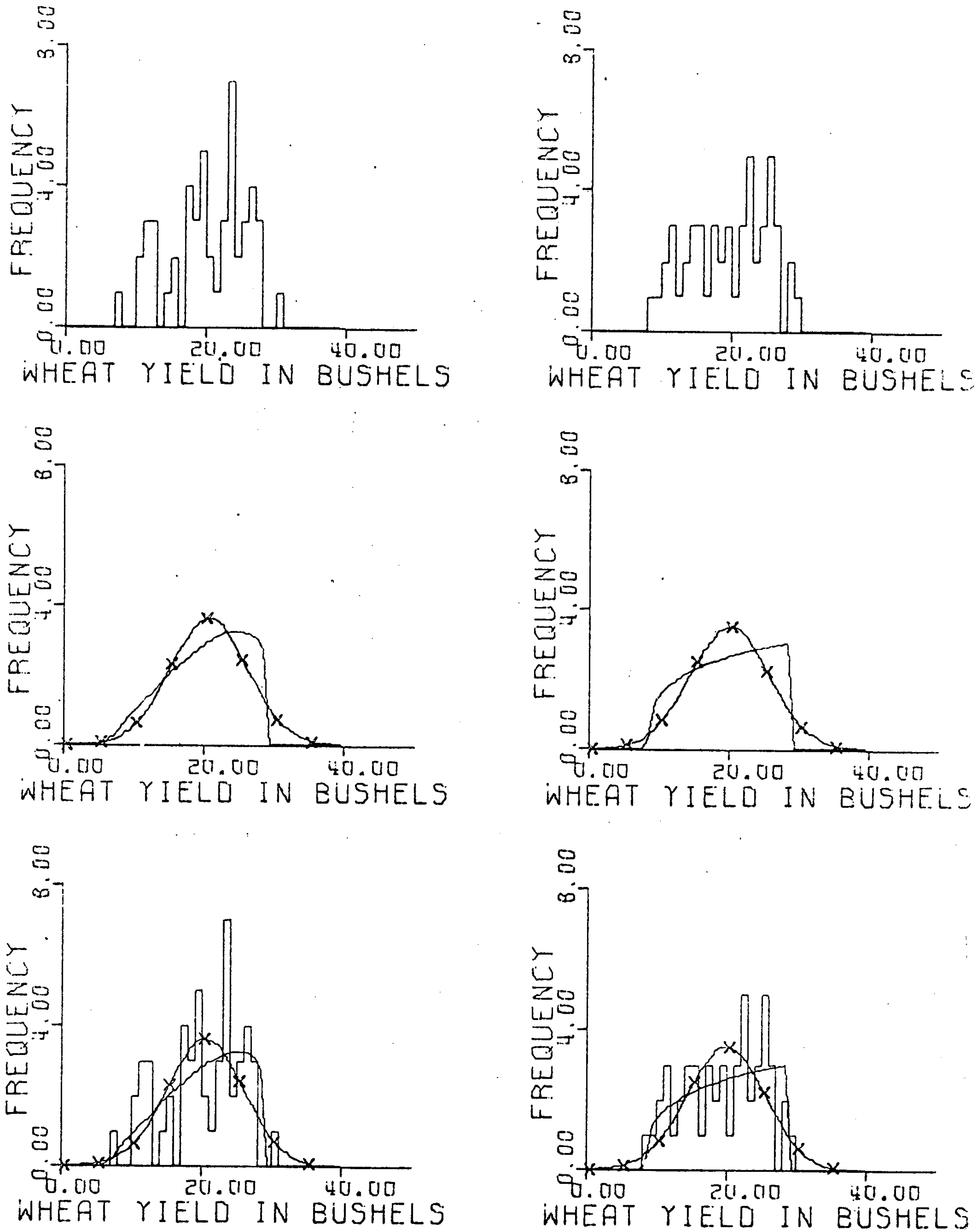
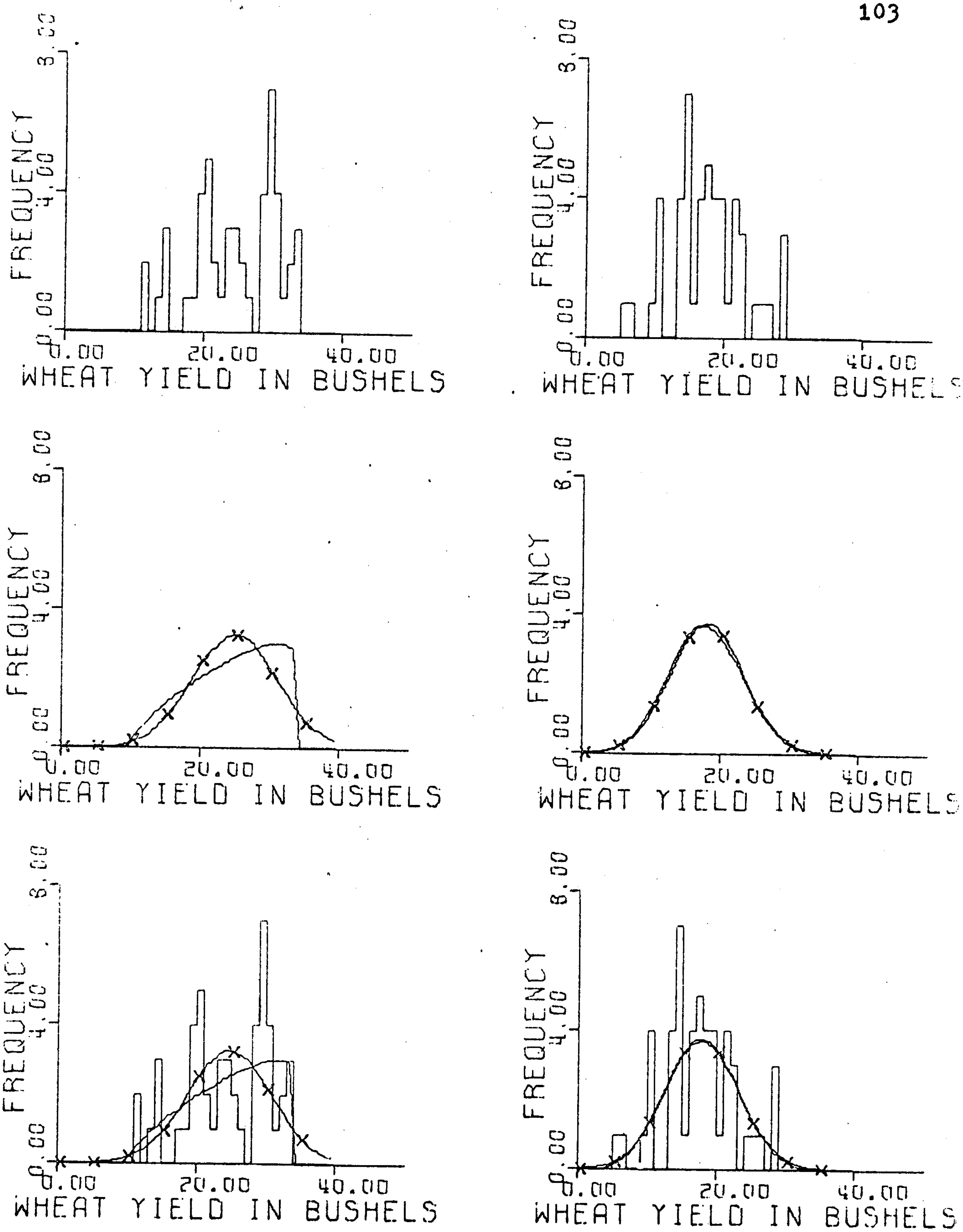


Figure 8 (continued)



CROP DISTRICT 11 CROP DISTRICT 12

Figure 8 (continued)



CROP DISTRICT 13

CROP DISTRICT 14

Figure 8 (continued)

IV.3. Test of Goodness of Fit

The chi-square test may be used to determine whether it is justifiable to approximate an observed frequency distribution by means of an estimated distribution. Formally, the hypothesis to be tested is as follows:

$$(41) \qquad H_0: \qquad f(x) = f^*(x)$$

where f(x) is the estimated distribution and $f^*(x)$ is the observed distribution. It is believed that this test can be applied to determine which distribution, for example, an estimated normal distribution or an estimated Pearson distribution, is the better one to represent the observed distribution of wheat yields.

The chi-square statistic, X^2, used to measure the discrepancy between the observed and the estimated distributions is calculated by the following formula:

$$(42) \qquad X^2 = \frac{(o_1-e_1)^2}{e_1} + \frac{(o_2-e_2)^2}{e_2} + - - - + \frac{(o_n-e_n)^2}{e_n}$$

where o_i is the observed frequency and e_i is the estimated frequency at the i-th interval of the distribution, providing the range of the distribution is divided into n intervals.

The observed distribution is seldom exactly equal to the estimated distribution. If it were, X^2 would be equal to zero. The value of X^2 increases as the discrepancy between the observed and the estimated distributions increases, and a large X^2 would lead to reject the hypothesis that the estimated distribution is equal to or consistent with the observed distribution. When chi-square statistics from two estimated distributions are compared, the smaller one indicates that the estimated distribution from which it came is the closer of the two to the observed distribution.

Using the observed and estimated frequency distributions, the chi-squares have been calculated for the 14 crop districts and are shown in Table 4. Column (2) of this table shows the chi-square statistic when the observed distribution is compared with an estimated normal distribution. Column (3) shows chi-square when the observed distribution is compared with an estimated Pearson distribution. It can be seen from the table that the chi-square statistic, when an estimated normal distribution is compared, is greater than that when an Pearson distribution is compared, except for crop district 5. For example for crop district 1, the chi-square statistic, when a normal distribution is compared, is 34.01, while it is 25.92 when an estimated Pearson distribution is compared. These results suggest that the estimated Pearson distribution is better than the estimated normal distribution in representing the observed distribution of wheat yields.

Table 4

Comparison of An Estimated Normal Distribution with An Estimated Pearson Distribution for Wheat Yields in Terms of X^2-Statistic

Crop District	X^2-Statistic			
	Normal Distribution		Pearson Distribution	
		d.f.		d.f.
1	34.01	37	25.92	35
2	23.98	34	16.58	35
3	26.90	30	24.11	32
4	29.26	30	25.71	32
5	24.20	29	37.43	31
6	19.61	31	16.98	33
7	30.65	32	19.05	34
8	29.18	34	21.05	34
9	30.78	32	22.16	33
10	38.03	34	26.86	33
11	34.51	31	24.66	33
12	20.48	29	13.61	34
13	48.31	27	31.11	31
14	40.81	36	40.05	35

Note: The number of intervals or classes used in calculating the X^2-statistic is determined by the frequency of the estimated distribution, if the frequency of the estimated distribution is less than 0.01, then that class is not counted. The computer program for calculating the X^2-statistics is included in Appendix I.

Source: Based on wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.

If the estimated Pearson distribution can represent the observed distribution of crop yields better than a normal distribution, it would imply that by using an estimated Pearson distribution the premium rate for crop insurance could be more accurately estimated than by using a normal distribution. In other words, if an estimated Pearson distribution were used the premium rate schedule would be more consistent with the risk involved in the operation of a crop insurance program. In the following section, the estimated distributions will be used as the statistical base for premium ratemaking. Premium rates calculated using different estimated distributions will be compared.

IV.4. Estimated Pure Premium Rate

As discussed in Chapter I, the pure premium rate is determined by the following formula,

$$P = \int_0^c (c - x) f(x) \, dx \tag{43}$$

where f(x) is the density function of the crop yield, x, and c is the insured yield per acre. The density function of the crop yield, for example wheat, can be estimated as shown in Chapter IV.2. The insured yield per acre can be determined using the current coverage level; such as 60, 70, and 80 percent of the average yield per acre. Therefore, the pure premium rate for the insured crop, e.g., wheat, can be estimated by integrating equation (43).

Using the estimated Pearson distribution and the estimated normal distribution of wheat yields in the 14 crop districts, pure premium rates for wheat have been calculated at different coverage levels and shown in Table 5.[52]

Column (2) of Table 5 shows the average wheat yield per acre for each crop district. It is the same as column (2) of Table 2. Column (3) of Table 5 indicates the coverage level, e.g., 60, 70, or 80 percent. Column (4) shows the insured wheat yields at different coverage levels for each crop district. The insured yields are calculated by multiplying the average wheat yield by the coverage level. For example, for crop district 1 at 60 percent coverage level, the insured yield is 10.21 bushels per acre. This is the average yield, 17.02 bushels per acre, times the coverage level, 60 percent. Column (5) of Table 5 shows the premium rate per acre calculated using equation (43) and an estimated normal distribution, while column (6) shows the premium rate using an estimated Pearson distribution. For example, in crop district 1 at 60 percent coverage, using an estimated normal distribution the premium rate is 0.4383 bushels per acre and using an estimated Pearson distribution the premium rate is 0.6564 bushels per acre. Column (7) shows the difference

[52] The Trapezoidal Rule, for approximate integration was used. It is discussed in detail in Appendix III.

Table 5

Comparison of Pure Premium Rates Calculated From An Estimated Normal Distribution and An Estimated Pearson Distribution for Wheat Yields
(bushels/acre)

Crop District (1)	Average Yield (2)	Coverage Level (3)	Insured Yield (4)	Pure Premium Rate: Normal (5)	Pearson (6)	Difference: (7)	(8) (7)/(6)
		%					%
1	17.02	60	10.21	0.4383	0.6564	-0.2181	-33.23
		70	11.91	0.7372	0.9721	-0.2349	-24.16
		80	13.62	1.1665	1.3941	-0.2276	-16.32
2	19.54	60	11.72	0.2620	0.3333	-0.0713	-21.39
		70	13.68	0.5105	0.6374	-0.1269	-19.90
		80	15.63	0.9165	1.0852	-0.1687	-15.54
3	19.78	60	11.87	0.1113	0.0743	0.0370	49.79
		70	13.85	0.2714	0.2438	0.0276	11.32
		80	15.82	0.5856	0.5884	-0.0028	-0.47
4	20.74	60	12.44	0.1266	0.0740	0.0526	71.08
		70	14.52	0.3020	0.2531	0.0489	19.32
		80	16.59	0.6400	0.6217	0.0183	2.94
5	20.86	60	12.52	0.0804	0.0633	0.0171	27.01
		70	14.60	0.2180	0.2238	-0.0058	-2.59
		80	16.69	0.5120	0.5587	-0.0467	-8.35
6	17.84	60	10.70	0.1167	0.0679	0.0488	71.87
		70	12.49	0.2731	0.2265	0.0466	20.57
		80	14.27	0.5703	0.5492	0.0211	3.84
7	19.90	60	11.94	0.1707	0.2558	-0.0851	-33.26
		70	13.93	0.3719	0.5381	-0.1662	-30.88
		80	15.92	0.7325	0.9396	-0.2071	-22.04

Source: Based on wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.

Table 5 (continued)

8	20.74	60	12.44	0.1877	0.2646	-0.0769	-29.06
		70	14.52	0.4033	0.5323	-0.1290	-24.23
		80	16.59	0.7854	0.9503	-0.1649	-17.35
9	20.46	60	12.28	0.1590	0.2369	-0.0779	-32.88
		70	14.32	0.3555	0.4983	-0.1428	-28.65
		80	16.37	0.7151	0.9051	-0.1900	-20.99
10	22.90	60	13.74	0.1754	0.2863	-0.1109	-38.73
		70	16.03	0.3936	0.5670	-0.1734	-30.58
		80	18.32	0.7943	1.0046	-0.2103	-20.93
11	20.66	60	12.39	0.1577	0.1955	-0.0378	-19.33
		70	14.46	0.3543	0.4519	-0.0976	-21.59
		80	16.53	0.7154	0.8667	-0.1513	-17.45
12	20.08	60	12.05	0.1937	0.1611	0.0326	20.23
		70	14.06	0.4095	0.4503	-0.0408	-9.06
		80	16.06	0.7868	0.9126	-0.1258	-13.78
13	24.96	60	14.98	0.1283	0.1358	-0.0075	-5.52
		70	17.47	0.3208	0.4004	-0.0796	-19.88
		80	19.97	0.7063	0.8661	-0.1598	-18.45
14	18.02	60	10.81	0.2013	0.1788	0.0225	12.58
		70	12.61	0.4101	0.3891	0.0210	5.39
		80	14.42	0.7643	0.7520	0.0123	1.63

between these two premium rates. A negative value in column (7) indicates that the premium rate is less using an estimated normal distribution than it is using an estimated Pearson distribution. The results of previous section indicated that using an estimated Pearson distribution the premium rate schedule would be more consistent with the risk involved in the operation of a crop insurance program. A negative value in column (7) implies that the premium rate per acre is underestimated using an estimated normal distribution for wheat yields. For the same reason, a positive value in column (7) would imply that the premium rate per acre is overestimated by using an estimated normal distribution for wheat yields.

As shown in column (7) bias in estimating the premium rate may range from 0.0526 to -0.2181 bushels per acre, 0.0489 to -0.2349 bushels per acre, and 0.0211 to -0.2276 bushels per acre for 60, 70, and 80 percent coverages, respectively. Column (8) shows the percentage of the underestimation, by a negative value, or overestimation, by a positive value, of the premium rate per acre in terms of the premium rate calculated using an estimated Pearson distribution. According to column (8) bias in estimating the premium rate can vary from 71.87 to -38.73 percent, 20.57 to -30.88 percent, and 3.84 to -22.04 percent of the premium rate using an estimated Pearson distribution for 60, 70, and 80 percent coverages.

Reviewing the figures in columns (7) and (8), it can be noticed that in most cases the premium rate has been underestimated when an estimated normal distribution is used. Persistent underestimation of the premium caused by using an estimated normal distribution may be explained below.

On the average for the 14 crop districts, the mean and standard deviation of wheat yields are about 20 and 5 bushels per acre respectively. Therefore, one standard deviation from the mean is located at the insurance coverage level of 80 percent. This is the maximum coverage level of the current insurance program. It is also the location of the inflection point when the wheat yields are assumed to be normally distributed. The conclusion derived in Section III.3.3. indicated that if the actual distribution of crop yields does not have an inflection point, then using a normal distribution could cause an underestimation of the probability of crop yield being below the level of the inflection point. This could be the reason for the underestimation of the premium rate in most cases as shown in Table 5.

However, overestimation of the premium rate did occur in some cases, e.g., districts 4, and 6. This is due to the fact that the infinite range of the normal distribution may overstate the lower end of a yield distribution and, therefore, causes overestimation of the probability of extremely low yields.

With reference to the results of this chapter, it may be concluded that the distribution of crop yields may be of any kind; a normal distribution may occur as a special case. Assuming that crop yields are normally distributed may simplify the computational task of estimating the premium rate. However, this simplification will bias the estimation of premium rates and this in turn will cause operational problems for the crop insurance program. These results suggest that Pearson distributions should be used, since they permit an estimate of the probability distribution without specifying the algebraic form of the distribution prior to the estimation process. Consequently, the administrators of the crop insurance program should be able to assess more accurately the risk involved in the operation of the program.

CHAPTER V

AN EXPERIENCE RATING SYSTEM FOR A CROP INSURANCE PROGRAM

In most of the currently operating crop insurance programs, the premium rate is charged on an area basis. Furthermore, aggregate data are used in estimating the premium rate. This chapter deals with the significance of individual yield distributions and suggests a method for developing an experience rating system for a crop insurance program. The experience rating system will reallocate the indemnity costs of the program among the insured farmers according to individual farmers' performance. First, the theoretical foundation of an experience rating system is discussed and significant differences in yield among farmers are statistically tested. Then, the current approaches to rate insured farmers are presented. Their theoretical basis and limitations are explained. Finally, a method is recommended which considers not only the frequencies of the indemnity claims but also their levels.

V.1. Theoretical Foundation

Crop insurance has been available for more than ten years in some areas of Manitoba. For crop insurance purposes the Province is divided into fourteen risk areas. Criteria used for the delination are climate, topography, and the history of crop production. The insurance premium for an individual farmer depends on the particular risk area in which his farm is located and on the estimated productivity of the land. Farmers in the same risk area and with the same soil productivity rating pay the same amount of premium for the same coverage per acre.[53] This policy implies that farms with the same soil productivity and in the same risk area would have the same yield expectation; i.e., the average yield is the same for all these farms and the variation in yield from this average is also the same for all. If these assumptions do not hold, the following situation may develop. If average yields are not the same for all farms in an area, farmers with low average yield receive larger indemnities than farmers with high average yield. Similarly, if the variations in yield differ among farms in an area, farmers with larger yield variation receive indemnities more often than the farmers with smaller yield variation. Therefore, charging the same premium to all farmers will discourage low risk farmers from participating in the program and will only encourage high risk farmers to buy insurance. Consequently, crop insurance operating on a voluntary

---

[53] Detailed explanation of the soil rating can be found in J.E.B. Campbell, "An Appraisal of Soil Classifications for Crop Insurance Purposes" (Winnipeg: Manitoba Crop Insurance Corporation, 1969).

basis will involve a group of farmers with lower average yield and greater variation in yields than average farmers. Difficulties would be experienced in attracting a large number of farmers, especially good farmers, and of maintaining a financially viable program. To test for differences in the expected yields of farms in the same risk area and with the same soil productivity, the technique of analysis of variance is used.

The data used in the analysis were obtained from the Manitoba Crop Insurance Corporation. The Corporation collected the information through crop insurance agents who complete a research questionnaire with each insured farmer. To test for differences in yield variation on different farm units within the same soil type rating, data from individual farms were used. Data for an individual farm were identified by the individual's insurance contract number and by the land description. These two criteria ensured that each year the data used in the analysis were collected on the same parcel of land and that it was operated by the same farmer.

Three risk areas in Manitoba, number 4, 7, and 13, were selected for the analysis. Figure 11 is a map of the crop insurance risk areas. Risk Area 4 is located in south central Manitoba. It is an area where crop yield variation is considered to be fairly

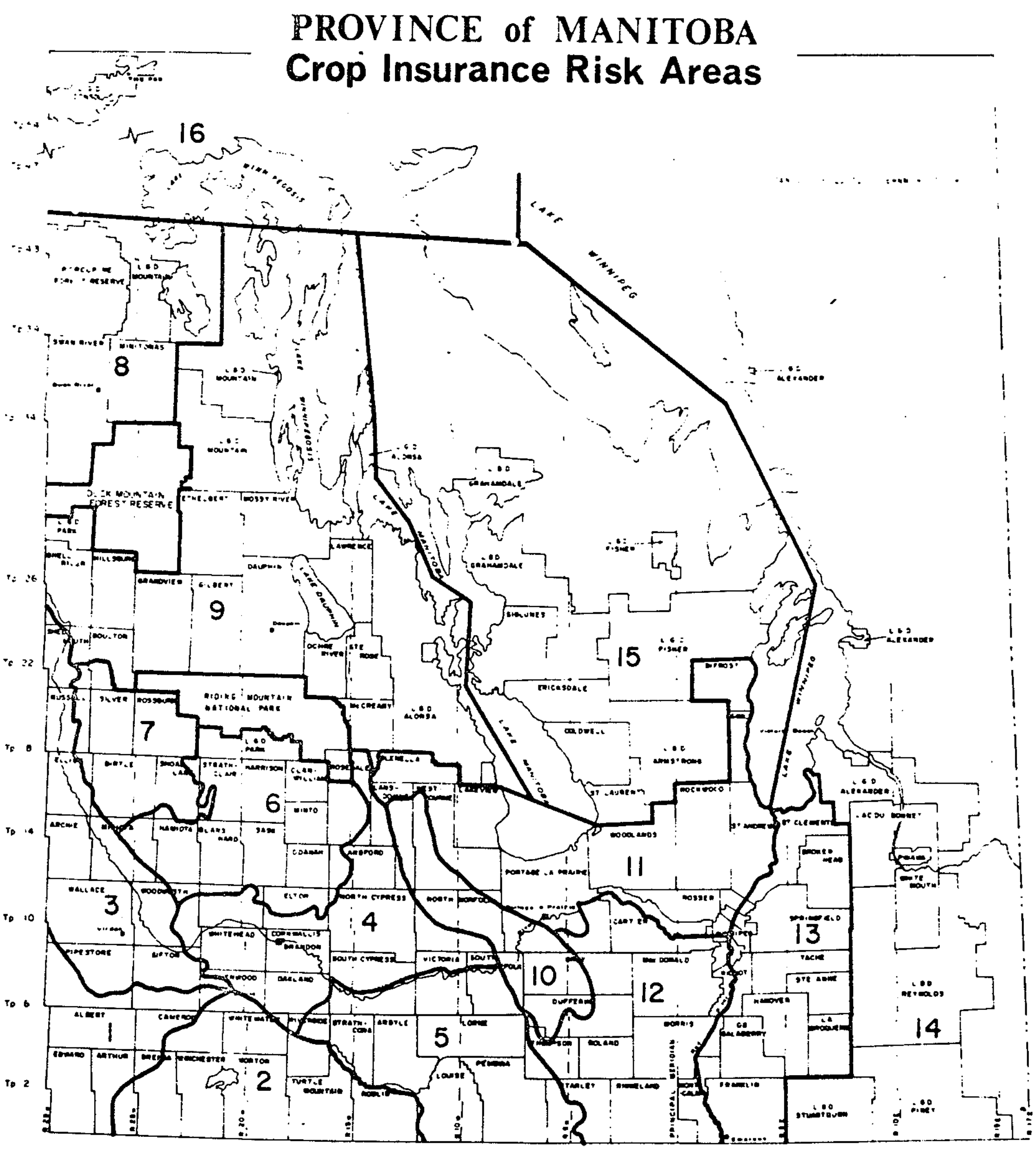


Figure 9. Manitoba Crop Insurance Risk Area (1970)

Source: Manitoba Crop Insurance Corporation, Winnipeg.

small.[54] Risk Area 7 is located in Western Manitoba, and is an area with frequent hail damage. Risk Area 13 is located in Eastern Manitoba, in the Red River Valley. The soils in this area contain a high percentage of clay, and conditions which are either too wet or too dry seriously handicap field operations. As a result, yields are rather unstable from year to year. Because of limitations in data available, yield data for only soil rating C for the three areas and rating F for Area 4 could be used in the analysis.

The sample from Risk Area 13 soil rating C is presented in Table 6.[55] From this table it can be seen that the average yield for 1966 through 1968 on the 24 farms in the sample was 22.22 bushels per acre. Average yields for 1966, 1967, and 1968 were 18.78, 25.88, and 22.00 bushels per acre, respectively. The average yield of the 24 farms over these three years has a range of 21.00 bushels per acre with a minimum of 12.33 for farms number 6 and 21 and a maximum of 33.33 for farm number 11.

The hypothesis to be tested by analysis of variance is that the means of the wheat yield distributions on

---

[54] This is according to the experience of the Manitoba Crop Insurance Corporation.

[55] The sample from Risk Areas 4 and 7 are presented in Appendix III.

Table 6

Wheat Yields of Sample Farms in Manitoba
Risk Area 13, Soil Rating C

Farm	Wheat Yield (bushels per acre) 1966	1967	1968	Average
1	25	25	24	24.67
2	20	18	20	19.33
3	20	20	20	20.00
4	14	18	17	16.33
5	22	23	30	25.00
6	15	5	17	12.33
7	10	42	3	18.33
8	18	30	27	25.00
9	18	30	25	24.33
10	12	37	20	23.00
11	30	40	30	33.33
12	30	37	25	30.67
13	18	28	27	24.33
14	28	28	35	29.33
15	23	25	25	24.33
16	20	25	30	25.00
17	25	25	40	30.00
18	13	16	9	12.67
19	18	25	20	21.00
20	15	35	11	20.33
21	11	16	10	12.33
22	14	30	18	20.67
23	20	28	30	26.00
24	12	18	15	15.00
Average	18.79	25.88	22.00	22.22

Source: Manitoba Crop Insurance Corporation, Winnipeg.

different farms with the same type of soil are the same, that is,

$$(44) \qquad H_0: \quad u_1 = u_2 = \ldots\ldots\ldots\ldots = u_i = \ldots\ldots = u_k$$

where u_i is the mean of the wheat yield distribution for the i-th farm and k is the number of farms.[56]

The results for different soil ratings and risk areas are shown in Tables 10 through 13. Using a one-tailed F-test, the hypothesis of equality of the means of wheat yield distributions among different farms was rejected at the one percent level of significance for each of the three risk areas and for the two soil productivity ratings. The conclusion is that the means of the wheat yield distributions on different farms on the same soil type are not equal. It implies that, within the same risk area, farmers using soil of the same productivity do not necessarily have the same distribution of crop yields. Since the crop insurance contract is with the individual farm, the premium rate should be based upon the distribution of crop yields on the individual farm. If this is unknown, it is necessary to estimate it and this can be handled by Pearson distributions.

---

[56] A more sophisticated model can be used in the analysis of variance. However, the model of two-way cross-classification with exactly one observation per cell serves the purpose of testing the hypothesis of equality of the means of wheat yield distributions among different farms. M.G. Kendall and A. Stuart, op. cit., Vol. III, 1966, p. 23.

Table 7

Test of Wheat Yield Variation for
Manitoba Risk Area 4, Soil Rating C

Analysis of Variance

Source of Variation	Sum of Squares	d.f.	Mean Square	F-ratio
Years	337.06	2	168.53	3.35
Farms	12761.44	105	121.54	2.42*
Residual	10544.31	210	50.21	
Total	23642.81	317		

*significant at 1% using a one-tailed test.

Source: Based on data from Manitoba Crop Insurance Corporation, Winnipeg.

Table 8

Test of Wheat Yield Variation for
Manitoba Risk Area 4, Soil Rating F

Analysis of Variance

Source of Variation	Sum of Squares	d.f.	Mean Square	F-ratio
Years	395.13	2	197.56	4.06
Farms	5016.38	49	102.38	2.10*
Residual	4774.25	98	48.72	
Total	10185.75	149		

*significant at 1% using a one-tailed test.

Source: Based on data from Manitoba Crop Insurance corporation, Winnipeg.

Table 9

Test of Wheat Yield Variation for
Manitoba Risk Area 7, Soil Rating C

Analysis of Variance

Source of Variation	Sum of Squares	d.f.	Mean Square	F-ratio
Years	310.44	2	155.22	4.24
Farms	2709.06	29	93.42	2.55*
Residual	2120.94	58	36.57	
Total	5140.44	89		

*significant at 1% using a one-tailed test.

Source: Based on data from Manitoba Crop Insurance Corporation, Winnipeg.

Table 10

Test of Wheat Yield Variation for
Manitoba Risk Area 13, Soil Rating C

Analysis of Variance

Source of Variation	Sum of Squares	d.f.	Mean Square	F-ratio
Years	603.86	2	301.93	7.16
Farms	2318.45	23	100.80	2.39*
Residual	1940.14	46	42.18	
Total	4862.45	71		

*significant at 1% using a one-tailed test.

Source: Based on data from Manitoba Crop Insurance Corporation, Winnipeg.

However, at the beginning of a crop insurance program there may not be enough yield information on an individual farm available to estimate the distribution of crop yields for setting the individual premium rate. Moreover, if the accumulated information was available, it might be found too costly and tedious to incorporate additional data to revise the estimated distribution and premium rate every year. In recognition of these factors, it seems justifiable for the insurance agency to set an initial premium rate and attach an experience rating table to it adjusting the premium as well as the corresponding insured yield according to the actual yields on the individual farm. By this approach, the premium charged by the insurance agency will gradually reflect the risk involved in the operation of the insurance scheme as well as the possible non-stationary changes in the yield distribution.

V.2. Current Approaches

Most crop insurance programs offer some kind of discounts, for example in Manitoba, for "continuous participation and good experience."[57] This discount gradually reduces the premium of the low risk farmer who makes modest indemnity claims and it also implies that superior farming practices frequently reduce the

[57]Manitoba Crop Insurance Corporation, Annual Report, 1969, op. cit., p. 3.

risk of crop losses. One of the arguments in favor of offering a discount to low risk farmers is that it is more equitable than charging a uniform premium rate irrespective of the loss experience of the individual farmer.

V.2.1. Experience Tables

The United States Crop Insurance Corporation has applied the good experience discount for all insured crops since 1960.[58] Policyholders earn a 5 percent discount on the premium after one or two consecutive years without a crop loss in terms of the insured yield, 10 percent after three or four years, 15 percent after five years, 20 percent after six years, and 25 percent after seven or more years.[59] A policyholder with seven or more years of good experience reduces to an equivalent of four years of good experience when he is paid a loss indemnity, that is, he will receive a 10 percent discount on his next year premium. Those with less than seven years good experience, for instance, say six years, will be reduced to an equivalent of three years of good experience when they are paid a loss indemnity.[60]

---

[58] W. Bar and R. Dougan, Crop Insurance, Agricultural Extension Service Publications (Ohio: The Ohio State University, 1960).

[59] G.A. Smith, E.L. Ladue, and R.S. Smith, op.cit., p. 18.

[60] W. Bar and R. Dougan, op. cit., p. 24.

The Manitoba Crop Insurance Corporation has applied a discount schedule for continuous participation and good experience since 1961. If the policyholder is in the program for one or several years without making a claim, then his premium will be discounted as shown in Table 11. For example, a farmer would be offered a discount of 2 percent of his premium if he did not claim indemnity for one year. The discount would increase to 40 percent of the premium if a farmer did not claim an indemnity for 9 years. However, an insured farmer may still be eligible for a discount following a loss year. A reclassification index (R.I.) has been designed for this purpose as follows;

$$\text{(45)} \qquad \text{R.I.} = \frac{\text{A - amount of indemnity paid}}{\text{B}}$$

where A is the cumulative total premium balance credited and includes the current year premium, and B is defined as

$$\text{(46)} \qquad \text{B} = \frac{\text{A}}{\text{number of no loss years}}$$

B is equivalent to an estimate of the net premium which would be received per no loss year by the Insurance Corporation. R.I. is designed to calculate an adjusted number of no loss years taking account of the current indemnity payment to the farmer. Once the R.I. has

Table 11

Discount Rate Schedule,
Manitoba Crop Insurance Corporation

Number of Years without Claim	Reclassification Index	Discount Rate (%)
0	less than 1.00	0
1	1.00 - - 1.99	2
2	2.00 - 2.99	3
3	3.00 - 3.99	5
4	4.00 - 4.99	8
5	5.00 - 5.99	12
6	6.00 - 6.99	17
7	7.00 - 7.99	23
8	8.00 - 8.99	30
9 or more	9.00 or over	40

Source: Manitoba Crop Insurance Corporation, Winnipeg.

been determined the discount for the following year can be read from Table 11. For example, a reclassification index between 1.00 and 1.99 is equivalent to one year without making a claim, thus a 2 percent discount of premium will be offered.[61]

The Ontario Crop Insurance Commission offers a constant discount schedule, allowing five percent discount for each year without making a claim to a maximum of 20 percent.[62]

Instead of offering a discount rate to the low risk farmers, the crop insurance program in Alberta has initiated a coverage adjustment plan whereby farmers who have had three or more loss years in the latest six-year period and have had an unfavorable loss to premium ratio, would have their coverage adjusted downward on the following basis:

Loss/premium Ratio	Coverage Reduction
1- 2.99	10%
3- 5.99	20%
6 and over	30%

[61] Manitoba Crop Insurance Corporation, "Administrative Regulations Re Experience Discount" (Winnipeg: The Corporation, 1964). (Mimeographed.)

[62] M.H. Yeh, "Crop Insurance in Canada, 1960-69" (Winnipeg: Department of Agricultural Economics, University of Manitoba, 1970), p. 3.

The coverage adjustment plan is a move toward individual coverage, and is another means of recognizing greater or lower than average risk of the individual farmer within a given area.[63]

V.2.2. Limitations

With reference to the determination of current discount rates, two drawbacks should be noted. Firstly, there is no theoretical foundation for the determination of these discount or adjustment rates. It cannot be claimed, theoretically, whether the discount rate schedule adjusts the premium to a more equitable one or not. In particular, no one knows whether the discount rate does the work of approximating the loss probability of an aggregate normal distribution to the loss probability of an actual individual distribution. It has to take the risk of being inequitable and let actual experience test the soundness of the discount rate schedule.

Secondly, the discount rate does not differentiate between a farmer with an average level of yield expectation and a farmer with a higher level of yield expectation. What needs to be noted here is that the impact of future yield experience on the shape and the level of a distribution depends not only the frequency of yields falling above

---

[63] Alberta Hail and Crop Insurance Corporation, Third Annual Report, for Fiscal Year Ending March 31, 1971 (Calgary: The Corporation, 1972).

the insured yield but also to a large extent upon the level of these future yields. A farmer with yield variation within a range of say 20 percent of the average yield for the area should probably not be treated the same as a farmer with yields which always fall above the area average yield. The latter farmer should probably get a greater discount than the former at the same level of coverage. Besides discounting premium rate, consideration should be given to offering increased coverage or a coverage bonus to those farmers who had been loss-free over several years. Furthermore, when a farmer frequently makes indemnity claims, the insurance corporation should either adjust the insured yield downward and/or increase the premium charge.

V.3. Proposed Approach

In the field of insurance, experience rating systems are generally classified into two group; retrospective, and prospective.[64] The prospective experience rating system is proposed for the crop insurance program. Following presentation of the rationale in favor using this system in crop insurance, hypothetical illustrative examples are given.

V.3.1. Theoretical Formulation

In the retrospective experience rating system,

---

[64] H.L.Seal, op. cit., pp. 56-89.

a risk-loaded premium P_1 is charged to the insured. Depending on individual experience, low risk insured are reimbursed by means of dividend or bonus payments. If P_1 is the loaded premium, then

$$P_1 = P + L \tag{47}$$

where P is the pure premium rate and L is the risk loading. In other words, under this kind of system, a more than adequate premium is initially charged to the insured so that a dividend can be returned to the contractholders based on their actual claims. As a result, there is no need to have an accurate estimate of the pure premium rate. It is only the continuation of surpluses arising from the size of L that prompts the payment of dividends. Because entitlement to a dividend requires that the insured continuously sign a contract with the insurer, this has the advantage of preventing intermittent participation. However, besides the receipt of dividends being purely an expectation, this system requires higher initial premium contributions which may deter farmers' willingness to participate in the insurance. Moreover, since the crop insurance contract is with the individual farmer, the risk of reduction on his farm only determines his premium. The characteristics of mutuality in crop insurance in terms of distributing uncertain crop losses at one point in time among insured farmers is undoubtedly true. However, this aspect should not be confused with the redistribution of income characterized by most other insurance schemes. Crop production involves the use of resources. As a result, crop insurance which causes a redistribution of incomes will distort the efficiency of resource allocation. Consequently, even though there is no need to have an accurate estimate of the pure premium, the insurance agency will at some stage have difficulty in determining the amount of dividend payments.

Frequently there is not enough yield information on individual farms for estimating an accurate individual premium at the start of an insurance scheme. On the other hand, the revisions of estimated distributions for determining premium rates may prove too costly to do every year. Under such a situation a prospective experience rating system, which considers how an initial estimate of the probability distribution is changed when additional information is gathered, seems more appropriate and should be used. That is, an initial premium rate, P_i, is calculated using a base distribution, $f(x)$. When additional yield observations, $x_{t+1}, \ldots, x_{t+n}$, are available, a new distribution $f(x \mid x_{t+1}, \ldots\ldots, x_{t+n})$, which may differ from the base distribution, is estimated. Then after n years, the new premium rate, P_n, where,

$$P_n = \int_0^{c'} (c' - x) \, f(x \mid x_{t+1}, \ldots\ldots, x_{t+n}) \, dx \tag{48}$$

is charged to the insured farmer. When the insured yield is calculated as a percentage of the average yield, additional yield information may imply that a new coverage, c', is also necessary. By comparing P_i and P_n the experience table can be established. In other words, the experience rating system will consider not only the frequency of the indemnity claim but also the level of actual realized yields on the individual farm. Under such a theoretical structure, it is possible for the insurance scheme to make consistent adjustments to the premium rate charged to the individual farmer. Reestimation of the yield distribution allows for trends as well as removing irregular fluctuations in the observed yields. Therefore, some unnecessary adjustments may be avoided. With respect to this problem, Lundberg recommended that an experience rating system should be implemented at about five years to avoid unnecessary fluctuations in the premium rate.[65]

Examples are given in the following paragraphs using Manitoba district 14 to illustrate the prospective experience rating system. For this district, the premium rates based on a normal distribution are close to those based on the Pearson distribution.[66] This permits

---

[65] Ibid., p. 65.

[66] See Table 4 on page 106.

observation of how these results compare with the discount rates that are currently offered in Manitoba.

V.3.2. Illustrative Examples

To illustrate changes in distribution, four different hypothetical cases were selected, farms with yields respectively as follows; (a) near the average yield for 5 years, (b) near the average for 10 years, (c) above the average for 5 years, and (d) above the average for 10 years.[67] The original distribution for district 14 and the four hypothetical cases are shown in Table 12. The average yield is 18.02 bushels per acre with a standard deviation of 5.26 bushels per acre. When yields are said to be near the average, it is assumed they are evenly distributed around 18.02 bushels per acre, between the intervals of 15-16 and 19-20 bushels, with yields in one or two years in each interval. Thus the frequency distribution is assumed to be that shown in column (3) of Table 12 for case (a) with yields near the average for 5 years. Similarly, column (4), (5), and (6) show distributions for cases (b), (c), and (d) respectively.

The basic statistics for estimating the parameters of a Pearson distribution have been calculated and are shown in Table 13. The result reveals possible

[67]The average yield refers to the average yield of the base or original distribution.

Table 12

Actual (1920-70) and Hypothetical Distributions of Wheat Yields, Crop District 14, Manitoba

Yield (bushels) (1)	Frequency Distribution: Actual (2)	Case (a) (3)	Case (b) (4)	Case (c) (5)	Case (d) (6)
below 5	0	0	0	0	0
5 - 6	1	1	1	1	1
6 - 7	1	1	1	1	1
7 - 8	0	0	0	0	0
8 - 9	0	0	0	0	0
9 -10	1	1	1	1	1
10 -11	4	4	4	4	4
11 -12	0	0	0	0	0
12 -13	0	0	0	0	0
13 -14	4	4	4	4	4
14 -15	7	7	7	7	7
15 -16	1	2	3	1	1
16 -17	4	5	6	4	4
17 -18	5	6	7	5	5
18 -19	4	5	6	4	4
19 -20	4	5	6	4	4
20 -21	1	1	1	2	3
21 -22	4	4	4	5	6
22 -23	3	3	3	4	5
23 -24	0	0	0	1	2
24 -25	1	1	1	2	3
25 -26	1	1	1	1	1
26 -27	1	1	1	1	1
27 -28	0	0	0	0	0
28 -29	3	3	3	3	3
29 -30	0	0	0	0	0
30 and above	0	0	0	0	0

Source: Based on wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.

Table 13

Basic Statistics for Estimating A Pearson Distribution

	Mean	u_2	u_3	u_4	S.D.	B_1	B_2	k
original data								
	18.02	27.62	15.75	2199.43	5.26	0.01	2.88	-0.03
(a) near the average for 5 years								
	18.02	25.28	14.45	1999.72	5.03	0.01	3.13	0.04
(b) near the average for 10 years								
	18.02	23.33	13.34	1833.95	4.83	0.01	3.37	0.02
(c) above the average for 5 years								
	18.47	27.33	-9.07	2064.76	5.23	0.01	2.76	-0.01
(d) above the average for 10 years								
	18.85	26.78	-28.77	1968.35	5.17	0.04	2.75	-0.05

Note: see text for the explanation of cases (a) through (d).

Source: Based on Wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.

changes in the distribution when additional data are collected. In the event that additional yield observations were near the average, the average would not, of course, change at all. However, the kurtosis or coefficient of peakness would increase if subsequent yields were concentrated near the average. The criterion k of these two cases indicates that a Type IV Pearson distribution which is skewed and has unlimited ranges at both ends should be adopted to represent the changed distributions. Using the basic statistics, two Type IV Pearson distributions were estimated. Because a Type IV Pearson distribution takes the form of equation (21), i.e.,

$$(21) \qquad y = y_0 \left(1 + x^2/a^2 \right)^{-m} e^{-v \tan^{-1}(x/a)} \qquad -\infty < X < \infty$$

with origin at va/r + mean

the parameters were estimated as follows;

$y_0 = 0.0221$

$a = 37.0772$

$m = 29.9525$

$v = -12.4790$

$r = 57.9050$

origin $= 10.0278$

for the case of yields near the average for 5 years, and

$y_0 = 0.0744$

$a = 21.0734$

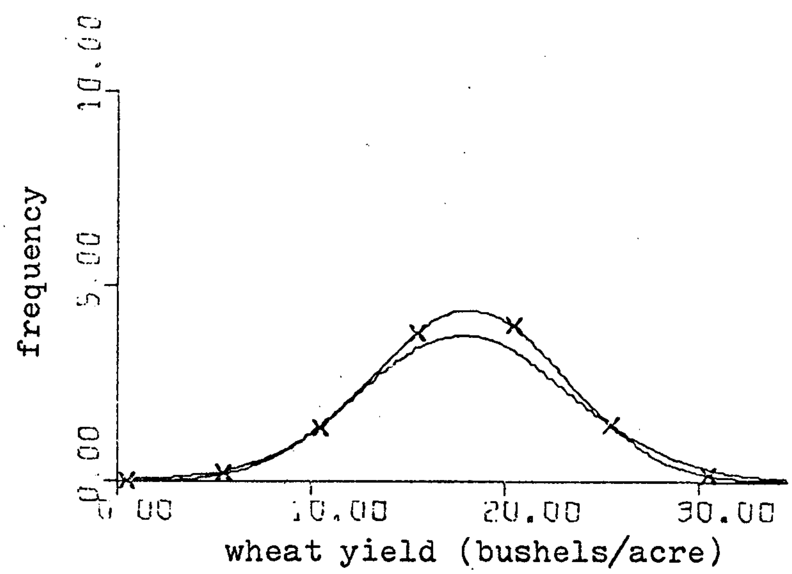
$$m = 11.1634$$
$$v = -2.5274$$
$$r = 20.3268$$
$$\text{origin} = 15.3948$$

for the case of yields near the average for 10 years. These two distributions and the base one are shown in Figure 10 where the base distribution is plotted by a smooth line and the changed distribution is shown by the line of crosses.

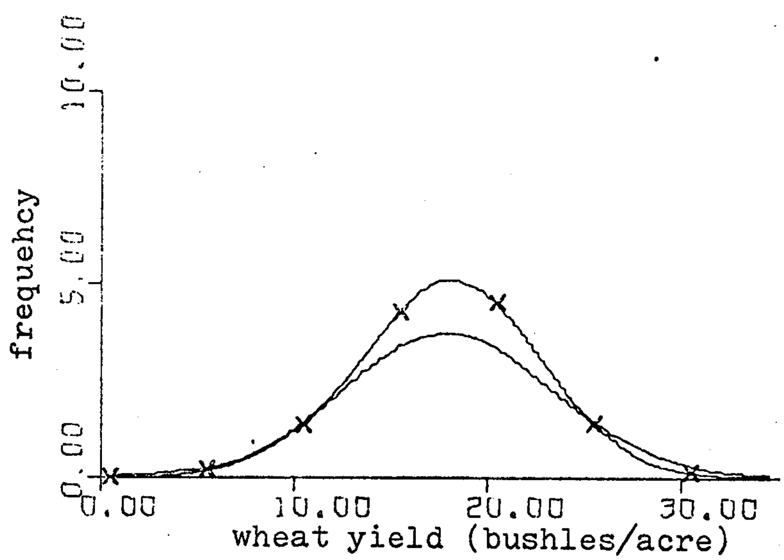
Using these two Type IV distributions, premium rates at different coverage levels were calculated and are shown in Table 14. Because of the assumption that subsequent yields will be near the average, the average yields do not change. Thus the insured yields under the two changed distributions are the same as before. In case (a), the premium rate for 60 percent coverage decreases by about 13.93 percent, if yields are near the average for 5 years. For 70 percent coverage the discount is 12.76 percent and for 80 percent coverage it is 10.64 percent.

Table 14 also shows that discounts of 25.17, 23.37, and 19.41 percent are offered for 60, 70, and 80 percent coverage levels respectively, if an insured farmer does not make a claim for 10 years and if his yields in these ten years are near the average--case (b).

The result implies that if an insured farmer



(a) additional yields around the average for 5 years

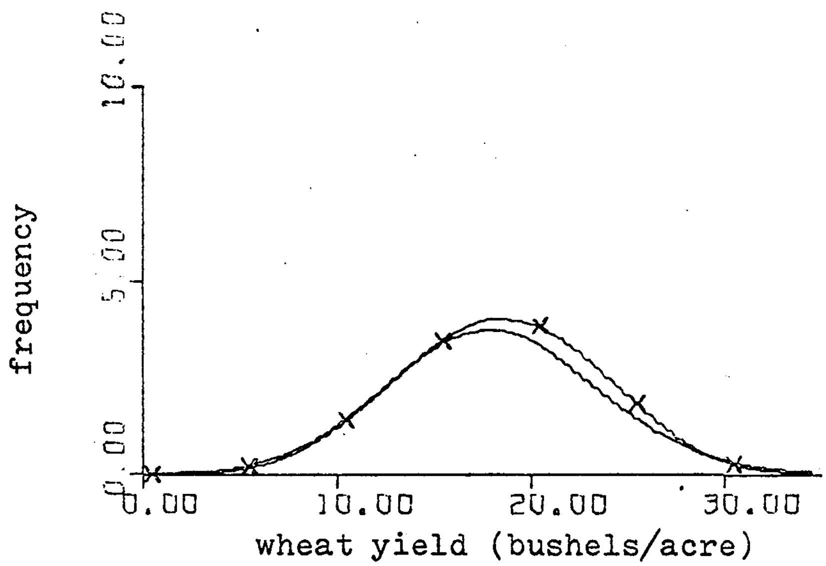


(b) additional yields around the average for 10 years

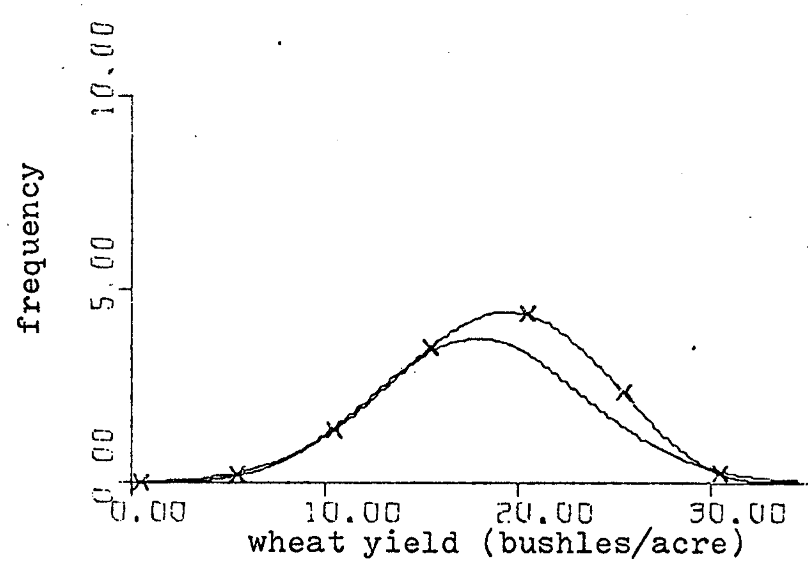
based distribution
changed distribution

Figure 10. Examples of Changed Distribution after Incorporation of Additional Data

Source: Based on wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.



(a) additional yields above the average for 5 years



(b) additional yields above the average for 10 years.

Figure 10 (continued)

Table 14

Comparison of Premium Rates--Implications for Experience Rating

Assumed Yield Distribution and Coverage Level	Insured Yield	Premium Rate	Experience Rate
	(bushels/acre)	(bushels/acre)	(%)
Original			
60%	10.81	0.1788	
70%	12.61	0.3891	
80%	14.41	0.7520	
(a) Near the Average for 5 Years			
60%	10.81	0.1539	-13.93
70%	12.61	0.3387	-12.96
80%	14.41	0.6720	-10.64
(b) Near the Average for 10 Years			
60%	10.81	0.1338	-25.17
70%	12.61	0.2982	-23.37
80%	14.41	0.6061	-19.41
(c) Above the Average for 5 Years			
60%	11.08	0.1892	5.81
70%	12.93	0.4002	2.85
80%	14.78	0.7602	1.09
(d) Above the Average for 10 Years			
60%	11.31	0.1941	8.52
70%	13.19	0.4029	3.55
80%	15.08	0.7573	0.69

Note: See text and Table 12 for explanation of cases (a) through (d)

Source: Based on wheat yield data (1921-70) from Department of Agriculture, Province of Manitoba, Winnipeg.

does not qualify for an indemnity for 5 or 10 years, the risk involved is reduced and a discount on the premium should be offered to that farmer. However, the discounts offered for different coverages should not be the same. According to the results, the higher is the insurance coverage, the lesser is the discount rate to be offered.

For cases (c) and (d), the average yields increased to 18.47 and 18.85 bushels per acre for the 5-year and 10-year cases respectively. Because yields were assumed to be above the average, the distribution changed from one which was slightly skewed to the right to one which was slightly skewed to the left.[68]

The criterion k shows that a Type I Pearson distribution is still able to represent the changed distribution. Using the basic statistics in Table 13 two Type I Pearson distributions were estimated. Because a Type I Pearson distribution takes the form of

(13) $$y = y_0 \left(1 + x/a_1 \right)^{m_1} \left(1 - x/a_2 \right)^{m_2} \quad -a_1 \leq x \leq a_2$$

with origin at the mode,

the parameters were estimated as follows;

$y_0 = 0.0738$

$m_1 = 10.7863$

$m_2 = 99.0624$

---

[68] The signs of u_3 have changed from positive to negative and B_1 is close to zero.

$$a_1 = 27.2444$$
$$a_2 = 22.8900$$
$$\text{origin} = 18.6720$$

for tne case of yields falling above the average for 5 years, and

$$y_o = 0.0741$$
$$m_1 = 8.7531$$
$$m_2 = 5.2245$$
$$a_1 = 27.3805$$
$$a_2 = 16.3428$$
$$\text{origin} = 19.5408$$

for the case of yields falling above the average for 10 years. These two distributions and the base one are shown in Figure 10, where the changed distributions are plotted by the line with crosses and the base distribution is plotted by a smooth line.

From these two new Type I Pearson distributions the insured yields at different coverage levels and the corresponding premium rates were calculated and are shown in Table 14. For 60, 70, and 80 percent coverage levels, the insured yields have increased from 10.81, 12.60, and 14.41 bushels per acre to 11.08, 12.93, and 14.78 bushels per acre, respectively, for case (c) in which yields are above the average for 5 years. Similarly, insured yields increased to 11.31, 13.19, and 15.08 bushels per acre for case (d) in which yields are above the average for 10 years.

Comparison of the new and original premiums shows that there should be no discount. Conversely, for 60, 70, and 80 percent coverage, premium rates should be increased by about 5.81, 2.85, and 1.09 percent, respectively of the original premium rates in case (c), with yields above the average for 5 years, and should increase by about 8.52, 3.55, and 0.69 percent of the original premium rates in case (d), with yields above the average for 10 years.

The reason for the increase in premiums is that an increase in the insured yield involves more risk. If the insured yield remains the same as before, new premiums may be less than before. However, if the insured yield were to remain the same, then the insurance program would not provide adequate protection to farmers with higher expected yields.[69]

Results shown in the final column of Table 14 suggest that the discounts now offered by the Manitoba Crop Insurance Corporation may not meet the needs of insurance involving different coverage levels and several crops. A single rate does simplify calculation of

---

[69] Records of yields from several years indicate that yields have increased. The original insured yields are now below the designated percentages of the average. Therefore, if the insurance program retains the same insured yield, the actual coverage will not be what was intended.

a premium rate and enable the insured farmers to understand easily what they are being charged. However, it may not be actuarially valid and it may therefore be contrary to the basic principle of an insurance program according to which the premium should be based upon the risk involved.

CHAPTER VI

SUGGESTIONS AND IMPLICATIONS

The findings of this study are generally consistent with the hypotheses that were developed in the first chapter. The results and their implications are summarized below.

The variability of crop yields has traditionally been a serious problem in the agricultural industry. Severe crop losses have led to distress on the part of the farmer, and have often prevented the expansion of the farm firm or have caused outright bankruptcy. Crop insurance, by guaranteeing a minimum level of return from the investment in crop production, may be chosen by farmers as a precaution against potentially disastrous yield uncertainty. It also increases the farmer's ability to withstand unfavorable economic outcomes resulting from severe reduction in crop yield, thus allowing the farmer to make better plans for his farm business without the threat of bankrupting the farm unit due to large loss. Therefore, for some farmers, crop insurance may generate higher expected incomes which may not be realized in a short period in the absence of crop insurance.

Crop yield in a single year, which cannot be predicted without error, is an uncertainty phenomenon.

However, risk is present when the probability distribution is established over a large number of observations either through calculation *a priori* or from statistics of past experience. When the probability distribution of yield outcomes is known, it is possible to estimate the number of years in which the yield will fall in each yield interval. Since the variability of the crop yield is measurable, the supply of crop insurance becomes possible.

Crop insurance is not a free good. Insured farmers have to pay a premium to be entitled to an indemnity in the event of crop loss. The institution which underwrites crop insurance has to charge a premium in order to recoup the cost of operating the scheme. Crop insurance programs generally operate toward the goal that they can attract a majority of farmers to participate in the program and be carried out under the basis of self-sustaining growth in the long-run. Fulfillment of these provisions necessitates an actuarially sound premium indemnity schedule for operating the crop insurance program. The actuarial requirement is that the premium paid by a farmer using the crop insurance should be equal to indemnities over a long period of time, and, all farmers facing similar probabilities for similar indemnities be assessed similar premiums.

An actuarially sound crop insurance program is one in which premium determination is based upon the actual probability distribution of crop yields. Under this condition, crop insurance program administrators are able to assess the actual loss probabilities of reductions in crop yields. For establishing the premium rate schedule it is inappropriate to use the assumption that crop yields are normally distributed, even though their actual distribution is unknown. The results of Chapter IV indicate that yield distributions can be of any type; the normal distribution may exist but appears to occur quite rarely. Pearson distributions are recommended for estimating crop yield distributions. With this technique, it is not necessary to assume any specific type of distribution, and conversely, the type of distribution can be determined empirically from any given set of data.

Crop yield distributions are affected not only by weather, topography, and soil productivity but also by special farm practices and individual managerial abilities. Therefore, aggregate yield data compiled from a district or a risk area are not appropiate for estimating the yield distribution to be used in premium determination. Farms in the same area and on the same soil type do not necessarily have the same crop yield distribution. This hypothesis has been supported by the results of Chapter V. These show that the mean yields on different farms are not the same. In the present situation there is not enough data to estimate a yield distribution for each individual farm which is necessary for determining an individual premium rate schedule. One solution to the problem of imperfect knowledge about the distribution is the establishment of an experience rating system. The experience rating system should consider not only the frequency of indemnity claim but also the level of actual realized yields on the individual farm by offering a discount or charging a penalty as well as adjusting the insurance coverage. However, determination of the experience rating table should be based upon proper revision of the distribution in light of additional data about the farmer(s) in question. To avoid penalizing insured farmers who had lower yields in earlier time periods, it is suggested that this data should be gradually excluded when additional data is obtained.

In summary, the study suggests that the crop insurance authorities should investigate other distributions besides the normal distribution. Knowledge of crop yield distributions is the core of a sound crop insurance program. The premium rate schedule is the most important factor in determining whether a voluntary insurance program can attract enough farmer participants for the purpose of distributing severe uncertain losses and avoid a financial deficit in the long-run. But only when the distributions of crop yields are accurately estimated, it is possible to construct a premium rate schedule with confidence and efficiency. Furthermore, accurate information concerning distributions of crop yields must be obtained before a crop insurance program can be fairly evaluated. Without this, one could not put much trust in the results of an evaluation.

In addition to being important for crop insurance, knowledge of crop yield distributions is also important for farm planning. The contrast among the estimated distributions suggests that a decision for maximizing profit and minimizing risk must be based not only on expected yields and variance but upon skewness and kurtosis as well. A production decision based only on the first two measures of the risk concerned is clearly inadequate. For example, when the yield distribution is not normal, the mean, the mode, and the median of the distributions are not equal. Then the mean and the variance of the distribution may not provide enough information about the most probable yield in the future. If the objective is to set a yield that has an equal chance of over- or under-predicting, then it is the median that should be used. Farmers cannot depend on the long-run average yield occurring each year. To make rational decisions, they must take account of the uncertainty they face not only in terms of variance, but also in terms of skewness and peakness in the yield distribution.

In some economic models, including those for simulation and stochastic programming, there are random variables. In these models, it is necessary to generate random variates before carrying out the analysis. Knowledge of the probability distributions of these random variables is very important if a valid analysis is to be made. Some variates can be generated according to an *a priori* assumed distribution. However, if the assumed distribution does not adequately represent the possible outcomes of the random variable, seriously biased results can be expected. The importance of being able to accurately estimate the unknown distribution of a random variable is again obvious.

BIBLIOGRAPHY

1. Alberta Hail and Crop Insurance Corporation. Third Annual Report, for Fiscal Year Ending March 31, 1971. Calgary: The Corporation, 1972.

2. Bailey, W. R. and L.A. Jones. Economic Considerations in Crop Insurance. ERS-447. Washington, D. C.: U. S. Department of Agriculture, 1970.

3. Bar, W. and R. Dougan. Crop Insurance. Agricultural Extension Service Publication. Ohio: Ohio State University, 1960.

4. Botts, R. R. and J. N. Boles. "Use of Normal Curve Theory in Crop Insurance Ratmaking," Journal of Farm Economics, 40:733-40, 1958.

5. Brakensiek D. L. and A. W. Zingg. Application of the Extreme Value Statistical Distribution to Annual Precipitation and Crop Yields. ARS 41-13. Washington: U. S. Department of Agriculture, 1957.

6. Buhlmann, H. Mathematical Methods in Risk Theory. New York: Springer-Verlag, 1970.

7. Canada Department of Agriculture. Annual Report of the Minister under the Crop Insurance Act, 1971/72. Ottawa: The Department, 1972.

8. ________. Federal Agricultural Legislation, Canada 1969. Economics Branch Publication No. 69/19. Ottawa: The Department, 1969.

9. Canada Department of Insurance. "Memorandum Concerning the Computation of the Coefficient of Variation from the Experience Data of Crop Insurance Programs Based on Individually Determined Coverages." Ottawa: The Department, 1970.

10. ________. "Memorandum Concerning a Simplified Method of Computing Premium Rates for Crop Insurance Programmes." Ottawa: The Department, 1967.

11. Crop Insurance Commission of Ontario. Third Annual Report. Fiscal Year Ended March 31, 1969. Toronto: Ontario Department of Agriculture and Food, 1969.

12. Day, R. H. "Probability Distribution of Field Crop Yields,"

Journal of Farm Economics, 47:713-41, 1965.

13. Denenberg, H. S., R. D. Eilers, J. J. Melone, and R. A. Zeltcn. Risk and Insurance. New Jersey: Prentice-Hall, 1964.

14. Dorn, W. S. and H. J. Greenberg. Mathematics and Computing. New York: John Wiley, 1967.

15. Elderton, W. P. Frequency Curves and Correlation. London: Cambridge University Press, 1938.

16. ________ and N. L. Johnson. Systems of Frequency Curves. London: Cambridge University Press, 1969.

17. Federal Task Force on Agriculture. Canadian Agriculture in the Seventies. Ottawa: Queen's Printer, 1970.

18. Feller, W. An Introduction to Probability Theory and Its Applications. Vol. I. New York: John Wiley, 1968.

19. Fisher, R. A. "On the Mathematical Foundation of Theoretical Statistics," Philosophical Transactions of the Royal Society of London, (a), 222:309-68, 1922.

20. Friedman M. and L. J. Savage. "The Utility Analysis of Choices Involving Risk," Journal of Political Economy, 56:279-304, 1948.

21. Gilson J. C. "Instability in Agriculture and Crop Insurance." Paper presented at the Farm Conference Week, March, 1962, University of Manitoba, Winnipeg.

22. Halcrow, H. G. "Actuarial Structures for Crop Insurance," Journal of Farm Economics, 31:418-43, 1949.

23. Heady, E. O. and J. L. Dillon. Agricultural Production Functions. Ames: Iowa State University Press, 1964.

24. Henderson J. M. and R. E. Quandt. Microeconomic Theory. New York: McGraw-Hill, 1958.

25. Hicks, J. R. Value and Capital. London: Oxford University Press, 1948.

26. Hogg, R. V. and A. T. Craig. Introduction to Mathematical Statistics. New York: MacMillion, 1965.

27. Kendall. M. G. and A. Stuart. The Advanced Theory of Statistics. 3 vols. London: Charles Griffin, 1958-66.

28. Knight, F. H. Risk, Uncertainty and Profit. Boston: Houghton Mifflin, 1921.

29. Manitoba Crop Insurance Committee. Crop Insurance in Manitoba. Winnipeg: Economic Survey Board, Province of Manitoba, 1940.

30. Manitoba Crop Insurance Corporation. "Administrative Regulations Re Experience Discount." Winnipeg: The Corporation, 1964. (Mimeographed.)

31. ________. "Method Used to Determine Theoretical Loss Cost per Acre." Winnipeg: The Corporation, 1968. (Mimeographed.)

32. ________. Annual Report. Winnipeg: The Corporation, annually.

33. Merrill, W. C. and K. A. Fox. Introduction to Economic Statistics. New York: John Wiley, 1970.

34. Mood, A. M. and F. A. Graybill. Introduction to the Theory of Statistics. 2d ed. New York: McGraw-Hill, 1965.

35. Ottoson, H. W. (ed.). Management Strategies in Great Plains Farming. Great Plains Council Publication No. 19. Lincoln, Nebraska: Univeristy of Nebraska, 1961.

36. Pearson, E. S. and H. O. Hartley. Biometrika Tables for Statisticians. Vol. I. London: Cambridge University Press, 1962.

37. Pearson, K. (ed.). Tables for Statisticians and Biometricians. Part. I. 3d ed. London: Biometrika Office, University College, 1945.

38. Pfefer, I. Insurance and Economic Theory. Homewood: Richard D. Irwin, 1956.

39. Ray, P. K. Principles and Practices of Agricultural Insurance. Calcutta: Bookland Private, 1958.

40. ________. Agricultural Insurance. Oxford: Pergamon, 1967.

41. Riegel, R. and J. S. Miller. Insurance Principles and Practices. New Jersey: Prentice-Hill, 1965.

42. Royal Commission on Agriculture and Rural Life, Province of Saskatchewan. Crop Insurance. Regina: Queen's Printer, 1965.

43. Seal, H. L. Stochastic Theory of a Risk Business. New York: John Wiley, 1969.

44. Smith, G. A., E. L. LaDue, and R. S. Smith. Crop Insurance Strategies in High Risk Production Areas. A. E. Res. 72-11. Ithaca: Department of Agricultural Economics, Cornell University, 1972.

45. Tetley, H. Actuarial Statistics. Vol. I. Statistics and Graduation. London: Cambridge University Press, 1964.

46. Tintner, G. "A Note on Economic Aspects of the Theory of Errors in Time Series," Quarterly Journal of Economics, 53:141-4, 1938.

47. ________. The Variate Difference Method. Cowles Commission Monomgraph No. 5. Bloomington, Indiana: Principia Press, 1940.

48. Willett, A. H. The Economic Theory of Risk and Insurance. Homewood: Richard D. Irwin, 1951.

49. Yeh, M. H. and R. Y. Wu. "Premium Ratemaking in an All Risk Crop Insurance Program," Journal of Farm Economics, 48:1580-6, 1966.

APPENDICES

I. A COMPUTER PROGRAM FOR ESTIMATING PEARSON DISTRIBUTION

II. THE APPROXIMATE INTEGRATION

III. DATA USED IN ANALYSIS OF VARIANCE

APPENDIX I

A COMPUTER PROGRAM FOR ESTIMATING PEARSON DISTRIBUTIONS

A computer program was written in Fortran IV programming language for estimating the actual distribution of crop yields using three main types of Pearson distribution as its base. The type of Pearson distribution which should be adopted for the unknown distribution is identified, and the following options are included;

1. Estimate a normal distribution for crop yields.
2. Calculate a chi-square statistic
3. Calculate pure premium rates using the estimated Pearson distribution, or the estimated normal distribution, at three insurance coverage levels; i.e., 60, 70, and 80 percent of the average yield.

To do this, the program was made to include one main program, seven subroutine subprograms, and nine function subprograms. Figure A-I-1 is a flow chart which shows the basic procedure. Section A.I.1. illustrates sequences of the card deck usig different compilers; Fortran G and WATFIV. Then the program is presented. Section A.I.2. is designed to explain the procedure for preparing input data set(s). The output of the program which are used in Chapters IV and V are shown in Section A.I.3. Extensions

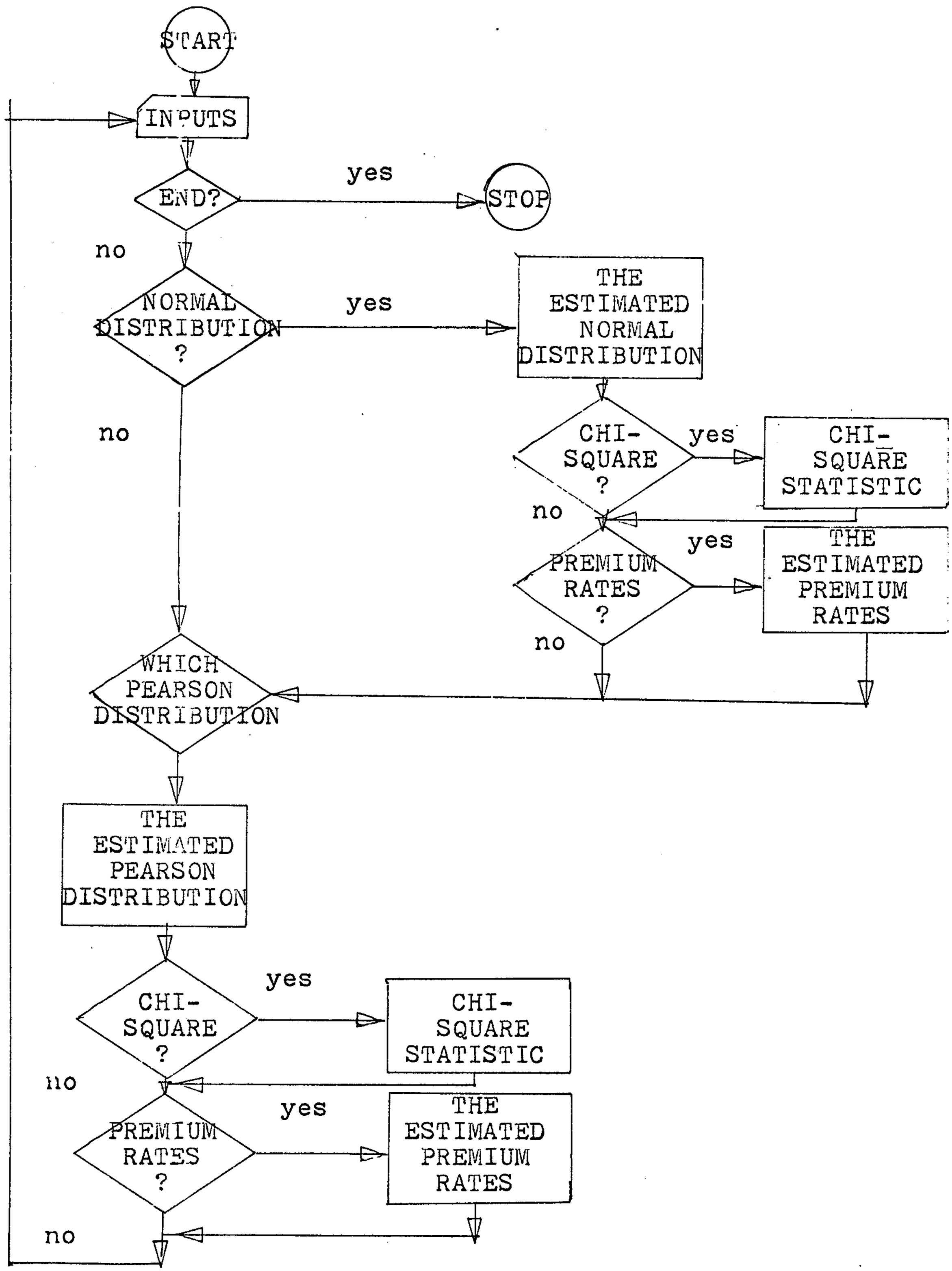


Figure A-I-1. The Flow Chart of The Computer Program for Estimating Pearson Distribution and limitations of the program are then discussed.

A.I.1. Sequences of the Card Deck

Using a Fortran G compiler the sequence of the card deck is as follows:

```
//PEARSON    JOB 'XXXX,XXXX,XXXX','YOUR-NAME"
//  EXEC FORTGCG
//FORT.SYSIN DD *
```

The Program Deck

```
/*
//GO.SYSIN DD *
```

The Input Data Set(s)

```
/*
```

Note: In the JOB card XXXX,XXXX,XXXX, is used to sepecify the account number, programmer's name, etc., depending local computer specifications.

using a WATFOR Compiler the sequence of the card deck should be prepared in the order of:

```
//PEARSON     JOB 'XXXX,XXXX,XXXX','YOUR-NAME'
//  EXEC WATFOR
//GO.SYSIN DD *
$JOB            YOUR-NAME

              The Program Deck

$ENTRY

              The Input Data Set(s)

$STOP
/*
```

The program deck is illustrated on the following pages.

```
$JOB          STEVEN-SUN,TIME=10
C      PROGRAM FOR ESTIMATING PEARSON DISTRIBUTION
C      THE FREQUENCY DISTRIBUTION WILL BE PRINTED WITH THE
C         MAXIMUM CLASS EQUAL TO THE GIVEN MCLASS
C      THE ESTIMATED PREMIUM RATE WILL BE CALCULATED IF
C        REQUIRED OTHERWISE NPREM IS GIVEN AS ZERO
C      THE PROGRAM CONSISTS OF A SUBPROGRAM TO ESTIMATE
C         A NORMAL DISTRIBUTION IF NORM IS NOT EQUAL TO 0
C
C      ****** MAIN PROGRAM STARTS FROM HERE ******
C
       DATA IW,IR/3,1/
       DIMENSION DATA(100),CLASS(50),IFREQ(50),NAME(20)
       INTEGER FORM(20)
C
   100 READ(IR,1) NAME,N,NCLASS,NORM,NPREM,MCLASS,ICHI
     1 FORMAT(20A4/6I5)
       IF(N.EQ.0) GO TO 101
       WRITE(IW,2) NAME
     2 FORMAT(1H1// 6X,'JOB NAME'/6X, 20A4)
       IF(NCLASS.NE.0) GO TO 43
       READ(IR,3) FORM
     3 FORMAT(20A4)
       READ(IR,FORM) (DATA(I),I=1,N)
       WRITE(IW,4) (DATA(I),I=1,N)
     4 FORMAT(/ 10X,'THE DATA SET IS:'/ (6X,10F6.1))
    11 AMAX=DATA(1)
       DO 10 I=2,N
       IF(DATA(I).GT.AMAX) AMAX=DATA(I)
    10 CONTINUE
       CLASS(1)=1.
       DO 20 I=2,50
       CLASS(I)=CLASS(I-1)+1.
       IF(AMAX.GT.CLASS(I-1).AND.AMAX.LE.CLASS(I)) GO TO 21
    20 CONTINUE
    21 NCLASS=I
       DO 22 I=1,NCLASS
       IFREQ(I)=0
    22 CONTINUE
       DO 42 I=1,N
       DO 40 J=1,NCLASS
       IF (DATA(I).LE.CLASS(J)) GO TO 41
    40 CONTINUE
    41 IFREQ(J)=IFREQ(J)+1
    42 CONTINUE
       GO TO 49
    43 READ(IR,3) FORM
       READ(IR,FORM) (IFREQ(I),I=1,NCLASS)
    49 WRITE(IW,7) (IFREQ(I),I=1,NCLASS)
     7 FORMAT(/ 10X,'THE OBSERVED FREQUENCY DISTRIBUTION IS',
      1':'/ (6X,10I6))
       NCLASS=NCLASS+1
       DO 70 I=NCLASS,MCLASS
       IFREQ(I)=0
    70 CONTINUE
       CALL SUB1(IFREQ,D,U2,U3,U4,SD,B1,B2,AKI,NCLASS)
       WRITE(IW,5) D,U2,U3,U4,SD,B1,B2,AKI
     5 FORMAT(/ 10X,'THE VALUES IN ORDER ARE:'/
      110X, 'MEAN, U2, U3, U4, SD, B1, B2, K'/(4F15.5))
       IF(NORM.EQ.1) CALL NORMAL(N,D,U2,MCLASS,NPREM,IFREQ,
```

```
     1ICHI)
      IF(AKI.LE.0.) CALL TYPE1(N,D,U2,U3,B1,B2,MCLASS,
     1NPREM,IFREQ,ICHI)
      IF(AKI.LT.1..AND.AKI.GT.0.) CALL TYPE4(N,D,U2,U3,B1,
     1B2,MCLASS,NPREM,IFREQ,ICHI)
      IF(AKI.GT.1.) CALL TYPE6(N,D,U2,U3,B1,B2,MCLASS,
     1NPREM,IFREQ,ICHI)
      GO TO 100
  101 WRITE(IW,102)
  102 FORMAT(1H1//10X,' **** END OF JOB **** ')
      STOP
      END
C
C
      SUBROUTINE SUB1(IFREQ,D,U2,U3,U4,SD,B1,B2,AKI,NCLASS)
      DIMENSION IFREQ(50),S2(50),S3(50),S4(50),S5(50),A(50)
      SUM=0.
      DO 50 I=1,NCLASS
      II=NCLASS-I+1
      A(I)=IFREQ(II)
      SUM=SUM+A(I)
   50 CONTINUE
      S2(1)=A(1)
      TS2=S2(1)
      DO 52 I=2,NCLASS
      S2(I)=S2(I-1)+A(I)
      TS2=TS2+S2(I)
   52 CONTINUE
      S3(1)=A(1)
      TS3=S3(1)
      DO 53 I=2,NCLASS
      S3(I)=S3(I-1)+S2(I)
      TS3=TS3+S3(I)
   53 CONTINUE
      S4(1)=A(1)
      TS4=S4(1)
      DO 54 I=2,NCLASS
      S4(I)=S4(I-1)+S3(I)
      TS4=TS4+S4(I)
   54 CONTINUE
      S5(1)=A(1)
      TS5=S5(1)
      DO 55 I=2,NCLASS
      S5(I)=S5(I-1)+S4(I)
      TS5=TS5+S5(I)
   55 CONTINUE
      TS3=TS3/SUM
      TS4=TS4/SUM
      TS5=TS5/SUM
      D=TS2/SUM
      V2=2.*TS3-D*(1.+D)
      V3=6.*TS4-3.*V2*(1.+D)-D*(1.+D)*(2.+D)
      V4=24.*TS5-2.*V3*(2.*(1.+D)+1.)-V2*(6.*(1.+D)*
     1(2.+D)-1.)-D*(1.+D)*(2.+D)*(3.+D)
      U2=V2-1./12.
      U3=V3
      U4=V4-0.5*V2+7./240.
      SD=SQRT(U2)
      B1=U3*U3/(U2*U2*U2)
```

```
          B2=U4/(U2*U2)
          AKI=B1*(B2+3.)*(B2+3.)/(4.*(4.*B2-3.*B1)*(2.*B2-3.*
         1B1-6.))
          RETURN
          END
C
C
          SUBROUTINE CHISQ(T,IT,N)
          DATA IW,IR/3,1/
          DIMENSION O(50),IT(50),T(50)
          DO 5 I=1,N
          O(I)=IT(I)
        5 CONTINUE
          DO 10 I=1,N
          IF(T(I).GT.0.01) GO TO 11
       10 CONTINUE
       11 N1=I
          DO 20 I=1,N
          II=N-I+1
          IF(T(II).GT.0.01) GO TO 21
       20 CONTINUE
       21 N2=II
          CHI=0.
          DO 25 I=N1,N2
          CHI=CHI+(O(I)-T(I))*(O(I)-T(I))/T(I)
       25 CONTINUE
          NCLASS=N-N1+1
          WRITE(IW,8) NCLASS,CHI
        8 FORMAT(/6X,'CHI-SQUARE TEST:',I3,' CLASSES ',
         1'EXAMINED, CHI-SQUARE IS',F9.4)
          RETURN
          END
C
C
          SUBROUTINE WRITE1(IW,TMEAN)
          WRITE(IW,5)
        5 FORMAT(/          16X,'% OF   ',35X,'PREMIUM/'/11X,
         1'  MEAN YIELD    INSURED YIELD    PREMIUM  INSURED',
         4' YIELD'/)
          RETURN
          END
C
C
          SUBROUTINE NORMAL(N,AMEAN,U2,MCLASS,NPREM,
         1IT,ICHI)
C
C         PROGRAM FOR ESTIMATING A NORMAL DISTRIBUTION
C          AND THE PREMIUM RATE
          DIMENSION Y(50),C(3),IT(50)
          DATA IW,IR/3,1/
          EXTERNAL EQNORM
          AC=2.*U2
          Y0=1./SQRT(6.2832*U2)
C
C         THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C         THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION
C
```

```
      X=.5-AMEAN
      Y0=Y0*N
      DO 10 I=1,40
      Y(I)=Y0*(1./EXP(X*X/AC))
      X=X+1.
   10 CONTINUE
      Y0=Y0/N
      WRITE(IW,3) (Y(I),I=1,MCLASS)
    3 FORMAT(/ 10X,'THE ESTIMATED NORMAL FREQUENCY DISTRI',
     1'BUTION IS:'/(6X,10F6.2))
      IF(ICHI.NE.0) CALL CHISQ(Y,IT,MCLASS)
C
C     THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C     THE ESTIMATED PREMIUM RATES
C     IF NPREM = 0 NO PREMIUM RATE IS CALCULATED
C
      IF(NPREM.EQ.0) GO TO 101
      CALL WRITE1(IW,AMEAN)
      AN=50.
      C(1)=60.
      C(2)=70.
      C(3)=80.
      A=0.-AMEAN
      B1=AMEAN*0.6-AMEAN
      B2=AMEAN*0.7-AMEAN
      B3=AMEAN*0.8-AMEAN
      DO 20 II=1,3
      COV=AMEAN*C(II)/100.-AMEAN
      FRA=EQNORM( A,Y0,AC,AMEAN,COV)
      FB1=EQNORM(B1,Y0,AC,AMEAN,COV)
      SUM1=SUMN( A,B1,EQNORM,Y0,AC,AMEAN,COV)
      H=(B1-A)/AN
      ARE1=(FRA+FB1+2.*SUM1)*(H/2.)
      IF(II.EQ.1) GO TO 19
      FB2=EQNORM(B2,Y0,AC,AMEAN,COV)
      SUM2=SUMN(B1,B2,EQNORM,Y0,AC,AMEAN,COV)
      H=(B2-B1)/AN
      ARE2=(FB1+FB2+2.*SUM2)*(H/2.)
      ARE1=ARE1+ARE2
      IF(II.EQ.2) GO TO 19
      FB3=EQNORM(B3,Y0,AC,AMEAN,COV)
      SUM3=SUMN(B2,B3,EQNORM,Y0,AC,AMEAN,COV)
      H=(B3-B2)/AN
      ARE3=(FB2+FB3+2.*SUM3)*(H/2.)
      ARE1=ARE1+ARE3
   19 COV=COV+AMEAN
      TEMP=ARE1/COV
      WRITE(IW,4) C(II),COV,ARE1,TEMP
    4 FORMAT(17X,F4.0,F15.2,2F15.4)
   20 CONTINUE
  101 RETURN
      END
C
C
      FUNCTION SUMN(A,B,EQNORM,Y0,AC,AMEAN,COV)
      AN=50.
      SUMN=0.
      H=(B-A)/AN
      DO 10 J=1,49
```

```
      AJ=J
      X=A+AJ*H
      SUMN=SUMN+EQNORM(X,YO,AC,AMEAN,COV)
   10 CONTINUE
      RETURN
      END
C
C
      FUNCTION EQNORM(A,YO,AC,AMEAN,COV)
      EQNORM=(COV-A)*YO*(1./EXP(A*A/AC))
      RETURN
      END
C
C
      SUBROUTINE TYPE1(N,AMEAN,U2,U3,B1,B2,MCLASS,NPREM,
     1IT,ICHI)
C
C     PROGRAM FOR ESTIMATING THE PARAMETERS OF TYPE I
C     PEARSON DISTRIBUTION AND THE PREMIUM RATE
C
      DIMENSION C(3),Y(50),IT(50)
      DATA IW,IR/3,1/
      EXTERNAL EQT1
      R=6.*(B2-B1-1.)/(6.+3.*B1-2.*B2)
      DUM=B1*(R+2.)*(R+2.)+16.*(R+1.)
      AA=0.5*SQRT(U2)*SQRT(DUM)
      BF=SQRT(B1/DUM)
      Z1=0.5*(R-2.+R*(R+2.)*BF)
      Z2=0.5*(R-2.-R*(R+2.)*BF)
      A2=(AA*Z2)/(Z1+Z2)
      A1=AA-A2
      ORIGIN=AMEAN-0.5*(U3/U2)*(R+2.)/(R-2.)
      IF( U3.LT.0.) GO TO 6
      TEMP=Z1
      Z1=Z2
      Z2=TEMP
      TEMP=A1
      A1=A2
      A2=TEMP
    6 GAM=GAMMA(Z1+Z2+2.)/(GAMMA(Z1+1.)*GAMMA(Z2+1.))
      IF(Z1.GT.0..AND.Z2.GT.0..OR.Z1.LT.0..AND.Z2.LT.0.)
     1 GO TO 30
      YO=GAM/AA**(Z1+Z2+1.)
      ORIGIN=ORIGIN-A1
      WRITE(IW,5)
    5 FORMAT(/  7X,'*** J-SHAPED TYPE I PEARSON DISTRIBUTION',
     1' IS USED ***'//10X,'    Y = YO * X**M1 * (A1+A2-X)**M2'/
     210X,'   WITH ORIGIN AT THE START OF THE DISTRIBUTION',
     3', WHERE')
      GO TO 34
   30 Z12=Z1+Z2
      YO=Z1**Z1*Z2**Z2/Z12**Z12*GAM/AA
      WRITE(IW,1)
    1 FORMAT(/  7X,'*** TYPE I PEARSON DISTRIBUTION IS USE',
     1'D ***'//13X,'Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2'/
     210X,'   WITH ORIGIN AT THE MODE, WHERE')
   34 WRITE(IW,2) YO,A1,Z1,A2,Z2,ORIGIN
    2 FORMAT(10X,'YO=',F15.4,' A1=',F15.4,' M1=',F15.4/
```

```
     210X,'A2=',F15.4,' M2=',F15.4,' ORIGIN=',F10.4)
C
C     THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C     THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION
C
      AX=0.5-ORIGIN
      A=-1.*A1
      Y0=Y0*N
      DO 40 I=1,MCLASS
      Y(I)=0.
      IF(Z1.LT.0..OR.Z2.LT.0.) GO TO 36
      IF (AX.GT. A.AND.AX.LT.A2) Y(I)=Y0*(1.+AX/A1)**Z1*(1.-
     1AX/A2)**Z2
      GO TO 39
   36 IF(AX.GT.0..AND.AX.LT.AA) Y(I)=Y0*AX**Z1*(AA-AX)**Z2
   39 AX=AX+1.
   40 CONTINUE
      Y0=Y0/N
      WRITE(IW,3) (Y(I),I=1,MCLASS)
    3 FORMAT(/ 10X,'THE ESTIMATED TYPE I FREQUENCY DISTRIB',
     1'UTION IS'/(6X,10F6.2))
      IF(ICHI.NE.0) CALL CHISQ(Y,IT,MCLASS)
C
C     THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C     THE ESTIMATED PREMIUM RATES
C     IF NPREM = 0 NO PREMIUM RATE IS CALCULATED
C
      IF(NPREM.EQ.0) GO TO 101
      CALL WRITE1(IW,AMEAN)
      AN=50.
      C(1)=60.
      C(2)=70.
      C(3)=80.
      B1=AMEAN*.6-ORIGIN
      B2=AMEAN*.7-ORIGIN
      B3=AMEAN*.8-ORIGIN
      DO 20 II=1,3
      COV=AMEAN*C(II)/100.-ORIGIN
      FRA=EQT1( A,Z1,Z2,A1,A2,Y0,COV)
      FB1=EQT1(B1,Z1,Z2,A1,A2,Y0,COV)
      SUM1=SUMT1( A,B1,EQT1,Z1,Z2,A1,A2,Y0,COV)
      H=(B1-A)/AN
      ARE1=(FRA+FB1+2.*SUM1)*(H/2.)
      IF(II.EQ.1) GO TO 19
      FB2=EQT1(B2,Z1,Z2,A1,A2,Y0,COV)
      SUM2=SUMT1(B1,B2,EQT1,Z1,Z2,A1,A2,Y0,COV)
      H=(B2-B1)/AN
      ARE2=(FB1+FB2+2.*SUM2)*(H/2.)
      ARE1=ARE1+ARE2
      IF(II.EQ.2) GO TO 19
      FB3=EQT1(B3,Z1,Z2,A1,A2,Y0,COV)
      SUM3=SUMT1(B2,B3,EQT1,Z1,Z2,A1,A2,Y0,COV)
      H=(B3-B2)/AN
      ARE3=(FB2+FB3+2.*SUM3)*(H/2.)
      ARE1=ARE1+ARE3
   19 COV=COV+ORIGIN
      TEMP=ARE1/COV
      WRITE(IW,4) C(II),COV,ARE1,TEMP
    4 FORMAT(17X,F4.0,F15.2,2F15.4)
   20 CONTINUE
```

```
  101 RETURN
      END
C
C

      FUNCTION SUMT1(A,B,EQT1,Z1,Z2,A1,A2,YO,COV)
      AN=50.
      SUMT1=C.
      H=(B-A)/AN
      DO 10 J=1,49
      AJ=J
      X=A+AJ*H
      SUMT1=SUMT1+EQT1(X,Z1,Z2,A1,A2,YO,COV)
   10 CONTINUE
      RETURN
      END
C
C

      FUNCTION EQT1(X,Z1,Z2,A1,A2,YO,COV)
      EQT1=C.
      IF(Z1.LT.0..CR.Z2.LT.0.) GC TO 5
      A=-1*A1
      IF(X.GT. A.AND. X.LT.A2) EQT1=(COV-X)*YO*(1.+X/A1)**Z1*
     1(1.-X/A2)**Z2
      GO TO 10
    5 AA=A1+A2
      IF(X.GT.C..AND.X.LT.AA) EQT1=(COV-X)*YO*X**Z1*(AA-X)**Z2
   10 RETURN
      END
C
C

      SUBROUTINE TYPE4(N,AMEAN,U2,U3,B1,B2,NCLASS,NPREM,
     1IT,ICHI)
C     PROGRAM FOR ESTIMATING THE PARAMETERS OF TYPE IV
C     PEARSON DISTRIBUTION AND THE PREMIUM RATE
C
      DIMENSION C(3),Y(5C),IT(5C)
      DATA IW,IR/3,1/
      EXTERNAL EQT4
      WRITE(IW,1)
    1 FORMAT(//10X,'*** TYPE IV PEARSON DISTRIBUTION IS US',
     1'ED ***'//10X,'    Y = YO * (1+X*X/A*A)**-M * EXP(-V*',
     2'ARCTAN(X/A))'/13X,'WITH CRIGIN AT V*A/R + MEAN, WHERE')
      R=6.*(B2-B1-1.)/(2.*B2-3.*B1-6.)
      AM=0.5*(R+2.)
      V=R*(R-2.)*SQRT(B1)/SQRT(16.*(R-1.)-B1*(R-2.)*(R-2.))
      IF(U3.GT.0.) V=-1.*V
      A= SQRT(U2/16.)*SQRT(16.*(R-1.)-B1*(R-2.)*(R-2.))
      YO=1./(A*FUNCT(R,V))
      ORIGIN=AMEAN+V*A/R
      WRITE(IW,2) YC,A,AM,V,R,ORIGIN
    2 FORMAT(10X,'YO=',F15.5,' A=',F15.5,' M=',F15.5/
     210X,'V=',F15.5,' R=',F15.5,' ORIGIN=',F15.5)
C
C     THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C     THE ESTIMATED TYPE IV FREQUENCY DISTRIBUTION
C
      X=C.5-CRIGIN
```

```
      YO=YO*N
      DO 40 I=1,MCLASS
      Y(I)=YO*(1.+X*X/(A*A))**(-1.*AM)*EXP(-1.*V*ATAN(X/A))
      X=X+1.
   40 CONTINUE
      YO=YO/N
      WRITE(IW,3) (Y(I),I=1,MCLASS)
    3 FORMAT(/ 10X,'THE ESTIMATED TYPE IV FREQUENCY DISTRIBU',
     1'TION IS'/ (6X,10F6.2))
      IF(ICHI.NE.0) CALL CHISQ(Y,IT,MCLASS)
C
C     THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C     THE ESTIMATED PREMIUM RATES
C     IF NPREM = 0 NO PREMIUM RATE IS CALCULATED
C
      IF(NPREM.EQ.0) GO TO 101
      CALL WRITE1(IW,AMEAN)
      AN=50.
      C(1)=60.
      C(2)=70.
      C(3)=80.
      AA=0.-ORIGIN
      B1=AMEAN*C(1)/100.-ORIGIN
      B2=AMEAN*C(2)/100.-ORIGIN
      B3=AMEAN*C(3)/100.-ORIGIN
      DO 20 II=1,3
      COV=AMEAN*C(II)/100.-ORIGIN
      FRA=EQT4(AA,AM,A,V,YO,COV)
      FB1=EQT4(B1,AM,A,V,YO,COV)
      SUM1=SUMT4(AA,B1,EQT4,AM,A,V,YO,COV)
      H=(B1-AA)/AN
      ARE1=(FRA+FB1+2.*SUM1)*(H/2.)
      IF(II.EQ.1) GO TO 19
      FB2=EQT4(B2,AM,A,V,YO,COV)
      SUM2=SUMT4(B1,B2,EQT4,AM,A,V,YO,COV)
      H=(B2-B1)/AN
      ARE2=(FB1+FB2+2.*SUM2)*(H/2.)
      ARE1=ARE1+ARE2
      IF(II.EQ.2) GO TO 19
      FB3=EQT4(B3,AM,A,V,YO,COV)
      SUM3=SUMT4(B2,B3,EQT4,AM,A,V,YO,COV)
      H=(B3-B2)/AN
      ARE3=(FB2+FB3+2.*SUM3)*(H/2.)
      ARE1=ARE1+ARE3
   19 COV=COV+ORIGIN
      TEMP=ARE1/COV
      WRITE(IW,4) C(II),COV,ARE1,TEMP
    4 FORMAT(17X,F4.0,F15.2,2F15.4)
   20 CONTINUE
  101 RETURN
      END
C
C
      FUNCTION FUNCT(R,V)
C     ***** THIS IS FOR CLACULATING F(R,V) ***********
C
      DIMENSION AREA(10)
      N=50
      A=0.
```

```
      B=3.1416
      DO 20 J=1,10
      X=A
      DX=(B-A)/N
      SUM=0.
      DO 10 I=2,N
      X=X+DX
      SUM=SUM+SIN(X)**R*EXP(V*X)
   10 CONTINUE
      AREA(J)=SUM*DX
      IF(J.EQ.1) GO TO 20
      IF(J.EQ.10) GO TO 100
      IF(AREA(J-1)-AREA(J).LT.0.01) GO TO 100
      N=N+50
   20 CONTINUE
  100 FUNCT=AREA(J)*EXP(-0.5*V*3.1416)
      RETURN
      END
C
C
      FUNCTION SUMT4(A,B,EQT4,AM,C,V,YO,COV)
      AN=50.
      SUMT4=0.
      H=(B-A)/AN
      DO 10 J=1,49
      AJ=J
      X=A+AJ*H
      SUMT4=SUMT4+EQT4(X,AM,C,V,YO,COV)
   10 CONTINUE
      RETURN
      END
C
C
      FUNCTION EQT4(X,AM,A,V,YO,COV)
      EQT4=(COV-X)*YO*(1.+X*X/(A*A))**(-1.*AM)*EXP(-1.*V*
     1ATAN(X/A))
      RETURN
      END
C
C
      SUBROUTINE TYPE6(N,AMEAN,U2,U3,B1,B2,MCLASS,NPREM,
     1IT,ICHI)
C
C     PROGRAM FOR ESTIMATING THE PARAMETERS OF TYPE VI
C     PEARSON DISTRIBUTION AND THE PREMIUM RATE
C
      DIMENSION C(3),Y(50),IT(50)
      DATA IW,IR/3,1/
      EXTERNAL EQT6
      WRITE(IW,1)
    1 FORMAT(//10X,'*** TYPE VI PEARSON DISTRIBUTION IS USE',
     1'D ***'//13X,'Y= YO * (1+X/A1)**-Q1 * (1+X/A2)**Q2'/
     210X,'    WITH THE ORIGIN AT THE MEAN, WHERE')
      R=6.*(B2-B1-1.)/(6.+3.*B1-2.*B2)
      TA=B1*(R+2.)*(R+2.)+16.*(R+1.)
      A= 0.5*SQRT(U2)*SQRT(TA)
      IF(U3.LT.0.) A=-1.*A
```

```
      Q2=0.5*(R+2.)+R*(R+2.)*SQRT(B1/TA)
      Q1=-.5*(R+2.)+R*(R+2.)*SQRT(B1/TA)
      TEMP=Q1-Q2-1.
      A1=A*(Q1-1.)/((Q1-1.)-(Q2+1.))
      A2=A*(Q2+1.)/((Q1-1.)-(Q2+1.))
      YO=(Q2+1.)**Q2*(TEMP-1.)**(Q1-Q2)*GAMMA(Q1)/
     1(A*(Q1-1.)**Q1*GAMMA(TEMP)*GAMMA(Q2+1.))
      ORIGIN=AMEAN
      WRITE(3,2) YO,A,Q1,Q2,A1,A2
    2 FORMAT(10X,'YO=',F15.5,' A=',F15.5,' Q1=',F15.5/
     210X,'Q2=',F15.5,' A1=',F15.5,' A2=',F15.5)
C
C     THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C     THE ESTIMATED TYPE VI FREQUENCY DISTRIBUTION
C
      X=0.5-ORIGIN
      YO=YO*N
      DO 40 I=1,MCLASS
      IF(ABS(X).LT.ABS(A)) GO TO 38
      Y(I)=YO*(1.+X/A1)**(-1.*Q1)*(1.+X/A2)**Q2
      GO TO 39
   38 Y(I)=0.
   39 X=X+1.
   40 CONTINUE
      YO=YO/N
      WRITE(3,3) (Y(I),I=1,MCLASS)
    3 FORMAT(/ 10X,'THE ESTIMATED TYPE VI FREQUENCY DISTRIB',
     1'UTION IS:'/ (6X,10F6.2))
      IF(ICHI.NE.0) CALL CHISQ(Y,IT,MCLASS)
C
C     THE FOLLOWING SECTION IS FOR CALCULATING AND PRINTING
C     THE ESTIMATED PREMIUM RATES
C     IF NPREM = 0 NO PREMIUM RATE IS CALCULATED
C
      IF(NPREM.EQ.0) GO TO 101
      CALL WRITE1(IW,AMEAN)
      AN=50.
      C(1)=60.
      C(2)=70.
      C(3)=80.
      AA=A
      IF(A.LT.0.) AA=0.-ORIGIN
      B1=AMEAN*0.6-ORIGIN
      B2=AMEAN*0.7-ORIGIN
      B3=AMEAN*0.8-ORIGIN
      DO 20 II=1,3
      COV=AMEAN*C(II)/100.-ORIGIN
      FRA=EQT6(AA,A1,A2,Q1,Q2,YO,COV)
      FB1=EQT6(B1,A1,A2,Q1,Q2,YO,COV)
      H=(B1-AA)/AN
      SUM1=SUMT6(AA,B1,EQT6,A1,A2,Q1,Q2,YO,COV)
      ARE1=(FRA+FB1+2.*SUM1)*(H/2.)
      IF (II.EQ.1) GO TO 19
      FB2=EQT6(B2,A1,A2,Q1,Q2,YO,COV)
      SUM2=SUMT6(B1,B2,EQT6,A1,A2,Q1,Q2,YO,COV)
      H=(B2-B1)/AN
      ARE2=(FB1+FB2+2.*SUM2)*(H/2.)
      ARE1=ARE1+ARE2
      IF (II.EQ.2) GO TO 19
      FB3=EQT6(B3,A1,A2,Q1,Q2,YO,COV)
```

```
      SUM3=SUMT6(B2,B3,EQT6,A1,A2,Q1,Q2,Y0,COV)
      H=(B3-B2)/AN
      ARE3=(FB2+FB3+2.*SUM3)*(H/2.)
      ARE1=ARE1+ARE3
   19 COV=COV+ORIGIN
      TEMP=ARE1/COV
      WRITE(IW,4) C(II),COV,ARE1,TEMP
    4 FORMAT(17X,F4.0,F15.2,2F15.4)
   20 CONTINUE
  101 RETURN
      END
C
C
      FUNCTION SUMT6(A,B,EQT6,A1,A2,Q1,Q2,Y0,COV)
      AN=50.
      SUMT6=0.
      H=(B-A)/AN
      DO 10 J=1,49
      AJ=J
      X=A+AJ*H
      SUMT6=SUMT6+EQT6(X,A1,A2,Q1,A2,Y0,COV)
   10 CONTINUE
      RETURN
      END
C
C
      FUNCTION EQT6(X,A1,A2,Q1,Q2,Y0,COV)
      EQT6=0.
      IF(ABS(X).LT.ABS(A)) GO TO 10
      EQT6=(COV-X)*Y0*(1.+X/A1)**(-1.*Q1)*(1.+X/A2)**Q2
   10 RETURN
      END
C
C
$ENTRY
```

A.I.2. Input Data Set(s)

In writing the program it was recognized that one may often want to estimate a number of distributions using different sets of data. Consequently, the program was designed to run more than one job at a time by repeating the input data set. If no further data set is to be read in, the input data set(s) ends with two blank cards. The card ordering of the input data set can be summarized as in Figure A-I-2. The preparation of the input data set(s) is described below in the order that they are supplied to the computer.

Title Card: This is the first card of an input data set. Any alphanumeric character may be punched in columns 2-80. This card is designed for the purpose of identifying the name of the input data set. The information on the card will be printed on the top of a new page following the "JOB NAME".

Parameter Card: This is the second card of any input data set. It contains the parameters or control numbers which have to be punched in integer form in the specified column(s) and right justified.

Name Used in the Program	Column(s)	Description
N	1-5	number of observations
NCLASS	6-10	if 0(zero) is punched, the observed yield data of N will be read in, otherwise, the observed frequency distribution with NCLASS should be punched in and right justified.

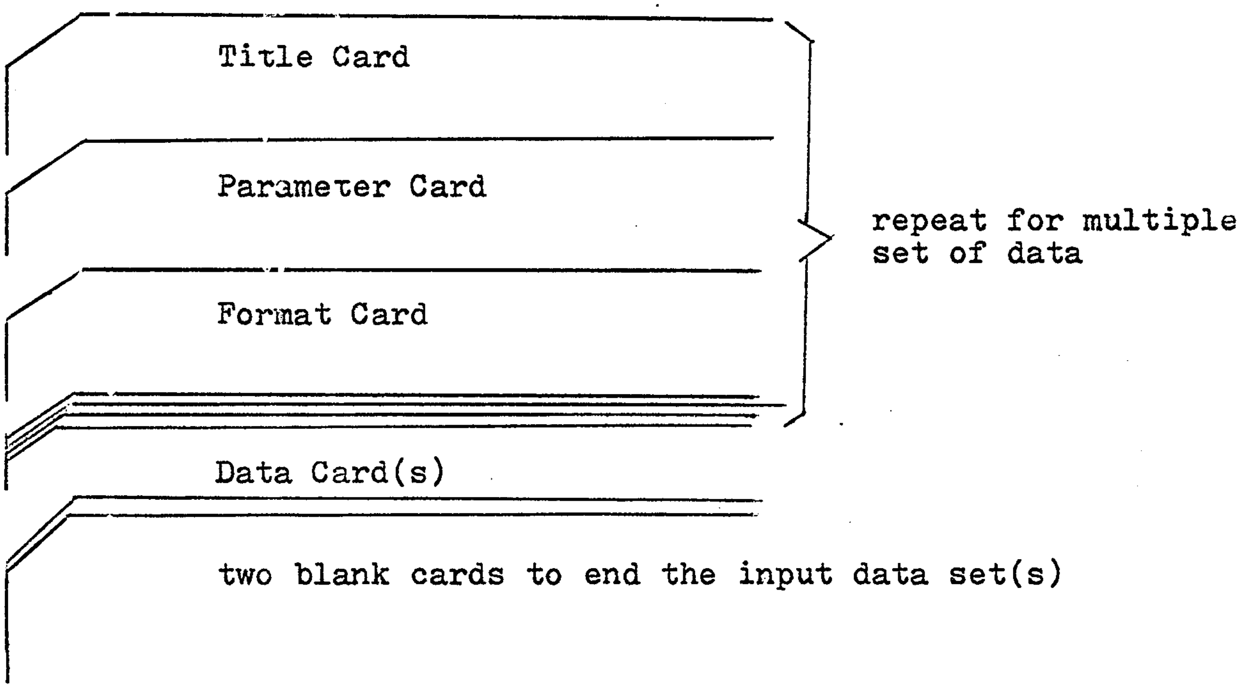


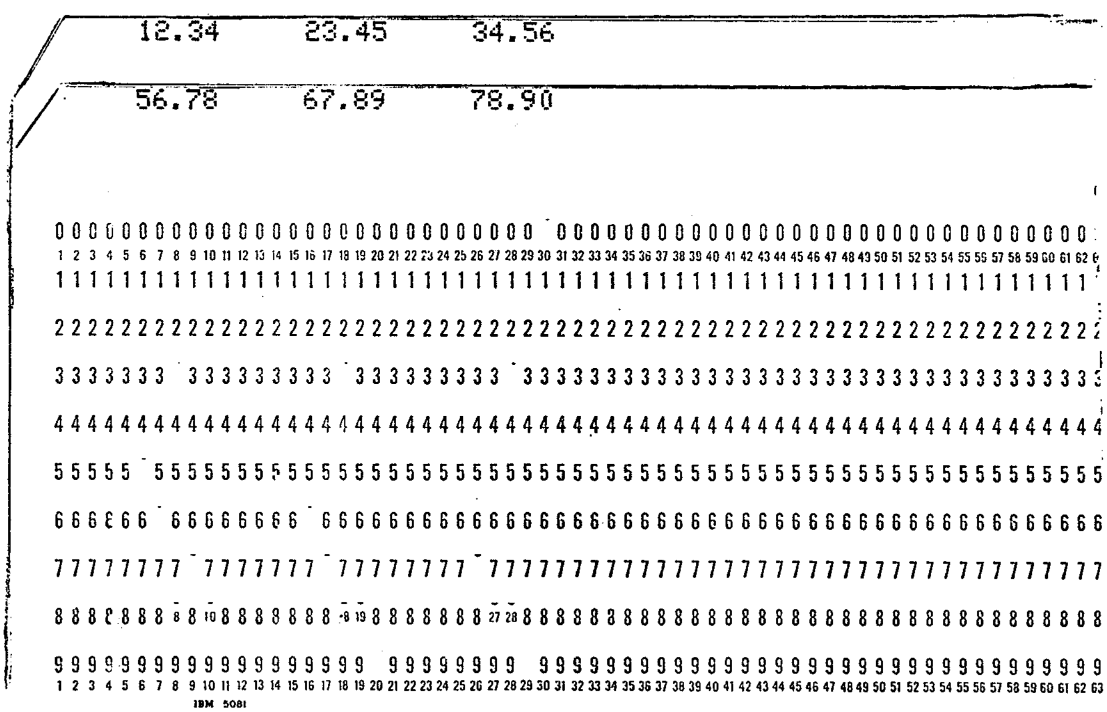
Figure A-I-2. SEQUENCE OF INPUT DATA SET(S)

NORM	15	if 0(zero) is punched, no normal distribution will be estimated. if 1(one) is punched, a normal distribution for crop yields will be estimated.
NPREM	20	if 0(zero) is punched, no premium rate will be estimated. if 1(one) is ounched, premium rates at three specified coverage levels will be estimated.
MCLASS	21-25	this is used to indicate the maximum number of classes or categories of the estimated frequency distribution to be printed.
ICHI	30	if 0(zero) is punched, no chi-square statistic will be calculated.

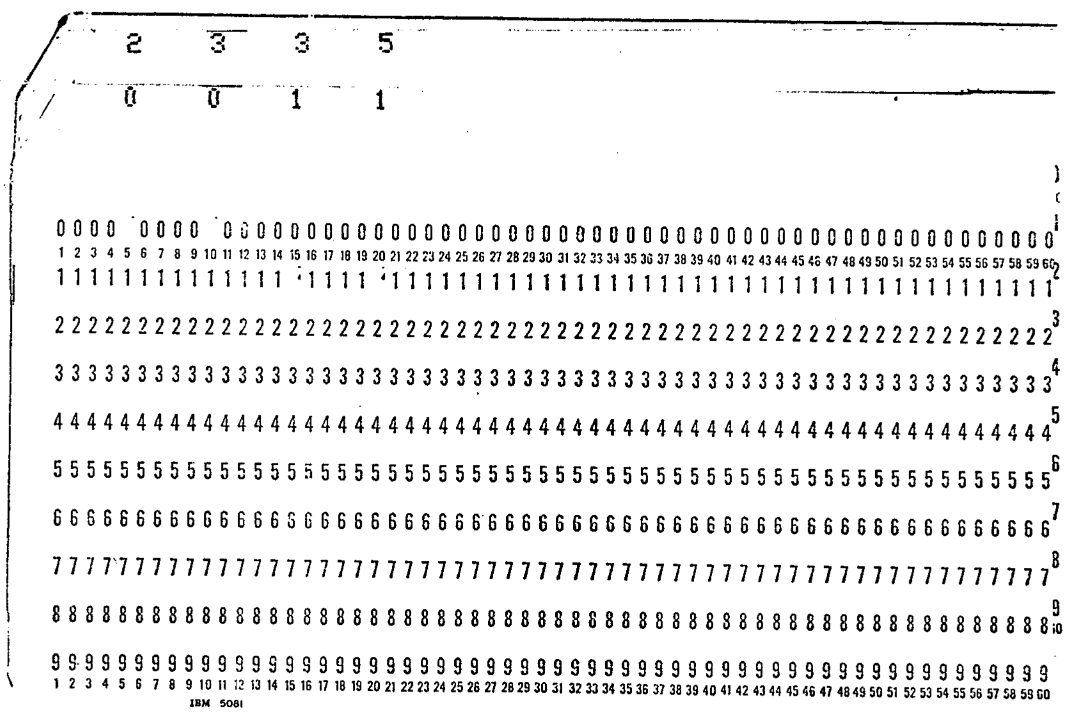
<u>Format Card</u>: This is used to describle the format of the data punched on card(s). Since the program allows reading in of the observed frequency distribution as

well as the observed yield data, two alternatives may be followed.

If observed yield data are to be read in, the format card has to be punched in the form of (aFb.c) and from column 1(one), where a is the number of observed yields punched on one card, b is the maximum number of columns available for each value including the decimal point, and c is the number of decimal places in each value. For example, if (3F10.2) is punched as the format card, it implies that 3 yield observations are punched on each card, and each value may occupy no more than 10 columns and will have 2 decimal places. For example:



If the observed frequency distribution is to be prepared for reading in, the format card has to be punched in the form of (dIe), where d is the number of frequencies to be read in from one card, and e is the maximum number of columns available for each frequency. All the frequencies have to be punched in the integer form and right justified. For example, (4I5) implies that there are four frequencies to be read in from each card and each frequency has a maximum of five columns to be punched in. For example:



Also, the frequencies have to be punched on the card in order, beginning with the first class or category of the frequency distribution, i.e., the frequency between 0-1 bushels per acre.

Data Card(s): The data card(s), prepared in the form of either observed yields (bushels per acre) or the frequency distribution, are then attached following the format card.

A.I.3. The Computer Output

The print-outs for the analyses of Chapters IV and V are shown as follows.

JOB NAME
MANITOBA CROP DISTRICT 1 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

5.7	17.7	9.8	16.4	18.5	23.1	16.7	20.7	13.2	14.2
1.7	10.8	3.5	0.6	9.2	5.0	12.4	9.8	14.6	19.9
17.6	20.6	24.5	20.0	16.1	16.5	14.8	17.7	13.2	18.2
16.9	16.9	21.2	11.9	21.0	26.7	15.9	18.6	20.1	21.0
8.0	26.2	19.1	24.5	22.9	24.1	17.1	24.5	28.3	21.4

THE OBSERVED FREQUENCY DISTRIBUTION IS:

1	1	0	1	1	1	0	1	0	3
1	1	1	2	3	1	6	4	3	3
5	2	1	1	4	0	2	0	1	

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

17.01999	42.09659	-161.63670	5189.75000
6.48819	0.35022	2.92855	-0.24179

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.12	0.18	0.25	0.35	0.48	0.64	0.83	1.05	1.30	1.57
1.86	2.14	2.41	2.65	2.85	2.99	3.06	3.07	3.00	2.86
2.66	2.42	2.15	1.87	1.58	1.31	1.06	0.83	0.64	0.48
0.36	0.25	0.18	0.12	0.08	0.05	0.03	0.02	0.01	0.01

CHI-SQUARE TEST: 40 CLASSES EXAMINED, CHI-SQUARE IS 34.0070

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.21	0.4383	0.0429
70.	11.91	0.7372	0.0619
80.	13.62	1.1665	0.0857

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0606	A1=	34.3610	M1=	4.7177
A2=	8.8611	M2=	1.2166	ORIGIN=	20.2339

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.22	0.29	0.38	0.47	0.58	0.71	0.85	1.01	1.18	1.36
1.55	1.75	1.95	2.16	2.35	2.54	2.70	2.84	2.95	3.02
3.03	2.98	2.86	2.66	2.37	1.98	1.50	0.93	0.31	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 40 CLASSES EXAMINED, CHI-SQUARE IS 25.9229

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.21	0.6564	0.0643
70.	11.91	0.9721	0.0816
80.	13.62	1.3941	0.1024

JOB NAME
MANITOBA CROP DISTRICT 2 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

9.3	17.8	9.6	16.3	18.7	24.7	15.6	21.7	12.1	20.6
6.5	16.5	11.8	9.0	7.8	7.3	16.1	14.5	15.3	18.5
20.5	26.6	25.0	21.8	16.8	20.5	17.0	23.0	15.6	19.3
18.0	22.6	27.7	12.6	22.0	22.3	23.6	24.1	25.3	23.3
11.2	26.5	18.5	27.2	24.0	25.7	25.0	30.2	26.8	23.3

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	1	2	1	2
0	2	2	0	1	3	5	2	3	1
3	3	3	4	4	2	3	2	0	0
1									

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

19.53999	35.84512	-80.01563	2907.75200
5.98708	0.13901	2.26307	-0.05896

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.02	0.04	0.06	0.09	0.14	0.21	0.31	0.44	0.61	0.82
1.07	1.35	1.67	2.00	2.34	2.65	2.93	3.14	3.28	3.33
3.29	3.16	2.95	2.68	2.36	2.03	1.70	1.38	1.09	0.84
0.62	0.45	0.32	0.22	0.15	0.10	0.06	0.04	0.02	0.01

CHI-SQUARE TEST: 40 CLASSES EXAMINED, CHI-SQUARE IS 23.9826

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	11.72	0.2620	0.0223
70.	13.68	0.5105	0.0373
80.	15.63	0.9165	0.0586

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0593	A1=	20.2384	M1=	1.2041
A2=	6.0937	M2=	0.3626	ORIGIN=	23.5058

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.02	0.17	0.34	0.53	0.72	0.91	1.11
1.30	1.49	1.67	1.85	2.03	2.19	2.34	2.49	2.62	2.73
2.82	2.90	2.95	2.96	2.94	2.87	2.74	2.50	2.08	0.91
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 37 CLASSES EXAMINED, CHI-SQUARE IS 16.5789

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	11.72	0.3333	0.0284
70.	13.68	0.6374	0.0466
80.	15.63	1.0852	0.0694

```
JOB NAME
MANITOBA CROP DISTRICT  3 WHEAT YIELD (BUSHELS/ACRE) 1921-70

   THE DATA SET IS:
  11.9   19.0    9.6   30.2   18.3   22.8   10.7   16.1   14.4   19.3
  13.6   15.0   13.9   19.7   11.0   13.4   21.2   16.2   20.4   20.6
  20.2   28.8   24.0   17.0   16.7   21.1   14.2   23.0   16.5   20.7
  22.4   22.9   23.5   13.9   18.9   26.5   19.0   22.7   21.8   22.6
  14.0   24.4   14.1   23.4   25.5   20.5   27.2   27.3   18.5   19.4

   THE OBSERVED FREQUENCY DISTRIBUTION IS:
     0      0      0      0      0      0      0      0      0      1
     2      1      0      5      4      0      5      0      5      3
     5      3      6      3      1      1      1      2      1      0
     1

   THE VALUES IN ORDER ARE:
   MEAN, U2, U3, U4, SD, B1, B2, K
 19.78000        24.16812         5.24219      1418.02800
  4.91611         0.00195         2.42772        -0.00128

   THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:
  0.00   0.00   0.01   0.02   0.03   0.06   0.11   0.18   0.29   0.46
  0.68   0.98   1.36   1.79   2.28   2.78   3.25   3.64   3.92   4.05
  4.01   3.82   3.48   3.05   2.56   2.06   1.59   1.18   0.84   0.57
  0.38   0.24   0.14   0.08   0.05   0.02   0.01   0.01   0.00   0.00

 CHI-SQUARE TEST: 37 CLASSES EXAMINED, CHI-SQUARE IS  26.8958

            % OF                                           PREMIUM/
        MEAN YIELD     INSURED YIELD       PREMIUM   INSURED YIELD

               60.            11.87          0.1113          0.0094
               70.            13.85          0.2714          0.0196
               80.            15.82          0.5856          0.0370

  *** TYPE I PEARSON DISTRIBUTION IS USED ***

        Y = YC * (1+X/A1)**M1 * (1-X/A2)**M2
        WITH ORIGIN AT THE MODE, WHERE
   YC=             0.0737 A1=         13.5882 M1=          2.5850
   A2=            14.9881 M2=          2.8513 ORIGIN=   19.5917

   THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS
  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.07   0.22   0.48
  0.82   1.22   1.65   2.09   2.52   2.91   3.23   3.47   3.63   3.68
  3.65   3.51   3.29   2.99   2.64   2.24   1.83   1.42   1.04   0.69
  0.41   0.21   0.07   0.01   0.00   0.00   0.00   0.00   0.00   0.00

 CHI-SQUARE TEST: 33 CLASSES EXAMINED, CHI-SQUARE IS  24.1098

            % OF                                           PREMIUM/
        MEAN YIELD     INSURED YIELD       PREMIUM   INSURED YIELD

               60.            11.87          0.0743          0.0063
               70.            13.85          0.2438          0.0176
               80.            15.82          0.5884          0.0372
```

JOB NAME
MANITOBA CROP DISTRICT 4 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

16.9	19.8	12.3	33.0	17.8	22.6	10.8	21.8	16.5	17.6
15.7	16.4	14.5	23.3	12.1	14.0	22.0	16.2	20.4	24.2
21.2	30.8	25.1	16.0	18.0	21.5	14.7	23.9	16.5	22.3
23.1	22.2	22.3	12.7	20.0	26.3	20.1	21.3	21.4	21.1
12.5	24.2	11.0	26.9	26.3	26.0	29.6	29.1	19.2	20.8

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	0	0	0	0
2	0	4	1	2	2	5	3	0	3
3	7	4	3	2	2	3	0	0	2
1	0	1							

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

20.73999	27.50967	18.61719	1874.73200
5.24497	0.01665	2.47724	-0.01156

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.00	0.00	0.01	0.02	0.03	0.06	0.10	0.16	0.25	0.38
0.57	0.81	1.11	1.47	1.87	2.31	2.74	3.14	3.47	3.70
3.80	3.76	3.59	3.31	2.94	2.52	2.08	1.66	1.27	0.94
0.67	0.46	0.31	0.20	0.12	0.07	0.04	0.02	0.01	0.01

CHI-SQUARE TEST: 37 CLASSES EXAMINED, CHI-SQUARE IS 29.2611

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.44	0.1266	0.0102
70.	14.52	0.3020	0.0208
80.	16.59	0.6400	0.0386

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0697	A1=	13.5697	M1=	2.5723
A2=	18.0813	M2=	3.4276	ORIGIN=	20.1760

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.12	0.32
0.61	0.97	1.37	1.79	2.21	2.59	2.92	3.18	3.37	3.47
3.48	3.41	3.27	3.05	2.78	2.47	2.13	1.79	1.44	1.12
0.82	0.57	0.36	0.21	0.10	0.04	0.01	0.00	0.00	0.00

CHI-SQUARE TEST: 33 CLASSES EXAMINED, CHI-SQUARE IS 25.7080

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.44	0.0740	0.0059
70.	14.52	0.2531	0.0174
80.	16.59	0.6217	0.0375

JOB NAME
MANITOBA CROP DISTRICT 5 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

16.7	20.6	12.8	17.9	18.9	23.7	11.0	21.5	15.5	18.8
18.3	14.9	14.8	25.3	11.2	16.2	23.2	19.0	21.5	26.2
22.3	31.9	26.0	25.0	21.4	25.0	16.5	23.9	20.6	24.7
23.1	27.1	23.7	12.3	15.0	26.9	20.3	25.7	20.0	20.1
16.1	25.4	11.4	20.9	24.6	18.8	25.4	26.4	17.2	17.0

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	0	0	0	0
1	2	2	0	3	1	5	2	5	1
5	3	1	5	4	5	3	1	0	0
0	1								

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

20.85999	23.11783	-16.70313	1234.67800
4.80810	0.02258	2.31025	-0.01199

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.00	0.00	0.00	0.01	0.01	0.03	0.05	0.09	0.15	0.25
0.41	0.62	0.92	1.29	1.73	2.23	2.75	3.25	3.68	3.99
4.14	4.11	3.91	3.57	3.11	2.60	2.09	1.60	1.17	0.83
0.56	0.36	0.22	0.13	0.07	0.04	0.02	0.01	0.00	0.00

CHI-SQUARE TEST: 36 CLASSES EXAMINED, CHI-SQUARE IS 24.2036

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.52	0.0804	0.0064
70.	14.60	0.2180	0.0149
80.	16.69	0.5120	0.0307

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0732	A1=	14.2970	M1=	1.9598
A2=	10.0577	M2=	1.3787	ORIGIN=	21.6541

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.26
0.53	0.85	1.20	1.58	1.97	2.34	2.69	3.01	3.27	3.47
3.60	3.66	3.63	3.51	3.30	3.00	2.62	2.16	1.63	1.07
0.51	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 32 CLASSES EXAMINED, CHI-SQUARE IS 37.4267

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.52	0.0633	0.0051
70.	14.60	0.2238	0.0153
80.	16.69	0.5587	0.0335

JOB NAME
MANITOBA CROP DISTRICT 6 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

9.4	20.6	11.7	15.9	13.7	16.5	11.0	16.3	14.3	17.0
14.9	13.9	14.2	17.4	9.6	11.0	18.4	16.9	18.3	21.2
20.6	25.7	20.0	17.0	17.7	18.3	13.7	19.2	16.0	17.3
16.4	18.0	24.1	8.2	16.0	25.4	17.4	22.3	14.5	23.4
11.0	21.0	11.1	20.3	24.3	17.7	25.9	27.2	20.7	18.1

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	0	0	1	2
3	2	0	3	4	3	6	6	4	2
5	1	1	1	2	3	0	1		

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

17.84000	20.97134	14.86328	1103.87600
4.57945	0.02395	2.50997	-0.01734

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.00	0.01	0.02	0.03	0.06	0.12	0.20	0.34	0.54	0.83
1.21	1.67	2.21	2.78	3.34	3.82	4.17	4.34	4.31	4.08
3.68	3.16	2.60	2.03	1.51	1.08	0.73	0.47	0.29	0.17
0.10	0.05	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 38 CLASSES EXAMINED, CHI-SQUARE IS 19.6123

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.70	0.1167	0.0109
70.	12.49	0.2731	0.0219
80.	14.27	0.5703	0.0400

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YC * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0804	A1=	11.7892	M1=	2.6847
A2=	16.6481	M2=	3.7913	ORIGIN=	17.2667

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.20	0.52	0.96
1.48	2.04	2.60	3.10	3.51	3.81	3.98	4.02	3.92	3.71
3.40	3.01	2.58	2.12	1.67	1.25	0.88	0.58	0.34	0.18
0.07	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 34 CLASSES EXAMINED, CHI-SQUARE IS 16.9834

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.70	0.0679	0.0063
70.	12.49	0.2265	0.0181
80.	14.27	0.5492	0.0385

JOB NAME
MANITOBA CROP DISTRICT 7 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

10.9	20.3	13.5	17.1	17.1	22.5	16.5	21.0	14.1	18.3
7.5	16.1	12.2	11.3	7.1	9.0	11.9	16.4	16.4	14.0
20.0	25.9	25.0	26.9	20.5	23.7	20.0	24.9	19.9	22.5
21.2	27.0	27.3	12.8	24.0	22.0	23.4	21.3	21.9	24.0
12.0	25.8	22.5	24.2	22.3	23.6	23.4	22.3	28.4	22.5

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	0	2	1	0
1	3	2	2	1	0	4	2	1	3
3	4	6	6	3	2	2	1	1	

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

19.89999	29.52676	-93.21484	2053.16100
5.43385	0.33754	2.35500	-0.12500

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.01	0.01	0.02	0.04	0.07	0.11	0.18	0.27	0.41	0.59
0.82	1.11	1.45	1.83	2.24	2.64	3.02	3.33	3.55	3.66
3.65	3.52	3.27	2.95	2.57	2.16	1.76	1.38	1.05	0.77
0.55	0.38	0.25	0.16	0.10	0.06	0.03	0.02	0.01	0.01

CHI-SQUARE TEST: 39 CLASSES EXAMINED, CHI-SQUARE IS 30.6461

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	11.94	0.1707	0.0143
70.	13.93	0.3719	0.0267
80.	15.92	0.7325	0.0460

*** J-SHAPED TYPE I PEARSON DISTRIBUTION IS USED ***

Y = Y0 * X**M1 * (A1+A2-X)**M2
WITH ORIGIN AT THE START OF THE DISTRIBUTION, WHERE

Y0=	0.0090	A1=	25.9575	M1=	0.7675
A2=	-3.9316	M2=	-0.1162	ORIGIN=	5.2161

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.12	0.38	0.60	0.80	0.99
1.17	1.34	1.51	1.68	1.86	2.03	2.20	2.37	2.55	2.74
2.93	3.13	3.35	3.60	3.89	4.26	4.88	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 35 CLASSES EXAMINED, CHI-SQUARE IS 19.0467

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	11.94	0.2558	0.0214
70.	13.93	0.5281	0.0379
80.	15.92	0.9396	0.0590

JOB NAME
MANITOBA CROP DISTRICT 8 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

10.7	16.2	19.5	30.2	19.6	23.4	11.1	20.8	12.5	17.9
9.9	16.3	13.2	14.9	6.0	11.9	15.2	17.6	21.7	17.8
19.5	25.6	24.0	22.0	18.9	23.9	19.5	23.5	18.0	18.6
21.1	26.0	25.1	13.5	22.0	27.6	24.5	26.6	25.2	24.4
10.7	26.8	19.5	27.6	24.7	26.0	26.7	28.7	27.3	23.2

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	1	0	0	0	1
2	2	1	2	1	1	2	4	2	5
1	4	0	5	3	5	3	3	1	0
1									

THE VALUES IN ORDER ARE:
MEAN, L2, L3, L4, SD, B1, B2, K

20.73999	32.90958	-90.56641	2668.03200
5.73669	0.23013	2.46347	-0.10627

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.01	0.01	0.02	0.04	0.06	0.10	0.16	0.24	0.36	0.51
0.71	0.95	1.24	1.57	1.92	2.29	2.65	2.96	3.22	3.40
3.47	3.45	3.32	3.10	2.81	2.46	2.10	1.74	1.39	1.08
0.82	0.60	0.43	0.29	0.20	0.13	0.08	0.05	0.03	0.02

CHI-SQUARE TEST: 39 CLASSES EXAMINED, CHI-SQUARE IS 29.1832

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.44	0.1877	0.0151
70.	14.52	0.4033	0.0278
80.	16.59	0.7854	0.0473

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0649	A1=	21.8994	M1=	1.7485
A2=	5.6094	M2=	0.4479	ORIGIN=	24.6219

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.02	0.08	0.17	0.29	0.42	0.58	0.75
0.93	1.13	1.33	1.53	1.74	1.95	2.16	2.36	2.55	2.73
2.89	3.03	3.14	3.21	3.25	3.22	3.12	2.92	2.55	1.85
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 37 CLASSES EXAMINED, CHI-SQUARE IS 21.0534

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.44	0.2646	0.0213
70.	14.52	0.5323	0.0367
80.	16.59	0.9503	0.0573

JOB NAME
MANITOBA CROP DISTRICT 9 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

12.7	16.0	10.3	14.9	15.0	18.5	10.8	20.5	12.4	16.8
11.5	16.8	14.7	18.0	5.8	13.9	18.8	17.4	23.3	22.1
22.4	27.3	25.0	25.0	23.3	24.0	21.0	26.8	18.6	21.5
22.1	27.1	25.7	12.5	23.0	25.5	22.6	25.1	21.5	22.5
11.7	26.9	19.8	26.0	21.6	23.5	27.0	28.1	24.5	23.0

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	1	0	0	0	0
2	2	3	1	3	1	2	2	3	1
2	3	7	4	3	4	3	2	1	

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

20.45999	29.80557	-91.18359	2139.95900
5.45945	0.31401	2.40885	-0.12436

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.00	0.01	0.02	0.03	0.05	0.09	0.14	0.22	0.33	0.49
0.69	0.95	1.26	1.62	2.01	2.42	2.81	3.15	3.43	3.60
3.65	3.59	3.41	3.13	2.78	2.39	1.98	1.59	1.24	0.93
0.67	0.47	0.32	0.21	0.13	0.08	0.05	0.03	0.02	0.01

CHI-SQUARE TEST: 38 CLASSES EXAMINED, CHI-SQUARE IS 30.7822

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.28	0.1590	0.0130
70.	14.32	0.3555	0.0248
80.	16.37	0.7151	0.0437

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YC * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YC=	0.0799	A1=	22.7365	M1=	1.0604
A2=	0.6850	M2=	0.0319	ORIGIN=	27.5910

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.10	0.28	0.46	0.64	0.82
1.01	1.20	1.39	1.58	1.77	1.96	2.15	2.34	2.53	2.72
2.90	3.09	3.27	3.44	3.61	3.77	3.91	3.99	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 35 CLASSES EXAMINED, CHI-SQUARE IS 22.1618

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.28	0.2369	0.0193
70.	14.32	0.4983	0.0348
80.	16.37	0.9051	0.0553

JOB NAME
MANITOBA CROP DISTRICT 10 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

15.6	26.5	16.7	12.9	18.3	22.5	18.5	21.6	15.2	18.8
14.6	22.3	18.7	17.7	6.5	15.9	11.7	20.7	23.4	14.9
21.5	30.5	30.0	28.0	23.3	26.0	22.2	27.8	25.7	21.7
25.6	26.1	28.4	14.1	26.0	29.6	23.5	22.3	27.6	26.6
8.8	28.2	30.1	30.0	25.3	28.5	28.5	25.9	32.1	26.3

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	1	0	1	0
0	1	1	0	3	3	1	1	4	0
1	3	4	3	0	6	4	3	4	3
2	0	1							

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

22.89999	37.08707	-141.48820	3624.00600
6.08991	0.39244	2.63478	-0.17441

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.00	0.01	0.01	0.02	0.03	0.06	0.09	0.13	0.20	0.29
0.41	0.57	0.76	1.00	1.27	1.57	1.89	2.21	2.52	2.80
3.03	3.19	3.27	3.26	3.16	2.99	2.75	2.46	2.15	1.82
1.50	1.21	0.95	0.72	0.53	0.39	0.27	0.19	0.12	0.08

CHI-SQUARE TEST: 38 CLASSES EXAMINED, CHI-SQUARE IS 38.0254

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	13.74	0.1754	0.0128
70.	16.03	0.3936	0.0246
80.	18.32	0.7943	0.0434

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0675	A1=	26.1619	M1=	1.7065
A2=	3.0775	M2=	0.2007	ORIGIN=	28.8082

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.02	0.06	0.12	0.20	0.29	0.39	0.51
0.64	0.78	0.92	1.08	1.24	1.40	1.58	1.75	1.93	2.11
2.29	2.46	2.64	2.80	2.96	3.10	3.23	3.32	3.37	3.35
3.20	2.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 37 CLASSES EXAMINED, CHI-SQUARE IS 26.8559

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	13.74	0.2863	0.0208
70.	16.03	0.5670	0.0354
80.	18.32	1.0046	0.0548

JOB NAME
MANITOBA CROP DISTRICT 11 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

12.6	20.0	11.9	25.2	12.9	18.6	11.5	19.2	14.2	17.1
19.2	18.6	19.6	21.0	10.5	15.6	15.8	18.9	27.6	17.8
19.5	26.9	24.0	24.0	17.7	24.7	12.7	23.9	21.1	22.7
22.1	22.1	11.4	10.2	17.9	23.5	20.9	25.9	25.2	24.3
7.4	27.3	26.3	23.4	26.9	27.6	27.0	23.9	30.6	23.9

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	0	1	0	0
2	3	3	0	1	2	0	4	3	5
2	1	3	7	2	3	4	3	0	0
1									

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

20.65999	30.14175	-67.41797	2050.16600
5.49015	0.16598	2.25659	-0.06774

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.00	0.01	0.02	0.03	0.05	0.08	0.13	0.21	0.31	0.46
0.66	0.90	1.20	1.55	1.94	2.34	2.73	3.08	3.36	3.55
3.63	3.59	3.43	3.18	2.84	2.46	2.06	1.67	1.31	0.99
0.73	0.52	0.36	0.24	0.15	0.09	0.06	0.03	0.02	0.01

CHI-SQUARE TEST: 38 CLASSES EXAMINED, CHI-SQUARE IS 34.5052

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.40	0.1577	0.0127
70.	14.46	0.3543	0.0245
80.	16.53	0.7154	0.0433

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YC * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YC=	0.0659	A1=	19.2891	M1=	1.0637
A2=	4.2303	M2=	0.2333	ORIGIN=	25.2274

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.33	0.56	0.79
1.01	1.23	1.45	1.66	1.87	2.06	2.26	2.44	2.61	2.77
2.91	3.04	3.15	3.23	3.28	3.30	3.25	3.10	2.75	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 34 CLASSES EXAMINED, CHI-SQUARE IS 24.6618

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.40	0.1955	0.0158
70.	14.46	0.4519	0.0312
80.	16.53	0.8637	0.0524

JOB NAME
MANITOBA CROP DISTRICT 12 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

11.2	22.4	12.4	9.9	14.3	13.9	11.4	19.9	14.2	17.5
15.9	19.0	15.4	19.1	8.6	14.3	19.0	17.6	26.0	13.9
19.5	28.8	26.0	27.0	22.3	22.5	16.6	25.9	17.7	23.7
25.0	24.1	22.7	11.5	15.5	29.1	20.9	21.5	21.1	23.6
10.2	25.7	11.0	21.6	24.8	26.6	27.0	28.4	26.0	23.0

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	0	0	1	1
2	3	1	2	3	3	1	3	2	3
1	3	5	2	3	5	3	0	2	1

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

20.07999	31.83096	-37.48438	1945.07100
5.64189	0.04357	1.91971	-0.01524

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.01	0.02	0.03	0.05	0.08	0.13	0.20	0.29	0.43	0.61
0.84	1.11	1.43	1.79	2.17	2.54	2.89	3.18	3.40	3.52
3.53	3.43	3.22	2.94	2.60	2.23	1.85	1.49	1.16	0.88
0.64	0.46	0.31	0.21	0.13	0.08	0.05	0.03	0.02	0.01

CHI-SQUARE TEST: 39 CLASSES EXAMINED, CHI-SQUARE IS 20.4804

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.05	0.1937	0.0161
70.	14.06	0.4095	0.0291
80.	16.06	0.7868	0.0490

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = Y0 * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

Y0=	0.0609	A1=	20.1738	M1=	0.2877
A2=	0.4620	M2=	0.0066	ORIGIN=	28.6716

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	1.32
1.61	1.80	1.96	2.09	2.20	2.30	2.39	2.47	2.54	2.61
2.67	2.73	2.79	2.84	2.89	2.94	2.98	3.02	3.05	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 32 CLASSES EXAMINED, CHI-SQUARE IS 13.6099

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	12.05	0.1611	0.0134
70.	14.06	0.4503	0.0320
80.	16.06	0.9126	0.0568

JOB NAME
MANITOBA CROP DISTRICT 13 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

20.3	26.0	17.9	14.5	20.0	23.6	14.0	20.4	20.2	19.2
22.0	22.7	24.4	23.7	15.0	20.5	14.6	18.4	33.5	20.7
19.5	30.0	30.0	25.0	26.0	31.1	29.8	28.7	23.7	28.8
33.6	33.1	21.7	11.6	25.0	29.7	19.8	30.6	30.7	30.0
12.0	30.8	29.4	28.5	26.3	32.7	30.6	29.5	32.9	28.4

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	0	0	0	0	0
0	2	0	1	3	0	0	1	1	4
5	2	1	3	3	2	1	0	4	7
4	1	2	3						

THE VALUES IN ORDER ARE:
MEAN, L2, L3, L4, SD, B1, B2, K

24.95999	37.07535	-87.73828	2972.51200
6.08895	0.15105	2.16249	-0.05769

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05	0.08	0.13
0.20	0.28	0.40	0.56	0.75	0.98	1.25	1.55	1.87	2.19
2.51	2.79	3.02	3.18	3.27	3.26	3.17	3.00	2.77	2.48
2.17	1.84	1.52	1.23	0.96	0.73	0.54	0.39	0.28	0.19

CHI-SQUARE TEST: 36 CLASSES EXAMINED, CHI-SQUARE IS 48.3074

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	14.98	0.1283	0.0086
70.	17.47	0.3208	0.0184
80.	19.97	0.7063	0.0354

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YC * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YC=	0.0605	A1=	21.9013	M1=	0.7588
A2=	3.6780	M2=	0.0928	ORIGIN=	31.7013

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.27	0.53	0.75	0.95	1.13	1.31	1.47	1.62	1.77	1.91
2.05	2.17	2.30	2.41	2.52	2.63	2.72	2.81	2.89	2.95
3.00	3.02	3.01	2.90	0.00	0.00	0.00	0.00	0.00	0.00

CHI-SQUARE TEST: 30 CLASSES EXAMINED, CHI-SQUARE IS 31.1156

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	14.98	0.1358	0.0091
70.	17.47	0.4004	0.0229
80.	19.97	0.8661	0.0434

JOB NAME
MANITOBA CROP DISTRICT 14 WHEAT YIELD (BUSHELS/ACRE) 1921-70

THE DATA SET IS:

11.0	18.4	13.6	14.7	14.6	17.0	10.6	17.8	13.4	16.7
22.2	19.9	22.0	17.3	9.4	14.2	14.7	14.2	24.4	15.1
11.0	18.4	13.6	14.7	14.6	17.0	10.6	17.8	13.4	16.7
28.9	29.0	17.2	6.5	20.0	22.9	19.7	21.6	18.3	19.6
5.7	26.5	17.2	18.2	20.5	21.7	22.1	28.4	25.7	21.6

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	1	1	0	0	1
4	0	0	4	7	1	4	5	4	4
1	4	3	0	1	1	1	0	3	

THE VALUES IN ORDER ARE:
MEAN, L2, L3, L4, SD, B1, B2, K

18.01999	27.61661	15.75000	2199.42800
5.25515	0.01178	2.88383	-0.03311

THE ESTIMATED NORMAL FREQUENCY DISTRIBUTION IS:

0.01	0.03	0.05	0.08	0.14	0.22	0.34	0.51	0.74	1.02
1.36	1.76	2.19	2.62	3.03	3.38	3.64	3.78	3.78	3.65
3.40	3.05	2.64	2.20	1.77	1.38	1.03	0.75	0.52	0.35
0.23	0.14	0.09	0.05	0.03	0.02	0.01	0.00	0.00	0.00

CHI-SQUARE TEST: 40 CLASSES EXAMINED, CHI-SQUARE IS 40.8133

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.81	0.2013	0.0186
70.	12.61	0.4101	0.0325
80.	14.42	0.7643	0.0530

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = YO * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

YO=	0.0747	A1=	28.4291	M1=	16.2248
A2=	41.5924	M2=	23.7373	ORIGIN=	17.7063

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.00	0.01	0.02	0.05	0.10	0.19	0.32	0.50	0.75	1.07
1.44	1.86	2.31	2.74	3.12	3.43	3.64	3.73	3.69	3.54
3.28	2.94	2.56	2.15	1.75	1.38	1.05	0.78	0.55	0.38
0.25	0.16	0.10	0.06	0.03	0.02	0.01	0.00	0.00	0.00

CHI-SQUARE TEST: 39 CLASSES EXAMINED, CHI-SQUARE IS 40.0534

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.81	0.1788	0.0165
70.	12.61	0.3891	0.0308
80.	14.42	0.7520	0.0522

JOB NAME
ANALYSIS OF DISCOUNT RATE CASE(A) NEAR AVERAGE FOR 5 YEARS

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	1	1	0	0	1
4	0	0	4	7	2	5	6	5	5
1	4	3	0	1	1	1	0	3	

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

18.01817	25.28016	14.45313	1999.72217
5.02794	0.01293	3.12903	0.04438

*** TYPE IV PEARSON DISTRIBUTION IS USED ***

Y = Y0 * (1+X*X/A*A)**-M * EXP(-V*ARCTAN(X/A))
WITH ORIGIN AT V*A/R + MEAN, WHERE
Y0= 0.02214 A= 37.07721 M= 29.95247
V= -12.47901 R= 57.90495 ORIGIN= 10.02772

THE ESTIMATED TYPE IV FREQUENCY DISTRIBUTION IS

0.01	0.02	0.03	0.06	0.10	0.17	0.28	0.45	0.69	1.01
1.42	1.91	2.45	3.01	3.53	3.98	4.28	4.42	4.38	4.16
3.79	3.33	2.82	2.30	1.81	1.38	1.02	0.73	0.51	0.35
0.23	0.15	0.09	0.06	0.03	0.02	0.01	0.01	0.00	0.00

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.81	0.1539	0.0142
70.	12.61	0.3387	0.0269
80.	14.41	0.6720	0.0466

JOB NAME
ANALYSIS OF DISCOUNT RATE CASE(E) NEAR AVERAGE FOR 10 YEARS

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	1	1	0	0	1
4	0	0	4	7	3	6	7	6	6
1	4	3	0	1	1	1	0	3	

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

18.01666	23.33316	13.34375	1833.94580
4.83044	0.01402	3.36852	0.01522

*** TYPE IV PEARSON DISTRIBUTION IS USED ***

Y = Y0 * (1+X*X/A*A)**-M * EXP(-V*ARCTAN(X/A))
WITH ORIGIN AT V*A/R + MEAN, WHERE

Y0=	0.07442	A=	21.07346	M=	11.16341
V=	-2.52735	R=	20.32683	ORIGIN=	15.39648

THE ESTIMATED TYPE IV FREQUENCY DISTRIBUTION IS

0.01	0.02	0.03	0.06	0.09	0.16	0.26	0.42	0.64	0.97
1.39	1.93	2.57	3.25	3.93	4.52	4.94	5.14	5.09	4.79
4.31	3.70	3.05	2.42	1.85	1.37	0.99	0.69	0.47	0.32
0.21	0.14	0.09	0.06	0.04	0.02	0.01	0.01	0.01	0.00

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	10.81	0.1338	0.0124
70.	12.61	0.2982	0.0236
80.	14.41	0.6061	0.0421

JOB NAME
ANALYSIS OF DISCOUNT RATE CASE(C) ABOVE AVERAGE FOR 5 YAERS

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	1	1	0	0	1
4	0	0	4	7	1	4	5	4	4
2	5	4	1	2	1	1	0	3	

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

18.47272	27.32999	-9.06641	2064.75977
5.22781	0.00403	2.76434	-0.00626

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = Y0 * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE
Y0= 0.0738 A1= 27.2444 M1= 10.7863
A2= 22.8901 M2= 9.0624 ORIGIN= 18.6720

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.01	0.01	0.03	0.06	0.12	0.20	0.33	0.50	0.73	1.03
1.38	1.78	2.21	2.66	3.08	3.46	3.77	3.97	4.06	4.02
3.85	3.57	3.19	2.76	2.29	1.83	1.39	1.01	0.70	0.45
0.27	0.15	0.08	0.03	0.01	0.00	0.00	0.00	0.00	0.00

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	11.08	0.1892	0.0171
70.	12.93	0.4002	0.0309
80.	14.78	0.7602	0.0514

JOB NAME
ANALYSIS OF DISCOUNT RATE CASE(D) ABOVE AVERAGE FOR 10 YEARS

THE OBSERVED FREQUENCY DISTRIBUTION IS:

0	0	0	0	0	1	1	0	0	1
4	0	0	4	7	1	4	5	4	4
3	6	5	2	3	1	1	0	3	

THE VALUES IN ORDER ARE:
MEAN, U2, U3, U4, SD, B1, B2, K

18.84999	26.77774	-28.76563	1968.34839
5.17472	0.04309	2.74507	-0.05127

*** TYPE I PEARSON DISTRIBUTION IS USED ***

Y = Y0 * (1+X/A1)**M1 * (1-X/A2)**M2
WITH ORIGIN AT THE MODE, WHERE

Y0=	0.0741	A1=	27.3806	M1=	8.7531
A2=	16.3428	M2=	5.2245	ORIGIN=	19.5408

THE ESTIMATED TYPE I FREQUENCY DISTRIBUTION IS

0.01	0.02	0.04	0.07	0.13	0.21	0.33	0.50	0.72	1.00
1.33	1.71	2.14	2.59	3.05	3.49	3.87	4.17	4.37	4.45
4.38	4.18	3.84	3.40	2.89	2.33	1.78	1.27	0.83	0.49
0.26	0.11	0.04	0.01	0.00	0.00	0.0	0.0	0.0	0.0

% OF MEAN YIELD	INSURED YIELD	PREMIUM	PREMIUM/ INSURED YIELD
60.	11.31	0.1941	0.0172
70.	13.19	0.4029	0.0305
80.	15.08	0.7573	0.0502

A.I.4. Limitations and Extensions of the Program

Limitations and extension of the computer program are summarized as follows;

1. The maximum number of yield observations to be read in is limited to one hundred. The maximum number of classes of the frequency distribution is limited to fifty. However, these two limitations can be extended by changing the DIMENSION cards of the main program and subroutine subprograms. For example, if the maximum number of observations to be read in increases to two hundred and the maximum number of classes of the frequency distribution increases to seventy-five, the DIMENSION cards to be changed are as follows;

   In the main program

   DIMENSION DATA(100),CLASS(50),IFREQ(50),NAME(20)

   should be replaced by

   DIMENSION DATA(200),CLASS(75),IFREQ(75),NAME(20)

   For example, in the subroutine subprogram

   DIMENSION Y(50),c(3),IT(50)

   should be replaced by

   DIMENSION Y(75),c(3),IT(75)

2. The input/output devices of the compiler are specified as 1 for card reader and 3 for printer. If the input/output devices are different,

for example the card reader is 5 and the printer is 6, the DATA cards in the main program and subroutine subprograms should be replaced by

DATA IW,IR/6,5/

for

DATA IW,IR/3,1/

3. The boundaries of each class of the frequency distribution are limited to one bushel, e.g., 1.01-2.00 bushels. The classes of the frequency distribution start from 0.01-1.00 bushels and then continue to the maximum. In order to save space in the print-out the boudaries of the frequency distribution are not printed. The estimated frequencies are measured at the middle point of each class.

APPENDIX II

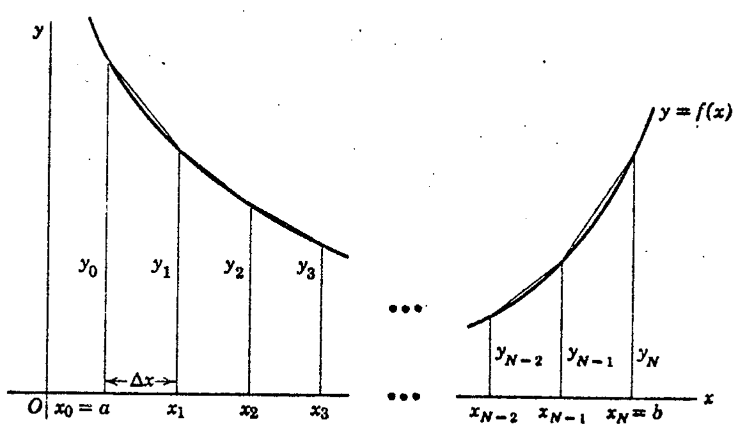
THE APPROXIMATE INTEGRATION

The approximate integration used in calculating premium rate is called the TRAPEZOIDAL RULE, This is based upon the simple idea of capping each strip with a line segment, joining the points on the curve corresponding to the end points of the strip, and then replacing the area under the curve by the area under the trapezoid.

For example, let y = f(x) be the function to be in integrated from x = a to x = b, or symbolically,

$$\int_a^b f(x)\,dx$$

To estimate this integral, the interval from x = a to x = b can be divided into N strips of width $\Delta x = (b - a)/N$. The points $x_0, x_1, x_2, \ldots\ldots\ldots, x_N$ shown along the x-axis in the following figure locate the x-coordinates of the sides of the dividing strips.



The area under the i-th line segment ($i = 1, 2, \ldots, N$) is equal to the area of a trapezoid with base Δx and sides y_{i-1} and y_i. The area of this trapezoid is equal to

$$\frac{\Delta x}{2} (y_{i-1} + y_i)$$

A simple formula results when the areas of the individual trapezoids are summed. From the first to the N-th trapezoid, the areas are successively given by

$$\frac{\Delta x}{2} (y_0 + y_1)$$

$$\frac{\Delta x}{2} (y_1 + y_2)$$

$$\frac{\Delta x}{2} (y_2 + y_3)$$

$$\vdots$$

$$\frac{\Delta x}{2} (y_{N-2} + y_{N-1})$$

$$\frac{\Delta x}{2} (y_{N-1} + y_N)$$

The formula, known as the trapezoidal rule, for estimating the integration follows;

$$\int_a^b f(x)\, dx = \frac{\Delta x}{2} (y_0 + 2y_1 + 2y_2 + \ldots + 2y_{N-1} + y_N)$$

In calculating the premium rate, the integration to be estimated is as follows:

$$\int_a^c (c - x) f(x) dx$$

where a = starting point of the yield distribution,

c = insured yield,

f(x) = probability density function of yield.

Therefore, if c_1 is the insured yield for coverage at 60 percent of the average yield, the premium rate P_{60} is as follows;

$$P_{60} = \int_a^{c_1} (c_1 - x) f(x) dx$$

If c_2 is the insured yield for coverage at 70 percent of the average yield, the premium rate P_{70} is;

$$P_{70} = \int_a^{c_2} (c_2 - x) f(x) dx$$

In the computer program P_{70} is estimated by

$$P_{70} = \int_a^{c_1} (c_2 - x) f(x) dx$$

$$+ \int_{c_1}^{c_2} (c_2 - x) f(x) dx$$

Similarly, if the insured yield for coverage at 80 percent of the average yield is c_3, the premium rate P_{80} is calculated by;

$$P_{80} = \int_{a}^{c_3} (c_3 - x) f(x) dx$$

$$= \int_{a}^{c_1} (c_3 - x) f(x) dx$$

$$+ \int_{c_1}^{c_2} (c_3 - x) f(x) dx$$

$$+ \int_{c_2}^{c_3} (c_3 - x) f(x) dx$$

Each integration is handled by calling two function subprograms, e.g., SUMT1 and EQT1, when the estimated distribution takes the form of a Type I Pearson distribution. The integration is then estimated by dividing the area under the curve into 50 strips.

APPENDIX III

DATA USED IN ANALYSES OF VARIANCE

TABLE A-III-1

WHEAT YIELDS OF SAMPLE FARMS IN
RISK AREA 4 SOIL RATING C

	WHEAT YIELD (BUSHELS PER ACRE)			
FARM	1966	1967	1968	AVERAGE
1	30.	30.	25.	28.33
2	20.	23.	25.	22.67
3	20.	25.	20.	21.67
4	25.	25.	25.	25.00
5	37.	20.	35.	30.67
6	21.	27.	25.	24.33
7	42.	38.	35.	38.33
8	30.	25.	26.	27.00
9	28.	29.	25.	27.33
10	29.	28.	33.	30.00
11	40.	30.	40.	36.67
12	40.	28.	22.	30.00
13	33.	35.	25.	31.00
14	30.	20.	12.	20.67
15	16.	25.	25.	22.00
16	32.	30.	25.	29.00
17	35.	40.	50.	41.67
18	35.	35.	30.	33.33
19	30.	40.	40.	36.67
20	21.	40.	30.	30.33
21	28.	35.	35.	32.67
22	30.	35.	25.	30.00
23	25.	30.	22.	25.67
24	32.	35.	25.	30.67
25	30.	40.	30.	33.33
26	25.	30.	30.	28.33
27	30.	30.	27.	29.00
28	35.	30.	30.	31.67
29	20.	30.	15.	21.67
30	30.	25.	20.	25.00

TABLE A-III-1 (CONTINUED)

31	30.	25.	30.	28.33
32	35.	36.	27.	32.67
33	30.	20.	4.	18.00
34	32.	25.	3.	20.00
35	30.	40.	7.	25.67
36	25.	28.	0.	17.67
37	25.	18.	34.	25.67
38	22.	18.	30.	23.33
39	35.	22.	20.	25.67
40	40.	40.	23.	34.33
41	25.	43.	21.	29.67
42	30.	15.	8.	17.67
43	30.	28.	12.	23.33
44	40.	30.	25.	31.67
45	25.	30.	0.	18.33
46	25.	20.	11.	18.67
47	30.	30.	12.	24.00
48	15.	40.	12.	22.33
49	20.	25.	31.	25.33
50	31.	32.	35.	32.67
51	30.	35.	20.	28.33
52	35.	6.	35.	25.33
53	25.	18.	30.	24.33
54	22.	25.	25.	24.00
55	19.	40.	20.	26.33
56	30.	40.	25.	31.67
57	22.	28.	21.	23.67
58	17.	12.	32.	20.33
59	35.	30.	30.	31.67
60	30.	37.	35.	34.00
61	20.	35.	23.	26.00
62	3.	14.	21.	12.67
63	8.	20.	35.	21.00
64	25.	30.	30.	28.33
65	30.	35.	40.	35.00
66	15.	15.	15.	15.00
67	40.	40.	30.	36.67
68	40.	45.	35.	40.00
69	27.	40.	30.	32.33
70	25.	30.	30.	28.33

TABLE A-III-1 (CONTINUED)

71	30.	35.	40.	35.00
72	30.	35.	25.	30.00
73	30.	25.	30.	28.33
74	38.	28.	45.	37.00
75	38.	34.	40.	37.33
76	30.	33.	40.	34.33
77	36.	36.	40.	37.33
78	35.	30.	40.	35.00
79	30.	40.	40.	36.67
80	30.	30.	45.	35.00
81	40.	40.	45.	41.67
82	30.	40.	30.	33.33
83	20.	30.	30.	26.67
84	35.	35.	30.	33.33
85	38.	33.	37.	36.00
86	30.	33.	33.	32.00
87	40.	35.	40.	38.33
88	40.	30.	30.	33.33
89	35.	35.	40.	36.67
90	35.	20.	30.	28.33
91	35.	34.	37.	35.33
92	35.	30.	37.	34.00
93	25.	45.	21.	30.33
94	25.	30.	30.	28.33
95	30.	35.	36.	33.67
96	48.	50.	40.	46.00
97	30.	30.	30.	30.00
98	20.	30.	35.	28.33
99	30.	35.	25.	30.00
100	32.	40.	28.	33.33
101	15.	20.	35.	23.33
102	20.	12.	14.	15.33
103	35.	32.	30.	32.33
104	30.	12.	15.	19.00
105	35.	35.	30.	33.33
106	33.	30.	32.	31.67
AVERAGE	29.25	30.28	27.77	29.10

SOURCE: MANITOBA CROP INSURANCE CORPORATION

TABLE A-III-2

WHEAT YIELDS OF SAMPLE FARMS IN
RISK AREA 4 SOIL RATING F

FARM	WHEAT YIELD (BUSHELS PER ACRE) 1966	1967	1968	AVERAGE
1	20.	23.	18.	20.33
2	45.	37.	24.	35.33
3	35.	40.	35.	36.67
4	30.	25.	32.	29.00
5	35.	28.	25.	29.33
6	40.	35.	29.	34.67
7	25.	28.	30.	27.67
8	25.	20.	25.	23.33
9	20.	15.	20.	18.33
10	25.	25.	25.	25.00
11	23.	35.	20.	26.00
12	27.	30.	21.	26.00
13	38.	40.	50.	42.67
14	30.	40.	35.	35.00
15	35.	40.	30.	35.00
16	20.	30.	20.	23.33
17	25.	32.	52.	36.33
18	25.	16.	52.	31.00
19	22.	24.	43.	29.67
20	30.	12.	30.	24.00
21	25.	20.	20.	21.67
22	24.	18.	17.	19.67
23	21.	27.	22.	23.33
24	25.	20.	25.	23.33
25	25.	22.	25.	24.00
26	20.	20.	18.	19.33
27	30.	12.	30.	24.00
28	15.	22.	40.	25.67
29	25.	30.	30.	28.33
30	26.	30.	30.	28.67

TABLE A-III-2 (CONTINUED)

31	25.	20.	20.	21.67
32	35.	23.	25.	27.67
33	18.	22.	42.	27.33
34	25.	15.	40.	26.67
35	38.	33.	40.	37.00
36	35.	18.	30.	27.67
37	28.	15.	30.	24.33
38	44.	29.	37.	36.67
39	43.	30.	35.	36.00
40	30.	20.	22.	24.00
41	12.	25.	20.	19.00
42	18.	32.	40.	30.00
43	25.	23.	12.	20.00
44	33.	30.	30.	31.00
45	22.	30.	30.	27.33
46	23.	18.	20.	20.33
47	25.	24.	30.	26.33
48	20.	15.	30.	21.67
49	32.	35.	35.	34.00
50	15.	20.	30.	21.67
AVERAGE	27.14	25.46	29.42	27.34

SOURCE: MANITOBA CROP INSURANCE CORPORATION

TABLE A-III-3

WHEAT YIELDS OF SAMPLE FARMS IN
RISK AREA 7 SOIL RATING C

	WHEAT YIELD (BUSHELS PER ACRE)			
FARM	1966	1967	1968	AVERAGE
1	30.	30.	10.	23.33
2	35.	30.	28.	31.00
3	25.	20.	16.	20.33
4	25.	30.	33.	29.33
5	30.	25.	30.	28.33
6	30.	28.	20.	26.00
7	33.	25.	45.	34.33
8	35.	30.	35.	33.33
9	35.	20.	30.	28.33
10	25.	25.	25.	25.00
11	50.	40.	40.	43.33
12	25.	28.	20.	24.33
13	25.	35.	33.	31.00
14	35.	35.	26.	32.00
15	25.	25.	25.	25.00
16	35.	45.	40.	40.00
17	25.	25.	10.	20.00
18	40.	20.	30.	30.00
19	33.	35.	40.	36.00
20	40.	35.	20.	31.67
21	28.	35.	40.	34.33
22	34.	25.	28.	29.00
23	20.	22.	16.	19.33
24	27.	15.	30.	24.00
25	32.	25.	15.	24.00
26	35.	30.	15.	26.67
27	30.	23.	30.	27.67
28	30.	25.	30.	28.33
29	40.	35.	25.	33.33
30	30.	25.	25.	26.67
AVERAGE	31.40	28.20	27.00	28.87

SOURCE: MANITOBA CROP INSURANCE CORPORATION