

THE EFFECTIVENESS OF HOMEWORK AND POST-
HOMEWORK TREATMENT ON ACHIEVEMENT
IN POLYNOMIALS

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"THE EFFECTIVENESS OF HOMEWORK AND POST-
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by
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A dissertation submitted to the Faculty of Graduate Studies of
the University of Manitoba in partial fulfillment of the requirements
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ABSTRACT

The purpose of this study was to examine the effectiveness of traditional homework and post-homework discussion on mathematic achievement in polynomials. A secondary purpose was to determine whether age and/or sex of the students were factors that affected achievement. The questions raised in this regard were:

- 1) Is there a significant difference in the achievement of students with homework compared to those who have none?
- 2) Is the process of doing the work at home the significant factor on achievement?
- 3) Is the post-homework discussion at school the more important process?
- 4) Is it the combination of "doing and discussing" that is significant?
- 5) Is the age of a student a factor in mathematic achievement?
- 6) Is the sex of a student a factor in mathematic achievement?

A review of literature indicated that a limited number of experiments had been done on the effectiveness of homework. Some of the studies were incondite, others were outdated, and those which were recent and well-designed had contradictory findings.

A sample of seventy-nine grade IX students was involved in the study. These students were members of

three intact classes taught by the vice-principal. One class was designated to be the control group receiving no homework, while the other two were the experimental groups. Both groups did homework; one received the answers only, while the other received answers and took part in extensive discussion in class.

The following control variables were obtained: I.Q. scores and scores on the Arithmetic Test and the Structure of the Number System Test from the Cooperative Mathematics Series.

The independent variables were the three treatments, age, and sex, while the dependent variable was achievement in polynomials measured on three teacher-prepared tests.

To allow for variation that existed among the groups in I.Q. and mathematic achievement and to allow for intact groups, the analysis of covariance statistical design was utilized.

Mean achievement scores did not differ significantly among the three treatment groups. Sex and age of the students were found to have no significant effect on mathematic achievement. The combination of any two or three of the factors (sex, age and treatment) had no significant effect on mathematic achievement.

Several conclusions appear warranted:

1. It is the doing of the work that assures a student a higher degree of success in polynomials. The location where the work is done is inconsequential.
2. Extensive post-homework discussion does not enhance achievement in the homework process. The provision of answers and student-initiated discussion, if any, is just as effective.
3. The topic of polynomials is appropriate for grade IX students.
4. Boys are not more mathematically inclined than girls.

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Chapter 1

THE NATURE OF THE INQUIRY

INTRODUCTION

A remark was made at a mathematics seminar that, to date, there was no conclusive evidence that assigning homework in mathematics was of any educational benefit (4).

Since the writer had always thought that assigning homework was a sound educational practice, this statement both surprised and intrigued him.

A review of literature was made and the validity of the above statement appeared to be irrefutable. The writer was further stimulated to undertake this study as a result of his own experience as a teacher. During parent-teacher interviews many parents had asked the writer to give their son or their daughter more home assignments, the rationale being that homework increased achievement in mathematics.

Similar situations have been reported by others. "Many people are urging a panacea to the problem--that more and ever more homework will help to solve the problems related to scholastic achievement in the classroom" (2:46).

Lacking knowledge of homework's effectiveness, an educator faces a dilemma. To agree to a parent's request

for homework means to adopt a practice which may have no educational value while, on the other hand, to refuse means to reject a practice which may have benefit.

In our educational system, homework for reinforcement is an accepted practice, yet, "it is surprising that research evidence concerning it is so limited" (16: 20). Students spend hours completing assignments; teachers spend further time and energy correcting them and admonishing homework delinquents. Yet, if traditional homework is not effective in increasing achievement, then we should look to homework to fulfill other objectives.

Homework assignments, for example, could be used for the completion of class work (1, 13, 17), for background to class discussion (1, 12), for the provision of individual differences (1, 12, 13, 17), for supplementary information pertaining to classroom work (12, 13), and for enhancement and enrichment of the general learning process (11, 12, 14). The limitations of these objectives should be realized and our educational practices and expectations altered accordingly.

THE NEED FOR THE STUDY

The relationship between homework and school success is still a prominent issue in education today. One author (5) found that the Education Index lists 500 titles on the subject of homework in the forty-year period 1928-1969. Of

these only twenty-four are experimental studies, while the rest may be described as anecdotal or polemic in nature.

June Mulry states that "very few experimental studies on the problem of homework are reported and many of those reported are limited in number, scope and quality" (8:49).

Ruth Strang, in the Encyclopedia of Educational Research, comments that "experimental studies in this field have been meager; they have also been faulty in research design" (15:675).

Some of the studies that were well-designed have been misinterpreted:

. . . Some studies whose research is well-constructed, are misinterpreted and used to support preconceived notions about the benefits or harmful effects of homework. The conclusions of these studies, usually unfavourable to homework, are often cited rather than the actual findings of the researcher (8:49).

Since 1969 only three experimental studies have been carried out and their combined results have not added significantly to our knowledge of the effectiveness of homework. The statement made by June Mulry is still true today--there is a dearth of well-designed homework studies: "in general, research indicates that greater study is needed before homework as an educational tool can be fully evaluated" (8:49).

The urgent need of research and the evaluation of the effectiveness of homework was apparent in 1960 when the Educational Policies Commission (3) classified homework as

one of the ten contemporary issues in elementary education. In 1966 the NEA Research Division (10) reported that research on the effectiveness of homework was still inconclusive. Four years later, in the thirty-third yearbook of the NCTM, it was stated that "perhaps the weakest area of the teacher planning is the homework assignment" (9:326). Today, the need to evaluate the homework practice is as pressing as it was in the past.

Early experiments on homework were broad-spectrum in nature, covering many academic subjects and many grades, while the more recent studies, which are fewer in number, tend to cluster in the elementary and high school grades. Homework at the junior high school level is an area that has been neglected in research.

Finally, most studies make no mention of what treatment was given the homework when it was returned to school. Only three of twenty-four experiments mentioned what was done. In two studies (5, 6) homework was corrected by the experimenters, and in one (7) the answers were provided by the parents as the pupils finished the questions.

Thus, in these experiments, it is difficult to evaluate whether homework alone was the determinant for any improvement in mathematic achievement. Furthermore, only one study (5) examined sex as an independent variable in the homework process, and no study has yet researched age

as a factor.

STATEMENT OF THE PROBLEM

The purpose of this study was to compare experimentally three instructional approaches as evidence of the effectiveness of homework and post-homework discussion on mathematic achievement. A control group received no homework while two experimental groups received homework assignments; one of the latter participated in extensive post-homework discussion while the other did not. For the remainder of the study, these three instructional approaches will be referred to as "the three treatments" and the three groups involved as "the three treatment groups."

A secondary purpose was to determine whether age and sex of the students were factors that affected achievement in the homework process.

The questions of interest to the study were the following:

- 1) Does traditional homework enhance the student's mathematic achievement?
- 2) Is the process of doing the work at home the significant factor in achievement?
- 3) Is the post-homework discussion at school the more important process?
- 4) Is it the combination of "doing and discussing" that is most necessary?
- 5) Is age a factor in the homework process?
- 6) Is sex a factor in the homework process?

HYPOTHESES

The following hypotheses were tested in this study.

1. There is no significant difference in the mean score of achievement among the three treatment groups with respect to their performance:

- (a) on a test of the addition and subtraction of polynomials;
- (b) on a test of the multiplication of polynomials;
- (c) on a test of the division of polynomials;
- (d) on a total of the three preceding tests.

2. There is no significant difference between the mean score of achievement of students who are older than or equal to the mean age of 14.4 years and that of the students who are younger than 14.4 years with respect to their performance:

- (a) on a test of the addition and subtraction of polynomials;
- (b) on a test of the multiplication of polynomials;
- (c) on a test of the division of polynomials;
- (d) on a total of the three preceding tests.

3. There is no significant difference between the mean score of achievement of male students and that of female students with respect to their performance:

- (a) on a test of the addition and subtraction of polynomials;

- (b) on a test of the multiplication of polynomials;
- (c) on a test of the division of polynomials;
- (d) on a total of the three preceding tests.

4. There is no significant difference in the mean score of achievement among students of one particular age-and-treatment combination and students of another age-and-treatment combination with respect to their performance:

- (a) on a test of the addition and subtraction of polynomials;
- (b) on a test of the multiplication of polynomials;
- (c) on a test of the division of polynomials;
- (d) on a total of the three preceding tests.

5. There is no significant difference in the mean score among students of one particular sex-and-treatment combination and students of another sex-and-treatment combination with respect to their performance:

- (a) on a test of the addition and subtraction of polynomials;
- (b) on a test of the multiplication of polynomials;
- (c) on a test of the division of polynomials;
- (d) on a total of the three preceding tests.

6. There is no significant difference in the mean score of achievement among students of one particular sex-and-age combination and students of another sex-and-age combination with respect to their performance:

- (a) on a test of the addition and subtraction of polynomials;
- (b) on a test of the multiplication of polynomials;
- (c) on a test of the division of polynomials;
- (d) on a total of the three preceding tests.

7. There is no significant difference in the mean score of achievement among students of one particular sex-and-age-and-treatment combination and students of another sex-and-age-and-treatment combination with respect to their performance:

- (a) on a test of the addition and subtraction of polynomials;
- (b) on a test of the multiplication of polynomials;
- (c) on a test of the division of polynomials;
- (d) on a total of the three preceding tests.

DEFINITIONS

For the purposes of this study, the following definitions apply:

- homework --describes the activities assigned to and undertaken by the pupils outside the regular class period.
- traditional homework--refers to homework for reinforcement--questions or drill designed to review text materials or concepts taught in the classroom.

DELIMITATIONS

The experiment was designed within the bounds of the following delimitations:

1. This study was confined to one junior high school and involved one mathematics teacher.
2. The results can only be generalized to the grade IX student population in that one school.

ASSUMPTIONS

There were several assumptions that had to be made in this study relating to students, instruments and procedures that were followed:

1. Students answered all questions on pre-tests and tests during the study to the best of their ability.
2. Occasional absences from class did not affect the criterion score of students significantly.
3. The students in this study were not significantly different from other grade IX junior high school students who study polynomials in Manitoba schools.
4. The students' responses on the various administered tests were not significantly affected by the time of day at which they were administered or the days on which they were tested.

LIMITATIONS

This study is subject to the following limitations:

1. I.Q. tests were administered to most students when they were in grade VI at two different schools. Students who came into the district after this were given I.Q. tests under different circumstances. Therefore some of the I.Q. scores used herein do not have exactly the same basis for comparison.

2. Achievement measurements during and after the unit on polynomials are limited by the validity and reliability of the teacher-prepared tests.

3. Since three classes presently in existence were chosen for the three treatments, true randomness of the samples was not present.

4. The measurement of achievement in mathematics is limited by the validity and reliability of the pre-experimental achievement test administered to the three groups.

5. Home conditions, if substantially different for two of the three groups, would have an effect on the study.

6. The general atmosphere of the school and any idiosyncracies of the teacher's personality in the classroom must be considered.

ORGANIZATION OF THE STUDY

In the remaining portions of this study, the writer

will review the literature in Chapter 2; describe the population, the research procedure and the experimental design in Chapter 3; present the experimental findings in Chapter 4; and discuss the conclusions and implications of the study in Chapter 5. The significant appendices and references are included at the end of the study.

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Chapter 2

A REVIEW OF LITERATURE

Homework is one of the most widely used and controversial teaching techniques. A controversy about its effectiveness has continued from the 1930's and still remains today.

In the late 1930's there was a reaction against the traditional practices of excessively long homework assignments, rote tasks, and the use of homework as punishment. There was also the concern about indications that homework fosters copying and necessitates parental help.

In the 1940's opinion against all homework crystallized, particularly at the elementary level. This was brought about by educational theory which emphasized the development of student interest and initiative in learning (20).

By the 1950's many educators opposed conventional homework and advocated that homework be optional and of a recreational variety (42). However, when Russia launched Sputnik in 1957, the twenty-five year trend towards less homework was reversed (17). Immediately after 1957, a movement back to an emphasis on subject matter ensued, and homework was viewed as a means to gain academic excellence (14, 24). Pressure from teachers (35, 52), from parents who felt

that homework assignments reflected "a quality education" (8,42) and from colleges (40) led Bond and Smith to report that homework assignments were increasing in length and number at an alarming rate in seventy-nine New York school districts (7). Thus homework, along with other educational practices, came under public and professional scrutiny.

Surveys of public school students (2, 10, 25, 48, 52) and college students (5) reveal that the majority of the groups surveyed favoured homework as being helpful in their studies. Teachers (10, 35, 52) and college professors (10) surveyed were strongly in favour of homework. In the 1961 NEA survey of teachers (35), 83.5 percent of the elementary school teachers were in favour of homework at the elementary level. Parents who were surveyed (10, 27, 52) were also in favour of homework. Two of these surveys (27, 52) revealed that many parents were also willing to help students with their assignments.

While the surveys strongly approved homework and the need for it, there was no agreement as to the amount, type, or process of assignment. Many articles on homework can be found; one researcher (21) found 500 titles listed under the general subject of homework in the Education Index but most of these are polemic in nature. Some of the topics, for example, are the amount of homework that should be assigned to different grades (4, 16, 30, 31, 35, 42, 44, 49, 53), the

type of homework that can be assigned (23, 36, 41, 53, 60), the manner in which it is assigned (16, 31, 52, 60), the establishment of homework policies in a school or school district (7, 17, 37, 44, 59), the home conditions that are conducive to the doing of homework (9, 25), and the teacher's role in assigning homework (7, 16, 33, 38, 39, 45). These examples are not exhaustive.

There are twenty-four experimental studies that have been carried out in the period 1933 to 1976. These can be divided into two groups, thirteen early studies (pre-1950) and eleven modern studies (post-1950).

The early experiments span a broad spectrum, encompassing many subjects and grades. Most of the modern studies are not only better designed and statistically more sophisticated but also more limited in their investigation of specific grades and topics.

Three of the studies from the thirties as described by Goldstein (18, 15, 51) are of special interest:

1. The Di Napoli experiment (1935), the most extensive study of homework, covered grades five to seven, nine subjects and three socio-economic groups. The results on achievement were statistically significant to a small degree in favour of the homework group in eight of the nine subjects at the fifth and sixth grade levels. In grade seven, the non-homework group had a slight edge.

2. The Crawford and Carmichael experiment (1947) encompassed grades five through eight in eight subjects. The results showed, in the two comparisons carried out among four classes one year apart, that twenty-nine of thirty-two scores and twenty-eight of thirty-two scores favoured the homework group. Perhaps the most significant finding was that the high school grades of those pupils who had attended El Segundo Elementary School after homework was abolished dropped more than the grades of the students who had homework at El Segundo.

3. Steiner (1934) carried out a limited experiment in a single seventh grade class. He found that the homework group had made clear gains in achievement in arithmetic and grammar, but in literature the two groups were the same.

One of the early studies, Foran and Weber's experiment (19) specifically examines arithmetic homework in grade seven in seven parochial schools. The study was of a cross-over design: Group A had no homework in the first term while Group B did. The reverse was true in the second term. The results on the achievement test in problem-solving revealed that both groups showed greater gains with homework than without, but these gains were small. In computation the difference was not as clear. Group A achieved higher computational levels of achievement when they had no homework, while Group B reacted just the opposite.

Anderson (3) studied two classes of eighth-grade

pupils in arithmetic, English and social studies. The two classes in a given subject had the same teacher. Achievement was measured on five tests administered after each unit of work was completed. The results clearly showed that achievement was higher for the homework group in all three subjects on all five tests.

The remaining early studies (11, 12, 34, 46, 55, 56, 58) reveal very little conclusive evidence about homework and achievement. All of them are open to serious criticism (20) relative to the experimental design or to the fragmentary data, or both.

Of the recent studies (post-1950), four were conducted at the high school level. McGill (32) did a study to determine whether homework affects (a) social studies achievement and (b) achievement in economics and American history. He designated each of sixteen classes in a New York City school as either an experimental or a control group.

None of the differences obtained from testing social studies abilities was statistically significant. In economics the homework classes had higher mean scores than the non-homework groups. However, in history, the homework classes did better during the first term but poorer in the second. The differences were not statistically significant.

Schneider (49) performed a similar experiment in economics at another New York City high school. There

appeared to be no significant difference between the homework class and the non-homework class on the same test.

Another study in American history was carried out by Schain (47) using a single class in a New York City high school. He used a cross-over design with two groups from the same class equal in I.Q.. The homework group in the first and second part of the experiment outperformed the non-homework group both in daily short-answer questions and on essay tests.

An analysis of the results revealed that the brighter students did well with or without homework. The average students scored much better when they were in the homework group, while the poor students were "lost" when they did no homework.

The only study in mathematics at the high school level was done by Hines (22) who investigated homework and its effects on achievement in plane geometry with seventeen paired pupils.

He found that every one of the comparisons favoured the homework group, though only nine were statistically significant at the five percent level, while three more became significant at the ten percent level. He concluded that written homework increased achievement in plane geometry by about one grade score.

Two experiments at the junior high school level are

described below:

1. Baughman and Pruitt compared two kinds of homework-- homework for reinforcement ("dull-type" assignments), and homework for enrichment (experimental, creative, or research assignments). Their findings revealed that the nature of homework made no appreciable difference in achievement gains for junior high school students in social studies.

2. Peterson (43) suggested four different homework patterns:

- (a) vertical or traditional,
- (b) oblique,*
- (c) spiral,
- (d) semi-oblique.**

He compared the vertical (traditional) homework pattern with the oblique. His findings showed that grade VIII students assigned homework by the oblique pattern achieved significantly better in mathematics than those students assigned traditional homework.

In the elementary grades there are five recent experiments.

* A pattern of homework whereby a teacher provides exploratory work on a future topic, immediate practice over the topic under discussion and reinforcement for still a third topic previously taught.

** As oblique but no effort is made to provide reinforcement of previous topic.

1. Koch (26) compared the achievement of three groups of grade sixes in computation and problem-solving. The first group had thirty minutes of homework per day, the second only fifteen minutes, while the third had none. He found that only the homework group with thirty minutes homework had statistically higher achievement scores in computation than the non-homework group, a finding which suggested that longer homework assignments were necessary for increased achievement. Achievement in problem-solving, however, was not increased by homework of any length.

2. Maertens (28) carried out an experiment in grade III arithmetic with three groups: a non-homework group, a teacher-assigned homework group, and a group which received its homework from the experimenter. He found that homework made no statistical difference on computational achievement and problem-solving ability. Also, he found that homework did not have any differential effect upon achievement with high, middle, or low intelligence groups.

3. Gray and Allison (21) studied the effect of homework on achievement in the four fundamental operations of fractions in grade VI. Two groups were formed, one homework and one non-homework. At the end of four weeks, treatments were reversed. No statistical difference in achievement in computation and fraction concepts was found between the two groups.

4. Maertens and Johnston (29) carried out a two-fold experiment with grades IV to VI. They investigated a homework pattern which involved the parents. Three groups were established. One group of children received knowledge of the results from their parents after completing each problem, while a second group received knowledge of the results from their parents only after completing the entire assignment. The third group was the non-homework group. After a six-week period, achievement in computation and problem-solving was tested. The two homework groups out-performed the non-homework group. The difference between the two homework treatments, however, showed little consistent variation. An attitude test administered to each group revealed no significant difference in attitude among the children. The causative factor for the success of the homework groups over the non-homework one is not clear. Perhaps it could be attributed to the homework practice alone, or to parental involvement, or to the combination of both factors.

Finally, an experimental study (50) was conducted to determine whether the kind of homework a teacher assigned made a difference in academic achievement in elementary school children, and whether homework such as collecting, observing, listening and constructing models improved reading ability.

The experiment was carried out in twenty-five schools

in Tampa, Florida with a total sample of 700 students in grades IV to VI from all socio-economic backgrounds. Two groups were organized: a control group with traditional homework and an experimental group which received individualized enrichment homework. The experimental period lasted sixteen weeks and homework was student-structured in the following subjects: reading, spelling, language, arithmetic, social studies and science.

It was reported that the reading ability of the experimental group improved over that of the control group. However, it is not recorded what statistical treatment was used in the comparison, nor whether the gain in reading ability was statistically significant.

At this time it is still unclear whether the assignment of homework is an educational practice that will enhance achievement in mathematics. Literature to date provides twenty-four studies in homework. Eight of the thirteen early studies (1, 11, 12, 34, 46, 55, 56, 58) from the 1930's have serious flaws in their design. The other five (3, 15, 18, 19, 51) are by nature broad-spectrum, covering many subjects and grades. All five support the premise that homework will increase achievement in arithmetic. However, the results are statistically significant only to a small degree.

Of the eleven recent experiments, three support the

effectiveness of homework, one in American history (47), one in plane geometry (26), and one in grade VI arithmetic (26). Four studies found that homework has no effect on achievement, two in American history and economics (32, 49), and two in elementary arithmetic (21, 28). One study (29), because of design, was inconclusive concerning homework or another home factor being responsible for the gain in student achievement. The remaining experiments were concerned with the kind of homework that was assigned. One found that achievement is not enhanced by enrichment homework over the traditional kind (5), while another (43) indicated that a semi-oblique pattern of organization of homework was more effective than the traditional kind, and the third (50) suggested that individualized enrichment homework in grades IV to VI increases reading ability.

SUMMARY

Thus, research to date has not provided enough evidence for educators to accept unilaterally the effectiveness of homework. Early wide-ranging, multi-grade, multi-subject experiments show some support for homework's effectiveness, but well-designed recent experiments which "zero in" on specific mathematic concepts are not unanimous in their findings.

Most participants of homework surveys (students,

teachers, parents, professors) felt that home assignments for reinforcement enhanced achievement, yet research has not borne this out. This anomaly--the difference between personal experience and experimental results--is difficult to explain.

Although many articles of opinion are published on homework, research in this area continues to be meager.

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Chapter 3

EXPERIMENTAL DESIGN

The purpose of this chapter is to describe the study in detail. It will include a description of the school, the population and sample, course and treatment, data collection procedures, and method of analysis.

THE SETTING

The study was conducted at a suburban junior high school in the St. James-Assiniboia School Division. The school is located in a middle and lower-middle class community. The organization of the school was of the standard graded form. The school timetable operated on the basis of a six-day cycle with eight forty-minute periods a day.

The department of mathematics consisted of the vice-principal and two teachers new to the school. The study involved only the vice-principal who taught three of five grade IX classes in the school. Prior to becoming an administrator, he had taught mathematics in high school for many years.

At the time the study was conducted, the principal had been in the school one year and was beginning his second year. The atmosphere which he created reflected an orderly and

businesslike conduct in the school.

POPULATION AND SAMPLE

The population of the school was four hundred forty-one. One hundred thirty-five students were enrolled in grade IX, of whom eighty-two students were in the sample.

Because the grade IX classes had already been established for some time, the selection of a random sample was not possible. However, in this school the students had been randomly assigned to five classes in an attempt to devise heterogeneous groups, except for one class in which twenty of the twenty-seven students had chosen the band-option and were, therefore, grouped together.

The three grade IX classes taught by the vice-principal were designated to participate in the experiment; one of these classes was the band-option class mentioned above. The students in each class in the sample ranged from low to high in terms of I.Q. and mathematic achievement.

COURSE DESCRIPTION AND TREATMENT

The factors common to the three treatment groups are outlined below.

1. Each class was taught the same mathematic content in the same period of time.
2. Each class had one double period every six days, the second period being used as a work period.

3. Exercises were assigned in two textbooks: Modern Elementary Algebra and Geometry, by Eugene D. Nichols (1971), and Modern Mathematics by David Keith, A. Wayne Kerr, and A. Murray McPherson (1970). A list of the assignments may be found in Appendix A.
4. Also, the teacher devised supplementary worksheets which were assigned to the students; these sheets may be found in Appendix B.
5. The topic "polynomials" was divided into three sections:
 - (A) addition and subtraction
 - (B) multiplication
 - (C) division and rational expressions.

After the concepts in each of these sub-topics were taught, a review followed and then a test was administered. The review sheets may be found in Appendix C and the tests in Appendix D.

6. Below is a list of the concepts emphasized in each of the sections (A), (B), and (C):

(A) Addition and Subtraction of Open Expressions

1. Adding and subtracting monomials
2. Adding and subtracting polynomials
 - (a) vertically
 - (b) horizontally
 - (i) addition of opposites
 - (ii) several expressions within parentheses
 - (iii) expressions with integral, fractional and decimal coefficients;

(B) Multiplication of Open Expressions

1. Multiplication of monomials
 - (a) any number of factors
 - (b) exponent theory covered
 - (i) definition of power, exponent and base
 - (ii) power of a power
 - (iii) power of a product
 - (iv) power of a quotient;

2. Multiplication of a monomial by a polynomial
 - (a) monomial by a binomial
 - (b) simplifying expressions involving addition or subtraction, for example, $2x(x-3) - 3x^2(2-x)$;
3. Multiplication of a polynomial by a polynomial
 - (a) binomial by a binomial
 - (b) square of a binomial
 - (c) binomial by a polynomial of three or four terms
 - (d) after students had mastered these products, they were incorporated with addition and subtraction, for example, $2x^2(-3x) - 2x(x-3) + (x-2)(3x-5)$;

(C) Division and Rational Expressions

- (a) division of monomials involving a reduction of fractions
- (b) multiplication and division of rational expressions
- (c) power of a quotient
- (d) quotient of a power
- (e) addition and subtraction of rational expressions
- (f) division of a polynomial by
 - (i) a monomial
 - (ii) a binomial.

VARIABLES USED IN THE STUDY

Covariates or Control Variables

Because the selection of a random sample was not possible in the study, the equivalence of the groups was in doubt. Thus, the following were used as covariates:

1. I.Q. scores as measured by the Large Thorndike Non-Verbal Tests administered to the pupils in grade VI obtained from school records. To the few students who entered the division after grade VI, the Otis Quick Scoring Mental Ability Test was administered;

2. achievement in mathematics as measured by two standardized tests of the Cooperative Mathematics Series:
 - (a) Arithmetic test
 - (b) Structure of the Number System test.The Arithmetic test may be found in Appendix E and the Structure of the Number test in Appendix F.

Independent Variables

The following were used as independent variables in the study:

1. No Homework Instructional Approach
The control group received no homework. Students were assigned questions which they did and corrected in class. If any homework was done at home, it was initiated by the student, not the teacher. This group, though covering exactly the same concepts as the two experimental groups, did not do as many questions as the latter.
2. Homework-with-Answers Instructional Approach
This experimental group received homework, which was corrected the following day in class. The correction of the questions involved only the furnishing of answers by the teacher. If a question was asked about the assignment, it was dealt with in the shortest possible way. At no time was there any teacher-initiated discussion of the homework assignment.
3. Homework-with-Answers-and-Discussion Instructional Approach
The final experimental treatment involved assigning homework, which was corrected the next day. Furthermore, correction involved an extensive discussion of each question assigned.
4. Sex
Sex of the students was determined from the school files.
5. Age
An analysis of the ages of the students in the sample

revealed that eighty-six percent were between 14.0 and 14.8 years. The histogram in Appendix G shows the age distribution of the students in the sample. Because of the clustering, the experimenter decided to treat this variable nominally. Two groups were formed on the basis of mean score of the group students who were less than the mean age 14.4 years became one group, and students who were older than, or equal to, the mean age formed the second group.

The design of the study is elucidated in Table 1.

Table 1
Research Design

Group	Pre-Test for Control Variables	Treatment	Tests in Polynomials
Control	Yes	No homework	Yes
Experimental #1	Yes	Homework with answers	Yes
Experimental #2	Yes	Homework with answers and discussion	Yes

Dependent or Criterion Variable

The criterion variable in this study was the mean achievement score obtained in polynomials. Measurement of this variable was accomplished by three teacher-prepared tests, the first on the addition and subtraction of polynomials, the

second on the multiplication of polynomials, and the third on division of polynomials and rational expressions. The three tests may be found in Appendix D.

COLLECTION OF DATA

Information about age, sex and I.Q. scores was obtained from student records in the school. The Cooperative Mathematic Series tests were administered to all groups December 18 and December 19. Students who had no I.Q. record were given the Otis Quick Scoring I.Q. test on December 22. Students who had missed any of the above tests were asked to write them on December 22 and 23. All tests were conducted under close supervision and complied with all test requirements. The experiment began on January 6 when school resumed after Christmas vacation.

DESIGN OF THE STUDY

Because the study involved intact groups, the experimenter decided to use the analysis of covariance for the statistical treatment. This is a useful method when studying the relation between a dependent variable and two or more groups representing an independent variable. This technique allows the researcher to equate statistically the independent variable groups with respect to one or more control variables which are relevant to the dependent vari-

iable. In other words, analysis of covariance allows the researcher to study the performance of several groups which are unequal with regard to an important variable as though they were equal in this respect.

In the experiment, the effect of the three treatments and the effect of age and sex on performance on polynomials were under scrutiny. The mean I.Q. and mathematic achievement scores were unequal for the three groups. Analysis of covariance statistically removed the effect of I.Q. and mathematic achievement of the three groups by adjusting the means of the dependent variable (achievement in polynomials). Thus, the effect of treatment, sex and/or age on the dependent variable can be studied without I.Q. and mathematic achievement confounding the results.

Each of the three classes was assigned to a treatment. Each of the treatments was further divided into two sub-groups, boys and girls, creating six sub-groups altogether. Then each of the six sub-groups was divided into two cells based on the mean age of the sample. Thus, twelve cells were formed. The composition of each cell is in Table 2.

ANALYSIS OF DATA

For each student in the study the following information was transferred to punch cards for computer analysis:

Table 2
Cell Composition

Sex	Age	No Homework	Homework with Answers	Homework with Answers and Discussion
Boys	Age < 14.4	n = 5	n = 10	n = 5
	Age ≥ 14.4	n = 4	n = 4	n = 7
Girls	Age < 14.4	n = 12	n = 7	n = 9
	Age ≥ 14.4	n = 7	n = 6	n = 6
	Sub Totals	28	27	27

1. treatment identification,
2. sex,
3. I.Q. score,
4. Cooperative Arithmetic test score,
5. Cooperative Structure of Number System test score,
6. addition and subtraction of polynomials test score,
7. multiplication of polynomials test score,
8. division of polynomials test score,
9. total of the three polynomials test scores.

A programmer assisted in the preparation of all data and control cards required for the computer programming. The data was analyzed by the computer at the University of Manitoba using a three-way analysis of covariance with three covariates. This programme yielded F ratios for each independent variable

and for each possible combination of independent variables. The F ratio indicated whether or not the adjusted means of the dependent variable were significantly different among the independent variables. The .05 level of significance was chosen as the rejection region for the hypothesis being tested.

Pearson product moment correlation coefficients were also obtained between the covariates and the dependent variable. These values indicated whether or not the covariates used in the study were related to the dependent variable or to each other. A two-tailed test at the .01 level of significance was chosen for the decision-making process.

SUMMARY

A sample of eighty-two students was drawn from the grade IX population. These students were already part of three intact classes taught by the vice-principal.

Each class was assigned to one of the following treatments:

1. a control group with no homework,
2. a homework group with answers,
3. a homework group with answers and discussion.

All groups covered essentially the same work in the same time. The duration of the experiment was approximately eight weeks.

Dependent and independent variables and the covariates were described; the collection of data and the design of the study were outlined.

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Chapter 4

ANALYSIS OF DATA

INTRODUCTION

In the first part of the chapter the sample is discussed, the correlation coefficients between the dependent variable and the covariates are compared, and the covariates for each source of variation are examined. Each hypothesis given in Chapter 1 is restated, and the related data analysis (means, adjusted means and F ratio) follows. Finally, a summary is provided which includes tables of means, adjusted means, and F ratios.

The following abbreviations will be used throughout the remainder of the thesis:

1. N-Hw No homework group,
2. Hw-A Homework-with-answers group,
3. Hw-AD Homework-with-answers-and-discussion group,
4. A-AT Arithmetic Achievement Test,
5. SNS-AT Structure of the Numbers System Achievement Test.

DESCRIPTIVE DATA CONCERNING THE SAMPLE

Table 3 records the number of students included in the study, and, within each treatment, the distribution of students by sex and age. During the course of the study, the original sample of eighty-two was reduced by three

students. Each treatment group lost one student over 14.4 years of age--a boy from the Hw-A, a boy from the N-Hw, and a girl from the Hw-AD. The two boys withdrew from school and the girl was sick over an extended period of time.

Table 3

The Distribution of Students by Sex
and Age in Each Treatment

Sex	Age	No Homework	Homework with Answers	Homework with Answers and Discussion
Boys	$A < 14.4$	5	10	5
	$A \geq 14.4$	3	4	6
Girls	$A < 14.4$	12	7	9
	$A \geq 14.4$	7	5	6
Sub-totals		27	26	26

The treatment of data employed the analysis of covariance design with three covariates or control variables. Table 4 compared the means of the covariates for students in each main source of variation. It shows the following:

Table 4

Summary of the Covariates of Students Grouped
According to the Main Sources of Variation

Main sources of variation	No. of students	Covariates		
		I.Q.	A-AT	SNS-AT
Treatment				
N-Hw	27	106.37	28.33	15.67
Hw-A	26	116.52	35.46	20.00
Hw-AD	26	108.38	32.19	15.46
Sex				
Boys	33	113.84	35.47	17.94
Girls	46	108.06	29.55	16.40
Age				
A < 14.4	48	113.94	33.82	18.75
A ≥ 14.4	31	104.94	28.90	14.35

Treatment

The mean scores on all three covariates for Hw-A are clearly higher than the mean scores in the other two treatments. The mean scores of the covariates are about equal for the Hw-AD and N-Hw.

Age

The mean scores of the younger students surpass those of the older students.

Sex

On all three independent variables, the boys have a higher mean score than the girls.

Similarly, the means of the covariates of the students grouped according to the combination of any two effects are found in Tables 5, 6 and 7.

Table 5

Covariates of the Students Grouped According to the Combination of Sex and Treatment

Source of Variation	No. of Students	Covariates		
		I.Q.	A-AT	SNS-AT
Boys in N-Hw	8	112.00	35.38	19.38
Boys in Hw-A	14	116.08	36.85	19.54
Boys in Hw-AD	11	112.55	33.91	15.00
Girls in N-Hw	19	104.00	25.37	14.11
Girls in Hw-A	12	117.15	34.08	20.46
Girls in Hw-AD	15	105.33	30.93	15.80

Sex and Treatment

Table 5 indicates that boys and girls of the Hw-A have higher mean scores than in the other groups. The girls in the Hw-AD and N-Hw obtained the lowest mean scores on the covariates, whereas the mean scores of the boys in Hw-AD and N-Hw are between these two extremes.

Table 6

Covariates of the Students Grouped According to the Combination of Age and Treatment

Source of Variation	No. of students	Covariates		
		I.Q.	A-AT	SNS-AT
Students < 14.4 in N-Hw	17	110.35	30.29	17.29
Students < 14.4 in Hw-A	17	118.82	37.59	22.24
Students < 14.4 in Hw-AD	14	112.36	33.86	16.29
Students \geq 14.4 in N-Hw	10	99.60	25.00	12.90
Students \geq 14.4 in Hw-A	9	112.44	31.44	15.78
Students \geq 14.4 in Hw-AD	12	103.75	30.25	14.50

Age and Treatment

Table 6 indicates that younger students in the Hw-A group have the highest mean scores on the three covariates, whereas older students in N-Hw have the lowest. In the other four groups the mean scores on the three covariates range between these two values.

Table 7

Covariates of the Students Grouped According
to the Combination of Age and Sex

Sources of Variation	No. of Students	Covariates		
		I.Q.	A-AT	SNS-AT
Boys < 14.4	20	116.50	36.90	19.35
Boys ≥ 14.4	13	109.42	33.08	15.58
Girls < 14.4	28	112.11	31.79	18.32
Girls ≥ 14.4	18	102.11	26.26	13.58

Age and Sex

Table 7 indicates that the younger boys and girls have higher mean scores than the older boys and girls. The boys' mean scores in each case are higher than the girls' mean scores.

In Table 8, the cells reveal broad differences. For instance, younger girls in Hw-A have the highest scores in I.Q., A-AT and SNS-AT of 124.86, 39.43, and 24.57 respectively, whereas older girls in N-Hw have the lowest mean scores on the three control variables of 99.29, 23.00 and 11.29 respectively.

The data in Tables 4, 5, 6, 7 and 8 indicate that there are differences among the compared cells. To compensate for the above differences and to accommodate intact classroom groups the analysis of covariance design was employed.

Table 8

Summary of the Covariates of Students Grouped
According to the Combination of Three Effects

Variation due to Combination of Three Effects	No. of Stu- dents	Covariates		
		I.Q.	A-AT	SNS-AT
Sex and Age and Treatment				
Boys < 14.4 in N-Hw	5	119.00	38.80	21.00
Boys > 14.4 in N-Hw	3	100.33	26.67	16.67
Girls < 14.4 in N-Hw	12	106.75	26.75	15.75
Girls > 14.4 in N-Hw	7	99.29	23.00	11.29
Boys < 14.4 in Hw-A	10	114.60	36.30	20.60
Boys > 14.4 in Hw-A	4	121.00	38.67	16.00
Girls < 14.4 in Hw-A	7	124.86	39.43	24.57
Girls > 14.4 in Hw-A	5	108.17	27.83	15.67
Boys < 14.4 in Hw-AD	5	117.80	36.20	15.20
Boys > 14.4 in Hw-AD	6	108.17	32.00	14.83
Girls < 14.4 in Hw-AD	9	109.33	32.56	16.89
Girls > 14.4 in Hw-AD	6	99.33	28.50	14.17

CORRELATION COEFFICIENTS

Table 9 presents the correlation coefficients among:

1. three covariates,
2. the three covariates and the dependent variable.

All the correlation coefficients are significant at the .01 level.

The correlation coefficients among the three covariates are high. For example, the correlation between I.Q. and A-AT is .780 and between A-AT and SNS-AT is .788. The higher value of the latter pair is to be expected because both tests belong to the Cooperative Mathematic Test Series.

Moderately high correlation coefficient values exist between the dependent variable and each of the control variables. This indicates that the control variables are significant independent variables in the study, and further confirms the necessity of equating these variables for the cells by the use of the analysis of covariance method.

Finally, the correlations among the achievement tests and the dependent variable are higher than between the dependent variable and I.Q. This finding indicates that a greater part of the variation in the dependent variable can be accounted for by the achievement tests rather than by I.Q. In Table 10 the percentage variance of the correlations results are presented. The A-AT and SNS-AT account for 50.55% and 51.41% respectively of the variability in the scores on the tests of polynomials, while I.Q. accounts for only 28.94% of the variability of the scores.

Table 9

Summary of the Correlation Matrices for
Polynomial Tests and Control Variables

	I.Q.	A-AT	SNS-AT	Addn. and Subtr.	Mult.	Div.	Total of Tests
I.Q.	1.000	.780	.726	.558	.472	.393	.538
A-AT		1.000	.788	.691	.615	.580	.711
SNS-AT			1.000	.700	.621	.606	.717

All correlation coefficients exceeding $r = .285$
are significant at the .01 level.

Table 10

Summary of the Percentage Variance Accounted
for by the Control Variables

	I.Q.	A-AT	SNS-AT	Addn. and Subtr.	Mult.	Div.	Total of Tests
I.Q.	100	60.84	52.70	31.14	26.34	21.14	28.94
A-AT		100	62.09	47.75	37.82	33.64	50.55
SNS-AT			100	49.00	44.15	43.09	51.41

However, it should be noted that 60.84% and 52.70% of the
scores in the A-AT and SNS-AT respectively can be accounted for
by I.Q.

ANALYSIS OF COVARIANCE FOR TREATMENT,
AGE AND SEX

The hypothesis of each main effect is dealt with below:

Hypothesis 1--Treatment

There is no significant difference in the mean scores of achievement among the three treatment groups with respect to their performance

- (a) on a test of the addition and subtraction of polynomials,
- (b) on a test of the multiplication of polynomials,
- (c) on a test of the division of polynomials,
- (d) on a total of the three preceding tests.

Table 11 gives the mean and adjusted mean scores on the polynomial tests obtained by each treatment group. Larger differences exist in the means than in the adjusted means in the tests. The larger differences in the means can be accounted for partly by initial differences in I.Q. and mathematic achievement of the groups and partly by differences in treatment. As can be seen, Hw-A means were adjusted down while N-Hw and Hw-AD means were adjusted up by the analysis of covariance, in order to equate the initial differences of the groups. Thus, the differences remaining in the adjusted mean are differences produced only by the treatments. The differences in the adjusted means are small among the three treatments.

Table 11

Comparison of Means and Adjusted Means of the Polynomial Tests for Students in the Three Treatments

Treat- ment	Addn. & Subt.		Mult.		Div.		Total of Tests		Covariates		
	M	AM	M	AM	M	AM	M	AM	I. Q.	A-AT	SNS-AT
N-Hw	22.22	23.81	19.35	21.01	19.06	20.34	59.91	64.66	106.37	28.33	15.67
Hw-A	25.39	23.25	24.42	22.25	20.21	18.55	69.98	63.80	116.61	35.46	20.00
Hw-AD	23.14	23.62	21.50	21.96	16.77	17.10	61.40	62.64	108.38	32.19	15.46

M - mean

AM - adjusted mean

Table 11 indicates that the scores on the adjusted means for the N-Hw exceed the homework groups except on the multiplication test.

Table 12 summarizes the data basic to the F ratio. Since none of the calculated F ratios exceeds the table value of 3.14 at the .05 level of probability, the above hypothesis must be accepted. In other words, the non-significant F ratio indicates that the differences among the adjusted means can be accounted for by chance alone more than five times in a hundred.

Table 12

Analysis of Covariance of the Polynomial Test Scores for Students in the Three Treatments

Tests	Degree of freedom		Mean square		F
	between	within	between	within	
Addn. & Subt.	2	64	2.532	43.424	.06
Mult.	2	64	13.285	61.640	.22
Div.	2	64	80.581	50.827	1.59
Total of Tests	2	64	31.156	377.612	.08

The table value of the F ratios at the .05 level of probability is 3.14.
None of the F ratios shown is significant at the .05 level.

The fact that there is no significant difference between the N-Hw and the homework groups indicates it is the doing of the work that counts rather than where it is done. Also the fact that there is no significant difference between Hw-A and Hw-AD suggests that post-homework discussion is not important for the enhancement of achievement in polynomials.

Hypothesis 2--Age

There is no significant difference between the mean score of achievement of students who are older than or equal to the mean age of 14.4 years and that of the students who are younger than 14.4 years with respect to their performance

- (a) on a test of the addition and subtraction of polynomials,
- (b) on a test of the multiplication of polynomials,
- (c) on a test of the division of polynomials,
- (d) on a total of the three preceding tests.

Table 13 contains the means and adjusted means for the tests on polynomials for the two age groups. The adjusted means indicate that the younger students did better than the older students in all tests.

Table 14 indicates that there is no significant difference in the adjusted mean scores at the .05 probability level since none of the F ratios exceed the table value of 3.99. The null hypothesis, therefore, must be accepted.

Table 13

Comparison of Means and Adjusted Means of the Polynomial Tests for Older and Younger Students

Age	Addn. & Subt.		Mult.		Div.		Total of Tests		Covariates		
	M	AM	M	AM	M	AM	M	AM	I.Q.	A-AT	SNS-AT
A < 14.4	25.56	24.19	23.32	22.07	20.07	19.02	68.53	64.80	113.94	33.92	18.75
A ≥ 14.4	20.47	22.60	19.26	21.19	16.53	18.16	56.26	62.04	104.94	28.90	14.35

M - mean

AM - adjusted mean

Table 14

Analysis of Covariance of the Polynomial Test
Scores for Older and Younger Students

Tests	Degree of Freedom		Mean Square		F
	between	within	between	within	
Addn. & Subt.	1	64	57.013	43.424	1.31
Mult.	1	64	17.575	61.640	.29
Div.	1	64	16.793	50.827	.33
Total of Tests	1	64	171.180	377.612	.45

The table value of the F ratio at the .05 level of probability is 3.99.

None of the F ratios is significant.

Hypothesis 3--Sex

There is no significant difference between the mean scores of achievement of male students and those of female students with respect to their performance

- (a) on a test of the addition and subtraction of polynomials,
- (b) on a test of the multiplication of polynomials,
- (c) on a test of the division of polynomials,
- (d) on a total of the three preceding tests.

Table 15 contains the means and adjusted means for the polynomial tests. The adjusted means indicate that boys tended

Table 15

Comparison of Means and Adjusted Means of the
Polynomial Tests for Boys and Girls

Sex	Addn. & Subt.		Mult.		Div.		Total of Tests		Covariance		
	M	AM	M	AM	A	AM	M	AM	I.Q.	A-AT	SNS-AT
Boys	25.31	23.81	23.64	22.19	19.53	18.20	68.47	64.00	113.84	35.47	17.94
Girls	22.37	23.40	20.43	21.41	18.11	19.01	60.48	63.52	108.06	29.55	16.40

M - means

AM - adjusted means

to outperform the girls to a small degree in polynomials.

Table 16 contains the data basic to the F ratio. None of the F ratios is significant. Therefore the null hypothesis must be accepted.

Table 16

Analysis of Covariance of the Polynomial
Test Scores for Boys and Girls

Tests	Degree of Freedom		Mean Square		F
	between	within	between	within	
Addn. & Subt.	1	64	4.034	42.434	.09
Mult.	1	64	14.418	61.640	.23
Div.	1	64	15.169	50.827	.30
Total of Tests	1	64	5.609	377.612	.01

The table value of the F ratio at the .05 level of probability is 3.99.
None of the F ratios is significant.

ANALYSIS OF COVARIANCE FOR THE COMBINATION OF
TWO FACTORS

Each hypothesis for the combination effect of two factors will be discussed in turn below.

Hypothesis 4--Combination of Age and Treatment

There is no significant difference in the means scores of achievement among students of one particular age-and-

treatment combination and students of another age-and-treatment combination with respect to their performance

- (a) on a test of the addition and subtraction of polynomials,
- (b) on a test of the multiplication of polynomials,
- (c) on a test of the division of polynomials,
- (d) on a total of the three preceding tests.

Table 17 provides a geometric interpretation of adjusted mean scores of the polynomial tests for student groups of a particular age and treatment. More complete information of means and adjusted means may be found in Appendix H.

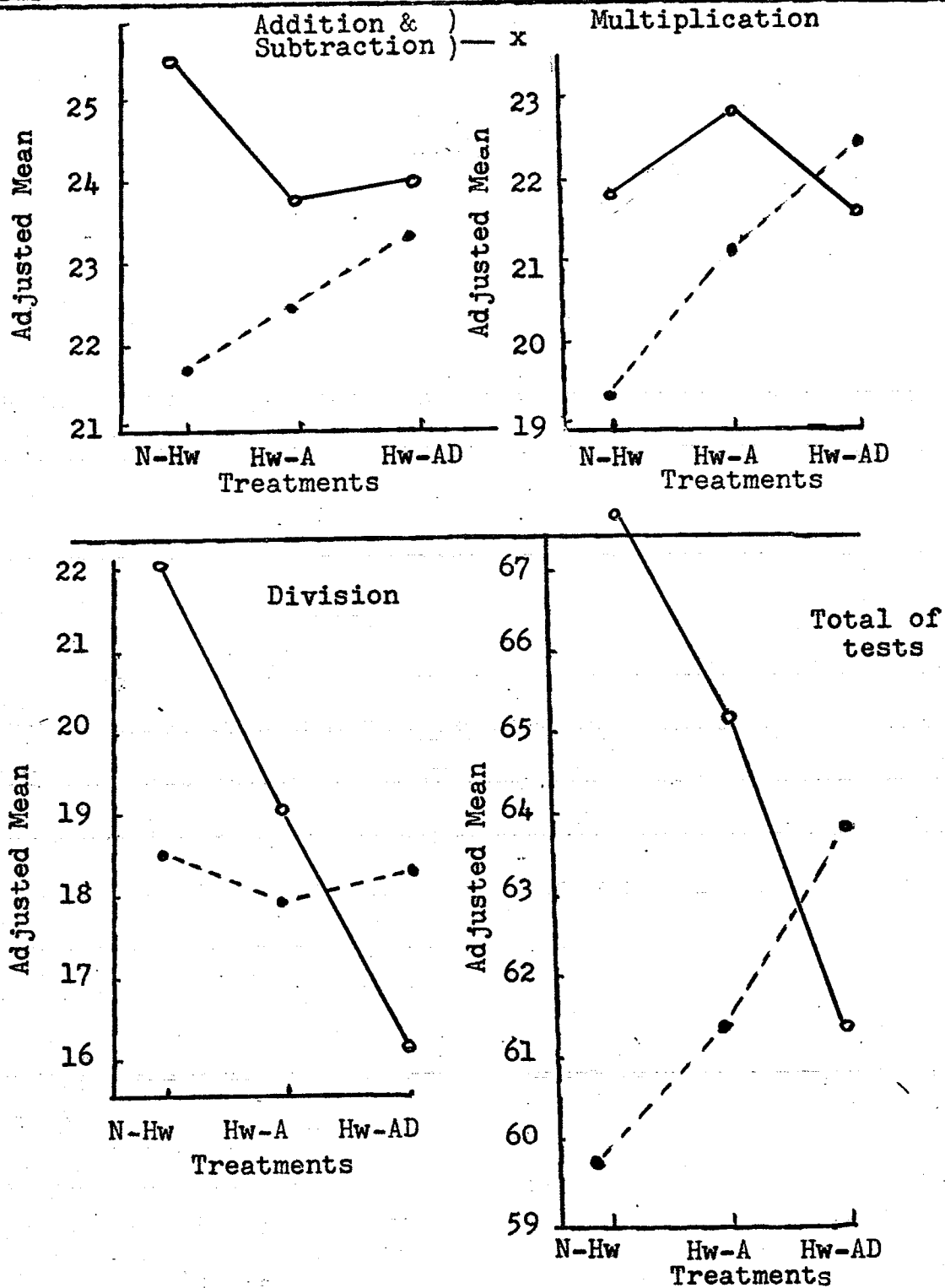
Table 17 shows that interaction effects occurred on two of the tests, multiplication and division, and on the total of the tests. Older students did better than younger students in the Hw-AD. This suggests that Hw-AD is the most effective method for older students. However, this interaction is not significant at the .05 level as revealed by the data in Table 18. This table shows that the calculated F ratios do not exceed the table value of 3.14. Therefore, the null hypothesis must be accepted.

Hypothesis 5--Combination of Sex and Treatment

There is no significant difference in the mean scores of achievement among students of one particular sex-and-treatment combination and students of another sex-and-

Table 17

Geometric Comparison of Adjusted Means of the Polynomial Tests for Combination Effect of Age and Treatment*



— $A < 14.4$

- - - $A \geq 14.4$

* James W. Popham, Educational Statistics Use and Interpretation (New York: McGraw-Hill, 1962), pp. 195-97.

treatment combination with respect to their performance

- (a) on a test of the addition and subtraction of polynomials,
- (b) on a test of the multiplication of polynomials,
- (c) on a test of the division of polynomials,
- (d) on a total of the three preceding tests.

Table 18

Analysis of Covariance of the Polynomial Test Scores
for Combination Effect of Age and Treatment

Tests	Degree of Freedom		Mean Square		F
	between	within	between	within	
Addn. & Subt.	2	64	18.52	43.42	.43
Mult.	2	64	20.81	61.64	.34
Div.	2	64	48.75	50.83	.96
Total of Tests	2	64	185.86	377.61	.49

The table value of the F ratio at the .05 level of probability is 3.14.

None of the F ratios is significant.

Table 19 provides a geometric interpretation of adjusted means of students of a particular sex and treatment. A complete table of means, adjusted means and covariates may be found in Appendix I. Table 19 indicates that there are three inter-

actions. On the addition and subtraction test and on the total of the three tests, the girls had higher mean achievement scores than the boys in Hw-AD. The other interaction occurred on the multiplication test where the girls did better than the boys in the N-Hw treatment.

However, these interactions are not statistically significant at the .05 level of probability. Table 20 presents the calculated F ratios. None of these ratios is larger than the table value of 3.14. Thus, the null hypothesis must be accepted.

Hypothesis 6--Combination Effect of Sex and Age

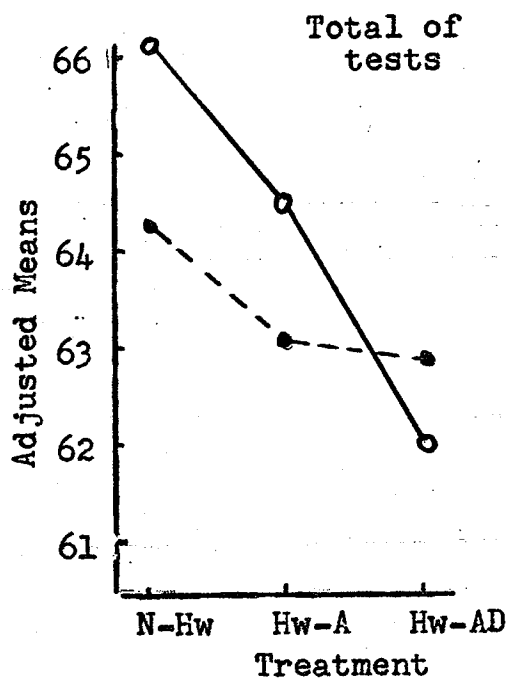
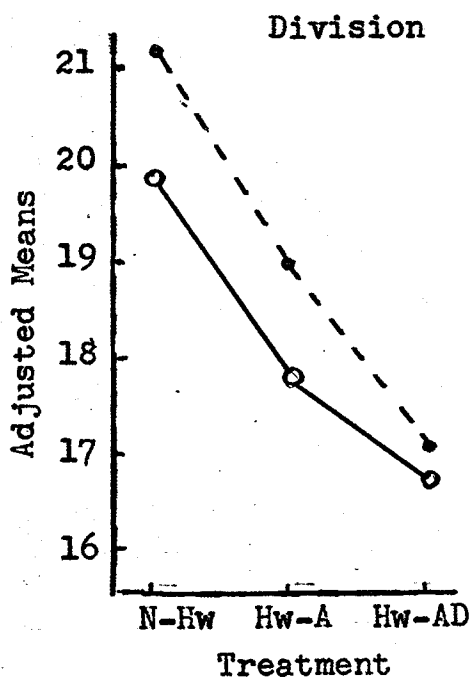
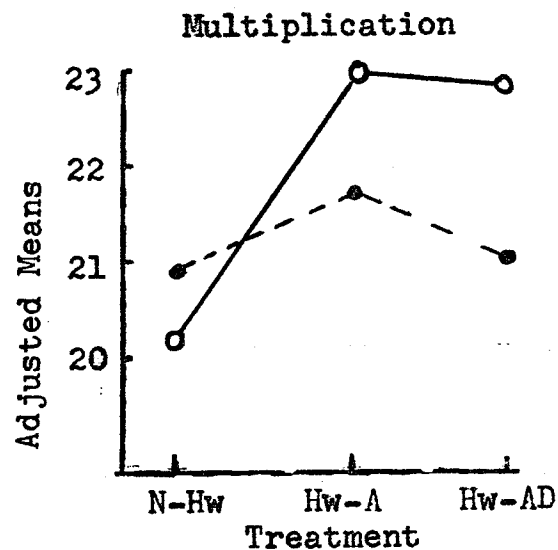
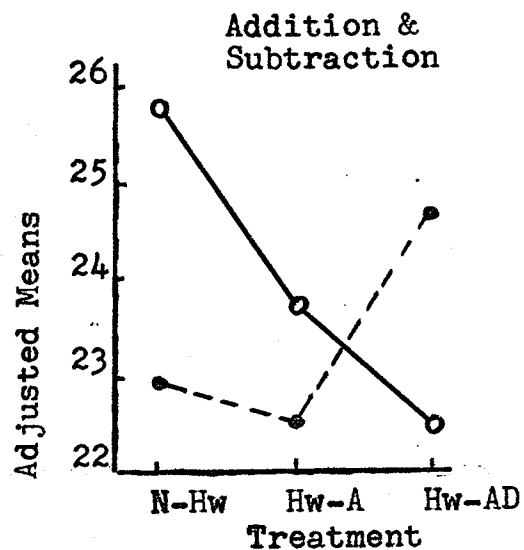
There is no significant difference in the mean scores of achievement among students of one particular sex-and-age combination and students of another sex-and-age combination with respect to their performance

- (a) on a test of the addition and subtraction of polynomials,
- (b) on a test of the multiplication of polynomials,
- (c) on a test of the division of polynomials,
- (d) on a total of the three preceding tests.

Table 21 is a geometric interpretation of the adjusted means of students of a particular sex and age. Full information pertaining to means and covariates may be found in Appendix J. Table 21 shows that there are two interaction

Table 19

Geometric Comparison of Adjusted Means of the
Polynomial Tests for Combination Effect
of Sex and Treatment



----- Girls

————— Boys

Table 20

Analysis of Covariance of the Polynomial Test Scores
for Combination Effect of Sex and Treatment

Tests	Degree of Freedom		Mean Square		F
	between	within	between	within	
Addn. & Subt.	2	64	48.43	43.42	1.12
Mult.	2	64	6.69	61.64	.11
Div.	2	64	-4.65	50.83	-.09
Total	2	64	18.18	377.61	.05

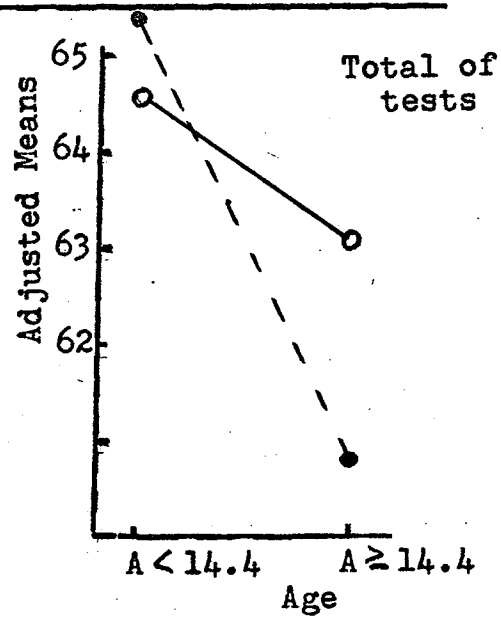
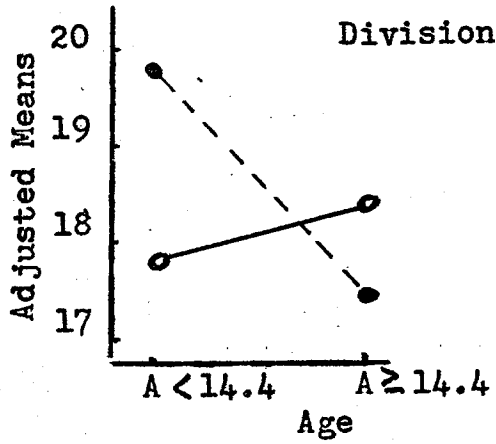
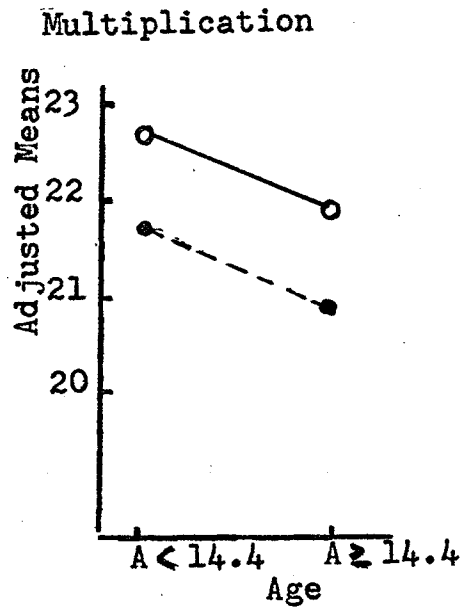
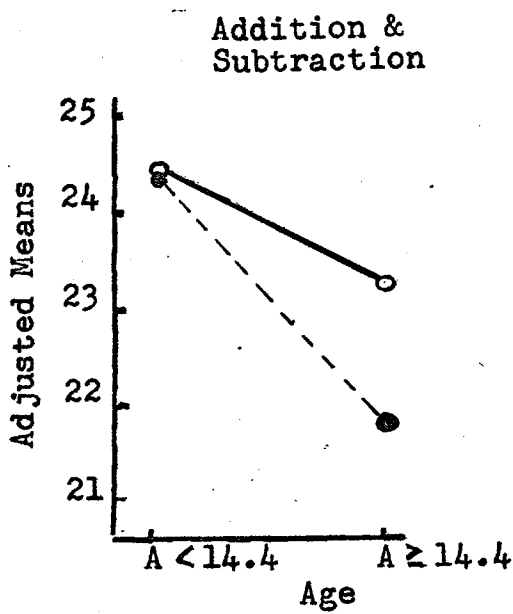
The table value of the F ratio at the .05 level of probability is 3.14.

None of the F ratios is significant.

effects: one on the division test, the other on the total of the tests. In each case, the older boys did better than the younger boys. These interactions, however, are not statistically significant at the .05 level.

Table 22 presents the calculated F ratios. None of these exceeds the table value of 3.99. Thus, the null hypothesis must be accepted.

Table 21
 Geometric Comparison of Adjusted Means of the
 Polynomial Tests for Combination Effect
 of Sex and Age



----- Girls
 _____ Boys

Table 22

Analysis of Covariance of the Polynomial Test Scores
for Combination Effect of Sex and Age

Tests	Degree of Freedom		Mean Square		F
	between	within	between	within	
Addn. & Subt.	1	64	4.14	43.42	.10
Mult.	1	64	-1.01	61.64	-.02
Div.	1	64	37.49	50.83	.74
Total of Tests	1	64	45.71	377.61	.12

The table value of the F ratio at the .05 level of probability is 3.99.
None of the F ratios is significant.

ANALYSIS OF COVARIANCE FOR THE COMBINATION
OF THREE FACTORS

Each hypothesis for the combination effect of three factors will be discussed in turn below.

Hypothesis 7--Combination of Sex, Age and Treatment

There is no significant difference in the means scores of achievement among students of one particular sex-and-age-and-treatment combination and students of another sex-and-age-and-treatment combination with respect to their performance

- (a) on a test of the addition and subtraction of polynomials,
- (b) on a test of the multiplication of polynomials,
- (c) on a test of the division of polynomials,
- (d) on a total of the three preceding tests.

Table 23 provides the means and covariates for all cells of a particular sex, age and treatment. None of the adjusted mean differences is statistically significant at the .05 level.

Table 24 presents the analysis of covariance of the adjusted mean scores of the groups. None of the F ratios exceeds the table value of 3.14. Therefore, the null hypothesis must be accepted.

SUMMARY

The size and distribution of the sample were described and correlations of the control and dependent variables were made.

Table 25 summarized the means, adjusted means and the number of students in each source of variation. Table 26 summarized the analysis of covariance for all sources of variation.

The results in the two tables indicate that there

Table 23

Comparison of Means and Adjusted Means of the Polynomial Tests
for Combination Effect of Age, Sex and Treatment

Tests	No Homework				Homework with Answers				Homework with Answers and Discussion				
	Boys		Girls		Boys		Girls		Boys		Girls		
	A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4	
Addn. & Subt.	M	30.90	25.50	22.33	15.29	27.30	23.17	27.36	21.00	21.50	22.50	25.83	21.08
Mult.	M	25.40	17.50	19.63	15.36	26.50	21.50	27.00	19.42	21.30	23.50	21.83	19.17
Div.	M	24.00	19.17	20.08	13.71	20.55	19.67	24.36	15.08	15.10	17.92	16.78	17.00
Total of Tests	M	80.40	60.17	60.38	44.36	74.25	64.33	78.71	55.50	57.90	63.92	64.44	57.25
Covariates	I.Q.	119.00	100.33	106.75	99.29	14.60	121.00	124.86	108.17	117.80	108.17	109.33	99.33
	A-AT	38.80	29.67	26.75	23.00	36.30	38.67	39.43	27.83	36.20	32.00	32.56	28.50
	SNS-AT	21.00	16.67	15.75	11.29	20.60	16.00	24.57	15.67	15.20	14.83	16.89	14.17

M = mean

Table 24

Analysis of Covariance of the Polynomial Test Scores
for Combination Effect of Age, Sex and Treatment

Tests	Degree of Freedom		Mean Square		F
	between	within	between	within	
Addn. & Subt.	2	64	21.21	43.42	.49
Mult.	2	64	15.88	61.64	.26
Div.	2	64	.22	50.83	.00
Total of Tests	2	64	63.64	377.61	.17

The table value of the F ratio at the .05 level of probability is 3.14.
None of the F ratios is significant.

are no significant differences traceable to treatment effects, sex effects, age effects, or interaction effects. Therefore, all null hypotheses were accepted.

Table 25

Summary of Means and Adjusted Means of the Polynomial Tests
Obtains for each Source of Variation

Source of Variation	Addn. & Subt.			Mult.			Div.			Total of Tests		
	N	M	AM	N	M	AM	N	M	AM	N	M	AM
	Treat- ment	27	22.22	23.81	27	19.35	21.01	27	19.06	20.34	27	59.91
--- N-Hw												
--- Hw-A	26	25.39	23.25	26	24.42	22.25	26	20.21	18.55	26	69.98	63.81
--- Hw-AD	26	23.14	23.62	26	21.50	21.96	26	16.77	17.10	26	61.40	62.64
Sex --- Boys	33	25.31	23.81	33	23.64	22.19	33	19.53	18.21	33	68.47	63.10
--- Girls	46	22.37	23.40	46	20.43	21.42	46	18.11	19.01	46	60.48	63.52
Age A < 14.4	48	25.56	24.19	48	23.32	22.07	48	20.07	19.02	48	68.53	64.80
A ≥ 14.4	31	20.47	22.60	31	19.26	21.19	31	16.53	18.16	31	56.26	62.04

N = number in cell

M = mean

AM = adjusted mean

Table 26

Summary of Analysis of Covariance of the Polynomial Tests
Using I.Q. and Mathematic Achievement as Covariates

Source of Variation	Addn. & Subt.			Mult.			Div.			Total of Tests		
	df	MS	F	df	MS	F	df	MS	F	df	MS	F
Treatment	2	2.53	.06	2	13.29	.22	2	80.58	1.59	2	31.16	.08
Sex	1	4.03	.09	1	14.42	.23	1	15.17	.30	1	5.61	.01
Age	1	57.01	1.31	1	17.56	.29	1	16.79	.33	1	171.18	.45
Treatment -- Sex	2	48.43	1.12	2	6.69	.11	2	-4.65	-.09	2	18.18	.05
Treatment --- Age	2	18.52	.43	2	20.81	.34	2	48.75	.96	2	185.86	.49
Sex --- Age	1	4.14	.10	1	-1.01	-.02	1	37.49	.74	1	45.71	.12
Treatment --- Sex --- Age	2	21.21	.49	2	15.88	.26	2	.23	.00	2	63.64	.17
Error (within)	64	43.42		64	61.64		64	50.83		64	377.61	
Total	75			75			75			75		

None of the above F ratios is significant at the .05 level.

Chapter 5

SUMMARY AND CONCLUSIONS

SUMMARY

The purpose of this study was to examine the effectiveness of traditional homework and post-homework discussion on mathematic achievement in polynomials. A secondary purpose was to determine whether age and sex of the students were factors that affected achievement. The questions raised in this regard were:

- (1) Is there a significant difference in the achievement of students with homework compared to those who have none?
- (2) Is the process of doing the work at home the significant factor on achievement?
- (3) Is the post-homework discussion at school the more important process?
- (4) Is it the combination of "doing and discussing" that is essential?
- (5) Is the age of a student a factor in mathematic achievement?
- (6) Is the sex of a student a factor in mathematic achievement?

A review of literature indicated that a limited number of experiments had been done on the effectiveness of homework. Some of the studies were inconclusive, others were outdated, and those which were recent and well-designed had contradictory findings.

A sample of seventy-nine grade IX students were involved in the study. These students were members of three intact classes taught by the vice-principal. One class was assigned to be the control group receiving no homework while the other two were the experimental groups. One experimental group received homework with answers while the other received homework with answers and extensive discussion in class.

The following control variables were obtained: I.Q. scores and scores on the Arithmetic Test and the Structure of Number System Test from the Cooperative Mathematics Series.

The independent variables were the three treatments, age, and sex, while the dependent variable was achievement in polynomials measured on three teacher-prepared tests.

To allow for variation that existed among the groups in I.Q. and mathematics achievement and to allow for intact groups, the analysis of covariance statistical design was utilized.

SUMMARY OF FINDINGS

1. Achievement scores did not differ significantly among the three treatment groups.

2. Sex of the students was found to have no significant effect on mathematic achievement.
3. Age of the students was found to have no significant effect on mathematic achievement.
4. The combination of any two or three of the above factors had no significant effect on mathematic achievement.

LIMITATIONS

As mentioned in Chapter 1, generalizations from the findings of this study are limited by the non-random sampling and intact classroom groups, the population from which the sample was chosen, the independent variables that were used and the mathematical topic used. The validity and reliability of the measuring instruments used must also be considered as limitations.

These limitations must be considered with respect to any conclusions based on the findings.

CONCLUSIONS

All students were taught the same concepts, with slightly varying amounts of work assigned to each class.

The following conclusions are apparent:

1. A comparison of achievement scores in polynomials indicates that those students who had homework did not excel those who had none, and vice versa. The comparison suggests:

- a) Doing the work assures the student of a higher degree of success in the topic; whether or not the assignments are completed at school or at home is inconsequential.
- b) An educator can be flexible; he may assign homework to vary his instructional programme or to allow for differences of character among the classes.
- c) A teacher has some justification in telling parents that if assignments are done in class, homework should not be necessary. The teacher can suggest other forms of homework (enrichment) if parents insist on homework.

2. The results of the study also show no level of superiority for either the homework group with answers or the homework group with answers and extensive discussion in terms of mathematic achievement. Thus, extensive discussion is not necessary in the homework process; the availability of answers and brief student-initiated discussion are all that is needed.

3. There is no significant difference in mathematic achievement between boys and girls. Thus, within the homework process there is no evidence to support a common belief that boys are more mathematically inclined than girls.

4. The fact that younger students did as well as older students on the tests in polynomials suggests that the topic of polynomials is appropriate for grade IX students.

RECOMMENDATIONS FOR FURTHER RESEARCH

1. A repetition of the present study to include more than one topic would perhaps support or clearly negate the trends suggested herein.
2. A follow-up study involving a larger sample drawn from a more inclusive population would

likely yield results that would enhance the reliability and validity of the present study.

3. The present study could be modified to treat I.Q. as an independent variable. Several researchers have suggested that high I.Q. students do not need homework while low I.Q. students require it.
4. Another investigation could examine the effect on achievement of homework patterns other than the traditional one.
5. A fifth possibility for research would be a study involving homework groups that receive more practice than the non-homework group to determine whether extra work enhances achievement.
6. The present study could be duplicated for a variety of topics in mathematics, for several grades and for other subjects.
7. A similarly designed study could be made using suitable standardized tests to measure the dependent variables.
8. The present study could be enlarged to four groups, the extra group being a non-homework group which receives no work at all. This design would determine with greater reliability that the doing of the work is the most important factor for success in the topic.

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APPENDIX A

A LIST OF ASSIGNMENTS

A. Addition and Subtraction

Baxter: P. 153 Nos. 3, 6, 7, 8
 P. 156 Nos. 4, 9
 P. 157 Nos. 10, 16

Nichols: P. 103 Nos. 8-26
 P. 109 Nos. 22-35
 P. 154 Part B, Nos. 21-40
 P. 157 Nos. 1-12
 P. 401 Nos. 4, 5, 6

B. Multiplication

1. Monomial by monomial

Nichols: P. 109 Nos. 38, 40, 42-54, 57, 58
 P. 113 Nos. 13-17, 28-31
 P. 153 Part A, Nos. 1-10

Exponent Theory:

Nichols: P. 70 Nos. 1-33, 38-52
 P. 72 Nos. 1-17
 P. 74 Nos. 1-44

2. Monomial by a polynomial

Nichols: P. 109 Nos. 36, 37, 39, 41, 55,
 56, 59, 61-81
 P. 119 Nos. 45-52
 P. 147 Nos. 74-87
 P. 157 Nos. 13-20

P. 158 Nos. 51-60

P. 159 Nos. 66, 67

3. Polynomial by a polynomial

Nichols: P. 372 Nos. 1-4

P. 374 Nos. 23-24, 41-46

P. 377 Nos. 1-16

Baxter* P. 174 Nos. 4, 5, 6

P. 177 Nos. 7-12

* extra questions done by homework groups only.

C. Division and Rational Expressions

1. Monomial divided by a monomial

Nichols: P. 182-3 Nos. 27-53

P. 189 Nos. 27-29

(a) Multiplication of Rational Expressions

Nichols: P. 174 Nos. 17-28

P. 178 Nos. 1-24

P. 189 Nos. 33-36, 39-41

(b) Division of Rational Expressions

Nichols: P. 188 Nos. 1-22, 26

(c) Addition and Subtraction of Rational Expressions

Nichols: P. 194 Nos. 23-57

(d) Complex Rational Expressions

Nichols: P. 198-99 Nos. 1-42

2. Polynomial divided by a binomial as per worksheet in Appendix B.
3. Polynomial divided by a binomial as per worksheet in Appendix B.

APPENDIX B

SUPPLEMENTARY EXERCISES

ALGEBRA WORKSHEET NO. 1

1) Circle like terms in each polynomial

a) $x + 3y - z + y$

b) $2ab - 3ac + 4dc - bc$

c) $ab - 2ab + 2cd - cd$

d) $y^2 - 2x^2 + 4^2 - 5y^2$

e) $2a^2b^2c^2 - 5a^2b^2c^2 + 8a^2b^2c^2$

f) $3m^2n^2 - 4mn^2 + 5m^2n^2 - 2m^2n^2$

g) $4 \cot - 9 \csc + 6 \cot$

h) $5m^5n^4x^3 - 6m^5n^4x^2 + 3m^4n^4x^3$

2) a) $8s$ b) $-2mno$ c) $12c^3d^2$ d) $-3m^2y^3$ e) $17x^2y^3$
 $-3s$ $-8mno$ $-10c^3d^2$ $-m^2y^3$ $-6x^2y^3$
 $12s$ $5mno$ $-c^3d^2$ $5m^2y^3$ x^2y^3

3) Add following monomials:

a) $5a + 2a + (-4a) =$

h) $\frac{1m}{2} + \frac{1m}{3} + \left(-\frac{1m}{6}\right) =$

b) $-8x^2 + (-12x^2) =$

i) $\frac{5n^2s}{8} + \left(-\frac{1n^2s}{2}\right) =$

c) $-3my + 11my =$

j) $\frac{-3mn}{7} + \left(\frac{-2mn}{7}\right) + \frac{1}{14}mn =$

d) $15rs^2 + (17rs^2) =$

k) $0.5m + (-2.3m) =$

e) $-17x^2y^4 + 13x^2y^3 =$

l) $2.32ab + (-4.1ab) =$

f) $7m + (-3m) + 2n =$

m) $2.57a + (-1.37a) =$

g) $-3m^2d + (-11m^2d) + m^2d =$

ALGEBRA WORKSHEET NO. 2

Simplify: Add where possible

a) $10x + 5x - 7 =$

b) $3m^2 - 4m^2 - 3b^2 =$

c) $-8m^2n - 6mn^2 - 3mn^2 =$

d) $-8d + 7c - 6d - 4c + d =$

e) $\frac{1}{2}d + 7m - 6m + 1\frac{1}{2}d =$

f) $0.5mn + 0.13mn + 1.7mn =$

g) $0.25m^2n + 0.5mn^2 + 0.63m^2n =$

h) $-3md + 4md^2 - 5md^2 =$

i) $-\frac{1}{3}m + \frac{1}{2}m - \frac{5}{6}m =$

j) $6m^2d^3 - 5m^3d^2 - 4m^2d^3 - 3m^3d^2 =$

k) $7m^2n - 6mn^2 - 5m^2n - 8mn^2 =$

ALGEBRA WORKSHEET NO. 3Polynomial Addition

$$\text{I } 1) \begin{array}{r} 3x + 4 \\ \underline{2x - 1} \end{array} \quad 2) \begin{array}{r} -8 + c \\ \underline{3 - 2c} \end{array} \quad 3) \begin{array}{r} 4r + 7s \\ \underline{4r - 7s} \end{array} \quad 4) \begin{array}{r} a - 3a + 5 \\ \underline{5a - 2} \end{array}$$

$$5) \begin{array}{r} 25x - 10y + z^2 \\ \underline{-12x + y - 3z} \end{array} \quad 6) \begin{array}{r} x^3 - x^2 + 2x \\ \underline{3x^2 - 4x} \end{array} \quad 7) \begin{array}{r} a^2 + b^2 - c^2 \\ -a^2 - 2b^2 + 3c^2 \\ \underline{8a^2 + 3b^2 - 7c^2} \end{array}$$

II Add:

$$a) (25x - 10y) + (5x - 2y) + (-3x - 5y) =$$

$$b) (x^3 + x^2 - 3x) + (3x^3 - 5x^2 + 4) + (-8x^2 - 5x + 10) =$$

$$c) (10y^2 + 5x - 6x + 8) + (8x^2 - 7x - 5y^2 - 7) =$$

$$d) (4x - 5y + 6) + (3x - 7y - 7) + (-10x + 12y - 2) =$$

$$e) (5m^2n - 3mn^2) + (6mn^2 - 3m^2n) + (-4m^2n - 6mn^2) =$$

$$f) (3y^2 + 5y) + (y^2 - 8y + 5) + (7y - 10) =$$

$$g) (2y^2 + 13y + 5) + (y^2 - 8y + 5) + (-3y^2 - 7y - 6) =$$

$$h) \left(\frac{3x}{4} + \frac{3y}{4} - \frac{1z}{2}\right) + \left(\frac{1x}{2} - \frac{1y}{8} - \frac{1z}{3}\right) + \left(\frac{2x}{3} - \frac{1y}{2} + \frac{1z}{6}\right) =$$

$$i) \left(\frac{x}{5} + \frac{y}{4} - \frac{z}{3}\right) + \left(\frac{3x}{2} - \frac{5y}{8} + \frac{7z}{5}\right) + \left(\frac{x}{10} - \frac{3y}{2} + \frac{5z}{6}\right) =$$

$$j) (1.45x + 2.7y) + (3.2x - 8.3y) + (4y - 2.02x) =$$

$$k) (1.45x^2 - y) + (3x^2 + x - y) + (-1.5x^2 + 0.5x + \frac{3y}{2}) =$$

ALGEBRA WORKSHEET NO. 4

Polynomial Subtraction (all "-" signs are "negatives")

I a) $\frac{-3xy}{5xy}$ b) $\frac{-3x^2}{-7x^2}$ c) $\frac{x^2y}{8x^2y}$ d) $\frac{15m}{-7m}$ e) $\frac{5mn}{-7mn}$ f) $\frac{-3x^2y}{-x^2y}$

g) $\frac{10x^3y^2}{-2x^3y^2}$ h) $\frac{-15x^4}{-18x^4}$ i) $\frac{9x^3}{-17x^3}$ j) $\frac{m^2n}{-m^2n}$ k) $\frac{-x^2}{-x^2}$

II a) $\frac{3xy - 7x - 3y}{-5xy + 5x - 2y}$ b) $\frac{9x^3 + 4x^2 + x}{-12x^3 - x^2 + 9x}$ c) $\frac{14x - 15y + 3}{-3x - 3y - 14}$

d) $\frac{4x - 7y + 3z}{-3x + 5y - 2z}$

III a) $(4x - 7y + 3z) - (3x + 5y - 2z) =$

b) $(4m + 5n - 3p) - (-2m + 4n + 2p) =$

c) $(5k^2 - 7k + 8) - (3k^2 + 5k - 7) =$

d) $(3x^2 - 5x - 3) - (-7x^2 + 4x - 5) =$

e) $(-4m + 8n - 5p) - (3m - 7n + 5p) =$

f) $4m - (3n + 2p) - (2m + 3n) - (2p - 6n) =$

g) $(45x^3 - 32y) - (-17x^3 - 83y) =$

h) $1 - (6x - 5y) - (-4y - 3z) - (z - 8y - 6x) =$

i) $(5a - 30b + 71c) - (4a + 3b - 5c) - (60a - 3b + 40c) =$

j) $(6m + 5n - 3k) - (5m + 5n + 2k) - (m - 3n - 5k) =$

ALGEBRA WORKSHEET NO. 5I. Subtraction (all "-" symbols represent negatives)

$$\begin{array}{r} \text{a) } 3.7x \\ +4.82x \\ \hline \end{array} \quad \begin{array}{r} \text{b) } 0.57x \\ 3.7x \\ \hline \end{array} \quad \begin{array}{r} \text{c) } 3.576m^2 \\ -7.2 m^2 \\ \hline \end{array} \quad \begin{array}{r} \text{d) } 1.007x \\ .890x \\ \hline \end{array}$$

$$\begin{array}{r} \text{e) } \frac{1y^3}{2} \\ \frac{1y^3}{6} \\ \hline \end{array} \quad \begin{array}{r} \text{f) } \frac{-2x}{3} \\ \frac{+1x}{4} \\ \hline \end{array} \quad \begin{array}{r} \text{g) } 0.3x - 0.6y \\ \frac{2x}{3} + \frac{1y}{3} \\ \hline \end{array} \quad \begin{array}{r} \text{h) } 3x + 2y - \frac{1z}{2} \\ -4x - 7y + \frac{3z}{2} \\ \hline \end{array}$$

II. Remove Parentheses and Simplify:

$$\text{a) } (5x - 3y) + (3x - 2y) + (-7x + 2y) =$$

$$\text{b) } (5x - 3) - (3x - 2) + (8x - 5) =$$

$$\text{c) } (7x^3 - 3x^2 - 5) - (6x^3 - 3x^2 - 6) + (7x^3 - 10) =$$

$$\text{d) } 5x - [2x - 3 - (3x - 5)] =$$

$$\text{e) } 7x - [5x - (3x - 7) + 2x + 7] =$$

$$\text{f) } 5x - 7 - (3x - 7) - (8 - 10x) =$$

$$\text{g) } 9y - \{9 - [8 - 6y - (7 - 2y) + 6] - 3\} =$$

$$\text{h) } 10x - \{17x + [8x - (5x - 30)] - 5\} =$$

$$\text{i) } x - [y - z] - [x - (y - z)] - [y - (z - x)]$$

ALGEBRA WORKSHEET NO. 6I. Remove Parentheses and Simplify:

a) $\left(\frac{1x}{2} + \frac{1y}{3} - \frac{1z}{9}\right) - \left(\frac{2x}{3} - \frac{1y}{2} - \frac{1z}{2}\right) =$

b) $(3.6x - 7.5y - 3.4z) - (-2.8x + 2.5y + 3.4z) =$

c) $x - \sqrt{y} - (z - 4p)] - \{4p - \sqrt{p} - x - (x - y)]\} =$

d) $\frac{1x}{2} - \left(\frac{1y}{3} - \frac{1z}{2}\right) - \left(\frac{1z}{3} - \frac{1y}{2} - \frac{1x}{2}\right) =$

e) $\sqrt{3p} - (4q + z)] - \sqrt{3p} - (-2z - 3q)] - \sqrt{2p} + (q - z)] =$

f) $\sqrt{a} - (b - c)] - \sqrt{b} + (c - a)] - \sqrt{c} - (a - b)] =$

g) $\sqrt{x} - (y - z)] + \sqrt{15x + (10y - 4z)] - \sqrt{30y - (3z - 2x)]} =$

Division of Open Expressions

I. Simplify all answers: Reduce to lowest terms if answer is a fraction.

$$a) \frac{50a^3bc^5}{-5a^2bc} = \quad d) \frac{7ab^2c}{2lab} = \quad g) \frac{(-3a^2b)^3}{9a^5b^2} =$$

$$b) \frac{-75x^7y^8z^9}{-15x^3y^4z^3} = \quad e) \frac{24mpx}{20xmy} = \quad h) \frac{-(-8b^2)^2}{-4b^3} =$$

$$c) \frac{-60a^4b^8c^{12}}{4a^2b^2c^2} = \quad f) \frac{8x^2y^9}{-4x^5y^4} = \quad i) \frac{-(-9a^2b^7)}{-3a^2b^7} =$$

II.

$$a) \frac{6x + 4x}{2x} = \quad f) \frac{12 + 18x}{-6} =$$

$$b) \frac{14t + 42}{3} = \quad g) \frac{7x^3y^3 - 14x^2y^2 + 21xy}{-7xy} =$$

$$c) \frac{7c - 14d}{-7} = \quad h) \frac{51x^3 - 34x^2 + 17x}{17x} =$$

$$d) \frac{9x^2y^3 - 18x^5y^6}{-3} = \quad i) \frac{91y^7 + 52y^5 - 39y^3}{13y^2} =$$

$$e) \frac{3x^2 - 6x + 27}{-3} = \quad j) \frac{(3x^2 - 2x)(3x - 3)}{3x} =$$

Hint: Multiply numerator out first.

Mathematics IX: Long Division

Copy and do the following questions in your notebook:

$$1) \quad 3x - 7 \overline{) 15x^2 - 41x + 14}$$

$$2) \quad 4x - 8 \overline{) 12x^3 - 28x^2 + 8x}$$

$$3) \quad 2x + 6 \overline{) 4x^2 + 2x - 30}$$

$$4) \quad 3x - 4 \overline{) 6x^3 - 23x^2 + 29x - 12}$$

$$5) \quad 2x - 7 \overline{) 2x^3 - 9x + 5x + 7}$$

$$6) \quad 2x - 4 \overline{) 8x^3 - 64}$$

$$7) \quad 3x + 5 \overline{) 27x^3 + 125}$$

APPENDIX C

REVIEW WORKSHEETS

Review:

I. Substitute in and evaluate according to ORDER OF OPERATIONS

a) $(3x - y) - 2(x - y) =$ when $x = -1$ and $y = -2$

b) $(3x - 2y + z)^2 =$ when $x = 0.1$, $y = 0.1$ and $z = 0.2$

c) $3x(x-y-z) - (x-y)^2 =$ when $x = \frac{1}{2}$, $y = -\frac{1}{2}$, $z = \frac{1}{3}$.

II. Subtract

a)
$$\begin{array}{r} 3.7x \\ -1.95x \\ \hline \end{array}$$

b)
$$\begin{array}{r} -3.8x \\ -5.73x \\ \hline \end{array}$$

c)
$$\begin{array}{r} \frac{1x}{2} \\ -\frac{5x}{6} \\ \hline \end{array}$$

d)
$$\begin{array}{r} -\frac{7x}{8} \\ -\frac{1x}{4} \\ \hline \end{array}$$

III. Simplify

a) $(3x - y + z) - (2x - 7y - 9z) + (x - z) =$

b) $(70x - 83y + 101) - (-22x + 71y + 129) =$

c) $\left(\frac{1x}{2} - \frac{23y}{12} - 2.3z\right) + \left(\frac{3x}{7} + \frac{1y}{2} + 1.45z\right) =$

d) $3x - \left\{7 - [7y - 3x + (7 - 8y)] + 3x\right\} =$

e) $2x - [7x - (3x + 1)] + [75 - (7 - 8x)] =$

f) $35x - (-75y) - (36x - 17y) + (-18y) =$

Math Grade Nine:

1. Simplify: Leave no brackets in answer. Perform necessary operation.

a) $(-0.4)(0.3)(0.2x) =$ _____

b) $-(-2) =$ _____

c) $(A + 2)A =$ _____

d) $5x - (3x - 2) =$ _____

e) $5x + (4x - 3) =$ _____

f) $x - y(z + w) =$ _____

g) $w(x + y + z) =$ _____

h) $-(x - 3) =$ _____

i) $(x + 2)(x + 3) =$ _____

j) $(3x)(2y)(-4) =$ _____

k) $-3(x - 3) =$ _____

l) $(.2x^2y^3)^2 =$ _____

II. If $A = 0.01$ and $B = 0.2$, then $3A^2 - 2AB + B^2 =$

(Substitute in values of A and B and evaluate)

Simplify fully: Perform necessary operation.

a) $(-3x^2)(2xy)(-7x^3y^3) =$ _____

b) $3x^2y^3(4x - 2y) =$ _____

c) $-2(6x)(4x) - x(7 - 3x) =$ _____

d) $3x^2 - x(3x + 2) =$ _____

e) $(2x - 4)(3x - 5) =$ _____

f) $\left(-\frac{2}{3}x^3\right)\left(-5x^2\right)\left(\frac{9}{2}x^4\right) =$ _____

g) $(2x - 3)^2 =$ _____

h) $\left(-\frac{2}{3}x^2\right)^3 =$ _____

i) $(2.1x^2)(.04x^3)^2 =$ _____

j) $\frac{1}{4}(8a + 12b) - 2(3b + a) =$ _____

k) $(x - 2)(3x - 2x + 3) =$ _____

l) $3 + 2(3 - 4x) - 4(-x - 1) - (2x + 1) =$ _____

m) $[(2x - 3)(2x + 3)] - [(x + 3)(x + 4)] =$ _____

n) $[\sqrt{3}a - 4(5a - 5b)] + [-2a - 4(a - b)] =$ _____

Review of Rational Expressions and Division

1. Simplify:

a) $\frac{-15x^6y^9z^{12}}{3x^2y^3z^4} =$

b) $\frac{3}{x} + \frac{x}{t} =$

c) $\frac{2c}{x^2} \div \frac{c}{x} =$

d) $\frac{3c}{x^2} \cdot \frac{x}{c^3} \cdot \frac{x^3}{c} =$

e) $\frac{2x-3}{2} - \frac{x+1}{3} =$

f) $\frac{\frac{2x}{x^2}}{\frac{x^2}{c^2}} =$

g) $\frac{15x^3 - 10x^2 + 5x}{-5x} =$

h) $\frac{3}{x-2} + \frac{2}{x+2} =$

i) $\frac{2x}{3} + \frac{2(2x-3)}{5} =$

j) $\frac{-24x^6y^9}{-8x^{10}y} =$

k) $\frac{(-2x^3y^2)^2}{4x^6y^4} =$

l) $\frac{38x^3y^6 - 19x^4y^8}{19x^2y^4} =$

m) $3x - 5 \sqrt{14x^2 - 34x + 15} =$

n) $2y - 3x \sqrt{8y^2 - 14xy + 3x^2} =$

o) $2x - 3 \sqrt{8x^3 - 27} =$

p) $\frac{1 - \frac{r}{s}}{2 + \frac{r}{s}} = \frac{(1 - \frac{r}{s})}{2 + \frac{r}{s}}$

APPENDIX D

TEACHER PREPARED TESTS

Algebra Test 1

Name _____

- 1) An open expression like $12x^2 + 3x - 3$ is called a(n) _____. An expression like $-2x^2 - 2y^2$ is called a(n) _____. The sum of these two expressions is _____.
- 2) The polynomial $-3x^2 - (-2)x - 3y^3 + 7y^3 - 13$ has _____ terms.
- 3) The opposite of $-2.3 + \frac{1}{2}x$ is _____.
- 4) Simplify: (leave no brackets in answer)
- a) $3x - (2t - 4) =$ _____
- b) $3 - (-7x + y) - 2t =$ _____
- c) $17x + (-7y + 3) =$ _____
- d) $8x^2 - 2x + 14x - 8 =$ _____
- 5) Add:
- a)
$$\begin{array}{r} 31x^2y \\ -82x^2y \\ \hline x^2y \end{array}$$
- b)
$$\begin{array}{r} 44.03 \\ -20.092m \\ \hline \end{array}$$
- c)
$$\begin{array}{r} -3x^3 \\ 5 \\ -2x^3 \\ \hline 3 \end{array}$$
- d)
$$\begin{array}{r} 3x - y + z \\ -x + y - 14z \\ \hline -10x - 5y + 7z \end{array}$$
- 6) Subtract:
- a)
$$\begin{array}{r} 18.3x - .001y \\ 7.92x - 8.2y \\ \hline \end{array}$$
- b)
$$\begin{array}{r} \frac{1}{2}x - 3y + 13z - 2 \\ -\frac{1}{2}x - 3y - 15z + 2 \\ \hline \end{array}$$

7) Simplify fully:

a) $(30x - 29y + 28) - (27x - 26y - 25) + (24x + 23y - 22) =$

b) $\left(\frac{1x}{2} - \frac{1y}{3} - \frac{1z}{5}\right) - (7x - 2y) - \frac{2x}{3} + \frac{1z}{4} =$

c) $7x - \left\{3y - \sqrt{2} + (7 - 8y) - \sqrt{z} - 2x\right\} =$

8) Evaluate: "Remember Order of Operations"

a) $3x(x-y)^2 - 2xy$ when $x = 0.1$ and $y = -0.2$

b) $(3x - y)^2 - (y - x)^2$ when $x = -\frac{1}{2}$ and $y = -\frac{2}{3}$

Multiplication Test 2

Name _____

Simplify fully

1) $\left(\frac{2}{3}\right)^4 =$

2) $(.05)^3 =$

3) $(mn^4)(mn^5)(m^2n) =$

4) $\left[-(-1)^4\right]^5 =$

5) $-3^2 =$

6) $a \cdot a^2 a^3 =$

7) $(a^3)^5 =$

8. $3(2x)(-5) =$

9. $(x - 7)^2 =$

10. $(7 - x) 3 =$

11) $-3x^2y^3 (5x^2y - 2xy^2) =$

12) $\left[(-.2)(-a)(-b)\right]^3$

13) $5 - 3(x - 2) =$

14) $(2x - 1)(3x^2 - 5x + 7) =$

15) $\frac{1}{2}b \left(\frac{1}{3}c + \frac{1}{5}b\right) =$

16) $2(a + b + 4c) - \frac{1}{2}(4c - 2b + 8a) =$

17) $3x - 2(3x + 2) =$

18) $(-.2)(.3x)(x + 2) =$

19) $(c + n)(4n^2 + 2nc - c^2) =$

20) $\left[-5 - 2x(3 - x)\right] - \left[(x - 2)(3 + 2x)\right] =$

21) $(.01x^2y^3)(-.2xy^2)^2 =$

Algebra Test 3

Name _____

Test on Division and Rational Expressions

I. Reduce the following fractions:

a) $\frac{40x^3}{60x^2} =$

b) $\frac{(-2x^2)^3}{(4x)^2} =$

II. Simplify fully by performing indicated operations.

a) $\frac{12c}{d} \cdot \frac{h}{3} \cdot \frac{d^2}{h^2} =$

b) $\frac{8A^5}{x^3y^3} \div \frac{12A^2}{x^2y} =$

c) $\frac{7}{b^3} \div b^2 =$

d) $\frac{5a}{3} - \frac{2a-9}{3} =$

e) $\frac{x+5}{3x} - \frac{1-x}{3x} + \frac{7x-4}{3x} =$

f) $\frac{a+4}{3} - \frac{2-4a}{2a} =$

g) $\frac{3x}{x-2} + \frac{5x}{x+2} =$

III. Simplify following complex fractions fully:

$$\text{h) } \frac{x - \frac{1}{2}}{4}$$

$$\text{i) } \frac{\frac{x}{3} - \frac{x}{5}}{\frac{1}{2}} =$$

IV. Divide following expressions:

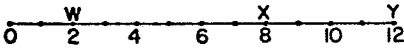
$$\text{a) } \frac{27xy - 18y^2 - 9y}{9y} =$$

$$\text{b) } 2x - 4 \overline{) 8x^2 - 10x - 12} =$$

$$\text{c) } 5x + 4 \overline{) 125x^3 + 64} =$$

APPENDIX E
COOPERATIVE MATHEMATICS TEST
ARITHMETIC

Arithmetic | 40 minutes

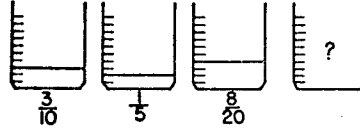
- 1 The result of a computation using only the numbers 8 and 7 is 15. In this computation, the number 15 is the
- A product
B sum
C quotient
D difference
E average
- 2 Which of the following describes how to find the average of a group of scores?
- F Find the sum of the scores and divide by 2.
G Find the sum of the scores and divide by the number of scores.
H Arrange the scores from lowest to highest and select the middle one.
J Take half the difference between the highest score and the lowest score.
K None of these
- 3
- | |
|-------------|
| 6428 |
| 974 |
| 86 |
| 7280 |
| 763 |
| <u>5407</u> |
- A 19,838
B 20,828
C 20,838
D 20,928
E 20,938
- 4 What is the inverse operation used to check division?
- F Addition
G Subtraction
H Multiplication
J Division
K None of these
- 5 What is the ratio of 1 inch to 1 yard?
- A 1 to 1
B 1 to 3
C 1 to 12
D 1 to 24
E 1 to 36
- 6 Which of the following is not evenly divisible by 8?
- F 6
G 8
H 40
J 72
K 104
- 7 Each of the numerals listed below represents a number of feet. Which numeral most nearly represents the height of an average American man?
- A .059
B 0.59
C 5.90
D 59.0
E 590
- Questions 8 and 9 refer to the line below.
- 
- 8 The point halfway between W and X would correspond to
- F 4
G $4\frac{1}{2}$
H 5
J $5\frac{1}{2}$
K 6
- 9 What number would correspond to point P if it is placed on the number line so that P is between X and Y, and W is between P and X?
- A 6
B $7\frac{1}{2}$
C 9
D 10
E No such point can exist.

Go on to the next page.

- 10 What is the greatest common divisor of 24, 40, and 120?
- F 2
G 4
H 8
J 10
K 12

- 11 Which of the following is not equal to $\frac{4}{9}$?
- A $\frac{2}{3}$
B $\frac{20}{45}$
C $\frac{8}{18}$
D $\frac{16}{36}$
E $\frac{12}{27}$

- 12 For which pair of the following operations are the rules for placing the decimal point in the answer the same?
- I. Addition
II. Subtraction
III. Multiplication
IV. Division
- F I and II
G I and III
H II and IV
J III and IV
K The rules are different for each operation.



13 Three of four identical measuring containers are filled as shown above. If all the liquid in the three containers is poured into the empty container on the right, what fractional part of this container will be filled?

- A $\frac{1}{10}$
B $\frac{12}{35}$
C $\frac{7}{10}$
D $\frac{9}{10}$
E 1
- 14 $\frac{1}{2}$ of 20 is the same as $\frac{1}{4}$ of
- F 5
G 10
H 40
J 60
K 80
- 15 What is the smallest number which can be divided evenly by each of the following numbers: 4, 6, 8?
- A 48
B 32
C 24
D 16
E 12

-4-

16 $\left(\frac{2}{3} \div \frac{1}{2}\right) \times \frac{1}{2} = (?)$

F $\frac{1}{6}$

G $\frac{3}{8}$

H $\frac{2}{3}$

J $\frac{3}{2}$

K $\frac{8}{3}$

- 17 A bank clerk reported that the number of \$100 bills in the vault was 10,003. About how much money is this?

- A \$1,000
 B \$10,000
 C \$100,000
 D \$1,000,000
 E \$10,000,000

- 18 $\frac{3}{40}$ is the same as

- F .0075
 G .0133
 H .075
 J .1333
 K .75

19

$$\begin{array}{r} 9\frac{4}{5} \\ +13\frac{1}{4} \\ \hline \end{array}$$

- A $22\frac{5}{9}$
 B $22\frac{9}{20}$
 C 23
 D $23\frac{1}{20}$
 E $23\frac{1}{5}$

20

$$\begin{array}{r} 36 \\ 52 \overline{)1872} \end{array}$$

To make the answer in the example above four times as large as it is, you could change the number 1872 to

- F 208
 G 468
 H 936
 J 3944
 K 7488

- 21 Which of the following will produce an even whole number no matter what whole number is put in place of Δ ?

- I. $2 \times \Delta + 1$
 II. $2 \times \Delta + 2$
 III. $2 \times \Delta + 3$

- A I only
 B II only
 C III only
 D I and II only
 E I and III only

- 22 Which of the following shows the correct meaning of 407?

- F $(4 \times \text{ten}) + (7 \times \text{one})$
 G $(4 \times \text{ten} \times \text{ten}) + (0 \times \text{ten}) + (7 \times \text{one})$
 H $(4 + 0 + 7) \times (\text{one hundred})$
 J $(4 \times \text{one}) + (0 \times \text{ten}) + (7 \times \text{ten} \times \text{ten})$
 K $(4 \times \text{one}) + (7 \times \text{ten})$

- 23 If the scale length of $4\frac{1}{2}$ inches represents an actual distance of 72 miles, how many miles does the scale length of 7 inches represent?

- A 2
 B 56
 C $74\frac{1}{2}$
 D 112
 E 504

Go on to the next page.

24

4 5 6 . 7 2 3 8
 ↑↑ ↑↑↑
 FG HJK

In the above numeral, which arrow points to the hundreds' place?

- F F
- G G
- H H
- J J
- K K

25 Which of the following is between $\frac{5}{6}$ and $\frac{7}{8}$?

- A $\frac{2}{3}$
- B $\frac{3}{4}$
- C $\frac{4}{5}$
- D $\frac{6}{7}$
- E $\frac{8}{9}$

26

$$340.292 \div 48.2 = (?)$$

- F 706
- G 76
- H 70.6
- J 7.6
- K 7.06

27 Jim started mowing the grass at 1:45 p.m. and finished at 2:15 p.m. How many minutes did Jim take to mow the grass?

- A 30
- B 70
- C 90
- D 180
- E 240

28 To reduce a fraction to lowest terms, what should be done to both numerator and denominator?

- F Each should be divided by 2.
- G Each should be multiplied by 2.
- H Each should be multiplied by the least common multiple.
- J Each should be divided by the greatest common divisor.
- K The same number should be subtracted from each.

29

$$3 + \sqrt{64} = (?)$$

- A 11
- B 19
- C 24
- D 35
- E $\sqrt{73}$

30 Between 8 a.m. and 3 p.m. the temperature rose 25° . If the temperature at 8 a.m. was 10° below zero, what was the temperature at 3 p.m.?

- F 25° above zero
- G 15° above zero
- H 5° above zero
- J 5° below zero
- K 35° below zero

31 A boy saves 18 dollars in 8 weeks. If he continues to save at the same rate, how many weeks will it take him to save 81 dollars?

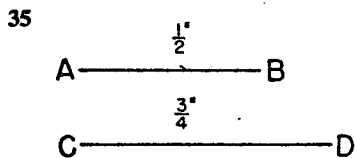
- A 13
- B 36
- C 40
- D 71
- E $182\frac{1}{4}$

-6-

- 32 When one whole number is divided by another whole number, which of the following is always true?
- F The divisor is smaller than the quotient.
 G The remainder is smaller than the divisor.
 H The quotient is smaller than the divisor.
 J The remainder is smaller than the quotient.
 K The dividend is smaller than the remainder.

- 33 Which of the following will never change the value of a number?
- I. Multiplying it by 1
 II. Dividing it by 1
 III. Multiplying it by its reciprocal
- A I only
 B II only
 C III only
 D I and II only
 E I and III only

- 34 Which of the following equals $7 \times (3 + 9)$?
- F $(7 \times 3) + (7 \times 9)$
 G $(7 \times 9) + (3 \times 9)$
 H $(7 \times 3) + (3 \times 9)$
 J 7×27
 K $21 + 9$



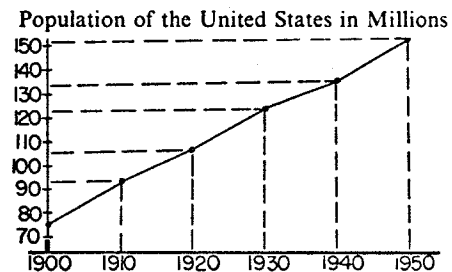
In the figure above,

$$\frac{\text{length of AB}}{\text{length of CD}} = (?)$$

- A $\frac{1}{2}$
 B $\frac{1}{3}$
 C $\frac{2}{3}$
 D $\frac{3}{2}$
 E $\frac{5}{3}$

- 36 Which of the following is **not** in descending order?
- F 4.04, 4.004, .404
 G 2.1, 1.2, 1.12
 H .06, .009, .10
 J 13.2, 12.3, 12.03
 K 736, 631, 367

Questions 37 and 38 refer to the graph below.



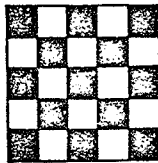
- 37 According to the graph above, the population of the United States in 1935 was about
- A 127,000
 B 1,270,000
 C 12,700,000
 D 127,000,000
 E 1,270,000,000
- 38 What was the average increase per year between 1900 and 1950?
- F 1,500
 G 15,000
 H 150,000
 J 750,000
 K 1,500,000

Go on to the next page.

39 What is the ratio of 2 gallons to 3 quarts?

- A 8 to 3
- B 3 to 8
- C 3 to 2
- D 2 to 3
- E 1 to 6

40



What per cent of the figure above is darkened?

- F 12
- G 25
- H 48
- J 50
- K 52

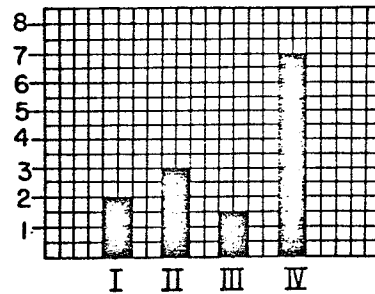
41 A cutting edge .004 inch thick is four times as thick as a second cutting edge. How many inches thick is the second cutting edge?

- A .001
- B .0032
- C .004
- D .016
- E .04

42 20% is equal to the fraction $\frac{?}{30}$

- F $\frac{2}{3}$
- G 6
- H 60
- J 150
- K 600

43



In the figure above, the two bars whose lengths have the ratio 2 to 1 are

- A II and III
- B IV and I
- C IV and III
- D I and III
- E IV and II

44 The advertisement for a sale reads: "All books reduced more than 20%." If two books each have the same sale price, which of the following must be true?

- F The original prices of both books were the same.
- G The original prices of both books were different.
- H The per cent reduction for both books was the same.
- J The sale price of each book is less than 80% of the original price.
- K The sale price of each book is more than 80% of the original price.

- 8 -

- 45 Which of the following multiplications will result in an odd number?
- I.
$$\begin{array}{r} 3049 \\ \times 6431 \\ \hline \end{array}$$
- II.
$$\begin{array}{r} 7002 \\ \times 3485 \\ \hline \end{array}$$
- III.
$$\begin{array}{r} 6543 \\ \times 3456 \\ \hline \end{array}$$
- IV.
$$\begin{array}{r} 8765 \\ \times 3497 \\ \hline \end{array}$$
- A I and III only
 B I and IV only
 C II and IV only
 D II, III, and IV only
 E I, II, III, and IV
- 46 A movie opened in a theatre on April 6 and was shown every day through April 27. On how many days was it shown?
- F 20
 G 21
 H 22
 J 23
 K None of these
- 47 A student has an average of 80 for three tests. What must he score on the next test in order to obtain an average of 84?
- A 80
 B 84
 C 88
 D 92
 E 96
- 48 Of 28 students in a class, 25 contributed to the Junior Red Cross and 16 to the March of Dimes. Every member of the class contributed to at least one of the two organizations. The number who contributed to both is
- F 3
 G 12
 H 13
 J 16
 K 25
- 49 On an arithmetic test, Bill got 32 as an answer to one problem. In working this problem, Bill's only mistake was multiplying by 4 in the last step when he should have divided by 4. What is the correct answer to the problem?
- A 2
 B 4
 C 8
 D 28
 E It cannot be determined from the information given.
- 50 Each of two whole numbers is greater than 1. If their product is an odd number, then their sum is
- F an odd number less than their product
 G an even number less than their product
 H an odd number greater than their product
 J an even number greater than their product
 K a number equal to their product

Look over your work on this test.

APPENDIX F

COOPERATIVE MATHEMATICS TEST:
THE STRUCTURE OF THE NUMBER SYSTEM

Structure of the Number System | 40 minutes

Note: In this test, if a number is expressed in any base except the usual base ten, it will have its base written after it and slightly below it.

For example: 235_{seven} means that 235 is the expression in base seven for the number being used in the question.

- 1 If a 3 by 4 by k rectangular box has the same volume as a 5 by 4 by 3 rectangular box, then $k = (?)$
- A 3
B 4
C 5
D 6
E 10

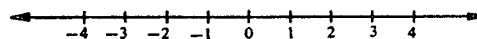
- 2 Which of the following are true?
- I. $15736 + 7243 = 7243 + 15736$
 II. $15736 \times 7243 = 7243 \times 15736$
 III. $15736 - 7243 = 7243 - 15736$
 IV. $15736 \div 7243 = 7243 \div 15736$
- F I and II only
 G I and III only
 H II and IV only
 J III and IV only
 K I, II, III, and IV

- 3 The greatest number which divides evenly into 6 and 10 and 15 is
- A 1
B 2
C 3
D 4
E 5

- 4 Which of the following multiples of 13 is also a multiple of 7?
- F 52
G 65
H 78
J 91
K 104

- 5 Which, if any, of the following numbers is greater than all the others?
- A 8×1
 B 2^3
 C 4^2
 D $\sqrt{64}$
 E None of these is greater than all the others.

6



If we represent two numbers, r and s , on the number line above, r is greater than s if

- F r lies to the right of zero
 G s lies to the left of zero
 H r is farther from zero
 J s lies to the right of r
 K r lies to the right of s
- 7 If the two middle digits of 6248 were interchanged, the number would be
- A made smaller by 200
 B made smaller by 180
 C unchanged
 D made larger by 180
 E made larger by 200

- 8 If x , y , and z are positive integers, which of the following is (are) true?
- I. $(x + y) + z = x + (y + z)$
 II. $(x \times y) \times z = x \times (y \times z)$
 III. $(x - y) - z = x - (y - z)$
- F I only
 G II only
 H III only
 J I and II only
 K I, II, and III

Go on to the next page.

4

- 9 How many of the integers from 1 through 20 which belong to the set of multiples of 2 also belong to the set of multiples of 4?
 A 2
 B 3
 C 4
 D 5
 E 10
- 10 If a and b are the same or different prime numbers, then which of the following may be concluded?
 F $a \times b$ cannot be a prime number
 G $a - b$ cannot be a perfect square
 H $a \div b$ cannot be a whole number
 J $a \times b$ is an odd number
 K $a + b$ is an odd number
- 11 How many factors does 24 have, besides 1 and 24?
 A 4
 B 6
 C 7
 D 8
 E 10
- 12 The problem $8 - (-5)$ can be interpreted on the number line as the distance from
 F 8 to 5
 G 5 to 8
 H -5 to -8
 J -8 to -5
 K -5 to 8
- 13 The inscription on the cornerstone of a building has the date MCMXXVIII. If the building is rebuilt 53 years later, what should the new inscription be?
 A MLMXXV
 B MCMLXXXI
 C MCMXXXVI
 D MDCCCLXXV
 E MLMXXXI
- 14 Express 26 as the sum of powers of 2.
 F 2^{13}
 G $2^7 + 2^6$
 H $2^4 + 2^3 + 2^1$
 J $2^4 + 2^3 + 2^2$
 K $2^4 + 2^3 + 2^2 + 2^1$
- 15 Which of the following numbers in base three is even?
 A 10_{three}
 B 101_{three}
 C 111_{three}
 D 1011_{three}
 E 1110_{three}
- 16
- | | | | |
|-------|---------|-------|---------|
| Row 1 | | Row 1 | |
| Row 2 | | Row 2 | |
| Row 3 | | Row 3 | |
| | | Row 4 | |
- In the figure above, on the left there are three rows of four dots each and on the right there are four rows of three dots each. The fact that the number of dots in each set is the same is an illustration of which principle?
 F Commutative principle of addition
 G Commutative principle of multiplication
 H Distributive principle of multiplication with respect to addition
 J Associative principle of addition
 K Associative principle of multiplication
- 17 Which of the following is true of the sum of any three consecutive integers?
 A It is divisible by 3
 B It is divisible by 5
 C It is odd
 D It is even
 E It is divisible by either 2 or 5

Go on to the next page.

- 18 Consider the sentence: " $\frac{1}{2}(x + y) = \frac{x}{2} + \frac{y}{2}$."
- By which of the following properties is this sentence true?
- F Distributive
G Associative
H Commutative
J Closure
K It is not always true.

- 19 In which base is the fraction $\frac{1}{3}$ written exactly as .3?
- A Six
B Eight
C Nine
D Ten
E Twelve

- 20 What is the difference, expressed in base ten, between 7_{twelve} and 14_{twelve} ?
- F 6
G 7
H 8
J 9
K 10

21

$$\begin{array}{r} 9_7 \\ 8_6 \\ 5_8 \\ 7_5 \\ 6_8 \\ 4_9 \\ \hline 4423 \end{array}$$

Above is an addition problem of six 3-digit numbers with the center digit omitted. What is the center digit, if it is the same for all six numbers?

- A 2
B 3
C 6
D 7
E 8

22

$$\begin{array}{r} 5006 \\ 7009 \\ \hline 45054 \end{array}$$



A student has just finished multiplying by 9 in the multiplication problem above. Which of the lettered arrows points to the position in which he should write the next nonzero numeral?

- F A
G B
H C
J D
K E

- 23 If $(987 \times 234) + (987 \times 573)$ were divided by 987, the result would be
- A 0
B a number between 234 and 573
C the sum of 234 and 573
D the sum of 234 and the product of 987 and 573
E the product of 234 and 573
- 24 Which of the following sets of whole numbers is (are) closed under addition?
- I. All those greater than 7
II. All those divisible by 5
III. All those less than 100
- F I only
G II only
H I and II only
J I, II, and III
K None

25

$$\frac{1}{2 \times 3} \quad \frac{1}{2 \times 5} \quad \frac{1}{3 \times 7}$$

What is the lowest common denominator of the fractions above?

- A 2×3
B $2 \times 3 \times 5$
C $2 \times 2 \times 3 \times 7$
D $2 \times 3 \times 5 \times 7$
E $2 \times 2 \times 3 \times 3 \times 5 \times 7$

Go on to the next page.

6

- 26 If $3 \cdot 5 \cdot 7 \cdot 17 \cdot 29 \cdot 31 \cdot j = 3 \cdot 5 \cdot 17 \cdot 29 \cdot 31 \cdot 77$, then $j =$ (?)
- F 7
G 11
H 15
J 21
K 33
- 27 A number is a multiplicative inverse of another number if the product of the two numbers is 1. Which of the following sets of numbers is the same as the set of its multiplicative inverses?
- A $\{1, 2, 3\}$
B $\{1, \frac{1}{2}\}$
C $\{1, 2, \frac{1}{2}\}$
D $\{2, 3, 5, \frac{1}{2}, \frac{1}{3}\}$
E $\{2, 3, \frac{2}{3}\}$
- 28 Which of the following does not represent a whole number?
- F $\frac{8-4}{2-2}$
G $\frac{8-4}{4-2}$
H $\frac{8+4}{4-2}$
J $\frac{8+4}{4+2}$
K $\frac{8+4}{2+2}$
- 29 In base five, what is the unit's digit of the product $3_{\text{five}} \times 3_{\text{five}}$?
- A 0
B 1
C 2
D 3
E 4
- 30 If x is an integer and $\frac{x+3}{2}$ is also an integer, then x could be
- F any negative integer
G any positive integer
H any even integer
J any odd integer
K any multiple of 3
- 31 In the product $3 \times 4 \times 5 \times 6 \times 7$, a decrease of which factor by 1 would result in the greatest change in the product?
- A 3
B 4
C 5
D 6
E 7
- 32 Consider the set of all positive even integers and the set of all positive odd integers. Which of the following is true?
- F Each set is closed under both multiplication and addition.
G Each set is closed under multiplication and neither is closed under addition.
H Each set is closed under multiplication, but only one set is closed under addition.
J Only one set is closed under multiplication, but each set is closed under addition.
K One of the sets is closed under multiplication only and the other is closed under addition only.
- 33 When the sum of thirty numbers is doubled, the result is 344,806. If one of the thirty numbers is changed from 17,232 to 15,232, what is the result when the new sum is doubled?
- A 304,806
B 340,806
C 342,806
D 344,406
E 345,806

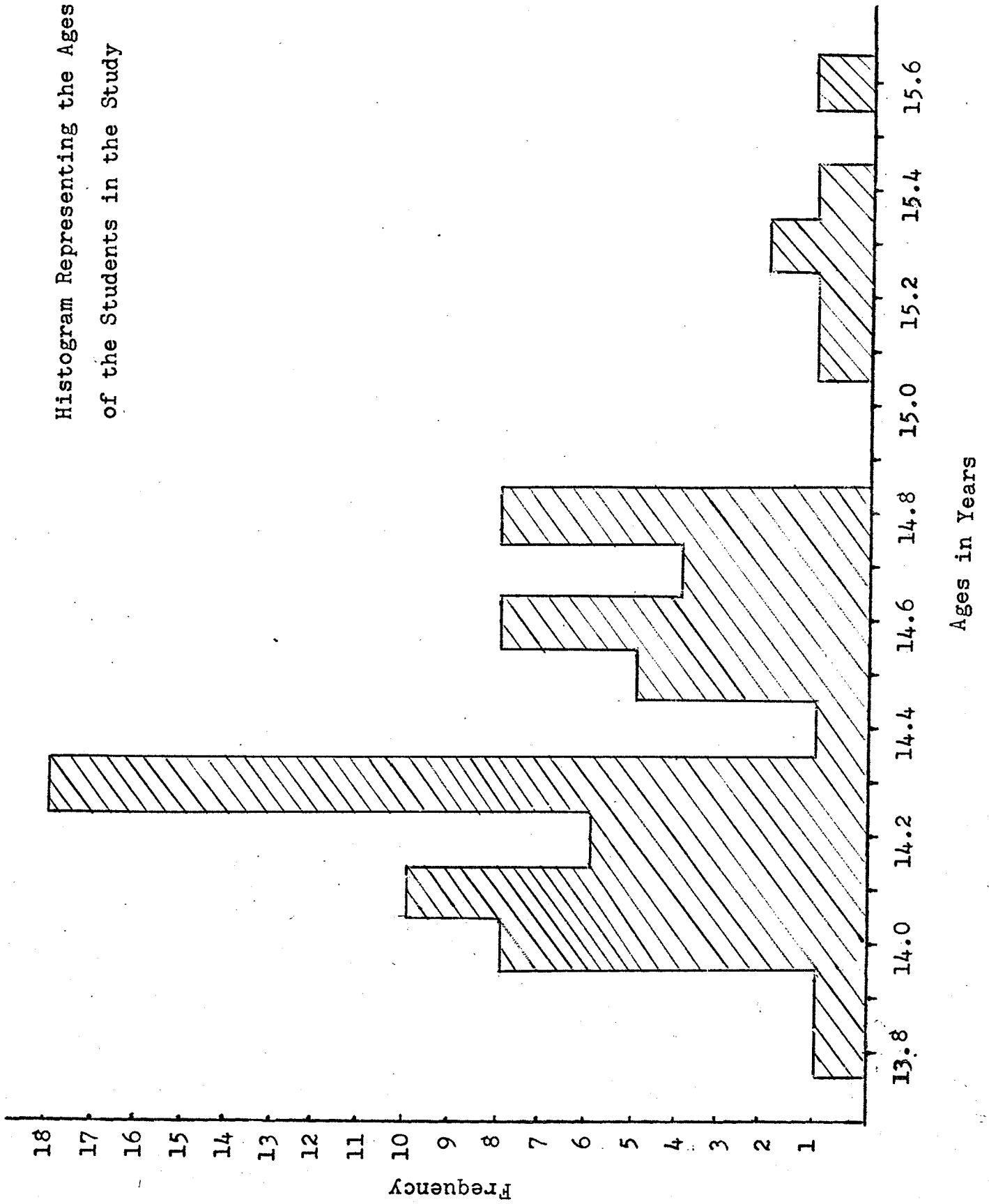
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- 34 If p and q are negative numbers, which of the following is true?
F $p - q$ is negative
G $p + q$ is negative
H $p \times q$ is negative
J $p \div q$ is negative
K None of these
- 35 If 5 is added to both the numerator and denominator of a positive fraction which is less than 1, its value is
A unchanged
B increased by 1
C increased by 5
D decreased
E nearer 1
- 36 Foghorn A sounds every 12 seconds and foghorn B every 14 seconds. They sound together at noon. How many seconds later will they next sound together?
F 24
G 26
H 72
J 84
K 168
- 37 If the remainder is 2 when the integer n is divided by 6, which of the following is a multiple of 6?
A n
B $2n$
C $3n$
D $4n$
E $5n$
- 38 What number can be divided by a third of itself to give 7 as an answer?
F 21
G $\frac{7}{3}$
H 0
J $\frac{3}{7}$
K No number
- 39 In which of the following bases is the computation $11 \times 101 = 1111$ correct?
I. Two
II. Five
III. Ten
A I only
B III only
C I and III only
D II and III only
E I, II, and III
- 40 If m and q represent integers, it is possible that $\frac{q^2}{m}$ is an integer when $\frac{q}{m}$ is not an integer. Which one of the following pairs of numbers illustrates this?
F $m = 2$ and $q = 6$
G $m = 3$ and $q = 5$
H $m = 4$ and $q = 9$
J $m = 7$ and $q = 21$
K $m = 9$ and $q = 12$

Look over your work on this test.

APPENDIX G
HISTOGRAM OF AGES OF
STUDENTS IN THE STUDY

Histogram Representing the Ages
of the Students in the Study



APPENDIX H

COMPARISON OF MEANS AND ADJUSTED MEANS OF THE
POLYNOMIAL TESTS FOR COMBINATION EFFECT
OF AGE AND TREATMENT

Comparison of Means and Adjusted Means of the
Polynomial Tests for Combination Effect
of Age and Treatment

Tests		No Homework		Homework with Answers		Homework with Answers and Discussion	
		A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4
Addn. & Subt.	M	24.85	17.75	27.32	21.72	24.29	21.79
	AM	25.28	21.47	23.6	22.36	23.96	23.25
Mult.	M	12.32	16.00	26.71	20.11	21.64	21.33
	AM	21.75	19.38	22.95	21.16	21.58	22.58
Div.	M	21.24	15.35	22.12	16.60	16.18	17.46
	AM	21.91	18.36	18.84	17.75	15.93	18.07
Total of Tests	M	66.27	49.10	76.09	58.44	62.11	60.58
	AM	67.86	59.59	65.08	61.23	61.36	63.97
Co- vari- ates	I.Q.	110.35	99.60	118.82	112.44	112.36	103.75
	A-AT	30.29	25.00	37.59	31.44	33.86	30.25
	SNS-AT	17.29	12.90	22.24	15.78	16.29	14.50

M - mean
AM - adjusted mean
A - age

APPENDIX I

COMPARISON OF MEANS AND ADJUSTED MEANS OF THE
POLYNOMIAL TESTS FOR COMBINATION EFFECT
OF SEX AND TREATMENT

Comparison of Means and Adjusted Means of the
Polynomial Tests for Combination Effect
of Sex and Treatment

Tests		N-HW		Hw-A		Hw-AD	
		Boys	Girls	Boys	Girls	Boys	Girls
Addn. & Subt.	M	28.13	19.74	26.35	24.42	22.05	23.93
	AM	25.98	23.03	23.79	22.51	22.31	24.58
Mult.	M	22.44	18.05	25.35	23.50	22.50	20.77
	AM	20.37	20.97	23.10	21.78	23.04	21.22
Div.	M	22.19	17.74	20.35	20.08	16.64	16.87
	AM	19.90	21.00	17.95	18.93	16.73	16.96
Total of Tests	M	72.81	54.47	71.96	68.00	61.18	61.57
	AM	66.18	64.28	64.50	63.07	61.93	61.87
Co- vari- ates	I.Q.	112.00	104.00	116.08	117.15	112.55	105.33
	A-AT	35.38	25.37	36.85	34.08	33.91	30.93
	SNS-AT	19.38	14.11	19.54	20.46	15.00	15.80

M - mean
AM - adjusted mean

APPENDIX J

COMPARISON OF MEANS AND ADJUSTED MEANS OF THE
POLYNOMIAL TESTS FOR COMBINATION EFFECT
OF SEX AND AGE

Comparison of Means and Adjusted Means of the
Polynomial Tests for Combination Effect
of Sex and Age

Tests		Boys		Girls	
		A < 14.4	A ≥ 14.4	A < 14.4	A ≥ 14.4
Addn. & Subt.	M	26.75	22.92	24.71	18.92
	AM	24.34	23.10	24.29	21.98
Mult.	M	24.93	21.50	22.18	17.84
	AM	22.68	21.72	22.77	20.68
Div.	M	20.05	18.67	20.09	15.18
	AM	17.81	18.49	19.98	17.82
Total of Tests	M	71.70	63.08	66.27	51.95
	AM	64.55	63.24	65.32	60.77
Co- vari- ates	I.Q.	116.50	109.42	112.11	102.11
	A-AT	36.90	33.08	31.79	26.26
	SNS-AT	19.35	15.58	18.32	13.58

M - mean
AM - adjusted mean
A - age