

THE UNIVERSITY OF MANITOBA

A BRIEF INVESTIGATION INTO THE ANALYSIS OF STATIC
AND DYNAMIC STRESSES IN RAILWAY TRACK
WITH APPLICATION TO THE CANADIAN RAIL TRANSPORT
PROBLEMS

by

Mohamed Ali Abdel Razik

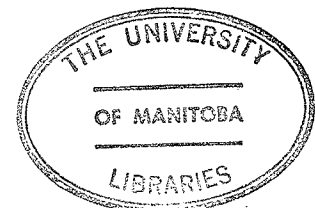
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A dissertation submitted to the Faculty of Graduate Studies of
the University of Manitoba in partial fulfillment of the requirements
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To
Hekmat and Hedda

ABSTRACT

At the present time, several problems confront the Canadian Railways development, which demand the full understanding of the stresses in the railway track. The analysis of these stresses is of a particular difficult nature and simple practical methods are always needed. The lack of publications in Canada about some of these procedures and their use in solving some Canadian planning and design difficulties, brought about the necessity for a self-contained guide on that concern.

Some of the problems confronting the Canadian railways today and in the future, and the use of the study of the track stresses in the proper decision-making for a solution to these problems, is the subject of Chapter 9, the last chapter. It is to be pointed out, however, that of necessity, this study has been limited in its scope, and only some obvious examples have been studied as fully as possible. Chapter 8 illustrates the application of the track stress analysis to a specific problem related to the grain movement economy in Canada.

The other parts of this thesis deal with the theories and the procedures for estimating the railway track stresses. The research in this field has been far more advanced and dynamic in Europe than in North America. In the last four decades, for example, series of publications have appeared in Germany which deal with these fundamentals and which conclude with useful formulas. Some of these have never been introduced in North America, although they are now in major use in the European Railway administrations. After a presentation of various theories and formulas that have been known since the beginning of the railways in the world, and briefly dealing with the theoretical and experimental analysis, in Chapters 2 to 6, Chapter 7 discusses the technical evaluation of the different procedures, and comparisons are made between the different methods. A useful table is established in the appendix for the convenience of the planner or designer, to give calculated values according to the recommended formulas. Chapter 2 deals briefly with a simplified elastic model of the track system as a basis for Chapter 3, which discusses the practical procedures of estimating the static bending moment exerted on railroad tracks. Chapter 3 also includes a historical review of the development of these techniques.

Chapter 4 complements Chapter 3, as it considers the dynamic effects resulting from train motion and vibration, which must be added to the bending moment equations presented in Chapter 3. Following the theoretical and imperial models in Chapter 3 and Chapter 4, Chapter 5 presents the analysis of stresses in each of the railway track elements. Chapter 5 also includes some relationships between different properties of the rail section, which may be useful to the railway engineer.

In Chapter 6 a brief discussion of the use of experimental techniques is presented with broad emphasis upon experiments that

lead to successful experimental stress analysis.

This study, in general, can be a useful guide for both the practicing engineer and the student or researcher in the subject of stresses in railway tracks and its application to the Canadian Railway problems.

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CHAPTER 1: INTRODUCTION

Railways belong to the early technologies which founded the massive industrialization of western Europe and played a major role in the development of North America. From a status of near-monopoly until the beginning of this century, railways have been relegated to only one among several competing modes of transportation. This drastic change which came about as a result of technological innovation, has been recognized in Western Europe and Japan over the past 20 years, but - as we shall see - has yet to be faced up to in Canada.

- J. Lukasiewicz in The Institutionalization of Canadian Rail's Obsolescence.

1. General Considerations

The near future of the Canadian Railways is most likely to be full of activity. The expanding economy of Canada has called upon those in control of railroads and other forms of transport to exert every effort to meet the demand for carrying freight and passengers.

Railways are the most important means of surface transportation in agricultural-industrial countries such as Canada. Despite the growth of other modes of transportation, all indications point to railroads as continuing to be the backbone of the transportation industry in the future of this country. The dynamically developing railroad industry, in keeping pace with the current "state of the art", serves the growing economy and population of Canada. For this reason, the decisions of design

and practice must be based on full economic costs, and modified only where safety governs.

The steel wheel on steel rail, which is the principle of guided surface transport, has four elements in it; the track, rolling stock, traction, and signalling. In all of these four elements of basic railway engineering, there are indications that a basis for modernization exists in Canada. The practices and procedures now in use are not the ultimate, for every phase of railroad engineering is subject to further study and improvement. However, the questions involved in the development in the first three of the elements mentioned above, are still basically:

(a) what rail speeds can be attained and what will they demand in track elements, rolling stock, and traction ?

(b) what rail tonnages can be carried and what effect will they have on track elements, rolling stock, and traction ?

The answer to these two questions is deeply related to the understanding of the stresses exerted on the railway track by the weight of the rolling stock, and the effect of the speed on such a stress. Thus, the study of these stresses forms the major part of the work reported here.

This study may furthermore be of particular interest at the present time, as the discussion of the national railway policy has been gaining momentum in Canada. Daily reports speak of:

- The railway electrification of the Canadian main lines,
- The constant tendency in railway practice to increase the loading and the speed,
- The overloading or overstressing in some railway lines,

- The speed limitations or restrictions in certain railway lines,
- The introduction of passenger fast service trains,
- The abandonment of the light branch lines,
- The relocation of some railway lines,
- CP Rail's granting 1/2 million dollars for track structure research,
- The placement of the first reinforced concrete ties in Canada.
- The introduction of the LRC (Light, Rapid, Comfortable) train in Canada, using light alloys in the structure of locomotives and passenger cars.
- The introduction of the Grain Hopper Aluminum Cars.

Thus, a major part of this thesis is devoted to bringing together the information available in the world on track stresses and related problems, and show how this information may be applied to Canadian problems.

2. Track Elements Considerations

The entire subject matter is emmanently suitable for a comprehensive treatment, which is the aim of the present work. The track structure of today is the result of years of experience and of trial and error. The railroad track as a complete structural unit has not evolved through any process of straight forward design, unlike other branches of structural engineering, such as, bridges and other steel structures, for instance. The undetermined character of the track components largely account for this situation. Very little is known of the strength and

forces in a roadbed. What is known, indicates some lack of uniformity in its strength, and other characteristics. The same holds true for the ballast. The ballast consists of randomly arranged stones with only limited grading, without any matrix or filler to keep each stone in constant relation to its neighbors. The ballast provide support to the tie by a very rudimentary form of mechanical interlocking of the stones themselves, which brings point-loading, friction, and indeed the shape of the stones into play. This interlocking slowly breaks down under traffic. Furthermore, the endless variations in the nature of the subgrade and the ballast materials, the methods used in their construction, the different topographical situations, the differences of rainfall, snowfall and drought from place to place and from year to year, leads to uncertainties which are not surprising.

Concerning the tie pressures distributed to the ballast and subgrade, it is known that ties possess the variability of all wood and are subject to further changes with weathering and use. The non-uniformity of ballast sections furthermore makes tie mechanics indeterminate.

The rail itself is difficult to analyze, especially in view of the non-uniformity of support and variable loading applied to it. Another great difficulty of railroad structure, is that one must deal with moving loads, especially with two load-kinds; the repeating load and the shock-type load. The effect of these loads can only be understood with great difficulty. Although the wheel loads of stationary vehicles are known quite accurately, they can only be estimated approximately for vehicles in motion.

Thus, uncertainty already prevails with regard to forces exerted upon and stresses induced in the rails. The determination of the stresses caused in the rails by the various forces, however, depends on values which fluctuate within wide limits and in certain cases are known only by their order of magnitude. This applies especially to the elastic deflection of the tie caused under the vertical loads transmitted through the rails.

Bearing in mind all the difficulties reported above about the railroad track stress analysis, one can understand not only the difference between the railroad track and any other engineering structure, but also the great uncertainty prevailing in the railway track structure when the action of its several elements work as a complete structural unit.

A great deal of research has been done on the difficulties mentioned above, and while many theories have been put forth, the majority of them have not been recognized as wholly applicable. In addition, the computation methods should be as simple as possible for practical application. The lack of information in the Canadian libraries about some of the computation of the railway track stresses was a motive for the present work. The primary concern of almost all papers published so far on the subject in the United States or Europe has been restricted to the consideration of the technical details. In the present work, however, a wider horizon is attempted and the practical use of this information is illustrated.

This thesis is therefore designed, not to fill a gap in the experience of the line-engineer, but for all those concerned with

practical work in the field of concern in Canada, specifically the planners and designers who are responsible for the improvement of our railway network. Moreover, in dealing with these factors, the thesis will point to the direction in which subsequent research may continue. Most of the problems of track stress analysis are dealt with from a practical viewpoint in the light of the most up-to-date knowledge available, in addition to the development of this knowledge. Furthermore, some problems confronting Canadian railways today, and in the future, and the use of the study of the track stress in the proper decision-making for a solution of these problems, is the secondary concern of this work.

This study concerns itself primarily with standard gauge railways, rails with expansion joints over wooden or concrete ties. The loads considered are only the vertical loads on the track elements. The study deals only with the track superstructure (rails, ties and ballast) in addition to the vertical pressure distribution on the subgrade underneath. The form of the track discussed is the intercity rail transport track. The effect of the traffic volume on the stresses is left for other papers and research.

In the course of this work, major help was derived from the numerous publications on the subject in the United States and overseas, including several monographs in German. In attempting to form a comprehensive unit of all this material, it has been found convenient to state the units of the physical terms used in the study in the two international systems of units according to

precise conversion factors. However, for the convenience of the reader, a conversion table is provided at the back of the Appendix, especially for the compound terms, such as bending moment, stress ...etc. It is important that the reader notice that the unit of weight "TON", is meant in Europe as 1000 Kilograms, while in North America it is 2000 Pounds.

In the Mathematical Notation of this text, two minor departures were made from the existing practice, in that Greek letters were avoided and also that subscripts were not used. The need for this arose from two facts;

- (1) the entire manuscript was filed and printed by the Computer and these characters are not available in its editing facilities.
- (2) the variability of the notations used by different engineering backgrounds.

The use of the English letters, whether individually or compound, however, was found to be a simple and effective way to achieve the purpose.

CHAPTER 2: PRELIMINARIES

THE ANALYSIS OF BEAMS ON ELASTIC FOUNDATION⁽¹⁾

The analysis presented in this chapter is restricted to the analysis of beams of unlimited length (the infinite beam), and loading is restricted to that of concentrated loading. The above restrictions are imposed because they are the closest theoretical assumptions to the reality of the case of rolling stock loading on a railway track. In addition, a brief analysis is given at the end of the chapter for beams of finite length with free ends, which are loaded by two symmetrical forces, as is the case which exists in a tie under the action of rail pressure.

Another assumption which is important for the mathematical analysis, is that the elastic foundation is continuous⁽²⁾, so that when the beam is deflected, the intensity of the continuously distributed reaction at any section is proportional to the deflection at that section. Mathematically,

$$p=k.y$$

(1)The material covered in this chapter is fairly standard "Strength of Materials" text book information. It was found convenient to report it here, because of the extensive dependence of the following chapters on this theory.

(2)The consistency of such an assumption with reality is discussed in Chapter 7.

b is the constant width of the beam (in or cm)

The well-known differential equation of deflection to be applied is [3]:

$$EI \frac{d^4y}{dx^4} = q \quad \dots\dots(2-1)$$

where,

E is the modulus of elasticity of the beam

I is the moment of inertia of the beam,

and

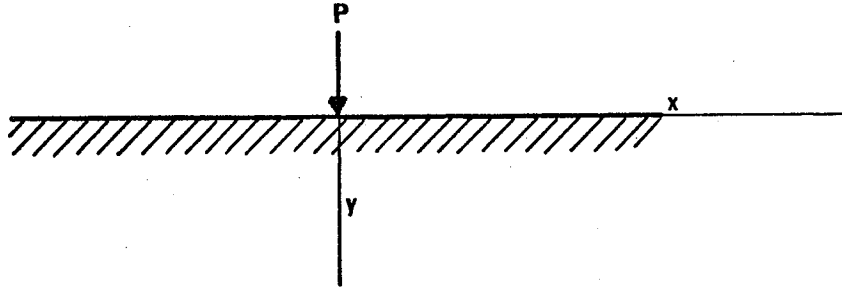
dx is the distance between two vertical cross sections on the beam under consideration.

For an unloaded portion of the beam, the only acting force will be a continuously distributed reaction from the foundation of intensity ky. then $q = -ky$ where q is the intensity of the load acting on the beam.

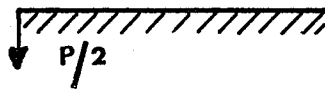
Hence the equation (2-1) becomes;

$$EI \frac{d^4y}{dx^4} = -ky \quad \dots\dots(2-2)$$

and represents the deflection curve of a beam supported on an elastic foundation. The general solution of Equation (2-2), can be derived using the boundary conditions which were introduced at the beginning of this chapter as restrictions, which are shown in Fig. (2-1) below.



A single concentrated load on an infinitely long bar, taking the origin of the coordinates at the point of application of the load



Only part of the bar to the right of the load need to be considered, due to symmetry.

Fig. (2-1)
A single concentrated load on an infinitely long bar

In this general solution, the deflection can be, finally, represented by [1],[2],[3]:

where,

$$y = (P \cdot v / 2k) \cdot (e^{-vx}) \cdot (\cos vx + \sin vx) \dots (2-3)$$

$$v = \sqrt{k / 4EI}$$

e is the exponential.

Taking the successive derivatives of y, with respect to x in Equation (2-3), we obtain the expressions for the slope (s), the bending moment (M), and the shearing force (Q) on the right side of the beam as;

$$dy/dx = s = -(P \cdot v^2 / k) \cdot (e^{-vx}) \cdot (\sin vx) \dots (2-4)$$