Studies of the Non-Coplanar $^{2}H(p,2p)n$ Reaction and Faddeev Type s-wave Model Theoretical Predictions.

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"STUDIES OF THE NON-COPLANAR $^{2}H(p,2p)n$ REACTION AND FADDEEV TYPE S-WAVE MODEL THEORETICAL PREDICTIONS"

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ABSTRACT

Calculations and measurements of proton-deuteron breakup cross sections have been made at 23.0 MeV and 39.5 MeV along constant-equal-relative energy loci such as those described by Jain, Rogers and Saylor. Exact three body calculations using s-wave local and separable nucleon-nucleon potentials yield cross section predictions which are quite sensitive to the model parameters in regions of destructive interference minima.

Two procedures were used to investigate the interference effects. In the first procedure the variation of the breakup cross section is studied for fixed values of the final state NN relative energies and a fixed value of the momentum of one of the emerging protons. By choosing particular equal relative energies between the two protons and the neutron one ensures that the model cross section is dominated by the $M(S_{ppn} = \frac{1}{2}, S_{pp} = 0)$ amplitude along most of the locus. In the second procedure the variation of the breakup cross section is studied for equal relative energies between all three pairs of nucleons with the two protons emerging with equal momenta symmetrically with respect to the incident beam direction. In this situation only the crucial $M(S_{ppn} = \frac{1}{2}, S_{pp} = 0)$ amplitude contributes.

At the higher energy two different loci were measured, one locus for each procedure while at the lower energy one locus for the second procedure was measured.

At 39.5 MeV the data along the constant-relative energy loci do not agree with the s-wave model calculations at the interference minimum nor for large values of the energy of the unobserved neutron. The trend of the data provides closer agreement for the predictions of the MTI-III potential, the most realistic of the three s-wave NN potentials in that it correctly predicts the $^3S_1$ and $^1S_0$ NN phases up to 300 MeV. For the lower incident energy the data agree with the prediction of the three s-wave models for large values of the neutron energy but in the region of the interference minimum the data are considerably larger than the MTI-III potential calculations. Since the s-wave potential model calculations at 23.0 MeV should give an adequate representation then the discrepancies at the minimum are assigned to the d-state component of the deuteron wave function. At the higher incident energy one is tempted to assign the further discrepancies as resulting not only from the tensor force but also the P-wave and D-wave NN interactions.

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CHAPTER 1

INTRODUCTION

1.1 The Nuclear Potential

The philosophy of science is the construction of models to explain certain observed effects and then test their ability to predict so far unobserved phenomena. The hope is that in the final analyses, these models will be a true picture of nature. Now the basic constituents of matter on a nuclear scale are the nucleons. If one knows their interaction and properties, one can employ non-relativistic quantum mechanics which permits the description of matter in terms of a wave-function ψ. The wavefunction ψ is calculable from the equation

$$(K + V)\Psi = E\Psi \tag{1}$$

Here the symbols K and V are the operators representing the kinetic energy and the potential due to the interaction respectively. However, the interaction is not known a priori and most efforts in nuclear physics, at present, are directed toward determining it. In other words the problems of nuclear physics have a perspective which is the opposite of the one encountered in atomic physics. Thus in nuclear physics one takes a phenomenological approach to the nucleon-nucleon interaction by satisfying first of all certain invariance requirements, spatial rotations and reflections, time reversal, and rotations in isotopic spin space. The interaction should be symmetric under the interchange of the two particles and depend only on their relative spatial separation. In the case of a particular model the parameters of the model are adjusted to match theoretical predictions to the experimental data. This procedure involves a certain arbitrariness and the problems involved will be discussed below.

Once a certain potential model is selected, one searches for the solutions of equation (1). The solutions for negative energies (E) are mapped onto the existing data for the ground states and excited states of stable nuclei, and for positive energies onto the scattering data. For this purpose and simplicity, one first attempts to study the two-nucleon problem.

Here the negative energy solution or bound state solution of equation (1) for a system of two nucleons must match the following experimental information:

1) A bound state exists only for a system consisting of a proton and a neutron, namely the deuteron. (There is no excited state.)
2) The deuteron is bound by 2.2246 MeV; its angular momentum and parity are $J^{\pi} = 1^{+}$.
3) The deuteron has a quadrupole moment (0.00282 barns).
4) The deuteron has a magnetic moment (0.8574 nm) slightly different than the sum of the magnetic moments (0.8796 nm) of the neutron and the proton.

Further, the positive energy solutions of equation (1) are mapped onto the existing elastic pp and np scattering data. These data indicate the existance of virtual states for the pp, np and nn systems in a singlet s-state (the deuteron ground state is an admixture of $^3S_1$ and $^3D_1$-states with the triplet s-state predominant).

The boundary conditions for the scattering process require that the wavefunction $\Psi^{(+)}$ which satisfies equation (1), taking into account spatial coordinates only, will have an asymptotic form:

$$\Psi^{(+)}(\bar{k},\bar{r}) \underset{r\to\infty}{=} e^{ikz} + f(k,\Theta)\frac{e^{ikr}}{r} \tag{2}$$

which is a description of the process in terms of an incident plane wave, e^{ikz}, and a scattered outgoing spherical wave whose amplitude is $f(k,\Theta)$, known as the scattering amplitude. The quantity Θ is the angle between the direction of the scattered wave and the incident wave. The desired interaction potential V is related to the scattering amplitude by the matrix element:

$$f(k,\Theta) = \frac{-2m}{4\pi\hbar^2} <\phi_f|V|\Psi_i^{(+)}> \tag{3}$$

Here the wavefunction $\Psi_i^{(+)}$ describes scattering from an initial state ϕ_i with momentum $\bar{k}_i$ such that $K\phi_i = E_i\phi_i$. The state ϕ_f is a plane wave characterized by a momentum vector $\bar{k}_f$. The quantity m is the nucleon mass.

As mentioned earlier, there is an arbitrariness in determining V from experimental data. Firstly, it is due to the fact that the quantity $|f(k,\Theta)|^2$ instead of $f(k,\Theta)$ is obtained from two-body scattering experiments. Secondly and the most important, the determination of V requires not only a knowledge of the scattering amplitude $f(k,\Theta)$ with $|\bar{k}_i| = |\bar{k}_f|$ but also with $|\bar{k}_i| \neq |\bar{k}_f|$. That is, the scattering occurs between states for which the magnitude of the initial and final momenta differ. To appreciate the latter point, it is convenient to define a quantity, the T matrix, which is proportional to the scattering amplitude

$$<\bar{k}_f'|T|\bar{k}_i> \equiv T(\bar{k}_f,\bar{k}_i) = -\frac{4\pi\hbar^2}{2m} f(\Theta,k_f',k_i)$$

where the scattering amplitude now depends on the initial and final momenta of the scattering states. The transition matrix T can be related directly to the interaction potential by the integral equation (Lippmann-Schwinger)

$$T(\bar{k}',\bar{k}) = V(\bar{k}',\bar{k}) + \int \frac{d^3k''}{(2\pi)^3} \frac{V(\bar{k}',\bar{k}'')\ T(\bar{k}'',\bar{k})}{E_k - E_{k''} + i\varepsilon} \tag{4}$$

The potential matrix $V(\bar{k}',\bar{k})$ is given by

$$V(\bar{k}',\bar{k}) = \int d^3r \, e^{i\bar{k}'\cdot\bar{r}} \, V(r) e^{i\bar{k}\cdot\bar{r}}$$

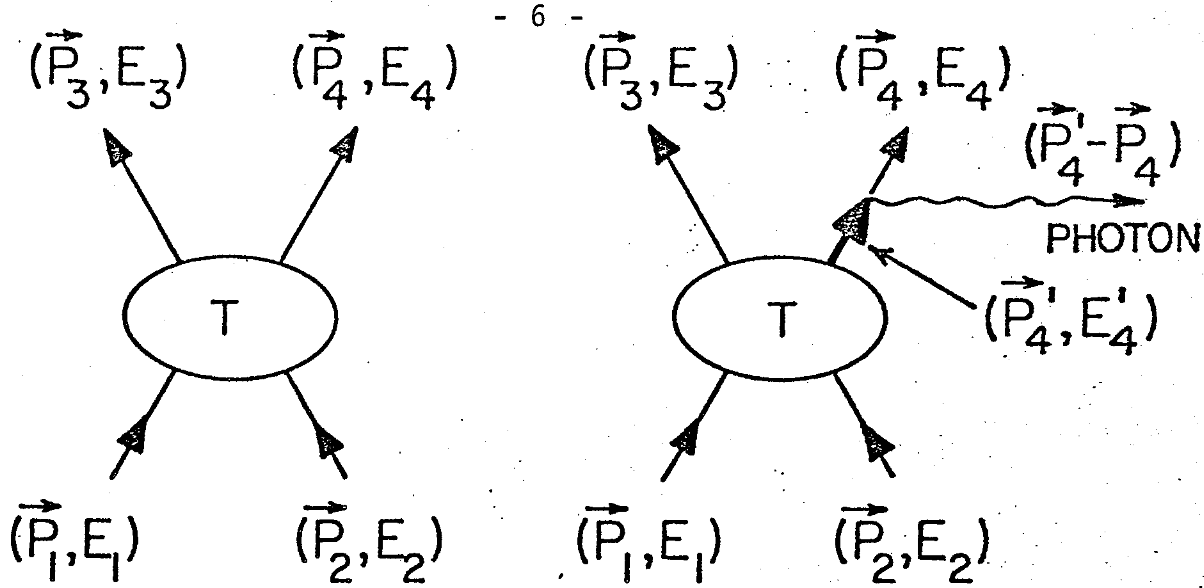
and is the Born approximation to the T matrix. In equation (4) a knowledge of the wavefunction $\Psi_i^{(+)}$ is no longer required. However, one now requires a knowledge of $T(\bar{k}',\bar{k})$ (or $f(\Theta,k',k)$) for all energies as the range of integration runs from zero to infinity. In other words, equation (4) reveals one of the central difficulties in nuclear physics. Reconstruction of the interaction potential matrix $V(\bar{k}',\bar{k})$ from $T(\bar{k}'\bar{k})$ will require a knowledge of the interaction not only "on-energy-shell", $k'=k$, but also when energy is not conserved $k' \neq k$ i.e. "off-energy-shell". But an analysis of elastic scattering data yields a value for $T(\bar{k}',\bar{k})$ only on the energy-shell because energy is conserved in the process.

Therefore numerous forms of the interaction potential exist which can reproduce the two nucleon elastic scattering data, (σ, P, D, A, R, A' R' and spin-spin correlations). A comprehensive review of the two-nucleon problem up to 1971 has been given by Moravcsik (Mo72). The properties of the force are found to be short-range, spin-dependent and non-local. As early as 1935 Yukawa (Yu35) showed that a strong short-range force could be interpreted as due to the exchange of finite mass quanta between the nucleons. These finite mass quanta are now known as the pi-mesons and provide the attactive tail of the nuclear force. In the intermediate range (0.3 - 1.0 fm) the nuclear force is mediated by two pion exchange including the ρ and ω mesons. At distances smaller than 0.3 fermis the nucleon-nucleon (NN) interaction has a repulsive core which has still to be explained satisfactorily in terms of heavy meson exchanges. Still the problem exists concerning the extent by which charge independence is broken, the form of the NN interaction off-energy-shell (scattering between states with momenta of different magnitude) and the importance of three-body and many-body forces.

In summary the information about the on-energy-shell behaviour of the NN interaction comes from studying the p-n bound state and NN scattering data. However, information relating to the off-energy-shell behaviour of the NN interaction can only be obtained from a study of many body systems. Therefore the system composed of three nucleons should be studied, the next simplest many body system.

1.2 Three Energy Regions

An interaction between two nucleons means the transfer of energy and momentum from one nucleon to another as shown in figure 1.2(1). When this occurs between initial and final states containing only two free nucleons, the total momentum and total energy E of the two nucleon system are conserved. If the centre of mass energy is kept below the pion-production threshold at ∿270 MeV, only two processes can occur, the elastic scattering of the two nucleons and bremsstrahlung production. In the former case one says an "on-energy-shell" process has occurred which as seen from equation (4) was insufficient to allow a full determination of the NN interaction potential. In the case of pp or np bremsstrahlung, the creation of a photon could occur in either the initial or final state. As an example the final state is considered in figure 1.2(lb) where nucleon (4') is off-energy-shell. Consequently only a "half-off-energy-shell" T matrix could be studied in this process. The calculation of nucleon-nucleon bremsstrahlung cross sections requires a knowledge of both the NN interaction and the electromagnetic interaction. The bremsstrahlung experiments are difficult to perform and the cross sections are small, of the order of $\mu b/sr^2$, compared to reactions involving three nucleons.



(a) $E_i = E_1 + E_2 \quad \bar{k}_i = \bar{P}_1 + \bar{P}_2$

$E_f = E_3 + E_4 \quad \bar{k}_f = \bar{P}_3 + \bar{P}_4$

(b) $E_i = E_1 + E_2 \quad \bar{k}_i = \bar{P}_1 + \bar{P}_2$

$E_f = E_3 + E_4' \quad \bar{k}_f = \bar{P}_3 + \bar{P}_4'$

NON-RELATIVISTIC T MATRIX ELEMENT

$$<\phi_f(\bar{k}_f) \,|\, T(E) \,|\, \phi_i(\bar{k}_i)> \equiv <\phi_f(\bar{k}_f) \,|\, V \,|\, \psi_i^{(+)}(E)>$$

<u>ENERGY REGIONS</u>

1) ON-SHELL $\quad k_i = k_f$ and $\dfrac{k_i^2}{2m} = \dfrac{k_f^2}{2m} = E$

2) HALF-OFF-SHELL $\quad k_i \neq k_f$ but $\dfrac{k_i^2}{2m} = E$ (or $\dfrac{k_f^2}{2m} = E$)

3) FULL-OFF-SHELL $\quad k_i \neq k_f$ and $E \neq \dfrac{k_i^2}{2m}$, $E \neq \dfrac{k_f^2}{2m}$

Figure 1.2(1) Definition of the three energy regions for the non-relativistic T matrix

Alternatively in the three-body system, internal collisions between any two of the nucleons can occur off-energy-shell, the difference in momentum and energy being absorbed by the other nucleon. While within the interaction region a nucleon (i) may be off-energy-shell i.e.

$$E_i^2 \neq \frac{p_i^2}{2m} .$$

After leaving the interaction region all three nucleons together will satisfy the energy and momentum conservation laws.

Thus knowledge about nuclear forces off-energy-shell is desirable because (a) the T matrix elements to describe nucleon-nucleon collisions inside the nucleus may change in the presence of other nucleons and (b) it can possibly provide information about the interaction potential at small distances.

Current research on the three nucleon problems is carried out in the hope of (a) removing the ambiguity caused by having families of potentials that fit the low-energy scattering data (e.g. by requiring the correct value for the ^3He, ^3H bound states) and (b) a general reduction in the uncertainty of the off-energy-shell behaviour of the NN interaction.

1.3 Three Particle Scattering Theory

Considerable complexity is involved in solving the three-body problem which contains multichannel processes, particle rearrangements and breakup reactions. A direct extension of the Lippmann-Schwinger integral equation, successful for the two-body scattering problem, to the three-nucleon system is found to be inadequate. This occurs because the formal solution of Schrödinger's equation (1) in the integral form

$$\psi_n = \phi_n - \frac{1}{K - E_n} V \psi_n \tag{4}$$

is no longer uniquely determined by the free term ϕ_n representing different channels (or boundary conditions). In other words the wave-function ϕ_n of the three nucleon system should represent one of the following four possible asymptotic states:

a) one state to describe the motion of three free nucleons:

$$\phi_{n_o} = \exp(i\bar{k}_1 . \bar{r}_1 + i\bar{k}_2 . \bar{r}_2 + i\bar{k}_3 . \bar{r}_3) \qquad (5)$$

and

b) three states to describe the motion of two bound nucleons and one free nucleon. For example consider nucleon 1 to be free:

$$\phi_{n_{23}} = \exp(i\bar{k}_1 . \bar{r}_1 + i\bar{K}_{23} \cdot \bar{R}_{23})\ \phi_{23}^{(\ell)}(\bar{r}_{23}) \qquad (6)$$

$$R_{23} = \frac{m_2\bar{r}_2 + m_3\bar{r}_3}{m_2 + m_3} \qquad \bar{r}_{23} = \bar{r}_2 - \bar{r}_3$$

The first and second terms in the exponential describe the free motion of nucleon 1 and of the centre of mass of the bound system of nucleons 2 and 3. The bound state wave function $\phi_{23}^{(\ell)}(\bar{r}_{23})$ may be obtained by solving Schrödinger's equation in which the potential V has been set equal to V_{23}, the interaction potential between nucleons 2 and 3.

The scattering state wavefunction ψ_n of equation (4) may then be obtained by applying the three-body resolvent operator G to the states ϕ_n

$$\psi_n = \underset{\varepsilon \to 0^+}{-i\varepsilon G(E_n + i\varepsilon)\phi_n} \qquad (7)$$

where $G(z) = (K + V - Z)^{-1}$, $V = V_{12} + V_{23} + V_{31}$. Expanding G(Z) in terms of the free Green's function $G_0(Z) = (K-Z)^{-1}$ and the interaction potential V:

$$G(Z) = G_0(Z) - G_0(Z)VG(Z)$$

$$G_0(Z) = \frac{1}{K - E - i\varepsilon} = (K-Z)^{-1}$$

allows equation (7) to be rewritten as (compare with the form of equation (2))

$$\psi_n = -i\varepsilon G_0(E_n+i\varepsilon)\phi_n - G_0(E_n+i0)V\psi_n \qquad (9)$$

Note no problem arises in the case where the wavefunction ϕ_n refers to three free nucleons (equation 5). Here the first term in equation (9) becomes

$$\underset{\varepsilon\to 0^+}{-i\varepsilon G_0}(E_{n_o} + i\varepsilon)\phi_{n_o} = \phi_{n_o} . \qquad (10)$$

However, for the wavefunctions (equation 6) involving bound states where for illustration nucleons 2 and 3 are considered to be bound, one obtains for the first term in equation (9)

$$\underset{\varepsilon\to 0^+}{-i\varepsilon G_0}(E_{n_{23}} + i\varepsilon)\phi_{n_{23}} = 0. \qquad (11)$$

Therefore equation (4) is no longer inhomogeneous and does not represent a scattering solution. Or equivalently a singularity does not occur when $Z = E_{n_{23}}$ and hence a unique solution to equation (4) can not be found representing the scattering of one free and two bound particles. However, one could resolve this difficulty by using the bound state Green's function

$$G_\alpha = (K + V_\alpha - Z)^{-1} \quad , \quad \alpha = 12, 23, \text{ or } 31$$

applied to the state function ϕ_{n_α} . But in the rearrangement collisions an analogous complication again appears. For example the resolvent G_{23} yields a homogeneous equation for $\psi_{n_{13}}$ or $\psi_{n_{23}}$.

$$\psi_{n_{13}} = G_{23}(E_{n_{23}} + i\varepsilon)V_{23}\psi_{n_{23}} \qquad (12)$$

That is, the solution no longer represents the collision process. To summarize, the simple generalization of the two-body formalism to the three-body problem fails (as shown by equations (11) and (12)) to provide unique solutions. Further requirements (Gl70) have to be imposed in order to select a unique solution. This defeats the fundamental purpose of an integral equation, namely the combination of a differential equation and its asymptotic boundary conditions into a single equation. Therefore in practice the above straight forward extension of the Lippmann-Schwinger equations in unsuitable for finding solutions.

A breakthrough came in 1960 with the publication of Faddeev's work (Fa60,61,62). The status of earlier work on rearrangement collisions was given by Ekstein (Ek56) and Gerjuoy (Ge58). Later work by others such as Lovelace (Lo64) and Sandhas (Sa72) have been an augmentation rather than a change of the original equations of Faddeev. The Faddeev equations are restricted to a sequence of two body interactions such that the consecutive interactions are between different pairs in the three body ensemble.

The Faddeev technique consisted of writing the three-body transition matrix T^{3B} as a sum of three terms

$$T^{3B} = \sum_{\alpha} T^{\alpha} = T^1 + T^2 + T^3$$

where T^1 is the sum of all diagrams contributing to T^{3B} in which nucleons 2 and 3 interacted last (see diagram 1.3(1e)). The Lippmann-Schwinger equation for the operator T^1 is of the form:

$$T^1 = V_{23} - V_{23} G_O T^{3B}$$

which as shown by Faddeev (Fa60) can be rewritten

$$T^1 = t_1^{\,3B} + t_1^{\,3B} \; G_O (T^2 + T^3)$$

The quantity $t_1^{\,3B}$ is a two particle T matrix in a three particle Hilbert space which finally appears as a product of the full two-body T matrix $t_1^{\,2B}$, valid on and off-energy-shell, times a momentum conserving delta-function for the third free particle. The various forms of the T matrices and the above equations are depicted in figure 1.3(1).

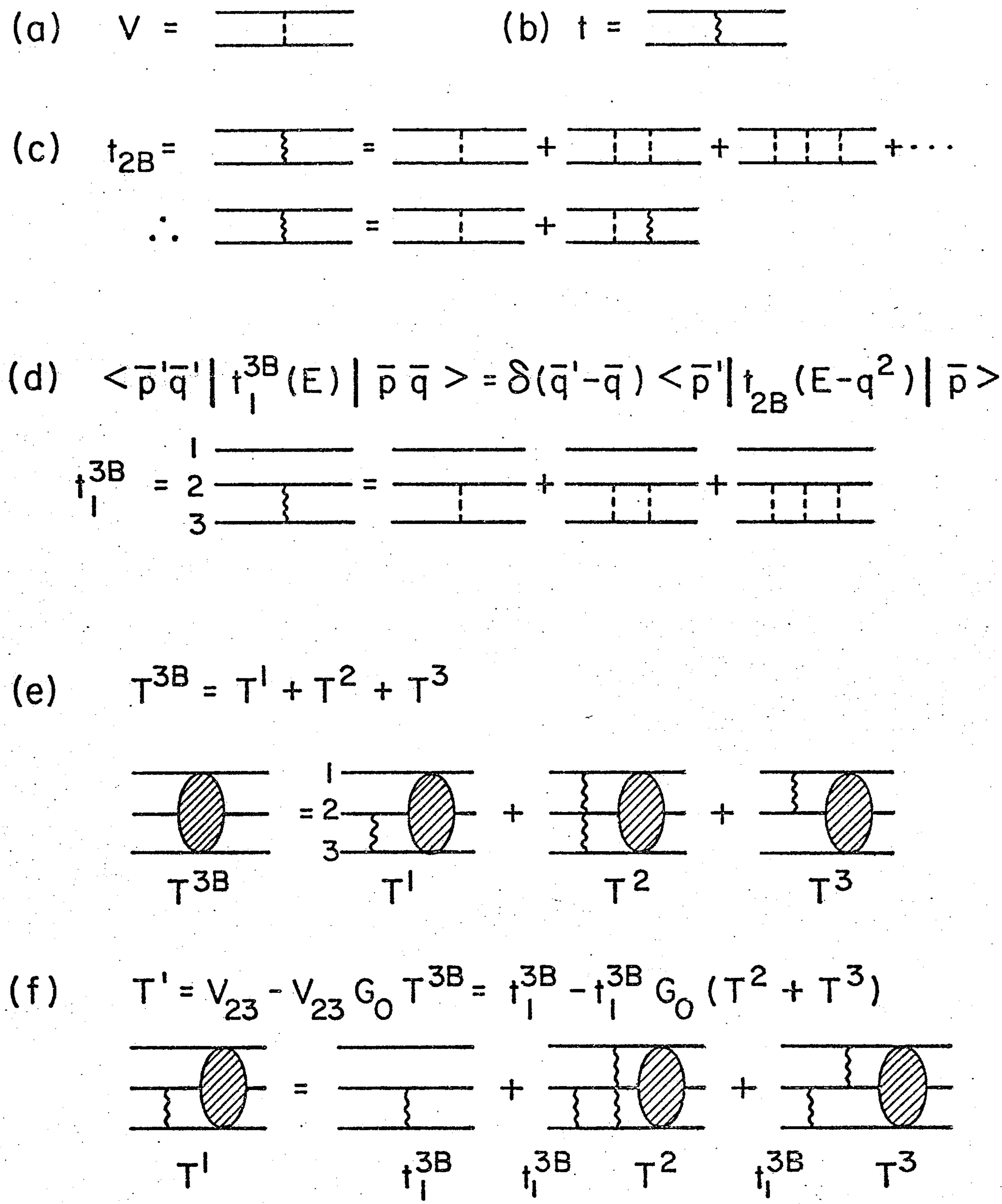


Figure 1.3(1) Diagrammatic representation of the T matrices. The quantity $\bar{p}$ is the relative momentum of the two interacting nucleons and $\bar{q}$ is the momentum of the third nucleon with respect to this two-nucleon subsystem.

Thus the formalism of Faddeev represents a mathematically sound theory for non-relativistic three particle systems. The equations contain the two-nucleon interaction via the t^{3B}. The three-body problem allows the possibility of testing various forms of t^{3B}, including assumptions about its off-energy-shell behaviour. An understanding of this behaviour is necessary because the dynamics of the off-energy-shell NN interaction is important in many body systems more complex than the deuteron. Further if the properties of the A = 3 nuclei, ^3H and ^3He could not be derived using two-nucleon potentials, one may need to postulate the existance of three-body potentials and investigate some of their properties.

1.4 Motivation for Present Experiment

In this work the three-body reaction selected was the breakup of deuterons induced by 23.0 MeV and 39.5 MeV protons. Up to a few years ago, studies of the breakup of deuterons by nucleons were mainly concerned with the enhancements in energy spectra due to the typical pseudo-two-body processes, the final state interactions (FSI) and quasifree scattering (QFS), see section 2.3. In these enhancement regions there was little difference in the quality of theoretical fits to the experimental data. These theoretical results were obtained using T matrices derived mainly from separable Yamaguchi potentials and a small number of calculations using local potentials. Obviously it would be more interesting if one could locate regions in phase space where large relative differences, say 40% at least, would exist between the various theoretical predictions. Two such regions were found and the results of an experimental investigation of each region will be given in chapter four.

The first region, suggested by Jain, Rogers and Saylor (Ja73), is a region of phase space dominated by the $M^{\frac{1}{2}0}$ scattering amplitude*. This is a spatial configuration in which the three nucleons have the highest probability of being close together. The second region of phase space was selected because breakup calculations by Kloet and Tjon (Kl73) for this region were particularly sensitive to the presence or absence of a repulsive core in their local potentials.

In the investigation of these regions, three forms for the NN interaction potential were used, the YY separable potential model of Jain and Doolen (Ja73a) and two local potential sets due to Malfliet and Tjon (Ma69). These potentials were fitted to the deuteron binding energy and the same zero energy scattering parameters. The Malfliet and Tjon potentials gave triton binding energies close to the experimental value. However the two local potential sets differ in that one contains repulsion in the triplet S-state while the other local potential set does not. The present experiment thus tests which of the three S-wave interaction potentials, if any, provide an adequate prediction of the experimental breakup cross sections. Conclusions concerning the off-energy-shell behaviour of the NN interaction cannot be inferred as these three potentials are not phase equivalent, i.e. their on-energy-shell behaviour differs.

In the next chapter a discussion is given for three-nucleon kinematics and a description of the various possible breakup processes.

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* $M^{\frac{1}{2}0}$ is the amplitude describing the scattering state in which the three nucleons have total spin ½ and the two identical nucleons have relative spin 0.

The local Malfliet-Tjon potentials are also presented, followed by a summary of the theoretical equations employed by Kloet and Tjon to calculate the three body breakup cross sections. References to the literature are given where full details of the calculations are available. The latter part of the second chapter contains a discussion on the inability of previous theoretical calculations to produce good fits to three-nucleon experimental data. This prompted the theoretical search for a phase space region sensitive to differences between the three s-wave models mentioned above.

Details of the experimental equipment, electronics and the data collection procedures and software are given in chapter three. An analysis of the data, their reliability and the conclusions drawn from this research are presented in the final chapter.

Chapter II

Theory

2.1 Kinematics of Systems with Three Nucleons in the Final State

The notation adopted is shown in figure 2.1(1) where the Z axis is defined to be parallel to the incident beam direction. The detected particles in this notation are 3 and 4, both protons in the present experiment, a study of the D(p,2p)n reaction. The undetected neutron is labeled 5. As the plane of scattering for the third particle was not changed, the detector to the right of the incident beam axis is referred to as the 'in-plane' counter telescope. The second counter telescope, for which the azimuthal angle ϕ_4 was in general not equal to 0 or 180 degrees, is called the 'out of plane' counter telescope.

For a particle of rest mass m the following relationship holds

$$E^2 = (pc)^2 + (mc^2)^2 \qquad (1)$$

where E^2 and p^2 are the squares of the total energy and three momenta respectively. For a reaction with three free particles in the final state $(1+2 \rightarrow 3+4+5)$, with the masses known, the kinematical variables will have nine degrees of freedom. The four relations due to conservation of energy and momentum between the initial (1+2) and the final (3+4+5) states will reduce the number of independent kinematical variables to five. Due to azimuthal independence, the angle ϕ_3 can be set equal to zero degrees. Then a measurement of the kinetic energies T_3 and T_4, the polar angles Θ_3 and Θ_4 and the azimuthal angular difference $\phi_3 - \phi_4$ will overdetermine the kinematics by one variable. Such an overdetermination serves to reduce the background and to remove a two-fold ambiguity. This ambiguity arises because the momentum of any nucleon in the final state can be