THE UNIVERSITY OF MANITOBA

A MULTIPERIOD LINEAR PROGRAMMING MODEL FOR FARM PLANNING UNDER UNCERTAINTY: A DRYLAND-IRRIGATED SITUATION

BY

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ABSTRACT

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by

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Problems of farm planning and decision making under uncertainty have been investigated in numerous studies of the farm firm, and a variety of models that take some account of uncertainty have been developed. The model used in this study is a contribution to the general class of models that attempt to handle the uncertainty dimension in decision making. The conceptual framework of the model owes much to the work of Boussard. Although the approach here was somewhat different, the basic approach was a variation on Boussard's model.

The major objectives of the study were: (1) to specify a decision model that made some allowance for uncertainty; (2) to assess the economic feasibility of irrigating, in the context of reducing uncertainty, in Southwestern Manitoba; (3) to evaluate the effects of financial reserves in decision making under uncertainty; and (4) to make an assessment of the effects of uncertainty on the efficiency of resource use.

A multiperiod linear programming model with subjective and/or objective considerations of the worst possible outcomes of different alternatives, was used to approximate the problem of farm firm planning under uncertainty. The locale for the study was the Souris River Basin in the southwestern part of Manitoba. A farm firm situation in the area

was specified and this firm constituted the basic resource situation for the applications of the multiperiod model.

The model incorporated both short-term and long-term considerations. The operator was presumed to have a long-run objective of maximizing expected additions to net worth and a short-term objective of being able to afford the unfavourable outcomes that might occur due to chance. This latter objective was incorporated in the model as a worst possible income constraint. A ten-year planning horizon was employed and the model had a total of 189 activities and 86 restraints in each period.

Production alternatives in the model included such crops as wheat, oats, barley, rapeseed and forage - both dryland and irrigated and both insured and uninsured. Yields for the irrigated crop production alternatives were developed from analysis of the long-term relationships between yield and moisture deficiency for some crops grown in the area.

To evaluate the model for purposes of making some allowance for uncertainty, solutions were prepared for what were called deterministic and stochastic cases. In the deterministic use, only the expected value criterion was employed. In the stochastic case, the worst possible income criterion was used as an additional decision criterion. In the latter case, the worst possible income constraint was varied parametrically from a lower level at which it was not an effective constraint, to an upper level beyond which an optimal plan was no longer feasible.

Taking differences in first period plans as most important, there were substantial differences between the deterministic and the stochastic optimal plans for the same resource situation. In the first case, solutions emphasized oilseed crops, without crop insurance and with heavy applications of fertilizer. In the latter case, the solutions placed more emphasis on wheat production, took advantage of crop insurance and specified a lower level of fertilizer use.

The irrigation system considered in the study was a capital-intensive centre-pivot system. The analysis indicated that, while this alternative would improve the expected gross margin of the firm, it would not be profitable to acquire it for the cropping alternatives that were considered in this study. At the same time, the opportunity to irrigate in years in which dryland yields were low because of moisture deficiency was found to be quite profitable. Irrigation would clearly reduce the uncertainty associated with dryland farming and would improve the worst possible income position in any one year. However, the average benefits from irrigating were not enough to justify the average costs of owning the irrigation system. Higher value crops and/or an irrigation system with a lower capital requirement would have been necessary to make irrigation competitive with dryland production.

Financial reserves were hypothesized as an important factor affecting the ability and willingness of farmers to afford risk and to adopt plans with more uncertain outcomes. The applications of the model with different levels of reserves--in the form of cash reserves and in the form of borrowable reserves--supported the hypothesis. The optimal plans incorporated more risky alternatives when reserves were ample than when they were limited.

In the formal sense, the model used in the study can accommodate farm-specific information concerning alternatives, assets, objectives and expectations. At the same time, it has obvious limitations. As examples, the model presented does not formally allow for diversification as a means of reducing uncertainty and the worst possible estimates were not statistically based. For practical application to actual farm situations, the suitability of the model would be improved by having fewer alternatives and a shorter planning horizon with consequent reduction in data requirements.

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CHAPTER 1

INTRODUCTION

THE PROBLEM AND THE OBJECTIVES

The basic problem that concerned this study was that of improving farm incomes and expanding economic opportunities in the Souris River valley in southwestern Manitoba. This is an area in which soil moisture is usually a limiting factor in crop production. A remedy for the moisture deficiency problem lies in the supplies of ground and surface water in the area and potentially available for farm use from ground wells and/or from a proposed set of storage reservoirs on the tributaries of the Souris.[1] This study contributes background material for the economic evaluation of both public and private participation in developing water supplies for irrigation.

The direct problem for this study was that of decision making by an individual firm and the overall objective was to investigate the effects of uncertainty on farm firm planning, decision making, and the efficiency of resource use. Specifically the study was initiated to evaluate the economic feasibility of changing from dryland farming to a combination of dryland and irrigation farming, taking account of the uncertainties of results in both cases.

Irrigation has the potential to reduce the crop yield uncertainties of dryland farming and to improve average yields. At the same time,

[1]At the time of this study, public investment had not been committed to developing the water supplies and private investments in irrigation facilities were not significant.

decisions concerning irrigating have to be evaluated in the context of the complete decision environment and in terms of the overall costs and benefits. There are, for example, other ways of reducing or accommodating dryland yield uncertainties--principally through crop insurance and financial reserves; other ways of improving expected dryland yields--principally through improved fertility and management; and other sources of uncertainty which are not substantially reduced by irrigating--principally price. This study specifically considered these factors in evaluating irrigation and explicitly recognized uncertainty with respect to yield, price and irrigation water requirements.

As shown in a later section, risk and uncertainty are economic facts of life in the decision environment of farms in the Souris Basin. They must be taken into account when decisions are made; many farm managers make allowances for them as a matter of course. Nevertheless, the problems of how best to allow for uncertain outcomes and how to incorporate lack of knowledge about different outcomes into decision criteria are difficult both in theory and in practice.[2] An instrumental objective of this study was to develop decision models for the farm firm that would approximate the real-life decisions of farmers in situations similar to those identified in this study.

This study is a successor to many other applied studies of the problems of decision making under uncertainty. Some of these studies that were concerned with farm situations are discussed in the third chapter. As for studies on the economics of irrigating in the Souris Basin, irrigation and dryland

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[2]For a sampling of theoretical and applied literature in this area, see G. J. Hahn, A Categorized Bibliography on Decision and Risk Analysis (Schenectady: General Electric Co., Report No. 69-C-189, 1969).

farming had been analyzed in studies by Finn[3] and by Singh.[4] Besides these studies on the Souris, irrigation farming in the Pembina Triangle area of Manitoba has been evaluated in earlier studies by Boyko[5] and by Iga.[6] The difference between this study and the earlier ones is that it takes account of uncertainty in the context of a multidimensional planning horizon, when comparing dryland and irrigation farming. The study by Boyko took specific account of the time horizon but not of uncertainty; that by Iga took specific account of uncertainty but not of time.[7] There remained the possibility that decisions and the economic feasibility of irrigating would be substantially affected by specific consideration of both time and uncertainty.

This study took advantage of existing economic theory in developing the decision models and in testing hypotheses. The models themselves relied in part on the contributions of McInerney, of Hazell and of Boussard.[8] They were slightly different from those of previous studies in their representation of the decision process. The main difference lay in the attempt to

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[3]G. J. Finn, "An Economic Feasibility Study of Irrigating from Groundwater in South Western Manitoba" (Unpublished Master's Thesis, University of Manitoba, 1971).

[4]R. H. Singh, "An Economic Analysis of Irrigation in the Souris River Basin in Manitoba" (Unpublished Master's Thesis, University of Manitoba, 1972).

[5]E. S. Boyko, "A Multi-Period Analysis of Capital Accumulation and Financing of Beginning Irrigation Farms in the Pembina River Basin" (Unpublished Master's Thesis, University of Manitoba, 1969).

[6]M. Iga, "Economic Evaluation of On-Farm Irrigation in the Morden-Winkler Area of Southern Manitoba" (Unpublished Ph.D. Dissertation, University of Manitoba, 1970).

[7]The studies by Finn and by Singh did not specifically consider uncertainty or the time dimension.

[8]Their contributions are discussed in Chapter 3 and are referenced in the periodicals section of the bibliography of this study.

incorporate both long-run and short-run considerations into a linear programming representation of a game theoretic decision rule.

Variability in Production and Returns in the Souris Basin[9]

The yield data for Crop District Number 1 shown in Table 1 provide some evidence of the degree of production uncertainty in the Souris Basin. The long-term coefficients of variations in yields for the four crops wheat, oats, barley and flax, range from 37 percent to 53 percent and as such indicate a fairly high degree of instability. Also when yields in District 1 were compared with those in the province as a whole, the District yields were lower and more variable.[10]

Table 1

Variability in Yields of Wheat, Oats, Barley and Flax in Crop District Number 1, 1932-1970[a]

Crop	Mean Yield[b]	Coeff. of Variation
	bushels per acre	percent
Wheat	12.07	37
Oats	18.74	53
Barley	15.09	47
Flax	6.30	44

[a]Yields in Crop District Number 1 were used here as a measure of yields in the Souris Basin. Most of the Basin in Manitoba is within District Number 1. See map on p. 5.

[b]Mean yields were computed from time series data with linear trends removed.

Source:
Calculated from data prepared for W. J. Craddock, Interregional Competition in Canadian Cereal Production, Special Study No. 12, Economic Council of Canada (Ottawa: Queen's Printer, 1970).

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[9]The Souris River, Crop District Number 1 and the specific area that concerned this study are identified on the map on page 5. For a general description of the area see Finn, p. 15 and Singh, pp. 3-4.

[10]Based on trend-free yields of wheat, oats, barley, flax and rye for the period 1952-1971.

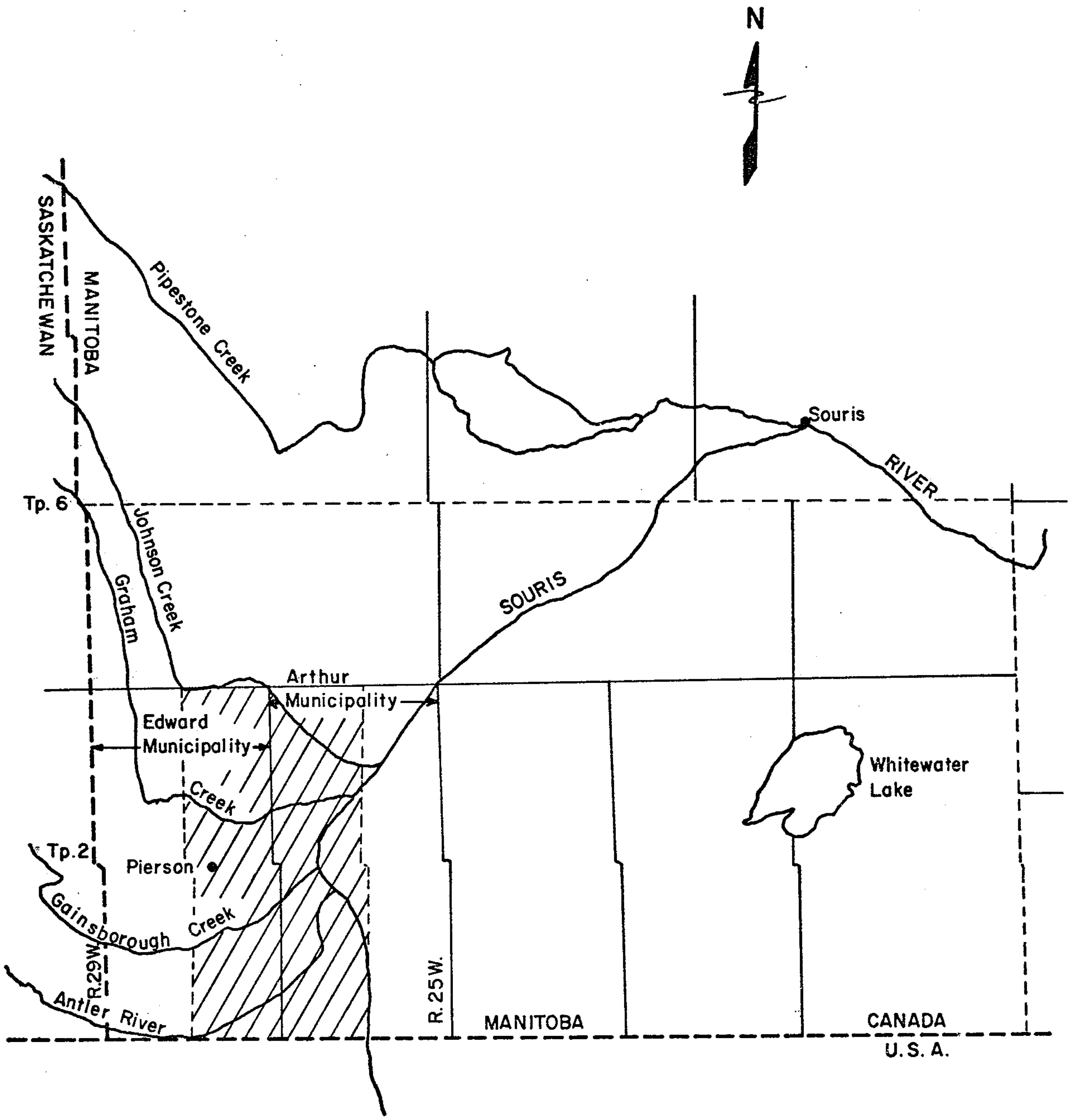

Figure I Map of Portion of Souris River and Tributaries in South-Western Manitoba, Boundaries of Crop District Number I and Area Used for Dryland Yield Estimation.

----- Boundary of Crop District Number I

//// Area Used for Dryland Yield Estimation

Variability in yields, in prices and in other facets of the farm decision environment have their ultimate effects on financial variables such as cash flow, cash income and net farm income. An indication of the degree of variation in the net income of a typical farm business in the Souris Basin was obtained from the time series of net incomes of all Manitoba farmers.[11] For the period 1960-1971, the average net farm income per year was 132 million dollars with a 26 percent coefficient of variation. For five years in which net incomes declined from the previous year, the average decrease was 30 percent; for six years in which it increased, the average increase was 150 percent. Similar evidence of variability in incomes could have been obtained from the records of individual farm businesses in the Souris Basin. The degree of variability would of course also vary from farm to farm.

Variation in net incomes to the degree just noted was enough to justify serious consideration of the different means of stabilizing them.[12] Among the means of stabilization are all-risk crop insurance, diversification, irrigation and the proposed and subsequently withdrawn Grain Stabilization Plan.[13] These different schemes are evidence that stability of income is one of the criteria that farmers use in evaluating alternatives. Whether it is an important and worthwhile criterion depends finally on the answers to two questions: (1) What would be the costs and benefits of stability of

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[11]See Appendix Table B.1.

[12]This was aside from the issue and the means of increasing net farm income.

[13]O. E.Lang, Minister Responsible for the Canadian Wheat Board, "Proposals for a Production and Grain Receipts Policy for the Western Grains Industry" (Mimeographed) 1970. (A revised plan was subsequently proposed and adopted in 1976).

earnings? (2) Who would pay the costs and who would receive the benefits? This study was concerned, in part, with estimating the benefits.

Uncertainty and Economic Efficiency

It is because of uncertainty that there is a need for the managerial role in agricultural production. Management is rewarded as a factor of production because the future is uncertain and because there are differences of opinion about what the future holds. The reward that it receives is in part a reward for making risky choices successfully.

Another side of risk, however, is its relationship to economic efficiency. If the return to entrepreneurship is somehow a private benefit of risk, misallocation of resources is both a private and a social cost of risk. As noted by Heady, there are two kinds of inefficiency which grow out of uncertainty:

> "(1) Precautions which are taken to meet uncertainty almost always necessitate a sacrifice; they either result in a less-than-maximum product from given resources or, conversely, do not allow a minimum cost for a given output. (2) Both the individual farmer and the consuming society sacrifice when production is geared to inaccurate expectations."[14]

If the precautions to meet uncertainty tend to restrict output below what it would be with perfect information, it could be expected that an increase in output would accompany a decrease in uncertainty. Whether economic, as well as technical efficiency, would be improved by such an adjustment was a question that concerned this study.

STATEMENT OF HYPOTHESES

The major objectives of the study are expressed in the following

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[14] E. O. Heady, Economics of Agricultural Production and Resource Use (Englewood Cliffs: Prentice-Hall Inc., 1952), p. 530.

hypotheses:

1. The problem of farm firm planning under uncertainty can be satisfactorily approximated in a multiperiod linear programming model that takes account of uncertainty through consideration of subjective and/or objective estimates of the worst possible outcomes of different alternatives.

2. Irrigation would be a profitable means of reducing the uncertainty of farming in the Souris River Basin in southwestern Manitoba.

3. The economic efficiency of resource use can be increased by reducing uncertainty.

4. The amount of financial reserves available to the firm is a factor affecting decisions under conditions of uncertainty.

GENERAL PROCEDURE

The hypotheses outlined in the previous section provided the general framework for the development of the study. To test these hypotheses, it was first necessary to specify a decision process for planning under uncertainty and it was then necessary to identify a farm situation that could be considered fairly representative of those in the Souris Basin. Then the planning and decision problems of this farm were placed within the framework of multiperiod linear programming. The programming model was formulated with specific consideration of the worst possible outcomes of the different alternatives that were available. Worst possible outcomes were conceived in terms of gross margins per unit and were separately identified for four main categories: (1) uninsured dryland crop production; (2) insured dryland crop production; (3) uninsured irrigated crop production; and (4) livestock production. These worst possible outcomes were regarded as contingencies to be provided for in decision making. Thus providing for them became a constraint on the decision rules used in the models. Within this overall framework, alternative dryland and irrigated opportunities were evaluated.

Definitions

Risk situations - cases in which the outcomes are uncertain but in which the decision maker has some information, subjective and/or objective, about the outcomes and their probabilities or possibilities. The distributions of outcomes may be regarded as continuous or discrete.

Uncertain situations - defined the same as risk situations above.[15]

Outcomes - the different possible results of a decision with respect to a stochastic variable; in a game theory context, the possible states of nature for a particular action.

Choice variable - a variable within the control of the firm; may be stochastic or deterministic.

Deterministic variable - a variable with only one possible outcome.

Stochastic variable - a variable with more than one possible outcome.

Expected income - forecast money returns above relevant costs; equivalent to expected net income, expected gross

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[15]These definitions of risk and uncertainty are not consistent with Knight but were in line with the contemporary viewpoint that uncertainty refers to the state of the decision maker's conception of his knowledge about future events. The case of pure uncertainty, in which the decision maker has no information whatever, either subjective or objective, about the possible outcomes and their probabilities, is practically beyond the reach of economic analysis and was not considered in this study. See F. H. Knight, Risk, Uncertainty and Profit (Boston: Houghton, Mifflin Co., 1921), Ch. VII.

margin or forecast income.

ORGANIZATION OF THESIS

There are eight chapters in this thesis. Chapter 1 has outlined the problem situation and the objectives, developed the hypotheses and briefly indicated the procedure of the investigations. The second chapter discusses theoretical aspects of utility and behaviour as well as those of the economic planning horizon.

The next chapter is a review of literature. It reviews the methodology and findings of studies with objectives and problems broadly similar to those of this study. In Chapter 4, some of the theoretical considerations that were involved in representing the decision process are discussed and the basic framework of the model is developed.

Chapters 5 and 6 are devoted to the development and specification of the decision model used in the analysis. Chapter 7 reports on the empirical applications of the model and their results.

Chapter 8 summarizes the study, and its findings and presents the conclusions and implications of the results. The appendices provide supporting data for the text and information on the development of the input-output data.

CHAPTER 2

I. UTILITY AND RATIONAL BEHAVIOUR

The framework for the economic behaviour of individuals is provided by utility theory and by the assumption of utility maximizing behaviour. The argument of utility theory is that rational individuals make their choices with the objective of maximizing utility. At one time, it was generally accepted that diminishing marginal utility was a necessary condition for rational behaviour and that utility was a cardinal quantity. While subsequent theoretical developments showed that neither of these conditions were necessary to identify utility-maximizing conditions, both concepts survived as issues.[1] The problem of explaining rational behaviour under uncertainty was one of the reasons.

The Riskless Choice

The Marshallian position was that rational choices among riskless alternatives were made with a view to maximizing utility by assigning numerical utility values to different goods and services and then selecting the appropriate bundle of goods. Choices among risky alternatives on the other hand were explained in terms of irrational behaviour. The principle of diminishing marginal utility could not accommodate, for example, a person participating in a fair game of chance and at the same time seeking to maximize utility. One play or repeated play of a fair game would not enhance

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[1] J. M. Henderson and R. E. Quandt, Microeconomic Theory (New York: McGraw-Hill Book Co., Inc., 1958), p. 8.

utility since the losses would have a higher disutility than the gains would have utility.[2]

While many of the economic choices that consumers and producers make are in fact riskless, a large class of situations for both groups has uncertain outcomes. Utility functions that purport to represent behaviour should take some account of the unsure as well as the sure prospects.

The Uncertain Choice

Modern day theories of behaviour in risky situations had their origins in Bernoulli's explanation of the so-called St. Petersburg paradox. Bernoulli observed that people faced with fair gambles were not willing to pay their mathematically expected values. This led to the hypothesis that "people were guided not by the mathematical expectations but by the moral expectation of success--the probabilities being weighted by the utility of income."[3] The notion that individuals attempt to maximize expected utility was derived from this hypothesis.

The idea of accommodating risky choices in the utility function was not seriously considered after Marshall until the work of von Neumann and Morgenstern.[4] Although the indifference curve analysis of riskless choices had established that it was not necessary to measure utility, von Neumann and Morgenstern showed that in the case of risky choices it was possible to

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[2] M. Friedman and L. J. Savage, "The Utility Analysis of Choices Involving Risk," The Journal of Political Economy, Vol. LVI, No. 4 (August, 1948), p. 280.

[3] M. Blaug, Economic Theory in Retrospect (Homewood: Richard D. Irwin Inc., 1968), p. 334.

[4] J. von Neumann and O. Morgenstern, Theory of Games and Economic Behaviour (Princeton: Princeton University Press, 1944).

obtain numerical estimates of utility. Without the assumption of diminishing marginal utility, and treating probabilities of different outcomes as long-run frequencies, they found mathematical principles to explain rational behaviour and derived from them the general characteristics of that behaviour. With a set of axioms about rational behaviour, they proved that a person who acted consistent with them would maximize expected utility.[5] Expected utility may be expressed in the simplest case as:

$$(2.1) \quad E(U) = \delta U[O^{o}(X)] + (1-\delta)U[O^{1}(X)]$$

where $E(U)$ = expected utility,

X = an action or decision,

O^{o}, O^{1} = possible outcomes of action X,

$U[O^{i}(X)]$ = utility of i-th outcome of action X, i=o , 1,

and δ, $1-\delta$ = probabilities of the outcomes of action X.

This theory of economic behaviour was developed in the context of game theory and in terms of selecting the best strategy to maximize expected utility. The objective of maximizing expected utility has become generally accepted and it was presumed for the managers of the farm situations that were analyzed in this study.

Representing the Utility Function Under Uncertainty

What general shape of function represents the relation between money income and utility for individuals such as farmers who both take chances and insure and who also make decisions in a risky environment? One representation of its shape is by Friedman and Savage[6] and is shown in Figure 2. The shape of this function is explained in terms of two different socio-economic classes. The two concave segments represent the two different

[5] Ibid., p. 31.

[6] Friedman and Savage, op. cit., p. 297.

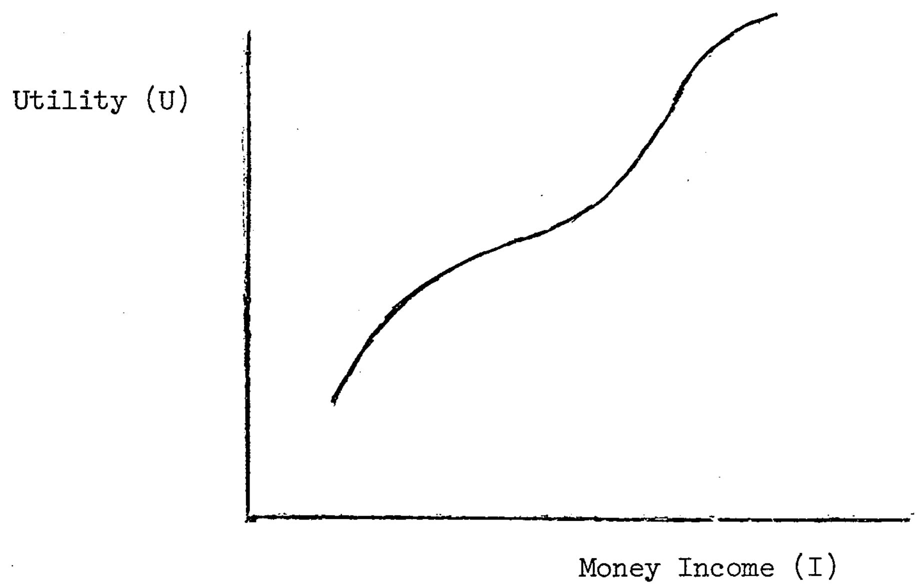

Figure 2

Illustration of Friedman-Savage Hypothesis
of Typical Shape of Utility Function

classes and the convex section represents the transition from the one income class to the other. The hypothesis here is that within an economic class, the marginal utility of income diminishes; but for choices that involve a chance of moving from one income class to the other, the marginal utility of income is increasing. This representation is consistent with the notion that a person may gamble at fair odds to change his economic status substantially, but will reject a fair gamble that would only marginally affect his relative status within his current economic class. This argument could be extended to encompass more than two income classes.

A similar but simpler representation of the relation between utility and income is shown in Figure 3. With current income I_o, this curve is consistent with insuring to protect it; the concave section to the left of I_o accords with the behaviour of people who will insure against large losses in income, or against losses that would jeopardize income-producing ability. The convex section of the curve, to the right of I_o, accords with the behaviour of people who will take long chances for big gains; over the convex section, the marginal utility of income is increasing. The utility curve changes back to the concave shape at higher income levels. This implies that the individual places an upper limit on the size of the gain for which he would take chances and that utility is bounded. As drawn in Figure 3, the utility curve is nearly linear around current income. This relation fits the behaviour of taking chances for small gains, but only at fair odds, and of refusing to insure against small losses.[7]

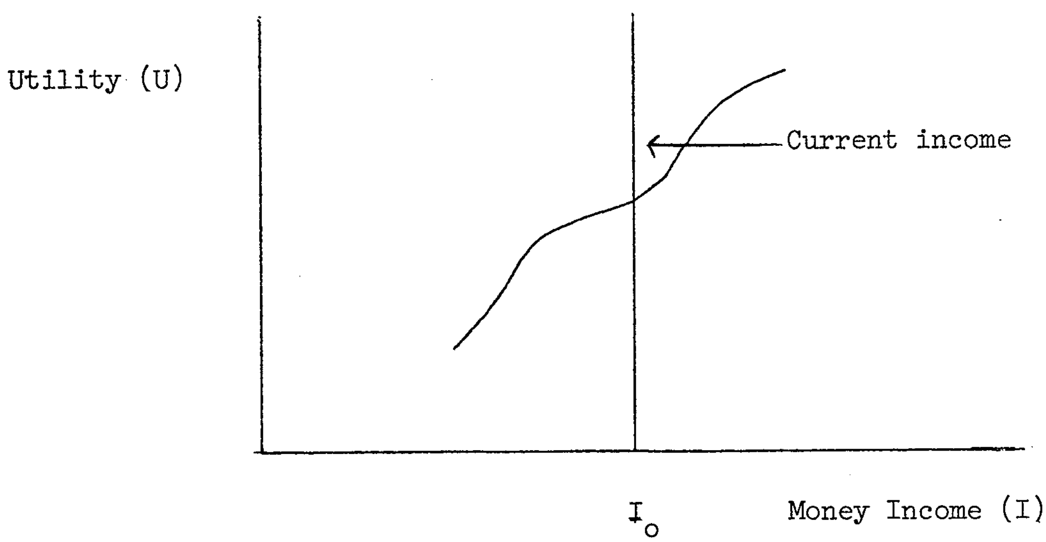

Figure 3

Illustration of Bradford-and-Johnson Representation of Typical Utility Curve

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[7] L. A. Bradford and G. L. Johnson, Farm Management Analysis (New York: John Wiley and Sons, Inc., 1953), pp. 408-410.

This conceptual representation of the relation between money income and utility accords with the facts that: (1) rational judicious people simultaneously insure and take chances and (2) formal gambling schemes, such as lotteries, have several prizes rather than one large prize.[8] It provides a theoretical base for a farmer's behaviour and decision making under uncertainty.

In farm firm decision studies such as this one, there is a practical problem in making use of the theoretical utility framework. To use it in a substantial way, some information about the form and the parameters of the utility function are necessary. Both the form and/or the parameters may be assumed or they may be measured. Alternately and more generally, specific representation of either one is avoided and it is assumed that a particular decision rule maximizes expected utility, whatever the true shape and the true parameters of the utility function. The utility framework for this study was a compromise of these alternatives. A general form of utility function was selected, and a particular decision rule was then assumed to maximize utility. This aspect of the study is discussed in Chapter 4.

II. THE PLANNING HORIZON

In some economic studies, the appropriate length of the planning horizon is defined by the problem. In others such as this, the planning horizon is one of the variables. In many cases of this kind, the question of the appropriate length of horizon is essentially ignored and the question is resolved by selecting some arbitrary length of, say ten, twelve or twenty periods.[9] This approach avoids the question of why a multiperiod planning

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[8] Bradford and Johnson, Ibid.

[9] Strictly speaking, the planning horizon is continuous and is not composed of discrete time periods. Discrete time periods are conventionally assumed for expositional purposes and for empirical applications.

horizon is appropriate and the question of how many periods are necessary. These questions are examined in some detail in this section.

The Concept of the Best First Move

The analysis of Modigliani[10] is considered the appropriate reference point for discussing the need for and the appropriate length of a multiperiod planning horizon. Following Modigliani's argument, the essential problem for planning purposes is that of deciding on the best first move. Later periods are relevant to this decision only if they affect it. If they do not or would not have any effect, it is not necessary to include them in the planning horizon. In this context, the appropriate length of planning horizon is one such that the first period decision is unaffected by the addition of one more period to the planning horizon. The actual length of planning horizon required to stabilize the first year decision would vary somewhat from problem to problem. Ordinarily one would expect it to be multiperiod at least.

It should be emphasized that the use of a multiperiod planning horizon does not obviate the need for annual planning and decision making. The fact that a multiperiod horizon is needed to decide what to do this year simply indicates that the annual planning process requires a multiperiod horizon. A manager who uses a multiperiod horizon makes plans for all of the periods in the planning horizon. However, in the first period, he can only implement the first period plan of the overall plan and he may never implement the plans for the later periods. After the first period has

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[10] F. Modigliani, "The Measurement of Expectations", paper presented at the American meeting of the Econometric Society in Boston, December, 1951, abstract in Econometrica, Vol. 20 (July, 1952), p. 482.

elapsed, he has the opportunity to develop another multiperiod plan. In developing this plan, the manager will be able to use information, including the results of the first period action, that was not available one period earlier. This new plan may or may not specify the same plan for the second period--now the first period--as did the plan made one year earlier.

The sequential aspect of planning with a multiperiod horizon is illustrated in Figure 4. This illustration assumes a period length of one year and a planning horizon of five periods. Assuming the current period is the year 1973, the years 1973-1977 inclusive make up the planning horizon; in 1974, the years 1974-1978 are the planning horizon; and so on. When the current period expires, a new planning horizon becomes relevant and each period in the horizon moves one period closer to the first period. The second period in the previous horizon now becomes the first; the third becomes the second; and so on. This gradual approach towards the current period means that each period is part of several different planning horizons. As it approaches current period status, both the quantity and the quality of the

Current Year or Period	Planning Year or Period							
	1973	1974	1975	1976	1977	1978	1979	1980
	Period in Planning Horizon							
1973	1	2	3	4	5			
1974		1	2	3	4	5		
1975			1	2	3	4	5	
1976				1	2	3	4	5

Figure 4

Illustration of Periods in Planning Horizons for Different Current Periods available information would be expected to improve; as a general rule, the information about the near term is more plentiful and more accurate than the information about the long term.

While Modigliani's thesis emphasizes the choice of the best first-period action, this does not imply that actions in later periods are regarded as unimportant. Nor does it imply that first-period actions may not have important implications for later periods; they usually do. What Modigliani's argument does recognize is that planned actions in periods after the first are just that; they are only intended. Given that there is some uncertainty about the outcome of first-period actions, intended actions in later periods are conditional. They are conditional on the outcome of the first period action being the expected outcome. They are also conditional on no new information becoming available about the later periods. As a general rule, neither of these conditions is perfectly satisfied in actual decision situations.

The Optimal Growth Path and the Turnpike Theorem

Von Neumann's[11] analysis provides a theoretical basis for discussing the appropriate length of the planning horizon and for attaining year-to-year stability in the optimal plan. Von Neumann showed that a model of the economy could be developed in which the economy eventually reached a position of a stable year-to-year growth rate. The turnpike theorem from Dorfman et al.[12] is concerned with the relation between the optimal growth path and the von Neumann efficient path of growth, called the turnpike. It has

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[11] J. von Neumann, "A Model of General Equilibrium," Review of Economic Studies, Vol. 13 (1945), pp. 1-9.

[12] R. Dorfman, P. A. Samuelson and R. M. Solow, Linear Programming and Economic Analysis (New York: McGraw-Hill Book Co., 1958), Chapter 12.

been interpreted as follows: Efficient growth can be achieved only if the resource endowment of the firm has a structure of the von Neumann vector.[13]

To understand the von Neumann vector and the optimal and efficient growth path in the context of a farm firm, the planning problem has to be seen in terms of (1) the objectives of the firm, (2) intertemporal efficiency of resource use in light of the firm's objectives and (3) the relation between the salvage value and the acquisition cost of farm assets. These factors are interrelated and all bear on the optimal planning horizon and time path of economic variables.

Consider the following simple case: A firm has endowments $A_1(0)$ and $A_2(0)$ of two assets at the beginning of the first period (period 0). Both assets can be used for consumption (i.e. withdrawn from the firm), and both can be used within the firm to produce more of A_1 and/or more of A_2 in the next period, and so on through to some final period T. The disposition of the assets within each period is represented by:

$$(2.2) \qquad C_i(t) + X_i(t) = A_i(t) \qquad i = 1, 2 \quad t = 0, 1, \ldots, T$$

where $C_i(t)$ = the amount of the i-th asset consumed in the t-th period,

$X_i(t)$ = the amount of the i-th asset not consumed in the t-th period and available for productive use in the next period,

(t) = is as previously defined.

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[13]J. M. Boussard, "Time Horizon, Objective Function and Uncertainty in a Multiperiod Model of Firm Growth," American Journal of Agricultural Economics, Vol. 53 (August, 1971), p. 470.

Assuming for the moment that the firm's objectives can be completely described in terms of the assets A_1 and A_2, the potentially attainable combinations of A_1 and A_2 in any time period are described by the production possibility curve--the locus of efficient possible combinations. One such locus for the final period T is illustrated in Figure 5 as the curve ABC. This curve is drawn convex and continuous and is consistent with constant returns to scale and diminishing marginal rates of transformation between A_1 and A_2.

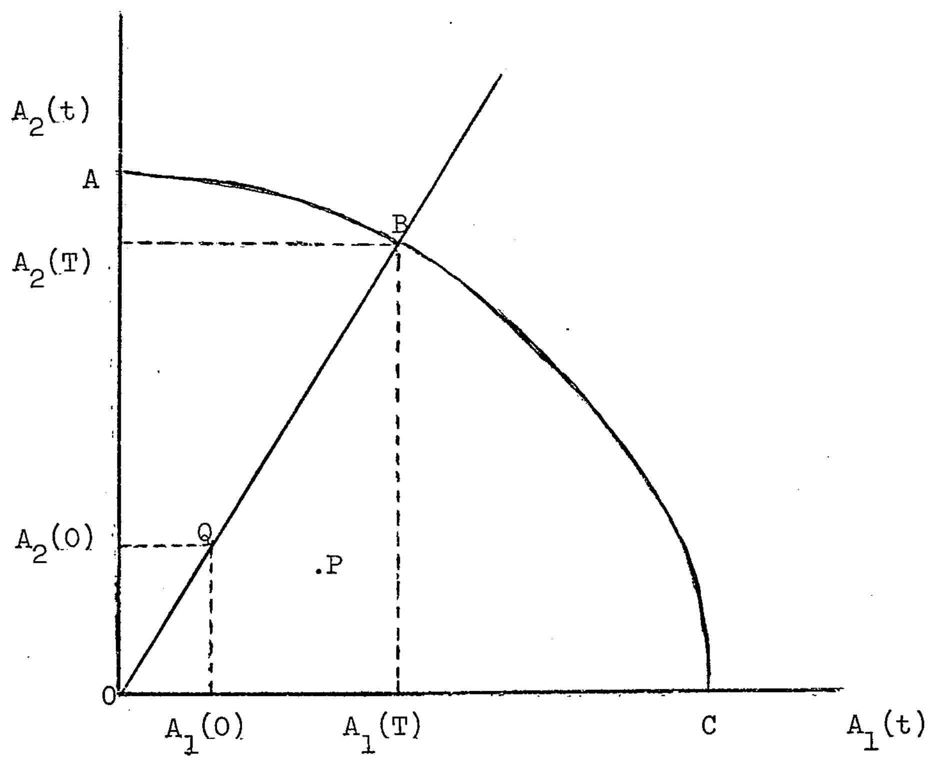

Figure 5

Illustration of Efficiency Locus and Efficient Growth Path for Two Capital Assets, A_1 and A_2.

Given perfect competition and the achievement of Pareto-optimal conditions, just one combination of assets will be most profitable and most efficient for the firm in each time period. With no change in technology, the asset structure would remain constant over time. Then assuming a long-run entrepreneurial objective of being somewhere on the production possibility frontier in the final period, there is one most efficient route to that frontier. Let that route be represented by the ray OB in Figure 5; it is the von Neumann growth path. Along this path from the origin, the assets A_1 and A_2 increase proportionally. This is the path of maximal efficient balanced growth in these two assets. If the firm's initial endowments, $A_1(0)$ and $A_2(0)$, were structured according to the proportions of the growth path, say at point Q in Figure 5, then they would have the structure of the von Neumann vector. The firm would be on the optimal efficient growth path from the beginning. If the initial endowments were of some other proportion, say that represented by point P, then the first move of the firm would be to try to alter this proportion to the von Neumann proportion. If feasible and profitable, the adjustment towards the path would occur over time. The firm might eventually reach a position of steady growth on the von Neumann path with no year-to-year changes in the optimal plan and a condition of intertemporal efficiency would be achieved.

This analogy between the firm and the conditions on the von Neumann path is not completely faithful to the latter. Strictly speaking, it required the assumption that capital assets, A_1 and A_2, and in the general case A_1, A_2 . . . , A_n, could be purchased and sold without limit to alter the asset holdings if necessary to the von Neumann structure. This is clearly an unrealistic assumption for some farm capital assets. Land is an important example in this category; practically, it cannot be added to or withdrawn from the firm without limit. Thus the conditions on the von Neumann path of a firm should only be regarded as efficient, within the bounds of some limiting capital asset. The assumption just referred to was that of perfectly competitive conditions. For the economy, the assumption ensured that the efficient von Neumann path would be reached as a result of profit-maximizing behaviour by each firm. Thus the time path would be independent of the planning horizon and of the terminal period weights of the capital assets. The position of the optimal path would be determined only by the input-output relationships in production.

Continuing with the two-asset case for final period T: If the ratio of the weights attached to the assets in this final period were the same as the slope of the efficiency locus at the point of interception by the efficient growth path, there would be no deviation from the path. If not, towards the end of the planning horizon there would be some deviation from the von Neumann path, to achieve the desired final proportions. This point is illustrated in Figure 6.

The curve ARSC represents the efficiency locus and the ray OR represents the efficient von Neumann path. Given the initial endowments at point P, and terminal asset weights for A_1 and for A_2 of $P_{A_1}^{1}$ and $P_{A_2}^{1}$, respectively, the firm would adjust its asset mix to follow a path from P to the von Neumann path at Q and would remain on it to R, the point of tangency. Given different terminal weights such as $P_{A_1}^{o}$ and $P_{A_2}^{o}$, the optimal path would depart from the efficient path before R to arrive at the tangency point S. With the same terminal period weights, but assuming a smaller value for T (i.e. a shorter planning horizon), it might be optimal for the firm to follow a less efficient but more profitable path directly from P to S and never attain the more efficient von Neumann path. It is for this reason that the Von Neumann path has been called the turnpike. It will be profitable for

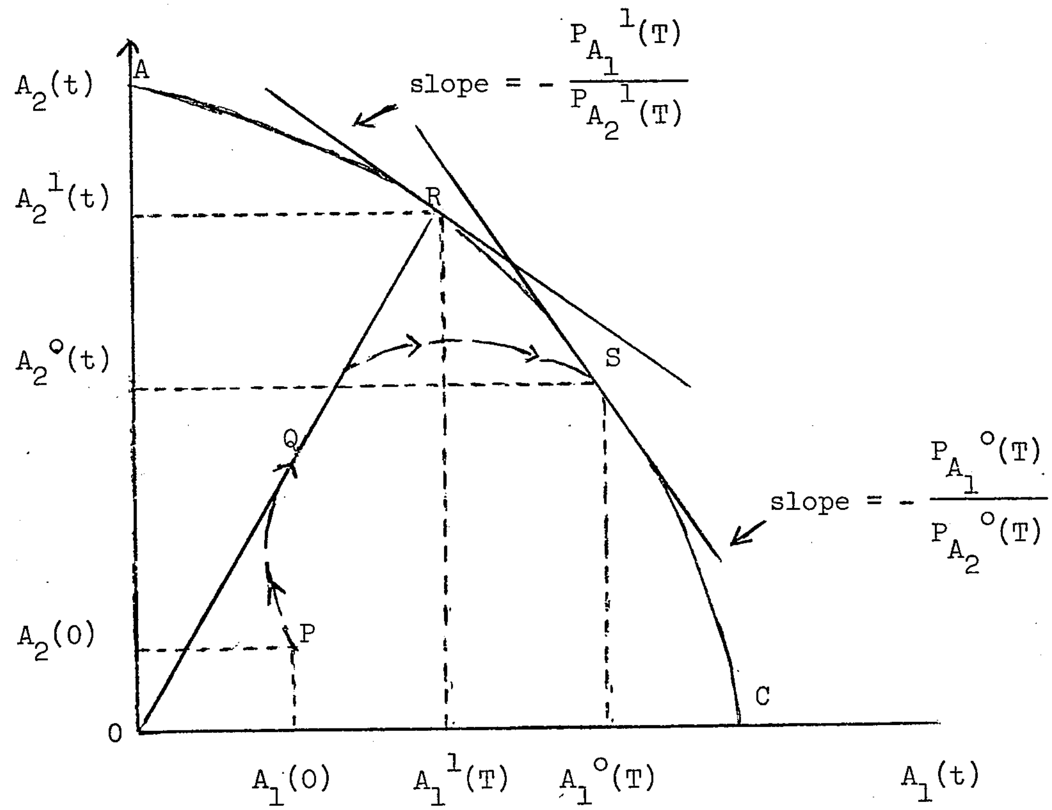

Figure 6

Illustration of Optimal Paths for an Initial Endowment of Capital Assets, A_1 and A_2, and Different Final Period Weights.[a]

[a]The ray OR is the von Neumann efficient path--also called the turnpike.

the firm to follow the von Neumann path--the turnpike--if the planning horizon is long enough. If not, it may be more profitable to follow a shorter but less efficient path that is not the von Neumann path but nevertheless optimal.

Salvage Value, Acquisition Cost, and the Turnpike

The problem of assigning values to assets at particular points in time has an important bearing on the achievement of year-to-year stability in asset structure. In this connection, the theory of fixed assets is relevant. According to the theory, a productive asset is economically fixed in its current use if its value in use is greater than its sale or salvage value and less than its acquisition cost. A capital loss would occur if more of this asset were acquired or if some of it were disposed. This situation is illustrated in Figure 7.

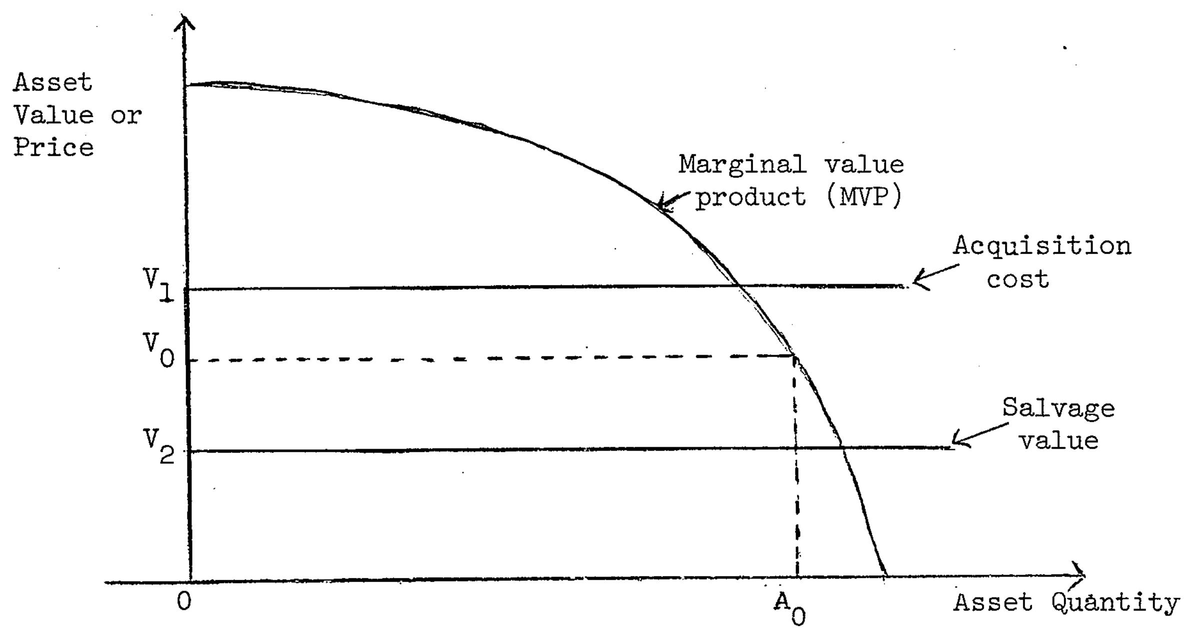

Figure 7

Illustration of Marginal Value, Acquisition Cost and Salvage Value of a Productive Asset

With quantity OA_0, the marginal value of the asset is OV_0. With acquisition cost OV_1, and salvage value OV_2, it would not be profitable either to increase or to decrease the quantity of the asset employed in the firm. The marginal earnings could fluctuate over the range $OV_1 - OV_2$ without any change in the quantity of the asset being economically justified. If the earnings came to exceed OV_1, it would pay to acquire more of the asset. If they fell below OV_2, then it would pay to reduce the holdings of the asset.

Fixed asset theory recognizes that there may be more than one price to consider in the valuation and allocation of an economic asset. In the context of growth in the stock of assets, differences among these values can be important. If an asset's salvage value is equal to its acquisition cost, it can be bought and sold with no capital loss and in response to shifts in its marginal value. Boussard has pointed out that if all of the assets of the firm are all perfectly liquid at every point in time, the firm would then be able to adjust its assets to the von Neumann asset structure in the very first period.[14] It could then follow the turnpike from the beginning to the point near the terminal period when it would be adjusted in accordance with the terminal period weights.

In general it is not the case that all farm assets are fully liquid and may be converted from one form to another with no capital gain or loss. The salvage value of farm assets such as improvements and equipment is often less than the value in use, which in turn is often less than the acquisition cost. Ceteris paribus this condition would hinder but would not preclude the eventual attainment of the turnpike. The greater the difference between

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[14] Boussard, op. cit., p. 475.

the salvage value and the acquisition cost, the greater the elapsed time before the turnpike could be reached. As an illustration, if the salvage value and acquisition cost of an asset were each equal to 1,000 dollars and the marginal value product was equal to 1,500 dollars, then more of the asset could be profitably acquired. However, if the marginal value product of the asset was again 1,500 dollars but the salvage value was 1,000 dollars and the acquisition cost was 2,000 dollars, a capital loss would arise if more of the asset were acquired and a capital loss would occur if the holdings of the asset were reduced. Thus inequality of the salvage value and the acquisition cost would normally lengthen the time required to adjust to the preferred asset structure. Whether or not a capital loss were worth taking would depend on the earnings of the salvage value of the asset weighed against the capital loss incurred in realizing the salvage value.

Summary

Theoretical considerations of the planning horizon for decision purposes and of the appropriate length of planning horizon support the observations that: (1) a planning horizon beyond the current period is appropriate if the optimal current period decision would be different if at least one more period were included; (2) the number of periods in the planning horizon should be such that the first period decision is independent of it; and (3) the particular prices used to value the assets in the final period are practically irrelevant to the first period decision, providing there are enough periods in the planning horizon.[15]

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[15]Although the average decision maker would not be aware of these theoretical arguments, he would know from his own experience that information about the near term should be considered in making current period decisions and that precise future prices are not particularly important for current decisions.

The discussion of this section supports no conclusion about the appropriate minimum number of periods in the planning horizon for a particular problem; there is no a priori way of knowing the number. Still, it is the minimum number of periods that is the important criterion. If it is not possible in an investigation to experiment with planning horizons of different lengths, it is important to choose one with little chance of not satisfying the theoretical minimum. In this study it was assumed that a planning horizon of ten periods would satisfy the minimum requirements.

CHAPTER 3

REVIEW OF FARM FIRM DECISION MODELS

Contemporary farm firm decision models fall naturally into four main groups according to the general class of decision problem handled. They are: (1) single period deterministic models, (2) single period stochastic models, (3) multiperiod deterministic models and (4) multiperiod stochastic models. The purpose of this chapter is to review these different models and to discuss some of the theoretical and practical issues in model selection that are of concern in this study. Decision models that employ an optimizing technique are emphasized with only brief reference to non-optimizing techniques such as simulation and budgeting. Single period models are defined as those with a planning horizon of just one period while multiperiod models have a planning horizon of two or more periods. Deterministic models are defined as those that treat all parameters as though they were single valued and specific account is not taken of within-period variation. Stochastic models are defined as those that do take specific account of within-period variation in the choice variables.

SINGLE PERIOD DETERMINISTIC MODELS

Within the framework of a single period and of deterministic outcomes, most of the farm decision models have been concerned with long-run rather than short-run problems. In general, they have dealt with the problem of determining the best long-run strategy, basis some average time period, and average values have usually been assumed for the coefficients of these models. Linear programming has probably been the most widely used technique of analysis for single period problems. A study by Johnson,[1] and the studies by Finn[2] and by Singh[3] are examples of the application of this particular technique. Simulation and budgeting have also been used.

There are two important criticisms of single period deterministic models. One concerns the credibility of the optimal long-run solution and the other concerns the attainability of this long-run solution. The question of credibility hinges on the fact that the models are deterministic and on the implicit assumption that the degree of uncertainty associated with the different choice variables is not a factor affecting the optimal decision. Ceteris paribus, this assumption is probably unrealistic. Concerning the attainability of a long-run optimum, single period models do not consider this question and cannot define the time path of economic variables to their long-run optimal values. Consequently, it is not possible to conclude from a single period model that the optimal solution is in fact attainable.

SINGLE PERIOD STOCHASTIC MODELS

Provision in the decision process for taking account of stochastic variation in choice variables goes some way toward making farm firm decision models more realistic. In recent years increased research effort has been committed to this end and several fairly satisfactory models have been developed. Two of the more important--quadratic and linear programming--

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[1]W. I. R. Johnson, "A Micro-Economic Analysis of Irrigation in the Morden-Winkler Area of Manitoba" (Unpublished Masters Thesis, University of Manitoba, 1963).

[2]Finn, op.cit.

[3]Singh, op. cit.

are discussed below with emphasis on linear programming models.

Quadratic Programming Models[4]

Unlike that of linear programming, the objective function in quadratic programming has two types of parameters--the level of expected income and the variance in expected income. The addition of variance to the objective function ensures that this dimension of expected income will be given some weight in determining the optimal plan. The importance of considering variance in developing the optimal plan depends on the degree of risk preference assumed. This point is illustrated by Iga's observation that "when risk preference is high variances and covariances do not play important roles for the determination of optimal activity combinations but they are important at high levels of risk aversion."[5]

Because quadratic programming employs a two-parameter objective function and also incorporates different degrees of risk preference, it has been found satisfactory for preparing optimal long-run plans for a fairly wide range of farmer objectives in terms of expected income and variance. The major problems that have been encountered in applying the technique are: (1) difficulties in obtaining satisfactory estimates of the variances and covariances of the net returns of the different choice variables and (2) difficulties in obtaining computer programs that are reliable and that do not seriously limit the size of the problem that can be reasonably handled.

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[4]For a sample of studies using quadratic programming models, see Iga, op. cit.; R. L. Batterham, "Investment and Financial Management in Farm Firm Growth" (Unpublished Ph.D. Dissertation), University of Illinois, 1971; and R. B. How and P. B. R. Hazell, Use of Quadratic Programming in Farm Planning Under Uncertainty, Department of Agricultural Economics, A.E. Res. 250, (Ithaca: New York State College of Agriculture, 1968).

[5]Iga, op. cit., p. 255.

Since the dimensions of the quadratic matrix increase rapidly as the number of choice variables expands, this has given impetus to the development of linear stochastic models.

Linear Programming Models

Several linear decision models have been developed that make allowance for uncertainty. Distinctive models using this approach have been presented by Boussard and Petit, by McInerney, by Hazell, by Maruyama and by Thomas et al.

Boussard and Petit Model.[6] The hypothesis of this model is that farmers want to maximize expected income, subject to the constraint that the focus of loss does not exceed the permissible loss. Following Shackle the focus of loss was defined as "the level of loss that a decision maker would be very surprised to reach in any eventuality."[7] The permissible loss was the difference between the expected income and some predetermined minimum acceptable income. Diversification was imposed on the optimal solution of this model by a requirement that the loss on any one enterprise could not exceed a certain percentage of the total permissible loss. This requirement was in contrast to the quadratic programming models such as the one used by Iga in which diversification occurred in response to the degree of risk preference. Nevertheless, Boussard and Petit concluded that this fairly simple model satisfactorily explained the behaviour of farmers in the Provence area of France.

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[6] J. Boussard and M. Petit, "Representation of Farmers' Behaviour Under Uncertainty with a Focus-Loss Constraint", Journal of Farm Economics, Vol. 49 (November, 1967), pp. 869-880. See also G. L. S. Shackle, Decision Order and Time in Human Affairs (Cambridge: Cambridge University Press, 1961).

[7] Ibid., p. 873.

McInerney Model.[8] The McInerney model was a conventional single period linear programming model except in terms of the objective function. Instead of the usual objective of maximizing expected income, McInerney used a criterion from the theory of games--the maximin criterion associated with Wald. Thus the optimal plan in this model was the one that maximized the minimum possible income. In the language of game theory, it was the plan with the highest minimum income, irrespective of the state of nature. It should be noted here that a decision strategy that maximizes the minimum or worst possible outcome does not imply that the decision maker expects the most unfavourable results to occur. It simply assumes that he wants to be prepared for the most unfavourable results.

The evidence of McInerney's study was that the strategic considerations of the maximin criterion could be successfully grafted onto the usual linear programming model. In an empirical application the optimal plan had only a slightly lower average gross margin than the plan based on expected gross margins. It had the additional advantage of being less risky and therefore could be considered to represent a superior strategy. Hazell[9] has shown that other criteria from game theory--the Laplace, the Savage regret and the Hurwicz--could be used as objective functions in conventional single period linear models. He also pointed out that the method of parametric programming could be used with each of these criteria to identify the optimal plans over a range of expected incomes.

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[8] J. P. McInerney, "Maximin Programming - An Approach to Farm Planning Under Uncertainty," Journal of Agricultural Economics, Vol. 18 (May, 1967), pp. 279-289.

[9] P. B. R. Hazell, "Game Theory--An Extension of Its Application to Farm Planning Under Uncertainty," Journal of Agricultural Economics, Vol. 21 (May 1970), pp. 239-252.

Hazell Model.[10] This model was developed primarily to provide a linear substitute for quadratic programming models. The objective function was specified as that of minimizing the total absolute deviations in income. The optimal plan was the one which gave the smallest possible difference between the expected gross margin and the actual gross margin. Again, with parametric variation of an expected gross margin constraint, efficient combinations of minimum absolute deviations and expected gross margin were identified.[11] In an empirical application, Hazell found that the model specified optimal plans very similar to those specified by a quadratic programming model of the same problem.

Maruyama Model.[12] Maruyama incorporated a variety of ways of taking account of uncertainty into a single period linear programming model. In its most comprehensive form, this model provided for specific consideration of uncertainty in the objective function, in the restraints and in the input-output coefficients. The decision maker was assumed to face a set of joint discrete outcomes of the stochastic choice variables. Again, using parametric variation of an income constraint, efficient combinations of expected income and worst possible income were generated. This model also

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[10] P. B. R. Hazell, "A Linear Alternative to Quadratic and Semi-variance Programming for Farm Planning Under Uncertainty," American Journal of Agricultural Economics, Vol. 53 (February, 1971), pp. 53-62.

[11] These combinations, and those in the McInerney game theory-linear programming formulations would be analagous to the E-V efficient combinations in quadratic programming.

[12] Y. Maruyama, "A Truncated Maximin Approach to Farm Planning Under Uncertainty with Discrete Probability Distributions," American Journal of Agricultural Economics, Vol. 54 (May, 1972), pp. 192-200.

had a provision to purchase resources to supplement those that would be in short supply with particular outcomes and it also had a provision to minimize the possible deviations of restraint values from their expected levels. In an empirical application to a mixed farm in Japan, Maruyama concluded that the model successfully handled uncertainty in all of the coefficients and generated information about the probabilities of ruin and of infeasibilities.

Thomas Model.[13] Thomas et al. developed a single period model that used separable programming to take account of the risk factor. Separable programming provides a means of approximating a nonlinear constraint by representing it as the sum of nonlinear functions of single arguments. In this model, variance in expected income was approximated by this kind of constraint. With parametric variation of the variance constraint, plans representing efficient combinations in terms of expected income and income variance were specified. In an empirical application, it was found that the approximated variance for a given expected income was quite similar to the estimated variance for the same income level using the quadratic form. It was concluded that this model represented an operational substitute to quadratic programming models.

MULTIPERIOD DETERMINISTIC MODELS

Setting the decision problem within a multiperiod time dimension while maintaining deterministic outcomes satisfies the criticism that a horizon of a single 'average' period ignores the adjustment process from a

[13] W. Thomas et al., "Separable Programming for Considering Risk in Farm Planning," American Journal of Agricultural Economics, Vol. 54 (May, 1972), pp. 260-266.

current to a long-run situation. As a result, an increasing number of farm firm studies have seen the decision problem within the framework of a multi-period or dynamic dimension. With a time dimension included, the optimal solution provides more information. Not only is the long-run solution specified as it is in the single period problem but in addition, the optimal time path to that long-run solution is also specified. The time span from the current to the long-run solution can be regarded as the transition period. In the context of the discussion of the previous chapter, this is the period of adjustment towards the turnpike.

The most widely used techniques of analysis in multiperiod deterministic models have been linear programming and simulation. Examples of the application of linear programming to this problem are studies by Boyko,[14] Martin and Plaxico,[15] Mitchell[16] and Boehlje and White.[17] A study by Harrison[18] provides an example of a simulation model of a multiperiod situation.

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[14]E. S. Boyko, "A Multi-Period Analysis of Capital Accumulation and Financing of Beginning Irrigation Farms in the Pembina River Basin" (Unpublished Master's Thesis, University of Manitoba, 1969).

[15]J. R. Martin and J. S. Plaxico, Polyperiod Analysis of Growth and Capital Accumulation of Farms in the Rolling Plains of Oklahoma and Texas, U.S. Department of Agriculture, Technical Bulletin No. 1381 (Austin, Texas: Economic Research Service, United States Department of Agriculture, 1969).

[16]R. Mitchell, "A Multiperiod Linear Programming Analysis of Net Income Goal Attainment in Manitoba Crop District No. 10" (Unpublished Master's Thesis, University of Manitoba, 1972).

[17]M. D. Boehlje and T. K. White, "A Production-Investment Decision Model of Farm Firm Growth," American Journal of Agricultural Economics, Vol. 51 (August 1969), pp. 546-563. See also M. D. Boehlje, "An Analysis of the Impact of Selected Factors on the Process of Farm Firm Growth" (Unpublished Master's Thesis, Purdue University, 1968).

[18]V. L. Harrison, "Management Strategies and Decision Processes for the Growth of Farm Firms" (Unpublished Ph.D. Dissertation, Purdue University, 1970).

In incorporating a time dimension into deterministic models, the most common approach has been to use a period length of one year. Of the studies referred to in the previous paragraph, the Martin and Plaxico study was the only one not using one-year periods. It used five-year periods and a planning horizon of six periods, thus encompassing thirty years. In general, it would appear that the greater the time in each period, the less the concern with short-term considerations and with the opportunities for decisions that occur within periods. In reality, the time frame for decision problems is continuous and the more a decision model departs from it, the less consideration it can give to short-term problems. In the Martin and Plaxico study, for example, there was an implicit assumption that resources could only be expanded every fifth year.

A variety of one-dimensional objective functions has been evaluated in different multiperiod linear models. Generally, the most widely used objective function has been that of maximizing the present value of an income stream. Boyko[19] for example, assumed an operator objective of maximizing the present value of net returns before taxes while Mitchell[20] assumed an objective of maximizing the present value of disposable income. The findings with respect to the effect of different objective functions on the optimal plan have not always been in agreement. Boehlje and White,[21] for example, compared the results of maximizing the present value of disposable income with that of maximizing terminal net worth and found substantial differences in production decisions and in farm business

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[19]Boyko, op. cit., p. 40.

[20]Mitchell, op. cit., p. 69.

[21]Boehlje and White, op. cit., p. 560.

organization. Martin and Plaxico[22] on the other hand found that the same plan was an optimum whether the objective was to maximize the present value of net returns or to maximize terminal net worth. They also found that the objectives of maximizing undiscounted net returns, maximizing discounted gross sales and maximizing acres of land operated were satisfied by the same plan as the two other objectives.

MULTIPERIOD STOCHASTIC MODELS

A number of models have been developed that incorporate some provision for taking account of stochastic variation with a multiperiod planning horizon. As with single period and deterministic models, the most widely used technique of analysis has been linear programming.

Johnson et al.[23] incorporated consideration of the variation in expected income into a conventional multiperiod linear programming model. This was accomplished by sampling from a distribution of the gross margins of two stochastic variables to obtain a pair of gross margin values for each period of a fifteen-period planning horizon. The optimal plan was the one that maximized the gross margin for this particular set of values. By repeated sampling and preparation of optimal plans, estimates of the means and standard deviations in income of the optimal plans at different points in time for different initial resource situations were obtained. This approach to the solution of stochastic programming problems is called the distribution method; it uses solutions of deterministic problems to

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[22] Martin and Plaxico, op. cit., pp. 30-33.

[23] S. R. Johnson, K. R. Tefertiller and D. S. Moore, "Stochastic Linear Programming and Feasibility Problems in Farm Growth Analysis," Journal of Farm Economics, Vol. 49 (November, 1967), pp. 908-919.

approximate the solution of the stochastic problem. This aspect of the model detracts somewhat from its usefulness for analysis of decision problems;[24] in the usual case, the decision must be taken without knowing the outcomes of the stochastic variables.

One of the simplest stochastic models was developed by Boussard.[25] He extended the single period focus-loss formulation to a multiperiod dimension. This model retained the essential feature of the single period model as far as taking account of uncertainty was concerned. The principal modification of the earlier model was a provision that a portion of the available financial reserves could be committed to the business to accommodate unfavourable outcomes. The practical effect of this provision was a higher level of uncertainty in the optimal plan than would have been the case without access to some financial reserves. Boussard experimented with planning horizons of different lengths and found that a seven-year horizon was fairly satisfactory. He also found that the beginning resource structure had an important effect on decisions and on the time path of economic variables.[26]

A multiperiod linear model that conceived of possible outcomes in terms of joint discrete probability distribution was proposed by Cocks.[27]

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[24]Ibid., p. 908. The authors indicated that their concern was with feasibility and growth problems rather than decision problems. While it is true that analysis of feasibility problems need not be concerned with decision problems, the converse is not true; decision analysis cannot ignore problems of feasibility.

[25]Boussard, op. cit., pp. 467-477.

[26]This finding is in agreement with everyday observation and with the findings of many other farm firm studies. See for example Boehlje and White, "A Production-Investment Decision Model of Farm Firm Growth", op. cit., p. 561.

[27]K. D. Cocks, "Discrete Stochastic Programming," Management Science, Vol. 15 (Sept., 1968), pp. 72-79.

Conceptually this model makes provision for stochastic outcomes in the coefficients of the objective function, in the restraints, and in the input-output coefficients in each period of the planning horizon. In an empirical application by Rae[28] there were three periods in the planning horizon and four months in each period. The optimal strategy for the entire planning horizon was determined, having considered every possible sequence of every possible outcome over all periods.[29] In this model the objective function that was maximized was expected utility; the money values of each outcome were weighted by their probabilities and by a utility function for money income.

Comparing the results of the stochastic formulation and a deterministic formulation of the same problem, it was found that the optimal strategies were different and that the stochastic formulation provided a higher level of expected income. The difference between the expected returns of the two solutions was an estimate of the potential payoff of taking explicit account of uncertainty and of obtaining information about the possible outcomes. A practical difficulty in using the Cocks model is that of dimensionality; with several stochastic choice variables, several periods and several outcomes per stochastic variable per period, this can be an important limitation.[30]

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[28] A. N. Rae, "An Empirical Application and Evaluation of Discrete Stochastic Programming in Farm Management," American Journal of Agricultural Economics, Vol. 53 (November, 1971), pp. 625-638.

[29] Viewed from the start of the first period, there were fifty possible outcomes of each decision.

[30] As an example, a problem with only three stochastic variables per period and three outcomes per period for each of these variables would require a total of twenty-seven activities in the second period to handle all possible combinations of outcomes.

Another model of a multiperiod stochastic problem combined single period parametric linear programming with decision tree analysis. This fairly simple model was developed by Yaron and Horowitz[31] as an attempt to integrate the problems of short-run and long-run planning under uncertainty. Optimal short-run plans were prepared for different resource situations for a greenhouse operation in Israel. Different outcomes for decisions made on the basis of these optimal short-run plans were imposed on the beginning set of resources, and short-run plans for the subsequent period were prepared for each new resource situation. Again different outcomes were imposed, and so forth. The optimal strategy was selected on the basis of the highest present value of the income streams of the feasible outcomes.

Chen[32] developed a multiperiod linear model that viewed the farm manager's objective as one of maximizing expected income subject to the constraint that the level of any one enterprise was constrained by a predetermined limit on the contribution of that enterprise to the variance in the expected income of the business. This model thus embodied some aspects of the Boussard model in terms of diversification. In an empirical application Chen found that similar decisions were implied by this model, by the single period quadratic programming model and by the single period model of Hazell.

SUMMARY

The foregoing review has sketched in a descriptive way some of the contemporary attempts to develop theories and to formulate quantitative

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[31] D. Yaron and U. Horowitz, "Short and Long Run Farm Planning Under Uncertainty-Integration of Linear Programming and Decision Tree Analysis," Canadian Journal of Agricultural Economics, Vol. 20 (July, 1972), pp. 17-30.

[32] J. T. Chen, "Modelling Responses to Uncertainty in Mathematical Programming Growth Models of the Farm Firm" (Unpublished Ph.D. Dissertation, University of Illinois, 1972).

approximations to the decision environment of the farm firm and to the decision criteria in farm firm management. It supports the observation that the effort to incorporate uncertainty and a time dimension into farm decision studies has been by and large successful. Also, the effort to accommodate a variety of decision criteria has been fairly successful. This is not to imply the existence of a universal and validated theory of the farm firm, nor of a universal decision model incorporating a theory. It does indicate, though, that specific account of variability and of time can be incorporated into simple as well as complex representations of a farm firm. It also indicated that decisions could be substantially different with specific consideration of uncertainty and of time, although the evidence with respect to time was not as clear cut.

As far as decision rules were concerned the literature indicated several different rules and strategies for choices under uncertainty. Some of them implied approximately the same decisions. Some were optimizing and some were not.

The simple rule of maximizing expected income, which is also the Laplace criterion from game theory,[33] could be considered an appropriate rule for decision making only when the financial resources of the firm are adequate to withstand any unfavourable results. Since this is not usually the case in farm firms--although it is for some decisions--this is not a particularly suitable rule for whole farm planning. Most of the alternatives to the single parameter objective function models have included a parameter that gives some weight to the dispersion (variance) in the outcomes of the

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[33]For details of the game theory criteria of Laplace, Wald, Hurwicz and Savage, see R. D. Luce and H. Raiffa, Games and Decisions, Introduction and Critical Survey (New York: John Wiley and Sons, Inc., 1957), Ch. 13.

choice variables. However, not all of the decision rules that do so are suitable for problems that have a time dimension, nor do they all have equivalent data requirements. As an example, the Rae formulation of the discrete stochastic programming model required estimates of all of the discrete outcomes and their probabilities of each choice variable in each period; the Boussard model on the other hand required an estimate of just one of the discrete outcomes for each stochastic variable and an estimate of the probability of that outcome was not required.

The use of the expected income criterion in both one- and two-dimensioned decision rules assumes that this is the variable whose expected level is important. In this connection, Cocks and Carter have discussed different objective functions and have concluded that the important difference between different functions primarily concerns the weighting of consumption goals versus wealth goals in different time periods.[34] Though the existence of a tradeoff between these goals is well known, the problem arises with the assigning of the weights.[35]

As far as this study is concerned, it was felt that the decision model should satisfy the following general criteria:

1. It should incorporate both short-run and long-run considerations into the decision process.

2. It should take account of a firm's ability to withstand unfavourable results in determining the optimal plan.

3. Ceteris paribus, it should be a simple representation of the decision process.

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[34]K. D. Cocks and H. O. Carter, "Micro Goal Functions and Economic Planning," American Journal of Agricultural Economics, Vol. 50 (May, 1968), pp. 406 and 408.

[35]Further discussed in the next chapter.

Specifically it was felt that there were theoretical and/or practical arguments for considering both short-run and long-run objectives, for giving consideration to available information about the short term and the long term in arriving at the current period decision and for recognizing financial reserves as an important determinant of the risk that a farmer is willing to assume.

Given the above requirements, the model for this study had to be multiperiod and stochastic. Of the available models of this kind, the Boussard model most nearly satisfied the conceptual and practical requirements of this study. The model that was developed for this study resembled the Boussard model in some important respects. There were some substantial differences however and the final model borrowed characteristics of three models--the focus-loss model of Boussard, the maximin model of McInerney and the expected income model. The particular decision model and the theoretical framework for it are developed in the next chapter.

CHAPTER 4

THEORETICAL CONSIDERATIONS AND FRAMEWORK OF ANALYSIS

As noted in previous chapters, this study was primarily concerned with decision making, uncertainty, and the transition from dryland to irrigation farming, if feasible. Accordingly, the model used to represent the decision process had to satisfy minimum requirements as follows:

1. It had to have decision rules, consistent with some information or set of assumptions about the utility of money income to the decision maker.

2. It had to have some provision for taking account of uncertainty in the decision process.

3. It had to have a time dimension. Comparative analysis of dryland and irrigation farming, without a time dimension, would not indicate if a profitable dryland-irrigation equilibrium position were attainable.

After considering the objectives of this study and the approaches and methodologies used in other studies, it was decided:

1. To adopt a lexicographic utility function.

2. To take account of uncertainty through making provision for the worst possible outcome.

3. To use the multiperiod linear programming technique of analysis.

Considerations involved in making these and other decisions about the models and the analysis are discussed below.

UTILITY FRAMEWORK OF ANALYSIS

In adopting a lexicographic function as the utility framework for this study, a number of factors were taken into consideration. First of all, it was felt that a lexicographic formulation closely approximated the actual decision process of many farmers. On the evidence of theory and the review of literature, as well as observation of the everyday behaviour of farmers, it was concluded that a multidimensional lexicographic objective function with the overall objective of maximizing expected utility was realistic for the farm situations that were to be analyzed.[1]

With a lexicographic ordering of utility, an individual is able to rank different objectives according to their subjective importance, but is not able to assign quantitative weights to the rankings. Assume the case of an individual who must choose between two courses of action, $0'$, and $0''$, Assume also that action $0'$ and action $0''$, respectively, satisfy n objectives $(0_1, 0_2, \ldots, 0_n)$ as represented by:

(4.1) $$0' = (0_1', 0_2', \ldots, 0_n')$$

(4.2) $$0'' = (0_1'', 0_2'', \ldots, 0_n'').$$

Then by lexicographic ordering, $U(0')$, the utility of action $0'$, is greater than $U(0'')$, the utility of action $0''$, if $0_1'$ is greater than $0_1''$, irrespective of whether the other objectives, 0_2 through 0_n, are individually better achieved by action $0'$ than by action $0''$. Thus the rule for selecting the appropriate action would be action $0'$, if $0_1'$ were greater than $0_1''$, and

[1]Jose Encarnacion, Jr., "Constraints and the Firm Utility Function," Review of Economic Studies, Vol. 31 (April, 1964), pp. 113-120. See also C. E. Ferguson, "The Theory of Multidimensional Utility Analysis in Relation to Multiple-Goal Business Behaviour: A Synthesis," Southern Economic Journal, Vol. 32 (October, 1965), pp. 169-175.

action O" if it were less. If the first and most important objective, O_1, were equally satisfied by either action, then the choice between them would be made on the basis of which one best satisfied the second objective, and so forth.

A general multidimensional lexicographic expected utility function can be represented, as a special case of a multidimensional expected utility function, as:[2]

$$(4.3) \qquad E\ (U) = \sum_{i=1}^{n} \alpha_i (p'U_i)$$

where $E\ (U)$ = expected utility,

p = a vector of probabilities, p^r, $r = 1, 2 \ldots, k$,

U_i = vectors of utilities U^r, $i = 1, 2 \ldots, n$,

k = the number of discrete outcomes,

n = the number of objectives or payoff factors,

and α_i = weights indicating tradeoffs between the n different objectives,

$i = 1, 2, \ldots n$, $\alpha_1 > 0$ and $\alpha_i = 0$ for $i = 2, 3, \ldots, n$.

An assumption of satisficing behaviour with respect to certain objectives was added to the lexicographic function. It was felt that this was a realistic assumption and consistent with some aspects of the behaviour of many farm managers. The theory of the satisficer[3] argues that a person may be motivated to satisfice rather than maximize. It is associated with behavioural rather than market theories of the firm. A number of farm firm studies, particularly those using the technique of linear programming, have

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[2]A. N. Rae, "Stochastic Programming, Utility and Sequential Decision Problems in Farm Management," American Journal of Agricultural Economics, Vol. 53 (August, 1971), pp. 455-457.

[3]H. A. Simon, Models of Man (New York: John Wiley and Sons, Inc., 1957).

had at least one satisficing objective. Boyko's study was an example.[4] It required that expected net income in each period be sufficient to satisfy at least a minimum level of expected consumption.

The lexicographic utility function can accommodate satisficing behaviour as follows: Assume that some of the objectives, within a hierarchy of objectives, must be achieved to some minimum levels, but the decision maker is indifferent to levels above these minimums; then expected utility would be maximized with respect to the most important of the remaining objectives, and subject to the constraints of satisfactory attainment of the satisficing objectives.

A conceptual representation of a lexicographic utility function, with satisficing behaviour, is shown in Figure 8. The argument of this function is that utility is a linear function of annual money income above the income level I_o. Below this level, the marginal utility of income

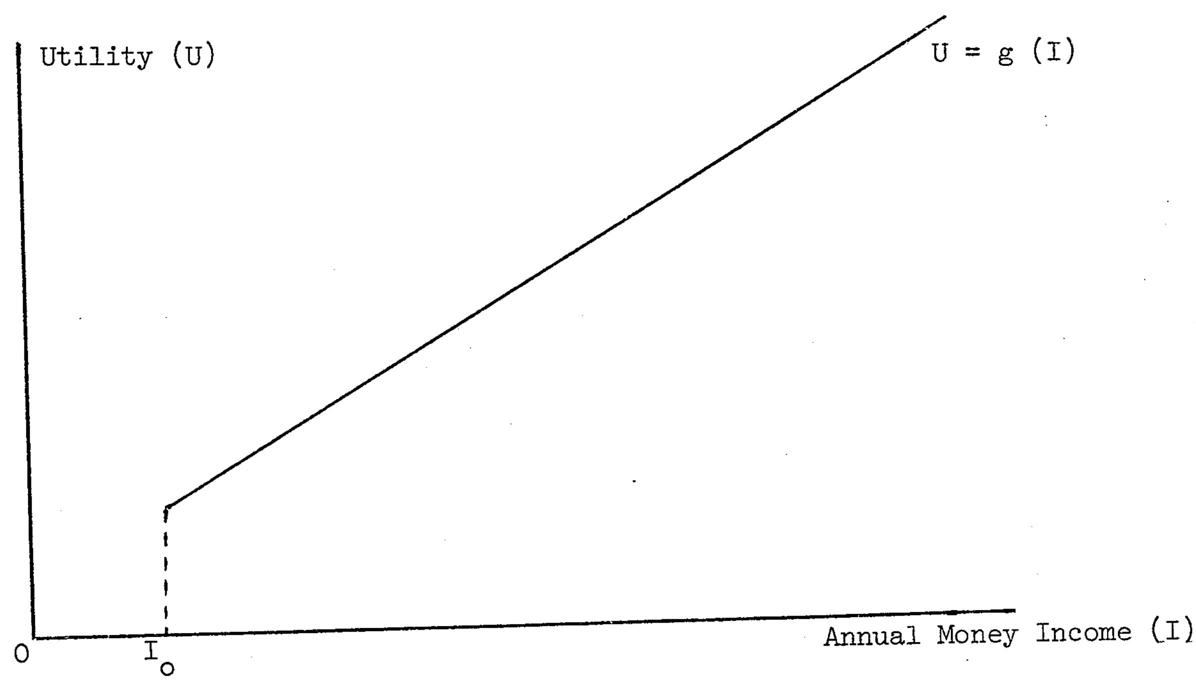

Figure 8

Representation of a Lexicographic Utility Function
With a Satisficing Objective

[4] Boyko, op. cit.

is infinitely negative; in other words, money income at any level below I_o has infinite disutility. In practical terms, this could represent a decision rule of an individual who would consider only those courses of action in which there was no substantial chance of income falling below a particular level. Then, among these courses of action, he could maximize utility by choosing the one with the highest expected income.

In this study, it was decided that the most important objective of the operator was to maximize an objective function, say forecast or expected income, subject to attainment of at least a minimum income in the near term of the planning horizon. This latter objective focused on subjective estimates of the worst possible outcomes from different courses of action. It was concerned with the provisions in the models for coping with uncertainty.[5] Thus the basic decision rule in most of the situations to be analyzed was: maximize expected income subject to satisficing or behaviourial constraints, and the usual resource constraints associated with farm production problems. As a proxy for the theoretical objective of maximizing expected utility, and expressed in equation (4.3), this rule assumed that expected utility was a linear function of expected income subject to achieving some minimum income every year. In the context of a lexicographic utility function with satisficing behaviour, the function may be represented as

(4.4) Maximize $E(U) = f\ (O_1, O_2)$

Subject to $O_2 \geq O_2^*$

where: $E(U)$ = expected utility,

O_1 = the expected income objective,

O_2 = the minimum or worst possible income objective, and

O_2^* = the minimum acceptable worst possible income.

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[5]See following section.

ALLOWING FOR UNCERTAINTY IN THE DECISION PROCESS

As previously indicated, it was decided to allow for uncertainty in decision making by incorporating consideration of the worst possible outcomes into the decision process. The reasons for selecting this approach are outlined below.

The Concept of the Worst Possible Outcome of a Stochastic Choice Variable

The idea that there is one outcome among the possible outcomes of a stochastic choice variable, which the decision maker regards as the worst possible, was developed by Shackle.[6] He postulated that the decision maker perceives a pair of outcomes--called a focus loss-focus gain pair--for each stochastic choice variable. Of this pair, the focus loss outcome is the decision maker's subjective estimate of the most unfavourable or worst possible event that could happen. It is regarded by the decision maker as an unlikely possibility, and as one whose actual occurrence would come as a surprise.

In this study, the worst possible approach conformed with a particular view on the typical long-run behaviour of managers of farm resources in uncertain situations. Here differences among farmers in the apparent degree of risk taken are partly, if not largely, explained by differences in the quantity and the mix of assets in the firm and available for use in the farm business. This view was supported by Heady who argued that few farmers diversify to reduce risk, at the expense of any substantial reduction

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[6] Shackle, op. cit.

in long run expected income.[7] The argument behind this opinion was that, in the long run, managers of farm resources tend to choose that mix of activities and enterprises for which they can handle all of the outcomes. In other words, within the limits of their resources, they make decisions primarily on the basis of expected values. One of the conditions for making decisions on the basis of expected outcomes is that the resources of the firm be adequate to handle all of the outcomes. The questions then were: "What set of resources would be adequate?", and "What is meant by 'handle all of the outcomes'?"

Dealing with the second part of the question first, the possible outcomes of a stochastic variable are distributed on either side of the expected value. On one side, the outcomes will be favourable relative to the expected value; on the other side, they will be relatively unfavourable. Favourable outcomes do not in general create serious management problems. While there would clearly be some exceptions, such as the case where very favourable yields create storage problems or cause serious labour shortages, the problems associated with favourable outcomes are the kinds of problems which have low costs of failing to anticipate them. In the vernacular, they are the kinds of problems that managers like to have, and managers of farm resources likely do not have any substantial aversion to outcomes that are more favourable than the expected.

The unfavourable outcomes of a stochastic variable require anticipation and can have high costs for failing to do so. These are the

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[7]Heady, op. cit., p. 521. This is not to argue that managers do not diversify to reduce year-to-year variation in income. Diversification would clearly be preferred to specialization, if it gave less year-to-year variation in income and equivalent expected income.

outcomes that must, so to speak, be handled by the resources available to the firm. Unfavourable outcomes, such as low yields, or prices, or the two together, or low yields two years in a row, can create serious managerial and financial problems for the firm. They can jeopardize the ability of the firm to continue in business, if the resources of the firm are not adequate to accommodate the unfavourable outcomes until more favourable outcomes occur. This view of unfavourable outcomes led to the adoption of a decision rule that maximized expected income but at the same time provided for the minimum acceptable or worst possible income. This amounted to planning on two levels--one level for the long run, based on expected incomes, and another level for the short run based on worst possible incomes. Again, this position was consistent with that of Heady who noted that the firm objective of staying in business, the survival objective, can only be a short-run objective.[8]

This method of providing for uncertainty closely paralleled that of McInerney who used a game theoretic approach to develop a maximin strategy with single period linear programming.[9] There were, however, some important differences. McInerney obtained estimates of all of the outcomes of each stochastic choice variable. The particular estimates used were the time series observations on the gross margins of each of the stochastic choice variables. The gross margins of the variables in a particular year were then regarded as one joint outcome. Then this outcome was treated as one of the possible outcomes, assuming one unit of each choice variable. The payoff matrix of joint outcomes, expressed as gross

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[8]Ibid., pp. 504-505.

[9]McInerney, op. cit.

margins per unit can be represented as:

Outcomes	Choice Variables			
	X_1	X_2	X_j	X_n
1	C_{11}	C_{12}.....	C_{1j}......	C_{1n}
2	C_{21}	C_{22}.....	C_{2j}......	C_{2n}
⋮	⋮	⋮	⋮	⋮
k	C_{k1}	C_{k2}.....	C_{kj}......	C_{kn}
⋮	⋮	⋮	⋮	⋮
T	C_{T1}	C_{T2}.....	C_{Tj}......	C_{Tn}

where: X_j = the jth choice variable, j = 1, 2 . . . , n,

C_{kj} = the kth outcome of the jth choice variable,

k = 1, 2, . . . , T.

Then, these joint payoffs were used to choose the optimal levels of the choice variables, according to the decision rule: Maximize the minimum gross margin, i.e.maximize the worst possible gross margin.

In this study, an estimate of only one outcome, besides the expected outcome, was required for each stochastic choice variable. This outcome was called the worst possible. The worst possible outcomes of the different stochastic choice variables, taken together, constituted a joint worst possible outcome. This outcome was then used in choosing the levels of the choice variables, according to the decision rule: Maximize the expected value of some variable, say income, subject to the constraint that the worst possible income in any one period be equal to or above some minimum. This rule was somewhat similar to the McInerney rule of maximizing the minimum gross margin. With parametric variation of the minimum income in this case, and of expected income in the McInerney case, each rule would generate efficient expected income-worst possible income pairs.

Other ways of allowing for uncertainty were considered, but on balance did not seem more appropriate. The method of quadratic programming was one possibility but this method is limited in the number of stochastic choice variables that can be handled in a multiperiod model.[10] Another possible method, generally referred to as discrete stochastic multiperiod programming, treats each joint outcome of the stochastic variables as deterministic, attaches a probability to it and maximizes expected returns or expected utility.[11] A disadvantage of using the approach in this study would have been one of problem size. For the kinds of problem situations to be analyzed--with several stochastic choice variables and possible outcomes per period and several periods in the planning horizon--the discrete stochastic programming method would have generated a very large matrix.

In concluding this section, two additional points should be mentioned. The first concerns the actual decision process of farmers in the study area. Choice of the worst possible approach as the appropriate one for allowing for uncertainty in this study did not imply a belief that farmers in the area used this decision rule. It did carry the implied assumption that decisions made according to this rule would approximate decisions actually made by farmers in the area in situations similar to those to be analyzed. The second point concerns the actual means of providing for the occurrence of the worst possible outcome. There are a variety of practical ways, some of them quite makeshift, for accommodating unfavourable outcomes. In the analysis, some of these different alternatives were

[10] In quadratic programming, the number of columns required to handle the nonlinear term in the objective function in each period is the square of the number of stochastic choice variables.

[11] See Cocks, op. cit. and Rae, op. cit.

considered.

TECHNIQUE OF ANALYSIS

Linear programming was chosen as the technique of analysis for this study. The main reasons were as follows: (1) it readily accommodated a time dimension, one of the important considerations in this study, and (2) it was an appropriate method of solving a constrained maximization problem of the kind found in this study.

In algebraic notations, the multiperiod linear programming model[12] is defined as:

(4.5) Maximize $Z = c_j^t \, x_j^t$

(4.6) Subject to $a_{ij_s}^{\;t} \, x_{j_s}^{\;t} \leqq\!\!\geqq b_{i_s}$

(4.7) $x_j^t \geqq 0$

where Z = the value of the objective function,

c_j^t = the value or weight of one unit of the j-th activity in the t-th time period,

$a_{ij_s}^{\;t}$ = the technical coefficient of the j-th activity in the t-th time period for the i-th resource in the s-th period,

b_{i_s} = the level of the i-th resource in the s-th time period

i = 1, 2, ... , m,

j = 1, 2, ... , n, and

s, t = 1, 2, ... , T.

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[12]The theoretical and mathematical structures of linear programming and linear economic models are available in many textbooks and are not repeated here. See for example G. Hadley, Linear Programming (Reading: Addison-Wesley Publishing Co., 1962). See also R. Dorfman, P. A. Samuelson and R. Solow, Linear Programming and Economic Analysis (New York: McGraw-Hill Book Co. Inc., 1958).

One of the assumptions of linear programming is that all of the choice variables are perfectly divisible and may take any positive real value within the constraints of the model. In actual farm situations some choice variables may only take integer values; practical examples are dryland and irrigation machines, livestock and in some cases farm land. The assumption of divisibility does not make any important difference in some situations. An important example is the case in which the problem is to define an optimal long-run plan without specifically considering the time dimension and the timing of decisions. In some, though not all of these cases, the optimal long-run plan may be essentially the same whether integer values are used or not.[13] In other cases, the optimal decision may depend very much on the assumption of divisibility, and forcing an integer value on a variable in the solution may change the decision substantially. This is most likely to occur when the planning horizon is multiperiod and when, as well as whether, to take a particular decision is important. The practical effect of imposing integer values on the solution is to extend the length of the planning horizon needed to establish a stable plan.[14] In other words, the transition period will be longer. An integer restraint on an item of equipment with a capital restraint that becomes non-limiting as capital is accumulated over time would be an example. A firm might not be able to afford an integer unit

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[13] Iga, op. cit., p. 244. See also W. J. Craddock, "Linear Programming Models for Determining Irrigation Demand for Water," Canadian Journal of Agricultural Economics, Vol. 19 (November, 1971), pp. 84-92.

[14] P. S. Barry, "Asset Indivisibility and Investment Planning: An Application of Linear Programming," American Journal of Agricultural Economics, Vol. 54 (May, 1972), p. 257.

until it has accumulated the necessary capital.[15] In this study, integer restraints were imposed on the acquisition of an irrigation system.

Single Stage and Multi-Stage Decision Processes. A multiperiod linear programming model represents a decision process that is conceived to be single stage; that is, decisions or plans for all periods are assumed to be made simultaneously and at a particular point in time. This conception of the decision process is in contrast to that represented in a dynamic programming model. In dynamic programming, the decision process is presumed to be multi-stage and decisions are presumed to be made in sequence. Provided one sees the decision problems in the context of decisions taken at different points in time, the decision process of dynamic programming (i.e. sequential decisions and recursive relationships) accords with the real world and with the way decisions are made.

The crux of dynamic programming is contained in Bellman's "Principle of Optimality" which states that:

> "An optimal policy has the property that whatever the initial state and initial decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."[16]

For practical purposes, managers of farms and other businesses attempt to operate by this principle. Given the state at any one point of time, managers try to determine, and carry out, an optimal policy, basis the information available at the time. Then given the outcome of that policy a new state exists and again a new policy is identified and

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[15] A similar situation was observed in the study of Boyko of developing land for irrigation; land was scheduled for development over a number of years, even though with adequate capital it would have been profitable to develop all of the irrigable land in the first period.

[16] R. E. Bellman, Dynamic Programming (Princeton: Princeton University Press, 1957), p. 83.

implemented; and similarly for later periods. The substantive difference between this procedure and the technique of linear programming is that linear programming specifies an optimal policy for all periods or stages in the planning horizon on the assumption that all of the information about future periods is known when the current period policy is established.

THE OBJECTIVE FUNCTION

The objective of the manager of the firm was assumed to be that of maximizing terminal net worth and it was further assumed that this objective would be approximated by maximizing the expected additions to net worth over the entire planning horizon. The reasoning behind the selection of this particular objective function is explained below.

It was felt that the three main possibilities for monetary objectives were: (1) expected income, (2) intended consumption, and (3) expected net worth. The question of which of these objectives would be most appropriate centred primarily on the nature of the consumption function.

If intended consumption spending could be regarded as a linear function of expected income, and if the balance after consumption could be assumed to be saved and added to net worth, then it would not matter which of these objectives were maximized. With a linear function, maximizing intended consumption or maximizing expected additions to net worth would be equivalent to maximizing expected income.[17] In other words, the same plan would be optimal and the same decision would be best, no matter which of these criteria were used as the objective function to be maximized. Given this relationship among these objectives when the consumption function is linear, the question was whether or not the assumption of a

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[17]For a derivation of proof that they are equivalent, see Boussard, op. cit., pp. 468-469.

linear consumption function was appropriate.

The practical alternative to a linear consumption function and a constant marginal propensity to consume--through time and through different income levels--was one that declined with increases in income and/or changed through time in some systematic way, and there was no compelling reason to expect either of these conditions. The empirical evidence from time series data on consumption expenditures supports a constant marginal propensity to consume with respect to income,[18] and there was no reason to expect the intended marginal propensity to consume to be more or less in, say, the fifth year than the first year of the planning horizon. There may well be reasons for planning more or less spending on consumption in particular years to accommodate anticipated costs such as clothing and education. Outside of these kinds of situations, it was felt realistic to assume a constant marginal propensity to consume, with respect to time and with respect to income. Accordingly, a linear consumption function was considered to be appropriate and therefore the objective of maximizing expected additions to net worth with a linear consumption function was also felt to be appropriate.

SUMMARY

To summarize, it is proposed to specify the decision process of the farm firm within the framework of a multiperiod linear programming model. The firm manager's utility function is presumed to be embodied in a lexicographic formulation, together with a satisficing objective. This utility function is assumed to be maximized by farm plans which maximize terminal

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[18] M. Friedman, A Theory of the Consumption Function (Princeton: Princeton University, 1957). See also M. K. Evans, Macroeconomic Activity, Theory, Forecasting and Control (New York: Harper and Row, 1969), Ch. 2 and Ch. 3.

net worth--a long-run objective--subject to the constraint that the worst possible income be equal to or greater than some predetermined level--a short-run objective. It has been argued that a farmer takes specific recognition of the uncertainty of the forecast income in formulating plans for the near term but uses only the expected income criterion in formulating his long term plans. It has also been argued that the financial resources of the firm may be an important determinant of the acceptable degree of variation in the expected income in the short run. It is proposed to investigate the effects of this argument in the analysis of this study. Details of the decision model for the analysis are presented in the next two chapters.

CHAPTER 5

MAJOR FINANCIAL COMPONENTS AND MECHANICS OF HANDLING UNCERTAINTY IN THE MODEL

This chapter outlines the major financial components of the model and the mechanics of incorporating uncertainty considerations in the decision process. As indicated in the previous chapter, it was decided to employ two parameters as decision criteria in the planning process--specifically the expected additions to terminal net worth and the worst possible income in any one year. The purpose of this chapter is to describe how the worst possible criterion was integrated into the model as a means of allowing for uncertainty.

The multiperiod linear programming planning model is designed to specify a plan for each period of a ten-period planning horizon, for a given set of resources and a given set of choice variables. This aspect of a multiperiod linear programming model is illustrated in Figure 9. The illustration is for the first two periods and similar components for later periods are assumed.

There is an initial set of resources available in each period, and

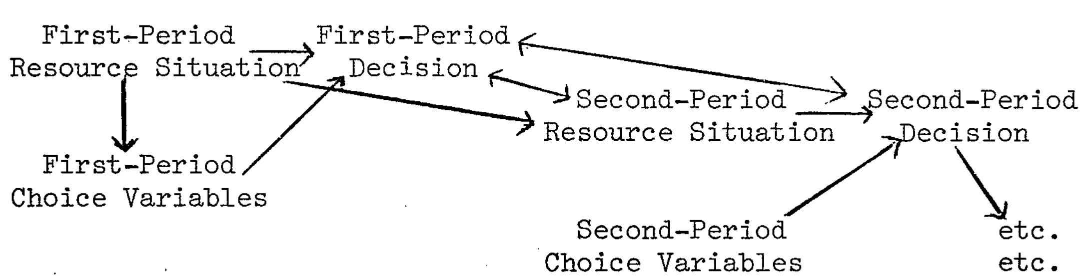

Figure 9

Illustration of Basic Information Required and Provided by Decision Model there is a set of choice variables in each period. These sets of data are the basic information required by the model. The basic information provided by the model are the decisions--first-period decisions, second-period decisions and so on. In addition, the model provides information about the expected resource situations in the second period, the third period and so on. In this particular model, the expected resource situation in the terminal period is the most important since the model is set up to maximize expected terminal net worth.[1]

Major Financial Components of the Model and the Flow of Financial Resources

Figure 10 shows, for one period, the major financial components of the model and illustrates the flow of financial resources from one period to the next. As shown in Figure 10, there is an initial set of resources--management, land, labour and capital--and a set of production and investment alternatives. Conceptually, the model computes expected gross income, expected costs, expected net income and finally the expected addition to the net worth of the firm in each period. The projected addition to net worth is really the expected end result of the decisions for the period. As shown on the right hand side of Figure 10, this amount is accumulated along with the expected additions to net worth in other periods to maximize the cumulative additions to net worth--i.e. the terminal net worth of the firm.

Most of the components of the computation of the expected addition to net worth are self-explanatory. As shown in the figure, operating

[1]It should be noted again that the optimal solution of a multiperiod model is obtained for all periods simultaneously. The optimal plan for each period cannot be interpreted as though the decisions are sequential.

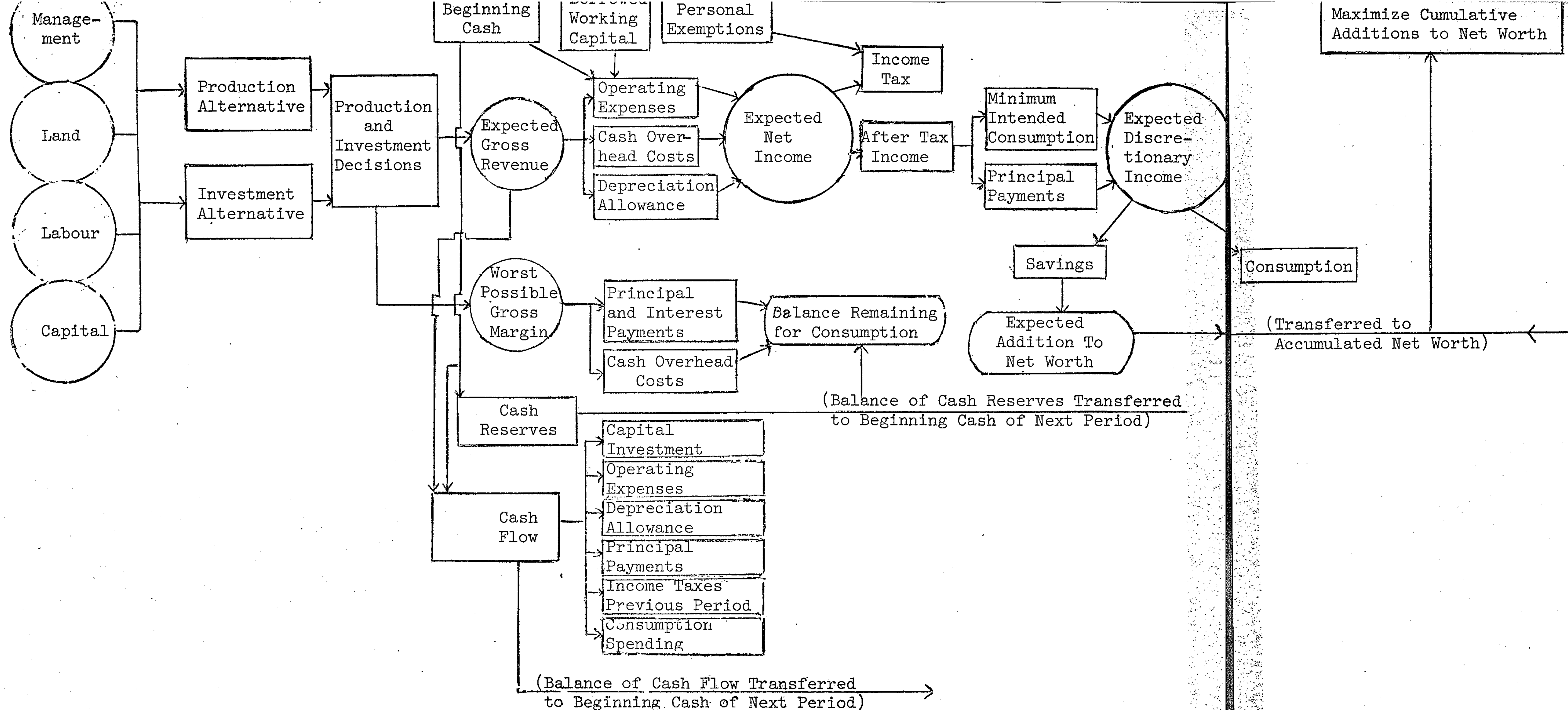

Figure 10

Schematic Representation of Major Financial Components and of the Flow of Financial Resources for One Period of the Model expenses for the period may be financed out of beginning cash on hand and/or by short-term borrowings from a commercial lender. Expected after-tax income is a function of the level of personal exemptions of the operator and of the level of income. Income tax activities are built into the model in each period to take account of the increasing marginal tax rates with increasing taxable income. Minimum intended consumption spending and principal payments on existing loans have first call, after taxes, on the net income of the firm. The amount of the minimum consumption spending is the intercept term of the consumption function. The balance of after-tax income remaining after subtracting minimum consumption and principal payments is allocated between savings, i.e. net worth, and consumption. The proportion going to each is determined according to the marginal propensity to consume.

Cash flow is identified in Figure 10 as a separate financial component of the model. The cash flow accounts for the net flow of funds from all sources and to all uses. In this sense it is different from the net income component which computes for accounting purposes the net change in the financial positions of the firm. Income tax payments are a charge against cash flow and are assumed to be made out of cash flow in the year following the year in which the income was earned. Cash flow can serve as a source of funds to purchase capital assets--specifically irrigation equipment, beef cattle and beef cattle housing. Depreciation charges, i.e. capital cost allowances, are shown as charges against cash flow on the assumption that this amount of money is actually withdrawn and used to replace capital equipment. The assumption in the analysis is that this is done each year. In practice, depreciation funds need not be withdrawn each and every year and many farmers at certain times use them for other purposes. In the event of unfavourable outcomes, farmers often allow a depreciation deficit to accumulate and use the annual depreciation allowance for other than equipment replacement. Finally, any balance remaining in cash flow at the end of the period is transferred to beginning cash on hand in the next period.

Worst Possible Gross Margin and Worst Possible Income

The estimated worst possible gross margin, basis a particular decision, is computed in each period. The computation is based on the estimated worst possible outcome of each production activity in the final plan for each period. In each period there is a requirement that the worst possible gross margin be sufficient to pay the cash overhead costs of the firm, the principal and interest payments on loans and provide some balance for consumption. As indicated in Figure 10, the estimated worst possible gross margin may be supplemented from cash reserves which in turn may be supplemented from cash on hand at the beginning of the period. In the analysis, the balance to be available for consumption spending is initially set at a negative level and is increased by parametric variation of the right-hand side of this restraint. Any balance of cash reserves remaining at the end of the period is transferred to beginning cash at the start of the next period.

The Worst Possible Income Constraint

The mechanics of taking account of uncertainty within the model and within the context of the financial components identified in Figure 10 are illustrated in Table 2. This table shows the main activities that are concerned with this aspect of the model. The key restraint concerning uncertainty and one of the two parameters used as decision criteria is the worst possible income constraint. This restraint is shown in row 4 of Table 2; it requires that the worst possible gross margin of the production alternatives in the optimal plan plus cash reserves less overhead costs and principal and interest payments be equal to or greater than some specified amount. In the illustration, this amount is set at five hundred dollars; in the actual analysis, different values for this restraint were assumed. To spell out this restraint more clearly it requires that:

Worst possible gross margin + Cash reserves - Cash overhead - Principal and interest payments ≥ Minimum acceptable income.

As indicated above and in Table 2, the argument of this restraint assumes that cash reserves can be used to supplement the worst possible gross margin from production activities. The "use cash reserves"

Table 2

Activities and Restraints Concerned with Allowing for Uncertainty in the Decision Process

Restraint Description	Activity Description					
	Production Activity[a]	Transfer Beginning Cash to Cash Reserves	Use Cash Reserves	Pay Cash Overhead Expenses	Pay Principal and Interest	Right Hand Side
	(1)	(2)	(3)	(4)	(5)	(6)
	....		dollars	....		
(1) Net Income	-50.			1.	.5	= 0
(2) Beginning Cash		1.				≤ 2000
(3) Cash Reserves		-1.	1.			≤ 3500
(4) Worst Possible Income	10		1.	-1.	-1.	≥ 500
(5) Cash Overhead					1.	= 2000
(6) Principal and Interest Payments					1.	= 3534

[a] Each production activity in the model has a coefficient in the net income row and in the worst possible income row. This is an illustrative example of this aspect of the production activities; the coefficients are hypothetical.

activity in column 3 of Table 2 takes cash from the cash reserves row to satisfy the worst possible restraint. Cash reserves in turn can be supplemented by the activity in column 2 to transfer beginning cash to cash reserves.

Reserves of course, if they are held at all, need not be held in cash; they can be held in the form of borrowable reserves at a lending agency. In many practical situations, farmers accommodate unfavourable outcomes by taking advantage of these borrowable reserves and increasing their short-term loans. Although not shown in Table 2, this alternative was considered in the analysis. In terms of satisfying the worst possible income constraint, it has the same effect as using cash reserves. However, there is an important difference; instead of drawing down cash reserves, this alternative increases the cash flow requirements in the next period or in some subsequent period because these kinds of loans must be repaid. In the analysis, it was assumed that these loans would have to be repaid out of income generated in the next period. In practice farmers may be able to spread the repayment of borrowed reserves over more than one period. In any case, the repayment of borrowed reserves would be expected to place a heavy requirement on the cash flow of the business in the year of repayment and could act as a restraint on the amount of borrowable reserves that could be profitably used.

In the model and as illustrated in Table 2, the restraints for cash overhead and for principal and interest payments, rows 5 and 6, respectively, were restraints that had to be met and were specified as equalities. Operation of the activities shown in columns 4 and 5 of the table satisfied these restraints. The activity to pay principal and interest in column 5 is shown to have a coefficient of less than one in the net income row. This simply indicates that some proportion of each dollar of principal and interest payments was interest and only this proportion was used in computing net income.[2]

As far as the right-hand sides of the restraints were concerned, the values shown in Table 2 were the actual values initially used in the first period of the model--with the exception of the worst possible income value that was mentioned previously. The operation was initially assumed to have beginning cash of 2,000 dollars, cash reserves of 3,500 dollars, cash overhead expenses of 2,000 dollars and principal and interest payments of 3,534 dollars.

In terms of what to expect in the analysis with this formulation of the model and with the two decision parameters of expected addition to net worth and worst possible income in any one period, it would be anticipated that higher values for the minimum income constraint would be associated with optimal plans which had smaller expected additions to net worth. If the worst possible income restraint was not binding, the optimal plan of course would be the same as the expected value or deterministic plan. In other words, if the only criterion for decision making was the total expected addition to terminal net worth, the optimal plan would be the deterministic plan. By assuming different values for beginning cash, and cash or borrowable reserves, and by including the worst possible income criterion as a binding restraint, it would be expected that the effects of these different values on decisions could be evaluated.

<u>One or Two Bad Years in a Row?</u> There is an important aspect of the use

[2]For illustration purposes, a coefficient of .5 was used in Table 2. In the empirical model, the proportion of interest in each dollar of principal and interest in each period depended on the initial terms of the loan (mortgage) and the number of years remaining until the loan (mortgage) would be paid off.

of cash reserves that should be explained. It concerns the occurrence of worst possible outcomes. Suppose that the worst possible outcome occurs in the first period and it is necessary to draw on cash reserves to satisfy the worst possible income constraint. If there were, say, 10,000 dollars of cash reserves at the start of the period and 3,000 were used, then 7,000 could be transferred to the next period. If the worst possible result was assumed to occur in the next period as well, then the cash reserves would be drawn down again; and similarly in subsequent periods. The issue here concerns the chances of worst possible or unfavourable outcomes two years in a row and the extent to which an operator should prepare for more than one 'bad' year in a row. If the operator decided that the chances of two unfavourable years in a row were not worth considering, then the cash reserves used in the second period should be available for use in the third period; those used in the third period should be available in the fourth period; and so on. Only in the first period should the cash reserves used be assumed to be unavailable for use in the next period.

The mechanics of handling different assumptions in this regard within the model are illustrated in Table 3. In this illustration, it is assumed that the operator is preparing for just one unfavourable year. The "use cash reserves" activity in the first period has a positive coefficient in the "cash reserves" row and a negative coefficient in the "worst possible income" row in the first period indicating that this activity uses up cash reserves and supplies the worst possible income. However, in the second period, the "use cash reserves" activity has an additional coefficient. This coefficient is in the "cash reserves" row of the third period; it is negative, thus indicating that this activity supplies cash reserves to the third period. Had the assumption been that the operator wanted to prepare for two bad years in a row, the "use cash reserves" activity in the second period would not have supplied cash reserves to the third period.

The distinction between these assumptions is fairly important. If the cash reserves are actually used, the effect should be quite different from the case in which they are available but are not used. In the analysis of Chapter 7, the usual assumption was that the operator prepares to actually use the cash reserves in the first period only. However, in one solution it was assumed that the operator does want to prepare for two bad years in a row and prepares to actually use the cash reserves in both the first and second periods. The decision strategy in these solutions would likely be more conservative than that in which the operator prepares for just one bad year.

Table 3

Submatrix of Input-Output Coefficients Concerned with the Use of Cash Reserves in the Linear Programming Model

	Activity Description Use Cash Reserves	
Restraint Description	Period 1	Period 2
Cash Reserves, Period 1	1.	
Worst Possible Income, Period 1	-1.	
Cash Reserves, Period 2		1
Worst Possible Income, Period 2		-1.
Cash Reserves, Period 3		-1.

Worst Possible Gross Margins of Production Alternatives

The worst possible income constraint in the model required estimates of the worst possible gross margins of each production alternative considered. While it was recognized that these estimates were necessarily partly subjective, the argument was that the typical farm manager does have some conception of the worst that could happen and in some cases has information in this area that is quite objective. Examples of this latter type of information are the guarantees that crop insurance coverage can provide in terms of worst possible gross incomes per acre and that futures hedging can provide in terms of price per unit of certain commodities.

For the empirical application of the model, it was decided to consider three variables as stochastic. In computing the worst possible gross margins per unit of the production activities, unfavourable outcomes in terms of crop yields, crop and livestock prices and irrigation water requirements per acre were considered.[3] As far as the latter variable was concerned, an unfavourable outcome was one that required a heavy application of water.

The worst possible gross margins were computed by obtaining estimates of the worst possible prices and yields and subtracting from this amount the operating or variable costs of production. This provided a dollar estimate of the worst that could occur on a per unit basis for each production possibility. In the final plans, the sum of these worst

[3]It was originally intended to consider marketing quotas as a source of variation, and worst possible marketing quotas per acre were estimated for crops subject to delivery quotas. (These estimates are reported in Table B. 7.) On further reflection, it was realized that price and marketing quota were highly correlated so marketing quota was not included as a separate variable.

possible gross margins was the amount that would be available, given the worst possible outcomes, to pay overhead expenses, principal and interest and to provide some balance for consumption.

CHAPTER 6

INITIAL RESOURCE SITUATION, MAJOR RESTRAINTS AND PROVISIONS IN THE MODEL, AND DATA DEVELOPMENT

In this chapter, the initial set of resources for a farm situation is identified, the major restraints and provisions of the model are described, and the general procedures for developing the data are outlined. In discussing the various parts and features of the model it is assumed that it is not necessary to refer specifically to each restraint and/or activity that may be concerned with incorporating a particular provision into the model. The mathematical formulation of the model, including a description of each variable and restraint, is presented in Appendix A. The matrix of input-output coefficients for the first period is presented in Appendix D.

Characteristics and Resources of Initial Dryland Farm Situation

The basic model for the analysis was developed and given an empirical application for a hypothetical ongoing farm situation located in the study area of the Souris Basin. This farm was the focus for the analysis and was the base resource situation to which the model was applied. Modifications in the resource supply of this farm permitted some evaluation of the effect of differences in the initial set of resources on the optimal farm plans. The managerial, physical and financial assets and liabilities which constituted the major planning restraints for this initial situation are described below.

Before proceeding to the description of the set of resources, some explanation of the general approach to this aspect of the study would be in order. As mentioned above, the farm situation that was analyzed was hypothetical; it did not represent an actual farm situation. Two main considerations influenced the identification of the resource set of this hypothetical farm. First, it was considered important that the basic resource endowment be, in some senses, fairly typical of farms in the Souris Basin of Manitoba. Secondly, it was considered important that the model be applied to the analysis of a specific farm situation. The reasoning with respect to this latter point was that each farm situation is quite unique for planning and decision purposes and that it would, therefore, be appropriate to assume a specific typical farm. Accordingly, the farm situation that was analyzed was considered fairly typical of many farms in the area in respect of farm size, machinery complement and labour supply but was not necessarily typical in respect of asset and debt structure.

In developing the hypothetical farm, the first step was the selection of a farm size. A review of census data indicated that farms of about one section in size were among the most common in the area so a farm of this size was selected. Then, in consultation with the agricultural representative for the area and with evidence from other sources, a fairly typical livestock, equipment and labour complement for this farm size was identified. Finally, a specific asset and debt structure was imposed on this typical complement of resources. To represent an ongoing situation, particular values were assumed for such items as amounts of cash on hand, readily accessible savings, and intermediate- and long-term loans. In assuming the values for these items, no particular attempt was made to ensure that they were typical of one section farms in the area. The argument here was that each one section farm is unique, at a specific point in time, in respect of items such as those just mentioned and that the values attached to these items constitute important constraints on the planning and decision environment of the operator. Accordingly, it was considered reasonable to assume particular values for certain financial assets and liabilities without regard to whether or not they were especially typical. At the same time, the values assumed were considered to be well within the range of possibilities for one section farms in the area.

In the process of identifying a specific farm situation, it was necessary to make fairly detailed assumptions concerning such characteristics as: (1) family size for income tax purposes, (2) operator age for mortgage insurance purposes and (3) amount, term and interest rate for intermediate- and long-term loans for purposes of determining the annual principal and interest payments in different periods of the planning horizon. In addition, specific values were assumed for the amount of grain in inventory at the start of the planning horizon and for the annual cash overhead expenses of the business. The values assigned to these items were realistic but it must be emphasized again that they were not for an actual farm situation.

Managerial and Family Characteristics. The hypothetical farm was assumed to be family-operated with a single proprietorship form of business organization. The operator was assumed to be forty years of age and to be married with no dependent children. He was assumed to have average managerial ability and to be capable of achieving the level of yield performance in crop production reflected in the average yields of Crop District Number 1, Province of Manitoba.

Physical Assets. The physical assets considered were land, livestock, equipment, improvements and the on-farm supply of labour. The supplies and characteristics of these assets are described below:

1. Land. The land resource of the farm was assumed to total 640 acres or one section. Of the total acreage, 550 acres, or approximately eighty-five percent, were improved and suitable for use in a regular rotation. The remaining acres were assumed to be unimproved and unproductive.

The first period supplies of stubble, fallow and forage acreages are shown in Table 4. The improved acreage of the farm was assumed to be comprised of soil zones G and H. These two soil productivity zones were the dominant ones in the study area and the proportions of the improved acreage of the study farm assigned to each soil zone was approximately in accordance with the proportions existing in the area.[4] Of the two soils, zone G is the more naturally productive; relative to the most productive

Table 4

First-Period Acreages of Stubble, Fallow and Forage Land for Initial Farm Situation

	Description	Acres
Soil Zone G:	Total improved land	200
	Stubble land	125
	Fallow land	50
	Forage land	25
Soil Zone H:	Total improved land	350
	Stubble land	225
	Fallow land	100
	Forage land	25

[4]According to records of the Manitoba Crop Insurance Corporation, soil zones G and H constituted about two-thirds and one-third, respectively, of the insured acreage in the study area.

soils, both soil zones are below average in productivity.[5]

The proportions of the improved land of each soil zone assigned to stubble and fallow were in accordance with usual practice in the area; most farmers in the area summerfallow about one-third of their cultivated acreage and crop the balance.[6] As shown in Table 4, 25 acres of each soil zone were assumed to be seeded to forage to support a small beef enterprise.

2. Livestock. A small beef cow enterprise was typical on the average one-section farm in the area and was assumed to exist on the hypothetical farm as well. The inventory at the start of the first period--calendar year 1973--was assumed to include twenty-five bred cows and heifers and five yearling heifers.

3. Equipment and Improvements. A set of equipment for a typical one section dryland farm in the area was identified.[7] It was assumed that this set of equipment would have adequate capacity and give satisfactory performance for all of the possible combinations of dryland production alternatives considered. Besides a dwelling for the family, improvements to provide winter shelter for the beef cow herd and to store equipment and grain were assumed.

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[5]Soil productivity zones are classified from A through J with soil zone A being most productive and soil zone J being least productive. See 1972 Rates and Coverages, Manitoba Crop Insurance Corporation (mimeo).

[6]From discussions with the agricultural representative in the area, Mr. R.E. Filteau, a fairly typical rotation is wheat-flax-fallow.

[7]With the co-operation of the agricultural representative, Mr. R. E. Filteau, the major components of an equipment complement were identified. They are reported in Table B.10 as part of the data base used in budgeting machine operating costs.

For the analysis, equipment was assumed to have a value of 10,600 dollars and improvements were assumed to have a value of 6,000 dollars. Using income tax rates for computing depreciation by the diminishing balance method, the annual depreciation on these items was in line with the depreciation expenses reported by similar sized farms.

4. Labour Supply. Six different categories of on-farm supply of operator and family labour were identified as possibly limiting resources at particular seasons and for particular tasks. The categories identified and the assumed labour supply for each were as follows:[8]

a) planting - 500 hours,

b) hay harvesting, first cut - 400 hours,

c) hay harvesting, second cut - 200 hours,

d) field crop harvesting I - 600 hours,

e) field crop harvesting II - 100 hours,

f) winter care and maintenance of beef cattle - 8 hours per day.

The categories of labour and the estimated supplies were rough approximations to the real situation and it is recognized that a planning and decision model that would take detailed account of labour as a potentially limiting resource would require more precision than is represented here. At the same time it was not expected that labour would be an important limiting resource on a one-section farm. To the extent that it would be, it was felt that the average farmer has considerable short-run flexibility in the amount of labour that he can supply and in the amount of labour required for different operations. He can, for example, work long hours

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[8] Adapted from: Finn, op. cit., p. 82; Singh, op. cit., p. 34; and from other sources.

for short periods of time and can adjust work methods to save time.[9]

Financial Assets and Liabilities. The major components of the asset and liability structure that was assumed for the farm business are shown in Table 5. As noted in an earlier section, a specific asset and liability structure was selected to represent an on-going operation and to tie in with the emphasis of the study on planning on the basis of a given financial position at a particular point in time.

The operator was assumed to have no short-term liabilities such as accounts payable or demand loans. However, debt commitments for intermediate- and long-term loans were assumed and were built into the model for the analysis. The terms and conditions for the intermediate-term loan were the prevailing ones for Farm Improvement Loans for equipment and those for the long-term loan were those prevailing for first-mortgages from Farm Credit Corporation. The Farm Improvement Loan was assumed to have one principal payment remaining while the principal payments on the mortgage extended over the entire planning horizon of the model.

Crop Production and Marketing Alternatives

Activities representing the production and/or marketing of wheat, oats, barley, flax, rapeseed, field peas, sunflowers, hay, pasture and corn silage were included in each period of the model. Different production choices were identified for each of these crops according to whether they were: (1) uninsured or insured, (2) dryland or irrigated,

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[9]This is not to argue that labour use and labour supplies are not important factors to consider for certain problems. Expansion alternatives, such as a substantial increase in farm size, are often conditional on the supply of labour and on improvements in the efficiency of labour use.

Table 5

Financial Assets and Liabilities of
Farm Situation, Basis January 1, 1973

Assets		Liabilities	
	Short Term		
Cash on hand	$2,000		
Crops held for sale[a]	3,690		
Marketable securities[b]	3,500		
	Intermediate Term		
Machinery and equipment	$10,600	Equipment loan[c]	$1,250
Cattle[d]	10,000		
	Long Term		
Farmland[e]	$53,000	Mortgage[f]	$26,923
Improvements	6,000		
		Net Worth	$60,617
Total Assets	$88,790	Total Liabilities	$88,790

[a]Assuming 1500 bushels of wheat at $1.76 per bushel and 1000 bushels of barley at $1.05 per bushel. The per bushel values assumed were the initial Canadian Wheat Board payments, basis Thunder Bay, for the 1972-73 crop year.

[b]Canada Savings Bonds or other readily marketable securities.

[c]Assuming an original loan of $5,000 in 1969 with annual payments of $1,250 and interest at $7\frac{3}{4}$ percent.

[d]30 beef females - 25 cows and 5 heifers.

[e]Land without improvements was valued at approximately $92 per acre. Yearbook of Manitoba Agriculture, 1971.

[f]Assuming original F.C.C. mortgage of $30,000 in 1965, amortized over 29 years, with interest rates of 5 percent on the first $20,000 and $6\frac{3}{8}$ percent on the balance. Including mortgage insurance, the annual payments were approximately $2,086.

(3) seeded on soil zone G or soil zone H, (4) seeded on stubble or on fallow and (5) seeded without commercial fertilizer, seeded with a light application of commercial fertilizer or seeded with a heavy application of commercial fertilizer.

All of these possibilities generated a fairly large number of crop production activities; there were for example twelve dryland uninsured wheat production activities. While this increased the size of the matrix, it was felt to be worthwhile to have all of these alternatives identified because they represented substantially different choices in terms of the two decision parameters of the model--expected contribution to net worth and worst possible income. Along this line, it was felt to be important to consider the use of a fairly heavy application of commercial fertilizer in crop production. Increased use of fertilizer is one method of improving expected yields and expected returns, although there may be costs in this regard in terms of the worst possible outcome. Another reason for including crop production choices with a heavier than normal application of fertilizer was to compare dryland and irrigated alternatives under approximately similar fertility levels.

Wheat, oats, barley, flax and rapeseed crops had marketing activities that were separate from their respective production activities. The reason for this was that farmers have delivery quotas on these crops and cannot always market as much as they might wish. In each period there was a separate selling activity for each of these crops which drew out of inventory or supply rows. Along with the supply rows for these crops, these activities allowed planned production in the short term to exceed or fall short of the expected delivery quotas, with the surplus (deficiency) going into (coming out of) storage.

Field peas and sunflowers were assumed to be marketed in the year of production and the activities for these crops represented both production and sale of these crops. Corn silage, hay and pasture activities produced intermediate products which could be used in the beef cattle enterprise. Hay and corn silage crops supplied forage for winter feeding of beef cattle while the pasture activities supplied forage for summer grazing.

Beef Cattle Production Alternatives

Two beef cattle production activities were specified in each period. One represented the livestock enterprise already on the farm--a beef cow-calf enterprise with the calves sold as feeder calves in the fall. The other was a beef cow-calf enterprise which represented the production of long yearlings. In this enterprise, the spring calves are wintered over and sold off pasture the next fall.

Capital-intensive beef operations such as feedlot production were not considered as possible choices in the model. The thinking was that since a capital-intensive irrigation system was being considered, only relatively capital-extensive beef cattle alternatives should be considered. With capital supplies likely to be rationed either internally or externally or both, this was felt to be a reasonable assumption.

Price Expectations Used

Since the usual price expectation models dealt with the expected price in just the first or current year, they were not fully appropriate for determining the prices to be used in this study. The methodology followed in obtaining the prices that were used in the different years was fairly subjective and required a number of assumptions. The principal assumptions about the prices of farm inputs and outputs are outlined below.

1. Farm Inputs. The prices of farm inputs were assumed to be non-stochastic and known with certainty. This was felt to be a reasonable assumption; except for trend, there had been relatively little year-to-year variation in the index of farm input prices in the period 1961-1971. The trend however was important; it amounted to 3.34 per year in the index of farm input prices, 1961 = 100, basis Western Canada, and explained 98 percent of the variation in the index during this period.[10] While all categories of farm inputs did not experience the identical steady trend in price in the period, they were all trending up and it was decided to ignore any differences in trend among categories of inputs. Operating cost budgets were prepared on the basis of estimated 1972 prices. The estimated costs in 1973 and subsequent periods were obtained by adding the appropriate allowance for trend to the estimated 1972 costs.

2. Farm Outputs. Prices of crops and cattle were regarded as stochastic variables in this study. In developing their prices for the different periods, the long-term--the years beyond the third--were considered separately from the short- or near-term--the first three years. For each year beyond the third, it was assumed, in all but two cases, that the ten-year average price, plus an allowance for any important trend was the best estimate of expected price.

(a) Crop Prices. In projecting expected prices of the different crops in the long term, the main consideration was whether or not there was any trend in price that should be included. From inspection of the time

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[10]Significant at the .005 percent level; based on linear regression of eleven observations. Data obtained from Catalogue 62-534, Statistics Canada, Farm Input Price Indexes 1961-1971 (Ottawa: Information Canada, 1971), p. 22.

series of prices, it was felt that trends could be safely ignored.[11] Cyclical and year-to-year variation in prices were felt to be so important that, for planning purposes, it would be unrealistic to attempt to incorporate any statistically significant trends into the expected long-run average price.

The estimates of the expected and the worst possible prices in the near term amounted to informed guesses about prices in each of those years. Although some objective information was evaluated and used in developing those estimates, the final estimates were quite subjective. The prices used were assumed to be the expectations held by a hypothetical decision maker at one point in time--specifically at the time of deciding what to do in the first year, calendar year 1973.[12]

Generally farmers have better information about near-term prices than about long-term prices and in practical situations it is probably not realistic to plan on the basis of say, ten-year average prices. A farmer may know, or feel that he knows, that the current year's price will be below or above the ten-year average. This is especially true when prices are cyclical in nature. For planning purposes, an individual farmer could specify some expected price and some worst possible price in the near term. While these estimates would also be subjective, they would incorporate into the planning process the farmer's own feelings about the kinds of outcomes that could occur.

(b) Beef Cattle Prices. There had been an important upward trend in

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[11]See Table B.2 in Appendix B.

[12]The estimates for expected and worst possible prices for crops are shown in Table B.3 in Appendix B.

beef cattle prices during the ten-year period 1962-1971.[13] However, it was felt unrealistic to expect the trend to continue indefinitely and to incorporate it into the expected long-run average. As a compromise, the five-year average price, basis 1968-1972, was used as the expected long-run price. The expected and worst possible prices for the near-term years were, as for crops, subjective estimates.[14]

Consumption Function

The planning model was set up to handle linear consumption functions, either proportional or non-proportional. The consumption function employed for the applications of the model was non-proportional and had the following form:

$$C_t = 3000 + .4\,(Y_{d_t} - 3000 - P_t) \qquad (6.1)$$

where: C_t = intended consumption in period t,

Y_{d_t} = expected after-tax income in period t, and

P_t = payments on principal (or forced savings) in period t.

While this function did not have an empirical foundation, it was felt to be a reasonable function for planning purposes for an individual farm. It specified that the family intends to spend a minimum of 3,000 dollars on family and consumption items[15] and that of any balance of after-tax income remaining after principal payments on existing loans, 40 percent

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[13]See Table B.4 in Appendix B.

[14]The estimates for expected and worst possible prices for beef cattle are shown in Table B.5 in Appendix B.

[15]Consumption expenditure records of a sample of farms in the Western Farm Business Association in Manitoba found average expenditures of $3,283 per family in 1968 on farms of about one section in size. See: Mitchell, op. cit., p. 166. The assumption in equation (6.1) of an MPC of .4 was considered reasonable in terms of total expected consumption spending. Although the function was biased in favour of saving, it did provide for some increase in consumption spending as income (after taxes and after principal payments) increased.

will be consumed and 60 percent will be saved or added to net worth. It was felt that the assumption that principal payments had first claim on after-tax income was practical and realistic. Generally principal payments represent a kind of forced saving and farmers can only realistically plan to consume out of any balance remaining after these payments are satisfied.

It was originally intended to also consider a proportional consumption function of the form:

$$C_t = b\,(Y_{d_t} - P_t) \qquad (6.2)$$

where b = a positive constant

and other variables are as previously defined.

In the final analysis, it was decided not to consider this functional form. Since this function implied that a plan could be optimal with no expected balance for consumption expenditure,[16] it was not felt to be realistic for planning purposes.

Irrigation System and Irrigation Water Requirements

Of the several different methods of applying irrigation water that could be considered as potentials in the Souris Basin, the one considered in the model was a centre pivot sprinkler system.[17] With this system, water is delivered to the centre of a quarter section and the lateral, supported on two-wheeled or tracked towers, is self-propelled in a circle around the central pivot point. It is suitable for irrigating high crops since the lateral is seven to nine feet above the ground. The net acreage irrigated by this system is 138 acres.

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[16]After principal payments out of after-tax income.

[17]H. C. Korven et al., Irrigation on the Prairies, Publication 1488 (Ottawa, Agriculture Canada, 1972), pp. 12-16.

The centre pivot sprinkler system is one of the most capital-intensive irrigation systems available. It requires a capital investment of more than 200 dollars per acre.[18] While it can be argued that a system that was less capital-intensive would be more appropriate to consider in the model, it was felt that farmers in the area, if they were going to adopt irrigation farming at all, would be most likely to adopt capital-intensive systems.

The estimated average water deficit on August 13 at the Pierson[19] weather station--6.95 inches--was taken as an estimate of the average net irrigation water requirements per acre for wheat, oats, barley, flax, rapeseed and pea crops. For the full season crops of sunflowers, corn silage, hay and pasture, the average deficit of 10.50 inches, basis September 20 at the Pierson weather station, was used as an estimate of average net irrigation water requirements per acre. To allow for inefficiencies in water use, the net requirements on August 13 and September 20 were increased, by twenty-five percent, to 8.69 inches and 13.12 inches per acre, respectively.

Restraints and Requirements in the Model

To enable an understanding of the restraints and requirements in each period, the purpose and function of the different restraints are described below.

A. Accounting Rows and Restraints

Net Farm Income. This was an accounting row that computed net

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[18] The estimated capital costs and operating costs per acre-inch for a centre pivot system are shown in Table B.8 in Appendix B.

[19] The location of the Pierson weather station is shown on the map on page 5.

farm or before-tax income. Activities which added to the revenue of the firm, such as wheat selling had a negative coefficient in this row, while those that represented expense items, such as cash overhead expenses, had a positive coefficient. This restraint was set equal to zero and was satisfied by transferring net farm income to the row that computed the expected addition to net worth.

Expected Discretionary Income. This was also an accounting row. It computed the expected addition to net worth in each period and accordingly played an important role in terms of the objective function. The operation of this row can be expressed as:

After-tax income - minimum intended consumption - principal payments + expected discretionary income = 0.

Income tax activities computed the after-tax income for this row and the minimum consumption expenditures and principal payments were deducted from the amount of after-tax income leaving a balance of expected discretionary income. A special activity apportioned this balance between consumption and addition to net worth and transferred the addition to net worth to a row that accumulated the additions to net worth from all periods.

Income Tax Brackets. Eleven rows and columns represented the different income tax brackets and marginal tax rates on net farm income according to the National and Province of Manitoba tax schedules in 1973. The submatrix of these activities and restraints concerned with income tax is shown in Table 6. As the farm operator had personal exemptions of 3,000 dollars (basis 1973 exemption levels), the first 3,000 dollars of net farm income was not taxable.

Cash Flow. This row accounted for the net inflow and outflow of

Table 6

Submatrix of Restraints and Activities for Income Tax, Period t

Restraint Description	Income Tax Bracket												Right Hand Side
	1	2	3	4	5	6	7	8	9	10	11		
	dollars												
Net Farm Income	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	=	0.
Discretionary Income	-1.	-.76285	-.74890	-.73495	-.72100	-.70705	-.67915	-.65125	-.62335	-.56755	-.51175	=	0.
Tax Bracket 1	1.											≤	3000.
Tax Bracket 2		1.										≤	500.
Tax Bracket 3			1.									≤	500.
Tax Bracket 4				1.								≤	1000.
Tax Bracket 5					1.							≤	1000.
Tax Bracket 6						1.						≤	2000.
Tax Bracket 7							1.					≤	2000.
Tax Bracket 8								1.				≤	2000.
Tax Bracket 9									1.			≤	2000.
Tax Bracket 10										1.		≤	3000.
Tax Bracket 11											1.	≤	10000.
Cash Flow[a]		.23715	.25110	.26505	.27900	.29295	.32085	.34875	.37665	.43225	.48825	≤	0.

[a]The cash flow restraint refers to period t+1. The coefficients in this row represent the marginal tax rates on income earned in period t with the income tax to be actually paid in period t+1.

funds from all sources and to all uses in each period. Activities which contributed to cash flow had a negative sign on the coefficient in this row; those which drew on cash flow had a positive sign. A transfer activity was provided to transfer any balance remaining in cash flow at the end of the period to the beginning cash row of the next period.

Accumulated Additions to Net Worth. This row was simply an accounting row that accumulated the expected additions to net worth from each period. A single activity to maximize the objective function drew from this row.

B. Financial Restraints

Beginning Cash. This restraint represented the amount of cash on hand at the beginning of the period. The total amount of cash used for operating expenses, or transferred to cash reserves, could not exceed this amount. In the first period, beginning cash was assumed to be 2,000 dollars.

Working or Operating Capital. This restraint required that the amount of operating capital used to pay the variable costs of the firm could not exceed the amount of working capital available. Working capital could be supplied from beginning cash or from short-term borrowings from a lending agency.

Short-Term Loan Limit. The amount of short-term funds borrowed for operating capital could not exceed the short-term loan limit. This limit was assumed to be set either by the farmer or by the lending agency or by both. The former represents internal credit rationing and the latter represents external credit rationing. Both kinds of rationing probably occur on most farms.

Consumption. As previously noted, the model required that the business generate enough income to provide a minimum of 3,000 dollars for consumption expenditure--assuming the non-proportional consumption function. This requirement was handled by an equality restraint set equal to 3,000 dollars.

Cash Overhead Expenses. A requirement that the cash overhead expenses be paid was also handled by an equality restraint. In the first period, cash overhead expenses were estimated to be 2,000 dollars. Expenses assumed to be covered by this amount included items such as property taxes, telephone, hydro, insurance and miscellaneous repairs.

Depreciating Capital Assets. Restraints were included to compute capital cost allowance on depreciating capital assets, i.e. equipment and improvements and to charge this allowance against income. The submatrix of restraints and activities that were concerned with capital cost allowance is shown in Table 7. Depreciable assets were identified by class for depreciation according to the diminishing balance method. It was assumed that the capital

Table 7

Submatrix of Restraints and Activities Concerned with Capital Assets and Capital Cost Allowance, First Period

Restraint Description	Capital Asset Class 10	Capital Asset Class 8	Capital Asset Class 6	Pay Capital Cost Allowance		Right Hand Side
	... dollars ...					
Net Income	.30	.20	.10		=	0.
Class 10 Capital Asset	1.				=	4200.
Class 8 Capital Asset		1.			=	6400.
Class 6 Capital Asset			1.		=	6000.
Capital Cost Allowance	-.30	-.20	-.10	1.	=	0.
Cash Flow				1.	=	0.
Class 10 Capital Asset[a]	-1.				=	0.
Class 8 Capital Asset[a]		-1.			=	0.
Class 6 Capital Asset[a]			-1.		=	0.

[a]Second-period restraints.

cost allowance in each period would be used to replace equipment to maintain the value of the different capital assets. The "pay capital cost allowance" activity in Table 7 drew out of the cash flow row to satisfy this assumption, and was forced to operate by the "capital cost allowance" row being set equal to zero. The rates for depreciating class 10, class 8, and class 6 capital assets were 30, 20 and 10 percent respectively; the coefficients representing these rates are shown in Table 7 as .30, .20 and .10 under "Capital Asset".

Capital To Purchase Irrigation Equipment, Cattle and Beef Cattle Housing. Separate restraints represent the capital supplies for purchasing irrigation equipment, expanding the beef enterprise and enlarging beef cattle housing facilities. In the event that any of these capital purchase activities came into the solution, capital supplies could be provided from cash, i.e. from cash flow or from borrowing activities for intermediate- or long-term capital. The effective restraints on these borrowing activities would be the ability of the firm to repay the principal out of expected cash flow and the worst possible income requirement.

Cattle Enterprise. A restraint was included that required the beef cattle enterprise to be maintained at least at the size assumed in the initial situation. This requirement was built into the model by requiring that the capital invested in cattle be at least equal to the initial capital investment of 10,000 dollars.

Cash Reserves. The amount of cash reserves used could not exceed the supply of cash reserves. Cash reserves not used to accommodate unfavourable outcomes earned interest at 8 percent and were transferred at the end of the period to beginning cash at the start of the next period. Thus in each period, the model allowed a choice between allocating beginning cash to working capital or to cash reserves.

Worst Possible Income. The purpose and operation of this restraint have been described in the previous chapter. To parametrically vary this restraint simultaneously in each period, a special restraint and a special activity were provided for this purpose. Expressed as a greater-than-or-equal-to restraint, a one-unit increase in the right hand side of this equation was satisfied by a one-unit increase in the special worst possible activity. At the same time, this activity increased by one unit the worst possible income in each period of the model. By this means, parametric variation of the worst possible income restraint in each period was achieved.

Principal and Interest Payments. This restraint required that the principal and interest on existing intermediate- and long-term loans be paid. It was concerned only with commitments which the farm operator was assumed to have at the start of the planning horizon. Specifically, the farmer was assumed to have commitments to repay an F.C.C. mortgage and an equipment loan. A "principal and interest" activity apportioned the total payments between interest and principal--charging interest against net farm income and principal payments against after-tax income.

C. Labour Restraints

The labour supplies described in the initial resource situation were restraints that could not be exceeded.

D. Land and Land-Related Restraints

Besides the land restraints identified as part of the initial resource situation, a number of other restraints were also concerned with land.

Summerfallow. A minimum acreage of land, of each soil zone--

specifically the amount of the initial situation--was required to be under fallow in each period. Irrigation activities reduced this minimum under the assumption that land under irrigation would not be summerfallowed.

Flaxseed Acreage. Flaxseed acreage was restricted to a maximum of approximately one-half of the total acreage of crops grown on stubble on each soil zone.

Irrigated Land. Supply rows of each soil zone were included to represent the amount of land serviced by an irrigation system. Purchase of an irrigation system supplied these rows and irrigation activities drew from them. It was assumed that the amount of each soil zone serviced by an irrigation system would be proportional to the amounts of each type of land on the farm.

Assigned Acreage and Assignable Acres. To represent the assigned acreage provisions for marketing wheat, oats, barley, flaxseed and rapeseed, an assigned acreage restraint was included. Each acre of each crop and of summerfallow supplied one assignable acre, and activities representing the sale of wheat, oats, barley, flaxseed and rapeseed drew on this supply. The assigned-acre requirement per bushel sold was a rough estimate derived from estimates of expected quota per assigned acre of the different crops.

Forage and Straw Requirements for Beef Enterprise. Supply rows for forage, pasture and straw[20] for the beef enterprise were included. These rows

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[20]The straw account was formulated on the implicit assumption that there would be straw available to harvest. This assumption ignored the possibility that there could be optimal solutions in which no straw was produced. In the empirical applications, this result did occur in some solutions.

served as temporary storage rows for these intermediate products that were supplied by respective crop production activities and used by the beef production activities.

E. Other Restraints

Supply Rows for Wheat, Oats, Barley, Flaxseed and Rapeseed. As previously indicated, supply or inventory rows were included for each of the above crops. Production activities supplied these rows and marketing activities drew from them. Activities to transfer wheat, oats and barley supplies to the next period were provided. It was assumed that rapeseed and flaxseed would not be carried over from one year to the next and could be sold in the year of production.

Irrigation System. Restraints were included that provided that if an irrigation system were purchased it would have to be an integer unit and that only one irrigation system could be purchased or planned for purchase in the ten-year planning horizon. The integer requirement on the purchase of an irrigation system was satisfied by using the separable algorithm of MPS/360[21] to incorporate the following non-linear restraint:

$$X - X^2 = 0 \quad (6.3)$$

where X represents the purchase of an irrigation system.

This equation is only satisfied by values for X of zero or one; if this restraint was satisfied either a complete irrigation system or no irrigation system would be purchased.[22]

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[21] MPS/360 stands for 'Mathematical Programming System/360' which is an IBM application program for linear and separable programming. The MPS package was used for the empirical application of the model.

[22] The author is indebted to Dr. Daryl Kraft, Department of Agricultural Economics, University of Manitoba, for proposing and developing this method of incorporating an integer requirement.

Irrigation Water. The expected irrigation water requirements, in acre-inches, of the irrigated crop production activities were represented in a separate row. A water supply or water pumping activity supplied this row and irrigated activities drew from it.

Activities

Most of the activities in the model have been mentioned in the discussion of the production alternatives and in the discussion of the restraints. Some additional information about certain activities is presented below.

Investment Activities. Investments in an irrigation system, additional cattle and cattle housing facilities, if economically feasible, could be made out of cash or by borrowing, or by some combination of the two. Separate activities were provided to transfer money from cash flow to the capital supply rows for these alternatives. For the borrowing activities, the repayment periods were assumed to be five years for cattle, ten years for beef cattle housing and fifteen years for the irrigation system. In each case, an interest rate of 8 percent was assumed and it was also assumed that principal was paid in equal annual installments with interest paid on the remaining balance.[23]

Use Cash Reserves. As previously indicated, the "use cash reserves" activity drew from the supply of cash reserves to accommodate unfavourable outcomes. The amount of cash reserves could be increased from year to year and the amount not used represented savings. No particular savings instrument was specified; practically, savings would likely be held as term deposits, investment certificates or Canada Savings Bonds.

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[23]The repayment schedules per dollar borrowed for these different terms are shown in Table B.16 in Appendix B.

Irrigation System Purchase. The activity in each period that represented purchase of an irrigation system supplied irrigation facilities and serviced irrigation land to all subsequent periods in the planning horizon.

Data Development

The major requirements for data for the model concerned the specification of the technical coefficients for the production activities and especially for the crop production activities. The general methodology followed in developing these coefficients is outlined below.

A. Crop Production Coefficients

Yields.[24] Yield data for the Province of Manitoba for Crop District Number 1 and from Manitoba Crop Insurance Corporation records for the east-half of the municipality of Edward and for the west-half of the municipality of Arthur were the main source of data for estimating expected yields, variance in expected yields and the worst possible yields for dryland production. Twenty-year average yields were used as the basis for forecasting expected yields and significant trends in the twenty-year average were incorporated into the projected yields.

Crop insurance records were used to estimate the relation between yields of crops seeded on stubble and fallow and with and without the application of commercial fertilizer. Previously developed fertilizer yield response equations for wheat and barley[25] were used to estimate the response

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[24] The development of the yield estimates is described in more detail in Appendix C, beginning on page 227.

[25] Developed by the Soil Science Department, University of Manitoba. See page 231.

of these two crops to a fairly heavy application of commercial fertilizer and to approximate the response of other crops.

In deriving the estimates of the irrigated yields, historical yield and climatic data were used to estimate the relation between yield and moisture. The technique of regression analysis was used to obtain estimates of the historical relation between the trend-free yields of wheat, oats, barley and flax, and the estimated water deficit. The latter variable is a measure of the net irrigation water requirements. Generally, the results of this approach were felt to give reasonable estimates of the irrigated yields of these crops and similar relationships between yield and moisture were generally assumed for the other crops considered.

In estimating the worst possible yields under irrigation, it was assumed that the variance in the irrigated yields would be the same as for the dryland yields. The reasoning here was that moisture deficiency does not account for all the year-to-year variation in dryland yields and farmers would still have considerable year-to-year variation even with irrigation. Since the expected irrigated yields were considerably above the expected dryland yields, the assumption of the same variance and a lower coefficient of variation did not seem unreasonable.[26, 27]

<u>Man Hours and Machine Hours</u>. Man and machine times were estimated for the different crop production activities by budgeting the time required with the given equipment set for the different operations. Typical sequences of

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[26] A study by Iga of irrigation in the Morden-Winkler area of Manitoba found that the variance of expected income would be less, but not substantially less, under irrigation than under dryland farming--implying that the variance in yields would not be substantially reduced by irrigating. See: Iga, op. cit., p. 195.

[27] The expected and worst possible yields for the different dryland and irrigated activities are shown in Appendix Tables C.3 and C.5.

field operations were identified for the different crops and these sequences were used in estimating the time required. Allowance was made for extra man and machine time for harvesting crops with higher fertility levels and higher expected yields. In preparing the estimates of the man and machine hours, the Farm Data Handbook of the Manitoba Department of Agriculture[28] was used as the source of information on field speeds, efficiency and so on.

Operating Costs Per Acre. Total operating costs per acre included such costs as seed, fertilizer, herbicides, insecticides, fuel and oil, machinery repairs, crop insurance and water application. Estimates of the fuel, oil and machinery repair costs per acre were prepared on the basis of the estimated machine times referred to above. For insured crops, a coverage level of 60 percent was selected since the majority of farmers who insure choose this coverage level.[29, 30]

Marketing Quotas. Expected and worst possible quotas per acre are shown in Appendix Table B.7 for wheat, oats, barley, flax, rye and rapeseed. While time series data[31] on quotas were evaluated in developing these estimates, they were primarily subjective.

B. Livestock Production Coefficients.

The coefficients for the two beef cow alternatives were developed

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[28]Manitoba Department of Agriculture, Farm Data Handbook (Winnipeg: Manitoba Department of Agriculture, Economics Branch, 1972).

[29]Personal communication, J. Forsberg, Manitoba Crop Insurance Corporation, February 1973.

[30]Data on the insured price and yield, on the insured gross per acre and on the cost of insurance for different crops is presented in Appendix Table C.6.

[31]See Appendix Table B.6.

primarily from data contained in the Farm Data Handbook of the Manitoba Department of Agriculture. Basic input-output data for these alternatives are found in Appendix Table B.14.

C. Near-Term Prices for Crops and Beef Cattle.

As previously indicated, the near term of the planning horizon was defined to be the first three years. The expected and worst possible prices used for these and later periods are shown for crops in Appendix Table B.3 and for beef cattle in Appendix Table B.5.

The procedure was to obtain a subjective estimate of the expected and worst possible prices for the first period and to project prices in the second and third years that represented a trend to the long-run average price and the long-run worst possible price in the fourth year. For years beyond the third, the worst possible price was assumed to be the lowest price of the past ten years. For the Board grains--wheat, oats and barley--the initial payment per bushel was taken as the worst possible price in the first year.

Summary

An initial resource situation and the major provisions of the model have been described in this chapter. The farm situation described represents a one-section farm in the Souris Basin with a complement of resources that is fairly typical of the region.

The major restraints and activities in the model have been outlined. They are intended to represent fairly realistic limitations and choices for a farmer with the resources assumed to be available on this farm. Besides dryland activities--both uninsured and insured--activities and restraints are included for an evaluation of irrigating a portion of the total farm acreage.

In the next chapter, the results of the empirical applications of the model are reported and discussed.

CHAPTER 7

EMPIRICAL ANALYSIS AND RESULTS

INTRODUCTION

The empirical analysis and results of the study are presented in this chapter. It will be recalled that the hypotheses of the study concerned: (1) farm firm planning with specific consideration of the worst possible outcome, (2) irrigation as a means of reducing uncertainty, (3) financial reserves as a means of accommodating uncertainty, and (4) reducing uncertainty as a means of increasing economic efficiency. Accordingly, the empirical applications of the model focused on these areas, with a view to obtaining information for acceptance or rejection of the hypotheses.

This chapter begins with a discussion and comparison of the expected and worst possible gross margins of the different production alternatives. This is followed by the results of applying the model to three different situations. The first situation is called the deterministic dryland case. In this situation, only dryland uninsured alternatives are considered and a single decision parameter--maximum expected additions to net worth--is employed. For comparison, solutions are obtained for the case in which all crops are considered as alternatives and for the case in which all crops except sunflowers are considered.

The second situation is called the stochastic dryland case. In this situation, dryland uninsured and dryland insured alternatives are considered, and two decision parameters--expected additions to net worth and worst possible income--are employed. Again, solutions are obtained with all crops considered as alternatives, and with all crops except sunflowers considered as alternatives.

The third situation is called the deterministic dryland-irrigated case. Dryland uninsured, dryland insured, and irrigated alternatives are considered in this situation. All crops are considered as alternatives and a single decision parameter--expected additions to net worth--is employed. Analysis of this situation provides a basis for evaluating the potential for irrigating.

In the remainder of the chapter, emphasis is on analysis of the effects of the supplies of cash and borrowable reserves, and of planning for one or two bad years in a row. Consideration is also given to the implications of higher crop prices. The chapter concludes with a discussion of the relation between the economic efficiency of resource use and uncertainty, and of means of coping with uncertainty after the event.

EXPECTED AND WORST POSSIBLE GROSS MARGINS OF DIFFERENT PRODUCTION ALTERNATIVES

Some discussion of the expected and worst possible gross margins of the different production alternatives will provide an appropriate introduction to the empirical applications of the model. An explanation of the reasons behind any differences in the gross margins is important to understanding the results of applying the model to different situations. Are there, for example, important differences in the worst possible outcomes of different production choices or reasonable explanations for any differences? These and other questions bear on the riskiness of different production decisions.

Apart from differences in resource requirements, the most important differences among the different production alternatives concerned the expected and worst possible outcomes. In terms of the two decision criteria--

expected additions to net worth and worst possible income--the expected gross margin per unit and the worst possible income gross margin per unit were potentially important determinants of the optimal farm organization.

The expected gross margins and the worst possible gross margins, per acre of the different cash crop production alternatives in the first period, are shown in Tables 8, 9 and 10. (The wheat data are in Table 8, the barley data are in Table 9, and the oat, flax, rapeseed, pea and sunflower data are in Table 10.) The expected gross margins per acre and the worst possible gross margins per acre were estimated by: (1) computing the expected gross and the worst possible gross per acre, and (2) subtracting from these respective figures the estimated variable costs of production. For the irrigated alternatives, the variable costs of production, given the worst possible moisture deficiency,[1] included the costs of applying the irrigation water required to remove the moisture deficiency. Given the worst possible moisture deficiency, worst possible gross margins for the irrigated alternatives were computed for two different assumptions about the worst possible irrigated yields. Under the first assumption, the worst possible irrigated yields and worst possible dryland yields were assumed to occur in the same year. In other words, if dryland yields were low due to inadequate moisture (i.e. if the worst possible dryland yields occurred), irrigated yields would also be low. Under the second assumption, if dryland yields were low due to inadequate moisture, the worst possible irrigated yields would be equal to the expected, or average, irrigated yields.[2]

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[1]The worst possible moisture deficiency for wheat, oats, barley, flax, rapeseed and peas was assumed to be 14 inches per acre; for hay, pasture and corn silage, it was assumed to be 20 inches per acre.

[2]The irrigated yields used in computing the worst possible gross margins are found in Appendix Table C.5. The estimated worst possible gross margins for these assumptions are shown in Tables 8, 9 and 10 under headings of "Worst Possible" and "Worst Possible'".

The crop production alternatives in the model represented three basic choices. These choices were: (1) dryland uninsured production, (2) dryland insured production and (3) irrigated production. Within each of these choices, different crops, different soil zones, different seedbeds and different fertility levels were possible. The same range of choices was not available for each crop;[3] nevertheless, the alternatives that were considered did represent a fairly broad range of choices. Comparison of these different choices, with respect to the expected gross margin and with respect to the worst possible gross margin, suggested some relationships that would have some bearing on decisions. They are discussed in the next section.

Discussion of the Gross Margins

A careful reading of the estimates for the expected and worst possible gross margins (Tables 8, 9 and 10) reveals some fairly consistent relationships, among the different crops, with respect to the effects of: (1) soil zone, (2) seedbed, (3) amount of fertilizer, (4) crop insurance and (5) irrigation. Some of these relationships are discussed in the following paragraphs.

Soil Zone. For those crops--wheat, oats, barley, peas and sunflowers--with yield estimates according to soil zone,[4] the expected gross margins and the worst possible gross margins are higher on soil zone G than on soil

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[3]For example, crop insurance was not available for some crops and some crops were restricted to either a stubble or a fallow seedbed. In addition, an equal number of fertility alternatives was not identified for each crop.

[4]Yield estimates according to soil zone were not available for the other crops and their yields were assumed to be the same on both soil zones. It is recognized, however, that, in practice, there should be some differences in the expected yield of each crop according to soil zone.

zone H. Since soil zone G is the more productive of the two zones, this result is hardly surprising; given the same treatment, one would expect a greater return from the more productive soil.

Stubble vs. Fallow. Given the same treatment, a dryland uninsured crop, seeded on fallow, has a higher expected gross margin and a higher worst possible gross margin than the same crop seeded on stubble. This result can be seen for wheat (Table 8) and for barley (Table 9). Irrespective of the fertility level and the soil zone, the expected and worst possible margins for these crops are superior on fallow.

With crop insurance, the expected gross margins for dryland crops on fallow are seen to be higher than those on stubble. However, the worst possible gross margins are not always higher. The reason is that the insured coverage per acre is the same for crops seeded on stubble as for crops seeded on fallow. Accordingly, the relative payoff from insuring is greater on the stubble seeded than on the fallow seeded crop.

One aspect of the choice between stubble and fallow seeded crops, that is not shown in the figures in Tables 8 and 9, is the cost of summer-fallowing. One of the costs of obtaining an improvement in the expected gross margin by this means is the necessity to forego one year's production on the land that is under fallow. Although the programming model takes account of these costs in developing an optimal plan, they are not reflected in the per acre figures in the tables.

Amount of Fertilizer. With respect to different fertilizer rates for the dryland alternatives, the estimates indicate that the expected gross margin per acre tends to increase, and the worst possible gross margin per acre tends to decrease, with higher fertilizer rates. In all cases, the expected gross margin per acre increased with higher fertilizer rates, and in all but one case,[5] the worst possible gross margin decreased.

The improvement in the expected gross margins with higher rates of fertilizer application indicated that the projected improvements in yields were more than enough to compensate for the extra fertilizer and harvesting costs. Although the amount of fertilizer specified for the "heavy fertilizer" alternatives was well above what most farmers in the area had been using, somewhat higher levels would have been required to exhaust all of the expected profit opportunities from using fertilizer.

The decrease in the worst possible dryland gross margin with higher rates of fertilizer use deserves some comment. This result was partly due to an assumption that the coefficient of yield variation would be the same with different rates of fertilizer. Accordingly, the expected variance in yield was greater with higher rates of fertilizer, and the worst possible yield was proportionately no better with higher, than with lower rates of fertilizer. As a result, the possible loss per acre was higher and the worst possible gross margin per acre was lower, with higher fertilizer rates.[6]

In contrast to the estimated worst possible results for the dryland alternatives with higher rates of fertilizer, the worst possible results for the irrigated alternatives improved with higher fertilizer inputs. For

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[5]The exception was dryland uninsured wheat, on fallow on soil zone G with a medium application of fertilizer; the worst possible gross margin in this case was greater than that for the same alternative without fertilizer.

[6]The assumption that the proportional variation around the mean yield would be as great with high as with low rates of fertilizer was considered reasonable. While it could be argued that more fertilizer should reduce the relative yield variation, the assumption in this study was that it would not.

Table 8

Expected and Worst Possible Gross Margins per Acre,[a] in First Period, for Dryland Uninsured, Dryland Insured and Irrigated Wheat, for Different Soil Zones, Seedbeds and Fertility Levels

Description	Dryland Uninsured		Dryland Insured		Irrigated[b]		
	Expected	Worst Possible	Expected	Worst Possible	Expected	Worst Possible	Worst Possible[c]
	dollars						
Soil zone G: stubble, no fertilizer	27.10	1.06	26.30	8.53	-	--	-
stubble, medium fertilizer	33.58	.63	32.78	5.74	38.70	6.22	25.14
stubble, heavy fertilizer	38.74	-2.77	37.95	-.46	47.80	9.35	33.45
fallow, no fertilizer	39.59	3.90	38.80	8.08	-	-	-
fallow, medium fertilizer	47.01	4.31	46.21	5.83	-	-	-
fallow, heavy fertilizer	58.19	2.60	57.39	.32	-	-	-
Soil zone H: stubble, no fertilizer	23.96	.32	23.16	7.03	-	-	-
stubble, medium fertilizer	29.82	-.30	29.82	4.24	33.79	3.11	20.41
stubble, heavy fertilizer	34.13	-3.75	34.13	-1.96	41.58	5.53	27.57
fallow, no fertilizer	35.67	3.33	35.67	7.03	-	-	-
fallow, medium fertilizer	42.72	3.06	42.72	4.33	--	-	-
fallow, heavy fertilizer	51.87	.95	51.87	-1.95	-	-	-

[a]The estimated gross margins are net of the variable costs of production (including, for the irrigated alternatives, the costs of applying water).

[b]A dash (-) indicates that this alternative was not considered for irrigation.

[c]The values in this column are the estimated worst possible gross margins under the assumption that the irrigated yields would, at the worst, be equal to the expected irrigated yields.

Table 9

Expected and Worst Possible Gross Margins per Acre,[a] in First Period, for Dryland Uninsured, Dryland Insured and Irrigated Barley, for Different Soil Zones, Seedbeds and Fertility Levels

Description	Gross Margin per Acre of Barley						
	Dryland Uninsured		Dryland Insured		Irrigated[b]		
	Expected	Worst Possible	Expected	Worst Possible	Expected	Worst Possible	Worst Possible,[c]
	....	....	dollars	....	....	....	....
Soil zone G: stubble, no fertilizer	23.74	-1.30	22.58	4.54	-	-	-
stubble, medium fertilizer	28.92	-3.42	27.76	.82	38.84	3.45	18.69
stubble, heavy fertilizer	32.15	-7.26	30.99	-4.61	46.38	5.23	23.93
fallow, no fertilizer	38.53	1.44	37.37	4.25	-	-	-
fallow, medium fertilizer	49.77	.66	48.62	1.09	-	-	-
fallow, heavy fertilizer	55.98	-2.96	54.82	-4.81	-	-	-
Soil zone H: stubble, no fertilizer	21.49	-1.70	20.33	3.61	-	-	-
stubble, medium fertilizer	26.31	-3.95	24.85	-.11	34.61	1.16	15.28
stubble, heavy fertilizer	28.73	-7.91	27.85	-5.54	33.84	2.35	19.67
fallow, no fertilizer	35.39	.63	34.23	3.42	-	-	-
fallow, medium fertilizer	45.46	-.15	44.30	.15	-	-	-
fallow, heavy fertilizer	50.69	-4.03	49.53	-5.77	-	-	-

[a] The estimated gross margins are net of the variable costs of production (including, for the irrigated alternatives, the costs of applying water).

[b] A dash (-) indicates that this alternative was not considered for irrigation.

[c] The values in this column are the estimated worst possible gross margins, under the assumption that the irrigated yields would, at the worst, be equal to the expected irrigated yields.

Table 10

Expected and Worst Possible Gross Margins per Acre,[a] in First Period, for Dryland Uninsured, Dryland Insured and Irrigated Oats, Flax, Rapeseed, Peas and Sunflowers, for Different Soil Zones and Fertility Levels

Description	Gross Margin per Acre						
	Dryland Uninsured		Dryland Insured		Irrigated[b]		
	Expected	Worst Possible	Expected	Worst Possible	Expected	Worst Possible	Worst Possible'[c]
	dollars						
Oats: soil zone G, no fertilizer	28.08	-2.70	27.07	4.32	-	-	-
soil zone G, medium fertilizer	35.56	-4.28	34.55	1.59	47.52	3.04	19.48
soil zone G, heavy fertilizer	39.00	-9.65	37.99	-4.92	56.16	3.61	23.84
soil zone H, no fertilizer	24.70	-3.07	23.69	3.13	-	-	-
soil zone H, medium fertilizer	31.45	-4.66	30.44	.51	40.59	.12	14.95
soil zone H, heavy fertilizer	33.85	-10.21	32.81	-6.08	48.31	-.02	18.22
Flax: soil zone G or H, no fertilizer	31.78	-.20	30.56	2.88	31.68	-5.22	11.03
soil zone G or H, heavy fertilizer	49.33	-1.08	48.11	-1.93	65.23	10.07	36.20
Rapeseed: soil zone G or H, medium fertilizer	50.20	-.75	48.59	.53	67.18	21.25	46.00
soil zone G or H, heavy fertilizer	62.32	-2.36	60.70	-3.82	82.92	27.94	58.36
Peas: soil zone G, medium fertilizer	24.66	-8.33	22.49	-1.90	58.47	-4.14	34.26
soil zone H, medium fertilizer	17.70	-9.76	15.53	-4.87	45.73	-9.50	22.30
Sunflowers: soil zone G, medium fertilizer	25.91	-8.29	24.59	-6.67	-	-	-
soil zone G, heavy fertilizer	27.96	-12.05	26.63	-11.80	55.35	-.74	21.00
soil zone H, medium fertilizer	24.49	-8.56	23.17	-6.67	-	-	-
soil zone H, heavy fertilizer	26.29	-12.35	24.97	-11.78	52.70	-2.15	18.69

[a]The estimated gross margins are net of the variable costs of production (including, for the irrigated alternatives, the costs of applying water).

[b]A dash (-) indicates that this alternative was not considered for irrigation.

[c]The values in this column are the estimated worst possible gross margins under the assumption that the irrigated yields would, at the worst, be equal to the expected irrigated yields.

those irrigated crops with two fertility alternatives--namely wheat, oats, barley, flaxseed and rapeseed--the worst possible gross margin with the higher fertilizer rate was higher than with the lower rate.

No Insurance vs. Insurance. Comparing the dryland uninsured with the dryland insured alternatives, the expected gross margins for the insured alternatives were lower than those for the uninsured alternatives, by the amount of the crop insurance premium. Accordingly, by the expected value criterion, the uninsured alternatives would be preferred.

The worst possible gross margins for the insured alternatives were higher in some cases, and lower in some cases, than those for the uninsured alternatives. Generally, the insured worst possibles were: (1) higher than the uninsureds with no fertilizer or with a medium application of fertilizer and (2) lower than the uninsureds with a heavy application of fertilizer. This result was largely due to the fact that the insured coverage per acre is the same regardless of how much fertilizer is applied. This makes the payoff from crop insurance relatively greater with low rates of fertilizer, and implies that the question of whether to insure is not as easily resolved with high as with low rates of fertilizer.

Irrigated vs. Dryland. Comparing the expected gross margins for the irrigated alternatives with those for the dryland alternatives, the expected gross margin with irrigation was superior in every case--given the same soil zone, seedbed and fertility level.

The objective of improving the worst possible gross margin of a particular dryland alternative through irrigation was generally satisfied by either of the assumptions about the worst possible irrigated yield. However, the degree of improvement was much greater under the assumption that the worst possible irrigated yield, given a dry year, would be equal to the expected irrigated yield.

Under the assumption that the worst possible irrigated yield[7] could occur, some dryland alternatives had superior worst possible gross margins. A case in point was dryland insured wheat, seeded on stubble on soil zone G with no fertilizer (Table 8). The worst possible gross margin in this case was estimated to be $8.53 per acre; for the irrigated case, with the same soil zone and seedbed but a medium level of fertilizer, the estimated worst possible was $6.22 per acre.

Implications for Decisions

The estimates for the gross margins suggest that there would be tradeoffs involved in choosing among the different alternatives according to the expected value and the worst possible criteria. There would appear to be enough differences in the expected and worst possible gross margins of the different production alternatives to have some implications for decisions. Some alternatives are superior according to the one criterion but are inferior according to the other. Alternatives with higher expected gross margins, for example, often have lower worst possible gross margins. Thus, while a particular alternative might be preferred because it had a higher expected return, the same alternative might carry a risk of a higher loss, if the worst possible result did occur.

The idea of tradeoffs, when two or more decision criteria are employed, is well known. Farmers, for example, are well aware of the tradeoffs that are involved in attempting to satisfy two or more competing objectives. While there is clearly more to decision making than evaluating tradeoffs

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[7]The estimated yields for this case are shown in Appendix Table C.5.

among different alternatives, it is an important aspect of the decision process.

Before proceeding to the empirical applications, it should be noted that the worst possible outcomes of the different alternatives are not strictly comparable in a probabilistic sense. By way of example, the estimated worst possible gross margins for the uninsured and the insured alternatives for a particular crop[8] did not carry the identical probability. While the probability of the worst possible gross margin of the uninsured alternative was small, there was still some chance, albeit a small one, that the actual gross margin would, in fact, be worse than the estimated worst possible gross margin. On the other hand, there was virtually no chance that the actual gross margin of the insured alternative could be worse than the estimated worst possible. Since crop insurance guarantees a minimum worst possible gross income per acre, the worst possible insured gross margin is, for practical purposes, known for certain when production decisions are made.

It should also be pointed out that the different production alternatives are not equally intensive in terms of resource use. An important difference in this regard concerns the working capital requirements. Generally, those alternatives with more risk (i.e. lower worst possible gross margins) would have higher variable costs of production, and accordingly, would require more operating capital. If working capital were a limiting resource, choices among the different production alternatives could turn on differences in the working capital requirements.

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[8]Given the same soil zone, the same seedbed and the same amount of fertilizer.

EMPIRICAL APPLICATIONS

Deterministic Dryland Case: Uninsured Crops

The first situation that was analyzed was called the deterministic dryland case. The purpose of analyzing this case was to specify optimal plans under the assumption that: (1) a single parameter was to be used as a criterion for decision making and (2) no specific consideration was to be given to uncertainty in decision making. Optimal plans prepared under these assumptions would provide a basis for comparison with plans prepared in which specific account was taken of uncertainty, and would serve as benchmark optimal plans for the particular farm situation.

As indicated earlier, the decision parameter in this case was expected additions to net worth and it required that the model choose the plan which maximized the total expected additions to net worth over the whole planning horizon. This case was said to be deterministic in that single-valued expectations were assumed for all of the coefficients, prices and restraints in the model. It could also be called the perfect knowledge case.

The only crop production activities considered in this case were the dryland uninsured activities. Insured activities were not considered here since they represented production choices which took some account of uncertainty. It was decided to prepare optimal plans for the deterministic dryland case for two different assumptions about alternative crops. Under the first assumption, all of the crops were considered as alternatives; these crops were wheat, oats, barley, flaxseed, rapeseed, sunflowers, field peas, hay, pasture and corn silage. Under the second assumption, sunflower production activities were excluded but all other crops were considered.

For these solutions, and for later solutions as well, flaxseed acreage was limited to a maximum of about one-half of the total acreage of field crops seeded on stubble on each soil zone. In addition, rapeseed and field pea acreages were restricted to a maximum of the amount of fallow land available on each soil zone; the fallow acreage represented about one-third of the total field crop acreage.

Optimal Farm Plans. Table 11 shows the optimal crop and livestock programs in each period for the case in which all crops were considered. Table 12 shows the same data for the case in which all crops except sunflowers were considered.

Comparing the two deterministic solutions,[9] there is considerably less emphasis on wheat and flax production, in the case in which all crops were considered, than in the case in which sunflowers were excluded. In the former case, wheat is not part of the cropping program in the first two periods, while in the latter case, almost 100 acres of wheat is indicated for each of these periods. Flax acreage is projected to be at the maximum permissible level in the last seven periods of the planning horizon when sunflowers are not an alternative. However, when flax had to compete with sunflowers, the projected flax acreage was considerably below the maximum permissible level in each of the last seven periods.

In both of these plans, the beef cattle enterprise was at the minimum permissible level in each period except the final period. In the final period, the beef cattle enterprise was projected to increase from a herd size of 25 cows to a herd size of 26 cows. These results indicate that the beef cattle alternatives were less profitable than the cash crop alternatives. While this finding was somewhat unexpected, it could have been a result of a requirement in the model that any expansion in the beef

[9]In the data reported in Tables 11 and 12, and in later tables as well, values have been rounded to the nearest whole number.

Table 11

Dryland Uninsured Deterministic Case: Acres of Crops and Numbers of Cows in Each Period of Planning Horizon in Optimal Plan With All Crops Considered[a]

Description of Enterprise	Unit	Period 1	2	3	4	5	6	7	8	9	10
Wheat: Zone G, fallow, heavy fertilizer	acre			10	50	50	50	50	50	50	50
Zone H, fallow, heavy fertilizer	acre				60	75	93	92	92	92	91
Barley: Zone H, fallow, medium fertilizer	acre								3	3	3
Zone H, fallow, heavy fertilizer	acre						2	2			
Flax: Zone H, stubble, heavy fertilizer	acre	101	101	103	110	110	110	110	110	54	46
Rapeseed: Zone G, fallow, heavy fertilizer	acre	50	50	40							
Zone H, fallow, heavy fertilizer	acre	100	100	100	40	25	5	5	5	6	6
Sunflowers: Zone G, stubble, medium fertilizer	acre										75
Zone G, stubble, heavy fertilizer	acre	75	75	75	75	75	75	75	75	75	
Zone H, stubble, medium fertilizer	acre							15	15	71	79
Zone H, stubble, heavy fertilizer	acre	24	24	22	15	15	15				
Beef Cows: Calves sold as feeder calves	head	25	25	25	25	25	25	25	25	25	26

[a]In the data reported in this table, and in later tables as well, activity levels have been rounded to the nearest whole number.

Table 12

Dryland Uninsured Deterministic Case: Acres of Crops and Numbers of Cows in Each Period of Planning Horizon in Optimal Plan With All Crops Except Sunflowers Considered

Description of Enterprise	Unit	Period 1	2	3	4	5	6	7	8	9	10
Wheat: Zone G, stubble, medium fertilizer	acre	75	75	75				20	20	20	20
Zone G, stubble, heavy fertilizer	acre				20	20	20				
Zone G, fallow, heavy fertilizer	acre			12	50	50	50	50	50	50	50
Zone H, stubble, medium fertilizer	acre	24	24	15	15	15	15		15	15	15
Zone H, fallow, heavy fertilizer	acre					18	32	56	83	83	82
Oats: Zone H, stubble, medium fertilizer	acre							15			
Barley: Zone H, fallow, medium fertilizer	acre								1	3	3
Flax: Zone G, stubble, heavy fertilizer	acre				55	55	55	55	55	55	55
Zone H, stubble, heavy fertilizer	acre	101	101	110	110	110	110	110	110	110	110
Rapeseed: Zone G, fallow, heavy fertilizer	acre	50	50	38							
Zone H, fallow, heavy fertilizer	acre	100	100	100	100	82	68	43	16	14	15
Beef Cows: Calves sold as feeder calves	head	25	25	25	25	25	25	25	25	25	26

enterprise was permanent; in other words, expansion of the beef enterprise in a particular period required that the expanded herd size be maintained in later periods.

In each of the deterministic dryland solutions, corn silage was found to be uncompetitive with hay as a means of providing the winter forage requirements for the beef herd.[10] The reason was that the production cost per unit of forage supplied by corn silage was higher than that for hay. Since the same relative relationship existed between corn silage and hay under irrigation, the corn silage activities were not included as alternatives in later solutions.

It is significant to note that, in both plans, there is an emphasis--especially in the near term--on the oilseed crops. While this could be partly explained by inappropriate specification of the restraints in the model, it is also possible that it is a consequence of this model being deterministic, and of making no allowance for the degree of uncertainty associated with different crops. It is also significant that a heavy application of fertilizer is indicated for most of the crops, and for most periods. Although there is a shift towards lower fertilizer rates in later periods of the horizon, the fact that the optimal plans indicate that a heavy application of fertilizer would be profitable, may also be a result of making no allowance for uncertainty in developing these plans. As suggested in the discussion of the worst possible gross margins, the within-crop risk increases with heavier applications of fertilizer and the deterministic model takes no account of this aspect of heavier fertilizer

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[10]Since hay and pasture acres were projected to remain constant, in all solutions, in all but the final period of the planning horizon, they are not reported in the tables.

inputs.

As noted above, and as shown in the tables, there are some year-to-year changes projected in each of the deterministic solutions. Even though there was no limiting resource whose supply could be increased through time, the plans did not stabilize in the first period, or in any other period. Although plans in certain periods were the same as in other periods, there was no one year beyond which the plans for all succeeding years were identical. The reason was that the technological matrix was not constant from year to year. As discussed in an earlier chapter, the achievement of a stable plan requires a constant technological matrix. This condition was not fulfilled by this matrix; some crop yield projections incorporated a trend, and a trend was also incorporated into the cost of purchased inputs. As a result, a stable, optimal growth path could not be defined.

In both plans, there is a fairly significant shift in the crop mix between the third and fourth years. This shift is primarily accounted for by differences in the relative prices of crops between the near term (the first three years) and the long term (the last seven years). The prices in the near term were subjective estimates and did not necessarily conform to the historical price ratios. As previously indicated, the prices that were used for the long term were ten-year averages, and accordingly represented historical price relationships among the different crops.

The expected additions to net worth and the expected discretionary income in each period of the planning horizon are shown in Table 13 for the two deterministic solutions. The expected additions to net worth were higher, when all crops were considered, than they were when sunflowers were excluded. With all of the crops included as alternatives, the expected additions to net worth totalled 25,925 dollars; with sunflowers excluded,

Table 13

Expected Additions to Net Worth and Expected Discretionary Income in Each Period of the Planning Horizon for Optimal Solutions for Two Dryland Uninsured Deterministic Cases

		Deterministic Case	
		All Crops Considered	All Crops Except Sunflowers Considered
		.... dollars	....
Expected Additions to Net Worth		25,925	23,658
Expected Discretionary Income:	Period 1	4,829	2,444
	Period 2	4,744	2,093
	Period 3	2,943	0
	Period 4	3,469	4,194
	Period 5	2,863	4,195
	Period 6	1,973	4,084
	Period 7	1,823	2,072
	Period 8	1,684	1,474
	Period 9	1,550	1,234
	Period 10	1,433	1,085
	Total	27,311	22,875

they totalled 23,658 dollars. In the plan with sunflowers as an alternative, there was no upper limit placed on the total sunflower acreage and sunflowers could come into the plan at whatever level was most profitable. As can be seen in Table 11, the total acreage devoted to sunflowers was substantial. Although the average, one-section farmer would be unlikely to allocate as much acreage to sunflowers as in the optimal plan in Table 11, the results do indicate that some acreage of sunflowers would improve the expected income position of the firm.

As far as expected discretionary incomes were concerned, there were some important differences between the two solutions. While the sum of the expected discretionary incomes in all periods was higher when all crops were considered as alternatives--27,311 dollars compared to 22,875 dollars--the plan for this situation did not have a higher expected discretionary income in each and every period. It did have an advanhage in the first three and the last three periods, but in the in-between periods, the plan without sunflowers had an advantage. The reason that the plan with sunflowers had higher expected discretionary incomes in the near-term periods was that the supply of assignable acres was relatively greater when sunflowers were included in the cropping program. Since sunflowers contributed to the supply of assignable acres, but did not draw on the supply for marketing purposes, wheat marketing quotas were relatively less restrictive in this case. This permitted the marketing of wheat out of inventory in the near-term periods, and accordingly, increased the near-term incomes. Without sunflowers as an alternative, wheat marketing had to be delayed. This delay had the effects of decreasing the near-term incomes, and of increasing the incomes in the middle periods of the planning horizon.

Expected discretionary incomes were projected to decline in the long term in both plans. In both cases, the projected discretionary income in the tenth period was less than half that in the first period. Similar results will be seen in later solutions as well. Declining incomes through time resulted from the cost-price relationships that were built into the model; these relationships are discussed later in this chapter.

Comments on Deterministic Solutions. Since neither of the deterministic solutions give any consideration to the uncertainty--or degree of risk--associated with different alternatives, they would not likely be adopted in real situations. Apart from uncertainty considerations, there may be other restraints and activities that should have been included in the model to make the optimal deterministic solutions more realistic. While any omissions in this regard are recognized as limitations on the direct application of the results of these and other solutions, the model was considered reasonably satisfactory for evaluating some of the effects of uncertainty in farm planning problems.

In this study, the primary emphasis is on the specification of the best first-period decision and on the question of what to do now. Although the multiperiod linear programming model specifies the optimal plan for each period of the planning horizon, it is the optimal first-period plan and the optimal first-period decision that are of most interest. The deterministic solutions have provided estimates of the best first-period decisions for two dryland cases,[11] using a single decision parameter and a multiperiod planning horizon. We turn now to the stochastic solutions for the dryland case; they also provide estimates of the best first-period decisions for

[11] Two dryland deterministic cases were analyzed in order to have two optimal plans to compare; in one case, all of the crops were considered as alternatives, and in the other case, sunflowers were excluded. Optimal plans, with and without sunflowers, were of interest since sunflowers had relatively unfavourable worst possible gross margins.

the same resource situation, using two decision parameters and a multi-period planning horizon.

Stochastic Dryland Case: Uninsured and Insured Crops

To give some consideration to uncertainty in arriving at an optimal decision, two decision parameters were now employed. They were: (1) expected additions to net worth and (2) the worst possible income in any one period. The worst possible income constraint was varied parametrically, from a lower level at which it was not a limiting constraint, to an upper level beyond which an optimal plan was no longer feasible. Operationally, the model maximized expected additions to net worth subject to the constraint that the worst possible income was satisfied.

As in the deterministic case, solutions were obtained for the situation in which all crops were considered as alternatives and for the situation in which all crops except sunflowers were considered. Besides the uninsured alternatives, the insured alternatives were included as production possibilities. The results of these applications are discussed below.

Optimal Farm Plans. The optimal cropping program in the first three periods[12] of the planning horizon, for different values of the worst possible income constraint, are shown in Table 14 for the case in which all crops were considered.

When the worst possible income constraint was set at -8,000 dollars, this constraint was not limiting and had no effect on the optimal decision. The optimal cropping program in this case was identical to the solution

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[12] Since the near-term periods, and especially the first period, were of most interest, only the first three periods are reported.

Table 14

Stochastic Dryland Case - Uninsured and Insured Crops: Acres of Crops in First Three Periods of Planning Horizon, for Different Worst Possible Incomes, for Optimal Plans With 11 Crops Considered

Description of Enterprise	Unit	Worst Possible Income Constraint																				
		$-8,000			$-2,500			$-1,500			$-500			$500			$1,500			$1,770[a]		
		Period			Period			Period			Period			Period			Period			Period		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Wheat: Uninsured, fallow, heavy fertilizer	acre			10											5							
Insured, stubble, no fertilizer	acre													55			135			200		34
Insured, stubble, medium fertilizer,	acre				95			99			75								6			
Insured, fallow, no fertilizer	acre																			46		
Insured, fallow, medium fertilizer	acre																					
Barley: Uninsured, fallow, medium fertilizer	acre														3				50		50	
Uninsured, fallow, heavy fertilizer	acre															2						
Insured, stubble, heavy fertilizer	acre																	22			75	
Flax: Uninsured, heavy fertilizer	acre	101	101	103	101	101	93	101	101	90	101	101	[illegible]	61	107	102		110	110			96
Insured, no fertilizer	acre																				94	
Rapeseed: Uninsured, heavy fertilizer	acre	150	150	140	150	150	150	150	150	150	150	150	[illegible]	150	142	148	9	45	18			
Insured, medium fertilizer	acre																141	105	82	100	150	100
Sunflowers: Uninsured, medium fertilizer	acre				4							16			93				84			41
Insured, heavy fertilizer	acre	99	99	99		99	107		99	110		83	[illegible]			98						
Insured, medium fertilizer	acre										24						65	68			31	29
Peas: Insured, medium fertilizer	acre																			4		

[a]An optimal plan was not feasible for a worst possible income constraint above $1,770.

reported in Table 11 for the deterministic dryland uninsured case, in which the expected additions to net worth were the only decision parameter. The computed worst possible income in the first period was a loss of 6,502 dollars.

When the worst possible income constraint was set at -2,500 dollars, the cropping program was altered and the first-period plan included 95 acres of insured wheat, seeded on stubble with a medium application of fertilizer. This wheat acreage replaced 95 of 99 acres of uninsured sunflowers, with a heavy application of fertilizer, which was indicated in the previous plan.

For the next level of the worst possible income constraint, -1,500 dollars, there were only marginal changes in the first-period plan. However, as the worst possible income constraint was increased to higher levels, there was a substantial shift in the optimal cropping program in the first period. At successively higher levels for the worst possible income, there is a shift in the first-period plans towards insured crops, towards lower levels of fertilizer use, towards wheat, and away from flaxseed and sunflowers.

When the worst possible income constraint is equal to 500 dollars, flax acreage and the amount of fertilizer applied to wheat are both reduced from the previous plan and the sunflower acreage is insured. At the next level, flaxseed acreage is eliminated, the acreage of insured sunflowers is reduced and the acreage of insured wheat is increased. At the highest level for the worst possible income constraint--1,700 dollars--all crops in the first period are insured and fertilizer use is at a minimum. In this case, wheat and rapeseed crops occupy nearly all of the field crop acreage and a small acreage of field peas makes up the balance. As far as fertilizer use in this case was concerned, each crop was produced with the lowest possible amount of fertilizer; for wheat, this meant that no fertilizer was applied, and for rapeseed and peas, this meant a medium application of fertilizer.

For the case in which sunflowers were excluded as a possible crop, Table 15 shows the optimal crop acreages in the first three periods for different values of the worst possible income constraint. The effects of parametric variation of the worst possible income constraint, with the objective of maximizing the expected additions to net worth, were much the same for this case as for the case in which sunflowers were considered.

For the lowest level of the worst possible income constraint, -8,000 dollars, the optimal solution was the same as the deterministic solution for the same case (Table 12). The computed worst possible income in the first period in this case was a loss of 4,767 dollars. As previously mentioned, the comparable figure for the case in which all crops were considered was a loss of 6,502 dollars. The difference between these figures suggests that the plan without sunflowers would be less risky, and therefore preferable, according to the criterion of the worst possible income. However, balanced against the disadvantage of a lower worst possible income with sunflowers, there would be the advantages of higher expected additions to net worth and higher near-term incomes.

Again, as in the previous case, there was a shift towards insured crops, lower levels of fertilizer use and increased wheat production, as the worst possible income constraint was raised. There is a heavy emphasis on wheat production as the worst possible income constraint becomes especially limiting. In the most limiting case, insured wheat, with no fertilizer, occupied 346 acres out of a total of 350 acres of field crops in the first period. A similar emphasis on wheat production was seen in the case in which all crops were considered. These results are in line with

Table 15

Stochastic Dryland Case - Uninsured and Insured Crops: Acres of Crops in First Thr[ee] Periods of Planning Horizon, for Different Worst Possible Incomes, for Optimal Plans With A[ll] Crops Except Sunflowers Considered

Description of Enterprise	Unit	$-8,000			$-2,500			$-1,500			$-500			$500			$1,500			$1,638[a]		
		Worst Possible Inco[me] Constraint																				
		Period			Period			Period			Period			Period			Period			Period		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Wheat: Uninsured, stubble, medium fertilizer	acre	99	99	90		99	96		97	92		75	9[illegible]		46							
Uninsured, fallow, heavy fertilizer	acre			12										189								
Insured, stubble, no fertilizer	acre										110						200	3	40	200	3	
Insured, stubble, medium fertilizer	acre				75			75								75						
Insured, fallow, no fertilizer	acre																			146		
Insured, fallow, medium fertilizer	acre				24			24														
Flax: Uninsured, heavy fertilizer	acre	101	101	110	101	101	99	101	101	94	90	101	8[illegible]	11	101	76		75	35		74	
Insured, no fertilizer	acre																					35
Barley: Uninsured, fallow, medium fertilizer	acre												[illegible]			3			50			45
Uninsured, stubble, medium fertilizer	acre																					3
Insured, stubble, no fertilizer	acre																	72	75		75	75
Rapeseed: Uninsured, heavy fertilizer	acre	150	150	138	150	150	150	150	150	151	150	150	16[illegible]	150	150	180		100	150			110
Insured, medium fertilizer	acre																128				150	2
Peas: Uninsured, medium fertilizer	acre															16						
Insured, medium fertilizer	acre																22			4		5

[a]An optimal plan was not feasible for a worst possible income constraint above $1,638.

what is generally well known to farmers in the Souris area, i.e. that wheat is a fairly "safe" crop.

The effects of different values of the worst possible income constraint on the expected additions to net worth, on the expected discretionary incomes and on the amount of cash reserves used, are shown in Table 16 for the case in which all crops were considered. Comparable data are shown in Table 17 for the case which excluded sunflowers.

Considering first the case of all crops, there was a difference of 11,001 dollars between the expected additions to net worth for the case in which the worst possible income was not a limiting constraint and the case in which it was most limiting. The value of the objective function was 25,925 dollars in the first case and 14,924 dollars in the second case.

Of the available cash reserves of 5,500 dollars,[13] the amount used to satisfy the worst possible income constraint ranged from zero for the unconstrained case to the full 5,500 dollars for the most constrained case. As the worst possible income constraint was increased, both cash reserves and production alternatives with higher worst possible gross margins were used to accommodate this constraint. By way of illustration of the latter adjustment, when the worst possible income constraint was raised from -8,000 dollars to -2,500 dollars, 95 acres of insured wheat on stubble with medium fertilizer replaced 95 acres of uninsured sunflowers with heavy fertilizer in the first period. As shown in Tables 8 and 10, there was a difference of about 17 dollars per acre between the worst possible gross margins of these two alternatives; so a shift of 95 acres to wheat represented an improvement in the worst possible income of just under 1,600 dollars.

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[13]Made up of 3,500 dollars of initial cash reserves and 2,000 dollars of beginning cash.

Table 16

Stochastic Dryland Case - Uninsured and Insured Crops: Cash Reserves Used in First Period, Expected Additions to Net Worth and Expected Discretionary Income for Each Period of Planning Horizon for Different Worst Possible Incomes, for Optimal Plans With All Crops Considered

Level of Worst Possible Income Constraint	Cash Reserves Used In First Period	Expected Additions to Net Worth	Expected Discretionary Income In Each Period									
			1	2	3	4	5	6	7	8	9	10
			dollars									
-8,000	0	25,925	4,829	4,744	2,943	3,469	2,836	1,973	1,823	1,684	1,550	1,433
-2,500	1,829	23,304	2,394	4,350	2,943	4,264	2,837	1,568	1,382	1,220	1,051	908
-1,500	2,772	22,545	2,236	4,237	2,943	3,982	2,837	1,439	1,245	1,076	902	754
- 500	4,027	21,776	2,707	4,136	2,826	3,247	2,837	1,294	1,090	915	736	582
500	5,500	20,699	4,422	3,776	2,264	2,906	1,521	1,099	915	735	549	388
1,500	5,500	18,193	4,349	3,258	1,717	1,210	1,084	924	746	560	366	183
1,770	5,500	14,924	2,598	2,997	1,237	1,000	768	256	80	13	0	0

The projected financial results, for the case in which sunflowers were not considered as an alternative crop, were similar to those for the case with sunflowers, but with lower values for the expected additions to net worth for each level of the worst possible income constraint. The figures for this situation appear in Table 17. The estimated expected additions to net worth ranged from 23,658 dollars to 14,002 dollars for the unconstrained and most constrained cases, respectively.

Comparison of the two different plans shows that the amount of cash reserves needed to provide for unfavourable outcomes was always less for the case in which sunflowers were not considered. Although this result was probably a result of several factors, an important factor was the fact that sunflowers, according to the criterion of the worst possible gross margin, were more risky than other crops. Accordingly, if reserves were available to accommodate the unfavourable outcomes, these reserves were used when sunflowers were an alternative.

It should be pointed out that in the case in which sunflowers were not considered, there were unused cash reserves at every level of the worst possible constraint. Why were the cash reserves not all used--at least in the most constrained case? The explanation would appear to lie in the fact that the model projected declining expected discretionary incomes through time. As seen in Table 17, the projected discretionary income was zero in the last three periods of the planning horizon for the most constrained case. This result would imply that the constraints, to satisfy the minimum consumption requirements and the principal payments on existing loans, were limiting in those periods. If so, the interest to be earned in later periods on unused cash reserves in the first period was probably a factor that the model considered in leaving some cash reserves unused in the first period.

While the expected discretionary incomes are generally shown to

Table 17

Stochastic Dryland Case - Uninsured and Insured Crops: Cash Reserves Used in First Period, Expected Additions to Net Worth and Expected Discretionary Income for Each Period of Planning Horizon for Different Worst Possible Incomes, for Optimal Plans with All Crops Except Sunflowers Considered

Level of Worst Possible Income Constraint	Cash Reserves Used in First Period	Expected Additions to Net Worth	Expected Discretionary Income in Each Period									
			1	2	3	4	5	6	7	8	9	10
dollars												
-8,000	0	23,658	2,444	2,093	0	4,194	4,195	4,084	2,702	1,474	1,234	1,085
-2,500	1,772	21,646	2,290	1,881	0	4,170	4,029	2,746	2,596	1,020	799	622
-1,500	2,772	20,865	2,235	1,769	0	4,264	3,920	2,746	1,946	860	646	464
- 500	3,400	19,987	2,587	1,869	0	3,551	3,805	2,746	1,288	714	508	319
500	3,764	18,639	3,119	2,001	368	2,906	2,837	1,975	852	564	357	162
1,500	4,293	16,348	3,320	2,469	831	1,726	1,216	763	637	359	0	0
1,638	3,407	14,002	1,877	2,178	760	1,611	702	218	65	0	0	0

decline as the worst possible income constraint was increased, there were some anomolies in this connection in the near term. In some cases, when the worst possible income constraint was higher, the expected discretionary incomes in the near-term periods were also higher. While this result occurred in both plans, it was most apparent in the case which considered all crops. This result was believed to be due to the interaction of several factors--including the availability of cash reserves, interest earned on unused reserves, declining gross margins through time, the assignable acreage restraint and the relative costs of reducing uncertainty by different means. While all of these factors were believed to contribute to the result, it was difficult to say exactly how the result came about.[14]

Comments on Stochastic Solutions. The optimal plans presented above are stochastic in the sense that they make some provision for the uncertainty of different production choices. They were prepared for a one-section farm situation that was assumed to have a supply of cash reserves, amounting to 5,500 dollars, that could be committed to providing for unfavourable outcomes.

The first-period plans for the stochastic solutions were substantially different from the first-period plans for the deterministic solutions. Moreover, they were more believable in terms of approximating the decisions that farmers in the Souris area would be likely to make. Generally, optimal first-period plans that were oriented to wheat production and medium fertilizer use would be more in line with actual decisions than those that were oriented to oilseeds and heavy fertilizer applications.

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[14]While one of the advantages of a multiperiod linear programming model is that it simultaneously evaluates the costs and benefits of all alternatives over the entire planning horizon, one of the disadvantages is that it complicates the explanations of some of the results.

The results suggest that uncertainty may be an important determinant of the level of fertilizer use on many farms. While heavy fertilizer inputs may be justified according to the expected value criterion, the ability to afford the unfavourable outcomes associated with these inputs would appear to be an important consideration.

Working capital was not an effective constraint in these solutions. While it could be argued that working capital is normally an important constraint for farm decisions, it can also be argued that working capital is not likely to be a serious restraint as long as the amount borrowed does not exceed the ability of the firm to repay the loan in the event of unfavourable outcomes. In identifying optimal plans that the firm could, in some sense afford in the worst possible situation, this model did take account of the supply of working capital.

Efficient Combinations: Expected Additions to Net Worth - Worst Possible Income.

The application of the model to the two dryland situations with the two decision parameters--expected additions to net worth and worst possible income--has shown that this approach can be used to generate optimal plans that represent efficient combinations in terms of these two parameters. Figure 11 has been drawn to show the different combinations for the two situations.

In each case, the range of combinations extends from a level of the worst possible income constraint that is not limiting to a level beyond which an optimal plan is not feasible. The shape of the curves indicates that there are tradeoffs involved in choosing between the two decision parameters. Those combinations with higher expected additions to net worth also have lower worst possible incomes.

The combinations represented in the figure are said to be efficient, in that for any value of the worst possible income parameter, some point on the line represents the maximum possible expected additions to net worth. Points below the line represent combinations that are inferior in terms of this parameter. In E-V analysis, the locus of points representing efficient combinations in terms of expected income and expected variance in income, is called the efficiency frontier; the curves in Figure 11 can also be called efficiency frontiers for the decision parameters used in this study.

In terms of choosing a point on the frontier and the associated optimal plan, the interpretation here is somewhat different than in E-V analysis. In this case, the farmer, in theory at least, would choose the combination that he could afford, whereas, in E-V analysis, he would choose the combination that was consistent with his risk preference and/or risk aversion. At the same time, with the decision parameters of this study, a particular combination of expected additions to net worth and worst possible income could also be interpreted in terms of risk preference and/or

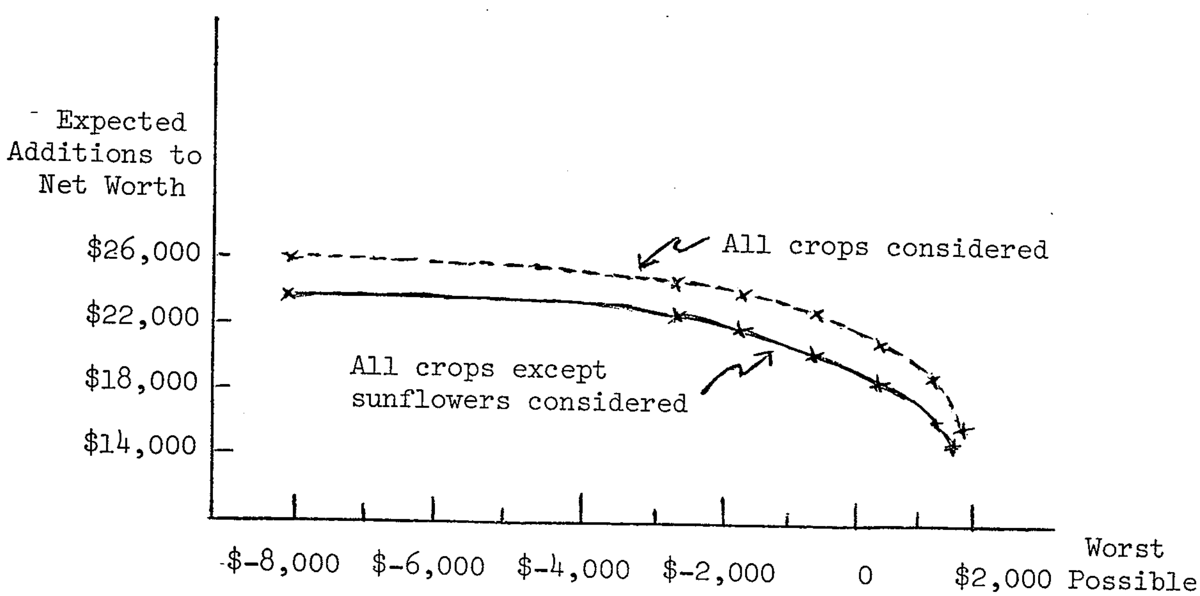

Figure 11

Efficient Combinations in Terms of Expected Additions to Net Worth and Worst Possible Income for Two Dryland Situations risk aversion. Farmers who would select a combination of a higher expected addition to net worth and a lower worst possible income could be said to prefer risk, when compared to farmers who would choose a plan with a lower expected addition to net worth and a higher worst possible income.

Deterministic Dryland - Irrigated Case: Uninsured, Insured and Irrigated Alternatives.

To assess the economic feasibility of acquiring an irrigation system, and to attempt to evaluate irrigation as a means of reducing uncertainty, the model was applied to the analysis of the case in which the irrigated alternatives were allowed to compete with the dryland uninsured and insured alternatives. A single decision parameter--expected additions to net worth--was used as the criterion for developing an optimal plan. The findings concerning this aspect of the study are discussed below.

Economic Feasibility of Irrigating. It will be recalled, from the previous chapter, that the model included an integer restraint on the purchase of an irrigation system. This restraint meant that irrigated crop production alternatives could not be included in the optimal plan unless an irrigation system were acquired, and an irrigation system could only be acquired in an integer unit. For decision purposes, it was thus a matter of acquiring a system that would irrigate 138 acres or of not purchasing any irrigation system at all.

Given this restraint in each period of the model, it was found that it would not be economically feasible to adopt irrigation. The optimal deterministic plan, when the irrigated alternatives were allowed to compete with the dryland alternatives, was identical to the optimal plan when only the dryland alternatives were allowed to compete. This latter plan was shown in Table 11. Accordingly for this situation, the hypothesis--that the opportunity to irrigate would have an important effect on decisions--would be rejected. A farmer in the Souris area would not find it profitable to acquire an irrigation system to irrigate the crops that were considered in the model.

There were three major reasons why the irrigated alternatives were found to be less profitable than the dryland alternatives. They were: (1) the integer requirement on the irrigation system, (2) the high capital cost of the irrigation system, and (3) the relatively low value of the crops considered for irrigation. If higher-valued crops, such as potatoes, sugar beets and vegetable crops, had been included as production alternatives under irrigation, or if the integer restraint were relaxed, or if the capital costs of the system were lower, the chances that irrigation would prove economically feasible would clearly be improved. However, from the point of view of an existing dryland farmer in the Souris area, the findings, or lack of findings, concerning irrigation suggest that the adoption of irrigation would require a shift from the traditional crops grown in the area, or a much lower capital investment per acre, or both.

The fact that it was found to be unprofitable to acquire an irrigation system, under the particular assumptions of this model, did not mean, of course, that there would be no return or benefit from irrigating 138 acres. It simply meant that the marginal improvement in the expected gross margin of the firm, with irrigation, would be insufficient to pay the costs of owning the system.

To obtain an estimate of the improvement in the expected additions to net worth under irrigation, a solution was obtained in which the capital costs of the irrigation system were not considered. The expected additions to net worth for this situation totalled 35,297 dollars. This compared with 25,925 dollars for the solution to the deterministic dryland case with uninsured alternatives. When the irrigated alternatives were included, the expected discretionary incomes were also higher in each period. Shown in Table 18, they ranged from a high of 5,566 dollars in the fourth period, to a low of 2,872 dollars in the tenth period. The comparable figures for the dryland uninsured case were 4,829 dollars in the first period and 1,433 dollars in the tenth period.

Flax, rapeseed and sunflowers were found to be the most profitable crops for irrigation. As shown in Table 18, with the exception of some irrigated peas in the third period, the irrigated acreage in each period was allocated to some combination of flax, rapeseed and sunflowers. A heavy application of fertilizer was indicated for all of the irrigated crops.

The difference between the expected additions to net worth for the dryland-irrigated case and for the dryland case amounted to 9,372 dollars--advantage the dryland-irrigated case. However, since the capital costs for irrigating were not considered in this case, the irrigation system could only be justified if the costs of owning the system did not exceed 9,372 dollars over a period of ten years.

Aside from operating costs, the most important costs of owning an irrigation system are depreciation and interest on the capital invested. These costs in turn are influenced by the expected life of the system and by the effective rate of depreciation of the system. In this respect, the model used in this study had an important limitation that should be mentioned. Although it did not affect the conclusion concerning the economic feasibility of acquiring an irrigation system, it bears on the question of the costs of owning a system and should be explained. The problem concerned assumptions in the calculation of the capital cost

Table 18

Expected Discretionary Income and Irrigated Crop Acreages in Each Period, for Optimal Plan for Deterministic Dryland-Irrigated Case,[a] With All Crops Considered and Without Capital Costs of Irrigation System

Period	Expected Discretionary Income	Irrigated Crop[b]			
		Flax	Rapeseed	Sunflowers	Peas
	dollars	acres			
1	3,487	-	90	48	-
2	4,610	-	39	99	-
3	3,369	-	9	103	26
4	5,566	43	-	95	-
5	5,497	46	-	92	-
6	5,350	46	-	92	-
7	3,101	35	-	103	-
8	4,015	46	-	92	-
9	3,036	40	-	98	-
10	2,872	36	-	102	-

[a]The expected additions to net worth for this case totalled $35,297.

[b]A heavy application of fertilizer was indicated for each of the irrigated crops in each period.

allowance in each period.

As previously mentioned, diminishing balance rates of depreciation were used in the model. However, with strict adherence to the diminishing balance method of depreciation, the model projected a declining capital investment in equipment and improvements. This was not considered realistic, so it was assumed that depreciation funds would be reinvested to maintain the stock value of equipment and improvements. While this was considered a reasonable assumption for the existing stock of equipment and improvements, it was not entirely satisfactory for evaluating the costs of owning an irrigation system. It had the effect of over-estimating the depreciation expenses and accordingly, it prejudiced the economic feasibility of acquiring an irrigation system.

Although the model overestimated the costs of owning the system, as previously mentioned, this did not affect the conclusion on the economic feasibility of irrigating. With projected returns (net of operating costs) of 9,372 dollars in ten years, the system would only be economically feasible with fairly generous assumptions about the effective life of the system and the effective depreciation rate. For example, even if the system were assumed to have an effective life of thirty years (and assuming zero salvage value), annual depreciation cost would average 970 dollars. Over a period of ten years, the total depreciation cost would be 9,700 dollars and, by itself, with no allowance for interest on capital invested, would be greater than the projected returns from irrigating.

It should be mentioned that the centre-pivot system considered in this study was assumed to be capable of irrigating an area of 138 acres. If it was assumed that the system could be moved back and forth between two fields, the system would have a capacity to irrigate 276 acres. Under this assumption, the system would be more nearly justified on economic grounds. Assuming that the gross margin from irrigating would double with a doubling of the irrigated acreage, the projected returns to the system would be 18,744 dollars over a period of ten years. Again assuming a thirty-year life and zero salvage value, depreciation and interest costs in ten years would total 21,340 dollars.[15] Thus, though the costs of irrigating would still exceed the estimated benefits, the difference between the estimated costs and the estimated benefits would be much less.

Improving the Worst Possible Income by Irrigating. One of the arguments in favour of irrigating is that it can reduce the uncertainty of dryland farming. One way of looking at the advantage of irrigating in this connection is in terms of the payoff from being able to irrigate in years in which the dryland yields are low, because of moisture deficiency. In other words, by how much would the worst possible dryland income be improved by being able to irrigate?

Solutions which provided estimates of this improvement were obtained for two different assumptions about irrigated and dryland yields. In the first case, it was assumed that unfavourable, or worst possible, dryland yields and worst possible irrigated yields would occur in the same year. In the second case, it was assumed that, if the worst possible dryland yields did occur in a particular year, then the irrigated yields in that year would be equal to their average or expected value. The estimated worst possible incomes in the first three periods of the planning horizon for these two assumptions are shown in Table 19, for the deterministic dryland-

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[15]Assuming an interest rate of 8 percent and an average investment of 14,554 dollars.

irrigated situation and for the deterministic dryland situation.

The figures of most interest in this table are those for the first period. For the dryland case, the estimated worst possible income in the first period is a loss of 6,002 dollars. For the dryland-irrigated case, with the assumption that the worst possible dryland and irrigated yields would occur in the first period, the worst possible income was estimated to be a loss of 3,077 dollars. For the same case, but assuming that the irrigated yields would be average if the dryland yields were worst possible, the worst possible income was estimated to be 705 dollars.

Of the two assumptions about dryland and irrigated yields in a dry year, the one which assumes average irrigated yields is the more realistic. Ordinarily, one would expect that if dryland yields were low because of dry weather, the yields with irrigation would be at least average. Accordingly, the most important comparison is between the worst possible irrigated income of 705 dollars in the first period and the worst possible dryland income of a loss of 6,002 dollars in the first period. The difference amounts to

Table 19

Estimated Worst Possible Incomes in First Three Periods of Planning Horizon for Optimal Plans for Alternative Assumptions About Worst Possible Dryland and Irrigated Yields

Situation Analyzed	Worst Possible Yield Assumption	Estimated Worst Possible Income		
		Period 1	Period 2	Period 3
		.... dollars		
Dryland, all crops	worst possible	-6,002	-5,488	-6,249
Dryland and irrigated, all crops	worst possible dryland and worst possible irrigated	-3,077	-3,817	-5,442
Dryland and irrigated, all crops	worst possible dryland and expected irrigated	705	711	1,179

6,707 dollars and is an estimate of the improvement in the worst possible income by irrigating in a dry year.

If the irrigation system could be used to irrigate more than 138 acres in any one year, the payoff from being able to irrigate in a dry year would be substantially greater than the above estimates suggest. Alternatively, if dryland yields were low for two years in a row because of dry weather, the payoff from being able to irrigate would also be substantially improved. Using the figures in Table 19, for example, the total improvement for this latter case would be 12,906 dollars--6,707 dollars in the first period plus 6,199 dollars in the second period.

The evidence that there can be a substantial payoff from being able to irrigate in dry years is by no means new information and is well known to farmers in the Souris area. However, the issue for decision purposes is the cost of achieving the potential benefits.

The benefits of irrigating will fluctuate from year to year with fluctuations in the relation between the irrigated and the dryland yields. In areas like the Souris, one would normally expect fairly wide year-to-year variation in the yield advantage with irrigation. There would, for example, be some years in which there was no moisture deficiency at all, or perhaps even a surplus of moisture, and in those years, dryland and irrigated yields should be much the same. At the other extreme, there would be some years in which the moisture deficiency would be such that, the dryland yields would be very low, and the irrigated yields would be well above average. Within these two extremes, a wide range of irrigated-dryland yield combinations would be possible. Thus, over a period of years, the benefits of irrigation could reasonably be expected to vary over quite a wide range--all the way from no benefit at all, to a very substantial benefit.

For decision purposes, the fact that the benefits of irrigation will fluctuate from year to year is not too important. If an irrigation system is to be available to capture the benefits of irrigating in the dry years--when the benefits are high--it must also be available in the not-so-dry years--when the benefits are not so high. Accordingly, it is really the expected or average benefits and costs that are most important in deciding whether or not to acquire an irrigation system. If the average benefits exceed the average costs, then ceteris paribus, it would be profitable to acquire the facility to irrigate. The fact that the benefits of irrigating would be irregularly distributed through time, while the costs of irrigating would be fairly regularly distributed, would not be an issue in evaluating the relative profitability of irrigating versus not irrigating.

This is not to argue, of course, that a farmer would be indifferent to the fluctuation in irrigation benefits from one year to the next. Clearly, if it were profitable on average to irrigate, he would prefer to acquire the system in a year in which the payoff would be high, i.e. in a dry year. Thus, there are really two questions concerning the acquisition of an irrigation facility; one is concerned with whether to acquire the facility and the other is concerned with when to acquire it.

On the question of acquiring an irrigation system to reduce the uncertainty of dryland farming, the conclusion would be that it would not be profitable. This does not mean, however, that irrigation would be ineffective as a means of reducing uncertainty. Irrigation would clearly reduce the yield uncertainty of dryland farming and, according to the criterion of the worst possible income, the payoff from irrigating in a dry year would be substantial. The problem lies with the costs of achieving a reduction in uncertainty by this means. As with many other means of reducing uncertainty, there are costs associated with irrigation. Although the returns in a single dry year would more than cover the costs of an irrigation system for that year, the average costs and average benefits would not justify the acquisition of an irrigation system to reduce uncertainty.

Effects of Supply of Cash Reserves

One of the arguments of this study, and one that is incorporated into the planning model, is that the supply of cash reserves is one of the determinants of the amount of risk that a farmer could, or would be willing to take. The argument is that the more reserves he had available to accommodate unfavourable outcomes, the more risk he could afford to take. A corollary of the argument is: the more of a given supply of reserves he were willing to commit to accommodating unfavourable outcomes, the more risk he could take. This argument does not imply that a producer would expect to use the cash reserves to provide for unfavourable outcomes; it simply implies that he would be preparing for them according to the criterion of the worst possible income.

To evaluate the effects of differences in the supply of cash reserves on the objective function and on the optimal decisions, optimal plans were prepared for different values of the cash reserve constraint in the first period. From an initial upper level of 6,500 dollars, the supply of cash reserves was parametrically varied down to a level of zero. Solutions were obtained for cash reserves of 6,500, 4,500, 2,500 and 500 dollars, and for the case in which it was assumed that there were no cash reserves at all. For each solution, the worst possible income constraint was set at -2,500 dollars and the first-period supply of beginning cash was set equal to zero. The solutions were obtained for the case in which dryland crops, both uninsured and insured, were considered as possibilities; sunflowers were excluded from the set of production alternatives.

The results, with different supplies of cash reserves, were much as expected and they conformed with the hypothesis that the available supply of reserves had an effect on the amount of risk, in terms of unfavourable outcomes, that a farmer could afford. As shown in Table 20, with successively smaller supplies of cash reserves, the expected additions to net worth declined and the first-period cropping program became more oriented toward the less risky alternatives. As the cash reserves decreased, the first-period program shifted to alternatives with higher worst possible gross margins, i.e. to wheat, crop insurance and less fertilizer. Thus the effects of different supplies of cash reserves, for a given worst possible income constraint, are broadly similar to the effects of different worst possible income constraints for a given supply of cash reserves.

Table 20

Effects of Supply of Cash Reserves on Expected Additions to Net Worth and on the Optimal Cropping Program in the First Period

Supply of Cash Reserves	Expected Additions to Net Worth	Crop Enterprises in First-Period Plan
6,500	22,058	Uninsured flax and rapeseed, heavy fertilizer; insured wheat, medium fertilizer
4,500	21,233	Uninsured flax and rapeseed, heavy fertilizer; insured wheat, medium fertilizer
2,500	20,398	Insured wheat, medium and no fertilizer
500	19,066	Insured wheat, no fertilizer
0	18,481	Insured wheat, no fertilizer

Effects of Supply of Borrowable Reserves

As mentioned in an earlier chapter, borrowable reserves are a practical alternative to cash reserves as a means of providing for unfavourable outcomes. If cash reserves are not available, access to borrowable reserves can serve the same purpose. Even if cash reserves are available, it may be preferable, at certain times, to take advantage of borrowable reserves rather than commit equity reserves to the same purpose.

The average farmer probably does maintain some borrowable reserves--either implicitly or explicitly--as a form of either internal or external credit rationing. From a farmer's point of view, borrowable reserves are probably more implicit than explicit. Though he would probably not arrange borrowable reserves with a lending agency, he would likely have a fairly good idea of how much he could borrow for special purposes. From a lender's point of view, borrowable reserves would tend to be more explicit. Since the lender must take the final decision on how much a farmer can borrow, he would have to consider, both the amount borrowed and the amount borrowable, more explicitly than the farmer would.

To provide a basis for comparing cash reserves and borrowable reserves as means of providing for unfavourable outcomes, a set of solutions was obtained in which it was assumed that there were no cash reserves available, but that borrowable reserves, up to a maximum of 5,500 dollars per year, were available. Special borrowing activities were set up to handle this assumption in the model.

In these solutions, as in previous solutions, the operator was assumed to be preparing for one bad year. Accordingly, reserves borrowed in the first period would have to be repaid but reserves borrowed in later periods would not have to be repaid.[16] The model required that any reserves borrowed in the first period be repaid out of cash flow in the second period. Optimal plans were prepared for the situation in which dryland uninsured and dryland insured alternatives were considered; all crops except sunflowers were considered as alternatives. Plans were prepared for three different values of the worst possible income constraint.

Some of the effects of using borrowable reserves rather than cash reserves can be seen in Table 21. For comparison purposes, the results for both the cash and the borrowable reserves case are set out in the table. Comparing the expected additions to net worth for the two cases, the advantage is in favour of cash reserves; for worst possible income constraint values of -2,500 dollars, -1,500 dollars and -500 dollars, the advantages for the cash reserves alternative are 1,153 dollars, 879 dollars and 825 dollars, respectively.

Some of the improvement in the expected additions to net worth, with cash reserves rather than borrowable reserves, can probably be explained in terms of the interest on unused reserves. In the model, and in practice too, cash reserves that are not used earn interest. On the other hand, borrowable reserves that are not used earn no interest. As seen in Table 21, the available reserves in the first period--both cash and borrowable--were not completely used in any of the solutions. For the cash reserves case, the surplus reserves in the first period were available for use in later periods and, to this extent, they represented a continuing productive asset in the business. On the other hand, surplus borrowable reserves in the first period

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[16] As discussed in Chapter 5, planning on the basis of one bad year implies that the operator expects no more than one bad year in a row. Therefore, for planning purposes, he would not expect to actually use the reserves borrowed in later periods.

Table 21

Estimated Expected Additions to Net Worth, Reserves Used and Operating Capital Borrowed in First Period, for Optimal Plans[a] with Cash Reserves and Borrowable Reserves of 5,500 Dollars, and for Different Values of Worst Possible Income Constraint

	With Cash Reserves of 5,500 Dollars			With Borrowable Reserves of 5,500 Dollars		
Worst Possible Income Constraint	Expected Additions to Net Worth	Cash Reserves Used In First Period	Operating Capital Borrowed in First Period	Expected Additions to Net Worth	Borrowable Reserves Used in First Period	Operating Capital Borrowed in First Period
			..........dollars..........			
-2,500	21,646	1,772	4,380	20,493	1,772	6,380
-1,500	20,865	2,772	4,380	19,986	2,434	4,435
- 500	19,987	3,400	4,053	19,162	3,383	6,044

[a]For the situation in which dryland, uninsured and insured, activities were considered and in which all dryland crops except sunflowers were considered.

were unproductive in later periods in that they could not earn interest.

It can also be seen in Table 21 that the amount of operating capital borrowed in the first period is greater, for the case of borrowable reserves, than for the case of cash reserves. This result was observed for each value of the worst possible income constraint that was considered, and was most noticeable for the constraint values of -2,500 dollars and -500 dollars. In each of these cases, the amount of operating capital borrowed in the first period was about 2,000 dollars greater, for the case of borrowable reserves. Since interest had to be paid on borrowed working capital, the differences in interest costs would account for part of the differences between the two cases in the expected additions to net worth.

Although not shown in the table, the amount of operating capital borrowed in the second period was also greater for the borrowable reserves case. The reason, for greater borrowings in both periods, was that borrowed reserves had to be repaid out of cash flow. This created a relative shortage of cash flow that was satisfied by increased borrowing of working capital. Since the working capital borrowing limit was high enough that it was not an effective restraint, this was not a problem in this model. Had the capital restraint been set at a lower level, it would have restricted the use of borrowable reserves.

A situation that has not been mentioned so far, and one that may occur on occasion, is one in which a farmer has neither cash nor borrowable reserves to draw on in the event of unfavourable outcomes. In this situation, the optimal cropping program would be one with as little downside risk as possible. In effect, this would mean emphasis on wheat production, insured crops and minimal fertilizer use. Even with this kind of cropping program, a farmer, in a situation like that analyzed in the model, would still face a serious income shortfall if the worst possible outcome did occur. While the probability of the worst possible outcomes of all crops occurring in the same year would be low, nevertheless producers without access to some kind of reserves for contingency purposes would be financially vulnerable if that unlikely event did occur.

Effects of Planning for Two Bad Years in a Row

In the solutions reported so far, the assumption has been that the manager did not expect more than one bad year in a row. In the solutions reported in Table 22, this assumption is compared with the assumption that the manager wished to prepare for two bad years in a row. As discussed in Chapter 5, this latter assumption meant that cash reserves that were scheduled to be used in the first and the second periods, would, in fact, be used to provide for unfavourable results in each of those periods. Consequently, the reserves required in each of these periods would be unavailable for use in subsequent periods.

Two points are apparent from the figures in Table 22. One is that the optimal strategy is more conservative when the operator prepares for two bad years in a row. This is reflected in the value of the objective function; for every value of the worst possible income constraint above -8,000 dollars, the expected additions to net worth are less for this strategy than for the strategy in which the operator prepares for one bad year. As the worst possible income constraint is increased, the difference becomes greater and an infeasible condition is reached earlier, under the more conservative strategy.

The other important observation is that the cash reserves used, in each of the first two periods, are less when the operator prepares for two bad years rather than one. This is in line with the differences in the expected additions to net worth. Given the same supply of cash reserves and

Table 22

Comparison of Planning for One or Two Bad Years in a Row: Cash Reserves Used and Expected Additions to Net Worth for Optimal Plans for Different Worst Possible Incomes, with All Dryland Crops Except Sunflowers Considered

Level of Worst Possible Income Constraint	One Bad Year			Two Bad Years in a Row		
	Cash Reserves Used: Period 1	Cash Reserves Used: Period 2	Expected Additions to Net Worth	Cash Reserves Used: Period 1	Cash Reserves Used: Period 2	Expected Additions to Net Worth
	dollars					
-8,000	0	0	23,658	0	0	23,658
-2,500	1,772	1,799	21,646	1,772	1,205	21,235
-1,500	2,772	2,798	20,865	2,497	1,877	20,014
-500	3,400	3,768	19,987	2,674	2,821	17,941
500, 374[a]	3,764	4,774	18,639	2,184	1,921	13,414

[a]For the assumption of two bad years in a row, an optimal plan was not feasible above a worst possible income of 374 dollars.

the same worst possible income constraint, the optimal strategy will have to be one that requires fewer cash reserves. Accordingly, other things being equal, production alternatives will be selected that have higher worst possible gross margins. Although not shown in this table, the optimal cropping programs in the first period for the higher worst possible income constraints did include alternatives with higher worst possible gross margins per acre.

Implications of Higher Product Prices

During the course of this study, the prices of all of the field crops that were considered as production alternatives in the model increased sharply. During 1973, and the first part of 1974, the prices of crops such as wheat,flaxseed and rapeseed reached all-time highs and, in some cases, more than tripled their normal values. Near-term and long-term price expectations that were used in the model were developed in the early part of 1973 and were reasonable and believable at that time. However, in the context of the early part of 1974, they did not have too much relevance in terms of estimates of the absolute returns from different production decisions. While one of the advantages of this model is that it uses current-period information for developing optimal farm plans, one of the disadvantages is that it must be continually updated to be relevant for different current periods. The model used in this study specifies an optimal first-period decision that applies to a particular first period. As time passes, a different period becomes the first period, new information becomes available and another optimal plan must be developed. Practically, this is what farmers do, and it can be argued that planning models for decision making should do the same. Nevertheless, one of the costs of having a model that is appropriate for a specific point in time, is that it is not, per se, appropriate for other points in time.

It will be recalled, from earlier sections of this chapter, that the model projects declining expected discretionary income through time. In each solution, expected discretionary incomes declined towards the end of the planning horizon and, in some solutions, expected discretionary incomes in the later periods were projected to be zero. It will also be recalled that the expected additions to net worth, over the ten periods in the planning horizon, were projected to be at a fairly low level. As an example, the estimated expected additions to net worth for the dryland deterministic case, with all crops considered, was just under 26,000 dollars; a projected increase in net worth of this order, over ten years, could best be described as modest.

The comparatively small increase in net worth, and the declining incomes through time, resulted primarily from the low product prices that were used in the model, and from the cost-price squeeze that was effectively built into the model by two main assumptions. These assumptions were: (1) that the costs of purchased inputs would increase through time and (2) that the long-term average product price would be constant through time. Although an upward trend in the yield of some crops was incorporated into the model, the improvement in productivity that was represented in these trends was not enough to counterbalance the upward trend in costs.

Had the product prices in the model been more in line with prices and price expectations as of early 1974, the model would still have projected a cost-price squeeze and declining expected discretionary incomes through time. With input prices trending higher, a cost price squeeze on income would only be avoided by projecting increasing product prices.[17] Given the relatively high prices of 1974, this was not considered a reasonable assumption.

To evaluate some of the effects and implications of higher crop prices, a new set of crop prices was imposed on the optimal solutions to two situations. These prices were called high prices and they were exactly twice the prices used in obtaining the initial optimal solutions.[18] The optimal solutions, to which these new prices were applied, were those prepared for: (1) the deterministic dryland case with all crops except sunflowers considered and (2) the deterministic dryland-irrigated case, with all crops considered and without considering the capital costs of the irrigation system. It will be recalled that the worst possible income constraint was not limiting in these cases.

The projected values, for the expected additions to net worth and for the expected discretionary incomes, for these two cases, for both the high prices and the low prices,[19] are shown in Table 23. With no change on the cost side, the expected additions to net worth increase substantially with the high crop prices. For the dryland case, expected additions to net worth increase from 23,658 dollars to 84,089 dollars; for the dryland-

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[17]The reference here is to the model used in this study. In practice, there are some other means of adjusting to, and mitigating the effects of, a cost-price squeeze. Historically, individual farmers have continually adjusted to the effects of a cost-price squeeze and have adopted various measures in an effort to compensate. For most farmers, increasing output, to improve resource productivity, has been the most important adjustment to the cost-price squeeze.

[18]It is difficult to say what price expectations should be used in a planning model in 1974. This is especially true for the long-term prices, since there are many unknowns concerning the long-run supply and demand for many commodities. In the absence of better information, a moving ten-year average might still be most appropriate.

[19]The crop prices used in the initial solutions are defined as low prices.

irrigated case, they increase from 35,297 dollars to 106,442 dollars. For the dryland case, this represents an increase of more than 3.5 times and, for the dryland-irrigated case, this represents an increase of more than 3 times.

In line with the much higher expected additions to net worth with the high crop prices, the expected discretionary incomes in each period are also much improved. Not surprisingly, the degree of improvement is greatest in the latter periods of the planning horizon in which the projected costs are highest. For the dryland case, discretionary income in the last period increased more than ten-fold; for the dryland-irrigated case, the increase was more than five-fold.

Apart from an important effect on the absolute level of returns, higher crop prices also have implications for the economic feasibility of irrigating. Comparing the expected additions to net worth with high prices, for the two situations reported in Table 23, there is an advantage of 22,353 dollars for the dryland-irrigated case over the dryland case. Although these two cases are not identical, in that sunflowers were not considered in the dryland case, these figures suggest that an irrigation system would be economically viable with the high prices.[20]

As far as worst possible incomes would be concerned under the assumption of high crop prices, they would be substantially improved as well. Although a new set of worst possible gross margins was not computed for the different production choices, one could be sure that providing for the worst possible would not be an important problem. Indeed, the typical one section

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[20]Assuming a thirty-year life for the system, and an interest rate of 8 percent per annum, capital costs (i.e. depreciation and interest) were previously estimated to total 21,340 dollars over ten years.

Table 23

Comparison of Low[a] and High[b] Crop Prices: Expected Additions to Net Worth and Expected Discretionary Income for Optimal Plan for the Deterministic Dryland Case[c] and for the Deterministic Dryland-Irrigated Case[d]

Description of Situation	Expected Additions to Net Worth	Expected Discretionary Income in Each Period									
		1	2	3	4	5	6	7	8	9	10
		........dollars........									
Deterministic Dryland Case:											
Low crop prices	23,658	2,444	2,093	0	4,194	4,195	4,084	2,702	1,474	1,234	1,085
High crop prices	84,089	12,436	11,392	8,673	15,211	15,194	15,075	12,867	11,312	11,086	10,978
Deterministic Dryland - Irrigated Case:											
Low crop prices	35,297	5,487	4,610	3,369	5,566	5,497	5,350	3,101	4,015	3,036	2,872
High crop prices	106,442	17,774	15,973	18,931	17,442	17,666	16,308	13,142	15,693	14,397	14,150

[a]Defined as price expectations, basis 1973, these were the prices that were used to obtain the optimal solutions.

[b]Defined as prices twice the value of the low prices, these prices were applied to the optimal low-price solutions to estimate the returns with the high crop prices.

[c]With all crops except sunflowers considered.

[d]With all crops considered and without considering capital costs of an irrigation system.

farmer, in the Souris area, would likely have surplus funds available to take on new investments, replenish equipment inventory, increase consumption and increase savings.

For decision purposes, the most important effect of assuming higher crop prices concerns the acquisition of an irrigation system. If crop prices were to continue at a historically high level, the irrigation alternative would deserve more serious consideration than it would with prices that were used in this study. However, the adoption of irrigation, under the assumption that the current high prices would continue indefinitely, would be a fairly risky undertaking.

The effect of high crop prices, on the absolute level of returns to a one-section farm in the Souris area, points up the degree to which both the level of returns and the year-to-year variation in returns are outside the effective control of the farm manager. Even with price increases substantially below those assumed for the high-price situation, any actions that the farmer could take, to improve the expected income and/or the worst possible income, would be secondary in their effects to those of the higher prices. The same situation would occur, in reverse, in the case of rapidly falling prices. To the extent that product prices are unpredictable in either the near term or the long term, but especially in the near term, farmers are well advised to be prepared for the role that chance plays in determining the outcomes of their decisions.

Improving Economic Efficiency by Reducing Uncertainty

For purposes of this study, the degree of uncertainty associated with a stochastic choice variable was deemed to be reduced if the worst possible outcome of that variable was improved. With the worst possible outcomes expressed in terms of gross margins, an improvement in the worst possible gross margin of a production alternative was taken, ceteris paribus, as a reduction in uncertainty. In the context of the firm, an improvement in the worst possible gross margin of the overall business was taken as a reduction in uncertainty.

One of the hypotheses of this study was that the economic efficiency of resource use could be improved by reducing uncertainty. Should the hypothesis be accepted or rejected? Is there enough information available to either accept or reject this hypothesis from the point of view of the individual farm firm? In the following paragraphs, this hypothesis and these questions are discussed.

Costs and Benefits of Reducing Uncertainty. As mentioned in the introductory chapter, there are both costs and benefits to be considered in terms of the economics of reducing uncertainty. For example, crop insurance will reduce uncertainty by improving the worst possible gross income per acre of the insured crop relative to the uninsured crop. Thus, one measure of the benefits of crop insurance is the amount of improvement in the worst possible gross income per acre. Balanced against this benefit is the cost of the insurance premium per acre. According to this view of benefits and costs, the crop insurance would be presumed to be worth the cost if the premium were less than the improvement in the worst possible gross margin. In many instances, crop insurance would be justified according to this criterion. However, while this may be one criterion to use in deciding whether or not to take out crop insurance, what, if anything, does it say about the economic efficiency of resource use? Since the expected value of an insured crop is reduced by the cost of the insurance, the expected return (expected net income) with insured crops is less than with uninsured crops. Accordingly, the economic efficiency of resource use would be reduced through the use of crop insurance.

Irrigation is a means of reducing the uncertainty of dryland farming and the worst possible gross margins, with irrigation, have been shown to be substantially higher than under dryland conditions. However, it has also been shown, given the particular assumptions of the model, that the costs of this method of reducing uncertainty in the Souris area would be difficult to justify. To be worth the cost, fairly generous assumptions about the expected life of an irrigation system would be required.

The economic efficiency of a firm could be said to be improved if the expected returns to a given set of resources were improved. Thus, if the economic productivity of these resources were being constrained by the degree of uncertainty associated with the different production choices, then a reduction in the degree of uncertainty (i.e. an improvement in the worst possible outcomes) would improve economic efficiency. However, we again come back to the question of the costs and benefits of reducing the degree of uncertainty.

The empirical applications of the model have demonstrated tradeoffs between expected returns and uncertainty. Those solutions that required a higher worst possible income projected lower expected additions to net worth. By selecting alternatives with less risk, a manager would be reducing the expected economic productivity of his set of resources. These results suggest that the objectives of improving economic efficiency and reducing uncertainty are incompatible and would indicate that the hypothesis should be rejected.

<u>Reducing the Costs of Reducing Uncertainty</u>. From the viewpoint of society, the value of reducing the degree of uncertainty in farming involves not only the overall costs and benefits but also the question of who will pay the costs and who will receive the benefits. However, from the viewpoint

of an individual farm manager, who is attempting to reduce uncertainty through his own actions, the latter questions are not relevant since he, himself, will be paying the costs and receiving the benefits. Given this assumption, what can he do to reconcile the two competing objectives?

The major reason for rejecting the hypothesis, on the basis of the empirical applications of the model, was that the costs of reducing uncertainty were too high. Putting it another way, the sacrifices in expected income, in order to have a higher worst possible income, were too great. Accordingly, are there practical means of reducing the sacrifice? There are, in fact, some ways in which managers may reduce uncertainty, at fairly reasonable cost, and through their own actions. Two such alternatives are outlined below; both are primarily concerned with management.

The first is concerned with the costs of improving the expected gross margin per acre through improving the expected yield per acre. In the empirical model, higher fertilizer rates were the only means of improving expected dryland yields of field crops that were considered. Though the higher fertilizer rates had the effect of improving the expected yield and the expected gross margin per acre, they also had the effect of increasing the variable costs of production per acre. It was largely because of these higher cash costs that those alternatives with higher fertilizer rates were found to have lower worst possible gross margins and were deemed to be more risky. In reality, farmers are often able to improve expected yields by improving their management. Better timing of field operations, improved seedbed preparation and superior variety selection are examples of relatively inexpensive means of improving expected yields. Practically, they have the effect of improving the expected gross margin for a given worst possible gross margin. This amounts to an improvement in economic efficiency with a reduction in uncertainty.

Another means of reducing the costs of reducing uncertainty is concerned with insured crop production. The Manitoba Crop Insurance Corporation grants increased coverage to farmers who insure, but who have recorded no claims for a certain number of years.[21] The amount of the increase depends on the number of successive years in which the farmer has made no claim and can reach a maximum of 127 percent of basic coverage. Farmers who are able to achieve a good experience record are thus able to improve their worst possible income and reduce uncertainty. Again, the costs of reducing uncertainty by this means are low since the managerial resource is the key input in achieving a good experience record.

An idea of the advantage of reducing the degree of uncertainty through a good experience discount may be gained from the data in Table 24. This table shows the estimated insured gross margin, by crop and in total, for three different good experience discounts, for the optimal first-period plan for one situation. This particular situation is the dryland stochastic case, with all crops except sunflowers considered, and with a level of the worst possible constraint that is most limiting. The good experience discounts considered were 100 percent, 112 percent and 124 percent of basic coverage. The first of these is the case in which there is no good experience discount and the coverage in this case is equal to the basic coverage of 60 percent of the average yield.

As seen in the table, the estimated advantage of the 112 percent coverage, over the 100 percent coverage, is 657 dollars. The estimated advantage of the 124 percent coverage, over the 100 percent coverage, is

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[21]Details on the policy for increased coverage can be found in mimeo material available from the Manitoba Crop Insurance Corporation, Winnipeg, Man. Besides increased coverage, reduced premiums are granted to farmers who insure but do not have claims. The benefits of this feature were not evaluated in the analysis.

1,314 dollars. These figures demonstrate that the worst possible income can be substantially improved through good experience discounts. A farmer, who is able to qualify for a good experience discount, can afford to choose more risky production alternatives or, to the extent that he was using reserves to provide for the worst possible, could free up reserves for other uses. Since good experience discounts are granted at no cost, providing, of course, that a farmer can qualify for them, they represent a means of simultaneously reducing uncertainty and improving the economic efficiency of resource use.

The two examples discussed above are illustrative of means of reducing uncertainty that do not require a substantial sacrifice in expected income. Since they rely heavily on the management input, it might be argued

Table 24

Estimated Worst Possible Gross Margins from Insured Crops for Three Different Good Experience Discounts for First-Period Cropping Program for Optimal Plan for Stochastic Dryland Case with All Crops Considered Except Sunflowers, and with Worst Possible Income Constraint Level of 1,638 Dollars

First-Period Cropping Program	Good Experience Discount[a]		
	100 Percent[a]	112 Percent	124 Percent
	Estimated Worst Possible Gross Margin		
	dollars		
200 acres wheat on stubble	1,529	1,900	2,271
146 acres wheat on fallow	1,075	1,352	1,629
4 acres peas on fallow	-8	1	10
Total	2,596	3,253	3,910

[a]With the 100 percent discount, the coverage is equal to the basic coverage of 60 percent of average yield, with the 112 percent discount, coverage is equal to 112 percent of the basic coverage, and with the 124 percent discount rate, coverage is equal to 124 percent of basic coverage.

that they are not alternatives for all farmers, on the grounds that improved management is not a ready alternative for each farm manager. While it is true that management cannot be upgraded as easily, say, as extra fertilizer can be purchased, improved management is an alternative for every farmer in the longer term.

To summarize, the objectives of higher expected returns and reduced uncertainty do not involve tradeoffs in all situations. Acceptance or rejection of the hypothesis, that the economic efficiency of resource use by the farm firm can be increased by reducing uncertainty, depends on the particular means of reducing uncertainty that is being considered.

Coping With Unfavourable Outcomes After the Event.

Farmers' actions to cope with uncertainty and with unfavourable outcomes can be classified into two main categories: (1) actions taken before the event, and (2) actions taken after the event. Included in the first category are those actions or precautions that are taken at the time of making a decision and before knowing the outcome of the decision. The kinds of precautions for uncertainty that have been the concern of this study fall in this category; formal insurance schemes, the explicit or implicit provision and maintenance of cash or borrowable reserves, selection of enterprises with comparatively low risk and similar precautions against unfavourable outcomes are all examples of actions taken to accommodate uncertainty in advance of the event.

The second category of actions to cope with uncertainty are those taken after the unfavourable event has occurred. As noted in a previous chapter, these actions are often makeshift but, nevertheless, they are an important aspect of how farmers cope with temporarily low incomes. Probably the most important of these kinds of actions concern the capital cost allowance, or, as it is usually called, depreciation expense. Every farmer knows that the capital cost allowance can be used for purposes other than replacing or improving equipment and improvements, and that this money can be used to accommodate unfavourable outcomes. Though he may not plan to use this money, it is available as a contingency against unfavourable or worst possible income.

In the farm situation analyzed in this study, the annual charges for depreciation amounted to 3,140 dollars. In the event of unfavourable outcomes, this money would likely be used--and would probably have to be used--to help pay cash overhead expenses, principal payments, and consumption expenditures. Given the assumptions of the model, the precautions against uncertainty taken before the event would, in most cases, not provide sufficient income to satisfy the minimum farm expenditures and still leave a reasonable balance for consumption. Accordingly, one would expect a farmer in a situation such as this to divert depreciation funds to these purposes. It must be emphasized, however, that the accumulation of what might be called a depreciation deficit, is only acceptable in the short term. If a business is to remain viable in the long term, the depreciation deficit must be paid. While farmers, who are nearing the end of their farming career, may profitably live off depreciation for several, or even many years, farmers with a longer planning horizon for their farming operations cannot afford to allow a depreciation deficit to accumulate beyond the short term.

Other means of coping with unfavourable outcomes after the event include temporary off-farm employment, reduced essential expenses, rescheduled principal payments, and temporary shelving of planned investments. All of these alternatives are not always open. Off-farm employment, for example, is not always an alternative in areas, like the Souris, in which there are few off-farm employment opportunities available in the non-farm and the farm-related sectors. For those off-farm opportunities that do exist, there is typically a wide variation in the ability of farm operators or their families to compete for them. Some farm families have skills that are readily marketable in a limited employment area, whereas other families do not. Those that are fortunately placed, in terms of marketable skills, are clearly less vulnerable than those whose off-farm opportunities are available at a lower rate of return, or not available at all.

The above discussion is not an exhaustive treatment of the means by which farmers cope with unfavourable outcomes after they happen and no mention has been made of actions taken by others to help farmers cope with unfavourable outcomes. Included in the latter category are programs, such as: (1) the advance payments programs of the Canadian Wheat Board, under which farmers may receive cash for farm-stored grain when opportunities for delivering grain on quota are restricted and (2) the policy of the Farm Credit Corporation that permits temporary deferral of principal payments on mortgages. The discussion does serve to emphasize, however, that a variety of alternatives is available to farmers to cope with temporarily low incomes. This is not to argue that these alternatives do not imply economic hardship. They do, and farmers who have had to resort to them can well attest to the constraints that they impose. Nevertheless, farmers, who feel that their best long-run economic opportunities lie in farming, have found it profitable to take advantage of these kinds of alternatives and they will likely continue to do so.

SUMMARY

The empirical applications of the model had four main purposes:

(1) evaluation of the model for problems of farm planning under uncertainty, (2) consideration of the economic feasibility of irrigating and of irrigation as a means of reducing uncertainty for a dryland farm situation in the Souris Basin, (3) evaluation of the importance and use of reserve in the context of farm planning under uncertainty and (4) consideration of reducing uncertainty as a means of improving economic efficiency. The major results are summarized in the following paragraphs.

The model was applied to the analysis of what were called deterministic and stochastic cases. In the deterministic case, a single parameter--expected additions to net worth--was used as a decision criterion. In the stochastic case, two parameters--expected additions to net worth and worst possible income--were used as decision criteria. The optimal first-period decisions for the deterministic and the stochastic cases were found to be substantially different.

For the particular assumptions of this study, an irrigation system was found to be infeasible according to the expected value criterion. Although irrigation would clearly reduce uncertainty, by improving the worst possible dryland yields and the worst possible income, it was found that the average costs of achieving a reduction in uncertainty by this means would generally exceed the average benefits.

Financial reserves were found to have an important influence on the optimal first-period decisions. When higher levels of reserves, either cash or borrowable, were assumed, the first-period decisions included alternatives with higher expected gross margins and lower worst possible gross margins. These results implied that the amount of risk that a farmer could assume, would be related to the amount of reserves that he had available.

Concerning the relation between uncertainty and the economic efficiency of resource use, the empirical applications found that, from an individual farmer's point of view, actions taken to reduce uncertainty generally had the effect of reducing economic efficiency.

CHAPTER 8

SUMMARY AND CONCLUSIONS

This study was concerned with the decision problems of the individual farm firm. The general objectives of the study were to investigate the effects of uncertainty on farm firm planning and decision making, and the relation between uncertainty and the efficiency of resource use. Within the framework of these general objectives, the study considered the costs and benefits of different means of reducing uncertainty. The alternative of irrigating was one of the means considered and the economic feasibility of irrigating was evaluated. An instrumental objective of the study was the specification of a decision model that would take some account of uncertainty in the decision process and provide a framework for farm planning under uncertainty.

The major objectives of the study were represented in the following hypotheses:

1. The problem of farm firm planning under uncertainty can be satisfactorily approximated in a multiperiod linear programming model that takes account of uncertainty through consideration of subjective and/or objective estimates of the worst possible outcomes of different alternatives.

2. Irrigation would be a profitable means of reducing the uncertainty of farming in the Souris River Basin in southwestern Manitoba.

3. The amount of financial reserves available to the firm is a factor affecting decisions under conditions of uncertainty.

4. The economic efficiency of resource use can be increased by

reducing uncertainty.

The locale for the study was the Souris River Basin in the southwestern part of the province of Manitoba. This is an area in which the supply of moisture is usually a limiting resource as far as crop yields are concerned, and farmers in the area typically experience considerable year-to-year variation in crop yields. Irrigation has the potential to reduce the year-to-year variation in dryland yields and to improve the average crop yield. At the same time, there are other means of reducing uncertainty and improving yields. Irrigation was evaluated against some of these other means of achieving the same objectives.

Problems of farm planning and decision making under uncertainty have been investigated in numerous studies of the farm firm, and a variety of models that take some account of uncertainty have been proposed. The model used in this study is another contribution to that general class of models that attempt to handle the uncertainty dimension in decision making. This study broke no theoretical ground; rather, existing theoretical concepts were employed in an attempt to develop a model that would have some practical application to everyday farm management problems. The conceptual framework of the model owes much to the work of Boussard[1] in the area of farm planning under uncertainty. Although the approach here was somewhat different, the basic framework was a variation on Boussard's model.

The theoretical model for the analysis reflected three major assumptions about the planning environment of the individual farm firm. The first was that the planning horizon for the firm is multidimensional and that it is appropriate for the firm manager to use a multiperiod planning

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[1]See Boussard and Petit, op. cit. and Boussard, op. cit.

horizon to make current-period decisions. In addition to being useful for specifying best current-period decisions, a model with a multiperiod horizon provides a projection, basis the information available in the current period, of the expected resource and income position of the firm in future periods.

In making the case for a multiperiod planning horizon, it has been argued, following Modigliani, that the problem for decision purposes is one of choosing the best first move. In line with this argument, the planning horizon of a decision model should be as a minimum, long enough that the first-period decision is independent of the length of the planning horizon. In this study, a planning horizon of ten periods, with one year in each period was assumed to be of sufficient length to develop a stable first-period plan.

The second major assumption that was reflected in the model concerned the utility function of the manager. This function was assumed to be multi-dimensional and to be adequately represented by a lexicographic ordering of utilities, together with a satisficing criterion. It was assumed that the most important of the different components of the utility function was expected income. This objective was represented in the model by an objective function that maximized expected additions to net worth over a planning horizon of ten years. The satisficing criterion concerned the minimum acceptable income in any one period and was incorporated in the model as a worst possible income constraint.

The third assumption concerned the satisficing criterion and the means of satisfying the worst possible income constraint. It was assumed that a manager has no substantial aversion to outcomes that are more favourable than expected but that he must take account of the unfavourable outcomes that can occur. In this connection, it has been argued that subjective and/or objective estimates of the worst possible outcomes of different production choices can provide an appropriate criterion for choosing among alternative risky production opportunities. In this same connection, it has been presumed that the amount of risk that a manager would be willing to assume would be related, in part, to the amount of risk that he could afford. The amount of risk that the manager could afford was assumed to be represented by the amount of reserves, either cash or borrowable, that he had available and that he was willing to commit to accommodating unfavourable outcomes.

The analytical model for this study used the multiperiod linear programming technique. With this technique, a time dimension and the two decision parameters--expected additions to net worth and the worst possible income--were satisfactorily incorporated into the model.

A hypothetical on-going, one-section, farm business served as the focus for the empirical applications of the model. A complement of managerial, physical and financial resources was identified as the initial resource situation for this farm. Although the set of resources that was assumed was in some senses typical of one-section farms in the Souris area of Manitoba, in other senses, it was atypical. The land base, machinery complement, and labour supply were believed to be fairly typical of one-section farms in the area. On the other hand, the financial assets and liabilities that were assumed for the farm were not typical. In this connection, the operator was assumed to have certain financial assets and liabilities that could be best described as farm specific. Examples are the amount of cash on hand, the amount of money invested in Canada Savings Bonds, and the amounts and terms of the first mortgage and equipment loan.

No attempt was made to ensure that the values assigned to these items were typical of one-section farms in the Souris area. The reasoning here was that, for planning purposes, each one-section farm is quite unique in respect of the values of these variables. Since the values of these variables were considered to be important for decision purposes, they were chosen without regard to whether or not they were especially typical. They were, however, considered to be well within the range of possible values for one-section farms.

Data for the empirical model were obtained from several sources. Among the more important sources of information were the Farm Data Handbook and the Yearbook of Manitoba Agriculture, both of which are published by the Manitoba Department of Agriculture. Records of the Manitoba Crop Insurance Corporation, for the adjoining municipalities of Edward (E 1/2) and Arthur (W 1/2) in southwestern Manitoba, were also an important source of basic data.

In terms of data development, the major requirement concerned estimates of the input-output coefficients for the production alternatives considered in the model. Coefficients for these alternatives were developed, in the main, by the method of budgeting. Cost and labour coefficients, for example, were developed by budgeting these requirements for the different components of the different alternatives.

Yield estimates for dryland crops were developed for two different soil zones, two different seedbeds and up to three fertility levels. Yield data for Manitoba Crop District Number 1, from crop insurance records for Edward and Arthur municipalities, and from soil fertility experiments, were used to estimate the yields for these different dryland alternatives.

The method used to develop the yield estimates for irrigated crops was somewhat different from the usual procedure. The general procedure here was to estimate the long-run relation between yield and moisture deficiency and, on the basis of this relation, to then estimate the expected yields with no moisture deficiency. The yield figures that resulted from this procedure were considered to be reasonably satisfactory estimates of irrigated yields. Although this method of estimating irrigated yields has several limitations, it would appear to be a promising approach for areas in which irrigation is contemplated as an alternative to dryland farming.

A capital-intensive irrigation system (centre-pivot) was considered for the irrigation alternatives. The model included an integer restraint on the acquisition of the irrigation system. It required that if an irrigation system were purchased, it would be a complete system.

To incorporate the worst possible criterion into the model, it was necessary to estimate the worst possible gross margin for each production alternative. Conceptually, these worst possible gross margins represented possible but unlikely events. Three variables--yield, price and irrigation water requirements--were regarded as stochastic, and estimates of the worst possible values of these variables were used to estimate the worst possible gross margins.

As previously indicated, the model was set up to use both an expected value criterion and a worst possible criterion in choosing among the different production alternatives. In the context of a probability distribution of possible gross margins, the estimates for the expected and worst possible gross margins represented two points on that distribution.

Summary of Results

The empirical applications of the model focused primarily on questions related to the hypothesis of the study. Specifically, they were concerned with: (1) evaluation of the conceptual model, in particular the worst possible income criterion, for purposes of farm planning under uncertainty, (2) consideration of the irrigation alternative as a means of reducing uncertainty and as a means of improving expected net returns, (3) evaluation of the effects of financial reserves in decision making under uncertainty and (4) assessment of the relation between efficiency and uncertainty. In addition, consideration was also given to the effects of planning for two bad years in a row. The results of the empirical analysis are summarized below.

To evaluate the model for purposes of making some allowance for uncertainty, solutions were prepared for what were called deterministic and stochastic cases. In the deterministic case, only the expected value criterion (maximum expected additions to net worth) was employed. In the stochastic case, the worst possible income criterion was used as an additional decision criterion. In the latter case, the worst possible income constraint was varied parametrically from a lower level, at which it was not an effective constraint, to an upper level beyond which an optimal plan was no longer feasible.

Taking differences in first-period plans as the most important, there were substantial differences between the deterministic and the stochastic optimal plans for the same resource situation. With the expected value criterion, the first period plan emphasized sunflowers, rapeseed and flaxseed crops, without crop insurance and with heavy applications of fertilizer. However, when the worst possible criterion was used in conjunction with the expected value criterion, the first-period plan placed less emphasis on the oilseed crops and more emphasis on wheat production, took advantage of crop insurance and specified a lower level of fertilizer use. The shift away from the oilseed crops, towards crop insurance and away from heavy applications of fertilizer, became more pronounced as the worst possible income constraint was raised to successively higher levels. When this constraint was most limiting, the field crop acreage was almost entirely allocated to insured wheat, without fertilizer or, with only a medium application of fertilizer.

Generally speaking, with higher values of the worst possible income constraints, the alternatives in the first-period plan of the optimal solutions had higher worst possible gross margins per acre. By this criterion, they were less risky than the alternatives in the first-period plan, for the deterministic case, or for the cases in which the worst possible income constraint was set at a lower level.

With parametric variation of the worst possible income constraint, the model generated optimal plans that could be called efficient in terms of the two criteria--expected additions to net worth and worst possible income. The plans with lower values of the worst possible income constraint had higher expected additions to net worth, and vice versa. These results were consistent with the usual observation that the opportunities that are more risky have higher expected returns.

The irrigation alternative was evaluated from two viewpoints: (1) as a means of improving the income position of the firm according to the expected value criterion and (2) as a means of reducing the uncertainty of dryland farming.

As a means of improving the income position of the firm, the irrigation alternative that was considered in this study was found to be infeasible. A dryland farmer in the Souris area would not find it profitable to acquire an irrigation system, of the kind considered in this study, to irrigate the crops that were considered in this study. While irrigation would clearly improve the expected gross margin of the firm, the improvement was found to be insufficient to justify the costs of owning the irrigation system. The estimated return to irrigating for ten years was less than ten thousand dollars while the estimated costs of owning the system for ten years were more than twenty thousand dollars.

From the point of view of reducing uncertainty and improving the worst possible income position of the firm, relative to dryland farming, the irrigation alternative would clearly be superior in years in which dryland yields were low because of moisture deficiency. The estimated improvement in the worst possible income amounted to about six thousand dollars in a year in which dryland yields were low and irrigated yields were average. This meant that if a farmer could irrigate in a dry year, he would be about six thousand dollars better off; or alternatively, he could afford to pay about six thousand dollars for the opportunity to irrigate and still be as well off as he would have been without irrigation. There would be no question, therefore, that it would pay very well for existing dryland farms in the Souris area to irrigate in the dry years, and that irrigation would reduce the uncertainty of dryland farming. At the same time, the empirical applications, with the expected value criterion, suggested that the average returns to the irrigation system would not be enough to justify the average costs of owning the system.

Financial reserves were hypothesized as an important factor affecting the ability and willingness of farmers to afford risk and to adopt plans with more uncertain outcomes. The analysis with different levels of reserves--in the form of cash reserves and in the form of borrowable reserves--supported the hypothesis and found that the amount of reserves could be a factor in decision making. When the reserves were assumed to be ample to provide for the worst possible outcomes, the optimal decisions included more risky production alternatives; as the supply of reserves was reduced, the optimal first period plan emphasized the less risky alternatives--i.e. placed more emphasis on wheat, crop insurance and reduced fertilizer inputs.

Concerning the relation between uncertainty and economic efficiency, the empirical applications of the model found that those solutions with a lesser degree of uncertainty (i.e. higher worst possible incomes) had lower expected additions to net worth. Assuming that, for a given set of resources, expected additions to net worth are a measure of the economic productivity and the economic efficiency of resource use, these findings indicated that economic efficiency would be reduced by actions taken to reduce uncertainty.

Concerning the effects of planning for one or two bad years in a row, the optimal strategy in the latter case was found to be more conservative. In other words, if a farmer wants to prepare for the possibility that he will face two bad years in a row, he will choose less risky alternatives in the first period, given the same set of resources, than he will if he wants to prepare for the possibility of only one bad year. These results were as expected. For decision purposes, the issue is really one of whether or not a manager should plan for the possibility of two bad years in a row. Generally, most farmers would most likely regard this possibility as so unlikely that they would not bother to prepare for it. While there may be times when a farmer has information that indicates that he should be prepared for that possibility, it would be fairly uncommon.

Assessment of Results

The findings of the empirical analysis suggest tentative acceptance of the hypothesis that the problem of farm firm planning under uncertainty can be satisfactorily approximated in a multiperiod linear programming model that takes account of uncertainty through consideration of subjective and/or objective estimates of the worst possible outcomes of different alternatives. However, although the model was found to have some capability for dealing with uncertainty, it has obvious limitations.

One of the most important limitations concerns the inability of the model to consider diversification as a means of reducing uncertainty. Diversification is usually seen as a means of reducing the year-to-year variation in income and this model was unable to take account of this benefit of diversification. In an earlier chapter, it was argued that farmers will diversify to achieve a smaller year-to-year variation in income, providing the sacrifice, in terms of expected income, is fairly small. The optimal solutions of linear programming models do indicate the marginal sacrifice from including additional enterprises in the solution, and in some of the solutions prepared for this study, the sacrifice was estimated to be fairly small. (For example, the costs of including barley in the first-period plan were estimated to be fairly low.) However, this does not obviate the limitation that this model could not handle, in a formal way, the diversification alternative.

The model used in this study has taken a somewhat unconventional view of farm planning under uncertainty in that it has emphasized the role of pure chance in determining the outcomes of decisions. Accordingly, it has emphasized making allowance for the role of chance in making a decision. At the same time, the model does allow the manager to use the best information (both subjective and objective) that he can obtain, about the expected and worst possible outcomes of different alternatives, in the near term and in the long term.

From a practical point of view, the idea of giving some consideration, within a multiperiod planning horizon, to the unfavourable outcomes would appear to make sense. This would be especially true in the case of making decisions to make substantial investments in land, equipment, or improvements. A realistic evaluation of the worst possible outcomes that could occur, in the near term and in the long term, would provide a good indication of whether or not the firm would be likely to encounter serious problems in carrying these investments.

The findings of the empirical analysis suggest rejection of a hypothesis that the opportunity to irrigate would have an important effect on decisions of one-section dryland farmers in the Souris River Basin. This finding was at variance with some other studies of the economic feasibility of irrigating in the Souris area and in other parts of Manitoba.

There were a number of reasons for the differences in the findings. First of all, this study was more concerned with irrigation in the context of existing dryland situations and in the context of reducing uncertainty than with drawing conclusions concerning the feasibility of irrigating in general. Accordingly, it did not consider high-value crops, such as potatoes and sugar beets, which are usually found to be quite profitable under irrigation. Secondly, this study specified an integer requirement on an irrigation system and considered a system with a high capital requirement and a low labour requirement. These two factors together meant that the returns from irrigation had to be relatively high to justify the system according to the expected value criterion. Thirdly, the dryland alternatives included choices representing heavy fertilizer applications. As long as these alternatives were profitable under dryland conditions, the advantage of irrigation was less than it would have been with lower fertilizer alternatives for dryland farming.

It would be wrong to conclude that irrigation would be an unprofitable alternative in the Souris area with different assumptions from those used in this study. The introduction of higher valued crops and lower capital costs for the system could well improve the advantage of irrigation, so that the irrigation alternative would be economically attractive.

The empirical findings, concerning the relation between economic efficiency of resource use and uncertainty, imply rejection of the hypothesis that economic efficiency would be improved by actions that a farmer might take to reduce uncertainty. Given the alternatives considered in the model, the analysis indicated that actions to reduce uncertainty had costs, in terms of the expected value criterion, and that economic efficiency would be reduced by choosing less risky alternatives.

However, as indicated in the previous chapter, producers do have some other means of reducing uncertainty without sacrificing expected income. Examples referred to included the ability to obtain a good experience discount on insured crops and the ability to improve the expected and the worst possible dryland yields without large increases in the variable costs of production. These, and similar means of reducing uncertainty, rely heavily on managerial ability, and accordingly, represent fairly inexpensive means of reducing uncertainty. Thus, although the hypothesis would be rejected on the basis of the alternatives specifically considered in the empirical model, the objectives of improving economic efficiency and reducing uncertainty are not incompatible in all respects. Acceptance or rejection of the hypothesis really depends on the alternatives that are being considered.

The question of who will bear the costs of reducing uncertainty is one of the considerations in developing agricultural policies and programs to reduce uncertainty. In this connection, the economic efficiency of resource use, by an individual farmer, would be enhanced the most if the costs of the program were borne by society in general, and would be enhanced the least if the costs were borne by each farmer individually. In the usual case, the costs are shared between the farmer and society. (The federal-provincial crop insurance program is an example of cost sharing by the federal and provincial governments and the farmer.) This principle will likely continue to be followed in government farm programs to reduce uncertainty.

The findings,concerning the effect of financial reserves on optimal plans, would suggest that some of the differences among farmers, in the amount of risk taken, may be explained by the amount and availability of financial reserves. The extent to which financial reserves could be deemed to influence decisions would depend in part on their liquidity; those reserves that were most liquid would be most likely to be available for accommodating unfavourable outcomes. For example, reserves held in the form of equity in land and improvements would be comparatively illiquid and consequently less available to provide for uncertain outcomes than reserves held in the form of savings or in the form of borrowable reserves.

Whether or not the hypothesis about financial reserves and their effects on decisions would be supported by empirical testing on a sample of farmers is, of course, speculative. Nevertheless, one would expect to find a positive correlation between the amount of risk taken and the financial assets, and especially the financial reserves, of the firm. A farmer must be able to afford the amount of risk associated with his decisions; otherwise, he could not count on staying in business. Accordingly, one would expect that those farmers who regularly took more risk would have the financial assets to afford them.

Limitations of the Study

This study was partly theoretical and partly applied. The theoretical aspect focused on the development of a model for farm planning under uncertainty. The applied aspect focused on the empirical applications of the model to particular problems. Both aspects of the study had important limitations; the major ones are outlined below.

Limitations of the Model

1. As previously indicated, the model did not take account of diversification as a means of reducing uncertainty. Generally, this shortcoming would bias the optimal solutions towards more specialization than would be the case in actual situations.

2. The model implicitly assumed that the worst possible outcomes of the different alternatives would occur simultaneously. Optimal plans were specified on this basis. The probability that these worst possible outcomes would occur in the same year is extremely small. From this standpoint, the optimal plans were probably more conservative than they would have been if the model had been able to take account of variance-covariance relationships among different production alternatives.

3. This model did not include debt-equity ratios as a factor for consideration in planning. Debt-equity ratios are indications of the ability of a business to withstand unfavourable outcomes and are commonly

used by lenders for this purpose. They were not included in this model since it was felt that planning, according to the worst possible criterion, would take adequate account of this aspect of the business. In retrospect, the model would have been more complete if debt-equity considerations had also been included.

Limitations of the Empirical Analysis

1. Undiscounted values were used in computing the value of the objective function. While this was a limitation on theoretical grounds, it was not felt to be a serious limitation in terms of defining the best first-period decision, nor necessarily an unreasonable assumption, in terms of how farmers evaluate income, earned at different points in time, when making decisions. (The model did incorporate an upward trend in input costs over time and this had the practical effect of discounting future income.)

2. The model did not include a trend in the replacement cost of equipment. Since there had been an important upward trend in the cost of equipment, this was a limitation on the empirical analysis. In not considering any trend in this category of inputs, the assumption was that any increase in the cost of replacement equipment would be approximately compensated by increased trade-in value of used equipment.

3. Changes in land values, over time, were not considered in the model. There had been a long-term upward trend in land values and this trend has been an important source of capital and net worth for many farmers. To the extent that the trend would continue, the model used in the analysis underestimated the expected additions to net worth. However, for purposes of decisions, within the framework of the alternatives that were considered, it was not considered important to take account of changes in land values.

4. This study did not consider the alternative of expanding the land base and the general problem of growth in the farm business. The model and the results would have been more generally applicable if this alternative had been considered.

5. As previously noted, the analysis did not consider the irrigation alternative in enough depth to support general conclusions about the economic feasibility of irrigating in the Souris area of Manitoba. A broader range of irrigation alternatives and irrigation systems would have to be considered to adequately evaluate the irrigation alternative.

Suitability of Multiperiod Model and Worst Criterion for Planning Individual Farms

The kind of model used in this study would appear to have some features that would make it appropriate for planning individual farms. In a formal sense, it can accommodate farm-specific information concerning alternatives, assets, objectives and expectations. These are the factors that farmers consider, either formally or informally, in decision making and it is possible that a model like the one used here could provide information that would be of value to the decision maker.

More specifically, arguments in favour of this kind of model are: (1) a multiperiod planning horizon is appropriate for some decision problems; (2) a multiperiod linear programming model can take account of both short-run and long-run considerations and objectives; (3) the concept of the worst possible outcomes of different alternatives should be easily grasped by most farmers and obtaining subjective and/or objective estimates of these outcomes should not be a serious problem;[2] and (4) the prudent

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[2]At least one study has found that farmers have a fairly good conception of unlikely, unfavourable outcomes. See Boussard and Petit, op. cit.

decision maker probably allows for the occurrence of chance outcomes and makes provision for those chance outcomes that are unfavourable. The worst possible criterion in a multiperiod model serves this latter purpose.

For practical purposes, an important disadvantage of a multiperiod model is that it can be quite cumbersome if several periods and several production alternatives per period are considered. Although a planning horizon of two, three or four periods might be adequate for many problems, and although the number of alternatives that a farmer would want to actively consider would probably be fairly small, the dimensionality problem might still be a limitation in practical applications.

Whether or not multiperiod linear programming models, with the worst possible criterion for allowing for uncertainty, will ever have widespread application to the planning of specific farms is difficult to say. Although the conceptual framework of the model is probably in line with the decision environment of farmers, farmers and their advisors can often approximate the information provided by this kind of model by simple hand budgeting. Since the costs of budgeting by hand are low, compared to the costs of budgeting with a formal model, this would be a factor that would work against widespread use of formal models of specific farms to aid decision making.

BIBLIOGRAPHY

BIBLIOGRAPHY

I. Books

Bellman, R. E. Dynamic Programming. Princeton, N.J.: University Press, 1957.

Blaug, M. Economic Theory in Retrospect. Homewood: Richard D. Irwin Inc.,1968.

Bradford, L.A. and G. L. Johnson. Farm Management Analysis. New York: John Wiley and Sons, Inc., 1953.

Dorfman, R., P. A. Samuelson and R. M. Solow. Linear Programming and Economic Analysis. New York: McGraw-Hill Book Co., 1958.

Evans, M. K. Macroeconomic Activity, Theory, Forecasting and Control.

Friedman, M. A Theory of the Consumption Function. Princeton: Princeton University Press, 1957.

Heady, E. O. Economics of Agricultural Production and Resource Use. Englewood Cliffs: Prentice-Hall Inc., 1952.

Henderson, J. M. and R. E. Quandt. Microeconomic Theory. New York: McGraw-Hill Book Co. Inc., 1958.

Johnson, G. L. et al. Managerial Processes of Midwestern Farmers. Ames: The Iowa State University Press, 1961.

Knight, F. H. Risk, Uncertainty and Profit. Boston: Houghton-Mifflin Co., 1921.

Luce, R. D. and H. Raiffa. Games and Decisions. New York: John Wiley and Sons, Inc., 1957.

Makeham, J. P., A. N. Halter and J. L. Dillon. Best-Bet Farm Decisions. Armidale: Department of Farm Management, University of New England, 1968.

Markowitz, H. M. Portfolio Selection. New Haven: Yale University Press, 1959.

Shackle, G. L. S. Decision, Order and Time in Human Affairs. Cambridge: Cambridge University Press, 1961.

__________, Expectations in Economics. Cambridge: Cambridge University Press. 1949.

Simon, H. A. Models of Man. New York: John Wiley and Sons, Inc., 1957.

Vickers, D. The Theory of the Firm: Production, Capital, Finance. New York: McGraw-Hill, Inc., 1968.

von Neumann, J. and O. Morgenstern. Theory of Games and Economic Behaviour. Princeton: Princeton University Press, 1944.

II. Periodicals

Barry, P. J. "Asset Indivisibility and Investment Planning: An Application of Linear Programming," American Journal of Agricultural Economics, Vol. 54 (May, 1972), pp. 255-259.

Boehlje, M. D. and T. K. White. "A Production-Investment Decision Model of Farm Firm Growth," American Journal of Agricultural Economics, Vol. 51 (August, 1969), pp. 546-563.

Boussard, J. M. "Time Horizon, Objective Function, and Uncertainty in a Multiperiod Model of Firm Growth," American Journal of Agricultural Economics, Vol. 53 (August, 1971), pp. 467-477.

__________, and M. Petit. "Representation of Farmers' Behaviour Under Uncertainty with a Focus-Loss Constraint," Journal of Farm Economics, Vol. 49 (November, 1967), pp. 869-880.

Cocks, K. D. "Discrete Stochastic Programming," Management Science, Vol. 15 (September, 1968), pp. 72-79.

__________, and H. O. Carter. "Micro Goal Functions and Economic Planning," American Journal of Agricultural Economics, Vol. 50 (May, 1968), pp. 400-410.

Craddock, W. J. "Linear Programming Models for Determining Irrigation Demand for Water," Canadian Journal of Agricultural Economics, Vol. 19 (November, 1971), pp. 84-92.

Encarnacion, Jose, Jr. "Constraints and the Firm Utility Function," Review of Economic Studies, Vol. 31 (April, 1964), pp. 113-120.

Ferguson, C. E. "The Theory of Multidimensional Utility Analysis in Relation to Multiple-Goal Business Behaviour: A Synthesis," Southern Economic Journal, Vol. 32 (October, 1965), pp. 169-175.

Fishburn, P. C. "Utility Theory," Management Science, Vol. 14 (January, 1968) pp. 335-378.

Freund, R. J. "The Introduction of Risk into a Programming Model," Econometrica, Vol. 24, No. 3 (1956).

Friedman, M. and L. J. Savage. "The Utility Analysis of Choices Involving Risk," The Journal of Political Economy, Vol. LVI, No. 4 (August, 1948), pp. 279-304.

Hazell, P. B. R. "Game Theory - an Extension of Its Application to Farm Planning Under Uncertainty," Journal of Agricultural Economics, Vol. 21 (May, 1970), pp. 239-252.

________, "A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning Under Uncertainty," American Journal of Agricultural Economics, Vol. 53 (February, 1971), pp. 53-62.

________, and R. B. How. "Obtaining Acceptable Farm Plans Under Uncertainty," in Policies, Planning and Management for Agricultural Development, Papers and Reports of the 14th International Conference of Agricultural Economists. Oxford: Institute for Agrarian Affairs, 1971, pp. 338-347.

Maruyama, Y. "A Truncated Maximin Approach to Farm Planning Under Uncertainty with Discrete Probability Distributions," American Journal of Agricultural Economics, Vol. 54 (May, 1972), pp. 192-200.

McInerney, J. P. "Linear Programming and Game Theory Models - Some Extensions," Journal of Agricultural Economics, Vol. 20 (May, 1969), pp. 269-278.

________, "Maximin Programming - An Approach to Farm Planning Under Uncertainty," Journal of Agricultural Economics, Vol. 18 (May, 1967), pp. 279-289.

Modigliani, F. "The Measurement of Expectations," paper presented at the American meeting of the Econometric Society in Boston, December, 1951, abstract in Econometrica, Vol. 20 (July, 1952), pp. 481-482.

Rae, A. N. "An Empirical Application and Evaluation of Discrete Stochastic Programming in Farm Management," American Journal of Agricultural Economics, Vol. 53 (November, 1971), pp. 625-638.

________, "Stochastic Programming, Utility and Sequential Decision Problems in Farm Management," American Journal of Agricultural Economics, Vol. 53 (August, 1971), pp. 448-460.

Thomas, W. et al. "Separable Programming for Considering Risk in Farm Planning," American Journal of Agricultural Economics, Vol. 54 (May, 1972), pp. 260-266.

von Neumann, J. "A Model of General Equilibrium," Review of Economic Studies, Vol. 13 (1945), pp. 1-9.

Yaron, D. and U. Horowitz. "Short and Long Run Farm Planning Under Uncertainty - Integration of Linear Programming and Decision Tree Analysis," Canadian Journal of Agricultural Economics, Vol. 20 (July, 1972), pp. 17-30.

III. Unpublished Works

Batterham, R. L. "Investment and Financial Management in Farm Firm Growth." Unpublished Ph.D. Dissertation, University of Illinois, 1971.

Boehlje, M. D. "An Analysis of the Impact of Selected Factors on the Process of Farm Firm Growth." Unpublished Master's Thesis, Purdue University, 1968.

Boyko, E. S. "A Multi-Period Analysis of Capital Accumulation and Financing of Beginning Irrigation Farms in the Pembina River Basin." Unpublished Master's Thesis, University of Manitoba, 1969.

Chen, J. T. "Modelling Responses to Uncertainty in Mathematical Programming Models of the Farm Firm." Unpublished Ph.D. Dissertation, University of Illinois, 1972.

de Yong, E. R. "The Determination of Fertilization Rates Under Conditions of Uncertainty." Unpublished Master's Thesis, University of Saskatchewan, 1971.

Finn, G. J. "An Economic Feasibility Study of Irrigating From Groundwater in South Western Manitoba." Unpublished Master's Thesis, University of Manitoba, 1971.

Harrison, V. L. "Management Strategies and Decision Processes for the Growth of Farm Firms." Unpublished Ph.D. Dissertation, Purdue University, 1970.

Iga, M. "Economic Evaluation of On-Farm Irrigation in the Morden-Winkler Area of Southern Manitoba." Unpublished Ph.D. Dissertation, University of Manitoba, 1970.

Johnson, W. I. R. "A Micro-Economic Analysis of Irrigation in the Morden-Winkler Area of Manitoba." Unpublished Master's Thesis, University of Manitoba, 1963.

Lang, O. E. "Proposals for a Production and Grain Receipts Policy for the Western Grains Industry." (Office of the Minister Responsible for the Canadian Wheat Board, 1970). (Mimeographed).

Mitchell, R. "A Multiperiod Linear Programming Analysis of Net Income Goal Attainment in Manitoba Crop District No. 10." Unpublished Master's Thesis, University of Manitoba, 1972.

Neilsen, J. G. "Relationship Between Farm Yields and Yields Predicted on the Basis of Soil and Fertilizer Nitrogen and Water Deficit." Unpublished Master's Thesis, University of Manitoba, 1972.

Shaykewich, C. F. "Progress Report on Climatic Analysis of Weather Stations in Manitoba." Winnipeg: Department of Soil Science, University of Manitoba, 1972. (Mimeographed).

Singh, R. H. "An Economic Analysis of Irrigation in the Souris River Basin in Manitoba." Unpublished Master's Thesis, University of Manitoba, 1972.

IV. Government, University and Business Documents

Craddock, W. J. Interregional Competition in Canadian Cereal Production. Special Study No. 12, Economic Council of Canada. Ottawa: Queen's Printer, 1970.

Hahn, G. J. A Categorized Bibliography on Decision and Risk Analysis. Schnectady, N.Y.: General Electric Co., Report No. 69-C-189, 1969.

How, R. B. and P. B. R. Hazell. Use of Quadratic Programming in Farm Planning Under Uncertainty. A. E. Res. 250, Ithaca, New York: Department of Agricultural Economics, New York State College: Agriculture, 1968.

Korven, H. C., W. E. Randall and A. D. Hutcheon. Irrigation on the Prairies. Publication 1488. Ottawa: Canada Department of Agriculture, 1972.

Manitoba Department of Agriculture. Farm Data Handbook. Winnipeg: Economic Branch, Manitoba Department of Agriculture.

Manitoba Department of Agriculture. Yearbook of Manitoba Agriculture, 1971. Winnipeg: Queen's Printer for Province of Manitoba, 1972.

Martin, J. R. and J. S. Plaxico. Polyperiod Analysis of Growth and Capital Accumulation of Farms in the Rolling Plains of Oklahoma and Texas. United States Department of Agriculture Technical Bulletin No. 1381. Austin, Texas: Economic Research Service, United States Department of Agriculture, 1969.

Statistics Canada. Farm Input Price Indexes, 1961-1971. Catalog 62-354 Occasional. Ottawa: Information Canada, 1971.

Wedemeyer, W. G. and T. L. Dobbs. Financing and Feasibility of Center-Pivot Sprinkler Irrigation Systems in Wyoming. Research Journal 72. Laramie: University of Wyoming. 1973.

APPENDIX A

THE MATHEMATICAL MODEL

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THE MATHEMATICAL MODEL

The general mathematical model used in the study is specified in this appendix. The complete model encompasses ten periods, with one year in each period. The notation for the mathematical formulation is presented first; this is followed by the specification of the model itself.

Notation

The notation used in describing the model is as follows:

Let t be a superscript denoting a time period for activities, $t = 1, 2, \ldots, 10$;

s be a subscript denoting a time period for restraints, $s = 1, 2, \ldots, 10$;

j be a subscript denoting the type of crop production activity, $j = 1, 2, \ldots, 136$, and within which,

$j = 1, 2, \ldots, 12$ for dryland uninsured wheat,
$j = 13, 14, \ldots, 18$ for dryland uninsured oats,
$j = 19, 20, \ldots, 30$ for dryland uninsured barley,
$j = 31, 32, 33, 34$ for dryland uninsured flax,
$j = 35, 36, 37, 38$ for dryland uninsured rapeseed,
$j = 39, 40$ for dryland uninsured peas,
$j = 41, 42, 43, 44$ for dryland uninsured sunflowers,
$j = 45, 46, \ldots, 50$ for dryland uninsured hay,
$j = 51, 52$ for dryland uninsured corn silage,
$j = 53, 54, \ldots, 58$ for dryland uninsured pasture,
$j = 59, 60, \ldots, 70$ for dryland insured wheat,
$j = 71, 72, \ldots, 76$ for dryland insured oats,
$j = 77, 78, \ldots, 88$ for dryland insured barley,
$j = 89, 90, 91, 92$ for dryland insured flax,
$j = 93, 94, 95, 96$ for dryland insured rapeseed,
$j = 97, 98$ for dryland insured peas,
$j = 99, 100, 101, 102$ for dryland insured sunflowers,
$j = 103, 104, 105, 106$ for irrigated wheat,
$j = 107, 108, 109, 110$ for irrigated oats,
$j = 111, 112, 113, 114$ for irrigated barley,
$j = 115, 116, 117, 118$ for irrigated flax,

j = 119, 120, 121, 122 for irrigated rapeseed,
j = 123, 124 for irrigated peas,
j = 125, 126 for irrigated sunflowers,
j = 127, 128, 129, 130 for irrigated hay,
j = 131, 132 for irrigated corn silage,
j = 133, 134, 135, 136 for irrigated pasture;

f be a subscript denoting the type of summerfallow activity,

f = 1 for summerfallow on soil zone G,
f = 2 for summerfallow on soil zone H;

q be a subscript denoting the type of forage establishment activity,

q = 1 for forage establishment on soil zone G,
q = 2 for forage establishment on soil zone H;

v be a subscript denoting the type of beef cattle production activity,

v = 1 for cow-calf with calves sold as feeder calves,
v = 2 for cow-calf with calves sold as long yearlings;

n, n' be subscripts denoting a particular quota crop selling activity and restraint, respectively,

n, n' = 1 for wheat,
n, n' = 2 for oats,
n, n' = 3 for barley,
n, n' = 4 for flax,
n, n' = 5 for rapeseed;

i, i' be subscripts denoting a particular separable activity and restraint, respectively, i, i' = 1, 2;

h be a subscript denoting the type of cash-capital investment activity,

h = 1 for irrigation system,
h = 2 for beef cattle housing,
h = 3 for beef cattle;

m be a subscript denoting the type of borrowed-capital investment activity,

m = 1 for irrigation system,
m = 2 for beef cattle housing,
m = 3 for beef cattle;

g, g' be subscripts denoting the capital class of a capital cost allowance activity and restraint, respectively,

g, g' = 1 for class 10,
g, g' = 2 for class 8,
g, g' = 3 for class 6;

r be a subscript denoting the category of labour,

$r = 1$ for planting labour,
$r = 2$ for hay harvest labour, first cut,
$r = 3$ for hay harvest labour, second cut,
$r = 4$ for crop harvest labout I,
$r = 5$ for crop harvest labour II,
$r = 6$ for daily winter labour;

u be a subscript denoting soil zone,

$u = 1$ for soil zone G,
$u = 2$ for soil zone H;

k, k' be subscripts denoting the particular tax bracket activity and restraint, respectively,

$k, k' = 1, 2, \ldots, 11$;

z be a subscript denoting the type of grain in the grain transfer activity,

$z = 1$ for wheat,
$z = 2$ for oats,
$z = 3$ for barley;

C_j^t denote the acres of the j-th crop production activity in the t-th period;

L_f^t denote the acres of the f-th summerfallow activity in the t-th period;

$\bar{L}_q^t$ denote the acres of the q-th forage-establishment activity in the t-th period;

U_v^t denote the number of beef cows in the v-th beef cattle activity in the t-th period;

Q_n^t denote the bushels of the n-th quota crop sold in the t-th period;

H^t denote the tons of straw harvested in the t-th period;

W^t denote the acre-inches of irrigation water applied in the t-th period;

I^t denote the number of units of an irrigation system acquired in the t-th period;

S_i^t denote the level of the i-th separable activity in the t-th period;

Y^t denote the number of beef-cow-housing units acquired in the t-th period;

G_h^t denote the amount of cash used for the h-th cash-capital investment in the t-th period;

$\bar{G}_m^t$ denote the amount of capital borrowed for the m-th borrowed-capital investment in the t-th period;

D_g^t denote the amount of capital in the g-th capital-cost-allowance activity in the t-th period;

$\bar{D}^t$ denote the amount of capital cost allowance in the t-th period;

K^t denote the amount of beginning cash allocated to working capital in the t-th period;

J^t denote the amount of beginning cash allocated to cash reserves in the t-th period;

B^t denote the amount of operating capital borrowed in the t-th period;

E^t denote the amount of cash reserves used to provide for the worst possible income in the t-th period;

F^t denote the amount of after-tax-after-principal-payments income withdrawn for minimum intended consumption spending in the t-th period;

V^t denote the amount of cash overhead expenses paid in the t-th period;

P^t denote the amount of principal and interest payments made on existing loans of intermediate- and long-term capital in the t-th period;

T_k^t denote the amount of income in the k-th tax bracket in the t-th period;

$\bar{Q}_z^t$ denote the number of bushels of the z-th grain transferred to period t + 1 in the t-th period;

N^t denote the amount of cash flow transferred to period t + 1 in the t-th period;

R^t denote the amount of cash reserves transferred to period t + 1 in the t-th period;

M^t denote the amount of expected discretionary income in the t-th period;

X denote the expected additions to net worth over the ten periods of the planning horizon;

A denote the level of the special activity, for parametric variation of the worst possible income in all periods;

c_{js}^{1t} denote the operating cost or gross margin in the s-th period per acre of the j-th crop production activity in the t-th period;

c_{fs}^{2t} denote the operating cost in the s-th period per acre of the f-th summerfallow activity in the t-th period;

c_{qs}^{3t} denote the operating cost in the s-th period per acre of the q-th forage-establishment activity in the t-th period;

c_{vs}^{4t} denote the gross margin in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

c_{ns}^{5t} denote the value in the s-th period per bushel of the n-th quota crop sold in the t-th period;

c_{s}^{6t} denote the operating cost in the s-th period per ton of straw harvested in the t-th period;

c_{s}^{7t} denote the operating cost in the s-th period per acre-inch of irrigation water applied in the t-th period;

c_{ms}^{8t} denote the interest cost in the s-th period per dollar of the m-th borrowed-capital investment activity in the t-th period;

c_{gs}^{9t} denote the capital cost allowance in the s-th period per dollar of the g-th capital-cost-allowance activity in the t-th period;

c_{s}^{10t} denote the interest cost in the s-th period per dollar of operating capital borrowed in the t-th period;

c_{s}^{11t} denote the interest cost in the s-th period per dollar of principal and interest payments on existing loans of intermediate- and long-term capital in the t-th period;

c_{s}^{12t} denote the interest earned in the s-th period per dollar of cash reserves in the t-th period;

a_{s}^{1t} denote the principal portion in the s-th period per dollar of principal and interest payments on existing loans of intermediate- and long-term capital in the t-th period;

a_{ks}^{2t} denote the after-tax income in the s-th period per dollar of net farm income in the k-th tax bracket in the t-th period;

b^{1t} denote the addition to net worth (saving) per dollar of expected discretionary income in the t-th period;

b^{2t} denote the addition to net worth (saving) per dollar of principal and interest payments on existing loans of intermediate- and long-term capital in the t-th period;

d_{js}^{1t} denote the worst possible gross margin in the s-th period per acre of the j-th crop production activity in the t-th period;

d_{qs}^{2t} denote the worst possible gross margin in the s-th period per acre of the q-th forage-establishment activity in the t-th period;

d_{vs}^{3t} denote the worst possible gross margin in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

d_{ms}^{4t} denote the principal and interest payment in the s-th period per dollar of the m-th borrowed-capital investment activity in the t-th period;

e_{js}^{1t} denote the working capital requirement in the s-th period per acre of the j-th crop production activity in the t-th period;

e_{fs}^{2t} denote the working capital requirement in the s-th period per acre of the f-th summerfallow activity in the t-th period;

e_{qs}^{3t} denote the working capital requirement in the s-th period per acre of the q-th forage-establishment activity in the t-th period;

e_{vs}^{4t} denote the working capital requirement in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

e_{s}^{5t} denote the working capital requirement in the s-th period per ton of straw harvested in the t-th period;

e_{s}^{6t} denote the working capital requirement in the s-th period per acre of irrigation water applied in the t-th period;

o_{js}^{1t} denote the cash flow requirement or the contribution to cash flow in the s-th period per acre of the j-th crop production activity in the t-th period;

o_{fs}^{2t} denote the cash flow requirement in the s-th period per acre of the f-th summerfallow activity in the t-th period;

o_{qs}^{3t} denote the cash flow requirement in the s-th period per acre of the q-th forage-establishment activity in the t-th period;

o_{vs}^{4t} denote the contribution to cash flow in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

o_{s}^{5t} denote the cash flow requirement in the s-th period per ton of straw harvested in the t-th period;

o_{s}^{6t} denote the cash flow requirement in the s-th period per acre-inch of irrigation water applied in the t-th period;

o_{ns}^{7t} denote the contribution to cash flow in the s-th period per bushel of the n-th quota crop sold in the t-th period;

o_{ms}^{8t} denote the cash flow requirement in the s-th period per dollar of the m-th borrowed-capital investment activity in the t-th period;

o_{s}^{9t} denote the cash flow requirement in the s-th period per dollar of working capital borrowed in the t-th period;

o_{ks}^{10t} denote the cash flow requirement in the s-th period per dollar of net farm income in the k-th income tax bracket in the t-th period;

o_{s}^{11t} denote the cash flow requirement (consumption) in the s-th period per dollar of expected discretionary income in the t-th period;

y^{1t} denote the coefficient in the cash reserve restraint in the s-th period for the activity to use cash reserves (to provide for the worst possible income) in the t-th period;

$p_{g's}^{1t}$ denote the increase in capital investment in the g'-th capital class in the s-th period per irrigation system acquired in the t-th period;

$p_{g's}^{2t}$ denote the increase in capital investment in the g'-th capital class in the s-th period per unit of beef cow housing acquired in the t-th period;

$p_{gg's}^{3t}$ denote the coefficient in the g'-th capital class in the s-th period per dollar of the g-th capital-cost-allowance activity in the t-th period;

z_{gs}^{1t} denote the capital cost allowance in the s-th period per dollar of the g-th capital-cost-allowance activity in the t-th period;

$\bar{a}_{vs}^{1t}$ denote the beef-cattle-capital requirement in the s-th period per beef cow in the v-th beef cattle activity;

$\bar{b}_{s}^{1t}$ denote the beef-cattle-housing-capital requirement in the s-th period per unit of beef cattle housing acquired in the t-th period;

$\bar{c}_{s}^{1t}$ denote the irrigation-system-capital requirement in the s-th period per irrigation system acquired in the t-th period;

$\bar{d}^{1t}_{jn's}$ denote the amount of the n'-th quota crop produced in the s-th period per acre of the j-th crop production activity in the t-th period;

$\bar{d}^{2t}_{qn's}$ denote the amount of the n'-th quota crop required in the s-th period per acre of the q-th forage-establishment activity in the t-th period;

$\bar{d}^{3t}_{vn's}$ denote the amount of the n'-th quota crop required in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

$\bar{d}^{4t}_{zn's}$ denote the coefficient for the n'-th quota crop restraint in the s-th period per unit of the z-th grain transfer activity in the t-th period;

$\bar{e}^{1t}_{js}$ denote the acre-inches of water required in the s-th period per acre of the j-th irrigated crop production activity in the t-th period;

$\bar{f}^{1t}_{vs}$ denote the amount of straw required in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

$\bar{g}^{1t}_{js}$ denote the amount of forage produced in the s-th period per acre of the j-th crop production activity in the t-th period;

$\bar{g}^{2t}_{vs}$ denote the amount of forage required in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

$\bar{h}^{1t}_{js}$ denote the amount of pasture produced in the s-th period per acre of the j-th crop production activity in the t-th period;

$\bar{h}^{2t}_{vs}$ denote the amount of pasture required in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

$\bar{i}^{1t}_{vs}$ denote the coefficient for the minimum cattle enterprise restraint in the s-th period for the v-th beef cattle activity in the t-th period;

$\bar{k}^{1t}_{jrs}$ denote the amount of the r-th category of labour required in the s-th period per acre of the j-th crop production activity in the t-th period;

$\bar{k}^{2t}_{frs}$ denote the amount of the r-th category of labour required in the s-th period per acre of the f-th summerfallow activity in the t-th period;

$\bar{k}^{3t}_{qrs}$ denote the amount of the r-th category of labour required in the s-th period per acre of the q-th forage-establishment activity in the t-th period;

$\bar{k}^{4t}_{vrs}$ denote the amount of the r-th category of labour required in the s-th period per beef cow in the v-th beef cattle activity in the t-th period;

$\bar{m}^{1t}_{ns}$ denote the assigned acreage requirement in the s-th period per bushel of the n-th quota crop sold in the t-th period;

n^{-1t}_{jus} denote the amount of fallow land of the u-th soil zone required in the s-th period per acre of the j-th crop production activity in the t-th period;

n^{-2t}_{fus} denote the amount of fallow land of the u-th soil zone supplied to the s-th period per acre of the f-th summerfallow activity in the t-th period;

o^{-1t}_{jus} denote the coefficient for the u-th soil zone minimum fallow restraint in the s-th period per acre of the j-th irrigated crop production activity in the t-th period;

o^{-2t}_{qus} denote the coefficient for the u-th soil zone fallow restraint in the s-th period per acre of the q-th forage-establishment activity in the t-th period;

p^{-1t}_{jus} denote the amount of stubble land of the u-th soil zone required or supplied in the s-th period per acre of the j-th crop production activity in the t-th period;

q^{-1t}_{jus} denote the amount of forage land of the u-th soil zone required in the s-th period per acre of the j-th crop production activity in the t-th period;

q^{-2t}_{qus} denote the amount of forage land of the u-th soil zone supplied to the s-th period per acre of the q-th forage-establishment activity in the t-th period;

r^{-1t}_{jus} denote the irrigable acreage of the u-th soil zone required in the s-th period per acre of the j-th irrigated crop production activity in the t-th period;

r^{-2t}_{us} denote the irrigable acreage of the u-th soil zone supplied to the s-th period per unit of irrigation system acquired in the t-th period;

$v^{-1t}_{ii's}$ denote the coefficient for the i'-th separable restraint in the s-th period for the i-th separable activity in the t-th period;

W^{*}_{s} denote the amount of the worst possible income restraint in the s-th period;

Y^{*} denote the amount of the special worst possible income restraint;

$T^{*}_{k'}$ denote the maximum income in the k'-th tax bracket in the s-th period;

B^{*}_{s} denote the supply of beginning cash in the s-th period;

K_s^* denote the working capital borrowing limit in the s-th period;

J_s^* denote the supply of cash reserves in the s-th period;

F_s^* denote the amount of the minimum intended consumption spending in the s-th period;

V_s^* denote the amount of cash overhead expenses in the s-th period;

P_s^* denote the amount of principal and interest payments on existing loans of intermediate- and long-term capital in the s-th period;

$D_{g's}^*$ denote the amount of capital investment in the g'-th capital class in the s-th period;

A_s^* denote the amount of capital investment in beef cattle in the s-th period;

$C_{n's}^*$ denote the supply of the n'-th quota crop in the s-th period;

I_s^* denote the number of irrigation systems that may be acquired in the s-th period;

$\bar{I}^*$ denote the number of irrigation systems that may be acquired;

U_s^* denote the minimum number of beef cows in the s-th period;

L_{rs}^* denote the supply of the r-th category of labour in the s-th period;

M_{us}^* denote the supply of summerfallowed land of the u-th soil zone in the s-th period;

N_{us}^* denote the minimum amount of summerfallowed land of the u-th soil zone in the s-th period;

Q_{us}^* denote the supply of stubble land of the u-th soil zone in the s-th period;

R_{us}^* denote the supply of forage land of the u-th soil zone in the s-th period;

V_{us}^* denote the maximum flax acreage on the u-th soil zone in the s-th period;

$S_{i's}^*$ denote the i'-th separable restraint in the s-th period;

Z denote the value of the objective function, i.e. the undiscounted expected additions to net worth.

Algebraic Formulation of the Model

Following the notations outlined in the above section, the objective of the model is to identify the combination of activities that will maximize:

$$Z = f(X) = X$$

subject to the following restraints:[1]

(1) net farm income accounting restraint in the s-th period.

$$\sum_{j=1}^{136} c_{js}^{1t} C_j^t + \sum_{f=1}^{2} C_{fs}^{2t} L_f^t + \sum_{q=1}^{2} c_{qs}^{3t} \bar{L}_q^t + \sum_{v=1}^{2} c_{vs}^{4t} U_v^t + \sum_{n=1}^{5} c_{ns}^{5t} Q_n^t$$

$$+ c_s^{6t} H^t + c_s^{7t} W^t + \sum_{t=1}^{s} \sum_{m=1}^{s} c_{ms}^{8t} \bar{G}_m^t + \sum_{g=1}^{3} c_{gs}^{9t} D_g^t + c_s^{10t} B^t$$

$$+ c_s^{11t} P^t + \sum_{k=1}^{11} T_k^t + C_s^{12t} R^t + V^t = 0;$$

(2) expected discretionary income accounting restraint in the s-th period

$$a_s^{1t} P^t + \sum_{k=1}^{11} a_{ks}^{2t} T_k^t + F^t + M^t = 0;$$

(3) expected additions to net worth accounting restraint

$$\sum_{t=1}^{10} (b^{1t} M^t + b^{2t} P^t) + X \leq 0;$$

(4) worst possible income requirement in the s-th period

$$\sum_{j=1}^{136} d_{js}^{1t} C_j^t + \sum_{q=1}^{2} d_{qs}^{2t} \bar{L}_q^t + \sum_{v=1}^{2} d_{vs}^{3t} U_v^t + \sum_{t=1}^{s} \sum_{m=1}^{3} d_{ms}^{4t} \bar{G}_m^t + E^t - V^t - P^t$$

$$- A \geq W_s^*;$$

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[1] Unless otherwise indicated (by summation over t or by definition within the restraint) t = s. Variables defined for a value of t = s - 1 are operative only for values of s $\geq$ 2.

(5) special worst possible income restraint for parametric variation of worst possible income requirement in all periods

$$A \geq Y^{*};$$

(6) Income restraint for the k'-th tax bracket in the s-th period

$$T_{k}^{t} \leq T_{k's}^{*};$$

(7) beginning cash on hand restraint in the s-th period

$$K^{t} + J^{t} - \underset{t=s-1}{N^{t}} - \underset{t=s-1}{R^{t}} \leq B_{s}^{*};$$

(8) working capital restraint in the s-th period

$$\sum_{j=1}^{136} e_{js}^{1t} C_{j}^{t} + \sum_{f=1}^{2} e_{fs}^{2t} L_{f}^{t} + \sum_{q=1}^{2} e_{qs}^{3t} \bar{L}_{q}^{t} + \sum_{v=1}^{2} e_{vs}^{4t} U_{v}^{t} + e_{s}^{5t} H^{t}$$

$$+ e_{s}^{6t} W^{t} - K^{t} - B^{t} \leq 0;$$

(9) working capital borrowing restraint in the s-th period

$$B^{t} \leq L_{s}^{*};$$

(10) cash flow accounting restraint in the s-th period

$$\sum_{j=1}^{136} o_{js}^{1t} C_{j}^{t} + \sum_{f=1}^{2} o_{fs}^{2t} L_{f}^{t} + \sum_{q=1}^{2} o_{qs}^{3t} \bar{L}_{q}^{t} + \sum_{v=1}^{2} o_{vs}^{4t} U_{v}^{t} + o_{s}^{5t} H^{t}$$

$$+ o_{s}^{6t} W^{t} + \sum_{n=1}^{5} o_{ns}^{7t} Q_{n}^{t} + \sum_{h=1}^{3} G_{h}^{t} + \sum_{t=1}^{s} \sum_{m=1}^{3} o_{ms}^{8t} \bar{G}_{m}^{t} + o_{s}^{9t} B^{t} + \sum_{k=1}^{11} o_{ks}^{10t} \underset{t=s-1}{T_{k}^{t}}$$

$$+ o_{s}^{11t} M^{t} + \bar{D}^{t} - K^{t} + F^{t} + V^{t} + P^{t} + N^{t} \leq 0;$$

(11) cash reserve restraint in the s-th period

$$- J^{t} + \sum_{t=s-1}^{s} y^{1t} E^{t} + R^{t} \leq J_{s}^{*};$$

(12) minimum consumption spending restraint in the s-th period

$$F^{t} = F_{s}^{*};$$

(13) cash overhead expense restraint in the s-th period

$$V^t = V_s^*;$$

(14) principal and interest payments restraint in the s-th period

$$P^t = P_s^*;$$

(15) capital restraint for the g'-th capital class in the s-th period

$$p_{g's}^{1t}\ I^t + p_{g's}^{2t}\ Y^t + \sum_{\substack{t=s-1}}^{s} p_{gg's}^{3t}\ D_g^t = D_{g's}^*; \quad g=g'$$

(16) capital cost allowance account in the s-th period

$$\sum_{g=1}^{3} z_{gs}^{1t}\ D_g^t + \bar{D}^t = 0;$$

(17) beef cattle investment restraint in the s-th period

$$\sum_{v=1}^{2} \bar{a}_{vs}^{1t}\ U_v^t - \sum_{t=1}^{s} (G_3^t + \bar{G}_3^t) = A_s^*;$$

(18) capital restraint for additional beef cattle housing in the s-th period

$$\bar{b}_s^{1t}\ Y^t - G_2^t - \bar{G}_2^t \leq 0;$$

(19) capital restraints for the purchase of an irrigation system in the s-th period

$$\bar{c}_s^{1t}\ I^t - G_1^t - \bar{G}_1^t \leq 0;$$

(20) supply restraint for the n'-th quota crop in the s-th period

$$\sum_{j=1}^{136} \bar{d}_{jn's}^{1t}\ C_j^t + \sum_{q=1}^{2} \bar{d}_{qn's}^{2t}\ \bar{L}_q^t + \sum_{v=1}^{2} \bar{d}_{vn's}^{3t}\ U_v$$

$$+ \sum_{t=s-1}^{s} \sum_{z=1}^{s} \bar{d}_{zn's}^{4t}\ \bar{Q}_z^t + Q_n^t \leq C_{n's}^*; \quad n=n'$$

(21) irrigation water restraints in the s-th period

$$\sum_{j=103}^{136} \bar{e}_{js}^{1t} C_j^t - W^t \leq 0;$$

(22) irrigation system purchase restraint in the s-th period

$$I^t \leq I_s^*;$$

(23) irrigation system purchase restraint

$$\sum_{t=1}^{10} I^t \leq \bar{I}^*;$$

(24) straw account in the s-th period

$$\sum_{v=1}^{2} \bar{f}_{vs}^{1t} U_v^t - H^t \leq H_s^*;$$

(25) forage account in the s-th period

$$\sum_{j=1}^{136} \bar{g}_{js}^{1t} C_j^t + \sum_{v=1}^{2} \bar{g}_{vs}^{2t} U_v^t \leq 0;$$

(26) pasture account in the s-th period

$$\sum_{j=1}^{136} \bar{h}_{js}^{1t} C_j^t + \sum_{v=1}^{2} \bar{h}_{vs}^{2t} U_v^t \leq 0;$$

(27) minimum cattle enterprise restraint in the s-th period

$$\sum_{v=1}^{2} \bar{i}_{vs}^{1t} U_v^t - \sum_{t=1}^{s} Y^t = U_s^*;$$

(28) labour restraint for the r-th category of labour in the s-th period

$$\sum_{j=1}^{136} \bar{k}_{jrs}^{1t} C_j^t + \sum_{f=1}^{2} \bar{k}_{frs}^{2t} L_f^t + \sum_{q=1}^{2} \bar{k}_{qrs}^{2t} \bar{L}_q^t + \sum_{v=1}^{2} \bar{k}_{vrs}^{3t} u_v^t \leq L_{rs}^*;$$

(29) assignable acres account in the s-th period

$$- \sum_{j=1}^{136} C_j^t - \sum_{j=1}^{2} L_f^t - \sum_{q=1}^{2} \bar{L}_q^t + \sum_{n=1}^{5} \bar{m}_{ns}^{1t} Q_n^t \leq 0;$$

(30) summerfallow land restraint of the u-th soil zone in the s-th period

$$\sum_{j=1}^{136} \bar{n}_{jus}^{1t} C_j^t + \sum_{\substack{t=s-1 \\ f=u}}^{s} \bar{n}_{fus}^{2} L_f^t \leq M_{us}^*;$$

(31) minimum summerfallow restraint of the u-th soil zone in the s-th period

$$\sum_{t=1}^{s} \sum_{j=103}^{136} \bar{o}_{jus}^{1t} C_j^t + \underset{f=1}{L_f^t} + \sum_{\substack{t=1 \\ q=u}}^{s} \bar{o}_{qus}^{3t} \bar{L}_q^t \geq N_{us}^*;$$

(32) stubble and restraint of the u-th soil zone in the s-th period

$$\sum_{t=s-1}^{s} \sum_{j=1}^{136} \bar{p}_{jus}^{1t} C_j^t + \underset{f=u}{L_f^t} \quad \underset{q=u}{\bar{L}_q^t} \leq Q_{us}^*;$$

(33) forage land restraint of the u-th soil zone in the s-th period

$$\sum_{j=1}^{136} \bar{q}_{jus}^{1t} C_j^t + \sum_{\substack{t=1 \\ q=u}}^{s} \bar{q}_{qus}^{2t} \bar{L}_q^t \leq R_{us}^*;$$

(34) flax restraint for the u-th soil zone in the s-th period

$$\sum_{\substack{j=31,\ 32, \\ 33,\ 34,\ 89,\ 90, \\ 91,\ 92,\ 115,\ 116, \\ 117,\ 118}} C_j^t \leq V_{us}^*;$$

(35) irrigable acreage restraint of the u-th soil zone in the s-th period

$$\sum_{j=103}^{136} \bar{r}_{jus}^{1t} C_j^t + \sum_{t=1}^{s} \bar{R}_{us}^{2t} I^t \leq 0;$$

(36) i'-th separable restraint in the s-th period

$$- I^t + \sum_{i=1}^{2} \bar{v}^{-1t}_{ii's} S^t_i = S^*_{i's}; \text{ and}$$

(37) non-negativity conditions

$$C^t_j, L^t_f, \bar{L}^t_q, U^t_v, Q^t_n, H^t, W^t, I^t, S^t_i, Y^t, G^t_h, \bar{G}^t_m, D^t_g, \bar{D}^t, K^t, J^t, B^t;$$

$$E^t, F^t, V^t, P^t, T^t_k, \bar{Q}^t_z, N^t, R^t, M^t, X, A \geq 0.$$

With respect to the coefficients in the model, it should be noted that they do not all have non-zero values. For example, each crop production activity does not require labour from each labour restraint. Accordingly, the specification of a coefficient in the algebraic model does not always mean that the coefficient is different from zero. Although it would have been possible to specify the mathematical model in such a way that only non-zero coefficients were included, incorporation of this feature would have required a more complex algebraic formulation. The formulation that has been presented provides the basic outline of the mathematical relationships in the model.

APPENDIX B

SUPPORTING DATA

Table B.1[a]

Net Farm Income, Province of Manitoba, by Year
and as a Percentage of Previous Year, 1960-1971

Year	Province of Manitoba	
	Net Farm Income	Net Farm Income as a Percentage of Previous Year
	millions of dollars	percent
1960	108	-
1961	57	53
1962	162	48
1963	107	66
1964	157	147
1965	169	108
1966	148	88
1967	154	104
1968	162	105
1969	114	70
1970	84	74
1971	158	188

[a]The data in this table give an indication of the amount of year-to-year variation in net farm incomes in Manitoba.

Source:

Manitoba Department of Agriculture, Yearbook of Manitoba Agriculture, 1971, (Winnipeg: Queen's Printer for the Province of Manitoba, 1972).

Table B.2

Prices Received by Manitoba Farmers, Dollars Per Unit, Eleven Crops, by Year, 1962-1971, and on Average, 1962-1971

Year	Wheat	Oats	Barley	Flax	Rye	Rapeseed	Peas	Mustard	Sunflowers	Hay	Corn Silage
	....			bushel	....			...... pound	....	....	ton ....
1962	1.70	.59	1.00	3.00	1.05	1.75	2.20	.045	.055	14.00	7.00
1963	1.71	.55	.92	2.85	1.30	2.50	2.30	.050	.044	15.00	7.00
1964	1.63	.60	1.02	2.95	1.05	2.70	1.90	.050	.050	16.00	6.00
1965	1.65	.71	1.05	2.69	1.04	2.45	1.90	.051	.058	16.50	7.00
1966	1.78	.75	1.10	2.70	1.08	2.45	2.25	.051	.060	16.50	7.00
1967	1.64	.69	.89	3.08	1.08	1.90	2.10	.054	.045	17.50	7.00
1968	1.36	.49	.79	2.84	1.00	1.88	2.03	.050	.050	18.50	7.00
1969	1.45	.55	.69	2.57	.87	2.40	1.87	.034	.055	16.50	7.00
1970	1.45	.50	.80	2.15	.85	2.35	1.75	.035	.060	16.50	7.00
1971	1.40	.50	.80	2.00	.70	2.15	1.75	.038	.055	16.50	7.00
Average	1.58	.59	.91	2.68	1.00	2.25	2.01	.053	.046	16.35	6.90

Source:

Yearbook of Manitoba Agriculture, 1971, (Winnipeg: Queen's Printer for Province of Manitoba, 1972) pp.42-53.

Table B.3

Estimated Expected and Worst Possible Crop Prices, Years One, Two, Three and Four[a]

Crop	Description of Price	Unit	Price per Unit Year 1	Year 2	Year 3	Year 4
			.... dollars	....		
Wheat	expected	bushel	1.90	1.79	1.68	1.58
	worst possible	bushel	1.76	1.62	1.49	1.36
Oats	expected	bushel	.89	.79	.69	.59
	worst possible	bushel	.65	.59	.54	.49
Barley	expected	bushel	1.29	1.16	1.03	.91
	worst possible	bushel	1.05	.93	.81	.69
Flax	expected	bushel	4.25	3.72	3.20	2.68
	worst possible	bushel	3.25	2.91	2.57	2.00
Rye	expected	bushel	1.50	1.33	1.16	1.00
	worst possible	bushel	1.09	.96	.83	.70
Rapeseed	expected	bushel	3.50	3.08	2.66	2.25
	worst possible	bushel	3.00	2.58	2.16	1.75
Peas	expected	bushel	2.18	2.12	2.06	2.01
	worst possible	bushel	2.00	1.91	1.83	1.75
Mustard	expected	pound	.060	.055	.050	.046
	worst possible	pound	.060	.051	.043	.035
Sun-flowers[b]	expected	pound	.053	.053	.053	.053
	worst possible	pound	.044	.044	.044	.044
Corn silage[b]	expected	ton	7.00	7.00	7.00	7.00
	worst possible	ton	7.00	7.00	7.00	7.00
Hay[b]	expected	ton	16.50	16.50	16.50	16.50
	worst possible	ton	15.00	15.00	15.00	15.00

[a]The expected and the worst possible prices of the fourth year were assumed for each succeeding year.

[b]Since cyclical variation in the prices of these crops was not as apparent as in most other crops, the same expected and the same worst possible prices were assumed in each year.

Table B.4

Prices Received, Dollars per Hundredweight, Basis Winnipeg Public Stockyards, 1963-1972, for Five Different Classes of Cattle

Year	Medium Cows	Good Feeder Steers	Good Feeder Heifers	Good Stock Steer Calves	Good Stock Heifer Calves
	.... dollars per hundredweight				
1963	15.92	23.20	19.63	25.85	21.79
1964	14.33	20.85	16.52	22.21	17.95
1965	13.67	22.05	18.08	23.20	19.15
1966	17.70	25.55	21.35	28.60	23.85
1967	18.35	26.55	22.45	30.40	24.95
1968	18.29	26.85	22.95	30.85	25.35
1969	22.03	31.75	26.81	37.77	30.97
1970	21.89	28.71	28.04	39.42	32.78
1971	21.70	33.75	28.43	41.80	33.23
1972	24.90	38.81	32.92	46.28	37.25
10-year average:	18.88	27.81	23.72	32.64	26.73
5-year average:					
1967-71	20.45	29.52	25.74	36.05	29.46
1968-72	21.76	31.97	27.83	39.22	31.92

Source:

Agriculture Canada, Livestock Market Review, (Ottawa: Markets Information Section, Livestock Division, Production and Marketing Branch, Agriculture Canada).

Table B.5

Estimated Expected and Worst Possible Beef Cattle Prices and Gross Revenues per Cow, Years One, Two, Three and Four

Category	Description of Price	Year 1	Year 2	Year 3	Year 4
		.... dollars			
	(a) Per Hundredweight, Six Cattle Classes				
Medium cow	expected	24.90	23.85	22.80	21.76
	worst possible	21.76	20.71	19.66	18.62
Good feeder steer	expected	38.81	36.53	34.25	31.97
	worst possible	31.97	29.69	27.41	25.13
Good feeder heifer	expected	32.92	31.22	29.52	27.83
	worst possible	27.83	26.13	24.43	22.74
Good feeder steer calf	expected	46.28	43.92	41.57	39.22
	worst possible	39.22	36.86	34.51	32.16
Good feeder heifer calf	expected	37.25	35.47	33.62	31.92
	worst possible	31.92	30.14	28.36	26.59
	(b) Per Cow, Two Cow-Calf Alternatives[a]				
Cow-calf I	expected	170.73	162.66	154.51	146.60
	worst possible	146.60	138.53	130.48	122.47
Cow-calf II	expected	231.67	219.54	207.40	195.31
	worst possible	195.31	183.17	171.03	158.94

[a]The expected and worst possible prices of the fourth year were assumed for each succeeding year.

Table B.6

Delivery Quota, Bushels Per Assigned Acre, and Per Specified Acre,[a] Basis July 31, Six Different Crops, by Year, 1963-1973

Year	Wheat	Oats	Barley	Flax	Rye	Rapeseed
	.... bushels per assigned acre					
1973	open[b]	open	open	open	open	open
1972	9	12	40	15	30	20
1971	8	20	30	12	20	40
	.... bushels per specified acre					
1970	5	5	5	open	5	open
1969	6	6	6	open	open	open
1968	7	7	7	open	open	open
1967	10	10	10	open	open	open
1966	open	open	open	open	open	open
1965	10	10	10	open	open	open
1964	9	9	9	open	open	open
1963	13	13	13	open	open	open

[a]In 1971 the quota system was changed from a 'specified' acre system to an assignable acre system. Under the former system, the Canadian Wheat Board indicated the quota per specified acre, and producers could fill their quota with wheat, oats, barley or rye. Under the latter system, the producer assigns quota acres to particular crops before the Canadian Wheat Board indicates the delivery quotas per assigned acre.

[b]An open quota indicates no restrictions on delivery.

Source:

Annual Reports of the Canadian Wheat Board and personal communication from T. Kascisa of Canadian Wheat Board, June 1973.

Table B.7

Estimated Expected and Worst Possible[a] Delivery Quotas, Bushels Per Assigned Acre, Six Quota Crops, Years One, Two, Three and Four[b]

Crop	Quota Description	Year			
		One	Two	Three	Four
		bushels			
Wheat	expected	open	9	8	10
	worst possible	9	8	5	5
Oats	expected	open	12	20	25
	worst possible	12	20	10	10
Barley	expected	open	40	30	20
	worst possible	40	30	15	10
Flax	expected	open	open	open	open
	worst possible	open	open	open	open
Rye	expected	open	open	open	open
	worst possible	open	open	open	open
Rapeseed	expected	open	open	open	open
	worst possible	open	open	open	open

[a]Worst possible delivery quotas were estimated but were not included in the empirical analysis.

[b]The expected quotas of the fourth year were assumed for each succeeding year.

Table B.8

Estimated Capital and Operating Costs for a Centre-Pivot Irrigation System, with 138 Acre Capacity

Item	Amount or Cost	
	1971[a]	1973[b]
	.... dollars	....
Capital Investment[c]	27,623	29,108
Operating cost per acre-inch of water applied	1.04	1.11

[a]From estimates reported by G.J. Finn, "An Economic Feasibility Study of Irrigating from Groundwater in South Western Manitoba", (Unpublished Master's Thesis, University of Manitoba, 1971), p. 74 and by R.H. Singh, "An Economic Analysis of Irrigation in the Souris River Basin in Manitoba", (Unpublished Master's Thesis, University of Manitoba, 1972), p. 103. See also W.G. Wedemeyer and T.L. Dobbs, Financing and Feasibility of Centre-Pivot Sprinkler Irrigation Systems in Wyoming, Research Journal 72 (Laramie: University of Wyoming, March 1973). p. 5.

[b]The estimated values for 1973 were obtained by adding an allowance for trend to the estimates for 1971.

[c]Includes the cost of a well, sprinkler system, motor and pump, pipe and miscellaneous capital costs.

Table B.9

Sequence of Field Operations Used in Estimating Machine Costs for Different Crops

Crop	Sequence of Operations[a]
Wheat, oats, barley or flax on stubble	Cs-D-Pd-Sp-Sw-Co-H
Wheat, barley, or rapeseed on fallow	Cs-D-Pd-Sp-Sw-Co-H-Ch
Peas	Sp-D-Pd-Sp-Sw-Co-H-Ch
Sunflowers	Sp-D-Pd-Co-H-Ch
Corn silage	Sp-D-Pl-Pa-Ha-H
Hay	M-R-B-H
Summerfallow	Ch-Cl-Cl-Cl-Cl

[a]Cs - cultivate with shovels
D - disc
Pd - press drill
Sp - spray
Sw - swath
Co - combine
Ch - cultivate (heavy duty)
H - haul and store
Pl - plant
Pa - pack
Ha - harvest
M - mow
R - rake
B - bale
Cl - cultivate (light duty)

Table B.10

Basic Data Used in Estimating Machine Operating Costs and Man-Time Per Acre, by Machine

Machine	Description	Field Efficiency	Field Speed	Machine Time	Man-Time	Repair Cost Per Hour	Repair Cost Per Acre	Fuel Cost Per Hour	Fuel Cost Per Acre	Maintenance Cost Per Hour	Maintenance Cost Per Acre	Total Cost Per Hour	Total Cost Per Acre
		percent	m.p.h.	hours per acre		....	....	....	....	dollars	....	....	....
Cultivator	14 ft. H.D.	75	5	.158	.182	.250	.046	-	-	.010	.002	.260	.048
Cultivator	20 ft. L.D.	82.5	4.5	.121	.139	.220	.027	-	-	.010	.002	.240	.029
Disc	16 ft.	82.5	5	.138	.159	.600	.083	-	-	.050	.007	.650	.090
Harrow	40 ft.	80	5.5	.052	.060	.140	.007	-	-	.010	.001	.150	.008
Press drill	14 ft.	65	4	.245	.339	.710	.174	-	-	.040	.010	.750	.184
Sprayer	45 ft.	65	4.5	.070	.091	.280	.020	-	-	.010	.001	.290	.021
Swather	14 ft. p.t.o.	82.5	4.5	.158	.182	.460	.073	-	-	.030	.005	.490	.078
Combine	s.p. (9500)	70	3.5	.238	.321	2.170	.516	1.150	.274	.120	.029	3.440	.819
Tractor	gas (40 h.p.)	-	-	1.000	1.000	.310	-	.890	-	.040	-	1.240	-
Tractor	diesel (80 h.p.)	-	-	1.000	1.000	.550	-	.880	-	.040	-	1.470	-
Truck	½ ton	-	-	-	-	.730	-	.630	-	.090	-	1.450	-
Truck	2 ton	-	-	-	-	.870	-	.920	-	.140	-	1.930	-
Front end loader	-	-	-	-	-	.130	-	-	-	.010	-	.140	-
Baler	p.t.o.	-	-	-	-	.420	-	-	-	.040	-	.460	-
Side rake	9 ft.	82.5	6.0	.167	.192	.170	.028	-	-	.010	.002	.180	.030
Mower	9 ft.	77.5	5.0	.185	.213	.270	.050	-	-	.010	.002	.280	.052
Packer	14 ft.	82.5	4.0	.245	.339	.060	.020	-	-	.010	.003	.070	.023

Source:

Developed from information in Farm Data Handbook, (Winnipeg: Manitoba Department of Agriculture, Economics Branch), 1970.

Table B.11

Seed Requirements and Estimated 1972 Seed Cost per Acre for Crops

Crop	Seed Requirement	Seed Cost[a]
	per acre	dollars per acre
Wheat	1.50 bushels	3.42
Oats	2.25 bushels	2.38
Barley	1.75 bushels	2.64
Flax	.57 bushels	2.58
Rapeseed	.10 bushels	1.30
Peas	2.75 bushels	6.58
Sunflowers	10.00 pounds	3.05
Corn silage	.21 bushels	3.48
Hay	12.00 pounds[b]	6.18

[a]The seed costs for wheat, oats, barley, flax, rapeseed, peas and sunflowers were estimated by valuing the seed at the ten-year average price and adding an allowance of $1.05 per acre for seed replacement, treatment and cleaning. Seed corn was priced at $16.00 per bushel.

[b]A seed mixture of 3 pounds of alfalfa and 9 pounds of brome grass and crested wheat grass was assumed. Alfalfa seed was priced at $.80 per pound and grass seed was priced at $.42 per pound.

Table B.12

Estimated Operating Costs per Acre for Crop Production Alternatives, Basis 1972[a]

Crop	Soil Zone	Seedbed	Fertilizer Application	Operating Costs: Machinery[b]	Fertilizer[c]	Sprays	Seed	Insurance[d]	Total
				dollars per acre					
Wheat	G	stubble	medium	3.30	2.70	.60	3.42	.77	10.79
Oats	G	stubble	medium	3.61	2.61	.60	2.38	.98	10.18
Barley	G	stubble	medium	3.30	3.60	.60	2.64	1.12	11.26
Flax	G	stubble	none	3.37	-	.60	2.58	1.18	7.73
Rapeseed	G	fallow	medium	3.69	4.05	3.25	1.30	1.56	13.85
Peas	G	fallow	medium	3.61	.90	5.25	6.58	2.10	18.44
Sunflowers	G	stubble	medium	3.04	4.15	5.25	3.05	1.28	16.77
Corn silage	G	stubble	medium	8.32	8.95	9.38	3.48	3.98	30.13
Hay	G	-	medium	3.78	2.70	-	-	-	6.48
Pasture	G	-	medium	.41	2.70	-	-	-	3.31

[a]The estimated operating costs per acre shown in this table are for selected crop production alternatives. Other alternatives in the model, for the same crops, had different costs for machinery and for fertilizer.

[b]Machinery operating costs were estimated by budgeting the variable costs for the different operations; for alternatives with expected yields above the crop district average, higher harvesting costs were incorporated into the estimated machinery costs; the assumption was that harvesting costs per acre were linearly related to yield. Extra costs for applying fertilizer to the heavily fertilized alternatives were included. Fuel prices of $.225 and $.260, per gallon of diesel and gasoline, respectively, were assumed.

[c]Nitrogen, phosphorous and potash nutrient prices of $.09, $.09 and $.05 per pound, respectively, were assumed. See Table C.3 and Table C.5 for application rates for the different alternatives.

[d]If insurable, and insured.

Table B.13

Estimated Seasonal Labour Requirements for Crop Production Alternatives[a]

Crop	Planting	Harvesting
	...... hours per acre	
Wheat on stubble	.82	.82
Oats on stubble	.82	.82
Barley on stubble	.82	.82
Flax on stubble	.82	.82
Rapeseed on fallow	.91	.84
Peas on fallow	.73	.86
Sunflowers on stubble	.73	.65
Corn silage on stubble	1.17	4.23
Hay	-	2.93

[a]The labour requirements were estimated by budgeting the man-hours required for the different operations. The figures shown in the table are for an average-yield alternative. Alternatives with higher-than-average yields had higher labour requirements for planting (to apply additional fertilizer) and for harvesting (to harvest additional yield).

Table B.14

Input-Output Coefficients for Two Beef Cow-Calf Alternatives, Per Cow Basis

Item	Unit	Cow-Calf I: Calves Sold as Feeder Calves	Cow-Calf II: Calves Sold as Long Yearlings
Inputs:			
Hay	ton	1.79	2.71
Barley	bushel	5.18	13.43
Straw-feed	ton	.24	.24
-bedding	ton	.84	1.24
Salt	pound	25.	40.
Mineral	pound	50.	75.
Winter labour	hour		
Operating costs:			
Salt and minerals	dollar	5.04	7.73
Veterinary medicine	dollar	3.10	4.50
A.I. fee	dollar	8.00	8.00
Trucking	dollar	3.12	4.48
Yardage and commission	dollar	1.88	2.98
Manure disposal	dollar	8.38	13.38
Miscellaneous	dollar	4.50	6.00
Total	dollar	34.02	43.07
Outputs:			
Cull cow	head	.18	.18
	pound	216.	216.
Feeder calf-steer	head	.35	-
	pound	140.	-
Feeder calf-heifer	head	.35	-
	pound	140.	-
Feeder steer	head	-	.33
	pound	-	248.
Feeder heifer	head	-	.33
	pound	-	248.

Source:

Developed from information in Farm Data Handbook (Winnipeg: Manitoba Department of Agriculture, Economics Branch, 1972).

Table B.15

1972 Marginal Income Tax Rates and After-Tax Income per Dollar of Marginal Income, Federal and Province of Manitoba Taxes Combined[a, b]

Total Taxable Income	Marginal Taxable Income	Marginal Tax Rate	After-Tax Income Per Dollar Marginal Taxable Income
dollars	dollars	percent	dollars
500	500	23.715	.76285
1,000	500	25.110	.74890
2,000	1,000	26.505	.73495
3,000	1,000	27.900	.72100
5,000	2,000	29.295	.70705
7,000	2,000	32.085	.67915
9,000	2,000	34.875	.65125
11,000	2,000	37.665	.62335
14,000	3,000	43.245	.56755
24,000	10,000	48.825	.51175
39,000	15,000	54.405	.45595
60,000	21,000	59,985	.40015
above 60,000	remainder	65.565	.34435

[a]Manitoba income tax for 1972 was 42.5 percent of "Basic Federal Tax" and the combined marginal rate was computed as 142.5 percent of the federal rate.

[b]Tax rates for 1973 and subsequent years were assumed to be the same as for 1972.

Source:

Computed from Federal and Manitoba tax rates and schedules for 1972.

Table B.16

Repayment Schedules for Principal and Interest, per Dollar Borrowed for Five, Ten and Fifteen Year Terms[a], with Equal Annual Principal Payments and Interest at 8 Percent on Remaining Balance

	Term of Loan					
	Five Years		Ten Years		Fifteen Years	
Year	Principal	Interest	Principal	Interest	Principal	Interest
	dollars					
1	.20	.080	.10	.080	.067	.0800
2	.20	.064	.10	.072	.067	.0746
3	.20	.048	.10	.064	.067	.0692
4	.20	.032	.10	.056	.067	.0638
5	.20	.016	.10	.048	.067	.0584
6			.10	.040	.067	.0530
7			.10	.032	.067	.0476
8			.10	.024	.067	.0422
9			.10	.016	.067	.0368
10			.10	.008	.067	.0314
11					.067	.0260
12					.067	.0206
13					.067	.0152
14					.067	.0098
15					.067	.0044

[a]In the analysis, it was assumed that payments would commence one year from the year of purchase.

APPENDIX C

ESTIMATION OF DRYLAND AND IRRIGATED YIELDS

APPENDIX C

ESTIMATION OF DRYLAND AND IRRIGATED YIELDS

Eleven crops were initially selected for consideration in this study. They were wheat, oats, barley, flax, rye, rapeseed, field peas, sunflowers, mustard, corn silage and hay.[1] This appendix reports on the procedure used to obtain estimates of the dryland and irrigated yields for the different crop production activities.

Expected Dryland Yields

Time-series data were used to estimate the expected and forecast dryland crop yields. The procedure followed in obtaining an estimate of the expected dryland yield for each crop is reported below.

The twenty-year average yield was computed for each of the eleven crops for Manitoba Crop District Number 1. These yield figures, for the period 1952-1971, are shown in Table C.1, along with the estimated standard deviation and coefficient of variation for the yield of each crop.

In preparing the estimates in Table C.1, twenty-year time series on yields were not available for all eleven crops basis Crop District Number 1.[2] In these cases, provincial data were adjusted for use as estimates for Crop District Number 1. The adjustments took the form of reducing the average

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[1] To reduce the number of activities, it was later decided to exclude rye and mustard crops from consideration in the empirical applications of the model.

[2] Crop District yields were available for wheat, oats, barley, flax and rye from data prepared for W.J. Craddock, Interregional Competition in Canadian Cereal Production, Special Study No.12, Economic Council of Canada (Ottawa: Queen's Printer, 1970).

yield and increasing the variance in yield.[3] The estimated standard deviations in Table C.1 were computed after removing any significant linear trend in yield from 1952 to 1971.

Table C.1

Estimated Twenty-Year Average Yield Per Acre, Standard Deviation and Coefficient of Yield Variation for Eleven Crops, Basis Manitoba Crop District Number 1, 1952-1971

	Yield per Acre			
Crop	Unit	Average	Standard Deviation	Coefficient of Variation
				percent
Wheat	bushels	21.31	4.20	20
Oats	bushels	37.99	9.34	25
Barley	bushels	28.94	6.22	21
Flax	bushels	8.75	2.77	32
Rye	bushels	19.59	4.37	22
Rapeseed	bushels	15.16	2.56	17
Field Peas	bushels	17.35	5.14	30
Sunflowers	pounds	652.	193.00	30
Mustard[a]	pounds	683.	170.00	25
Corn Silage	tons	5.44	.98	18
Hay	tons	1.70	.37	22

[a]Based on observations for seventeen years 1955-1971.

[3]Comparison of the provincial data and the Crop District Number 1 data for wheat, oats, barley, flax and rye indicated lower yields and higher variance in Crop District Number 1 than in the province as a whole. The same relationship was assumed to hold for the yield of other crops in Crop District Number 1.

To the extent that there was any significant trend in the twenty-year average yields, the estimated yields shown in Table C.1, underestimate the expected twenty-year average yield for future years. Accordingly, a twenty-year moving average was computed for each of the eleven crops and this value was tested for a linear trend. Shown in Table C.2, significant positive trends were found for all crops except flaxseed, field peas, mustard and hay.

The trend in the twenty-year average yield was added to the estimated average yield for the period 1952-1971 to obtain an estimate of the twenty-year average yield for the periods 1963-1982, 1964-1983, 1965-1984, and so on. The forecast twenty-year average yield for the period 1963-1982 was taken as the expected yield in 1973; that for 1964-1983 was taken as the expected yield for 1974; and similarly for other years. The projected average yields, basis 1963-1982, are shown in Table C.2.

Dryland Yields According to Soil Zone, Seedbed and Amount of Fertilizer. Records of the Manitoba Crop Insurance Corporation, soil test records from the Soil Testing Service of the Manitoba Department of Agriculture and experimental data from fertility experiments conducted by the Department of Soil Science, University of Manitoba, were used to estimate crop yields according to soil zone (G or H), according to cultural practice (stubble or fallow seedbed), and according to applications of commercial fertilizer (none, light or heavy). The crop insurance records used were for the E ½ of Edward and the W ½ of Arthur municipalities for the years 1963 through 1971. These records were used to obtain estimates of the relation between the yields of stubble seeded and fallow seeded crops and between the yields of crops seeded with and without commercial fertilizer.

The soil test records were used to estimate the average soil nitrogen available for crops seeded on stubble, and for crops seeded on fallow in the study area. To obtain estimates of the response of wheat and barley crops to

Table C.2

Estimated Trend in Twenty-Year Average Yield Per Acre and Estimated Average Yield Per Acre, 1963-1982, by Crop

Crop	Unit	Trend in Twenty-Year Average Yield[a]	Estimated Average Yield 1963-1982
		per acre per year	per acre
Wheat	bushel	.10	23.11
Oats	bushel	.35	43.15
Barley	bushel	.27	33.01
Flax	bushel	-	8.75
Rye	bushel	.26	23.13
Rapeseed	bushel	.17	17.03
Field Peas	bushel	-	17.35
Sunflowers	pound	8.	767.
Mustard	pound	-	683.
Corn Silage	ton	.10	6.96
Hay	ton	-	1.70

[a]A dash (-) indicates no significant trend.

a fairly heavy application of commercial fertilizer, the following crop yield prediction equations were employed:[4]

A. Wheat

$$(C.1) \quad Y = 50.1 \log_{10} (19.6 + 0.975X_2 - 0.00235X_1^2 + 0.510X_2 - 0.0007X_2^2 + 0.00537X_1X_2) - 56.20$$

where Y = forecast yield of wheat,

X_1 = soil nitrogen

and X_2 = fertilizer nitrogen.

B. Barley

$$(C.2) \quad Y = 75.7 \log_{10} (6.35 + 1.95X_1 - 0.0115X_1^2 + 0.733X_2 - 0.00122X_2^2 - 0.00276X_1X_2) - 82.5$$

where Y = forecast yield of barley,

X_1 = soil nitrogen

and X_2 = fertilizer nitrogen.

Although the forecast estimated yields from these equations exceeded the actual yields reported by farmers in the area, (for the same amount of fertilizer), the forecast response to nitrogen fertilizer corresponded fairly well with the actual response. An example will illustrate the point. The forecast response to 15.5 pounds of nitrogen for barley seeded on stubble was 7.08 bushels per acre. The actual response, basis crop insurance records, to 15.5 pounds of nitrogen for barley seeded on stubble on soil zone G was 6.90 bushels per acre. The similarity between the responses to the same amount of fertilizer implied that the production functions had the same shape but represented different technologies. Thus, to obtain estimates of the actual

[4] J. Neilsen, "Relationship Between Farm Yields and Yields Predicted on the Basis of Soil and Fertilizer Nitrogen and Water Deficit", (Unpublished Masters Thesis, University of Manitoba, 1972), p. 20.

farm yield response to heavier applications of fertilizer[5] than those reported in crop insurance records, the forecast yields from the prediction equations were adjusted proportionately.

Since fertilizer response equations were not available for crops other than wheat and barley, it was assumed that the response to fertilizer would be the same for oats, flax, rye and rapeseed, as for wheat and barley. For the other crops the yield response to fertilizer was estimated on evidence from a variety of sources.

The estimated expected dryland crop yields for 1973, according to soil zone, seedbed and amount of fertilizer are shown in Table C.3. They represent the dryland crop production alternatives in the model.

Expected Irrigated Yields

Expected irrigated yields were based on information derived from time series data on yield and water deficit. Water deficit, also called available soil water, is a measure of the combined effect of seasonal moisture and temperature. It is a cumulative measure, given certain assumptions,[6] of soil moisture available for crop production on a particular date. A positive value for water deficit indicates a surplus, a negative value indicates a deficit and a value of zero indicates neither surplus nor deficit. Average water deficits on four different dates are shown in Table C.4 for the Pierson weather station which is located on the western edge of the study area within the Souris Basin.[7] They showed a fairly wide year-to-year variation in water deficit and suggested that time series data on water deficits and crop yields could be used to estimate the expected improvement in yields with irrigation.

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[5]It was considered important to include alternatives with heavy applications of commercial fertilizer to get a fair comparison of the relative payoffs from fertilizing and irrigating.

[6]See C.F. Shaykewich, "Progress Report on Climatic Analysis of Weather Stations in Manitoba" (Winnipeg: Department of Soil Science, University of Manitoba, 1972). (Mimeographed), p. 1.

[7]The location of the Pierson weather station is identified on the map on page 6.

Table C.3

Estimated Expected and Worst Possible Yields per Acre for Dryland Crop Production Activities for 1973, According to Soil Zone, Seedbed and Fertilizer Inputs

Crop	Soil Zone	Seedbed	Fertilizer per Acre[a]	Estimated Yield Per Acre: Expected	Estimated Yield Per Acre: Worst Possible
				.... bushels	....
Wheat:	G	stubble	0-0-0	18.14	4.90
	G	stubble	12-18-0	23.12	6.24
	G	stubble	50-30-30	29.45	7.95
	G	fallow	0-0-0	25.06	6.77
	G	fallow	6-23-0	30.67	8.28
	G	fallow	50-30-30	40.29	10.88
	H	stubble	0-0-0	16.59	4.48
	H	stubble	12-18-0	21.14	5.71
	H	stubble	50-30-30	26.92	7.27
	H	fallow	0-0-0	22.91	6.19
	H	fallow	6-23-0	28.05	7.57
	H	fallow	50-30-30	36.82	9.94
Oats:	G	stubble	0-0-0	38.84	5.83
	G	stubble	13-16-0	50.64	7.60
	G	stubble	50-30-30	62.32	9.35
	H	stubble	0-0-0	35.04	5.26
	H	stubble	13-16-0	45.67	6.85
	H	stubble	50-30-30	56.20	8.43
Barley:	G	stubble	0-0-0	23.64	5.20
	G	stubble	17-23-0	30.54	6.72
	G	stubble	50-30-30	37.45	8.24
	G	fallow	0-0-0	35.44	7.80
	G	fallow	10-24-0	47.10	10.36
	G	fallow	50-30-30	56.89	12.52
	H	stubble	0-0-0	21.90	4.82
	H	stubble	17-23-0	28.29	6.22
	H	stubble	50-30-30	34.68	7.63
	H	fallow	0-0-0	32.82	7.22
	H	fallow	10-24-0	43.61	9.59
	H	fallow	50-30-30	52.68	11.59
Flax:	G and H	stubble	0-0-0	9.07	2.00
	G and H	stubble	50-0-0	14.60	3.21
Rye:	G	stubble	0-0-0	23.75	5.23
	G	stubble	50-30-30	38.08	8.38
	H	stubble	0-0-0	22.29	4.90
	H	stubble	50-30-30	35.73	7.86

Table C.3 (continued)

Crop	Soil Zone	Seedbed	Fertilizer per Acre[a]	Estimated Yield Per Acre: Expected	Estimated Yield Per Acre: Worst Possible
				.... bushels	
Rapeseed:	G and H	fallow	25-20-0	17.97	3.95
	G and H	fallow	70-20-0	22.08	4.86
Peas:	G	fallow	0-10-0	19.06	4.19
	H	fallow	0-10-0	15.78	3.97
				.... pounds	
Sunflowers:	G	stubble	15-20-20	791	174
	G	stubble	50-30-35	932	205
	H	stubble	15-20-20	763	168
	H	stubble	50-30-35	899	198
Mustard:	G and H	fallow	25-20-0	721	159
	G and H	fallow	70-20-0	851	187
				.... tons	
Corn Silage:	G and H	stubble	50-30-35	6.96	1.53
Hay:	G and H	sod	0-0-0	1.70	.37
	G and H	sod	15-15-0	2.20	.48
	G and H	sod	30-30-0	2.50	.55

[a]In pounds of N, P_2O_5 and K_2O applied per acre. Fertilizer rates were specified on the basis of information from various sources. The intermediate rates for wheat, oats and barley were taken from crop insurance records for the study area and were the average amounts applied by farmers who used fertilizer.

Table C.4

Means, Standard Deviations and Coefficients of Variation for Water Deficit on Four Different Dates at Pierson Weather Station[a]

Date	Mean	Standard Deviation	Coefficient of Variation
	inches	inches	percent
July 2	-1.82	2.66	146
Aug. 13	-6.95	4.06	58
Sept. 17	-10.50	4.92	47
Sept. 30	-11.12	5.07	46

[a]Computed from available weather records for the years 1932-1970, inclusive; there were a minimum of 35 observations on each date.

Source:

C.F. Shaykewich, "Progress Report on Climatic Analysis of Weather Stations in Manitoba" (Winnipeg: Department of Soil Science, University of Manitoba, 1972). (Mimeographed)

Regression equations were fitted to thirty-nine observations on trend-free yields of wheat, oats, barley and flax in Manitoba Crop District Number 1 and on the average water deficit on August 13 at the Pierson, Waskada, Boissevain, Deloraine, Virden and Brandon weather stations. All of these weather stations were either within or just outside the boundaries of Crop District Number 1. The estimated relationships and supporting statistical measures were:

(C.3) $Y_1 = \underset{(3.67)}{11.96} - \underset{(.4806)}{.9460X_1} - \underset{(.0323)}{.1161X_1^2}$ $R^2 = .50$ $F = 18.14$

(C.4) $Y_2 = \underset{(2.04)}{24.88} - \underset{(.0206)}{.1331X_1^2}$ $R^2 = .53$ $F = 41.94$

(C.5) $Y_3 = \underset{(1.60)}{19.10} - \underset{(.0161)}{.0868X_1^2}$ $R^2 = .44$ $F = 28.99$

(C.6) $Y_4 = \underset{(.61)}{7.95} - \underset{(.0061)}{.0358X_1^2}$ $R^2 = .48$ $F = 34.43$

where Y_1 = trend-free wheat yield,

Y_2 = trend-free oat yield,

Y_3 = trend-free barley yield,

Y_4 = trend-free flax yield

and X_1 = average water deficit, on August 13, at Pierson, Waskada, Boissevain, Deloraine, Virden and Brandon weather stations.[8]

For these four crops, these equations provided an estimate of the long-term relation between water deficit and the trend-free dryland yields with the intercept term representing the trend-free average dryland yield.

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[8]The bracketed values in the equations are the standard errors of the coefficients.

Taking the first derivative of each equation with respect to water deficit and solving for water deficit provided a water deficit value that was used to estimate the trend-free, water deficit-free yield for each crop. The ratio of this yield to the trend-free average yield was taken as an estimate of the ratio of the irrigated yield to the dryland yield without fertilizer. The ratio of the maximum trend-free yield to the average water deficit-free, trend-free yield, was taken as an estimate of the ratio of irrigated yields to dryland yields with fertilizer. For these four crops, these ratios averaged 1.25 and 1.44, respectively. With minor modifications in the latter ratio for heavier applications of fertilizer, these ratios were used to obtain estimates of the irrigated yields. The estimated irrigated yields for different crops, and according to soil zone, seedbed and fertilizer levels, are shown in Table C.5.

Comparison of these estimates of irrigated yields with estimates used in other studies[9] suggested that the approach used here was reasonable satisfactory. While the estimates were not identical to those in other studies, they were quite similar and they would suggest that the use of climatic and dryland yield data may be appropriate for economic analysis of irrigated alternatives in areas in which irrigated yield data are not available.

Worst Possible Dryland and Irrigated Yields

In addition to an estimate of the expected yield for each dryland and each irrigated alternative, an estimate of a worst possible or unfavourable yield was needed to compute the worst possible gross margin for each alternative. The procedure followed is outlined below.

Dryland Insured Crops. The insured yield per acre, based on the sixty percent coverage level, was taken as the worst possible yield for the dryland

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[9]See for example Iga, op. cit., Finn, op. cit., and Singh, op. cit.

Table C.5

Estimated Expected and Worst Possible Yields Per Acre for Irrigated Crop Production Activities for 1973, According to Soil Zone, Seedbed and Fertilizer Inputs

Crop	Soil Zone	Seedbed	Fertilizer per Acre	Estimated Yield Per Acre: Expected	Estimated Yield Per Acre: Worst Possible
				.... bushels	
Wheat:	G	stubble	12-18-0	31.22	20.47
	G	stubble	50-30-30	39.72	26.03
	G	fallow	6-23-0	49.24	34.98
	G	fallow	50-30-30	62.18	43.44
	H	stubble	12-18-0	28.54	18.71
	H	stubble	50-30-30	36.31	23.79
	H	fallow	6-23-0	45.00	31.96
	H	fallow	50-30-30	56.84	39.72
Oats:	G	stubble	13-16-0	76.11	50.81
	G	stubble	50-30-30	93.92	62.79
	H	stubble	10-16-0	68.64	45.83
	H	stubble	50-30-30	84.68	56.61
Barley:	G	stubble	17-23-0	46.27	31.75
	G	stubble	50-30-30	56.78	38.97
	G	fallow	10-24-0	64.59	42.19
	G	fallow	50-30-30	83.80	56.75
	H	stubble	17-23-0	42.85	29.40
	H	stubble	50-30-30	52.57	36.08
	H	fallow	10-24-0	59.80	39.06
	H	fallow	50-30-30	69.72	44.67
Flax:	G and H	stubble	0-0-0	11.43	6.43
	G and H	stubble	50-0-0	20.92	12.88
Rye:	G	stubble	0-0-0	29.62	18.27
	G	stubble	50-30-30	53.99	35.79
	H	stubble	0-0-0	27.80	17.15
	H	stubble	50-30-30	50.66	33.58
Rapeseed:	G and H	fallow	25-20-0	25.82	17.57
	G and H	fallow	70-20-0	31.74	21.60
Peas:	G	fallow	0-15-0	35.64	16.44
	H	fallow	0-15-0	39.51	13.61

Table C.5 (continued)

Crop	Soil Zone	Seedbed	Fertilizer per Acre	Estimated Yield Per Acre: Expected	Estimated Yield Per Acre: Worst Possible
				.... pounds	....
Sunflowers:	G	stubble	50-30-35	1472	978
	H	stubble	50-30-35	1420	944
Mustard:	G and H	fallow	70-20-0	1348	916
				.... tons	....
Corn Silage:	G and H	stubble	100-30-35	12.93	
Hay:	G and H	sod	15-15-0	3.80	2.73
	G and H	sod	30-30-0	4.68	3.46

[a]Pounds of N, P_2O_5 and K_2O applied. Generally, though not in all cases, the fertilizer rates for the irrigated alternatives were the same as those for the dryland alternatives.

insured alternatives. Table C.6 presents basic data on coverage and costs for the insured crops. As the Manitoba Crop Insurance Corporation does not distinguish between crops seeded on stubble or on fallow, nor between different fertility levels, in determining coverage rates, the insured coverage is shown in Table C.6 only according to soil zone.

Dryland Uninsured Crops. Estimates of the worst possible dryland uninsured yields were obtained on the basis of unlikely, but possible, water deficits. The specific water deficit values used were: 14 inches, basis August 13, for wheat, oats, barley, flax, peas, rapeseed, mustard and rye, and 20 inches, basis September 30, for sunflowers, corn silage, hay and pasture.[10]

The dryland yields associated with the worst possible water deficits were estimated from the relation between the worst possible dryland yields and the expected dryland yields found in equations C.3 through C.6. Expressed as percentages, the worst possible dryland yields ranged from 27 percent to 15 percent of the expected dryland yields.[11] On average, the worst possible dryland uninsured yields were 22 percent of the expected dryland yields and this relationship was used in computing the worst possible dryland yields for crops other than wheat, oats, barley and flax. The estimated worst possible dryland uninsured yields were reported in Table C.3.

Irrigated Crops. The estimated worst possible yields for the different irrigated alternatives were reported in Table C.5. They were estimated by computing the expected value of the area under the distribution of irrigated yields below 60 percent of the mean. Although the distribution of irrigated yields was not known, it was assumed to be normal. The coefficient of variation of irrigated yields of each crop was assumed to be the same as that for the dryland yields.

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[10]Assuming a normal distribution, there was about a 10 percent chance of a water deficit of 12 inches or more on August 13, and of a water deficit of 18 inches or more on September 30.

[11]The percentages were as follows: wheat--27 percent, oats--15 percent, barley--22 percent and flax--22 percent.

Table C.6

Insured Price Per Unit, Insured Yield and Gross Per Acre and Insurance Cost Per Acre, Basis Sixty Percent Coverage, Soil Zones G and H, by Crop for Risk Area Number 1, Province of Manitoba

Crop	Soil Zone	Insured Price Per Unit[a]	Insured Yield Per Acre[b]	Insured Gross Per Acre	Insurance Cost Per Acre
		dollars		.... dollars	....
Wheat	G	1.50	11.20	16.80	.77
	H	1.50	10.20	15.30	.77
Oats	G	.60	19.50	11.70	.98
	H	.60	17.50	10.50	.98
Barley	G	.85	14.50	12.32	1.12
	H	.85	13.40	11.39	1.12
Flax	G	2.50	4.80	12.00	1.18
	H	2.50	4.80	12.00	1.18
Rye	G	.94	11.50	10.81	.44
	H	.94	10.80	10.15	.44
Rapeseed	G	2.50	7.30	18.25	1.56
	H	2.50	7.30	18.25	1.56
Peas	G	1.75	9.70	16.98	2.10
	H	1.75	8.00	14.00	2.10
Mustard	G	2.13	6.50	13.84	1.56
	H	2.13	6.50	13.84	1.56
Sunflowers	G	.04	340.	12.80	1.28
	H	.04	320.	12.80	1.28

[a]Prices are per bushel for all crops except sunflowers which are per pound.

[b]Yields are in bushels for all crops except sunflowers which are in pounds.

Source:

1972 Rates and Coverage, (Winnipeg: Manitoba Crop Insurance Corporation) (Mimeographed).

APPENDIX D

ACTIVITIES, RESTRAINTS AND MATRIX OF INPUT-OUTPUT COEFFICIENTS IN FIRST PERIOD

Table D.1

Description of Activities in the Model in the First Period

Activity Number	Code Name	Description of Activity	Unit
Dryland Uninsured Crop Production Activities:			
1	WD1	Wheat, soil zone G, stubble, no fertilizer	acre
2	WD2	Wheat, soil zone G, stubble, medium fertilizer	acre
3	WD3	Wheat, soil zone G, stubble, heavy fertilizer	acre
4	WD4	Wheat, soil zone G, fallow, no fertilizer	acre
5	WD5	Wheat, soil zone G, fallow, medium fertilizer	acre
6	WD6	Wheat, soil zone G, fallow, heavy fertilizer	acre
7	WD7	Wheat, soil zone H, stubble, no fertilizer	acre
8	WD8	Wheat, soil zone H, stubble, medium fertilizer	acre
9	WD9	Wheat, soil zone H, stubble, heavy fertilizer	acre
10	WD10	Wheat, soil zone H, fallow, no fertilizer	acre
11	WD11	Wheat, soil zone H, fallow, medium fertilizer	acre
12	WD12	Wheat, soil zone H, fallow, heavy fertilizer	acre
13	OD1	Oats, soil zone G, stubble, no fertilizer	acre
14	OD2	Oats, soil zone G, stubble, medium fertilizer	acre
15	OD3	Oats, soil zone G, stubble, heavy fertilizer	acre
16	OD4	Oats, soil zone H, stubble, no fertilizer	acre
17	OD5	Oats, soil zone H, stubble, medium fertilizer	acre
18	OD6	Oats, soil zone H, stubble, heavy fertilizer	acre
19	BD1	Barley, soil zone G, stubble, no fertilizer	acre
20	BD2	Barley, soil zone G, stubble, medium fertilizer	acre
21	BD3	Barley, soil zone G, stubble, heavy fertilizer	acre
22	BD4	Barley, soil zone G, fallow, no fertilizer	acre
23	BD5	Barley, soil zone G, fallow, medium fertilizer	acre
24	BD6	Barley, soil zone G, fallow, heavy fertilizer	acre
25	BD7	Barley, soil zone H, stubble, no fertilizer	acre
26	BD8	Barley, soil zone H, stubble, medium fertilizer	acre
27	BD9	Barley, soil zone H, stubble, heavy fertilizer	acre
28	BD10	Barley, soil zone H, fallow, no fertilizer	acre
29	BD11	Barley, soil zone H, fallow, medium fertilizer	acre
30	BD12	Barley, soil zone H, fallow, heavy fertilizer	acre
31	FD1	Flax, soil zone G, stubble, no fertilizer	acre
32	FD2	Flax, soil zone G, stubble, heavy fertilizer	acre
33	FD3	Flax, soil zone H, stubble, no fertilizer	acre
34	FD4	Flax, soil zone H, stubble, no fertilizer	acre
35	RD1	Rapeseed, soil zone G, fallow, medium fertilizer	acre
36	RD2	Rapeseed, soil zone G, fallow, heavy fertilizer	acre
37	RD3	Rapeseed, soil zone H, fallow, medium fertilizer	acre
38	RD4	Rapeseed, soil zone H, fallow, heavy fertilizer	acre
39	PD1	Peas, soil zone G, fallow, medium fertilizer	acre
40	PD2	Peas, soil zone H, fallow, medium fertilizer	acre

Table D.1 (continued)

Activity Number	Code Name	Description of Activity	Unit
41	SD1	Sunflowers, soil zone G, stubble, medium fertilizer	acre
42	SD2	Sunflowers, soil zone G, stubble, heavy fertilizer	acre
43	SD3	Sunflowers, soil zone H, stubble, medium fertilizer	acre
44	SD4	Sunflowers, soil zone H, stubble, heavy fertilizer	acre
45	HD1	Hay, soil zone G, no fertilizer	acre
46	HD2	Hay, woil zone G, medium fertilizer	acre
47	HD3	Hay, soil zone G, heavy fertilizer	acre
48	HD4	Hay, soil zone H, no fertilizer	acre
49	HD5	Hay, soil zone H, medium fertilizer	acre
50	HD6	Hay, soil zone H. heavy fertilizer	acre
51	CSD1	Corn silage, soil zone G, medium fertilizer	acre
52	CAD2	Corn silage, soil zone H, medium fertilizer	acre
53	PAD1	Pasture, soil zone G, no fertilizer	acre
54	PAD2	Pasture, soil zone G, medium fertilizer	acre
55	PAD3	Pasture, soil zone G, heavy fertilizer	acre
56	PAD4	Pasture, soil zone H, no fertilizer	acre
57	PAD5	Pasture, soil zone H, medium fertilizer	acre
58	PAD6	Pasture, soil zone H, heavy fertilizer	acre
Dryland Insured Crop Production Activities:			
59	WID1	Wheat, soil zone G, stubble, no fertilizer	acre
60	WID2	Wheat, soil zone G, stubble, medium fertilizer	acre
61	WID3	Wheat, soil zone G, stubble, heavy fertilizer	acre
62	WID4	Wheat, soil zone G, fallow, no fertilizer	acre
63	WID5	Wheat, soil zone G, fallow, medium fertilizer	acre
64	WID6	Wheat, soil zone G, fallow, heavy fertilizer	acre
65	WID7	Wheat, soil zone H, stubble, no fertilizer	acre
66	WID8	Wheat, soil zone H, stubble, medium fertilizer	acre
67	WID9	Wheat, soil zone H, stubble, heavy fertilizer	acre
68	WID10	Wheat, soil zone H, fallow, no fertilizer	acre
69	WID11	Wheat, soil zone H, fallow, medium fertilizer	acre
70	WID12	Wheat, soil zone H, fallow, heavy fertilizer	acre
71	OID1	Oats, soil zone G, stubble, no fertilizer	acre
72	OID2	Oats, soil zone G, stubble, medium fertilizer	acre
73	OID3	Oats, soil zone G, stubble, heavy fertilizer	acre
74	OID4	Oats, soil zone H, stubble, no fertilizer	acre
75	OID5	Oats, soil zone H, stubble, medium fertilizer	acre
76	OID6	Oats, soil zone H, stubble, heavy fertilizer	acre
77	BID1	Barley, soil zone G, stubble, no fertilizer	acre
78	BID2	Barley, soil zone G, stubble, medium fertilizer	acre
79	BID3	Barley, soil zone G, stubble, heavy fertilizer	acre
80	BID4	Barley, soil zone G, fallow, no fertilizer	acre
81	BID5	Barley, soil zone G, fallow, medium fertilizer	acre
82	BID6	Barley, soil zone G, fallow, heavy fertilizer	acre
83	BID7	Barley, soil zone H, stubble, no fertilizer	acre
84	BID8	Barley, soil zone H, stubble, medium fertilizer	acre
85	BID9	Barley, soil zone H, stubble, heavy fertilizer	acre

Table D.1 (continued)

Activity Number	Code Name	Description of Activity	Unit
86	BID10	Barley, soil zone H, fallow, no fertilizer	acre
87	BID11	Barley, soil zone H, fallow, medium fertilizer	acre
88	BID12	Barley, soil zone H, fallow, heavy fertilizer	acre
89	FID1	Flax, soil zone G, stubble, no fertilizer	acre
90	FID2	Flax, soil zone G, stubble, heavy fertilizer	acre
91	FID3	Flax, soil zone H, stubble, no fertilizer	acre
92	FID4	Flax, soil zone H, stubble, heavy fertilizer	acre
93	RID1	Rapeseed, soil zone G, fallow, medium fertilizer	acre
94	RID2	Rapeseed, soil zone G, fallow, heavy fertilizer	acre
95	RID3	Rapeseed, soil zone H, fallow, medium fertilizer	acre
96	RID4	Rapeseed, soil zone H, fallow, heavy fertilizer	acre
97	PID1	Peas, soil zone G, fallow, medium fertilizer	acre
98	PID2	Peas, soil zone H, fallow, medium fertilizer	acre
99	SID1	Sunflowers, soil zone G, stubble, medium fertilizer	acre
100	SID2	Sunflowers, soil zone G, stubble, heavy fertilizer	acre
101	SID3	Sunflowers, soil zone G, stubble, medium fertilizer	acre
102	SID4	Sunflowers, soil zone H, stubble, heavy fertilizer	acre
Irrigated Crop Production Activities:			
103	WI1	Wheat, soil zone G, stubble, medium fertilizer	acre
104	WI2	Wheat, soil zone G, stubble, heavy fertilizer	acre
105	WI5	Wheat, soil zone H, stubble, medium fertilizer	acre
106	WI6	Wheat, soil zone H, stubble, heavy fertilizer	acre
107	OI1	Oats, soil zone G, stubble, medium fertilizer	acre
108	OI2	Oats, soil zone G, stubble, heavy fertilizer	acre
109	OI3	Oats, soil zone H, stubble, medium fertilizer	acre
110	OI4	Oats, soil zone H, stubble, heavy fertilizer	acre
111	BI1	Barley, soil zone G, stubble, medium fertilizer	acre
112	BI2	Barley, soil zone G, stubble, heavy fertilizer	acre
113	BI5	Barley, soil zone H, stubble, medium fertilizer	acre
114	BI6	Barley, soil zone H, stubble, heavy fertilizer	acre
115	FI1	Flax, soil zone G, stubble, no fertilizer	acre
116	FI2	Flax, soil zone G, stubble, heavy fertilizer	acre
117	FI3	Flax, soil zone H, stubble, no fertilizer	acre
118	FI4	Flax, soil zone H, stubble, heavy fertilizer	acre
119	RI1	Rapeseed, soil zone G, fallow, medium fertilizer	acre
120	RI2	Rapeseed, soil zone G, fallow, heavy fertilizer	acre
121	RI3	Rapeseed, soil zone H, fallow, medium fertilizer	acre
122	RI4	Rapeseed, soil zone H, fallow, heavy fertilizer	acre
123	PI1	Peas, soil zone G, fallow, heavy fertilizer	acre
124	PI2	Peas, soil zone H, fallow, heavy fertilizer	acre
125	SI1	Sunflowers, soil zone G, stubble, heavy fertilizer	acre
126	SI2	Sunflowers, soil zone H, stubble, heavy fertilizer	acre
127	HI1	Hay, soil zone G, medium fertilizer	acre
128	HI2	Hay, soil zone G, heavy fertilizer	acre
129	HI3	Hay, soil zone H, medium fertilizer	acre
130	HI4	Hay, soil zone H, heavy fertilizer	acre

Table D.1 (continued)

Activity Number	Code Name	Description of Activity	Unit
131	CSI1	Corn silage, soil zone G, heavy fertilizer	acre
132	CSI2	Corn silage, soil zone H, heavy fertilizer	acre
133	PAI1	Pasture, soil zone G, medium fertilizer	acre
134	PAI2	Pasture, soil zone G, heavy fertilizer	acre
135	PAI3	Pasture, soil zone H, medium fertilizer	acre
136	PAI4	Pasture, soil zone H, heavy fertilizer	acre
Other Production and Marketing Activities:			
137	SMFG	Summer fallow, soil zone G	acre
138	SMFH	Summer fallow, soil zone H	acre
139	EFG	Establish forage, soil zone G	acre
140	EFH	Establish forage, soil zone H	acre
141	CAT1	Beef enterprise I - calves sold as feeder calves	beef cow
142	CAT2	Beef enterprise II - calves sold as long yearlings	beef cow
143	SQW	Sell wheat	bushel
144	SQO	Sell oats	bushel
145	SQB	Sell barley	bushel
146	SQF	Sell flax	bushel
147	SQB	Sell rapeseed	bushel
148	HS	Harvest straw for cattle enterprise	ton
149	BUYH2O	Apply irrigation water	acre-inch
Capital Investment and Financial Activities:			
150	BUYSYS	Purchase irrigation system	system
151	SQUARE1	Separable activity I - concerned with first linear segment of non-linear restraint to handle integer restraint on purchase of irrigation system	.25 system
152	SQUARE2	Separable activity II - concerned with second linear segment of non-linear restraint to handle integer restraint on purchase of irrigation system	.75 system
153	BUYIMP	Expand cattle housing	head
154	TCBYIR	Use cash to purchase irrigation system	dollar
155	TCBYIM	Use cash to expand cattle housing	dollar
156	TCBYC	Use cash to purchase additional cattle	dollar
157	BBYSYS	Borrow to purchase irrigation system	dollar
158	BBYIMP	Borrow to expand cattle housing	dollar
159	BBYC	Borrow to purchase cattle	dollar
160	DEP10	Depreciate capital class 10	dollar
161	DEP8	Depreciate capital class 8	dollar
162	DEP6	Depreciate capital class 6	dollar
163	PDD	Pay depreciation expenses from cash flow	dollar
164	TBC	Allocate beginning cash to working capital	dollar
165	TBCCR	Allocate beginning cash to cash reserves	dollar

Table D.1 (continued)

Activity Number	Code Name	Description of Activity	Unit
166	BWC	Borrow working capital	dollar
167	ACR	Use cash reserves	dollar
168	C	Pay minimum consumption expenditure	dollar
169	CO	Pay cash overhead expenditures	dollar
170	PIP	Pay principal and interest	dollar
Income Tax Activities:			
171	I1	Pay income tax - tax bracket 1	dollar
172	I2	Pay income tax - tax bracket 2	dollar
173	I3	Pay income tax - tax bracket 3	dollar
174	I4	Pay income tax - tax bracket 4	dollar
175	I5	Pay income tax - tax bracket 5	dollar
176	I6	Pay income tax - tax bracket 6	dollar
177	I7	Pay income tax - tax bracket 7	dollar
178	I8	Pay income tax - tax bracket 8	dollar
179	I9	Pay income tax - tax bracket 9	dollar
180	I10	Pay income tax - tax bracket 10	dollar
181	I11	Pay income tax - tax bracket 11	dollar
Year-to-Year Transfer Activities:			
182	TWNY	Transfer surplus wheat to next period	bushel
183	TONY	Transfer surplus oats to next period	bushel
184	TBNY	Transfer surplus barley to next period	bushel
185	TCF	Transfer surplus cash flow to next period	dollar
186	TCRNY	Transfer cash reserve balance to next period	dollar
Other Activities:			
187	TNW	Transfer addition to net worth to net worth accumulation row	dollar
188	MAX	Maximize expected additions to net worth	dollar
189	TAWPI	Increase worst possible income constraint in each period	dollar

Table D.2

Matrix of Input-Output Coefficients for Activities and Restraints in the First Period

Restraint No.	Code Name	Description of Restraint	Type of Restraint	Right Hand Side	Unit	Activity No. 1	2	3	4	Row No.
					Activity Description	Dryland Uninsured Wheat				
						G, stub. no f.	G, stub. med. f.	G, stub. heav. f.	G, fall. no f.	
					Unit	acre	acre	acre	acre	
1	OBJ	Expected additions to net worth	N	-	dollar					1
2	NFI	Net farm income accounting	E	0	dollar	7.56	10.35	17.22	8.02	2
3	DI	After-tax and discretionary income accounting	E	0	dollar					3
4	NW	Additions to net worth accounting	L	0	dollar					4
5	WPI	Minimum worst possible income	G	-8000	dollar	1.06	1.63	-2.57	3.50	5
6	AWPI	Special worst possible constraint for parametric soln.	G	0	dollar					6
7	I1	Maximum income in tax bracket 1	L	3000*	dollar					7
8	I2	Maximum income in tax bracket 2	L	500*	dollar					8
9	I3	Maximum income in tax bracket 3	L	500*	dollar					9
10	I4	Maximum income in tax bracket 4	L	1000*	dollar					10
11	I5	Maximum income in tax bracket 5	L	1000*	dollar					11
12	I6	Maximum income in tax bracket 6	L	2000*	dollar					12
13	I7	Maximum income in tax bracket 7	L	2000*	dollar					13
14	I8	Maximum income in tax bracket 8	L	2000*	dollar					14
15	I9	Maximum income in tax bracket 9	L	2000*	dollar					15
16	I10	Maximum income in tax bracket 10	L	3000*	dollar					16
17	I11	Maximum income in tax bracket 11	L	10000*	dollar					17
18	BC	Beginning cash on hand	L	2000	dollar					18
19	WC	Working capital	L	0	dollar	7.56	10.35	17.22	8.02	19
20	WCBL	Working capital borrowing limit	L	10000*	dollar					20
21	CF	Cash flow accounting	L	0	dollar	7.56	10.35	17.22	8.02	21
22	CR	Cash reserves	L	3500	dollar					22
23	C	Minimum intended family consumption spending	E	3000*	dollar					23
24	CO	Cash overhead expenses	E	2000**	dollar					24
25	PIP	Principal and interest payments	E	3512**	dollar					25
26	CA10	Investment in class 10 capital asset	E	4200	dollar					26
27	CA8	Investment in class 8 capital asset	E	6400	dollar					27
28	CA6	Investment in class 6 capital asset	E	6000	dollar					28
29	DD	Capital cost allowance on capital assets	E	0	dollar					29
30	CATM	Capital investment in cattle	E	10000	dollar					30
31	IMPM	Capital to purchase additional cattle housing	L	0	dollar					31
32	IEM	Capital to purchase irrigation system	L	0	dollar					32
33	W	Wheat supply	L	1500	bushel	-18.14	-23.12	-29.45	-25.06	33
34	O	Oat supply	L	0	bushel					34
35	B	Barley supply	L	1000	bushel					35
36	F	Flax supply	L	0	bushel					36
37	R	Rapeseed supply	L	0	bushel					37
38	WR	Irrigation water supply	L	0	acre-inch					38
39	IS	Irrigation system purchase limit	L	1*	system					39
40	ISL	Irrigation system purchase limit, all periods	L	1*	system					40
41	S	Straw supply	L	0	ton					41
42	FOR	Forage supply in hay equivalents	L	0	ton					42
43	PAST	Pasture supply in hay equivalents	L	0	ton					43
44	CAT	Minimum cattle enterprise	E	.25*	beef cow					44
45	LA1	Planting labour	L	500*	hour	.82	.82	.91	.82	45
46	LA2	Hay harvest labour-first cut	L	400*	hour					46
47	LA3	Hay harvest labour-second cut	L	200*	hour					47
48	LA4	Crop harvest labour I	L	500*	hour	.82	.82	.94	.86	48
49	LA5	Crop harvest labour II	L	100*	hour					49
50	LA6	Daily winter labour	L	8*	hour					50
51	AA	Assigned acres	L	0	acre	-1.	-1.	-1.	-1.	51
52	FSG	Summerfallowed land, soil zone G	L	50	acre				1.	52
53	FSH	Summerfallowed land, soil zone H	L	100	acre					53
54	FRG	Minimum summerfallowed land, soil zone G	G	50	acre					54
55	FRH	Minimum summerfallowed land, soil zone H	G	100	acre					55
56	SSG	Stubble land, soil zone G	L	125	acre	1.	1.	1.		56
57	SSH	Stubble land, soil zone H	L	225	acre					57
58	FORAG	Forage land, soil zone G	E	25	acre					58
59	FORAH	Forage land, soil zone H	E	25	acre					59
60	FLAXG	Maximum flax acreage, soil zone G	L	55*	acre					60
61	FLAXH	Maximum flax acreage, soil zone H	L	110*	acre					61
62	ILG	Land served by irrigation system, soil zone G	L	0	acre					62
63	ILH	Land served by irrigation system, soil zone H	L	0	acre					63
64	ROW 1	Separable restraint I	E	0	unit					64
65	ROW 2	Separable restraint II	E	0	unit					65
66	BC	Beginning cash on hand (Period 2)	L	0	dollar					66
67	CF	Cash flow (Period 2)	L	0	dollar					67
68	CR	Cash reserves (Period 2)	L	0	dollar					68
69	CA10	Investment in class 10 capital asset (Period 2)	E	0	dollar					69
70	CA8	Investment in class 8 capital asset (Period 2)	E	0	dollar					70
71	CA6	Investment in class 6 capital asset (Period 2)	E	0	dollar					71
72	W	Wheat supply (Period 2)	L	0	bushel					72
73	O	Oat supply (Period 2)	L	0	bushel					73
74	B	Barley supply (Period 2)	L	0	bushel					74
75	FSG	Summerfallowed land, soil zone G (Period 2)	L	0	acre					75
76	FSH	Summerfallowed land, soil zone H (Period 2)	L	0	acre					76
77	SSG	Stubble land, soil zone G (Period 2)	L	0	acre	-1.	-1.	-1.	-1.	77
78	SSH	Stubble land, soil zone H (Period 2)	L	0	acre					78
79	CAT	Minimum cattle enterprise (Period 2)	E	0	beef cow					79
80	CATM	Capital investment in cattle (Period 2)	E	10000*	dollar					80
81	NFI	Net farm income accounting (Period 2)	E	0	dollar					81
82	WPI	Minimum worst possible income (Period 2)	G	-8000*	dollar					82
83	FRG	Minimum fallow land, soil zone G (Period 2)	G	50*	acre					83
84	FRH	Minimum fallow land, soil zone H (Period 2)	G	100*	acre					84
85	FORAG	Forage land, soil zone G (Period 2)	E	25*	acre					85
86	FORAH	Forage land, soil zone H (Period 2)	E	25*	acre					86

Table D.2 (continued)

Restraint No.	Activity No. / Activity Description / Unit	5	6	7	8	9	10	11	12	13	14	15	16	17	Row No.
		Dryland Uninsured Wheat								Dryland Uninsured Oats					
		G,fall. med.f.	G,fall. heav.f.	H,stub. no f.	H.stub. med f.	H,stub. heav.f.	H,fall. no f.	H,fall. med.f.	H,fall. heav.f.	G,stub. no f.	G,stub. med.f.	G,stub. heav.f.	H,stub. no f.	H,stub. med.f.	
		acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		11.15	18.36	7.56	10.35	17.02	7.86	10.58	18.09	6.49	9.51	16.46	6.49	9.20	2
3															3
4															4
5		4.31	2.60	.32	-.30	-3.75	3.33	3.06	.95	-2.70	-4.28	-9.65	-3.07	-4.66	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		11.15	18.36	7.56	10.35	17.02	7.86	10.58	18.09	6.49	9.51	16.46	6.49	9.20	19
20															20
21		11.15	18.36	7.56	10.35	17.02	7.86	10.58	18.09	6.49	9.51	16.46	6.49	9.20	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33		-30.67	-40.29	-16.59	-21.14	-26.92	-22.91	-28.05	-36.82						33
34										-38.84	-50.64	-62.32	-35.04	-45.67	34
35															35
36															36
37															37
38															38
39															39
40															40
41															41
42															42
43															43
44															44
45		.82	.91	.82	.82	.91	.82	.82	.91	.82	.82	.91	.82	.82	45
46															46
47															47
48		.96	1.13	.82	.82	.89	.82	.91	1.07	.82	.89	1.00	.82	.84	48
49															49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52		1.	1.												52
53							1.	1.	1.						53
54															54
55															55
56										1.	1.	1.			56
57				1.	1.	1.							1.	1.	57
58															58
59															59
60															60
61															61
62															62
63															63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77		-1.	-1.							-1.	-1.	-1.			77
78				-1.	-1.	-1.	-1.	-1.	-1.				-1.	-1.	78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	18	19	20	21	22	23	24	25	26	27	28	29	30	Row No.
Activity No.		18	19	20	21	22	23	24	25	26	27	28	29	30	
Activity		Dryland Uninsured Barley													
Description		H,stub. heav.f.	G,stub. no f.	G,stub. med.f.	G,stub. heav.f.	G,fall. no f.	G.fall. med.f.	G,fall. heav.f.	H,stub. no f.	H,stub. med.f.	H,stub. heav.f.	H,fall. no f.	H,fall. med.f.	H,fall. heav.f.	
		acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		16.19	6.76	10.48	16.16	7.19	10.99	17.41	6.76	10.48	16.01	6.95	10.80	17.27	2
3															3
4															4
5		-10.21	-1.30	-3.42	-7.26	1.14	.66	-2.96	-1.70	-3.95	-7.91	.63	-.15	-4.03	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		16.19	6.76	10.48	16.16	7.19	10.99	17.41	6.76	10.48	16.01	6.95	10.80	17.27	19
20															20
21		16.19	6.76	10.48	16.16	7.19	10.99	17.41	6.76	10.48	16.01	6.95	10.80	17.27	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33															33
34		-56.20													34
35			-23.64	-30.54	-37.45	-35.44	-47.10	-56.89	-21.90	-28.29	-34.68	-32.82	-43.61	-52.68	35
36															36
37															37
38															38
39															39
40															40
41															41
42															42
43															43
44															44
45		.91	.82	.82	.91	.82	.82	.91	.82	.82	.91	.82	.82	.91	45
46															46
47															47
48		.94	.82	.82	.88	.85	1.00	1.12	.82	.82	.84	.82	.96	1.07	48
49															49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52						1.	1.	1.							52
53												1.	1.	1.	53
54															54
55															55
56			1.	1.	1.										56
57		1.							1.	1.	1.				57
58															58
59															59
60															60
61															61
62															62
63															63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77			-1.	-1.	-1.	-1.	-1.	-1.							77
78		-1.							-1.	-1.	-1.	-1.	-1.	-1.	78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	31	32	33	34	35	36	37	38	39	40	41	42	43	Row No.
Activity No.		31	32	33	34	35	36	37	38	39	40	41	42	43	
Activity		Dryland Uninsured Flax				Dryland Uninsured Rapeseed				Dry. Uninsured Peas		Dry. Uninsured Sunflowers			
Description		G,stub. no f.	G,stub. heav.f.	H,stub. no f.	H,stub. heav.f.	G,fall. med.f.	G,fall. heav.f.	H,fall. med.f.	H,fall. heav.f.	G,fall. med.f.	H,fall. med.f.	G,stub. med.f.	G,stub. heav.f.	H,stub. med.f.	
		acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		6.77	12.72	6.77	12.72	12.70	17.48	12.70	17.48	-24.66	-17.70	-25.91	-27.96	-24.49	2
3															3
4															4
5		-.20	-1.08	-.20	-1.08	-.75	-2.36	-.75	-2.36	-8.33	-9.76	-8.29	-12.05	-8.56	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		6.77	12.72	6.77	12.72	12.70	17.48	12.70	17.48	16.89	16.70	16.01	21.44	15.95	19
20															20
21		6.77	12.72	6.77	12.72	12.70	17.48	12.70	17.48	-24.66	-17.70	-25.91	-27.96	-24.49	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33															33
34															34
35															35
36		-9.07	-14.60	-9.07	-14.60										36
37						-17.97	-22.08	-17.97	-22.08						37
38															38
39															39
40															40
41															41
42															42
43															43
44															44
45		.82	.91	.82	.91	.91	.91	.91	.91	.73	.73	.73	.82	.73	45
46															46
47															47
48		.84	1.10	.84	1.10	.84	.96	.84	.96	.86	.86				48
49												.65	.72	.65	49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52						1.	1.			1.					52
53								1.	1.		1.				53
54															54
55															55
56		1.	1.									1.	1.		56
57				1.	1.									1.	57
58															58
59															59
60		1.	1.												60
61				1.	1.										61
62															62
63															63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77		-1.	-1.			-1.	-1.			-1.		-1.	-1.		77
78				-1.	-1.			-1.	-1.		-1.			-1.	78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	44	45	46	47	48	49	50	51	52	53	54	55	56	Row No.
Activity No.		44	45	46	47	48	49	50	51	52	53	54	55	56	
Activity			Dryland Uninsured Hay						Dry.Unins.Corn S.		Dryland Uninsured Pasture				
Description		H,stub. heav.f.	G, no.f.	G, med.f.	G, heav.f.	H, no f.	H, med.f.	H, heav.f.	G,stub. med.f.	H,stub. med.f.	G, no f.	G, med.f.	G, heav.f.	H, no f.	
		acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		-26.29	2.88	6.70	10.56	2.88	6.70	10.56	31.13	31.13	.32	3.31	6.10	.32	2
3															3
4															4
5		-12.35	2.67	1.35	-.23	2.67	1.35	-.23	-20.42	-20.42	5.23	3.89	1.95	5.23	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		21.36	2.88	6.70	10.56	2.88	6.70	10.56	31.13	31.13	.32	3.31	6.10	.32	19
20															20
21		-26.29	2.88	6.70	10.56	2.88	6.70	10.56	31.13	31.13	.32	3.31	6.10	.32	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33															33
34															34
35															35
36															36
37															37
38															38
39															39
40															40
41															41
42			-1.70	-2.20	-2.50	-1.70	-2.20	-2.50							42
43											-1.70	-2.20	-2.50	-1.70	43
44															44
45		.82		.09	.09		.09	.09	1.17	1.17		.09	.09		45
46			2.93	3.36	2.93	2.93	3.36	2.93							46
47					1.67			1.67							47
48															48
49		.72							4.23	4.23					49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52															52
53															53
54															54
55															55
56									1.						56
57		1.								1.					57
58			1.	1.	1.						1.	1.	1.		58
59						1.	1.	1.						1.	59
60															60
61															61
62															62
63															63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77									-1.						77
78		-1.								-1.					78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	Activity No. 57	58	59	60	61	62	63	64	65	66	67	68	69	Row No.
		Activity Description		Dryland Insured Wheat											
		H, med.f.	H, heav.f.	G,stub. no f.	G,stub. med.f.	G,stub. heav.f.	G,fall. no f.	G,fall. med.f.	G,fall. heav.f.	H,stub. no f.	H,stub. med.f.	H,stub. heav.f.	H,fall. no f.	H,fall. med.f.	
		acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		3.31	6.10	8.36	11.15	18.01	8.81	11.95	19.16	8.36	11.15	17.82	8.66	11.38	2
3															3
4															4
5		3.89	1.95	8.53	5.74	-.46	8.08	5.83	-.46	7.03	5.59	-1.96	7.03	4.33	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		3.31	6.10	8.36	11.15	18.01	8.81	11.95	19.16	8.36	11.15	17.82	8.66	11.38	19
20															20
21		3.31	6.10	8.36	11.15	18.01	8.81	11.95	19.16	8.36	11.15	17.82	8.66	11.38	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33				-18.14	-23.12	-29.45	-25.06	-30.67	-40.29	-16.59	-21.14	-26.92	-22.91	-28.05	33
34															34
35															35
36															36
37															37
38															38
39															39
40															40
41															41
42															42
43		-2.20	-2.50												43
44															44
45		.09	.09	.82	.82	.91	.82	.82	.91	.82	.82	.91	.82	.82	45
46															46
47															47
48				.82	.82	.94	.86	.96	1.13	.82	.82	.89	.82	.91	48
49															49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52							1.	1.	1.						52
53													1.	1.	53
54															54
55															55
56				1.	1.	1.									56
57										1.	1.	1.			57
58															58
59		1.	1.												59
60															60
61															61
62															62
63															63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77				-1.	-1.	-1.	-1.	-1.	-1.						77
78										-1.	-1.	-1.	-1.	-1.	78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	70	71	72	73	74	75	76	77	78	79	80	81	82	Row No.
	Activity No.	70	71	72	73	74	75	76	77	78	79	80	81	82	
	Activity		Dryland Insured Oats						Dryland Insured Barley						
	Description	H,fall. heav.f.	G,stub. no f.	G,stub. med.f.	G,stub. heav.f.	H,stub. no f.	H,stub. med.f.	H,stub. heav.f.	G,stub. no f.	G,stub. med.f.	G,stub. heav.f.	G,fall. no f.	G,fall. med.f.	G,fall. heav.f.	
	Unit	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		18.89	7.50	10.52	17.47	7.50	10.21	17.21	7.92	11.64	17.32	8.35	12.14	18.57	2
3															3
4															4
5		-1.95	4.32	1.59	-4.92	3.13	.51	-6.08	4.54	.82	-4.61	4.25	1.09	-4.81	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		18.89	7.50	10.52	17.47	7.50	10.21	17.21	7.92	11.64	17.32	8.35	12.14	18.57	19
20															20
21		18.89	7.50	10.52	17.47	7.50	10.21	17.21	7.92	11.64	17.32	8.35	12.14	18.57	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33		-36.82													33
34			-38.84	-50.64	-62.32	-35.04	-45.67	-56.20							34
35									-23.64	30.54	-37.45	-35.44	-47.10	-56.89	35
36															36
37															37
38															38
39															39
40															40
41															41
42															42
43															43
44															44
45		.91	.82	.82	.91	.82	.82	.91	.82	.82	.91	.82	.82	.91	45
46															46
47															47
48		1.07	.82	.89	1.00	.82	.84	.94	.82	.82	.88	.85	1.00	1.12	48
49															49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52												1.	1.	1.	52
53		1.													53
54															54
55															55
56			1.	1.	1.				1.	1.	1.				56
57						1.	1.	1.							57
58															58
59															59
60															60
61															61
62															62
63															63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77			-1.	-1.	-1.				-1.	-1.	-1.	-1.	-1.	-1.	77
78		-1.				-1.	-1.	-1.							78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (Continued)

Restraint No.	Unit	Activity No. 83	84	85	86	87	88	89	90	91	92	93	94	95	Row No.
	Activity	Dryland Insured Barley						Dryland Insured Flax				Dryland Insured Rapeseed			
	Depreciation	H,stub. no f.	H,stub. med.f.	H,stub. heav.f.	H,fall. no f.	H,fall. med.f.	H,fall. heav.f.	G,stub. no f.	G,stub. heav.f.	H,stub. no f.	H,stub. heav.f.	G,fall. med.f.	G,fall. heav.f.	H,fall. med.f.	
		acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		7.92	11.64	17.16	8.11	11.96	18.43	7.99	13.94	7.99	13.94	14.31	19.10	14.31	2
3															3
4															4
5		3.61	-.11	-5.54	3.42	.15	-5.77	2.88	-1.93	2.88	-1.93	.53	-3.82	.53	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		7.92	11.64	17.16	8.11	11.96	18.43	7.99	13.94	7.99	13.94	14.31	19.10	14.31	19
20															20
21		7.92	11.64	17.16	8.11	11.96	18.43	7.99	13.94	7.99	13.94	14.31	19.10	14.31	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33															33
34															34
35		-21.90	-28.29	-34.68	-32.82	-43.61	-52.68								35
36								-9.07	-14.60	-9.07	-14.60				36
37												-17.97	-22.08	-17.97	37
38															38
39															39
40															40
41															41
42															42
43															43
44															44
45		.82	.82	.91	.82	.82	.91	.82	.91	.82	.91	.91	.91	.91	45
46															46
47															47
48		.82	.82	.84	.82	.96	1.07	.84	1.10	.84	1.10	.84	.96	.84	48
49															49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52												1.	1.		52
53					1.	1.	1.							1.	53
54															54
55															55
56								1.	1.						56
57		1.	1.	1.						1.	1.				57
58															58
59															59
60								1.	1.						60
61										1.	1.				61
62															62
63															63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77								-1.	-1.			-1.	-1.		77
78		-1.	-1.	-1.	-1.	-1.	-1.			-1.	-1.			-1.	78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	96	97	98	99	100	101	102	103	104	105	106	107	108	Row No.
Activity No.		96	97	98	99	100	101	102	103	104	105	106	107	108	
Activity			Dry. Insured Peas		Dryland Insured Sunflowers					Irrigated Wheat			Irrigated Oats		
Description		H,fall. heav.f.	G,fall. heav.f.	H,fall. heav.f.	G,stub. med.f.	G,stub. heav.f.	H,stub. med.f.	H,stub. heav.f.	G,stub. med.f.	G,stub. heav.f.	H,stub. med.f.	H,stub. heav.f.	G,stub. med.f.	G,stub. heav.f.	
		acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		19.10	-22.49	-15.53	-24.59	-26.63	-23.17	-24.97	10.97	18.02	10.79	17.76	10.57	17.78	2
3															3
4															4
5		-3.82	-1.90	-4.87	-6.67	-11.80	-6.67	-11.78	6.22	9.35	3.11	5.53	3.04	3.61	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		19.10	19.06	18.87	17.33	22.77	17.27	22.68	10.97	18.02	10.79	17.76	10.57	17.78	19
20															20
21		19.10	-22.49	-15.53	-24.59	-26.63	-23.17	-24.97	10.97	18.02	10.79	17.76	10.57	17.78	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33									-31.22	-39.72	-28.54	-36.31			33
34													-76.11	-93.92	34
35															35
36															36
37		-22.08													37
38									8.69	8.69	8.69	8.69	8.69	8.69	38
39															39
40															40
41															41
42															42
43															43
44															44
45		.91	.73	.73	.73	.82	.73	.82	.82	.91	.82	.91	.82	.91	45
46															46
47															47
48		.96	.86	.86					.96	1.12	.92	1.06	1.14	1.30	48
49					.65	.72	.65	.70							49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52			1.												52
53		1.		1.											53
54									.50	.50			.50	.50	54
55											.50	.50			55
56					1.	1.			1.	1.			1.	1.	56
57							1.	1.			1.	1.			57
58															58
59															59
60															60
61															61
62									1.	1.			1.	1.	62
63											1.	1.			63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77			-1.		-1.	-1.			-1.	-1.			-1.	-1.	77
78		-1.		-1.			-1.	-1.			-1.	-1.			78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	109	110	111	112	113	114	115	116	117	118	119	120	121	Row No.
	Activity No.	109	110	111	112	113	114	115	116	117	118	119	120	121	
	Activity Description	Irrigated Oats		Irrigated Barley				Irrigated Flax				Irrigated Rapeseed			
		H,stub. med.f.	H,stub. heav.f.	G,stub. med.f.	G,stub. heav.f.	H,stub. med.f.	H,stub. heav.f.	G,stub. no f.	G,stub. heav.f.	H,stub. no f.	H,stub. heav.f.	G,fall. med.f.	G,fall. heav.f.	H,fall. med.f.	
	Unit	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		10.25	17.40	11.20	17.22	11.02	16.99	7.25	14.03	7.25	14.03	13.54	18.52	13.54	2
3															3
4															4
5		.12	-.02	3.45	5.23	1.16	2.35	-5.22	10.07	-5.22	10.07	21.25	27.94	21.25	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		10.25	17.40	11.20	17.22	11.02	16.99	7.25	14.03	7.25	14.03	13.54	18.52	13.54	19
20															20
21		10.25	17.40	11.20	17.22	11.02	16.99	7.25	14.03	7.25	14.03	13.54	18.52	13.54	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33															33
34		-68.64	-84.68												34
35				-46.27	-56.78	-42.85	-52.57								35
36								-11.43	-20.92	-11.43	-20.92				36
37												-25.82	-31.74	-25.82	37
38		8.69	8.69	8.69	8.69	8.69	8.69	8.69	8.69	8.69	8.69	8.69	8.69	8.69	38
39															39
40															40
41															41
42															42
43															43
44															44
45		.82	.91	.82	.91	.82	.91	.82	.91	.82	.91	.91	.91	.91	45
46															46
47															47
48		1.06	1.24	.98	1.12	.94	1.06	.94	1.40	.94	1.40	1.04	1.18	1.04	48
49															49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52												1.	1.		52
53														1.	53
54				.50	.50			.50	.50						54
55		.50	.50			.50	.50			.50	.50				55
56				1.	1.			1.	1.						56
57		1.	1.			1.	1.			1.	1.				57
58															58
59															59
60															60
61															61
62				1.	1.			1.	1.			1.	1.		62
63		1.	1.			1.	1.			1.	1.			1.	63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77				-1.	-1.			-1.	-1.			-1.	-1.		77
78		-1.	-1.			-1.	-1.			-1.	-1.			-1.	78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Unit	122	123	124	125	126	127	128	129	130	131	132	133	134	Row No.
	Activity No.	122	123	124	125	126	127	128	129	130	131	132	133	134	
	Activity Description		Irrigated Peas		Irr. Sunflowers		Irrigated Hay				Irr. Corn Silage		Irr. Pasture		
		H,fall. heav.f.	G,fall. heav.f.	H,fall. heav.f.	G,stub. heav.f.	H,stub. heav.f.	G, med.f.	G, heav.f.	H, med.f.	H, heav.f.	G,stub. heav.f.	H,stub. heav.f.	G, med.f.	G, heav.f.	
	Unit	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	acre	
1															1
2		18.52	-58.47	-45.73	-55.35	-52.70	12.77	17.07	12.77	17.07	40.95	40.95	6.10	8.89	2
3															3
4															4
5		27.94	-4.14	-9.50	-.74	-2.15	7.74	15.62	7.74	15.62	7.20	7.20	12.65	20.81	5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		18.52	19.23	18.60	22.67	22.56	12.77	17.07	12.77	17.07	40.95	40.95	6.10	8.89	19
20															20
21		18.52	-58.47	-45.73	-55.35	-52.70	12.77	17.07	12.77	17.07	40.95	40.95	6.10	8.89	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33															33
34															34
35															35
36															36
37		-31.74													37
38		8.69	8.69	8.69	13.12	13.12	13.12	13.12	13.12	13.12	13.12	13.12	13.12	13.12	38
39															39
40															40
41															41
42							-3.80	-4.68	-3.80	-4.68	-4.31	-4.31			42
43													-3.80	-4.68	43
44															44
45		.91	.82	.82	.82	.82	.09	.09	.09	.09	1.17	1.17	.09	.09	45
46							3.90	4.56	3.90	4.56					46
47							2.64	3.30	2.64	3.30					47
48		1.18	1.26	1.11											48
49					.99	.96					6.07	6.07			49
50															50
51		-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	51
52			1.												52
53		1.		1.											53
54					.50						.50				54
55						.50						.50			55
56					1.						1.				56
57						1.						1.			57
58							1.	1.					1.	1.	58
59									1.	1.					59
60															60
61															61
62			1.		1.		1.	1.			1.		1.	1.	62
63		1.		1.		1.			1.	1.		1.			63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77			-1.		-1.						-1.				77
78		-1.		-1.		-1.						-1.			78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Activity No. / Activity Description / Unit	135	136	137	138	139	140	141	142	143	144	145	146	147	Row No.
	Activity Description	Irr. Pasture H, med.f.	H, heav.f.	Fallow soil G	Fallow soil H	Establ. forage soil G	Establ. forage soil H	Cattle enter- prise I	Cattle enter- prise II	Sell Wheat	Sell oats	Sell barley	Sell flax	Sell rapeseed	
	Unit	acre	acre	acre	acre	acre	acre	beef cow	beef cow	bushel	bushel	bushel	bushel	bushel	
1															1
2		6.10	8.89	1.26	1.26	15.62	15.50	-133.29	-184.28	-1.90	-.89	-1.29	-4.25	-3.50	2
3															3
4															4
5		12.65	20.81			-4.28	-4.66	40.72	46.07						5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		6.10	8.89	1.26	1.26	15.62	15.50	37.44	47.39						19
20															20
21		6.10	8.89	1.26	1.26	15.62	15.50	-133.29	-184.28	-1.90	-.89	-1.29	-4.25	-3.50	21
22															22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30								400.00	550.00						30
31															31
32															32
33										1.					33
34						-25.32	-22.83				1.				34
35								5.18	13.43			1.			35
36													1.		36
37														1.	37
38		13.12	13.12												38
39															39
40															40
41								1.08	1.48						41
42								1.79	2.71						42
43		-3.80	-4.68					2.41	3.14						43
44								1.00	1.40						44
45		.09	.09			.82	.84								45
46															46
47															47
48						.89	.84								48
49															49
50								.07	.10						50
51		-1.	-1.	-1.	-1.	-1.	-1.			.07	.04	.04	.17	.09	51
52															52
53															53
54				1.		.50									54
55					1.		.50								55
56				1.		1.									56
57					1.		1.								57
58															58
59		1.	1.												59
60															60
61															61
62															62
63		1.	1.												63
64															64
65															65
66															66
67															67
68															68
69															69
70															70
71															71
72															72
73															73
74															74
75				-1.											75
76					-1.										76
77															77
78															78
79															79
80															80
81															81
82															82
83						.50*									83
84							.50*								84
85						-1.*									85
86							-1.*								86

Table D.2 (continued)

Restraint No.	Activity No. / Activity Description / Unit	148 Harvest straw (ton)	149 Apply Irrigat. water (acre-inch)	150 Purchase irrigation system (system)	151 Separable activity I-irr.sys. (.25 sys.)	152 Separable activity II-irr.sys. (.75 sys.)	153 Expand cattle housing (beef cow)	154 Use cash to buy irr.sys. (dollar)	155 Use cash cattle housing (dollar)	156 Use cash to buy cattle (dollar)	157 Borrow to buy irr.sys. (dollar)	158 Borrow, cattle housing (dollar)	159 Borrow to buy cattle (dollar)	160 Deprec. capital class 10 (dollar)	Row No.
1															1
2		1.45	1.10											.30	2
3															3
4															4
5															5
6															6
7															7
8															8
9															9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18															18
19		1.45	1.10												19
20															20
21		1.45	1.10					1.	1.	1.					21
22															22
23															23
24															24
25															25
26														1.	26
27				-29108.											27
28							-300.								28
29														-.30	29
30										-1.			-1.		30
31							300.		-1.			-1.			31
32				29108.				-1.			-1.				32
33															33
34															34
35															35
36															36
37															37
38			-1.												38
39				1.											39
40				1.											40
41		-1.													41
42															42
43															43
44							-1.								44
45															45
46															46
47															47
48															48
49															49
50															50
51															51
52															52
53															53
54															54
55															55
56															56
57															57
58															58
59															59
60															60
61															61
62				-48.00*											62
63				-90.00*											63
64				-1.	.25	.75									64
65				-1.	.50	.50									65
66															66
67											.147**	.18**	.28**		67
68															68
69														-1.	69
70															70
71															71
72															72
73															73
74															74
75															75
76															76
77															77
78															78
79							-1.*								79
80										-1.*			-1.*		80
81											.08**	.08**	.08**		81
82											-.147**	-.18**	-.28**		82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Activity No. / Activity Description / Unit	161 Deprec. capital class 8 (dollar)	162 Deprec. capital class 6 (dollar)	163 Pay deprec. expense (dollar)	164 Allocate Beg. Cash to work. capital (dollar)	165 Allocate Beg. Cash to cash reserves (dollar)	166 Borrow working capital (dollar)	167 Use cash reserves (dollar)	168 Pay min. consump. expend. (dollar)	169 Pay cash overhead (dollar)	170 Pay princ. and interest (dollar)	171 Pay Income Tax bracket 1	172 Pay Income Tax bracket 2	173 Pay Income Tax bracket 3	Row No.
1															1
2		.20	.10				.08			1.	.469	1.	1.	1.	2
3									1.		.531	-1.	-.76285	-.74890	3
4											-.531				4
5								1.		-1.	-1.				5
6															6
7												1.			7
8													1.		8
9														1.	9
10															10
11															11
12															12
13															13
14															14
15															15
16															16
17															17
18					1.	1.									18
19					-1.		-1.								19
20							1.								20
21				1.	-1.		.08		1.	1.	1.				21
22						-1.		1.							22
23									1.						23
24										1.					24
25											1.				25
26															26
27		1.													27
28			1.												28
29		-.20	-.10	1.											29
30															30
31															31
32															32
33															33
34															34
35															35
36															36
37															37
38															38
39															39
40															40
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42															42
43															43
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58															58
59															59
60															60
61															61
62															62
63															63
64															64
65															65
66															66
67													.23715	.25110	67
68															68
69															69
70		-1.													70
71			-1.												71
72															72
73															73
74															74
75															75
76															76
77															77
78															78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Activity No. / Activity Description / Unit	174	175	176	177	178	179	180	181	182	183	184	185	186	Row No.
	Activity	Pay Income Tax								Transfer wheat to next yr.	Transfer oats to next yr.	Transfer barley to next yr.	Transfer cash flow next yr.	Transfer cash res. next yr.	
	Description	bracket 4	bracket 5	bracket 6	bracket 7	bracket 8	bracket 9	bracket 10	bracket 11						
	Unit	dollar	dollar	dollar	dollar	dollar	dollar	dollar	dollar	bushel	bushel	bushel	dollar	dollar	
1															1
2		1.	1.	1.	1.	1.	1.	1.	1.					-.08	2
3		-.73495	-.72100	-.70705	-.67915	-.65125	-.62335	-.56755	-.51175						3
4															4
5															5
6															6
7															7
8															8
9															9
10		1.													10
11			1.												11
12				1.											12
13					1.										13
14						1.									14
15							1.								15
16								1.							16
17									1.						17
18															18
19															19
20															20
21													1.	-.08	21
22														1.	22
23															23
24															24
25															25
26															26
27															27
28															28
29															29
30															30
31															31
32															32
33										1.					33
34											1.				34
35												1.			35
36															36
37															37
38															38
39															39
40															40
41															41
42															42
43															43
44															44
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58															58
59															59
60															60
61															61
62															62
63															63
64															64
65															65
66													-1.	-1.	66
67		.26505	.27900	.29295	.32085	.34875	.37665	.43245	.48825						67
68															68
69															69
70															70
71															71
72										-1.					72
73											-1.				73
74												-1.			74
75															75
76															76
77															77
78															78
79															79
80															80
81															81
82															82
83															83
84															84
85															85
86															86

Table D.2 (continued)

Restraint No.	Activity No. / Activity Description / Unit	187 Transfer additions to net worth (dollar)	188 Maximize additions to net worth (dollar)	189 Increase worst possible income (dollar)	Row No.
1			1.		1
2					2
3		1.			3
4		-.60	1.		4
5				-1.	5
6				1.	6
7					7
8					8
9					9
10					10
11					11
12					12
13					13
14					14
15					15
16					16
17					17
18					18
19					19
20					20
21		.40			21
22					22
23					23
24					24
25					25
26					26
27					27
28					28
29					29
30					30
31					31
32					32
33					33
34					34
35					35
36					36
37					37
38					38
39					39
40					40
41					41
42					42
43					43
44					44
45					45
46					46
47					47
48					48
49					49
50					50
51					51
52					52
53					53
54					54
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57					57
58					58
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62					62
63					63
64					64
65					65
66					66
67					67
68					68
69					69
70					70
71					71
72					72
73					73
74					74
75					75
76					76
77					77
78					78
79					79
80					80
81					81
82				-1.*	82
83					83
84					84
85					85
86					86

EXPLANATORY NOTES FOR TABLE D.2

1. Under "Type of Restraint": (a) the letter N signifies that this row represents the objective function of the model, (b) the letter E signifies an equality restraint, (c) the letter L signifies a less than or equal restraint and (d) the letter G signifies a greater than or equal to restraint.

2. The column headings for the crop production activities were abbreviated somewhat in order to save space. The abbreviations were as follows: (a) the letter G means soil zone G, (b) the letter H means soil zone H, (c) "stub." means stubble seedbed, (d) "fall." means fallow seedbed, (e) "no f." means no fertilizer, (f) "med. f." means medium fertilizer and (g) "heav. f." means heavy fertilizer.

3. Restraint number 4 does not appear as a separate restraint in each period. It is an accounting restraint to accumulate expected additions to net worth, and, accordingly, it encompasses all periods. Similarly, activity number 188 does not appear in each period; it is a maximizing activity that transfers the accumulated additions to net worth to the objective function.

4. Restraint number 4 and activity number 189 also appear only once in the model. They are concerned with parametric variation of the worst possible income restraint in each period.

5. Some activities have a single or double asterisk beside one or more coefficients. A single asterisk indicates that the same coefficient appears in the same restraint in later periods. A double asterisk indicates that this activity has a coefficient in the same restraint in later periods, though not the same coefficient.