A RESONANT FREQUENCY TRACKING SYSTEM

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by

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ABSTRACT

A study of a resonant frequency tracking system to be used for the measurement of currents in HVDC transmission lines is presented. The tracking system was used to lock an oscillator to the resonant frequency of a YIG filter operating in the 10 to 11 GHz range. The resonant frequency of the YIG filter is dependent on the surrounding magnetic field and therefore on the line current.

A theoretical analysis of the resonant frequency tracking system was done for both linear and nonlinear operation. A model of the tracking system was designed and an analysis of the system response was performed using computer techniques to obtain the system response.

A prototype of the tracking system was constructed and experiments were done to determine the time and frequency responses and the operating range. The results corresponded very well with those predicted by theory. With an open loop gain of about 100,000 a bandwidth of 20 kHz and a rise time of 0.03 ms. were obtained. Because of the large open loop gain of the system the tracking accuracy for frequency changes from the nominal point was of the order of 10^{-5}.

It was concluded that a resonant frequency tracking system of the type described here can be used effectively in measurement systems. No reason was seen why larger bandwidths and faster rise times could not be obtained.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

A_d - detector constant

F_c - difference between VCO frequency and rest frequency

F_m - low frequency oscillator frequency

F_o - YIG filter center frequency

F_r - difference between YIG filter frequency and rest frequency

G - open loop transfer function

G_s - small signal gain of nonlinear part

G_{VCO} - transfer function of VCO and driver

G'_{VCO} - normalized G_{VCO}

G_1 - transfer function of bandpass amplifier

G'_1 - low pass equivalent of bandpass amplifier

G_2 - transfer function of low pass amplifier

H_a - YIG crystal-anistropy field

H_o - external magnetic field at YIG sphere

K_o - open loop DC gain

K_1 - gain of the product detector

K_2 - DC gain of the VCO and driver

K_3 - gain of the bandpass amplifier

K_4 - gain of the low pass amplifier

L - linearized equation for the nonlinear part of the system

P_o - microwave power at the detector

Q - quality factor of the YIG filter

Q' - loaded quality factor of the YIG filter

R - real part of Z_F

S_Γ - slope of the reflection coefficient with respect to $\triangle F$

X - twice $\triangle F$ divided by F_o

Z_F - impedance of the YIG filter

Z_o - source impedance feeding YIG filter

k_o - open loop DC gain with $\triangle F = 0$

v - voltage at detector output

v_1 - fundamental frequency voltage at detector output

v_2 - voltage at product detector output

v_3 - input voltage to driver

Γ - reflection coefficient

Γ_o - reflection coefficient at filter center frequency

γ - gyromagnetic ratio

$\triangle f$ - FM deviation

$\triangle F$ - difference between the VCO and YIG filter frequencies

$\triangle F_{max}$ - maximum $\triangle F$ for stable operation

$\triangle F_o$ - one-half of YIG filter bandwidth

Chapter 1

INTRODUCTION

With the recent introduction of high voltage direct current power transmission systems the problem of measuring large direct currents at high potentials has become important. The current measuring equipment presently being used suffers from the disadvantages of high cost, slow response, and poor accuracy.

The problem of high cost is directly related to the problem of isolating the measuring equipment from the high voltage of the power line. Similar difficulties caused by extreme environment exist in several other situations where measurements must be taken.

A measuring technique which may prove useful here has been suggested for determining temperatures inside nuclear reactors[2]. This system uses a resonant cavity as a sensing device. Measurements are taken using a frequency tracking loop that locks onto the temperature dependent resonant frequency of the cavity. The advantages of interest here are the passive nature of the cavity, eliminating the need for power supply connections to the sensing device, and the possibility of eliminating all electrical connections to the sensing device by using microwave antennas or dielectric waveguide as the link to the part of the system at ground potential.

Unfortunately no practical way has been found to use a resonant cavity for current measurement. However, with the introduction of the YIG filter, a microwave resonator whose resonant frequency can be used as a measure of the strength of a magnetic field, this problem has been resolved. Since the magnetic field is directly proportional to the power line current in the vicinity of the conductor the YIG filter can be used for current measurements. Like the resonant cavity, the YIG filter is a passive device requiring no power supply. YIG filters also offer the potential for much higher speed and greater accuracy than the current measuring devices presently in use, and because of the ease with which they allow high and low voltages to be isolated from one another a system using YIG filters should be much less costly.

The purpose of this research was to develop a theoretical analysis of a resonant frequency tracking system which will allow a YIG filter to be used as a current transducer.

Chapter 2

REVIEW OF LITERATURE

Several measurement systems have been developed in which resonant frequency tracking loops have been used. While all of these loops have been used to lock an oscillator to the resonant frequency of a tuned cavity, in each application a slightly different technique has been used.

In a system used for measuring the Q factors of resonators, a tracking loop has been used to lock a voltage controlled oscillator to a point on the resonant curve of the resonator [1]. A low frequency sine wave is used to frequency modulate the VCO. The VCO output is then fed into the resonator, where the signal is amplitude modulated by the frequency response characteristic of the resonator. The phase of this amplitude modulation, relative to the phase of the modulating signal, is then zero degrees or 180 degrees depending on whether the VCO frequency is higher or lower than the resonant frequency of the tuned cavity. The AM signal is then demodulated by means of synchronous detection in a high frequency mixer. A lock-in amplifier generates a positive or negative voltage, depending on the phase of the demodulated signal relative to the phase of the initial signal, and this voltage is used to tune the VCO. As a result of this the VCO is locked to the resonant frequency of the cavity. Synchronous detection is used in this system for reasons related to the Q factor measurement problem.

A similar tracking loop has been developed to measure temperatures inside a nuclear reactor [2]. Changes in temperature cause a tuned cavity to expand or contract, changing its resonant frequency. A resonant frequency tracking loop is then used to observe the cavity's resonant frequency, and thereby determine its temperature. This tracking loop is different from the one described above in that it uses a diode square detector rather than a synchronous detector. In this system the tracking loop is required to work only at very low speeds, since measurement of the cavity's response to a step change in temperature is reported to yield a time constant of 0.5 second.

Another system has been developed to measure the density of liquid hydrogen using an open ended cavity as a sensor [3]. The resonant frequency tracking loop used here uses an envelope detector to demodulate the AM signal. The lock-in amplifier in this loop uses a digital system consisting of a level detector and phase detector so that either zero or a fixed positive or negative voltage is applied to the VCO. Unfortunately this causes a low frequency oscillation of the loop about the resonator frequency. Another interesting feature of this loop is the seek and lock system. When the loop becomes unlocked, that is when the VCO frequency is far enough from the resonator frequency so that no AM appears on the carrier at the resonator output, the loop switches into its seek mode. In this mode of operation the VCO is swept through the frequency range of interest at a rate sufficiently slow to allow the loop to re-lock itself when the VCO and resonator frequencies coincide.

A great deal of study has been devoted to phase and frequency lock-in systems in the form of phase locked loops [4] [5]. While the results of this work are not directly applicable to resonant frequency tracking loops, they can be useful in developing a qualitative understanding of such systems. In addition to this the properties of some components, such as phase detectors, have already been studied in great detail because of their use in phase locked loops.

While the YIG filter is a relatively recent development, its properties have been studied and described in considerable detail [6] [7] [8], and manufactured units which can easily be put to use are available. The YIG resonator itself is a small, highly polished sphere of single crystal yttrium-iron-garnet. It is operated at the ferrimagnetic resonance where, because of the small linewidth very high Q factors can be obtained. The YIG filter operates in a magnetic field, where the resonant frequency of the filter is directly proportional to the field strength.

While several resonant frequency tracking loops have been developed, all have been used to measure static or relatively slowly varying resonator frequencies. The availability of YIG filters for use as measuring elements, with their ability to change frequencies at much higher speeds than tuned cavities, has made important the development and analysis of a tracking loop capable of operating at higher speeds.

Chapter 3

THEORETICAL CONSIDERATIONS

3.1 PRINCIPLE OF OPERATION

In the current measurement system shown in figure 3.1 the YIG reflection filter is held in a fixed position with respect to the transmission line so that its resonant frequency will vary according to the strength of the magnetic field surrounding the line. The resonant frequency is given by the equation [6]

$$F_o = (H_o + H_a)\,\gamma \qquad (3.1)$$

where H_o is the magnetic field at the YIG crystal due to the transmission line current, H_a is the crystal anisotropy field, and γ is the gyromagnetic ratio.

In the system a low frequency oscillator is used to frequency modulate the high frequency voltage controlled oscillator. The FM signal is fed to the YIG filter and then to an envelope detector. In figure 3.2 the detector outputs are shown for VCO frequencies higher than, lower than, and equal to the YIG filter's resonant frequency. For higher VCO frequency the low frequency oscillator signal and the detector output signal are in phase, and for lower VCO frequency they are 180 degrees out of phase. When the VCO is at the YIG filter's resonant frequency the detector output contains the second and higher harmonics of the frequency modulation, but does not contain the fundamental. The amplitude of the fundamental frequency signal at the detector output depends on the slope of the YIG filter characteristic. The fundamental frequency signal is amplified by a bandpass amplifier and fed to a

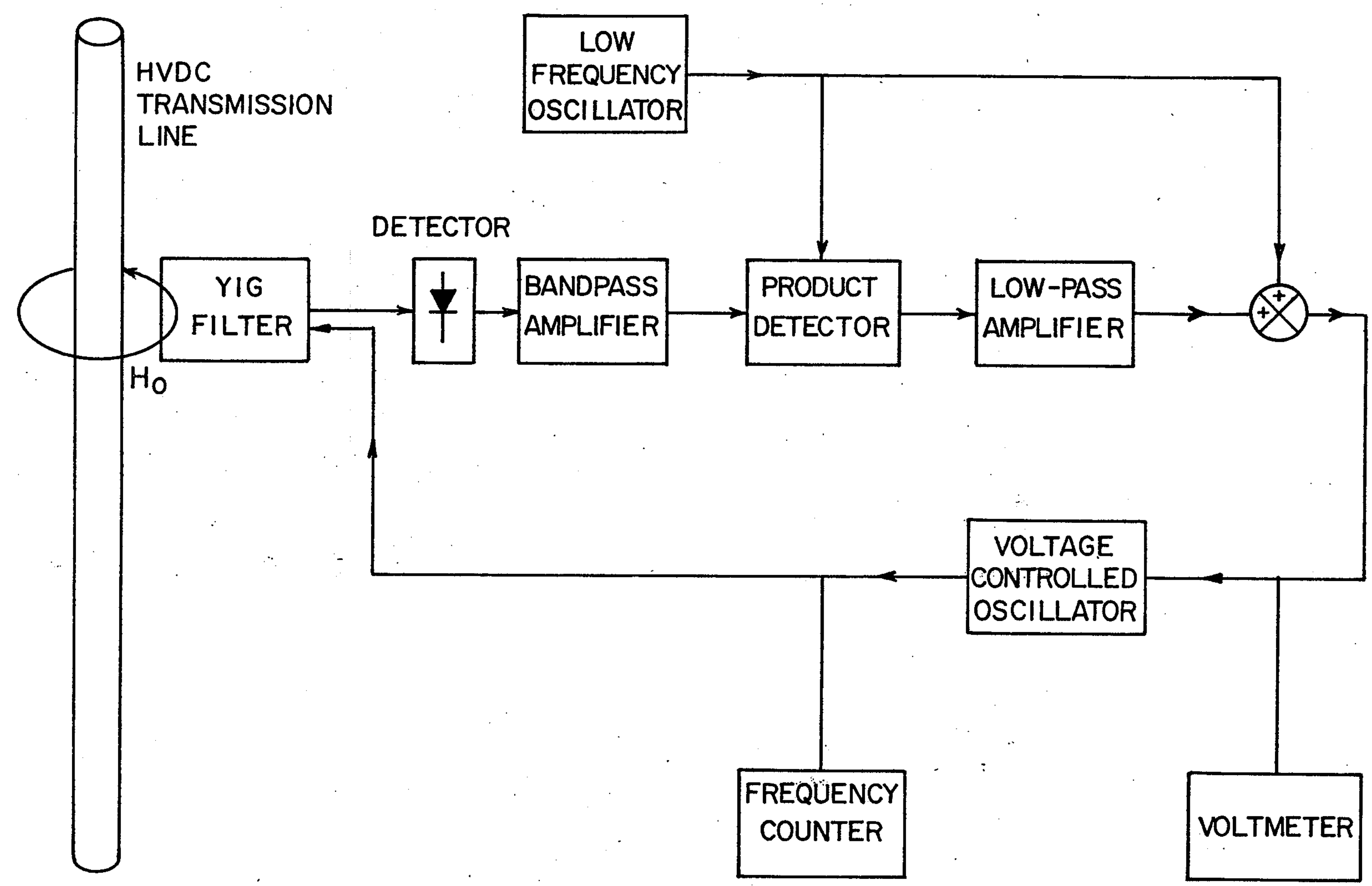

FIGURE 3.1: CURRENT MEASUREMENT SYSTEM

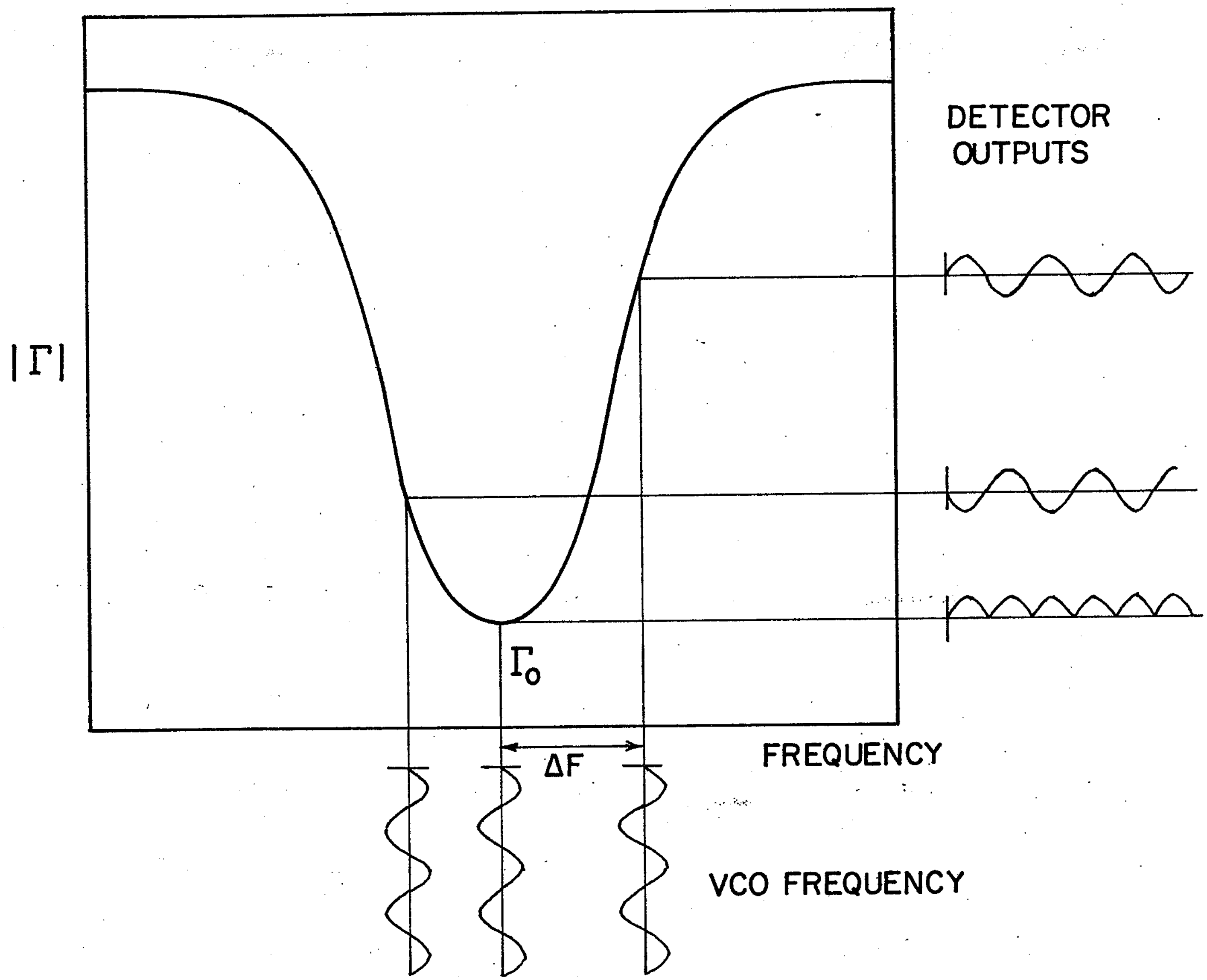

FIGURE 3.2: DETECTOR OUTPUT WAVEFORMS

product detector. The product detector output contains a DC voltage which is positive or negative depending on the relative phase of the input signal to the phase of the low frequency oscillator. The amplitude of this DC voltage is proportional to the amplitude of the input signal. The higher mixing products generated in the product detector are removed by the low pass amplifier. The DC voltage which remains is fed to a summing block and then to the voltage controlled oscillator where it is used to tune the VCO to the resonant frequency of the YIG filter.

The tracking loop can be divided into two parts. The part of the loop consisting of the YIG filter and detector behaves nonlinearly, while the rest can easily be made to behave linearly.

3.2 ANALYSIS OF THE NONLINEAR PART OF THE SYSTEM

Figure 3.2 is a reflection filter characteristic represented in terms of reflection coefficient versus frequency, where Γ is the reflection coefficient, Γ_o is the reflection coefficient at resonance, and Δ_F is the difference between the VCO and YIG resonator frequencies. It can readily be seen that for small FM deviation, Δf; the square law detector's output is

$$v = A_d P_o [\Gamma(\Delta F) + S_\Gamma(\Delta F)\Delta f \cos\omega_m t]^2 \cos^2 \omega_c t \qquad (3.2)$$

where A_d is the detector constant, P_o is the signal power at the detector, $S_\Gamma(\Delta F)$ is the slope of the reflection coefficient curve, ω_c is the modulating frequency, and ωc is the microwave carrier radian frequency. The fundamental frequency component of the detector output is

$$v_1 = A_d P_o [\Gamma(\Delta F) S_\Gamma(\Delta F)\Delta f \cos\omega_m t] \qquad (3.3)$$

The reflection coefficient can be written as

$$\Gamma = \frac{Z_F - Z_o}{Z_F + Z_o} \tag{3.4}$$

where Z_F is the resonator impedance and Z_o is the source impedance connected to the resonator. The resonator impedance can be written as

$$Z_F = R + jRQX \tag{3.5}$$

where R is the real part of Z_F, Q is the unloaded quality factor of the resonator and X is

$$X = \frac{2\Delta F}{F_o} \tag{3.6}$$

F_o is the center frequency of the YIG filter.

Combining equations (3.4) and (3.5) gives

$$\Gamma = \frac{(R + jRQX) - Z_o}{(R + jRQX) + Z_o} = \frac{R - Z_o + jRQX}{R + Z_o + jRQX} \tag{3.7}$$

Now substituting with the loaded quality factor Q'

$$\Gamma = \frac{\Gamma o + jQ'X}{1 + jQ'X} \tag{3.9}$$

The absolute value of Γ is

$$|\Gamma| = \sqrt{\frac{\Gamma o^2 + Q'^2X^2}{1 + Q'^2X^2}} \tag{3.10}$$

The loaded Q factor is

$$Q' = \frac{F_o}{2\Delta F_o} \tag{3.11}$$

where ΔF_o is one-half of the 3dB bandwidth of the filter.

Multiplying equations (3.11) and (3.6)

$$Q'X = \frac{\Delta F}{\Delta F_o} \tag{3.12}$$

For convenience the frequency scale in figure (3.2) can be normalized by dividing by ΔF_o. Equation (3.3) becomes

$$v_1 = A_d P_o [\Gamma (\frac{\Delta F}{\Delta F_o}) S_\Gamma (\frac{\Delta F}{\Delta F_o}) \frac{\Delta f}{\Delta F_o} \cos\omega_m t] \qquad (3.13)$$

Now taking the derivative of Γ with respect to $Q'X$ gives the slope

$$S_\Gamma = \frac{d|\Gamma|}{d(Q'X)} = \sqrt{\frac{1+(Q'X)^2}{\Gamma_o + (Q'X)^2}} \frac{2Q'X [1 - \Gamma_o^2]}{[1 + (Q'X)^2]^2} \qquad (3.14)$$

Multiplying (3.14) by (3.10) gives

$$|\Gamma| S_\Gamma = \frac{Q'X}{[1 + (Q'X)^2]^2} [1 - \Gamma_o^2] \qquad (3.15)$$

Substituting (3.12) into (3.15)

$$|\Gamma| S_\Gamma = \frac{\frac{\Delta F}{\Delta F_o}}{[1 + (\frac{\Delta F}{\Delta F_o})^2]^2} [1 - \Gamma_o^2] \qquad (3.16)$$

The expression for the detector output can now be found by substituting equation (3.16) into equation (3.13)

$$v_1 = A_d P_o \frac{\frac{\Delta F}{\Delta F_o}}{[1 + (\frac{\Delta F}{\Delta F_o})^2]^2} [1 - \Gamma_o^2] [\frac{\Delta f}{\Delta F_o}] \cos\omega_m t \qquad (3.17)$$

This equation is a nonlinear function of ΔF, the difference in frequency between the YIG filter and the VCO.

3.3 THE LINEARIZED SYSTEM

The nonlinear part of equation (3.17) is

$$\frac{\frac{\Delta F}{\Delta F_o}}{[1 + (\frac{\Delta F}{\Delta F_o})^2]^2}$$

For small ΔF this can be approximated by

$$\frac{\frac{\Delta F}{\Delta F_o}}{[1 + (\frac{\Delta F}{\Delta F_o})^2]^2} = \frac{\Delta F}{\Delta F_o} \tag{3.18}$$

Comparison of these two functions in figure 3.3 show that for $\frac{\Delta F}{\Delta F_0}$ greater than 0.2 equation (3.18) is no longer a good approximation

Equation (3.17) can now be rewritten as

$$v_1 = A_d P_o [\frac{\Delta F}{\Delta F_o}] [1 - \Gamma_o^2] [\frac{\Delta f}{\Delta F_o}] \cos\omega_m t \tag{3.19}$$

The system shown in figure 3.1 can be simplified by modeling the bandpass amplifier as a low pass amplifier placed after the product detector in the signal path. In this new arrangement the voltage at the output of the product detector is

$$v_2 = K_1 A_d P_o [1 - \Gamma_o^2] [\frac{\Delta f}{\Delta F_o}] \quad [\frac{\Delta F}{\Delta F_o}] \tag{3.20}$$

where K_1 is the gain of the product detector. Rewriting this as a transfer function for the linear approximation of the YIG filter and product detector.

$$L (\frac{\Delta F}{\Delta F_o}) = \frac{v_2(\frac{\Delta F}{\Delta F_0})}{(\frac{\Delta F}{\Delta F_o})} = K_1 A_d P_o [1 - \Gamma_o^2] \quad [\frac{\Delta f}{\Delta F_o}] \tag{3.21}$$

The linearized system is shown in figure 3.4. The free running frequency of the VCO is assumed to be the same as the frequency of the YIG filter with zero transmission line current. $F_r(s)$ in figure 3.4 is the difference between the YIG filter frequency and the free running frequency and $F_s(s)$ is the difference between the VCO frequency and

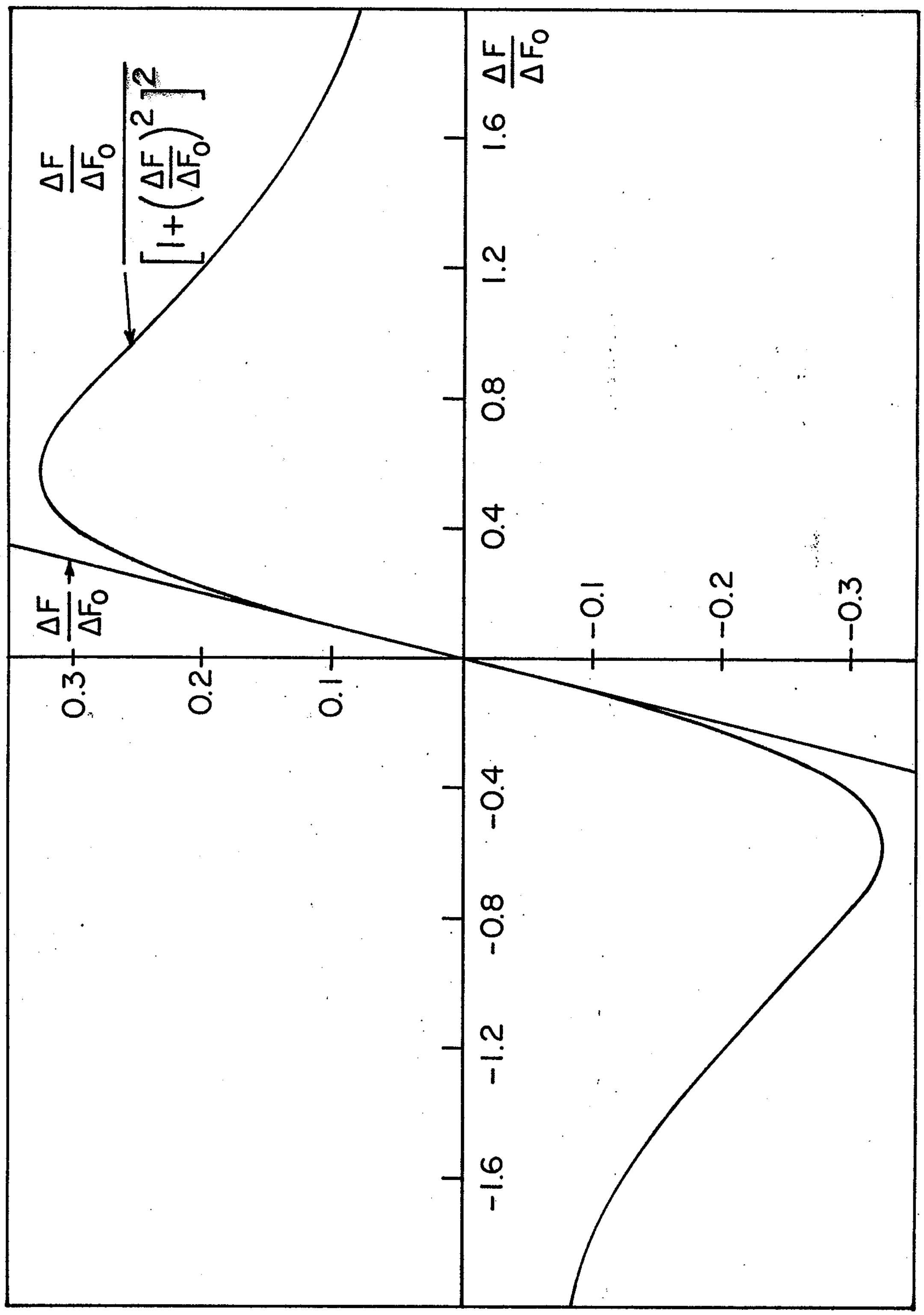

FIGURE 3.3: NONLINEAR CHARACTERISTIC

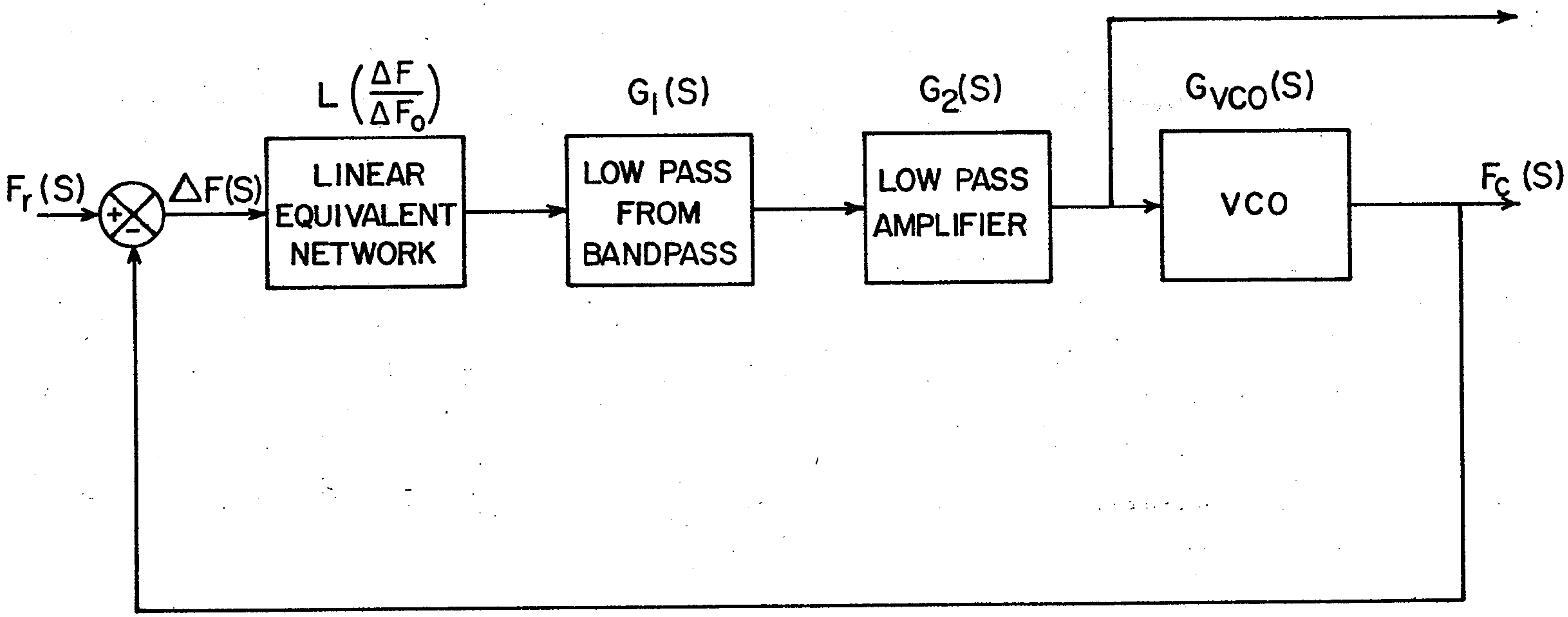

FIGURE 3.4: LINEARIZED SYSTEM

the free running frequency.

The usual relationships for control system apply here. The open loop transfer function is

$$\frac{Fc(s)}{\Delta F(s)} = L(s)\ G_1(s)\ G_2(s)\ G_{VCO}(s) \tag{3.22}$$

the closed loop transfer function is

$$\frac{Fc(s)}{Fr(s)} = \frac{[L(s)\ G_1(s)\ G_2(s)\ G_{VCO}(s)]}{1 + [L(s)\ G_1(s)\ G_2(s)\ G_{VCO}(s)]} \tag{3.23}$$

and the normalized error function is

$$\frac{\Delta F(s)}{Fr(s)} = \frac{1}{1 + [L(s)\ G_1(s)\ G_2(s)\ G_{VCO}(s)]} \tag{3.24}$$

These equations are only useful for small Δ F(s). This condition exists when changes of the YIG filter frequency are not larger than the linear part of the response characteristic, or when the input is large but changes at a rate no faster than the slew rate of the tracking loop.

It can readily be seen from figure 3.3 that for large ΔF the gain of the loop decreases. For this reason if the loop is designed to be stable using the linearized approximation it will also be stable in the nonlinear case.

3.4 NONLINEAR OPERATION

When ΔF, the difference between the YIG filter frequency and the VCO frequency is small, the system operates in an approximately linear manner. When Δ F is larger, however, the system operation becomes nonlinear because of the shape of the YIG filter resonance characteristic. From figure 3.3 it can be seen that nonlinear operation becomes noticable if the condition $\frac{\Delta F}{\Delta F_0} > 0.2$ exists.

Large errors occur under two possible sets of circumstances. One of these is when a large static offset of the system's operating frequency from the free running frequency of the VCO occurs. The relatively large tuning voltage then required for the VCO must then be generated by a correspondingly large frequency difference between the VCO and the YIG filter.

A large frequency error can also occur when a large change in the YIG filter frequency occurs at a rate faster than the slew rate of the system. Because the tracking loop is not capable of changing the VCO frequency as fast as the YIG filter frequency is changing, a frequency error occurs.

Figure 3.3 shows the transfer characteristic of the nonlinear part of the system. This transfer characteristic is divided into three regions. One of these is the central part where the slope is positive, while in the other two parts the slope is negative.

In the region of positive slope it is possible for the tracking loop to lock onto the YIG filter. If the loop is operating in this region, any increase of the YIG filter frequency causes a decrease in the detector output voltage. This causes the voltage applied to the VCO to increase, raising the VCO frequency. Thus the VCO follows any change in the operating frequency of the YIG filter.

As the frequency error increases the slope of the characteristic decreases. This means that the open loop gain of the system is reduced. This decrease in gain slows down the time response of the loop.

When the point where

$$\frac{\Delta F}{\Delta F_0} = \frac{1}{\sqrt{3}} \qquad (3.25)$$

is reached, the slope of the transfer characteristic and the open loop gain are zero. Beyond this point the slope is negative, which means that the open loop gain is also negative. In this region a decrease in the YIG filter frequency would cause an increase in the frequency of the VCO.

The small signal gain, Gs, for any given value of $\frac{\Delta F}{\Delta F_O}$ is

$$G_S = \frac{d}{d(\frac{\Delta F}{\Delta F_O})}\left[\frac{\frac{\Delta F}{\Delta F_O}}{[1 + (\frac{\Delta F}{\Delta F_O})^2]^2}\right] = \frac{1 - 3(\frac{\Delta F}{\Delta F_O})^2}{[1 + (\frac{\Delta F}{\Delta F_O})^2]^2} \qquad (3.26)$$

Figure 3.5 is a graph of G_S versus $\frac{\Delta F}{\Delta F_O}$. From this it can be seen that for values of $\frac{\Delta F}{\Delta F_O} > \frac{1}{\sqrt{3}}$ the loop goes into positive feedback. It can also be seen from figure 3.5 that outside the region where the loop is approximately linear the gain decreases rapidly. Therefore, if the linear approximation of the tracking loop is stable, then the loop will also be stable during nonlinear operation for $\frac{\Delta F}{\Delta F_O} < \frac{1}{\sqrt{3}}$.

The YIG filter could change frequency very rapidly without becoming unlocked if the magnitude of the change remained small.. For a very fast change the frequency error would be equal to the magnitude of the change. Therefore, the maximum allowable magnitude for a small rapid change is

$$\Delta F_{max} = \frac{\Delta F_O}{\sqrt{3}} \qquad (3.27)$$

Larger changes of the YIG filter frequency must occur slowly enough to be tracked by the system. The slew rate of the system is determined by the bandwidths of the low pass and bandpass amplifiers, and by the open loop gain.

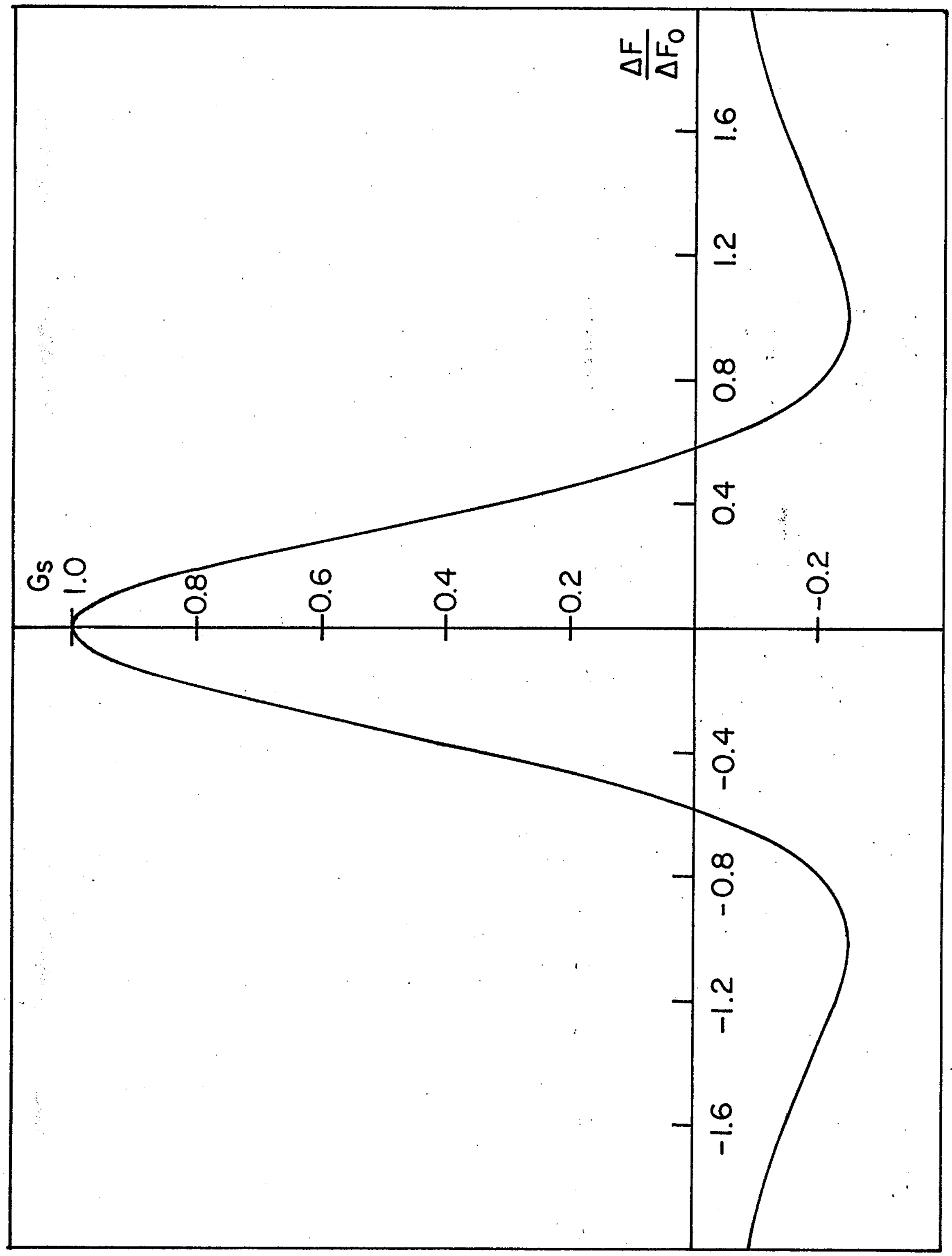

FIGURE 3.5: SMALL SIGNAL GAIN

For static offsets of the YIG filter from the rest frequency of the system, a static frequency error occurs. This error is equal to

$$\frac{\Delta F}{Fr} = \frac{1}{1 + K_0} \qquad (3.28)$$

where K_0 is the DC open loop gain. In this system K_0 decreases for larger values of ΔF. When a static offset of this sort occurs, it degrades several of the operating characteristics of the system.

Since an offset already exists, the ability of the system to follow small rapid changes is reduced if those changes occur in the same direction as the offset. The total frequency error must be less than $\frac{\Delta F_0}{\sqrt{3}}$ if the system is to remain locked.

When a static offset exists the open loop gain is reduced. This causes the slew rate of the system to drop. Therefore the ability of the loop to follow any further change in the YIG filter frequency will be impaired.

The frequency offset also determines the maximum range of frequency over which the system can operate. This maximum range can be determined by substituting

$$K_0 = \frac{k_0 \frac{\Delta F}{\Delta F_0}}{[1 + (\frac{\Delta F}{\Delta F_0})^2]^2} \qquad (3.29)$$

, where k_0 is the DC gain of the loop with $\Delta F = 0$, into equation (3.28) to get

$$F_{r\,max} = \Delta F_{max} \left[1 + \left[\frac{k_0(\frac{\Delta F}{\Delta F_0})}{[1 + (\frac{\Delta F}{\Delta F_0})^2]^2} \right] \right] \qquad (3.30)$$

now substituting $\frac{\Delta F}{\Delta F_0} = \frac{1}{\sqrt{3}}$ and

$$\Delta F_{max} = \frac{\Delta F_o}{\sqrt{3}} \tag{3.31}$$

one gets

$$F_{r\,max} = \frac{\Delta F_o}{\sqrt{3}} \left[1 + k_o \frac{3\sqrt{3}}{16} \right] \tag{3.32}$$

3.5 EXAMPLE OF A RESONANT FREQUENCY TRACKING SYSTEM

Theoretical analysis was performed on the system initially proposed for the breadboard model. The example described here is based on that proposal.

The VCO input port used for frequency modulation has a first order low pass characteristic with the cut off at 300 kHz. The driver for this input can be designed to have a much higher cut off frequency than this. Thus the transfer function for the VCO and driver can be written as

$$G_{vco}(s) = \frac{F_c(s)}{V_3(s)} = \frac{K_2}{\frac{S}{6\pi \times 10^5} + 1} \tag{3.33}$$

where K_2 is the gain at DC and $V_3(s)$ is the input voltage to the driver. Normalizing equation (3.33) to F_o gives

$$G_{vco}(s) = \frac{\frac{F_c(s)}{\Delta F_o}}{V_3(s)} = \frac{\frac{K_2}{\Delta F_o}}{\frac{S}{6\pi \times 10^5} + 1} \tag{3.34}$$

The bandpass amplifier has a center frequency of 300 kHz and a bandwidth of 100 kHz. In order to obtain sufficient attenuation of undesired signals in the loop a sixth order filter was used. In the linearized system of figure 3.4 this bandpass filter is converted to a lowpass filter placed after the product detector in the signal path. The transfer function of this can be approximated by

$$G'_1(S) = \frac{K_3}{\left[\frac{S}{\pi \times 10^5} + 1\right]^6} \qquad (3.35)$$

where K_3 is the gain amplifier and product detector.

The lowpass amplifier must remove undesirable mixing products and have a high DC gain in order to obtain the best operating characteristics for the tracking loop. A first order filter with a very low cut off frequency was used. This is an approximation of an integrator. The transfer function is

$$G_2(s) = \frac{K_4}{\frac{S}{10} + 1} \qquad (3.36)$$

where K_4 is the DC gain of this stage. For this system example the open loop transfer function, for small frequency errors which change at rates slower than the slew rate, is

$$G(s) = \frac{K_0}{\left[\frac{S}{10} + 1\right]\left[\frac{S}{\pi \times 10^5} + 1\right]^6\left[\frac{S}{6 \times 10^5} + 1\right]} \qquad (3.37)$$

This was obtained by substitution in equation (3.22) with equations (3.34), (3.35), (3.36), and (3.21). The gain has been lumped together into one variable, K_0.

$$K_0 = K_1 A_d P_0 (1 - \Gamma_0^2) \left(\frac{\Delta f}{\Delta F_0}\right)] \left[\frac{K_2}{\Delta F_0}\right] [K_3] [K_4] \qquad (3.38)$$

The closed loop transfer function and the normalized error function can be found by similar substitution into equation (3.23) and (3.24).

3.6 COMPUTER ANALYSIS OF THE LINEARIZED EXAMPLE

Computer techniques were used to obtain the frequency and step responses for the example system described in the previous section. A simple electronic network whose transfer function was the same as equation 3.27 was analyzed using the Electronic Circuit Analysis Program (ECAP). The simple addition of a feedback path allowed closed loop responses to be analyzed. Details of the circuit model used for the ECAP analysis are given in the appendix.

Using this technique the open loop response, the closed loop response, and the transient response were obtained for gains of 1,000, 5,000, 10,000, and 20,000.

The open loop phase and frequency characteristics are shown in figure 3.6. The first order pole at 1.59 hertz is not visible in the graph.

Figures 3.7 to 3.10 show the phase and amplitude response of the closed loop for four different open loop gains. For low gains the frequency response is poor, while for high gain there is excessive peaking and a very rapid phase change.

Figure 3.11 shows the time responses to a step change in the YIG filter frequency. It can be seen that with an open loop gain of 1,000 the slew rate is far too slow, but with a gain of 20,000 there is excessive ringing. Higher gain would be preferable because it would improve the operating range and linearity of the system; but because the open loop phase shift in figure 3.6 reaches 180° at 30 KHZ, the gain at this frequency must be less than one if the system is to be stable. Because the first pole of the low pass amplifier is at a much lower frequency, the DC open loop gain can be increased if the frequency of