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COMPUTER SOLUTIONS OF ELECTRICAL TRANSIENTS ON POWER TRANSMISSION LINES

by

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ABSTRACT

Two digital computer programs have been developed to calculate transients which result on "electromagnetically" and "electrostatically" balanced multiconductor transmission lines. The first of the two programs determines the impulse response of the system based on Bergeron's Method of Characteristics and includes the effect of the frequency dependence of the transmission line parameters. To simplify the calculations, the phase quantities are converted to modal quantities by virtue of the theory of natural modes. The impulse responses are obtained by applying an FFT technique to a modification of the Fourier transform. The second of the two programs, using the results of the first, performs the network analysis from which the transient voltages are calculated. Although the computer programs are unable to handle transients on untransposed lines, the theory involved for such a system is presented.

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CHAPTER 1

INTRODUCTION

The use of digital computers to calculate transient voltages and currents on power lines has been expanding over the past decade or so. Prior to its advent, transient analysis had been carried out on transient network analysers (TNA's) in which a model of the transmission system is comprised of a finite number of π sections. Typical disturbances are then applied to the model and transient voltages and currents are recorded photographically from oscilloscope patterns. These devices are still used today, however, the digital computer has added a new dimension in the field of transmission line transient analyses.

Electric transients in power transmission lines involves travelling waves. For this reason, two graphical techniques had been developed to assist in transient calculations, viz., the Bewley lattice diagram and Bergeron's method. Both methods have been computer programmed, however, Bergeron's method appears to be more conducive to programming.

A computer algorithm for Bergeron's method had initially been developed by Frey and Althammer[7] This algorithm was applied to the analysis of transients on lossless transmission lines. Branin[8] developed a more viable algorithm based on Bergeron's method in terms of simple equivalent networks at the input and output terminals of the line. Moreover it engendered a particularly facile way of tying in the transmission line analysis to whatever networks terminated the two ends of the line. Using this algorithm, Dommel[1] created an all-purpose computer program which analysed transients on single-and multi-phase transmission lines. The resultant power system network could then be solved by the nodal admittance matrix method. Lossy transmission lines were included in the program, however, these losses were independent of frequency.

Uram et al.[26,27] developed a computer program which calculated transients on a balanced three-phase transmission line. The technique was that of forming the partial differential equations of this transmission system in terms of the positive and zero sequence parameters and then using the Laplace transform to reduce these equations to ordinary differential equations. After performing a transformation of coordinates to the mutually coupled differential equations, a solution in the frequency domain was obtained. The first two terms of the binomial expansion of the propagation constant and characteristic impedance were retained from which the inverse Laplace transform of the functions could be calculated. The limitation of this method was the lack of representation of the positive and zero sequence parameters over a range of frequencies.

Day et al.[10,11,12] investigated numerical integration techniques for solving Fourier integrals; these Fourier integrals arising from the use of the Fourier transform in the solution of power system transients. This investigation resulted in what is known as the modified Fourier transform. It involved the use of a contour of integration shifted below the real axis as opposed to the normal contour taken along the real axis.

Wedepohl and Mohamed,[28] by utilizing the steady state solution of the wave equation as well as the virtues of the modified Fourier integral and theory of natural modes, were able to calculate transients on symmetrical and unsymmetrical multiconductor systems. This approach allowed them to include for the frequency dependence of the transmission line parameters.

Budner[29] developed a computer program which included for the effect of frequency dependence of the transmission line parameters by introducing the concept of impulse response. The steady state two-port equations were the basis of his analysis, i.e., the admittance parameters were transformed into the time domain thus allowing the port currents to be evaluated by convolving the time domain admittance parameters with the port voltages. However, one inherent difficulty with this approach was the existence of multiple reflections of the applied Dirac impulse; these reflections being due to the use of the short-circuit admittance parameters. This resulted in the necessity of retaining the impulse response data over several propagation times. The difficulty was rectified by Snelson[3] who introduced the idea of forward and backward impulse response functions. The basis of this idea was Bergeron's Method of Characteristics. This approach required less storage requirements and computation time because of fewer multiple reflections of the applied impulse.

The development of the computer programs in this thesis follows the approach of Snelson. Two programs have essentially been written, viz., a program to calculate the impulse response of the system and a program to perform the actual network analysis. The calculation of the impulse response is simplified by the use of the theory of natural modes. Incorporated into the program is the effect of conductor and earth losses; the earth losses being calculated from Carson's equations.[2] The modified inverse Fourier transform is used to calculate the impulse response functions. Once the impulse responses are determined, they are used as input to the network analysis program where the convolution process is performed.

These programs are applicable to "electromagnetically" and "electrostatically" balanced multiconductor systems and are not applicable to untransposed lines. However, some of the idiosyncrasies associated with the computer programming of the latter system are presented at the end of Chapter 5.

As a test for the programs, a transient analysis is carried out on a bipolar line. More specifically, a single-pole closure with the remaining terminals open-circuited is analysed followed by the calculation of voltages which result from a fault occurring at the midpoint of the positive pole for various terminal conditions. All calculations were performed on IBM 370 and Xerox Sigma 9 computers.

CHAPTER 2

THEORETICAL DEVELOPMENT

2.1 Introduction

The calculation of transients on transmission lines which are above an assumed perfectly conducting earth is well known.[1,17] The problem becomes very complex, however, if a lossy conductor above a finitely conducting earth is considered. In this particular case, the transmission line parameters are frequency dependent and a surge propagating along the line will distort somewhat; the distortion being due to the attenuation of the high frequency components contained in the surge.

In determining the transients on such a line, one method is to calculate the impulse response of the line and then use convolution in the time domain to determine the response to an arbitrary wave.

This theory can be extended to an n-conductor system above a lossy earth. The analysis of such a system is simplified by performing a matrix transformation on phase quantities to yield modal quantities.[5,30] In the case of perfectly balanced lines, the transformation matrix is frequency independent, hence the problem is manageable.

When calculating the transients, Bergeron's Method of Characteristics is used. This method is amenable to computer programming for systems with distributed elements since reflection factors are non-existent.

In the present chapter Bergeron's Method will be described afterwhich the derivation of impulse response functions for a single-phase line will be given. A brief explanation of the theory of natural modes and its application to perfectly balanced systems follows. The derivation of the equivalent circuits for the bipolar line and lumped elements (inductor, capacitor, and resistor) as well as an explanation of the network analysis concludes the chapter.

2.2 Bergeron's Method of Characteristics

The transmission line equations for the single phase line are

$$-\frac{\partial v}{\partial x} = L\frac{\partial i}{\partial t} + Ri \ , \tag{1}$$

$$-\frac{\partial i}{\partial x} = C\frac{\partial v}{\partial t} + Gv \ . \tag{2}$$

Bergeron's Method applies to lossless lines where R and G are zero, and L and C are independent of frequency. Subject to these limitations, there are conditions at each end of the line at times t and t-τ independent of the terminating networks (Fig. 1). The general solution to (1) and (2) (with R, G=0) are given by D'Alembert, namely,

$$v(x,t) = f_1(x-ut) + f_2(x+ut), \tag{3}$$

$$Zi(x,t) = f_1(x-ut) - f_2(x+ut). \tag{4}$$

where $Z=\sqrt{L/C}$. From (3) and (4) we get

$$v(x,t) + Zi(x,t) = 2f_1(x-ut), \tag{5}$$

$$v(x,t) - Zi(x,t) = 2f_2(x+ut). \tag{6}$$

The arbitrary functions $f_1(x-ut)$ and $f_2(x+ut)$ are dependent on the arguments x-ut and x+ut, respectively, and represent travelling waves in the forward and backward direction on the line. When in (5) x-ut is a constant, $v(x,t) + Zi(x,t)$ is a constant and similarly in (6) when x+ut is a constant, $v(x,t) - Zi(x,t)$ is a constant. Thus x-ut = constant and x+ut = constant are called characteristics of the differential equations.

The significance of (5) and (6) may be visualized in the following manner: let a fictitious observer travel along the line in a forward direction at a velocity u. Then x-ut and consequently v+Zi along the transmission line will be constant for him. If the travel time to get from one end of the line to the other is τ, then the expression v+Zi encountered by the observer when he leaves node k at time $t-\tau$ must be the same when he arrives at node m, that is,

$$v_m(t) - Zi_m(t) = v_k(t-\tau) + Zi_k(t-\tau). \tag{7}$$

Similarly

$$v_k(t) - Zi_k(t) = v_m(t-\tau) + Zi_m(t-\tau). \tag{8}$$

The following reference directions and nomenclature will be used throughout (Fig. 1):

a) Waves travelling into the port will be characterized as forward waves and will be denoted by

$$F_k(t-\tau) = v_k(t-\tau) + Zi_k(t-\tau), \tag{9}$$

$$F_m(t-\tau) = v_m(t-\tau) + Zi_m(t-\tau). \tag{10}$$

b) Waves travelling out of the port will be characterized as backward waves and will be denoted by

$$B_m(t) = v_m(t) - Zi_m(t), \tag{11}$$

$$B_k(t) = v_k(t) - Zi_k(t). \tag{12}$$

Using this notation (7) and (8) become

$$B_m(t) = F_k(t-\tau), \tag{13}$$

$$B_k(t) = F_m(t-\tau). \tag{14}$$

The interpretation of (13) and (14) are as follows: a forward wave leaving node k at time $t-\tau$ $\left(F_k(t-\tau)\right)$ arrives at node m at time t as a backward wave ($B_m(t)$) and similarly, a forward wave leaving node m at time $t-\tau$ ($F_m(t-\tau)$) becomes a backward wave at node k ($B_k(t)$) τ seconds later.

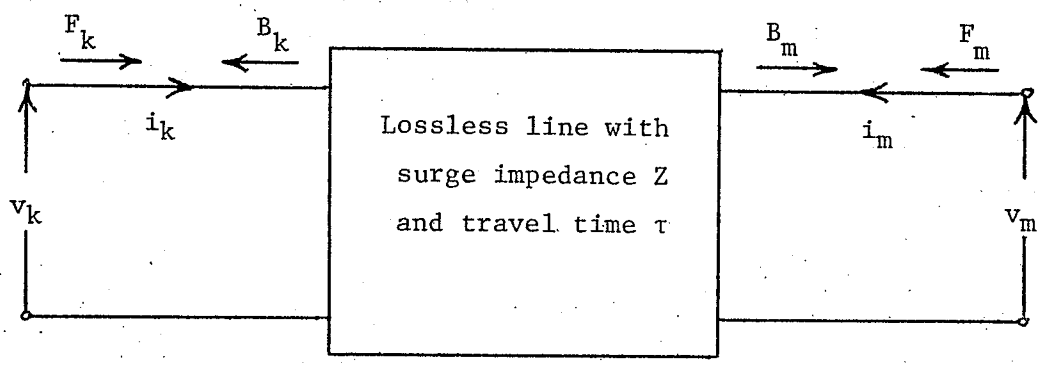

Fig. 1

From (7) and (8), if the conditions a travel time earlier are known, the currents at time t can be solved for, i.e.,

$$i_m(t) = \frac{1}{Z}v_m(t) - \frac{1}{Z}(v_k(t-\tau) + Zi_k(t-\tau)), \tag{15}$$

$$i_k(t) = \frac{1}{Z}v_k(t) - \frac{1}{Z}(v_m(t-\tau) + Zi_m(t-\tau)). \tag{16}$$

After substituting (9), (10), (13) and (14) into (15) and (16),

$$i_m(t) = \frac{1}{Z}v_m(t) - \frac{1}{Z}B_m(t), \tag{17}$$

$$i_k(t) = \frac{1}{Z}v_k(t) - \frac{1}{Z}B_k(t). \tag{18}$$

It is evident from (17) and (18) that the transmission line can be replaced by a current source in parallel with a resistance Z. This allows a solution to be obtained for the voltages and currents at time t in the network consisting of the ends of the transmission line and the components connected to them (Fig. 2).

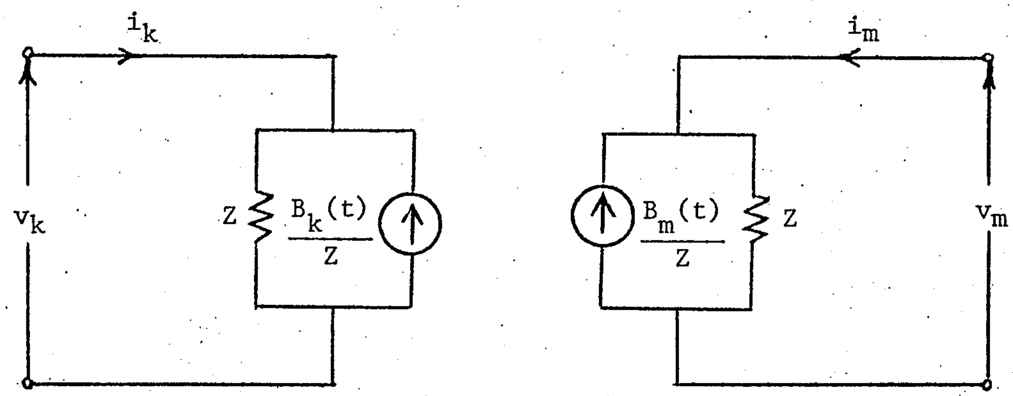

Fig. 2: Equivalent circuit of lossless single phase line.

Up to this point, emphasis has been placed on lossless systems. Likewise, the equivalent circuit of Fig. 2 had been developed on that basis. The equivalent circuit, however, is still applicable to lossy systems with frequency dependent resistance and inductance. This is evident if one considers the arbitrary input waveform to be decomposed into pulses of width Δt. As each pulse is injected into the system, it will initially see a lossless line (surge impedance) and as these pulses propagate to the end of the line, they will distort somewhat. The sum of these pulses at the far end of the line will determine the response of the system to the arbitrary waveform. This phenomenon leads to the concept of impulse response and convolution as derived in the next section.

2.3 Impulse Response Functions for a Single-Phase Line with Frequency Dependent Parameters

The transmission line equations in the frequency domain (Fourier transform plane) are

$$-\frac{dV}{dx} = \left(R(\omega) + j\omega L(\omega)\right) I, \tag{19}$$

$$-\frac{dI}{dx} = \left(G + j\omega C\right) I. \tag{20}$$

In (20) it is assumed that the frequencies encountered are not high enough to render G and C frequency dependent. The complex surge impedance and propagation constant are defined respectively by

$$Z(\omega) = \sqrt{\frac{R(\omega) + j\omega L(\omega)}{G + j\omega C}}, \tag{21}$$

$$\gamma(\omega) = \sqrt{\left(R(\omega) + j\omega L(\omega)\right)\left(G + j\omega C\right)}. \tag{22}$$

If a Dirac impulse is applied to the line, the high frequencies contained in the impulse dominate. Taking the limit of (21) as $\omega \to \infty$, viz.,

$$Z_1 = \lim_{\omega\to\infty} \sqrt{\frac{R(\omega) + j\omega\,(L_c(\omega) + L_g)}{G + j\omega C}} \tag{23a}$$

where $L_c(\omega)$ is the inductance of the conductor and L_g is the inductance due to the geometry. Since $R(\omega)$ varies as $\sqrt{\omega}$ and L_c as $1/\sqrt{\omega}$, Z_1 becomes

$$Z_1 = \sqrt{\frac{L_g}{C}} \tag{23b}$$

Therefore, in the time domain the forward and backward characteristics become

$$F_k = v_k + Z_1\, i_k \tag{24}$$

$$B_k = v_k + Z_1\, i_k \tag{25}$$

$$F_m = V_m + Z_1\, i_m \tag{26}$$

$$B_m = v_m - Z_1 i_m. \qquad (27)$$

This is a repetition of (9), (10), (11) and (12) as developed for the lossless case. Thus Bergeron's Method still applies since the line appears lossless. Taking the Fourier transform of (24) to (27) yields

$$\overline{F}_k = V_k + Z_1 I_k, \qquad (28)$$

$$\overline{F}_m = V_m + Z_1 I_m, \qquad (29)$$

$$\overline{B}_k = V_k - Z_1 I_k, \qquad (30)$$

$$\overline{B}_m = V_m - Z_1 I_m, \qquad (31)$$

where $\overline{F}_k$, $\overline{F}_m$, $\overline{B}_k$ and $\overline{B}_m$ denotes the transforms of the characteristic equations.

The solution to (19) and (20) are known to be

$$V_k = V_m \cosh\big(\gamma(\omega)d\big) - I_m Z(\omega)\sinh\big(\gamma(\omega)d\big), \qquad (32)$$

$$I_k = -I_m \cosh\big(\gamma(\omega)d\big) + \frac{V_m}{Z(\omega)} \sinh\big(\gamma(\omega)d\big). \qquad (33)$$

If (28) to (33) are solved simultaneously by eliminating V_k, I_k, V_m and I_m, then

$$\overline{B}_k = \overline{A}_1 \overline{F}_m + \overline{A}_2 \overline{F}_k, \qquad (34)$$

$$\overline{B}_m = \overline{A}_1\overline{F}_k + \overline{A}_2\overline{F}_m \tag{35}$$

where

$$\overline{A}_1 = \frac{1}{\cosh(\gamma d) + \frac{1}{2}\left(\frac{Z_1}{Z} + \frac{Z}{Z_1}\right)\sinh(\gamma d)}\ , \tag{36}$$

$$\overline{A}_2 = -\frac{1}{2}\left(\frac{Z_1}{Z} - \frac{Z}{Z_1}\right)\sinh(\gamma d)\overline{A}_1\ . \tag{37}$$

Transforming $\overline{A}_1(j\omega)$ and $\overline{A}_2(j\omega)$ into the time domain and since (34) and (35) are in the frequency domain, convolution must be performed in the time domain to obtain $B_k(t)$ and $B_m(t)$, i.e.,

$$B_k(t) = \int_{-\infty}^{\infty} a_1(u)F_m(t-u)du + \int_{-\infty}^{\infty} a_2(u)F_k(t-u)du\ , \tag{38}$$

$$B_m(t) = \int_{-\infty}^{\infty} a_1(u)F_k(t-u)du + \int_{-\infty}^{\infty} a_2(u)F_m(t-u)du\ . \tag{39}$$

The infinite integrals of (38) and (39) can be reduced to integrals with finite limits because of the nature of $a_1(t)$ and $a_2(t)$, i.e., if in (34) and (35) F_m is zero ($v_m + i_mZ_1=0$) and F_k is the Dirac impulse function then $\overline{F}_k$ and $\overline{F}_m$ are 1 and 0 respectively. This yields $\overline{B}_k = \overline{A}_2$ and $\overline{B}_m = \overline{A}_1$. In the time domain, therefore, $B_k(t) = a_2(t)$ and $B_m(t) = a_1(t)$ where $a_1(t)$ is the backward characteristic at the remote end and $a_2(t)$ is the backward characteristic at the sending end due to an impulse of forward characteristic. $a_1(t)$ and $a_2(t)$ are zero for $t<0$ and since $F_m = 0$ (the line is match terminated and multiple reflections do not exist) we get

$$B_k(t) = \int_{\tau}^{t_1} a_1(u) F_m(t-u)\,du + \int_0^{t_2} a_2(u) F_k(t-u)\,du \;, \tag{40}$$

$$B_m(t) = \int_{\tau}^{t_1} a_1(u) F_k(t-u)\,du + \int_0^{t_2} a_2(u) F_m(t-u)\,du \tag{41}$$

where t_1 and t_2 are times when $a_1(t)$ and $a_2(t)$ are reasonably insignificant.

To evaluate (40) and (41) numerically the integrals are converted to summations, i.e.,

$$B_k(t) = \sum_{n=n_1}^{n=n_2} a_1(n\Delta t) F_m(t-n\Delta t)\Delta t + \sum_{n=1}^{n=n_3} a_2(n\Delta t) F_k(t-n\Delta t)\Delta t, \tag{42}$$

$$B_m(t) = \sum_{n=n_1}^{n=n_2} a_1(n\Delta t) F_k(t-n\Delta t)\Delta t + \sum_{n=1}^{n=n_3} a_2(n\Delta t) F_m(t-n\Delta t)\Delta t. \tag{43}$$

The physical interpretation of (42) and (43) follows. If an arbitrary wave is applied at node k it is decomposed into pulses of width Δt which represent the forward waves into the system. Each pulse propagates to node m giving rise to $B_m(t)$ and at the same time contributing to $B_k(t)$ because of reflections from the frequency dependent distributed resistances along the line. When pulses from node k arrive at node m, forward waves are generated (the magnitude of which depends on the type of network attached to the transmission line) which are injected into the port at node m and in turn propagate to node k. This gives rise to $B_k(t)$ and at the same time contributes to $B_m(t)$. This cycle continues until the transient is damped out.

$a_1(t)$ and $a_2(t)$ are calculated by numerically inverting $\overline{A}_1$ and $\overline{A}_2$. This is done by applying a modified Fast Fourier Transform technique[9,18] to (36) and (37). This technique is presented in Appendix C. The inversion of $\overline{A}_1$ and $\overline{A}_2$ requires a knowledge of R and L at various frequencies. $R(\omega)$ and $L(\omega)$ are obtained by using Carson's formulae (see Appendices A and B).

The forward and backward impulse response functions as given, can be extended to multi-conductor systems, however, this requires the concept of natural modes.

2.4 The Theory of Natural Modes[5,30]

The transients on a multiphase transmission line can be analyzed by the method of natural modes. This method takes an arbitrary wave propagating along a multiphase system and decomposes it into n independent waves (n referring to the number of conductors) known as modes. Each mode has its own propagation constant. The theory of the method follows.

If (19) and (20) are written in matrix form we get

$$\frac{-d[V^{(p)}]}{dx} = [Z^{(p)}][I^{(p)}] , \tag{44}$$

$$\frac{-d[I^{(p)}]}{dx} = [Y^{(p)}][V^{(p)}] \tag{45}$$

where, respectively,

$$[Z^{(p)}] = [R^{(p)}] + j\omega[L^{(p)}], \tag{46}$$

$$[Y^{(p)}] = j\omega[C^{(p)}] \tag{47}$$

with the superscript p indicating phase quantities. $[V^{(p)}]$ and $[I^{(p)}]$ are column vectors of length n and $[Z^{(p)}]$ and $[Y^{(p)}]$ are symmetric n x n matrices. (44) and (45) can be solved simultaneously to yield two independent equations in $[V^{(p)}]$ and $[I^{(p)}]$, i.e.,

$$\frac{d^2[V^{(p)}]}{dx^2} = [P][V^{(p)}], \tag{48}$$

$$\frac{d^2[I^{(p)}]}{dx^2} = [P]^T[I^{(p)}] \tag{49}$$

where $[P] = [Z^{(p)}][Y^{(p)}]$ and $[P]^T$ is the transpose of [P] and is equal to $[Y^{(p)}][Z^{(p)}]$. It is next assumed that $[V^{(p)}]$ is linearly related to a new set of voltages $[V^{(m)}]$ and $[I^{(p)}]$ is linearly related to a new set of currents $[I^{(m)}]$, i.e.,

$$[V^{(p)}] = [S][V^{(m)}], \tag{50}$$

$$[I^{(p)}] = [Q][I^{(m)}]. \tag{51}$$

The matrices [S] and [Q] in, respectively, (50) and (51) are to be determined.

Substituting (50) and (51) into respectively (48) and (49) yields

$$\frac{d^2[V^{(m)}]}{dx^2} = [S]^{-1}[P][S][V^{(m)}], \tag{52}$$

$$\frac{d^2[I^{(m)}]}{dx^2} = [Q]^{-1}[P]^T[Q][I^{(m)}]. \tag{53}$$

Inspection of (52) (and similarly for (53)) shows that if $[S]^{-1}[P][S] = [\gamma^2]$ is diagonal, then n independent equations are obtained with the i^{th} equation being

$$\frac{d^2V_i^{(m)}}{dx^2} = \gamma_i^2 V_i^{(m)} . \tag{54}$$

(54) has the solution

$$V_i^{(m)} = \exp(-\gamma_i x)V_s^{(m)} + \exp(\gamma_i x)V_r^{(m)} . \tag{55}$$

In matrix notation this is

$$[V^{(m)}] = [\exp(-\gamma x)][V_s^{(m)}] + [\exp(\gamma x)][V_r^{(m)}] \tag{56}$$

where $[\exp(-\gamma x)]$ and $[\exp(\gamma x)]$ are diagonal matrices. Transforming (56) into phase quantities yields

$$[V^{(p)}] = [S][\exp(-\gamma x)][S]^{-1}[V_s{}^{(p)}] +$$

$$+ [S][\exp(\gamma x)][S]^{-1}[V_r{}^{(p)}] \quad . \tag{57}$$

The process of diagonalization to obtain a solution for $[V^{(p)}]$ in [48] implies that

$$\det[P - \gamma^2] = 0. \tag{58}$$

The solution of (58) yields n eigenvalues (γ^2). Corresponding to each eigenvalue is its eigenvector S_i which is obtained by solving

$$PS_i = \gamma_i{}^2 S_i \quad . \tag{59}$$

The n eigenvectors form the desired [S] matrix. Each column of [S] is a modal voltage the numerical values of which describe the voltage distribution of that mode on the various conductors. A similar analysis for $[I^{(p)}]$ shows, as expected, that the eigenvalues are the same.

For completely balanced systems a few standard transformation matrices exist. Multiphase systems with distributed parameters are said to be "electromagnetically balanced" if the self impedances of all phases are equal and all mutual impedances are equal. Such lines are also said to be "electrostatically balanced" if the capacitances to ground of all phases are equal and all capacitances between phases are equal. Therefore from (44) and (45) the series and admittance matrices respectively become