"Transient Performance of

Variable-Reluctance Stepping Motors"

A Thesis

Presented To

The Faculty of Graduate Studies

The University of Manitoba

In partial fulfillment

of the requirements for the degree of

MASTER OF SCIENCE

IN

ELECTRICAL ENGINEERING

By

RAVINDER KUMAR GUPTA

MAY, 1975

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ABSTRACT

Multistep operation of a stepping motor can be quite different from single step operation. This thesis presents a study of the ultimate performance limitations, of a stepping motor when supplied by a series of pulses for multiple stepping and for single step operation. The results have been obtained by simulation studies on Analog and Digital Computers. The effects of machine and load parameters on the limiting performance have been obtained which should prove valuable both for the designers of stepping motors and application engineers.

ACKNOWLEDGEMENT

The author would like to express his appreciation and deepest gratitude to Professor R.M. MATHUR for suggesting the thesis topic, his esteemed guidance in the preparation of this thesis, and financial support.

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LIST OF SYMBOLS

Main Symbols

i	instantaneous current, A.
J	moment of inertia of the system, $Kg\text{-}m^2$
K	number of pulses per second, p.p.s.
K_1	damping coefficient, N-m-s/rad
K_2	stiffness coefficient, N-m/rad.
L	self inductance of stator windings, H.
L_{max}	maximum self inductance, H.
L_{min}	minimum self inductance, H.
L_v	variable part of the inductance, H.
P	number of stator phases
R	resistance of each stator winding, Ω.
t	time, S.
T	number of teeth in each stator phase or rotor.
T_a	approximated equivalent time constant, s.
T_L	load torque, N-m.
TR	resultant motor torque, N-m.
V	instantaneous voltage, V.
V_{max}	maximum voltage, V.
w	width of each pulse, s.
θ	instantaneous position, rad.
θ_s	step angle, rad.

Subscripts

A,B,C,1,2,P...	stator phases
SS,MS,OUT	single-step, multiple-step, pull-out

CHAPTER 1

INTRODUCTION

In recent years the use of pulse technology in the field of automatic control has rapidly increased. In fact, the digital control of the tooling machines is being used intensively and the number of tooling machines not employing digital control is continuously declining.

A stepping motor is basically a D-A convertor since it is capable of rotating through a predetermined angle with the application of prescribed number of electrical pulses. The input is usually in the form of rectangular voltage pulses. The rotational angle and the angular velocity of rotation depends, respectively, upon the pulses fed and the pulse rate (p.p.s.)

Basically, there are two types of stepping motors.

(i) Variable-reluctance stepping motors,

and (ii) Permanent-magnet stepping motors.

The variable-reluctance stepping motors usually consist of three or more phases. The stator windings are housed on salient poles which form electromagnetic structures on excitation. When one or more stator windings are excited or energized, the rotor takes a position to provide a path of minimum reluctance for the magnetic flux. When the windings in the stator are energized one after another, in a particular sequence, a continuous motion takes place. Excitation of more than one winding, sequentially, may be adopted to obtain a smaller step angle.

A permanent-magnet stepping motor consists of a permanent magnet rotor. The stator windings are wound on salient poles, as in a variable reluctance machine. When any one of the stator windings is energized, the interaction between the magnetic flux set up by a stator coil and the permanent magnet rotor produces torque causing the rotor to move in such a way that the magnetic moment of the permanent magnet aligns with the stator winding magnetic field.

A number of papers have been published on the operation and analysis of stepping motors. The published work suffers from one drawback or another owing to the simplistic assumptions made by the authors and hence, a need exists to analyse the dynamic performance of a stepping motor under transient conditions and to obtain the limits of satisfactory operation by simple analytical expressions. The study undertaken in this thesis is aimed to bridge this gap. O'Donahue[1] has developed a linear second-order transfer function for the analysis and approximately describes the dynamics of the stepping motor for a single step operation, which is unsuitable for the multi-step operation Kuo, Singh and Yackel[2] have approximated the inductance waveforms, for a particular motor, as the function of rotor position. Their treatment lacks generality. Also, the adjustment of a particular constant in the torque equation just to make the practical results match with the theoretical results seems arbitrary, since its validity has neither been explained nor mentioned. Venkataratnam, Sarkar and Palani[3] have neglected inductance in comparison with the resistance of the stator windings. This is equivalent to assuming constant currents in the stator windings. But how far they have been able to achieve this ideal current source practically has not been indicated. The load-torque has also been neglected in comparison with inertial and frictional torques. Robinson and Taft[4] have also done the analysis assuming the constant current source. Venkataratnam and Mouli[5] have replaced the rectangular voltage with the sinusoidal input for the convenience of analytical treatment

The purpose of this thesis is to develop the models, both analog and digital, to suit the performance of a variable-reluctance, (VR) stepping motor. In Chapter II, both analog and digital models for a simple three-phase motor and a general digital-model for a P-phase motor are developed. Because of the limitations of components available on the analog computer, the response simulated is primarily for a single pulse. However, the simulated results serve to predict the motor performance, when the last pulse is applied continuously to hold the rotor at a desired position, to eliminate the requirement of detenting windings, after having achieved the incremental motion as desired, by supplying a series of pulses or one pulse at a time, depending upon the number of phases and number of teeth present and the angle of rotation required. The schematic diagram, for simulating the multi-step operation of the VR stepping motor is given in Fig. 2.2.2. The motor is simulated first by using Fourier series for both voltage and inductance waveforms, second, by generating exact voltage waveforms in Fourier series and finally, by generating the exact waveforms both for voltages and inductances of an idealized motor. Depending upon the shape of the supply voltage and inductance waveforms of a particular motor, dependent on the shape of the rotor slots and stator pole shape, one of the above computer programs may be used to economize the computational time.

In addition to the development of the above simulation techniques, in Chapter III expressions for an absolute upper limit of frequency, the maximum pull-in frequency as applicable to single-step operation and multiple-stepping and for the pull-out frequency in terms of load torque, inertia and damping are derived. The operational performance is discussed in Chapter IV.

Safe pull-in frequency in combination with the pull-out frequency may be used as the performance criterion for the selection of a motor for a particular application.

Self-inductance versus position curve, experimentally determined for a particular motor, is given in Appendix 3 in support of the theoretical model.

CHAPTER 2

ANALYSIS OF THE VARIABLE RELUCTANCE STEPPING MOTOR

2.1. The analysis of a simple three phase VR stepping motor

The type of motors being analyzed here have the following specific features, detailed briefly with their operation.

(i) Each unit is identical in construction to others and a P-phase motor can easily be assembled by putting P-modular units together. All the rotors are aligned along the rotor axis. However, poles of the stators of phase A, phase B and phase C are mis-aligned by 360/PxT degrees. Each rotor has as many teeth (T) as a stator.

(ii) Torque may be increased by having more than one unit per phase.

(iii) Highest stepping rate can be achieved by using thin discs solidly attached to the shaft instead of using conventional rotors, because of the low inertia.

To briefly explain the principle of operation of a VR motor, assume the rotor of phase A to be in a position of minimum reluctance, as shown by solid lines in Fig. 2.1.1. When the phase B is excited, the motor moves one step angle in a clockwise direction to provide a path a minimum reluctance for the magnetic flux of B. If the stator coils are excited in sequence A→B→C, the motor would move clockwise executing one step angle by excitation of one phase. It may be rotated in anti-clockwise direction by exciting the phases A→C→B, with the same step. Smaller step angles may be obtained by energization sequence of A→AB→B→BC→C→CA or A→AC→C→CB→B→BA depending upon the direction one wants the motor to move.

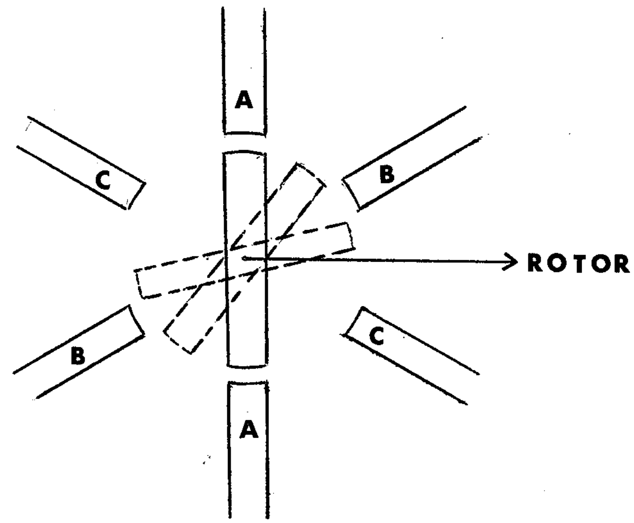

Fig.2.1.1. THREE-PHASE TWO-POLE MACHINE

The assumptions made for the analysis are:

(i) Inductances of the stator windings are functions of rotor position (θ) only. This is equivalent to assuming that no saturation occurs and flux linkages are directly proportional to the exciting currents. The inductance waveforms are at first approximated by triangular waveforms.

(ii) The voltage input pulses are of rectangular shape (which is usually the case).

(iii) On account of the nature of machines being analyzed, no mutual coupling exists between coils of different phases. This assumption is reasonable even for a single stator machine so long as only one coil is excited at one time.

(iv) The overall shape and magnitudes of inductances of all coils are identical. This assumption results from the identical geometry of all units of a multi-stack machine.

The above waveforms are approximated by the Fourier series to allow a generality in the analysis which would permit adequate representation of non-rectangular voltage pulses and non-triangular inductance waveforms, by including an appropriate number of harmonics for predicting the dynamic response.

The inductance and voltage waveforms for the above motor are drawn in Figs. 2.1.2 and 2.1.3.

Since these waveforms are periodic and satisfy the Dirichlet's conditions that they are continuous except for a possibility of finite number of maxima and minima, they can be represented by infinite series.

As derived in appendix (1), the inductance and voltage waveforms for phases A, B and C are:

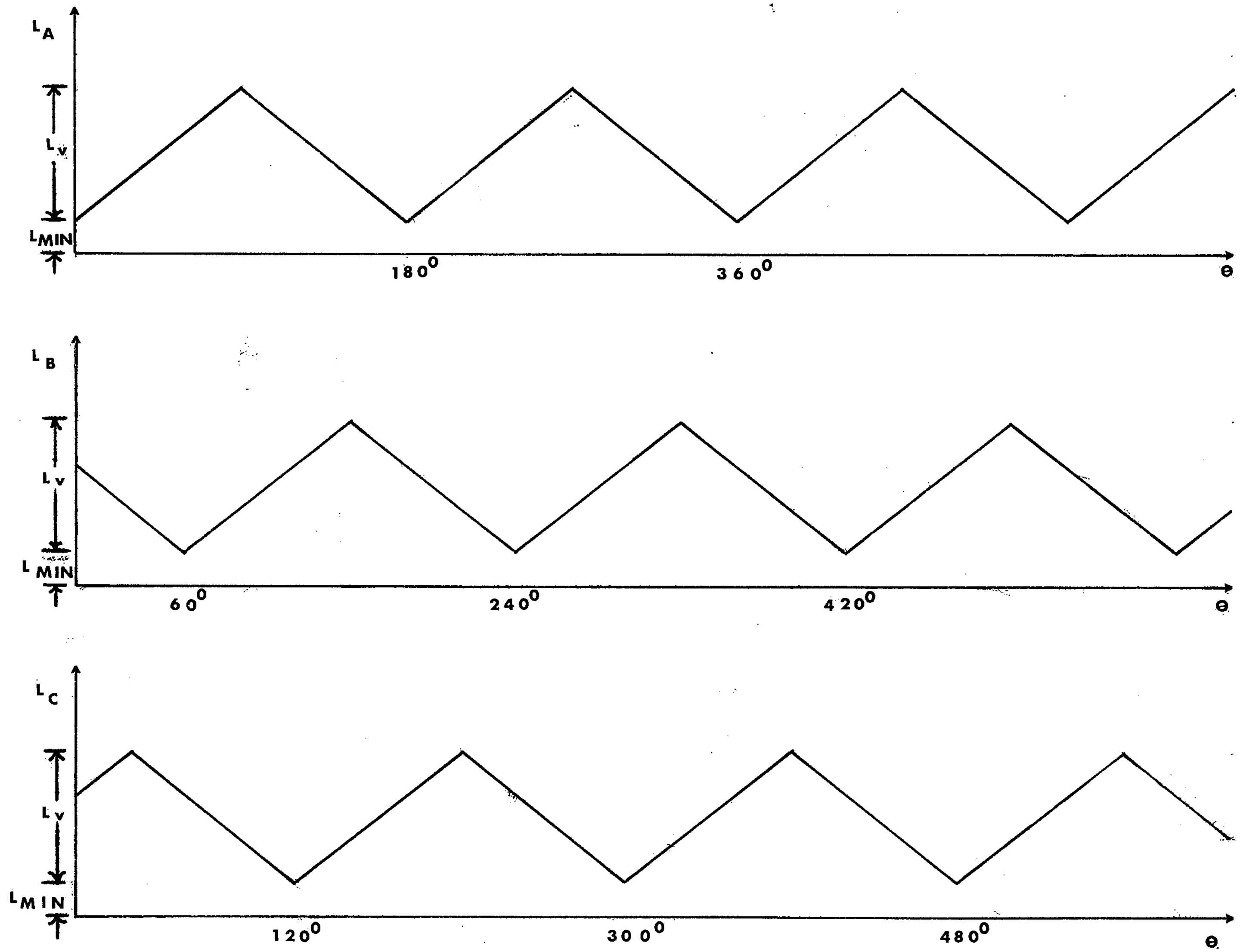

Fig.2.1.2. INDUCTANCE WAVEFORMS FOR A TWO-POLE THREE-PHASE MACHINE

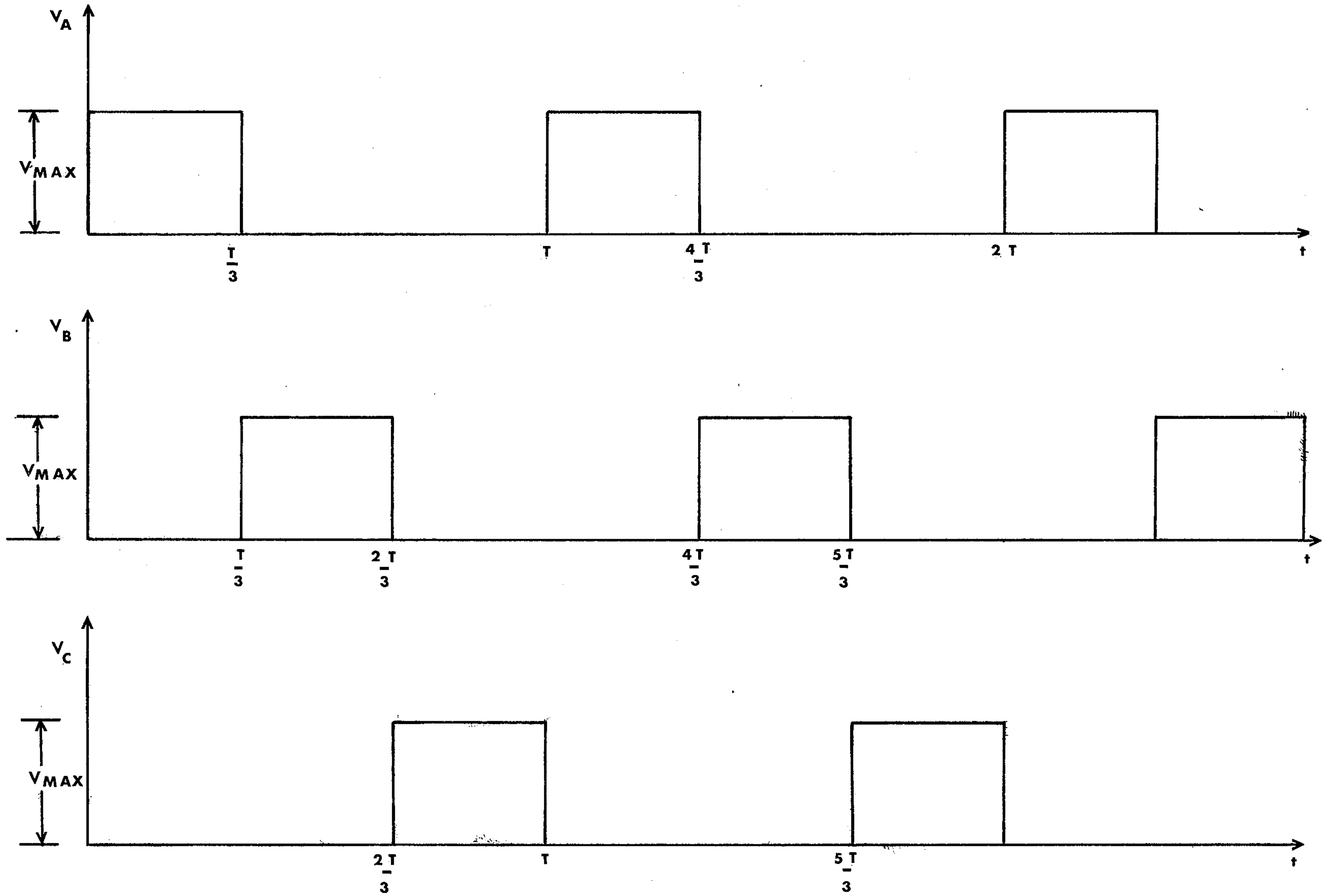

Fig. 2.1.3. VOLTAGE WAVEFORMS FOR A THREE-PHASE MACHINE

$$L_A(\theta) = L_{min} + \frac{L_v}{2} + \frac{2L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[(-1)^n - 1 \right] \cos(2n\theta) \quad ......2.1.1.$$

$$L_B(\theta) = L_{min} + \frac{L_v}{2} + \frac{2L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[(-1)^n - 1\right] \{\cos 2n(\theta - \pi/3)\} .2.1.2.$$

and
$$L_C(\theta) = L_{min} + \frac{L_v}{2} + \frac{2L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[(-1)^n - 1\right] \{\cos 2n(\theta - \tfrac{2\pi}{3})\} .2.1.3$$

Also,

$$V_A(t) = \frac{V_{max}}{3} + \frac{V_{max}}{\pi} \sum_{n=1}^{\infty} \left[\frac{1}{n} \sin(\frac{2n\pi}{3}) \cos(\frac{2n\pi t}{T}) + \frac{1}{n}(1-\cos\frac{2n\pi}{3}) \sin \frac{2n\pi t}{T} \right] \quad ...2.1.4$$

$$V_B(t) = \frac{V_{max}}{3} + \frac{V_{max}}{\pi} \sum_{n=1}^{\infty} \left[\frac{1}{n} \sin(\frac{2n\pi}{3}) \cos \frac{2n\pi(t-T/3)}{T} + \frac{1}{n}(1-\cos\frac{2n\pi}{3}) \sin \frac{2n\pi(t-T/3)}{T} \right] \quad ....2.1.5$$

and

$$V_C(t) = \frac{V_{max}}{3} + \frac{V_{max}}{\pi} \sum_{n=1}^{\infty} \left[\frac{1}{n} \sin(\frac{2n\pi}{3}) \cos \frac{2n\pi(t-2T/3)}{T} + \frac{1}{n}(1-\cos\frac{2n\pi}{3}) \sin \frac{2n\pi(t-2T/3)}{T} \right] \quad ....2.1.6$$

The voltage equation for any one stator coil is

$$V = i(t)\, R + L(\theta) \frac{di(t)}{dt} + i(t) \frac{dL(\theta)}{d\theta} \cdot \frac{d\theta(t)}{dt} \quad .... 2.1.7$$

$L(\theta) \dfrac{di(t)}{dt}$ is the transformer voltage and $i(t) \dfrac{dL(\theta)}{d\theta} \cdot \dfrac{d\theta(t)}{t}$ is the back e.m.f. acting in opposition to the applied voltage.

or $\quad V = i(t) \left[R + \dfrac{dL(\theta)}{d\theta} \cdot \dfrac{d\theta(t)}{dt}\right] + L(\theta) \dfrac{di(t)}{dt}$

let $\quad V = y, \quad R + \dfrac{dL(\theta)}{d\theta} \cdot \dfrac{d\theta(t)}{dt} = R + \dfrac{dL}{dt} = A$ and $i = x$

$\therefore \quad y = Ax(t) + L(\theta) \dfrac{dx(t)}{dt}$

or $\quad y - Ax(t) = L(\theta) \dfrac{dx(t)}{dt}$

or $\quad \displaystyle\int \frac{1}{L(\theta)}\, dt = \int \frac{dx(t)}{y - Ax(t)}$

Assuming that during a small interval of time or the step of integration, θ does not change appreciably i.e., $L(\theta)$ remains constant and so does A.

$$\therefore \quad \frac{t}{L} = -\frac{1}{A} \log (y - Ax) + Z \qquad \text{.......... 2.1.8.}$$

where Z is the constant of integration.

At t = o, x (t) = o

$$\therefore \quad Z = \frac{1}{A} \log (y)$$

Substituting back in equation 2.1.8., we get

$$\frac{At}{L} = \log (y/(y - Ax))$$

or $$\frac{y}{y-Ax} = e^{At/L}$$

or $$x = \frac{y}{A} (1 - e^{-At/L})$$

Therefore, in terms of the original parameters, we have

$$i = \frac{V}{R + \frac{dL(\theta)}{d\theta} \cdot \frac{d\theta(t)}{dt}} \left[1 - e^{-(R + \frac{dL(\theta)}{d\theta} \cdot \frac{d\theta(t)}{dt}) t/L} \right]$$

.... 2.1.9.

Torque of the motor can be represented by the sum of the torques of all the three independent phases

$$\therefore \quad TR = T_A + T_B + T_C$$

$$= \frac{1}{2} (i_A)^2 \frac{dL_A(\theta)}{d\theta} + \frac{1}{2} (i_B)^2 \frac{dL_B(\theta)}{d\theta} + \frac{1}{2} (i_C)^2 \frac{dL_C(\theta)}{d\theta}$$

.... 2.1.10.

$\frac{dL_A}{d\theta}(\theta)$, $\frac{dL_B}{d\theta}(\theta)$, and $\frac{dL_C}{d\theta}(\theta)$ can be obtained directly from the respective inductance equations, 2.1.1., 2.1.2. and 2.1.3., as follows:

$$\frac{dL_A(\theta)}{d\theta} = \frac{4L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} [1 - (-1)^n] \sin(2n\theta) \qquad \text{......... 2.1.11.}$$

$$\frac{dL_B(\theta)}{d\theta} = \frac{4L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} [1 - (-1)^n] \sin 2n(\theta - \pi/3) \qquad \text{... 2.1.12.}$$

and $$\frac{dL_C(\theta)}{d\theta} = \frac{4L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} [1 - (-1)^n] \sin 2n(\theta - \frac{2\pi}{3}) \qquad \text{... 2.1.13.}$$

Instantaneous speed and position i.e., $\frac{d\theta(t)}{dt}$ and $\theta(t)$ is obtained by using the equation of motion:

$$TR = J \frac{d^2\theta}{dt^2} + K_1 \frac{d\theta}{dt} + K_2\theta + T_L \qquad \ldots\ldots\ldots \; 2.1.14.$$

Values of $i_A(t)$, $i_B(t)$ and $i_C(t)$ can be obtained using equation 2.1.9. by substitution of the respective values of $\frac{dL(\theta)'s}{d\theta}$, $\frac{d\theta(t)}{dt}$, $L(\theta)$'s, and $v(t)$'s. Substitution of $i_A(t)$, $i_B(t)$, $i_C(t)$, $\frac{dL_{A(\theta)}}{d\theta}$, $\frac{dL_{B(\theta)}}{d\theta}$ and $\frac{dL_{C(\theta)}}{d\theta}$ in equation 2.1.10. results in the instantaneous torque.

The equations 2.1.1. - 2.1.14. describe the transient operation of a stepping motor for multiple-step or single-step operation for positional control.

The choice of the number of harmonics to be included for voltage and inductance waveforms depends upon the supply voltage and the shape and depth of the rotor slots respectively. Inclusion of first three or four harmonics for the inductance waveforms is usually good enough. However, if higher accuracy is desired, tests for measurements of inductance should be conducted at different rotor positions if the pertinent data is not available from the manufacturer, and the number of harmonics included should be such that the simulated inductance waveforms match as closely as possible the measured ones.

The results from simulation studies are discussed in Chapter 4.

2.2. Analog model for the motor

Analog simulation is undertaken to gain an insight into the effects of various load parameters because it is very easy to vary these parameters on an analog computer as compared to a digital computer. As indicated earlier electrically and magnetically the electrical circuits of the three phases are isolated and hence each phase has its independent set of governing equations. However, the torques of the three independent phases add algebraically. A block diagram, for the prediction of the response of a variable reluctance stepping motor, when supplied by a series of pulses is shown in Fig. 2.2.2.

Because of the limited number of computing elements available on the analog computer, the actual simulation was done only for one step response. As the results obtained from the analog computer agree with the results obtained from the digital computer, for single step operation, it is logical to assume that analog model results will be comparable to the results obtained from the digital computer, when the motor is supplied with a series of pulses.

The actual simulation schematic diagram for single step operation is given in Fig. 2.2.1.

The two equations governing the dynamics of the motor are:

$$J \frac{d^2\theta}{dt^2} + K_1 \frac{d\theta}{dt} + K_2\theta + T_L = TR$$

For the results in this thesis K_2 is neglected, hence above equation is rewritten as:

$$J \frac{d^2\theta}{dt^2} + K_1 \frac{d\theta}{dt} + T_L = TR \qquad \text{.......... 2.2.1.}$$

Also, $$V = iR + L\frac{di}{dt} + i\frac{dL}{d\theta} \cdot \frac{d\theta}{dt} \qquad \text{.......... 2.2.2.}$$

<u>SCALING:</u> The scaling procedure for the analog simulation is illustrated by one example.

<u>TIME SCALING:</u> Consider a case, when 100 p.p.s. are supplied. Therefore, duration of one pulse $= \frac{1}{100} = 0.01$ sec. = w. This is the real time. If T represents the computer time and h the time scale factor

$$\therefore \quad t = \frac{T}{h}$$

Choosing h = 100, T = th = 0.01 x 100 = 1 sec.

Substituting $t = \frac{T}{h}$ in equations 2.2.1., and 2.2.2., we get

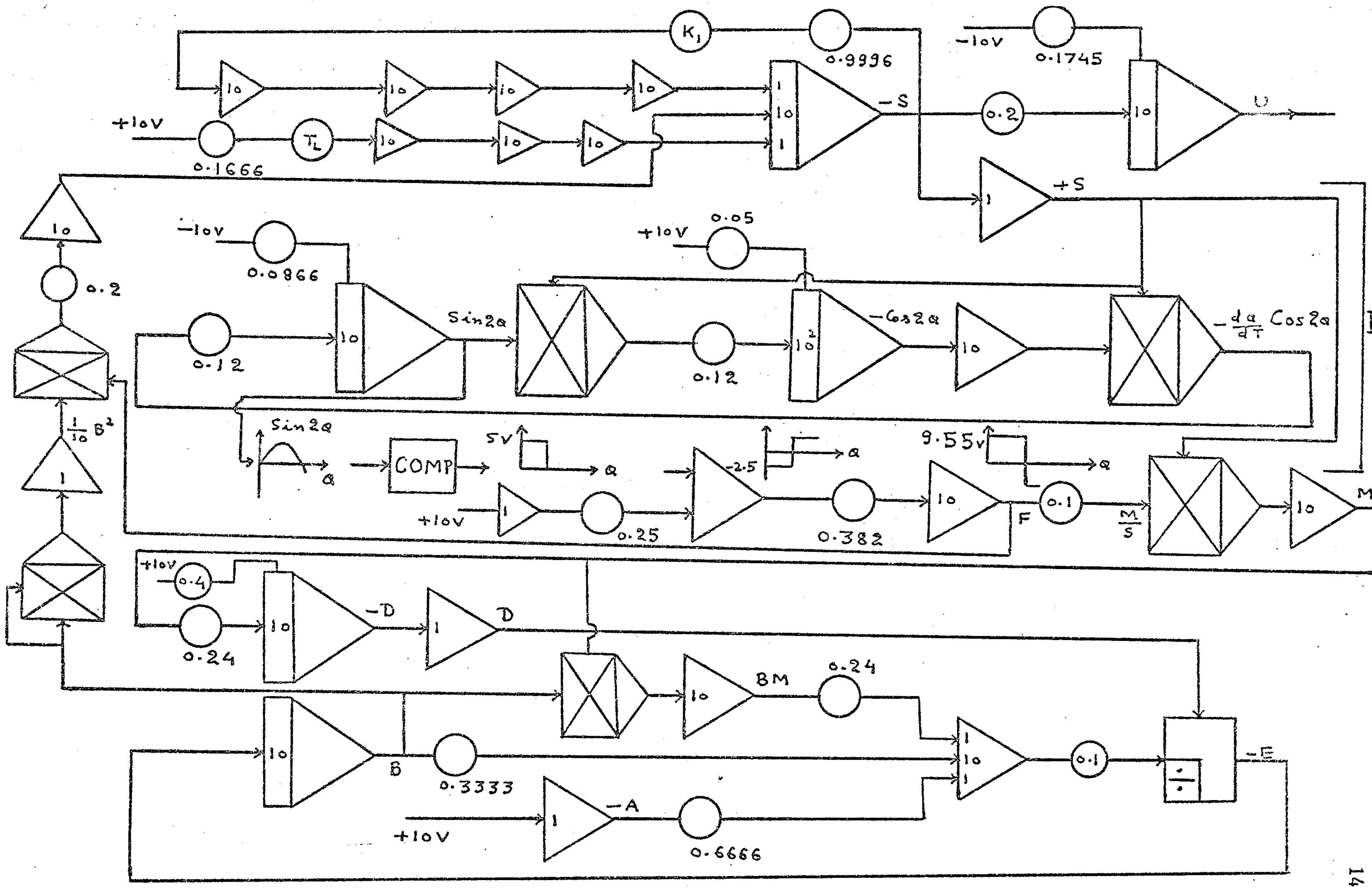

Fig.2.2.1. SCHEMATIC DIAGRAM FOR SINGLE-PHASE SIMULATION

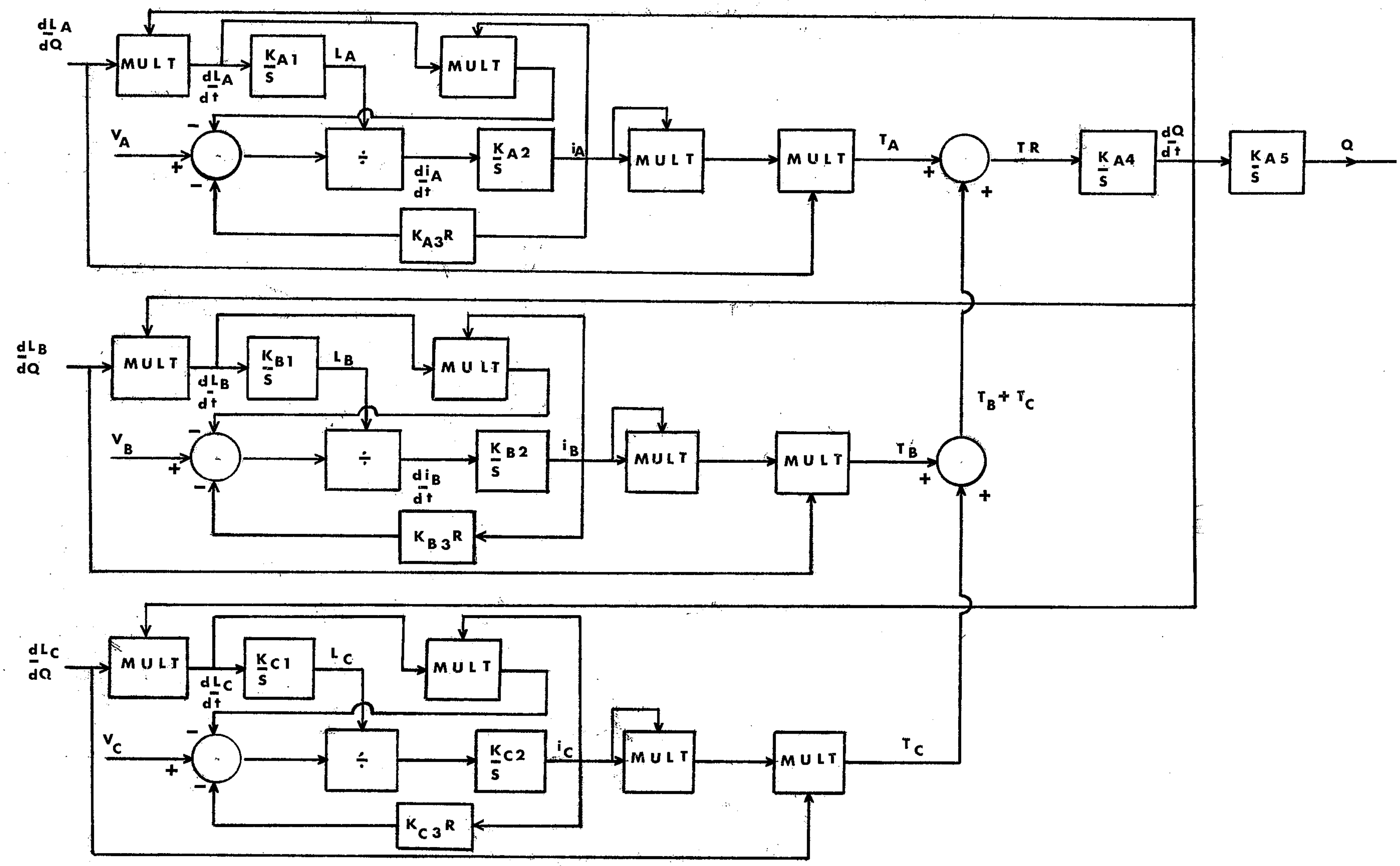

Fig. 2.2.2. BLOCK DIAGRAM FOR A THREE-PHASE MACHINE

$$Jh^2 \frac{d^2\theta}{dT^2} + K_1 h \frac{d\theta}{dT} + T_L = TR = \frac{1}{2} i^2 \frac{dL}{d\theta} \quad \ldots\ldots\ldots \; 2.2.3.$$

and $$V = iR + Lh \frac{di}{dT} + ih \frac{dL}{dT} \quad \ldots\ldots\ldots \; 2.2.4.$$

MAGNITUDE SCALING:

In the following equations, subscripted K's represent the magnitude scale factors for respective variables. Cm is the maximum computer voltage, i.e., 10 volts.

$$K_\theta = \frac{\max|\theta|}{Cm} = \frac{3}{10} = 0.3 \text{ rad/V}$$

$$\therefore \theta = 0.3U \text{ radians}$$

$$K_{\frac{d\theta}{dT}} = \frac{\max|d\theta/dT|}{cm} = \frac{6}{10} = 0.6 \text{ rad/s-V}$$

$$\frac{d\theta}{dT} = 0.6S \text{ rad/sec}$$

$$K_{\frac{d^2\theta}{dT^2}} = \frac{\max|d^2\theta/dT^2|}{cm} = \frac{10^3}{10} = 10^2 \text{ rad/s}^2\text{-V}$$

$$\therefore \frac{d^2\theta}{dT^2} = 10^2 G \text{ rad/s}^2$$

$$K_V = \frac{\max|V|}{cm} = \frac{20}{10} = 2 \text{ V/V}$$

$$\therefore V = 2A \text{ volts}$$

$$K_i = \frac{\max|i|}{cm} = \frac{20}{10} = 2 \text{ A/A}$$

$$\therefore i = 2B \text{ amps}$$

$$K_{\frac{dL}{dT}} = \frac{\max\left|\frac{dL}{d\theta}\right| \cdot \left|\frac{d\theta}{dT}\right|}{cm} = \frac{6 \times 10^{-3} \times 6}{10} = 0.36 \times 10^{-2} \text{ H/s-V}$$

$$\therefore \frac{dL}{dT} = 0.36 \times 10^{-2} M \text{ H/s}$$

$$K_L = \frac{\max|L|}{cm} = \frac{1.5 \times 10^{-2}}{10} = 1.5 \times 10^{-3} \text{ H/V}$$

$\therefore \quad L = 1.5 \times 10^{-3} D \quad H$

$$K_{\frac{di}{dT}} = \frac{2 \times 10^2}{10} = 20 \text{ A/s-V}$$

$\therefore \quad \frac{di}{dT} = 20\ E \quad A/s \qquad A/s$

$$K_{\frac{dL}{d\theta}} = \frac{\max|dL/d\theta|}{cm} = \frac{6 \times 10^{-3}}{10} = 6 \times 10^{-4} \text{ H/rad-V}$$

$\therefore \quad \frac{dL}{d\theta} = 6 \times 10^{-4}\ F \quad H/rad$

U, S, G, A, B, M, D, E and F represent computer voltages for $\theta, \frac{d\theta}{dT}, \frac{d^2\theta}{dT^2}$, V, i, $\frac{dL}{dT}$, L, $\frac{di}{dT}$, and $\frac{dL}{d\theta}$ respectively.

From the above, we can write the following equations:

$$U = 0.2 \times 10 \int S\, dT + U(o), \qquad \text{........ 2.2.5.}$$

$$S = \int (0.2\, B^2 F - 0.9996 \times 10^4\, SK_1 - 0.1666 \times 10^3\, T_L)\, dT \qquad \text{........ 2.2.6.}$$

$$E = \frac{1}{D}\left[0.6666A - 0.3333 \times 10\, B - 0.24\, BM\right] \qquad \text{........ 2.2.7.}$$

$$B = 10 \int E\, dT + B(o) \qquad \text{........ 2.2.8.}$$

$$D = 0.24 \times 10 \int M\, dt + D(o) \qquad \text{........ 2.2.9.}$$

$$\frac{dL}{d\theta} = \frac{dL}{dT} / \frac{d\theta}{dT} = 6 \times 10^{-3} \frac{M}{S} = 6 \times 10^{-4} F$$

$$\therefore \quad \frac{M}{S} = 0.1\ F$$

Equations 2.2.5., 2.2.6., 2.2.7., 2.2.8., and 2.2.9. are simulated as usual on an Analog computer. Since $\frac{dL}{d\theta}(\theta)$ changes its sign every ninety degrees, the second harmonic of sinθ is generated as shown, (marked as II) in Fig. 2.2.1. and accomplish the objective of generating $\frac{dL}{dQ}$ (Q) using a comparator.

The parameters chosen for this example are given in Appendix 2.

The analog model refers to the rectangular voltage pulses and triangular inductance waveforms.

The schematic and block diagrams given in Figs. 2,2.1. and 2.2.2.

are considerably different from the ones published by Kuo, Singh and Yackel[2]. It appears that they have assumed inductances constant. Since no simulation results have been shown, those block diagrams[2] are probably only of theoretical importance.

A discussion of the results is taken up in Chapter 4.

2.3 General analysis of a variable-reluctance stepping motor

The motor considered here is similar to the one described in section 2.1. except that it consists of P number of units for P phases in place of three units for three phases. The rotors of all units are aligned axially and each unit is staggered by 360/TXP degrees on the stator. A three phase ten tooth machine is shown in Fig. 2.3.1.

The units may be excited, one at a time or in other possible combinations to achieve step angles smaller than the one, when only one coil is excited at a time.

The inductance and voltage waveforms are shown in Figs. 2.3.2. and 2.3.3. respectively. The inductance waveforms are periodic functions of θ (mechanical angle) with a period of 2α, where,

$$\alpha = \frac{P}{2} \cdot \theta_s \qquad \text{........ 2.3.1.}$$

and $$\theta_s = \frac{360}{P \cdot T}$$

The number of teeth on the stator unit are equal to the number of rotor teeth for the type of motors analyzed in this thesis.

The voltage waveforms are periodic. If K is the number of pulses per second, $W = \frac{1}{K}$ and since there are P phases, the period of the voltage waveforms is PW, when only one unit is excited at a time.

The inductance and voltage waveforms described above are represented by Fourier Series according to Appendix I. Therefore, we have

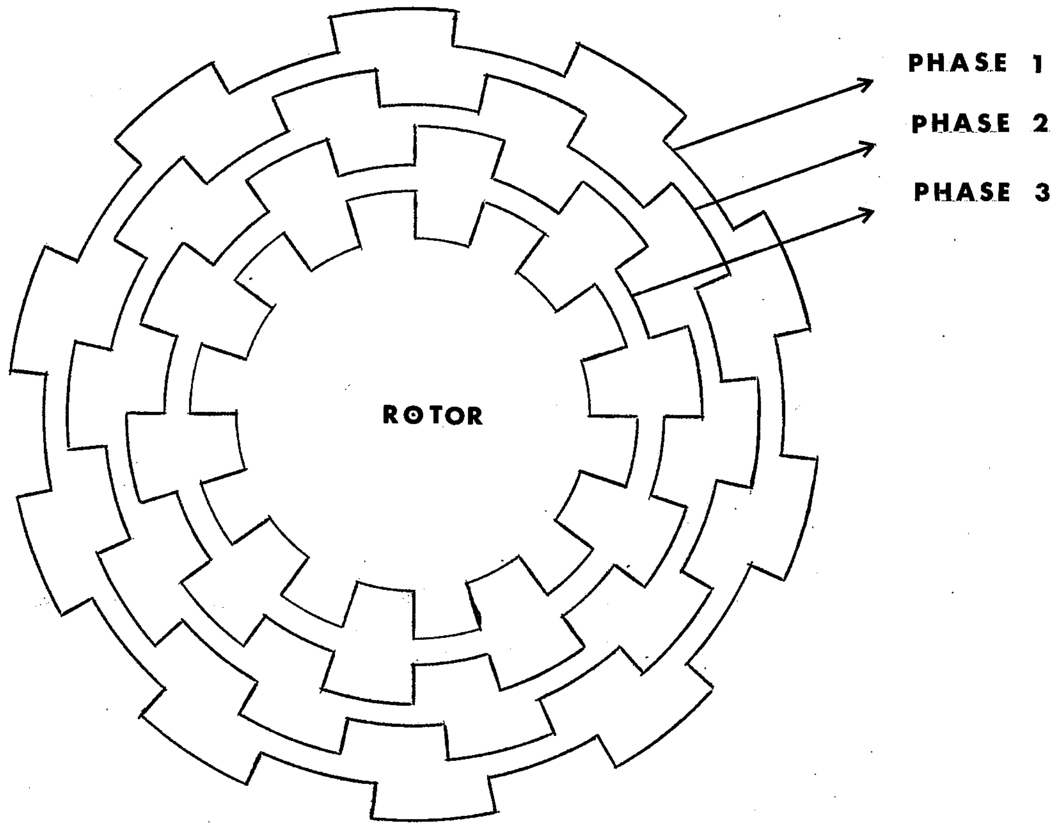

Fig.2.3.1. THREE – PHASE TEN – TOOTH MACHINE

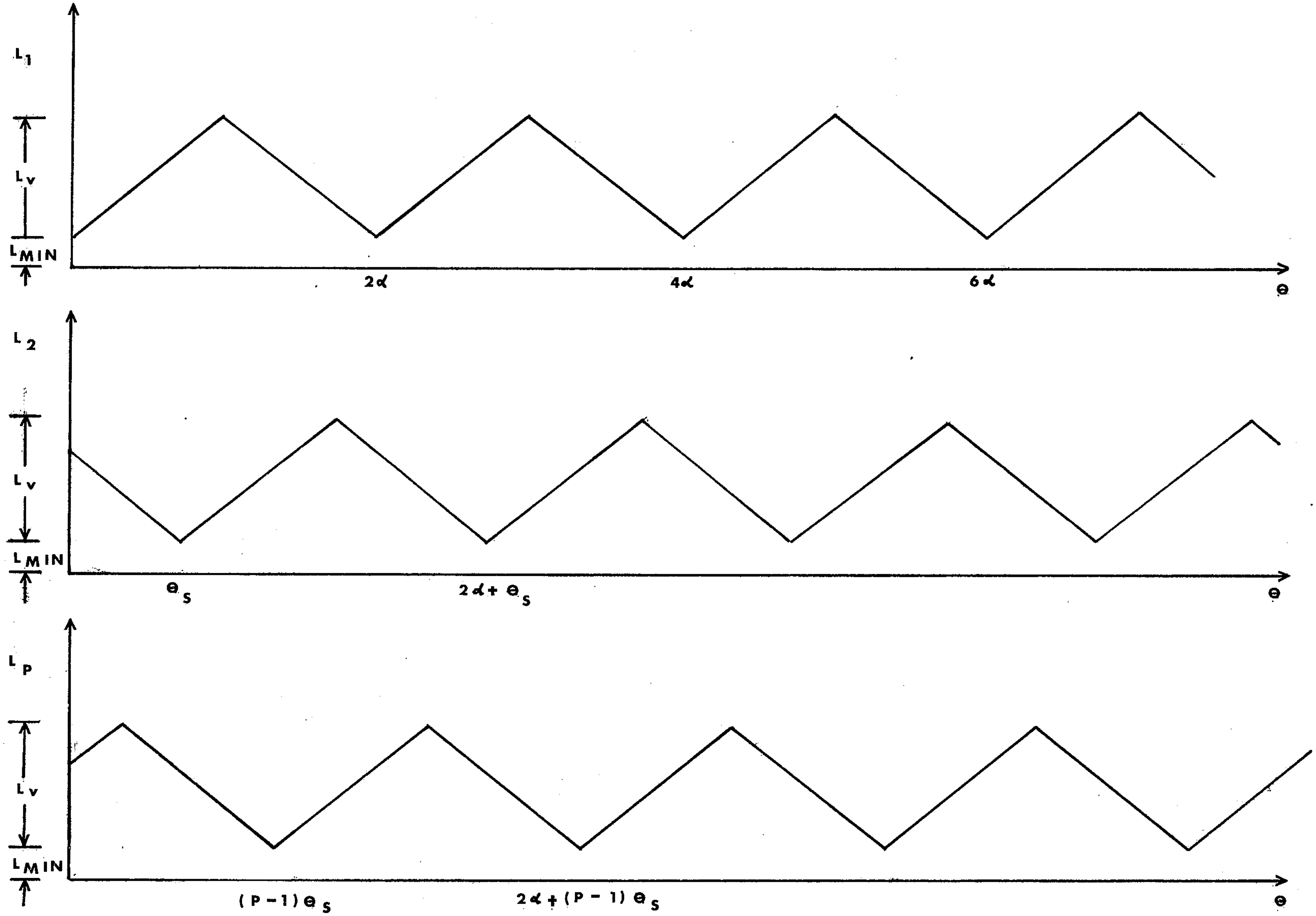

Fig.2.3.2. INDUCTANCE WAVEFORMS FOR A MULTI-STACK MACHINE

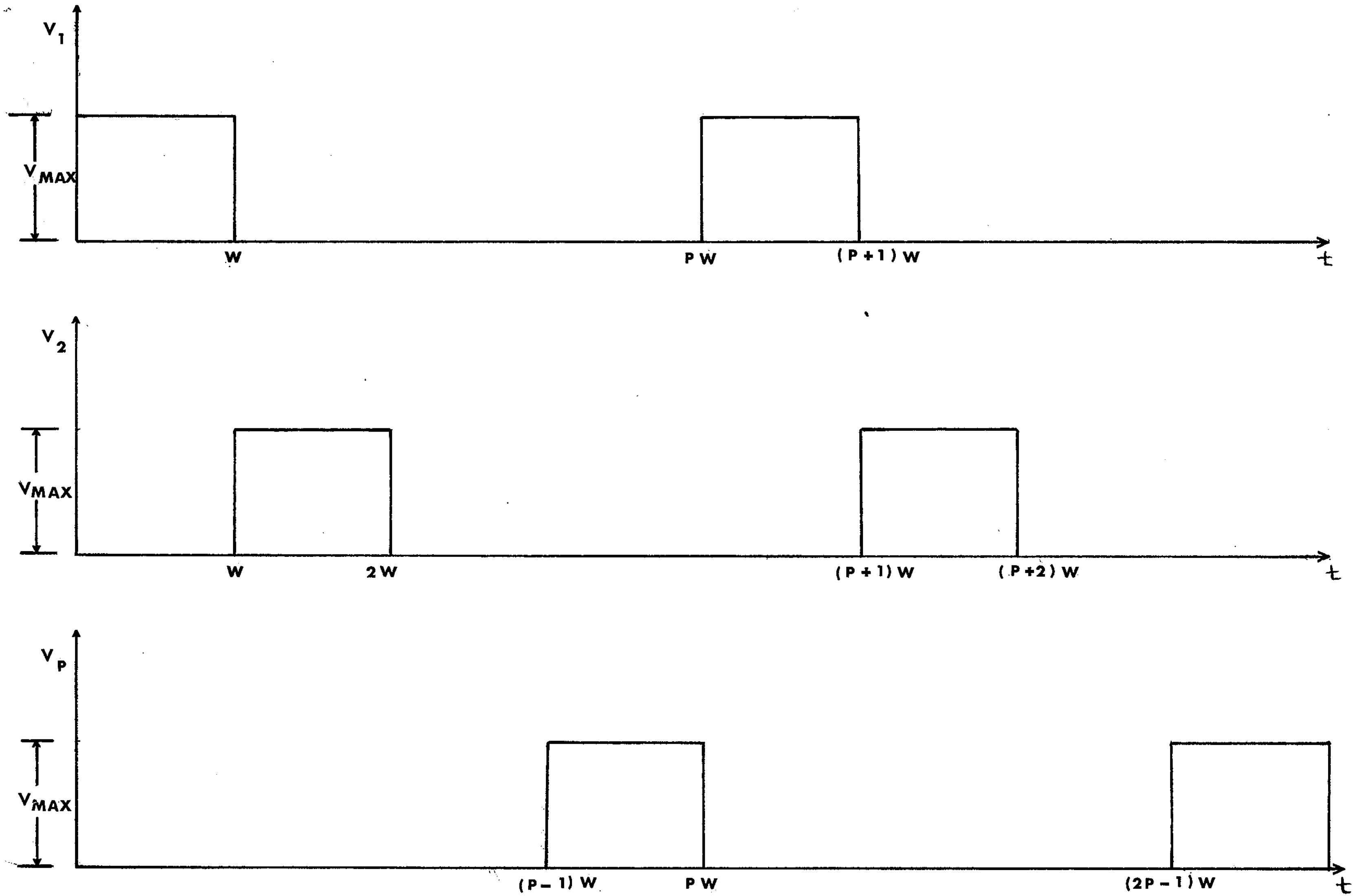

Fig.2.3.3. VOLTAGE WAVEFORMS FOR A MULTI-STACK MACHINE

$$L_1(\theta) = \frac{L_v}{2} + \frac{2L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \{(-1)^n - 1\} \cos \{\frac{n\pi(\theta)}{\alpha}\} + L_{min}$$

.... 2.3.3.

$$L_2(\theta) = \frac{L_v}{2} + \frac{2L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \{(-1)^n - 1\} \cos \{\frac{n\pi}{\alpha} (\theta-\theta_s)\} + L_{min}$$

.... 2.3.4.

and $$L_p(\theta) = \frac{L_v}{2} + \frac{2L_v}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \{(-1)^n - 1\} \cos \{\frac{n\pi}{\alpha} (\theta-(P-1)\theta_s)\} + L_{min}$$

.... 2.3.5.

Also,

$$V_1(t) = \frac{V_{max}}{P} + \sum_{n=1}^{\infty} \left[\frac{V_{max}}{n\pi} \{\sin (\frac{2n\pi}{P}) \cos (\frac{2n\pi t}{Pw}) + (1 - \cos \frac{(2n\pi)}{P}) \sin(\frac{2n\pi t}{Pw})\} \right]$$

.... 2.3.6.

$$V_2(t) = \frac{V_{max}}{P} + \sum_{n=1}^{\infty} \left[\frac{V_{max}}{n\pi} \{\sin (\frac{2n\pi}{P}) \cos (\frac{2n\pi(t-w)}{P}) + (1 - \cos \frac{(2n\pi)}{P}) \sin (\frac{2n\pi(t-w)}{Pw})\} \right]$$

.... 2.3.7.

and $$V_p(t) = \frac{V_{max}}{P} + \sum_{n=1}^{\infty} \left[\frac{V_{max}}{n\pi} \{\sin (\frac{2n\pi}{P}) \cos (\frac{2n\pi(t-(P-1)w)}{Pw}) + (1 - \cos (\frac{2n\pi}{P})) \sin (\frac{2n\pi(t-(P-1)w)}{Pw})\} \right]$$

.... 2.3.8.

The current in any one of the stator coils, when excited by voltage is given by

$$i = \frac{V}{R + \frac{dL}{d\theta} \cdot \frac{d\theta}{dt}} \left[1 - e^{-(R + \frac{dL}{d\theta} \cdot \frac{d\theta}{dt}) t/L} \right] \quad \text{... 2.3.9.}$$

as derived in section 2.1.

Torques produced by all phases can be added algebraically to give the resultant torque. Therefore,

$$TR = \frac{1}{2} (i_1)^2 \frac{dL_1(\theta)}{d\theta} + \frac{1}{2}(i_2)^2 \frac{dL_2(\theta)}{d\theta} + \ldots\ldots + \frac{1}{2} (i_p)^2 \frac{dL_p(\theta)}{d\theta} \quad \text{........ 2.3.10}$$

$\frac{dL_1(\theta)}{d\theta}$, $\frac{dL_2(\theta)}{d\theta}$, ... $\frac{dL_p(\theta)}{d\theta}$ are obtained directly from equations 2.3.3., 2.3.4., and 2.3.5., by differentiating w.r.t.θ. Therefore,

$$\frac{dL_1(\theta)}{d\theta} = \frac{2L_v}{\pi\alpha} \sum_{n=1}^{\infty} \frac{1}{n} \{1 - (-1)^n\} \sin \frac{n\pi\theta}{\alpha} \quad \text{........ 2.3.11.}$$

$$\frac{dL_2(\theta)}{d\theta} = \frac{2L_v}{\pi\alpha} \sum_{n=1}^{\infty} \frac{1}{n} \{1-(-1)^n\} \sin \frac{n\pi(\theta-\theta_s)}{\alpha} \quad \text{....... 2.3.12.}$$

and $$\frac{dL_P(\theta)}{d\theta} = \frac{2L_v}{\pi\alpha} \sum_{n=1}^{\infty} \frac{1}{n} \{1 - (-1)^n\} \sin \left(\frac{n\pi(\theta-(P-1)\theta_s)}{\alpha}\right). \quad \text{2.3.13.}$$

Instantaneous values of speed and position, i.e., $\frac{d\theta(t)}{dt}$ and $\theta(t)$ are obtained by using the equation

$$TR = J \frac{d^2\theta}{dt^2} + K_1 \frac{d\theta}{dt} + K_2\theta + T_L \quad \text{..... 2.3.14}$$

Values of $i_1(t)$, $i_2(t)$, ... $i_P(t)$ are obtained by substituting respective instantaneous values of $\frac{dL(\theta)}{d\theta}$'s, $\frac{d\theta(t)}{dt}$'s, $L(\theta)$'s and $V(t)$'s in equation 2.3.9. Torque is obtained using equation 2.3.10.

Hence the performance characteristics are predicted for a particular motor, by proper substitution of the number of teeth and phases of the motor together with its electrical and mechanical parameters. The computer programs are capable of simulating multi-tooth and multi-phase machines. It is necessary because there is a growing trend to go in for more than three phase machines.

The simulation results are discussed in Chapter 4.