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SOME NEW ASPECTS OF CONGRUENCE TRANSFORMATION AS APPLIED TO RC LADDER NETWORKS

by

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with love

To my parents

my wife Nivin

and my son Tarek

ABSTRACT

In a canonic RC ladder network realized from specified two-port parameters a proper transformation is applied to generate a family of equivalent networks in which the total resistance, total capacitance and the open-circuit voltage gain change continuously with respect to a single variable. A straight forward method is then developed to minimize cost function in ladder networks. The cost function is formed by weighting each of the three components - total resistance, total capacitance and the gain - and combining them.

The thesis also develops a systematic method of realizing RC low-pass ladder networks with nonuniformly lossy capacitors. The method is based on applying a proper congruence transformation to the capacitance and conductance matrices of the canonic network. The transformation matrix is constructed in such a way that the capacitors of the transformed network have shunt conductances while the driving-point impedance is kept invariant and the transmission characteristics of the two-port are preserved. Application of the results to other types of ladder structures is also investigated. In addition, the synthesis of RC low-pass ladder networks with equal-valued capacitors is studied. Realizability conditions are determined such that the transformed network includes as many equal capacitors as possible without increasing the total number of capacitors. Explicit formulas for the element values of the equivalent network are given throughout the thesis.

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Adel Ahmed Ali

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

SYMBOLS AND ABBREVIATIONS

A	Transformation matrix
A^t	Transpose of matrix A
a_i	The diagonal elements of A
a_{ij}	The (i, j)th element of A
a_i, b_i	Polynomial coefficients
B	Transformation matrix
C	Capacitance matrix
C'	Capacitance matrix of the transformed network
C_T	Total capacitance
c_{ij}	The (i, j)th element of C
c'_{ij}	The (i, j)th of C'
c_k	The k-th capacitor of the network
c'_k	The k-th capacitor of the transformed network
diag( )	Diagonal matrix
E	Vector of the node voltages
E'	Vector of node voltage in the transformed network
$F(\cdot)$	Cost function
G	Conductance matrix
G'	Conductance matrix of the transformed network
G_T	Total conductance
G_{ij}	The (i, j)th element of G
G'_{ij}	The (i, j)th of G'
G_k	The k-th conductor of the network
G'_k	The k-th conductor of the transformed network
g'_k	The k-th shunt conductor of the transformed network

H	Open-circuit voltage gain
I	Vector of equivalent source currents
I'	Vector of equivalent source current in the transformed network
I_n	Unit matrix of order n
$K, K(\cdot)$	Relative gain
k	Positive constant
N	Canonical network
N'	Transformed network
R	Resistance matrix
R'	Resistance matrix of the transformed network
R_k, r_k	k-th resistor
S	Elastance matrix
S_k	The k-th elastance
s	Complex frequency variable
Y	Node admittance matrix
Y'	Node admittance matrix of the transformed network
y_{ij}	The (i, j)th element of Y
y'_{ij}	The (i, j)th element of Y'
Z	Mesh impedance matrix
Z'	Mech impedance matrix of the transformed network
z_{ij}	The (i, j)th element of Z
z'_{ij}	The (i, j)the element of Z'
β, β_i	Normalized variables; $0 \leq \beta \leq 1$
γ_k	$\gamma_k = \sum_{i-k}^{n} \frac{1}{C_i}$
δ_i	Loss factor
θ_k	$\theta_k = \sum_{i=k}^{n} G_i$
σ_k	$\sigma_k = \sum_{i=k}^{n} R_i$

CHAPTER I

INTRODUCTION

In circuits designed for low frequency applications the use of inductors is undesirable mainly for the fact that they are bulky, nonlinear dissipative and uneconomical. There are other disadvantages: where specifications are precise, the usual synthesis techniques for lossless networks cannot be used because of the inherent resistance of the inductors. There is, however, a great need for precise performance at low frequency, especially in the control field where accurate compensations are required. Thus, the use of a network containing only resistors and capacitors is very important.

This thesis deals with three important problems in RC synthesis: i) precorrecting for capacitor dissipations, ii) minimizing duly defined cost function, and iii) designing RC ladder networks with equal capacitors.

With the advent of thin film and integrated circuit techniques, design criteria have changed significantly. In particular, the new techniques have led to less emphasis on the number of elements in the network and to more interest in design methods which take into account the higher losses that occur with thin-film capacitors at the present state of development [1]. Another important problem that faces the designer of RC circuits fabricated by integrated-circuit techniques is the size of the circuit chip. As the chip size increases, yield decreases, and cost rises. In some manufacturing processes, the area required by a resistor or a capacitor is proportional to its value [1]. Hence, for a small chip size, it is necessary to minimize the total resistance and capacitance in the circuit. The use of equal capacitors in integrated circuit design is advantageous in other ways as well: such networks are more easily made and tested; they also minimize mask layout errors, and errors in the network functions caused by temperature variations [1].

The following chapters treats these design problems. The basic approach to them is the application of the matrix transformation technique. The results of these studies reveal some new implications of the equivalent network theory.

Generating equivalent networks is one of the central concerns in network theory. Methods of realizing equivalent networks are quite important, both from theoretical and practical reasons.

It is well known that two networks differing in their topology and elements may be completely equivalent if the relations between variables at a selected subset of their terminals are concerned.

Cauer [2] showed in 1929 that congruence transformation applied to the matrix representing a realizable network can generate another realizable network in such a manner that a specified set of input and transfer admittances will be invariant. He also pointed out that these transformations would preserve the positive semidefinite character of the parameter matrices. Since this latter property is both necessary and sufficient for realization of networks containing ideal transformers, he therefore established a basic method for generating equivalent networks.

If ideal transformers are not used, however, the positive semidefinite character of the parameter matrices is no longer sufficient to guarantee realizability. Unfortunately, the general conditions that are both necessary and sufficient for this transformation procedure to yield realizable parameter matrices are not known. This lack of information has therefore limited the practical applications of Cauer's method.

Guillemin [3], [4], was the first to recognize the great importance of the normal form for Cauer's approach in generating equivalent two-element-kind networks. The normal form ties together the so-called methods of direct transformation and matrix synthesis. The former refers to Cauer's method of transforming a given network directly into an equivalent one, while the latter refers to Guillemin's procedure of generating networks from the normal form by reversing the normal coordinate transformation procedure. Since direct transformation can always be viewed as a conversion from the network to the normal form, followed by a conversion from the normal form to an equivalent network, it is clear that the normal form introduced by Guillemin is the common feature of equivalent networks produced by linear transformations.

Duda [5], and Schneider [6] applied Guillemin's procedure to generate equivalent realizations of RC driving-point impedances.

The theory of continuously equivalent networks, which was introduced by Schoeffler [7], [8], is an extension of the Howitt transformation [9], for deriving equivalent networks; however, it allows all elements in a network to vary continuously as functions of single parameter. The method is quite general, flexible, and easily programmed. Schoeffler and others [10]-[16], have applied the method to generate equivalent networks for the purpose of minimizing sensitivity.

The main objective of this thesis is the investigation of a special class of congruence transformations for novel features which facilitate the solution of the stated problems. The application of the transformation technique for the stated objectives is new, and to the best knowledge of the author, the same problems has not been previously treated in the literature.

The thesis consists of six chapters, including the introduction. In Chapter II, the general linear transformation is discussed and then congruence transformation is presented in detail. The equilibrium equations for the low-pass ladder networks are set out and the transformation matrix is constructed to generate equivalent networks within the ladder topology. Element values are given explicitly in terms of those of the canonic network. The results are then directly extended to high-pass and other types of ladder networks using the topological duality and the GC:CG:GC transformation.

The minimization of cost functions is dealt with in Chapter III. The relevant quantities - total resistance, total capacitance, and the gain - are combined to form a penalty function which has to be minimized to produce the optimum network. In this chapter a family of n-canonic ladder networks is shown to have some interesting properties. We then generate a one-parameter family of continuously equivalent networks having useful characteristics. In the two preceding cases, the optimum network will be found uniquely. Explicit formulas for the element values of the transformed networks are derived and illustrative examples are provided.

Chapter IV develops a systematic method for the compensation of lossy capacitors. Uniform and nonuniform loss are treated and explicit formulas for the element values of the precompensated networks are given.

Chapter V discusses the possiblity of realizing ladder networks with equal capacitors. It shows that a canonic network can be transformed into an equivalent network of equal-valued capacitors under certain specified conditions. This chapter provides element values and examples to illustrate the synthesis procedure.

Chapter VI then outlines some concluding remarks. Finally, six appendices set out the mathematical proofs of realizability conditions and the construction of transformation matrices.

CHAPTER II

EQUIVALENT TRANSFORMATION OF RC LADDER NETWORKS

2.1 Introduction

Ladder networks are one of the simplest and most important network structures. Because of their versatile properties, they are the most widely used configurations. The transmission zeros, for example, can easily be realized in such networks by creating poles of series impedances and shunt admittances. Zero shifting and tuning are accomplished by adjusting a readily accessible element in a simple way. They are also characterized by their high reliability and low sensitivity to element variations [17].

The main objective of this chapter is to develop a general transformation technique that can be used in devleoping a continuously equivalent family of RC ladder networks. Network matrices are subjected to congruence transformation in such a way that one of the driving point functions remains unchanged, while the transmission characteristics of the two-port are preserved. But first it is useful to introduce the general theory of linear transformations and discuss the physical interpretation of congruence transformation. It is essential to identify the proper transformation matrices for the two-element-kind ladder networks. Then, as a result of the transformation of the parameter matrices, it is possible to explicitly establish the formulas for element values.

2.2 Equivalent Network Theory

The earliest theory of equivalent networks was based upon congruence transformations of loop impedance matrices. The basic idea of the theory is to keep the port variables fixed while modifying the internal loop variables to obtain a desired structure. Since constant-congruence transformations on impedance matrices keep the internal energy invariant, they can be realized with transformers. Using such transformations in the manner developed by Howitt, Cauer and others [18]-[21], we will generate equivalent ladder networks. However, as pointed out by Brune [22], not all equivalent realizations can be determined by the method.

Theorem

Given an $n + p$-port network N, described by the admittance matrix Y, consider an $n + p$-port N' described by Y' such that

$$Y' = B\,Y\,A, \tag{2.1}$$

where

$$B = \begin{bmatrix} I_n & B_{12} \\ 0 & B_{22} \end{bmatrix} \begin{matrix} \}n \\ \}p \end{matrix}, \quad \text{(columns: } \underbrace{}_{n}\ \underbrace{}_{p}\text{)}$$

$$A = \begin{bmatrix} I_n & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{matrix} \}n \\ \}p \end{matrix} \quad \text{(columns: } \underbrace{}_{n}\ \underbrace{}_{p}\text{)} \tag{2.2}$$

and

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{matrix} \}n \\ \}p \end{matrix} \quad \text{(columns: } \underbrace{}_{n}\ \underbrace{}_{p}\text{)} \tag{2.3}$$

For nonsingular B and A, Y and Y' have the same nxn admittance matic Y_n

$$Y_n = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}. \tag{2.4}$$

which is obtained by opening the last p-ports of the network. The theorem can be readily proved by using (2.1)-(2.4).

Corollary (Howitt)

If A is a constant matrix and B is the transpose of A then

$$Y' = A^t Y A \tag{2.5}$$

is called a congruence transformation.

It should be noted that B and A of (2.1) may actually be rational or irrational functions of the complex frequency variable s [23].

Since the congruence transformation is applied to generate equivalent ladder networks, it is useful to physically interpret the transformation. Consider a network with n independent node pairs. Such a network is described by a set of n equations of the form

$$I = Y E \tag{2.6}$$

where Y is the nxn admittance matrix of the network, E is the nxl column matrix of node voltages and I is the nxl column matrix of equivalent source currents. If this network is imbedded in a 2n-port network of ideal transformers, another n-port results in which the variables of the original and the new network are related by a linear transformation:

$$E = A E'$$

$$I' = A^t I \tag{2.7}$$

Where A is an nxn nonsingular constant matrix and E' and I' are the new voltage and current variables [24]. The last equation follows from the energy conservation of the transformer network. The new variables are related by the equation

$$I' = Y' E' \tag{2.8}$$

where

$$Y' = A^t Y A \tag{2.9}$$

From the method of construction it is clear that the resulting admittance matrix is physically realizable. The usefulness of the theory stems from the possibility of interpreting (2.9) as the equilibrium equations of an equivalent transformerless network. In this case, not all of the elements will be positive in general, a fact that has limited the applications of the theory [25]. By proper choice of the transformation matrix A, certain driving-point and/or transfer functions can be kept invariant. For example, if the k-th row of A is the k-th unit vector (all zeros except for the k-th entry which is unity), the driving point impedance at the k-th terminal pair is invariant to the transformation. If two rows are so chosen, the driving point impedance at each port is invariant, as is the transfer impedance between the two ports. Thus, it is possible to maintain desired transfer functions invariant to the transformation.

2.3 Low-Pass RC Ladder Networks

2.3.1 z_{11}, z_{12} are specified

Let us consider an RC two-port with two of the open-circuit impedance parameters:

$$z_{11} = \frac{\Sigma_o^m a_i s^i}{\Sigma_o^p b_i s^i}, \qquad z_{12} = \frac{k}{\Sigma_o^p b_i s^i} \tag{2.10}$$

Since the transmission zeros are all at infinity, the synthesis of z_{11} in the first Cauer form, which will be called the reference network, gives the low-pass ladder realization shown in Figure 1.a, where $C_1 = o$ for $p=m$ and $C_1 \neq o$ for $p=m+1$, with $n=m+1$ in either case, $b_o \neq o$ is assumed throughout.

The node admittance matrix Y of N is given by

$$Y = G + sC \tag{2.11}$$

where G and C are the nxn conductance and capacitance matrices, respectively, and s is the complex frequency variable.

$$G = \begin{bmatrix} G_1 & -G_1 & 0 & \cdot & \cdot & \cdot & 0 \\ -G_1 & G_1+G_2 & -G_2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & & & & & & \\ 0 & 0 & \cdot & \cdot & G_{n-2}+G_{n-1} & -G_{n-1} & \\ 0 & 0 & & & -G_{n-1} & G_{n-1}+G_n & \end{bmatrix} \tag{2.12}$$

and

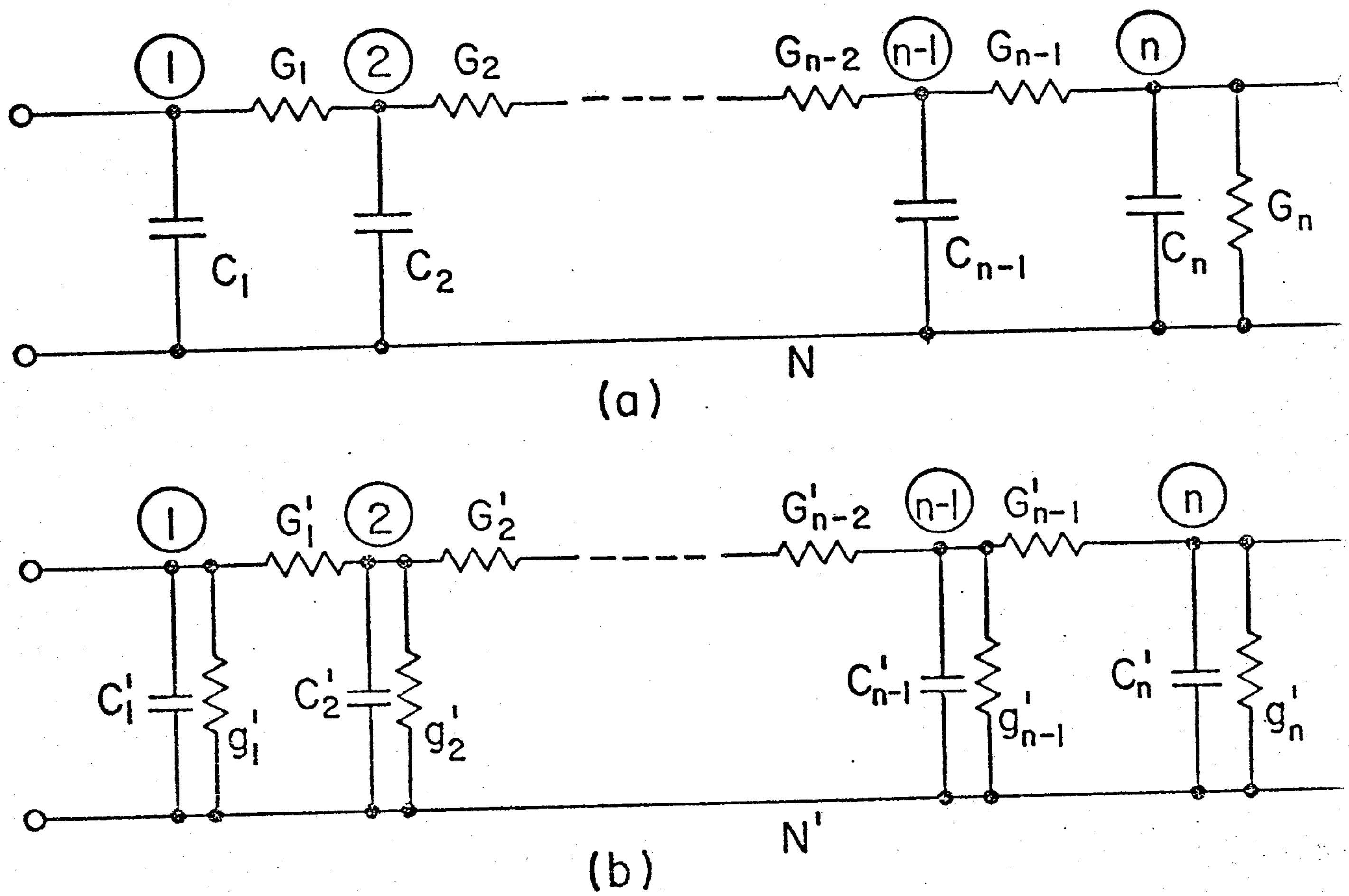

Figure 1 Low-pass realization of specified (z_{11}, z_{12})

(a) Canonic network.
(b) Equivalent network.

$$C = \text{diag}\ (C_1,\ C_2,\ \ldots,\ C_n) \tag{2.13}$$

In standard matrix notation

$$Y = [y_{ij}] \qquad i,\ j = 1,\ 2,\ \ldots,\ n \tag{2.14}$$

$$G = [G_{ij}] \qquad i,\ j = 1,\ 2,\ \ldots,\ n$$

$$C = [C_{ij}] \qquad i,\ j = 1,\ 2,\ \ldots,\ n \tag{2.15}$$

We now apply the congruence transformation on Y to obtain a new network N' corresponding to Y' as given by (2.9). The transformed network N' of Figure 1.b has $z'_{11} = z_{11}$; $z'_{12} = kz_{12}$ and each capacitor in N' has a shunt conductance. To keep z_{11} invariant, and preserve the ladder topology of N' characterized by Y' as

$$Y' = G' + sC' \tag{2.16}$$

it is required that the matrix A has the form (Appendix A)

$$A = \text{diag}\ (1,\ a_2,\ a_3,\ \ldots,\ a_n) \tag{2.17}$$

Substituting (2.14) in (2.9) we get

$$Y' = [a_i a_j y_{ij}] \qquad i,\ j = 1,\ 2,\ \ldots,\ n \tag{2.18}$$

$$G' = [a_i a_j G_{ij}] \qquad i,\ j = 1,\ 2,\ \ldots,\ n \tag{2.19}$$

$$C' = [a_i a_j C_{ij}] \qquad i,\ j = 1,\ 2,\ \ldots,\ n \tag{2.20}$$

In order to determine the element values in N' consider the node admittance matrix Y' of the equivalent ladder shown in Figure 1.b.

$$Y' = G' + sC' \tag{2.21}$$

where

$$G' = \begin{bmatrix} g_1'+G_1' & -G_1' & 0 & . & . & 0 \\ -G_1' & G_1'+G_2'+g_2' & -G_2' & . & . & 0 \\ . & . & . & . & & \\ & & & & & \\ 0 & 0 & 0 & . \; . & G_{n-2}'+G_{n-1}'+g_{n-1}' & -G_{n-1}' \\ 0 & 0 & 0 & . \; . & -G_{n-1}' & G_{n-1}'+g_n' \end{bmatrix} \quad (2.22)$$

and

$$C' = \text{diag}\ (C_1', C_2', \ldots, C_n') \quad (2.23)$$

Comparing the main diagonal elements of (2.19) and (2.22), we write

$$G_{k-1}'+G_k' + g_k' = a_k^2\ (G_{k-1} + G_k) \qquad k = 1, 2, \ldots, n \quad (2.24)$$

where

$$G_o = G_o' = 0$$

Since

$$G_k' = a_k\ a_{k+1}\ G_k \quad \text{and} \quad G_{k-1}' = a_{k-1}\ a_k\ G_{k-1} \quad (2.25)$$

we identify g_k' from (2.24) as

$$g_k' = a_k^2\ \{G_k(1- \frac{a_{k+1}}{a_k}) - G_{k-1}\ (\frac{a_{k-1}}{a_k} - 1)\} \qquad k = 1, 2, \ldots, n \quad (2.26)$$

where

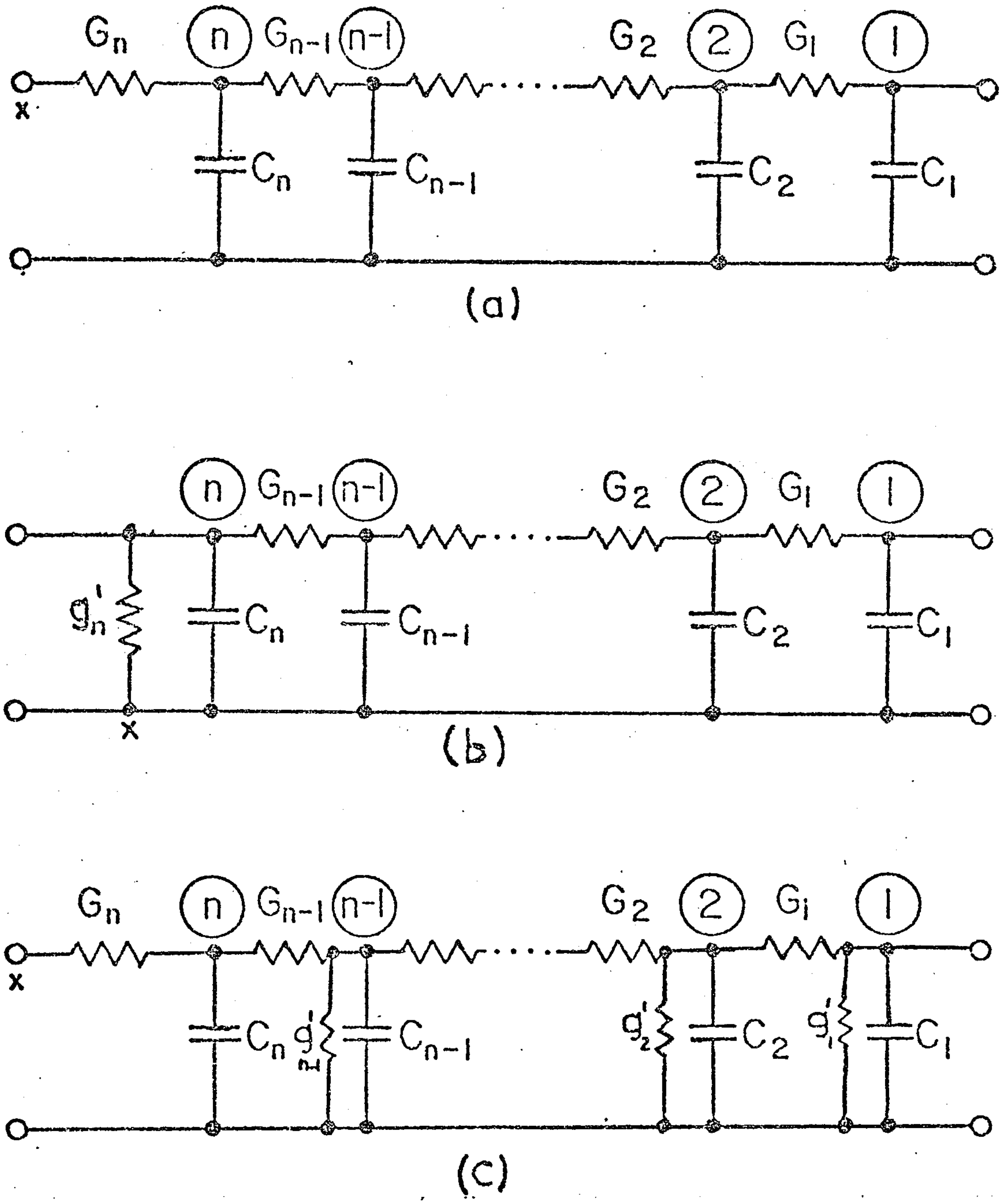

Figure 2 Low-pass realization of specified (y_{22}, y_{12}).

(a) Canonic network.
(b) Intermediate stage network.
(c) Equivalent network.

$$a_o = a_{n+1} = G_o = G_{n+1} = 0$$

Similarly, from (2.20) and (2.23) we write

$$C_k' = a_k^2 C_k \qquad k = 1, 2, \ldots, n \qquad (2.27)$$

Equations (2.25)-(2.27) give explicit formulas for the element values of the transformed network in terms of the element values of the reference network and the transformation parameters a_k. The transformation matrix A is chosen to meet the design criteria.

2.3.2 y_{12}, y_{22} are specified

The network shown in Figure 2.a is the canonic realization of

$$y_{22} = \frac{\sum_{o}^{m} a_i s^i}{\sum_{o}^{q} b_i s^i}, \qquad y_{12} = \frac{-k}{\sum_{o}^{q} b_i s^i} \qquad (2.28)$$

where $C_1 = o$, for $m=q$ and $n=q+1$. Since y_{22} is the driving-point admittance with terminal x grounded, we have the same situation as in Figure 1.a except that the input and output ports are now interchanged. Thus, we can directly apply the transformation method developed in the preceeding section to obtain the element values of the transformed network. Those values are given by (2.25)-(2.27) and the equivalent network is shown in Figure 2.c.

2.4 High-Pass Ladder Networks

2.4.1 y_{11}, y_{12} are specified

When the RC two-port short circuit admittance parameters y_{11} and y_{12} are given as

$$y_{11} = \frac{\Sigma_o^m a_i s^i}{\Sigma_o^p b_i s^i} , \qquad y_{12} = -\frac{k s^m}{\Sigma_o^p b_i s^i} \tag{2.29}$$

we can realize y_{11} in the Cauer second form as shown in Figure 3.a. The network can be described by the mesh impedance matrix of

$$Z = R + \frac{1}{s} S \tag{2.30}$$

where R and S are the nxn resistance and elastance matrices, respectively.

$$R = \begin{bmatrix} R_1 & -R_1 & 0 & \cdot & \cdot & & 0 \\ -R_1 & R_1+R_2 & -R_2 & \cdot & \cdot & & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & R_{n-2}+R_{n-1} & & -R_{n-1} \\ 0 & 0 & 0 & \cdot & -R_{n-1} & & R_{n-1}+R_n \end{bmatrix} \tag{2.31}$$

and

$$S = \text{diag}\ (\frac{1}{C_1}, \frac{1}{C_2}, \ldots, \frac{1}{C_n}) = \text{diag}\ (S_1, S_2, \ldots, S_n) \tag{2.32}$$

As a result of the topological duality existing between the low-pass ladder of Figure 1.a and the high-pass ladder of Figure 3.a, we observe the similarity between resistance and conductance matrices given by (2.12) and (2.31), and between capacitance and elastance matrices, given by equations (2.13) and (2.32), respectively. Thus, the congruence transformation

$$Z' = A^t Z A \tag{2.33}$$

can be applied to give the network of Figure 3.b with impedance matrix Z'.

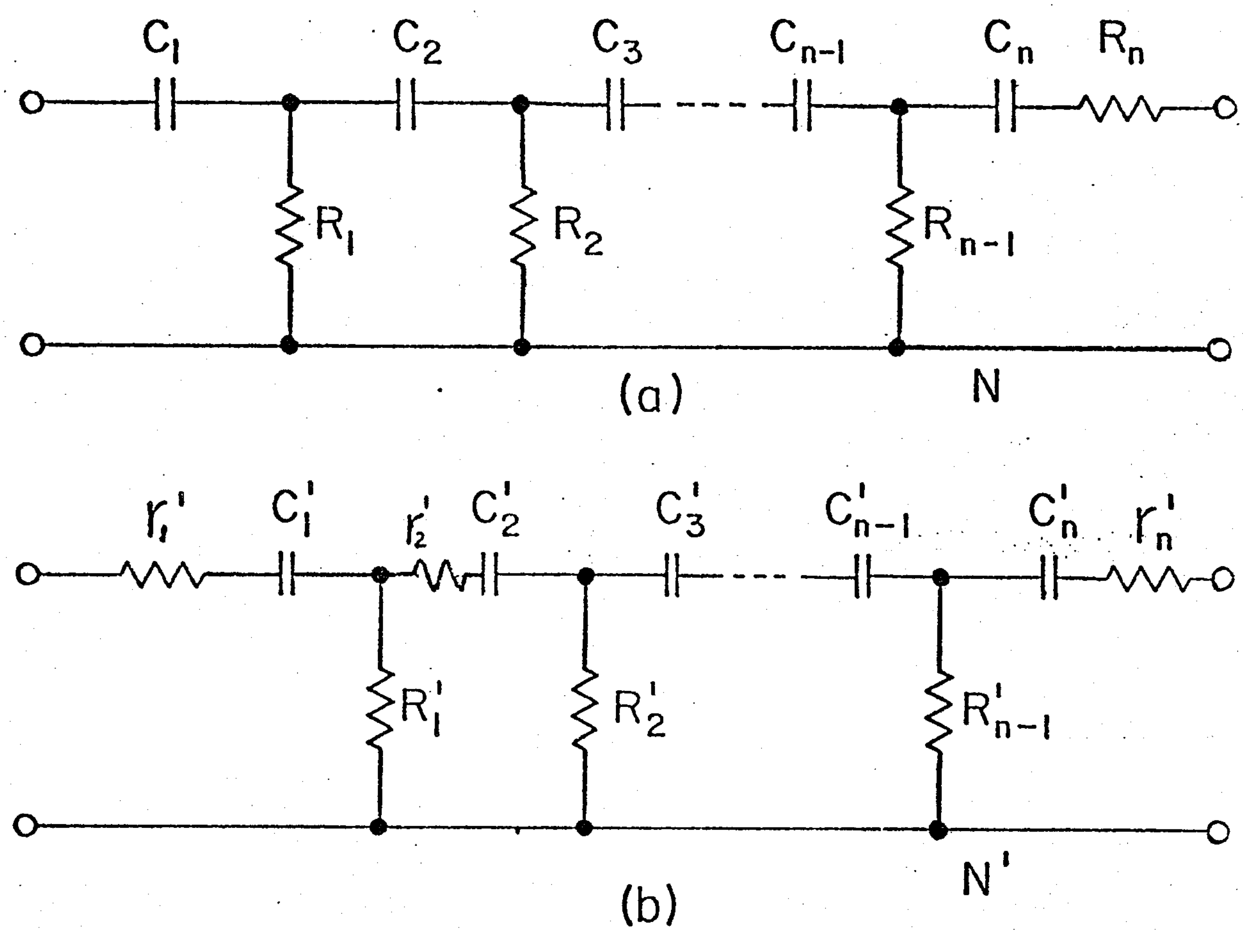

Figure 3 High-pass realization of specified (y_{11}, y_{12}).

(a) Canonic network.
(b) Equivalent network.