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SCATTERING BY CYLINDRICAL REFLECTORS AND THE EFFECTS OF DIELECTRIC LOADING

by

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MASTER OF SCIENCE

To my parents

with deep gratitude

ABSTRACT

The physical optics approximation is utilized to improve the convergence of the numerical solution using the moment method. As a result, the range of the application of the moment method is extended to objects of large to very large electrical dimensions. The advantage of this method over the conventional moment method is verified by its application to large conducting circular cylindrical reflectors and accurate solutions in relatively less computer time are obtained. The effects of variously shaped dielectric loadings on the radiation characteristics of cylindrical reflectors are also investigated and parameters most important for the overall behavior of such reflectors are determined. Finally, the effects of surface deformations on the radiation patterns of some commonly used reflectors are studied and is shown that by a proper choice of the parameters involved, directive beams of desired shape can be obtained.

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CHAPTER ONE

INTRODUCTION

The underlying differential equations, describing the scattering and diffraction phenomena, have long been fully understood. The greatest difficulty in the solution of many problems of electromagnetic theory is usually not the analytical reduction of the original vectorial problem to a set of scalar wave equations, but rather obtaining the solution of such reduced equations. The method of separation of variables serves quite well for the study of some simple electromagnetic wave problems. The wave equation $\nabla^2\psi + k^2\psi = 0$ is separable only in eleven coordinate systems. But a useful solution, in the presence of an object, can only be found if the object surface coincides with one of the coordinate surfaces. This is a necessity, since the solution is unique if it satisfies the boundary conditions, and it can be applied if the above condition is satisfied. However, even if the boundary surfaces coincide with one of the coordinate surfaces, the solution obtained using the separation of vaiables, may not be practically useful. This is due to the fact that the eigen functions of some of these coordinate systems cannot readily be computed or the resulting series are slowly convergent. Therefore, exact analytical solution of electromagnetic boundary value problems are restricted to a small group of objects with simple geometrical shapes. For this reason, attempts have been made to develop approximate solutions, which could provide useful solutions for variety ofthe problems, under certain conditions. Examples are the low and the high frequency approximations, variational methods as well as the geometrical theory of diffraction. However, one of the drawbacks of these methods is that, their application to each new scattering problem requires a great deal of thought and ingenuity to estimate their accuracy of the solution and to include all necessary rays or effects. In addition all these methods tend to be less accurate in the intermediate frequency range, where the dimensions of the objects are of the same order of the frequency wavelength.

On the other hand, numerical methods may also be used to study the electromagnetic scattering problems by an application of the boundary conditions in a finite set of boundary points. These methods have received increasing attention in recent years due to the availability of the digital computers. They are, however, most useful in the low and intermediate frequency ranges, for which the required computing time is not excessive. In application of these methods to the scattering problems, usually the original differential or the integral equations are transformed into a set of simultaneous equations. A solution for the problem is then obtained by the solution of these resulting linear equations.

Two distinct approaches have been employed to obtain a set of linear equations for the scattering problems. One is based on the modal expansion of the scattered field in terms of a complete set of certain modal functions with unknown coefficients. These unknown coefficients are then found by a truncation of the modal series and the application of the boundary conditions in a finite number of boundary points and solving the resulting simultaneous equations. Alternatively, the scattered field may also be found in terms of an integral equation of certain unknown quantities, which depending on the object, may be the induced charge and current densities on conductor or the polarization currents in a dielectric region. A solution for such an integral equation is usually obtained by an expansion of the unknown quantities in terms of a series of selected functions with unknown coefficients or a direct numerical evaluation of the integral equations.

The integral equation method was first used by Mei and Van Bladel[1] for determination of the fields scattered by perfectly conducting rectangular cylinders. The unknown quantity of the integrand was the induced current on the cylinder, which was expanded in a set of discontinuous step functions and the boundary conditions were enforced at N different points on the contour C of the scatterer. The integral equation was then approximated by a weighted sum of N sampled values of the current distribution and the resulting set of N linear equations were to give the required unknown current. In a later paper by Andreasen[2], a more refined form of this method was used with better approximations for the current distribution and the numerical integration(parabolic approximation) to determine the scattered field of a set of conducting cylinders of arbitrary cross sections.

Following these papers, the application of this method has been extended to various two and three-dimensional antenna and scattering problems. The publications in this area are numerous, but may be summarized as; scattering from bodies of revolution[3], scattering from cylinders with arbitrary surface impedance[4], scattering by a dielectric cylinder of arbitrary cross section using a volume integral formulation[5,6], straight and circular wires with arbitrary excitation and loading[7,8], scattering from conducting loops and solution of circular loop antennas[9] and the conical-equiangular spiral antennas[10].

The results of these and numerous other works in this area are used by Harrington[11,12] to present a unified approach for solving field problems using digital computers. More recently Mittra et al have summarized all these numerical methods in their book[13] and have provided useful discussions on their advantages and limitations. The method is also considered by Wallenberg[14], where he has described it in terms of the concept of generalized network parameters.

In application of these methods to scattering bodies with sharp edges, difficulties arise when certain field components become infinite and an accurate evaluation of these components require their careful treatments. One way, which has been used by Shafai[15,16,17] is to relate these singularities at the sharp edges to the geometry of the scatterer using a transformation method. The resulting integral then has a new unknown function, which is regular and readily obtainable. In another approach, utilized by Abdelmessih and Sinclair[18], the exact behavior of the fields at the edges was described by the Meixner edge conditions[19] and its contribution was included in solving the integral equation. However, the simplest approach is to ignore the contribution of the edge currents by placing the sampling points near but not on the edge itself. This approach has been applied to the problem of scattering by a perfectly conducting rectangle[20] and has shown that for step sizes less than $\lambda/10$, the resulting error in the computed induced current, due to a mesh size, will not exceed two per cent.

Of prime importance to the user of the moment method is how to solve the integral equation easily, accurately and rapidly. A limitation of this method, when used to analyze scattering problems, is that the scattering body cannot be longer than a few wavelengths, since large bodies result in large matrices which require excessive computer storage and running time. In addition, aside from the excessive computation time, the accumulation of error, while inverting large matrices, greatly impaires the accuracy of the solution thus obtained. The best one can do then, is to seek for a way to improve the rate of the convergence of this method. In a paper by Kinzel[21], a sectioning method is suggested for treating the two-dimensional scattering problems which involve large bodies. In this method, the scatterer is divided into a number of small sections where a section might typically be of the order of one wavelength long. Each section is then divided into more subsections. The next step is to compute the current distribution on three adjacent sections, using the method of moment, and ignoring the contribution of the current distribution over the remaining sections. The calculated approximate current distributions are stored for only the middle sections. The procedure is continued for determining the current distribution over the entire contour of the scatterer. This technique essentially assumes that the current on one section is significantly affected only by the currents in the adjoining sections, which is only a good approximation if the sections are not too small.

One of the drawbacks of this method is in completely neglecting the contribution from the edge current distribution, which is quite significant, on the other sections. Moreover, the computational time saved by this method becomes significant only when the number of sections is much larger than nine. Therefore, to extend the application of the method of moment to objects with large cross sectional dimensions, relative to the wavelength, a further modification of the method is required which does not suffer from the drawbacks given for the sectioning method.

For problems of conducting objects, the physical optics approximation usually provides an approximate value for the induced currents. This would suggest the use of the concept of "a priori knowledge of the solution" for improving the rate of the convergence of the moment method. In a recent paper[22], this technique has been applied to the case of two-dimensional scattering from a perfectly conducting strip illuminated normally by a plane wave. This method is essentially a combination of the physical optics approximation and the point matching technique. On the illuminated side of the conducting surface, the physical optics current has a value,

$$\tilde{J}_{po} = 2\hat{n} \times H_i$$

Thus, assuming the total current in the form of,

$$\tilde{J} = \tilde{J}_{po} + \tilde{J}_{d}$$

the J_{po} is readily known from the incident field of the source and J_d is the only unknown current to be determined. However, the difference current J_d is due to the surface discontinuities and the shadow region and is localized to the edges of the scatterer. Thus, as the scatterer size increases, the contribution of the difference current decreases, but due to its localized nature can always be determined by choosing enough matching points near the edges.

In chapter three of this thesis, the above method is applied to the problem of two-dimensional scattering from circular cylindrical reflectors in the presence of an electric line source. The improvement in the rate of the convergence is demonstrated for a medium size reflector and finally the method is applied for treating large reflectors which are not solvable by a direct moment method.

Ordinarily, the ultimate goal in the antenna design and studies is to seek a way of controling the radiation characteristics of the antenna, in a predetermined manner. This task is usually achieved by introduction of some passive elements in the path of rays, for phase correction. For example, reflectors, dielectric materials or metal plates are quite suitable for this purpose, since they perform the same basic function, that is, a modification of the phase.

So far, a number of theoretical and experimental investigations have been carried out to study the possible control of antenna characteristics by dielectrically loading the conventional antennas and reflectors. Among these works are; the dielectric loaded axial slot antennas on a circular cylinder, which is solved analytically within certain simplifying assumptions, for investigation of the external slot admittance[23], investigations on the optimum dielectric thickness for maximum power radiation[24], experimental studies of the variously shaped dielectric inserts on the radiation patterns of loaded horns and corner reflectors[25,26,27], the effects of the symmetrical loading of a horn aperture with E-plane dielectric slabs for obtaining high aperture efficiencies[28], radiation by dielectric-loaded spherical, circular cylindrical antennas, wedges and corner reflectors[29] and resonance effects due to dielectric loading in cylindrical and spherical slots[30,31].

Due to the scarcity of quantitative theoretical data on the effects of various parameters involved in scattering from dielectrically loaded cylindrical reflectors, chapter four of this work is oriented to study numerically the general scattering properties of this class of reflectors. The advantages of the numerical method are based on the fact that the approximate solution tends to converge to the exact solution as the number of matching points increases and has the flexibility of treating problems of arbitrary cross sections and two-dimensional source distributions.

Another method of controling the radiation characteristics of the antennas, as stated before, is by changing the geometrical structure of the antenna system. Reflectors and metal plates are quite suitable for this purpose, and if done judiciously, it is possible to shape the radiation pattern of the radiating element in a desired fashion.

So far, most of the information available for conventional reflectors, has been obtained through the use of ray theory, which is only applicable if the characteristic dimension of the reflector is much greater than the wavelength. However, for dimensions of the order of the wavelength, this approach most likely is not able to yield results with acceptable accuracy. The purpose of the fifth chapter is to investigate the radiation characteristics of a few commonly used types of reflectors of moderate cross section sizes, by means of numerical technique. It also studies the possibilities of beam shaping by surface deformations, and improving the radiation characteristics of these types of reflectors in a desired manner.

In summary, this thesis considers the numerical investigation of the scattering properties of circular cylindrical reflectors. The integral equation approach has been adopted and the problem is solved by the method of moment. In chapter two, the problem is rigorously formulated in the form of a set of integral equations. The general form of the equations are then reduced to the forms describing the two-dimensional case. Particular attention is paid to the case of transverse magnetic(TM)scattering which is the general topic of this study. Chapter three deals with the numerical processing of the integral equations. The advantages and limitations of the method of moment are described and the concept of using "a priori knowledge of the solution" is applied for extending the range of applicability of the numerical technique to large reflectors without excessive storage and computer running time. Chapter four is intended to investigate the general scattering properties of dielectrically loaded cylindrical reflectors. Since all the effects cannot be observed in one geometry, a number of selected geometries are investigated. One of the aims of this chapter is the determination of the most effective parameters for investigation of overall radiation characteristics of such loaded reflectors. A knowledge of the effects of the parameters, would in fact, be of great help to the designer to obtain a predetermined radiation pattern. Chapter five is intended to show the possibilities of beam shaping by deforming the reflecting surfaces of the conventional reflectors. Certain geometries are selected and the radiation patterns are compared against each other. The computed results are discussed in chapter six.

CHAPTER TWO

INTEGRAL EQUATION FORMULATION OF SCATTERING PROBLEMS

2.1 Introduction

From the point of view of classical theory,any precise treatment of electromagnetic scattering and diffraction phenomena,generally involves the complete solution of a boundary value problem. However, there are few cases in which this rigorous approach is productive. The best one can do then, is to find an approximate solution to the original problem.

In recent years, due to the high speed and large storage capacity of digital computers,considerable interest has been shown in the use of numerical solution techniques to the evaluation of scattering and radiation problems. The scattering properties of complex bodies can be computed with very high accuracy through the application of numerical techniques. However,the differential equation formulation is not efficient for this purpose. Of the many approaches available,considering the adaptability of the formulation for computer use,the integral equation representation has proven to be the most efficient form for the computer solution.

It is the purpose of this chapter to reduce the original vectorial boundary value problems associated with the electromagnetic scattering phenomena,to a set of integral equations,most suitabale for numerical solution using a digital computer.

2.2- Integral representation of the EM fields.

Consider the region V, illustrated in figure 2.1, bounded by the surfaces S_1 and S_2.

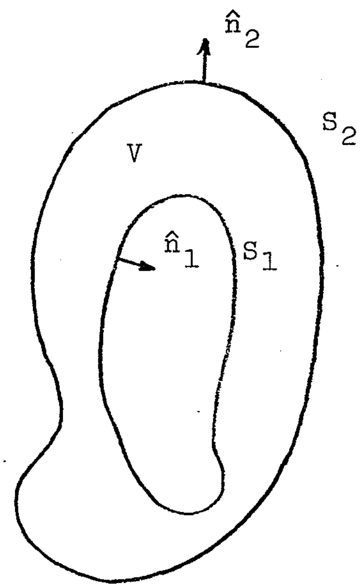

Fig. (2.1) The general representation.

It is assumed that constitutive parameters of region V differ from those of the medium in the surrounding space. Vectors $\hat{n}_1$ and $\hat{n}_2$ are the unit vectors normal to the bounding sufaces, directed out of the region V.

Considering the time harmonic case, in a linear, isotropic and homogeneous region, the field quantities must obey Maxwell's equations,

$$\nabla \times \tilde{H} = j\omega\varepsilon \tilde{E} + \tilde{J} \qquad (2.1)$$

$$\nabla \times \tilde{E} = -j\omega\mu \tilde{H} - \tilde{K} \qquad (2.2)$$

$$\nabla \cdot \tilde{E} = \rho/\varepsilon \qquad (2.3)$$

$$\nabla \cdot \tilde{H} = m/\mu \qquad (2.4)$$

where,

E electric field intensity.

H magnetic field intensity.

J electric current density.

K magnetic current density.

ρ electric charge density.

m magnetic charge density.

ε permittivity of the medium.

μ permeability of the medium.

complemented by the relationships defining the conservation of charge, given by,

$$\nabla \cdot \tilde{J} = - j\omega\rho \qquad (2.5)$$

$$\nabla \cdot \tilde{K} = - j\omega m \qquad (2.6)$$

where an exp(jωt) time dependence is assumed and suppresed throughout for convinience. A simple vector manipulation of the equations 2.1 and 2.2 leads to,

$$\nabla \times \nabla \times \tilde{E} - k^2 \tilde{E} = - j\omega\mu \, \tilde{J} - \nabla \times \tilde{K} \tag{2.7}$$

$$\nabla \times \nabla \times \tilde{H} - k^2 \tilde{H} = - j\omega\varepsilon \, \tilde{K} + \nabla \times \tilde{J} \tag{2.8}$$

where,

$$k = \omega\sqrt{\varepsilon\mu} \tag{2.9}$$

using the vector Green's theorem,

$$\int_V (\tilde{A} \cdot \nabla \times \nabla \times \tilde{B} - \tilde{B} \cdot \nabla \times \nabla \times \tilde{A}) \, dv = \int_S (\tilde{B} \times \nabla \times \tilde{A} - \tilde{A} \times \nabla \times \tilde{B}) \cdot \hat{n} \, ds \tag{2.10}$$

which is the result of the vector identity,

$$\nabla \cdot (\tilde{A} \times \nabla \times \tilde{B}) = \nabla \times \tilde{A} \cdot \nabla \times \tilde{B} - \tilde{A} \cdot \nabla \times \nabla \times \tilde{B} \tag{2.11}$$

and application of the divergence theorem,it is possible to transform the vector wave equations 2.7 and 2.8 into a set of integral equations.[32, 13] The equation 2.10 is valid for any two arbitrary continuous vector functions of position with continuous first and second derivatives within the region V and on the bounding surface S. The approach adopted is to assume $\tilde{E}$ and $\psi\hat{a}$ for $\tilde{A}$ and $\tilde{B}$ respectively. Here $\hat{a}$ is a unit vector of arbitrary orientation and $\psi\,\hat{a}$ is the Green's function representing the magnetic vector potential of a point current source in a homogenous space and is given by,

$$\Psi = \exp(-jk|\tilde{r}-\tilde{r}'|)/|\tilde{r}-\tilde{r}'| \qquad (2.12)$$

with $\tilde{r}$ and $\tilde{r}'$ representing the positional vectors of the observation and source points respectively. The discontinuity of the field on the boundary surfaces marking the abrupt change of the constitutive parameters of two different media and the singularity of (ψ) at $\tilde{r}=\tilde{r}'$ restrict the usefulness of the equation 2.10. However the latter difficulty can be avoided by making explicit use of the vector differential equation for the Green's function given by,

$$\nabla \times \nabla \times (\Psi\hat{a}) - k^2(\Psi\hat{a}) - \nabla\,\nabla\cdot(\Psi\hat{a}) = 4\pi\delta(\tilde{r}-\tilde{r}')\hat{a} \qquad (2.13)$$

where (δ) represents the Dirac's Delta function.

In order to overcome the problem of the discontinuous nature of the field on the boundary surfaces, we surround the observation point P, by a small sphere S_4 of radius r_o and consider the portion V_r of V which is bounded by the surfaces S_1, S_2, S_3 and S_4, figure 2.2.

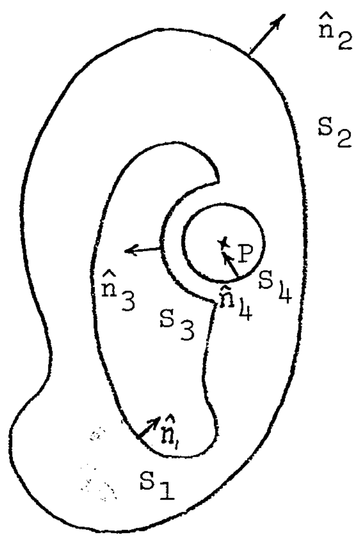

Fig. 2.2 Notation for Green's theorem

Substituting $\tilde{E}$ and $\Psi\,\hat{a}$ for $\tilde{A}$ and $\tilde{B}$ respectively in 2.11 leads to,

$$\int_{V_r} \{(\Psi\cdot\hat{a})\cdot\nabla'\times\nabla'\times\tilde{E} - \tilde{E}\cdot\nabla'\times\nabla'\times(\Psi\cdot\hat{a})\}\,dv' =$$

$$\Sigma_i \int_{S_i} \{\tilde{E}\times\nabla'\times(\Psi\cdot\hat{a}) - (\Psi\cdot\hat{a})\times\nabla'\times\tilde{E})\}\cdot\hat{n}'\,ds'$$

(2.14)

where primes are used to indicate vector operations in source coordinates and $\hat{n}_i$'s are the normal unit vectors to the bounding surfaces of like subscript, directed out of the region V_r and can take on the representations $\hat{n}_1, \hat{n}_2, \hat{n}_3$, and $\hat{n}_4$. After extensive vector manipulations[32] and making use of the equations 2.1, 2.3 and 2.7, the equation 2.14 reduces to,

$$\int_{V_r} \{ j\omega\mu \tilde{J} \Psi + \tilde{K} \times \nabla'\Psi - (\rho/\varepsilon) \nabla'\Psi \} dv' =$$

$$\sum_i \int_{S_i} \{ j\omega\mu(\hat{n}' \times \tilde{H})\Psi - (\hat{n}' \cdot \tilde{E})\nabla'\Psi\} ds' \qquad (2.15)$$

In the limit, when $(r_o = |\tilde{r} - \tilde{r}'|)$ approaches zero, the contribution of the first term of the surface integral over the surface S_4 goes to zero, since S_4 decreases as r_o^2 and (ψ) varies as $1/r_o$. The contribution of the second term over surfaces S_4 and S_3 is shown to be[13],

$$I = - \tilde{E}(\tilde{r}) \{ 4\pi - \Omega \} \qquad (2.16)$$

where (Ω) represents the absolute value of the solid angle subtended by a surface like S at $\tilde{r}$, in the limit as r vanishes. The value of (Ω) depends on the location of the observation point P and is given by,

$$\Omega = \begin{cases} 2\pi & \text{on the bounding surface} \\ 0 & \text{anywhere within V} \end{cases} \qquad (2.17)$$

Application of the foregoing argument to 2.15 results in an integral equation representing the electric field at any point within region V in terms of the sources existing in V and the fields on the bounding surfaces, namely,

$$\tilde{E}(\tilde{r}) = -(T/4\pi)\int_V (j\omega\mu\ \tilde{J}\Psi + \tilde{K} \times \nabla'\Psi - \rho/\varepsilon\ \nabla'\Psi)\ dv'$$

$$-(T/4\pi)\int_{S_1+S_2} \{\ j\omega\mu(\hat{n}'\times\ \tilde{H})\Psi - (\hat{n}'\times \tilde{E})\times \nabla'\Psi$$

$$-\ (\hat{n}'\cdot\ \tilde{E})\nabla'\Psi\ \}\ ds' \qquad (2.18)$$

where,

$$T = 1/\{1 - \Omega/4\pi\}$$

and keeping in mind that an infinitesimal region surrounding point P is to be excluded.

By a similar argument or using the duality of Maxwell's equations, one arrives at an expression for magnetic field at any point within V given by,

$$\tilde{H}(\tilde{r}) = -(T/4\pi)\int_V (-j\omega\varepsilon\ \tilde{K}\Psi + \tilde{J} \times \nabla'\Psi + m/\mu\ \nabla'\Psi)\ dv'$$

$$+(T/4\pi)\int_{S_1+S_2} \{\ j\omega\varepsilon(\hat{n}'\times \tilde{E})\Psi + (\hat{n}'\times \tilde{H})\times \nabla'$$

$$+\ (\hat{n}'\cdot\ \tilde{H})\nabla'\Psi\ \}\ ds' \qquad (2.19)$$

For a source free region, the equations 2.18 and 2.19 reduce to,

$$\tilde{E}(\tilde{r}) = -(T/4\pi) \int_{S_1+S_2} \{ j\omega\mu(\hat{n}' \times \tilde{H})\Psi - (\hat{n}' \times \tilde{E}) \times \nabla'\Psi - (\hat{n}' \cdot \tilde{E})\nabla'\Psi \} ds' \quad (2.20)$$

$$\tilde{H}(\tilde{r}) = (T/4\pi) \int_{S_1+S_2} \{ j\omega\varepsilon(\hat{n}' \times \tilde{E})\Psi + (\hat{n}' \times \tilde{H}) \times \nabla'\Psi + (\hat{n}' \cdot \tilde{H})\nabla'\Psi \} ds \quad (2.21)$$

which represent the effect of sources located outside the region V. Considering the analogy of the integrands of the volume and surface integrals in 2.18 and 2.19, one can represent the effect of sources existing outside volume V by surface distribution of charges and currents on the bounding surface S, namely,

$$\hat{n}' \times \tilde{H} = \tilde{J}_s \quad (2.22)$$

$$- \hat{n}' \times \tilde{E} = \tilde{K}_s \quad (2.23)$$

$$\varepsilon(\hat{n}' \cdot \tilde{E}) = \rho_s \quad (2.24)$$

$$\mu(\hat{n}' \cdot \tilde{H}) = m_s \quad (2.25)$$

where subscript s denotes surface distribution.

For an unbounded region (when S2 in figure 2.1 recedes to infinity) and assuming the sources in V being confined to a region of finite extent, the radiation condition requires that the contribution of the surface integral over S_2 be independent of the bounded sources, that is,

$$\tilde{E}(\tilde{r}) = T\,\tilde{E}_i(\tilde{r}) - (T/4\pi)\int_V \{j\omega\mu\,\tilde{J}\,\Psi + \tilde{K}\times\nabla'\Psi$$

$$-(\rho/\varepsilon)\nabla'\Psi\}\,dv' - (T/4\pi)\int_{S_1}\{j\omega\mu\,\tilde{J}_s\,\Psi + \tilde{K}_s\times\nabla'\Psi$$

$$-(\rho_s/\varepsilon)\nabla'\Psi\}\,ds' \qquad (2.26)$$

$$\tilde{H}(\tilde{r}) = T\,\tilde{H}_i(\tilde{r}) + (T/4\pi)\int_V \{-j\omega\varepsilon\,\tilde{K}\,\Psi + \tilde{J}\times\nabla'\Psi$$

$$+(m/\mu)\nabla'\Psi\}\,dv' + (T/4\pi)\int_{S_1}\{-j\omega\varepsilon\,\tilde{K}_s\Psi + \tilde{J}_s\times\nabla'\Psi$$

$$+(m_s/\mu)\nabla'\Psi\}\,ds' \qquad (2.27)$$

where use has been made of the equations 2.22 to 2.25 and $\tilde{E}_i$, $\tilde{H}_i$ are the fields due to the sources (if any) lying out of the surface S_1.