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ELECTROMAGNETIC BEAM SCATTERING AT A PLANAR INTERFACE

by

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A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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TO MY MOTHER

My Brother Samir, Sisters Magda,

Marriam and Samara

ABSTRACT

The aspects of electromagnetic beam wave scattering at a planar interface separating two lossless, homogeneous, isotropic dielectric media are considered. A general procedure is presented wherein the reflected and the transmitted fields for any well defined, symmetric and collimated beam can be thoroughly analysed. The fields are expressed as exact integral representations in terms of a continuous plane wave spectrum, where the spectral density functions play a substantial role. Particular emphasis is given to the Gaussian profile which represents the dominant beam mode in the radiation produced by a laser oscillator.

The range of regular incidence, i.e. below the critical angle of total internal reflection is considered first. Newly identified complex Gaussian beam modes are found to result from the interaction process. The existence of an angular beam shift, of both the reflected and transmitted fields, has been verified by virtue of these higher order beam modes. The different aspects of this angular beam shift are analysed and discussed. At polarizing incidence, it is shown that there still exists a reflected field whose characteristics are analysed and described in terms of these higher order reflected beam modes. By considering the problem for beams with non-Gaussian profiles, such as a Cauchy beam or a truncated plane wave, the generality of the reported phenomena is established. In particular, it is found that the angular beam shift is a characteristic of the reflection or refraction process for any well defined, symmetric and collimated beam.

Two aspects are considered for the total internal reflection regime. The Goos-Hänchen shift is analysed, in the range far beyond the critical angle, along with some aspects of the transient behaviour of a Gaussian beam upon total internal reflection. The penetration of the field in the rarer medium, due to a Gaussian beam that is incident at or beyond the critical angle, is also considered. In particular, the field features in the rarer medium, with its different wave species are thoroughly and carefully analysed. While the obtained results are in agreement with previous predictions, they are, as expected, in contradiction with geometrical optics predictions.

RÉSUMÉ

Considerons les propriétés d'un faisceau électromagnétique dans le plan de séparation de deux milieux diélectriques sans pertes, homogènes et isotropiques. Afin de permettre une analyse complète du champ réfléchi et du champ transmis, une méthode générale est introduite; elle est applicable à tout faisceau symétrique et collimaté correctement défini. Les champs sont exprimés par des intégrales exactes qui introduisent un spectre d'ondes planes, dans lequel les fonctions de densité spectrale ont un role important. Le cas du faisceau avec un profil Gaussien est traité en détail, êtant donné qu'un oscillateur laser rayonne un faisceau dont le mode dominant a un profil Gaussien.

En premier lieu, l'incidence à un angle plus petit que l'angle critique de réflection totale est considérée. Il est etabli que les modes complexes recemment découverts résultant de l'interaction entre les faisceaux. L'existence d'un déplacement angulaire des faisceaux réfléchi et transmis a été vérifiée en vertu des modes d'ordres élevés. Les différents aspects de ce déplacement angulaire sont analysés et discutés. Il est démontré qu'un champ réfléchi existe, même lorsque l'angle d'incidence est polarisant ; les caractéristiques de ce champ réfléchi sont analysées et décrites en fonction des modes d'ordres élevés. La généralité du phénomène rapporté est établie en considérant des faisceaux qui n'ont pas un profil Gaussien, tels que le faisceau de Cauchy ou qu'une onde plane tronquée. En particulier, il est démontré que le déplacement angulaire est un phènomène caractéristique de la réflection ou de la réfraction de tout faisceau symétrique et collimaté correctement défini.

Dans le cas de réflection totale, deux aspects sont considérés. Le déplacement de Goos-Hänchen est analysé pour un angle d'incidence plus grand que l'angle critique de réflection totale et une analyse du comportement en régime transitoire d'un faisceau Gaussien à incidence critique est présentée. En particulier, les propriétés du champ dans le milieu le moins dense, ainsi que les différentes sortes d'ondes du champ, sont analysées en détail. Bien que les résultats obtenus sont en accord avec les récentes prédictions, ils sont bien entendu en contradiction avec les résultats fournis par l'optique géométrique.

ZUSAMMENFASSUNG

Probleme der elektromagnetischen Strahlenbündelstreuung an einer ebenen Grenze, die zwei verlustlose, homogene, isotrope dielektrische Medien trennt, wird untersucht. Ein allgemeiner Lösungsweg wird angegeben mit dem die reflektierten und gebrochenen Felder für beliebig klar definierte, symmetrische und kollimierte Bündel gründlich analysiert werden können. Die Felder werden als exakte Integrale über kontinuierliche Spektren ebener Wellen ausgedrückt, wobei die spektralen Dichtefunktionen eine wesentliche Rolle spielen. Besondere Betonung wird der Gaußschen Verteilung geschenkt, da diese die dominante Wellenmode im Strahlungsfelde eines Lasers darstellt.

Der reguläre Einfallsbereich, d.h. jener unterhalb des kritischen Winkels der totalen inneren Reflexion,wird zuerst untersucht. Neu identifizierte komplexe Gaußsche Bündelmoden wurden als Ergebnis der Bündelstreuung gefunden. Die Existenz einer winkelabhängigen Versetzung des reflektierten sowie auch des gebrochenen Strahlenbündels wird mit Hilfe dieser neuen Bündelmoden höherer Ordnung nachgewiesen. Die verschiedenen Eigenschaften dieser winkelabhängigen Strahlversetzung werden untersucht und besprochen. Für polarisierenden Einfall wird gezeigt, daß ein reflektierter Strahl eben doch existiert und dessen Eigenschaften mit Hilfe dieser reflektierten Bündelmoden höherer Ordnung analysiert und beschrieben werden können. Indem das Problem für Strahlenbündel ohne Gaußsche Verteilung, wie z.B. das Cauchy Bündel oder die begrenzte ebene Welle, untersucht wurde, konnte die Allgemein-

gültigkeit der berichteten Phänomene erstellt werden. Im besonderen zeigte sich, daß die winkelabhängige Strahlversetzung eine charakteristische Eigenschaft des Reflexions und Brechungsprozesses eines klar definierten, symmetrischen und kollimierten Strahlenbündels darstellt.

Im Bereich der totalen inneren Reflexion werden zwei Probleme behandelt. Die Goos-Hänchen Verschiebung wird untersucht im Bereiche oberhalb des kritischen Winkels der Totalreflexion zusammen mit einigen Problemen des Impulsverhaltens eines Gaußschen Bündels unter total reflektierendem Einfall. Das Eindringen des Feldes in das dünnere Medium für ein Gaußsches Bündel für Einfall bei oder oberhalb kritischen Winkels wird ebenfalls untersucht. Im besonderen werden die Feldeigenschaften im dünneren Medium mit ihren verschiedenen Wellenarten gründlich und sorgfältig bearbeitet. Obwohl die erarbeiteten Ergebnisse mit kürzlich gestellten Mutmaßungen übereinstimmen; stehen solche, wie erwartet, nicht im Einklang mit Annahmen der geometrischen Optik.

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Yahia M. M. Antar

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LIST OF SYMBOLS

Unless otherwise stated, the symbols most commonly used in this thesis have the following meaning.

Greek Alphabet:

α_0	Far field diffraction angle for the fundamental Gaussian beam mode
α_t	Transmitted beam parameter defined by the quotient $(1/k_2 w_t)$
α_r	Reflected beam parameter defined by the quotient $(1/k_1 w_r)$
α_1, α_2	Phase factors for the transmitted field at critical incidence defined in equation (4.5d)
$\overline{\alpha}_1$	Constant for the beam field of equation (2.1)
$\alpha_{r_1}, \alpha_{r_2}, \alpha_{r_3}$	Parameters for the reflected field defined in equation (3.58d)
$\alpha_{i_{1/2}}$	Parameters for the incident field defined in equation (3.58a)
ψ_{refl}	Total reflected field
ψ_{inc}	Incident field
ψ_{trans}	Total transmitted field for regular refraction
$\psi(x_i, 0)$	Scalar amplitude of the field at the aperture plane as defined in equation (3.1)
ψ_{rn}	Reflected field n^{th} component for regular reflection $(n \geq 0)$
$\psi_{m,n}$	Higher order Gaussian conventional beam modes as defined in equation (2.10)

ψ_{tn}	Transmitted field n^{th} component for regular refraction ($n \geq 0$)
$\psi(r,z)$	Normalized fundamental Gaussian beam modes defined in equation (2.9)
$\hat{\psi}_n$	Complex eigensolutions to the paraxial wave equation and given by equation (3.37a)
ψ_t	Transmitted field for the Gaussian beam at and around critical incidence
$\psi_{tA};\psi_{tN}$	Transmitted field values at critical incidence as defined in Table 4.2
$\overline{\Delta x_r}$	The displacement of the reflected beam centre at the interface in the x_r direction
$\overline{\overline{\Delta x_r}}$	Location of the maximum for the first reflected field component
$\overline{\Delta x_{rB}}$	The displacement of the reflected field amplitude maximum at the Brewster angle θ_B
$\overline{\Delta\theta_r}$	Angular beam deflection for the reflected field
$\overline{\Delta\theta_t}$	Angular beam deflection for the transmitted field
$\overline{\Delta\theta_{rB}}$	Angular deflection of the field maximum at the Brewster angle θ_B
$\frac{\partial}{\partial x}$	First partial derivative
$\frac{\partial^2}{\partial x^2}$	Second partial derivative
∇^2	Laplacian operator
$\delta_{n,m}$	Kronecker delta (= 0 is $n \neq m$; = 1 if $n = m$)
$\delta(t)$	Dirac delta function
δ	A parameter related to the deviation of the beam incidence angle θ_i from the critical angle θ_c as is defined in equation (4.4)

$\rho(\gamma)$	Fresnel reflectance as defined in equation (3.8a) and (3.8b)
ω	Frequency of operation in radians
Ω	Complex phase parameter for the transmitted beam at critical incidence defined in equation (4.5c)
ζ_i, ξ_i	Transverse and longitudinal coordinates used in equation (2.1) for the trapezoidal beam description
λ_1	Wave length in medium (1) with wave number k_1
β	Complex phase constant for the transmitted beam at critical incidence defined in equation (4.5e)
β_1	Wave number in the z direction for medium (1) with k_1
β_2	Wave number in the z direction for medium (2) with k_2
β_i'	Normalized distance in the transverse direction for the trapezoidal beam defined in equation (2.1)
β_r	Phase parameter defined in equation (3.11c)
$\overline{\beta_r}$	Phase parameter for the reflected field of a limited plane wave defined in equation (3.58d)
β_i	Phase parameter for the incident field defined in equation (3.42)
β_t	Phase parameter for the transmitted field defined in equation (3.45)
$\overline{\beta_i}$	Phase parameter for the incident field due to a limited plane wave defined in equation (3.58a)
$\mathcal{L}^{-1}$	Inverse Laplace transform

$\sum_n$	Overall summation from n = 0 to n = ∞
$\varepsilon_1, \varepsilon_2$	Dielectric constants for medium (1) and medium (2) respectively
$\phi(z)$	Phase shift for the fundamental beam mode as in equation (2.9)
$\phi_{n,m}(z)$	Phase shift for the higher beam modes as defined in equation (2.12)
$\phi(\gamma)$	Spectral density function as defined in equation (3.4)
$\hat{\phi}_n(x), \hat{\phi}^*_n(x)$	The bi-orthogonal set to $\hat{\psi}_n(x,z)$, and its complex conjugate, as defined in equation (3.38)
γ	wave number in the x direction for any angle θ, and equals $k_1 \sin\theta$
γ_i	Wave number in the x direction for the incidence angle θ_i
γ_c	Wave number in the x direction at the critical angle θ_c
γ_g	Wave number in the x direction at grazing incidence ($\theta_i = \pi/2$)
$\Gamma(n)$	The Gamma function
θ_i	Angle of incidence
θ_r	Reflection angle
θ_t	Refraction angle
θ_c	Critical angle of total internal reflection
θ_B	Brewster's angle
$\overline{\theta}_i$	Phase function for the trapezoidal beam in equation (2.1)
σ	Integration variable used in equation (3.11c)
τ	Parameter defined in equation (4.3)
τ_t	Integration variable defined in equation (3.32a)

Latin Alphabet (Upper Case)

A	Half of the width for the extension of the limited plane wave
A_t	Normalizing parameter for the Cauchy profile transmitted field as is defined in equation (3.45d)
A_n	Normalizing constant for the nth reflected complex beam mode as defined in equation (3.35c)
A_L	Normalization constant for the lateral wave field as in equation (2.15)
$A_\perp$	Normalizing factor for the trapezoidal beam in equation (2.1)
$A(\beta_i')$	Amplitude function for the trapezoidal beam of equation (2.1)
$B_n(\theta_i)$	Coefficients in the expansion of the reflectance around γ_i as defined in equation (3.11b), $n \geq 0$
C_n	Normalizing constant for the nth refracted complex beam mode as defined in equation (3.35d)
D	Parameter related to the beam shift for the Cauchy profile defined in equation (3.51)
D_c	Lateral beam displacement in the positive x direction at critical incidence (The Goos-Hänchen shift)
D_n	Parabolic cylinder function or Weber function
$D_\perp$	The Goos-Hänchen shift for normal polarization at $\theta_i >> \theta_c$
$D_\parallel$	The Goos-Hänchen shift for parallel polarization at $\theta_i >> \theta_c$
E_y^i	Electric field for the trapezoidal beam in equation (2.1)

F_0	Normalizing constant for the incident field amplitude distribution at the aperture
$\bar{F}(s)$	Analytic continuation of the reflected field expression in the complex s plane for $\theta_i >> \theta_c$
F_r	Correction factor for the reflected field of a Gaussian beam as defined in equation (3.20b)
F_{rb}	Correction factor for the Cauchy beam reflected field as defined in equation (3.50c)
F_{01}	A parameter for the reflected field of the Cauchy profile, defined in equation (3.51)
$G(\gamma)$	Phase function related to the reflection coefficient for $\theta_i >> \theta_c$, defined in equation (3.59)
$G_n(\theta_i)$	Coefficients in the expansion of $G(\gamma)$ around γ_i as defined in equation (3.60)
$G(\bar{r})$	Free space Green's function
$H_{n,m}$	Hermite polynomial as defined in equation (3.16b)
$\bar{H}(t)$	Heaviside unit step function
I_n	Modified Bessel function of order n and of the first kind
J_n	Bessel function of order n
K_n	Constant resulting from the biorthogonal product in equation (3.39)
L, L_1, L_2	paths along the rays describing the lateral wave field in Fig.2.3
$O(x)$	Order of magnitude of
$P(z)$	Complex phase shift associated with a conventional beam mode in equation (2.5)

$Q(z)$	Complex beam parameter as defined in equation (2.6)
$\bar{Q}\ ,\bar{Q}_0$	Complex parameters for the complex beam modes defined in equation (3.37b)
Q_r,Q_t	Parameters for the reflected and refracted field as defined in equations (3.44c) and (3.45c) respectively
R_n	Remainder in the series expansion of the error function
$R(z)$	Radius of curvature of the beam wavefront as in equation (2.7b)
$S_{n,m}$	Lommel's functions
S_c	Displacement of the transmitted beam amplitude maximum from the centre of the incident beam at $z = 0$, and for critical incidence
$T(\gamma)$	Fresnel transmittance as defined in equation (3.8c)
$T_n(\theta_t)$	Coefficients in the series expansion of the reflectance as defined in equation (3.32b)
$U(x,y,z)$	Complex amplitude function for a conventional beam mode as expressed by equation (2.5)

Latin Alphabet (Lower Case)

a_i,a_r,a_t	Complex parameters for the incident, reflected and refracted fields for the Cauchy profile which are defined in equations (3.4c), (3.44c) and (3.45c) respectively
$\bar{a}$	Dislocation of the origin in complex space
$a_{\perp}$	Correction factor for the trapezoidal beam

$\hat{a}, \hat{a}_0$	Constants defined in equation (3.64b)
b	Beam width for the incident Cauchy profile at the waist $(z_i, x_i = 0)$
b_t	Modified beam width at the virtual waist of the transmitted field for the Cauchy profile
b_c	Complex beam width for the Cauchy beam defined in equation (3.51)
b_n	Ratio of the coefficient of the expansion $B_n(\theta_i)/B_0(\theta_i)$
$\bar{c}(z), \bar{c}^*(z)$	Complex parameter for the complex beam modes, and its complex conjugate, as defined in equation (3.37b)
c	A parameter for the reflected field of the Cauchy profile, defined in equation (3.51)
c_1	Speed of light in medium (1)
$\mathrm{erf}(\gamma)$	The error function of argument γ
$f(\theta_i)$	The second coefficient in the expansion of $r(\gamma)$ around the branch point $\gamma = \gamma_c$ as is defined by equation (4.5j)
$g(x_i, \underline{w})$	Amplitude distribution function for the beam
$g_n(t_r)$	Modified beam mode of order n, defined in equation (3.17)
$\hat{g}_n$	Coefficients of the power series expansion of β_2 around the branch point $\gamma_c = k_2$ as given by equation (4.5h)
$\bar{g}_n$	Coefficients in the series expression of the expansion of β_2 in powers of $\tau^{\frac{1}{2}}$ around the branch point $\gamma_c = k_2$ as defined in equation (4.5g)
h_1	Height of the incident beam waist above the interface as shown in Fig.3.1
h_2	Height of the virtual aperture for the transmitted beam above the interface as shown in Fig.3.3

k_1, k_2	Wave numbers in medium (1) and medium (2) respectively
ℓ	The distance between the origin of the transmitted beam coordinates and the z axis as shown in Fig.3.3
$\bar{\ell}$	Phase factor defined in equation (3.50b)
n	Refractive index
$n!$	Factorial n
$P_0, P_{11}, P_{12}, P_{22}, P_3$	Coefficients in the expansion of equation (3.21)
$r(\gamma)$	Correction factor for the reflectance $\rho(\gamma)$ defined in equation (4.1b)
r_1	Distance in cylindrical coordinates
$\bar{r}$	Distance from the observation point to the complex origin as defined in (2.12)
s	Complex frequency variable used in Laplace transformation
t	Time variable
t_r	Distance along the x_r direction normalized to the complex reflected beam width w_r
t_t	Distance along the x_t direction normalized to the complex transmitted beam width w_t
t_0	Different time variable related to t by the relation $t_0 = t - z_r/c_1$
v_1, v_2	Constants defined in equation (3.67b)
w	Beam width for the Gaussian profile away from the waist (in *chapter four* w replaces w_0)
w_0	Beam width for the Gaussian profile at the waist
w_i	Complex beam width for the incident beam at any constant plane $z_i \neq 0$ as is defined in equation (3.14b)

w_r	Complex beam width for the reflected beam at any constant plane $z_r \neq 0$ as is defined in (3.15b)
w_t	Complex beam width for the refracted beam at any constant plane $z_t \neq 0$ (for regular refraction) defined in equation (3.31a)
w_0'	Modified transmitted beam width at the virtual aperture in Fig.3.3 as is defined in equation (3.31b)
$\bar{w}$	Positive real constant in the complex s plane
x,y,z	Cartesian coordinates
x_i, z_i	Incident beam coordinates as shown in Fig.3.1 and Fig.3.3
x_r, z_r	Reflected beam coordinates as shown in Fig.3.1 and Fig.3.3
x_t, z_t	Transmitted beam coordinates as shown in Fig.3.3 and as defined by equations (3.29)

chapter one

INTRODUCTION

When a plane wave strikes a plane interface separating two homogeneous, lossless, isotropic dielectrics, it gives rise to reflected and refracted components in addition to the incident wave (Fig.1.1). The reflection and refraction phenomena depend on the dielectric constants of the media ε_1 and ε_2, the range of the incidence angle θ_i, and on the polarization. If incidence is from the first medium, with ε_1, to the second medium, with ε_2, then regular reflection and refraction occurs for the whole range of θ_i, i.e. $0<\theta_i<\pi/2$, if $\varepsilon_1<\varepsilon_2$. In case ε_1 is greater than ε_2, regular reflection and refraction takes place in the range $0<\theta_i<\theta_c$, where θ_c is the critical angle of total internal reflection and is defined by $\sin^{-1}\theta_c = (\varepsilon_2/\varepsilon_1)^{\frac{1}{2}}$. In the range $\theta_i \geq \theta_c$ there is total reflection, and no propagating field is transmitted in the less dense medium $(\varepsilon_2<\varepsilon_1)$. For the case of parallel polarization, total transmission occurs at the Brewster angle $\theta_B = \tan^{-1}(\varepsilon_2/\varepsilon_1)^{\frac{1}{2}} < \theta_c$, and the reflected field vanishes.

If the incident field is a beam wave, then there are different phenomena involved due to the limited nature of the field. These differences appear for the regular reflection and refraction ranges, as well as the total internal reflection regime. The study described here deals with these different aspects. Investigation of the problem for beam waves is of more interest and practical importance since beams describe more realistically than plane waves or point sources the fields associated with large aperture antennas or laser optical systems.

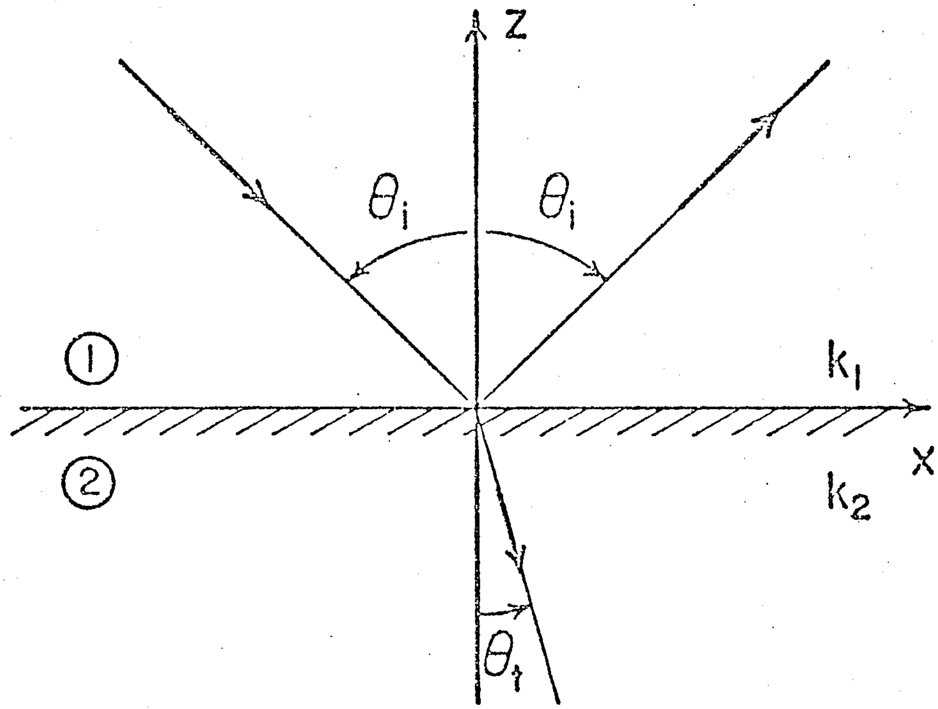

Fig. 1.1 PLANE WAVE INTERACTION

Snell's Law: $k_1 \sin\theta_i = k_2 \sin\theta_t$

Brewster's angle : $\theta_B \equiv \tan^{-1}(k_2/k_1)$

Critical angle : $\theta_c = \sin^{-1}(k_2/k_1)$

Most of the earlier studies were concerned with the phenomena of reflection and refraction of a light beam incident upon a prism in the range of the critical angle of total reflection. Three important features of the problem were extensively investigated. Total internal reflection of a beam at a dielectric interface gives rise to the lateral beam displacement (Fig.1.2), or the Goos-Hänchen shift as well as secondary effects, which have found many applications in several fields. A comprehensive review of most investigations regarding this area was given in a dissertation by Lotsch [37], and the interested reader is referred to that work for earlier bibliography. However, research into this area was continued after that without interruption, to clarify some unresolved questions regarding effects around the critical angle. Horowitz and Tamir [27,29], and Horowitz [28,30] very recently developed an elegant approach for the treatment of these phenomena, that, in addition to clarifying the critical angle effects, provided some new results and explained the relation between different features involved in the interaction process. In view of the new development in laser technology and the importance attached to laser beams and their applications in optical fields, the Gaussian beam which describes the fundamental mode in laser oscillators gained substantial interest. The Gaussian profile was also considered by Ra et al [47] and Bertoni et al [7], who employed a mathematical model for the Gaussian beam as a complex ray [16] to discuss the reflected field as well as the penetration of energy in the rarer medium upon total internal reflection.

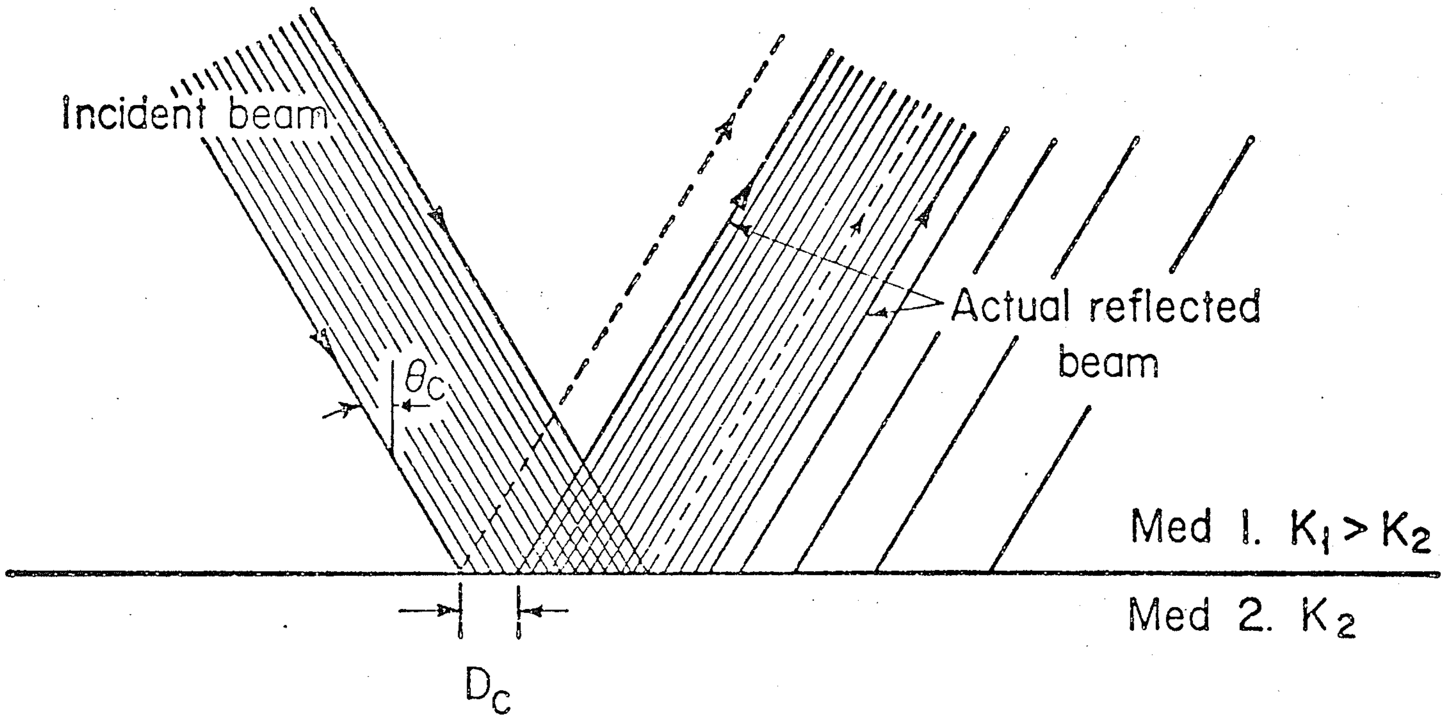

Fig. 1.2 The lateral displacement D_c of a light beam at a dielectric interface for total internal reflection ($\theta_i = \theta_c$)

However, a common feature in nearly all of these studies is that the case of regular incidence, i.e. excluding total internal reflection, has not been treated thoroughly or rigorously along with the phenomena related to it. In the design of optical circuit elements, such as couplers, filters, etc., which are used in beam guiding systems for transmission of light beams or millimeter waves, the properties of beam interaction at regular incidence are becoming of increasingly higher practical importance, and need to be investigated. In response to the above need, we first consider these aspects in the dissertation under presentation.

In order to study the propagation and scattering of beam waves in detail, knowledge of the aspects of beam propagation as well as thorough understanding of the different available analytical approaches for beam wave representations are essential. A trapezoidal incident beam was utilized in earlier studies, where small correction terms, that account for diffraction effects due to the boundedness of the incident beam were introduced [36]. Beams have also been analysed in terms of beam modes, where the fundamental mode, which is the dominant one in the coherent radiation generated by laser oscillators, has a Gaussian profile [34]. A mathematical model for this Gaussian profile in terms of complex rays was also introduced very recently [16]. A brief review of these different techniques is provided in the first part of *chapter two*.

The first feature of the present work deals with beam interaction for regular incidence. In order to analyse the phenomena involved and to gain meaningful insight in their aspects we adopt the modal analysis of beam waves. This approach was also utilized before in the analysis of total internal reflection of beam waves [27,28].

All symmetric collimated beams can be analysed in terms of an integral representation of continuous plane wave spectra. These representations describe the different characteristics of different beam profiles through their respective spectral density functions. These spectral density functions are normally concentrated and also symmetric about a central wave number. Moreover, the spectral density functions have a maximum at that wave number which corresponds to the central component of the spectrum and which has the same direction of propagation as that of the beam. The process of beam reflection and refraction is affected to an extreme degree by the characteristics of their spectral densities.

These aspects of the spectral functions are quite essential and at the same time rewarding in the present work, where a general formulation is being presented in Section 3.1 for beams that are symmetric and well defined, but with arbitrary cross-section. The reflected and transmitted fields are expressed as exact integral representations of plane wave spectra. Approximate explicit expressions for different field components, that are rather simple to evaluate analytically, are presented. In most cases, this approach leads to results that