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NEW CONFIGURATIONS FOR RC ACTIVE FILTERS

BY

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Abstract

This thesis deals with new RC active synthesis methods that yield low sensitivities.

The new synthesis techniques are developed from two new RC active network structures together with their dual counterparts. The structures are designed for the realization of voltage and current transfer function. Low sensitivities are obtained by large loop gain and/or optimum polynomial decompositions.

Apart from the low sensitivity feature, the proposed structures have many other figures of merit. The transfer function does not contain the driving-point parameters of the two-ports, and hence the realization of two RC two-ports is simple and straightforward. As for cascade realization, buffer amplifiers may not be needed. Since the sensitivities considered are not dependent on the zeros of the transfer function, pole-zero pairing is quite arbitrary. Also, the structures are conditionally stable and capable of realizing any rational transfer function except all-pass functions.

Acknowledgements

The author wishes to express his deep gratitude to Dr. H. K. Kim for his untiring supervision and encouragement.

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Contents

1 Introduction

Until recently, RC active filters were not considered of any practical use because of their high sensitivity and possible self-oscillation. A variation of the active gains due to temperature, bias-level, humidity, aging, or tolerance margin might completely change the network charateristics. The purpose of this study is to find some solutions to the sensitivity problems, particularly the sensitivity to the active elements.

Modern active network synthesis is usually considered to have started with the publication of a synthesis method using a negative-impedance converter (NIC) by Linvill in 1954 [14]. Synthesis techniques employing NIC's [11], [17], [18] are based on the idea of decomposing any arbitrary polynomial into the difference of two polynomials each having negative real roots. Although Horowitz decomposition [10] yields minimum pole sensitivity, it is not usually optimum for other kinds of sensitivities. For a biquadratic function, it can be easily shown that the quality factor sensitivity for Horowitz decomposition is approximately proportional to twice the quality factor, i.e., 2Q, which, in turn, indicates the limited value of the techique in the realization of high-Q networks.

Although the sensitivity is improved in RC-gyrator realization [11], [17], [18],the practical implementation of an ideal gyrator is not so economical as compared with a controlled source or a NIC. Another drawback is that the Calahan decomposition [4] which is optimal in RC-gyrator realization, is applicable only to a special class of polynomials.

On the other hand, most of RC active synthesis techniques using controlled sources have the undesirable feature that the sensitivity increased as the quality factor of the transfer function increases. It is shown [20] that some of the sensitivities are proportional to Q or Q^2 and inversely proportional to the active gains. Thus, low sensitivity realization requires high value of gains. Consequently, the methods are frequency limited.

This thesis is an attempt to circumvent some of the existing difficulties in realizing active networks. The main part of the work is in Chapter III. Two new RC active network structures together with their duals are presented. Methods of synthesis that yield stable networks with low sensitivities are developed.

Chapter II is a review of some basic concepts of sensitivity. Since the classical definition of root sensitivity [19] fails to exist for a multiple root, a modified definition is suggested and some consequences are derived.

2 Sensitivity Measures

The most important problem in RC active network synthesis is the sensitivities of the network due to parameter variations. Quantitative estimates of the effect of prarameter changes are given by various sensitivity measures. In this chapter, the three commonly used sensitivity measures, viz., the network function sensitivity, the root sensitivity, and the quality factor sensitivity are introduced. Since the classical definition [19] of root sensitivity fails to exist for the case of a multiple root, a modified version is suggested. Some sensitivity minimizations are also discussed.

2.1 NETWORK FUNCTION SENSITIVITY [8], [17]

For convenience in this work, the concepts of return difference of a single loop feedback system are discussed first. It has been noted by Bode [3] that any network function of an active linear time-invariant circuit can be expressed in a bilinear form in terms of a single network parameter, thus it can be represented by the fundamental single loop feedback model as in Fig. 2.1. For such a system, the closed loop transmittance is given by

$$T = \frac{X_2}{X_1} = t_o + \frac{kt_{13}t_{42}}{1 - kt_{43}}. \tag{2.1}$$

where

X_1, X_2, X_3, X_4 and k are the input signal, output signal, controlling signal, controlled signal and controlling parameter, respectively, and

$$t_{43} = \left.\frac{X_3}{X_4}\right|_{X_1=0}, \text{ the transfer function of the feedback path,}$$

kt_{43} is called the loop transmittance,

$$t_o = \left.\frac{X_2}{X_1}\right|_{k=0}, \text{ the leakage transmittance,} \tag{2.2}$$

$$t_{13} = \left.\frac{X_3}{X_1}\right|_{X_4=0},$$

$$t_{42} = \left.\frac{X_2}{X_4}\right|_{X_1=0}.$$

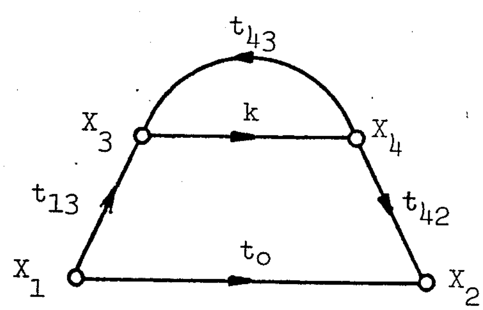


Fig.2.1 The fundamental single loop feedback model.

Definition 2.1.1 Return Ratio and Return Difference

The return ratio with respect to k of the system shown in Fig.2.1 is defined as the negative of loop transmittance

$$R_k = -kt_{43} \tag{2.3}$$

and the return difference with respect to k is defined as

$$F_k = 1 - kt_{43} = 1 + R_k \ . \tag{2.4}$$

The return difference with respect to a parameter k is a quantitative measure of the amount of feedback acting around k. If the return difference is greater than unity, the feedback is said to be negative, otherwise it is a positive feedback. The return difference can also be interpreted physically as follows. Break the branch of interest, k, as shown in Fig.2.2. Transmit a signal of unit strength at point a' while keeping

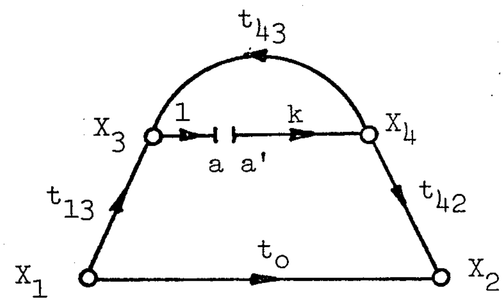


Fig.2.2 Physical interpretation of return difference.

the input $X_1 = 0$. The difference between the transmitted signal and the signal returned at point a is the return difference in terms of k.

Definition 2.1.2 Null Return Difference

The null return difference is defined as the return difference obtained under the condition that the input X_1 is adjusted such that the output signal X_2 reduces to zero.

It follows from the definition that

$$F_k^o = 1 - kt_{43} + \frac{kt_{13}t_{42}}{t_o} \ . \tag{2.5}$$

Simple manipulations among expressions (2.1), (2.2), (2.3), (2.4) and (2.5) show that

$$T = t_o \frac{F_k^o}{F_k} . \tag{2.6}$$

Network function sensitivity is a measure of the effect on the network function due to variations of network parameters. The widely used definition is the one given as the inverse of Bode's sensitivity function [3].

Definition 2.1.3 Network Function Sensitivity

The network function sensitivity, S_k^T, of the network function T with respect to the network parameter k is defined as

$$S_k^T = \frac{d \ln T}{d \ln k} = \frac{k}{T} \cdot \frac{dT}{dk} . \tag{2.7}$$

Differentiating expression (2.1) with respect to k gives

$$\frac{dT}{dk} = \frac{t_{13} t_{42}}{(1 - k t_{43})^2} . \tag{2.8}$$

Thus

$$S_k^T = \frac{k}{T} \cdot \frac{t_{13} t_{42}}{(1 - k t_{43})^2}$$

$$= \frac{1}{T F_k} (T - t_o)$$

$$= \frac{1}{F_k} (1 - \frac{t_o}{T}) . \tag{2.9}$$

It follows from equation (2.9) that the network function sensitivity is inversely proportional to the return difference if there is no leakage transmittance, i.e.,

$$S_k^T = \frac{1}{F_k} \quad \text{for} \quad t_o = 0. \tag{2.10}$$

Also, equations (2.6) and (2.9) give the following explicit relation between S_k^T, F_k and F_k^o

$$S_k^T = \frac{1}{F_k} - \frac{1}{F_k^o}. \tag{2.11}$$

Expression (2.9) suggests three basic ways of reducing the network function sensitivity [17]:

(1) design the network in such a way that the leakage transmittance is almost equal to the closed loop transmittance over the frequency band of interest,

(2) design the network with a return difference having poles at the frequencies where the sensitivity is to be improved,

(3) design the network with a large return difference over the frequency band of interest.

All of these schemes have certain difficulties in practical implementations.

In the ensuing chapter, the structures of zero leakage transmittance are developed so that the network function sensitivity is inversely proportional to a large return difference.

2.2 ROOT SENSITIVITY

Root sensitivity is a measure of the effect on a zero or a pole of a network function due to variations of network parameters.

Let a polynomial be written as

$$f(s,k) = f_1(s) + kf_2(s) \tag{2.12}$$

where $f_1(s)$ and $f_2(s)$ are component polynomials in s, k is the parameter of interest.

Let $s(k)$ be a zero of $f(s,k)$.

Let s_o be the limit of $s(k)$ as k approaches to its nominal value k_o, i.e.,

$$\lim_{k \to k_o} s(k) = s_o \ , \tag{2.13}$$

in other words, s_o is a zero of $f(s,k_o)$.

The conventional definition of root sensitivity is the one introduced by J. G. Truxal and I. M. Horowitz [19]

$$S_{k_o}^{s_o} = k \left. \frac{ds(k)}{dk} \right|_{k \to k_o} . \tag{2.14}$$

Despite of the fact that the incremental changes in the zeros due to an incremental change in k are finite, it has been shown in [11], [13], [17] that the root sensitivity defined by expression (2.14) becomes infinite for multiple zeros. To resolve this dilemma, a modified definition is suggested in the following.

For convenience, it is necessary to introduce the following concept of "proper" multiplicity of a zero with respect to the parameter k.

Let the multiplicity of s_o be m.

Definition 2.2.1

The multiplicity m is said to be "proper" with respect to k if $f_2(s)$ (and consequently $f_1(s)$) has no zero at $s = s_o$, otherwise "improper".

A proper multiplicity with respect to k has the property that all the m zeros at $s = s_o$ are affected by a variation of k. On the other hand, if the multiplicity is improper with respect to k, there exist some zeros which remain at $s = s_o$ regardless of the variation of k. As an example, consider

$$f(s,k) = f_1(s) + kf_2(s)$$

$$= s^3+4s^2+4s+1 + k(s^2+3s+2)\,. \qquad (2.15)$$

At $k = 1$, $s = -1$ is a zero of multiplicity 2. Since $f_2(s)$ also vanishes at $s = -1$, the multiplicity is improper with respect to k. Indeed, if k changes from 1 to 1.1, $f(s,1.1)$ still has a zero at $s = -1$, only the other zero shifts to the new location $s = -1.05$.

It should be noted that a proper multiplicity can always be derived from an improper one by redefining the function as follows

$$\hat{f}(s,k) = \hat{f}_1(s) + k\hat{f}_2(s) = \frac{f(s,k)}{(\; s - s_o\;)^b} \qquad (2.16)$$

where b is the multiplicity of the zero at $s=s_o$ of $f_2(s)$ (also of $f_1(s)$). Hence, without loss of generality, it is assumed that the multiplicity with respect to k is proper.

Since $s(k)$ is a zero of $f(s,k)$, it follows from expression (2.12) that

$$f_1(s(k)) + kf_2(s(k)) = 0$$

or

$$f_1(s(k)) + k_o f_2(s(k)) + (k-k_o)f_2(s(k)) = 0$$

$$f(s(k),k_o) = -(k-k_o)f_2(s(k))$$

$$\frac{1}{k-k_o} = \frac{-f_2(s(k))}{f(s(k),k_o)} . \qquad (2.17)$$

Multiplying both sides of equation (2.17) by $[\ s(k)-s_o\ ]^m$ yields

$$\frac{[\ s(k)-s_o]^m}{k-k_o} = \frac{-[\ s(k)-s_o\ ]^m f_2(s(k))}{f(s(k),k_o)} . \qquad (2.18)$$

Since $s(k) \to s_o$ as $k \to k_o$, it follows from equation (2.18) that

$$\lim_{k\to k_o} \frac{[\ s(k)-s_o\ ]^m}{k-k_o} = \lim_{s(k)\to s_o} \frac{-[\ s(k)-s_o\ ]^m f_2(s(k))}{f(s(k),k_o)}$$

$$= -f_2(s_o) \lim_{s(k)\to s_o} \frac{[\ s(k)-s_o\ ]^m}{f(s(k),k_o)} . \qquad (2.19)$$

By assumption, the multiplicity of the zero at $s=s_o$ is proper with respect to k, $f_2(s_o)$ does not vanish. Since s_o is a zero of multiplicity m of $f(s,k_o)$, the ℓ-th derivative $f^{(\ell)}(s_o,k_o)$ vanishes for all $\ell = 0,1,\ldots,m-1$. Thus applying L'Hospital's rule to evaluate the limit of the right-hand side of equation (2.19) yields

$$\lim_{k\to k_o} \frac{[\ s(k)-s_o\ ]^m}{k-k_o} = \frac{-m!f_2(s_o)}{f^{(m)}(s_o,k_o)} . \qquad (2.20)$$

Let

$$\Delta s = s(k) - s_o$$

and $$\Delta k = k - k_o,$$

then, equation (2.20) can be rewritten as

$$\lim_{\Delta k \to 0} \frac{(\Delta s)^m}{\Delta k} = \frac{-m! f_2(s_o)}{f^{(m)}(s_o, k_o)} . \tag{2.21}$$

Although the root sensitivity defined by expression (2,14) becomes infinite for a multiple zero, the limit on the left-hand side of equation (2.21) is finite. This suggests the following modified definition of root sensitivity.

Definition 2.2.2

The root sensitivity of a zero at $s=s_o$ of multiplicity m is defined as

$$\mathcal{S}_{k_o}^{s_o, m} = \lim_{\Delta k \to 0} k \frac{(\Delta s)^m}{\Delta k}. \tag{2.22}$$

For $m=1$, expression (2.22) agrees with the definition given by expression (2.14)

$$\mathcal{S}_{k_o}^{s_o, 1} = S_{k_o}^{s_o}. \tag{2.23}$$

It follows from expressions (2.21) and (2.22) that

$$\mathcal{S}_{k_o}^{s_o, m} = \frac{-k_o m! f_2(s_o)}{f^{(m)}(s_o, k_o)}. \tag{2.24}$$

Now, consider the function

$$F_2(s) = \frac{-k_o f_2(s)}{f(s, k_o)}. \tag{2.25}$$

Since $F_2(s)$ has a pole of order m at $s=s_o$, the Laurent expansion of $F_2(s)$ about $s=s_o$ gives

$$F_2(s) = \sum_{i=1}^{m} \frac{a_{-i}}{(s-s_o)^i} + g_2(s) \tag{2.26}$$

where $g_2(s)$ is the regular part of the expansion and

$$a_{-i} = \frac{1}{(m-i)!} \lim_{s \to s_o} \frac{d^{m-i}}{ds^{m-i}} [(s-s_o)^m F_2(s)], \tag{2.27}$$

particularly

$$a_{-m} = \frac{-k_o m! f_2(s_o)}{f^{(m)}(s_o,k_o)}. \tag{2.28}$$

Similarly, defining

$$F_1(s) = \frac{f_1(s)}{f(s,k_o)} \tag{2.29}$$

yields

$$F_1(s) = \sum_{i=1}^{m} \frac{b_{-i}}{(s-s_o)^i} + g_1(s)$$

and

$$b_{-m} = \frac{m! f_1(s)}{f^{(m)}(s_o,k_o)} . \tag{2.30}$$

Since s_o is a zero of $f(s,k_o)$,

$$f_1(s_o) + k_o f_2(s_o) = 0,$$

or

$$f_1(s_o) = -k_o f_2(s_o). \qquad (2.31)$$

Thus, it follows from equations (2.28), (2.30), and (2.31) that

$$a_{-m} = b_{-m} .$$

Hence, it is concluded that the root sensitivity defined by expression (2.22) is equal to the coefficient of the term $1/(s-s_o)^m$ in the Laurent expansion about $s=s_o$ of $F_2(s)$ or $F_1(s)$.

In the case of simple root, a_{-1} and b_{-1} are called the residues of $F_2(s)$ and $F_1(s)$ at $s=s_o$, respectively. This is denoted by

$$S_{k_o}^{s_o} = \mathscr{S}_{k_o}^{s_o,1} = \mathrm{Res}[F_2(s),s_o] = \mathrm{Res}[F_1(s),s_o]. \qquad (2.32)$$

Equation (2.32) agrees with the result obtained in [11], [13], [17].

2.3 QUALITY FACTOR SENSITIVITY [12], [17]

Consider the following biquadratic function

$$T = \frac{a_2 s^2 + a_1 s + a_o}{s^2 + 2\sigma s + \sigma^2 + \omega^2} \qquad (2.33)$$

with pole pair $(p,p^*) = -\sigma \pm j\omega$.

The quality factor as defined by Mitra [17] is

$$Q = \frac{\omega}{2\sigma} \qquad (2.34)$$

and the quality factor sensitivity is defined by

$$S_k^Q = \frac{d\ \ln Q}{d\ \ln k} = \frac{k}{Q} \cdot \frac{dQ}{dk} . \qquad (2.35)$$

It follows from expresions (2.34) and (2.35) that

$$S_k^Q = \frac{\sigma d\omega - \omega d\sigma}{\omega\sigma} \cdot \frac{k}{dk} . \qquad (2.36)$$

Since $p = -\sigma + j\omega$ is simple,

$$S_k^p = k\frac{dp}{dk} = (-d\sigma + j d\omega)\frac{k}{dk} . \qquad (2.37)$$

Thus

$$\frac{S_k^p}{p} = \frac{-d\sigma + jd\omega}{-\sigma + j\omega} \cdot \frac{k}{dk} .$$

Therefore

$$\mathrm{Im}\left[\frac{S_k^p}{p}\right] = \frac{\omega d\sigma - \sigma d\omega}{\omega^2 + \sigma^2} \cdot \frac{k}{dk} . \qquad (2.38)$$

From expressions (2.36) and (2.38), the exact relation between S_k^Q and S_k^p can be shown as

$$S_k^Q = -\frac{\omega^2 + \sigma^2}{\omega\sigma}\ \mathrm{Im}\left[\frac{S_k^p}{p}\right] . \qquad (2.39)$$

From expression (2.39), it is observed that to minimize the quality factor sensitivity it is necessary to minimize the imaginary part of S_k^p/p.

A decomposition method that gives zero quality factor sensitivity [12] for the sum case and the difference case has been developed.

Although extensive investigations have been made on pole sensitivity minimization [4], [5], [9], [10], only meager success has been recorded for the simultaneous minimization of more than one sensitivity measure. It is observed that if

$$S_k^p = mp$$

where m is a real number, then

$$\mathrm{Im}\left[\frac{S_k^p}{p}\right] = 0$$

and hence the quality factor sensitivity is zero. This suggests a possible synthesis technique which produces zero quality factor sensitivity with prescribed pole sensitivity. Polynomial decomposition techniques that make pole sensitivity proportional to the pole are being developed by the author.

3 The New Configurations and Methods of Synthesis

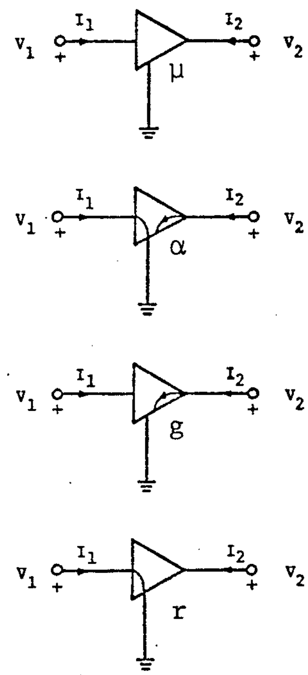
It is well known that feedback improves sensitivity if it is applied properly; the ideal zero-sensitivity system ([8], p.405) is a representative example. In this chapter, based on the feedback theory, two RC active network structures suitable for the realization of voltage and current transfer functions are presented. Methods of synthesis, sensitivity optimizations and stability are discussed. Also, the dual couterparts are developed.

3.1 THE NEW RC ACTIVE NETWORK STRUCTURES AND THEIR DUAL COUNTERPARTS

The structures presented in the following are developed by using controlled sources. The controlled sources are assumed to be ideal and are defined as in Fig. 3.1. For convenience, voltage-controlled voltage source is abbreviated to VVS, current-controlled current source to CCS, voltage-controlled current source to VCS and current-controlled vottage source to CVS.

3.1.1 The New Network Structures

Two structures are presented: structure A is designed for the realization of voltage transfer ratio and structure B for that of current transfer ratio. The feedback configuration of the structures is the voltage-shunt



(a) $$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

(b) $$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

(c) $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(d) $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Fig.3.1 Symbol and Definition of controlled sources:
(a) voltage-controlled voltage source (VVS),
(b) current-controlled current source (CCS),
(c) voltage-controlled current source (VCS),
(d) current-controlled voltage source (CVS).

feedback type[1]. The feedback network is a passive RC two-port.

3.1.1.1 Structure A: Voltage Transfer Function Realizarion

Structure A shown in Fig.3.2 consists of one CCS with gain $-\alpha$, one VVS with gain μ, one resister R connected at the output-port of the CCS, one input RC two-port and one feedback RC two-port. The signal-flow graph representation of the circuit is given in Fig.3.3, from which the transfer

1 J. Millman and C. C. Halkias, Electronic Devices and Circuits. McGraw-Hill, 1967, p.491.

voltage ratio is obtained as

$$T = \frac{V_2}{V_1} = \frac{-\alpha\mu R y_{12a}}{1+\alpha\mu R y_{12b}} . \tag{3.1}$$

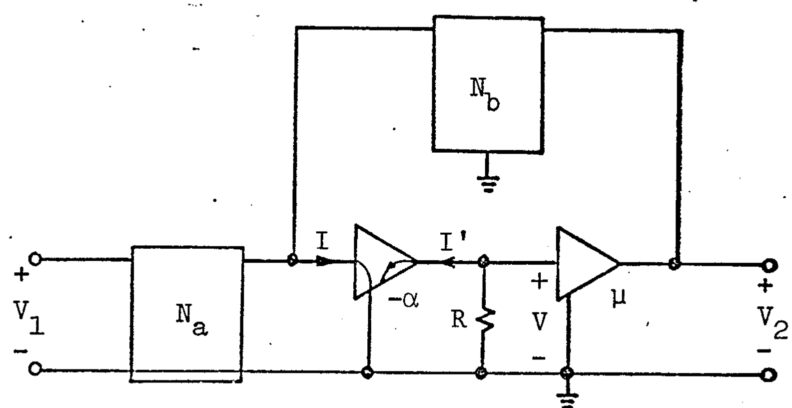


Fig.3.2 Structure A: a realization of voltage transfer ratio.

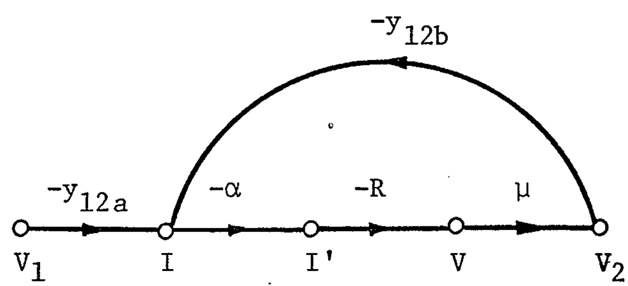


Fig.3.3 Signal-flow graph representation of structure A.

3.1.1.2 Structure B: Current Transfer Function Realization

Fig. 3.4 shows the structure for a current transfer ratio realization. The structure consists of one CCS with gain $-\alpha$, one VVS with gain μ, one resistor R terminated in the output-port of the CCS, one output RC two-port and one feedback RC two-port. The signal-flow graph representation of structure B is given in Fig.3.5, from which the transfer current ratio is obtained as

$$T = \frac{I_2}{I_1} = \frac{-\alpha\mu R y_{12a}}{1+\alpha\mu R y_{12b}} . \tag{3.2}$$

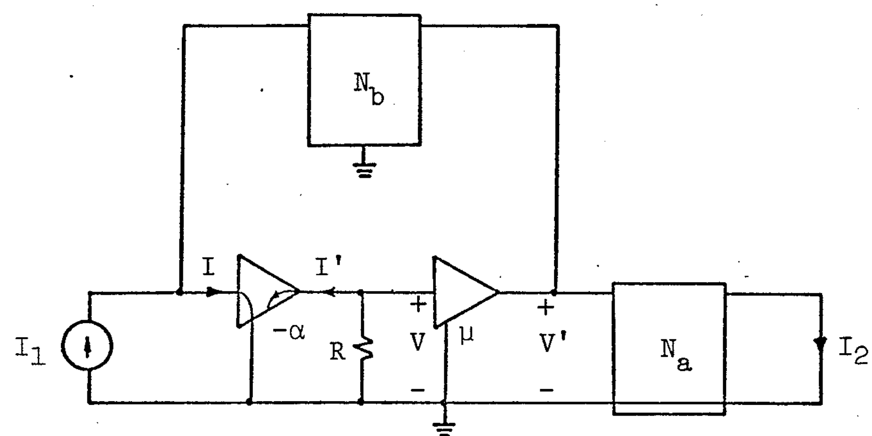


Fig.3.4 Structure B: a realization of current transfer ratio.

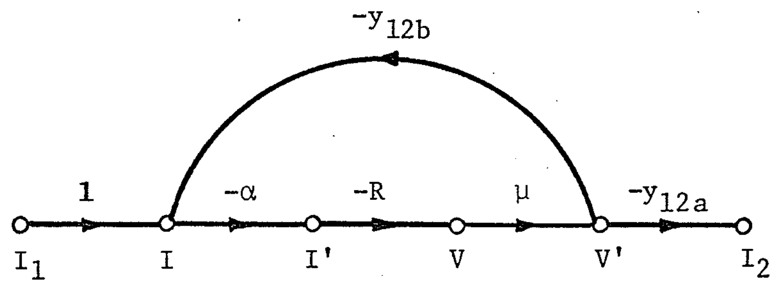


Fig.3.5 Signal-flow graph representation of structure B.

3.1.2 The Dual Counterparts

The usefulness of duality concept cannot be overemphasized. Two n-port networks are said to be dual if the voltage vector of one network is identical to the current vector of the other. In this section, the dual structures are derived by using the concept of duality between port-variables and port-parameters rather than the conventional concept of single-element to single-element duality.

3.1.2.1 The Dual of Structure A

Fig.3.6 is obtained through the dual operation on every discernible two-port of structure A. The signal-flow graph representation of the network is given in Fig.3.7. By the principle of duality, it follows that

$$\hat{T} = \frac{\hat{I}_2}{\hat{I}_1} = \frac{-\hat{\alpha}\hat{\mu}\hat{G}\hat{z}_{12a}}{1+\hat{\alpha}\hat{\mu}\hat{G}\hat{z}_{12b}}. \tag{3.4}$$

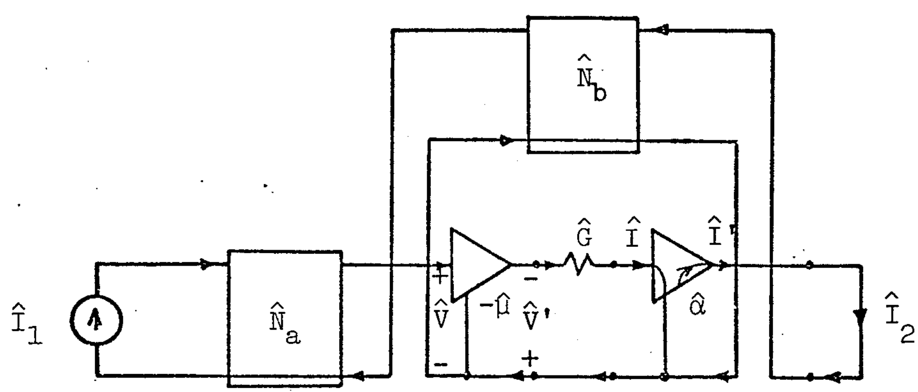


Fig.3.6 The dual of structure A.

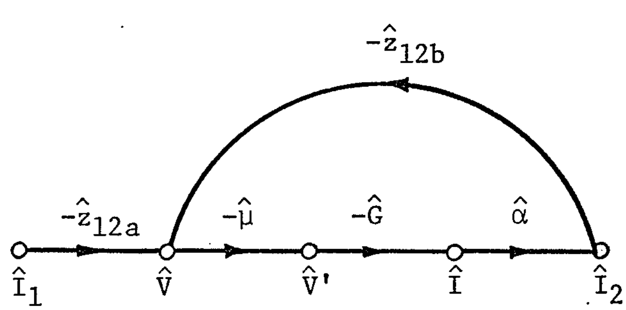


Fig.3.7 Signal-flow graph representation of Fig.3.6.

It will be shown later that a large $\hat{G}$ is desirable from the sensitivity point of view. However, the value of $\hat{G}$ is pratically limited by the nonideal input impedance of the CCS. This shortcoming is diminished by modifying the circuit as shown in Fig. 3.8, where the VVS and the CCS are replaced by two VCS with gain $-g_1$ and g_2, respectively, and the conductance $\hat{G}$ is replaced by a resistance R. Simple analysis gives the following transfer ratio

$$T = \frac{I_2}{I_1} = \frac{-g_1 g_2 R z_{12a}}{1+g_1 g_2 R z_{12b}} . \tag{3.5}$$

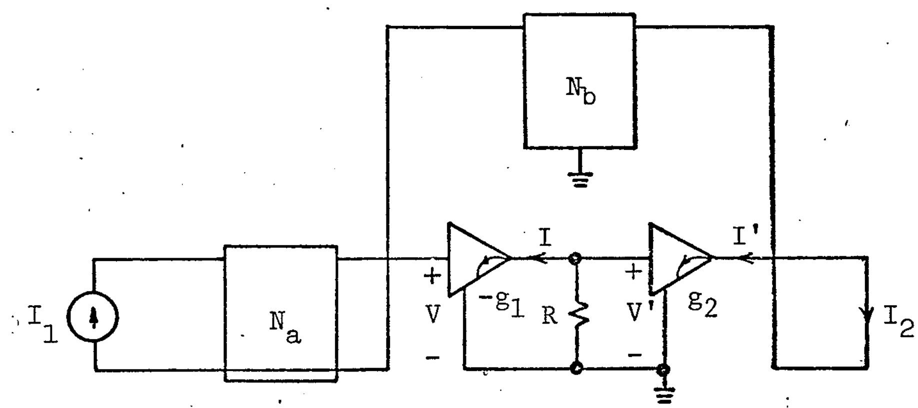


Fig.3.8 Structure C: the modified structure of Fig.3.6.

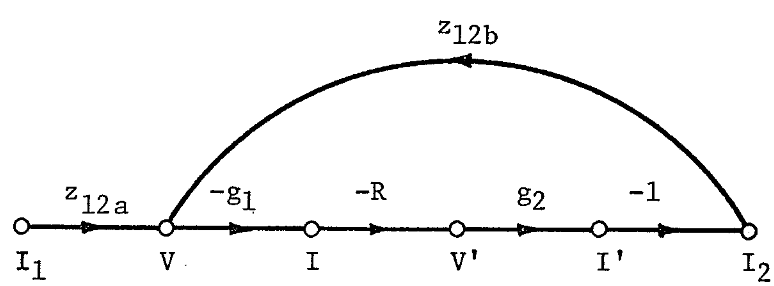


Fig.3.9 Signal-flow graph representation of structure C.

3.1.2.2 The Dual of Structure B

Fig.3.10 and Fig.3.11 show the dual of structure B and its signal-flow graph representation, respectively. Again, by the principle of duality, it follows that

$$\hat{T} = \frac{\hat{V}_2}{\hat{V}_1} = \frac{-\hat{\alpha}\hat{\mu}\hat{G}\hat{z}_{12a}}{1+\hat{\alpha}\hat{\mu}\hat{G}\hat{z}_{12b}} . \tag{3.6}$$

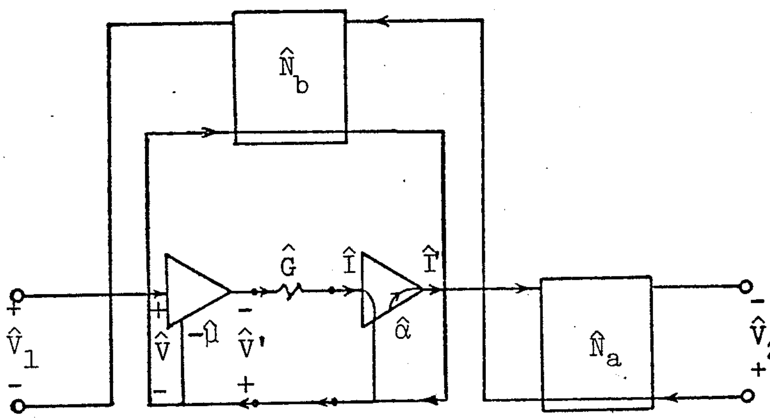


Fig.3.10 The dual of structure B.

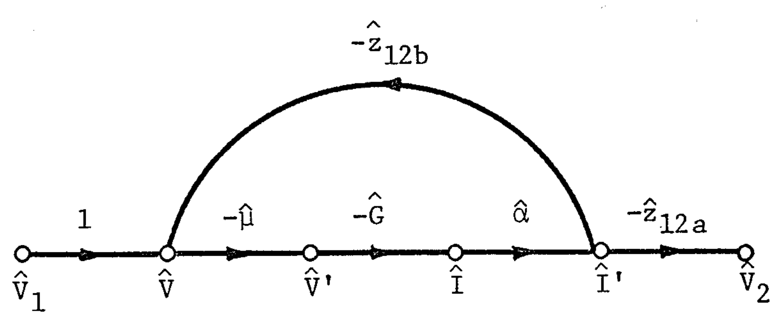


Fig.3.11 Signal-flow graph representation of fig.3.10.

The modified structure is shown in Fig.3.12 and its signal-flow graph representation in Fig.3.13. Simple analysis gives the following transfer function

$$T = \frac{V_2}{V_1} = \frac{-g_1 g_2 R z_{12a}}{1+g_1 g_2 R z_{12b}}. \tag{3.7}$$

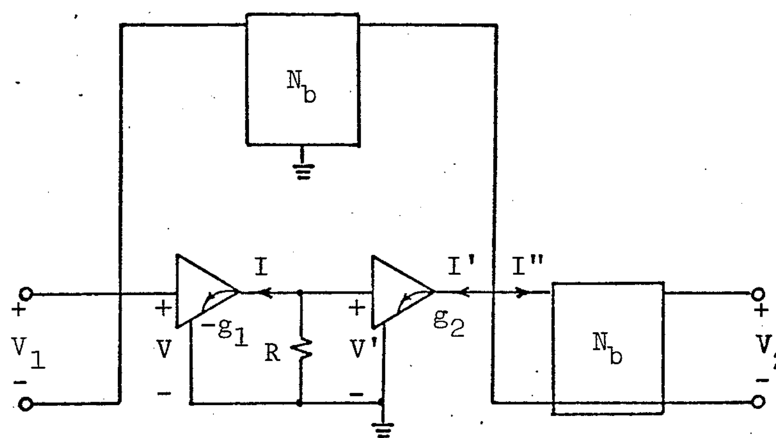


Fig.3.12 Structure D: the modified structure of Fig.3.10.

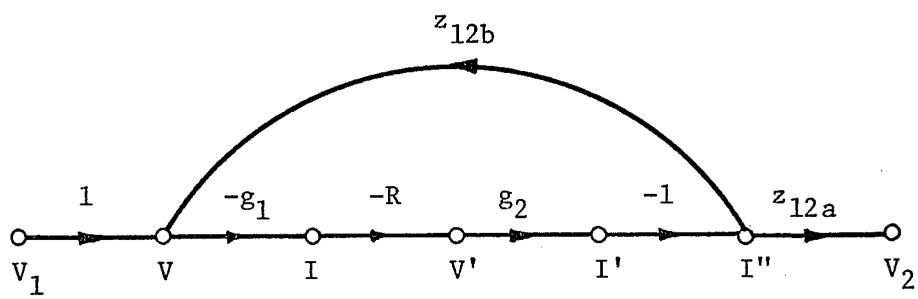


Fig.3.13 Signal-flow graph representation of structure D.

3.2 METHOD OF SYNTHESIS I

In the following, a method is developed for realizing transfer function using structures A and B. Realization using structures C and D can be achieved in a similar way.

3.2.1 The Method

For simplicity, let k represent product $\alpha\mu R$. Equations (3.1) and (3.2) can be rewritten in the following single expression

$$T = \frac{-ky_{12a}}{1+ky_{12b}} = \frac{-y_{12a}}{\frac{1}{k} + y_{12b}} . \tag{3.8}$$

Let

$$T = \frac{-N(s)}{D(s)} \tag{3.9}$$

be the transfer function to be realized.

Let Q(s) be a monic polynomial having only negative real zeros and satisfying the following degree condition

$$Q(s)^o \geqslant \max[N(s)^o, D(s)^o] - 1$$

where $Q(s)^o$, $N(s)^o$ and $D(s)^o$ denote the degrees of Q(s), N(s), and D(s), respectively.

Equation (3.9) can be rewritten as

$$T = \frac{N(s)/Q(s)}{-D(s)/Q(s)} . \tag{3.10}$$

From Equations (3.8) and (3.10), it follows that

$$-y_{12a} = \frac{N(s)}{Q(s)} \tag{3.11}$$

and

$$-y_{12b} = \frac{1}{k} + \frac{D(s)}{Q(s)} . \tag{3.12}$$

To realize expression (3.9), it is sufficient to realize the two passive RC two-ports, N_a and N_b, having short circuit transfer admittances given by expressions (3.11) and (3.12), respectively.

Since the transfer function does not contain any driving-point parameters the synthesis is simple and straightforward. For the second order case, N_a and N_b of a specified $-y_{12}$ are tabulated in the literature [2], [13], [17].

Furthermore, it is noted that the zeros and the poles of the transfer function are realized by the zeros of the short-circuit transfer admittances of N_a abd N_b, respectively; thus they can be located anywhere in the complex s-plane except on the positive real axis. Hence, it is concluded that the proposed structures are capable of realizing any rational function except all-pass functions.

3.2.2 Sensitivity Consideration

It is clear from expression (3.8) that the leakage transmittance is zero and the return difference for k is

$$F_k = 1 + ky_{12b} . \tag{3.13}$$

Therefore the network function sensitivity with respect to k is given by expression (2.10)

$$S_k^T = \frac{1}{F_k} = \frac{1}{1 + ky_{12b}} . \tag{3.14}$$

Let the companion network N_b be realized with the nominal value k_o of k, then it follows from expression (3.12) that

$$-y_{12b} = \frac{1}{k_o} + \frac{D(s)}{Q(s)} . \tag{3.15}$$

Substituting expression (3.15) into expression (3.14) yields

$$S_{k_o}^{T} = \frac{-Q(s)}{k_o D(s)} . \tag{3.16}$$

Substituting expressions (3.11) and (3.15) into expression (3.8) gives

$$T = \frac{-k_o kN(s)}{k_o Q(s) - k[Q(s) + k_o D(s)]} = \frac{-k_o kN(s)}{f(s,k)} .$$

Thus, the pole sensitivity can be obtained from expression (2.32)

$$S_{k_o}^{p} = (s-p)F_1(s)\Big|_{s=p} = \frac{(s-p)k_o Q(s)}{f(s,k_o)}\Big|_{s=p} = \frac{-(s-p)Q(s)}{k_o D(s)}\Big|_{s=p} \tag{3.17}$$

where p is a pole.

Now, consider a biquadratic function having the normalized deniminator

$$D(s) = s^2 + 2\sigma s + 1$$

with pole pair (3.18)

$$(p,p^*) = -\sigma \pm j\sqrt{1 - \sigma^2} .$$

The quality factor sensitivity is given by expression (2.39)

$$S_{k_o}^{Q} = \frac{-1}{\sigma\sqrt{1-\sigma^2}} \operatorname{Im}\left[\frac{S_{k_o}^{p}}{p}\right] \tag{3.19}$$

or using expressions (3.17) and (3.19), it can be shown further that

$$S_{k_o}^{Q} = \frac{1}{k_o\sigma\sqrt{1-\sigma^2}} \operatorname{Im}\left[\frac{Q(p)}{2jp\sqrt{1-\sigma^2}}\right] . \tag{3.20}$$

Examination of expressions (3.15), (3.17) and (3.20) reveals that the sensitivities can be made small by

(i) Choosing a suitable polynomial for Q(s) such that the sensitivities are low in the frequency band of interest.

(ii) Designing with a large gain k_o which can be made large by taking a large value for R while keeping the active gains finite or unity if it is so desired.

It is illustrated in Fig. 3.14 that the desensitization is possible by choosing a proper polynomial for Q(s).

Let a second-order D(s) be defined as expression (3.18), and let

$$Q(s) = \prod_{i=1}^{\ell} (s + \alpha_i) \tag{3.21}$$

where α_i's are positive.

(a) Desensitization of $|S_{k_o}^{T}|$

Fig.3.14 shows the zero plot of D(s) and Q(s) where s=jω is the frequency of interest, d_1, d_2 and n_i are the distances from s to p, p* and $-\alpha_i$, respectively, δ_i is the distance from $-\alpha_i$ to p.

Expressions (3.15), (3.18) and (3.21) give

$$S_{k_o}^{T} = \frac{-\prod_{i=1}^{\ell} (s + \alpha_i)}{k_o(s^2 + 2\sigma s + 1)} . \tag{3.22}$$

Therefore

$$|S_{k_o}^T| = \frac{\prod_{i=1}^{\ell} n_i}{k_o d_1 d_2} . \tag{3.23}$$

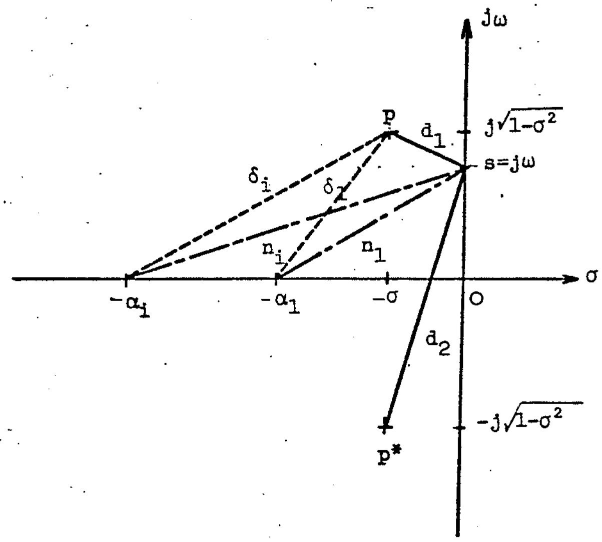


Fig.3.14 Zero plot of D(s) and Q(s).

Expression (3.23) indicates that to minimize $|S_{k_o}^T|$, it is desirable to have n_i as small as possible, i.e., $s = -\alpha_i$ should be located as close to the origin as possible. If within the frequency band of interest, the n_i's can be so chosen that they are all less than unity, then it is desirable to take more than one factor for Q(s), otherwise one factor corresponding to the smallest n_i is optimal.

(b) Desensitization of $|S_{k_o}^p|$

From expressions (3.17), (3.18) and (3.21), it follows that

$$S_{k_o}^p = \frac{-\prod_{i=1}^{\ell} (p + \alpha_i)}{2k_o j\sqrt{1 - \sigma^2}} .$$

Therefore $$|S^{p}_{k_o}| = \frac{\prod_{i=1}^{\ell} \delta_i}{2k_o\sqrt{1 - \sigma^2}}. \tag{3.24}$$

Since the smallest δ can be obtained at $s = -\alpha = -\sigma$, $Q(s) = s + \sigma$ is an optimum choice. Furthermore, since $\delta_{min} = \sqrt{1 - \sigma^2}$ is less than unity, it is always possible to find more than one factor for $Q(s)$ with δ_i less than unity. Hence, $|S^{p}_{k_o}|$ can be further improved. However, certain compromises should be made between the sensitivity and the realization of the passive companion networks. To choose $Q(s) = s + \sigma$ may cause an unduly large spread of element values for high Q poles. It should also be noted that economy would not allow too many factors for $Q(s)$ because this would require a large number of passive elements.

(c) Desensitization of $|S^{Q}_{k_o}|$

For the first order $Q(s) = s + \alpha$, expression (3.20) gives

$$S^{Q}_{k_o} = \frac{\alpha\sigma - 1}{k_o\sigma\sqrt{1 - \sigma^2}}. \tag{3.25}$$

Therefore the quality factor sensitivity can be made equal to zero simply by choosing

$$\alpha = \frac{1}{\sigma}, \tag{3.26}$$

i.e., $$Q(s) = s + \frac{1}{\sigma}. \tag{3.27}$$

If $Q(s) = (s + \alpha_1)(s + \alpha_2)$, then also from expression (3.20) it can be shown that

$$S^{Q}_{k_o} = \frac{(1 + \alpha_1\alpha_2)\sigma - (\alpha_1 + \alpha_2)}{k_o\sigma\sqrt{1 - \sigma^2}}. \tag{3.28}$$

Thus $S_{k_o}^{Q}$ will be zero if α_1 and α_2 are so chosen that

$$\alpha_1 = \frac{\alpha_2 - \sigma}{\alpha_2\sigma - 1} . \tag{3.29}$$

Since both α_1 and α_2 are required to be positive, α_2 cannot take the value in the open interval $(\sigma, \frac{1}{\sigma})$.

In conclusion, the optimum solution can be obtained by

(1) Choosing $Q(s) = s + \frac{1}{\sigma}$, so that the quality factor sensitivity is zero.

(2) Choosing a sufficiently large gain k_o, so that the pole sensitivity and the network function sensitivity become very low.

This will be illustrated by an example in Section 3.2.4.

3.2.3 Stability Analysis

A linear time-invariant circuit is asymptotically stable if all the natural frequencies are restricted to the open left-half complex frequency s-plane, i.e., the numerator of the return difference must be strictly Hurwitz.

The return difference for k is given by expression (3.13)

$$F_k = 1 + R_k$$

where

$$R_k = k y_{12b} \tag{3.30}$$

is the return ratio.

Substituting expression (3.15) into expression (3.30) yields

$$R_k = ky_{12b} = -k \frac{Q(s)/k_o + D(s)}{Q(s)} . \tag{3.31}$$

For convenience, let D(s) be of second-order as defined in expression (3.18) and

$$Q(s) = s + \alpha \qquad (\alpha > 0).$$

Expression (3.31) can be rewritten in terms of the phantom zeros z and z* as follows

$$R_k = -k \frac{(s+z)(s+z^*)}{s+\alpha} \tag{3.32}$$

where

$$(z,z^*) = -(\sigma+\frac{1}{2k_o}) \pm \sqrt{(\sigma+\frac{1}{2k_o})^2 - (1+\frac{\alpha}{k_o})} . \tag{3.33}$$

The roots of F_k are the pole pair (p,p^*) at the nominal value k_o of k. As k varies, they will move along the root locus as shown in Fig.3.15.

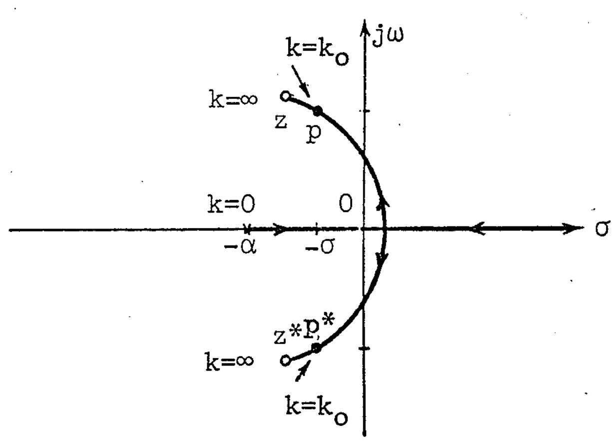


Fig.3.15 Root locus of the return difference.

If α satisfies

$$\alpha > k_o[(\sigma + \frac{1}{2k_o})^2 - 1], \tag{3.34}$$

then the phantom zeros will always be complex conjugate and lie to the left of the pole pair (p,p^*). Hence the circuit is always stable for $k > k_o$. If k moves in the decreasing direction from k_o, there are two cases to be considered.

Case 1 : $\alpha < Q$, where Q is the quality factor. The complex path of the root locus intersects the $j\omega$-axis at

$$k_u = \frac{Qk_o}{Q + k_o} . \tag{3.35}$$

Thus the circuit will oscillate at $k = k_u$ with the oscillating frequency $s = j\sqrt{1-2\alpha\sigma}$, and becomes unstable for any value of

$$k \leqslant k_u = \frac{Qk_o}{Q + k_o} . \tag{3.36}$$

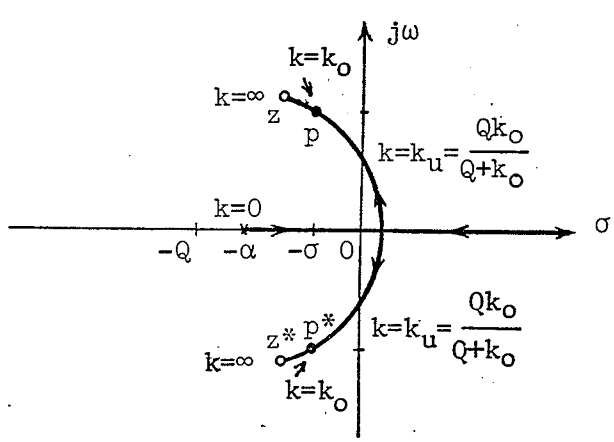


Fig.3.16 Root locus of the return difference for case $\alpha < Q$.

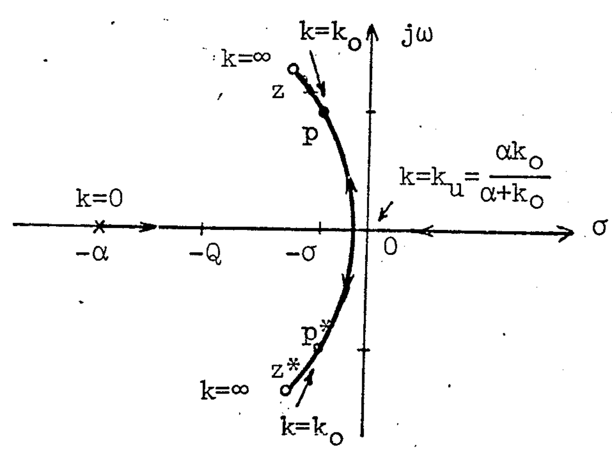


Fig.3.17 Root locus of the return difference for case $\alpha > Q$.

Case 2 : $\alpha > Q$. The complex path of the root locus does not intersect the $j\omega$-axis. However, the circuit becomes unstable for

$$k \leqslant k_u = \frac{\alpha k_o}{\alpha + k_o} , \qquad (3.37)$$

since one of the roots migrates to the right half plane of the complex frequency s-plane.

Consider the ratio

$$\frac{\Delta k_u}{k_o} = \frac{k_u - k_o}{k_o} = \begin{cases} \dfrac{-k_o}{Q + k_o} & \text{for case 1} \\[2ex] \dfrac{-k_o}{\alpha + k_o} & \text{for case 2} \end{cases}$$

In any case

$$\frac{\Delta k_u}{k_o} \rightarrow -1 \qquad \text{for} \quad k_o \gg \max(\alpha , Q).$$

Under practical operation, this situation can hardly occur. Therefore the system is highly stable for large value of k_o.

In general, for any given $\mathring{D}(s)$ and $Q(s)$, the same root locus technique can be applied to investigate the stability. Any value of k that shifts some of the zeros of the return difference to the right-half (including the $j\omega$-axis) complex frequency s-plane will make the circuit unstable.

3.2.5 Example : biquadratic function realization with zero quality-factor sensitivity and prescribed pole and network function sensitivities.

Consider the following biquadratic function

$$T = \frac{V_2}{V_1} = \frac{-N(s)}{D(s)} = -\frac{a_2s^2+a_1s+a_o}{s^2+2\sigma s+1} \tag{3.38}$$

where the denominator has been normalized. The most commonly used transfer functions derived from the biquadratic function are the low-pass, high-pass, band-pass and the elliptic functions:

$$T_\ell = -\frac{a_o}{s^2+2\sigma s+1} \qquad \text{(low-pass)}, \tag{3.39}$$

$$T_h = -\frac{a_2s^2}{s^2+2\sigma s+1} \qquad \text{(high-pass)}, \tag{3.40}$$

$$T_b = -\frac{a_1s}{s^2+2\sigma s+1} \qquad \text{(band-pass)}, \tag{3.41}$$

$$T_e = -\frac{a_2s^2+a_o}{s^2+2\sigma s+1} \qquad \text{(elliptic)}. \tag{3.42}$$

Let T_ℓ, T_h, T_b and T_e be realized as transfer voltage ratios with the prescribed sensitivities:

$$S_{k_o}^{Q} = 0$$

and

$$|S_{k_o}^{T(j)}| \;,\; |S_{k_o}^{P}| \leqslant m \tag{3.43}$$

where m is a prescribed number.

From expression (3.27), a choice of

$$Q(s) = s + \frac{1}{\sigma}$$

yields

$$S_{k_o}^{Q} = 0.$$

Expression (3.15) gives

$$S_{k_o}^{T} = \frac{-Q(s)}{k_o D(s)} = \frac{-(s+\frac{1}{\sigma})}{k_o(s^2+2\sigma s+1)} .$$

Therefore

$$|S_{k_o}^{T}(j)| = \frac{\sqrt{1+\sigma^2}}{2\sigma^2 k_o} . \tag{3.44}$$

Expression (3.17) gives

$$S_{k_o}^{p} = \frac{-(s-p)Q(s)}{k_o D(s)} = \frac{-(s+\frac{1}{\sigma})}{2k_o j\sqrt{1-\sigma^2}} .$$

Therefore

$$|S_{k_o}^{p}| = \frac{\sqrt{1+\sigma^2}}{2\sigma k_o\sqrt{1-\sigma^2}} . \tag{3.45}$$

Hence, to satisfy (3.43), it is necessary to choose

$$Q(s) = s + \frac{1}{\sigma}$$

(3.46)

and

$$k_o \geqslant \max\left(\frac{\sqrt{1+\sigma^2}}{2\sigma^2 m}, \frac{\sqrt{1+\sigma^2}}{2\sigma m\sqrt{1-\sigma^2}} \right) .$$

It follows from expressions (3.11) and (3.15) that

$$-y_{12a} = \frac{N(s)}{s+\frac{1}{\sigma}} ,$$

and

$$-y_{12b} = \frac{1}{k_o} + \frac{s^2+2\sigma s+1}{s+\frac{1}{\sigma}} .$$

The complete realization is shown in Fig.3.18.

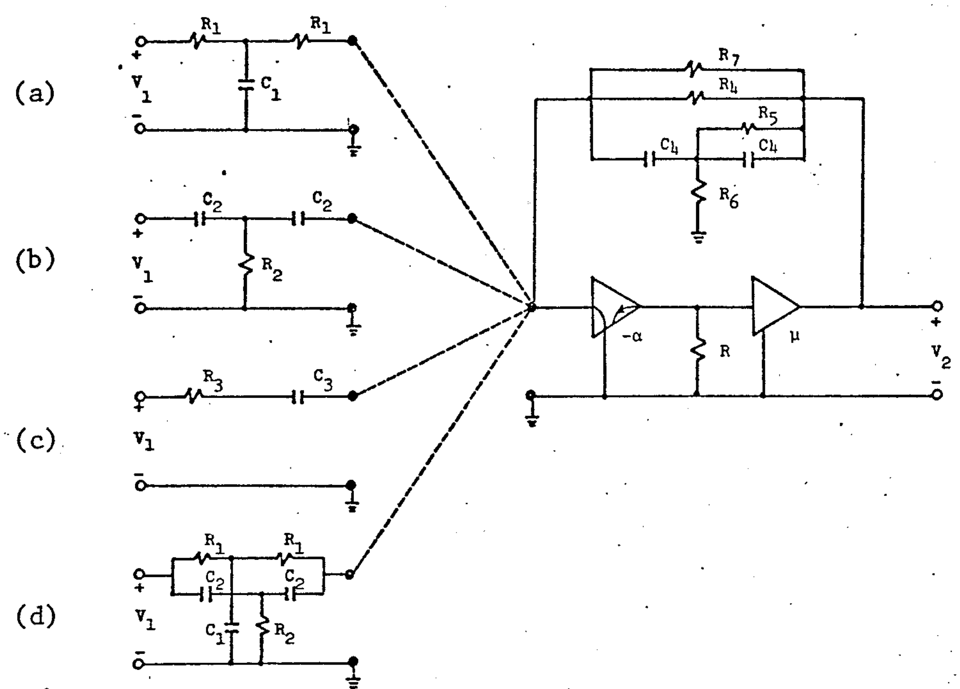


Fig.3.18 Realization of (a) low-pass,
(b) high-pass,
(c) band-pass,
(d) elliptic.

Element values :

$\alpha = \alpha_o$	$R_1 = 1/2\sigma a_o$		
$\mu = \mu_o$	$R_2 = \sigma/4a_2$		
$R = R_o$	$R_3 = 1/a_1$		
$C_1 = 4\sigma^2 a_o$	$R_4 = 1/\sigma$		(3.47)
$C_2 = 2a_2$	$R_5 = 1/2(1-\sigma)$		
$C_3 = \sigma a_1$	$R_6 = \sigma/2(2-\sigma^2)$		
$C_4 = 2$	$R_7 = k_o = \alpha_o\mu_o R_o$		

Since $\alpha = \frac{1}{\sigma} > Q$, it follows from (3.37) that the circuit is stable for

$$k > \frac{\alpha k_o}{\alpha + k_o} .$$

It can be shown that the quality factor of the circuit is given by

$$Q = \frac{\sqrt{kA[kD-(1-kG_7)EF]}}{kB-(1-kG_7)E} \tag{3.48}$$

where

$$\begin{aligned} k &= \alpha\mu R , \\ A &= R_6 C_4^2 , \\ B &= R_6 C_4 (G_5 + 2G_4) , \\ D &= G_4 (1 + R_6 G_5) , \\ E &= 2R_6 C_4 , \\ F &= (1 + R_6 G_5)/2R_6 C_4 . \end{aligned} \tag{3.49}$$

Using definition (2.35)

$$S_x^Q = \frac{x}{Q}\frac{dQ}{dx} ,$$

it can be easily shown that

$$\begin{aligned} S_{\alpha_o}^Q &= S_{\mu_o}^Q = S_{R_o}^Q = S_{R_1}^Q = S_{R_2}^Q = S_{G_7}^Q \\ &= S_{R_3}^Q = S_{C_1}^Q = S_{C_2}^Q = S_{C_3}^Q = S_{C_4}^Q = 0 , \end{aligned}$$

$$S_{G_4}^Q = \frac{G_4}{2Q^2} \frac{(1-R_6G_5)(G_5-2G_4)}{R_6(G_5+2G_4)^3} ,$$

$$S^{Q}_{R_6} = \frac{R_6}{2Q^2} \frac{-G_4}{R_6^2(G_5+2G_4)^2} ,$$

$$S^{Q}_{G_5} = \frac{G_5}{2Q^2} \frac{G_4(2G_4-G_5R_6-2)}{R_6(G_5+2G_4)^3} .$$

For a high-Q pole pair, $\sigma \ll 1$. Expression (3.47) gives

$$R_6 = \sigma/4 ,$$

$$G_5 = 2 ,$$

$$G_4 = \sigma .$$

Therefore

$$S^{Q}_{G_4} \simeq \frac{1}{Q^2} ,$$

$$S^{Q}_{R_6} \simeq \frac{-1}{Q^2} ,$$

$$S^{Q}_{G_5} \simeq \frac{-2}{Q^2} .$$

The above calculations show that the circuit in Fig.3.18 has very low Q-sensitivity with respect to the variations of both passive and active elements.

3.2.5 Higher Order Realization

As it was mentioned earlier that the proposed structures are capable of realizing any rational transfer function except the class of all-pass functions. In constrast to most other synthesis methods, the proposed one still yields small sensitivities even for direct higher order realization. However, cascading of second-order sections is recommended for the following advantages:

(1) The ease in the realization of biquadratic companion passive network.

(2) The new structures have either high input impedance or low output impedance, so buffer amplifiers may not be needed.

(3) Since there is no leakage transmission, expression (3.14) shows that the network function sensitivity is independent of the transmission zeros. Consequently, pole-zero pairing is not necessary from the sensitivity point of view. However, this fact provides a freedom in pole-zero pairing to optimize some other specifications.

(4) Stability analysis is simpler for a lower order system than the higher ones.

3.3 METHOD OF SYNTHESIS II

If the element R in structures A and B is replaced by an impedance Z(s), then the corresponding transfer function becomes

$$T(s) = \frac{-\alpha\mu y_{12a}}{Y(s) + \alpha\mu y_{12b}} . \tag{3.50}$$

Similarly, a replacement of R by Z(s) in structures C and D yields the corresponding transfer function

$$T(s) = \frac{-\, g_1 g_2 z_{12a}}{Y(s) + g_1 g_2 z_{12b}} . \tag{3.51}$$

Expressions (3.50) and (3.51) suggest the following alternative synthesis method.

Let the transfer function to be realized be

$$T(s) = N(s)/D(s) . \tag{3.52}$$

Choose a polynomial $Q(s)$ having only negative real roots and satisfying the following degree condition

$$Q(s)^{o} \geqslant \max[\, N(s)^{o}, D(s)^{o}] - 1$$

where $Q(s)^{o}$, $N(s)^{o}$ and $D(s)^{o}$ denote the degrees of $Q(s)$, $N(s)$ and $D(s)$, respectively.

Dividing the numerator and denominator of (3.52) by $Q(s)$ gives

$$T(s) = \frac{\frac{N(s)}{Q(s)}}{\frac{D(s)}{Q(s)}} = \frac{\frac{N(s)}{Q(s)}}{\frac{D_1(s)}{Q(s)} + \frac{D_2(s)}{Q(s)}} . \tag{3.53}$$

From (3.50) and (3.53), it follows that

$$y_{12a} = \frac{-1}{\alpha\mu} \frac{N(s)}{Q(s)} ,$$

$$Y(s) = \frac{D_1(s)}{Q(s)} , \tag{3.54}$$

and

$$y_{12b} = \frac{1}{\alpha\mu} \cdot \frac{D_2(s)}{Q(s)} .$$

From (3.51) and (3.53),

$$z_{12a} = \frac{-1}{g_1 g_2} \cdot \frac{N(s)}{Q(s)} ,$$

$$Y(s) = \frac{D_1(s)}{Q(s)} , \tag{3.55}$$

and

$$z_{12b} = \frac{1}{g_1 g_2} \cdot \frac{D_2(s)}{Q(s)} .$$

It should be noted that $D(s)$ must be decomposed in such a way that (3.54) and (3.55) are RC-realizable. The proposed structures are very flexible as far as decomposition techniques are concerned. Firstly, since the zeros of $D_2(s)$ are to be realized by the zeros of a transfer immittance, they can be located anywhere in the complex frequency s-plane except on the positive real axis. Secondly, the sign of $D_2(s)$ is also immaterial; y_{12b} or z_{12b} can always be made passive RC realizable by appropriate choice of the signs of the active gains. Consequently, a large number of existing decomposition techniques can be apllied, some are listed as follows:

In [10], I. M. Horowitz presents a RC:-RC decomposition technique that minimizes coefficient sensitivities.

In [4], D. A. Calahan discusses a RC:RL decomposition technique that gives minimum pole sensitivities.

In [5], S. S. Haykim suggests a decomposition technique that yields prescribed pole sensitivity.

In [9], N. M. Herbst presents some decomposition techniques to minimize pole sensitivity.

In [12], H. K. Kim and C. S. Phan propose a decomposition technique which gives zero quality factor sensitivity.

The following is an example illustrating the synthesis method.

Example: Realize the band-pass filter given by

$$T(s) = \frac{V_2}{V_1} = \frac{N(s)}{D(s)} = \frac{-s}{s^2 + 2\sigma s + 1}$$

with zero-quality factor sensitivity.

According to [12], for the sum-decomposition,

$$D_1(s) = s(s + \sigma) ,$$

and

$$D_2(s) = \frac{\sigma}{k_o} \left(s + \frac{1}{\sigma}\right) .$$

The pole sensitivity is

$$S^p_{k_o} = \frac{1}{2} p = \frac{1}{2} (-\sigma + j\sqrt{1 - \sigma^2}) .$$

and the quality factor sensitivity is

$$S_{k_o}^{Q} = \frac{-1}{\sigma\sqrt{1-\sigma^2}} \; \mathrm{Im}\{ \frac{S_{ko}^{p}}{p} \}$$

$$= \frac{-1}{\sigma\sqrt{1-\sigma^2}} \; \mathrm{Im}\{ \frac{\frac{1}{2}p}{p} \} = 0 .$$

Thus, choosing

$$Q(s) = s$$

and

$$g_1 = g_2 = 1 ,$$

expression (3.35) gives

$$z_{12a} = \frac{-1}{g_1 g_2} \cdot \frac{N(s)}{Q(s)} = 1 ,$$

$$Y(s) = \frac{D_1(s)}{Q(s)} = \frac{s(s+\sigma)}{s} = s + \sigma ,$$

and

$$z_{12b} = \frac{1}{g_1 g_2} \cdot \frac{D_2(s)}{Q(s)} = \frac{\sigma(s + 1/\sigma)}{s} = \sigma + 1/s .$$

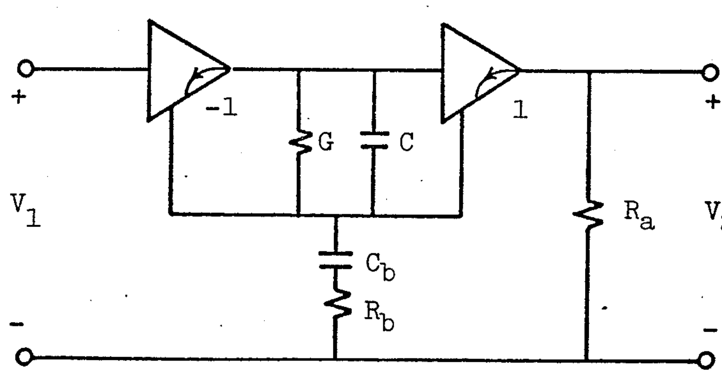


Fig.3.19 A realization of $\frac{V_2}{V_1} = \frac{-s}{s^2 + 2\sigma s + 1}$.

Using structure D with R being replaced by Z(s), The complete realization is obtained as shown in Fig. 3.19.

Element values are:

$$C = 1 \,,$$

$$C_b = 1 \,,$$

$$R_a = 1 \,,$$

$$G = \sigma \,,$$

$$R_b = \sigma \,.$$

It can be shown that

$$Q = \frac{\sqrt{CC_b g_1 g_2}}{C_b(G + g_1 g_2 R_b)} \,.$$

Thus

$$S^Q_{g_1} = S^Q_{g_2} = S^Q_{R_a} = 0 \,,$$

$$S^Q_{C_b} = S^Q_G = S^Q_{R_b} = -\frac{1}{2} \,,$$

$$S^Q_C = \frac{1}{2} \,.$$

4 Conclusions

Two new RC network structures together with their dual counterparts have been proposed. The structures are suitable for realizing voltage and current transfer functions. Accordingly, two new synthesis techniques have been developed in the proposed structures so as to give low sensitivities. In terms of quality factor sensitivity due to the variations of both active and passive elements, the proposed techniques exhibit some notable advantages over the existing network configurations as illustrated in Table 4.1.

Besides the low sensitivity merit, the proposed structures have the following distinctive features. The passive companion RC two-ports are required to satisfy only one two-port parameter, viz., y_{12}. A large loop gain can be obtained from a large value of a resistor and therefore realization with finite active gains is possible. Consequently, the proposed structures are useful over a wide frequency range. Furthermore, structure A derives its output from the output-port of a VVS and structure B has as its input-port the input-port of a CCS, hence buffer amplifiers may be eliminated in cascade realization of second order sections. Although the development in Chapter III has dealt mainly with the second-order functions, the extension to any higher-order function excluding the all-pass functions, is straight-

Filter Type		Controlled Sources	$\left\| S^{Q}_{R,C} \right\|$	$\left\| S^{Q}_{K} \right\|$	
Sallen & Key	+ve	1	$\propto Q$	$\propto \frac{Q}{k}$	
	-ve	1	$\leq \frac{1}{2}$	$\propto \frac{Q^2}{k}$	
Hamilton&Sedra Single OA [7]		1	≤ 1	$\frac{4Q}{k}$	
Moschytz [21] MSFEN		2	1	$\frac{4Q}{k}$	
Kerwin, Huelsman &Newcomb [22] State Variable		3	$\leq \frac{1}{2}$	$\geq \frac{2Q}{k}$	
Hamilton&Sedra Double OA [7]		2	$\propto q_R$	$\frac{.4Q}{k}$	$q_R=3$
Moschytz [21] HSFEN		3	$\propto q_R$	$\frac{.2Q}{k}$	$q_R=5$
Method I		2	$< \frac{2}{Q^2}$	0	
Method II		2	$\leq \frac{1}{2}$	0	

(q_R is the quality factor of the twin-T network in the cited methods.)

Table 4.1 Comparison table[1].

forward. Since pole-zero pairing is not necessary from sensitivity point of view, it may be ueed to optimize some other specifivations.

Method of synthesis II also reveals that the roots of one of the component polynomials in the polynomial decomposition are no longer restricted to the negative real axis. The relaxation of this constraint provides some freedom in the development of the decomposition techniques

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1 The table is excerpted partially from Fig.9 in [7], the last two rows are added showing the results obtained in Chapter III.

that minimize simultaneously two or more sensitivity criteria. Some work along this direction remains to be done. In Chapter II, a simple method has been suggested to develop a decomposition technique that gives zero quality factor sensitivity with prescribed pole sensitivity.

Finally, it is also worthwhile to mention here that the modified definition of root sensitivity as suggested in Section 2.2 is more general than the conventional one [19]. The obvious advantage of this definition is that it remains finite, while the conventional one fails to exist for the case of multiple roots. As for the case of simple roots, the modified definition agrees with the conventional one.

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