

THE EFFECTS OF VARIATION OF THE STRESS  
GRADIENT ON FLEXURAL FATIGUE STRENGTH

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by  
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## ABSTRACT

A literature survey and experimental tests were carried out to derive some information regarding the effect of variation of the stress gradient in specimens subjected to reversed flexural bending. The information from the literature review indicates that the presence of a stress gradient increases the fatigue strength of bending specimens relative to that for axial specimens. It is responsible for the large size effect found in bending and may be a necessity for size effect to occur in any mode of repeated loading. Experimental tests conducted for this report indicate that as the stress gradient declines because of a thickness change, with a constant fibre stress, the fatigue strength declines proportionally.

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## TABLE OF CONTENTS

	PAGE
ABSTRACT. . . . .	i
ACKNOWLEDGEMENTS. . . . .	ii
LIST OF TABLES. . . . .	v
LIST OF FIGURES . . . . .	vi
CHAPTER	
1 INTRODUCTION. . . . .	1
1.1 Introduction . . . . .	1
1.2 Statement of Problem . . . . .	2
1.3 Scope of Thesis. . . . .	2
2 REVIEW OF LITERATURE. . . . .	4
2.1 Introduction . . . . .	4
2.2 Fatigue Life . . . . .	4
2.3 Size Effect. . . . .	11
2.4 Notches and Surface Effects. . . . .	18
2.5 Specimen Shape . . . . .	22
2.6 Cracking and Cumulative Damage . . . . .	30
2.7 Summary. . . . .	34
3 EXPERIMENTAL STUDY. . . . .	36
3.1 Introduction . . . . .	36
3.2 Material and Specimens . . . . .	36
3.3 Specimen Preparation . . . . .	39
3.4 Testing Equipment. . . . .	47
3.5 Test Procedure . . . . .	49
4 EXPERIMENTAL RESULTS. . . . .	53
4.1 Introduction . . . . .	53
4.2 Results. . . . .	53
4.3 Analysis and Discussion of Results . . . . .	57
4.4 Summary. . . . .	62

CHAPTER	PAGE
5 SUMMARY OF THESIS . . . . .	63
5.1 Introduction . . . . .	63
5.2 Conclusions . . . . .	63
5.3 Recommendations and Suggestions for Further Study . . . . .	64
BIBLIOGRAPHY . . . . .	66

## LIST OF TABLES

TABLE	PAGE
2.1 Results of Tests of Specimens Selected to Cycles of Reversed Axial Stress (Tension- Compression) . . . . .	9
2.2 Effect of Shape on Fatigue Life (Dolan) . . . . .	23
2.3 Effect of Shape on Fatigue Life (Kawamoto and Nishioka) . . . . .	24
3.1 Chemical Composition of 2024-T4 Aluminum Alloy. .	37
3.2 Mechanical Properties of 2024-T4 Aluminum Alloy . . . . .	38
3.3 Tensile Data From Median Fatigue Specimens - Hounsfield Tensometer . . . . .	38
3.4 Polishing and Etching Solutions . . . . .	43
4.1 Cycles to Failure (Stress 40,000 psi) . . . . .	54
4.2 Cycles to Failure (Stress 36,000 psi) . . . . .	55

## LIST OF FIGURES

FIGURE	PAGE
2.1 Fatigue Data Under Axial Loading and Rotating Bending for 4340 Steel . . . . .	5
2.2 S - N Curves for Direct Stressing and Rotating Bending ( $R = -1$ ) of 4340 Steel . . . . .	7
2.3 Prediction of Rotating Bending Fatigue from Reversed Strain - Cycling Data for 4340 Steel .	10
2.4 Fatigue Curves for Various Diameter Specimens of Mild Steel, Axial Tests. . . . .	13
2.5 The Size Effect in Rotating Bending of Notch Free Carbon Steel Specimens . . . . .	14
2.6 Size Effect - Bending, Axial, and Torsion Loads. . . . .	17
2.7 Effect of Notching on Strength in Axial and Rotating Bending Fatigue Tests. . . . .	20
2.8 Effect of Surface Finish on Endurance Limits of Steel . . . . .	21
2.9 Fatigue Strength of Beams of Various Cross-Sectional Shapes. . . . .	26
2.10 Idealized Cyclic Stress-Strain Curves for a Strain Softening Material . . . . .	28

FIGURE	PAGE
2.11 S - N Curves for Rotating Bending Fatigue Tests of Structural Steel Specimens of Diamond and Round Cross-Sections . . . . .	29
2.12 Cycles to Failure at Strain C. . . . .	31
2.13 The Effects of Overstressing . . . . .	33
3.1 Tensile Specimens for Hounsfield Tensometer. . . . .	40
3.2 Microstructure of 1/4" Bending Specimen. . . . .	41
3.3 Microstructure of Axial and 1/8, 3/8 and 1/2 Bending Specimen . . . . .	42
3.4 Test Specimens - 2024-T4 Aluminum Alloy. . . . .	44
3.5 Test Specimens - 2024-T4 Aluminum Alloy. . . . .	46
3.6 Sonntag Fatigue Testing Machine. . . . .	48
3.7 Axial Loading Fixture. . . . .	50
3.8 Bending Fixture. . . . .	51
4.1 Graphical Representation of Results. . . . .	56



## CHAPTER I

### INTRODUCTION

#### 1.1 Introduction

The problem of fatigue appeared with the advent of the industrial revolution which brought with it many forms of machinery. Machinery meant that some form of repeated loading in the structural members was inevitable, and as a result led to inexplicable equipment failures; failure of parts designed under what would be considered safe static design criteria.

Since its early recognition and study by men such as Wohler [1] to the present with its formidable amount of research, even now failure by fatigue is the cause of ninety per cent of in-service failures.

From the year 1950 there has been an average of a fifteen per cent per year increase in the number of papers published on the various aspects of fatigue. At present a great deal is known about the causes and effects of fatigue and yet, there remains an enormous amount to be learned and utilized in the future. The technical progress in outer space, hydrospace, and general living demands that more be known and more be understood. It was for this reason that this thesis project was undertaken: to add, if possible to

that knowledge or if definite results could not be achieved, to give impetus to others who may read this to explore deeper in this direction.

Much has been written and most tests when conducted in fatigue are in either the axial or bending mode, with torsion a poor third. With respect to inter-relation between the two, it is generally accepted, by men such as Manson [2], Yokobori [9], etc., on the basis of comparison, that tests conducted in bending will show a longer fatigue life than those conducted under axial loading for a given stress level. The question of what causes this disparity, if there is a disparity, is the object of much speculation.

In the thesis the author has attempted to explore this difference, but in general has confined himself to tests which would give possible insight into the effects of the variation of the stress gradient on fatigue life of a specimen subjected to bending. Not much, if anything, is known about the effects, other than there exists a feeling among researchers in this field, that this difference between the two testing methods is related to the stress gradient.

## 1.2 Statement of the Problem

The area to be investigated is the effect of variation of the stress gradient on fatigue life in completely reversed flexural bending. It will be an experimental approach.

## 1.3 Scope of Thesis

The thesis is divided into five chapters.

Chapter 1 is an introduction to fatigue and a general statement of what is to be explored in the thesis.

Chapter 2 is a literature review of representative literature available associated and dealing specifically with the subject under consideration.

Chapter 3 is an experimental program carried to obtain experimental information about the point under consideration.

Chapter 4 is a discussion of results and their relation to previously discussed reference material.

Chapter 5 is a summary of the thesis and recommendations for further study in this area.

A bibliography is presented at the end of the thesis.

## CHAPTER 2

### REVIEW OF LITERATURE IN BENDING FATIGUE

#### 2.1 Introduction

The literature review to follow, will deal mainly with bending fatigue strength and factors which have a marked influence on this strength and to a degree characterize its performance. It will cover bending fatigue strength in all ranges of fatigue life, that is both high and low cycle. Bending fatigue properties, where the author deems desirable, will be discussed relative to axial fatigue performance under similar conditions or situations. This will define the differences and similarities between certain aspects of bending fatigue strength and axial fatigue strength characteristics.

#### 2.2 Fatigue Life

It is generally conceded that fatigue strength in bending, regardless of type, flexural or rotational, will be greater than that obtained by axial fatigue tests conducted at the same nominal stress level. This does not appear to apply to the same degree over the complete stress-cycles to failure spectrum. The most pronounced difference between the two forms of testing occurs under one million cycles according to Manson [2], as shown in Figure 2.1. The work of Sachs and

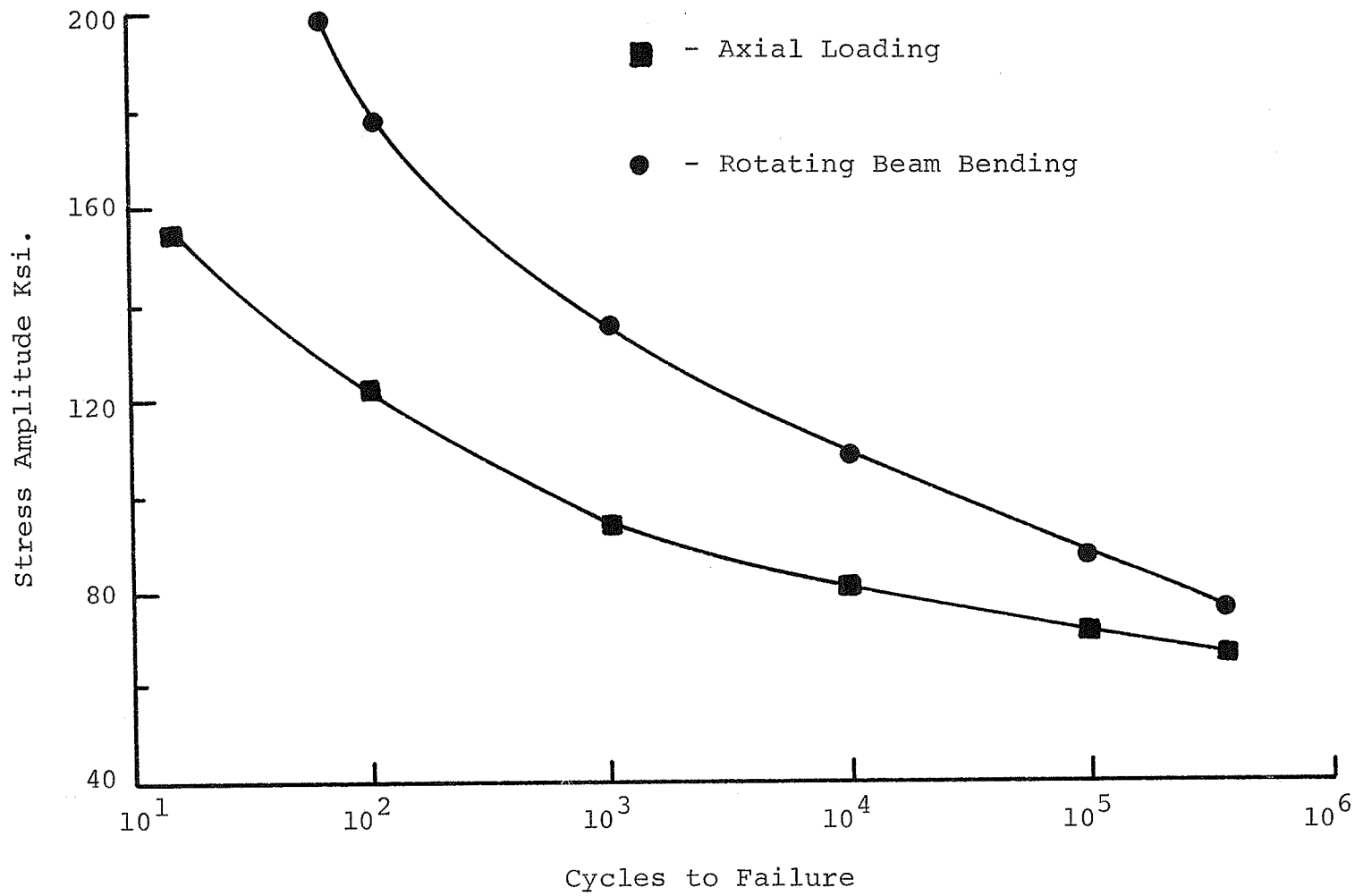


Fig. 2.1 Fatigue Data Under Axial Loading and Rotating Bending for 4340 Steel [2]

Scheven [3] in a similar area substantiates this conclusion. Some of their results are shown in Figure 2.2. This was found to apply to both notched and unnotched specimens.

Mechanical testing has shown that the nominal bending strength of ductile materials was considerably higher than the ultimate tensile strength. Tests conducted by Sachs and Scheven on high strength steels confirmed that the bend-strength ratio; ie. ratio of strength in bending to strength in tension, is usually higher than unity. This was found to apply not only to smooth specimens but also to low cycle fatigue strength values of notched specimens.

Sachs and Scheven found that the ratio between bending and direct fatigue test results in the high cycle region is almost unity. In Figure 2.1 the curves apparently converge and one could speculate on the possibility of the bending-direct fatigue ratio approaching unity, particularly in the million plus range. But this, based on Manson's work involves only guessing, as the curve is not drawn beyond a million cycles.

Work done by Moore and Jasper [4] at the University of Illinois in the high cycle region of the S-N curve indicates that for steels of both ductile and brittle types the endurance limit as determined from bending tests was not equal to that determined via axial tests. Their results indicate at least a thirty per cent reduction in endurance limit when determined via axial tests. A portion of their results can

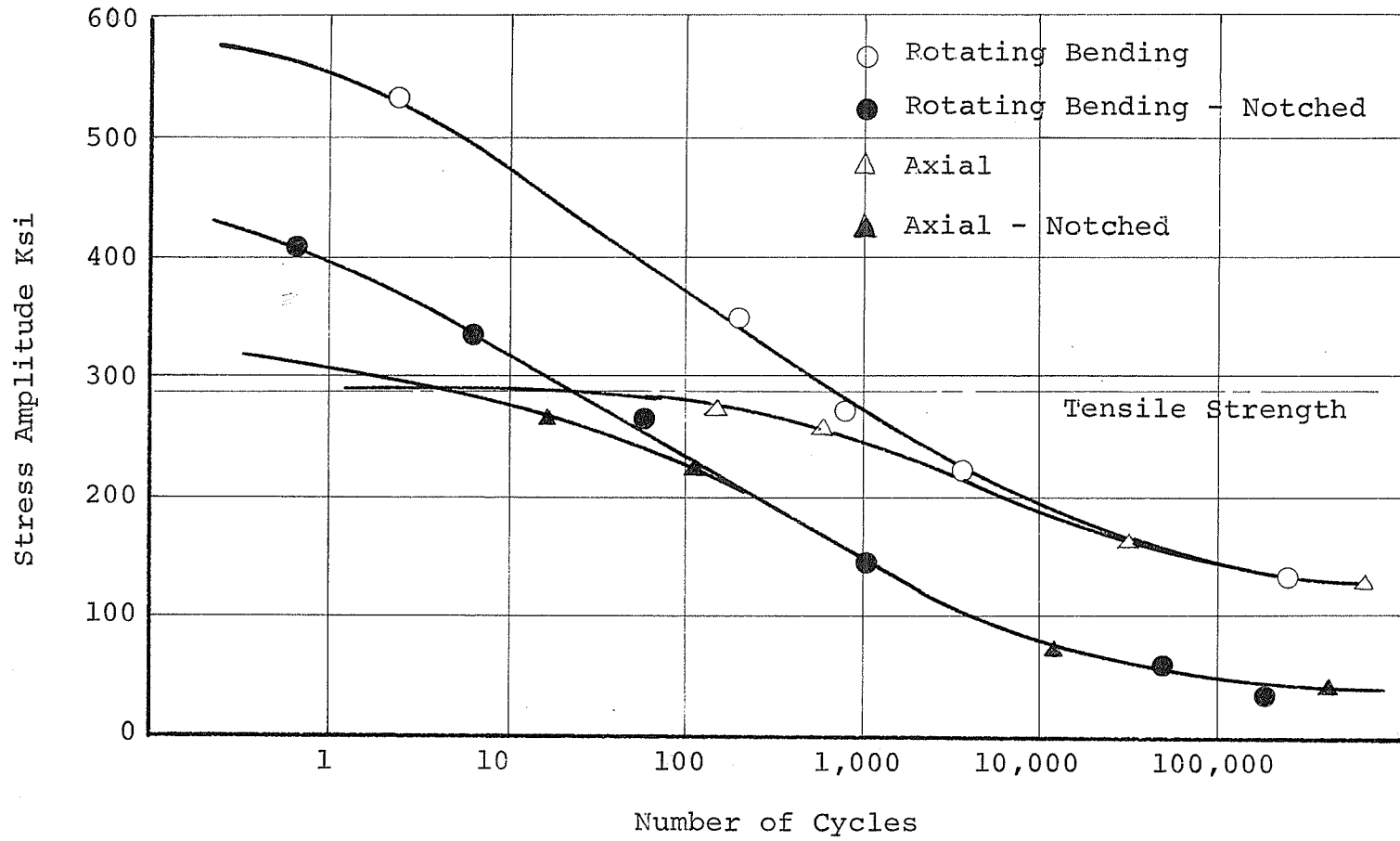


Fig. 2.2 S-N Curves for Direct Stressing and Rotating Bending ( $R = -1$ ) of 4340 Steel [3]

be seen in Table 2.1. Manson and Hirschberg [5] support this contention. They state that the endurance limit found from axial tests will be in the neighbourhood of sixty-five per cent that found by bending tests.

Lipson and Juvinal [6] suggest that the difference in bending and axial fatigue strengths could be safely accounted for by assuming that axial fatigue life should be estimated as eighty-five per cent or less when designing for repeated axial loads from bending fatigue strength data.

Moore and Jasper attribute this to the difference in areas under load. Since the stress gradient is uniform in the axial specimens and fatigue being a statistical problem the chance of stress raisers to promote cracking is therefore greater in an axially loaded specimen.

Manson in his discussion of the difference, in the low cycle range only, states that the difference in fatigue strengths shown in Figure 2.1 is not as great as it would appear. He bases this on the fact that the formula used to calculate bending stresses assumes elasticity, which is not accurate, as strain in the low cycle region is in the plastic range. He concludes that one of the main reasons that bending and axial fatigue data do not agree in this region is that in reality one does not know what the true strain on the surface is for a given applied bending moment. The predicted values in Figure 2.3 are based



TABLE 2.1  
 RESULTS OF TESTS OF SPECIMENS SELECTED TO CYCLES  
 OF REVERSED AXIAL STRESS (TENSION--COMPRESSION)[4]

Designation	Endurance Limit lb. per sq. in.		Ratio of Endurance Limits Ea;Ef
	Axial Testing Machine (Tension- Compression) Ea	Rotating- Beam Testing Machine (Flexure) Ef	
1.20 carbon, normalized	28,000	50,000	0.56
0.37 carbon, normalized, longitudinal	21,000	33,000	0.64
sorbitic, longitudinal	33,000	57,000	0.58
0.93 carbon, normalized	20,000	30,500	0.66
sorbitic	36,000	56,000	0.64
3.50 nickel, treatment A	44,000	64,000	0.69
treatment B	38,000	63,000	0.60
treatment C	42,000	64,000	0.66
treatment D	36,500	54,000	0.68
0.02 carbon, as received	17,000	26,000	0.65
0.49 carbon, normalized	20,000	33,000	0.61
0.53 carbon, normalized	24,000	36,000	0.67
Wrought iron, longitudinal	16,000	23,000	0.70
Average			0.64

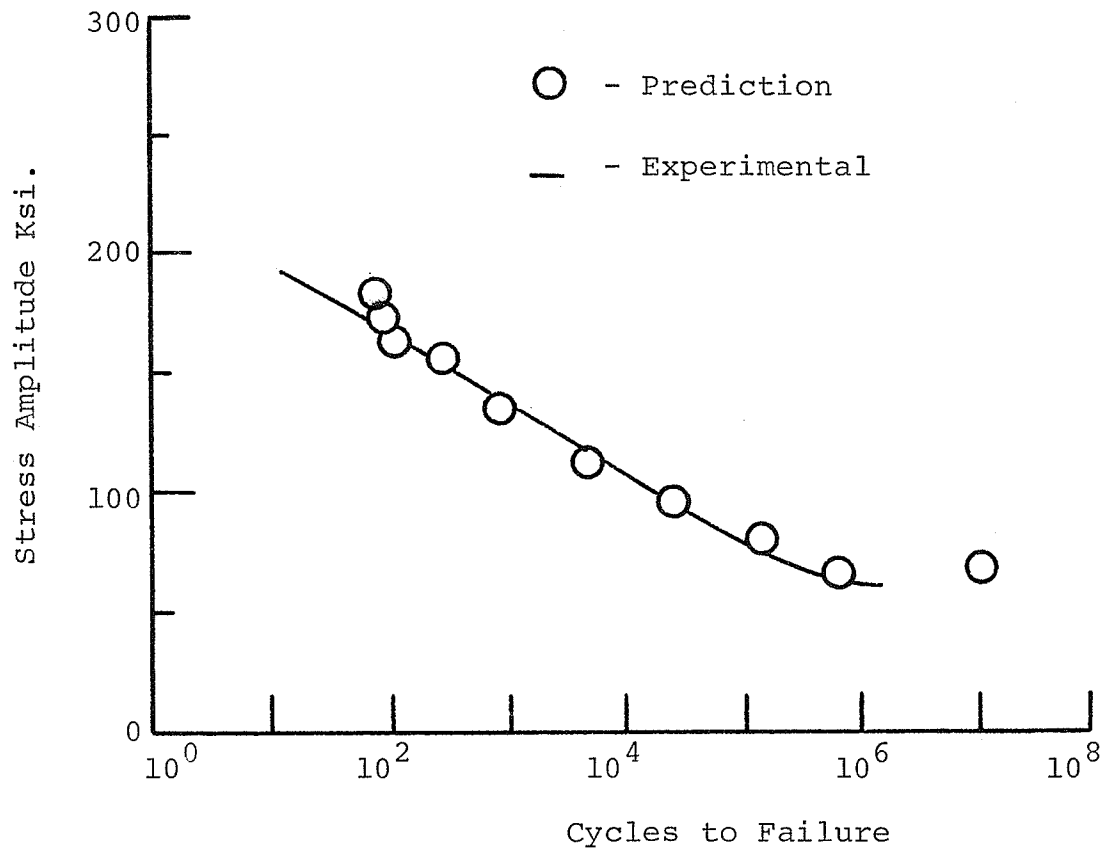


Fig. 2.3 Prediction of Rotating-Bending Fatigue from Reversed Strain-Cycling Behaviour for 4340 Steel [2]

on axial fatigue data, and the experimental results are based on specimens tested using the calculated actual strain values, not just the nominal strain values calculated by the elastic criterion.

Using Manson's criterion, assuming it to be correct, and the information by Sachs, Scheven, Moore and Jasper, the bending fatigue strength is close to that found from axial fatigue tests throughout the full stress-cycle to failure range. The difference which does exist can be attributed to something involving the stress or strain gradient of which Manson states not very much is known.

### 2.3 Size Effect

Dieter's [8] results in his research on size effect are contradictory, but generally show a decline in the fatigue limit with an increase in specimen dimensions. Yokobori [9] states that there is ample evidence, both from recent experiments and earlier work beginning in the nineteen thirties that fatigue strength declines as size increases. His research indicates that size effect can be attributed to several factors, of which there are five main points. These are listed below:

(i) Defects are present on the surface regardless of the finish. These defects act as notches and are considered to be statistically distributed and as a consequence the larger the surface area the larger the probability of such a defect being present.

(ii) When the stress gradient is high, as it is in specimens with a notch, the growth processes often have some influence. After the sub-microcracks (cracks not detectable under a microscope) form, the process of growth and of formation alternate. If a crack once grows to certain length from the surface, then the crack successively propagates in steps with which are associated decreases in the free energy of the system. Eventually rupture occurs.

(iii) The positions in a material from which a sample is taken for testing has some effect. In as-rolled materials, for example, the center part is weaker because of segregation, e.g., phosphorous and manganese in as-rolled steels.

(iv) Heat treatment and working have differing effects, at times, on different parts of one of the specimens. The surface and the center are an obvious example of two such parts, where there may be a slightly different structure due to differing rates of cooling or differences as listed in part three.

(v) The heating rate, during testing, of specimens varies according to the specimen sizes. If the cycling rate is the same, a thicker specimen experiences a higher temperature. This heating effect can be loosely attributed to the effects of internal friction. An example of size effect in axial fatigue is shown in Figure 2.4.

Kawamoto and Harumoto [10] used the guidelines listed by Yokobori, to establish a consistent set of data, as shown in Figure 2.5. From this figure one can see a tendency for

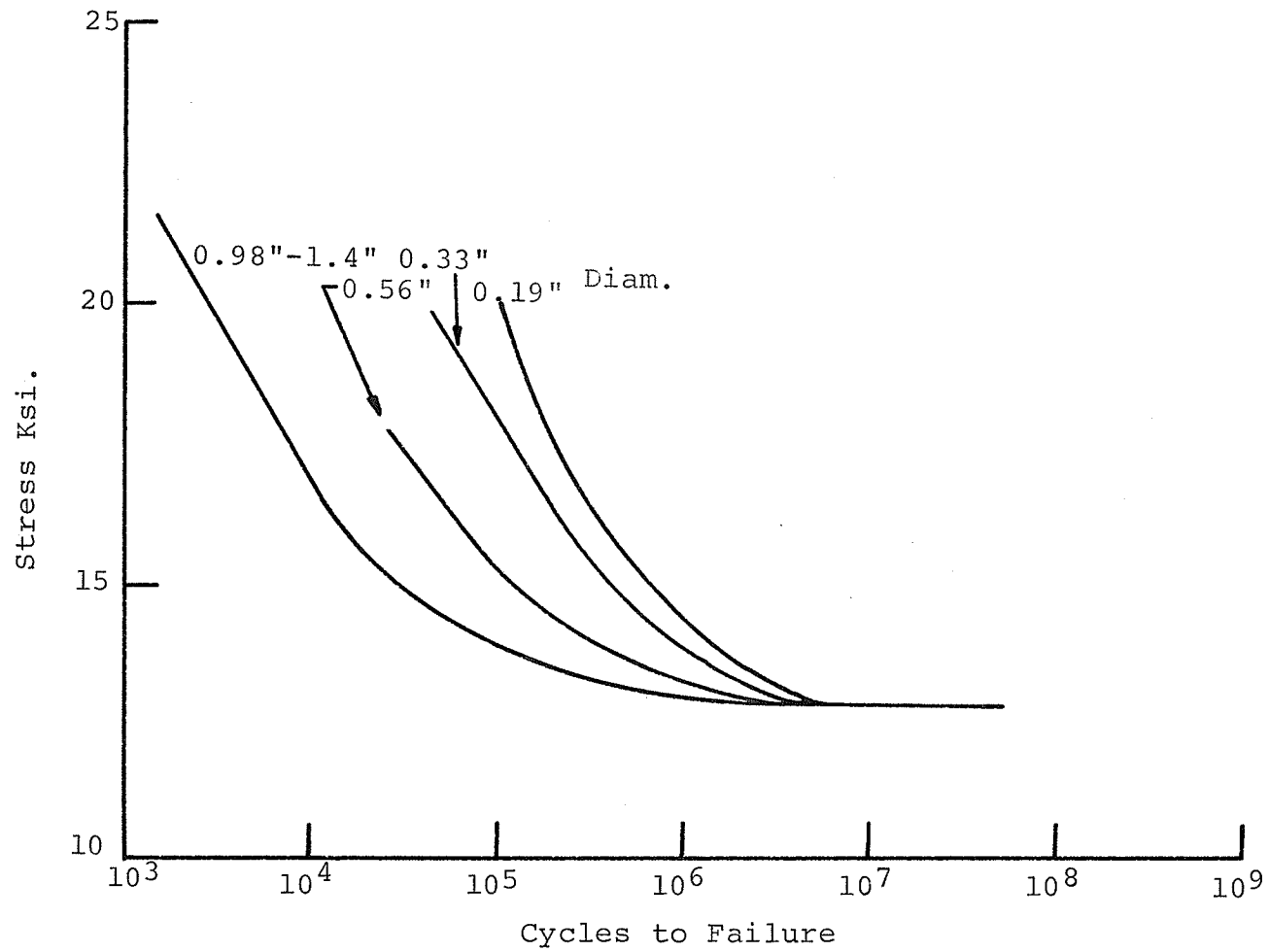
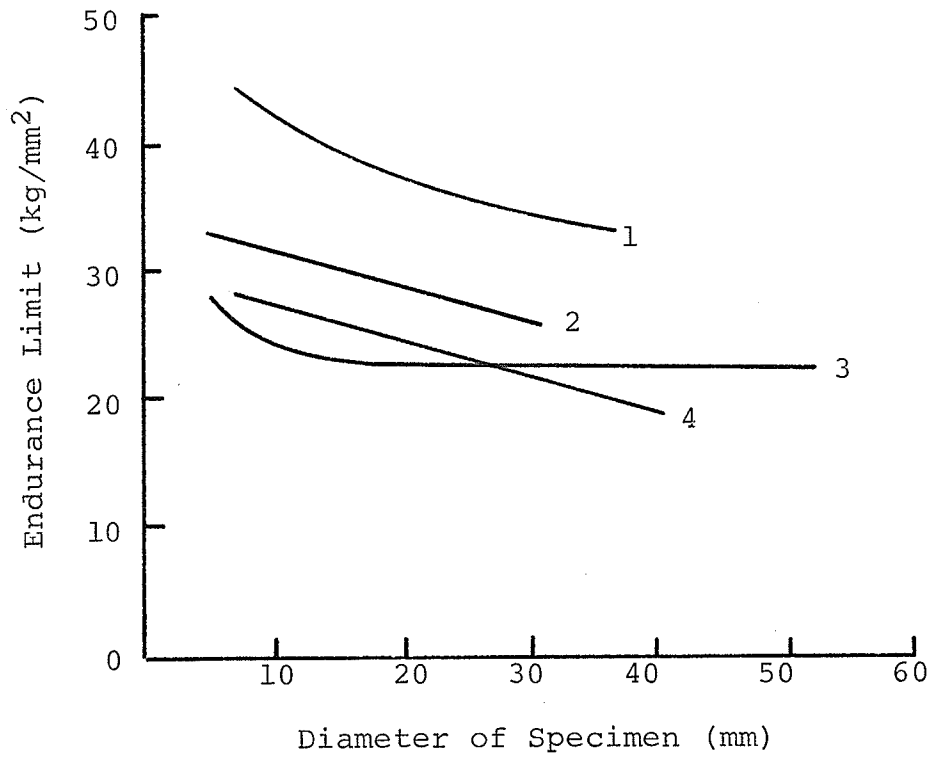


Fig. 2.4 Fatigue Curves for Various Diameter Specimens of Mild Steel; Axial Tests [1]



1.  $C = 0.46\%$ ,  $uts = 86 \text{ Kg/mm}^2$
2.  $C = 1.0\%$ , normalized,  $uts = 85 \text{ Kg/mm}^2$
3.  $C = 0.34\%$ ,  $uts = 61.8 \text{ Kg/mm}^2$
4.  $C = 0.1\%$ , normalized,  $uts = 85 \text{ Kg/mm}^2$

Fig. 2.5 The Size Effect in Rotation Bending of Notch-Free Carbon Steel Specimens [10]

the endurance limit to decrease as the diameter increases. According to the results noted by Yokobori the endurance limit of carbon steels tested in tension and compression decrease markedly as size increases. Phillips [11] states that this behavior does not extend to mild and alloy steels under similar conditions, which show little or no size effect. Studies by Yokobori, indicate that the size effect based on diameter are not as marked in tension-compression loading as for torsion or bending loads. There is also, as noted by Weibull [12], a size effect in length and fatigue strength decreases as specimen length increases.

Polakowski and Rippling [13] state that a thirty per cent reduction in fatigue life is possible in going from a 0.5 inch diameter to a 10 inch diameter specimen. This concurs with Yokobori in the formulation that size effect is very pronounced in bending. No explanation of the reversed bending behaviour is offered.

Dieter indicates that no size effect has been found for smooth plain carbon steel specimens with diameters ranging from 0.2 inch diameter to 1.4 inch diameter when tested in axial tension compression loading. When a notch was introduced a definite size effect took place. The notch introduced a stress gradient. This would, according to Dieter, support the idea that a size effect in fatigue was due to the existence of a stress gradient in the specimen. The fact that a large specimen with a shallow stress gradient has a lower fatigue strength is consistent with the idea,

that a critical value of stress must be exceeded over a certain finite depth of material for failure to occur.

In pursuing the point further, Dieter states that changing the size of a fatigue specimen usually results in a variation of two factors. First, an increase in diameter increases the volume or surface area. The change in amount of surface area is significant in that fatigue failures usually start at the surface. Second, for plain or notched specimens loaded in repeated bending or torsion an increase in diameter means a decrease in stress gradient across the diameter and an increase in volume which is highly stressed. To Dieter this appears to be a more realistic criterion of size effect than simply the ratio of the change in surface area. The importance of the stress gradient in size effect may help explain why correlation between laboratory results and in service results is not good.

Lipson and Juvinall [6] state that the size effect is negligible, as advanced by Dieter and Yokobori, in some cases and all cases of axial loading is less than that for bending or torsion. They also state that this is probably due to the zero stress gradient associated with axial loads as shown in Figure 2.6.

It appears based on the various statements of the authors noted in this section regarding size effect, that the stress or strain gradient plays a large role in the occurrence or nonoccurrence of size effects. The introduction



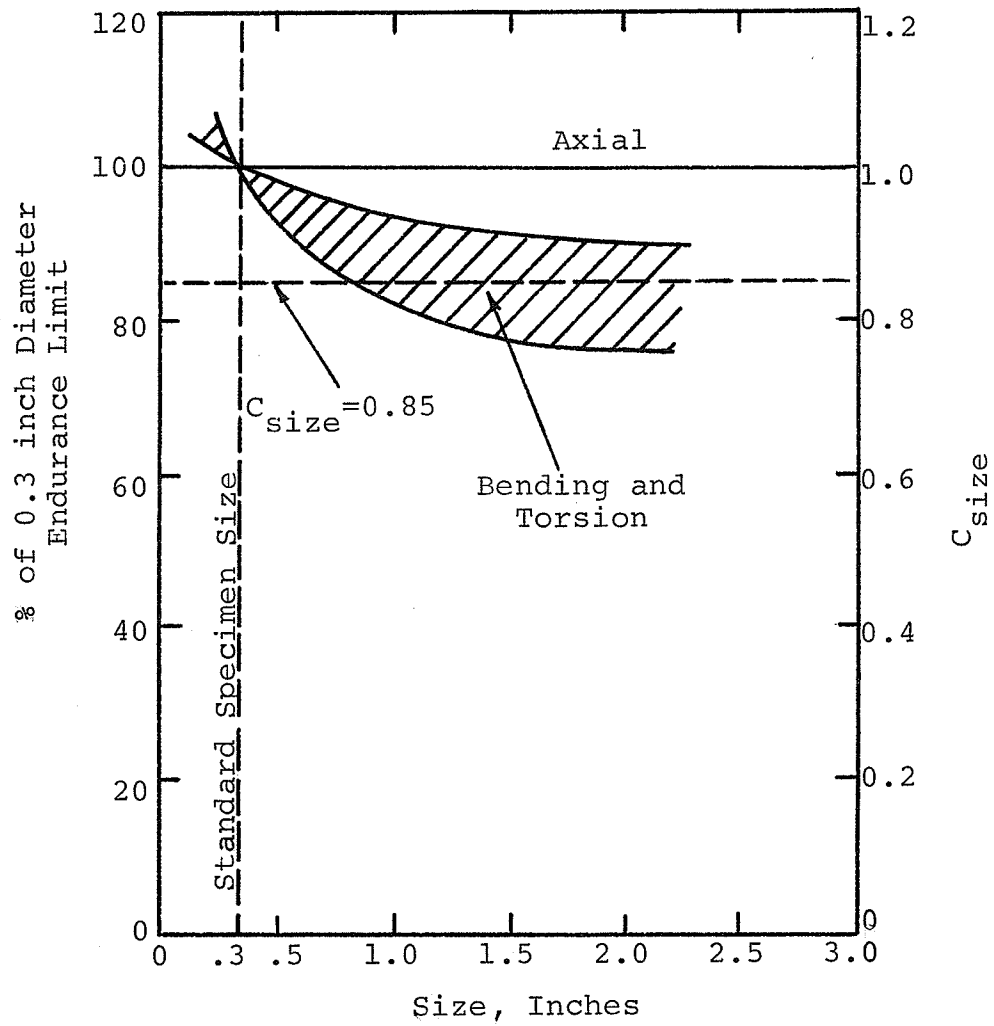


Fig. 2.6 Size Effect - Bending, Axial, and Torsional Loads [6]

of a stress gradient by a notch in an axial specimen in some cases is enough to trigger a size effect in a material which showed none in a smooth condition. The lower the stress gradient in bending fatigue the greater the tendency to reduce life in two different sizes of specimens for a given nominal stress. According to Dieter, in particular, the stress gradient is instrumental in causing a size effect. This based on the statement that it is apparent that there is a finite depth over which a critical value of stress must act for failure to occur. The depth over which this critical stress must act is greater in a larger specimen than for a smaller one.

#### 2.4 Notch Sensitivity and Surface Effects

Although it is probably not too desirable to separate the above topics from size effect, as they are interrelated and not easily distinguishable in some cases, for clarity it is advantageous to do so here.

Sachs and Scheven [3] in their research have indicated that there is only a minimal difference between behaviour of notched specimens in bending and notched specimens in axial loading. The overall behaviour of either notched bending or notched axial specimens shows a marked reduction in life with respect to unnotched specimens. The same characteristics are present in both, the high and low cycle regions. As shown in Figure 2.2, the fatigue strength of the notched specimens may equal or exceed that of the unnotched

specimens between one hundred to a thousand cycles. According to Dieter [8], having geometrically similar notches will not produce the same stress gradient across the grains of the specimens, if the specimens are of different diameters. Thus, tying in size effect, the effects of the notch will be greater in the large diameter specimen because of stress gradient effects. Yokobori [9] indicates that there is a marked size effect in notched specimens, and as mentioned by Dieter in Section 2.3, it is possible that size effect is a result of a stress gradient being present. This stress gradient can come from notches or surface defects, such as scratches, pits, etc..

It has been found by Dieter [8] and several other authors, that surface treatment such as carburizing and nitriding, which improves fatigue performance of a material in most cases, have a greater effect on material being subjected to repeated bending or torsional loads than for material subjected to repeated axial loading. The greatest percentage increase in fatigue strength was found when notched fatigue specimens were treated with the forementioned processes. As recorded previously, the presence of a notch introduces a stress gradient. The amount of strengthening also depends on the diameter of the part and depth of treatment. Figure 2.7 shows the effect of notches on fatigue life in axial and bending modes.

In general, surface effects, as shown in Figure 2.8,

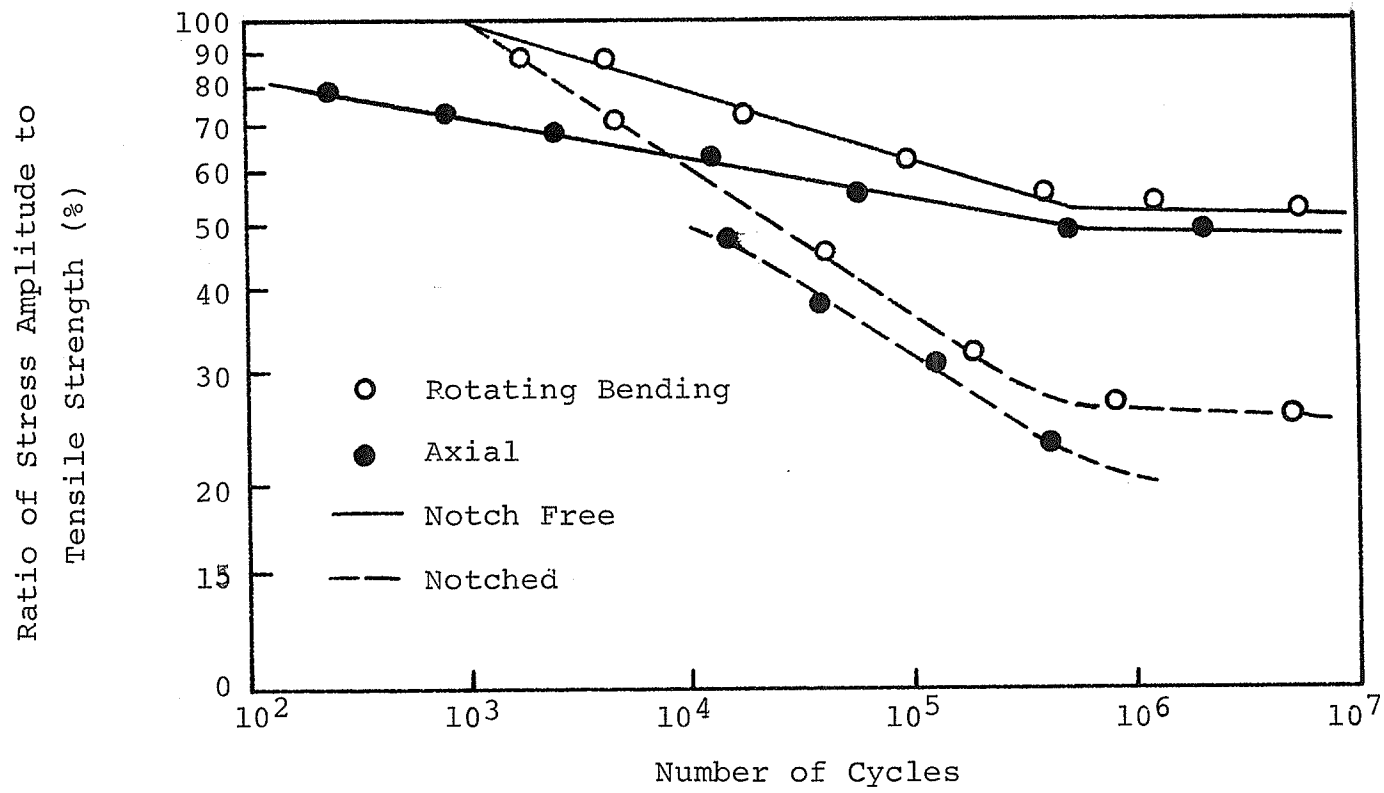


Fig. 2.7 Effect of Notching on Strength in Axial and Rotation Bending Fatigue Tests [9]

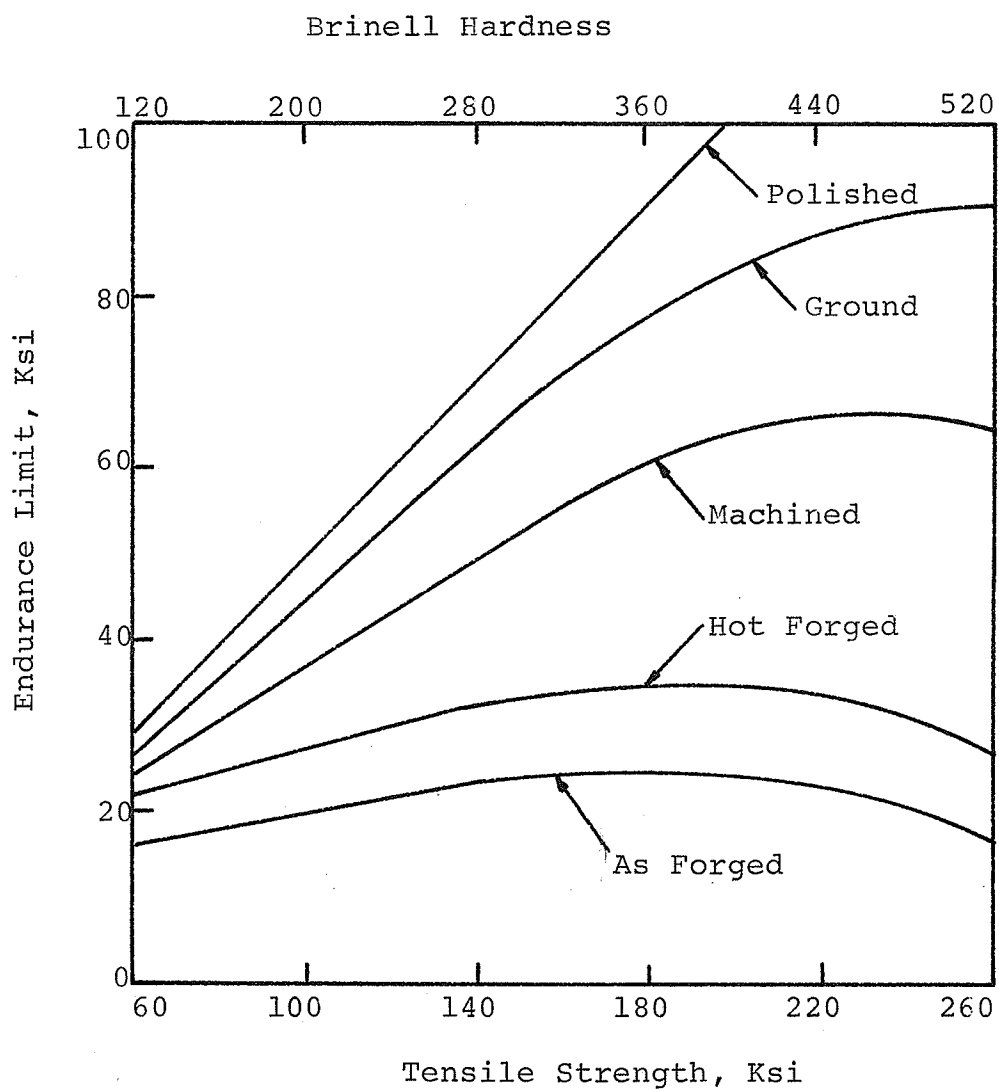


Fig. 2.8 Effect of Surface Finish on Endurance Limit of Steel [6]

play an important role in fatigue life, whether it be axial, bending, or torsional. Defects in the surface and notches combine to reduce fatigue life. However, their influence is not equal, but depends on the nature of loading and therefore the stress gradient present in the specimen. It appears that the bending and torsional specimens would be more sensitive to these effects because of the existence of an inherent stress gradient and its effect on the performance characteristics of specimens subjected to such loading. It could be thus concluded, based on information in Sections 2.2 and 2.3 that because of a stress gradient reduction, i.e., less of a gradient in a large specimen for a given nominal stress, that the surface effects would be more pronounced in bending or torsion than for an axial specimen with its uniform stress gradient.

### 2.5 Specimen Shape

Dolan [14] and Kawamoto and Nishioka [15] have shown that bending fatigue strength is strongly influenced by the shape of the specimen cross-section. Results obtained by Dolan, as shown in Table 2.2 show that the fatigue strength varies according to the specimen shape. The specimen with the circular section has the longest fatigue life, with the diamond section next. Table 2.3 contains the results obtained by Kawamoto and Nishioka for specimens of 0.1% annealed carbon steel. Their results agree with Dolan's.

TABLE 2.2

## EFFECTS OF SHAPE ON FATIGUE LIFE (DOLAN)[14]

## MAYARI - R. Steel

Shape of Section	Endurance Limit (psi)	Ratio of Endurance Limit to that of Circular Specimen
circular (lathe turned)	50,000	1.00
diamond (milled)	48,000	0.96
square (milled)	46,000	0.92
modified diamond	44,000	0.88

## SAE 4340 Steel

circular (lathe turned)	90,000	1.00
circular (milled)	89,000	0.99
diamond (milled)	83,000	0.92
modified diamond	82,000	0.91
square (milled)	80,500	0.89

TABLE 2.3

## EFFECT OF SHAPE ON FATIGUE LIFE

(Kawamoto and Nishioka) [15]

Shape of Section	Endurance limit kg/mm <sup>2</sup>	Ratio of endurance limit to that of circular specimen
circular	20.4	1.00
cross	20.0	0.98
rectangular	19.8	0.97
round tube	19.7	0.96
I	18.7	0.91



The specimen shapes as used are shown in Figure 2.9.

Yokobori [9] attributes the difference in fatigue strength to various causes as listed below:

(i) Preferential stress accretion--A certain region or volume of the specimen is subject to stresses in excess of a specified percentage of the maximum. The occurrence of these is determined probabilistically and their integrated effect is supposed responsible for fracture.

(ii) Inelastic behaviour--Local effects at the outermost parts of a beam differ according to the shape of the cross-section.

(iii) Stress concentration--The effect of stress raisers near the surface is greater than that of those located internally. Moreover, plastic deformation in grains or aggregates situated near surfaces occurs more easily because of the smaller constraint in the lateral direction.

(iv) Manufacture--The processes themselves produce differences in residual stresses and the extent of initial work hardening in beams of different shapes.

According to Dolan [14] the last effect is small whereas the influence of inelastic behaviour is noticeable. In his view, microscopically nonuniform inelastic strains are sensitive to the shape of the cross-section of a beam in the manner encountered in yielding.

Raghavan [15] on conducting similar tests as Dolan found that for rotating bending that the diamond specimen was

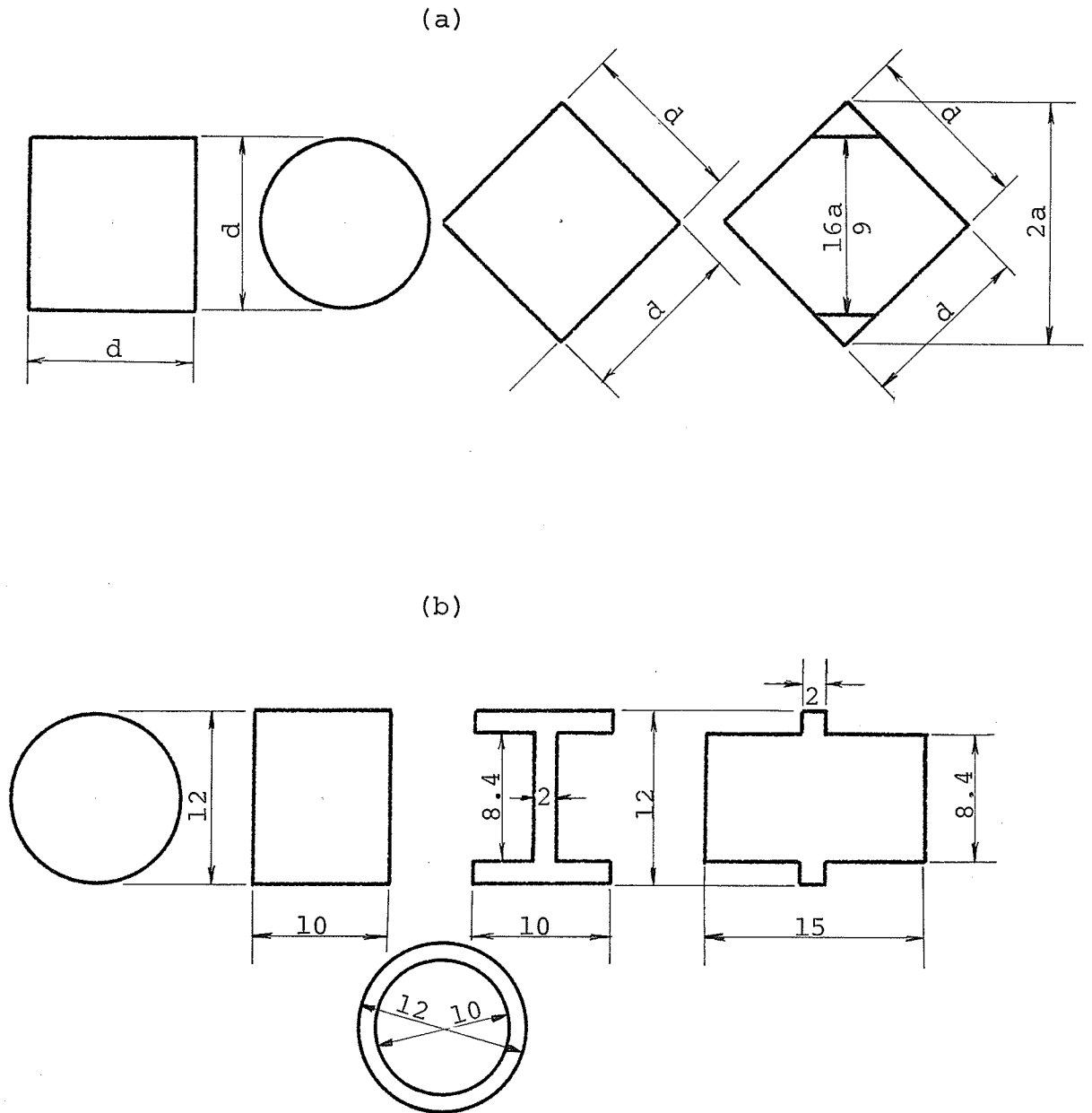


Fig. 2.9 Beams of various cross-sections

(a) Dolan [14]

(b) Kawamoto and Nishioka [15]

stronger in fatigue than the round specimen. He attributes the difference to size effect, polishing technique and type of testing machine. However, for specimens under vibration loading Raghavan found that the diamond shaped specimens versus the round specimens, in one case agreed with Dolan and in the next case did not. He attributes this also to size effect, polishing technique, and testing machine.

Blatherwick and Lazan [17] concluded from tests that the cyclic secant modulus changed as a result of change in the distribution of bending stress during low-cycle fatigue testing, as shown in Figure 2.10. Furthermore, the degree of change depended on the cross-section. In general a shape having a greater portion of its area remote from the neutral axis was more sensitive to the change than one with most of its area bulk near the neutral axis. The stress redistribution resulting from changing the cyclic secant modulus affects fatigue life. The authors do not, however, give any actual fatigue results. Some obtained by Raghavan are shown in Figure 2.11 and also in previously mentioned Tables 2.2 and 2.3.

Lazan and Blatherwick found regardless of the specimen shape, that the results of the fatigue tests would also vary according to the mode of loading. Maximum life was obtained for constant stress amplitude, then constant moment amplitude to finally constant strain amplitude. This was found for strain softening materials with the reverse being true for

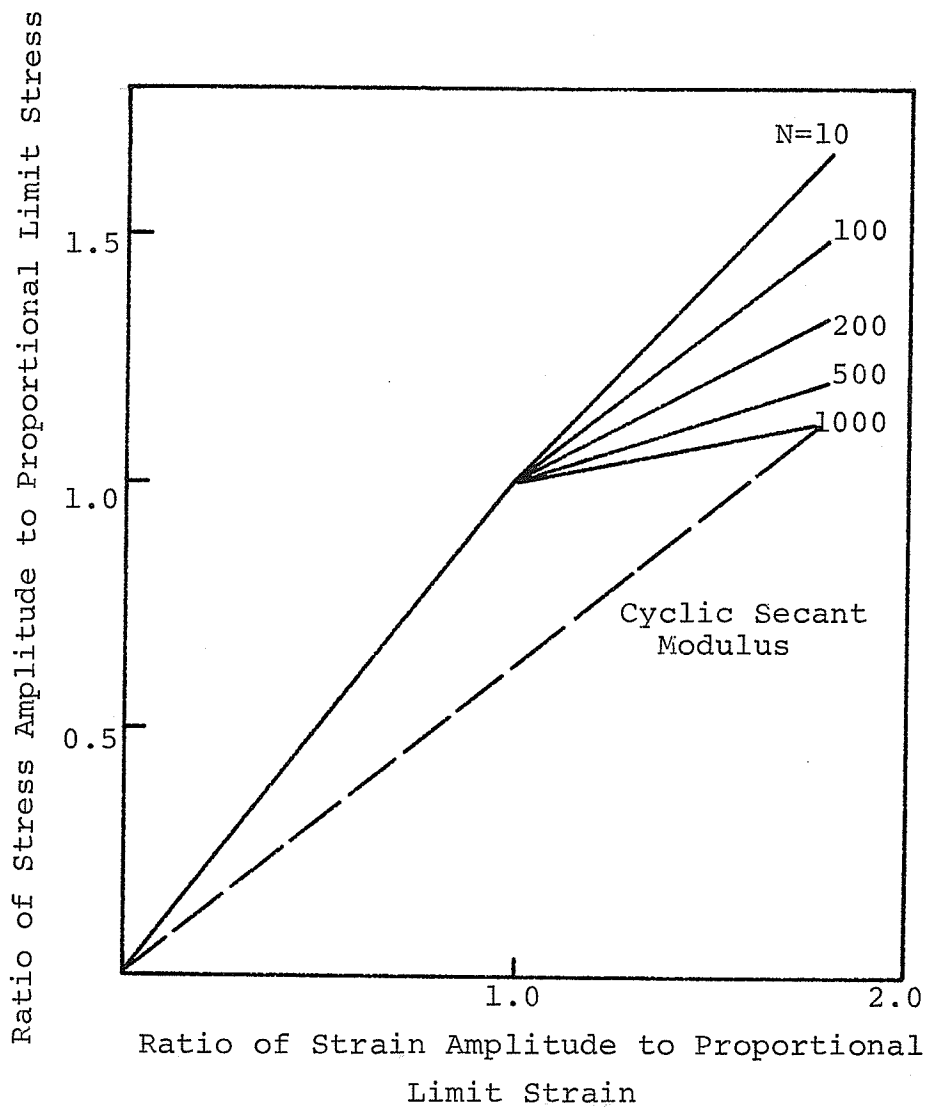


Fig. 2.10 Idealized Cyclic Stress-Strain Curves for a Strain-Softening Material [17]

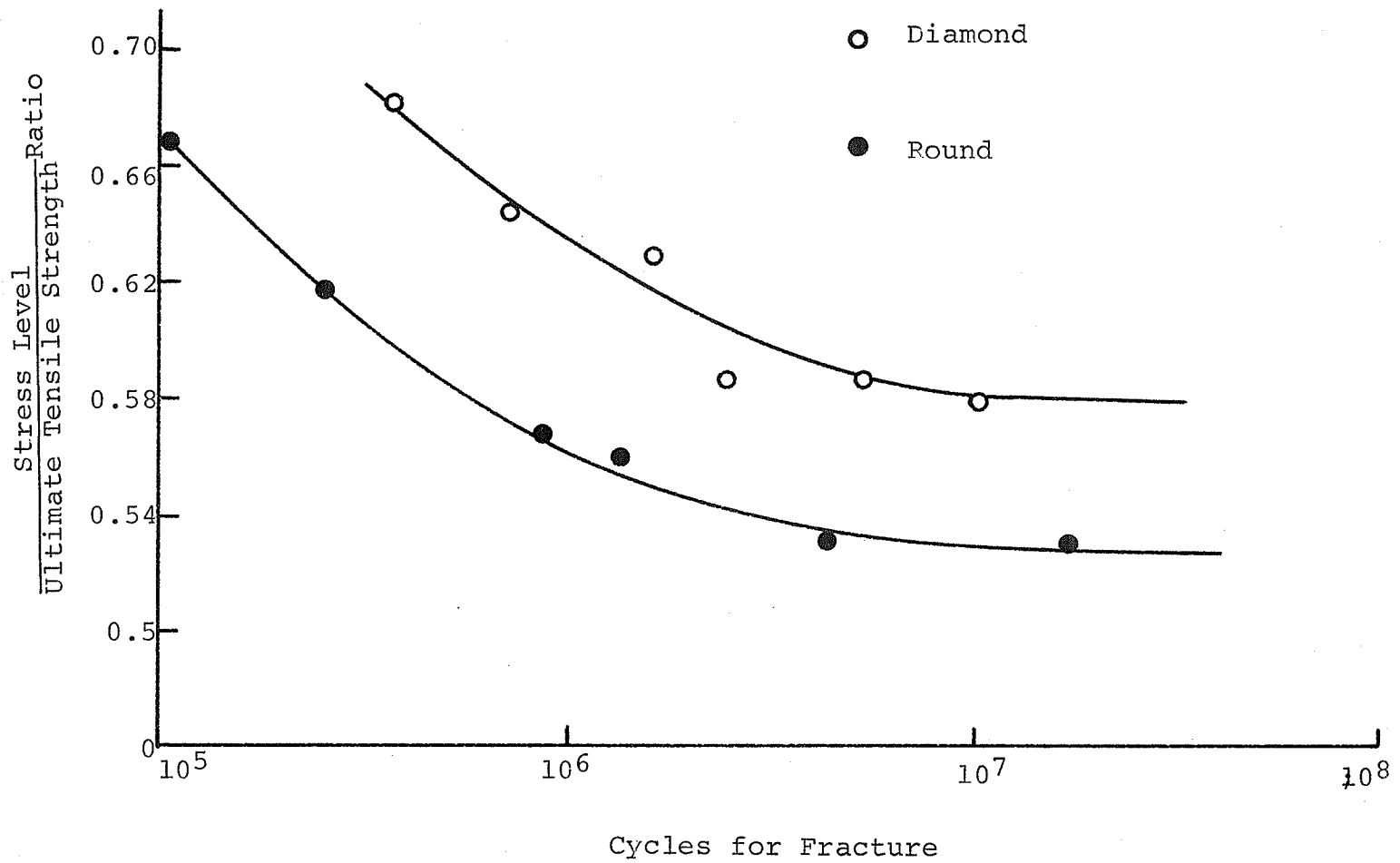


Fig. 2.11 S-N Curves for Rotating Bending Fatigue Tests of Structural Steel Specimens of Diamond and Round Cross-Sections [16]

strain hardening materials.

The link between specimen shape and the stress gradient is that a shape with most of its area out beyond the neutral axis will effectively place most of its area or volume in the high stress region of the stress gradient. This is comparable to a reduction of the stress gradient by an increase in size which places a greater area of material subjected to high bending stresses in the region beyond the infinite life region of the S-N curve. It would seem as if it were equivalent to size effect only arrived at by a different means.

## 2.6 Cracking and Cumulative Damage

Manson [2] in his discussion of bending fatigue strength versus that of axial fatigue strength indicates that the crack stage in bending is longer than that for axial conditions. He moves on to state that the assumption that failure occurs soon after the crack starts is somewhat conservative for bending tests. This effect in axial specimens is due to saturation hardening of the entire cross-section. It does not occur in bending where all levels in the specimen are subject to different degrees of working. In addition he states that the linear damage rule does not generally apply to bending tests.

Low [18] found from results of tests on aluminum alloys in bending that neither the cumulative damage theory nor any form modified by a constant factor held. His results are shown in Figure 2.12.

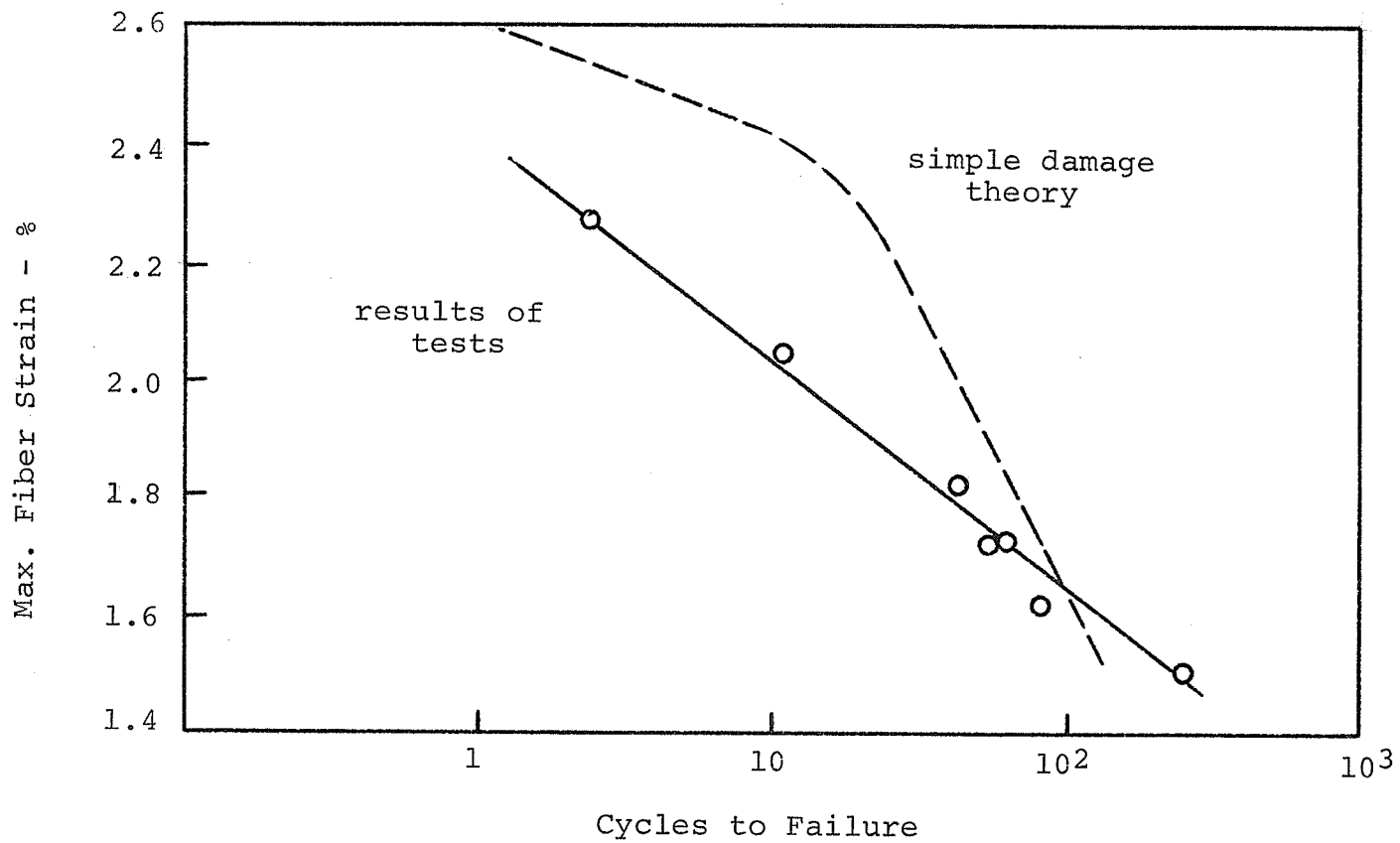


Fig. 2.12 Cycles to Failure at Strain C [18]

Work has been done on cumulative damage in bending as noted in the bibliography. Manson, Nachtigall, and Freche [19] proposed a new relation for cumulative damage in bending based on observed experimental behaviour to overcome the difficulty in applying theories based on axial fatigue performance criterion. The authors make one thing clear; this theory applies only to bending and not other forms of fatigue loading. Later investigations into the same by Manson, Nachtigall, Ensign, and Freche [20] proposed additional concepts to improve accuracy but it apparently still applies only to bending fatigue.

Results on damage interrelation between the three major forms of tests, suggest that damage in one mode may have no effect on the fatigue life in another mode in certain cases. Nishihara and Kawamoto [21] showed that the effects of overstressing in bending had no effect on the torsional endurance limit, as shown in Figure 2.13. The latter suggests that the damage occurred with a preferred orientation in the specimen.

From the minimal information available it appears that the strain or stress gradient has an effect on crack propagation and the effects of cumulative damage. Because of the stress gradient cracking is slower and cumulative damage theories do not apply due to their being based on axial data and performance. No information is available on the effect of the stress gradient variation on cracking rate or damage



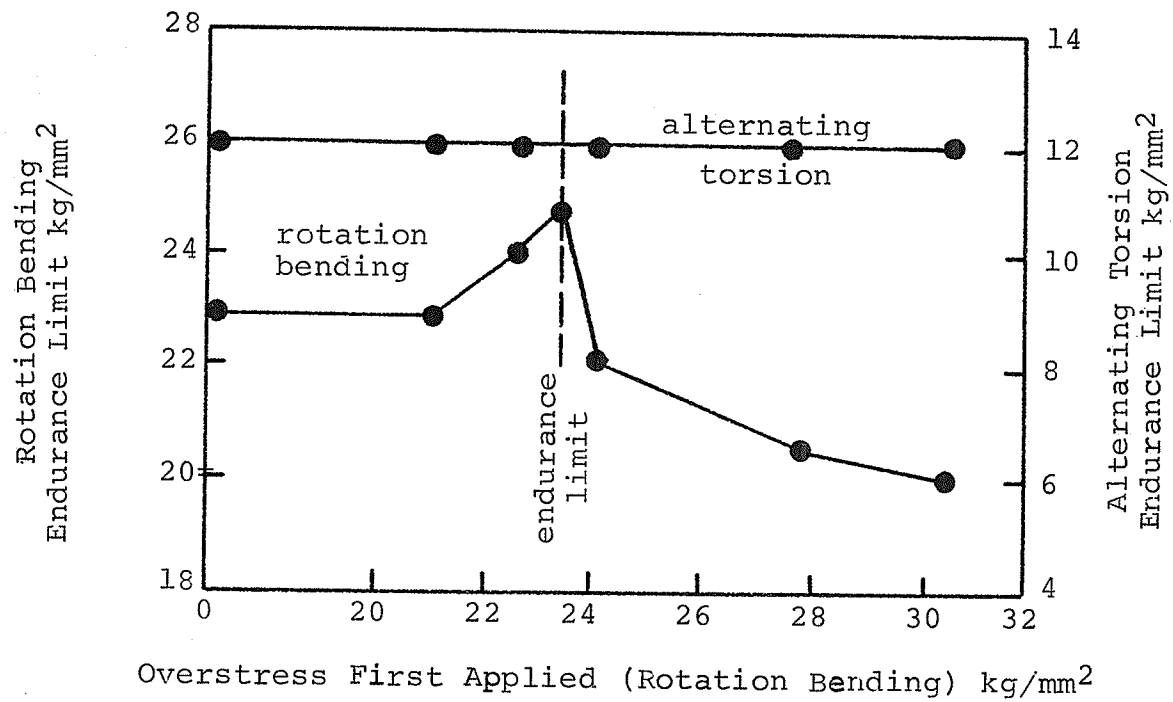


Fig. 2.13 The Effect of Overstressing [21]

due to working. In particular, does the cracking rate increase and the cumulative damage theories apply more closely when the stress gradient in the specimens approaches that of a uniform condition?

## 2.7 Summary

The fatigue strength in bending and the influential variables which affect it can be summarized in the following point form:

(a) Fatigue strength in bending is greater in the low cycle region than in the high cycle region when measured against axial fatigue strength. The difference in fatigue life between the two modes in the low cycle region may be more apparent than real. However, the overall fatigue strength in bending is greater than that found for axial loading in all regions of the S-N curve. The increase in life in bending can be attributed to the presence of the stress gradient.

(b) The presence of a stress gradient appears to cause a greater size effect than found in axial tests, whether the specimens are notched or not.

(c) Notches and surface effects are more critical to fatigue strength in bending than for axial tests.

(d) Bending specimen shapes have a very great influence on the fatigue strength, compared to axial sensitivity in the same area.

(e) The stress gradient affects growth rates and the

validity of cumulative damage theories. The effect of variation of the stress gradient on either one of these is not known.

## CHAPTER 3

### EXPERIMENTAL STUDY

#### 3.1 Introduction

As indicated in the literature review, the effect of variation of the stress gradient for a given nominal stress in bending, due to a specimen thickness change will result in a decline of fatigue life as the stress gradient becomes smaller. An experimental program was carried out to ascertain the validity of the information gained from the literature review as to the effects of the stress gradient. The experiments were conducted in the Department of Civil Engineering Laboratory the University of Manitoba.

#### 3.2 Material and Specimens

The material selected for the experiments was 2024-T4 aluminum alloy purchased as bar stock having the following dimensions:  $1/4"$  x  $1"$ ,  $3/8"$  x  $1"$ , and  $1/2"$  x  $1"$ , in twelve foot lengths. The chemical composition as given by the manufacturer is shown in Table 3.1.

Tensile tests were conducted upon random samples of the material after heat treatment. The mechanical properties as determined by these tests are listed in Table 3.2. The material for the Hounsfield test specimens was cut from the portion of the fatigue specimens located in the clamps of

TABLE 3.1  
CHEMICAL COMPOSITION OF 2024 ALUMINUM ALLOY

Copper . . . . .	3.8-4.9 %
Iron . . . . .	0.50
Magnesium. . . . .	1.2-1.8
Managanese . . . . .	0.30-0.90
Silicon. . . . .	0.50
Zinc . . . . .	0.10
Chromium . . . . .	0.10
Other elements . . . . .	0.15 total

All percentages are by weight, and except where a range is given, all percentages are maximum value.

TABLE 3.2  
MECHANICAL PROPERTIES

Ultimate Tensile Strength. . . . .	80,500 psi.
Yield Strength 0.2%. . . . .	49,500 psi.
Hardness - Vickers Pyramid No. . . . .	135
Young's Modulus. . . . .	10,000,000 psi.

TABLE 3.3  
TENSILE DATA FROM MEDIAN FATIGUE SPECIMENS  
- HOUNSFIELD TENSOMETER

Size nominal	UTS psi	YS psi	EL %	RA %
1/8 x 1/4	83,400	61,100	15	19
1/4 x 1/2	65,300	49,000	21	25
3/8 x 3/4	85,000	59,800	16	18
1/2 x 1	83,400	58,500	16	18

Nomenclature: UTS - ultimate tensile strength

YS - Yield strength 0.2% offset

EL - elongation % - gage length .632"

RA - reduction in area %

the bending fixture and therefore not subjected to mechanical working during the machines operation. Table 3.3, page 38 lists the tensile properties of the material from the specimens nearest the median of each group tested. The Hounsfield specimens used for these tests are shown in Figure 3.1.

Micrographs of the material used in the experiments are shown in Figures 3.2 and 3.3. These micrographs show the structure of the aluminum after heat treatment. The electropolishing agents and etchants used to produce the structures shown, are listed in Table 3.4.

### 3.3 Specimen Preparation

The bar stock was machined into the configurations shown in Figure 3.4. As the material was Alclad it was necessary to remove 0.040 inches off the surface of the specimens numbered 2 through 4 in Figure 3.4 and 0.030 inches off the surface of the axial specimens. The operation also removed the accompanying diffusion layer. This was done to ensure uniformity of surface conditions among the specimens. The test pieces in their final form are shown in Figure 3.5. The specimens were polished longitudinally with 400 silicon carbide paper to remove scratches left on the surface as a result of milling.

Following fabrication the specimens were solution heat treated to the T4 condition. This was done by Bristol Aerospace Limited. The specifications for the heat treatment are listed below:

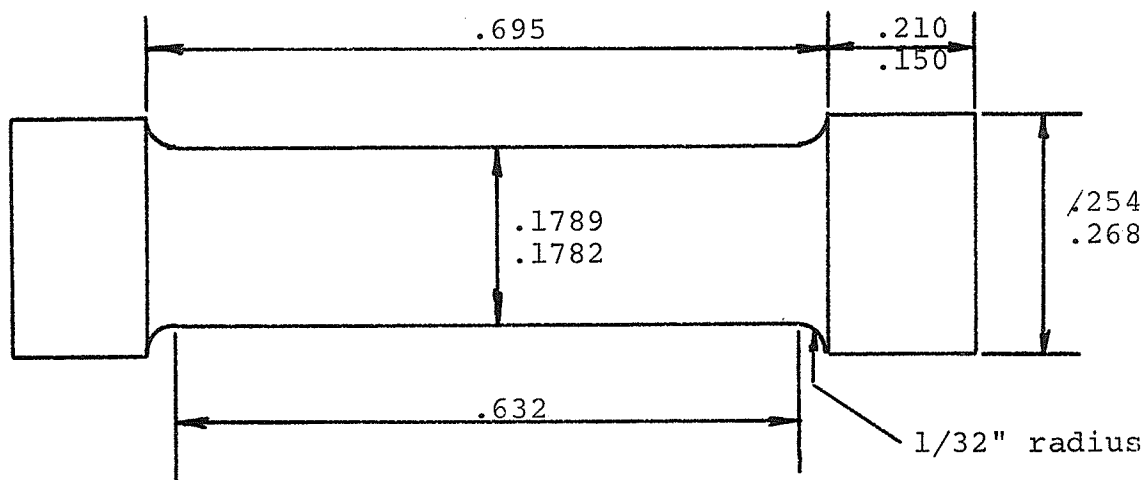


Fig. 3.1 Tensile Specimen for  
Hounsfield Tensometer





Fig. 3.2 Microstructure of 1/4"  
Bending Specimen



Fig. 3.3 Microstructure of Axial and 1/8,  
3/8 and 1/2" Bending Specimens

TABLE 3.4  
POLISHING AND ETCHING SOLUTIONS

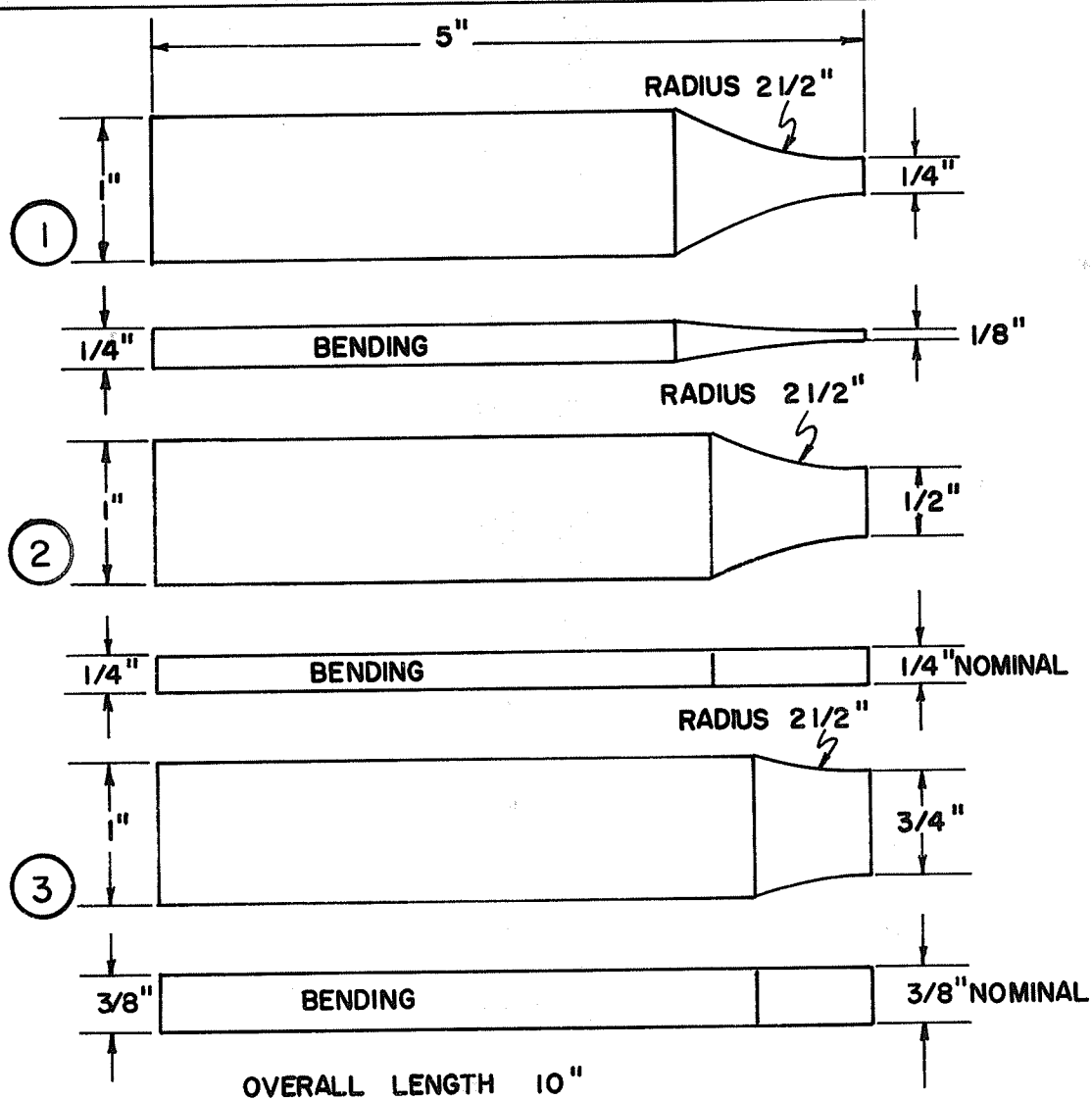
Electrolytics for electropolishing:

1.	Perchloric acid . . . . .	.2 parts
	Glacial Acetic acid . . . . .	7 parts
	Current density - amp./sq. dm. . . . .	3-5
	Time. . . . .	5-15 min.
2.	Methyl alcohol. . . . .	2 parts
	Nitric acid (conc.) . . . . .	1 part
	Current density - amp./sq. dm. . . . .	31-93
	Time. . . . .	4-7 min.

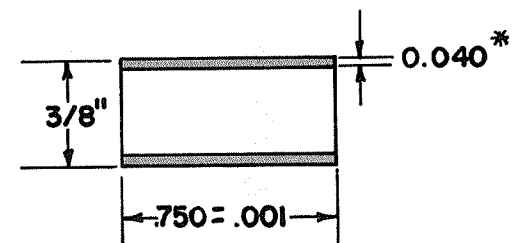
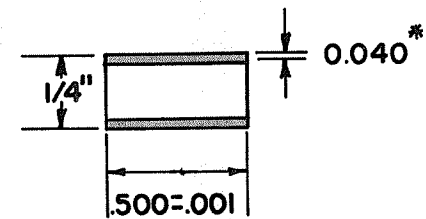
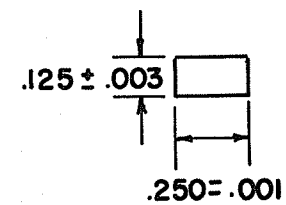
Etching reagent - Keller's Etch

Hydrofluoric acid (conc.) . . . . .	1.0 ml.
Hydrochloric acid (conc.) . . . . .	1.5 ml.
Nitric acid (conc.) . . . . .	2.5 ml.
Water . . . . .	95 ml.

Immerse for 10 to 20 seconds and wash in a stream of warm water.



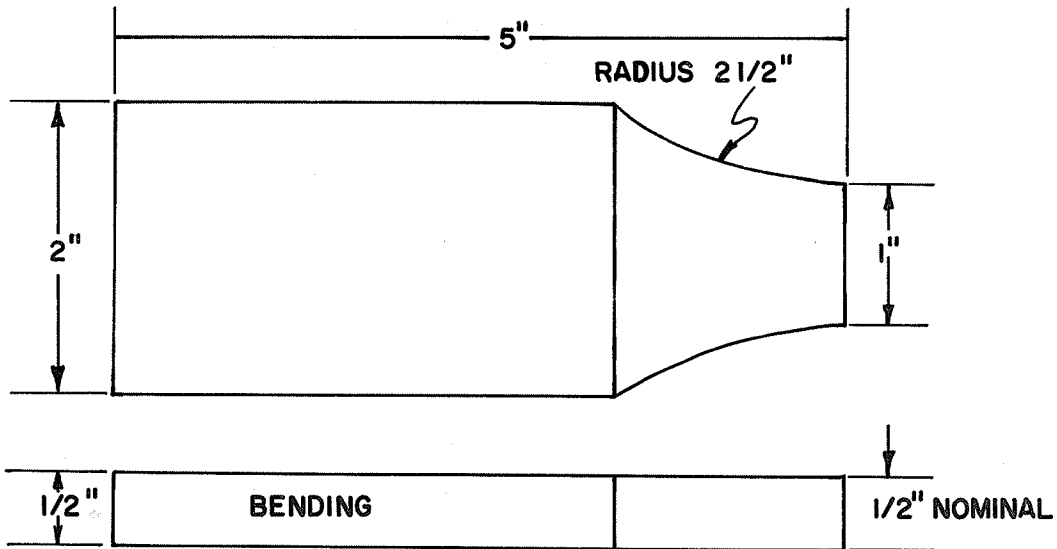
ACTUAL X - SECTIONS



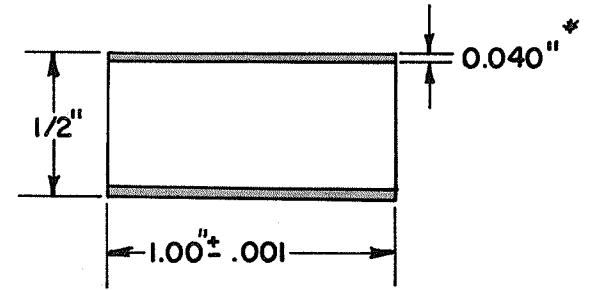
\* SEE TEXT ON SPECIMENS

**FIG. 34 TEST SPECIMENS**

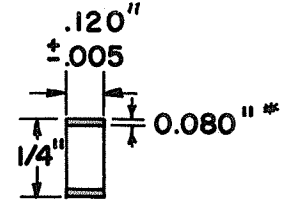
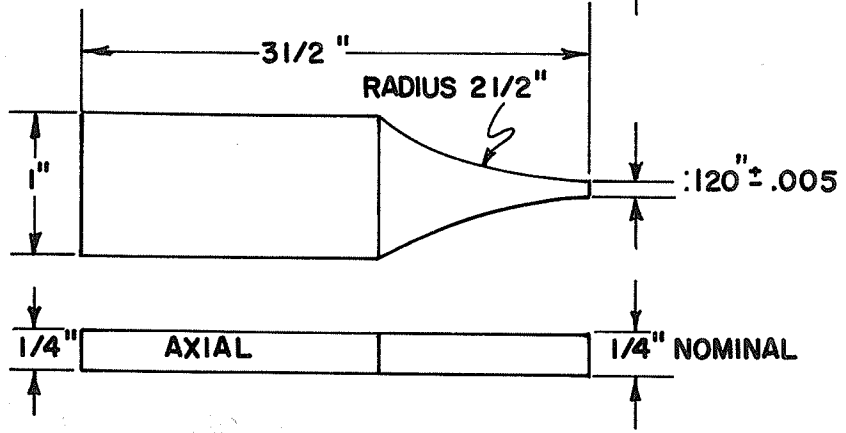
④



ACTUAL X-SECTIONS



⑤



\* SEE TEXT ON SPECIMEN

OVERALL LENGTH	4	10"
	5	7"

FIG. 3.4 TEST SPECIMENS

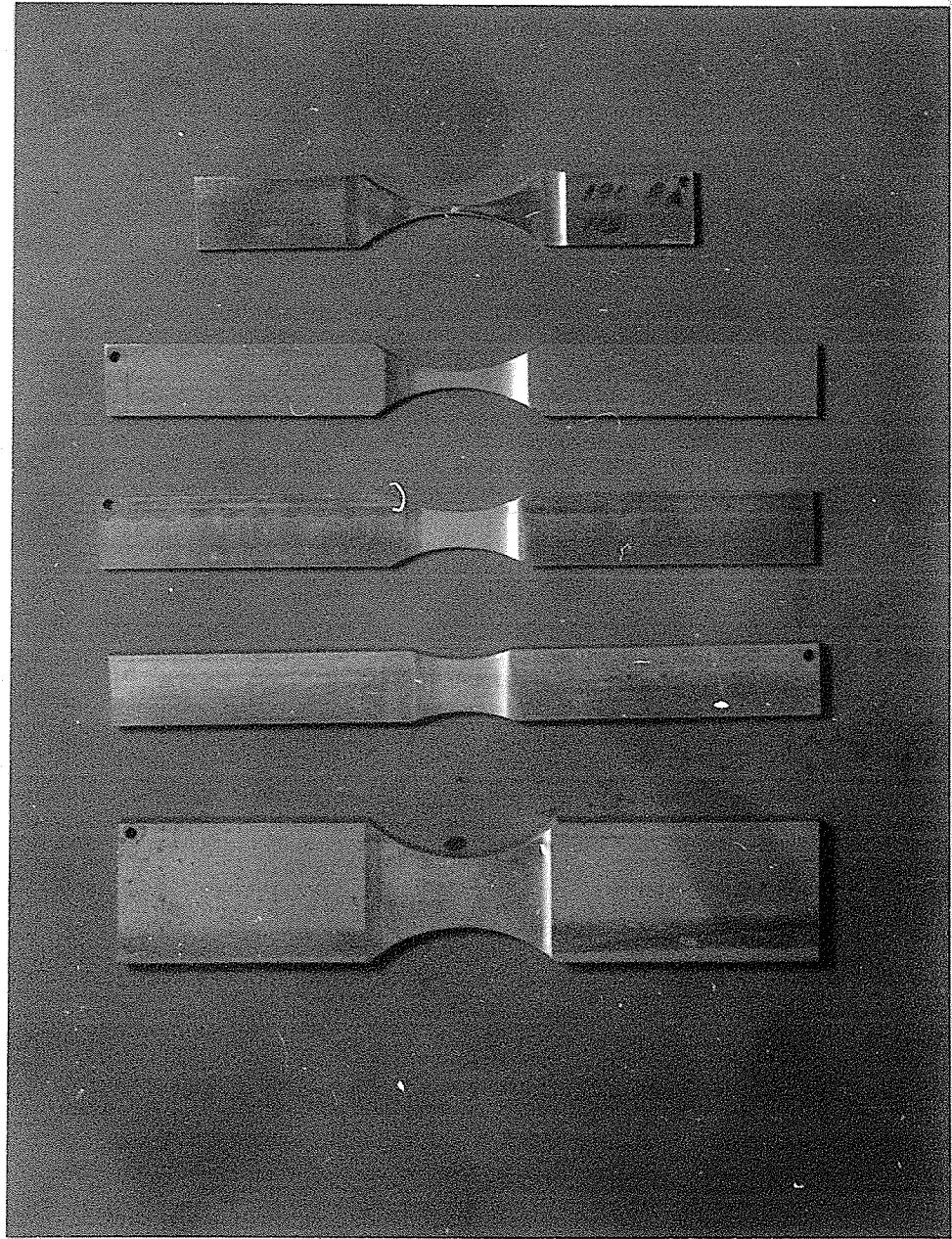


Fig. 3.5 Test Specimens - 2024-T4 Aluminum

- 920 F for 45 min. in a salt bath
- cold water quench
- precipitation hardening at room temperature for  
48 hours

The specimens were then polished, this time using 600 silicon carbide paper to remove the corrosion caused by the salt bath. They were buffed to a high gloss using a cloth buffing wheel and a suitable polishing compound; Tripoli. The test pieces were cleaned to remove surface residue left as a result of polishing. The final polishing and cleaning was done immediately prior to testing.

#### 3.4 Testing Equipment

A Sonntag Fatigue Testing Machine, model number SF-1U with automatic preload, was used to fatigue the specimens described in Section 3.3. It is a constant load type of machine. The maximum dynamic load is one thousand pounds fully reversed, at a speed of eighteen hundred rpm. The machine can be preloaded to a value of a thousand pounds in either tension or compression. The testing device was calibrated at the factory for the speed quoted and yields the correct dynamic load only at this speed. If a different speed were used the machine would have to be recalibrated. The factory calibration was checked by use of strain gages for both the axial and bending fixtures and was found to be correct. A photograph of the machine is shown in Figure 3.6.

As both axial and bending tests were required, two

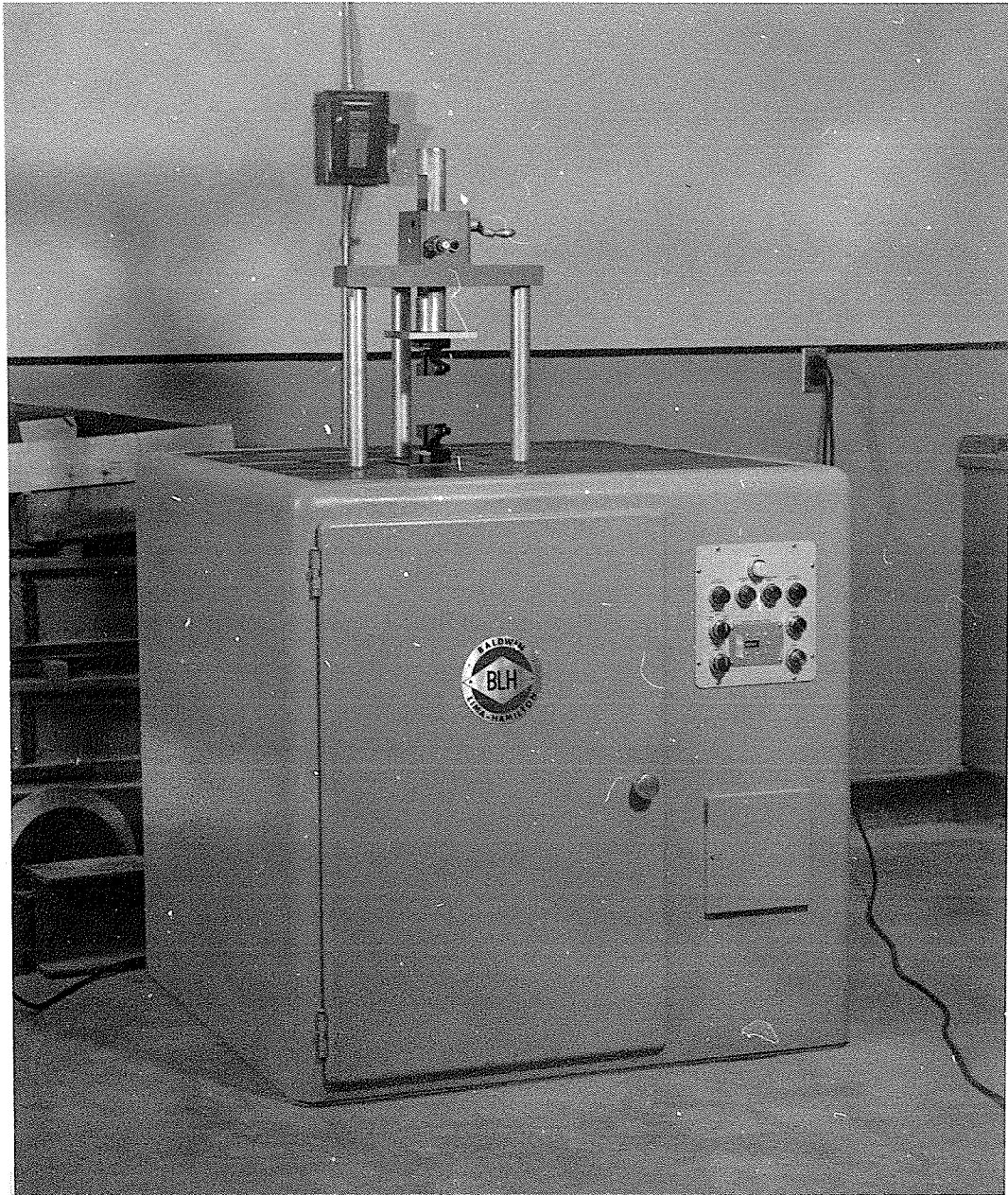


Fig. 3.6 Sonntag Fatigue Testing Machine



separate loading fixtures were used. One was the axial loading fixture shown in Figure 3.7. Due to the requirement of fully reversed loading for this experiment the maximum allowable load which could be used was the thousand pound dynamic load. To gain the stresses used in the tests required an axial specimen of very small cross-section as shown in Figure 3.4 and the upper most specimen in Figure 3.5. The bending fixture shown in Figure 3.8 was the type used for the bending tests. The thousand pound dynamic load limitation was not a problem in the bending tests. The limitation imposed by the machine was a maximum allowable amplitude of  $\pm .37$  inches due to the location of the amplitude activated shutdown switches.

### 3.5 Test Procedure

Two stress levels were selected to allow a reasonable fatigue life differential, and also a tolerable run duration time for both sets of specimens. The stress levels chosen were 40,000 psi and 36,000 psi. According to accumulated information this would have given run duration times of approximately 50,000<sup>+</sup> and 150,000<sup>+</sup> cycles respectively for the material used. From the results one can see that these estimates were not accurate.

In both sets, the axial specimens were run first, the basic assumption being that they would break sooner than the small bending specimens and allow an estimate of the approximate run duration at each stress level. The bending

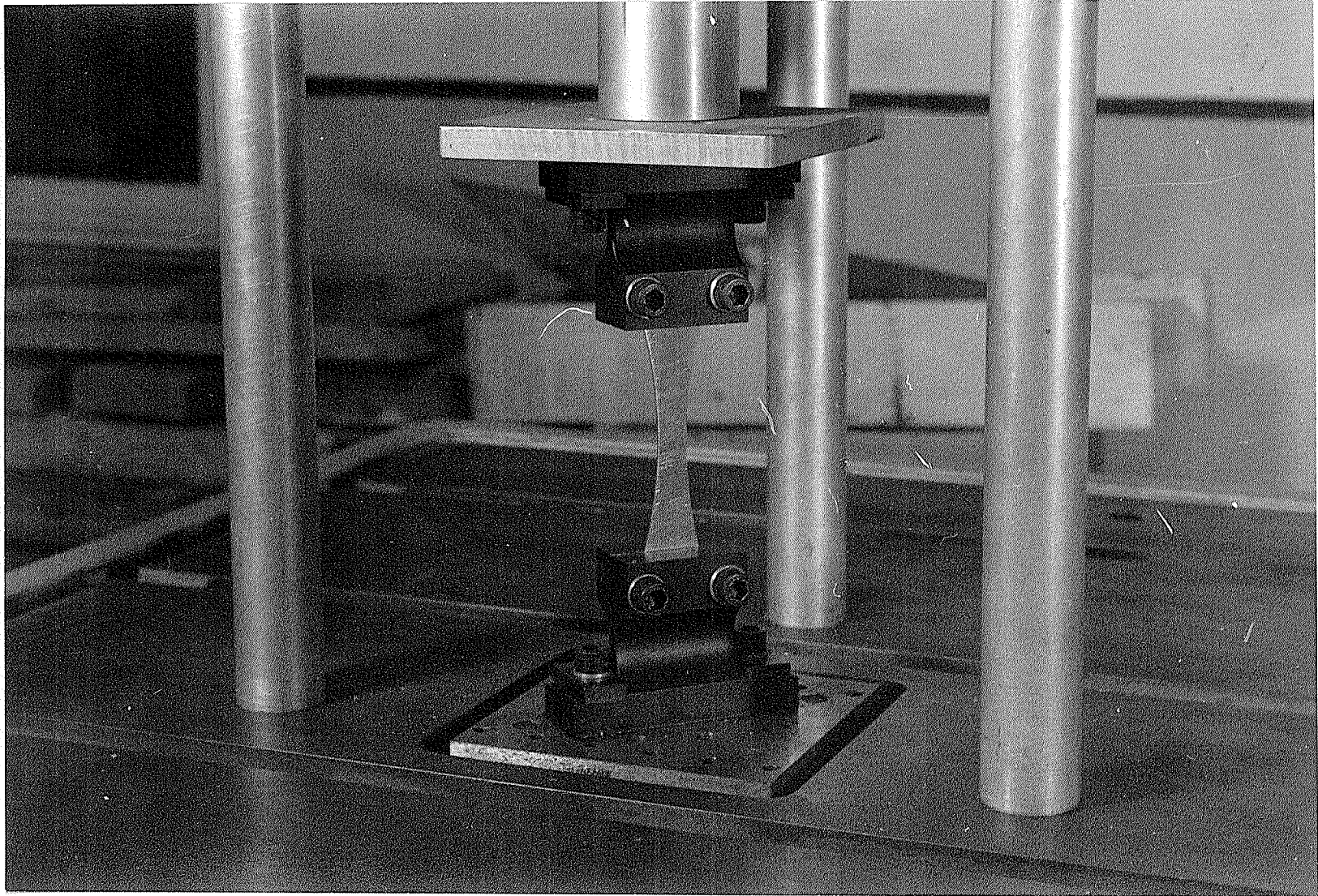


Fig. 3.7 Axial Loading Fixture

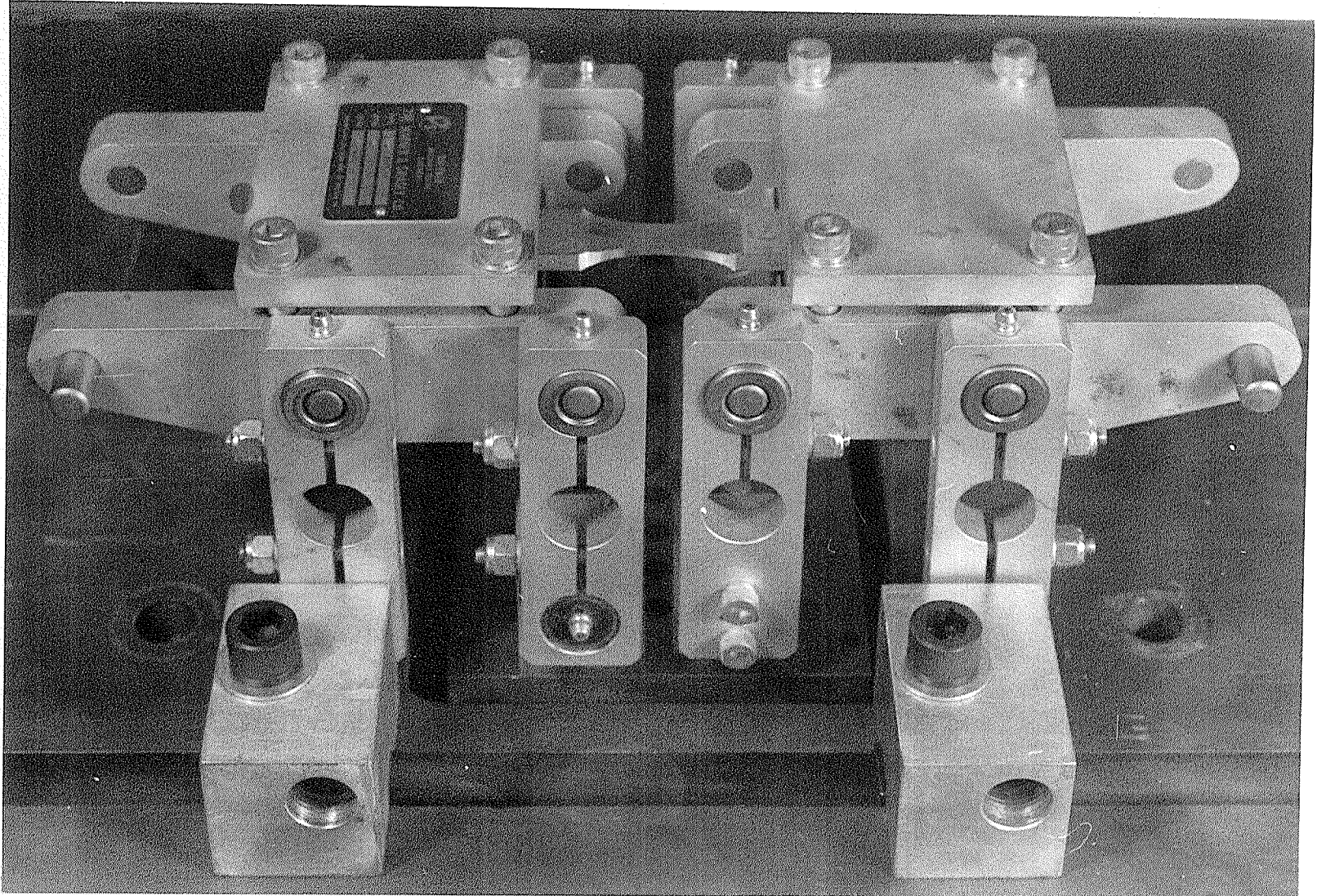


Fig. 3.8 Bending Fixture

specimens were then run in sequence, from the smallest to the largest. The number of specimens in each size group was six, with a total of thirty specimens both axial and bending to be run at each stress level.

All specimens were cleaned and dimensioned prior to testing and the load required to obtain the desired stress value was calculated for a given fixture configuration. The specimen life as recorded for these tests is the number of cycles to specimen separation and not to when the first cracks appear.

## CHAPTER 4

### EXPERIMENTAL RESULTS

#### 4.1 Introduction

This chapter will discuss the results as tabled in Section 4.2. It will cover all aspects, including the tensile test data and the microstructure examination. In addition it will review the validity of the results and speculate on the influence of certain variables on these results.

#### 4.2 Results

The results of the experimental testing are shown in Tables 4.1 and 4.2 and are presented graphically in Figure 4.1. The results of the bending fatigue tests, with one exception, agreed with the prediction of a reduction in fatigue strength as the stress gradient decreases due to a size increase. The reduction in fatigue life from the smallest to the largest specimen was eighty-eight per cent at the forty thousand psi stress level and two hundred and thirty per cent at the thirty-six thousand psi stress level. The thickness change in going from the smallest to the largest cross-section was four hundred per cent.

As no axial tests were run for the specimens for each of the sizes used in bending comparative data between size

TABLE 4.1  
CYCLES TO FAILURE

Stress 40,000 psi

Type A or B	Size Nominal	Test - Cycles to Failure						Median
		1	2	3	4	5	6	
A	.120 x .250	162,000	126,000	428,000	439,000	198,000	145,000	180,000
B	1/8 x 1/4	192,000	169,000	125,000	128,000	173,000	230,000	171,000
B	1/4 x 1/2	109,000	148,000	121,000	171,000	104,000	118,000	119,500
B	3/8 x 3/4	112,000	225,000	323,000	235,000	244,000	263,000	239,000
B	1/2 x 1	78,000	103,000	90,000	93,000	96,000	81,000	91,500

A - Axial

B - Bending

TABLE 4.2  
CYCLES TO FAILURE

Stress 36,000 psi

Type A or B	Size Nominal in.	Test - Cycles to Failure						Median
		1	2	3	4	5	6	
A	.120 x .250	614,000	160,000	868,000	198,000	382,000	1,083,000	637,000
B	1/8 x 1/4	130,000	754,000	1,316,000	368,000	226,000	842,000	561,000
B	1/4 x 1/2	227,000	209,000	205,000	321,000	266,000	204,000	218,000
B	3/8 x 3/4	250,000	297,000	566,000	1,351,000	1,001,000	951,000	758,000
B	1/2 x 1	265,000	124,000	188,000	167,000	129,000	171,000	169,000

A - Axial

B - Bending

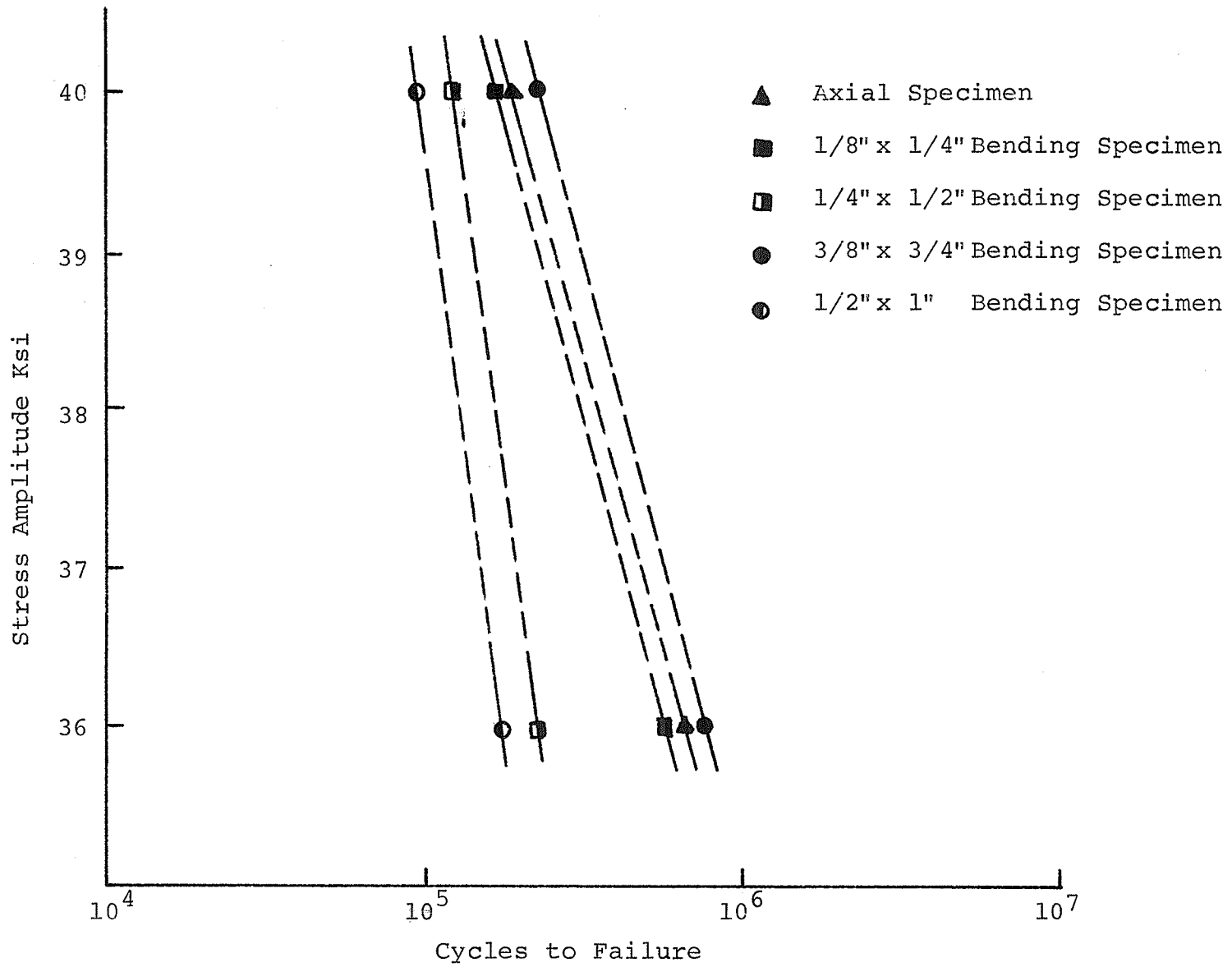


Fig. 4.1 Graphic Representation of Results



effect in axial loading and bending is not available. This was impossible to achieve as no fatigue machine large enough to do axial tests on the large bending specimens was available. The way in which one could accurately estimate the effect of the stress gradient on fatigue life in bending would be to run an identical set of axial specimens at the same stress levels. A comparative percentage reduction could then be made between the two modes of loading.

Based on information in Chapter 2 one might speculate that as the stress gradient is decreased the fatigue strength will decrease as the gradient approaches that of a uniform loading condition. It is probable that if axial tests were made with specimens identical to those used in bending they would show a size effect due to surface defects and subsequent stress gradient establishment. The size effect would not be as great as that found for the bending specimens due to their inherent stress gradient.

It is reasonable that the size effect was increased due to the change in the stress gradient in going from the smallest to the largest specimen for a given nominal stress, when compared to the size effect which would have occurred if the specimens were tested axially.

#### 4.3 Analysis & Discussion of Results

From the results it can be noted that two inconsistencies exist. One is the axial fatigue test results and

second, the bending fatigue results for the three-eighths inch bending specimens.

The axial tests, the author believes, can be attributed to size effect. It is known that laboratory specimens because a size differential between them and their in-service counterparts will have a different fatigue life. As the area required to achieve the desired stress had to be as small as possible because of dynamic load limitations the probable effect was that the specimen life was extended. From axial performance data the fatigue life of these specimens should have been much less than that of the equivalent size bending specimen. Since the area of the axial specimens was approximately two-thirds that of the smallest bending specimen size effect may have overridden the normal tendency for reduced life in axial tests.

The three-eighths inch bending specimens did not follow the trend set by the other bending samples. If they had, the results would have been situated between those of the half- and quarter-inch test piece results. This disparity cannot be attributed to a difference in specimen static properties. As one can note from Table 3.3, the mechanical properties of the three-eighth inch bending specimens are no different than those of the majority of results listed. An examination of the microstructure of the three-eighths inch bending specimen revealed no visible structural differences when compared to the microstructure

of the half and one-eighth inch bending specimens. The quarter-inch bending specimens, although apparently different in microstructure had a fatigue life which followed the trend set by the majority of bending specimen results. There is a possibility that the three-eighth specimens had a structural difference not revealed in the micrographs which may have influenced results.

The other possibility, however, is that geometric similarity after final shaping was not close enough to the rest of the specimens in other size ranges. This the author believes may be the cause of its extended life. It was observed that during the fatiguing process, the other specimens tended to form numerous visible cracks on the surface even in areas not subjected to anything near maximum stress. This was not the case for the three-eighths specimens. They cracked in only a few places and nowhere near the degree of the half inch specimen. Usually only one or two major cracks formed and fracture was through these, whereas the others had a profusion of surface cracks.

The microstructures can be seen in Figures 3.2 and 3.3. The structure is highly elongated due to some form of working during the fabrication process. The intention was to have a recrystallized or a consistent structure after heat treatment. The microscopic examination was conducted after the specimens were in the process of being fatigued, too late to correct the structural variation. The structure

of the quarter inch bending specimen was partially recrystallized although still elongated. The effect of this variation is difficult to determine as the results for these specimens follow the general trend.

The major effect of the structure not being recrystallized, based on information by Kaufin, Lyst, and Miller [22] is that the fatigue strength would be greater than that for the fully recrystallized material. This strengthening effect would not be as great in the transverse direction as in the longitudinal direction relative to the elongated structure orientation. This would account for the longer fatigue lives for all specimens when compared with published data for fatigue life of this material.

Based on the preceding, it could be speculated that the quarter inch bending specimens, would have had a lower fatigue life than if their structure had been identical to the rest of the test samples. The degree of difference between the two structures was not that great and the author doubts whether a large difference would have been made in fatigue life even if the structure had been the same as the rest.

The results of the tensile tests on samples from the median specimens in each group, with the exception of the quarter inch specimen, show no marked difference and are within reasonable proximity of each other in both ultimate tensile strength and yield strength values. It is apparent

that the grain structure may have had significant effect on the tensile properties of the material of the quarter inch specimens.

Although it is speculation, the author believes that the microstructure of the material although not quite what was desired did not play that great a role in influencing the fatigue results. The possible exception may be the quarter inch bending specimen which probably would have yielded a greater fatigue strength, but how much greater is unknown.

In summing this section, the results show a large size effect, existing in bending. Variables, whose effects are not really known and can only be guessed have entered the test results. The variables which have entered could have been eliminated or reduced. They may not have altered the results but their absence would have removed any elements of doubt as to their effect. Only two sets of bending fatigue results can be said to be free of influential variables; those of the half and one-eighth inch specimens. Here a definite decline in fatigue strength exists to the degree mentioned in Section 4.2. The remaining two sets of bending results have variables which may influence their behaviour and hence their usefulness in substantiating the difference between the previously mentioned two variable free sets. From the experiments conducted, the effect of variation of the stress gradient to a thickness change for a given nominal

stress is to cause a large size effect, the degree of which is greater than that found for axial loading.

#### 4.4 Summary

There are several variables which have entered the experiment and whose effects are not known. The performance of the one-eighth and half inch bending specimens substantiate what was advanced in the literature review. But these are the only two sets free of variables and do not comprise a large enough set of data to allow more than rudimentary picture of stress gradient effects.

The conclusion which can be reached from the tabled results is that, overlooking the performance of the three-eighths inch bending specimen, there exists a large effect in bending. To attribute all of this to the effect of the different stress gradients is hard to say as there is no basis for comparison. If Dieter's [8] idea is correct that a stress gradient is required for a size effect to occur irrespective of the mode of loading then all of the reduction in life between the four sizes can be attributed to the stress gradient variation.

## CHAPTER 5

### SUMMARY OF THESIS

#### 5.1 Introduction

This chapter is a summary of the results obtained through the literature survey and experimentally and will dwell on the major points of each. It will also specify some recommendations on how to improve the experimental approach and suggestions for addition probing into this area.

#### 5.2 Conclusions

The results as obtained from the literature survey are listed as follows:

The stress gradient appears instrumental and possibly a necessity for a size effect to occur. This requirement may be needed in all forms of repeated loading. Notched axial specimens, particularly those of reasonable hardness show a pronounced size effect even though in the smooth condition the size effect may be nil. This is possibly due to the setting up of a strain or stress gradient because of the notch.

A stress gradient is present from the beginning in bending and torsional loading. It has been found that bending specimens exhibit a pronounced size effect when compared with axial specimens. It would appear that as the stress gradient

becomes less for a given nominal stress due to a size increase the fatigue life drops off. Authors have advanced the idea that as the stress gradient declines under the conditions defined previously the area placed under the high stress beyond the endurance limit increases. This would reduce the amount of material in the infinite life region of the S-N curve and presumably be a cause for a decrease in life.

The results of the experiments, show a reduction in fatigue strength as the stress gradient declines due to an increase in thickness. As no axial specimens were run of identical size no comparison between life reduction in the two modes could be made.

From the information in the literature review and experimental study, the effect of the stress gradient variation is to produce what is currently termed size effect. The stress gradient, must be present for such an effect to occur. In bending for a given constant fiber stress, the stress gradient, when it declines as dimensions increase, will reduce fatigue life. The reduction in fatigue life due to thickness changes will be greater in bending and in torsion than in axial loading.

### 5.3 Recommendations and Suggestions for Further Study

To improve the experimental portion of the thesis the following suggestions have been made:

- (a) A material, if aluminum, should be free of aluminum cladding. The aluminum cladding was not difficult to account



for, but was a nuisance when machining, and in specimen design because the diffusion layer which accompanies the cladding varies in depth according to the thickness of the material and is not uniform on both sides of the specimen.

(b) Geometric similarity should be achieved throughout all specimen sizes. The use of nonclad material would have made this easier.

(c) A set of bending specimens and also a set of identical axial specimens should be tested. This would allow a more direct and meaningful comparison to determine the effect of the stress gradient.

(d) The material should be consistent in structure. The means to achieve this would be to use recrystallized material.

As a further study it would be interesting to explore the rate of cracking as a function of the stress gradient in bending. In the area of fatigue damage, it would be interesting to investigate whether, as the stress gradient approaches that of a uniform load, if the accuracy of some of the damage theories improves.

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