CONSOLIDATION AS A DELAYED YIELD FACTOR IN CONFINED AQUIFER RESPONSE

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ABSTRACT

During pumping from elastic artesian aquifers confined by or containing interbeds of fine-grained sediments the Theis assumption of instantaneous release of water from the aquifer system is not satisfied. A mathematical model using the Terzaghi consolidation theory to describe the release of water from these fine-grained compressible units was developed and its response compared to the Theis model as well as delayed yield and leakage models presented by Boulton and Hantush.

Using typical consolidation parameters for the fine-grained interbed and boundary units the consolidation model indicated that the rate of water release is significant even during short term pumping test periods. The resemblance of the type curves from this model to those of the Hantush-Walton leaky aquifer model could lead to a serious overestimation of the aquifer capacity if the delayed release of water during the terminal consolidation process is mistaken for a continuous source of recharge.

During the initial potions of pumping tests the time-drawdown curves for the consolidation model closely follow those of the Boulton and Hantush models of delayed yield. However, deviations at longer periods of time are generally observed because of different boundary conditions for the models.

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LIST OF MAJOR SYMBOLS

a_v	Coefficient of compressibility	L^2F^{-1}
b	Thickness of the main aquifer	L
b',b"	Thickness of the compressible layers	L
c_v	Coefficient of consolidation	L^2T^{-1}
D =	$\sqrt{T/\alpha S'}$ A leakage factor	L
e_o	Initial void ratio	—
E_c	Bulk modulus of compression	L^{-2}
h	Hydraulic head in the main aquifer	L
h',h"	Hydraulic heads in the compressible layers	L
H	Length of drainage path in the compressible layers	L
$H(u,\beta) =$	$\int_u^\infty \frac{e^{-y}}{y} \mathrm{erfc}(\beta\sqrt{u} / \sqrt{y(y-u)})dy$	—
$J_o(x)$	Bessel function of the first kind of zero order	—
K	Hydraulic conductivity of the main aquifer	LT^{-1}
K',K"	Vertical hydraulic conductivities of the compressible layers	LT^{-1}
N =	$\pi^2 c_v/4H^2$	T^{-1}
n =	$T\lambda^2/S$	T^{-1}
q	Total specific discharge from a unit prism of the compressible layer	L^3T^{-1}

q_L	Rate of leakage added to the main aquifer	L^3T^{-1}
Q	Constant rate of discharge from a pumped well	L^3T^{-1}
r	Distance from pumped well to observation well	L
s	Drawdown in an observation well	L
S	Storage coefficient of the main aquifer	—
S', S''	Storage coefficients of the compressible layers	—
S_s	Specific storage of the compressible layer	L^{-1}
t	Time	T
T	Transmissibility	L^2T^{-1}
T_v =	$c_v t/H^2$ Time constant	—
u	Excess hydrostatic pore pressure	FL^{-2}
u =	$r^2S/4\pi Tt$	—
U	Degree of consolidation	—
V	Total volume withdrawn during pumping	L^3
V_L	Total volume withdrawn from leakage during pumping	L^3
$W(u)$	$\int_u^\infty \frac{e^{-y}}{y}\,dy$ Well function for nonleaky aquifer	—
x	Horizontal coordinate axis	L

y	Horizontal coordinate axis	L
	Variable of integration	—
z	Vertical coordinate axis	L
α	Reciprocal of 'delay index'	T^{-1}
β	$= r\lambda/4$	—
γ_w	Unit weight of water	FL^{-3}
δ	$= 1 + (S' + S'')/S$	—
ε	$= \dfrac{w^2}{w^2 + 1} \exp\,(\alpha\eta\, t(w^2+1))$	—
η	$= \dfrac{S + S'}{S}$	—
λ	$\sqrt{\dfrac{K'/b'}{T} \dfrac{S'}{S}} + \sqrt{\dfrac{K''/b''}{T} \dfrac{S''}{S}}$	L^{-1}
μ_1	$= \dfrac{\alpha\eta t(1 - w^2)}{2}$	—
μ_2	$= \dfrac{\alpha t \sqrt{\eta(1 + w^2)^2 - 4\eta w}}{2}$	—
ν	$= \dfrac{S'}{S + S'}$	—
σ	Applied stress	FL^{-2}
τ	Time	T

INTRODUCTION

In the development of the Theis (1935) non-equilibrium equation describing the response of a confined aquifer during pumping the following conditions are assumed:

1) The aquifer is homogeneous and isotropic.

2) The aquifer is infinite in areal extent and uniform in thickness.

3) The aquifer is fully confined.

4) The flow is horizontal, therefore being independent of the vertical position in the aquifer.

5) A well of infinitesimal diameter penetrates the full thickness of the aquifer and discharges at a constant rate.

6) The release of water from storage is instantaneous with a decline of head.

The time-drawdown response of an aquifer deviates from the Theis solution when one or more of the above conditions are not fulfilled.

This thesis considers the effect of noninstantaneous release of water from storage in confined aquifer systems, a phenomena which is a major deviation from the sixth assumption. An understanding of this process is necessary if geohydrologists are to be able to correctly appraise the value of short term pump tests as applied to several types of aquifer systems.

In this thesis the noninstantaneous release of water from confined aquifer systems refers to the slow seepage of water from fine-grained interbeds or boundary units of the aquifer caused by rapidly changing pore water pressures in the coarser grained aquifer units. This is a process of consolidation and has been referred to as delayed yield (Boulton, 1955), a term which has also been applied to the noninstantaneous yield of water from unconfined aquifers caused by a time lag in desaturation of fine-grained materials as the result of a decline of the water table. The latter type of delayed yield is not dealt with in detail in this report. Delayed yield is obtained from storage within the aquifer system and should be contrasted with the noninstantaneous release of water in leaky aquifer systems where the water is derived from a recharge source adjacent to the aquifer.

Because so little attention has been given to the role of delayed yield in confined aquifer systems it was considered appropriate to review the previous work on the topic in the initial portion of this thesis. Subsequent chapters are devoted to a qualitative and quantitative evaluation of mathematical response models developed by previous workers. These models are compared and contrasted to a model developed by the author using an analytical procedure programmed for a digital computer.

MECHANISMS OF DELAYED YIELD

Consolidation of Low Permeability Beds

When a load is applied to a volume of soil the volume will decrease because of four conditions. Firstly, air bubbles in the soil will be compressed by the applied load if the soil is incompletely saturated. Secondly, deformation of the soil structure by the movement of the soil grains over one another will decrease the volume of the voids of the soil. Thirdly, the water in the pores of the soil will be compressed and, fourthly, the soil grains themselves will be compressed by the increased stress.

The compressibility of soil grains and water is negligible compared to the compressibility of the bulk soil structure. Since only saturated, confined aquifer systems are under consideration, compression of the air bubbles in the soil can also be considered as negligible.

When a load is applied to a soil system the system will compact, causing a decrease in the volume of its voids. The fluids occupying the voids of the soil are then forced out. The low permeability of a saturated fine-grained soil system prevents the immediate escape of pore water.

Initially most of the stress from the applied load is transferred to the water occupying the voids causing an excess pore water pressure. The excess pressure dissipates quickly at the drainage surface of the fine-grained unit and a pressure gradient is set up between the center and the drainage surface. Water, therefore, flows slowly out of the fine-grained beds and the void ratio of the system gradually decreases. The flow of water continues until the excess pore water pressure is completely dissipated and the stress from the applied load is transferred to the soil matrix.

The gradual compaction of the voids and the corresponding expulsion of water from a saturated fine-grained compressible soil system by an applied load is referred to in the soil mechanics literature as consolidation. A quantitative theory of consolidation was first proposed by Terzaghi (1925) and has later been described extensively in numerous soil mechanics texts (for example, Terzaghi and Peck, 1948; Taylor, 1948; Jumikis, 1962; Scott, 1963).

The consolidation theory of soil mechanics is in many respects similar to the theory describing groundwater flow in an elastic artesian aquifer. Both theories consider transient flow phenomena in saturated porous media. The continuity equation in conjunction with Darcy's law is used to describe flow through an elemental volume in either a fine-grained compressible layer or a coarse-grained aquifer. When applying the law of conservation of mass to an elemental volume of a compressible layer the change of mass within the volume is ascribed to compression of the soil structure only because the compressibility of water in fine-grained sediments is negligible compared to the compressibility of the soil structure. However, the compressibility of coarse granular sediments of an aquifer is generally the same order of magnitude as the compressibility of water. When applying the law of conservation of mass to an elemental volume of aquifer the change in mass within the volume is caused by expansion of water as well as compression of the soil structure. Therefore the expression for the change in fluid mass within an elemental volume of a fine-grained compressible layer is similar to the expression for specific storage of an adjacent aquifer differing only in that the compressibility of water has been neglected.

The differential equation describing one dimensional consolidation of a horizontal saturated fine-grained unit of infinite areal extent with a stress applied in the z direction and with single or double drainage is (Scott, 1963):

$$\frac{K'(1 + e_0)}{\gamma_w \quad a_v} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial t} . \qquad (1)$$

The equation is usually reduced to one dimension because components of flow in the x and y directions are negligible in most cases of consolidation.

In the derivation of the Terzaghi consolidation theory the stress is assumed to be applied instantaneously so that the second term on the right hand side of equation (1) vanishes and the equation becomes:

$$\frac{K'(1 + e_0)}{\gamma_w \quad a_v} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} . \qquad (2)$$

The coefficient of compressibility, a_v, is usually determined from graphs of void ratio versus effective stress obtained from laboratory tests on samples of the consolidation material. The void ratio-effective stress diagram from a typical laboratory test is shown in figure (1a). The initial portion of the graph represents the recompression of the soil sample up to some maximum stress to which it has been subjected in the past. The straight line portion to the right of the break in the void ratio-effective stress diagram represents virgin compression of the soil. The relationship of void ratio to effective stress is approximately exponential in this range. However, if a small pressure increment is applied to a compressible soil the change in void ratio is sufficiently small that for ease of mathematical analysis the approximation may be made that compression is a linear function of pressure for both portions of the graph (Fig. 1b). Therefore a_v is taken as the average slope of the void ratio-effective pressure diagram for the small increment of pressure being considered. This approximation is particularly valid for geohydrologic purposes because the drawdowns resulting from most pump tests correspond to a stress increase of less than 1 kgm /cm^2 on the compressible layer.

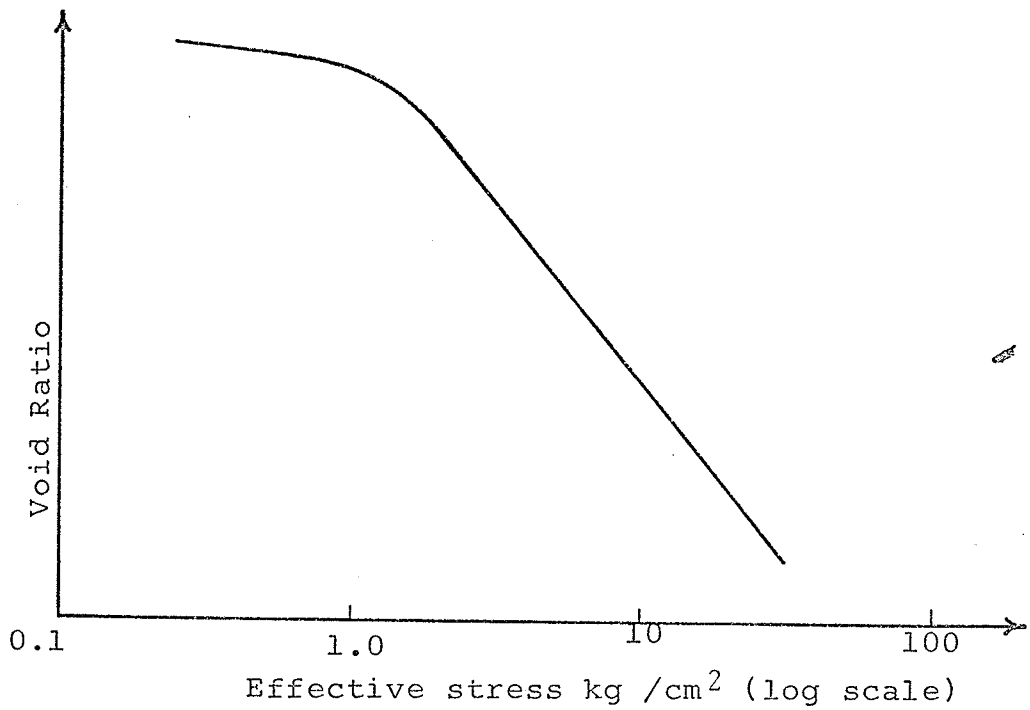


Fig. 1a Void ratio-effective stress curve for a fine-grained compressible soil.

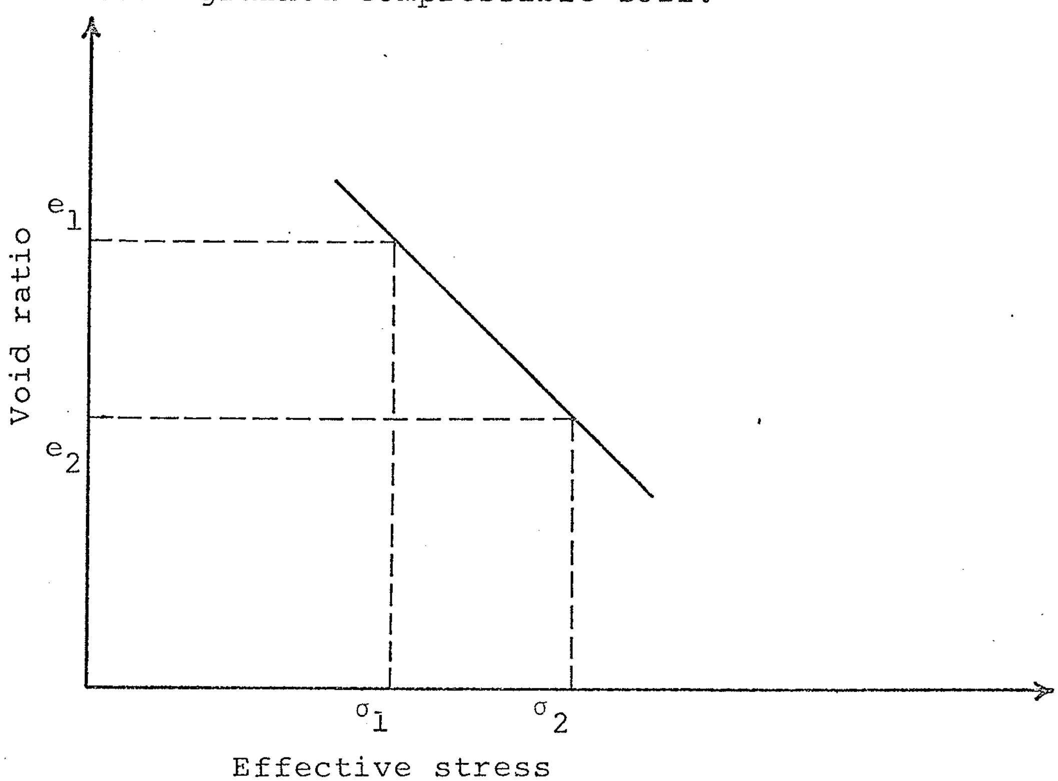


Fig. 1b Void ratio-effective stress relationship for a small pressure increment.

The constant terms on the left hand side of equation (2) are combined into a single constant called the coefficient of consolidation,

$$C_v = \frac{K' (1 + e_o)}{\gamma_w a_v} . \qquad (3)$$

The coefficient of consolidation representing the ratio of the hydraulic conductivity (permeability) of the medium to its compressibility or water storage capacity, is analogous to the hydraulic diffusivity, T/S, used by geohydrologists. The coefficient influences the response of a compressible layer to an applied load in the same manner as the ratio T/S influences the response of a groundwater system subjected to pumping. The larger the compressibility or smaller the permeability, the greater is the time required to re-establish steady flow conditions in the compressible layer.

Domenico and Mifflin (1965) derived an expression relating the coefficient of consolidation for a compressible bed to the specific storage of the bed. If the base of the aquifer is taken as the elevation datum, the Theis equation becomes

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{K'}{Ss} \frac{\partial u}{\partial t} . \qquad (4)$$

Comparison of equation (4) with equation (2) leads to the equality

$$C_v = \frac{K'(1 + eo)}{a_v \quad \gamma w} = \frac{K'}{S_s} \qquad (5)$$

where

$$S_s = \frac{a_v \ \gamma_w}{1 + e_o} \qquad (6)$$

Therefore

$$S_s = \frac{K'}{C_v}$$

and

$$S' = S_s b' = \frac{K' \ b'}{C_v} \qquad (7)$$

where S' is the coefficient of storage as applied to the entire thickness of the compressible layer.

In confined aquifer systems the confining beds are frequently composed of clay, silt or even sands with a high content of silt and clay-size matrix material. These silt-clay and silty sand layers are much more compressible than clean sands and gravels. The consolidation properties of these finer grained materials have been well documented in the soil mechanics literature (Jumikis, 1962). Fine-grained interbeds which may occur within the main body of the aquifer are capable of consolidation producing delayed yield during pumping in the same manner as the confining beds of many aquifer systems. In fact, Jacob (1940) noted:

> "it appears that the chief source of the water derived from storage "within" an artesian aquifer is probably the contiguous and intercalated clay beds and that because of the low permeability of the clays there is a time-lag between the lowering of pressure within the aquifer and the appearance of that part of the water which is derived from storage in those clays."

Because of the extensive test drilling programs of Provincial Government agencies in recent years it has become apparent that interbedding of thin fine-grained beds within the aquifer body is a particularily common feature of glacial drift aquifers in Western Canada (Cherry, personal communication, 1968).

In an aquifer system the stress causing the excess pore water pressure in the fine-grained beds is caused by a lowering of the pressure at the surface of the clay beds rather than by application of a load on the surface of these beds. The lower pressure is produced when the aquifer potentiometric surface is lowered by pumping.

The situation in an aquifer system is a dynamic one with a continuous variation of head distribution with time as compared to the quasistatic case used in the Terzaghi consolidation theory where load increments are assumed to be applied instantaneously. The transient nature of conditions must be taken into account for a complete analysis of the delayed yield response of confined aquifer systems.

In the geohydrologic literature very few investigators have considered the delayed yield mechanism either qualitatively or quantitavely. Babushkin, Prokhorov and Saar (1961) conducted a field study of the release of water from consolidating silt-clay beds in an artesian aquifer system in the U.S.S.R. Boulton (1955) suggested that Terzaghi's theory of consolidation may be applicable to the problem of delayed yield but for ease of mathematical analysis adopted a simple empirical expression for the delayed release of water from fine-grained beds. Hantush (1960) has used a differential equation similar to that of consolidation theory to describe the release of water from storage in fine-grained beds during a period of aquifer pumping.

Desaturation Time Lag

Although this thesis is primarily concerned with the delayed yield from confined aquifer systems, delayed yield of unconfined aquifers is considered very briefly in order to distinguish clearly between the two mechanisms and thus avoid the confusion that is inherent in some of the literature on this subject.

As the water table recedes in a uniform sand residual bodies of water will be left in the sand above the zone of maximum capillary rise. With increasing decline of water table the vapor pressure surrounding the higher residual bodies decreases. The residual bodies of water must decrease in size to maintain equilibrium with their surroundings and water will be transferred by diffusion from the residual bodies to the saturated zone (Smith, 1961).

. If the water table recedes below a saturated fine sand, silt or clay unit, it will not drain immediately. The low permeability of the unit causes the water to be released from the interstices slowly. There is slow leakage from the unit down to the lowered water table (Rasmussen and Andreasen, 1959). The gravity drainage of sediments has been studied in the laboratory (Johnson et al, 1961: Prill et al, 1965) and Johnson et al (1961) have presented a bibliography on the subject.

With unconfined aquifers the slow gravity drainage of fine-grained sediments caused by a decline of water table during pumping produces a delayed yield effect. The problem of delayed gravity drainage during pumping has been noted by numerous authors (Theis, 1935; Wenzel, 1942; Walton, 1960; Ineson, 1963; Prickett, 1965; Stallman, 1965). An equation originally derived by Boulton (1955) which includes delayed yield response of fine-grained beds in confined aquifer systems is now commonly used to account for desaturation delayed yield from unconfined aquifers (Boulton, 1963; Prickett, 1965).

MATHEMATICAL MODELS OF DELAYED YIELD AQUIFER SYSTEMS

Introduction

The Theis equation (Eq. 4) can be written in cylindrical coordinates as

$$T \left(\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} \right) = S \frac{\partial h}{\partial t}. \qquad (8)$$

Although Theis (1935) originally developed this equation by using the analogy between radial flow in a confined aquifer and heat flow in an infinite slab, Jacob (1940) was largely responsible for deriving it from basic principles. Jacob showed the storage coefficient, S, to be composed of two terms. One term accounted for water derived from storage by expansion of the confined water. The other term accounted for the water derived from storage by the bulk compression of the aquifer skeleton. The expansion of the individual mineral grains composing the aquifer skeleton is very small and was neglected.

Jacob noted:

> "In the actual case, however, (an additional) term is required to account for the water derived from storage in the adjacent and included clay

> beds. This term would in general be a function of the rate and perhaps also the magnitude of the decline of pressure, of the thickness and distribution of the intercalated clay-beds and of the permeability and modulus of compression of the clay."

However, because of the complexity of the mathematical treatment of such time lag effects Jacob made the following assumptions:

> "if there are a sufficient number of clay laminae interbedded with the sand so that the release of stored water from the clay is virtually instantaneous, the required additional term may, for all practical purposes, be considered to be independent of both the rate and the magnitude of the decline of pressure and independent of the permeability of the clay."

Therefore Jacob did not pursue further the problem of delayed yield and the process was virtually ignored by geohydrologists for many years.

Boulton Model

Although the effects of the delayed release of water from sediments was recognized in the early years of well hydraulics, Boulton (1955) was the first to propose a theory to account for delayed yield during pump tests of confined aquifers.

In the development of his theory Boulton assumed that the water derived from storage, due to an increment of drawdown, consists of two components:

1) A volume of water released instantaneously from storage

$$S \quad \Delta h \tag{9}$$

2) A delayed yield per unit area at any time t due to an increment of drawdown Δ h during the interval $\Delta\tau$ ($\tau < t$)

$$\Delta h \quad \alpha S' \quad e^{-\alpha(t-\tau)} \tag{10}$$

where α is an empirical constant and S' is the total volume of water released from storage by delayed yield per unit drawdown. In equation (10) Boulton assumed that the rate of delayed yield due to an increment of drawdown Δ h is proportional to Δ h.

Boulton did not directly indicate by what reasoning he arrived at his empirical exponential expression for the amount of delayed yield at time, t, but implied that it was a suitable approximation. Laboratory tests indicate that for many fine-grained soils the rate of consolidation for an increment of load decreases exponentially with time. If the delayed yield is derived from the consolidation of fine-grained beds the volume of water released as delayed yield per increment of load should decrease exponentially with time. The amount of water drained by gravity drainage from a column of sand under laboratory conditions also has an exponential relationship with time (Johnson et al, 1963; Prill et al, 1965). This may provide some basis for the use of an exponential relationship for an approximation of the delayed yield process in both confined and unconfined aquifers.

Based on the analysis of data showing the relation between the depth to water and the volume of water stored above the water table for an observation well during a pump test Stallman (1967) has shown that the specific yield of the sediments is constant with time. Therefore he has indicated that Boulton's exponential expression may not be applicable. Assuming that Boulton's expression is valid, however, the rate of change of drawdown with time during a pump test may be represented by

$\partial h/\partial\tau$ where τ is the time since pumping was started. The increase in drawdown Δh occurring during the time interval $\Delta\tau$ is then given by

$$\frac{\partial h}{\partial \tau}\, d\tau \quad . \tag{11}$$

Substituting (11) for Δh in (10) gives

$$\alpha S' \quad \frac{\partial h}{\partial \tau} \quad e^{-\alpha(t-\tau)} \quad d\tau \quad . \tag{12}$$

The rate of delayed drainage per unit area at time, t, since the start of pumping is then

$$\alpha S' \int_0^t \frac{\partial h}{\partial \tau}\, e^{-\alpha(t-\tau)}\, d\tau. \quad . \tag{13}$$

The right hand side of the differential equation (8) for the unsteady radial flow to a pumped well in an elastic artesian aquifer represents the rate of instantaneous yield of water from storage per unit area due to an increase in drawdown at time, t. To the right hand side of equation (8) Boulton added the expression (13) for the rate of delayed drainage per unit area at time, t, giving

$$T\left(\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\right) = S\frac{\partial h}{\partial t} + \alpha S'\int_0^\infty \frac{\partial h}{\partial \tau}\, e^{-\alpha(t-\tau)}\, d\tau \, . \tag{14}$$

Using standard mathematical techniques Boulton (1963) has presented a general solution for equation (14) as follows

$$s = \frac{Q}{4\pi T}\int_0^\infty \frac{2}{w}\left[1 - e^{-\mu_1}\left(\cosh \mu_2 + \frac{\alpha\eta t\,(1-w^2)}{2\mu_2}\sinh \mu_2\right)\right] J_o\left(\frac{r}{\nu D}\right) dw, \tag{15}$$

where

$$\eta = \frac{S + S'}{S} \, ,$$

$$\mu_1 = \frac{\alpha\eta t\,(1+w^2)}{2} \, ,$$

$$\mu_2 = \frac{\alpha t\sqrt{\eta^2(1+w^2)^2 - 4\eta w^2}}{2} \, ,$$

$$\nu = \sqrt{\frac{\eta - 1}{\eta}} = \sqrt{\frac{S'}{S + S'}}$$

$$D = \sqrt{\frac{T}{\alpha S'}} \, .$$

In developing this equation Boulton made the following assumptions:

1) The flow is horizontal. The vertical components of flow within the aquifer due to leakage from delayed drainage are negligible.

2) The aquifer is infinite in areal extent and uniform in thickness.

3) The aquifer and associated beds are homogeneous and isotropic.

4) A well of infinitesimal diameter penetrates the full thickness of the aquifer and discharges at a constant rate.

5) For an unconfined aquifer the drawdown is small in comparison to the thickness of the aquifer.

Preliminary analysis of pump test data indicated that η is a large finite number. Therefore Boulton assumed that $\eta = \infty$ is a reasonable approximation and reduced equation (15) to the following form

$$S = \frac{Q}{4\pi T} \int_0^\infty 2\, J_o\left(\frac{rw}{D}\right) \left[1 - \frac{1}{w^2+1} \exp\left(\frac{-\alpha t w^2}{w^2+1}\right) - \varepsilon\right] \frac{dw}{w}, \tag{16}$$

where

$$\varepsilon = \frac{w^2}{w^2 + 1} \quad \exp \quad \{-\alpha\eta t \ (w^2+1)\}. \tag{17}$$

The function ε in equation (17) vanishes when $t >> 0$ but is finite as t tends to zero and ηt approaches a finite value. For small values of t equations (16) and (17) give

$$S = \frac{Q}{4\pi T} \int_0^\infty 2 \, Jo\left(\frac{rw}{D}\right) \frac{w^2}{w^2 + 1} \{1 - \exp[-\alpha\eta t(w^2+1)]\} \frac{dw}{w} \tag{18}$$

Boulton noted that equation (18) is identical (except for notation) to the equation obtained by Hantush and Jacob (1955) for unsteady radial flow in an infinite leaky artesian aquifer.

Boulton (1963) has prepared type curves for the solution of his equation. Since two ranges of time must be considered two families of type curves result. The family of type curves for small values of time obtained from equation (18) and denoted the "Type A curves" have been plotted from data given by Hantush (1956) for leaky artesian aquifers.

"Type B curves" for large values of time were obtained from equation (16) for assumed values of the parameters αt and r/D. These values have been tabulated by Boulton (1963).

Illustrative Cases of the Boulton Model

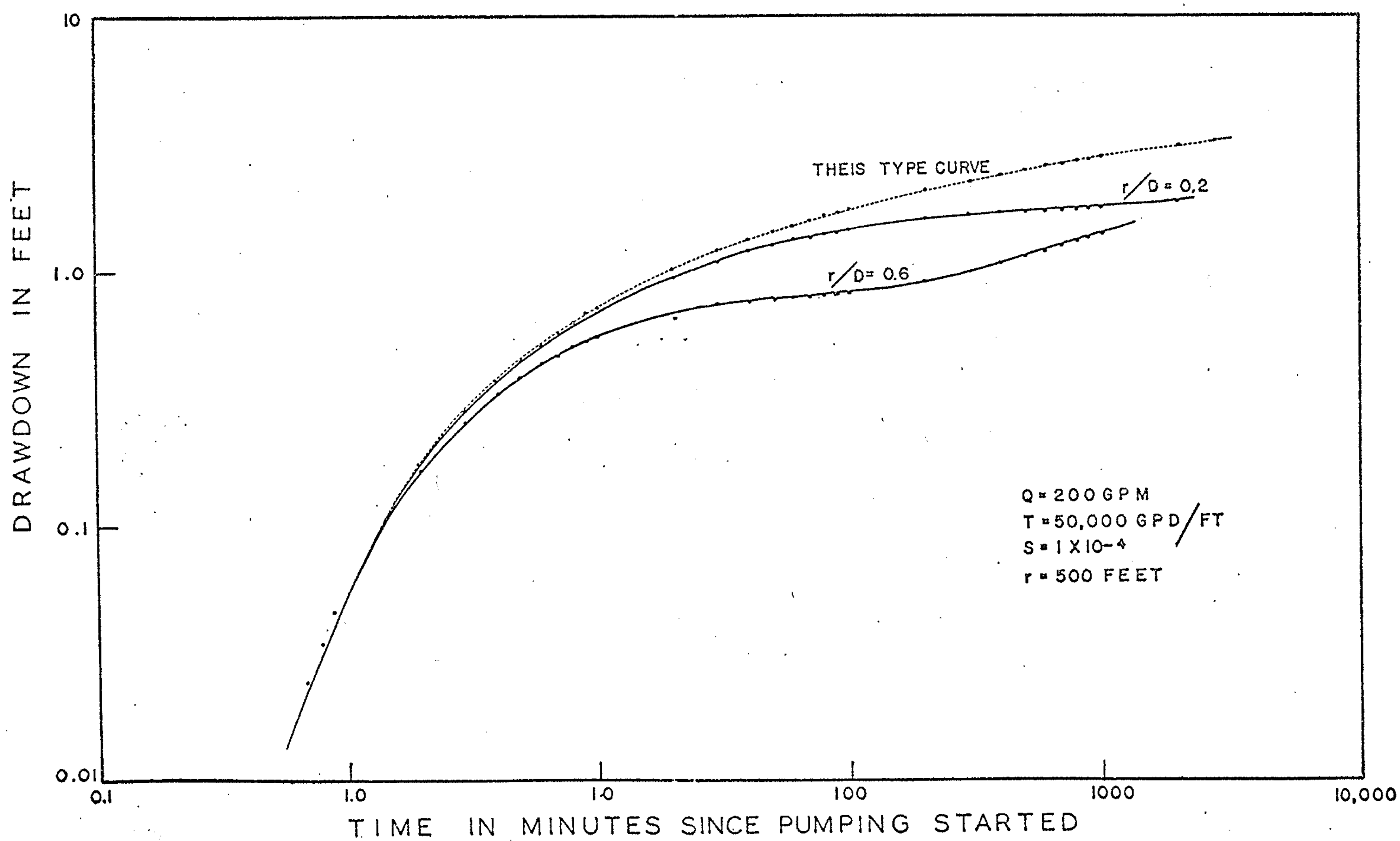
Using the tables presented by Boulton (1963) equations (16) and (18) were solved for an idealized aquifer having a transmissibility of 50,000 U.S. gpd/ft and a storage coefficient of 1×10^{-4}. An observation well is located 500 ft from a well pumping at a rate of 200 U.S. gpm. The storage coefficient of the associated compressible beds is 2.08×10^{-3}.

The curves for several values of r/D are plotted in figure (2). For the values of r/D chosen, a corresponding value for α can be obtained. Because α is an arbitrary constant the values of α cannot successfully be converted to the corresponding soil mechanics parameters for comparison with models presented later in this thesis.

Initially the permeability of the compressible materials seems to be the factor controlling delayed yield in the Boulton model.

For small values of time the curves from the Boulton model follow the Hantush-Walton leaky type curves which have been determined for various permeabilities in the confining layers. For a particular r/D value a change in S' does not affect the time-drawdown graph from the Boulton model. After a period of equilibrium the drawdown in the Boulton model begins to increase again. In this range of time the storage in the compressible materials has more affect on the volume of delayed yield.

Boulton's model is not a geometrically defined model such as later models discussed in this thesis. Boulton presented his theory for application to both confined and unconfined aquifers. For the artesian case the delayed yield incorporated in the storage coefficient, S', in equation (15) can represent the release of water from one or more interbeds and/or semipervious compressible confining layers. For the nonartesian case the delayed release of water by the slow percolation of the water through fine-grained sediments is accounted for in S'. Therefore, an understanding of the



DRAWDOWN RESPONSE CURVES FROM BOULTON MODEL *FIG. 2* aquifer boundary conditions cannot be gained by using the type curves derived from the Boulton model to analyse pump test data.

Hantush Model

Hantush (1960) has presented a more rigorous analysis of the phenomena of delayed yield. He restricted his analysis to three models of elastic artesian aquifers confined by semipervious compressible strata. In two of his models the semipervious units were overlain and underlain by more permeable saturated units. However, only the case where the confining beds are bounded by impermeable strata is of direct interest here (Fig. 3). Leakage to the aquifer is obtained from a reduction of storage in the compressible finer grained confining beds. Hantush assumed that flow is vertical in the confining layers and radial in the main aquifer.

Following Hantush (1960) the differential equation of flow for the main aquifer is

$$T \left(\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\right) + K' \frac{\partial h}{\partial z}(r, z_1, t) - K'' \frac{\partial h_2}{\partial z}(r, b'', t) = S\frac{\partial h}{\partial t} \tag{19}$$

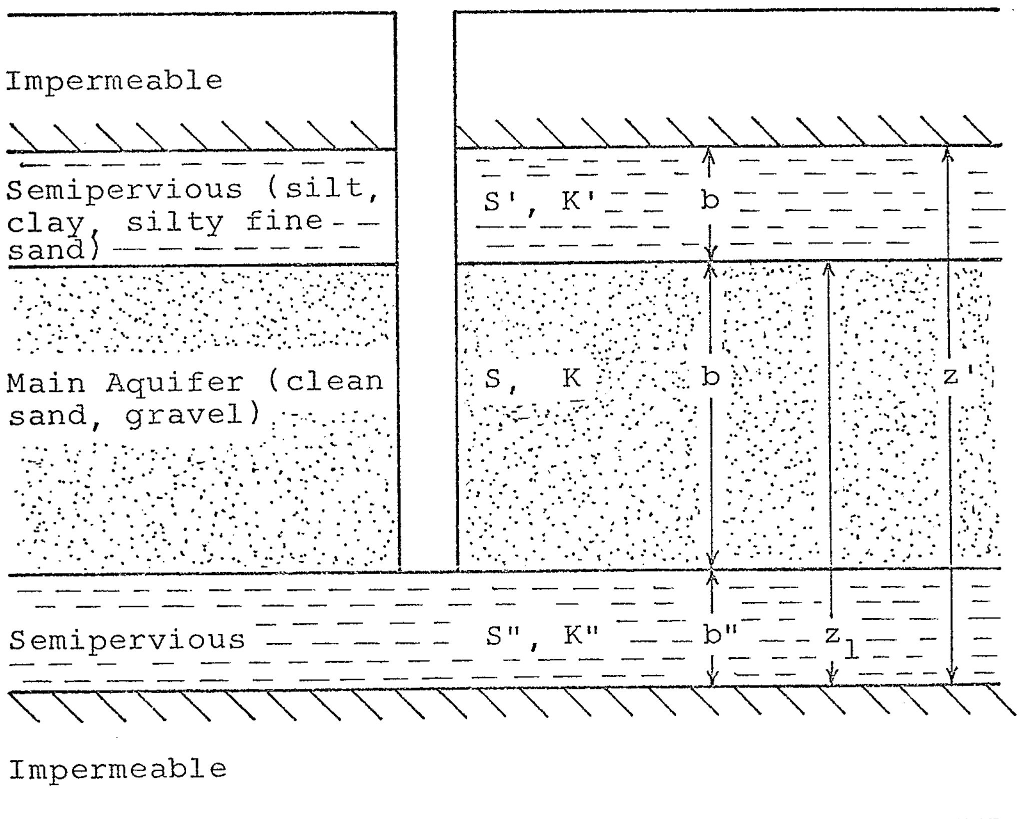


Figure 3 Schematic model for delayed yield.

where the terms

$$K' \frac{\partial h_1}{\partial z}(r, z_1, t) \text{ and } K'' \frac{\partial h_2}{\partial z}(r, b'', t)$$

account for the influx of water from the semi-pervious compressible confining beds.

The boundary conditions are

$$\lim_{r \to o} r \frac{\partial h}{\partial r}(r, t) = -\frac{Q}{2 \pi T} \tag{19a}$$

$$h(\infty, t) = 0 \tag{19b}$$

and the initial condition is

$$h(r, 0) = 0. \tag{19c}$$

For the upper semipervious layer the differential equation of flow is

$$\frac{\partial^2 h_1}{\partial z^2} = \frac{S'}{K' b'} \frac{\partial h_1}{\partial t}. \tag{20}$$

The boundary conditions are

$$\frac{\partial h_1}{\partial z}(r, z^1, t) = 0 \tag{20a}$$

$$h_1(r, z_1, t) = h(r, t) \tag{20b}$$

and the initial condition is

$$h_1(r, z, o) = 0 \tag{20c}$$

where $z_1 = b'' + b$

$z' = b'' + b + b'$.

Boundary condition (20a) states that the change in drawdown across the top surface of the upper semipervious layer is zero since the upper surface of this layer is bounded by an impermeable bed. Boundary condition (20b) states that the drawdown for any radius, r, and time, t, at the lower surface of the semi-pervious layer is equal to the drawdown in the main aquifer for that radius and time.

The initial condition (20c) indicates that the drawdown is zero everywhere in the compressible bed before pumping is started in the main aquifer.

For the lower aquifer the differential equation of flow is

$$\frac{\partial^2 h_2}{\partial z^2} = \frac{S''}{K'' \quad b''} \frac{\partial h_2}{\partial t}. \tag{21}$$

The boundary conditions are

$$\frac{\partial h_2}{\partial z} (r, o, t) = 0 \tag{21a}$$

$$h_2 \ (r, b'', t) = h \ (r, t) \tag{21b}$$

and the initial condition is

$$h_2 \; (r, z, o) \; = 0. \qquad (21c)$$

The boundary conditions (21a) and (21b) and the initial condition (21c) for the lower semipervious layer are the same as those for the upper semipervious layer except that the upper surface adjoins the main aquifer and the lower surface is bounded by the impermeable bed in the lower semipervious layer.

The differential equations used by Hantush to describe one dimensional flow in the upper and lower semipervious layers are analogous to equation (2), the differential equation developed by Terzaghi to describe one dimensional consolidation of fine-grained soils. The constant terms, $S'/K'b'$ and $S''/K''b''$, in equations (20) and (21) are analogous to the coefficient of consolidation, c_v, used in consolidation theory.

Pumping from the main aquifer creates an excess pore pressure in the semipervious layers. The dissipation of this pore pressure is analogous to the decay of the excess pore pressure during the consolidation process.

Because the confining layers are bounded on one side by impermeable beds, flow from either the upper or lower semipervious layer takes place in one direction only. Therefore, the boundary conditions for the Hantush model are similar to conditions for consolidation with single drainage where one surface of the compressible bed is bounded by impermeable material.

However, the boundary conditions for the Hantush model differ from those for consolidation by providing for continuous loading of the semipervious layers.

In Terzaghi's consolidation theory individual loads are assumed to be applied instantaneously and full consolidation to take place before the application of successive loads. Each load is then analyzed separately. Since Terzaghi's initial development of the consolidation theory, solutions of the consolidation problem under various loading schemes have been presented (Schiffman, 1958; Lumb, 1963). However, none is specifically suitable for the type of loading associated with aquifer systems during pumping.

To solve the boundary value problem Hantush considered two ranges of time. For t less than both $b'S'/10K'$ and $b''S''/10K''$ the solution is given by:

$$s = \frac{Q}{4 \pi T} \int_{u}^{\infty} \frac{e^{-y}}{y} \text{ erfc } \left\{ \frac{\beta \sqrt{u}}{\sqrt{y (y - u)}} \right\} dy$$

$$s = \frac{Q}{4 \pi T} \quad H (u, \beta), \tag{22}$$

where

$$\beta = r\lambda/4 ,$$

$$u = \frac{r^2}{4T} \frac{S}{t} ,$$

$$\lambda = \sqrt{\frac{K'/b'}{T} \quad \frac{S'}{S}} + \sqrt{\frac{K''/b''}{T} \quad \frac{S''}{S}}$$

Hantush also developed an expression for the rate of leakage, q_L, induced by pumping an aquifer. The expression is

$$q_L = Q [1 - e^{nt} \text{ erfc } (\sqrt{nt})], \tag{23}$$

where

$$n = T \lambda^2 /S.$$

The total volume of leakage occurring within the range of time from start of pumping to time t less than both $b'S'/10K'$ and $b''S''/10K''$ is obtained by integrating (23) with respect to t. The result is

$$V_L = V [1 - 2 / \sqrt{n \pi t} + q_L /Qnt], \tag{24}$$

where V = Qt is the total volume of water pumped during the range of time specified above.

For large values of time the lower limit of time is taken greater than both 10b'S'/K' and 10b"S"/K". The solution is

$$s = \frac{Q}{4\pi T} \int_{u\delta_2}^{\infty} \frac{e^{-y}}{y}\, dy$$
$$s = \frac{Q}{4\pi T} \quad W(u\,\delta_2), \tag{25}$$

where

$$\delta_2 = 1 + (S' + S'')/S .$$

The leakage is given by

$$q_L = Q\,(S' + S'')\,/\,(S + S' + S''). \tag{26}$$

The other cases of leakage from compressible confining beds considered by Hantush include leakage derived from aquifers above and/or below the semipermeable confining beds in addition to leakage derived from consolidation of the beds themselves. It should be noted that for a decline of artesian pressure, leakage will be derived only from storage within the semi-pervious layers until such time as the pore pressure decrease has been transmitted through the full thickness of the layers. At this point a gradient causing induced recharge through the semipervious layers is initiated. Prior to the commencement of the induced leakage the aquifer system behaves as the case described above and all three cases are described by equation (22).

Illustrative Cases of the Hantush Model

Several examples for Hantush's model have been chosen to illustrate the effect of delayed yield on confined aquifer response. These examples are compared to the response of an aquifer adhering to the Theis assumptions listed on page 1. Aquifer and confining bed parameters representative of the geologic materials indicated in figure (3) were chosen for the examples.

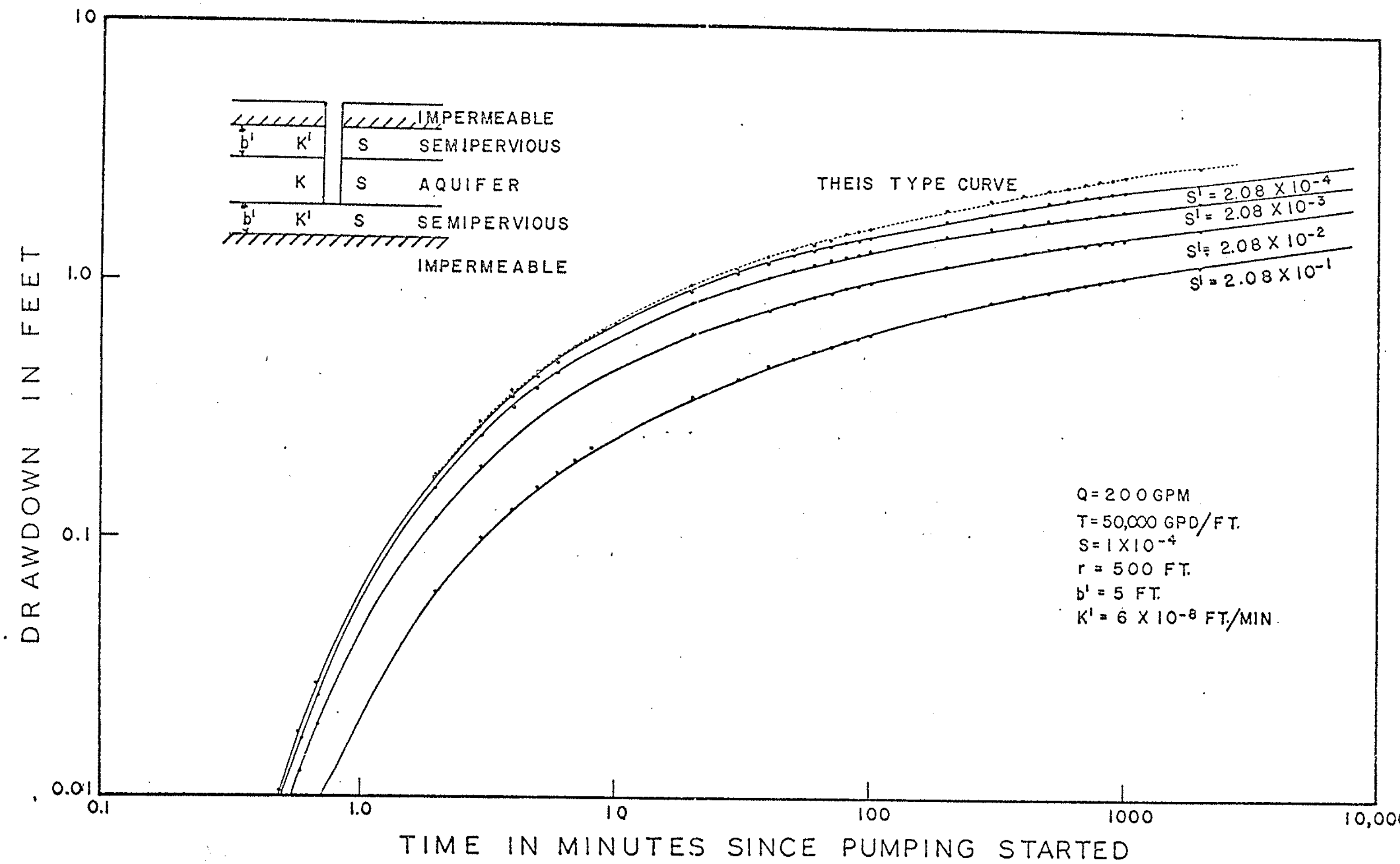
An idealized horizontal aquifer of infinite areal extent with a transmissibility of 50,000 U.S. gpd /ft is assumed to be bounded above and below by semipervious compressible beds having a vertical permeability of 6×10^{-8} ft/min. Each of the upper and lower beds is assumed to be 5 feet thick. An observation well is located 500 feet from a pumping well discharging at a rate of 200 U.S. gpm (Fig. 4a).

Using a table of values for the function

H (u, β), Hantush's equation (22) was solved for various values of the storage coefficient of the confining beds while keeping the storage coefficient of the main aquifer constant at 1×10^{-4} (Fig. 4b). A second set of curves for values of the storage coefficient of the main aquifer equal to 1.0, 0.1 and 0.01 times the storage coefficient of the confining layers was obtained for a confining-bed storage coefficient of 2.08×10^{-3} (Fig. 5). For the values of the parameters chosen, the time limits for equation (22) occur within the time range used for most pump tests.

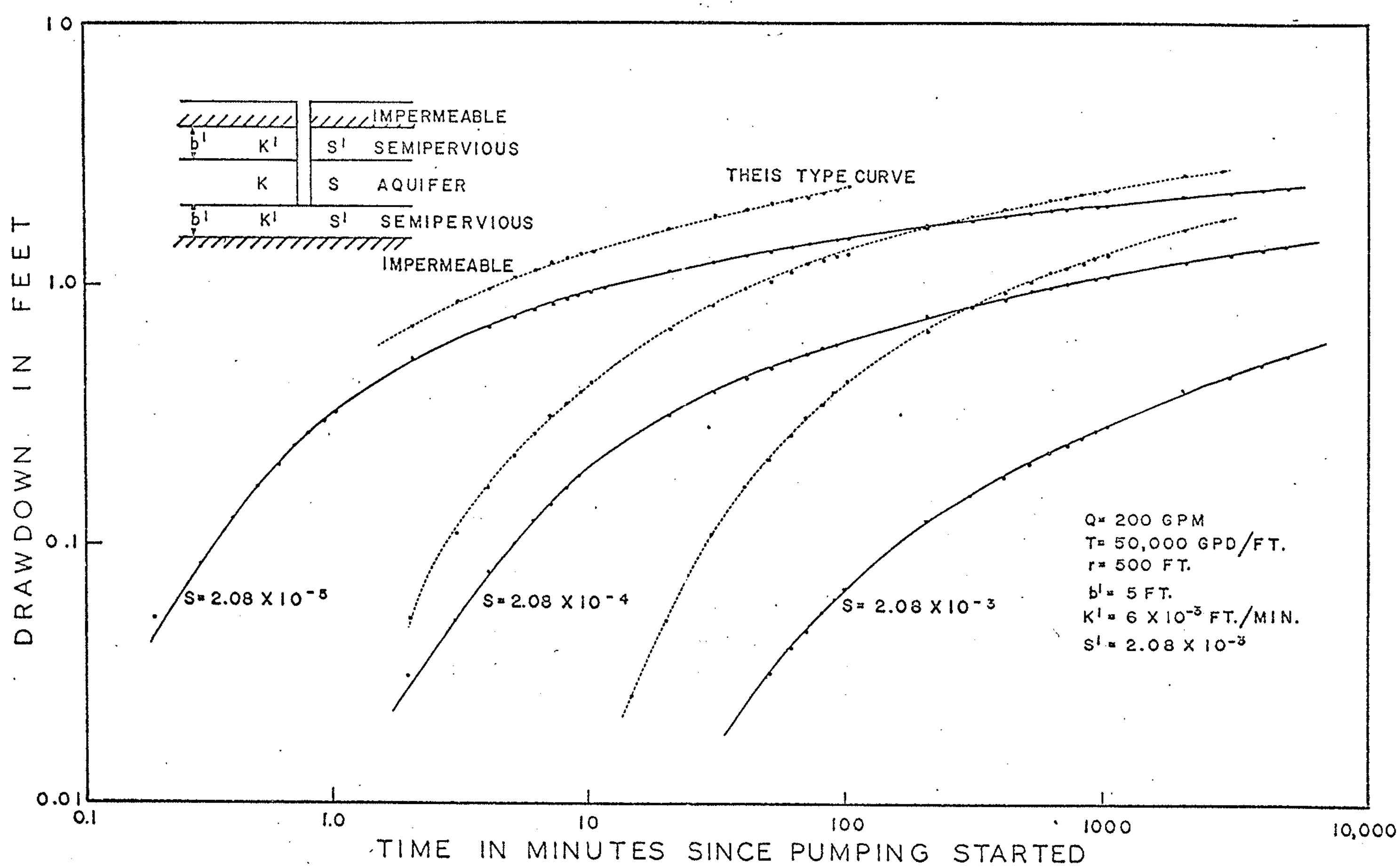
The curves for various storage coefficients in the semipervious layers (Fig. 4) and the curves for various storage coefficients in the main aquifer (Fig. 5) resemble the Theis type curve and probably would be assumed to fit the Theis model if the data were obtained from field tests. These curves do not show the double curvature indicated by Boulton's model.

The Hantush models do not include the situation of thin semipervious compressible beds



DRAWDOWN RESPONSE CURVES FOR AQUIFER SYSTEMS WITH DIFFERENT STORAGE COEFFICIENTS IN SEMIPERVIOUS LAYERS

FIG. 4



DRAWDOWN RESPONSE CURVES FOR AQUIFER SYSTEMS WITH DIFFERENT STORAGE COEFFICIENTS IN MAIN AQUIFER

FIG. 5

within an aquifer. In such a case drainage would occur from both faces of the interbed thus causing more rapid consolidation and therefore a more immediate effect on the time-drawdown curves.

An Analytical Model Based on Consolidation Theory

Introduction

To study the effect of thin semipervious compressible interbeds within an aquifer in terms of an unique geologic framework independent of any arbitrary constants such as α in the Boulton model, the author has developed an analytical approximation model based on consolidation theory. The model was also adjusted to solve for the case where the aquifer is bounded above and below by semipervious compressible confining beds. The reasoning on which the model is based is given below.

The solution given by the Theis equation for the drawdown at radius, r, in an elastic artesian aquifer can be adjusted to take into account the delayed yield quantity of water released from storage in thin compressible interbeds. The compression of a saturated fine-grained unit caused by a decrease in the volume of its voids is equal to the volume of water expelled per unit area from the layer. The quantity of water released from compressible interbeds can be determined from consolidation theory as presented in the soil mechanics literature.

Quantitative Development

Jumikis (1962) has shown that the degree of consolidation or amount of compression of a fine-grained bed is equal to the 'effective pressure area' for the bed. The 'effective pressure area' is determined from a depth-pressure diagram (Fig. 6) for the compressible layer. The curved line in figure (6) shows the proportion of an initial applied load being carried by the water occupying the voids of the soil as compared to that carried by the soil solids in the compressible layer at a time, t, following application of the load. This line is referred to as an isochrone. The distribution of the excess pore water pressure throughout the thickness of the compressible layer immediately after loading is given by the zero isochrone, t = 0. In this case the initial distribution of excess pore water pressure is uniform throughout the thickness of the layer.

In their study of the release of water from low permeability sediments in relation to land subsidence Domenico and Mifflin (1965) considered a compressible layer to be bounded above and below by highly permeable aquifers. For a decline of head in the aquifer at one boundary of the compressible layer they derived an equation giving total volume of water released per unit area of the compressible layer as the product of the specific storage and the effective pressure area developed across the compressible layer. The initial pore pressure distribution shown in figure (7) represents a hydrostatic case. Although in nature non-hydrostatic distribution of pore pressure across interbeds often occurs prior to aquifer pumping, the hydrostatic case is adequate for the purposes

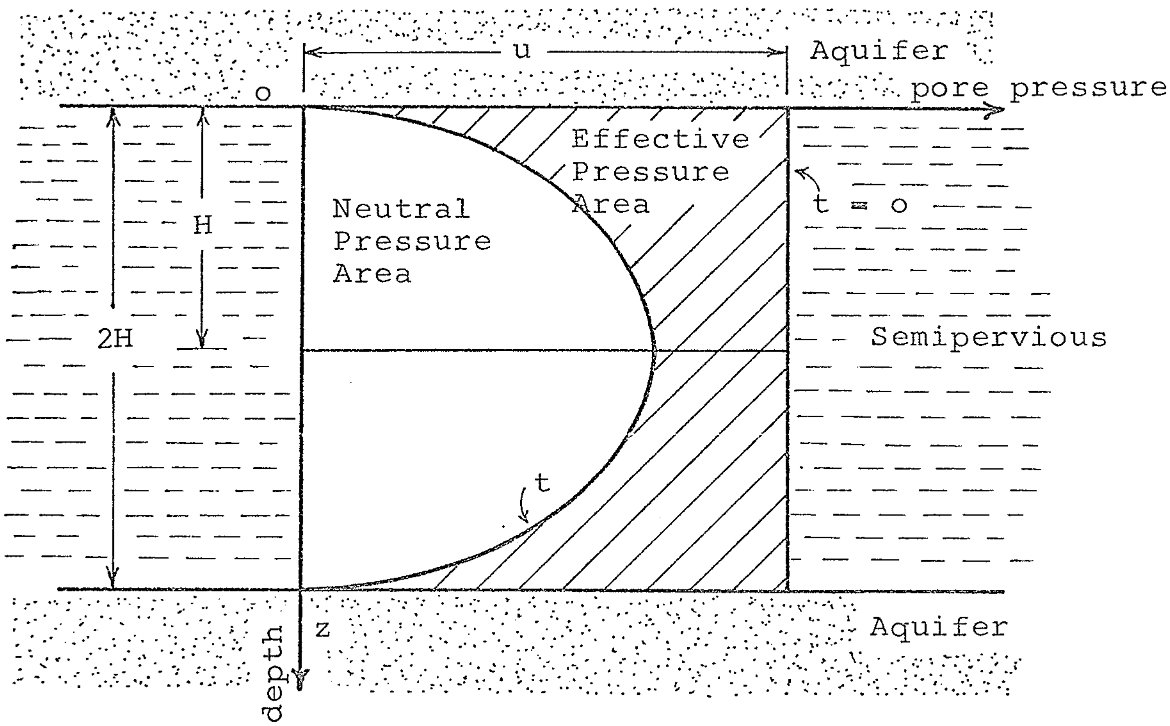


Figure 6 Depth-pressure diagram for a compressible layer during consolidation following an initial applied stress (After Jumikis, 1962).

of this analysis. The head in the aquifer at the other boundary of the compressible interbed is assumed to remain unchanged. The equation is

$$q_L = \frac{\gamma_w}{E_c} \quad \frac{\Delta h}{2} \quad b' \tag{27}$$

where the specific storage, $\gamma\, w/E_c = K'/c_v$.

Domenico and Mifflin have shown that if the heads in the aquifers above and below the compressible layer are lowered by an equal amount (Fig. 8) the equation for the total volume of water released is twice as much or,

$$q_L = \frac{\gamma_w}{E_c} \quad \Delta h \; b' . \tag{28}$$

Equation (28) gives the total volume of water released per unit area when consolidation of a prism of compressible interbed is complete under a reduction in artesian head. During the length of time of a typical pump test (20 - 70 hours), consolidation is not complete. Therefore, the average degree of consolidation of the layer must be determined before calculating the volume of water which has been released from the compressible layer. The average degree of consolidation is a ratio of the amount of consolidation which has taken place to the total

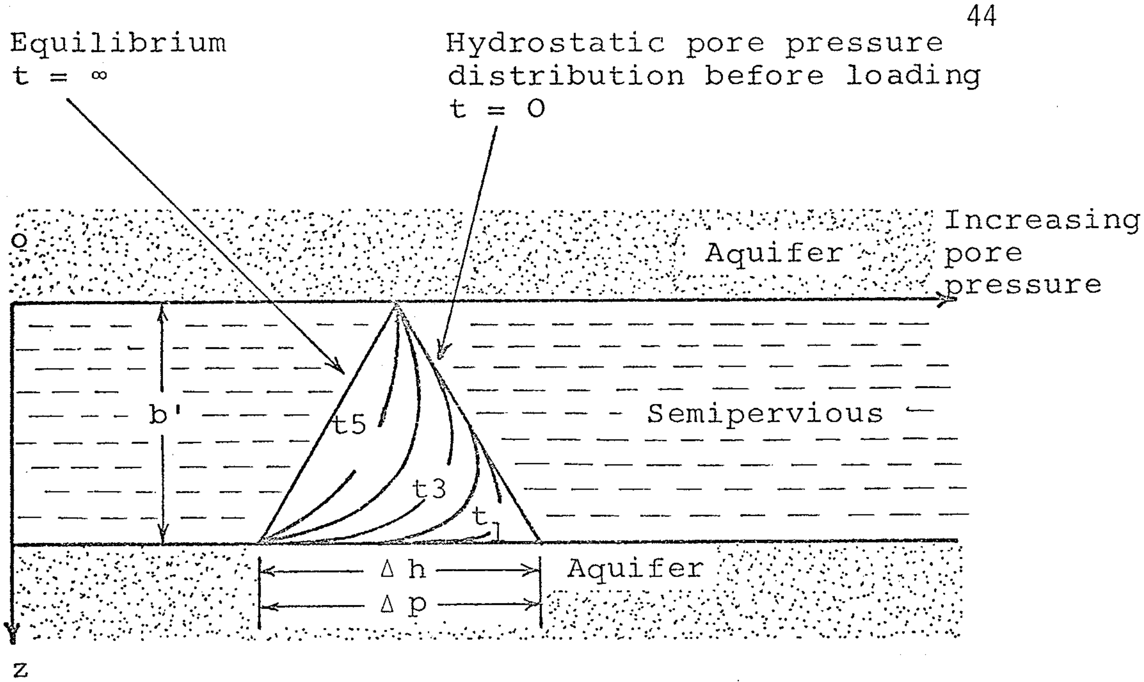


Figure 7 Depth-pressure diagram for a compressible layer when the head is lowered in one bounding aquifer.

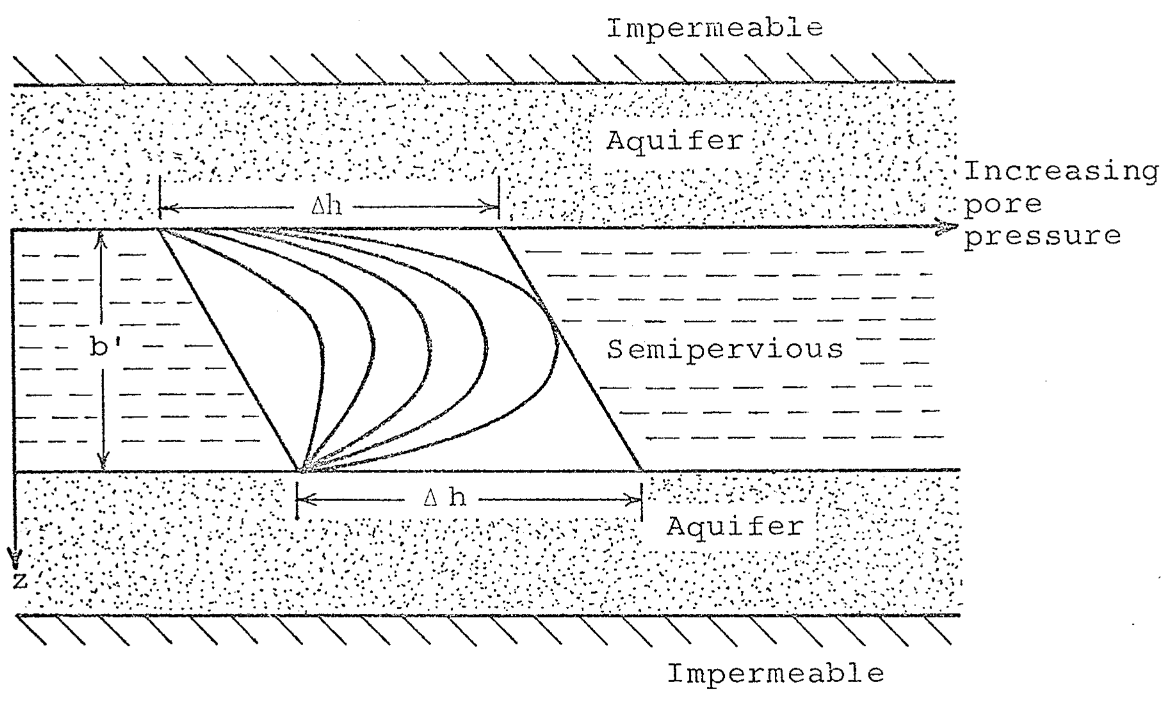


Figure 8 Depth-pressure diagram for a compressible layer when the head is lowered in both bounding aquifers.

amount of consolidation possible under the applied load. When the equation for total volume of water released is multiplied by the average degree of consolidation the volume of water released to time, t, is determined.

The following equation gives the average degree of consolidation for a infinite, horizontal, compressible bed with vertical drainage at one or two faces (Taylor, 1948; Jumikis, 1962; Scott, 1963):

$$U = 1 - \frac{8}{\pi^2} \sum_{m=o}^{\infty} \frac{e^{-(\pi^2 (2m+1)^2 /4) T}}{(2m+1)^2} \qquad (29)$$

$$U = 1 - \frac{8}{\pi^2} \left(e^{-Nt} + \frac{1}{9} e^{-9Nt} + \frac{1}{25} e^{-25Nt} + \ldots \right)$$

where

$$N = \frac{\pi^2}{4} \frac{C_v}{H^2} .$$

This equation can be used to determine the average degree of consolidation for the boundary and initial conditions shown in figures (7) and (8).

The drawdown in an elastic artesian aquifer containing fine-grained interbeds can be approximated during an initial increment of time in a pump test by the Theis solution of equation

(8):

$$s = \frac{Q}{4\pi T} \int_{r^2 S/4Tt}^{\infty} \frac{e^{-x}}{x} \, dx. \qquad (30)$$

This decline of artesian pressure in the aquifer produces an excess hydrostatic pressure within the compressible interbeds in the aquifer and the beds begin to consolidate.

During the next increment of time the amount of water yielded per unit area of the compressible bed will be equal to the average degree of consolidation of the compressible bed at the end of this increment of time multiplied by the total specific discharge of the bed under the load applied during the initial increment of time. The drawdown in the aquifer will also be increasing because of pumping during this second increment of time. Therefore the actual drawdown in the aquifer at the end of the second increment of time will be equal to the drawdown given by equation (30) less the amount of the water released by delayed yield. The increase in drawdown between the end of the first time increment and the end of the second time increment produces an additional load on the compressible interbed.

At the end of the third time increment the load applied during the first time increment will have consolidated the compressible bed for two time increments and the load applied during the second time increment will have consolidated the bed for one time increment. The total amount of water released from the compressible bed at the end of the third increment is the sum of the amounts of water released by consolidation of the bed under each of the loads. This value is used to correct the Theis drawdown at the end of the third time increment to arrive at the actual drawdown in the aquifer at the end of this time.

By the above process the drawdowns at the end of N time increments can be calculated, and a drawdown versus time curve can be plotted to show the result of pumping an aquifer containing a compressible interbed. By considering small time increments the continuous flow conditions in the main aquifer and in the compressible layer are approximated by the above approach.

A lowering of the aquifer head in the models presented by Hantush (1960) produces a triangular effective pressure area across a compressible confining layer at the boundary of the aquifer which is similar to the triangular effective pressure area developed across an interbed subjected to a head decline at only one drainage face. Equation (27) should therefore be suitable for calculating the total volume of water released per unit area from the compressible confining layer. The layer need not be bounded on both sides by permeable aquifers but may be confined at one face by impermeable material. It follows that when equation (27) is used to describe the total volume of water released per unit area from two confining layers of equal thickness and having the same consolidation properties it becomes

$$q = \frac{\gamma_w}{E_c} \Delta h \, b' \tag{31}$$

which is identical to equation (28). Therefore during pumping from an infinite horizontal aquifer the total volume of water released per unit area from compressible semipervious confining layers of equal thickness is equivalent to the total volume of water released from a compressible interbed of the same thickness as either of the confining layers.

The expression for the average degree of consolidation given in equation (29) can be used for a compressible bed which is overlain and underlain by aquifers and which has a triangular initial pore pressure distribution. Therefore the equations used in the consolidation model for the case of a compressible interbed are also applicable to the Hantush model for the case where the two compressible confining layers are overlain and underlain by other permeable aquifers which are not subjected to pumping (Fig. 9). The time drawdown curves obtained for delayed yield from two confining layers overlain and underlain by aquifers should therefore be the same as the curves obtained for a compressible interbed.

A different equation for the average degree of consolidation is required for the case shown in figure (10) where the compressible beds are located at the boundary of the aquifer and are overlain and underlain by impermeable material (Domenico and Mifflin, 1965). The average degree of consolidation for a compressible bed with a triangular initial stress distribution where one surface of the bed is an impermeable surface has been tabulated by Jumikis (1962) for a range of time constants. However, the mathematical expression for the average degree of consolidation for this case is not readily available. Therefore the analysis of delayed yield made in this thesis has been restricted to the case where the compressible layer is an interbed within the aquifer.

Illustrative Cases of the Analytical Consolidation Model

Using equations (28) and (29) and the Theis nonequilibrium equation, time drawdown curves for an elastic artesian aquifer containing an interbed of compressible material were obtained with the aid of a digital computer. In the model a pumping well discharges at 200 U.S. gpm from an aquifer having a transmissibility

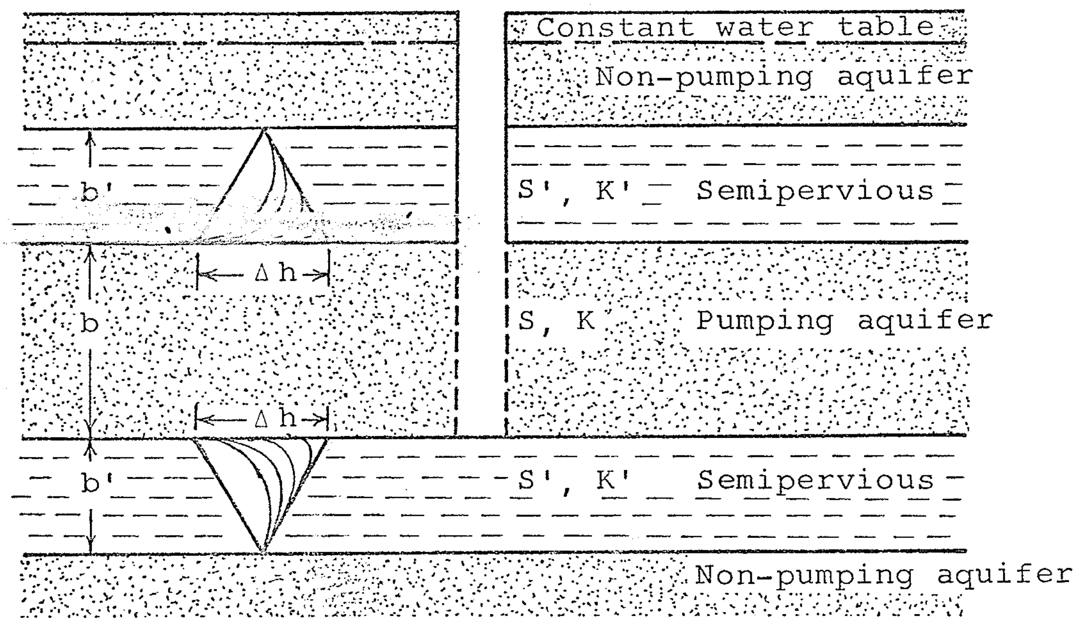


Figure 9 Depth-pressure diagram for compressible confining layers which are overlain and underlain by aquifers.

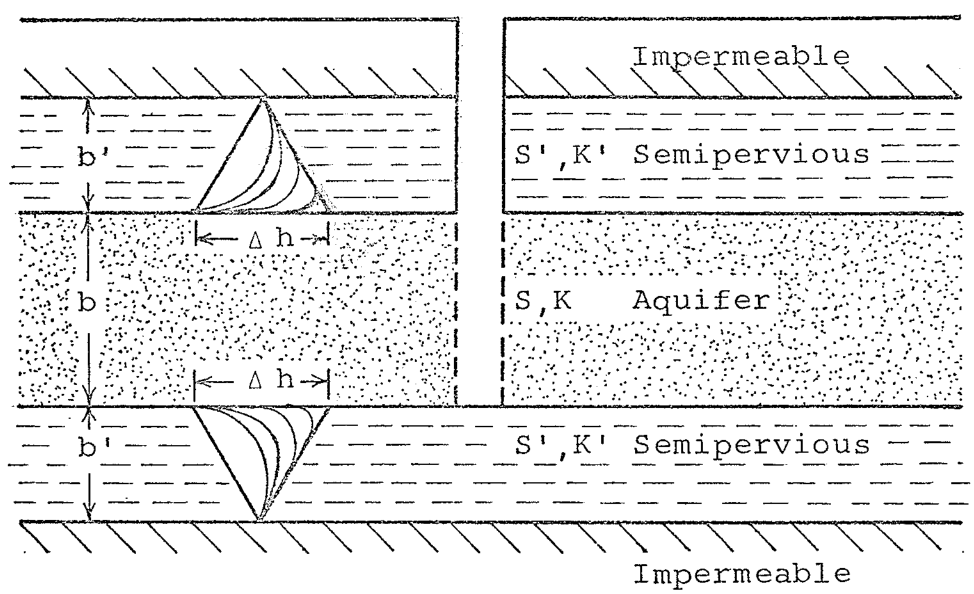


Figure 10 Depth-pressure diagram for compressible confining layers which are underlain and overlain by impermeable material.

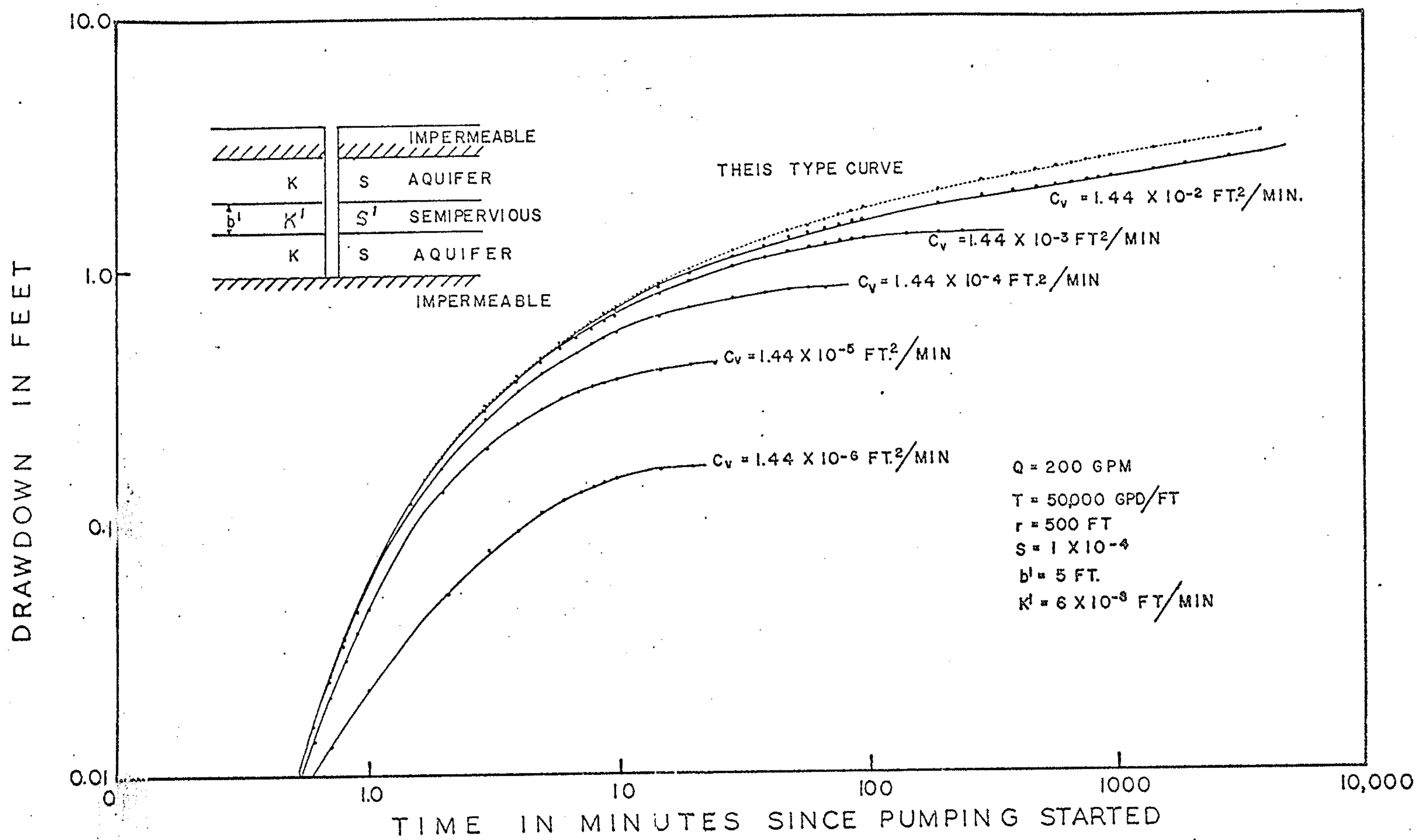
of 50,000 U. S. gpd/ft. The compressible interbed is assumed to be 5 feet thick and has a constant vertical permeability of 6×10^{-8} ft/min. An observation well is located 500 feet from the pumping well.

Time increments of 0.1 minutes were used for obtaining time drawdown curves which reached equilibrium conditions within four hours from start of pumping. Time increments of 1 minute were used for curves for which a longer time was required for equilibrium conditions to be reached. A set of curves for values of the coefficient of consolidation of the compressible bed was obtained (Fig. 11). The storage coefficient of the main aquifer for this computation was assumed to be 1×10^{-4}. Using a coefficient of consolidation of 1.44 $\times 10^{-4}$ ft^2/min another set of curves for values of the storage coefficient of the main aquifer equal to 1.0, 0.1, and 0.01 times the storage coefficient of the compressible bed was also computed (Fig. 12). The coefficient of storage of the compressible bed for a given coefficient of consolidation was determined by equation (7).

The curves for various coefficients of consolidation for the range of coefficients considered all reach equilibrium within four hours after the start of pumping. The computer computations indicate that the drawdowns continue to remain constant for the remainder of the two day pump test.

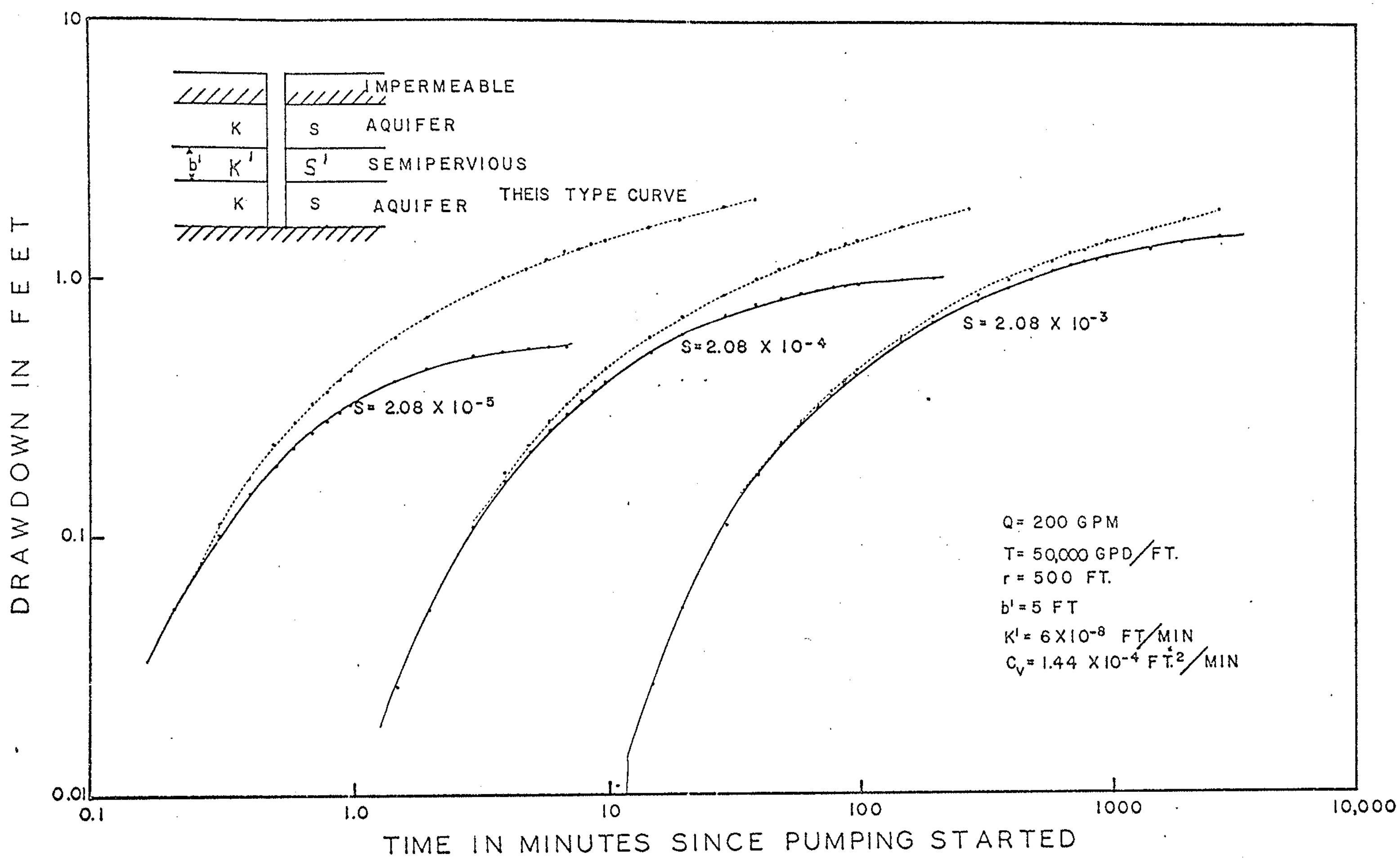
These computed curves very closely resemble the type curves for nonsteady state leaky artesian aquifers presented by Walton (1962) from values tabulated by Hantush (1956). The aquifer constants for the consolidation model were recalculated by fitting the curves in figure (11) to the leaky type curves. The constants originally used to obtain the time drawdown curves from the consolidation model were not returned by this curve fitting technique. Therefore, although time drawdown curves for delayed yield conditions may resemble leaky conditions, attempts to fit these curves to the Hantush-Walton leaky type curves would usually lead to erroneous results.

The time drawdown curves for various values of storage in the main aquifer also



DRAWDOWN RESPONSE CURVES FOR AQUIFER SYSTEMS WITH DIFFERENT COEFFICIENTS OF CONSOLIDATION IN SEMIPERVIOUS LAYERS

FIG. 11



DRAWDOWN RESPONSE CURVES FOR AQUIFER SYSTEMS WITH DIFFERENT STORAGE COEFFICIENTS IN MAIN AQUIFER

FIG. 12

resemble the leaky artesian aquifer curves. The curves for smaller values of the storage coefficient deviate from the Theis curve more rapidly than the curves for larger values. This is to be expected since a smaller coefficient of storage results in a more rapid drawdown of the potentiometric surface thus inducing the delayed leakage from the compressible beds.

The rate of drawdown for the model decreases as delayed yield from the compressible beds becomes effective and the drawdown eventually becomes constant and remains constant for the rest of the pump test. However, if extremely long pump tests were conducted, complete consolidation of the compressible beds would be attained and the drawdown in the aquifer would begin to increase again.

DISCUSSION OF DELAYED YIELD MODELS

The aquifer parameters presented in the above illustrative examples were chosen as being typical of many field situations. Computations were made using other aquifer parameters also falling with the general range of typical aquifer occurrence. The qualitative nature of the results was similar to the illustrative cases presented and therefore they are not presented here. The coefficients of consolidation and permeability for the semipervious beds were representative values for fine-grained materials as selected from the soil mechanics literature. They are believed to be representative of many interbed and boundary bed units commonly associated with confined aquifer systems in nonindurated deposits of glacial and fluvial origin.

For use in the Boulton and Hantush models, the coefficients of consolidation were converted to the equivalent storage coefficients for the semipervious compressible layers by equation (7). The curves obtained from the Boulton, Hantush and consolidation models deviate from the Theis response curve towards smaller values of drawdown. This indicates that significantly more water is being released from the aquifer system than would be the case if all the Theis assumptions were fulfilled.

Initially the drawdown response of all three models agree reasonably well. However, after a period of time ranging from a few minutes to about 100 minutes, depending on the values of the parameters chosen, the drawdowns of the Boulton and consolidation models tend to deviate more rapidly from the Theis model than the drawdowns of the Hantush model. The curves for the Boulton and the consolidation models reach equilibrium conditions similar to the Hantush-Walton leaky artesian case.

After a period of less than one day depending on the value of r/D chosen, the drawdowns in the Boulton model again begin to increase whereas the drawdowns in the consolidation model continue to remain constant.

Boulton has based his theoretical development predominantly on physical principles applicable to artesian aquifers only. He considered compressible beds being consolidated within an aquifer and included his expression for delayed yield in the differential equation for the nonsteady radial flow in an infinite artesian aquifer.

In the application of his theory to field data Boulton then utilized data from pumping tests in unconfined aquifers to illustrate the validity of his theory. The use of the empirical constant α enabled Boulton to apply his theory to both confined and unconfined aquifers. However, values for the term, r/D, which includes the empirical constant α had to be arbitrarily selected and the values could not be correlated with the values of the parameters used in the consolidation and Hantush models.

The lack of a physical basis is the most serious limitation of Boulton's equation. Different processes are operative in the artesian and nonartesian cases of aquifer tests particularily with respect to delayed yield. Any theoretical development attempting to describe flow for both artesian and nonartesian cases will be subject to limitations. A mathematical model explaining the phenomenon of delayed yield should be based on a quantitative description of the actual physical processes involved.

In contrast to the Boulton model the rate of release of water from the compressible beds in the consolidation model equals the rate of pumping from the aquifer and it would appear that equilibrium conditions remain until consolidation of the compressible beds is nearly complete under the applied load.

The consolidation process is predominantly inelastic. If a compressible bed is allowed to consolidate completely under an applied load the bed will not expand to its original volume upon removal of the load. The pore space or storage of the compressible layer is reduced by the consolidation process and a permanent loss of water is associated with the reduction in pore volume. The leakage will decrease as the semipervious beds consolidate and delayed yield will not be a source of recharge when consolidation is complete.

As already noted the curves from the Boulton and consolidation models deviate from the nonequilibrium type curve more rapidly than the curves for the Hantush model. The Boulton model cannot be successfully compared with the Hantush or consolidation models because of the empirical constant, α. However, it is possible to compare the results from the consolidation model with those from the Hantush model. The equations used by Hantush are analogous to those used in the Terzaghi consolidation theory and the same results should be obtained for similar boundary conditions.

Two boundary situations are of interest from the cases analysed by Hantush. In case (1) the semipervious layers at the boundary of the aquifer are overlain and underlain by impermeable material (Fig. 10). This Hantush model was considered to be the most representative of delayed yield as defined in this thesis and was the case used to illustrate Hantush's theory of delayed yield. In case (2) the semipervious layers at the boundary of the aquifer are overlain and underlain by other permeable aquifers (Fig. 9).

Hantush noted that the two cases behave in the same manner until the pore pressure decrease produced by pumping has been transmitted through the full thickness of the semipervious layers. Up to a certain time limit both cases are described by equation (22). For the parameters chosen in this thesis the time limit for which the two cases are similar is greater than the length of most pump tests. Therefore the results for the illustrative case (1) should also be applicable to case (2).

If each of the semipervious beds at the boundaries of the aquifer are of the same thickness as the interbed and if the permeable aquifers above and below the semipervious confining layers are not subjected to pumping then the boundary conditions controlling the consolidation of a compressible interbed in the consolidation model should be identical to the boundary conditions for the Hantush case (2). Since this second case is the same as the first case for the parameters chosen in this thesis the results from the consolidation model should be comparable to the results for case (1) illustrating Hantush's theory.

The drawdown curves for the two models are different, those of the consolidation model deviate from the Theis type curve more rapidly than those for the Hantush model. Therefore the consolidation model describes a more rapid release of consolidation water than the Hantush model.

As noted in the theoretical development of the consolidation model the expression for the average degree of consolidation given in equation (29) can be used for a compressible bed which is overlain and underlain by aquifers and which has a triangular initial pore pressure distribution. If the equation for the average degree of consolidation for a compressible bed which is bounded on one side by impermeable material and which has a triangular initial pore pressure distribution was available for use in the consolidation model, the time drawdown curve would be expected to more closely approach that of the Hantush case (1).

Jumikis (1962) presented tables of values for the average degree of consolidation of compressible beds over a range of time constants. The tables were prepared by solving the expressions for the average degree of consolidation appropriate for various boundary conditions. In each case a similar length of drainage path was used.

By calculating the time constants taking into account the length of drainage path for each of the two cases considered by Hantush the average degree of consolidation in each of the cases can be determined from the tables presented by Jumikis. For small values of time, t, the average degrees of consolidation for the two cases are approximately the same. For longer times the average degrees of consolidation differ. These observations support Hantush's analysis that the two cases are the same up to a certain time limit and diverge for larger values of time.

The volume of consolidation leakage released per unit area of a compressible interbed can be determined from the computer results for the time at which equilibrium conditions are reached in the aquifer. For a compressible interbed with a coefficient of consolidation of 1.44×10^{-4} ft^2/min in an aquifer having a transmissibility of 50,000

U.S. gpd/ft and a storage coefficient of 1×10^{-4} the consolidation leakage accounts for approximately 47.5 per cent of the total volume of water pumped from the aquifer up to the time equilibrium conditions are attained. In this time only 4.4 per cent of the total possible delayed yield has been released from storage in the compressible bed.

After equilibrium conditions are attained the rate of consolidation leakage from the compressible beds equals the rate of pumping from the well. The drawdown remains constant until the beds have consolidated under the applied stress, at which time drawdown of the piezometric surface will again occur. This conclusion is supported by Boulton (1955) who states "the rate of delayed yield must be so slow that only a small part of the total delayed drainage occurs during the test".

CONCLUSIONS

The analysis of the delayed yield models has indicated that fine-grained semipervious interbeds and/or fine-grained semipervious confining units are undoubtedly often capable of yielding appreciable quantities of water during a short term pump test. This influx of water to the aquifer can result in considerably smaller drawdowns at a given time than would occur if the Theis assumptions were applicable to the aquifer system. The character of the time drawdown curve obtained from an aquifer system affected by semipervious units is determined by several factors. These can be summarized as follows:

1) the presence of semipervious interbeds
   a) coefficient of consolidation and permeability of interbeds,
   b) number of interbeds,
   c) thickness of interbeds,
2) the presence of semipervious confining beds
   a) coefficient of consolidation and permeability of confining beds,

b) thickness of the semipervious confining beds,

c) the boundary conditions above and below the confining beds.

It is apparent that many different shapes of time drawdown curves may be obtained from aquifers having various combinations of the above factors. Such time drawdown curves could lead to serious misinterpretation of the character of the aquifer and its boundaries if the nature or presence of the semipervious interbeds or confining beds are not recognized or located during the geologic test drilling of the aquifer.

The character of the delayed yield units cannot be identified or determined from the time drawdown curves because a unique solution cannot be obtained for a particular combination of the factors listed above. Several different combinations of the factors can produce the same time response curves.

It has previously been mentioned that the time drawdown responses of the Boulton and consolidation models can yield curves identical to the Hantush-Walton leaky artesian curves.

It can also be shown that the effects of delayed yield as described by all three models can also yield drawdown curves identical to those produced by partial hydrologic barriers of the type analysed by Nind (1965). Walton (1966) has indicated that drawdown response curves controlled by induced infiltration from streams can closely resemble leaky artesian conditions and therefore may be similar to the delayed yield curves presented in this thesis.

A careful interpretation of delayed yield time drawdown curves is necessary to avoid an erroneous understanding of the boundary conditions affecting an aquifer system. Equilibrium conditions may be maintained in an aquifer for several days or weeks while the compressible layers consolidate under an applied load. The possibility that a delayed yield effect can easily be misinterpreted as a recharge effect during a short term pump test is a particularly serious one because of the irreversible nature of the consolidation process. Once consolidation of the compressible layer is complete delayed yield is no longer a source of recharge to the aquifer. Therefore the capacity of the aquifer may be grossly overestimated. The correct boundary conditions must be ascertained before determining the safe yield of an aquifer system.

In nonindurated confined aquifer systems long-term pump tests, the placement of piezometers in fine-grained boundary or inter-bed units and a thorough knowledge of the geologic setting are the only ways to avoid misinter-pretation of delayed yield effects. The similar effects produced by delayed yield and other boundary conditions on confined aquifer response during pump tests places severe limit-ations on the standard curve-fitting method of aquifer analysis.

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APPENDIX

Computer Program for Consolidation Model

The digital computer program used in this thesis was run on an IBM 360-65 computer at the University of Manitoba Computer Center. The program language is Fortran IV.

Computer times varied between four and ten minutes depending on the number of time increments required for equilibrium conditions to be reached. The program was limited by the amount of storage available in the computer and had to be restricted to a time period of two days for time increments of one minute or a period of four and one half hours for time increments of 0.1 minutes. Double precision variables have been used where necessary to eliminate round-off errors in series addition; otherwise real variables have been used in the program.

Unless the corrected drawdown for a time increment increases, the drawdown in the aquifer is assumed to remain constant and the difference in drawdown between the beginning and the end of the time increment is zero. A decrease in drawdown in the aquifer is not recorded. This provision was necessary to avoid errors introduced by large negative increments of drawdown caused by the oscillatory convergence of the digital program towards equilibrium conditions. Since drawdowns in an aquifer during most pump tests show either a steady increase or remain in equilibrium, neglecting the negative drawdowns should not produce appreciable errors.

Definition of Program Variables

N	Number of time increments.
DEL H(I), DEL S, S	Theis drawdown at the end of the i th increment (These variables are identical but were assigned different names to distinguish them in the main and subprograms).
CDEL H(I)	Theis drawdown at the end of the i th time increment corrected for delayed yield effects.
DIFF (I), CHANGE	Increase in corrected drawdown during a time increment.
DELTA (I) DDELTA (double precision)	Correction for delayed yield at the end of the i th time increment.

CORR DCORR (double precision)	Correction for delayed yield for a particular time increment.
COEFF CV (double precision)	Coefficient of consolidation for a compressible layer.
PERM	Vertical hydraulic conductivity of compressible layer.
AVER (I)	Average degree of consolidation of a compressible layer at the end of the i th time increment.
H	Thickness of the compressible layer.
STOR	Storage coefficient of aquifer.
TRANS	Transmissibility of aquifer.
Q	Pumping rate.
R	Distance from pumping well to observation well.

Terms Defined in Program

VALUE	k'/c_v
U	$\frac{2693\ r^2 S}{T\ t}$
WU	$-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!}$

CONST	$\frac{114.6\ Q}{T}$
C	$\frac{\pi^2 T}{4}$
T	$\frac{c_v t}{(H/2)^2}$
EXPON DEXPON (double precision)	$\frac{1}{x}\ e^{-Cx^2}$
SUM DSUM (double precision)	$\frac{e^{-(\pi^2 (2m+1)^2/4)T}}{(2m+1)^2}$
QTOTAL	$\frac{\gamma_w}{E_c}\ \Delta\ h\ b'$
QTIME	QTOTAL * AVER (I) Total amount of water yielded at the end of the i th increment of time per unit area of the compressible layer.

```
      DIMENSION TIME(2890),DEL H(2890),DIFF(2890),CDEL(2890),DELTA(2890)
      COMMON N, STOR, TRANS, Q, R, COEFF, H, AVER(2890), VALUE, I
      DOUBLE PRECISION PERM, CV, DDELTA, DCORR
      READ(5,20) PERM, CV
   20 FORMAT(2D15.0)
      VALUE = PERM/ CV
      COEFF = CV
      READ(5,12) H
      READ(5,12) R
   12 FORMAT(9F7.0)
      READ(5,14) STOR, TRANS, Q
   14 FORMAT(3F10.0)
      READ(5,50) N
   50 FORMAT(I5)
      I = 1
      TIME(I) = 0.1
      CALL CONE(TIME(I), DEL H(I))
   47 WRITE(6,60)
   60 FORMAT(1H1)
      WRITE(6,30) TIME(I), R, DEL H(I)
      NM1 = N - 1
      DO 48 I = 1,NM1
      CALL AVCON(TIME(I), DEL H(1), DELTA(I))
      TIME(I+1) = TIME(I) + 0.1
      CALL CONE(TIME(I+1), DEL H(I+1))
      IF(I-1) 100,43,100
  100 IM1 = I - 1
      DDELTA = DELTA(I)
      DO 42 K = 1,IM1
      IF(DIFF(K)) 44,42,44
   44 CORR = DIFF(K) * VALUE * H * AVER(I-K)/STOR
      DCORR = CORR
      DDELTA = DDELTA + DCORR
   42 CONTINUE
      DELTA(I) = DDELTA
      L = 0
  112 L = L + 1
      IF(DIFF(I-L)) 112, 112, 111
  111 IF(L-1) 113, 113, 114
  113 IF((DEL H(I+1) - DEL H(I)) - (DELTA(I) - DELTA(I-1))) 110,43,43
  110 DIFF(I) = 0.0
      CDEL(I) = DEL H(I+1) - (DELTA(I-1) + DEL H(I+1) - DEL H(I))
      GO TO 40
  114 LM1 = L-1
      IF((DEL H(I+1)-DEL H(I-LM1))-(DELTA(I) - DELTA(I-L))) 115,43,43
  115 DIFF(I) = 0.0
      CDEL(I) = DEL H(I+1)-(DELTA(I-L)+DEL H(I+1)-DEL H(I-LM1))
      GO TO 40
   43 CDEL(I) = DEL H(I+1) - DELTA(I)
      IF(I .EQ. 1) DIFF(1) = CDEL(I) - DEL H(I)
      IF(I-1) 101,40,101
  101 DIFF(I) = CDEL(I) - CDEL(I-1)
   40 WRITE(6,30) TIME(I+1),R,DEL H(I+1),DELTA(I),CDEL(I),DIFF(I),L
   30 FORMAT(2F7.1,4F15.6,I5)
   48 CONTINUE
      CALL EXIT
      END
```

```
      SUBROUTINE CONE(TIME1, S)
      COMMON N, STOR, TRANS, Q, R, COEFF, H, AVER(2890), VALUE, I
      U=(2693.0 * R * R * STOR)/(TRANS * TIME1)
      IF (9.0 - U) 1002, 1002, 1001
 1001 WU = -0.5772 - ALOG(U) + U
      TERM = U
      DO 10 M = 2,40
      DENOM = M
      TERM = -TERM*U/DENOM
      WU = WU + TERM/DENOM
      IF(ABS(TERM/DENOM) .LT. 1.0E-6) GO TO 11
   10 CONTINUE
   11 CONST = 114.6 * Q/TRANS
      S = CONST * WU
      GO TO 1000
 1002 S = 0.0
 1000 RETURN
      END


      SUBROUTINE AVCON(TIME2, CHANGE, DEL S)
      COMMON N, STOR, TRANS, Q, R, COEFF, H, AVER(2890), VALUE, I
      DOUBLE PRECISION DSUM, DEXPON
      T = COEFF * TIME2/(H/2.0 * H/2.0)
      C = 9.8696044 * T/4.0
      DSUM = 0.0
      X = 1.0
    3 EXPON = EXP(-X * X * C)/(X * X)
      IF(EXPON .LT. 0.00001) GO TO 2
      DEXPON = EXPON
      DSUM = DSUM + DEXPON
      X = X + 2.0
      IF(X .GT. 399.0) GO TO 2
      GO TO 3
    2 SUM = DSUM
      AVER(I) = 1.0 - 0.8105707 * SUM
      QTOTAL = CHANGE * VALUE * H
      QTIME = QTOTAL * AVER(I)
      DEL S = QTIME/STOR
   71 RETURN
      END
```