UNIVERSITY OF MANITOBA

SOLUTIONS FOR VOLUMETRIC AND DEVIATORIC COMPONENTS OF STRESS IN ELASTIC FOUNDATIONS

A Thesis
Presented to
The Faculty of Graduate Studies and Research
In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
DHANAPALA GODAGE
September 1968

ACKNOWLEDGEMENTS

The author wishes to express his gratitude to the following:

Professor A. Baracos, Department of Civil Engineering, for his suggestion of the topic of the investigation and under whose direction this study was undertaken;

Professor L. Domaschuk, Department of Civil Engineering, for his guidance during the investigation and whose encouragement stimulated the author's interest in the field of study;

Professor A. M. Lansdown, Head of the Department of Civil Engineering, for obtaining permission to use the computer;

The Canadian Commonwealth Scholarship and Fellowship Committee for awarding the scholarship, and the External Aid Office for the financial support and cooperation;

The author's parents for their support and encouragement, which shall be remembered at all times.

September 1968

University of Manitoba D. Godage

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOTATION

a	Length of rectangle
b	Breadth of rectangle, half-width of strip load
j,k	Variables related to strip load
m	Variable related to line load
m,n	Variables related to rectangular load
p,q	Intensity of load
s	Shear stress
$s_{1,2,3}$	Deviator stress components
$s_{xy,yz,zx}$	Shearing stresses on faces of elemental volume, first subscript indicates the direction perpendicular to plane on which stress acts, second gives the direction in which it acts (fig. 1)
x,y,z	Distances in coordinate directions
$A,B,..,H,J,K$	Functions related to rectangular load
I	Influence value for line load
L,M,N	Influence values for strip load
$\bar{s}_d$	Resultant deviatoric stress component
U	Pore pressure
V	Newtonian potential
X,Y,Z	Coordinate directions
α,β	Pore-pressure parameters
α	Angle
γ	Unit weight of soil
ϵ	Bulk strain

$\epsilon_{x,y,z}$	Normal strains in X,Y,Z directions
λ,μ	LAME's elastic constants
ν	Poisson's ratio
σ	Total normal stress
σ	Effective normal stress
$\sigma_{x,y,z}$	Normal stresses in X,Y,Z directions
σ_m	Mean normal stress
σ_c	Preconsolidation stress
$\sigma_{1,2,3}$	Principal stresses; major, intermediate and minor
χ	Boussinesq's logarithmic potential

SUMMARY

The results of extensive computations are presented to determine the complete stress pattern beneath a uniform rectangular load, an infinite line load and an infinite strip load on an elastic, homogeneous, isotropic half space.

The solution to the rectangular load is presented in tabular form consisting of ten functions. Simple formulae are given to combine these functions by which any component of stress may be determined for any value of Poisson's ratio. The solutions for the volumetric and deviatoric components of stress in the case of infinite line load and infinite strip load are presented in graphical form for Poisson's ratio of 0.5. In addition, tabulations of influence values are given when Poisson's ratio is other than 0.5.

An analysis of pore pressure observations obtained with respect to a loaded grain elevator was carried out based on a general formula which relates excess pore pressure to volumetric and deviatoric components of stress. The analysis consisted of computing two pore pressure parameters, one based on the change in volumetric component and the other based on the change in deviatoric component of stress. The values of these parameters were compared with a set obtained in a laboratory investigation. There was no close agreement between the two sets of parameters.

CHAPTER I

INTRODUCTION

The state of stress acting at a point in the interior of an elastic, homogeneous, isotropic solid can be defined by three normal stress components and three shear stress components, acting on mutually perpendicular planes. This system can be represented by two components, namely the volumetric component and the resultant deviatoric component. The common approach in many soil mechanics problems is that the effects of the intermediate principal stress are not included, and an obvious question arises concerning the effect which a variation of the value of the intermediate principal stress has on many soil mechanic problems like the mode and conditions of failure.

In conventional triaxial testing, the major and minor principal stresses alone are used, the cell pressure and the axial stress representing the minor and the major principal stresses respectively. The lack of ability to vary the intermediate principal stress independently, in tests, seems to be the biggest handicap to the study of effects of the intermediate principal stress.

A more complete study of stress conditions would require the determination of the complete stress pattern in the medium underlying the surface supporting the load. The purpose of this study was to develop influence charts for all stress components and present them in a manner that would permit easy determination of the complete stress pattern in the soil for such loading cases as uniformly distributed load over a rectangular area, uniform infinite line load and uniform infinite strip load.

For the case of a uniform load acting over a rectangular area, solutions for the stress components were obtained for points directly below the corner of the rectangle. The results are presented in terms of ten functions, in tabular form, which are used in conjunction with a set of simple formulae to determine stress components for any value of Poisson's ratio. Due to the non-symmetry of the area, it was not possible to develop influence charts for the volumetric component and the resultant deviatoric component of stress directly. However, these two stress components can be easily computed knowing all the stress components.

For the cases of infinite line load and infinite strip load, solutions are given in the form of stress influence factors to determine the volumetric and resultant deviatoric components of stress. The influence factors are presented in graphical form as well as in tabular form.

CHAPTER 2

THEORY

2.1 Basic Assumptions

It is usually assumed for purposes of analysis that the total stresses developed in a soil under load can be computed on the basis of the classical theory of elasticity.[3]

The stress distribution theory that has been developed as part of the mathematical theory of elasticity sets out to give an account of the stress and strain produced in an elastic solid by the pressure applied to part of its surface. The material of the solid is taken to be homogeneous and isotropic, and the solid is taken to be bounded by an infinite plane, and otherwise unlimited. As applied to soil mechanics, the solid can be taken to represent the soil, the pressed areas to represent the foundation areas of supporting structures like buildings, and the pressures to represent the weight per unit area of such structures supported on the soil.

The distribution of contact pressure on the base of foundations is unknown, but for simplicity the pressure is assumed to be uniform. The uniform load is an approximation to the footing reaction which is a function of the flexibility of the footing.

Because of the linear nature of the theory of elasticity employed, the stresses at a point due to a more complicated loading configuration can be obtained by superposition of stresses computed on the basis of rectangular component areas which, together, make up the actual area.

2.2 Volumetric and Deviatoric Components of Stress

The application of load at the surface of the soil results in development of stresses in the soil. The magnitude and distribution of these stresses must be known as they are related to the stress-deformation characteristics i.e. the deformation modulii of the soil.

In the conventional elastic theory, it is assumed that only normal stresses contribute to volumetric strain of an element. Thus, the volumetric strain of a volume element is proportional to the mean normal stress (σ_m) or the average of the three principal stresses. Hence, the mean normal stress is referred to as the equivalent hydrostatic component or the volumetric component of stress.

If the hydrostatic component is subtracted from each of the principal stresses in turn, the differences are termed the components of the deviator stress (s_1, s_2, s_3). The three components may be represented by their resultant (S_d), which Domaschuk[7] terms the resultant deviatoric component of stress. The two components can be expressed mathematically as:

$$\sigma_m = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$$

$$S_d = \sqrt{s_1^2 + s_2^2 + s_3^2}$$

where $s_1 = \sigma_1-\sigma_m$, $s_2 = \sigma_2-\sigma_m$ and $s_3 = \sigma_3-\sigma_m$.
The various components are shown in Figure 1.

The hydrostatic component is associated with the volume change while the resultant deviatoric component is associated with shearing or distortional deformation. In addition, there is a volume change during shearing, referred to as dilation or dilatancy.

2.3 Solution to Rectangular Loaded Area

A general solution to the problem of stresses produced in a semi-infinite elastic solid by pressure on part of the boundary was developed by J. Boussinesq in 1885, Love[1]. It is applicable to any form of boundary of the pressed area, and to any law of distribution of pressure over the area. The stresses are expressed in terms of the space derivatives of a certain function, due to what Boussinesq called "le potentiel logarithmique à trois variables", which is defined as a double integral taken over the pressed area. The difficulty in evaluating the integral has been an obstacle to the development of a numerical solution in some special cases.

The application of the general solution to the important case of a rectangular area under uniform pressure was reported by Love[1]. In terms of the Boussinesq's logarithmic potential χ, and the Newtonian potential V, the stress components are:

$$\sigma_x = \frac{1}{2\pi}\left| \frac{\lambda}{\lambda+\mu}\frac{\partial V}{\partial Z} - \frac{\mu}{\lambda+\mu}\frac{\partial^2\chi}{\partial x^2} - Z\frac{\partial^2 V}{\partial x^2} \right|$$

$$\sigma_y = \frac{1}{2\pi}\left| \frac{\lambda}{\lambda+\mu}\frac{\partial V}{\partial Z} - \frac{\mu}{\lambda+\mu}\frac{\partial^2\chi}{\partial y^2} - Z\frac{\partial^2 V}{\partial y^2} \right|$$

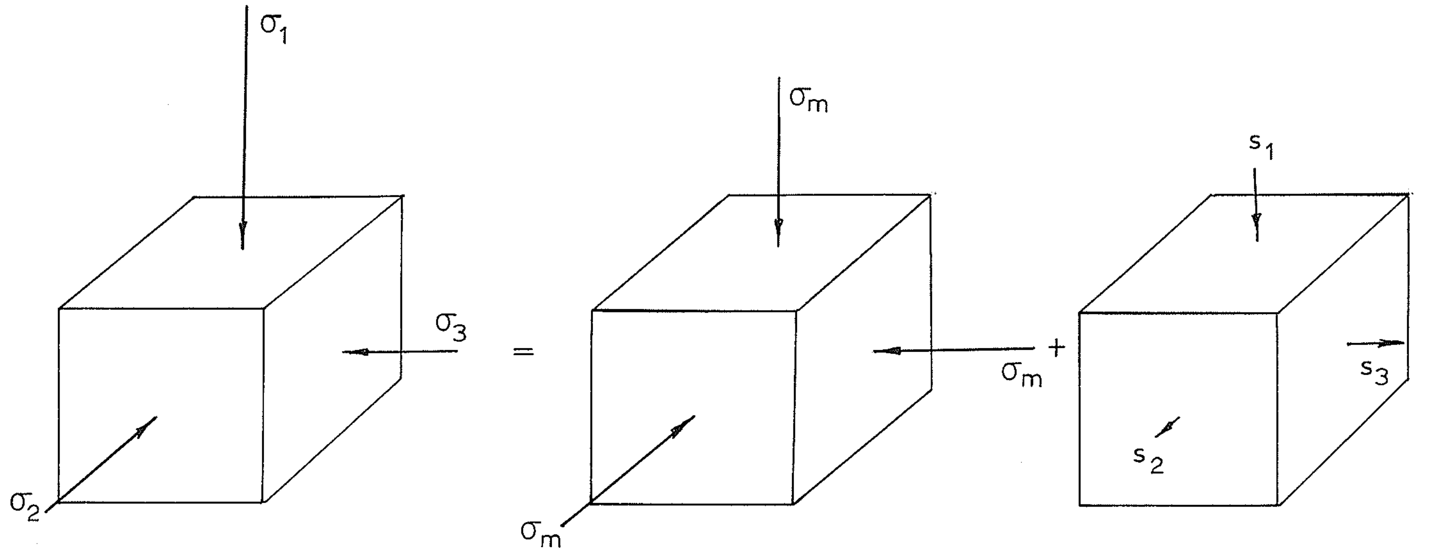
$$\sigma_z = \frac{1}{2\pi}\left| \frac{\partial V}{\partial Z} - Z\frac{\partial^2 V}{\partial Z^2} \right| \tag{1}$$

$$S_{yz} = -\frac{1}{2\pi} Z\frac{\partial^2 V}{\partial y\partial z}, \qquad S_{zx} = -\frac{1}{2\pi} Z\frac{\partial^2 V}{\partial z\partial x}$$

$$S_{xy} = -\frac{1}{2\pi}\left| \frac{\mu}{\lambda+\mu}\frac{\partial^2\chi}{\partial x\partial y} + Z\frac{\partial^2 V}{\partial x\partial y} \right|$$

where λ and μ are Lame's elastic constants.

The derivatives of χ and V appearing in the expressions for stresses have been evaluated for a general point (x,y,z) in the solid with respect to a right-handed set of cartesian coordinates, the positive sense of z being downwards, as indicated in Figure 2. The expressions for these

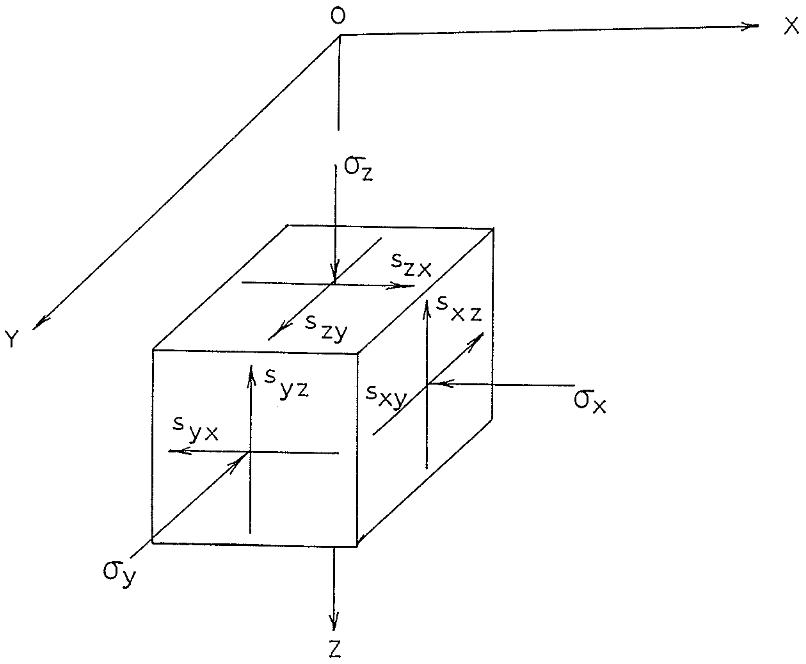


(1) Principal Stress System

(2) Volumetric Stress Component

(3) Deviator Stress Components

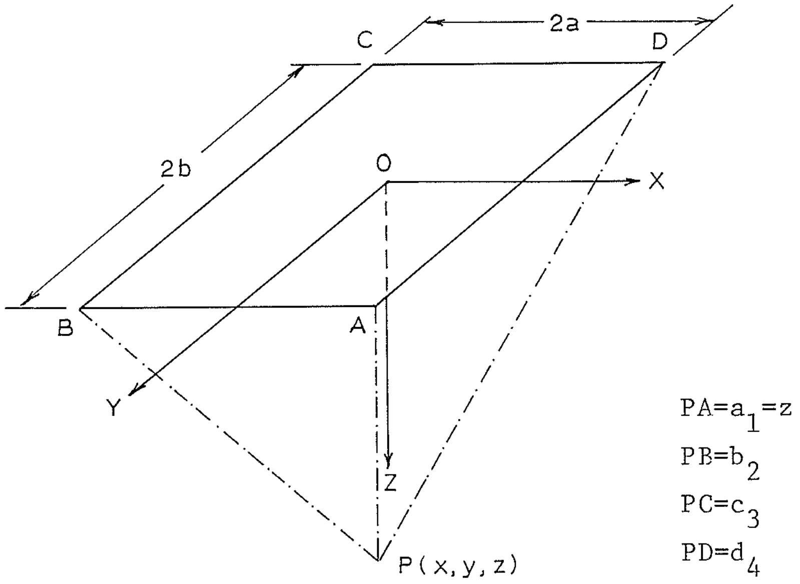
Fig. 1 Nomenclature for Stress System



All stresses are positive as shown above.

Fig. 2 Sign Convention and Notation for Axes and Stresses derivatives were obtained by Love[1], and they are given in the Appendix E. These expressions were applied to a point P (Figure 3) beneath the corner A of the rectangle to obtain the following equations (2):

$$
\begin{gathered}
\frac{\partial^2 \chi}{\partial x^2} = - \left| \tan^{-1}\frac{b}{a} - \tan^{-1}\frac{z}{c_3}\cdot\frac{b}{a} \right| \\
\frac{\partial^2 \chi}{\partial y^2} = - \left| \tan^{-1}\frac{a}{b} - \tan^{-1}\frac{z}{c_3}\cdot\frac{a}{b} \right| \\
\frac{\partial V}{\partial z} = \sin^{-1}\frac{4ab}{\sqrt{4a^2+z^2}\cdot\sqrt{4b^2+z^2}} \\
\frac{\partial^2 \chi}{\partial x \partial y} = - \log\frac{2z\cdot(z+c_3)}{(z+b_2)\cdot(z+d_4)} \\
\frac{\partial^2 V}{\partial x^2} = \frac{4ab}{c_3(4a^2+z^2)} \\
\frac{\partial^2 V}{\partial y^2} = \frac{4ab}{c_3(4b^2+z^2)} \\
\frac{\partial^2 V}{\partial z^2} = - \left| \frac{4ab}{c_3(4a^2+z^2)} + \frac{4ab}{c_3(4b^2+z^2)} \right| \\
\frac{\partial^2 V}{\partial x \partial z} = - \left| \frac{2b}{z\cdot d_4} - \frac{2bz}{c_3(4a^2+z^2)} \right| \\
\frac{\partial^2 V}{\partial y \partial z} = - \left| \frac{2a}{z\cdot b_2} - \frac{2az}{c_3(4b^2+z^2)} \right| \\
\frac{\partial^2 V}{\partial x \partial y} = - \left| \frac{1}{z} - \frac{1}{b_2} + \frac{1}{c_3} - \frac{1}{d_4} \right|
\end{gathered}
\tag{2}
$$



$PA = a_1 = z$

$PB = b_2$

$PC = c_3$

$PD = d_4$

Fig. 3 Notation for Uniformly Loaded Rectangular Area

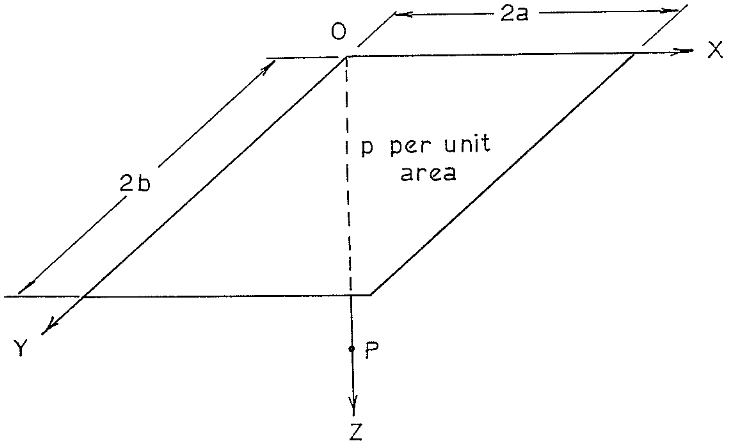


Fig. 4 Position of Point P with Respect to the Rectangular Area

where:

$$b_2 = \sqrt{z^2 + 4a^2}$$

$$c_3 = \sqrt{4a^2 + 4b^2 + z^2}$$

$$d_4 = \sqrt{z^2 + 4b^2}$$

If the rectangular area of length '2a', and breadth '2b' was situated in the positive quadrant of the xy-plane with one corner at the origin as in Figure 4, then the equations for stress components at the origin for any depth z are given by equations (1), but with sign of 's_{xz}' and 's_{yz}' changed.

It can be shown using equations (1) that the mean normal stress (σ_m) has the following relationship:

$$\sigma_m = \frac{1 + \nu}{\pi} \cdot \frac{\partial v}{\partial z} \tag{3}$$

However, it is not possible to obtain a simple equation for the resultant deviatoric component even though it can be stated in terms of the six components of stress defined above. Therefore, the resultant deviatoric component can be computed from the stresses obtained using the infuence charts for stresses, without first determining the principal stresses, using the following equation:

$$s_d = \sqrt{2/3 \cdot (\sigma_x + \sigma_y + \sigma_z)^2 - 2(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - s_{xy}^2 - s_{yz}^2 - s_{xz}^2)} \tag{4}$$

2.4 Line Load of Infinite Length

A load intensity q per unit length of a straight line of infinite extent acting on the surface of the solid produces a state of plane strain. The stresses at any point P (x,y,z) as in Figure 5 are given by Gray[5] as:

$$\begin{aligned} \sigma_x &= \frac{2q}{\pi} \cdot \frac{x^2 z}{R^4} \\ \sigma_y &= \frac{2q\nu}{\pi} \cdot \frac{z}{R^2} \\ \sigma_z &= \frac{2q}{\pi} \cdot \frac{z^3}{R^4} \\ s_{xz} &= \frac{2q}{\pi} \cdot \frac{xz^2}{R^4} \end{aligned} \qquad (5)$$

where,

$$R^2 = x^2 + z^2$$

Using these expressions to derive the mean normal stress (σ_m) and the resultant deviatoric component (S_d), the following expressions are obtained:

$$\sigma_m = \frac{2q}{3\pi} \frac{z}{R^2} (1 + \nu) \qquad (6)$$

$$s_d = \frac{2q}{\pi} \frac{z}{R^2} \sqrt{2/3.(1 - \nu + \nu^2)} \qquad (7)$$

2.5 Uniformly Loaded Infinite Strip

The stresses at any point P (x,y,z) as in Figure 6 due to a uniformly distributed load q per unit area over a strip of constant width 2b and of infinite length have been given by Gray[5], and are represented by:

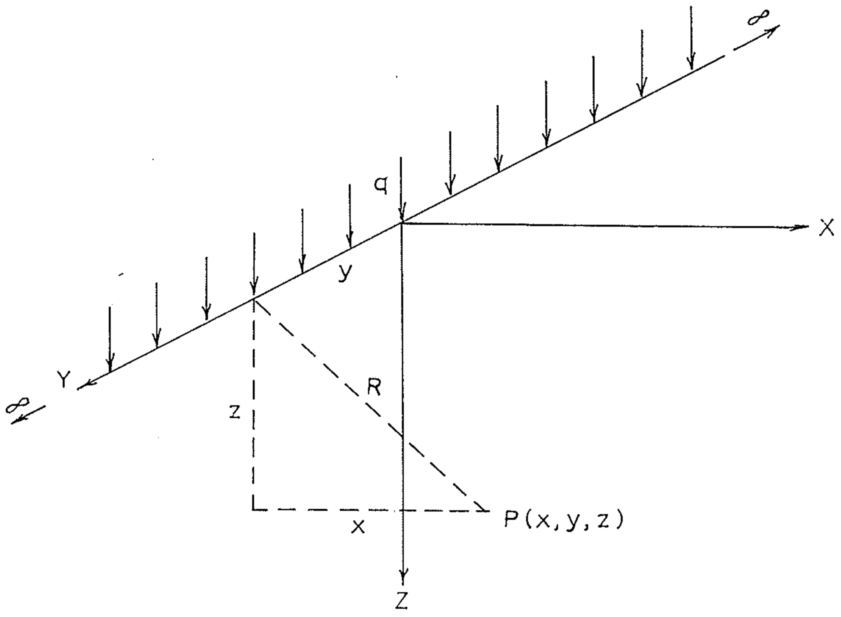


Fig. 5 Problem of Infinite Line Load

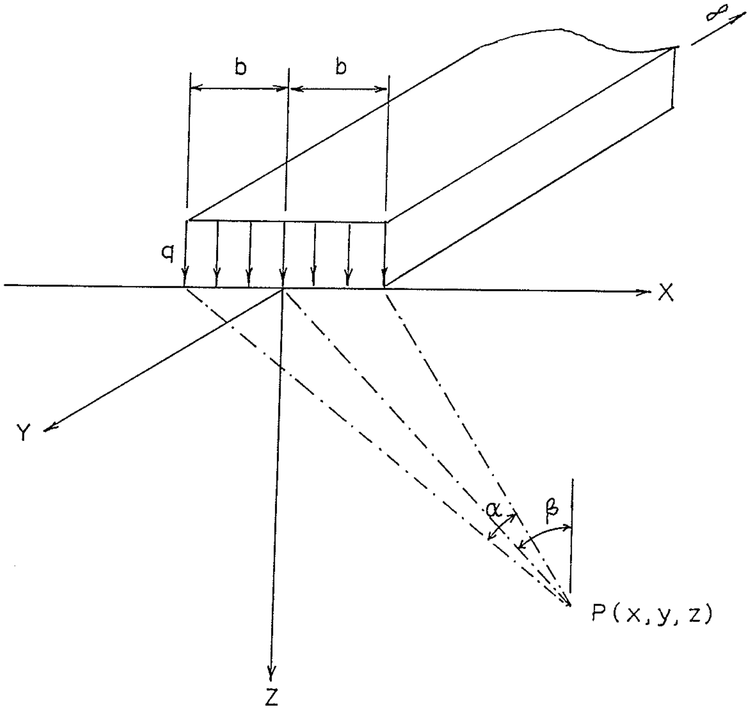


Fig. 6 Infinitely Long Strip Loading

$$\sigma_x = \frac{q}{\pi} \left| \alpha - \sin \alpha \cos 2 \beta \right|$$

$$\sigma_z = \frac{q}{\pi} \left| \alpha + \sin \alpha \cos 2 \beta \right|$$

$$\sigma_y = \frac{2q}{\pi} \nu \alpha \tag{8}$$

$$s_{xz} = \frac{q}{\pi} \sin \alpha \sin 2 \beta$$

The quantities in the equations (8) refer to Figure 6.

The mean normal stress (σ_m) and the resultant deviatoric component of stress (S_d) are derived using equations (8) as:

$$\sigma_m = \frac{2q}{3\pi} (1 + \nu) \alpha \tag{9}$$

$$S_d = \frac{q}{\pi} \sqrt{2/3.\alpha^2 (1 - 2\nu)^2 + 2 \sin^2 \alpha} \tag{10}$$

2.6 Total Stresses

In the preceding derivations the unit weight (γ) of the material has been neglected. Therefore, the computations furnish only the stresses produced by surface loads. In order to get the total stresses in the material with unit weight (γ), the stresses due to surface loads have to be combined with those produced by the weight of the supporting material. The stresses due to body forces are:

$$\sigma_z = z \gamma$$

$$\sigma_x = \sigma_y = K_o z \gamma \tag{11}$$

where, K_o = coefficient of earth pressure at rest.

Thus, the expression for total mean normal stress (σ_m) becomes:

$$(\sigma_m)_{\text{total}} = (\sigma_m)_{\text{surface}} + (\sigma_m)_{\text{body}}$$

and the total resultant deviatoric component (S_d) is:

$$(S_d)_{\text{total}} = (S_d)_{\text{surface}} + (S_d)_{\text{body}} .$$

CHAPTER 3

NUMERICAL SOLUTIONS FOR VOLUMETRIC AND DEVIATORIC COMPONENTS OF STRESS

The purpose of this study was to develop influence charts for the determination of stresses under such cases of loading as a uniform load over a rectangular area, an infinite line load and an infinite strip load. The mathematical solutions to these problems were obtained in the previous chapter.

The first step in the numerical approach is the conversion of the equations given in Chapter 2 into a more systematic form.

3.1.1 Development of Equations for Rectangular Load

The integral terms appearing in equations (1) are substituted by the expressions (2). In equations (1), certain functions repeat themselves and are independent of Poisson's ratio. These functions were separated and named as "Functions A, B, ..., H, J and K".

For the sake of convenience in computation, the three variables a, b, z appearing in equations (2) are reduced to two, namely m and n, by expressing a and b as functions of z . Using the substitution,

$2a = mz$ and

$2b = nz$,

the functions A, B etc. can be expressed in terms of m and n as follows:

$$
\begin{aligned}
A &= \frac{1}{2\pi} \cdot \sin^{-1} \frac{mn}{\sqrt{m^2 + 1}\,\sqrt{n^2 + 1}} \\
B &= \frac{1}{2\pi} \cdot \frac{mn}{\sqrt{m^2 + n^2 + 1}} \cdot \left(\frac{1}{m^2 + 1} + \frac{1}{n^2 + 1} \right) \\
C &= \frac{1}{2\pi} \left| \tan^{-1} \frac{n}{m} - \tan^{-1} \left(\frac{n}{m} \cdot \frac{1}{\sqrt{m^2 + n^2 + 1}} \right) \right| \\
D &= \frac{1}{2\pi} \cdot \frac{mn}{(m^2 + 1)\sqrt{m^2 + n^2 + 1}} \\
E &= \frac{1}{2\pi} \left| \tan^{-1} \frac{m}{n} - \tan^{-1} \left(\frac{m}{n} \cdot \frac{1}{\sqrt{m^2 + n^2 + 1}} \right) \right| \\
F &= \frac{1}{2\pi} \cdot \frac{mn}{(n^2 + 1)\sqrt{m^2 + n^2 + 1}} \\
G &= \frac{1}{2\pi} \left| \frac{m}{\sqrt{m^2 + 1}} - \frac{m}{(n^2 + 1)\sqrt{m^2 + n^2 + 1}} \right| \\
H &= \frac{1}{2\pi} \left| \frac{n}{\sqrt{n^2 + 1}} - \frac{n}{(m^2 + 1)\sqrt{m^2 + n^2 + 1}} \right| \\
J &= \frac{1}{2\pi} \left| 1 - \frac{1}{\sqrt{m^2 + 1}} - \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{m^2 + n^2 + 1}} \right| \\
K &= \frac{1}{2\pi} \cdot \log_e \frac{2(1 + \sqrt{m^2 + n^2 + 1})}{(1 + \sqrt{m^2 + 1})(1 + \sqrt{n^2 + 1})}
\end{aligned}
\tag{12}
$$

These ten functions were evaluated for a range of values of m and n, and were tabulated in Tables 1 through 10. The computations were made using an IBM 360 computer.

The following simple formulae combine these tabulated functions and Poisson's ratio to give the components of stress.

$$\sigma_z = p(A + B) \tag{13}$$

$$\sigma_x = p\left|2\nu A + (1 - 2\nu)C - D\right| \tag{14}$$

Table 1 .Function "A"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00158	0.00311	0.00455	0.00588	0.00708	0.00815	0.00909	0.00990	0.01060	0.01121	0.01218	0.01290
0.2	0.00311	0.00612	0.00897	0.01160	0.01398	0.01609	0.01794	0.01955	0.02094	0.02214	0.02407	0.02551
0.3	0.00455	0.00897	0.01316	0.01702	0.02051	0.02362	0.02635	0.02872	0.03079	0.03256	0.03542	0.03756
0.4	0.00588	0.01160	0.01702	0.02202	0.02656	0.03060	0.03416	0.03726	0.03996	0.04229	0.04605	0.04886
0.5	0.00708	0.01398	0.02051	0.02656	0.03205	0.03695	0.04128	0.04506	0.04835	0.05121	0.05582	0.05928
0.6	0.00815	0.01609	0.02362	0.03060	0.03695	0.04264	0.04767	0.05208	0.05592	0.05926	0.06467	0.06875
0.7	0.00909	0.01794	0.02635	0.03416	0.04128	0.04767	0.05333	0.05831	0.06266	0.06645	0.07261	0.07727
0.8	0.00990	0.01955	0.02872	0.03726	0.04506	0.05208	0.05831	0.06380	0.06862	0.07282	0.07966	0.08487
0.9	0.01060	0.02094	0.03079	0.03996	0.04835	0.05592	0.06266	0.06862	0.07385	0.07842	0.08590	0.09161
1.0	0.01121	0.02214	0.03256	0.04229	0.05121	0.05926	0.06645	0.07282	0.07842	0.08333	0.09140	0.09758
1.2	0.01218	0.02407	0.03542	0.04605	0.05582	0.06467	0.07261	0.07966	0.08590	0.09140	0.10047	0.10748
1.4	0.01290	0.02551	0.03756	0.04886	0.05928	0.06875	0.07727	0.08487	0.09161	0.09758	0.10748	0.11518
1.6	0.01345	0.02659	0.03918	0.05099	0.06191	0.07185	0.08083	0.08886	0.09600	0.10234	0.11292	0.12120
1.8	0.01386	0.02742	0.04041	0.05262	0.06392	0.07424	0.08357	0.09194	0.09941	0.10605	0.11718	0.12595
2.0	0.01418	0.02806	0.04137	0.05389	0.06549	0.07611	0.08572	0.09436	0.10209	0.10898	0.12056	0.12973
2.5	0.01472	0.02914	0.04298	0.05603	0.06815	0.07926	0.08936	0.09848	0.10666	0.11399	0.12639	0.13631
3.0	0.01505	0.02978	0.04394	0.05731	0.06973	0.08115	0.09155	0.10096	0.10943	0.11703	0.12996	0.14037
4.0	0.01539	0.03047	0.04496	0.05866	0.07142	0.08317	0.09390	0.10362	0.11240	0.12032	0.13384	0.14481
5.0	0.01555	0.03080	0.04546	0.05933	0.07225	0.08416	0.09505	0.10493	0.11387	0.12194	0.13577	0.14704
6.0	0.01565	0.03098	0.04574	0.05969	0.07271	0.08471	0.09569	0.10566	0.11469	0.12285	0.13686	0.14829
8.0	0.01574	0.03117	0.04602	0.06007	0.07318	0.08527	0.09634	0.10641	0.11553	0.12378	0.13796	0.14958
10.0	0.01578	0.03126	0.04615	0.06024	0.07340	0.08554	0.09665	0.10676	0.11592	0.12421	0.13848	0.15018
50.0	0.01586	0.03141	0.04638	0.06055	0.07378	0.08599	0.09718	0.10736	0.11660	0.12497	0.13939	0.15124
100.0	0.01586	0.03141	0.04638	0.06056	0.07379	0.08601	0.09719	0.10738	0.11662	0.12499	0.13942	0.15127

Table 1 .Function "A"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.01345	0.01386	0.01418	0.01472	0.01505	0.01539	0.01555	0.01565	0.01574	0.01578	0.01586	0.01586
0.2	0.02659	0.02742	0.02806	0.02914	0.02978	0.03047	0.03080	0.03098	0.03117	0.03126	0.03141	0.03141
0.3	0.03918	0.04041	0.04137	0.04298	0.04394	0.04496	0.04546	0.04574	0.04602	0.04615	0.04638	0.04638
0.4	0.05099	0.05262	0.05389	0.05603	0.05731	0.05866	0.05933	0.05969	0.06007	0.06024	0.06055	0.06056
0.5	0.06191	0.06392	0.06549	0.06815	0.06973	0.07142	0.07225	0.07271	0.07318	0.07340	0.07378	0.07379
0.6	0.07185	0.07424	0.07611	0.07926	0.08115	0.08317	0.08416	0.08471	0.08527	0.08554	0.08599	0.08601
0.7	0.08083	0.08357	0.08572	0.08936	0.09155	0.09390	0.09505	0.09569	0.09634	0.09665	0.09718	0.09719
0.8	0.08886	0.09194	0.09436	0.09848	0.10096	0.10362	0.10493	0.10566	0.10641	0.10676	0.10736	0.10738
0.9	0.09600	0.09941	0.10209	0.10666	0.10943	0.11240	0.11387	0.11469	0.11553	0.11592	0.11660	0.11662
1.0	0.10234	0.10605	0.10898	0.11399	0.11703	0.12032	0.12194	0.12285	0.12378	0.12421	0.12497	0.12499
1.2	0.11292	0.11718	0.12056	0.12639	0.12996	0.13384	0.13577	0.13686	0.13796	0.13848	0.13939	0.13942
1.4	0.12120	0.12595	0.12973	0.13631	0.14037	0.14481	0.14704	0.14829	0.14958	0.15018	0.15124	0.15127
1.6	0.12772	0.13289	0.13703	0.14427	0.14878	0.15376	0.15627	0.15769	0.15915	0.15984	0.16105	0.16108
1.8	0.13289	0.13842	0.14287	0.15071	0.15563	0.16111	0.16389	0.16548	0.16711	0.16788	0.16924	0.16928
2.0	0.13703	0.14287	0.14758	0.15596	0.16126	0.16721	0.17025	0.17199	0.17379	0.17464	0.17614	0.17619
2.5	0.14427	0.15071	0.15596	0.16542	0.17151	0.17849	0.18213	0.18424	0.18644	0.18750	0.18936	0.18942
3.0	0.14878	0.15563	0.16126	0.17151	0.17822	0.18605	0.19021	0.19265	0.19522	0.19647	0.19870	0.19877
4.0	0.15376	0.16111	0.16721	0.17849	0.18605	0.19514	0.20013	0.20313	0.20637	0.20797	0.21088	0.21098
5.0	0.15627	0.16389	0.17025	0.18213	0.19021	0.20013	0.20572	0.20915	0.21294	0.21485	0.21842	0.21854
6.0	0.15769	0.16548	0.17199	0.18424	0.19265	0.20313	0.20915	0.21291	0.21715	0.21934	0.22353	0.22367
8.0	0.15915	0.16711	0.17379	0.18644	0.19522	0.20637	0.21294	0.21715	0.22205	0.22466	0.22996	0.23015
10.0	0.15984	0.16788	0.17464	0.18750	0.19647	0.20797	0.21485	0.21934	0.22466	0.22759	0.23382	0.23406
50.0	0.16105	0.16924	0.17614	0.18936	0.19870	0.21088	0.21842	0.22353	0.22996	0.23382	0.24550	0.24644
100.0	0.16108	0.16928	0.17619	0.18942	0.19877	0.21098	0.21854	0.22367	0.23015	0.23406	0.24644	0.24776

Table 2. Function "B"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00312	0.00606	0.00868	0.01090	0.01269	0.01408	0.01511	0.01586	0.01638	0.01673	0.01708	0.01717
0.2	0.00606	0.01178	0.01688	0.02120	0.02468	0.02739	0.02941	0.03087	0.03189	0.03257	0.03326	0.03343
0.3	0.00868	0.01688	0.02419	0.03040	0.03542	0.03932	0.04224	0.04435	0.04582	0.04681	0.04781	0.04805
0.4	0.01090	0.02120	0.03040	0.03821	0.04455	0.04949	0.05319	0.05587	0.05774	0.05900	0.06026	0.06055
0.5	0.01269	0.02468	0.03542	0.04455	0.05198	0.05777	0.06213	0.06528	0.06749	0.06897	0.07044	0.07075
0.6	0.01408	0.02739	0.03932	0.04949	0.05777	0.06425	0.06912	0.07266	0.07513	0.07679	0.07842	0.07874
0.7	0.01511	0.02941	0.04224	0.05319	0.06213	0.06912	0.07439	0.07822	0.08090	0.08269	0.08443	0.08473
0.8	0.01586	0.03087	0.04435	0.05587	0.06528	0.07266	0.07822	0.08227	0.08509	0.08696	0.08877	0.08902
0.9	0.01638	0.03189	0.04582	0.05774	0.06749	0.07513	0.08090	0.08509	0.08800	0.08993	0.09175	0.09195
1.0	0.01673	0.03257	0.04681	0.05900	0.06897	0.07679	0.08269	0.08696	0.08993	0.09189	0.09369	0.09381
1.2	0.01708	0.03326	0.04781	0.06026	0.07044	0.07842	0.08443	0.08877	0.09175	0.09369	0.09537	0.09530
1.4	0.01717	0.03343	0.04805	0.06055	0.07075	0.07874	0.08473	0.08902	0.09195	0.09381	0.09530	0.09502
1.6	0.01713	0.03335	0.04792	0.06036	0.07051	0.07842	0.08433	0.08854	0.09137	0.09312	0.09439	0.09389
1.8	0.01704	0.03316	0.04763	0.05998	0.07002	0.07783	0.08363	0.08773	0.09045	0.09209	0.09314	0.09241
2.0	0.01692	0.03294	0.04730	0.05953	0.06946	0.07716	0.08284	0.08683	0.08943	0.09096	0.09179	0.09085
2.5	0.01666	0.03240	0.04650	0.05847	0.06814	0.07557	0.08100	0.08473	0.08709	0.08837	0.08872	0.08733
3.0	0.01645	0.03199	0.04588	0.05764	0.06711	0.07435	0.07958	0.08311	0.08528	0.08638	0.08637	0.08463
4.0	0.01619	0.03147	0.04510	0.05661	0.06582	0.07280	0.07778	0.08107	0.08300	0.08385	0.08338	0.08119
5.0	0.01605	0.03119	0.04468	0.05604	0.06511	0.07196	0.07680	0.07995	0.08174	0.08246	0.08172	0.07928
6.0	0.01597	0.03102	0.04443	0.05571	0.06470	0.07146	0.07622	0.07929	0.08100	0.08164	0.08074	0.07815
8.0	0.01588	0.03085	0.04416	0.05536	0.06426	0.07093	0.07561	0.07859	0.08022	0.08077	0.07971	0.07694
10.0	0.01584	0.03076	0.04404	0.05519	0.06405	0.07068	0.07531	0.07826	0.07984	0.08035	0.07920	0.07636
50.0	0.01576	0.03061	0.04381	0.05489	0.06368	0.07023	0.07479	0.07766	0.07917	0.07961	0.07831	0.07532
100.0	0.01576	0.03061	0.04381	0.05488	0.06367	0.07022	0.07478	0.07764	0.07915	0.07959	0.07828	0.07529

Table 2. Function "B"-continued

M/N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.01713	0.01704	0.01692	0.01666	0.01645	0.01619	0.01605	0.01597	0.01588	0.01584	0.01576	0.01576
0.2	0.03335	0.03316	0.03294	0.03240	0.03199	0.03147	0.03119	0.03102	0.03085	0.03076	0.03061	0.03061
0.3	0.04792	0.04763	0.04730	0.04650	0.04588	0.04510	0.04468	0.04443	0.04416	0.04404	0.04381	0.04381
0.4	0.06036	0.05998	0.05953	0.05847	0.05764	0.05661	0.05604	0.05571	0.05536	0.05519	0.05489	0.05488
0.5	0.07051	0.07002	0.06946	0.06814	0.06711	0.06582	0.06511	0.06470	0.06426	0.06405	0.06368	0.06367
0.6	0.07842	0.07783	0.07716	0.07557	0.07435	0.07280	0.07196	0.07146	0.07093	0.07068	0.07023	0.07022
0.7	0.08433	0.08363	0.08284	0.08100	0.07958	0.07778	0.07680	0.07622	0.07561	0.07531	0.07479	0.07478
0.8	0.08854	0.08773	0.08683	0.08473	0.08311	0.08107	0.07995	0.07929	0.07859	0.07826	0.07766	0.07764
0.9	0.09137	0.09045	0.08943	0.08709	0.08528	0.08300	0.08174	0.08100	0.08022	0.07984	0.07917	0.07915
1.0	0.09312	0.09209	0.09096	0.08837	0.08638	0.08385	0.08246	0.08164	0.08077	0.08035	0.07961	0.07959
1.2	0.09439	0.09314	0.09179	0.08872	0.08637	0.08338	0.08172	0.08074	0.07971	0.07920	0.07831	0.07828
1.4	0.09389	0.09241	0.09085	0.08733	0.08463	0.08119	0.07928	0.07815	0.07694	0.07636	0.07532	0.07529
1.6	0.09253	0.09083	0.08908	0.08513	0.08211	0.07824	0.07609	0.07480	0.07343	0.07277	0.07158	0.07154
1.8	0.09083	0.08894	0.08700	0.08265	0.07933	0.07506	0.07266	0.07123	0.06970	0.06896	0.06762	0.06758
2.0	0.08908	0.08700	0.08488	0.08018	0.07656	0.07191	0.06929	0.06772	0.06603	0.06520	0.06373	0.06368
2.5	0.08513	0.08265	0.08018	0.07468	0.07045	0.06494	0.06179	0.05988	0.05782	0.05680	0.05496	0.05490
3.0	0.08211	0.07933	0.07656	0.07045	0.06572	0.05949	0.05587	0.05366	0.05123	0.05003	0.04784	0.04777
4.0	0.07824	0.07506	0.07191	0.06494	0.05949	0.05215	0.04778	0.04504	0.04199	0.04045	0.03758	0.03748
5.0	0.07609	0.07266	0.06929	0.06179	0.05587	0.04778	0.04286	0.03971	0.03613	0.03429	0.03077	0.03065
6.0	0.07480	0.07123	0.06772	0.05988	0.05366	0.04504	0.03971	0.03625	0.03224	0.03013	0.02600	0.02586
8.0	0.07343	0.06970	0.06603	0.05782	0.05123	0.04199	0.03613	0.03224	0.02759	0.02506	0.01984	0.01965
10.0	0.07277	0.06896	0.06520	0.05680	0.05003	0.04045	0.03429	0.03013	0.02506	0.02223	0.01607	0.01584
50.0	0.07158	0.06762	0.06373	0.05496	0.04784	0.03758	0.03077	0.02600	0.01984	0.01607	0.00450	0.00356
100.0	0.07154	0.06758	0.06368	0.05490	0.04777	0.03748	0.03065	0.02586	0.01965	0.01584	0.00356	0.00225

Table 3. Function "C"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00079	0.00156	0.00232	0.00304	0.00373	0.00438	0.00498	0.00555	0.00607	0.00655	0.00741	0.00814
0.2	0.00154	0.00306	0.00454	0.00596	0.00731	0.00858	0.00977	0.01088	0.01191	0.01287	0.01457	0.01602
0.3	0.00223	0.00443	0.00658	0.00864	0.01061	0.01246	0.01420	0.01583	0.01734	0.01874	0.02125	0.02340
0.4	0.00284	0.00564	0.00838	0.01101	0.01353	0.01591	0.01815	0.02025	0.02220	0.02402	0.02728	0.03009
0.5	0.00335	0.00667	0.00990	0.01303	0.01602	0.01887	0.02155	0.02407	0.02643	0.02862	0.03258	0.03600
0.6	0.00377	0.00750	0.01115	0.01469	0.01808	0.02132	0.02438	0.02727	0.02998	0.03251	0.03709	0.04108
0.7	0.00410	0.00816	0.01214	0.01601	0.01973	0.02329	0.02667	0.02986	0.03287	0.03570	0.04084	0.04533
0.8	0.00435	0.00866	0.01290	0.01702	0.02099	0.02481	0.02845	0.03190	0.03517	0.03825	0.04386	0.04880
0.9	0.00453	0.00903	0.01345	0.01776	0.02193	0.02595	0.02979	0.03345	0.03692	0.04021	0.04623	0.05157
1.0	0.00466	0.00927	0.01382	0.01827	0.02258	0.02675	0.03075	0.03457	0.03821	0.04167	0.04803	0.05371
1.2	0.00477	0.00950	0.01418	0.01876	0.02324	0.02758	0.03177	0.03581	0.03967	0.04336	0.05023	0.05643
1.4	0.00476	0.00949	0.01417	0.01877	0.02328	0.02767	0.03194	0.03607	0.04004	0.04387	0.05104	0.05759
1.6	0.00467	0.00932	0.01393	0.01847	0.02294	0.02731	0.03157	0.03571	0.03971	0.04359	0.05091	0.05766
1.8	0.00454	0.00907	0.01356	0.01800	0.02238	0.02667	0.03087	0.03496	0.03895	0.04281	0.05017	0.05702
2.0	0.00440	0.00878	0.01313	0.01744	0.02169	0.02588	0.02998	0.03400	0.03792	0.04174	0.04906	0.05592
2.5	0.00400	0.00799	0.01196	0.01591	0.01981	0.02368	0.02749	0.03124	0.03492	0.03854	0.04554	0.05221
3.0	0.00363	0.00725	0.01085	0.01444	0.01801	0.02154	0.02504	0.02849	0.03190	0.03527	0.04182	0.04814
4.0	0.00301	0.00602	0.00903	0.01202	0.01500	0.01797	0.02092	0.02384	0.02675	0.02962	0.03529	0.04082
5.0	0.00256	0.00512	0.00767	0.01022	0.01276	0.01529	0.01781	0.02032	0.02281	0.02529	0.03020	0.03503
6.0	0.00222	0.00443	0.00664	0.00885	0.01106	0.01326	0.01545	0.01764	0.01981	0.02198	0.02628	0.03054
8.0	0.00174	0.00348	0.00523	0.00697	0.00870	0.01044	0.01217	0.01390	0.01562	0.01734	0.02077	0.02417
10.0	0.00143	0.00287	0.00430	0.00573	0.00716	0.00859	0.01002	0.01144	0.01287	0.01429	0.01712	0.01994
50.0	0.00031	0.00062	0.00094	0.00125	0.00156	0.00187	0.00218	0.00250	0.00281	0.00312	0.00374	0.00437
100.0	0.00016	0.00032	0.00047	0.00063	0.00079	0.00095	0.00110	0.00126	0.00142	0.00158	0.00189	0.00221

Table 3. Function "C"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.00877	0.00932	0.00979	0.01072	0.01142	0.01237	0.01300	0.01343	0.01400	0.01435	0.01555	0.01570
0.2	0.01727	0.01835	0.01928	0.02115	0.02254	0.02444	0.02568	0.02655	0.02769	0.02839	0.03079	0.03110
0.3	0.02525	0.02685	0.02824	0.03102	0.03309	0.03594	0.03779	0.03909	0.04079	0.04185	0.04544	0.04591
0.4	0.03252	0.03462	0.03645	0.04012	0.04286	0.04664	0.04911	0.05084	0.05310	0.05451	0.05930	0.05993
0.5	0.03897	0.04155	0.04380	0.04833	0.05173	0.05642	0.05949	0.06165	0.06448	0.06624	0.07222	0.07300
0.6	0.04455	0.04757	0.05023	0.05559	0.05961	0.06520	0.06887	0.07146	0.07484	0.07695	0.08412	0.08506
0.7	0.04926	0.05270	0.05573	0.06188	0.06651	0.07298	0.07724	0.08024	0.08417	0.08663	0.09499	0.09609
0.8	0.05315	0.05698	0.06036	0.06724	0.07246	0.07978	0.08461	0.08803	0.09251	0.09532	0.10487	0.10612
0.9	0.05629	0.06046	0.06416	0.07174	0.07752	0.08566	0.09106	0.09488	0.09991	0.10306	0.11380	0.11521
1.0	0.05875	0.06324	0.06723	0.07545	0.08176	0.09069	0.09664	0.10087	0.10643	0.10993	0.12185	0.12342
1.2	0.06201	0.06701	0.07150	0.08086	0.08814	0.09856	0.10557	0.11057	0.11719	0.12136	0.13565	0.13753
1.4	0.06354	0.06893	0.07381	0.08411	0.09222	0.10400	0.11201	0.11776	0.12541	0.13024	0.14687	0.14907
1.6	0.06386	0.06953	0.07470	0.08574	0.09457	0.10756	0.11650	0.12296	0.13160	0.13709	0.15606	0.15856
1.8	0.06336	0.06921	0.07459	0.08621	0.09563	0.10968	0.11948	0.12661	0.13622	0.14234	0.16362	0.16644
2.0	0.06233	0.06828	0.07379	0.08584	0.09573	0.11071	0.12129	0.12906	0.13959	0.14633	0.16991	0.17304
2.5	0.05853	0.06450	0.07012	0.08271	0.09338	0.11010	0.12232	0.13148	0.14412	0.15235	0.18157	0.18548
3.0	0.05421	0.06000	0.06552	0.07813	0.08911	0.10688	0.12029	0.13057	0.14506	0.15464	0.18935	0.19404
4.0	0.04620	0.05143	0.05650	0.06839	0.07917	0.09757	0.11229	0.12408	0.14141	0.15329	0.19843	0.20468
5.0	0.03977	0.04442	0.04896	0.05982	0.06992	0.08784	0.10286	0.11536	0.13450	0.14815	0.20288	0.21067
6.0	0.03473	0.03887	0.04293	0.05276	0.06208	0.07905	0.09379	0.10646	0.12659	0.14148	0.20490	0.21423
8.0	0.02754	0.03089	0.03420	0.04232	0.05017	0.06496	0.07843	0.09056	0.11102	0.12717	0.20521	0.21757
10.0	0.02275	0.02554	0.02831	0.03515	0.04184	0.05468	0.06671	0.07786	0.09749	0.11379	0.20303	0.21835
50.0	0.00499	0.00561	0.00624	0.00779	0.00935	0.01245	0.01555	0.01863	0.02475	0.03079	0.12275	0.17336
100.0	0.00252	0.00284	0.00315	0.00394	0.00473	0.00630	0.00787	0.00944	0.01258	0.01570	0.07308	0.12387

Table 4. Function "D"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00156	0.00308	0.00451	0.00583	0.00702	0.00808	0.00901	0.00981	0.01051	0.01111	0.01208	0.01280
0.2	0.00299	0.00589	0.00864	0.01118	0.01347	0.01552	0.01732	0.01889	0.02025	0.02143	0.02332	0.02474
0.3	0.00418	0.00824	0.01210	0.01567	0.01892	0.02183	0.02439	0.02664	0.02860	0.03030	0.03305	0.03512
0.4	0.00507	0.01002	0.01473	0.01911	0.02311	0.02671	0.02991	0.03272	0.03519	0.03734	0.04084	0.04350
0.5	0.00567	0.01121	0.01650	0.02145	0.02599	0.03010	0.03378	0.03705	0.03992	0.04244	0.04658	0.04975
0.6	0.00600	0.01187	0.01749	0.02278	0.02767	0.03212	0.03614	0.03972	0.04290	0.04571	0.05035	0.05395
0.7	0.00611	0.01209	0.01785	0.02328	0.02834	0.03298	0.03720	0.04099	0.04437	0.04738	0.05242	0.05636
0.8	0.00604	0.01198	0.01771	0.02315	0.02824	0.03294	0.03724	0.04113	0.04464	0.04778	0.05309	0.05729
0.9	0.00587	0.01164	0.01722	0.02255	0.02757	0.03223	0.03653	0.04045	0.04400	0.04721	0.05268	0.05706
1.0	0.00561	0.01114	0.01651	0.02166	0.02653	0.03108	0.03530	0.03918	0.04272	0.04594	0.05149	0.05598
1.2	0.00500	0.00994	0.01476	0.01942	0.02386	0.02807	0.03201	0.03568	0.03908	0.04220	0.04768	0.05224
1.4	0.00437	0.00869	0.01293	0.01705	0.02101	0.02479	0.02837	0.03174	0.03489	0.03783	0.04306	0.04751
1.6	0.00379	0.00754	0.01123	0.01483	0.01832	0.02168	0.02488	0.02792	0.03080	0.03350	0.03839	0.04262
1.8	0.00328	0.00653	0.00974	0.01288	0.01594	0.01890	0.02175	0.02447	0.02706	0.02952	0.03402	0.03799
2.0	0.00284	0.00567	0.00847	0.01121	0.01389	0.01650	0.01902	0.02145	0.02377	0.02599	0.03010	0.03378
2.5	0.00204	0.00407	0.00608	0.00806	0.01002	0.01194	0.01381	0.01563	0.01740	0.01911	0.02234	0.02532
3.0	0.00151	0.00301	0.00451	0.00599	0.00746	0.00890	0.01032	0.01171	0.01307	0.01440	0.01694	0.01933
4.0	0.00091	0.00181	0.00272	0.00362	0.00451	0.00539	0.00627	0.00713	0.00799	0.00883	0.01046	0.01204
5.0	0.00060	0.00120	0.00180	0.00239	0.00299	0.00358	0.00416	0.00474	0.00532	0.00589	0.00701	0.00810
6.0	0.00042	0.00085	0.00127	0.00169	0.00211	0.00253	0.00295	0.00337	0.00378	0.00419	0.00500	0.00579
8.0	0.00024	0.00049	0.00073	0.00097	0.00121	0.00145	0.00169	0.00193	0.00217	0.00241	0.00288	0.00335
10.0	0.00016	0.00031	0.00047	0.00063	0.00078	0.00094	0.00109	0.00125	0.00141	0.00156	0.00187	0.00217
50.0	0.00001	0.00001	0.00002	0.00003	0.00003	0.00004	0.00004	0.00005	0.00006	0.00006	0.00008	0.00009
100.0	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00002	0.00002	0.00002

Table 4. Function "D"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.01334	0.01376	0.01408	0.01462	0.01494	0.01528	0.01545	0.01554	0.01564	0.01568	0.01575	0.01576
0.2	0.02581	0.02663	0.02727	0.02834	0.02898	0.02966	0.02999	0.03017	0.03036	0.03045	0.03060	0.03061
0.3	0.03669	0.03789	0.03883	0.04042	0.04137	0.04238	0.04288	0.04316	0.04344	0.04357	0.04379	0.04380
0.4	0.04553	0.04709	0.04832	0.05040	0.05165	0.05299	0.05365	0.05402	0.05439	0.05457	0.05487	0.05488
0.5	0.05218	0.05408	0.05557	0.05812	0.05965	0.06131	0.06213	0.06258	0.06305	0.06327	0.06365	0.06366
0.6	0.05674	0.05893	0.06066	0.06363	0.06544	0.06741	0.06838	0.06893	0.06948	0.06974	0.07020	0.07021
0.7	0.05945	0.06188	0.06382	0.06719	0.06926	0.07152	0.07264	0.07327	0.07392	0.07422	0.07475	0.07477
0.8	0.06061	0.06326	0.06538	0.06910	0.07140	0.07394	0.07521	0.07593	0.07666	0.07701	0.07761	0.07763
0.9	0.06057	0.06339	0.06566	0.06969	0.07221	0.07501	0.07642	0.07722	0.07804	0.07843	0.07911	0.07913
1.0	0.05962	0.06257	0.06497	0.06926	0.07198	0.07503	0.07657	0.07746	0.07836	0.07879	0.07955	0.07957
1.2	0.05601	0.05912	0.06169	0.06638	0.06943	0.07291	0.07471	0.07575	0.07682	0.07734	0.07823	0.07826
1.4	0.05126	0.05442	0.05707	0.06201	0.06530	0.06915	0.07118	0.07236	0.07359	0.07419	0.07523	0.07526
1.6	0.04626	0.04938	0.05203	0.05709	0.06055	0.06469	0.06692	0.06824	0.06962	0.07029	0.07148	0.07152
1.8	0.04146	0.04447	0.04708	0.05215	0.05571	0.06007	0.06248	0.06391	0.06543	0.06618	0.06751	0.06755
2.0	0.03705	0.03992	0.04244	0.04745	0.05104	0.05557	0.05812	0.05965	0.06131	0.06213	0.06360	0.06365
2.5	0.02804	0.03050	0.03272	0.03734	0.04084	0.04553	0.04832	0.05007	0.05201	0.05299	0.05480	0.05486
3.0	0.02156	0.02362	0.02552	0.02961	0.03286	0.03746	0.04035	0.04224	0.04440	0.04552	0.04765	0.04772
4.0	0.01355	0.01498	0.01634	0.01942	0.02203	0.02608	0.02889	0.03086	0.03329	0.03462	0.03732	0.03742
5.0	0.00916	0.01019	0.01118	0.01347	0.01552	0.01889	0.02143	0.02332	0.02581	0.02727	0.03045	0.03057
6.0	0.00657	0.00732	0.00806	0.00981	0.01142	0.01418	0.01639	0.01812	0.02054	0.02205	0.02562	0.02576
8.0	0.00381	0.00427	0.00472	0.00580	0.00683	0.00871	0.01032	0.01169	0.01380	0.01525	0.01934	0.01952
10.0	0.00248	0.00278	0.00308	0.00380	0.00451	0.00583	0.00702	0.00808	0.00981	0.01111	0.01545	0.01568
50.0	0.00010	0.00011	0.00013	0.00016	0.00019	0.00025	0.00032	0.00038	0.00050	0.00062	0.00225	0.00285
100.0	0.00003	0.00003	0.00003	0.00004	0.00005	0.00006	0.00008	0.00010	0.00013	0.00016	0.00071	0.00113

Table 5. Function "E"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00079	0.00154	0.00223	0.00284	0.00335	0.00377	0.00410	0.00435	0.00453	0.00466	0.00477	0.00476
0.2	0.00156	0.00306	0.00443	0.00564	0.00667	0.00750	0.00816	0.00866	0.00903	0.00927	0.00950	0.00949
0.3	0.00232	0.00454	0.00658	0.00838	0.00990	0.01115	0.01214	0.01290	0.01345	0.01382	0.01418	0.01417
0.4	0.00304	0.00596	0.00864	0.01101	0.01303	0.01469	0.01601	0.01702	0.01776	0.01827	0.01876	0.01877
0.5	0.00373	0.00731	0.01061	0.01353	0.01602	0.01808	0.01973	0.02099	0.02193	0.02258	0.02324	0.02328
0.6	0.00438	0.00858	0.01246	0.01591	0.01887	0.02132	0.02329	0.02481	0.02595	0.02675	0.02758	0.02767
0.7	0.00498	0.00977	0.01420	0.01815	0.02155	0.02438	0.02667	0.02845	0.02979	0.03075	0.03177	0.03194
0.8	0.00555	0.01088	0.01583	0.02025	0.02407	0.02727	0.02986	0.03190	0.03345	0.03457	0.03581	0.03607
0.9	0.00607	0.01191	0.01734	0.02220	0.02643	0.02998	0.03287	0.03517	0.03692	0.03821	0.03967	0.04004
1.0	0.00655	0.01287	0.01874	0.02402	0.02862	0.03251	0.03570	0.03825	0.04021	0.04167	0.04336	0.04387
1.2	0.00741	0.01457	0.02125	0.02728	0.03258	0.03709	0.04084	0.04386	0.04623	0.04803	0.05023	0.05104
1.4	0.00814	0.01602	0.02340	0.03009	0.03600	0.04108	0.04533	0.04880	0.05157	0.05371	0.05643	0.05759
1.6	0.00877	0.01727	0.02525	0.03252	0.03897	0.04455	0.04926	0.05315	0.05629	0.05875	0.06201	0.06354
1.8	0.00932	0.01835	0.02685	0.03462	0.04155	0.04757	0.05270	0.05698	0.06046	0.06324	0.06701	0.06893
2.0	0.00979	0.01928	0.02824	0.03645	0.04380	0.05023	0.05573	0.06036	0.06416	0.06723	0.07150	0.07381
2.5	0.01072	0.02115	0.03102	0.04012	0.04833	0.05559	0.06188	0.06724	0.07174	0.07545	0.08086	0.08411
3.0	0.01142	0.02254	0.03309	0.04286	0.05173	0.05961	0.06651	0.07246	0.07752	0.08176	0.08814	0.09222
4.0	0.01237	0.02444	0.03594	0.04664	0.05642	0.06520	0.07298	0.07978	0.08566	0.09069	0.09856	0.10400
5.0	0.01300	0.02568	0.03779	0.04911	0.05949	0.06887	0.07724	0.08461	0.09106	0.09664	0.10557	0.11201
6.0	0.01343	0.02655	0.03909	0.05084	0.06165	0.07146	0.08024	0.08803	0.09488	0.10087	0.11057	0.11776
8.0	0.01400	0.02769	0.04079	0.05310	0.06448	0.07484	0.08417	0.09251	0.09991	0.10643	0.11719	0.12541
10.0	0.01435	0.02839	0.04185	0.05451	0.06624	0.07695	0.08663	0.09532	0.10306	0.10993	0.12136	0.13024
50.0	0.01555	0.03079	0.04544	0.05930	0.07222	0.08412	0.09499	0.10487	0.11380	0.12185	0.13565	0.14687
100.0	0.01570	0.03110	0.04591	0.05993	0.07300	0.08506	0.09609	0.10612	0.11521	0.12342	0.13753	0.14907

Table 5. Function "E"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.00467	0.00454	0.00440	0.00400	0.00363	0.00301	0.00256	0.00222	0.00174	0.00143	0.00031	0.00016
0.2	0.00932	0.00907	0.00878	0.00799	0.00725	0.00602	0.00512	0.00443	0.00348	0.00287	0.00062	0.00032
0.3	0.01393	0.01356	0.01313	0.01196	0.01085	0.00903	0.00767	0.00664	0.00523	0.00430	0.00094	0.00047
0.4	0.01847	0.01800	0.01744	0.01591	0.01444	0.01202	0.01022	0.00885	0.00697	0.00573	0.00125	0.00063
0.5	0.02294	0.02238	0.02169	0.01981	0.01801	0.01500	0.01276	0.01106	0.00870	0.00716	0.00156	0.00079
0.6	0.02731	0.02667	0.02588	0.02368	0.02154	0.01797	0.01529	0.01326	0.01044	0.00859	0.00187	0.00095
0.7	0.03157	0.03087	0.02998	0.02749	0.02504	0.02092	0.01781	0.01545	0.01217	0.01002	0.00218	0.00110
0.8	0.03571	0.03496	0.03400	0.03124	0.02849	0.02384	0.02032	0.01764	0.01390	0.01144	0.00250	0.00126
0.9	0.03971	0.03895	0.03792	0.03492	0.03190	0.02675	0.02281	0.01981	0.01562	0.01287	0.00281	0.00142
1.0	0.04359	0.04281	0.04174	0.03854	0.03527	0.02962	0.02529	0.02198	0.01734	0.01429	0.00312	0.00158
1.2	0.05091	0.05017	0.04906	0.04554	0.04182	0.03529	0.03020	0.02628	0.02077	0.01712	0.00374	0.00189
1.4	0.05766	0.05702	0.05592	0.05221	0.04814	0.04082	0.03503	0.03054	0.02417	0.01994	0.00437	0.00221
1.6	0.06386	0.06336	0.06233	0.05853	0.05421	0.04620	0.03977	0.03473	0.02754	0.02275	0.00499	0.00252
1.8	0.06953	0.06921	0.06828	0.06450	0.06000	0.05143	0.04442	0.03887	0.03089	0.02554	0.00561	0.00284
2.0	0.07470	0.07459	0.07379	0.07012	0.06552	0.05650	0.04896	0.04293	0.03420	0.02831	0.00624	0.00315
2.5	0.08574	0.08621	0.08584	0.08271	0.07813	0.06839	0.05982	0.05276	0.04232	0.03515	0.00779	0.00394
3.0	0.09457	0.09563	0.09573	0.09338	0.08911	0.07917	0.06992	0.06208	0.05017	0.04184	0.00935	0.00473
4.0	0.10756	0.10968	0.11071	0.11010	0.10688	0.09757	0.08784	0.07905	0.06496	0.05468	0.01245	0.00630
5.0	0.11650	0.11948	0.12129	0.12232	0.12029	0.11229	0.10286	0.09379	0.07843	0.06671	0.01555	0.00787
6.0	0.12296	0.12661	0.12906	0.13148	0.13057	0.12408	0.11536	0.10646	0.09056	0.07786	0.01863	0.00944
8.0	0.13160	0.13622	0.13959	0.14412	0.14506	0.14141	0.13450	0.12659	0.11102	0.09749	0.02475	0.01258
10.0	0.13709	0.14234	0.14633	0.15235	0.15464	0.15329	0.14815	0.14148	0.12717	0.11379	0.03079	0.01570
50.0	0.15606	0.16362	0.16991	0.18157	0.18935	0.19843	0.20288	0.20490	0.20521	0.20303	0.12275	0.07308
100.0	0.15856	0.16644	0.17304	0.18548	0.19404	0.20468	0.21067	0.21423	0.21757	0.21835	0.17336	0.12387

Table 6. Function "F"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00156	0.00299	0.00418	0.00507	0.00567	0.00600	0.00611	0.00604	0.00587	0.00561	0.00500	0.00437
0.2	0.00308	0.00589	0.00824	0.01002	0.01121	0.01187	0.01209	0.01198	0.01164	0.01114	0.00994	0.00869
0.3	0.00451	0.00864	0.01210	0.01473	0.01650	0.01749	0.01785	0.01771	0.01722	0.01651	0.01476	0.01293
0.4	0.00583	0.01118	0.01567	0.01911	0.02145	0.02278	0.02328	0.02315	0.02255	0.02166	0.01942	0.01705
0.5	0.00702	0.01347	0.01892	0.02311	0.02599	0.02767	0.02834	0.02824	0.02757	0.02653	0.02386	0.02101
0.6	0.00808	0.01552	0.02183	0.02671	0.03010	0.03212	0.03298	0.03294	0.03223	0.03108	0.02807	0.02479
0.7	0.00901	0.01732	0.02439	0.02991	0.03378	0.03614	0.03720	0.03724	0.03653	0.03530	0.03201	0.02837
0.8	0.00981	0.01889	0.02664	0.03272	0.03705	0.03972	0.04099	0.04113	0.04045	0.03918	0.03568	0.03174
0.9	0.01051	0.02025	0.02860	0.03519	0.03992	0.04290	0.04437	0.04464	0.04400	0.04272	0.03908	0.03489
1.0	0.01111	0.02143	0.03030	0.03734	0.04244	0.04571	0.04738	0.04778	0.04721	0.04594	0.04220	0.03783
1.2	0.01208	0.02332	0.03305	0.04084	0.04658	0.05035	0.05242	0.05309	0.05268	0.05149	0.04768	0.04306
1.4	0.01280	0.02474	0.03512	0.04350	0.04975	0.05395	0.05636	0.05729	0.05706	0.05598	0.05224	0.04751
1.6	0.01334	0.02581	0.03669	0.04553	0.05218	0.05674	0.05945	0.06061	0.06057	0.05962	0.05601	0.05126
1.8	0.01376	0.02663	0.03789	0.04709	0.05408	0.05893	0.06188	0.06326	0.06339	0.06257	0.05912	0.05442
2.0	0.01408	0.02727	0.03883	0.04832	0.05557	0.06066	0.06382	0.06538	0.06566	0.06497	0.06169	0.05707
2.5	0.01462	0.02834	0.04042	0.05040	0.05812	0.06363	0.06719	0.06910	0.06969	0.06926	0.06638	0.06201
3.0	0.01494	0.02898	0.04137	0.05165	0.05965	0.06544	0.06926	0.07140	0.07221	0.07198	0.06943	0.06530
4.0	0.01528	0.02966	0.04238	0.05299	0.06131	0.06741	0.07152	0.07394	0.07501	0.07503	0.07291	0.06915
5.0	0.01545	0.02999	0.04288	0.05365	0.06213	0.06838	0.07264	0.07521	0.07642	0.07657	0.07471	0.07118
6.0	0.01554	0.03017	0.04316	0.05402	0.06258	0.06893	0.07327	0.07593	0.07722	0.07746	0.07575	0.07236
8.0	0.01564	0.03036	0.04344	0.05439	0.06305	0.06948	0.07392	0.07666	0.07804	0.07836	0.07682	0.07359
10.0	0.01568	0.03045	0.04357	0.05457	0.06327	0.06974	0.07422	0.07701	0.07843	0.07879	0.07734	0.07419
50.0	0.01575	0.03060	0.04379	0.05487	0.06365	0.07020	0.07475	0.07761	0.07911	0.07955	0.07823	0.07523
100.0	0.01576	0.03061	0.04380	0.05488	0.06366	0.07021	0.07477	0.07763	0.07913	0.07957	0.07826	0.07526

Table 6. Function "F"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.00379	0.00328	0.00284	0.00204	0.00151	0.00091	0.00060	0.00042	0.00024	0.00016	0.00001	0.00000
0.2	0.00754	0.00653	0.00567	0.00407	0.00301	0.00181	0.00120	0.00085	0.00049	0.00031	0.00001	0.00000
0.3	0.01123	0.00974	0.00847	0.00608	0.00451	0.00272	0.00180	0.00127	0.00073	0.00047	0.00002	0.00000
0.4	0.01483	0.01288	0.01121	0.00806	0.00599	0.00362	0.00239	0.00169	0.00097	0.00063	0.00003	0.00001
0.5	0.01832	0.01594	0.01389	0.01002	0.00746	0.00451	0.00299	0.00211	0.00121	0.00078	0.00003	0.00001
0.6	0.02168	0.01890	0.01650	0.01194	0.00890	0.00539	0.00358	0.00253	0.00145	0.00094	0.00004	0.00001
0.7	0.02488	0.02175	0.01902	0.01381	0.01032	0.00627	0.00416	0.00295	0.00169	0.00109	0.00004	0.00001
0.8	0.02792	0.02447	0.02145	0.01563	0.01171	0.00713	0.00474	0.00337	0.00193	0.00125	0.00005	0.00001
0.9	0.03080	0.02706	0.02377	0.01740	0.01307	0.00799	0.00532	0.00378	0.00217	0.00141	0.00006	0.00001
1.0	0.03350	0.02952	0.02599	0.01911	0.01440	0.00883	0.00589	0.00419	0.00241	0.00156	0.00006	0.00002
1.2	0.03839	0.03402	0.03010	0.02234	0.01694	0.01046	0.00701	0.00500	0.00288	0.00187	0.00008	0.00002
1.4	0.04262	0.03799	0.03378	0.02532	0.01933	0.01204	0.00810	0.00579	0.00335	0.00217	0.00009	0.00002
1.6	0.04626	0.04146	0.03705	0.02804	0.02156	0.01355	0.00916	0.00657	0.00381	0.00248	0.00010	0.00003
1.8	0.04938	0.04447	0.03992	0.03050	0.02362	0.01498	0.01019	0.00732	0.00427	0.00278	0.00011	0.00003
2.0	0.05203	0.04708	0.04244	0.03272	0.02552	0.01634	0.01118	0.00806	0.00472	0.00308	0.00013	0.00003
2.5	0.05709	0.05215	0.04745	0.03734	0.02961	0.01942	0.01347	0.00981	0.00580	0.00380	0.00016	0.00004
3.0	0.06055	0.05571	0.05104	0.04084	0.03286	0.02203	0.01552	0.01142	0.00683	0.00451	0.00019	0.00005
4.0	0.06469	0.06007	0.05557	0.04553	0.03746	0.02608	0.01889	0.01418	0.00871	0.00583	0.00025	0.00006
5.0	0.06692	0.06248	0.05812	0.04832	0.04035	0.02889	0.02143	0.01639	0.01032	0.00702	0.00032	0.00008
6.0	0.06824	0.06391	0.05965	0.05007	0.04224	0.03086	0.02332	0.01812	0.01169	0.00808	0.00038	0.00010
8.0	0.06962	0.06543	0.06131	0.05201	0.04440	0.03329	0.02581	0.02054	0.01380	0.00981	0.00050	0.00013
10.0	0.07029	0.06618	0.06213	0.05299	0.04552	0.03462	0.02727	0.02205	0.01525	0.01111	0.00062	0.00016
50.0	0.07148	0.06751	0.06360	0.05480	0.04765	0.03732	0.03045	0.02562	0.01934	0.01545	0.00225	0.00071
100.0	0.07152	0.06755	0.06365	0.05486	0.04772	0.03742	0.03057	0.02576	0.01952	0.01568	0.00285	0.00113

Table 7. Function "G"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00023	0.00090	0.00191	0.00315	0.00449	0.00584	0.00712	0.00828	0.00932	0.01022	0.01167	0.01272
0.2	0.00046	0.00176	0.00374	0.00616	0.00879	0.01143	0.01394	0.01624	0.01828	0.02007	0.02293	0.02500
0.3	0.00066	0.00254	0.00541	0.00892	0.01274	0.01658	0.02024	0.02360	0.02660	0.02922	0.03343	0.03650
0.4	0.00084	0.00323	0.00687	0.01134	0.01622	0.02114	0.02585	0.03018	0.03405	0.03745	0.04293	0.04693
0.5	0.00098	0.00381	0.00811	0.01340	0.01920	0.02506	0.03069	0.03588	0.04054	0.04465	0.05129	0.05617
0.6	0.00111	0.00428	0.00913	0.01511	0.02168	0.02835	0.03477	0.04071	0.04607	0.05080	0.05850	0.06418
0.7	0.00121	0.00467	0.00996	0.01650	0.02370	0.03104	0.03813	0.04472	0.05068	0.05597	0.06459	0.07101
0.8	0.00128	0.00497	0.01061	0.01761	0.02533	0.03322	0.04087	0.04801	0.05448	0.06024	0.06969	0.07675
0.9	0.00134	0.00521	0.01113	0.01849	0.02663	0.03497	0.04308	0.05067	0.05758	0.06374	0.07391	0.08155
1.0	0.00139	0.00539	0.01154	0.01919	0.02766	0.03636	0.04485	0.05281	0.06008	0.06660	0.07737	0.08552
1.2	0.00146	0.00565	0.01211	0.02016	0.02911	0.03834	0.04738	0.05591	0.06374	0.07078	0.08253	0.09151
1.4	0.00150	0.00581	0.01246	0.02076	0.03002	0.03959	0.04900	0.05790	0.06611	0.07352	0.08598	0.09557
1.6	0.00152	0.00591	0.01268	0.02115	0.03060	0.04039	0.05004	0.05920	0.06766	0.07534	0.08829	0.09835
1.8	0.00154	0.00598	0.01282	0.02139	0.03097	0.04091	0.05072	0.06005	0.06869	0.07655	0.08986	0.10026
2.0	0.00155	0.00602	0.01291	0.02155	0.03122	0.04126	0.05118	0.06063	0.06939	0.07738	0.09095	0.10159
2.5	0.00156	0.00607	0.01304	0.02177	0.03154	0.04172	0.05179	0.06140	0.07034	0.07851	0.09245	0.10348
3.0	0.00157	0.00610	0.01309	0.02185	0.03168	0.04191	0.05205	0.06173	0.07076	0.07901	0.09313	0.10435
4.0	0.00157	0.00611	0.01312	0.02192	0.03178	0.04205	0.05224	0.06198	0.07106	0.07938	0.09364	0.10501
5.0	0.00157	0.00612	0.01313	0.02194	0.03181	0.04210	0.05230	0.06205	0.07115	0.07949	0.09380	0.10522
6.0	0.00158	0.00612	0.01314	0.02195	0.03182	0.04211	0.05232	0.06208	0.07119	0.07953	0.09387	0.10530
8.0	0.00158	0.00612	0.01314	0.02195	0.03183	0.04212	0.05233	0.06210	0.07121	0.07956	0.09391	0.10536
10.0	0.00158	0.00612	0.01314	0.02195	0.03183	0.04213	0.05234	0.06211	0.07122	0.07957	0.09392	0.10538
50.0	0.00158	0.00612	0.01314	0.02195	0.03183	0.04213	0.05234	0.06211	0.07122	0.07958	0.09393	0.10539
100.0	0.00158	0.00612	0.01314	0.02195	0.03183	0.04213	0.05234	0.06211	0.07122	0.07958	0.09393	0.10539

Table 7. Function "G"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.01347	0.01402	0.01441	0.01502	0.01533	0.01561	0.01572	0.01577	0.01581	0.01582	0.01584	0.01584
0.2	0.02650	0.02758	0.02838	0.02959	0.03021	0.03076	0.03097	0.03107	0.03115	0.03118	0.03121	0.03121
0.3	0.03871	0.04032	0.04150	0.04330	0.04423	0.04505	0.04537	0.04552	0.04564	0.04569	0.04573	0.04573
0.4	0.04984	0.05195	0.05350	0.05588	0.05711	0.05820	0.05863	0.05883	0.05899	0.05905	0.05911	0.05911
0.5	0.05972	0.06232	0.06423	0.06717	0.06869	0.07005	0.07058	0.07082	0.07102	0.07110	0.07118	0.07118
0.6	0.06834	0.07138	0.07364	0.07711	0.07892	0.08054	0.08117	0.08146	0.08170	0.08179	0.08188	0.08188
0.7	0.07572	0.07919	0.08176	0.08575	0.08783	0.08970	0.09044	0.09078	0.09106	0.09116	0.09127	0.09127
0.8	0.08197	0.08583	0.08870	0.09317	0.09552	0.09764	0.09847	0.09886	0.09918	0.09930	0.09942	0.09942
0.9	0.08722	0.09144	0.09458	0.09951	0.10211	0.10447	0.10541	0.10584	0.10620	0.10633	0.10647	0.10647
1.0	0.09160	0.09614	0.09954	0.10490	0.10774	0.11033	0.11136	0.11184	0.11224	0.11238	0.11254	0.11254
1.2	0.09827	0.10337	0.10721	0.11333	0.11662	0.11965	0.12086	0.12143	0.12191	0.12208	0.12226	0.12227
1.4	0.10287	0.10840	0.11262	0.11938	0.12307	0.12650	0.12789	0.12855	0.12909	0.12929	0.12951	0.12951
1.6	0.10605	0.11193	0.11644	0.12375	0.12778	0.13158	0.13313	0.13387	0.13449	0.13472	0.13496	0.13496
1.8	0.10827	0.11442	0.11917	0.12693	0.13125	0.13538	0.13709	0.13791	0.13859	0.13885	0.13912	0.13913
2.0	0.10983	0.11620	0.12113	0.12926	0.13385	0.13827	0.14012	0.14101	0.14176	0.14205	0.14235	0.14235
2.5	0.11209	0.11880	0.12405	0.13284	0.13790	0.14292	0.14508	0.14614	0.14705	0.14739	0.14777	0.14777
3.0	0.11314	0.12004	0.12547	0.13465	0.14003	0.14548	0.14788	0.14909	0.15013	0.15054	0.15098	0.15099
4.0	0.11397	0.12103	0.12662	0.13619	0.14192	0.14788	0.15063	0.15204	0.15331	0.15382	0.15440	0.15440
5.0	0.11424	0.12136	0.12701	0.13674	0.14261	0.14884	0.15178	0.15333	0.15477	0.15536	0.15606	0.15606
6.0	0.11434	0.12149	0.12716	0.13696	0.14291	0.14927	0.15233	0.15397	0.15553	0.15618	0.15698	0.15699
8.0	0.11441	0.12157	0.12727	0.13712	0.14312	0.14960	0.15276	0.15450	0.15620	0.15694	0.15792	0.15792
10.0	0.11443	0.12160	0.12730	0.13717	0.14319	0.14971	0.15291	0.15469	0.15646	0.15725	0.15835	0.15836
50.0	0.11445	0.12162	0.12732	0.13720	0.14324	0.14979	0.15303	0.15485	0.15671	0.15758	0.15908	0.15912
100.0	0.11445	0.12162	0.12732	0.13720	0.14324	0.14979	0.15303	0.15485	0.15671	0.15758	0.15909	0.15914

Table 8. Function "H"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00023	0.00046	0.00066	0.00084	0.00098	0.00111	0.00121	0.00128	0.00134	0.00139	0.00146	0.00150
0.2	0.00090	0.00176	0.00254	0.00323	0.00381	0.00428	0.00467	0.00497	0.00521	0.00539	0.00565	0.00581
0.3	0.00191	0.00374	0.00541	0.00687	0.00811	0.00913	0.00996	0.01061	0.01113	0.01154	0.01211	0.01246
0.4	0.00315	0.00616	0.00892	0.01134	0.01340	0.01511	0.01650	0.01761	0.01849	0.01919	0.02016	0.02076
0.5	0.00449	0.00879	0.01274	0.01622	0.01920	0.02168	0.02370	0.02533	0.02663	0.02766	0.02911	0.03002
0.6	0.00584	0.01143	0.01658	0.02114	0.02506	0.02835	0.03104	0.03322	0.03497	0.03636	0.03834	0.03959
0.7	0.00712	0.01394	0.02024	0.02585	0.03069	0.03477	0.03813	0.04087	0.04308	0.04485	0.04738	0.04900
0.8	0.00828	0.01624	0.02360	0.03018	0.03588	0.04071	0.04472	0.04801	0.05067	0.05281	0.05591	0.05790
0.9	0.00932	0.01828	0.02660	0.03405	0.04054	0.04607	0.05068	0.05448	0.05758	0.06008	0.06374	0.06611
1.0	0.01022	0.02007	0.02922	0.03745	0.04465	0.05080	0.05597	0.06024	0.06374	0.06660	0.07078	0.07352
1.2	0.01167	0.02293	0.03343	0.04293	0.05129	0.05850	0.06459	0.06969	0.07391	0.07737	0.08253	0.08598
1.4	0.01272	0.02500	0.03650	0.04693	0.05617	0.06418	0.07101	0.07675	0.08155	0.08552	0.09151	0.09557
1.6	0.01347	0.02650	0.03871	0.04984	0.05972	0.06834	0.07572	0.08197	0.08722	0.09160	0.09827	0.10287
1.8	0.01402	0.02758	0.04032	0.05195	0.06232	0.07138	0.07919	0.08583	0.09144	0.09614	0.10337	0.10840
2.0	0.01441	0.02838	0.04150	0.05350	0.06423	0.07364	0.08176	0.08870	0.09458	0.09954	0.10721	0.11262
2.5	0.01502	0.02959	0.04330	0.05588	0.06717	0.07711	0.08575	0.09317	0.09951	0.10490	0.11333	0.11938
3.0	0.01533	0.03021	0.04423	0.05711	0.06869	0.07892	0.08783	0.09552	0.10211	0.10774	0.11662	0.12307
4.0	0.01561	0.03076	0.04505	0.05820	0.07005	0.08054	0.08970	0.09764	0.10447	0.11033	0.11965	0.12650
5.0	0.01572	0.03097	0.04537	0.05863	0.07058	0.08117	0.09044	0.09847	0.10541	0.11136	0.12086	0.12789
6.0	0.01577	0.03107	0.04552	0.05883	0.07082	0.08146	0.09078	0.09886	0.10584	0.11184	0.12143	0.12855
8.0	0.01581	0.03115	0.04564	0.05899	0.07102	0.08170	0.09106	0.09918	0.10620	0.11224	0.12191	0.12909
10.0	0.01582	0.03118	0.04569	0.05905	0.07110	0.08179	0.09116	0.09930	0.10633	0.11238	0.12208	0.12929
50.0	0.01584	0.03121	0.04573	0.05911	0.07118	0.08188	0.09127	0.09942	0.10647	0.11254	0.12226	0.12951
100.0	0.01584	0.03121	0.04573	0.05911	0.07118	0.08188	0.09127	0.09942	0.10647	0.11254	0.12227	0.12951

Table 8. Function "H"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.00152	0.00154	0.00155	0.00156	0.00157	0.00157	0.00157	0.00158	0.00158	0.00158	0.00158	0.00158
0.2	0.00591	0.00598	0.00602	0.00607	0.00610	0.00611	0.00612	0.00612	0.00612	0.00612	0.00612	0.00612
0.3	0.01268	0.01282	0.01291	0.01304	0.01309	0.01312	0.01313	0.01314	0.01314	0.01314	0.01314	0.01314
0.4	0.02115	0.02139	0.02155	0.02177	0.02185	0.02192	0.02194	0.02195	0.02195	0.02195	0.02195	0.02195
0.5	0.03060	0.03097	0.03122	0.03154	0.03168	0.03178	0.03181	0.03182	0.03183	0.03183	0.03183	0.03183
0.6	0.04039	0.04091	0.04126	0.04172	0.04191	0.04205	0.04210	0.04211	0.04212	0.04213	0.04213	0.04213
0.7	0.05004	0.05072	0.05118	0.05179	0.05205	0.05224	0.05230	0.05232	0.05233	0.05234	0.05234	0.05234
0.8	0.05920	0.06005	0.06063	0.06140	0.06173	0.06198	0.06205	0.06208	0.06210	0.06211	0.06211	0.06211
0.9	0.06766	0.06869	0.06939	0.07034	0.07076	0.07106	0.07115	0.07119	0.07121	0.07122	0.07122	0.07122
1.0	0.07534	0.07655	0.07738	0.07851	0.07901	0.07938	0.07949	0.07953	0.07956	0.07957	0.07958	0.07958
1.2	0.08829	0.08986	0.09095	0.09245	0.09313	0.09364	0.09380	0.09387	0.09391	0.09392	0.09393	0.09393
1.4	0.09835	0.10026	0.10159	0.10348	0.10435	0.10501	0.10522	0.10530	0.10536	0.10538	0.10539	0.10539
1.6	0.10605	0.10827	0.10983	0.11209	0.11314	0.11397	0.11424	0.11434	0.11441	0.11443	0.11445	0.11445
1.8	0.11193	0.11442	0.11620	0.11880	0.12004	0.12103	0.12136	0.12149	0.12157	0.12160	0.12162	0.12162
2.0	0.11644	0.11917	0.12113	0.12405	0.12547	0.12662	0.12701	0.12716	0.12727	0.12730	0.12732	0.12732
2.5	0.12375	0.12693	0.12926	0.13284	0.13465	0.13619	0.13674	0.13696	0.13712	0.13717	0.13720	0.13720
3.0	0.12778	0.13125	0.13385	0.13790	0.14003	0.14192	0.14261	0.14291	0.14312	0.14319	0.14324	0.14324
4.0	0.13158	0.13538	0.13827	0.14292	0.14548	0.14788	0.14884	0.14927	0.14960	0.14971	0.14979	0.14979
5.0	0.13313	0.13709	0.14012	0.14508	0.14788	0.15063	0.15178	0.15233	0.15276	0.15291	0.15303	0.15303
6.0	0.13387	0.13791	0.14101	0.14614	0.14909	0.15204	0.15333	0.15397	0.15450	0.15469	0.15485	0.15485
8.0	0.13449	0.13859	0.14176	0.14705	0.15013	0.15331	0.15477	0.15553	0.15620	0.15646	0.15671	0.15671
10.0	0.13472	0.13885	0.14205	0.14739	0.15054	0.15382	0.15536	0.15618	0.15694	0.15725	0.15758	0.15758
50.0	0.13496	0.13912	0.14235	0.14777	0.15098	0.15440	0.15606	0.15698	0.15792	0.15835	0.15908	0.15909
100.0	0.13496	0.13913	0.14235	0.14777	0.15099	0.15440	0.15606	0.15699	0.15792	0.15836	0.15912	0.15914

Table 9. Function "J"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00001	0.00004	0.00010	0.00016	0.00022	0.00029	0.00035	0.00041	0.00046	0.00051	0.00058	0.00063
0.2	0.00004	0.00017	0.00037	0.00061	0.00087	0.00113	0.00138	0.00160	0.00180	0.00198	0.00227	0.00247
0.3	0.00010	0.00037	0.00078	0.00129	0.00185	0.00241	0.00294	0.00344	0.00388	0.00426	0.00488	0.00534
0.4	0.00016	0.00061	0.00129	0.00214	0.00306	0.00400	0.00490	0.00573	0.00648	0.00714	0.00820	0.00898
0.5	0.00022	0.00087	0.00185	0.00306	0.00440	0.00576	0.00707	0.00829	0.00939	0.01037	0.01195	0.01313
0.6	0.00029	0.00113	0.00241	0.00400	0.00576	0.00756	0.00931	0.01094	0.01242	0.01374	0.01591	0.01752
0.7	0.00035	0.00138	0.00294	0.00490	0.00707	0.00931	0.01149	0.01354	0.01541	0.01709	0.01986	0.02195
0.8	0.00041	0.00160	0.00344	0.00573	0.00829	0.01094	0.01354	0.01600	0.01826	0.02029	0.02367	0.02625
0.9	0.00046	0.00180	0.00388	0.00648	0.00939	0.01242	0.01541	0.01826	0.02088	0.02326	0.02725	0.03032
1.0	0.00051	0.00198	0.00426	0.00714	0.01037	0.01374	0.01709	0.02029	0.02326	0.02596	0.03054	0.03409
1.2	0.00058	0.00227	0.00488	0.00820	0.01195	0.01591	0.01986	0.02367	0.02725	0.03054	0.03618	0.04063
1.4	0.00063	0.00247	0.00534	0.00898	0.01313	0.01752	0.02195	0.02625	0.03032	0.03409	0.04063	0.04589
1.6	0.00067	0.00262	0.00567	0.00955	0.01399	0.01871	0.02350	0.02818	0.03264	0.03679	0.04409	0.05004
1.8	0.00070	0.00273	0.00590	0.00996	0.01462	0.01959	0.02466	0.02963	0.03439	0.03885	0.04675	0.05327
2.0	0.00072	0.00281	0.00608	0.01027	0.01509	0.02025	0.02552	0.03072	0.03571	0.04041	0.04881	0.05580
2.5	0.00075	0.00293	0.00635	0.01074	0.01581	0.02127	0.02687	0.03243	0.03781	0.04292	0.05215	0.05998
3.0	0.00076	0.00299	0.00649	0.01099	0.01619	0.02180	0.02758	0.03334	0.03893	0.04427	0.05399	0.06234
4.0	0.00078	0.00305	0.00661	0.01120	0.01652	0.02228	0.02823	0.03417	0.03997	0.04553	0.05573	0.06460
5.0	0.00078	0.00307	0.00666	0.01129	0.01665	0.02247	0.02848	0.03450	0.04038	0.04603	0.05644	0.06553
6.0	0.00079	0.00308	0.00668	0.01133	0.01671	0.02255	0.02860	0.03465	0.04057	0.04627	0.05677	0.06598
8.0	0.00079	0.00308	0.00670	0.01136	0.01676	0.02263	0.02870	0.03478	0.04073	0.04647	0.05705	0.06636
10.0	0.00079	0.00309	0.00671	0.01137	0.01678	0.02265	0.02873	0.03483	0.04079	0.04654	0.05715	0.06650
50.0	0.00079	0.00309	0.00671	0.01138	0.01680	0.02268	0.02877	0.03488	0.04086	0.04661	0.05727	0.06665
100.0	0.00079	0.00309	0.00671	0.01138	0.01680	0.02268	0.02877	0.03488	0.04086	0.04662	0.05727	0.06665

Table 9. Function "J"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	0.00067	0.00070	0.00072	0.00075	0.00076	0.00078	0.00078	0.00079	0.00079	0.00079	0.00079	0.00079
0.2	0.00262	0.00273	0.00281	0.00293	0.00299	0.00305	0.00307	0.00308	0.00308	0.00309	0.00309	0.00309
0.3	0.00567	0.00590	0.00608	0.00635	0.00649	0.00661	0.00666	0.00668	0.00670	0.00671	0.00671	0.00671
0.4	0.00955	0.00996	0.01027	0.01074	0.01099	0.01120	0.01129	0.01133	0.01136	0.01137	0.01138	0.01138
0.5	0.01399	0.01462	0.01509	0.01581	0.01619	0.01652	0.01665	0.01671	0.01676	0.01678	0.01680	0.01680
0.6	0.01871	0.01959	0.02025	0.02127	0.02180	0.02228	0.02247	0.02255	0.02263	0.02265	0.02268	0.02268
0.7	0.02350	0.02466	0.02552	0.02687	0.02758	0.02823	0.02848	0.02860	0.02870	0.02873	0.02877	0.02877
0.8	0.02818	0.02963	0.03072	0.03243	0.03334	0.03417	0.03450	0.03465	0.03478	0.03483	0.03488	0.03488
0.9	0.03264	0.03439	0.03571	0.03781	0.03893	0.03997	0.04038	0.04057	0.04073	0.04079	0.04086	0.04086
1.0	0.03679	0.03885	0.04041	0.04292	0.04427	0.04553	0.04603	0.04627	0.04647	0.04654	0.04661	0.04662
1.2	0.04409	0.04675	0.04881	0.05215	0.05399	0.05573	0.05644	0.05677	0.05705	0.05715	0.05727	0.05727
1.4	0.05004	0.05327	0.05580	0.05998	0.06234	0.06460	0.06553	0.06598	0.06636	0.06650	0.06665	0.06665
1.6	0.05479	0.05854	0.06151	0.06651	0.06938	0.07219	0.07337	0.07394	0.07443	0.07461	0.07480	0.07480
1.8	0.05854	0.06276	0.06613	0.07189	0.07527	0.07864	0.08008	0.08079	0.08139	0.08161	0.08186	0.08186
2.0	0.06151	0.06613	0.06985	0.07632	0.08019	0.08411	0.08582	0.08667	0.08740	0.08767	0.08798	0.08798
2.5	0.06651	0.07189	0.07632	0.08425	0.08920	0.09445	0.09686	0.09808	0.09916	0.09958	0.10004	0.10005
3.0	0.06938	0.07527	0.08019	0.08920	0.09501	0.10144	0.10452	0.10613	0.10759	0.10816	0.10882	0.10883
4.0	0.07219	0.07864	0.08411	0.09445	0.10144	0.10966	0.11390	0.11625	0.11850	0.11943	0.12054	0.12055
5.0	0.07337	0.08008	0.08582	0.09686	0.10452	0.11390	0.11902	0.12199	0.12498	0.12628	0.12793	0.12794
6.0	0.07394	0.08079	0.08667	0.09808	0.10613	0.11625	0.12199	0.12545	0.12909	0.13075	0.13297	0.13299
8.0	0.07443	0.08139	0.08740	0.09916	0.10759	0.11850	0.12498	0.12909	0.13369	0.13597	0.13937	0.13941
10.0	0.07461	0.08161	0.08767	0.09958	0.10816	0.11943	0.12628	0.13075	0.13597	0.13871	0.14326	0.14331
50.0	0.07480	0.08186	0.08798	0.10004	0.10882	0.12054	0.12793	0.13297	0.13937	0.14326	0.15504	0.15580
100.0	0.07480	0.08186	0.08798	0.10005	0.10883	0.12055	0.12794	0.13299	0.13941	0.14331	0.15580	0.15710

Table 10. Function "K"

M \ N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	-.00000	-.00001	-.00002	-.00004	-.00006	-.00008	-.00010	-.00012	-.00014	-.00016	-.00020	-.00023
0.2	-.00001	-.00004	-.00010	-.00016	-.00024	-.00032	-.00041	-.00049	-.00057	-.00064	-.00078	-.00089
0.3	-.00002	-.00010	-.00021	-.00035	-.00052	-.00070	-.00088	-.00106	-.00124	-.00140	-.00170	-.00195
0.4	-.00004	-.00016	-.00035	-.00060	-.00088	-.00119	-.00150	-.00181	-.00211	-.00240	-.00291	-.00335
0.5	-.00006	-.00024	-.00052	-.00088	-.00130	-.00176	-.00222	-.00269	-.00314	-.00357	-.00435	-.00502
0.6	-.00008	-.00032	-.00070	-.00119	-.00176	-.00237	-.00301	-.00364	-.00426	-.00486	-.00594	-.00687
0.7	-.00010	-.00041	-.00088	-.00150	-.00222	-.00301	-.00382	-.00464	-.00544	-.00621	-.00762	-.00884
0.8	-.00012	-.00049	-.00106	-.00181	-.00269	-.00364	-.00464	-.00565	-.00663	-.00759	-.00934	-.01087
0.9	-.00014	-.00057	-.00124	-.00211	-.00314	-.00426	-.00544	-.00663	-.00781	-.00895	-.01106	-.01291
1.0	-.00016	-.00064	-.00140	-.00240	-.00357	-.00486	-.00621	-.00759	-.00895	-.01027	-.01274	-.01492
1.2	-.00020	-.00078	-.00170	-.00291	-.00435	-.00594	-.00762	-.00934	-.01106	-.01274	-.01591	-.01875
1.4	-.00023	-.00089	-.00195	-.00335	-.00502	-.00687	-.00884	-.01087	-.01291	-.01492	-.01875	-.02223
1.6	-.00025	-.00099	-.00216	-.00372	-.00558	-.00765	-.00987	-.01218	-.01450	-.01681	-.02124	-.02531
1.8	-.00027	-.00106	-.00234	-.00402	-.00604	-.00831	-.01075	-.01328	-.01586	-.01842	-.02339	-.02799
2.0	-.00029	-.00113	-.00248	-.00428	-.00643	-.00886	-.01148	-.01421	-.01700	-.01979	-.02523	-.03032
2.5	-.00032	-.00125	-.00275	-.00475	-.00715	-.00988	-.01284	-.01596	-.01916	-.02239	-.02878	-.03486
3.0	-.00034	-.00133	-.00292	-.00505	-.00763	-.01056	-.01376	-.01713	-.02062	-.02416	-.03122	-.03803
4.0	-.00036	-.00142	-.00313	-.00541	-.00819	-.01136	-.01483	-.01853	-.02236	-.02629	-.03419	-.04194
5.0	-.00037	-.00147	-.00324	-.00561	-.00849	-.01179	-.01541	-.01928	-.02331	-.02744	-.03582	-.04412
6.0	-.00038	-.00149	-.00330	-.00572	-.00867	-.01204	-.01576	-.01972	-.02387	-.02813	-.03680	-.04543
8.0	-.00039	-.00152	-.00337	-.00584	-.00885	-.01231	-.01612	-.02020	-.02448	-.02888	-.03786	-.04686
10.0	-.00039	-.00154	-.00340	-.00590	-.00895	-.01245	-.01631	-.02044	-.02477	-.02924	-.03839	-.04757
50.0	-.00040	-.00157	-.00346	-.00601	-.00912	-.01269	-.01664	-.02088	-.02533	-.02993	-.03937	-.04890
100.0	-.00040	-.00157	-.00347	-.00601	-.00912	-.01270	-.01665	-.02089	-.02535	-.02995	-.03940	-.04895

Table 10. Function "K"-continued

M \ N	1.6	1.8	2.0	2.5	3.0	4	5	6	8	10	50	100
0.1	-.00025	-.00027	-.00029	-.00032	-.00034	-.00036	-.00037	-.00038	-.00039	-.00039	-.00040	-.00040
0.2	-.00099	-.00106	-.00113	-.00125	-.00133	-.00142	-.00147	-.00149	-.00152	-.00154	-.00157	-.00157
0.3	-.00216	-.00234	-.00248	-.00275	-.00292	-.00313	-.00324	-.00330	-.00337	-.00340	-.00346	-.00347
0.4	-.00372	-.00402	-.00428	-.00475	-.00505	-.00541	-.00561	-.00572	-.00584	-.00590	-.00601	-.00601
0.5	-.00558	-.00604	-.00643	-.00715	-.00763	-.00819	-.00849	-.00867	-.00885	-.00895	-.00912	-.00912
0.6	-.00765	-.00831	-.00886	-.00988	-.01056	-.01136	-.01179	-.01204	-.01231	-.01245	-.01269	-.01270
0.7	-.00987	-.01075	-.01148	-.01284	-.01376	-.01483	-.01541	-.01576	-.01612	-.01631	-.01664	-.01665
0.8	-.01218	-.01328	-.01421	-.01596	-.01713	-.01853	-.01928	-.01972	-.02020	-.02044	-.02088	-.02089
0.9	-.01450	-.01586	-.01700	-.01916	-.02062	-.02236	-.02331	-.02387	-.02448	-.02477	-.02533	-.02535
1.0	-.01681	-.01842	-.01979	-.02239	-.02416	-.02629	-.02744	-.02813	-.02888	-.02924	-.02993	-.02995
1.2	-.02124	-.02339	-.02523	-.02878	-.03122	-.03419	-.03582	-.03680	-.03786	-.03839	-.03937	-.03940
1.4	-.02531	-.02799	-.03032	-.03486	-.03803	-.04194	-.04412	-.04543	-.04686	-.04757	-.04890	-.04895
1.6	-.02895	-.03216	-.03497	-.04051	-.04444	-.04937	-.05214	-.05383	-.05567	-.05660	-.05833	-.05839
1.8	-.03216	-.03587	-.03914	-.04568	-.05039	-.05637	-.05979	-.06188	-.06419	-.06535	-.06754	-.06761
2.0	-.03497	-.03914	-.04286	-.05036	-.05585	-.06292	-.06701	-.06955	-.07235	-.07377	-.07646	-.07656
2.5	-.04051	-.04568	-.05036	-.06007	-.06742	-.07724	-.08313	-.08687	-.09107	-.09324	-.09740	-.09754
3.0	-.04444	-.05039	-.05585	-.06742	-.07643	-.08890	-.09664	-.10166	-.10743	-.11046	-.11637	-.11658
4.0	-.04937	-.05637	-.06292	-.07724	-.08890	-.10594	-.11720	-.12484	-.13403	-.13903	-.14921	-.14958
5.0	-.05214	-.05979	-.06701	-.08313	-.09664	-.11720	-.13148	-.14157	-.15422	-.16137	-.17668	-.17726
6.0	-.05383	-.06188	-.06955	-.08687	-.10166	-.12484	-.14157	-.15379	-.16969	-.17904	-.20014	-.20097
8.0	-.05567	-.06419	-.07235	-.09107	-.10743	-.13403	-.15422	-.16969	-.19111	-.20459	-.23851	-.23998
10.0	-.05660	-.06535	-.07377	-.09324	-.11046	-.13903	-.16137	-.17904	-.20459	-.22152	-.26898	-.27126
50.0	-.05833	-.06754	-.07646	-.09740	-.11637	-.14921	-.17668	-.20014	-.23851	-.26898	-.46126	-.49789
100.0	-.05839	-.06761	-.07656	-.09754	-.11658	-.14958	-.17726	-.20097	-.23998	-.27126	-.49789	-.56952

$$\sigma_y = p\left|2\nu A + (1 - 2\nu)E - F\right| \qquad (15)$$

$$s_{yz} = - pG \qquad (16)$$

$$s_{xz} = - pH \qquad (17)$$

$$s_{xy} = p\left|(1 - 2\nu)K + J\right| \qquad (18)$$

$$\sigma_m = 2/3 \cdot pA(1 + \nu) \qquad (19)$$

3.1.2 Formulae for Strains

The same functions A, B etc. tabulated can be used in conjunction with the following relations to obtain the linear strain components $\epsilon_x, \epsilon_y, \epsilon_z$ in x,y,z directions and the bulk strain ϵ which is the sum of the three linear strains.

$$\epsilon_z = p\,\frac{1 + \nu}{E}\left|(1 - 2\nu)A + B\right| \qquad (20)$$

$$\epsilon_x = p\,\frac{1 + \nu}{E}\left|(1 - 2\nu)C - D\right| \qquad (21)$$

$$\epsilon_y = p\,\frac{1 + \nu}{E}\left|(1 - 2\nu)E - F\right| \qquad (22)$$

$$\epsilon = p\,\frac{2(1 + \nu)}{E} \cdot (1 - 2\nu)A \qquad (23)$$

3.1.3 Sign Convention for Superposition of Stresses

In using the tables to determine the stresses at a point due to a more complicated load configuration, the choice of correct sign depending on the orientation of the figure becomes very important. This is applicable to shear stresses only, since the normal stresses are always positive, being compressive. The tables are prepared for the orientation of the rectangle relative to the x and y axis, with the side 2a along the x axis (Fig. 4).

The tables are used to compute the stresses beneath the corner situated at the origin of the axis system (Fig. 4). The stresses thus computed should carry the sign according to the sign convention shown in Fig. 7. The figure indicates the sign attached to each stress when the load rectangle is situated in the respective quadrant.

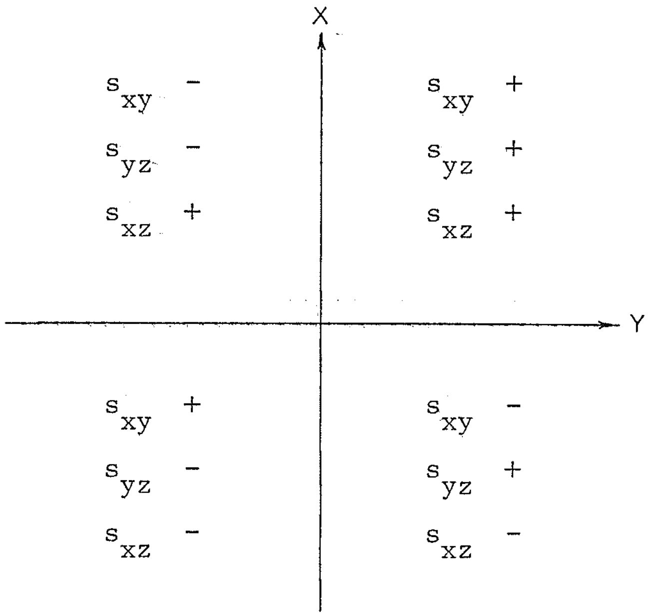


Fig. 7 Sign Convention for Shear Stresses

3.1.4 Resultant Deviatoric Stress Component

With the six stress components known at a point, the stresses on any plane through that point can be determined, in particular, the normal stresses on those planes on which no shear stresses act, which are the principal stresses σ_1, σ_2 and σ_3. It will be seen that the determination of these principal stresses is a tedious matter involving the solution of a cubic equation. This can be avoided by using the equation (4).

Unlike the mean normal stress (σ_m) , no simple relation can be found for the resultant deviatoric component (S_d) . In the particular case with $\nu = 0.5$, the resultant deviatoric component (S_d) becomes:

$$S_d = p\sqrt{2(B^2 + G^2 + H^2 + J^2 - DF)} \tag{24}$$

where, B,D,F,G,H and J should be obtained by summing up the individual values attached to each of the rectangles when used on a composite load configuration, provided the sign convention given in Figure 7 is adhered to.

3.2 Results for the Infinite Line Load

Equations (6) and (7) in Chapter 2 express the mean normal stress and the resultant deviatoric stress respectively in terms of Poisson's ratio and the linear dimensions x and z. It is seen from these equations that, for a given Poisson's ratio, the two stress components depend only on the ratio z/R^2 (Figure 5). It can be shown that the ratio z/R^2 is a constant for a circle in xz-plane having the centre on z-axis, passing through the point of application of the load. Therefore, the constant σ_m lines and constant S_d lines are circles as shown in Figures 8 and 9 respectively.

In addition, the influence values are tabulated in terms of a single variable 'm', where $m = x/z$. Using this relation $m = x/z$, the equations (6) and (7) reduces to:

$$\sigma_m = \frac{q}{z} \cdot \frac{1 + \nu}{3} \cdot I \tag{25}$$

$$S_d = \frac{q}{z} \cdot \sqrt{2/3 \cdot (1 - \nu + \nu^2)} \cdot I \tag{26}$$

where, $I = \frac{2}{\pi} \cdot \frac{1}{m^2 + 1}$

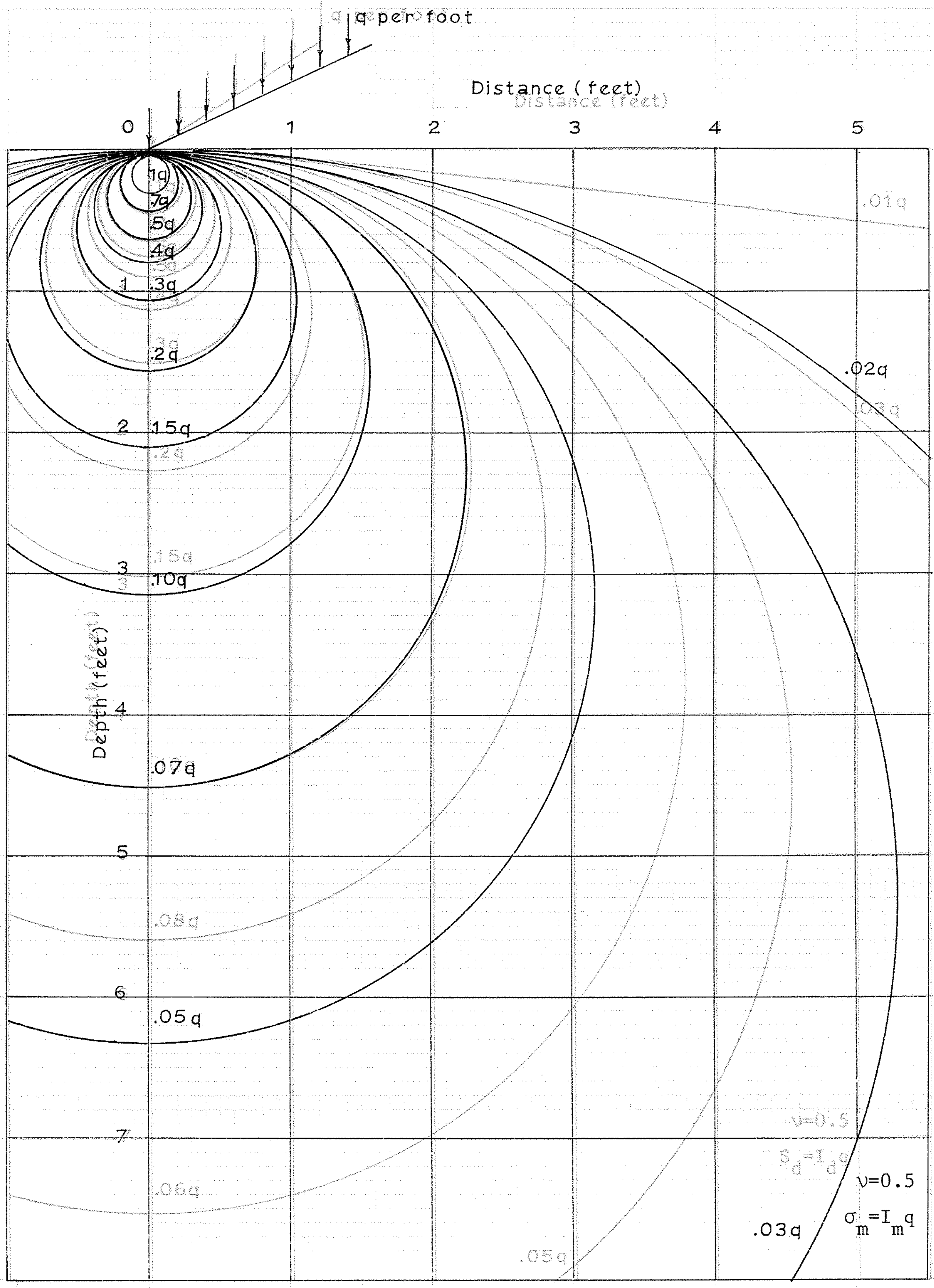


Fig. 8 Influence Factors for Increase in Volumetric Stress beneath an Infinite Line Load

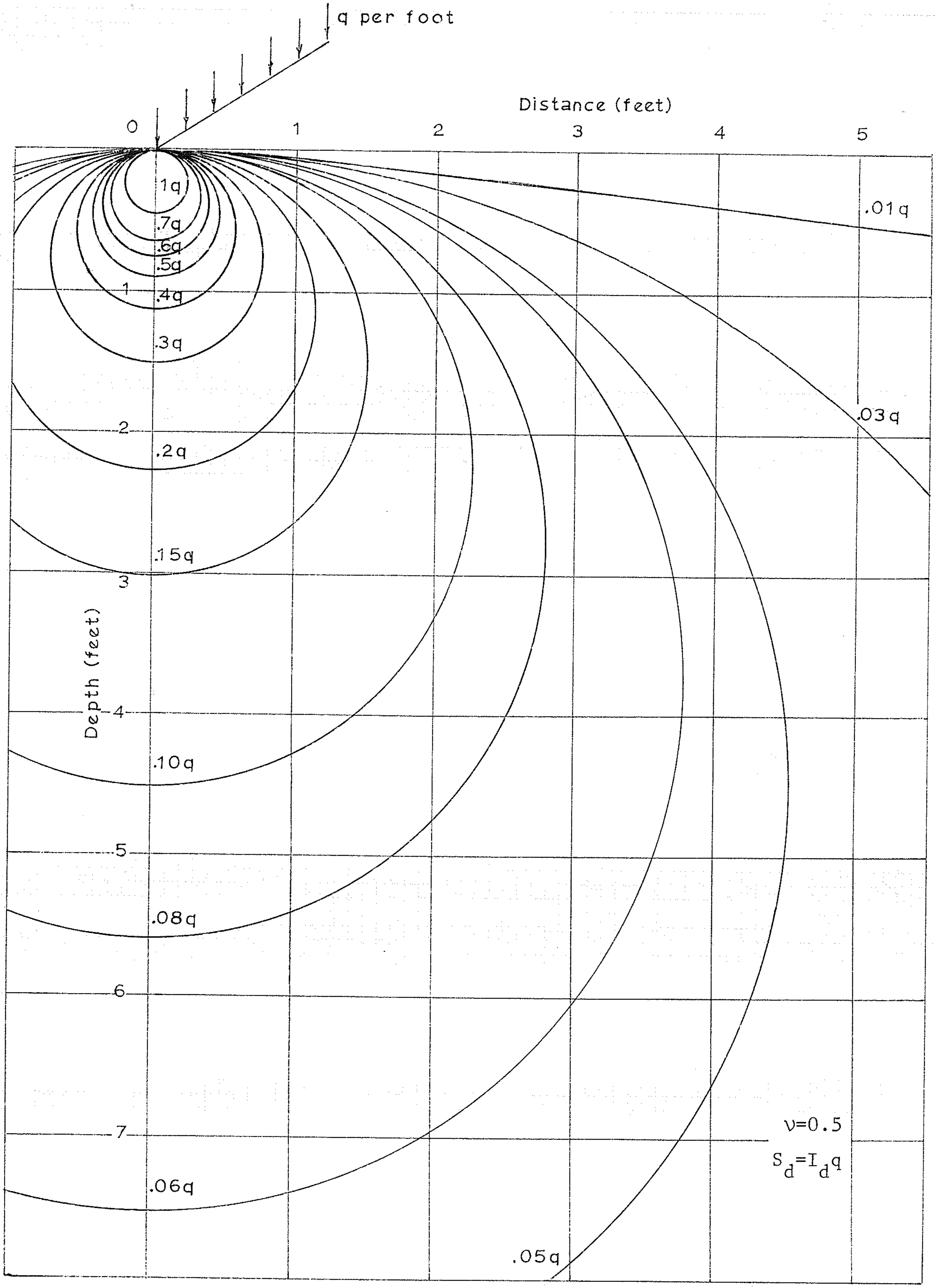


Fig. 9 Influence Factors for Increase in Resultant Deviatoric Stress beneath an Infinite Line Load

The values of I are tabulated in Table (11) for various values of 'm'.

3.3 Influence Values for Strip Load

The equations for mean normal stress and resultant deviatoric stress due to a uniformly loaded strip of infinite extension were derived and are given in Chapter 2 by equations (9) and (10). It is seen from these equations that equal σ_m values and equal S_d values both depend on the angle α shown in Figure 6.

These equations can be rewritten in the following form:

$$\sigma_m = q \cdot (1 + \nu) \cdot L \tag{27}$$

$$S_d = q\sqrt{(1 - 2\nu)^2 M + N} \tag{28}$$

where,

$$L = \frac{2}{3} \cdot \frac{\alpha}{\pi} \tag{29}$$

$$M = \frac{2}{3}\left(\frac{\alpha}{\pi}\right)^2 \tag{30}$$

$$N = 2\left(\frac{\sin\alpha}{\pi}\right)^2 \tag{31}$$

The problem of determining influence values thus reduces to the evaluation of α, for different points in the grid. For the sake of simplification, the following substitution was used in the computations:

$$\begin{aligned} x &= jb \text{ , and} \\ z &= kb \end{aligned} \tag{32}$$

where, b = half-width of the strip.

The mathematics of the development of the solution for evaluation of α is given in Appendix A.

TABLE II INFLUENCE VALUES FOR LINE LOAD OF INFINITE LENGTH

m	I	m	I	m	I	m	I
0.0	0.63662	3.0	0.06366	6.0	0.01721	9.0	0.00776
0.1	0.63032	3.1	0.06000	6.1	0.01666	9.5	0.00698
0.2	0.61214	3.2	0.05664	6.2	0.01614	10.0	0.00630
0.3	0.58406	3.3	0.05354	6.3	0.01565	11.0	0.00522
0.4	0.54881	3.4	0.05069	6.4	0.01517	12.0	0.00439
0.5	0.50930	3.5	0.04805	6.5	0.01472	13.0	0.00374
0.6	0.46810	3.6	0.04560	6.6	0.01429	14.0	0.00323
0.7	0.42726	3.7	0.04334	6.7	0.01387	15.0	0.00282
0.8	0.38818	3.8	0.04123	6.8	0.01348	16.0	0.00248
0.9	0.35172	3.9	0.03927	6.9	0.01310	17.0	0.00220
1.0	0.31831	4.0	0.03745	7.0	0.01273	18.0	0.00196
1.1	0.28806	4.1	0.03575	7.1	0.01238	19.0	0.00176
1.2	0.26091	4.2	0.03415	7.2	0.01205	20.0	0.00159
1.3	0.23666	4.3	0.03266	7.3	0.01173	21.0	0.00144
1.4	0.21507	4.4	0.03127	7.4	0.01142	22.0	0.00131
1.5	0.19588	4.5	0.02996	7.5	0.01112	23.0	0.00120
1.6	0.17883	4.6	0.02873	7.6	0.01083	24.0	0.00110
1.7	0.16366	4.7	0.02757	7.7	0.01056	25.0	0.00102
1.8	0.15015	4.8	0.02648	7.8	0.01029	26.0	0.00094
1.9	0.13810	4.9	0.02545	7.9	0.01004	27.0	0.00087
2.0	0.12632	5.0	0.02449	8.0	0.00979	28.0	0.00081
2.1	0.11767	5.1	0.02357	8.1	0.00956	29.0	0.00076
2.2	0.10901	5.2	0.02270	8.2	0.00933	30.0	0.00071
2.3	0.10121	5.3	0.02188	8.3	0.00911	31.0	0.00066
2.4	0.09417	5.4	0.02111	8.4	0.00890	32.0	0.00062
2.5	0.08781	5.5	0.02037	8.5	0.00869	33.0	0.00058
2.6	0.08204	5.6	0.01967	8.6	0.00849	34.0	0.00055
2.7	0.07679	5.7	0.01901	8.7	0.00830	35.0	0.00052
2.8	0.07202	5.8	0.01838	8.8	0.00812	50.0	0.00025
2.9	0.06765	5.9	0.01778	8.9	0.00794	90.0	0.00008

The influence values L,M and N are tabulated for a range of values of j and k , and are given in Tables 12 through 14.

When Poisson's ratio $\nu = 0.5$, the equations (27) and (28) reduce to;

$$\sigma_m = \frac{3}{2} q \ L \qquad (33)$$

$$S_d = q\sqrt{N} \qquad (34)$$

For convenience, $\sqrt{N}$ is also tabulated in Table 15.

The angle α of equations (29) through (31) is constant if point P (Figure 6) lies on a circle having the centre on the Z-axis and passing through the edges of strip load of width 2b. Hence, analytically, the contours of equal σ_m and equal S_d are circles as shown in Figures 10 and 11 respectively.

TABLE 12 INFLUENCE VALUES 'L' FOR STRIP FOOTING OF INFINITE LENGTH

k/j	0	.3	.5	.8	1.0	1.5	2.0	3.0	4.0	5.0	8.0	10
0.0	.66667	.66667	.66667	.66667	.33333	0	0	0	0	0	0	0
0.3	.54297	.53262	.51010	.42307	.30174	.08934	.04070	.01571	.00843	.00528	.00202	.00128
0.5	.46989	.45713	.43172	.35658	.28135	.12478	.06334	.02560	.01390	.00875	.00335	.00214
0.8	.38030	.36881	.34790	.29657	.25259	.14907	.08788	.03886	.02163	.01376	.00533	.00341
1.0	.33333	.32379	.30694	.26761	.23494	.15420	.09839	.04640	.02693	.01694	.00663	.00424
1.5	.24956	.24419	.23494	.21403	.19678	.15038	.11017	.06042	.03654	.02415	.00975	.00629
2.0	.19678	.19375	.18854	.17666	.16667	.13816	.11017	.06828	.04403	.03011	.01265	.00824
3.0	.13655	.13542	.13343	.12881	.12478	.11238	.09839	.07200	.05199	.03817	.01764	.01178
4.0	.10397	.10345	.10252	.10033	.09839	.09215	.08457	.06828	.05359	.04189	.02142	.01474
5.0	.08378	.08350	.08300	.08181	.08075	.07724	.07279	.06244	.05199	.04272	.02401	.01708
8.0	.05278	.05271	.05258	.05227	.05199	.05103	.04974	.04640	.04240	.03817	.02659	.02076
10.0	.04230	.04226	.04220	.04204	.04189	.04138	.04070	.03886	.03654	.03393	.02591	.02126

TABLE 13 INFLUENCE VALUES 'M' FOR STRIP FOOTING OF INFINITE LENGTH

k\j	0	.3	.5	.8	1.0	1.5	2.0	3.0	4.0	5.0	8.0	10
0.0	.66667	.66667	.66667	.66667	.16667	0	0	0	0	0	0	0
0.3	.44222	.42552	.39030	.26848	.13657	.01197	.00248	.00037	.00011	.00004	.00001	.00000
0.5	.33119	.31345	.27958	.19073	.11873	.02335	.00602	.00098	.00029	.00011	.00002	.00001
0.8	.21694	.20403	.18155	.13193	.09570	.03333	.01159	.00226	.00070	.00028	.00004	.00002
1.0	.16667	.15726	.14132	.10742	.08280	.03567	.01452	.00323	.00104	.00043	.00007	.00003
1.5	.09342	.08944	.08280	.06872	.05808	.03392	.01820	.00548	.00200	.00087	.00014	.00006
2.0	.05808	.05631	.05332	.04681	.04167	.02863	.01820	.00699	.00291	.00136	.00024	.00010
3.0	.02797	.02751	.02671	.02489	.02335	.01894	.01452	.00778	.00405	.00218	.00047	.00021
4.0	.01622	.01605	.01577	.01510	.01452	.01274	.01073	.00699	.00431	.00263	.00069	.00033
5.0	.01053	.01046	.01033	.01004	.00978	.00895	.00795	.00585	.00405	.00274	.00086	.00044
8.0	.00418	.00417	.00415	.00410	.00405	.00391	.00371	.00323	.00270	.00218	.00106	.00065
10.0	.00268	.00268	.00267	.00265	.00263	.00257	.00248	.00226	.00200	.00173	.00101	.00068

TABLE 14 INFLUENCE VALUES 'N' FOR STRIP FOOTING OF INFINITE LENGTH

k\j	0	.3	.5	.8	1.0	1.5	2.0	3.0	4.0	5.0	8.0	10
0.0	0	0	0	0	.20264	.0	.0	.0	.0	.0	.0	.0
0.3	.06140	.07066	.09169	.16852	.19818	.03384	.00736	.00111	.00032	.00013	.00002	.00001
0.5	.12969	.14116	.16211	.20022	.19072	.06235	.01753	.00293	.00087	.00034	.00005	.00002
0.8	.19288	.19703	.20169	.19662	.17469	.08460	.03281	.00672	.00210	.00085	.00013	.00005
1.0	.20264	.20223	.19952	.18382	.16211	.08944	.04053	.00954	.00312	.00129	.00020	.00008
1.5	.17267	.16894	.16211	.14507	.12969	.08583	.04988	.01599	.00595	.00261	.00043	.00018
2.0	.12969	.12691	.12206	.11085	.10132	.07443	.04988	.02026	.00860	.00405	.00072	.00031
3.0	.07295	.07191	.07010	.06593	.06235	.05172	.04053	.02245	.01192	.00648	.00140	.00062
4.0	.04488	.04446	.04373	.04202	.04053	.03587	.03052	.02026	.01265	.00779	.00206	.00098
5.0	.02998	.02979	.02945	.02866	.02795	.02568	.02292	.01704	.01192	.00810	.00258	.00131
8.0	.01228	.01225	.01219	.01205	.01192	.01149	.01093	.00954	.00798	.00648	.00317	.00193
10.0	.00795	.00793	.00791	.00785	.00779	.00761	.00736	.00672	.00595	.00514	.00301	.00203

TABLE 15 INFLUENCE VALUES '$\sqrt{N}$' FOR STRIP FOOTING OF INFINITE LENGTH, (when $\nu=0.5$ $S_d=\sqrt{N}\cdot q$)

k\j	0	.3	.5	.8	1.0	1.5	2.0	3.0	4.0	5.0	8.0	10
0.0	.00000	.00000	.00000	.00000	.45016	.0	.0	.0	.0	.0	.0	.0
0.3	.24780	.26582	.30281	.41051	.44518	.18396	.08581	.03329	.01788	.01121	.00428	.00273
0.5	.36013	.37571	.40263	.44746	.43672	.24970	.13239	.05417	.02946	.01855	.00712	.00454
0.8	.43918	.44388	.44910	.44342	.41796	.29086	.18114	.08197	.04581	.02917	.01131	.00723
1.0	.45016	.44970	.44668	.42874	.40263	.29907	.20132	.09765	.05584	.03590	.01406	.00900
1.5	.41553	.41102	.40263	.38087	.36013	.29296	.22334	.12645	.07713	.05111	.02068	.01333
2.0	.36013	.35624	.34937	.33294	.31831	.27282	.22334	.14235	.09274	.06366	.02683	.01747
3.0	.27009	.26816	.26477	.25677	.24970	.22741	.20132	.14982	.10918	.08053	.03738	.02497
4.0	.21184	.21085	.20912	.20500	.20132	.18939	.17469	.14235	.11248	.08828	.04535	.03124
5.0	.17314	.17259	.17161	.16928	.16718	.16025	.15140	.13055	.10918	.09001	.05083	.03619
8.0	.11081	.11066	.11040	.10976	.10918	.10721	.10456	.09765	.08936	.08053	.05627	.04398
10.0	.08914	.08906	.08892	.08859	.08828	.08723	.08581	.08197	.07713	.07168	.05482	.04502

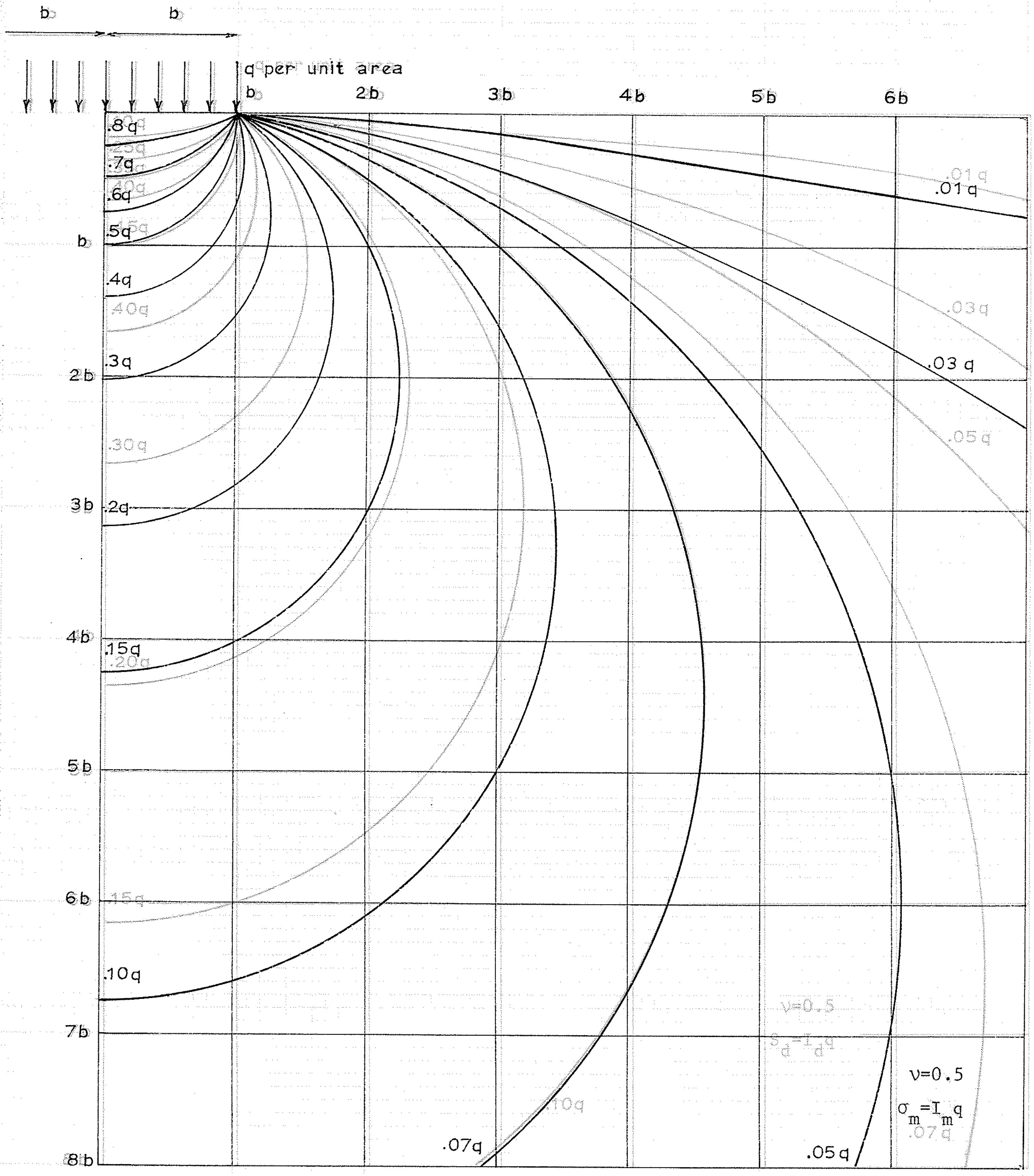


Fig. 10 Influence Factors for Increase in Volumetric Stress beneath an Infinite Strip Load

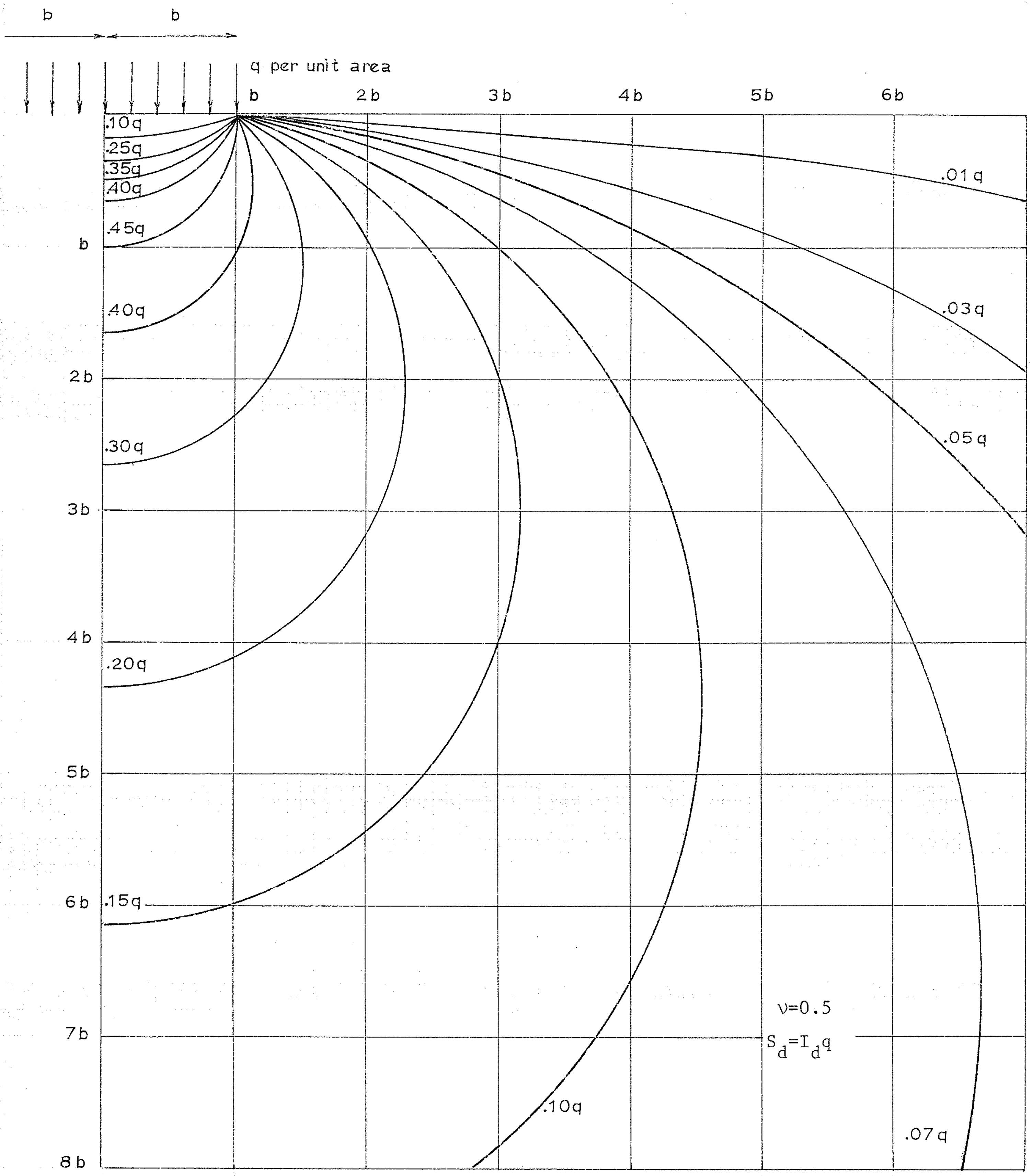


Fig. 11 Influence Factors for Increase in Resultant Deviatoric Stress beneath an Infinite Strip Load

CHAPTER 4

PRACTICAL APPLICATION OF STRESS INFLUENCE CHARTS

In the study of many soil mechanic problems like stress-deformation characteristics, mode and conditions of failure, the effects of the intermediate principal stress σ_2 are not included. In other instances, like in the triaxial test, σ_2 is made equal to σ_3 .

The conventional stress-strain diagrams are obtained for triaxial tests by plotting σ_1/σ_3 and $(\sigma_1 - \sigma_3)$ versus axial strain. The point of failure is sometimes ambiguous, as these two curves may differ considerably. In the study of failure conditions of cohesionless soils, Scott[3] has shown that the application of σ_2 not equal to σ_1 or σ_3 will affect the stresses on the potential failure plane. Kirkpatrick, Kjellmann, as reported by Scott,[3] have shown experimentally that the failure conditions are altered when σ_2 is greater than σ_3 as compared with the case when they are equal. Thus it would be wise to include the intermediate principal stress in failure criteria.

The study of soil deformation under stress, as in settlement problems, requires the relationship between stress and strain. To describe the behaviour of a Hookean elastic material, two elastic constants such as Young's modulus (E) and Poisson's ratio (ν) are required. It is also found that two stress deformation relationships exist, one relating volume change to the hydrostatic stress and the other relating shearing deformation to the shearing stress. The volumetric strain is taken to be linearly propotional to the volumetric stress, the constant of propotionality being the bulk modulus, K. The shear relation is also linear, the constant of propotionality being the shear modulus, G. Thus it is possible to replace the two elastic constants E and ν by bulk modulus and shear modulus. Soils do not exhibit constant elastic parameters, and due to the difficulty in determining the elastic parameters E and ν the latter parameters are preferred. An advantage in using the parameters, G and K is that they can be investigated separately since each is associated with a separate component of physical behaviour. Another limitation of the use of Young's modulus in soils is that, in the absence of any other practical method, the elastic modulus is determined from a plot of stress-difference versus axial strain as obtained in the case of a triaxial test. However, bulk modulus and shear modulus make use of the complete stress and strain pattern.

Another property of interest in clays is the excess pore pressure distribution in the soil due to applied stress. The excess pore pressure produced by an applied load has been related to volumetric and deviatoric components of stress by two parameters α and β (Domaschuk[7]).

Thus many of the soil mechanic problems can be solved on the basis of two stress components associated with the volume change and shear deformation, to include the complete stress pattern. These solutions can be simplified by using the prepared influence charts for volumetric and deviatoric stresses under different loadings. As an illustration, the following pore pressure analysis was carried out.

A method of predicting the excess pore pressure induced in clays due to imposed boundary loads has been presented by Domaschuk[7]. In this approach, the excess pore pressure is separated into two components, one associated with a change in the mean normal stress $(\Delta\sigma_m)$ and the other associated with a change in the resultant deviatoric stress (ΔS_d). The excess pore pressure (ΔU) is given by:

$$\Delta U = \beta \cdot \Delta\sigma_m + \alpha\, \Delta S_d \tag{35}$$

where, α and β are pore pressure parameters.

In order to use some of the stress influence charts developed, it was decided to analyze the pore pressure data obtained with respect to a grain elevator under load.

4.1 Description of Structure

The grain elevator is located in an area where the soil is composed of two feet of top soil, about 55 feet of stratified clay and silty clay, at least 4 feet of soft glacial till and then dense glacial till.

The elevator itself is a timber structure having an area of 38 feet by 80 feet in plan. It is approximately 110 feet high at its highest point, and has a nominal capacity of 150,000 bushels of grain.

The building is founded on two reinforced concrete slabs 35 feet by 38.5 feet by 2 feet thick. The tops of the slabs are located about 6 inches above the ground level. The 10 feet space between the slabs (Figure 12) is occupied by a 14.5 feet deep boot tank. Beneath the slabs are ten strip footings, 3 feet wide by 1.5 feet deep, and reinforced concrete bearing walls, 1.5 feet thick and 4 feet high, which run the width of the building. The space between the walls was backfilled with compacted gravel prior to slab construction.

The behaviour of the foundation has been studied in detail and reported by J. E. Peters[6]. One of the conclusions of the study is that the nature of the foundation changed from strip footings to rectangular

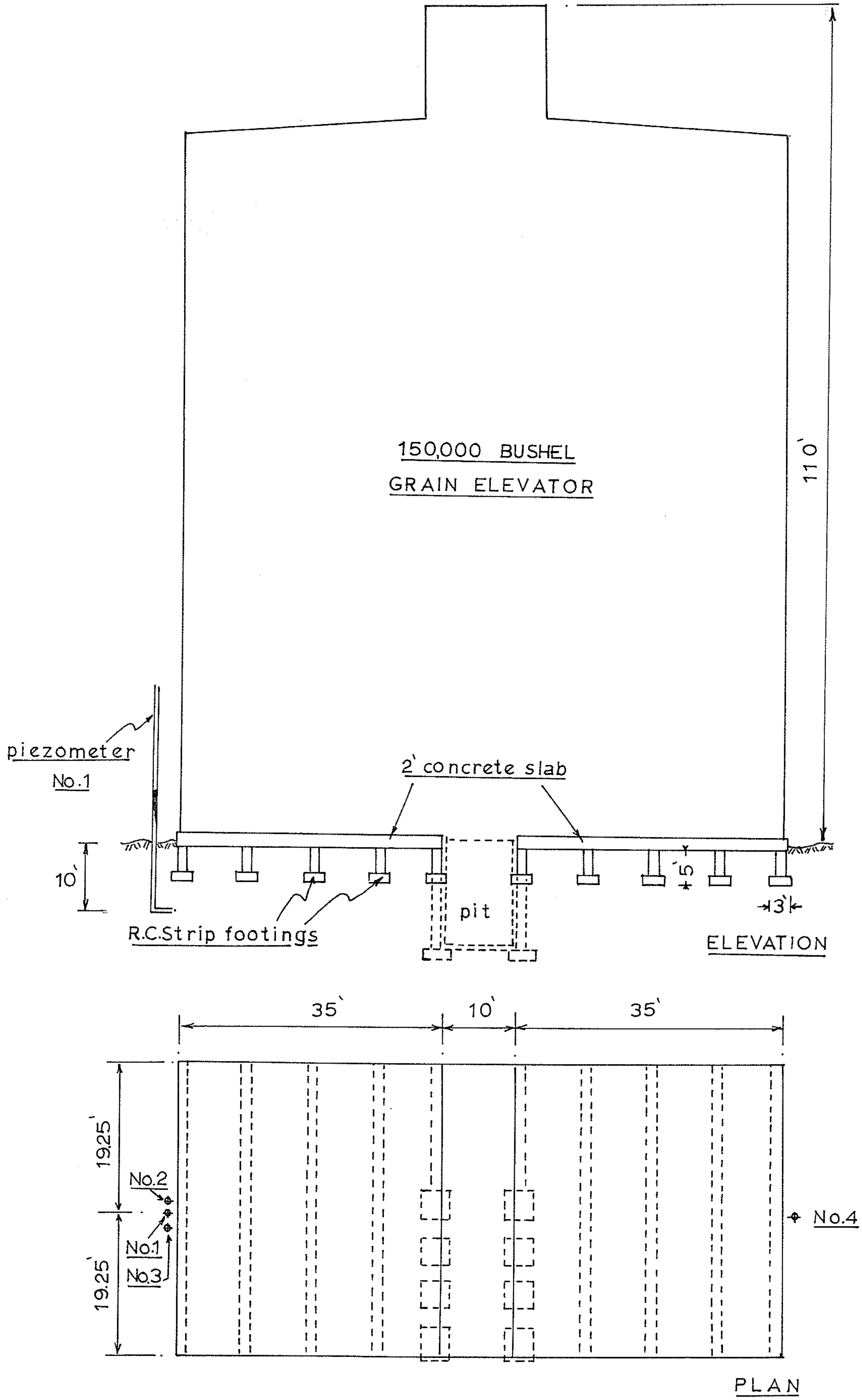

150,000 BUSHEL
GRAIN ELEVATOR
110'
piezometer
No.1
2' concrete slab
10'
5'
3'
pit
R.C.Strip footings
ELEVATION
35'
10'
35'
19.25'
19.25'
No.2
No.1
No.3
No.4
PLAN

FIG.12

footings during the initial loading period. The conclusion has been based on a detailed analysis of settlement and bearing values.

4.2 Instrumentation

Four piezometers were installed, three on the west end and one on the east end of the elevator (Figure 12). They consisted of a piezometer tip connected to a polyethylene tubing which extends to the surface. Piezometer Nos. 1, 2 and 3 were installed on the west end at 10 feet, 25 feet, and 40 feet below the ground surface. Piezometer No. 4 was embedded in glacial till which is a water bearing stratum. The reading on this piezometer was taken as the water table height at that point.

Initial pore pressure readings were taken by using an electric moisture-sensing probe lowered into the polyethylene tube. However, when water levels became high, pressure gauges were installed. A more detailed account of this is given by Peters[6].

4.3 Load and Pore Pressure Data

The loading data and the pore pressure observations were recorded over a period of time. The variations of pore pressure readings with time for piezometer No. 1, 2 and 3 are shown in Figure 13 along with the variation of total load over the same period of time. The loading data are recorded in Appendix C, Table 22. They include the dead load of 2315 kips of the structure. The pore pressure observations for piezometer No. 1, 2 and 3 are tabulated in Appendix C, Tables 19 through 21.

4.4 Stress Computation

Considering the foundation to behave as two rectangular portions

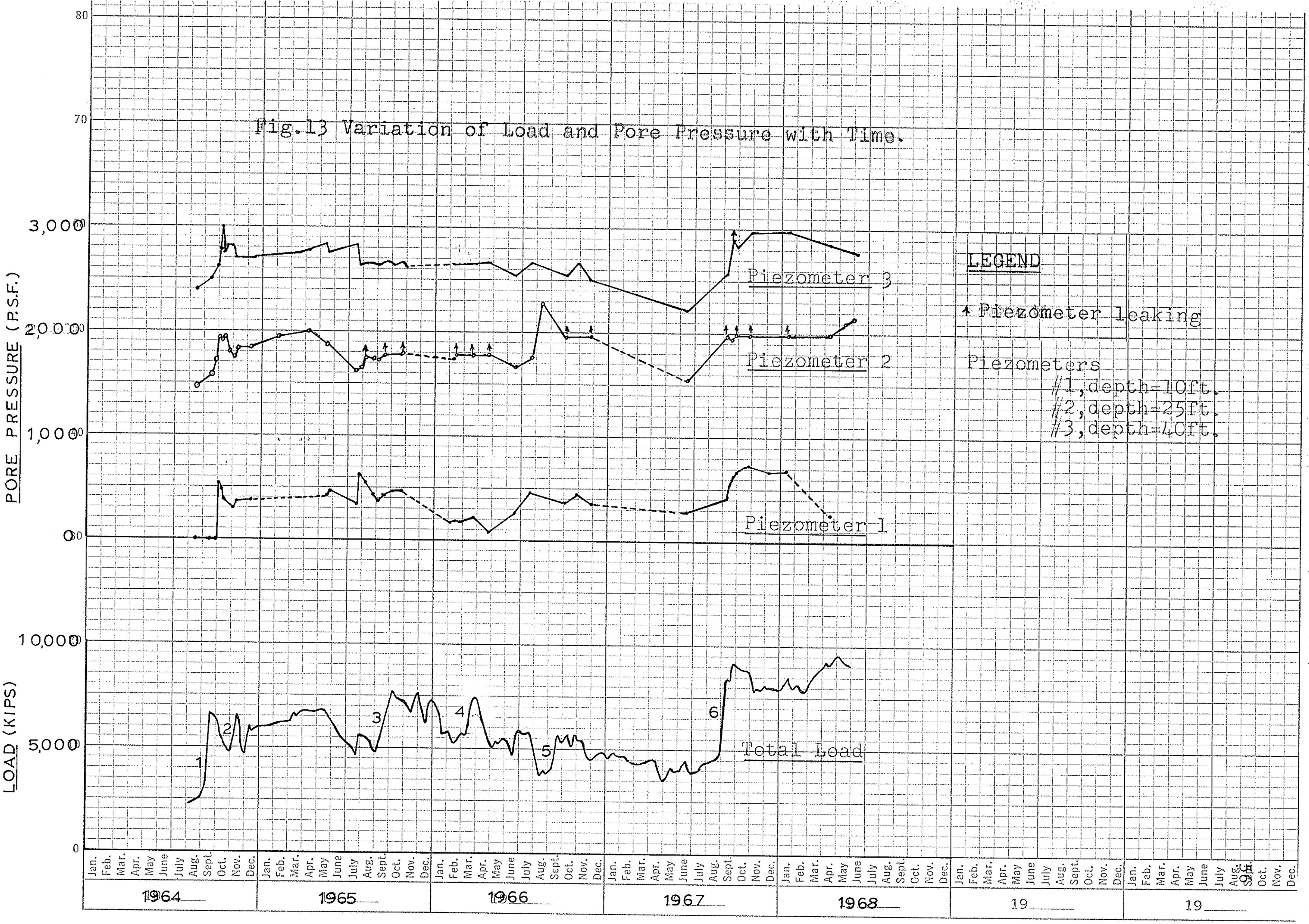


Fig. 13 Variation of Load and Pore Pressure with Time.

separated by a strip of 10 feet in width, the stresses at depth z below the surface, where piezometers were installed, can be calculated.

In order to determine the stresses at P, as shown in Figure 14, the area was divided into two halves and each half into three component rectangles I, II and III as shown in Figure 14. The stresses at the point P are obtained by summation of individual values according to the following equation:

$$\text{Stress at P} = \sum(\text{Stress due to I} - \text{stress due to II} + \text{stress due to III})$$

The mean normal stress and the resultant deviatoric component of stress alone are needed for the pore pressure analysis. As an illustrative problem, the method of computation is given in detail.

The loaded area is placed with respect to the X- and Y-axis as shown in Figure 15 with the point P at the origin and the longer side along the X-axis. Alternately, it may be placed with the longer side along the Y-axis. With the position shown in Figure 15, it can be seen that $s_{xy} = s_{yz} = 0$. This is due to the symmetry of the area about the X-axis, with the two rectangles (Figure 15) having equal and opposite values for s_{xy} and s_{yz}.

Assuming the material to be incompressible with $\nu = 0.5$, equations (13) through (19) reduced to:

$$\begin{aligned} \sigma_z &= p(A - B) \\ \sigma_x &= p(A - D) \\ \sigma_y &= p(A - F) \\ s_{xz} &= -pH \end{aligned} \tag{36}$$

$$\sigma_m = pA \tag{36a}$$

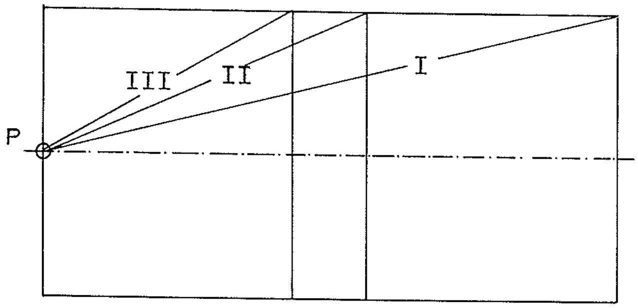


Fig. 14 Division of Rectangular Foundation for Stress Computation

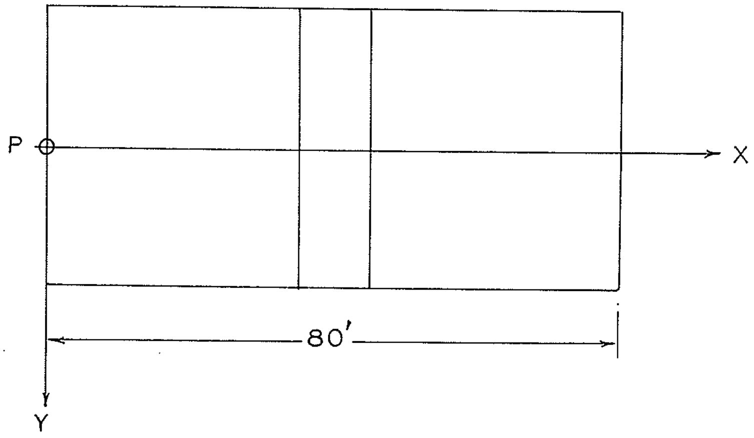


Fig. 15 Position of Loaded Area

Also, the resultant deviatoric component of stress (S_d) given by equation (24) can be reduced to the following form:

$$S_d = p\sqrt{2(B^2 + H^2 - DF)} \tag{37}$$

The influence numbers A,B,D,F and H were obtained from Tables 1 through 10. The determination of the individual values is given in Appendix D. By substituting the values of A,B,D,F and H into equations (36a) and (37), the mean normal stress (σ_m) and the resultant deviatoric stress (S_d) were obtained in terms of the load intensity p .

For the point P at a depth of 10 feet, the following expressions resulted:

$$\sigma_m = 0.334 \text{ p} \tag{38}$$

$$S_d = 0.394 \text{ p} \tag{39}$$

Similarly, for the point at 25 feet below the surface, the following equations apply:

$$\sigma_m = 0.184 \text{ p} \tag{40}$$

$$S_d = 0.322 \text{ p} \tag{41}$$

And, for the point at 40 feet depth, the two equations are given as:

$$\sigma_m = 0.114 \text{ p} \tag{42}$$

$$S_d = 0.222 \text{ p} \tag{43}$$

The details of the computations for the three points are given in Appendix D.

4.5 Pore Pressure Analysis

The change in pore pressure due to an imposed boundary load is given by equation (35). It should be recognized that the equation contains two parameters α and β ; hence, at least one parameter must be known to investigate the other for a set of known data. The pore pressure parameter β has been investigated for samples of Lake Agassiz clay by means of isotropic compression tests, and the ratio of change in pore pressure to the change in volumetric stress was found to be a constant, which is the parameter β , equal to 0.91 (Domaschuk[7]).

In this analysis a value of 0.91 was assumed for the parameter β . The material was assumed to be incompressible and perfectly elastic.

The variation of the total load and pore pressure readings over a period of time is shown in Figure 13. The loads are applied over a duration of time but not instantaneously. An appreciable load change is recorded at very few points. Hence, the increments at points 1 through 6, shown on the load curve in Figure 13, were considered.

At each of these points the total load change was taken and divided by the sum of the areas of the two rectangular foundation slabs to determine the intensity of loading. By substituting these values into equations (38) through (43), the change in mean normal stress $(\Delta\sigma_m)$ and the change in resultant deviatoric stress (ΔS_d) were computed for each point. These values are given in Tables 16 through 18 for the three piezometers.

The pore pressure changes at points 1 through 6 for the three piezometers are recorded in Tables 16 through 18. The pore pressure increment due to an increase in the volumetric stress, denoted by ΔU_m in the tables, was computed assuming a β value of 0.91. This increment was then subtracted

TABLE 16

Results for Piezometer #1, Depth=10 ft.

Region in load curve	1	2	3	4	5	6
$\Delta\sigma_m$ psf.	500	208	345	207	194	550
ΔS_d psf.	590	246	408	244	229	650
ΔU psf.	520	100	100	35	-	275
ΔU_m psf.	455	190	314	188	175	500
ΔU_d psf.	65	-90	-214	-153	-	-225
α	0.11	-0.37	-0.50	-0.63	-	-0.35
$\overline{\sigma}_m$ psf.	1180	1440	1410	1710	-	1200
$\overline{\sigma}_m/\sigma_c$	0.12	0.14	0.14	0.17	-	0.12

TABLE 17

Results for Piezometer #2, Depth=25 ft.

Region in load curve	1	2	3	4	5	6
$\Delta\sigma_m$ psf.	275	115	190	114	107	304
ΔS_d psf.	480	200	332	199	187	530
ΔU psf.	360	90	45	-	-	-
ΔU_m psf.	250	105	173	103	96	274
ΔU_d psf.	110	-15	-128	-	-	-
α	0.23	-0.08	-0.38	-	-	-
$\bar{\sigma}_m$ psf.	1470	1650	1820	-	-	-
$\bar{\sigma}_m/\sigma_c$	0.20	0.23	0.25	-	-	-

TABLE 18

Results for Piezometer #3, Depth=40 ft.

Region in load curve	1	2	3	4	5	6
$\Delta\sigma_m$ psf.	171	71	118	70	66	188
ΔS_d psf.	332	139	230	137	129	366
ΔU psf.	490	0	70	-	-	145
ΔU_m psf.	155	65	110	-	-	170
ΔU_d psf.	335	-65	-40	-	-	-25
α	1.0	-0.47	-0.17	-	-	-0.07
$\overline{\sigma}_m$ psf.	2120	2410	2630	-	-	2400
$\overline{\sigma}_m/\sigma_c$	0.34	0.39	0.42	-	-	0.35

from the measured pore pressure increment to obtain the contribution due to an increase in deviator stresses. These components are designated by ΔU_d in the tables.

It is seen that the components of pore pressure due to an increase in deviator stresses are negative except at the load increment 1. The negative components imply positive dilatancy under shearing strains. The average effective mean normal stress was computed at each point including the body forces. These values are denoted by $\bar{\sigma}_m$ in the tables. The preconsolidation pressures at various depths were determined by laboratory consolidation tests and were reported by Peters[6]. The preconsolidation pressures at 10 feet, 25 feet, and 40 feet are given to be 5 tsf., 3.6 tsf. and 3.1 tsf. respectively.

The ratio of confining stress to preconsolidation stress was computed at each point, denoted by $\bar{\sigma}_m/\sigma_c$. The over consolidation ratio of the soil exceeds a value of 2.5 at all points. For soils having the over-consolidation ratio greater than 2.5, positive dilatancy is expected at small changes in deviatoric stresses as reported by Domaschuk[7], for a sample of blue silty clay.

Values of the α parameter based on the pore pressure increment attributed to changes in deviator stresses, were computed and are given in the Tables 16 through 18. The relationship between changes in pore pressure and changes in deviator stresses is non-linear, the α value is not constant and depends on the magnitude of the deviatoric stress. Hence, it would be proper to investigate the relationship between the pore pressure and the deviator stress rather than the α values at each point.

The observed relationship between the changes in deviator stresses and changes in pore pressure is shown in Figure 16 for the three piezometers. The approximate value of the ratio of the present confining pressure to the preconsolidation pressure $(\bar{\sigma}_m/\sigma_c)$ is fixed to each point.

For the sake of discussion, the curves given by Domaschuk[7] based on laboratory tests for two soil samples from Lake Agassiz basin having $\bar{\sigma}_m/\sigma_c$ ratio of 0.2 were reproduced in Figure 16. The field results are seen to be higher than those obtained in the laboratory, though there exists agreement in a qualitative sense.

The inconsistent pattern of the results may be due to various assumptions used in the analysis with regard to the theory and observations. It is realised that the presence of firmer strata at about 55 feet below will modify the stress values to some extent even though no estimate could be made.

The change in pore pressure due to a change in the deviator stress is positive for all three piezometers at the initial loading. Since, the nature of the foundation changed from strip footings to rectangular footings during this initial loading period, as indicated by Peters[6], the assumption of the rectangular footing in the analysis would be incorrect. The actual nature of the foundation at this point is not known.

Some of the load increments considered occured over a period of about one month. But, in the analysis, the increment is assumed to be instantaneous with no dissipation of excess pore pressure. However, if there was dissipation, the observed pore pressure would be less, and this would result in larger implied negative values for the pore pressure component attributed to changes in deviator stresses. To illustrate this,

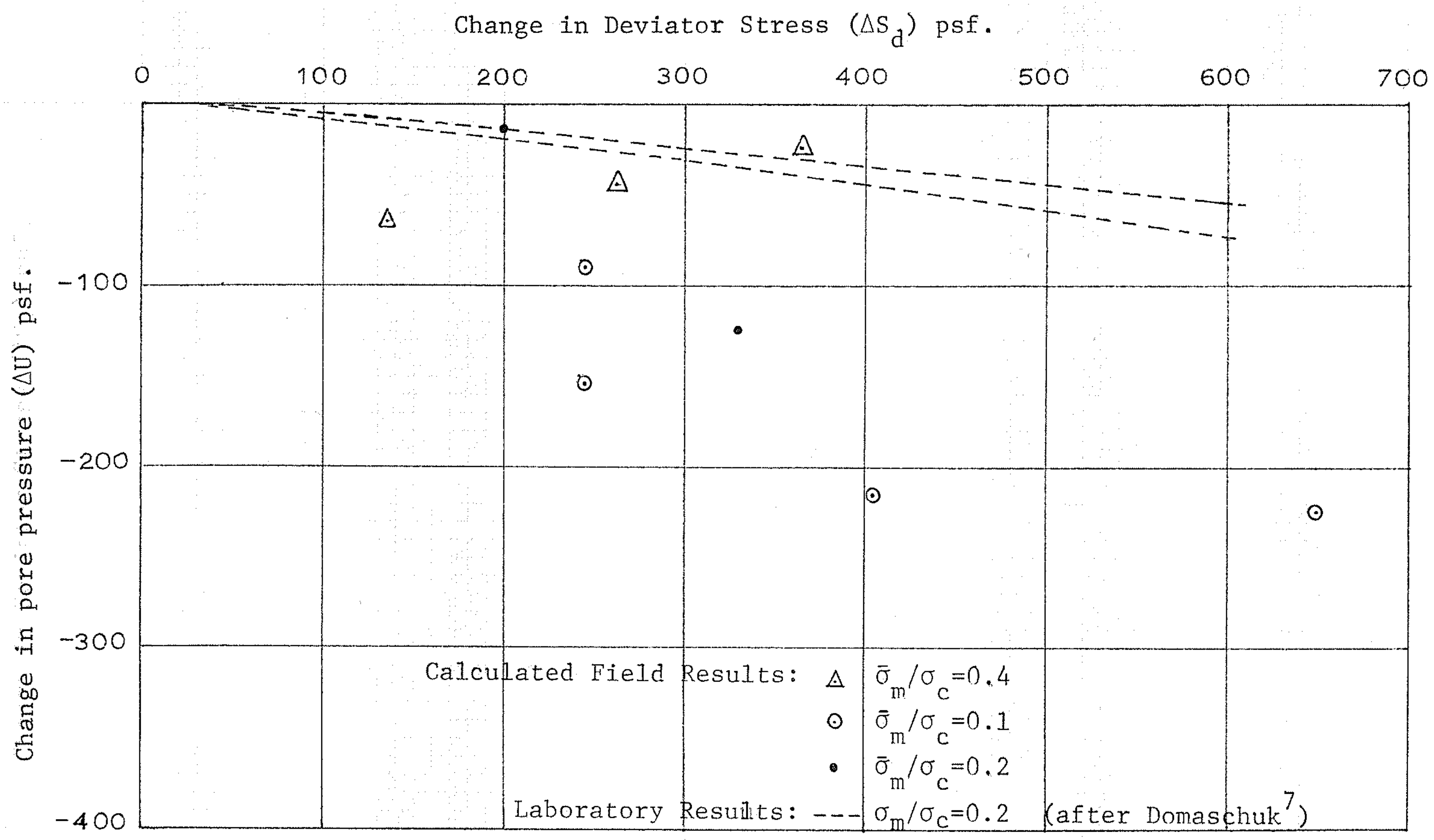


Fig. 16 Relationship between Change in Pore-Pressure and Change in Resultant Deviatoric Stress

consider the load increment indicated by 3 in Figure 13 which occured over a period of about one month. The piezometer No. 2 recorded a pore pressure increment of 45 psf. If there was 25% dissipation, the actual pore pressure increment would be 60 psf. Pore pressure change associated with the volumetric stress is 173 psf. Therefore, the pore pressure change associated with the deviator stress is -113 psf., as compared with -128 psf. given in Table 17. Furthermore, 65% dissipation at this point will make the results agree with those given by Domaschuk[7], in Figure 16. Thus pore pressure dissipation may be one reason why the field values obtained were higher than the laboratory values.

CHAPTER 5

CONCLUSIONS

The influence charts developed for different cases of loading are based on the usual assumptions of the mathematical theory of elasticity. These results will provide useful reference to anyone concerned with the distribution of stress induced by surface loads. These charts and tabulations cut down elaborate calculations which would otherwise be needed in stress calculation problems.

Many of the soil mechanic problems such as settlement, pore pressure, pavement systems and failure criteria can be solved using the concept of separating the effects due to volumetric and deviatoric components of stress. The influence charts and tabulations given will simplify the solutions to these problems.

In the illustrative problem of pore pressure analysis, the stresses were computed using the tabulations developed for the case of uniform load distributed over rectangular area. An attempt to compare the field results obtained here with those reported from laboratory tests by Domaschuk[7] may not be justifiable as the pore pressure parameter α would depend on the stress history of the particular soil. There is no evidence as to whether repetition of loading and unloading has any effect on these parameters, which was the case with the field problem. Further investigation will have to be undertaken with due thought to this fact before any conclusion can be made.

A recommendation for further study would be the investigation into shear modulus, G and bulk modulus, K for various soils. Since, they are associated with the deviatoric and volumetric components of stress respectively. These two parameters can be used in load-deflection and settlement calculations, or they can be used to calculate Young's modulus and Poisson's ratio. It would be necessary to investigate whether the volumetric strain and shear strain are functions of both volumetric stress and deviatoric stress.

BIBLIOGRAPHY

1. Love, A.E.H., "The Stress Produced in a Semi-infinite Solid by Pressure on Part of the Boundary." Philosophical Transactions of the Royal Society of London, Series A, Vol. 228, pp. 377-388, 1929.
2. Love, A.E.H., Mathematical Theory of Elasticity, Cambridge University Press, 1927.
3. Scott, R.F., Principles of Soil Mechanics, Chapter 6 and 7, Addison Wesley Publishing Company, 1963.
4. Harr, M.E., Foundations of Theoretical Soil Mechanics, Chapter 2, McGraw-Hill Book Company, New York, 1966.
5. Gray Hamilton, "Stress Distribution in Elastic Solids", Proceedings of the First International Conference on Soil Mechanics and Foundation Engineering, Vol. 2, pp. 157-168, 1936.
6. Peters, J.E., Settlement and Bearing Capacity Studies of a 150,000 Bushel Grain Elevator", M.Sc. Thesis, University of Manitoba, 1966.
7. Domaschuk, L., "An Analysis of Porewater Stress Through a Separation of the Volumetric and Deviatoric Components of Stress", Preprint of a paper presented at the Twenty-First Canadian Soil Mechanics Conference, Winnipeg, Manitoba, 1968.

APPENDIX A

A.1 Development of Influence Values for Strip Loading

The influence values L,M and N (3.3) require the evaluation of angle α (Fig. 6) for any point P (x,y,z).

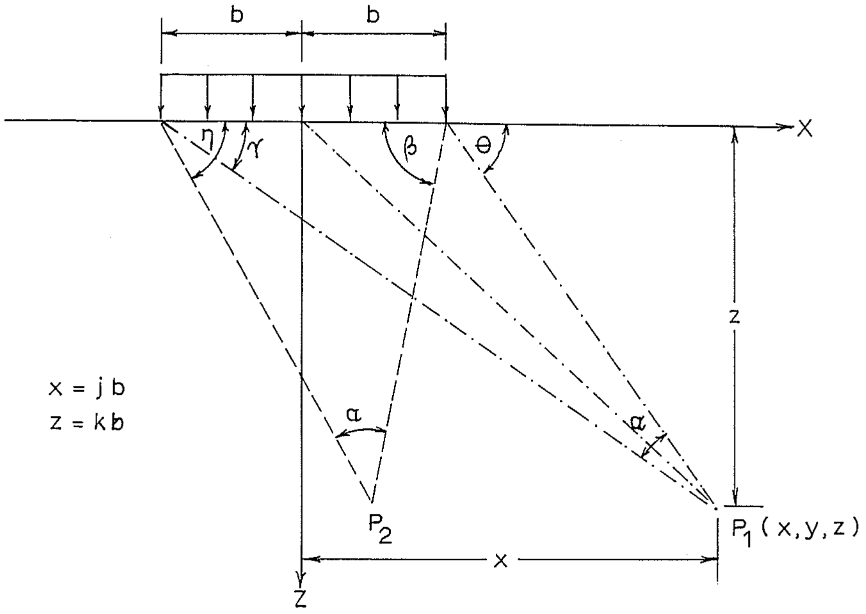


Fig. 17 Notation for Strip Load Problem

For points P (x,y,z) where $x > b$, such as P_1 ,

$$\begin{aligned}\alpha &= \theta - \gamma \\ &= \tan^{-1}\frac{z}{x-b} - \tan^{-1}\frac{z}{x+b} \\ &= \tan^{-1}\frac{k}{j-1} - \tan^{-1}\frac{k}{j+1}\end{aligned}$$

For points P where $x < b$, such as P_2 ,

$$\begin{aligned}\alpha &= \pi - \beta - \eta \\ &= \pi - \tan^{-1}\frac{z}{b-x} - \tan^{-1}\frac{z}{x+b} \\ &= \pi - \tan^{-1}\frac{k}{1-j} - \tan^{-1}\frac{k}{j+1}\end{aligned}$$

Since, the space is symmetrical about z-axis only the positive quadrant is considered.

For the case when $z = 0$ and $x < b$, $\alpha = \pi$; and when $z = 0$ and $x = b$, $\alpha = \pi/2$.

The computer program used on IBM 360/65 is given in the next section.

In the computer program, in addition to L,M,N, the values of $\sqrt{N}$ and 1.5L which appear in equations (33) and (34) are also calculated. These are represented by AI,BI,CI,DI and AII in the program.

A.2 Computer Program

Note: AI, BI, CI, DI and AII represent L, M, N, $\sqrt{N}$ and 1.5L respectively of section (3.3)

```
C     INFLUENCE VALUES BASED ON X=JB Z=KB
       DIMENSION A1(12,12),A2(12,12),A(12,12),S(12,12)
       DIMENSION AI(12,12),BI(12,12),CI(12,12),DI(12,12)
       DIMENSION AII(12,12)
       REAL J(12,12),K(12)
     8 FORMAT('1',///,16X,'TABLE',10X,'INFLUENCE VALUES ''AI'' FOR STRIP
      9FOOTING OF INFINITE LENGTH')
     9 FORMAT(/ ,2X,'K',1X,'J',6X,'0',7X,'.3',7X,'.5',7X,'.8',6X,'1.0',6X,'1.5',6
      1,'1.5',6X,'2.0',6X,'3.0',6X,'4.0',6X,'5.0',6X,'8.0',7X,'10')
    10 FORMAT('1',///,16X,'TABLE',10X,'INFLUENCE VALUES ''BI'' FOR STRIP
      9FOOTING OF INFINITE LENGTH')
    11 FORMAT('1',///,16X,'TABLE',10X,'INFLUENCE VALUES ''CI'' FOR STRIP
      9FOOTING OF INFINITE LENGTH')
    12 FORMAT('1',///,16X,'TABLE',10X,'INFLUENCE VALUES ''DI'' FOR STRIP
      9FOOTING OF INFINITE LENGTH')
       WRITE(6,8)
       WRITE(6,9)
       DO 1 N=1,12
     1 READ(5,2) K(N),(J(N,M),M=1,12)
     2 FORMAT(13F3.1)
       DO 3 N=1,12
       DO 4 M=1,12
       A1(N,M)=0.
       A2(N,M)=0.
       A (N,M)=0.
        S(N,M)=0.
       AI(N,M)=0.
       BI(N,M)=0.
       CI(N,M)=0.
       DI(N,M)=0.
       IF(J(N,M).LE.1.AND.K(N).EQ.0) GO TO 20
       A2(N,M)=ATAN(K(N)/(J(N,M)+1))
       IF(J(N,M).EQ.1) GO TO 15
       A1(N,M)=ABS(ATAN(K(N)/(J(N,M)-1)))
       GO TO 21
    15 A1(N,M)=3.14159/2.
```

```
      A(N,M)=A1(N,M)-A2(N,M)
      GO TO 18
   21 IF (J(N,M).LT.1) GO TO 17
      A(N,M)=A1(N,M)-A2(N,M)
      GO TO 18
   17 A(N,M)=3.14159-A1(N,M)-A2(N,M)
      GO TO 18
   20 A(N,M)=3.14159
      IF(J(N,M).EQ.1) A(N,M)=A(N,M)/2.
   18 AI(N,M)=2.*A(N,M)/3.14159/3.
      AII(N,M)=A(N,M)/3.14159
      ~~IF(A(N,M).EQ.0) GO TO 13~~
      BI(N,M)=2./3.*(A(N,M)/3.14159)**2
      GO TO 14
   ~~13 BI(N,M)=99999999~~
   14 S(N,M)=SIN(A(N,M))
      DI(N,M)=(2.**0.5)*S(N,M)/3.14159
    4 CI(N,M)=2.*S(N,M)**2/(3.14159**2)
      WRITE(6,5)K(N),(AI(N,M),M=1,12)
    5 FORMAT(/ ,1X,F5.1,12(2X,F7.5))
    3 CONTINUE
      WRITE(6,10)
      WRITE(6,9)
      DO 7 N=1,12
    7 WRITE(6,5)K(N),(BI(N,M),M=1,12)
      WRITE(6,11)
      WRITE(6,9)
      DO 6 N=1,12
    6 WRITE(6,5)K(N),(CI(N,M),M=1,12)
      WRITE(6,12)
      WRITE(6,9)
      DO 19 N=1,12
   19 WRITE(6,5) K(N),(DI(N,M),M=1,12)
   23 FORMAT('1')
      WRITE(6,23)
      DO 22 N=1,12
   22 WRITE(6,5) K(N),(AII(N,M),M=1,12)
      RETURN
      END
```

APPENDIX B

B.1 Lames's Elastic Constants

In the transforming the equations (1) to equations (13) through (18), the Lame's constants λ, μ were expressed in terms of Elastic Modulus, E and Poisson's ratio, ν in the following manner:

$$\mu = G = \frac{E}{2(1 + \nu)}$$

$$\lambda = \frac{\nu E}{(1 - 2\nu)(1 + \nu)} \qquad (\text{Gray}^5)$$

$$\frac{\lambda}{\lambda + \mu} = \frac{1}{1 + \mu/\lambda} = 2\nu$$

$$\frac{\mu}{\mu + \lambda} = \frac{1}{1 + \lambda/\mu} = (1 - 2\nu)$$

APPENDIX C
TABLE 19

Record for Piezometer #1

	Height of water above tip (ft.)	Pore Pressure psf.
1964		
Aug. 18	0	0 *
Sept. 17	0	0 *
Sept. 30	0	0 *
Oct. 3	8.32	519
Oct. 5	7.66	478
Oct. 7	6.20	386
Oct. 25	11.72	732
Oct. 29	4.29	268
Nov. 2	4.91	306
Nov. 7	5.83	380
Dec. 10	6.08	380
1965		
Feb. 6	14.99	936
Mar. 27	15.26	952
Apr. 8	15.12	945
May 12	15.13	945
May 19	6.73	421
May 21	7.66	479
July 20	5.30	331
July 27	9.80	612
Aug. 5	8.70	544
Aug. 25	6.80	425
Sept. 3	5.90	368
Sept. 18	6.70	418
Oct. 2	7.40	462
Oct. 30	7.30	456
1966		
Feb. 6	2.90	181
Feb. 12	3.10	194
Feb. 26	2.90	181
Mar. 21	3.45	216
Apr. 30	1.48	92
June 18	4.15	259
July 23	7.50	468
Oct. 1	5.85	365
Oct. 29	6.90	431
Nov. 26	5.45	340
1967		
June 16	4.60	287
Sept. 9	7.10	443
Sept. 21	9.70	605
Sept. 23	9.90	618
Sept. 29	10.10	630
Oct. 28	11.50	718
Dec. 9	10.58	660

* less than

TABLE 20

Record for Piezometer #2

Date	Height of water above tip (ft.)	Pore Pressure psf.
1964		
Aug. 18	23.75	1482
Sept. 17	25.40	1585
Sept. 30	27.40	1710
Oct. 3	31.08	1940
Oct. 5	30.69	1920
Oct. 6	31.08	1940
Oct. 25	28.85	1805
Nov. 2	28.10	1755
Nov. 7	29.51	1840
Dec. 10	29.51	1840
1965		
Feb. 6	31.08	1940
Apr. 8	32.00	1999
May 19	30.05	1880
July 20	25.99	1620
July 27	26.50	1655
Aug. 5	28.40m	1772m
Aug. 25	27.90	1742
Sept. 3	27.70	1730
Sept. 18	28.40m	1772m
Oct. 30	28.40m	1772m
1966		
Feb. 6	28.05	1750
Feb. 12	28.40m	1772m
Mar. 21	28.40m	1772m
Apr. 30	28.40m	1772m
June 18	26.70	1670
July 23	28.20	1760
Oct. 1	31.50m	1970m
Nov. 26	31.50m	1970m
1967		
June 16	24.36	1520
Sept. 21	31.50m	1970m
Sept. 23	30.90	1930
Sept. 29	31.50m	1970m
Oct. 28	31.50m	1970m
1968		
Jan. 13	31.50m	1970m
Apr. 10	31.75	1980
May 11	33.71	2100
June 4	33.86	2117

m piezometer leak

TABLE 21

Record for Piezometer #3

Date	Height of water above tip (ft.)	Pore Pressure psf.
1964		
Aug. 18	38.58	2405
Sept. 17	40.20	2507
Sept. 30	42.00m	2620m
Oct. 3	44.58	2780
Oct. 5	44.19	2760
Oct. 6	48.13	3000
Oct. 7	44.19	2760
Oct. 25	44.32	2763
Oct. 28	44.19	2760
Nov. 2	43.27	2700
Nov. 7	43.40	2703
Dec. 10	43.52	2720
1965		
Mar. 27	44.05	2750
Apr. 8	44.35	2765
May 12	45.11	2820
May 19	44.05	2750
July 20	45.27	2829
July 27	41.90	2620
Aug. 5	42.45	2640
Aug. 25	42.20	2635
Sept. 3	41.90	2620
Sept. 18	42.60	2660
Oct. 30	43.00	2680
1966		
Feb. 6	42.50	2653
Feb. 12	42.37	2640
Mar. 31	42.50	2653
Apr. 30	42.76	2670
June 18	40.40	2520
July 23	42.60	2660
Oct. 1	40.85	2550
Oct. 29	42.60	2660
Nov. 26	40.00	2498
1967		
June 16	35.56	2220
Sept. 9	41.50	2590
Sept. 21	46.50m	2900m
Sept. 23	43.80	2733
Sept. 29	45.50	2840
Oct. 28	47.50	2960
1968		
Jan. 13	47.50	2960
Apr. 10	45.50	2840
June 4	44.50	2780

m piezometer leak

TABLE 22

Load Data

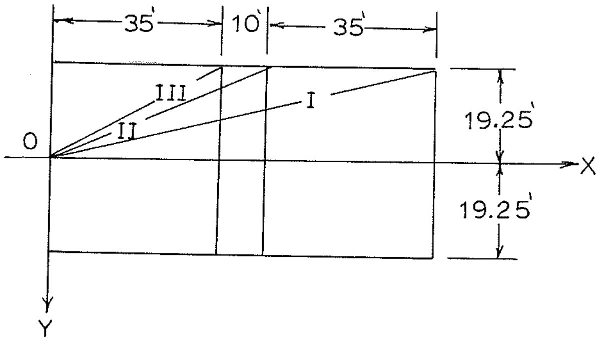
N.B. Only the pertinent data are given in the table

Date	Load Kips
1964	
Aug. 7	2315
Aug. 14	2360
Aug. 21	2422
Aug. 28	2483
Sept. 4	2497
Sept. 11	3434
Sept. 18	5455
Sept. 24	6528
Oct. 2	6232
Oct. 9	5501
Oct. 16	5339
Oct. 23	4961
Oct. 30	4815
Nov. 6	5650
Nov. 13	6500
Nov. 20	5061
Nov. 27	4661
1965	
July 16	4879
July 23	4642
July 30	5646
Aug. 13	5465
Aug. 20	5269
Aug. 27	4836
Sept. 3	4866
Sept. 10	5227
Sept. 17	5502
Sept. 24	6378
Oct. 1	7069
Oct. 8	7658
Oct. 15	7389
Oct. 22	7351
Oct. 29	7423
1966	
Feb. 4	5840
Feb. 11	5544
Feb. 18	5286
Feb. 25	5436
Mar. 4	5713
Mar. 11	5713
Mar. 18	6948
Mar. 25	7268
Apr. 1	7384
Apr. 7	6991
Apr. 15	6467
Apr. 22	6125
Apr. 29	5522
Aug. 19	3683
Aug. 26	3798
Sept. 2	3925
Sept. 9	4060
Sept. 16	4944
Sept. 23	5629
Sept. 30	5266
1967	
Aug. 4	4372
Aug. 11	4542
Aug. 18	4542
Aug. 25	4618
Sept. 1	6116
Sept. 8	7973
Sept. 15	8464
Sept. 22	8429
Sept. 29	9065
Oct. 6	8973
Oct. 13	8917
Oct. 20	8887

APPENDIX D

The influence value at P was calculated according to the following equation;

Influence value at $P = I_{I} - I_{II} + I_{III}$



Influence value at a depth of 40 feet

Considering half of the total area, as in the above figure, the m and n values were obtained.

Rectangle	m=a/z	n=b/z
I	$\frac{80}{40} = 2.0$	$\frac{19.25}{40} = .481$
II	$\frac{45}{40} = 1.125$	.481
III	$\frac{35}{40} = 0.875$	.481

Using tables 1 through 10 at these values of m and n the influence values were calculated as follows:

A = .06317 - .05222 + .04587 = .05682

B = .06747 - .06786 + .06500 = .06461

D = .01335 - .02395 + .02753 = .01693

F = .05412 - .04394 + .03828 = .04846

H = .06208 - .04721 + .03811 = .05298

These values are for half of the total area. Hence, they have to be summed up for the complete ractangle. It can be seen from Fig. 7 that the H value has the same sign in both quadrants. Therefore, doubling the above values for the complete rectange:

A = .11364

B = .12922

D = .03386

F = .09692

H = .10596

Therefore, $\sigma_m = A\ p = .11364\ p$

$$S_d = \sqrt{2(B^2 - H^2 - DF)}\ p = .222\ p$$

Influence values at a depth of 25 feet

The same procedure was followed in this case, too with the new values for m and n .

Rectangle	m=a/z	n=b/z
I	$\frac{80}{25} = 3.2$	$\frac{19.25}{25} = .77$
II	$\frac{45}{25} = 1.8$	.77
II	$\frac{35}{25} = 1.4$	.77

A = .09865 - .08943 + .08259 = .09181

B = .08166 - .08650 + .08773 = .08289

D = .01053 - .02365 + .03073 = .01761

F = .07125 - .06285 + .05701 = .06541

H = .09362 - .08384 + .07503 = .08481

The values were doubled to take into account the total area, Using these values;

σ_m = .18362 p

S_d = .322 p

Influence vales at a depth of 10 feet

Rectangle	m=a/z	n=b/z
I	$\frac{80}{10} = 8.0$	$\frac{19.25}{10} = 1.925$
II	$\frac{45}{10} = 4.5$	1.925
III	$\frac{35}{10} = 3.5$	1.925

A = .17128 - .16640 + .16204 = .16692

B = .06741 - .07183 + .07535 = .07093

D = .06286 - .05852 + .05503 = .05937

F = .00455 - .01332 + .02030 = .01153

H = .12514 - .12472 + .12399 = .12441

Therefore, σ_m = .33384 p

S_d = .394 p

APPENDIX E

Derivatives of χ and V

- taken from Love[1] (1929) -

$$\frac{\partial^2\chi}{\partial x^2} = p\left[\tan^{-1}\frac{b-y}{a-x} + \tan^{-1}\frac{b+y}{a-x} - \tan^{-1}\frac{z(b-y)}{(a-x)\,a_1} - \tan^{-1}\frac{z(b+y)}{(a-x)\,d_4}\right.$$
$$\left. + \tan^{-1}\frac{b-y}{a+x} + \tan^{-1}\frac{b+y}{a+x} - \tan^{-1}\frac{z(b-y)}{(a+x)\,b_2} - \tan^{-1}\frac{z(b+y)}{(a+x)\,c_3}\right],$$

$$\frac{\partial^2\chi}{\partial y^2} = p\left[\tan^{-1}\frac{a-x}{b-y} + \tan^{-1}\frac{a+x}{b-y} - \tan^{-1}\frac{z(a-x)}{(b-y)\,a_1} - \tan^{-1}\frac{z(a+x)}{(b-y)\,b_2}\right.$$
$$\left. + \tan^{-1}\frac{a-x}{b+y} + \tan^{-1}\frac{a+x}{b+y} - \tan^{-1}\frac{z(a-x)}{(b+y)\,d_4} - \tan^{-1}\frac{z(a+x)}{(b+y)\,c_3}\right],$$

$$\frac{\partial V}{\partial z} = -p\,\Omega,$$

$$\Omega = 2\pi - \cos^{-1}\frac{(a-x)(b-y)}{\sqrt{\{(a-x)^2+z^2\}}\sqrt{\{(b-y)^2+z^2\}}} - \cos^{-1}\frac{(a-x)(b+y)}{\sqrt{\{(a-x)^2+z^2\}}\sqrt{\{(b+y)^2+z^2\}}}$$
$$-\cos^{-1}\frac{(a+x)(b-y)}{\sqrt{\{(a+x)^2+z^2\}}\sqrt{\{(b-y)^2+z^2\}}} - \cos^{-1}\frac{(a+x)(b+y)}{\sqrt{\{(a+x)^2+z^2\}}\sqrt{\{(b+y)^2+z^2\}}}.$$

$$\frac{\partial^2\chi}{\partial x\,\partial y} = p\log\frac{(z+a_1)(z+c_3)}{(z+b_2)(z+d_4)}.$$

$$\frac{\partial^2 V}{\partial x^2} = -p\left\{\frac{a-x}{(a-x)^2+z^2}\left(\frac{b-y}{a_1} + \frac{b+y}{d_4}\right) + \frac{a+x}{(a+x)^2+z^2}\left(\frac{b-y}{b_2} + \frac{b+y}{c_3}\right)\right\}.$$

$$\frac{\partial^2 V}{\partial y^2} = -p\left\{\frac{b-y}{(b-y)^2+z^2}\left(\frac{a-x}{a_1} + \frac{a+x}{b_2}\right) + \frac{b+y}{(b+y)^2+z^2}\left(\frac{a-x}{d_4} + \frac{a+x}{c_3}\right)\right\}.$$

$$\frac{\partial^2 V}{\partial z^2} = p\left\{\frac{a-x}{(a-x)^2+z^2}\left(\frac{b-y}{a_1}+\frac{b+y}{d_4}\right)+\frac{a+x}{(a+x)^2+z^2}\left(\frac{b-y}{b_2}+\frac{b+y}{c_3}\right)\right.$$

$$\left.+\frac{b-y}{(b-y)^2+z^2}\left(\frac{a-x}{a_1}+\frac{a+x}{b_2}\right)+\frac{b+y}{(b+y)^2+z^2}\left(\frac{a-x}{d_4}+\frac{a+x}{c_3}\right)\right\}.$$

$$\frac{\partial^2 V}{\partial x\,\partial z} = p\left\{\frac{z}{(a-x)^2+z^2}\left(\frac{b-y}{a_1}+\frac{b+y}{d_4}\right)-\frac{z}{(a+x)^2+z^2}\left(\frac{b-y}{b_2}+\frac{b+y}{c_3}\right)\right\};$$

$$\frac{\partial^2 V}{\partial y\,\partial z} = p\left\{\frac{z}{(b-y)^2+z^2}\left(\frac{a-x}{a_1}+\frac{a+x}{b_2}\right)-\frac{z}{(b+y)^2+z^2}\left(\frac{a-x}{d_4}+\frac{a+x}{c_3}\right)\right\}$$

$$\frac{\partial^2 V}{\partial x\,\partial y} = p\left(\frac{1}{a_1}-\frac{1}{b_2}+\frac{1}{c_3}-\frac{1}{d_4}\right).$$