OPTIMALLY INSENSITIVE ACTIVE RC FILTER DESIGN WITH MINIMUM NUMBER OF ELEMENTS

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LIST OF FIGURES AND TABLES

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OPTIMALLY INSENSITIVE ACTIVE RC FILTER DESIGN WITH MINIMUM NUMBER OF ELEMENTS

CHAPTER I

Introduction

The use of resistors, capacitors, and active elements in the synthesis of filter networks has received increasing attention in recent years. Resistors and capacitors are used in filters because they are smaller, cheaper, and more nearly ideal elements than are inductors. One type of active element often used in filter design is the negative impedance converter (NIC). [1] A NIC is a two-port device which offers an input impedance of $-Z_L$ at one port when an impedance Z_L is connected to the other port.

The various methods of synthesis of active filters employing NIC's and RC networks are outlined in the work of Linvill [2], Yanagisawa [3], and Kinariwala [4]. Two of such methods will be discussed in the following chapters, namely the ones due to Yanagisawa and Linvill. Yanagisawa's method is perhaps the simplest to use while Linvill's method is the basic method from which others have evolved.

Yanagisawa's method uses a negative impedance converter of the current inversion type (INIC), while Linvill's method uses either an INIC or a negative impedance converter of the voltage inversion type (VNIC).

Both methods require a selection of a divisor polynomial. It has been shown that the sensitivity of the filter responses to changes in the converter characteristic and in the element values depends on the polynomial selected. These changes alter the pole and zero locations of the transfer function for the synthesized network. The changes in the parameter of an active element have by far more serious effect on the responses of the filter. Horowitz [5] proposed that a unique decomposition exists such that the magnitude of the sensitivities of all the roots of the polynomial is minimized with respect to the variations in the parameter of the active element. Holt and Stephenson [6] investigated the possibility of reducing the number of passive elements by varying the choice of divisor polynomial in Linvill and Yanagisawa configurations. Antoniou [7] uses optimum decomposition and also reduces the number of passive elements by studying various configurations for the passive part of the network.

There are many other different techniques of synthesizing a given network function in active RC synthesis. In practice, therefore, some criteria, other than the magnitude and the phase characteristics, provide the decisive factors in choosing a specific method. The main objective of this investigation is to satisfy two criteria of practical importance simultaneously in the realization of various types of low-pass filters.

To be more specific, this work employs the optimum decomposition [5] [8] [9] [10] for minimum sensitivity and in addition, minimized the number of elements by choosing proper gain constant for the transfer function. Realized networks are the low-pass filters of the maximially flat magnitude, the maximially flat phase, and the equal ripple responses. Network configurations are shown and the element values are tabulated for the second, third, and fourth order functions respectively.

CHAPTER II

Optimum Synthesis in Yanagisawa Configurations

This method is also known as the parallel RC active network synthesis. It employs a negative impedance converter of the current inversion type and two RC networks. Often the two RC networks are chosen in the form of inverse L networks as shown in Fig. 1.

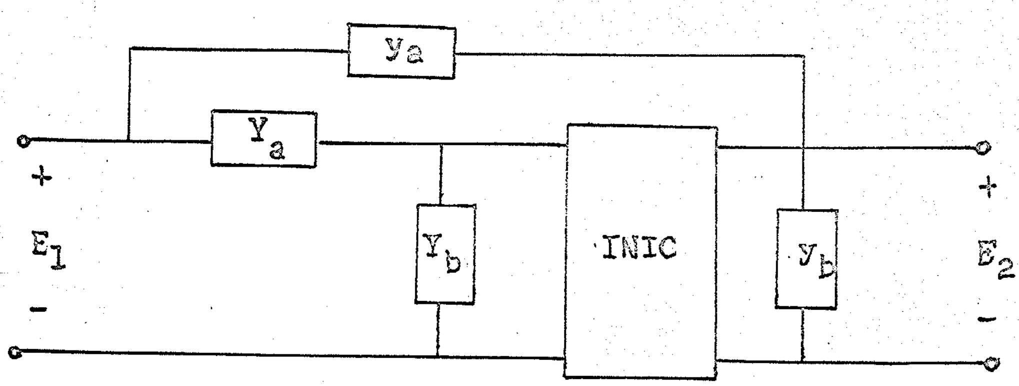

Fig. 1 Yanagisawa Network

Analysis of Fig. 1 yields that

$$\frac{E_2}{E_1} = \frac{y_a - Y_a}{(y_b - Y_b) + (y_a - Y_a)} = \frac{N(s)}{D(s)} \qquad D(s) \neq 0 \qquad (2\text{-}1)$$

This can also be written as

$$\frac{E_2}{E_1} = \frac{N(s)}{D(s) - N(s) + N(s)} \qquad (2\text{-}2)$$

Let an arbitrary polynomial be formed.

$$Q(s) = \prod_{i=1}^{n-1} (s + \sigma_i)$$

where the (n-1) points σ_i are selected on the negative real axis and n is the highest order of the numerator or denominator polynomial in

(2-1). Dividing both numerator and denominator by Q(s) gives

$$\frac{E_2}{E_1} = \frac{\frac{N(s)}{Q(s)}}{\frac{D(s)-N(s)+N(s)}{Q(s)}} \tag{2-4}$$

Comparing (2-1) and (2-4) we find,

$$y_a - Y_a = \frac{N(s)}{D(s)} \tag{2-5}$$

$$y_b - Y_b = \frac{D(s)-N(s)}{Q(s)} \tag{2-6}$$

Each of the expressions in (2-5) and (2-6) can be expanded in the form

$$\frac{N(s)}{Q(s)} = k_a^{(\infty)}s + k_a^{(o)} + \sum_i \frac{s}{s+\sigma_i} k_a^{(i)} = y_a - Y_a \tag{2-7}$$

$$\frac{D(s)-N(s)}{Q(s)} = k_b^{(\infty)}s + k_b^{(o)} + \sum_i \frac{s}{s+\sigma_i} k_b^{(i)} = y_b - Y_b \tag{2-8}$$

By associating the terms with positive $k_a^{(\infty)}, k_a^{(o)}$ and $k_a^{(i)}$ with y_a, and the terms with negative $k_a^{(\infty)}, k_a^{(o)}$ and $k_a^{(i)}$ with Y_a, and similarly for y_b and Y_b, all the four admittances can be identified directly from (2-7) and (2-8), and the transfer function synthesis is reduced to that of four d.p. admittances.

I The Second Order Case

The general second order all-pole voltage transfer function can be written in the following form,

$$\frac{E_2}{E_1} = \frac{H}{s^2 + as + b} = \frac{N(s)}{D(s)} \tag{2-9}$$

Horowitz's decomposition of D(s) is as follows

$$D(s) = s^2+as+b = (s+\sqrt{b})^2 - (2\sqrt{b}-a)s$$
$$= a^2(s) - b_o s b^2(s)$$

Thus divisor polynomial $Q(s) = a(s)b(s) = (s+\sqrt{b})$.

From (2-5) and (2-6)

$$y_a - Y_a = \frac{H}{s+\sqrt{b}} = \frac{H}{\sqrt{b}} - \frac{\frac{H}{\sqrt{b}}s}{s+\sqrt{b}} \qquad (2\text{-}10)$$

$$y_b - Y_b = \frac{s^2+as+b-H}{s+\sqrt{b}} = s - \frac{(2b-a\sqrt{b}-H)s}{(s+\sqrt{b})\sqrt{b}} + \frac{b-H}{\sqrt{b}} \qquad (2\text{-}11)$$

To minimize the number of elements, let

$$2b - a\sqrt{b} - H = 0$$

or $\quad H = 2b - a\sqrt{b}$

Substituting $H=2b-a\sqrt{b}$ into (2-10)and (2-11), gives

$$y_a - Y_a = (2\sqrt{b} - a) - \frac{(2\sqrt{b}-a)s}{s+\sqrt{b}}$$

$$y_b - Y_b = s + a - \sqrt{b}$$

We shall consider two cases.

Case I: $\quad \sqrt{b} < a < 2\sqrt{b}$

The admittances are identified as follows

$$y_a = 2\sqrt{b}-a = \frac{1}{R_1}$$

$$Y_a = \frac{(2\sqrt{b}-a)s}{s+\sqrt{b}} = \frac{1}{\frac{1}{2\sqrt{b}-a} + \frac{1}{(2-\frac{a}{\sqrt{b}})s}} = \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$y_b = s+(a-\sqrt{b}) = C_3 s+\frac{1}{R_3}$$

where

$$R_1 = \frac{1}{2\sqrt{b}-a} \qquad C_2 = 2-\frac{a}{\sqrt{b}} = \frac{1}{\sqrt{b}R_2}$$

$$R_2 = \frac{1}{2\sqrt{b}-a} = R_1 \qquad C_3 = 1$$

$$R_3 = \frac{1}{a-\sqrt{b}}$$

The network realization is shown in Fig. 2

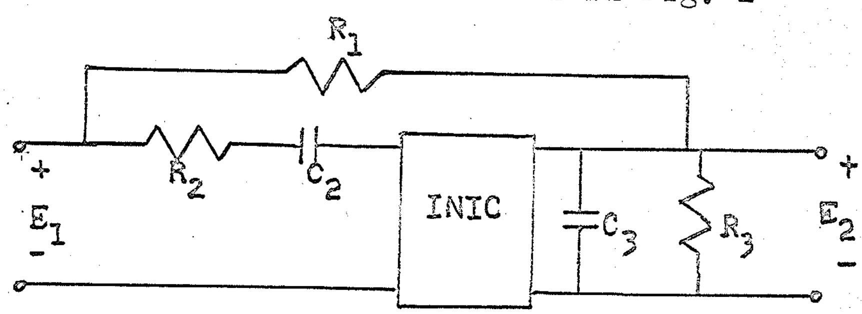

Fig. 2 Realization of 2nd order function, case (1) $\sqrt{b} < a < 2\sqrt{b}$

Case 2: $a < \sqrt{b}$

The admittances are identified as follows

$$y_a = 2\sqrt{b}-a = \frac{1}{R_1}$$

$$Y_a = \frac{(2\sqrt{b}-a)s}{s+\sqrt{b}} = \frac{1}{R_2+\frac{1}{C_2 s}}$$

$$y_b = s = C_3 s$$

$$Y_b = \sqrt{b}-a = \frac{1}{R_4}$$

where $R_1 = \dfrac{1}{2\sqrt{b}-a}$ $C_2 = 2-\dfrac{a}{\sqrt{b}} = \dfrac{2\sqrt{b}-a}{\sqrt{b}} = \dfrac{1}{\sqrt{b}\,R_2}$

$R_2 = R_1$ $C_3 = 1$

$R_3 = \dfrac{1}{\sqrt{b}-a}$

The network realization is shown in Fig. 3.

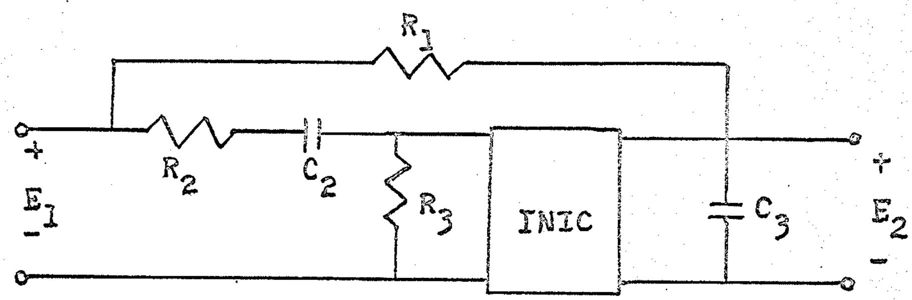

Fig. 3 Realization of 2nd order function, case (2), $a < \sqrt{b}$

Applying the above results to actual filter design, it is seen that the Bessel polynomial, Butterworth polynomial and Chebyshev polynomials ($\frac{1}{2}$db, 1db) fall into case (1) while the Chebyshev polynomial (2db, 3db) fall into case (2). The results are tabulated in Table 1 and Table 2, on the following page.

Table I Element Values for the Optimum Yanagisawa Network of Second Order Butterworth, Chebyshev ($\frac{1}{2}$db, 1db) and Bessel Responses.

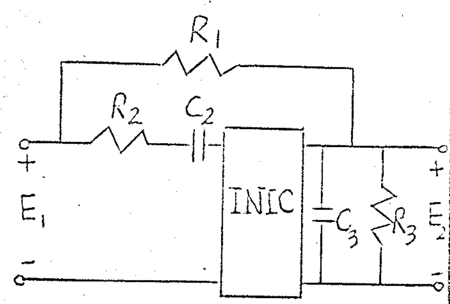

FIG. 2

	Butterworth	Chebyshev ($\frac{1}{2}$db)	Chebyshev (1db)	Bessel
R_1	1.706	0.9652	0.998	2.154
R_2	1.706	0.9652	0.998	2.154
R_3	2.42	5.129	20.83	0.7886
C_2	0.586	0.8416	0.9543	0.268
C_3	1	1	1	1
H	0.586	1.276	1.052	0.804

ohms, farads

Table 2 Element Values for the Optimum Yanagisawa Network of Second Order Chebyshev (2db, 3db) Responses.

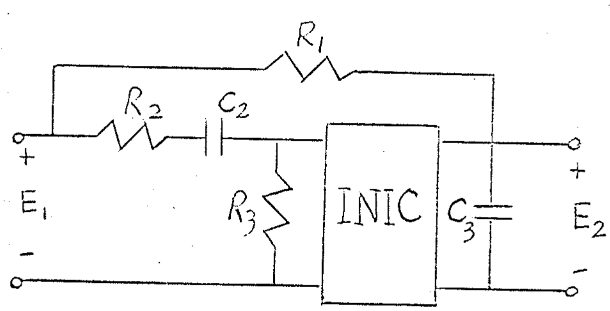

FIG. 3

	Chebyshev (2db)	Chebyshev (3db)
R_1	0.9309	1.058
R_2	0.9309	1.058
R_3	5.984	9.165
C_2	1.877	1.124
C_3	1	1
H	0.9746	0.795

ohms, farads

II The Third Order Case

The general third order all pole voltage transfer function can be written in the following form:

$$\frac{E_2}{E_1} = \frac{H}{(s^2+as+b)(s+c)} = \frac{N(s)}{D(s)} \qquad (2\text{-}12)$$

Horowitz's decomposition of (s^2+as+b), as in the second order case, is

$$(s^2+as+b) = (s+\sqrt{b})^2 - (2\sqrt{b}-a)s$$
$$= a^2(s) - b_o s b^2(s).$$

Thus Horowitz's decomposition of D(s) is,

$$(s^2+as+b)(s+c) = (s+c)(s+\sqrt{b})^2 - (2\sqrt{b}-a)s(s+c)$$

Therefore, the divisor polynomial

$$P(s) = (s+c)(s+\sqrt{b})$$

From (2-5) and (2-6)

$$y_a - Y_a = \frac{H}{(s+\sqrt{b})(s+c)}$$

$$= \frac{H}{c\sqrt{b}} + \frac{\frac{H}{\sqrt{b}(\sqrt{b}-c)}s}{s+\sqrt{b}} - \frac{\frac{H}{c(\sqrt{b}-c)}s}{s+c} \qquad (2\text{-}13)$$

$$y_b - Y_b = \frac{(s^2+as+b)(s+c)-H}{(s+\sqrt{b})(s+c)}$$

$$= \frac{bc-H}{c\sqrt{b}} + \frac{\frac{(2b-a\sqrt{b})(c-\sqrt{b})-H}{\sqrt{b}(\sqrt{b}-c)}s}{s+\sqrt{b}} + \frac{\frac{H}{c(\sqrt{b}-c)}s}{s+c} + s \qquad (2\text{-}14)$$

In order to minimize the number of elements, let

$$(2b-a\sqrt{b})(c-\sqrt{b})-H = 0$$

or $$H = (2b-a\sqrt{b})(c-\sqrt{b})$$

Substituting H into (2-13) gives

$$y_a - Y_a = \frac{\left(\frac{2b-a\sqrt{b}}{c}\right)s}{s+c} - \frac{(2\sqrt{b}-a)s}{s+\sqrt{b}} - \frac{(2b-a\sqrt{b})(\sqrt{b}-c)}{c\sqrt{b}}$$

Therefore,

$$y_a = \frac{1}{\frac{c}{2b-a\sqrt{b}} + \frac{c^2}{(2b-a\sqrt{b})s}} = \frac{1}{R_1 + \frac{1}{C_1 s}}$$

$$Y_a = \frac{(2b-a\sqrt{b})(\sqrt{b}-c)}{c\sqrt{b}} + \frac{(2\sqrt{b}-a)s}{s+\sqrt{b}} = \frac{1}{R_2} + \frac{1}{R_3 + \frac{1}{C_3 s}}$$

where $$R_1 = \frac{c}{2b-a\sqrt{b}} \qquad C_1 = \frac{2b-a\sqrt{b}}{c^2} = \frac{1}{cR_1}$$

$$R_2 = \frac{c\sqrt{b}}{(2b-a\sqrt{b})(\sqrt{b}-c)} = \frac{R_1\sqrt{b}}{(\sqrt{b}-c)} \qquad C_3 = \frac{2\sqrt{b}-a}{\sqrt{b}} = \frac{1}{\sqrt{b}R_3}$$

$$R_3 = \frac{1}{2\sqrt{b}-a} = \frac{R_1\sqrt{b}}{c}$$

Substituting H into (2-14) gives

$$y_b - Y_b = \frac{bc-(2b-a\sqrt{b})(c-\sqrt{b})}{c\sqrt{b}} + \frac{\frac{(2b-a\sqrt{b})(c-\sqrt{b})}{(\sqrt{b}-c)c}s}{s+c} + s$$

Therefore,

$$y_b = \frac{bc+(2b-a\sqrt{b})(\sqrt{b}-c)}{c\sqrt{b}} + s = \frac{1}{R_4} + C_4 s$$

$$Y_b = \frac{\dfrac{2b-a\sqrt{b}}{c}\,s}{s+c} = \frac{1}{R_5+\dfrac{1}{C_5 s}}$$

where

$$R_4 = \frac{\sqrt{b}\,c}{bc+(2b-a\sqrt{b})(\sqrt{b}-c)} \qquad\qquad C_4 = 1$$

$$= \frac{\sqrt{b}}{b+\dfrac{(2b-a\sqrt{b})(\sqrt{b}-c)}{c}} = \frac{\sqrt{b}}{b+\dfrac{(\sqrt{b}-c)}{R_1}}$$

$$R_5 = \frac{c}{2b-a\sqrt{b}} = R_1 \qquad\qquad C_5 = \frac{2b-a\sqrt{b}}{c^2} = C_1$$

The network realization is shown in Fig. 4

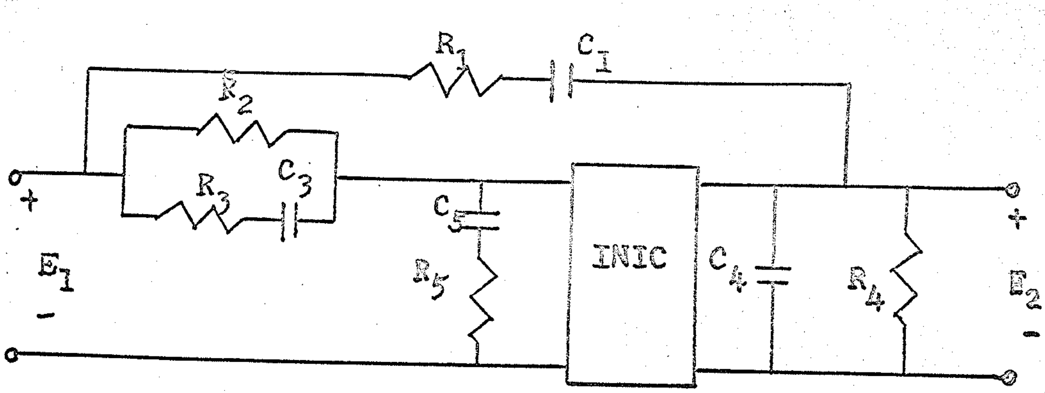

Fig. 4 Realization of General Third Order all pole voltage transfer function.

The synthesis of the following third order all pole voltage transfer function have been worked out.

(a) Bessel Polynomial

(b) Chebyshev Polynomials ($\frac{1}{2}$db, 1db, 2db, 3db)

The results are tabulated in Table 3.

Table 3 Element Values for the Optimum Yanagisawa Network of Third Order Bessel and Chebyshev ($\frac{1}{2}$db, 1db, 2db, 3db) Responses.

Type / Elements	Bessel	Chebyshev $\frac{1}{2}$db	Chebyshev 1db	Chebyshev 2db	Chebyshev 3db
R_1 (Ohms)	0.5734	0.3878	0.3301	0.2592	0.1907
R_2	6.626	0.9354	0.6543	0.4363	0.2721
R_3	0.6277	0.6622	0.6662	0.661	0.6375
R_4	0.38	0.4678	0.396	0.3093	0.217
R_5	0.5734	0.3878	0.3301	0.2592	0.1907
C_1 (Farad)	0.751	4.12	6.132	10.46	17.54
C_3	6.6266	1.413	1.505	1.608	1.68
C_4	1	1	1	1	1
C_5	0.751	4.12	6.132	10.46	17.54
H	-0.8907	-0.7151	-0.7527	-0.8143	-1.025

An attempt is made to use the above result to synthesize a Butterworth low-pass filter. The divisor was found to be $(s+1)^2$. Therefore, from (2-5)

$$y_a - Y_a = \frac{H}{(s+1)^2}$$

which is not an RC admittance.

One solution to this problem is to make the divisor

$$Q(s) = (s+1)(s+1 \pm \Delta x)$$

In this case Δx is made as small as possible to achieve mimimum number of elements or minimum sensitivity, whichever is desired.

III The Fourth Order Case

The general fourth order all pole-voltage transfer function can be written in the following form:

$$\frac{E_2}{E_1} = \frac{H}{s^4+as^3+bs^2+cs+d} = \frac{N(s)}{D(s)} \qquad (2\text{-}15)$$

Horowitz's decomposition gives

$$s^4+as^3+bs^2+cs+d = (s+y_1)^2(s+y_2)^2 - B_o s(s+y_3)^2 \qquad (2\text{-}16)$$

To solve for y_1, y_2, and y_3 in terms of the unknown constants a, b, c, and d means solving four nonlinear equations. This task is so unwieldly that the general analysis does not merit the laborious work. Instead, only the polynomials of interest will be decomposed individually.

A) Butterworth Polynomial

$$\frac{E_2}{E_1} = \frac{H}{(s^2+0.765s+1)(s^2+1.848s+1)}$$

Horowitz's decomposition gives [9]

$$Q(s) = (s+1.909)(s+0.524)(s+1)$$

From (2-5)

$$y_a - Y_a = \frac{H}{(s+1.909)(s+0.524)(s+1)}$$

$$= H - \frac{0.416\ Hs}{s+1.909} - \frac{2.895Hs}{s+0.524} + \frac{2.311Hs}{s+1}$$

From (2-6)

$$y_b - Y_b = \frac{(s+1.909)(s+0.524)}{s+1} - \frac{2.253s(s+1)}{(s+1.909)(s+0.524)} - (y_a - Y_a)$$

$$= s+1-H+\frac{(0.433-2.311H)s}{s+1} + \frac{(0.4161H-1.479)s}{s+1.909} + \frac{(2.895H-0.774)s}{s+0.524}$$

In order to minimize the number of elements, H can be chosen to be

(i) $H = \frac{1.479}{0.4161} = 3.554$

(ii) $H = \frac{0.433}{2.311} = 0.1874$

(iii) $H = \frac{0.774}{2.895} = 0.2674$

(i) $H = 3.554$,

$$y_a = 3.554 + \frac{8.214s}{s+1} = \frac{1}{R_1} + \frac{1}{\frac{1}{C_2 s} + R_2}$$

$$Y_a = \frac{1.479s}{s+1.909} + \frac{10.29s}{s+0.524} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{9.516s}{s+0.524} = C_5 s + \cfrac{1}{R_6 + \cfrac{1}{C_6 s}}$$

$$Y_b = 2.554 + \frac{7.781s}{s+1} = \frac{1}{R_8} + \cfrac{1}{R_7 + \cfrac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while the element values are tabulated in Table 4.

(ii) $H = 0.1874$

$$y_a = 0.1874 + \frac{0.433s}{s+1} = \frac{1}{R_1} + \cfrac{1}{R_2 + \cfrac{1}{C_2 s}}$$

$$Y_a = \frac{0.07797s}{s+1.909} + \frac{0.5425s}{s+0.524} = \cfrac{1}{R_4 + \cfrac{1}{C_4 s}} + \cfrac{1}{R_3 + \cfrac{1}{C_3 s}}$$

$$y_b = s + 0.8127 = C_5 s + \frac{1}{R_6}$$

$$Y_b = \frac{1.401s}{s+1.909} + \frac{0.2315s}{s+0.524} = \cfrac{1}{R_7 + \cfrac{1}{C_7 s}} + \cfrac{1}{R_8 + \cfrac{1}{C_8 s}}$$

The network realization is shown in Fig 5(b) while the element values are tabulated in Table 4.

(iii) $H = 0.2674$

$$y_a = 0.2673 + \frac{0.618s}{s+1} = \frac{1}{R_1} + \cfrac{1}{R_2 + \cfrac{1}{C_2 s}}$$

$$Y_a = \frac{0.1113s}{s+1.909} + \frac{0.774s}{s+0.524} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+0.7327 = C_5 s + \frac{1}{R_6}$$

$$Y_b = \frac{0.185s}{s+1} + \frac{1.366s}{s+1.909} = \frac{1}{R_7 + \frac{1}{C_7 s}} + \frac{1}{R_8 + \frac{1}{C_8 s}}$$

The network realization is shown in Fig. 5(b) while the element values are tabulated in Table 4.

(B) Bessel Polynomial

$$\frac{E_2}{E_1} = \frac{H}{s^4+10s^3+45s^2+105s+105}$$

From Horowitz's decomposition, using Calahan's procedure [8] ~~gives~~,

$$Q(s) = (s+14.921)(s+2.745)(s+3.273)$$

From (2-5)

$$y_a - Y_a = \frac{H}{(s+14.921)(s+2.745)(s+3.273)}$$

$$= 0.007197H - \frac{0.0004835Hs}{s+14.921} + \frac{0.04968Hs}{s+3.273} - \frac{0.05595Hs}{s+2.745}$$

From (2-6)

$$y_b - Y_b = s + 3.1289 - 0.007197H + \frac{(2.431-0.04968H)s}{s+3.273}$$

$$+ \frac{(0.0004835H-24.337)s}{s+14.921} + \frac{(0.05595H-\text{[illegible]}12)s}{s+2.745}$$

In order to minimize the number of elements, H can be shown to be (i) $H = \frac{2.431}{0.04968} = 48.94$

(ii) $H = \frac{1.112}{0.05595} = 19.87$

(iii) $H = \frac{24.337}{0.0004835} = 50327$

(i) $H = 48.94$

$$y_a = 0.3522 + \frac{2.431s}{s+3.273} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.02367s}{s+14.921} + \frac{2.738s}{s+2.745} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$Y_b = \frac{24.313s}{s+14.921} = \frac{1}{R_8 + \frac{1}{C_8 s}}$$

$$y_b = s + 2.7767 + \frac{1.626s}{s+2.745} = C_5 s + \frac{1}{R_6} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig 5(c) while the element values are tabulated in Table 4.

(ii) $H = 19.87$

$$y_a = 0.143 + \frac{0.9873s}{s+3.273} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.009608s}{s+14.921} + \frac{1.111s}{s+2.745} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+2.9859+\frac{1.4438s}{s+3.273} = C_5 s + \frac{1}{R_6} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

$$Y_b = \frac{24.327s}{s+14.921} = \frac{1}{R_8 + \frac{1}{C_8 s}}$$

The network realization is shown in Fig. 5(c) while the element values are tabulated in Table 4.

(iii) H = 50327

$$y_a = 362.2 + \frac{2500s}{s+3.273} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{24.34s}{s+14.921} + \frac{2816s}{s+2.745} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+ \frac{2814.9s}{s+2.745} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 359.07 + \frac{2497.6s}{s+3.273} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while the element values are tabulated in Table 4.

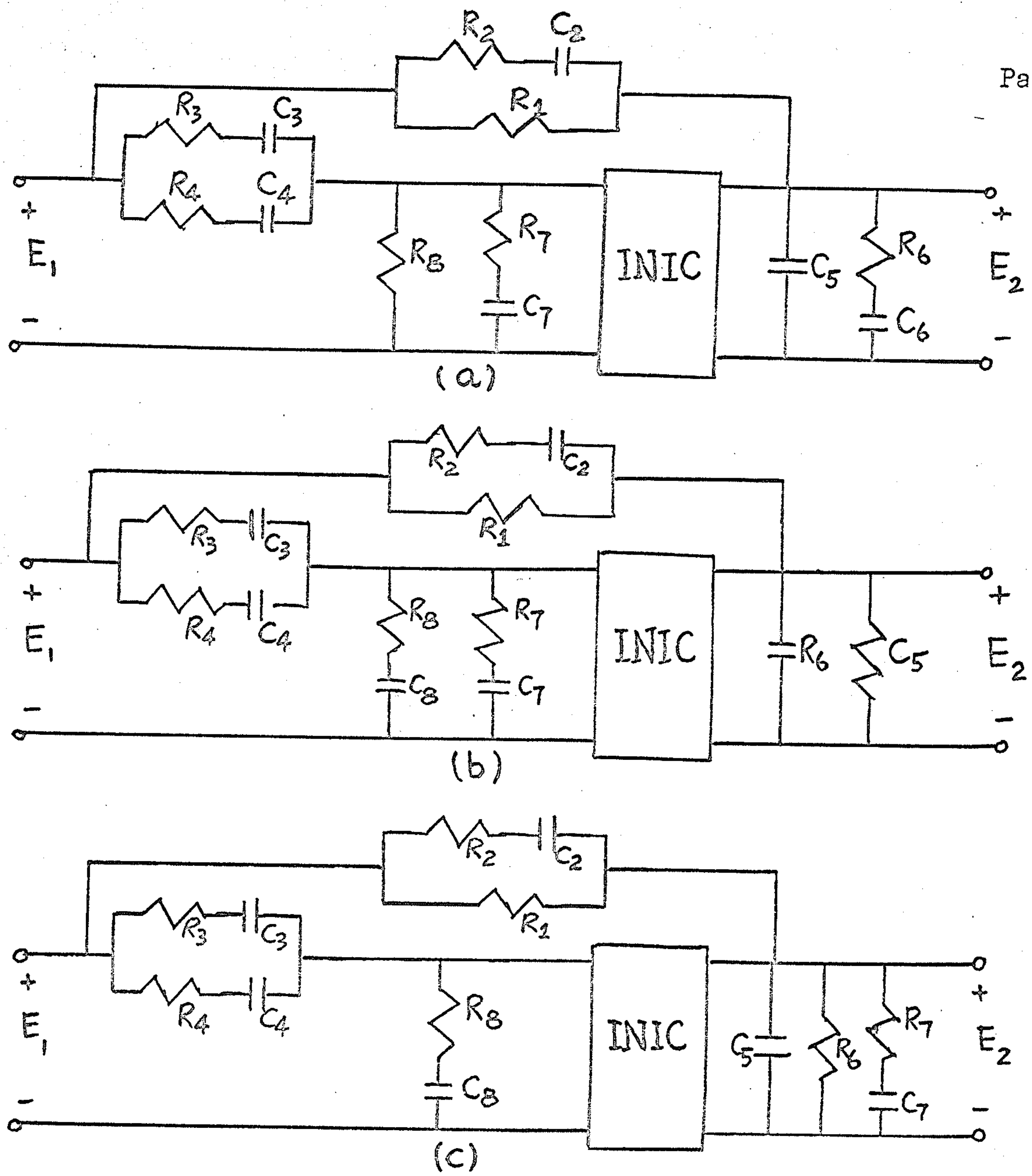

Fig. 5 (a), (b), (c), Realization of Fourth Order All-Pole Voltage Transfer Functions

Table 4 Element Values for the Optimum Yanagisawa Network of Fourth Order Butterworth's and Bessel's Responses.

Type / Element	Butterworth			Bessel		
	H=0.2674	H=0.1874	H=3.554	H=50327	H=19.87	H=48.94
Fig. No.	5(b)	5(b)	5(a)	5(a)	5(c)	5(c)
R_1	3.741	5.338	0.2813	0.00276	6.693	2.84
R_2	1.618	0.2309	0.1217	0.000399	1.013	0.4113
R_3	8.987	12.85	0.6762	0.0411	104.1	42.26
R_4	1.292	1.844	0.09718	0.000355	0.881	0.3653
R_6	1.365	1.23	0.1051	0.0003552	0.3349	0.3601
R_7	5.405	0.7138	0.1285	0.0004	0.6926	0.615
R_8	0.731	4.32	0.3915	0.002785	0.04111	0.04113
C_2	0.618	4.331	8.214	709.6	0.3016	0.7428
C_3	0.05828	0.04084	0.7748	1.631	0.000644	0.001586
C_4	1.477	1.035	19.63	989.9	0.3907	0.9624
C_5	1	1	1	1	1	1
C_6	-	0.3663	18.16	989.5	-	-
C_7	0.185	0.7239	7.781	763.1	0.4412	0.5716
C_8	0.7156	-	-	-	1.63	1.629

ohms, farads.

(C) Chebyshev $\frac{1}{2}$db

$$\frac{E_2}{E_1} = \frac{H}{(s^2+0.351s+1.064)(s^2+0.845s+0.356)}$$

Horowitz's decomposition gives [9]

$$Q(s) = (s+0.292)(s+2.111)(s+0.732)$$

From (2-5)

$$y_a - Y_a = \frac{H}{(s+0.292)(s+2.111)(s+0.732)}$$

$$= 2.216H - \frac{4.279Hs}{s+0.292} - \frac{0.1888Hs}{s+2.111} + \frac{2.251Hs}{s+0.732}$$

From (2-6)

$$y_b - Y_b = \frac{(s+0.292)(s+2.111)}{s+0.732} - \frac{3.61s(s+0.732)}{(s+0.292)(s+2.111)} - y_a + Y_a$$

$$= s + 0.8421 - 2.216H + \frac{(0.7289-2.251H)s}{s+0.732} + \frac{(4.279H-0.8737)s}{s+0.292} + \frac{(0.1888H-2.736)s}{s+2.111}$$

In order to minimize the number of elements, H can be chosen to be (i) $H = \frac{0.7289}{2.251} = 0.3238$

(ii) $H = \frac{0.8737}{4.279} = 0.2042$

(iii) $H = \frac{2.736}{0.1888} = 14.49$

(i) $H = 0.3238$

$$y_a = 0.7174 + \frac{0.7289s}{s+0.732} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.06113s}{s+2.111} + \frac{1.386s}{s+0.292} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+0.1247 + \frac{0.5123s}{s+0.292} = C_5 s + \frac{1}{R_6} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

$$Y_b = \frac{2.6749s}{s+2.111} = \frac{1}{R_8 + \frac{1}{C_8 s}}$$

The network realization is shown in Fig 5(c) and element values are tabulated in Table 5

(ii) $H = 0.2042$

$$y_a = 0.4525 + \frac{0.4597s}{s+0.732} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.8737s}{s+0.292} + \frac{0.03856s}{s+2.111} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+0.3896 + \frac{0.2692s}{s+0.732} = \frac{1}{R_7 + \frac{1}{C_7 s}} + \frac{1}{R_6} + C_5 s$$

$$Y_b = \frac{2.6974s}{s+2.111} = \frac{1}{R_8 + \frac{1}{C_8 s}}$$

The network realization is shown in Fig. 5(c) and element values are tabulated in Table 5.

(iii) $H = 14.49$

$$y_a = 32.11 + \frac{32.63s}{s+0.732} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{62.01s}{s+0.292} + \frac{2.736s}{s+2.111} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{61.136s}{s+0.292} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 31.268 + \frac{31.901s}{s+0.732} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) and element values are tabulated in Table 5.

(D) Chebyshev Polynomial 1db

$$\frac{E_2}{E_1} = \frac{H}{(s^2+0.219s+0.987)(s^2+0.674s+0.279)}$$

Horowitz's decomposition gives [9] ,

$$Q(s) = (s+0.253)(s+2.075)(s+0.677)$$

From (2-5)

$$y_a - Y_a = \frac{H}{(s+0.253)(s+2.075)(s+0.677)}$$
$$= 2.813H - \frac{5.117Hs}{s+0.253} - \frac{1.517Hs}{s+2.075} + \frac{2.492Hs}{s+0.677}$$

From (2-6)

$$y_b - Y_b = \frac{(s+0.253)(s+2.075)}{(s+0.677)} - \frac{3.698s(s+0.677)}{(s+0.253)(s+2.075)} - y_a + Y_a$$
$$= s + (0.7755 - 2.813H) + \frac{(0.8755-2.492H)s}{s+0.677}$$
$$+ \frac{(5.117H-0.86)s}{s+0.253} + \frac{(1.517H-2.838)s}{s+2.075}$$

In order to minimize the number of elements, H can be chosen to be (i) $H = \frac{0.8755}{2.492} = 0.3514$

(ii) $H = \frac{0.86}{5.117} = 0.1681$

(iii) $H = \frac{2.838}{1.517} = 1.872$

(i) $H = 0.3514$

$$y_a = 0.9886 + \frac{0.8755s}{s+0.677} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{1.797s}{s+0.253} + \frac{0.5328s}{s+2.075} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{0.937s}{s+0.253} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 0.2131 + \frac{2.3052s}{s+2.075} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while the element values are tabulated in Table 5.

(ii) $H = 0.1681$

$$y_a = 0.4728 + \frac{0.4189s}{s+0.677} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.86s}{s+0.253} + \frac{0.255s}{s+2.075} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+0.3027 + \frac{0.4566s}{s+0.677} = \frac{1}{R_7 + \frac{1}{C_7 s}} + \frac{1}{R_6} + C_5 s$$

$$Y_b = \frac{2.583s}{s+2.075} = \frac{1}{R_8 + \frac{1}{C_8 s}}$$

The network realization is shown in Fig. 5(c) while the element values are tabulated in Table 5

(iii) H = 1.872

$$y_a = 5.265 + \frac{4.663s}{s+0.677} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{9.575s}{s+0.253} + \frac{2.383s}{s+2.075} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{8.715s}{s+0.253} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 4.4895 + \frac{3.7875s}{s+0.677} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while the element values are tabulated in Table 5, on the following page.

(E) Chebyshev 2db

$$\frac{E_2}{E_1} = \frac{H}{(s^2+0.21s+0.929)(s^2+0.506s+0.222)}$$

Horowitz decomposition gives [9]

$$Q(s) = (s+0.218)(s+2.032)(s+0.636)$$

From (2-5)

$$y_a - Y_a = \frac{H}{(s+0.218)(s+2.032)(s+0.636)}$$

$$= 3.464H - \frac{5.887Hs}{s+0.218} - \frac{0.1732Hs}{s+2.032} + \frac{2.602Hs}{s+0.636}$$

Table 5 Element Values for the Optimum Yanagisawa Network of Fourth Order Chebyshev ($\frac{1}{2}$db, 1db) Responses.

Type / Element	Chebyshev $\frac{1}{2}$db			Chebyshev 1db		
	H=14.49	H=0.2042	H=0.3238	H=1.872	H=0.1681	H=0.3514
Fig. No.	5(a)	5(c)	5(c)	5(a)	5(c)	5(a)
R_1	0.03114	2.21	1.394	0.19	2.114	1.012
R_2	0.03065	2.175	1.372	0.2144	2.387	1.142
R_3	0.01613	1.145	16.35	0.1044	1.162	0.5563
R_4	0.3656	25.93	0.7219	0.3524	3.922	1.877
R_6	0.01636	2.627	8.02	0.1148	3.304	1.067
R_7	0.03134	3.801	1.952	0.264	2.19	0.4338
R_8	0.03199	0.3708	0.3739	0.2227	0.3872	4.692
C_2	45.61	0.628	0.9957	6.89	0.6187	1.293
C_3	212.3	2.992	0.02896	37.84	3.399	7.105
C_4	1.296	0.01827	4.744	1.368	0.1228	0.2567
C_5	1	1	1	1	1	1
C_6	209.3	-	-	34.44	0.6745	3.703
C_7	43.58	0.3594	1.754	5.595	0.1865	1.111
C_8	-	1.277	1.267	-	-	-

ohms, farads.

From (2-6)

$$y_b - Y_b = \frac{(s+0.218)(s+2.082)}{s+0.636} - \frac{3.885s(s+0.636)}{(s+0.218)(s+2.082)} - y_a + Y_a$$

$$= s + 0.7137 - 3.464H + \frac{(0.9503-2.602H)s}{s+0.636} +$$

$$\frac{(5.887H-0.872)s}{s+0.218} + \frac{(0.1782H-3.013)s}{s+2.082}$$

In order to minimize the number of elements, H can be chosen to be (i) $H = \frac{0.9503}{2.602} = 0.3652$

(ii) $H = \frac{0.872}{5.887} = 0.1481$

(iii) $H = \frac{3.013}{0.1782} = 16.9$

(i) $H = 0.3652$

$$y_a = 1.265 + \frac{0.9503s}{s+0.636} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{2.15s}{s+0.218} + \frac{0.06508s}{s+2.082} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{1.278s}{s+0.218} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 0.5513 + \frac{2.9479s}{s+2.082} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while element values are tabulated in Table 6.

(ii) $H = 0.1481$

$$y_a = 0.5131 + \frac{0.3853s}{s+0.636} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.872s}{s+0.218} + \frac{0.02639s}{s+2.082} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+0.2006 + \frac{0.565s}{s+0.636} = C_5 s + \frac{1}{R_6} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

$$Y_b = \frac{2.9866s}{s+2.082} = \frac{1}{R_8 + \frac{1}{C_8 s}}$$

The network realization is shown in Fig. 5(c) while the element values are tabulated in Table 6.

(iii) $H = 16.9$

$$y_a = 58.57 + \frac{43.98s}{s+0.636} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{99.54s}{s+0.218} + \frac{3.013s}{s+2.082} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{98.66s}{s+0.218} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 57.856 + \frac{43.03s}{s+0.636} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while the element values are tabulated in Table 6.

(F) Chebyshev Polynomial 3db

$$\frac{E_2}{E_1} = \frac{H}{s^4+0.58157s^3+1.1691s^2+0.40476s+0.17698}$$

Horowitz's decomposition using Calahan's procedure [8] gives,

$$Q(s) = (s+2.0975)(s+0.2005)(s+0.6171)$$

From(2-5),

$$y_a - Y_a = \frac{H}{(s+2.0975)(s+0.2005)(s+0.6171)}$$

$$= 3.853H - \frac{0.1792Hs}{s+2.0975} - \frac{6.662Hs}{s+0.2005} + \frac{2.627Hs}{s+0.6171}$$

From(2-6)

$$y_b - Y_b = \frac{(s+2.0975)(s+0.2005)}{(s+0.6171)} - \frac{4.014s(s+0.6171)}{(s+2.0975)(s+0.2005)} - y_a - Y_a$$

$$= s + 0.6818 - 3.853H + \frac{(0.9992-2.627H)s}{s+0.6171} +$$

$$\frac{(6.662H-0.8903)s}{s+0.2005} + \frac{(0.1792H-3.1237)s}{s+2.0975}$$

In order to minimize the number of elements, H can be shown to be

(i) $H = \frac{0.9992}{2.627} = 0.3804$

(ii) $H = \frac{0.8903}{6.662} = 0.1337$

(iii) $H = \frac{3.1237}{0.1792} = 17.43$

(i) $H = 0.3804$

$$y_a = 1.466 + \frac{0.9992s}{s+0.6171} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.06817s}{s+2.0975} + \frac{2.534s}{s+0.2005} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{1.6437s}{s+0.2005} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 0.7842 + \frac{3.055s}{s+2.0975} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while the element values are tabulated in Table 6.

(ii) $H = 0.1337$

$$y_a = 0.5152 + \frac{0.3514s}{s+0.6171} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{0.02396s}{s+2.0975} + \frac{0.8907s}{s+0.2005} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s+0.1666 + \frac{0.6478s}{s+0.6171} = C_5 s + \frac{1}{R_6} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

$$Y_b = \frac{3.0997s}{s+2.0975} = \frac{1}{R_8 + \frac{1}{C_8 s}}$$

The network realization is shown in Fig. 5(c) while the element values are tabulated in Table 6.

(iii) $H = 17.43$

$$y_a = 67.17 + \frac{45.79s}{s+0.6171} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{C_2 s}}$$

$$Y_a = \frac{3.124s}{s+2.0975} + \frac{116.1s}{s+0.2005} = \frac{1}{R_3 + \frac{1}{C_3 s}} + \frac{1}{R_4 + \frac{1}{C_4 s}}$$

$$y_b = s + \frac{115.21s}{s+0.2005} = C_5 s + \frac{1}{R_6 + \frac{1}{C_6 s}}$$

$$Y_b = 66.488 + \frac{44.78s}{s+0.6171} = \frac{1}{R_8} + \frac{1}{R_7 + \frac{1}{C_7 s}}$$

The network realization is shown in Fig. 5(a) while the element values are tabulated in Table 6.

Table 6 Element Values for the Optimum Yanagisawa Network of Fourth Order Chebyshev (2db, 3db) Responses.

Type / Element	Chebyshev 2db			Chebyshev 3db		
	H=16.9	H=0.1481	H=0.3652	H=17.43	H=0.3804	H=0.1337
Fig. No.	5(a)	5(c)	5(a)	5(a)	5(a)	5(c)
R_1	0.01707	1.948	0.7904	0.01489	0.6823	1.941
R_2	0.02274	2.595	1.052	0.02184	1	2.846
R_3	0.01004	1.147	0.465	0.3201	14.67	41.74
R_4	0.3319	37.88	15.37	0.008612	0.3946	1.123
R_6	0.01014	4.985	0.7824	0.00868	0.6083	6.002
R_7	0.02324	1.811	0.3392	0.02233	0.3273	1.544
R_8	0.01728	0.3348	1.813	0.01504	1.275	0.3226
C_2	69.16	0.6058	1.494	74.2	1.619	0.5693
C_3	456.6	4	9.863	1.489	0.0325	0.01142
C_4	1.447	0.01268	0.03126	579.2	12.64	4.442
C_5	1	1	1	1	1	1
C_6	452.6	-	5.862	574.6	8.198	-
C_7	67.66	0.8884	1.416	72.56	1.456	1.05
C_8	-	1.434	-	-	-	1.478

ohms, farads.

From Tables 1 - 6, the following observations are made:

(1) The minimum number of passive elements required is $(4n-3)$, where n is the order of the transfer function synthesized.

(2) In the 4th order case , even a choice of three H's is available, only one or two of the given values are of practical use because high gain gives unrealistic capacitors and resistors. On the average, only gains of $H<1$ are of practical use.

(3) The output impedance is not zero, making it impossible to cascade such realization without the using of isolating amplifiers.

CHAPTER III

Optimum Synthesis in Linvill Configurations

In this chapter, we will develop an optimum synthesis in Linvill [2] configurations. As shown in Fig. 6, there are two passive RC two-ports and a NIC between them, all in cascade.

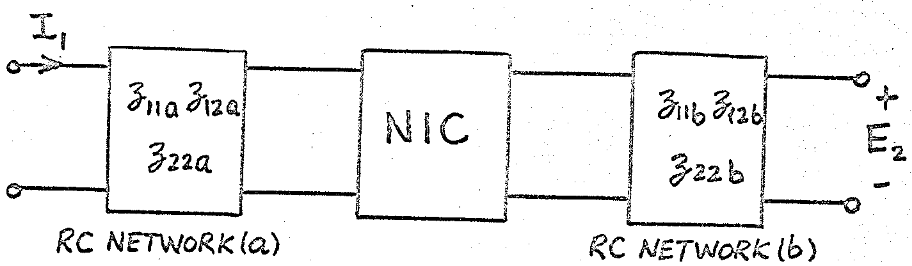

Fig. 6 Basic Linvill Network

Analysis of Fig. 6, shows that for a VNIC, with gain K= -1,

$$\left.\frac{E_2}{I_1}\right|_{I_2=0} = Z_{21} = \frac{-z_{12a}z_{12b}}{z_{11b}-z_{22a}} = \frac{N(s)}{D(s)} \qquad D(s)\neq 0 \qquad (3\text{-}1)$$

For an INIC, with gain K= -1,

$$\left.\frac{E_2}{I_1}\right|_{I_2=0} = Z_{21} = \frac{z_{12a}z_{12b}}{z_{11b}-z_{22a}} = \frac{N(s)}{D(s)} \qquad D(s)\neq 0 \qquad (3\text{-}2)$$

In the following synthesis, an INIC will be used. As in Yanagisawa's method, an arbitray polynomial Q(s) is chosen, where

$$Q(s) = \prod_{i=1}^{n}(s+\sigma_i) \qquad (3\text{-}3)$$

where n is not less than the degree of N(s) or D(s), whichever is greater. For the expression D(s)/Q(s), it can be expanded in the form,

$$\frac{D(s)}{Q(s)} = \sum_{i=1}^{n} \frac{A_i}{(s+\sigma_i)} \tag{3-4}$$

comparing (3-4) with (3-2), then

$$\sum_{i=1}^{n} \left(\frac{A_i}{s+\sigma_i}\right) = z_{11b} - z_{22a} \tag{3-5}$$

The terms of positive residues are associated with z_{11b} and those of negative residues with z_{22a}. From the expression N(s)/Q(s), N(s) can be divided into two factors in any convenient manner, ie,

$$N(s) = N_1(s)N_2(s)$$

where

$$z_{12a} = \frac{N_1(s)}{Q_1(s)} \quad ; \qquad z_{12b} = \frac{N_2(s)}{Q_2(s)}$$

Q_1 and Q_2 include all poles of z_{22a} and z_{11b} respectively. Thus we identify two sets of RC two-ports parameters.

3-1 The Second Order Case

A general second order all-pole transfer impedance function can be written as follows:

$$Z_{21} = \frac{H}{s^2 + as + b} = \frac{N(s)}{D(s)}$$

As in Yanagisawa's method, D(s) can be decomposed as

$$D(s) = (s+\sqrt{b})^2 - (2\sqrt{b}-a)s$$

From (3-3), divisor polynomial $Q(s) = s(s+\sqrt{b})$

From (3-5)

$$z_{11b} - z_{22a} = \frac{s^2+as+b}{s(s+\sqrt{b})} = 1 + \frac{\sqrt{b}}{s} - \frac{(2\sqrt{b} - a)}{s+\sqrt{b}}$$

Therefore,

$$z_{11b} = 1 + \frac{\sqrt{b}}{s} \quad , \quad z_{22a} = \frac{2\sqrt{b} - a}{s + \sqrt{b}}$$

Since all zeros are at infinity, the two networks can be synthesized by the Cauer method to give simple RC ladders. The gain constant H is assumed to be unity first before synthesis. The realization is shown in Fig. 7

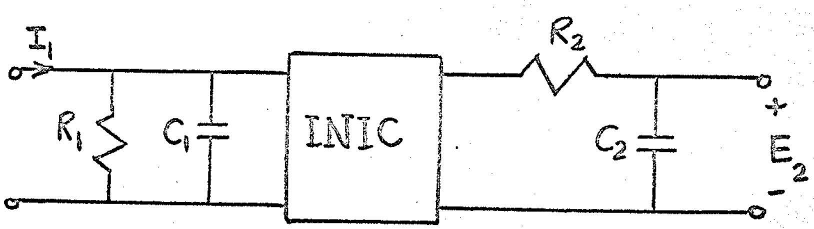

Fig. 7 Realization of the General Second Order All-Pole Transfer Impedance Functions

where $R_1 = 2 - \frac{a}{\sqrt{b}}$ $\qquad C_1 = \frac{1}{2\sqrt{b} - a}$

$R_2 = 1$ $\qquad C_2 = \frac{1}{\sqrt{b}}$

Analysis of Fig. 7 gives $H = 2b-a\sqrt{b}$.

Thus the optimal gain constant $H = 2b-a\sqrt{b}$. Networks having characteristics described by Butterworth, Chebyshev ($\frac{1}{2}$db, 1db, 2db, 3db) and Bessel Polynomials are synthesized and the results tabulated in Table 7.

Table 7 Element Values for the Optimum Linvill Network of Second Order Butterworth, Chebyshev ($\frac{1}{2}$db, 1db, 2db, 3db) and Bessel Responses.

Type / Element	Bessel	Butter-Worth	Chebyshev $\frac{1}{2}$db	Chebyshev 1db	Chebyshev 2db	Chebyshev 3db
R_1	0.268	0.586	0.842	0.955	0.993	0.586
R_2	1	1	1	1	1	1
C_1	2.155	1.701	0.9649	0.9977	1.262	1.707
C_2	0.576	1	0.8121	0.9523	1.253	1
H	0.804	0.586	1.277	1.052	0.6322	0.586

ohms, farads.

3-2 The Third Order Case

The general third order all-pole transfer impedance function can be given by

$$\frac{E_2}{I_1} = \frac{H}{(s+c)(s^2+as+b)} \tag{3-6}$$

The divisor polynomial Q(s) is found to be

$$Q(s) = s(s+c)(s+\sqrt{b})$$

As in the second order case

$$z_{11b} - z_{22a} = \frac{(s+c)(s^2+as+b)}{s(s+c)(s+\sqrt{b})} = 1 + \frac{\sqrt{b}}{s} - \frac{2\sqrt{b}-a}{s+\sqrt{b}} \qquad (3\text{-}7)$$

$$z_{12a} z_{12b} = \frac{H}{s(s+c)(s+\sqrt{b})} \qquad (3\text{-}8)$$

Due to the cancellation of the (s+c) term in (3-7), an extra term of $x/(s+c)$ has to be introduced in (3-7) in order to facilitate the synthesis yielding

$$z_{11b} - z_{22a} = 1 + \frac{\sqrt{b}}{s} + \frac{x}{s+c} - \left(\frac{2\sqrt{b}-a}{s+\sqrt{b}} + \frac{x}{s+c} \right)$$

where x has to be real and positive in order to keep the driving point functions z_{11b} and z_{22a} positive real.

$$\therefore \; x > 0 \qquad (3\text{-}9)$$

The following network is realized,

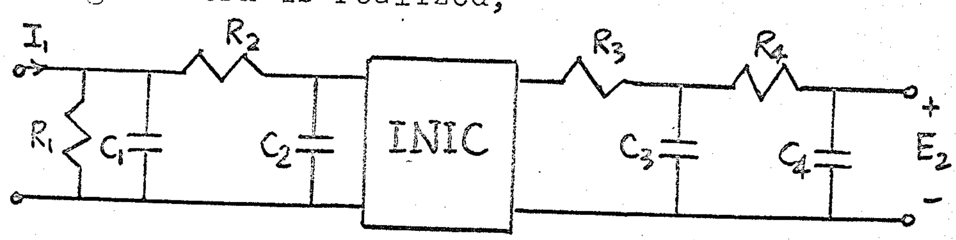

Fig. 8 Realization of Third Order All-Pole General Transfer Impedance Functions

where

$$\left.\begin{aligned}
R_1 &= \frac{x(2\sqrt{b}-a)(c^2+b-2c\sqrt{b})}{\sqrt{b}c(2b-a\sqrt{b}+cx)}, & C_1 &= \frac{(2b-a\sqrt{b}+cx)^2}{x(2\sqrt{b}-a)(c^2+b-2c\sqrt{b})(2\sqrt{b}-a+x)} \\
R_2 &= \frac{(2\sqrt{b}-a+x)^2}{2b-a\sqrt{b}+cx}, & C_2 &= \frac{1}{2\sqrt{b}-a+x} \\
R_3 &= 1, & C_3 &= \frac{1}{\sqrt{b}+x} \\
R_4 &= \frac{(\sqrt{b}+x)^2}{bx}, & C_4 &= \frac{x}{\sqrt{b}(\sqrt{b}+x)} \\
& & H &= \frac{1}{C_1C_2C_3R_2R_3R_4}
\end{aligned}\right\} \quad (3\text{-}10)$$

Since all of elements in (3-10) have to be positive, the following restrictions exist for x,

that is $2b-a\sqrt{b}+cx>0$

or $x>\frac{\sqrt{b}}{c}(a-2\sqrt{b})$ (3-11)

and $2\sqrt{b}-a+x>0$

or $x>(a-2\sqrt{b})$ (3-12)

and $x>0$ (3-13)

x must satisfy the most stringent condition. An attempt is also made to minimize the values of capacitances.

The capacitance $C_2\to 0$ as $x\to\infty$ (3-14)

To minimize the capacitance C_1

$$\frac{dC_1}{dx} = \frac{d\left[\frac{(2b-a\sqrt{b}+cx)^2}{x(2\sqrt{b}-a)(c^2+b-2c\sqrt{b})(2\sqrt{b}-a+x)}\right]}{dx}=0$$

or

$$x= \sqrt{b}-\tfrac{1}{2}a-c^2\pm\tfrac{1}{2}\sqrt{4b+a^2+4c^4+4ac^2+16bc-4a\sqrt{b}-8\sqrt{b}c^2-8\sqrt{bc}} \qquad (3\text{-}15)$$

Comparing (3-14) and (3-15), the latter one gives a more reasonable value for x. Networks having characticstics described by the Butterworth, Chebyshev ($\frac{1}{2}$db, 1db, 2db, 3db) and Bessel Polynomials are synthesized and the results tabulated in Table 8.

Table 8 Element Values for the Optimum Linvill Network of Third Order Butterworth, Chebyshev ($\frac{1}{2}$db, 1db, 2db, 3db) and Bessel Responses.

Type / Elements	Bessel	Butter-worth	Chebyshev $\frac{1}{2}$db	Chebyshev 1db	Chebyshev 2db	Chebyshev 3db
x	10.922	2	1.1278	0.811	1.1558	0.4622
R_1	0.005644	0	0.2195	0.3295	0.8909	0.7778
R_2	5.272	4.5	3.063	2.816	3.849	3.144
R_3	1	1	1	1	1	1
R_4	2.509	4.5	3.747	4.042	4.294	4.897
C_1	69.61	∞	5.873	5.043	3.3916	2.979
C_2	0.08063	0.3334	0.3787	0.4327	0.3746	0.501
C_3	0.07407	0.3334	0.4552	0.5531	0.4768	0.7255
C_4	0.3137	0.6667	0.4804	0.4499	0.5856	0.3659
H	0.08466	4.5	0.157	0.162	0.292	0.195

Ohms, Farad

3-3 The Fourth Order Case

As in Chapter 2 for Yanagisawa's method, the Linvill method is worked out for specific polynomials.

I Butterworth Polynomial

$$\frac{E_2}{I_1} = \frac{H}{s^4+2.613s^3+3.414s^2+2.613s+1}$$

divisor polynomial $\quad Q(s)=s(s+1.909)(s+0.524)(s+1)$

$$z_{11b} - z_{22a} = \frac{s^4+2.613s^3+3.414s^2+2.613s+1}{s(s+1.909)(s+0.524)(s+1)}$$

$$= 1 + \frac{0.9996}{s} + \frac{0.4345}{s+1} - \frac{1.478}{s+1.909} - \frac{0.7755}{s+0.524}$$

$$z_{11b} = 1 + \frac{0.9996}{s} + \frac{0.4345}{s+1}$$

$$z_{22a} = \frac{1.478}{s+1.909} + \frac{0.7755}{s+0.524}$$

Network realization is shown in Fig. 9

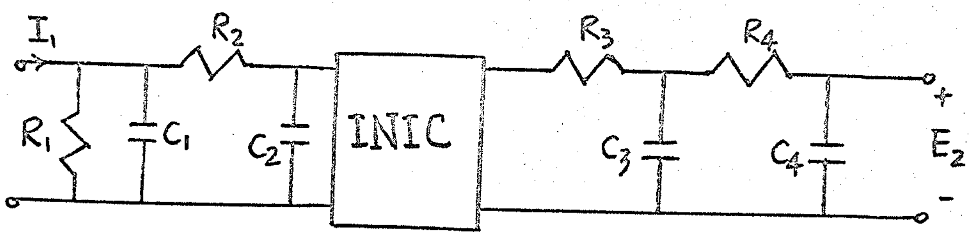

Fig. 9 Realization of Fourth Order All-Pole Transfer Impedance Functions

The element values are tabulated in Table 9

II Bessel Polynomial

$$\frac{E_2}{I_1} = \frac{H}{s^4+10s^3+45s^2+105s+105}$$

Divisor Polynomial $Q(s)=s(s+14.921)(s+2.745)(s+3.273)$

$$z_{11b} - z_{22a} = 1 + \frac{0.7832}{s} + \frac{0.3776}{s+3.273} - \frac{7.046}{s+14.921} - \frac{0.3327}{s+2.745}$$

$$z_{11b} = 1 + \frac{0.7832}{s} + \frac{0.3776}{s+3.273}$$

$$z_{22a} = \frac{7.046}{s+14.921} + \frac{0.3327}{s+2.745}$$

Network realization is shown in Fig. 9 while element values are tabulated in Table 9

III Chebyshev Polynomial ($\frac{1}{2}$db)

$$\frac{E_2}{I_1} = \frac{H}{s^4+1.1974s^3+1.7169s^2+1.0255s+0.3791}$$

Divisor polynomial $Q(s) = s(s+0.292)(s+2.111)(s+0.732)$

$$z_{11b} - z_{22a} = 1 + \frac{0.8402}{s} + \frac{0.8235}{s+0.732} - \frac{0.872}{s+0.292} - \frac{2.73}{s+2.111}$$

$$z_{11b} = 1 + \frac{0.8402}{s} + \frac{0.8235}{s+0.732}$$

$$z_{22a} = \frac{0.872}{s+0.292} + \frac{2.73}{s+2.111}$$

Network realization is shown in Fig. 9 while element values are tabulated in Table 9.

IV Chebyshev Polynomial 1db

$$\frac{E_2}{I_1} = \frac{H}{s^4+0.9528s^3+1.4539s^2+0.74262s+0.2756}$$

Divisor Polynomial $\quad Q(s)= s(s+0.253)(s+2.075)(s+0.677)$

$$z_{11b} - z_{22a} = 1 + \frac{0.7755}{s} + \frac{0.8808}{s+0.677} - \frac{0.8675}{s+0.253} - \frac{2.843}{s+2.075}$$

$$z_{11b} = 1 + \frac{0.7755}{s} + \frac{0.8808}{s+0.677}$$

$$z_{22a} = \frac{0.8675}{s+0.253} + \frac{2.843}{s+2.075}$$

Network realization is shown in Fig. 9 and element values are tabulated in Table 9.

V Chebyshev Polynomial 2db

$$\frac{E_2}{I_1} = \frac{H}{s^4+0.71621s^3+1.2565s^2+0.5168s+0.20577}$$

Divisor polynomial $\quad Q(s)= s(s+0.218)(s+2.082)(s+0.636)$

$$z_{11b} - z_{22a} = 1 + \frac{0.7128}{s} - \frac{0.9183}{s+0.218} - \frac{0.5742}{s+2.082} + \frac{0.9486}{s+0.636}$$

$$z_{11b} = 1 + \frac{0.7128}{s} + \frac{0.9486}{s+0.636}$$

$$z_{22a} = \frac{0.9183}{s+0.218} + \frac{0.5742}{s+2.082}$$

Network realization is shown in Fig. 9 and element values are tabulated in Table 9.

VI Chebyshev Polynomial 3db

$$\frac{E_2}{I_1} = \frac{H}{s^4+0.58157s^3+1.1691s^2+0.40476s+0.17698}$$

Divisor polynomial $\quad Q(s) = s(s+2.0975)(s+0.2005)(s+0.6171)$

$$z_{11b} - z_{22a} = 1 + \frac{0.682}{s} + \frac{0.7805}{s+0.6171} - \frac{3.134}{s+2.0975} - \frac{0.8819}{s+0.2005}$$

$$z_{11b} = 1 + \frac{0.682}{s} + \frac{0.7805}{s+0.6171}$$

$$z_{22a} = \frac{3.134}{s+2.0975} + \frac{0.8819}{s+0.2005}$$

Network realization is shown in Fig. 9 while element values are tabulated in Table 9 on the following page.

Table 9 Element Values for the Optimum Linvill Network of Fourth Order Butterworth, Chebyshev ($\frac{1}{2}$db, 1db, 2db, 3db) and Bessel Responses.

Type / Items	Butter-worth	Bessel	Chebyshev			
			$\frac{1}{2}$db	1db	2db	3db
H	0.6824	8.393	0.8051	0.6868	0.5951	0.6191
R_1	0.6824	0.07994	2.123	2.492	2.893	3.498
R_2	1.572	0.5135	2.156	2.306	1.596	2.389
R_3	1	1	1	1	1	1
R_4	4.733	1.091	4.592	3.896	8.164	4.44
C_1	2.1	4.389	1.277	1.229	0.7123	1.142
C_2	0.4437	0.1355	0.2776	0.2695	0.67	0.249
C_3	0.6972	0.8615	0.601	0.6038	0.6019	0.6837
C_4	0.3032	0.4151	0.589	0.8099	0.2832	0.7827

ohms,farads.

CHAPTER IV

In this thesis, an optimum synthesis technique has been developed. Horowitz's decomposition for Butterworth polynomials, Chebyshev polynomials which exhibit $\frac{1}{2}$, 1, 2, 3 db ripples in the pass-band, and Bessel polynomials are tabulated in the appendix where the first four parts are due to Hakim [9] .

The minimum number of elements required in Yanagisawa realization is (4n-3) for nth order function. Linvill network requiries 2n elements for even n, and 2(n+1) elements for odd n, but it is more sensitive than Yanagisawa network. Both methods give realized networks with gain less than unity. As for the choice of NIC's, Yanagisawa's method uses an INIC while Linvill's method uses either an INIC or VNIC. In the types of transfer functions realized, Yanagisawa's method realizes the voltage transfer function while the transfer impedance function is realized by Linvill's method. The nature of decomposition of the two methods are as follows. In Yanagisawa's method, both numerator and denominator of the function are decomposed uniquely in a sense that all the admittances are identified after decomposition. There is no unique decomposition of the numerator in Linvill's method and sometimes extra terms have to be used. The number of network structures in Yanagisawa's method increases with the order of the function while in Linvill's method, there is only one network structure for a function of a given order.

Optimum networks and element values are derived for various responses of low-pass transfer functions of order two to four. The synthesis technique developed can be applied to the nth order function. Further problems related to this thesis are the experimental verification of the sensitivity comparison and the test of stability as related to the terminations of the negative immittance converter.

APPENDIX I

Table A1

Horowitz's decomposition of Butterworth Polynomials,

$$D_2(s) = a^2(s) - b_o s b^2(s)$$

$D(s)=D_1(s)D_2(s)$	$D_1(s)$	$a(s)$	b_o	$b(s)$
$(s^2+1.414s+1)$	1	$(s+1)$	0.586	1
$(s+1)(s^2+s+1)$	$(s+1)$	$(s+1)$	1.000	1
$(s^2+0.765s+1)(s^2+1.848s+1)$	1	$(s+1.909)(s+0.524)$	2.253	$(s+1)$

Table A2

Horowitz's decomposition of Chebyshev Polynomials($\frac{1}{2}$ db ripple)

$$D_2(s)=a^2(s) - b_o s b^2(s)$$

$D(s)=D_1(s)D_2(s)$	$D_1(s)$	$a(s)$	b_o	$b(s)$
$(s^2+1.426s+1.516)$	1	$(s+1.231)$	1.036	1
$(s+0.626)(s^2+0.626s+1.142)$	$(s+0.626)$	$(s+1.069)$	1.510	1
$(s^2+0.351s+1.064)$ $(s^2+0.845s+0.356)$	1	$(s+0.292)$ $(s+2.111)$	3.610	$(s+0.732)$

Table A3

Horowitz's decomposition of Chebyshev Polynomials (1 db ripple)

$$D_2(s)=a^2(s) - b_o s b^2(s)$$

$D(s)=D_1(s)D_2(s)$	$D_1(s)$	$a(s)$	b_o	$b(s)$
$(s^2+1.098s+1.103)$	1	$(s+1.050)$	1.002	1
$(s+0.494)(s^2+0.494s+0.994)$	$(s+0.494)$	$(s+0.997)$	1.501	1
$(s^2+0.279s+0.987)$ $(s^2+0.674s+0.279)$	1	$(s+0.253)$ $(s+2.075)$	3.698	$(s+0.677)$

Table A4

Horowitz's decomposition of Chebyshev Polynomials (2 db ripple)

$$D_2(s)= a^2(s) - b_o s b^2(s)$$

$D(s)=D_1(s)D_2(s)$	$D_1(s)$	$a(s)$	b_o	$b(s)$
$(s^2+0.804s+0.637)$	1	$(s+0.798)$	0.792	1
$(s+0.369)(s^2+0.369s+0.886)$	$(s+0.369)$	$(s+0.941)$	1.513	1
$(s^2+0.210s+0.929)$ $(s^2+0.506s+0.222)$	1	$(s+0.218)$ $(s+2.082)$	3.885	$(s+0.636)$

Table A5

Horowitz's decomposition of Chebyshev Polynomials (3 db ripple)

$$D_2(s) = a^2(s) - b_o s b^2(s)$$

$D(s)=D_1(s)D_2(s)$	$D_1(s)$	$a(s)$	b_o	$b(s)$
$(s^2+0.645s+0.708)$	1	$(s+0.841)$	1.037	1
$(s+0.299)(s+0.1493+j904)$	$(s+0.299)$	$(s+0.9336)$	1.587	1
$(s+0.085\pm j0.946)$ $(s+0.206\pm j0.392)$	1	$(s+2.0975)$ $(s+0.2005)$	4.014	$(s+0.617$

Table A6

Horowitz's decomposition of Bessel Polynomials

$$D_2(s) = a^2(s) - b_o s b^2(s)$$

$D(s)=D_1(s)D_2(s)$	$D_1(s)$	$a(s)$	b_o	$b(s)$
(s^2+3s+3)	1	$(s+1.732)$	0.464	1
$(s^3+6s^2+15s+15)$	$(s+2.322)$	$(s+2.542)$	1.7466	1
$(s+2.896\pm j0.8672)$ $(s+2.1038\pm j2.6574)$	1	$(s+14.921)$ $(s+2.745)$	25.449	$(s+3.273)$

APPENDIX II

Horowitz's decomposition [5]

Given a polynomial $D(s)$, it can be decomposed to have the following form,

$$D(s)= a^2(s) -sb^2(s) \tag{I}$$

such that the magnitude of the sensitivities of all the roots of $D(s,x)$ are minimized with respect to variation in the parameter x. This decomposition is called the Horowitz decomposition.

Procedure for Horowitz decomposition

1. Determine the roots of the polynomial $D(s^2)$. $D(s^2)$ is obtained by replacing s with s^2 in $D(s)$. Let $F(s)$ contain the left-half-plane roots of $D(s^2)$, and $F(-s)$ contain the right-half-plane roots of $D(s^2)$. Then

$$D(s^2)= F(s)F(-s) \tag{II}$$

2. Let $F(s)=a(s^2)+sb(s^2)$, where $a(s^2)$ is the even part of $F(s)$, and $sb(s^2)$ is the odd part. Then $F(-s)=a(s^2)- sb(s^2)$.

3. Multiplying $F(s)$ into $F(-s)$, gives

$$D(s^2)= a^2(s^2) - s^2b^2(s^2) \tag{III}$$

4. Subtituting s for s^2 into (III), the decomposition is found to be,

$$D(s)= a^2(s) - s\, b^2(s)$$

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