INTERACTION OF SHEAR WALLS AND MULTI-STOREY FRAMES SUBJECTED TO LATERAL LOADS.

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SYNOPSIS

There are many approximate methods of calculating the distribution of lateral forces and their effects on the resisting elements in a building braced with shear walls. The author's purpose is to analyze the data from his experiments on a ten-storey, reinforced concrete model locared in the Civil Engineering Laboratory of the University of Manitoba, and compare these results with three of the many methods of analysis available, namely,

(1) Equivalent Column Method,

(2) B. Cardan's Method,

(3) F. R. Khan's Method,

Throughout the experiments, Electrical Strain Gages and Mechanical Dial Strain Gages were used, and a series of horizontal deflections and various deformations of the resisting elements of the model building were measured.

CONTENTS

INTRODUCTION

The behavior of shear walls and rigid frames when under external forces are basically different. For this reason, it is of great interest to observe their complicate interactions while they are tied together within a building. In the past, we knew little about their interactions and, normally, shear walls were designed to resist all lateral forces while the rigid frames were assumed to carry only vertical loads.

In general we have been considering a shear wall as a vertical cantilever or a deep beam which has a web composed of the shear wall and flange of the face walls, and the formular of its horizontal deflection was then;

$$\triangle = \int \frac{Mm'dx}{EI} + {}^{6}/_{5}\int \frac{Vv'dx}{AG}$$

and the bending moment and shearing stress within a shear wall were computed by the ordinary statical mechanics.

By using the above assumption, we could indeed come to a quick conclusion in designing. However, the calculated behavior of the structure departs in many ways from its true picture, as the above assumption can hardly be the actual distribution of the stresses within this kind of structure. We have been using this assumption only because of the lack of better understanding in this area.

Due to the high speed efficiency of the electronic digital computer, which has been a great help in the analysis of shear wall-rigid frame interaction, since its introduction into structural engineering, engineers have made many important discoveries on design theories in the past five years. For instance:

(1) Equivalent Column Method

(2) Method by F. R. Khan & J A. Sbarounis

(3) Method by P. L. Gould

(4) Method by B. Cardan

(5) Method by M.G. Tamhankar; J.P.Jain; & G. S. Ramaswamy.

From these new theories, the result of an improved analysis will indicate a reduction of reinforcement in the shear wall. More important, it will also indicate that a shear wall which combines with rigid frames when under external forces, does not behave like a perpendicular cantilever; shear wall and frames will try to obstruct each other from taking their natural free deflected shapes. As a result of the redistribution of forces between them, the frame will restrain or pull back the shear wall in the upper storeys, while in the lower regions of the building the opposite will occur.

We have then to deal with the problems of the stiffness ratio and the selection and computation of the rigidity of the link members between shear walls and rigid frames, all of them are necessary conditions for an economic and sound structure.

First in this thesis, I want to introduce three analytical methods for the interaction of shear walls and rigid frames; their theories and the computed results from their applications to a chosen model building.

Secondly, I want to show the deflected shapes of the model building while subject to lateral loads by means of placing mechanical Dial Strain Gages at the building side; and the partial structural deformations and deflections by using Electrical Strain Gages on columns, slabs and shear walls. The results are plotted into curves.

Thirdly, with the help of a computer IBM(7094) I made a few charts which can be used directly for the determination of shear walls structures, mainly for their sizes.

It is my sincere hope that the findings in this thesis would prove to useful to designers in their future practice.

CHAPTER I. INTRODUCTION TO THREE ANALYTICAL METHODS OF SHEAR WALLS STRUCTURES.

Analysis of shear walls which combined with rigid frames in multistorey buildings subject to lateral loads.

I.I Equivalent Column Method----- by W.W.Frischmenn, S.S.Prabhu, J.F.Toppler

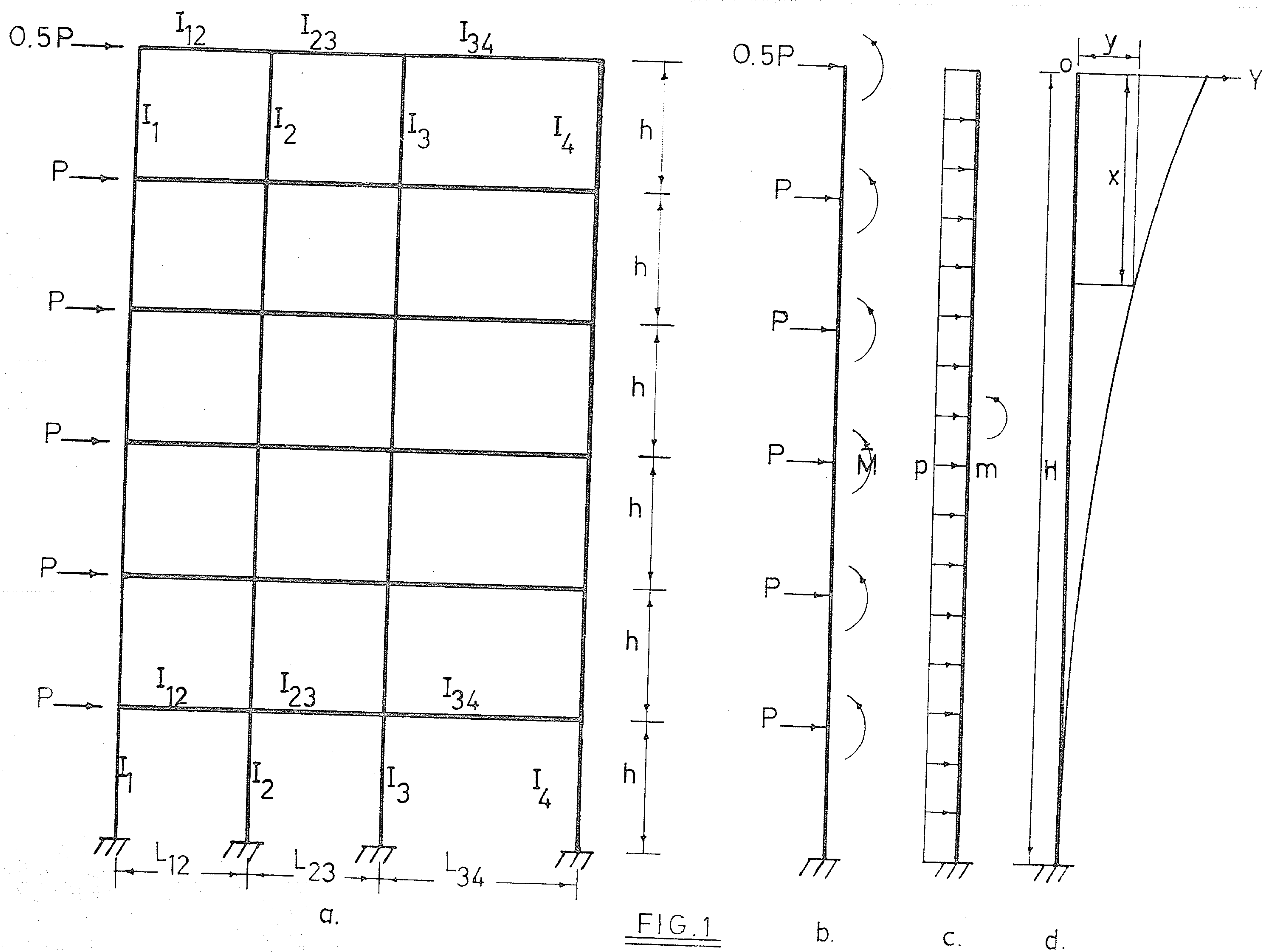
The method consists of replacing the frame by an equivalent column whose stiffness equals the sum of all the columns stiffness, and by restraints, applied at each floor. equivalent to the sum of the beam stiffnesses. The condition of equilibrium of all external and internal forces acting on the equivalent column is expressed in a second degree differential equation.

(1) All frames are fully fixed at the bases of the columns.

(2) All columns deflect, due to the horizontal loads, and remain parallel to each other, that is, it is assumed that there is no elastic shortening of the beams.

(3) The sectional properties of columns and beams in each bay are constant for the full height of the structure.*

(4) The storey height is constant throughout the


0.5P
I_{12}
I_{23}
I_{34}
I_1
I_2
I_3
I_4
P
P
P
P
P
h
h
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h
L_{12}
L_{23}
L_{34}
a.
0.5P
P
P
P
P
P
M
b.
p
m
c.
o
y
Y
x
H
d.

FIG. 1

height of the building.

(5) Axial shortening of columns and deflection due to shearing are ignored.

(6) The modulus of elasticity E of the concrete is assumed to be constant throughout the height of the building.

ANALYSIS OF THE METHOD. Consider Fig.1a, a frame which has horizontal loads applied at panel points. The equivalent column (Fig.1b.) is obtained by the summation of the moments of inertia of the columns, that is,

$$I = I_1 + I_2 + I_3 + I_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The beams are replaced by the restraint moments $\bar{M}$ applied at each floor level. The values of $\bar{M}$ depend on the stiffnesses of beams at each floor. For unit rotation at each end, the restraints are $K_{12}, K_{21}, K_{23}, K_{32}$,

where

$$K_{12} = K_{21} = \frac{kEI_{12}}{l} \quad ; \qquad K_{23} = K_{32} = \frac{kEI_{23}}{l} \quad ;$$

and $k = 6$, for prismatic beams. Then the total restraint at each floor,

$$K = K_{12} + K_{21} + K_{23} + K_{32} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

To simplify the solution, the system of forces, shown in Fig.1b. is distributed over the whole length of the equivalent column Fig.1c.

Hence $p = P/h$ ....(3)

and $\gamma = K/h$ ....(4)

Using the system of co-ordinates shown in Fig.1c. the deflection of the equivalent cantilever due to the horizontal loading may be desctibed as $y = f_1(x)$ and the uniformly-distributed restraint moments, acting opposite, as $\bar{m} = f_2(x)$. However, from Fig.1c.

$$\bar{m} = -\frac{\gamma dy}{dx}$$

The equation for all the moments acting on the equivalent column may be written as

$$M = \frac{px^2}{2} - \int \bar{m} \cdot dx$$

Differentiating with respect to x,

$$\frac{dM}{dx} = px - \bar{m}$$

and, substituting $\bar{m}$ from equation (5)

$$\frac{dM}{dx} = px + \frac{dy}{dx}$$

The final equation is

$$\frac{d^2M}{dx^2} = p + \frac{\gamma d^2y}{dx^2} \quad \text{........................(6)}$$

Since $\frac{d^2y}{dx^2} = \frac{M}{EI}$; and equation (6) then become

$$\frac{d^2M}{dx^2} - \frac{\gamma}{EI}M = p \quad \text{..................(7)}$$

let $$\alpha = \left(\frac{\gamma}{EI}\right)^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Then the solution of the differential equation may be expressed in the general form of

$$M = A \sinh \alpha x + B \cosh \alpha x - \frac{p}{\alpha^2} \quad \ldots\ldots\ldots (9)$$

from which the particular solution is found to be

$$\bar{M}_{(x)} = -\frac{ph}{\alpha} \frac{\sinh(\frac{1}{2}h)}{\frac{1}{2}h} \left(\frac{H - \sinh H}{\cosh H} \cos \alpha x \right.$$

$$\left. + \cosh \alpha x \right) + phx \quad \ldots\ldots (10)$$

when $x = 0$

$$\bar{M}_{(0)} = -\frac{p}{2} \left| \frac{\alpha H - \sinh \alpha H}{\cosh H} \sinh(\tfrac{1}{2}\alpha h) \right.$$

$$\left. + \cosh(\tfrac{1}{2}\alpha h) \right| + \frac{px^2}{8} \quad \ldots\ldots (11)$$

Let $X_1 = \dfrac{ph}{\alpha} \dfrac{\sinh(\frac{1}{2}\alpha h)}{\frac{1}{2}h}$;

$X_2 = \dfrac{\alpha H - \sinh \alpha H}{\cosh H}$; $\quad X_3 = \dfrac{p}{\alpha^2}$

Then the restraint moments may finally be expressed as

$$\bar{M}_{(x)} = - X_1 (X_2 \cosh \alpha x + \sinh \alpha x) + phx \quad \ldots (12)$$

$$\bar{M}_{(0)} = - X_3 \quad X_2 \sinh(\tfrac{1}{2}\alpha h) + \cosh(\tfrac{1}{2}\alpha h) - 1$$

$$+ \frac{1}{8} ph^2 \ldots\ldots\ldots (13)$$

The bending moments M_p due to p are calculated by simple statics. Then $M_{f(x)} = M_p(x) + \bar{M}(x)$,

from which the bending moment on each column is

$$M_1 = \frac{I_1}{I} M_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and the bending moment on each beam is

$$\bar{M}_{12} = \frac{K_{12}}{K} \bar{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

NOTATION

E = modulus of elasticity in bending

I_i = moment of inertia of column or shear wall.

I_{ij} = moment of inertia of beam or connecting member

h = storey height

H = total height of structure

l_{ij} = span of beam, or centre to centre distance between shear walls.

k = stiffness of beam or connecting member

K_{ij} = stiffness of beam or connecting member

P = horizontal load applied at panel points

$p = P/h$ = horizontal load per unit height of column

$\gamma = K/h$ = restraint per unit height of column

$= \left(\frac{\gamma}{EI}\right)^{\frac{1}{2}}$

x = co-ordinate, measured down from top of structure

$\dot{M}$ = restraint moment in beam or connecting member.

M_p = moment due to P.

M_f = moment in column or shear-wall.

NOTE * If the sectional properties of the beams (in each bay) or columns vary with the height of the building, an average value may be taken. This should yield a reasonably accurate answer (5%-10%) and it is left to the designer to decide whether further analysis is necessary.

I.2 B.Cardan's Method

In a multistorey concrete building with shear walls, the lateral loads are resisted by a combination of shear walls and rigid frames. There is a method which is based on a few simple assumptions; with regard to the properties of the building, then expresses the angle deflection of the wall at all points by a second degree differential equation, taking into account the effect of bending and shear.

The basic approach in this method is to

(1) assume that all the forces (lateral load and reactions from parallel frames connected to the wall either directly or through the diaphragm,) are continuously distributed throughout the full height of the wall.

(2) assume that the properties of the wall and the frames are constant for the full height of the building.

The conditions for equilibrium of all external and internal forces acting on the wall will then lead to a second degree differential equation with the rotation of the wall at all points as the unknowns. This equation can be solved, and all the related forces will be found.

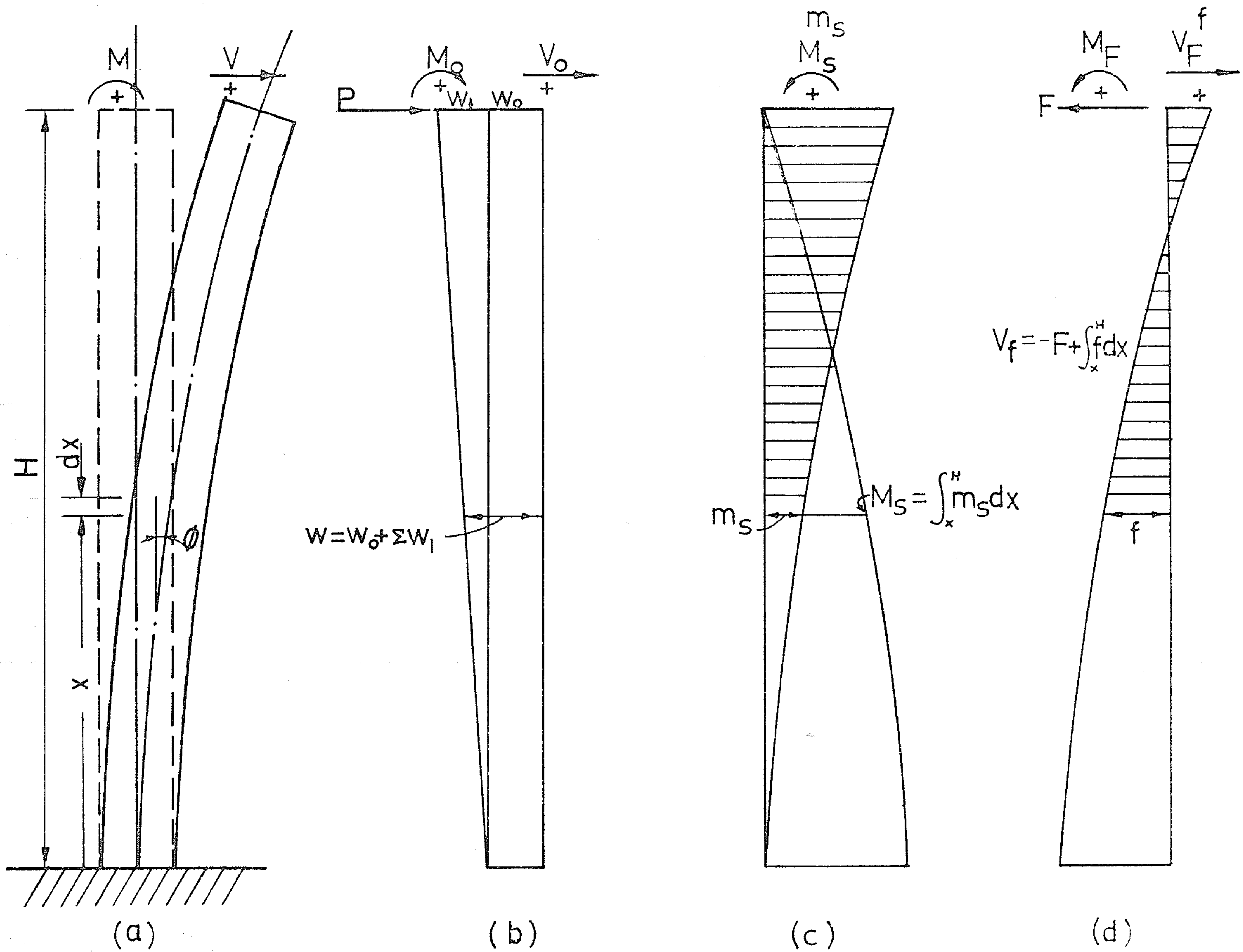

M
V
+
+
H
dx
x
(a)
P
M_o
V_o
+
w_1
w_o
$w = w_o + \Sigma w_1$
(b)
m_s
M_s
+
m_s
$M_s = \int_x^H m_s dx$
(c)
M_F
V_F^f
F
+
$V_f = -F + \int_x^H f dx$
f
(d)

FIG. 2.

ANALYSIS.

First consider the general case of a wall shown in Fig.2, which is subjected to all loads and reactions. The wall is deflected as shown in Fig.2a. The total angle deflection at any point is

$$\phi = \phi_b + \phi_s$$

By differentiation,

$$\frac{d^2\phi}{dx^2} = \frac{d^2\phi_b}{dx^2} + \frac{d^2\phi_s}{dx^2} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

The exterior load consists of a uniform load w_o, a triangular load w_1, and concentrated load P at the top. This combination will cover most wind and seismic loads. (Fig.2b). Let it be assumed that reacting on the wall at the center line is a series of moments m_s,(Fig.2c). Every inch of wall height is subjected to a moment m_s, but this varies from point to point. The total moment M_s at point x is the sum of all moments m_s is the effect of spandrel beams, slabs or girders resting on the wall and is an elastic function of ϕ_b and ϕ_s.

$$m_s = k_1\phi_b + k_2\phi_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

also

$$M_s = \frac{H}{x} m_s dx, \text{ and } \frac{dM_s}{dx} = - m_s \ldots.(3)$$

For later use m_s may be written as a function of V_o and ϕ.

$$m_s = K_1(\phi - \phi_s) + K_2\phi_s$$

$$= K_1\phi - (K_1 - K_2)\frac{3V}{AE}$$

$$= K_1\phi - (K_1 - K_2)\frac{3V_o}{AE}$$

$$+ (K_1 - K_2)\frac{3}{AE}K_3\phi$$

By using the Equ. (5) and (7) which follow.

Also assumed reacting on the wall is a series of elastic horizontal forces f, per inch of wall height, with an additional concentrated force F at the top (Fig.2d). As will be shown later, these forces are the reactions from rigid frames, and the total shear at point s is a function of ϕ.

$$V_f = - K_3\phi \quad \text{..........................(5)}$$

The other formulas for f and F are

$$V_f = - F + \int_x^H f dx = - K_3\phi$$

$$\frac{dV_f}{dx} = - f = - K_3\frac{d\phi}{dx}$$

$$\frac{df}{dx} = K_3\frac{d^2\phi}{dx} \quad \text{..................(6)}$$

$$\frac{dM_f}{dx} = V_f$$

where M_f is the total moment at point s due to F and f. The shear distortion ϕ_s can be expressed by $\phi_s = 1.2\ V/AE$, where generally $G = 0.4\ E$, (the factor 1.2 can vary between 1.0 to 1.5 or more, but is often taken as 1.2)

We have,

$$\phi_s \quad \frac{3V}{AE} = \frac{3V_o}{AE} + \frac{3V_f}{AE} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

$$\frac{d\phi_s}{dx} = -\frac{3w}{AE} - \frac{3K_3}{AE}\frac{d\phi}{dx}$$

$$\frac{d^2\phi_s}{dx^2} = -\frac{3}{AE}\frac{dw}{dx} - \frac{3}{AE}K_3\frac{d^2\phi}{dx^2} \quad \ldots\ \ldots\ldots(8)$$

The bending distortion ϕ_b can be expressed by

$$\frac{d\phi_b}{dx} = \frac{M}{EI} = \frac{1}{EI}(\ M_o - M_s - M_f\ )\ldots\ldots(9)$$

$$\frac{d^2\phi_b}{dx^2} = \frac{1}{EI}(\ -V_o + m_s - V_f\ )\ldots\ldots\ldots\ldots(10)$$

By applying Equ.(4) and (5) to Equ.(10) and substituting Equ.(8) and (10) in Equ.(1), it can be show that

$$\frac{d^2\phi}{dx^2} - B\phi = -CV_o - D\frac{dw}{dx} \quad \ldots\ldots\ldots\ldots(11)$$

where B, C, and D, are abbreviations(see notation). V_o and w are functions of s only.

Equ.(11) show the mathematical form

$$\frac{d^2y}{dx^2} - Cy = f(x), \quad \text{where } f(x)=ax^2+bx+c \qquad \ldots\ldots\ldots(12)$$

This has the solution $y = F(x) + C_1 e^{cx} + C_2 e^{-cx}$ where $F(x)$ is any function that satisfies Equ.(12).

Once Equ.(11) is solved, the wall shears, moments etc., may be found from the preceding formulas.

They are:

$$M = EI\frac{d\phi}{dx} + \frac{3EI}{AE}w \qquad \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

$$V = V_o - K_3\phi \qquad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

$$\phi_s = \frac{3V}{AE} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

$$\phi_b = \phi - \phi_s \qquad \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

$$m_s = K_1\phi_b + K_2\phi_s \qquad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

$$f = K_3\frac{d\phi}{dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

$$F = K_3\phi \;(\;\xi = 1\;) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$V_f = -K_3\phi \qquad \ldots\ldots\ldots\ldots\ldots\ldots(20)$$

$$y = \int_o^x \phi\, dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

Notation

A = cross sectional area of shear wall

I = moment of inertia of shear wall

H = height of shear wall

h = floor to floor height

I_b = moment of inertia of spandrel beam

I_s = moment of inertia of spandrel beams per inch of wall height, or I_b/h

E = modulus of elasticity of concrete in bending

G = modulus of rigidity of concrete in shear

P = concentrated exterior horizontal force at top of wall

w = exterior horizontal load on wall per inch of height

V_o = shear in wall due to w and P

M_o = moment in wall due to w and P

m_s = elastic moment reaction per inch of wall height

M_s = elastic moment reaction in wall due to m_s

f = elastic horizontal reaction force at top of wall

V_f = wall shear due to f and F

M_f = moment in wall due to f and F

V = total wall shear

M = total wall moment

x = coordinate measured upward from base of wall

ξ = x/H

ϕ = slope of deflected wall due to all force

ϕ_b = slope of deflected wall due to bending only

ϕ_s = slope of deflected wall due to shear only

K_1 and K_2 = constant, defined as $m_s = K_1\phi_b + K_2\phi_s$

K_3 = constant, defined as $V_f = -K_3\phi$

y = horizontal deflection of wall

$= 1 + 3K_3/AE$

$B = \beta\ (K_1 - K_2) + K_2 + K_3/\beta EI$

$C = 1 + 3(\ K_1 - K_2)/AE\ /\beta EI$

$D = 3/\beta AE$

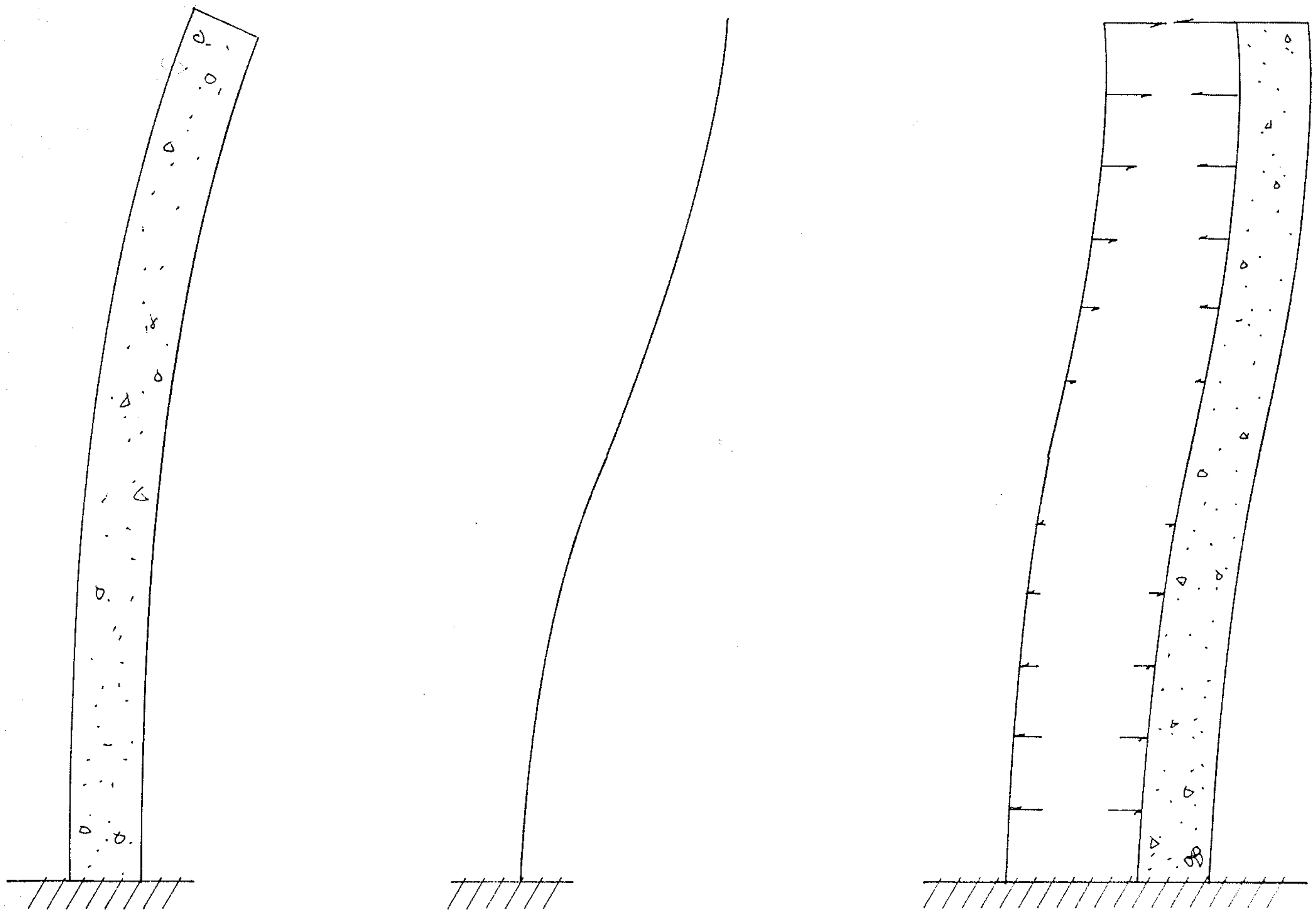
$= H\sqrt{B}$

$= CH^2 + 2D - 2C/B$

I.3 F.R.Khan's Method

The development of the method depends on the conflicting physical characteristics of the two system; a rigid frames system and a shear walls system. If the frame alone is considered to take the full lateral load, it would develop moments in columns and beams to resist the total shear at each storey while the effects of overturning would normally be considered secondary and in most cases, it is negligible. In resisting all lateral loads, a frame would deflect as in(Fig.3a). The floor would remain essentially level even though the joints would rotate, if a shear wall, on the other hand, is considered to resist all the lateral loads, it would develop moments at each floor equal to the overturning moment at that level and the deflected shape as in (Fig.3b). If a shear wall and a frame exist in a building, each one will try to obstruct the other from taking its natural free deflected shape as shown in (Fig. 3c).

The analysis is performed in two stages. In the first stage of analysis of a structure, it is necessary to determine the deflected shape and the amount of lateral load distributed to the walls and frame, respectively,at each storey, and the structure is separated into two distinct systems.



(a) Free wall

(b) Free frame

(c) Combined wall & frame

FIG. 3.

(1) System "W"(wall) $S_{syst.w.} = S_{wall}$

(2) System "F"(frames) $S_{syst.f.} = S_{col.} + S'_{bm.} + S''_{link\ bm.}$

FIRST STAGE OF ANALYSIS: (Solution by Iteration).

The equilibrium of the total structure requires that the following conditions be satisfied:

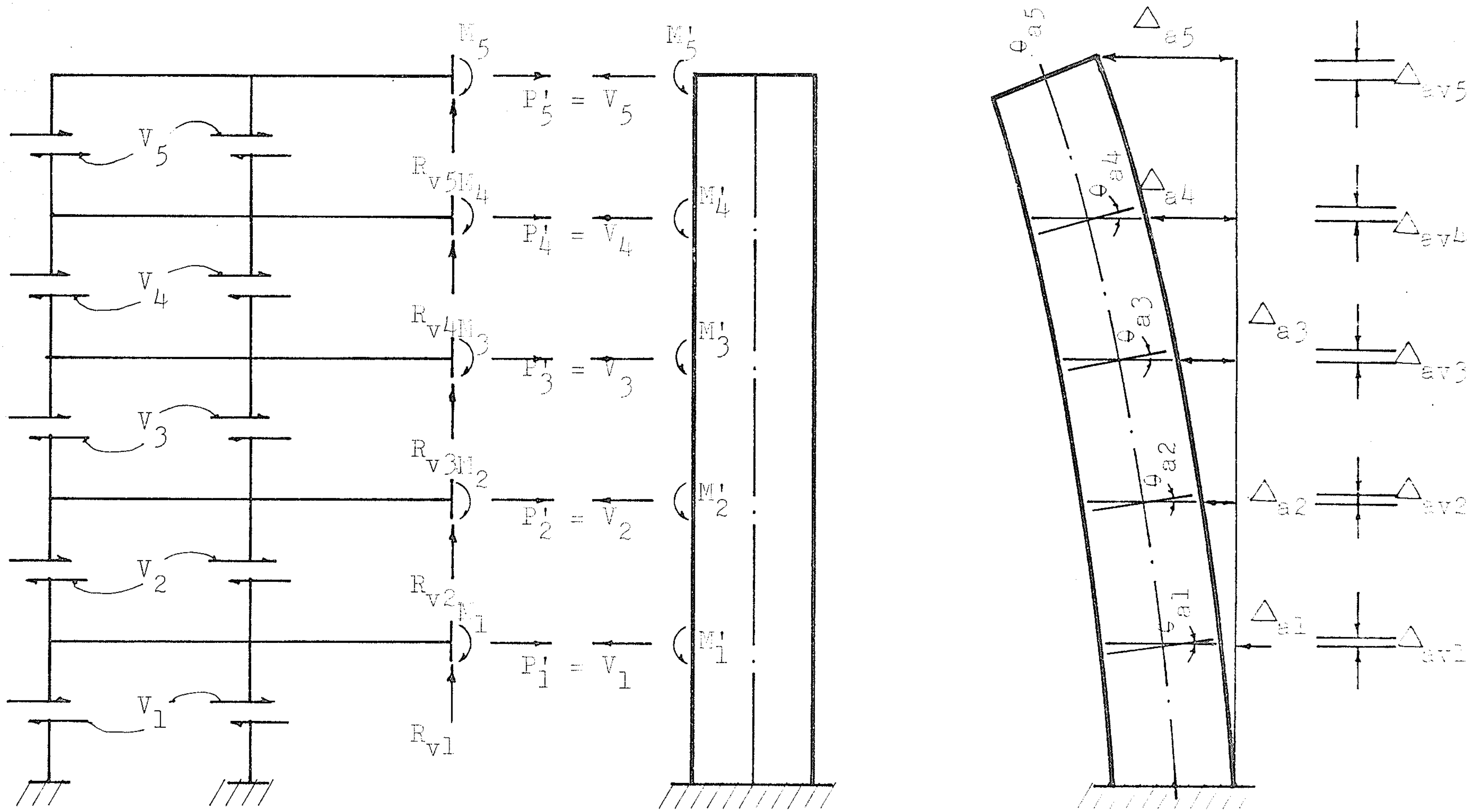
(1) Deflection in System W and System F must be the same at corresponding levels.

(2) "Link" members connecting system "F" to System "W" must undergo the same rotations and vertical translations as those of System W at their points of connection.

(3) Horizontal shear, V_w, developed in System W plus the horizontal shear, V_f, developed in System F must be equao to the total external shear, V_t, at every storey.

The foregoing three requirements of compatibility and equilibrium can be achieved by the following steps of analysis:

a) Apply the total external loads on the idealized structure(Fig.4) at each floor level, consider the System W alone to carry all the external loads in the first step. By conjugate beam method, the slopes and deflections of System W at each floor level are determined (Fig.5). The vertical movements of the connecting points with System W



Forces and Moments after moment distribution.

Forces and Moments from syst. F. applied to shear wall system.

$$M' = R_v L_s + M$$

FIG. 4.

are computed by multiplying the slope at each level by the distance from the neutral axis of the wall to the connecting point

$$\triangle_{fv} = L_s \, \theta_f \quad \text{.........................(1)}$$

b) This is the first cycle of iteration. For quick convergence, a final deflected shape could be assumed or approximated from Reference 4. However, the final deflected shape is assumed to be the same as the free deflected shape of System W, which would mean that, in the first cycle, initial deflection and rotation at any floor are equal the free deflection of wall at the same level, respectively.

$$\triangle_{ii(1)} = \triangle_{fi}$$

$$\theta_{ii(1)} = \theta_{fi} \quad \text{.........................(2)}$$

c) System F is forced to undergo the assumed deflections at each floor. Also, the connecting members at each floor requires that it must have the same rotations and vertical translations as System W at their points of connection with System F. As long as we have the storey deflections and rotations at the connecting points, the fixed-end moments (for uniformly section members) can be computed by

$$FM_{biw} = \left(\frac{2EI_{bi}}{L_b}\right) \left[2 + 3\left(\frac{L_s}{L_b}\right)\right] \theta_i \text{............(3)}$$

$$FM_{bif} = \left(\frac{2EI_{bi}}{L_b} \right) \left[1 + \frac{3(L_s)}{L_b} \right] \theta_i \quad ..(4)$$

d) After force-fitting System F to System W, the total shears in each storey of System F as well as moments and reactions applied on System W by the connecting links are computed. The shears generated by force-fitting can be used directly in the next step.

e) All shears, forces, and moments generated by force-fitting System F are applied to the isolated free System W (Fig.4). At each storey, M and R_V should be replaced by a moment

$$M' = M + R_V L_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

then, the deflections and rotations of System W at the i^{th} floor, at the end of the first cycle, would be expressed as

$$\triangle_{ei(1)} = \triangle_{fi} - \triangle_{ai(1)} \quad \ldots\ldots\ldots\ldots\ldots(6)$$

$$\theta_{ei(1)} = \theta_{fi} - \theta_{ai(1)} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

For a stable condition the assumed initial deflections at any floor "i" at the beginning of the n^{th} cycle, $\triangle_{ii(n)}$ must be the same as the end deflections, $\triangle_{ei(n)}$, at the completion of the n^{th} cycle. However, in many cases the first cycle, $\triangle_{ei}$ is negative, indicating that the iteration is divergent. The generalization of this method of solution therefore depends on the use of a proper "force-convergence-correction" to be applied to the initial deformations of the n^{th} cycle, $\Delta_{ii(n)}$ and $\theta_{ii(n)}$, to obtain the initial trial deformations of the $(n+1)^{th}$ cycle, $\Delta_{ii(n+1)}$ and $\theta_{ii(n+1)}$.

f) The convergence correction is derived from the hypothesis that in each cycle the movement of System W at each floor with respect to its free deflected shape is lineally proportional to the movement of System F with respect to the vertical line. Therefore, it can be shown that if at the n^{th} cycle the initial trial values at the i^{th} floor were $\Delta_{ii(n)}$ and $\theta_{ii(n)}$ and the end values were $\Delta_{ei(n)}$ and $\theta_{ei(n)}$, the initial trial values at the $(n+1)^{th}$ cycle should be

$$\Delta_{ii(n+1)} = \Delta_{ii(n)} + \frac{\Delta_{ei(n)} - \Delta_{ii(n)}}{1 + \left[\dfrac{\Delta_{fi} - \Delta_{ei(n)}}{ii(n)}\right]} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

$$\theta_{ii(n+1)} = \theta_{ii(n)} + \frac{\theta_{ei(n)} - \theta_{ii(n)}}{1 + \left[\dfrac{\theta_{fi} - \theta_{ii(n)}}{\theta_{ii(n)}}\right]} \quad \ldots\ldots\ldots\ldots\ldots (9)$$

g) Values of Δ and θ obtained by Eq. (8) and (9) are used as initial values for the next cycle, and the procedure is repeated beginning with the second step outlined previously.

h) At the end of each cycle, $\triangle_{ei}$ and $\triangle_{ii}$ should be checked until the convergency is within a specific tolerance, (5% -10%)

SECOND STAGE OF ANALYSIS:

After convergence of the iteration solution has been achieved, the final deflected shape of the structure is used to distribute moments and shear to every member in each bent of the structure.

At a column line that contains no shear walls, a set of fixed end column moments obtained from the difference in storey deflection can be apportioned to all the members by moment distribution. No sidesway correction is needed because the bent is in its final deflected shape.

If a shear wall is contained in a bent, it can be treated separately from the frame segment. With a known deflected shape and EI, the moment at any floor, i, can be obtained from

$$M_i = \left(\frac{EI_{si}}{h^2}\right) \left(\triangle_{i+1} - 2\triangle_i + \triangle_{i-1}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Notation

E = modulus of elasticity

G = shear modulus

H = total height of a structure

h_i = height of i^{th} storey

h' = height of hypothetical wall to simulate elastic foundation

I_{si} = moment of inertia of shear wall at i^{th} storey

L = span of slab

L_b = span of link beam

L_s = distance from the neutral axis to the extreme fibers of a shear wall

l = bay-width

l_e = effective width of slab

M_i = moment applied on the shear wall by the connecting link at floor, i.

M_i' = total moment applied on the shear wall by the connecting link at floor i

R_{vi} = vertical reaction of the link beam at the shear wall at floor i

S_b = sum of stiffnesses of all beams in the simplified frame is equal $S_b' + S_b''$

S_b' = sum of stiffnesses of all beams

S_b'' = sum of stiffnesses of all link beams

S_c = sum of stiffnesses of all columns

S_s = sum of stiffnesses of all shear walls

S_c/S_b = column-beam stiffness ratio at first storey

S_s/S_c = wall-column stiffness ratio at first storey

t = thickness of slab

V_f = shear in frame

V_{fx} = shear in frame at height x/H

V_t = total applied shear

V_{tx} = total applied shear at a height of x/H

V_{wx} = shear in wall at a height x/H

Δ_{fi} = free deflection of wall at floor, i

Δ_i = deflection at i^{th} floor

$\Delta_{ei(n)}$ = net deflection at i^{th} floor at end of n^{th} cycle of iteration

$\Delta_{ii(n)}$ = deflection at i^{th} floor at beginning of n^{th} cycle of iteration

$\Delta_{ai(n)}$ = the negative of deflection of system wall

Δ_{vi} = vertical movement of the shear wall at floor, i

$\theta_{ei(n)}$ = rotation in shear wall at i^{th} floor at the end of n^{th} cycle

$\theta_{ii(n)}$ = rotation in shear wall at i^{th} floor at the beginning of n^{th} cycle

$\theta_{ai(n)}$ = The negative rotation of system wall

$\theta_{i,n}$ = joint rotation in frame at i^{th} floor at n^{th} column line

Note: Subscript (i) denotes storey or floor number

CHAPTER II. THE MODEL, THE TESTS AND TESTING RESULTS, FLAT SLAB.

Three approximate methods in calculating the effects of lateral loads on shear wall of a multistorey reinforced concrete building have been introduced in chapter one. But most of these methods make simplying assumptions such as constant storey height, constant cross section of walls and columns, or assume a certain shear distribution on the walls. However, these assumptions lead to a simpler solution of the analysis of shear wall design. Unfortunately, none of the authors has put up any experimental results to verify their method. In this chapter, the experimental results from a ten-storey reinforced concrete model are shown, together with the results which are calculated by using previously described methods.

II.1 The Model, The Tests and Their Results

The data of all the tests in this thesis are obtained by applying lateral forces to a ten-storey reinforced concrete model (scale 1" = 1'-4"). All dimensions of the model are shown in Fig.5 to Fig.10. For its detailed data, material and manufacturing procedure, refer to " Model analysis of a ten storey building" (Master thesis, 1965. Dept. of Civil Engineering, Univ.

of Manitoba. By Robert Petri.) However, I would like to outline the facts concerning my work as follows:

a) The foundation of the model:

The model is fixed on an eight inch thick concrete plate on which vertical loads are applied throughout the tests. In order to prevent the basement rotation of the model.

b) The test method:

In applying lateral forces to the model, an increment of $\frac{1}{4}$ of the model design load is used. Besides lateral forces, there is no extra vertical load applied to the model except for its own weight. The lateral loading systen is shown in Fig.11.

c) The measurement of strain:

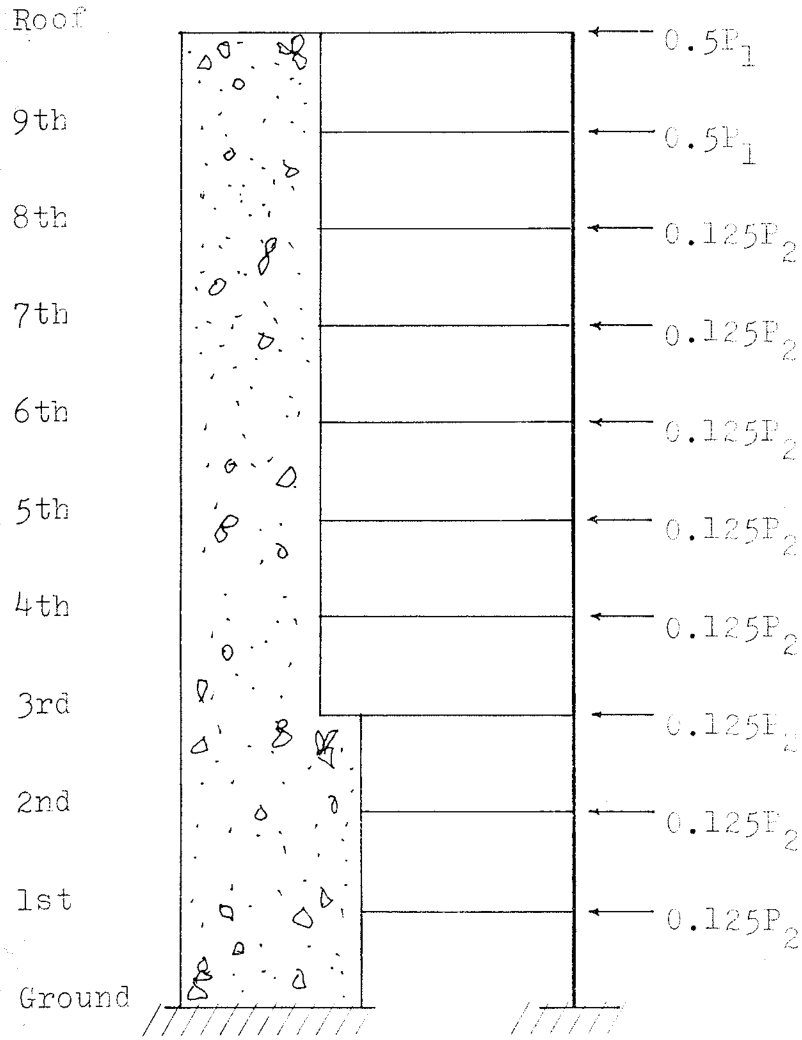
To measure the horizontal deformation, Mechanical Strain Gages are placed on each floor leved. Strains can be read when forces are added.

For the strains of slabs, Electrical Strain Gages (SR-4) are placed on the slab surfaces, as shown in Fig.6 , and the same kind of gage is used to measure strains of shear wall and columns, as shown in Fig.7.

d) The results:

The difference of each strain reading of a certain point is the net strain of that point, and the results are plotted as shown in Fig.15 to Fig.40. To clarify the strain curves are shown here plotted from loads of the first group only.

Some other phenomena worth mentioning due to various loads, will be pointed out in the concluding chapter.



Model testing load increaments

Test 1.	$P_1 = 80^{\#}$	$P_2 = 320^{\#}$
Test 2.	$P_1 = 120^{\#}$	$P_2 = 480^{\#}$
Test 3.	$P_1 = 160^{\#}$	$P_2 = 640^{\#}$

FIG. 5. TESTING LOAD SYSTEM FOR THE MODEL.

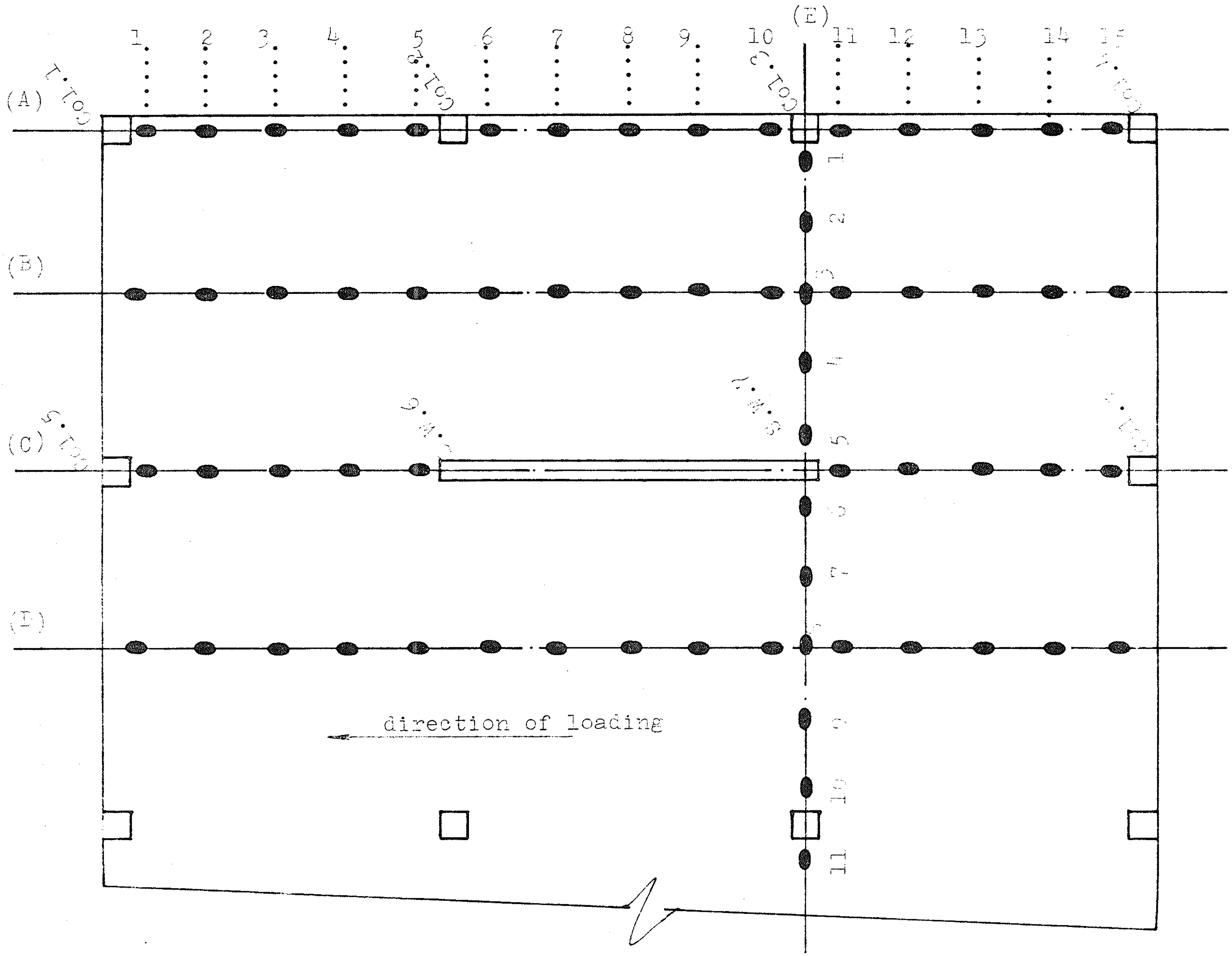


FIG. 6. The location of electrical strain gages on plan view of floor slab at 9th, 5th, 2nd floor level.

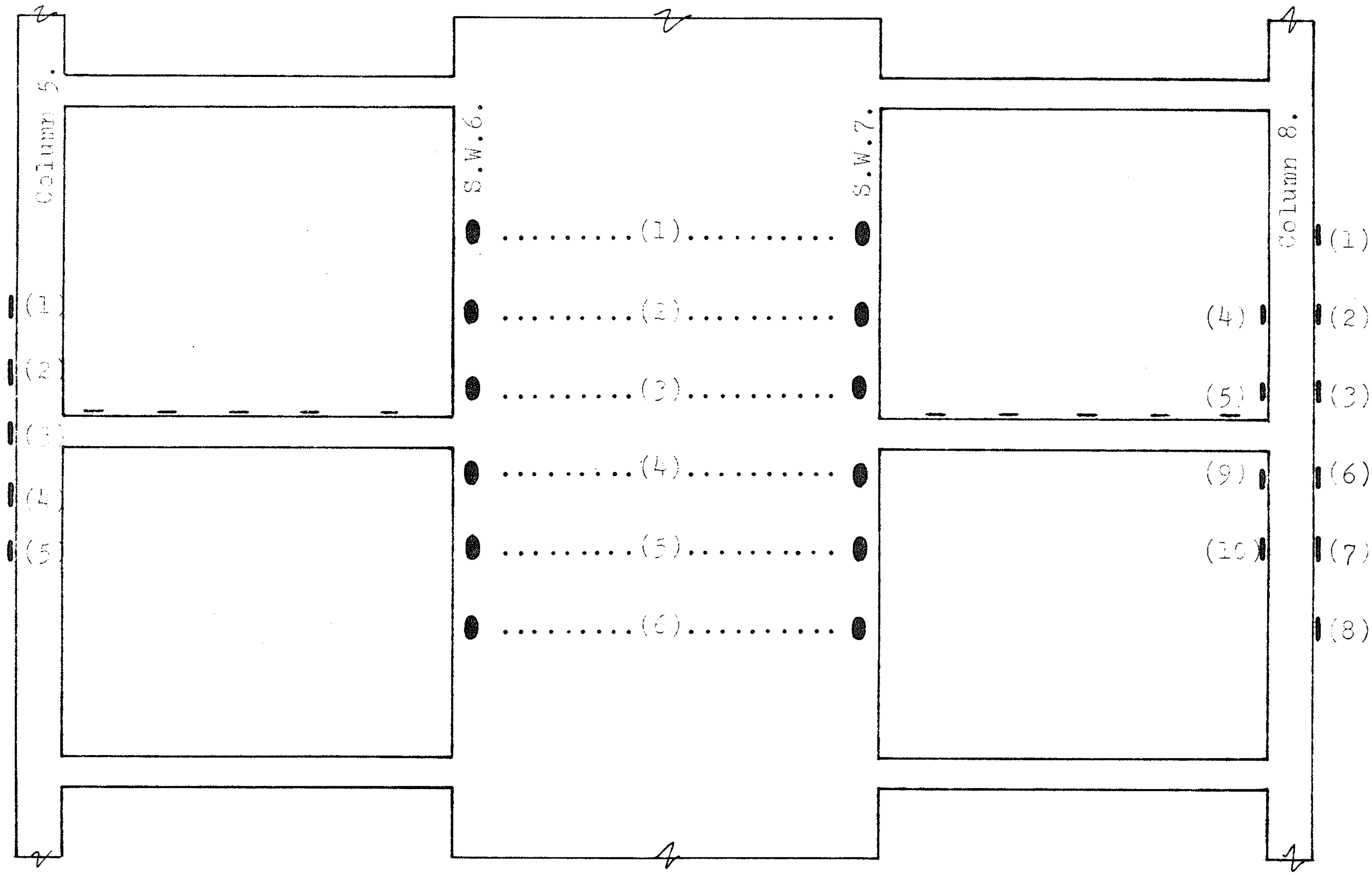


FIG. 7. LOCATION OF ELECTRICAL STRAIN GAGES ON SHEAR WALL AND COLUMNS AT 9th, 5th, 2nd FLOOR LEVEL.

HORIZONTAL DEFLECTION OF THE MODEL MEASURED BY MECHANICAL DIAL STRAIN GAGES

FLOOR	TEST 1 10^{-3}	TEST 2 10^{-3}in.	TEST 3 10^{-3}in.
ROOF	47.06	71.70	96.00
9th	40.74	62.00	82.70
8th	34.50	52.70	70.40
7th	28.43	43.40	58.00
6th	22.64	35.04	46.02
5th	17.23	26.20	35.02
4th	12.28	18.70	25.10
3rd	7.90	12.10	16.10
2nd	4.11	6.25	8.40
1st	0.99	1.51	2.01
GROUND	0.00	0.00	0.00

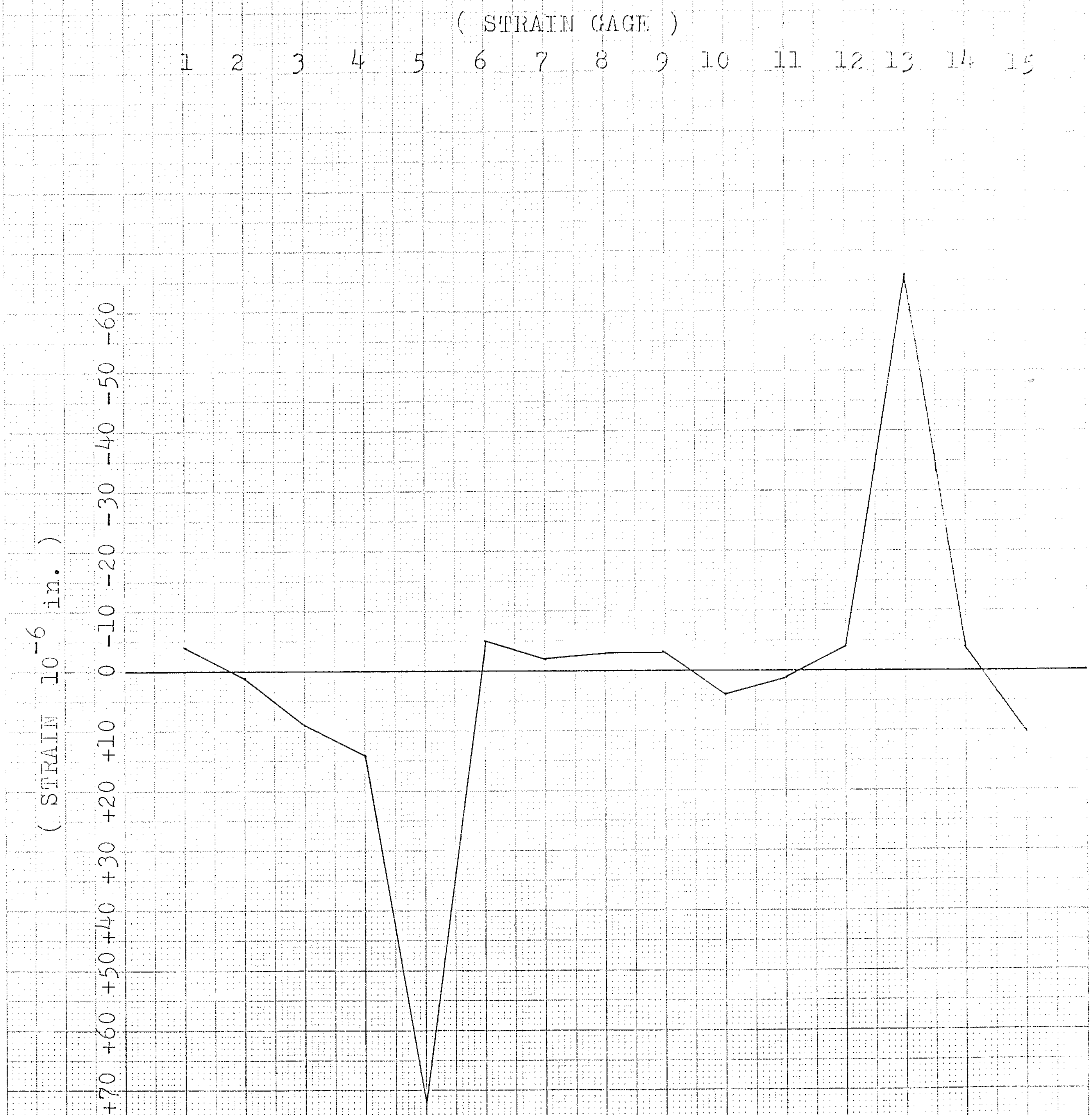


FIG. 8. STRAINS OF 9^{th} FLOOR SLAB ALONG LINE-A

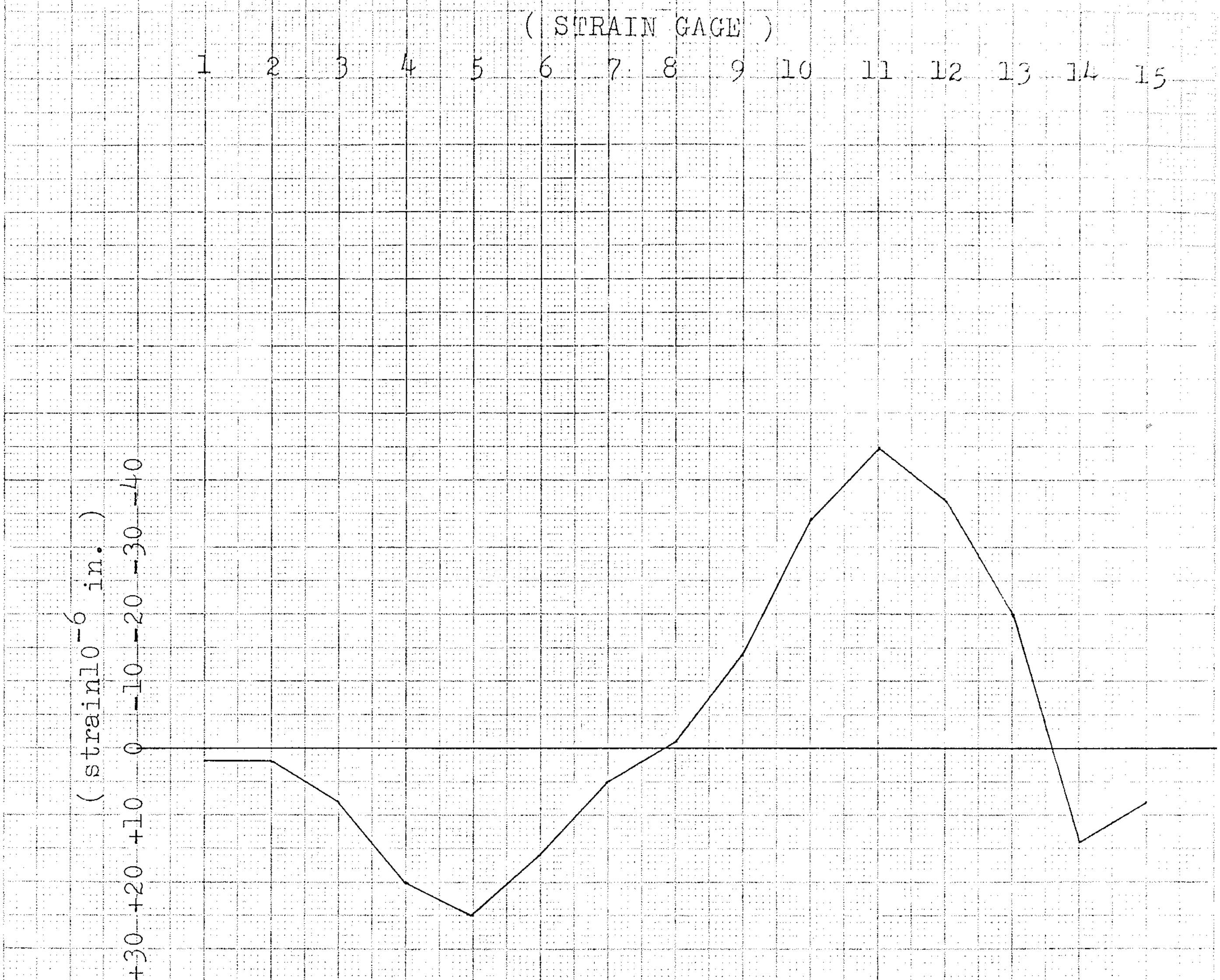


FIG. 9. STRAINS OF 9^{th} FLOOR SLAB ALONG LINE-B.

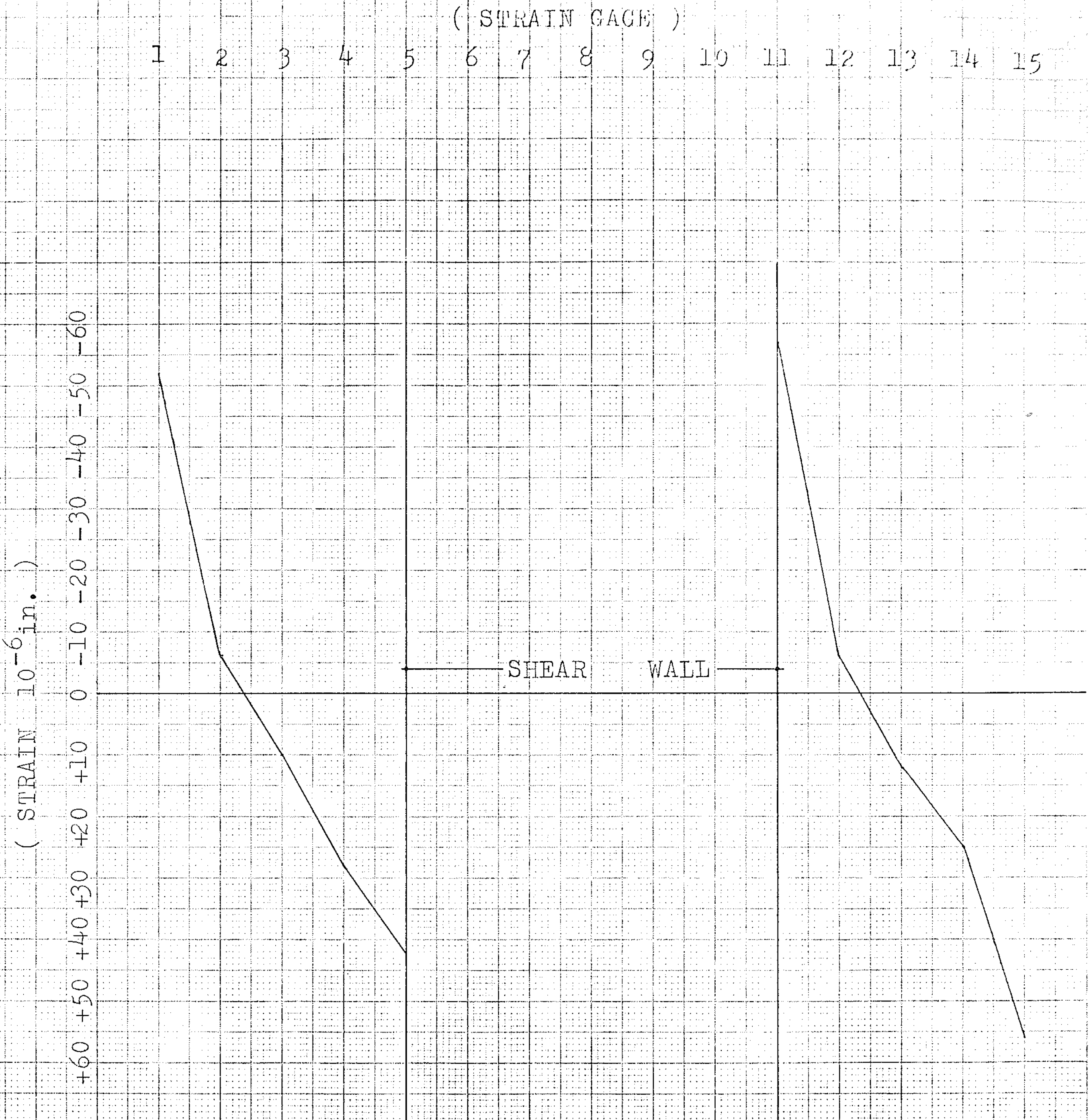


FIG. 10. STRAINS OF 9^{th} FLOOR SLAB ALONG LINE-C.

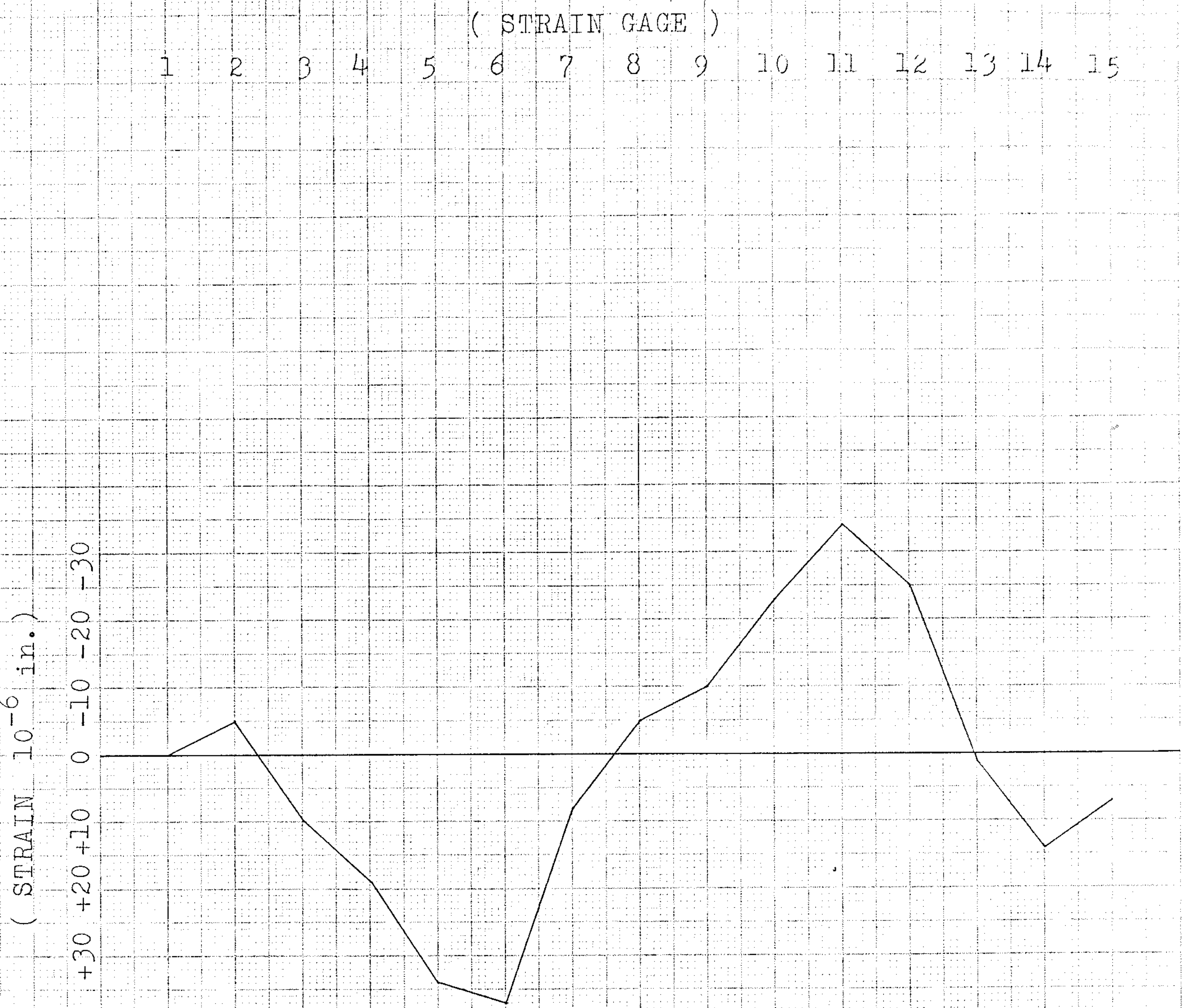


FIG. 11. STRAINS OF 9^{th} FLOOR ALONG LINE-D.

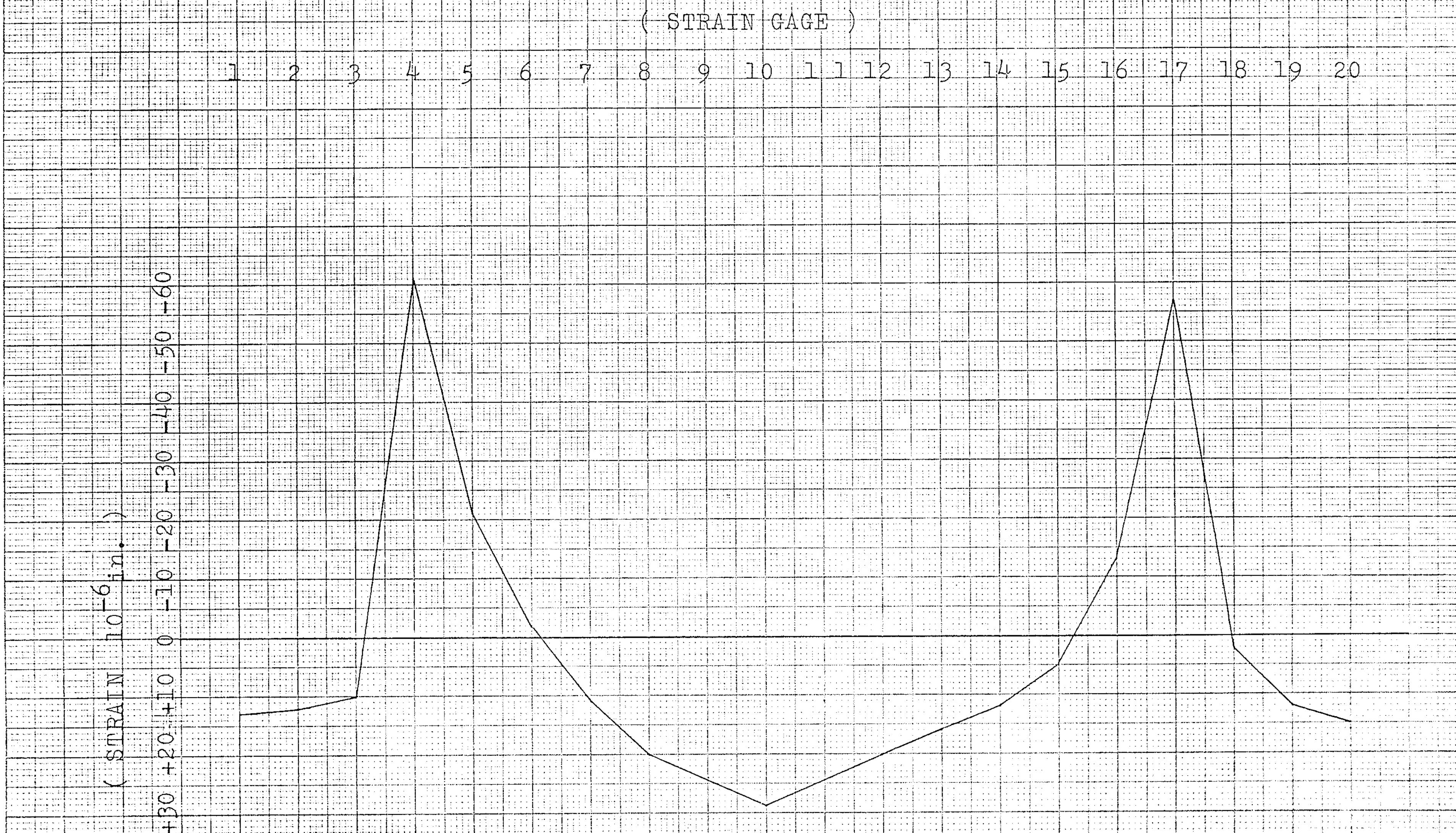


FIG. 12, STRAINS OF 9th FLOOR ALONG LINE-E.

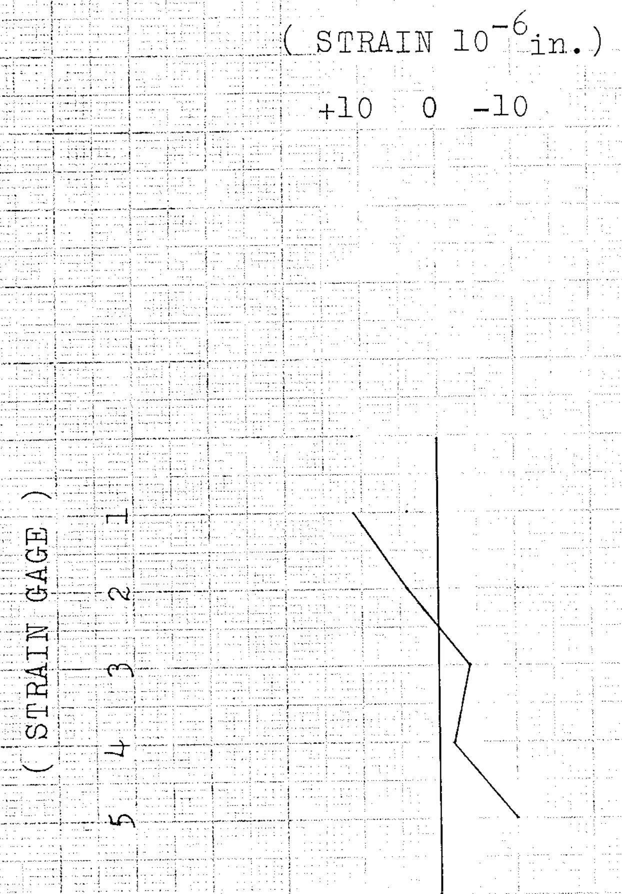


FIG. 13. STRAINS OF COL.4 AT 9^{th} FLOOR LEVEL.

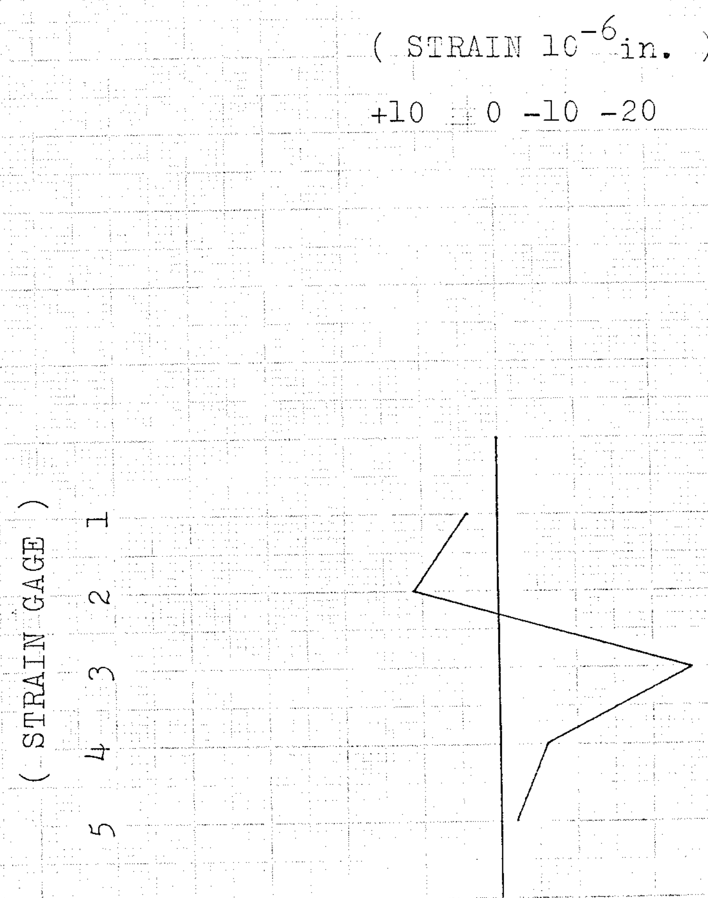


FIG. 14. STRAINS OF COL.5 AT 9^{th} FLOOR LEVEL.

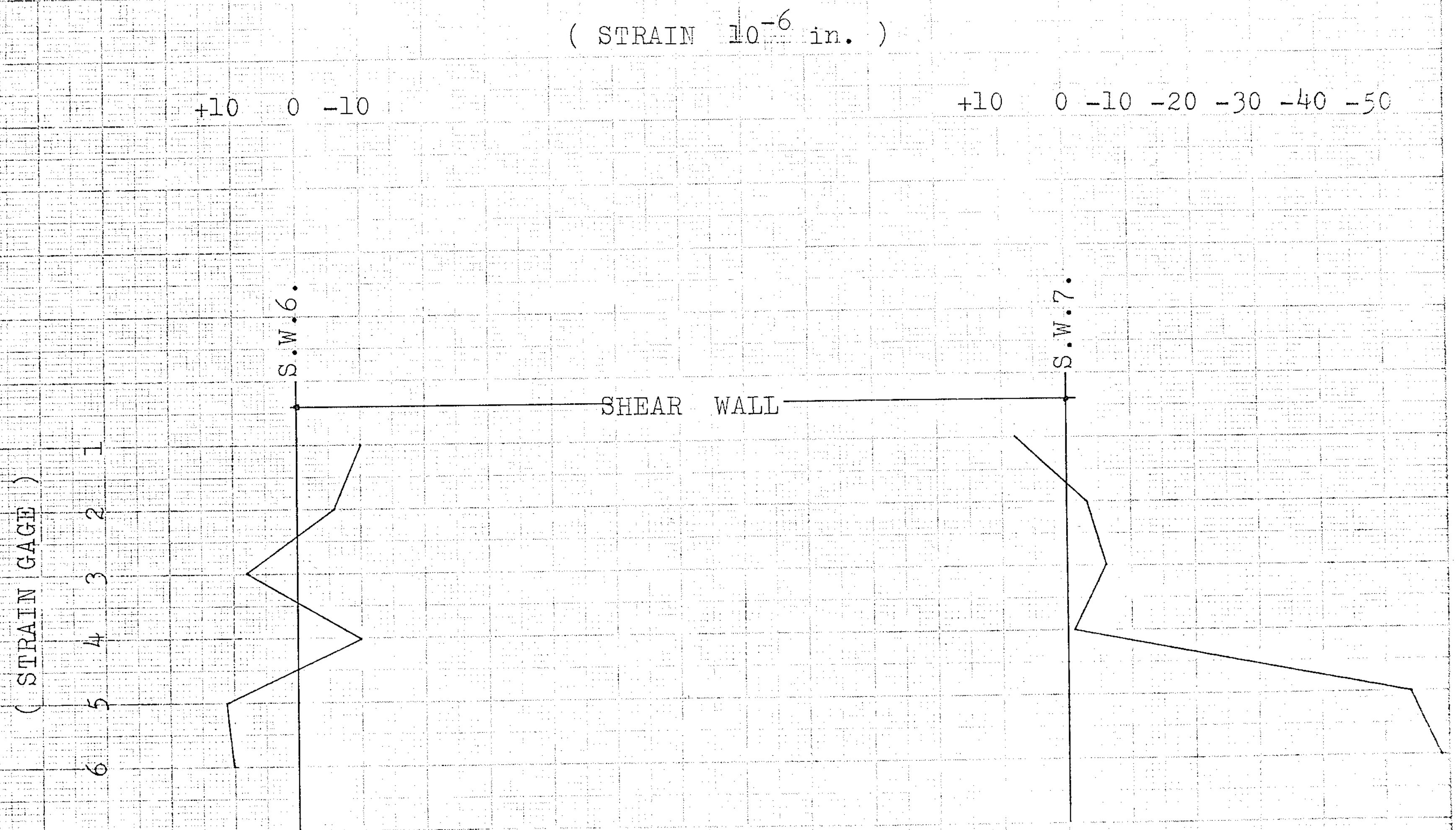


FIG. 15. STRAINS OF SHEAR WALL AT 9^{th} FLOOR LEVEL.

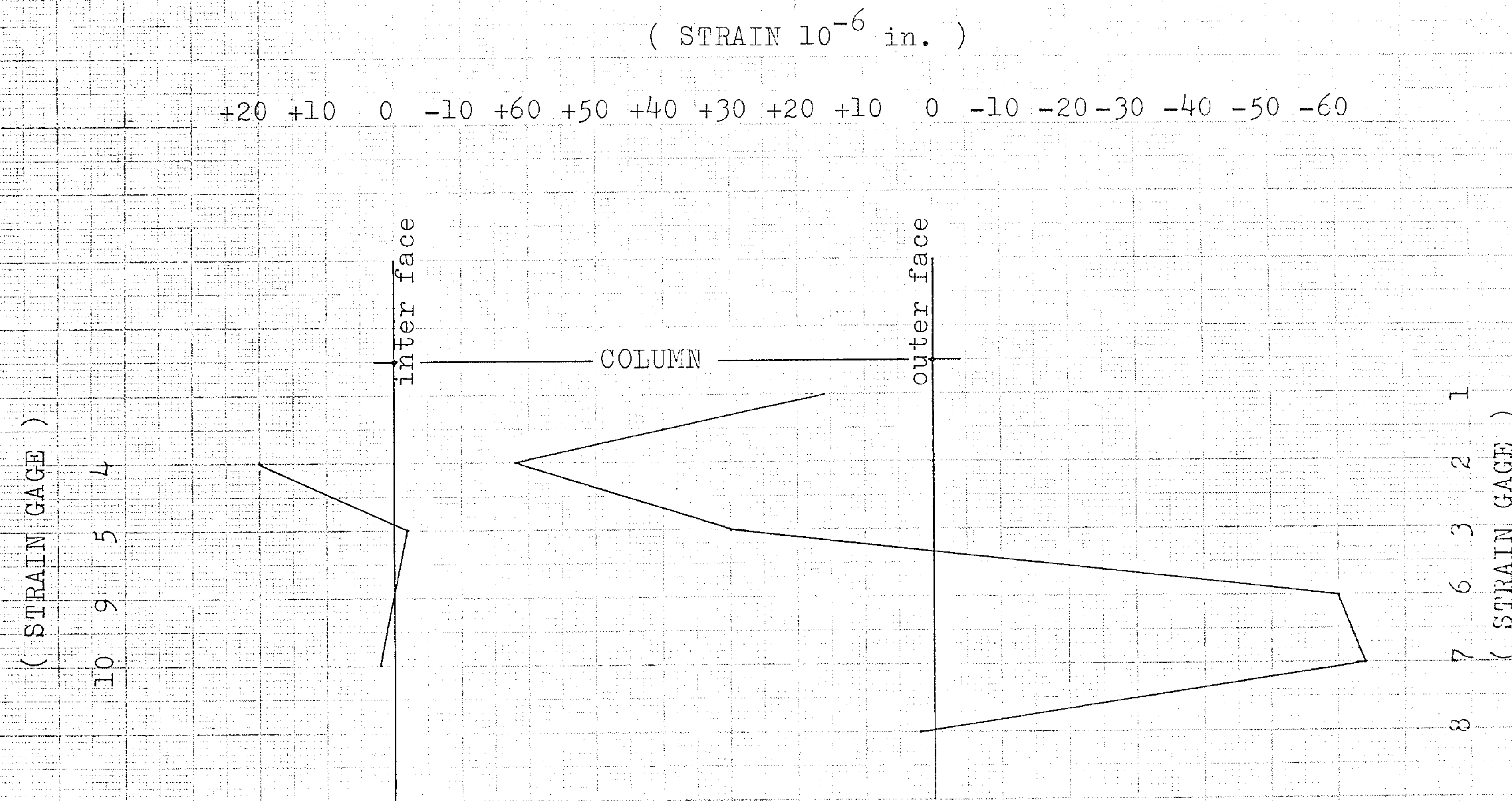


FIG. 16. STRAINS OF COL.8. AT 9^{th} FLOOR LEVEL.

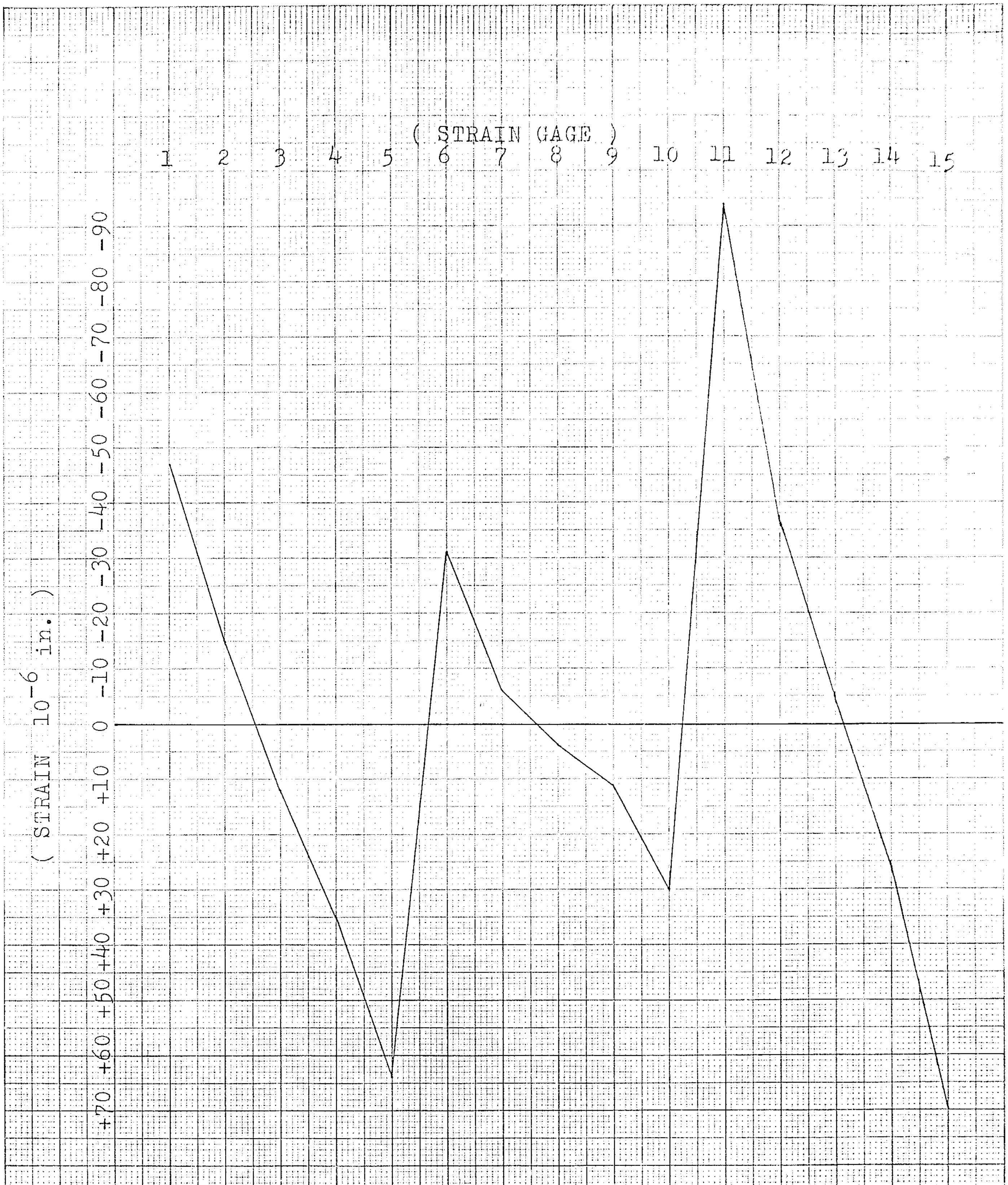


FIG. 17. STRAINS OF 5^{th} FLOOR SLAB ALONG LINE-A.

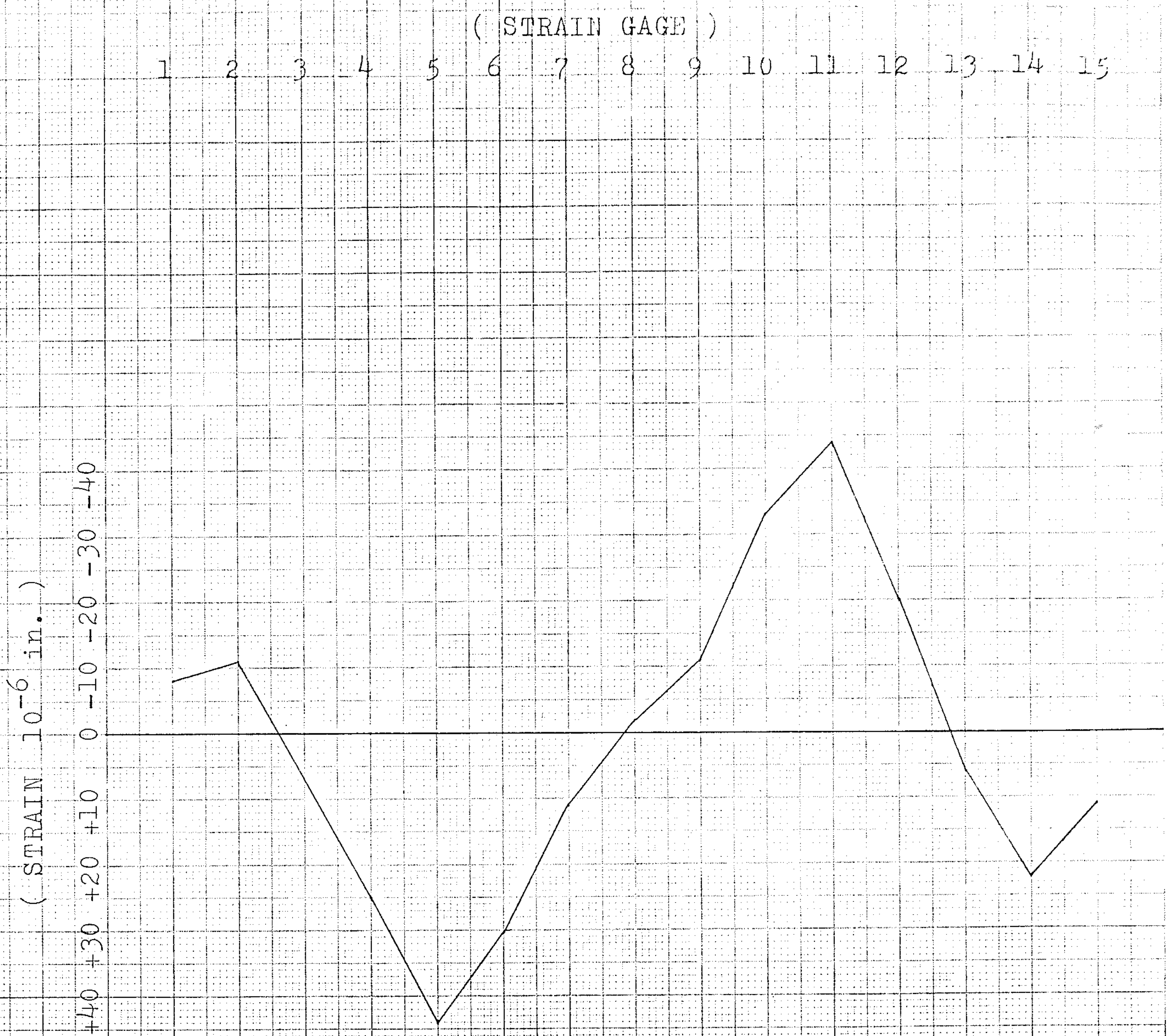


FIG. 18. STRAINS OF 5^{th} FLOOR SLAB ALONG LINE-B.

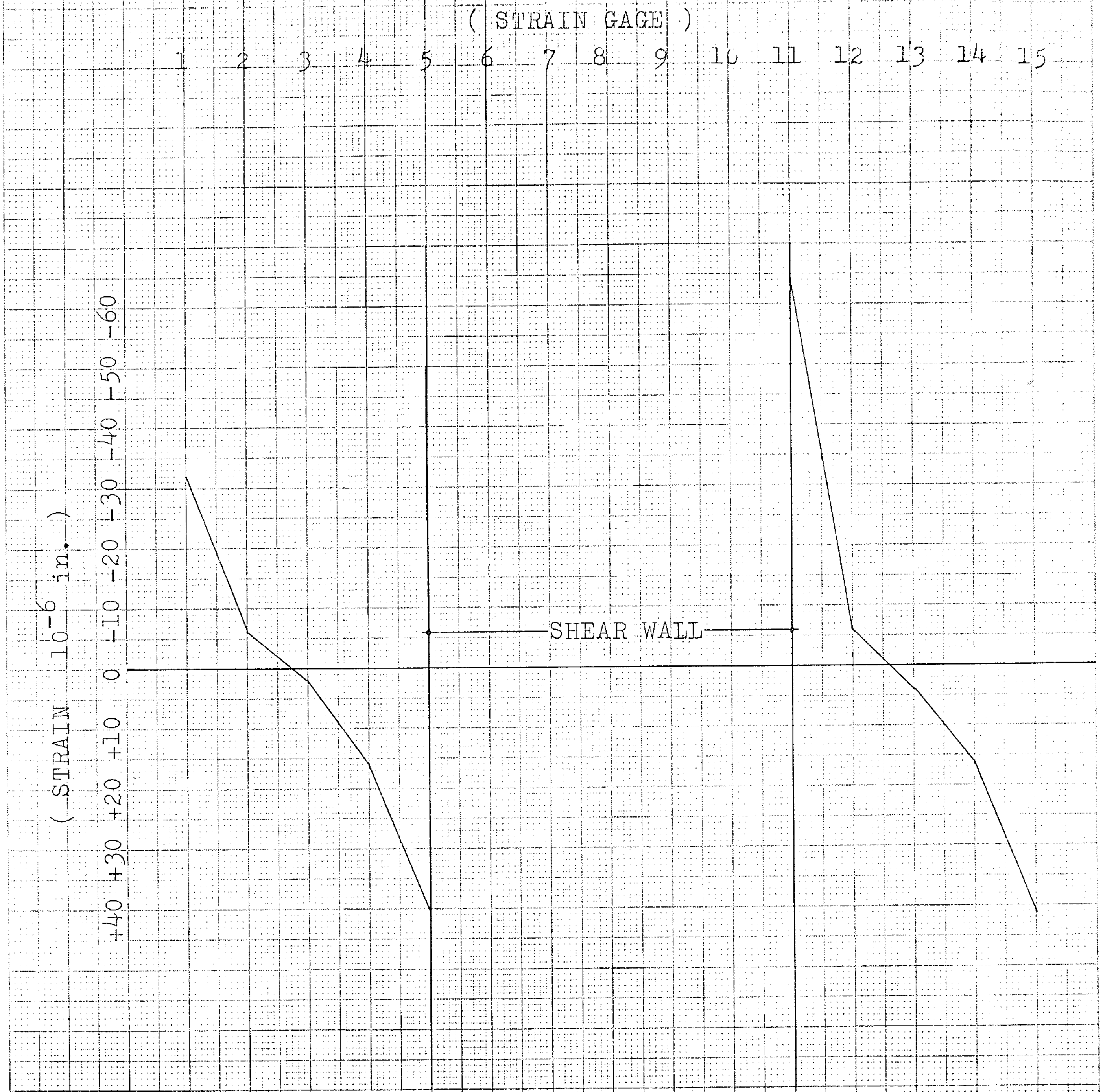


FIG. 19. STRAIN OF 5th FLOOR SLAB ALONG LINE-C.

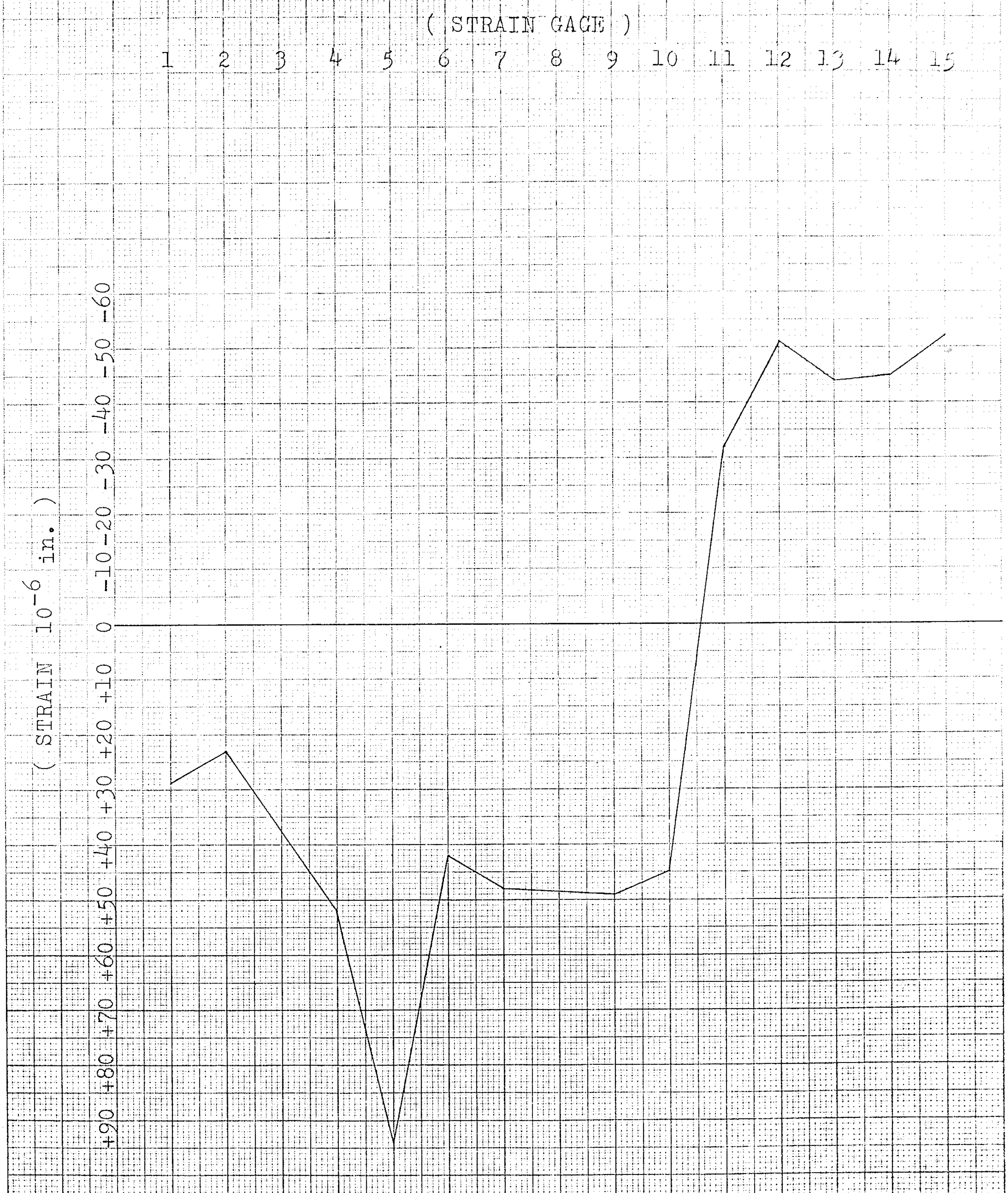


FIG. 20. STRAINS OF 5^{th} FLOOR SLAB ALONG LINE-D.

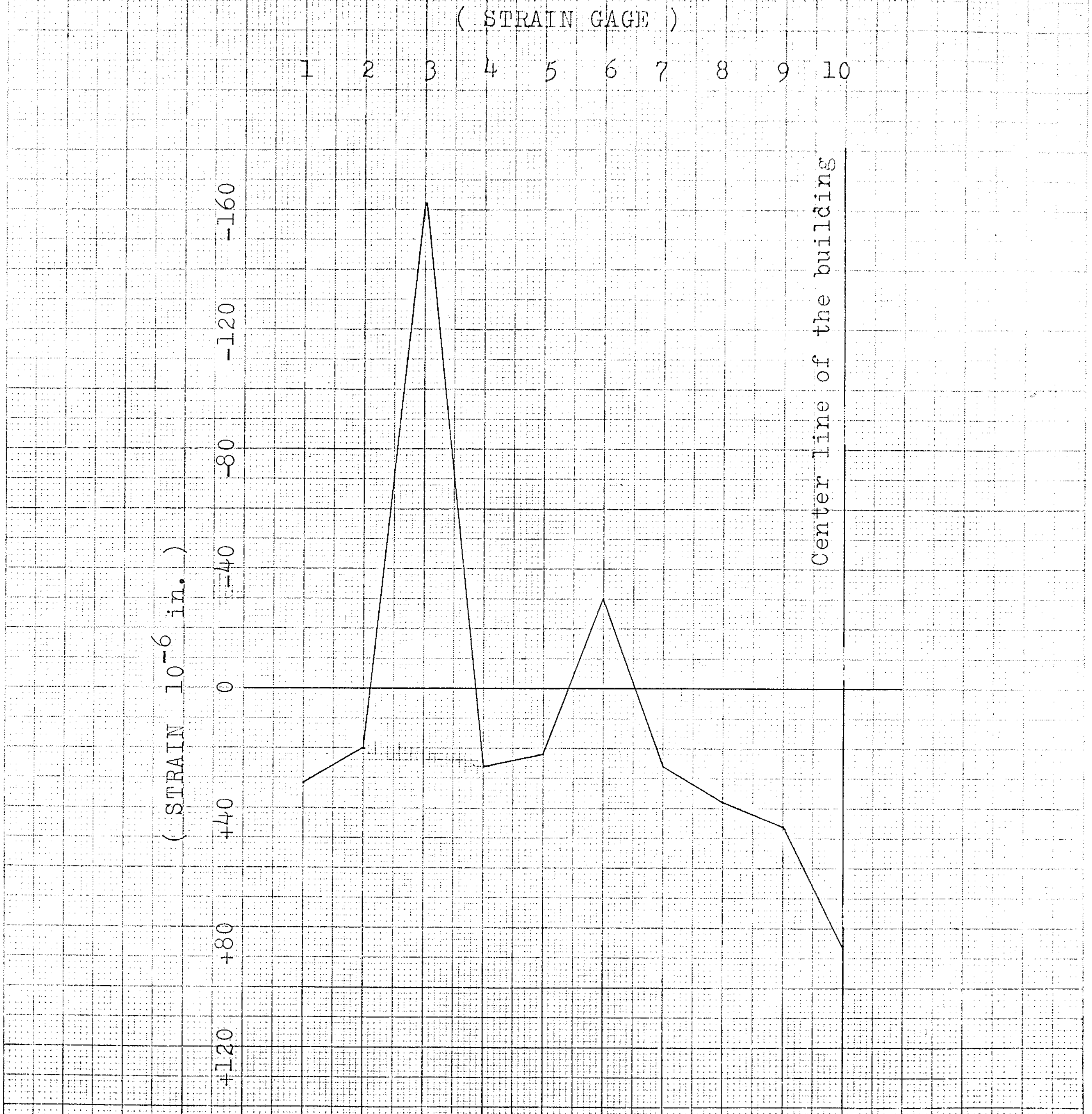


FIG. 21. STRAINS OF 5^{th} FLOOR SLAB ALONG LINE-E.

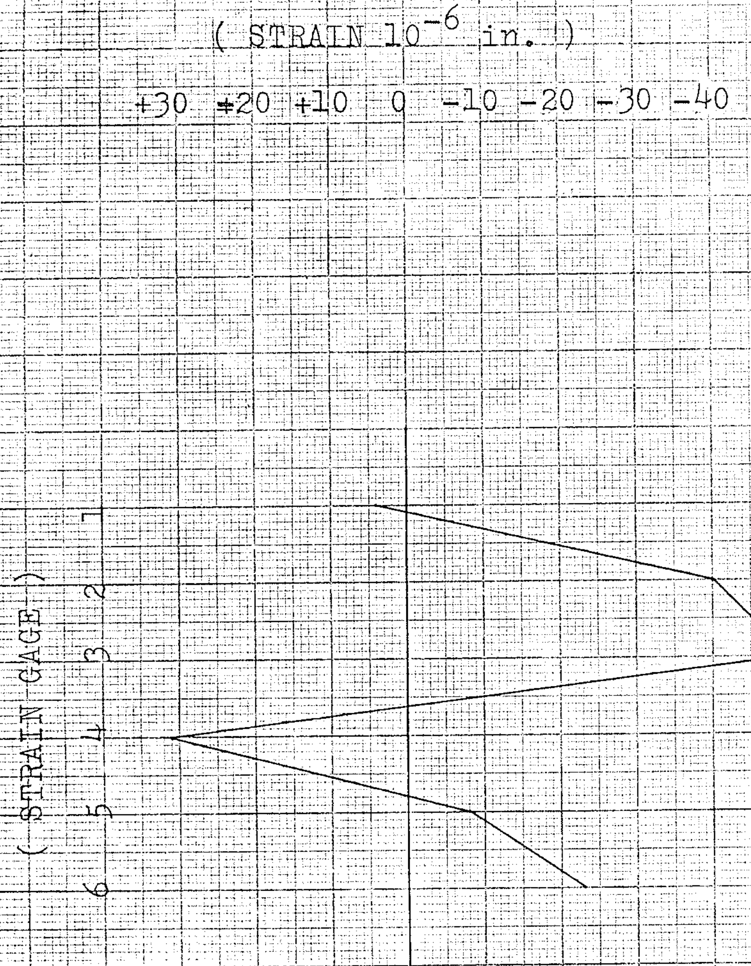


FIG.22. STRAINS OF COL.4 AT 5th FLOOR LEVEL.

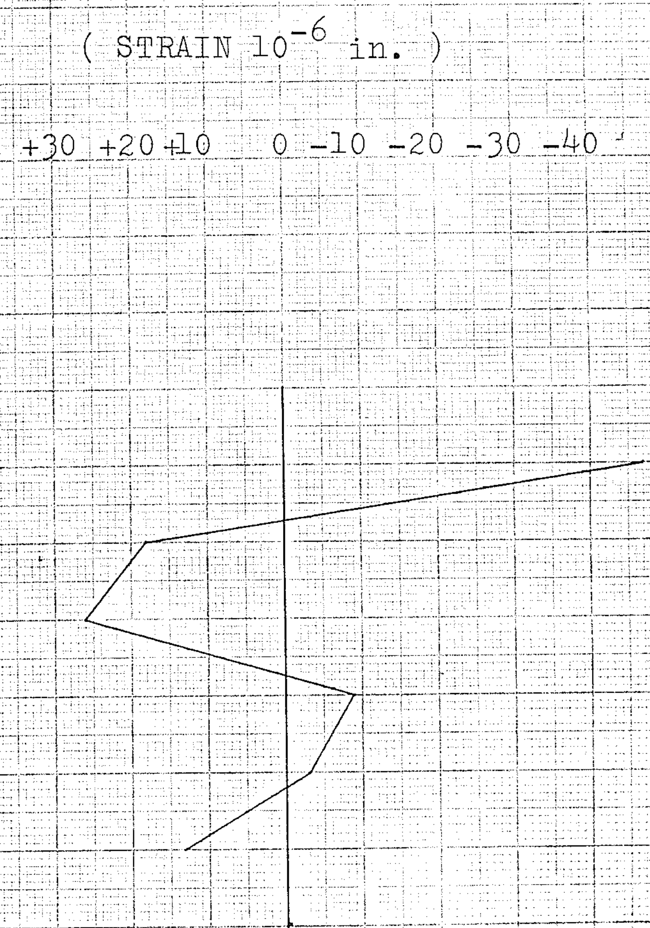


FIG. 23. STRAIN OF COL.5 at 5th FLOOR LEVEL.

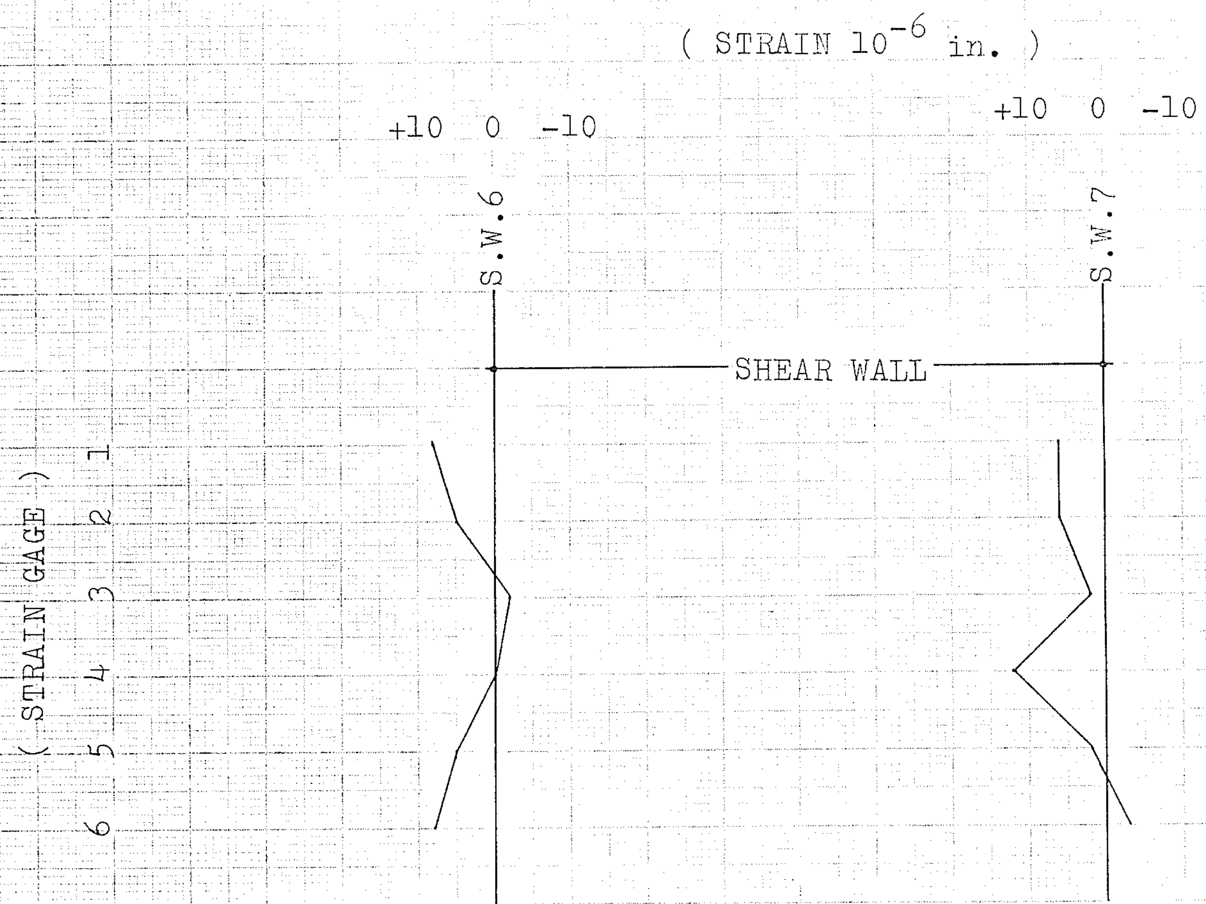


FIG. 24. STRAINS OF SHEAR WALL AT 5^{TH} FLOOR.

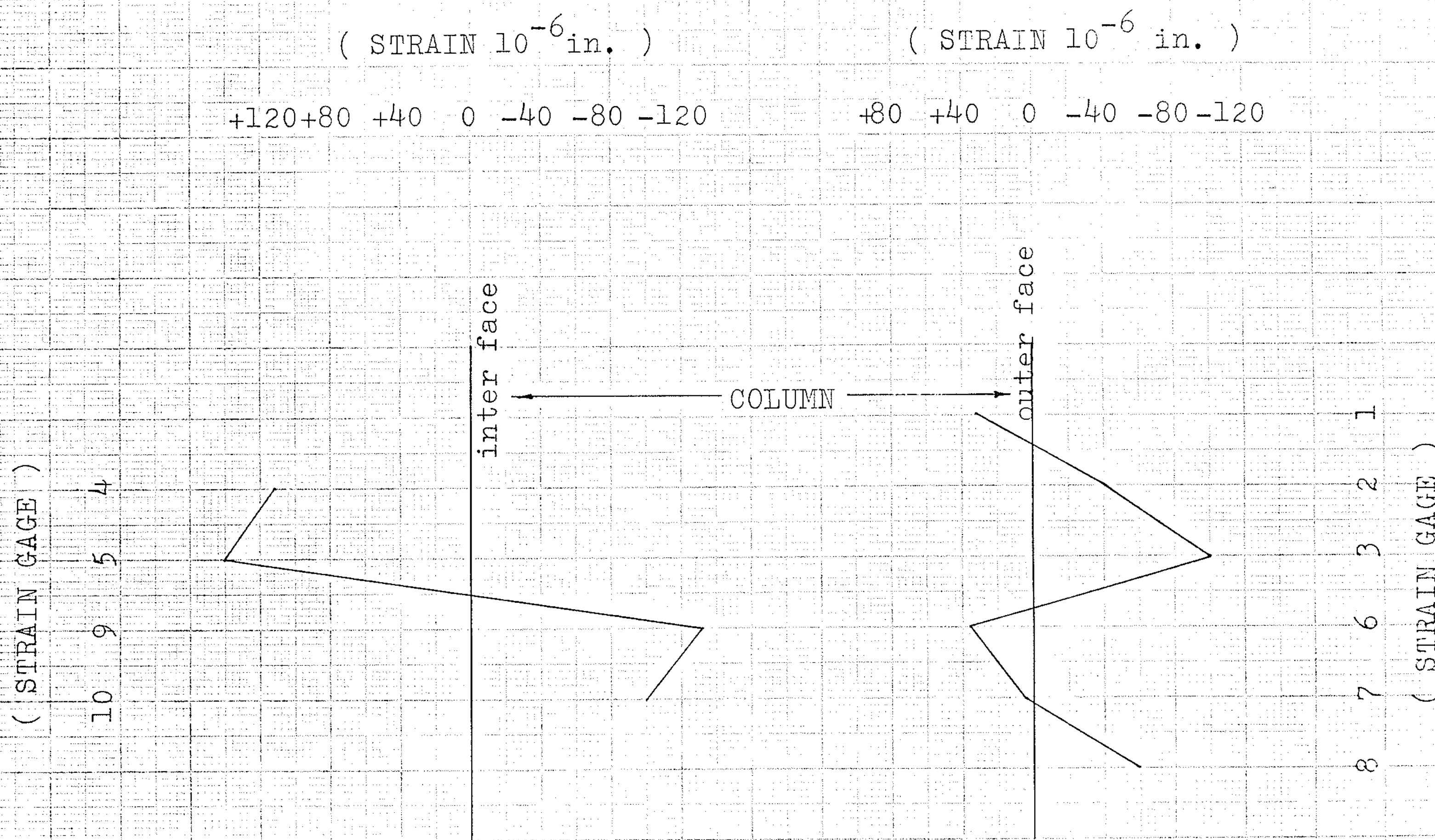


Fig. 25. STRAINS OF COL.8 AT 5^{th} FLOOR LEVEL.

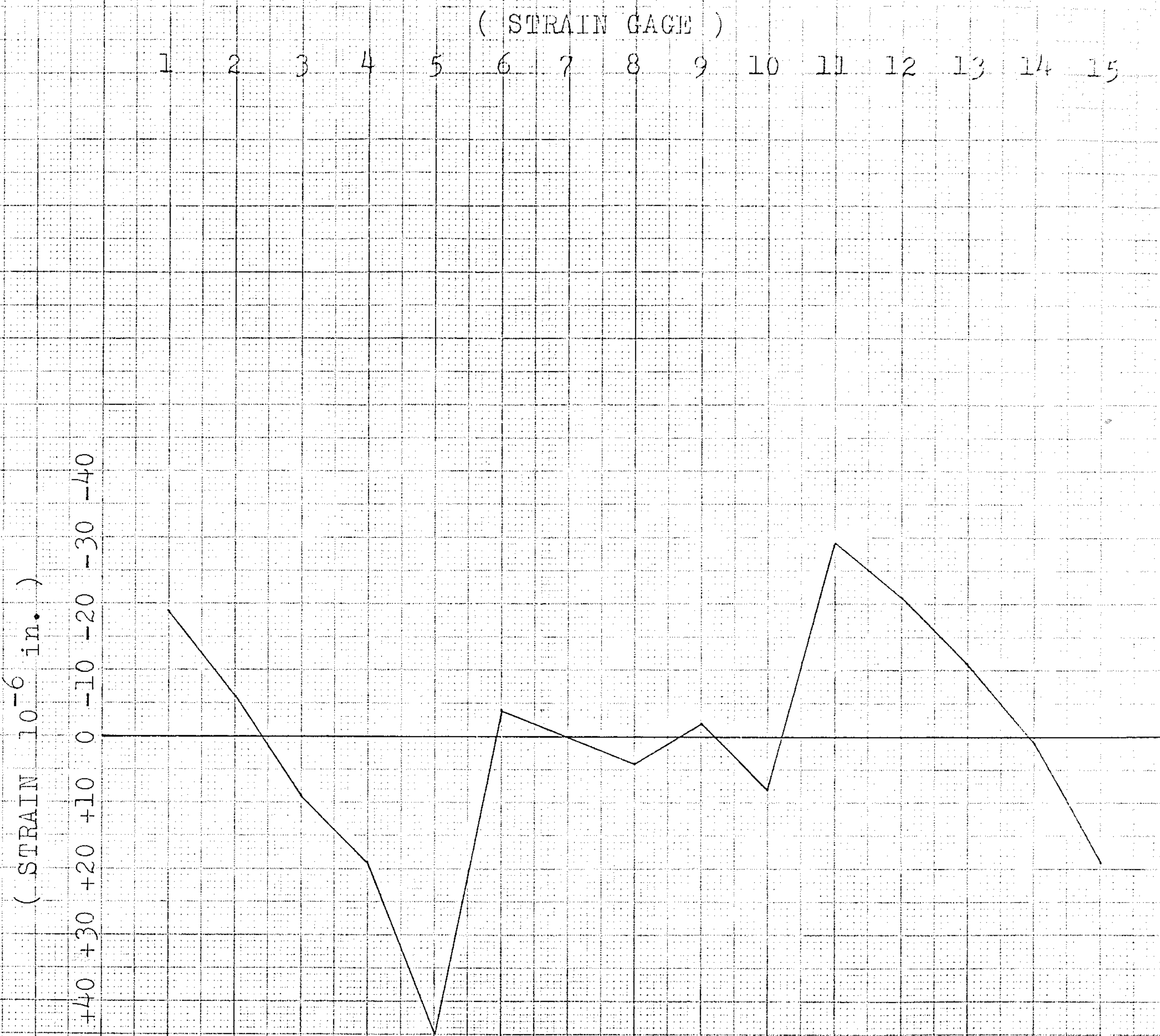


FIG. 26. STRAINS OF 2^{nd} FLOOR SLAB ALONG LINE-A

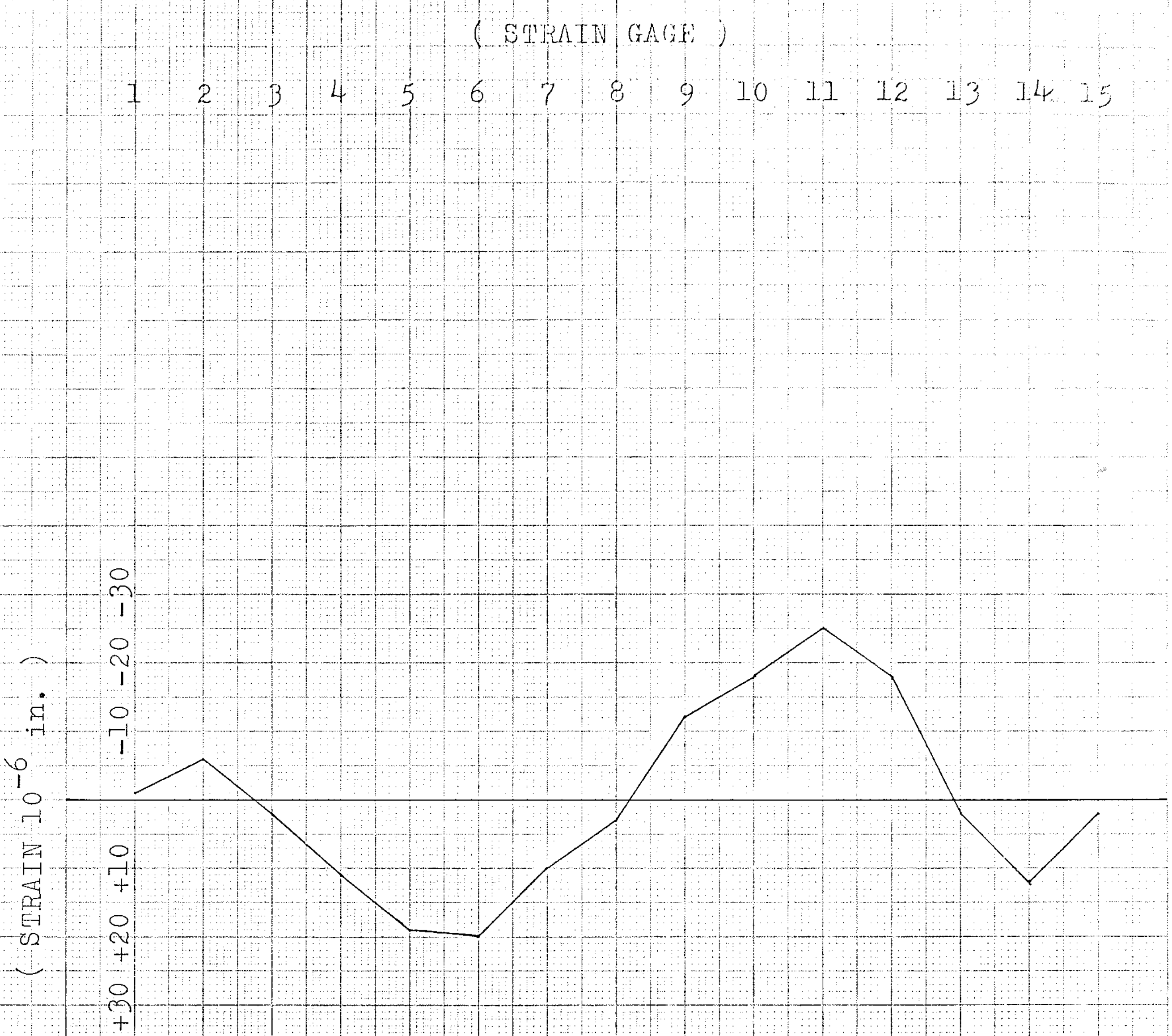


FIG. 27. STRAINS OF 2nd FLOOR SLABM ALONG LINE-B.

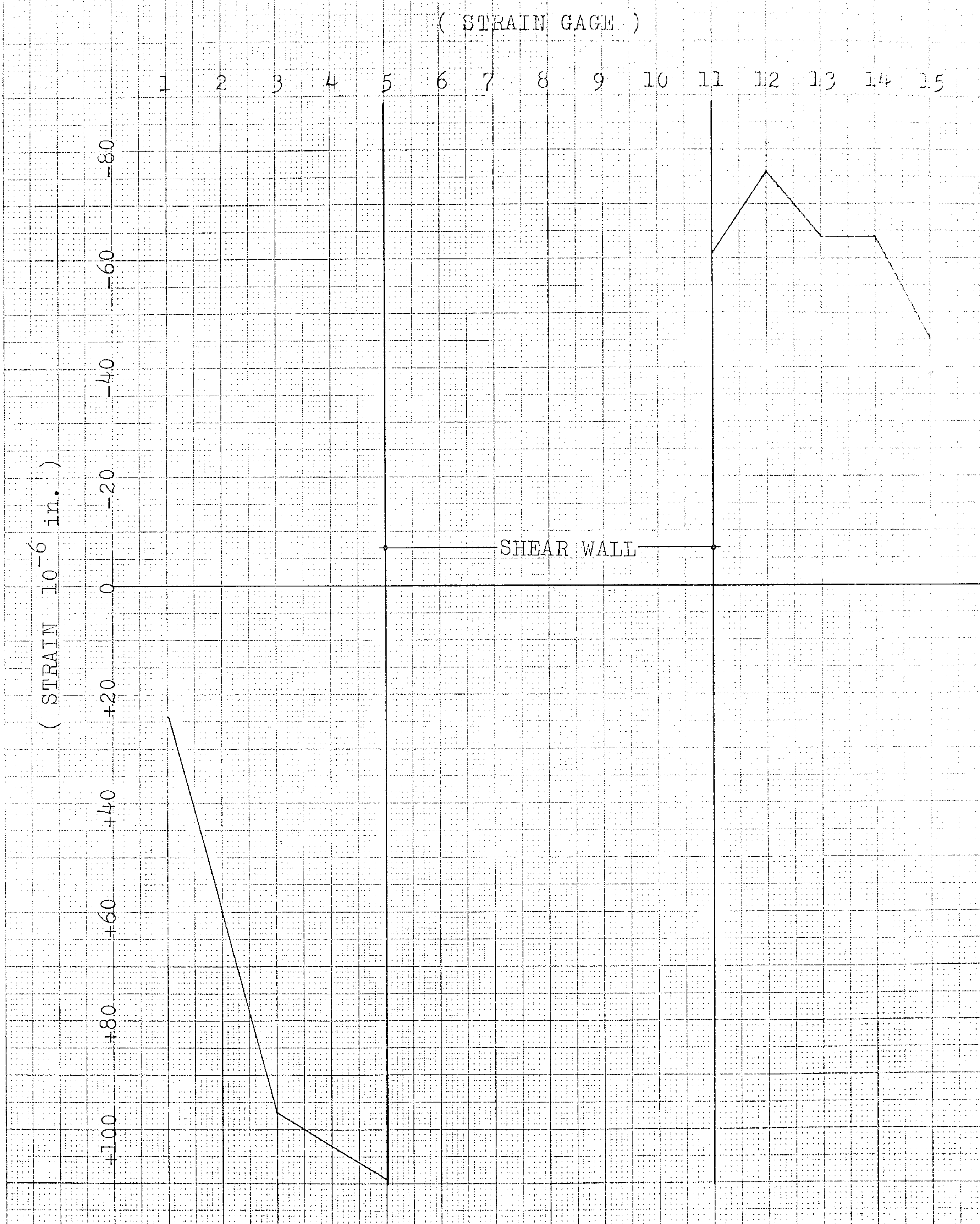


FIG. 28. STRAINS OF 2nd FLOOR SLB. ALONG LINE-C.

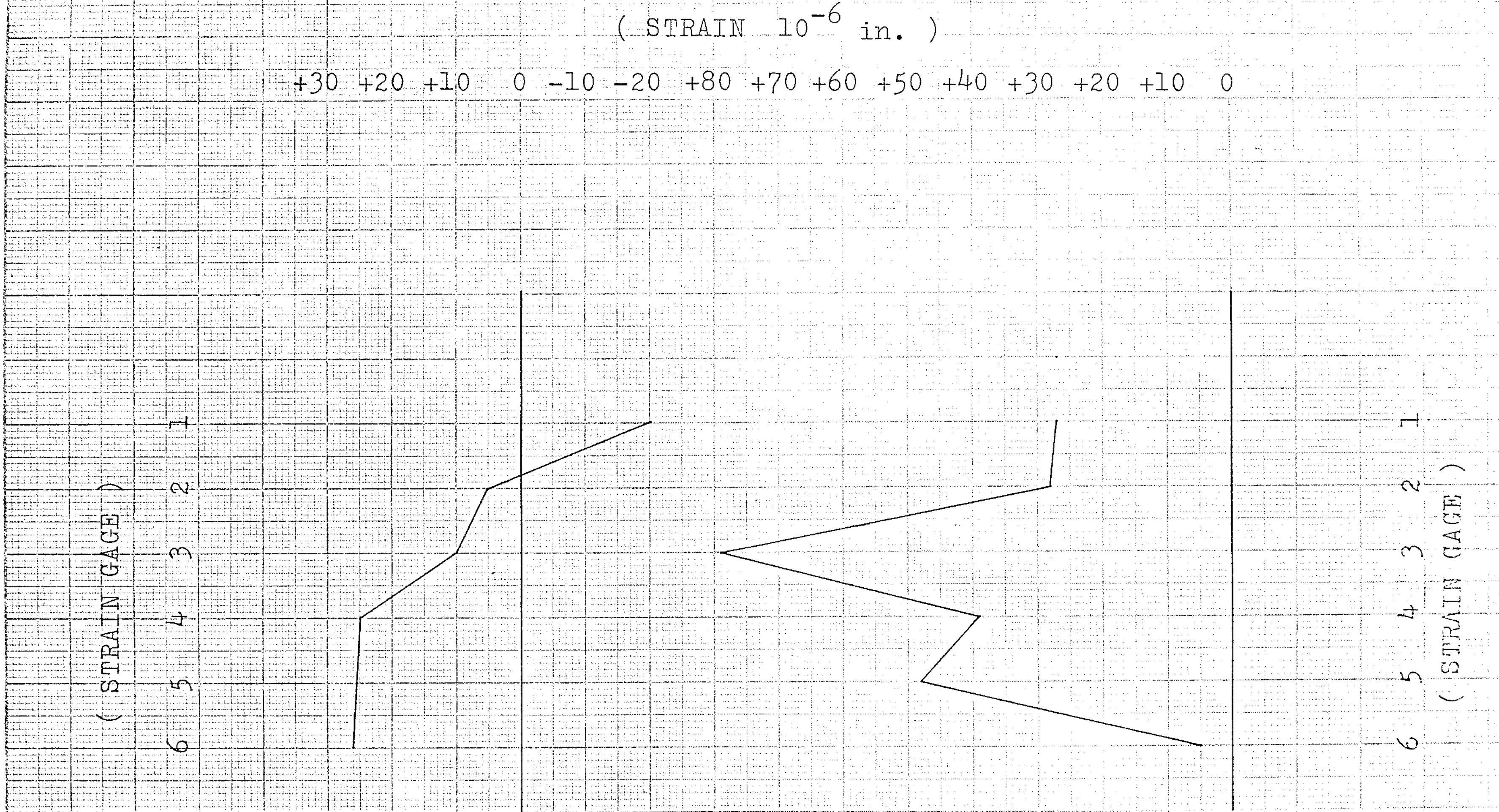


FIG. 29. STRAINS OF COL.4 AT 2nd FLOOR LEVEL.

FIG. 30. STRAINS OF COL.5. AT 2nd FLOOR LEVEL.

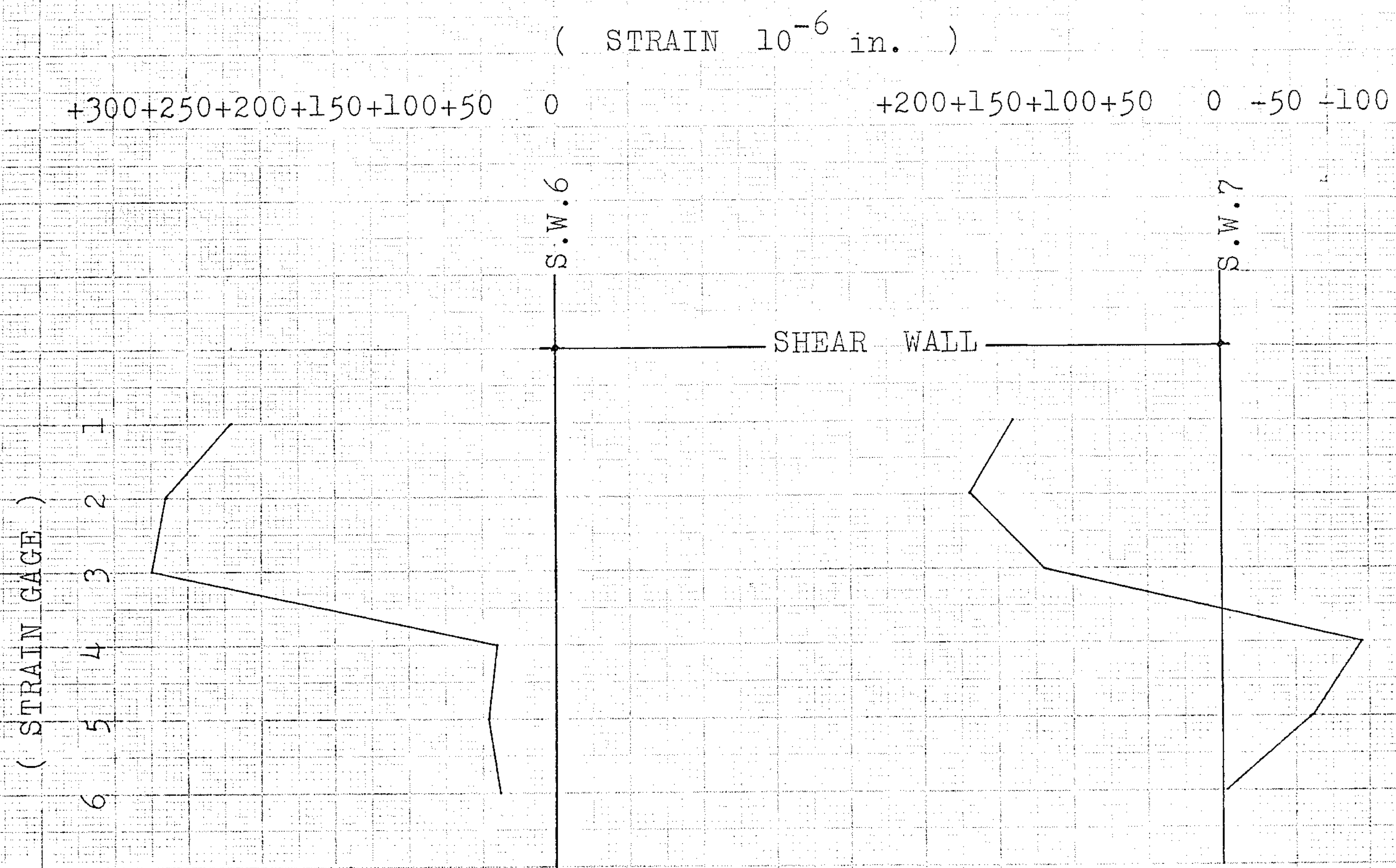


FIG. 31. STRAINS OF SHEAR WALL AT 2^{nd} FLOOR LEVEL.

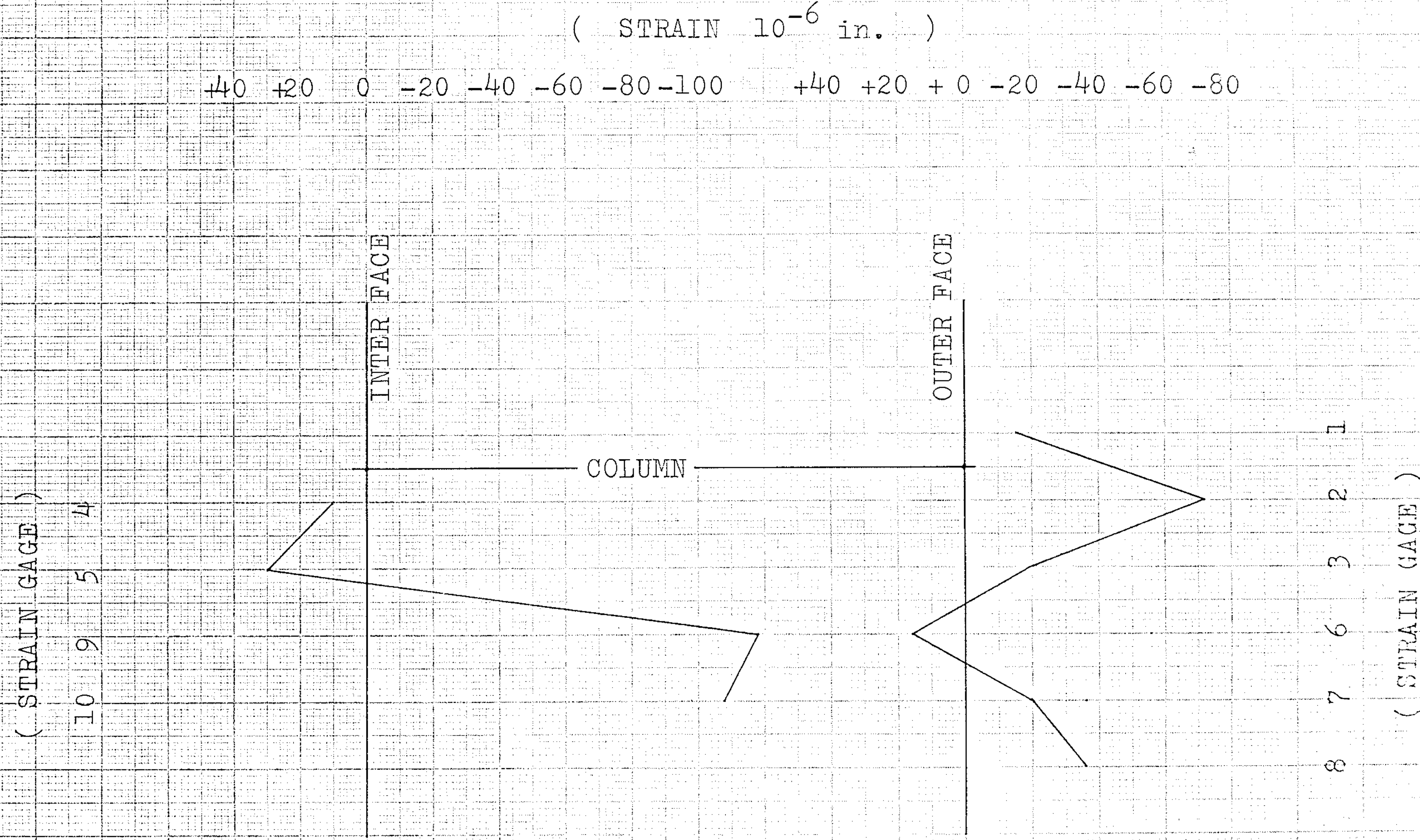


FIG. 32. STRAINS OF COL.8. AT 2nd FLOOR LEVEL.

II.2 FLAT SLAB IN SHEAR WALLS - FRAMES STRUCTURES.

While shear walls are not considered to carry the lateral forces alone, the analysis of a wall-frame structure will then come up to question, that is, how is the lateral force transfered from the column to wall, and what is the designed stiffness of the link member between the wall and column? In other words, the ratio of the stiffness between link member and column, and, link member and shear wall becomes one of the main purposes of study in such structural analysis. If the link member is a beam or a girder, the stiffness of link member is usually represented by I_b/L**.

** Where I_b is the moment of inertia of beam which has flange equal to one quarter of the distance from the centre of the beam to that of the adjacent beam. If the link member is a flat plate slab, many designers may intuitively assume that only a part of the slab width will act as a beam element, together with the columns to resist the forces of the frame action, but such assumption is not based on any theory or tentative data.

ESTIMATE THE STIFFNESS OF FLAT SLAB.

In estimating the stiffness of flat slab, a group of tentative data was introduced by Khan in 1964. This tentative data as plotted into some curves which show the relationship of L, Lc, t, d, of a model. These curves can be used as references for the estimation of the stiffness of flat plate slab. The boundary conditions of the model are:

1) Consider a part of the slab floor bounded by lines of inflection perpendicular to the direction of the moment and by centerlines of the bays parallel to the direction of the moment.

2) Simply supported along the lines of inflection. The other two parallel sides were unsupported with a distributed variable moment sufficient to produce zero slope at these boundaries.

FROM OBSERVATION, DEFORMATIONS OF FLAT SLAB UNDER LATERAL LOAD TESTS.

(1) Deflection of slab which along line B are similar to line D.

(2) The deflection of slab which between column at middle panels are small compare to the end panels.

(3) Points of contraflexure of slab are not right at the mid-span.

CHAPTER II.3 THE COMPARISON BETWEEN DATAS OF TEST RESULTS AND THAT OF THE THREE ANALYTICAL METHODS.

The numerous methods, " exact, " " iteration, " or " approximate, " have been published on the subject treated in the design of shear wall-frame structure. They are either lengthy and cumbersome or highly mathematical, and so are not easily adopted in practice.

It is also found that the results often vary with different methods employed. In the following page, the auther has put up experimental results and a comparision of the analytical results also.

NOTE: The tentative data of bending moment in shear wall are calculated by using the formula,

$$M_i = \frac{6EI}{h^2} (\Delta_{i+1} - 2\Delta_i + \Delta_{i-1})$$

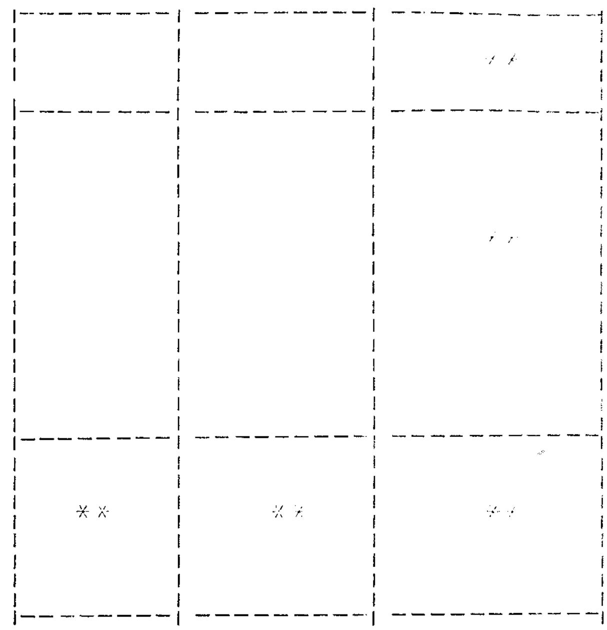
A COMPARISON OF THE ASSUMPTIONS OF THE PREVIOUSLY DESCRIBED METHODS:

ASSUMPTION	EQU.COL.	KHAN,S	CARDAN,S
1) All frames are fully fixed at the bases of the columns.	**	**	**
2) The sectional properties of columns and beams in each bay are constant for the full height of the structure.	**		**
3) No elastic shortening of the beams.	**	**	**
4) Storey height is constant throughout the height of the building.	**	**	**
5) Axial shortening of cols. and deflection due to shear is ignored.	**	**	**
6) E (modulus of elasticity of concrete) is assumed to be constant throughout the height of the building.	**	**	**
7) The 90 degree angle at the wall face between wall and girder will remain 90 degree after deflection due to bending.			**

8) Points of inflection in columns are at mid-height.

9) Point of inflection in girders not adjacent to the wall are at mid-span and on the horizontal line between adjacent joints.

10) Horizontal forces are continuously distributed through-out the full height of the wall.



A COMPARISON OF THE RESULTS OBTAINED BY MODEL RESULTS AND BY THREE ANALYTICAL METHODS.

FLOOR	HOR.DEFL.	BENDING MOMENT				
	TENTATIVE. 10^{-3} in.	EXPERI. in-lb	EQU.COL. in-lb	KHAN'S in-lb	CARDAN'S in-lb	FREE.WALL. in-lb
ROOF	47.06	+ 70	+ 97	+ 102	+ 14	0
9^{th}	40.74	- 650	- 316	- 620	- 782	- 316
8^{th}	34.50	- 1432	- 580	- 1382	- 1564	- 948
7^{th}	28.43	- 2520	- 1262	- 2340	- 2237	- 1897
6^{th}	22.64	- 3266	- 2265	- 3050	- 3119	- 3162
5^{th}	17.23	- 4180	- 3580	- 3876	- 3901	- 4743
4^{th}	12.28	- 4830	- 5200	- 4580	- 4674	- 6641
3^{rd}	7.90	- 5765	- 7150	- 5379	- 5457	- 8854
2^{nd}	4.11	- 6542	- 9460	- 6090	- 6230	-11384
1^{st}	0.99	- 7412	-12080	- 6875	- 7012	-14230
GROUND	0.00	- 8650	-16700	- 8100	- 8230	-18970

CHAPTER III.

In many multistorey concrete buildings shear walls are provided to resist the lateral load due to wind or seismic action. This is according to the recuirement of either or both the architectural and structural design. Normally, a good arrangement of the shear wall location and a good adjustment of the shear wall size will lead to a more economical structural design.

Generally, shear walls will be located around the service core or stair cases. The ideal location of shear walls are symmetrically distributed in the plan of the building. The reason for such location is to eliminate the effect of torsional moment caused by the external lateral forces.

Besides the location, the sizes of shear walls in a building is another design problem for the designer. As far as the designer is concerned, a shear wall is much deeper in depth compared to column, girder and beam. Shear wall have the characteristics of a deep beam. As the concentration of shear force is at the shear wall, the shear deflection of wall should not be ignored. (The experimental testing result shows that the deflection of an over size shear wall is slightly bigger than the result of the theoretical analysis.) Therefore it should be taken into account if the shear wall size is beyond a certain amount of the length. Such limitation of shear wall is discussed as follows in this chapter.

III.1 THE DERIVATION OF HEIGHT AND BASE WIDTH (H. L_s) EQUATION OF SHEAR WALL.

Due to the fact that the bulding action will be the undesirable wall action, should the ratio of shear walls and frames (S_s/S_f) be too great (ref. to chapter II) and that we have not been able to obtain desirable results of the shear wall structures from the three analytical methods mentioned, it is of primary importance for us to discuss the sizes of shear wall.

Let us look at the formula of deep beam deflection:

$$\Delta = \int \frac{Mm'dx}{EI} + 1.2 \int \frac{Vv'dx}{AG} \qquad (1)$$

If we wish the building structure design to be unaffected by the shear deflection, the ratio should be:

$$0.05 \Delta_{bending} = \Delta_{shear}$$

therefore

$$0.05 \int \frac{Mm'dx}{EI} = 1.2 \int \frac{Vv'dx}{AG} \quad ....(2)$$

assuming w is uniformly load, to find the deflection of the top of the building,

$$G = 0.4\ E \qquad I = \frac{bl^3}{12}$$

$$A = bL_s$$

then (2) becomes

$$d^2 = (12)^2 \times 0.05H^2 \qquad \text{or,}$$

$$d = H\sqrt{0.05} \quad \text{in ft.} \qquad ....(3)$$
$$= 12 \times H \times \sqrt{0.05} \quad \text{in inch}$$

Eq .(3) is derived from the deflection relation. Therefore, when designing a shear wall, we should consider the effect of its depth upon bending moment.

then:

$$L_s = 2 \times \sqrt{12 \times H \times \sqrt{0.05}} \quad \text{in inch}$$

or

$$L_s = 2 \times \sqrt{H \quad \sqrt{0.05}} \quad \text{in ft.} \qquad ....(4)$$

Simplifying

$$L_s = 3.276\sqrt{H} \qquad \text{in inch} \qquad ....(5)$$
$$= 0.948\sqrt{H} \qquad \text{in ft.}$$

III.2 THE DERIVATION OF THE WALL THICKNESS AND LATERAL LOAD (b_s, w) EQUATION OF SHEAR WALL

ASSUMPTIONS:

a) Sum of the shearing stresses at the bases of walls and columns is equal to the total external lateral applied force.

b) The stiffness ratio of wall-column systems is equal to 60. (Ref. to Khan's tentative datas.)

c) $b_c = 1.5\ b_s$

Analysis;

From assumption (a)

$$V_{total} = wH = V_c + V_s \qquad \text{.........(1)}$$

The average shearing stress formula is,

$$u = \frac{V}{bd} = 60\text{Psi}$$

then,

$$\frac{V_s}{b_s L_s} = \frac{V_c}{b_c^2} = 60\text{Psi}$$

$$V_s = 60 \times b_s \times L_s \text{ lbs.} \qquad \text{..........(2a)}$$

$$V_c = 60 \times b_c^2 \text{ lbs} \qquad \text{...........(2b)}$$

The shear distributed at the bases of wall and column can be expressed by the general

$$V_s = V_{total} \frac{S_s}{S} \quad \ldots\ldots\ldots\ldots\ldots(3a)$$

$$V_c = V_{total} \frac{S_c}{S} \quad \ldots\ldots\ldots\ldots\ldots(3b)$$

and using the meaning of Assumptions of (b) and (c).

when the total stiffness of shear wall equal to,

$$S_s = I_s/H = b_s L_s^2 / 12H \ldots\ldots\ldots(4a)$$

Then, the total stiffness of columns becomes

$$S_c = \frac{5.07\ b_s^4\ 60}{12\ \frac{H}{n}}$$

$$= \frac{304.2\ b_s^4\ n}{12\ H} \quad \ldots\ldots\ldots\ldots\ldots(4b)$$

By applying Eq.(2a) in Eq.(1)

$$V_c = wH - 60\ b_s L_s \quad \ldots\ldots\ldots\ldots\ldots(5)$$

Substituting Eq.(2a) in Eq.(3b)

$$V_c = \frac{\frac{wH \times 304.2\ b_s^4\ n}{12\ H}}{\frac{b_s L_s^3 + 304.2\ b_s^4\ n}{12\ H}} \quad \ldots\ldots\ldots\ldots(6)$$

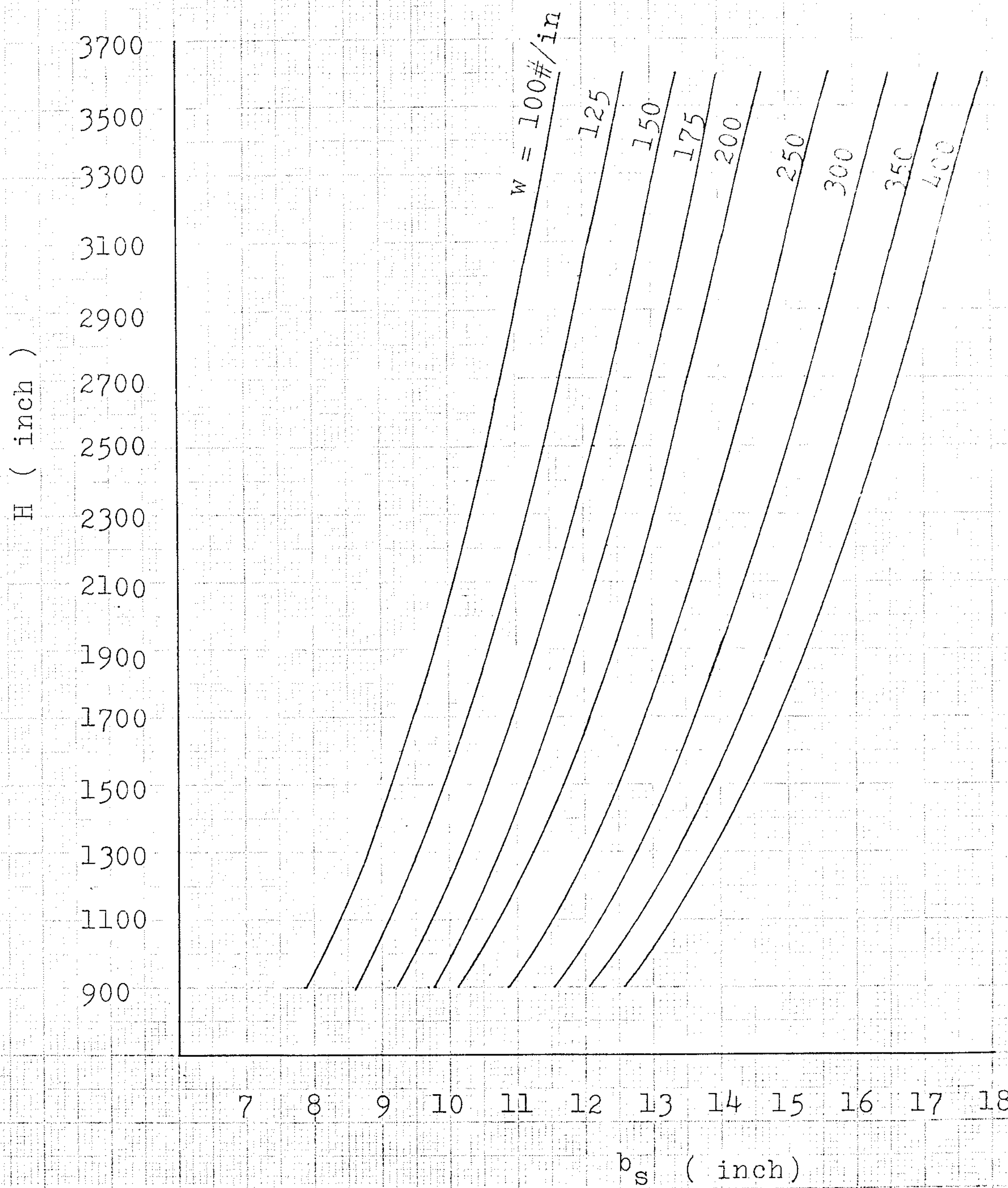
then, combined Eq.(5) and Eq.(6), we have,

$$wH - 60\ b_s L_s = \frac{wH \times 304.2\ b_s^4\ n}{b_s L_s^3 + 304.2\ b_s^4\ n}$$

The final equation becomes:

$$wHL_s^2 = 18252\ b_s^4\ n + 60\ b_s L_s^3 \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

A chart has been made in the next page, by using different building heights (from 900 inches to 3600 inches) combined with a group of different values of lateral force. (from 100 lbs. per linear inch to 400 lbs. per linear inch.)



3700
3500
3300
3100
2900
2700
2500
2300
2100
1900
1700
1500
1300
1100
900
H (inch)
w = 100#/in
125
150
175
200
250
300
350
400
7
8
9
10
11
12
13
14
15
16
17
18
b s (inch)

FIG. 33.

NOTATION

b_c Base of column.

b_s Base of shear wall

d Desired length of shear wall, based on deflection.

L_s Desired length of shear wall, based on bending moment.

G Modulus of elasticity of concrete in shear.

E Modulus of elasticity in bending.

H Total height of the building, in inches.

I_c Sum of the moment of Inertia of columns.

I_s Sum of the moment of Inertia of shear walls.

n Number of floors of the building.

V_c Sum of shear forces in columns.

V_s Sum of shear forces in shear walls.

w Unit lateral load, in pounds per inch.

u Allowable unit shearing stress in columns, beams and shear walls.

DISCUSSION

In Fig.(8;9;10;11;) are shown the deformation of a slab loaded with lateral force. Consider four parellel strips of slab (Fig.6.) L-A, L-B, L-C, L-D. L-C contains a shear wall so that it is more rigid than L-A. On the other hand, L-B is similar to L-D. This verifies that the horizontal displacement of any point at the same level will not be constant, and the reference for the effective slab width made by Khan is partially true and can be applied to the slab between columns only.

Assumption (2) (4) in page 62, also does not correspond with the actual structures (very few buildings have constant cross-section of wall, beam, and column). If the average values of beams....etc, is emplyed in practice, an adjustment should be made for the shear wall moment, especially for the upper protion of the wall.

The corrections in moments for axial deformations of walls and columns were small as compared with primary moment.** This is true only when the frame is low, and consists of just a few storeys. With tall multi-storey building frames this correction has an important value on the final moments in the beams, and therefore, should not be neglected in the computations.

A second degree differential equation is employed in the methods by W. W. Frishmann and by B. Cardon. The constants such as X_1, X_2, X_3, and B,C,I; respectively, must be determined very accurately, for such constant will give big mistakes in the final results if a slight error have been made by any means, therefore, a slide rule, is not suitable for this kind of calculation.

According to the deflected shapes of the different kind of buildings which are shown in Ref. No.3, The stiffness ratio S_s/S_f will reduce the horizontal displacement in the upper storeys of the building.

The equations in chapter three depend on the shear deflection which is equal to five percent of the bending moment deflection. These equations are applicable to the primary design. The equation are so simple they can be used in practice. Furthermore, if the depth of a shear wall is beyond the limitation as the equation indicates, the shear deformation has to be included during the process of the structural analysis of the shear wall. A study of this field is presented in reference No.3.P.305,...(Secondary deflection.)

CONCLUSION

The numerous methods of the analysis of shear wall have been described in this paper. They are either lengthy or highly mathematical. Taking into account the effect of bending and shear by using the angle deflection of the wall at all points with a second degree differential equation is not a practical method. The results vary greatly with different tries by using a slide rule . It is the opinion of the author that rather than use the equivalent stiffnesses of the link members to make it possible to apply direct moment distribution to this kind of structure analysis that use be made of the angle deflection of the shear wall.

In processing model tests it was found very difficult to prevent the base rotation of the model. In order to solve this problem, the model should be completely fixed into the ground.

Some of the electrical strain gages were not sufficiently put in correctly on the model, such as E-16 to E-30, and none of the electrical strain gage along Line F. That means, the values of the slab deformation along Line E do not make sense since there is no comparision in this case.

Also there is a note for the person interested in the model analysis that the quantity of electrical strain gages put onto the model should be reduced to a minimum number. The strain reading must be recorded throughout every gage of the whole model before release of the testing load. The author found it was not easy to run through his work in his case.

PRE VIEW OF THE ANALYSIS OF SHEAR WALLS IN MULTI STOREY REINFORCED CONCRETE BUILDINGS.

Shear walls often remain in the multi-storey reinforced concrete buildings, which are assigned to resist the lateral loads due to the wind or earth-quake. More mult-storey buildings will be built in future because of the concentration of population in many cities. Hence more and more engineers will become involve be in the shear wall design in their practices, therefore the improved methods of the shear wall analysis have to be easily applicable in general practice. In the author's opinion , the tendency of the analysis of shear wall will be:

1) Simplifying the procedures of design methods. (The highly mathematical and the electronic digital

computer works instead by the most commonly know " moment distribution " method and slide rule.

2) More tentative charts will be made which can be used directly on the practical problems.

3) The stiffness of link member may be represented by an equivalent stiffness, (The value of the equivalent stiffness with respect to the different types of frames.)

BIBLIOGRAPHY

1) Frischmann, W.W., " Multi-storey Frames and Inter-connected Shear-walls subjected to Lateral Loads." Constructional and Engineering, June 1963, PP.227-234.

2) Cardan, B. "Concrete shear walls combined with rigid frames in multistory buildings subject to lateral loads." Journal of the A.C.I., Proceedings V.58, September 1961, No.3.PP.299-316.

3) Khan,F. R.,"Interaction of Shear walls and Frames." Journal of the str.dicision, Proceedings of A.S.C.E. V.90 No.ST3, June 1964.PP.285-335.

4) Chen,L. K., "Wind Stresses in Multi-storey Frames with Shear Walls." University of Hong Kong Symposium on the Design of High Buildings, 1967.