

A STUDY OF DIAGONAL TENSION FAILURE  
IN REINFORCED CONCRETE BEAMS

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A Thesis

Presented to

The Faculty of Graduate Studies and Research

The University of Manitoba

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In Partial Fulfillment  
of the Requirements for the Degree of  
Master of Science in Civil Engineering

\* \* \* \* \*

by

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February, '1968

ACKNOWLEDGEMENTS

I wish to acknowledge the valuable guidance and keen interest taken by Dr. A.M. LANSDOWN throughout my work, both during the experimental stages and while writing a report on it. His advice was always a source of inspiration.

I would also like to acknowledge the assistance by the Civil Engineering Laboratory Staff, Mr. Edward Lemke and Mr. Harry Brooks in particular, during the experimental stages of the work.

Pouring concrete is an enormous job in itself and I would also like to thank Mr. John Towle, Mr. Lawrence Iife and Mr. Somsak Teerachaichayuti for their help during the pouring and testing of the beams.

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SYNOPSIS

This Thesis presents a study of diagonal tension failure in reinforced concrete beams, with particular emphasis on the effect of the variation of the size and distribution of shear reinforcement on the shear strength of concrete beams. Results obtained from tests of six simply supported slabs tested by the author are reported.

An attempt was made to keep the effective weight of web reinforcement as constant for all the beams and vary the size of stirrups from No. 4 to No. 2 in different patterns. Wire meshes were also used as shear reinforcement in the last beam. They consisted of verticals 0.156 in. dia. wires and horizontals 0.115 in. dia. wires. The longitudinal tension steel consisted of 6 No. 7 bars in all the beams. Compressive strength of concrete cylinders varied from 3400 psi to 4200 psi.

The beams were of rectangular cross-section, 8" wide x 18" deep and 8 ft. long. They were loaded at third points and the load was increased in 5000 lb. increments to failure. Deflections were recorded at five points along the length of beams.

Only beam 5, with No. 3 inclined stirrups failed by flexure. Beam 3, with No. 2 stirrups showed a secondary bond failure. All other beams failed by diagonal tension.

The propagation of cracks and crack pattern was marked on the beams, photographs were taken and curves were drawn for load and deflection. Results of tests were analysed to determine the suitability of web reinforcement.

It was found difficult to draw any definite conclusions on the basis of test results due to insufficient number of tests. It appeared,

however, that the inclined stirrups were more effective than the vertical stirrups. No. 3 stirrups also showed satisfactory behavior, but results were difficult to be interpreted for No. 2 stirrups because of their relatively brittle behavior and higher yield strength, and also due to the bond failure of this beam.

The wire meshes seemed very effective in resisting diagonal tension, but because of insufficient data for comparison, it is suggested that further tests be carried out to develop a better understanding of their behavior.

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NOTATIONS

- $A_s$  = area of tension reinforcement  
 $A_v$  = area of web reinforcement (area of two legs of a stirrup)  
 $a$  = shear span or distance from plane of the nearest concentrated load point to plane of the support  
 $\alpha$  = angle between inclined web bars and longitudinal axis of member  
 $b$  = width of a rectangular beam  
 $C$  and  $C'$  = resultants of compression stresses in concrete  
 $\Delta C$  = increment of compressive force  
 $c$  = principal compressive stress  
 $d$  = effective depth of tension reinforcement or diameter of concrete cylinder  
 $f$  = horizontal fiber stress  
 $f'_c$  = compressive strength of 6 in. x 12 in. concrete control cylinders  
 $f_s$  = stress in tension reinforcement at failure  
 $f_v$  = critical tensile stress of web reinforcement  
 $f_y$  = yield strength of reinforcement  
 $I$  = moment of inertia of cross section about neutral axis  
 $j$  = ratio of internal moment arm to effective depth  
 $K = (\sin \alpha + \cos \alpha) \sin \alpha$   
 $l$  = length of control cylinders  
 $M$  = bending moment  
 $M_u$  = maximum bending moment at failure  
 $M_{f1}$  = maximum flexural moment calculated for  $f_y = 50$  Ksi  
 $O$  = perimeter of reinforcing bar  
 $P$  = applied load  
 $P_c$  = initial cracking load  
 $P_u$  = ultimate load

$p = \frac{A_s}{bd}$  = steel ratio of longitudinal tension reinforcement

$p_w = \frac{A_s}{b'd}$  , where  $b'$  = width of web in I - and T - sections

$Q$  = statical moment about neutral axis of that portion of cross-section beyond the layer containing a particle.

$r = \frac{A_v}{bs}$  - ratio of web reinforcement or reduction factor

$s$  = spacing of stirrups in a direction parallel to the longitudinal reinforcement

$T$  and  $T'$  = tension forces in tension reinforcement

$\Delta T$  = increment of tension in tensile reinforcement

$t$  = principal tensile stress or diagonal tension

$u$  = bond stress of reinforcement

$V$  = total shear at a section

$V_c$  =  $V$  at the formation of initial diagonal tension cracks

$V_u$  = total ultimate shear

$V'_u$  = ultimate shear carried by web reinforcement

$V'$  = excess shear over that carried by concrete

$v$  = nominal shearing stress

$v_c$  = shear stress carried by concrete

$v_s$  = safe shearing stress

$v_u$  = nominal ultimate shearing stress as a measure of diagonal tension

$\phi$  = capacity reduction factor

CHAPTER I  
INTRODUCTION

1.1 Historical Background

It may be noted in the history of concrete and reinforced concrete that basic concepts were at times understood correctly, even though incompletely, by early pioneers. It is interesting to note that much of our present knowledge on the shear behaviour and mode of failure of concrete members and the factors that influence them the most, was developed in the first decade of the 20th century.

The first published study of web reinforcement dates back to 1899 when W. Ritter (1-H) presented a study based on the "truss analogy", in a basically sound manner. Ritter made it completely clear that in his opinion, the stirrups were stressed in tension. He thus recognized the existence of diagonal tension as the cause for shear failure.

The first laboratory tests specifically dealing with shear in concrete beams were reported by E. Mörsch in 1902 (2-H). One year later he published another paper which indicated diagonal tension as the cause of shear failures in reinforced concrete beams (3-H).

For nearly a decade horizontal shear versus diagonal tension was subject to considerable discussion. Some significant work was done by A.N. Talbot (14-H, 15-H, 19-H, 20-H, 25-H), and E. Moritz (124-H). The literature gives no clear indication of the origin of diagonal tension as a criterion of shear failures in reinforced concrete members. This is reasonable since the early designers of reinforced concrete structures did not publish their design methods.

In 1903 the American J.S. Sewell (6-H) suggested that tests indicated a formation of cracks along the lines of principal tensile stress in reinforced concrete beams. He advocated the design of stirrups

to resist these principal stresses, which later became known as diagonal tension.

A paper by Morsch (13-H) published in 1907 presented further developments pertaining to diagonal tension. The recommendations suggested by him were clear and complete indeed, considering the time at which they were presented. He suggested that stirrups should not be used alone to carry heavy shear, but always together with longitudinal bent-up bars. The first report by A.N. Talbot was planned with a view to getting data bearing on the establishment of principles. Among the facts that Talbot recognized were the various modes of failure for reinforced concrete beams. He also recognized that diagonal tension cracking was a function not only of shear but of moment and depth of the beam as well. In 1909, Talbot presented a study of web stresses (25-H), and pointed out that the shearing strength of concrete is a function of concrete strength, of span length, and of percentage of longitudinal reinforcement. His recommendations were adopted by the American Joint Committee on Concrete and Reinforced Concrete a few years later.

After 1910, and for the next three decades, a large number of concrete members were tested and studied in attempting a solution to the shear problems. In general the tendency was to study the effectiveness of the web reinforcement and bent-up bars, and considerable effort was devoted to types and methods of web reinforcement. The development of the specifications and design also followed the trend of research done during this period. This was such a tricky subject that many revisions were made in the building codes of different countries.

In the late twenties E. Morsch (45-H) repeated again that the diagonal tension failures should never occur in reinforced concrete structures because of their very nature, the failure being sudden.



Structures should be designed in such a manner that failure of flexural members will be due to moment, not shear, thereby taking advantage of the favourable redistribution of the moments and forces often possible in monolithic structures.

Prior to the mid 1940's the emphasis was on the working stress design in the design of reinforced concrete structures. With the advent of more complex structural systems, in particular monolithic or continuous structures, the inadequacy of working stress design became particularly apparent, and consequently after the mid 1940's the tendency was on determining the ultimate load capacity. Due to this shift in design philosophy, Joint Committee ACI-ASCE 327 was formed as a sub-committee of the ASCE committee on Masonry and Reinforced Concrete in 1944. It immediately commenced an extensive test programme on the shear resistance of reinforced concrete members. This committee continued its work until 1959, and made recommendations for allowable stresses for various cases of reinforcement. Still the basic problem of shear failures was not fully solved from the theoretical point of view.

The Committee 326 of ACI found that classical theories of diagonal tension were not adequate, and they came to a clear realization that shear and diagonal tension is a complex problem involving many variables. However, the goal of a complete understanding and of a fully rational solution to the problem of computing diagonal tension strength could not be attained. In view of this, they abandoned the classical procedures in favour of a logical, though empirical, approach which takes into account the major variables affecting diagonal tension strength as shown by the test results.

In the course of other findings, the committee concluded that the ultimate strength in shear of beams with web reinforcement was influenced

by four major factors:

- a) compressive strength of concrete  $f'_c$
- b) ratio of tensile reinforcement  $p$
- c) ratio relating moment to shear  $M/V$  at the section considered
- d) ratio, distribution, and yield point of web reinforcement.

A very large volume of work has been carried out in this field since 1960. This includes a number of tests on simple beams, continuous beams, slabs and footings with, and without, web reinforcement.

Dr. G.N.J. Kani (66, 71) conducted a number of tests on concrete beams, simply supported, with two-third point loading and just with tension reinforcing. General tests were carried out with varying the shear span  $a/d$ , and with different steel ratios of  $p=A_s/bd$ . His conclusions were very interesting. He suggested that the shear strength of a rectangular reinforced concrete beam does not depend on the concrete strength. Instead relative beam strength is a much more suitable indicator of the beam strength than the ultimate shear strength.

The problem of shear and diagonal tension has concerned structural concrete research workers for more than sixty years to-date. More than three thousand beams and frames have been tested and reported. Although in some areas, definite advancement has been made towards the understanding of the behaviour of the concrete members subjected to shear, there is still much to be learned before the problems may be considered solved.

## 1.2 Introduction

Beams of reinforced concrete, more than those of metal, have a potential weakness in the region where loads are transferred to the supports through the web. Vertical stress caused by the end shear

combines with longitudinal stress from bending in the beam and produces tensile and compressive stress components. The tensile component is the critical one as concrete while strong in compression is comparatively weak in resisting tension. This tensile component is diagonal tension. Since the end shear is used as a measure of its magnitude, diagonal tension is commonly called shearing stress. Reinforcing steel, generally in the form of stirrups, is used to resist this tension.

A theoretical analysis of the stresses in the web of a reinforced concrete beam is at best only approximate, and it becomes necessary to rely to a considerable extent on the results of tests for the determination of the resistance of a beam for varying concrete strengths, ratio of web reinforcement, and other factors. Design of web reinforcement at present is based on semi-rational methods, and there is need for additional test data to evaluate the factors which influence the resistance of a beam to diagonal tension, and for a general expression which will indicate the shear capacity of any reinforced concrete beam.

It is known that the capacity of a reinforced concrete beam to resist failure by diagonal tension is influenced by a number of factors.

1. compressive strength of concrete.
2. amount, distribution and yield strength of web reinforcement.
3. size, number and yield strength of bars used as tensile reinforcement.
4. ratio of the effective depth of the beam to shear span (distance from the plane of the nearest concentrated load point to the plane of support).

### 1.3 Object of the Work

As has been outlined in the preceding paragraphs, diagonal tension failures still present a challenge to the researchers. Many variables have been investigated to date to determine their influence on the

shear behaviour of the structures. These include compressive strength of concrete, amount, distribution and yield strength of web reinforcement, ratio of the effective depth of beam to shear span etc. The present dissertation deals with the effect of size and distribution of shear reinforcement on the ultimate strength of reinforced concrete beams. An attempt has been made to study this single variable keeping everything else as constant. The present ACI code is silent about the effect of distribution of shear reinforcement. The object of the present work was to use approximately the same amount of steel as shear reinforcement and find out the most effective pattern. For this purpose different size stirrups were used as well as inclined stirrups and a uniformly distributed pattern of wire meshes across the width of the beam was also attempted.

The results of the various tests are compared with the work already done in this connection; and to determine the ductile behaviour of the beam in case a wire mesh is used as shear reinforcement. The existing specifications for ultimate strength design are also reviewed in the light of test results and inferences drawn.

#### 1.4 Future Research

This field of determining the variables affecting the shear strength is a very vast field. Although a lot of work has been done in this area a great deal of work still remains. The ultimate strength behaviour of concrete structures is a more realistic approach, and any closer understanding of different variables involved will be a great help in simplifying design techniques. This present work is, in fact, of preliminary nature and the same could be carried out for continuous beams, beam - and -slab composite systems, footings etc.

It has been considered most advisable to be conservative in the design of shear reinforcement due to the very nature of sudden failure

due to shear. If a method is devised by which the ductility of concrete is enhanced, there will be sufficient deformation before failure and the structure would continue to carry a sufficient portion of the ultimate load for a long period. This was the reason why homogeneously spread wire-meshes were tried across the width of the beam. (Chapter VI). The work on this aspect could still be carried on further.

The work was carried out on 8 ft. long simply supported rectangular beams, with third point loading. The effect of size and distribution of shear reinforcement can also be carried out on different shape beams and especially on deep beams which still present a challenge to the present day researchers. The literature available on deep beam studies is scanty and there exist many possibilities of this work being carried out in further detail.

### 1.5 Shear Strength of Concrete

The shear strength of concrete is large but this value is significant only in rare cases, since shear must ordinarily be limited to much lower values in order to protect the concrete against diagonal tension stresses. Various tests on concrete have described the shear strength of concrete to be from 35% to 80% (15-H) of the compressive strength. Diagonal tension stresses are often referred to as shear stresses, which in fact is a misnomer. As we are primarily interested in diagonal tension, we shall consider this aspect of the concrete strength.

Homogeneous beams for diagonal stress can be analyzed by well established relationships. Reinforced concrete beams, prior to the formation of cracks, probably have stresses quite similar to those of a homogeneous beam. The behaviour, after fracture has occurred, is a separate problem and will be discussed later. Concrete is very well

known to be weak in tension. Shear stresses and compressive stresses combine to give rise to diagonal tension. Whenever these diagonal tension stresses exceed the capacity of concrete, reinforced or plain, a diagonal tension failure results.

Since the diagonal tensile stress at the neutral axis is equal to the unit shear, the unit shear is used as a measure of the diagonal tension. The unit shear used is itself a nominal or average stress, based on the assumption that the concrete carries no tensile bending stress. It ignores the complications at the moment cracks where the shear concentrates above the crack. The conventional shear stress formula,  $v=V/bjd$ , presents an over-simplified concept, and ignores numerous variables which have to be taken into account. The variables being still under study, the American Concrete Institute and National Building Code of Canada have changed the formula to  $v = \frac{V}{bd}$  (65) to make it a little more realistic.

A diagonal tension failure appears to be the least complicated when it occurs away from the concentrated load and reactions. In short spans, when the shear becomes large enough, the diagonal tension near the neutral axis leads to the formation of a crack at approximately a  $45^\circ$  slope. This crack crowds the shear resistance into a smaller depth and by thus increasing the stresses, tends to be self propagating. In longer spans the diagonal crack is more apt to develop as a growth or extension of a vertical moment crack which turns into an inclined crack in the neighbourhood of the neutral axis. In neither case does such a diagonal crack usually proceed immediately to failure. Instead, it encounters resistance as it moves up into the zone of compression and becomes flatter. Finally it results in a sudden failure. It is this type of sudden cracking which is to be avoided.

## 1.6 Web Reinforcement

If the concrete alone were left to resist diagonal tension stresses, it could usually fail at relatively small value of the loads. In such cases web reinforcement is required. The most commonly used web reinforcement consists of vertical stirrups, usually in U-shape or closed at the top also. Sometimes diagonal stirrups are also employed. The vertical stirrup carries no significant stress until after a diagonal crack forms. It has little effect upon the shear at which such crack forms. Once a diagonal crack opens, vertical stirrups act in tension to carry load from one side of the crack to the other. The beam cannot fail by further opening of the diagonal crack until the stirrup stress passes the yield-point value. Even up to failure a portion of the shear is still carried by the concrete, apparently by the compression concrete above the crack. Diagonal or inclined stirrups are aligned more nearly with the principal tension stresses in the beam. They share in carrying this tension and slightly delay the formation of diagonal tension cracks. Because of the awkwardness of field welding, inclined stirrups are rarely used.

## 1.7 Levels of Approach in Concrete

The three different approaches commonly employed are: large scale, structural level and atomic or molecular level. The first named, i.e. large scale, is used for phenomenological studies where a particular phenomenon has to be studied to amplify factors which could not be studied at a smaller scale. On the other hand, molecular level studies are more recent in their approach.

Because of the very heterogeneous nature of concrete, it presents many difficulties in establishing theoretical approaches and rational formulae. Molecular approach is not suitable for concrete as it consists

of particles having a very large variation in their size and because of the difficulties encountered in such studies. Large scale studies are also not very common in most cases because of their being very expensive. Structural level is the most suitable approach for concrete as the variables involved can be exactly analyzed to establish rational design methods. Most of the laboratory studies are made on structures or structural elements smaller than the actual size of the structures. However, the usual scale ratio is  $1/2$  or  $1/4$ th and many of the variables involved in studying structures are not appreciably affected by this scale ratio. This scale ratio is essential due to the limitations of equipment or space. The scale ratio for the present study was approximately  $1/2$ . As most of the other studies on shear strength of concrete have also been made at this scale, it was possible to compare and correlate the results with some previous studies.

### 1.8 Outline of the Thesis

This Thesis consists of ten Chapters. Introduction to the problem of shear and diagonal tension, with the historical background, is given in the first Chapter. This Chapter briefly describes as to how the concepts were gradually developed. A historical bibliography is given at the end of the Chapter.

Chapter II is mainly devoted to the existing research and design specifications. A brief account of the research carried out after 1945 to-date is given starting with Moretto's (1) research on the strength of welded stirrups. Design specifications are also discussed very briefly to consider the effect of research on design procedures and specifications for web reinforcement.

Chapter III on shear, diagonal tension and bond is primarily an effort to present the problems in a very simple form with the classical



approaches for the calculations of diagonal tension and bond stresses. The difference between stress trajectories of a plain concrete beam and a reinforced concrete beam is explained with the help of stress trajectory diagrams.

Chapter IV is very important from the phenomenological point of view in that it gives a detailed account of the nature of diagonal tension cracking. The behavior of a reinforced concrete beam before and after the initial diagonal cracking is explained. The propagation of diagonal cracks with the stresses generated within the material after a major crack has formed are also discussed.

Chapter V is a brief description of why web reinforcement is essential. Different types of web reinforcements are given with a brief discussion of which type of web reinforcement is more effective.

Chapters VI and VII describe the test program and the results of the tests. A detailed account of the specimens, materials, fabrication, curing etc., is given in Chapter VI. Load-deflection tables are given in Chapter VII and curves for midspan deflections are also plotted. Results of compression tests and tensile splitting tests on cylinders are given, as well as the results of the tests on web reinforcing steel.

Analysis of the results is given in Chapter VIII. A comparison of web reinforcements used in the beam specimens is made. Total effective steel resistance is also calculated for all the specimens. A comparison of deflections for different beams is also made. Results are analyzed with the help of these tables.

Chapter IX consists of discussion of the results and a comparison with the work carried out elsewhere. Some of the problems encountered during the testing are also discussed as well as their effect on the test results.

References are given to the work done by Clark (2), Moretto (1), Kani (71), Rensaa (32), and Moody (15) etc., to compare some of the results of this study.

Summary, conclusions and suggestions for future research are given in Chapter X.

HISTORICAL BIBLIOGRAPHY

References to historical bibliography in the thesis are marked with an 'H' along with the reference number, in order to distinguish them from the bibliography given at the end of the thesis; e.g. Reference 1 will be marked as 1-H.

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## CHAPTER II

### REVIEW OF EXISTING RESEARCH AND DESIGN SPECIFICATIONS

#### 2.1 General

The historical background to the problem of shear has already been given in Chapter I. However, the research carried out lately in this field was not discussed in detail. A lot of work was done in the 1950's and afterwards in this field, and it will be appropriate to give an outline of existing approaches to the problem of shear and diagonal tension and the gradual advancement in research programs.

#### 2.2 Recently Completed Research

Moretto (1) in 1945 reported on an investigation of the strength of welded stirrups in R.C. beams. Variables including the size and inclination of the stirrups, type of concrete, and ratio of longitudinal reinforcement were studied. He developed equations for the yield point shear and ultimate shear for beams with web reinforcement. The equation for the safe shearing stress was given as:

$$v_s = Krf_s + 0.02f_c' + 2000 p \text{ - - - - - (1)}$$

where  $r$  = ratio of web reinforcement

$$K = (\sin \alpha + \cos \alpha) \sin \alpha$$

$v_s$  = safe shearing stress

$\alpha$  = inclination of the stirrups to the horizontal

$f_s$  = steel stress in stirrups, assumed to be 20,000 psi

Stirrups inclined at  $45^\circ$  and  $67.5^\circ$  were also tried, and it was found that  $67.5^\circ$  inclined stirrups were better than the vertical or the  $45^\circ$  stirrups. Generally it was noticed that welded stirrups resisted about 20% higher stresses in diagonal tension.

Clark (2,3) reported further studies of diagonal tension in R.C. Beams in 1951. One of the results was that the position of the loads on a beam influences considerably its shear capacity. The major

variables of this study included the depth/span ratio, ratio of web reinforcement, effect of varying the tensile reinforcement and the strength of concrete. Moretto's equation was further improved when an empirical formula was developed which indicated that the shear resistance of the beams varies as the square root of the percent of web reinforcement and linearly as the compressive strength of the concrete multiplied by a factor representing the ratio of the effective depth of the beam to the distance from the plane of the nearest concentrated load point to the plane of the support. He gave his equation for resistance to diagonal tension as:

$$v_c = 7000p + (0.12f'_c)d/a + 2500\sqrt{r} \quad \text{--- --- --- --- --- (2)}$$

Zowyer and Siess (10) in 1954 reported on the ultimate strength in shear of simply supported Prestressed concrete beams without web reinforcement. He evaluated different parameters and found that the limits between shear and flexural failures were not always well defined. The ultimate strength of the beams failing in shear can be expressed by the equations of the same type as the ultimate capacity of a beam failing in flexure in compression. The equations were modified empirically to take account of the compression zone resulting from the inclined cracks and the concentration of the angle change at the critical section.

A very detailed study of the shear strength of reinforced concrete beams was reported by Moody (9, 12, 14, 15) in 1954 and 1955. Tests on the simple beams without web reinforcement indicated that the strength of beams with large  $a/d$  ratios may be governed by the load causing first cracking, whereas the strength of shorter beams is governed by the load causing the destruction of the compressive zone of concrete. It was

noted that the formation of diagonal tension cracks in the region of maximum shear, led to a new distribution of internal stresses which prevailed until failure. At failure the compression zone of concrete was destroyed above the diagonal tension crack and adjacent to one of the loading blades. The ratio of ultimate load to cracking load decreased as the ratio of the shear span to the effective depth increased.

Tests of restrained beams with web reinforcement indicated that the magnitude of the cracking load depended primarily on the dimensions of the cross-section and on the strength of concrete. It was found to be practically independent of the web reinforcement. The magnitude of the ultimate load depended on the amount and type of web reinforcement, which increased with the amount of web reinforcement, and was higher for beams with diagonal stirrups than for the vertical stirrups. He presented extensive analytical studies to predict the shear strength of reinforced concrete beams. He divided the problem into two categories:

1. Load at which the diagonal tension cracks form.
2. Load at which the compression zone of concrete is destroyed.

He gave the following expressions:

a) for  $f'_c$  varying from 1000 to 5000 psi;

$$v_c = \frac{V_c}{7/8bd} = 0.12(1-0.1 M/Vd)f'_c \left(1 - \frac{f'_c}{10,000}\right) \quad \text{--- (3a)}$$

where  $V$  = maximum shear

$V_c$  =  $V$  at the formation of initial diagonal tension cracking

$M$  = bending moment at critical section

- b) for  $f'_c$  varying from 5000 to 6000 psi:

$$v_c = (v_c)_{f'_c = 5000} \quad \text{--- (3b)}$$

For second cracking load:  $v'_c$  he gave the expression:

a)  $f'_c$  varying from 2000 to 5000 psi:

$$v'_c = 0.20 f'_c \left( 1 - \frac{f'_c}{10,000} \right) \quad \text{--- (4a)}$$

where  $v'_c$  = second cracking load

b)  $f'_c$  varying from 5000 to 6000 psi:

$$v'_c = (v'_c) \quad \text{--- (4b)}$$

$$f'_c = 5000$$

There seems to be convincing evidence that shear failure usually occurs before the yielding of the tension reinforcement, and the ultimate shear load depends primarily on the strength of concrete and to a lesser degree on the percentage of longitudinal reinforcement. For ultimate shear strength Moody obtained semi-rational equations in terms of the ultimate moment at the section where failure occurred.

Mogens Lorentsen (69) reported in 1965 on the theory for combined action of bending moment and shear in reinforced and prestressed concrete beams. He described the influence of bond on the strength of reinforced and prestressed concrete beams. Test results described in the paper show that the presence of bond may induce shear failure. On the basis of this observation, a theory for predicting the shear strength of concrete beams is introduced. According to the theory the shear is carried partly by the beam action, partly by arch action. It is shown that the shear strength of beams without web reinforcement may be expressed as a function of the strength of the crack lamellas, the shear span, and the flexural cracking moment. The most important parameter in the author's theory is the lamella strength  $K$ .

W. J. Krefeld and C.W. Thurston (70) made some tests to study the contribution of longitudinal steel to shear resistance of reinforced concrete beams. An hypothesis of the mechanism of shear failure of

reinforced concrete beams is developed from studies of the behavior of several conventional beams tested with special instrumentation, and a number of beams especially constructed to show the contributing resistance to external shear of the longitudinal steel acting in conjunction with the embedding concrete. Some data are provided to show the effects of bar size, bar spacings, depth of cover below the bars, and concrete strength on such dowel resistance.

The conclusion from these data and analysis were subsequently used in the development of an expression for the shearing strength of simply supported reinforced concrete beams, with and without stirrups.

Reference has already been made to the studies by Dr. G.N.J. Kani (66, 71) in the first Chapter. His conclusions are briefly presented here:

- 1) The shear strength of rectangular, reinforced concrete beams does not depend on the concrete strength within the entire range of  $f'_c = 2500$  to  $5000$  psi and  $p = 0.50$  to  $2.80$  percent.
- 2) While the shear stress at failure, for small  $a/d$  ratios is up to 760% higher than for large  $a/d$  values, the variation of the relative beam strength exhibits smaller variations in the order of 100%. Contrary to belief, the nominal shear strength turned out to be very remote from being a constant characteristic of a certain grade of concrete. He advocated that the relative beam strength is a much more suitable indicator of the beam strength than the ultimate shear strength.
- 3) From tests, the beam strength varied between 50% and 100% of the flexural capacity of the cross-section, the exact strength depending on the combination of  $a/d$  and  $p$ . If the ultimate strength of the beam is expressed by  $M_u = r.M'_{fl}$ , where  $M_u$  = maximum bending moment at failure and  $M'_{fl}$  = maximum flexural moment using  $f_y = 50$  ksi, the reduction

factor  $r$  would vary between 0.50 and 1.00. The reduction factor,  $r$ , which depends on  $p$  and  $a/d$  or  $M/Vd$ , may be determined by formula or from a table. The problem of shear strength would become an investigation of and a search for the type and quantity of web reinforcing to increase the reduction factor  $r$  to 1.00.

### 2.3 Design Specifications

The design specifications also followed the pattern of research. The limited knowledge regarding the exact effect of different variables on the mechanism of failure in shear is reflected in the design specifications. The German and European Specifications of the 1950's (11) were based on the concept that diagonal tension failures should never be allowed to occur and web reinforcement should mainly consist of the bent-up bars, which should be designed for the total shear whenever web reinforcement is at all necessary. In the United States and Canada, the basic view pertaining to the design of web reinforcement had been that of the allowable stresses and working loads. The web reinforcement was designed only for the excess shear over that assumed to be carried by the concrete. The 1951 American Concrete Institute Code (67-H) gave the allowable value of shear as a measure of diagonal tension as  $0.03f'_c$  for beams with no web reinforcement. The shearing unit stress,  $v$ , as a measure of diagonal tension, in R.C. flexural members was computed by the well known classical formula:

$$v = \frac{V}{b_j d} \quad \text{----- (5)}$$

The area of steel required in stirrups placed perpendicular to the longitudinal reinforcement was computed by the formula:

$$A_V = \frac{V' s}{f_V \cdot j d} \quad \text{----- (6)}$$

where  $V'$  = excess shear over that carried by the concrete.



For bent-up bars:

a) single bars:

$$A_v = \frac{V'}{f_v \sin \alpha} \quad \text{----- (7a)}$$

b) series of parallel bent bars:

$$A_v = \frac{V'_s}{f_v j d (\sin \alpha + \cos \alpha)} \quad \text{----- (7b)}$$

There was, however, a shift in the design philosophy and a joint committee ACI-ASCE 327 (16) was formed as a sub-committee on Masonary and reinforced concrete in 1944 to study the adequacy of the ultimate strength theory and design. Later on ACI-ASCE committee 326 was established for further work in ultimate strength design concepts in the field of shear resistance. A large volume of work was done during 1950-1960, and ACI Code of 1956 (17) was presented with some modifications. The maximum value of 90 psi was added to the shear stress of  $0.03 f'_c$  permitted without web reinforcement. Limits were laid out for members with web reinforcement.

ACI Committee 326 came up with an expression for finding the external shear at diagonal tension cracking of the section considered:

$$\frac{V_u}{b d \sqrt{f'_c}} = 1.9 + (2500 \text{ psi}) \frac{p V d}{M \sqrt{f'_c}} \quad \text{but not greater than 3.5} \quad \text{--- (8)}$$

This was a major change as shear strength was considered a function of  $\sqrt{f'_c}$  and not a direct function of the cylinder strength of concrete as in the previous code.

It also recommended for the ultimate shearing stress,  $v_u$ , the following formula:

$$v_u = v_c + K r f_y \quad \text{----- (9)}$$

The ACI Code of 1963 (65) (Present Code) presented design procedures for both the working stress and ultimate strength design to resist diagonal tension failures. The committee on shear investigation found that the classical formula for nominal shear strength (eq. 5) was an oversimplified concept and at best was an approximation. It decided to replace the formula by:

$$v = \frac{V}{bd} \quad \text{----- (10)}$$

Since the actual distribution of shear stress has not yet been fully clarified, the use of average stress seems to be more logical, since only approximations are being attempted for design and the refinement involving moment arm is unwarranted.

The code also stipulated that the shear stress,  $v_c$ , permitted on an unreinforced web shall not exceed  $1.1\sqrt{f'_c}$ . This is a more conservative specification than the value of  $.03f'_c$  permitted by ACI Code of 1956.

The formula (6) was also changed to:

$$A_v = \frac{V \cdot s}{f_v \cdot d} \quad \text{----- (11)}$$

for the design of stirrups. The equation (7b) was also changed correspondingly, replacing  $jd$  by  $d$ .

For series of parallel bars:

$$A_v = \frac{V \cdot s}{f_v d (\sin \alpha + \cos \alpha)} \quad \text{----- (12)}$$

The equation (7a) remained unchanged.

The following equations were stipulated for ultimate strength method of design:

$$v_u = V_u/bd \quad \text{----- (13)}$$

The shear stress,  $v_c$ , carried by an unreinforced web not to exceed  $2\phi\sqrt{f'_c}$  or  $\phi(1.9\sqrt{f'_c} + 2500 \frac{p_w V_d}{M})$  - - - - - (14)

The equations for the design of stirrups are of similar form:

a) Vertical stirrups:  $A_v = \frac{V_u \cdot s}{\phi f_y d}$  - - - - - (15)

b) Single bent bar:  $A_v = \frac{V_u}{\phi f_y \sin \alpha}$  - - - - - (16)

( $V_u \neq 3\phi b d \sqrt{f'_c}$ )  
 c) Series of parallel bars:  $A_v = \frac{V_u s}{f_y d (\sin \alpha + \cos \alpha)}$  - - - - (17)

The chapter on ultimate strength design for shear and diagonal tension in the present code is new in entirety since the previous codes contained no such criteria for shear. The major philosophy is to produce members for which ultimate strength tends toward being governed by flexure rather than shear, so that the members will have a ductile character.

## CHAPTER III

### SHEAR, DIAGONAL TENSION, AND ANCHORAGE IN BEAMS

#### 3.1 General

A short description of how the concepts of shear and diagonal tension were developed has already been given in the first Chapter. The intent of the present Chapter is to briefly describe the classical methods for determining these stresses and all the knowledge which has been accumulated since then.

As already pointed out, unit shearing stress can be computed from the principles of mechanics by the equation:

$$v = \frac{VQ}{Ib} \quad , \text{ for an elastic homogeneous beam}$$

Due to the heterogeneous nature of reinforced concrete beams, they show a very different behaviour when subjected to shearing forces. In order to develop a simple formula, it is assumed that no longitudinal tension is carried by the concrete and there is no slipping between concrete and steel.

#### 3.2 Shear Formula

The classical shear formula is developed as follows:

Consider an element of the beam with length  $dx$  shown in Fig. 1 (a).

The forces acting on this element are the resultants of compressive forces  $C$  and  $C'$ , the tensile forces  $T$  and  $T'$ , and the vertical shear  $V$ .

The increment of tension,  $\Delta T$ , tends to pull the lower part of the element to the right, whereas the increment of compression,  $\Delta C$ , tends to push the upper part to the left. These external forces must be in equilibrium. Hence,  $\Delta T$  in steel can be computed by taking moments at A.

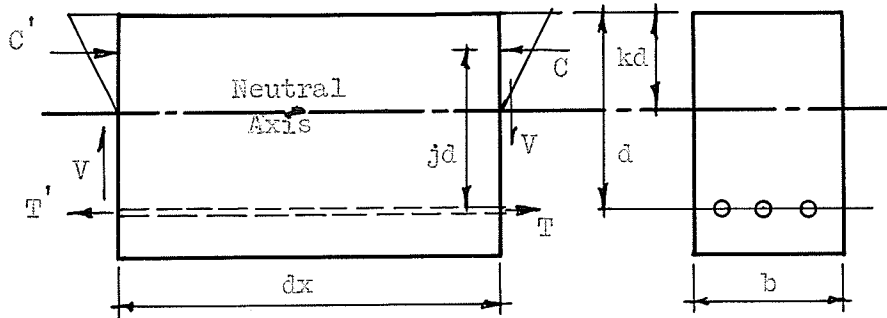


FIG. 1 (a)

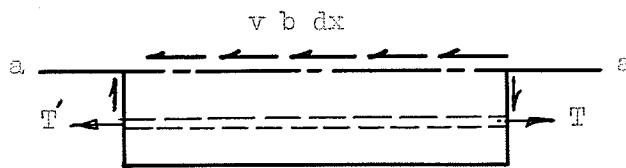


FIG. 1 (b)

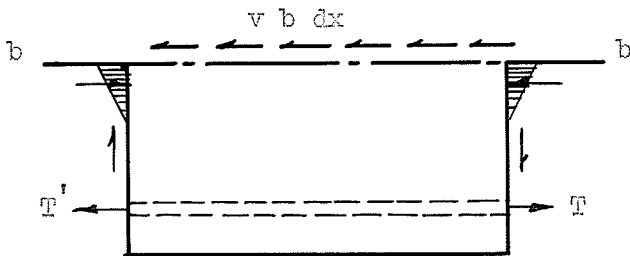


FIG. 1 (c)



FIG. 1 (d)

FIG. 1 Element of Beam Subjected to Shearing Stress

$$(\Delta T) jd = V(dx)$$

$$\Delta T = \frac{V(dx)}{jd}$$

Fig. 1 (b) shows a portion of the element of Fig. 1 (a) cut by the horizontal section a-a between the neutral axis and the tension steel. The increment  $\Delta T$  is resisted by the horizontal shearing force  $v b dx$  along a-a.

From Equilibrium:

$$\Delta T = v \cdot b(dx)$$

$$v = \frac{\Delta T}{b(dx)} = \frac{V(dx)}{jd(b \cdot dx)}$$

or 
$$v = \frac{V}{bjd}$$

which gives the equation of nominal shearing stress. However, this gives a constant unit shearing stress between the tension reinforcement and the neutral axis.

The shearing stress in the compression zone is smaller than the nominal shearing stress because there also exists the increment of compression. Fig. 1 (c) is a portion of the element of Fig. 1 (a) cut by the horizontal section b-b above the neutral axis. The formula  $v = \frac{VQ}{Ib}$  may be used for computing the shearing stress in the compressive zone. Fig. 1 (d) shows a typical distribution of unit shearing stress in a cross section of a reinforced concrete beam.

Due to the over-simplifications of this concept of distribution of shear stress, the formula has already been changed in the latest codes, ACI - 1963 (65) and National Building Code - 1965 (68) to  $v = \frac{V}{bd}$ , the refinement 'jd' being unnecessary due to the approximate nature of the value obtained.

### 3.3 Bond Stresses

For concrete and steel to work together in a beam, it is necessary

that the stresses be transferred between the two materials. The term "bond" is used to describe the means by which slip between concrete and steel is prevented or minimized. Thus bond stress is the intensity of an adhesive force between the surfaces of the steel reinforcement and the concrete.

Wherever the tensile or compressive stresses in a bar change, bond stresses must act along the surface of the bar to produce the change.

Referring to Fig. 1 (a)

$$\text{bond force} = u(\Sigma O)(dx)$$

This must equal  $\Delta T$ , the increment of tension.

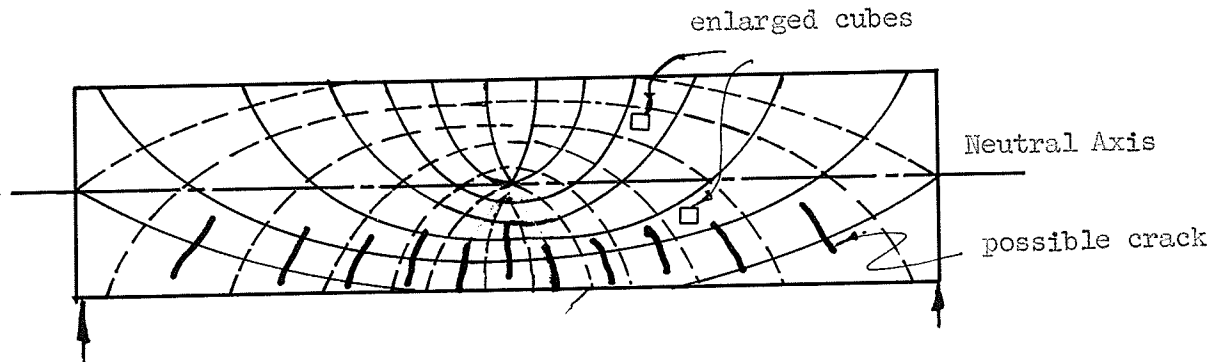
$$\& \quad u = \frac{v}{\Sigma O \cdot j d} \quad \therefore u(\Sigma O)(dx) = T = \frac{V dx}{j d}$$

which is the nominal bond stress in reinforced concrete beams, assuming uniform distribution of stress. The present code also gives the same value.

Bond stress distribution is usually much more complex. The limitations of the formula shall not be discussed here. Measurement and distribution of bond stresses along reinforcing bars are discussed by Mains (73-H).

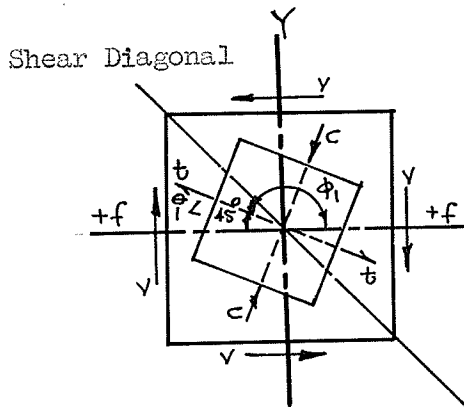
### 3.4 Diagonal Tension in Plain Concrete Beams

Consider the simply supported beam of Fig. 2 (a) which is subjected to shearing and bending stresses. Figs. 2 (b) and 2 (c) show infinitely small cubes of concrete enlarged below and above the neutral axis. Shearing stresses  $v$ , of equal intensity are acting vertically and horizontally as shown. Horizontal fiber stress,  $f$ , acts normal to the vertical axis. The horizontal stress is in tension, or in compression, depending on whether it is below or above the neutral axis. The combined effects of these stresses gives the principal stresses shown in Figs. (b) and (c). The principal tensile stress  $t$  and the principal

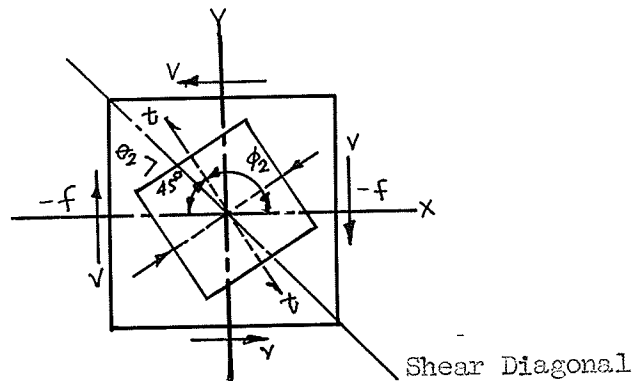


a) PLAIN CONCRETE BEAM - - Stress Trajectories

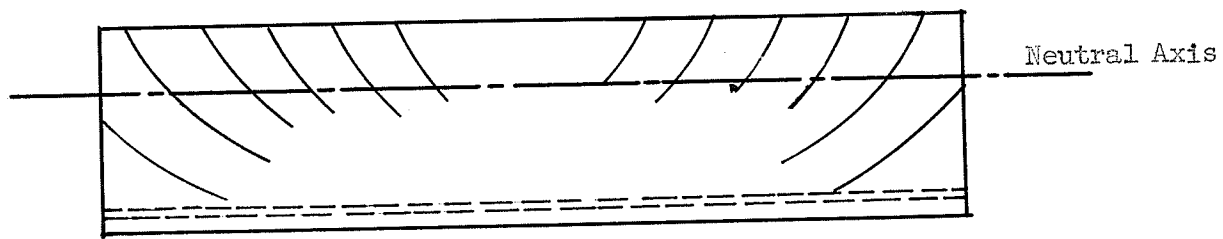
(---) Direction of Principal Tensile Stress  
 (---) Direction of Principal compressive stress



b) Enlarged Cube  
(below N.A.)



c) Enlarged Cube  
(above N.A.)



REINFORCED CONCRETE BEAM  
 (Principal Tensile Stress Trajectories)

FIG. 2: STRESS TRAJECTORIES



compressive stress  $c$  can be computed by the following equations:

$$t = \frac{f}{2} + \sqrt{\frac{f^2}{4} + v^2}$$

$$c = \frac{f}{2} - \sqrt{\frac{f^2}{4} + v^2}$$

$f$  is positive for a tensile fiber stress, and negative for a compressive fiber stress. The principal tensile stress in concrete beams is referred to as diagonal tension.

The directions of principal stresses are given by:

$$\tan \phi = -\frac{2v}{f}$$

Two values of  $\phi$ , differing by 90 degrees will satisfy this equation.

This agrees with the fact that principal stresses occur orthogonally at each point.

From mechanics, the principal tensile stress will always lie within the principal  $45^\circ$  angle subtended by the shear diagonal and the algebraically greater normal stress.

As represented by the Figs. 2 (b) and 2 (c), the inclination of diagonal tension of any particle below the neutral axis is less than  $45^\circ$  whereas that above the neutral axis is more than  $45^\circ$ . Along the neutral axis, the direction of diagonal tension of any particle must be  $45^\circ$  with the horizontal since the fiber stress given by the last equation becomes zero. Two families of stress trajectories at right angles to each other can be drawn for the whole beam. Tangents and Normals to these stress trajectories give the directions of principal stresses.

Analysis of the equation for principal tensile stress shows that in any section of a simply supported homogeneous beam, the intensities of diagonal tensions below the neutral axis are much larger than above the neutral axis. The probability of crack formation, hence, lies below the neutral axis and in the directions normal to the principal

stress trajectories.

### 3.5 Diagonal Tension in R.C. Beams

The longitudinal steel provides the concentration of tension in steel reinforcement and the magnitude of tensile fiber stresses cannot be obtained very correctly in the reinforced concrete beams. An approximation of stress trajectories may be obtained as shown in Fig. 2 (d). This shows the stress trajectories for a uniformly loaded, uncracked, R.C. beam without web reinforcement.

In order to simplify the conceptions of these inclined stresses and for convenience in designing web reinforcement, the nominal shearing stress is arbitrarily used as a measure of diagonal tension in common practice. Diagonal tension is a much more complicated problem, involving many variables, and cannot be described by simple exact relationships.

CHAPTER IVTHE NATURE OF DIAGONAL TENSION CRACKING4.1 General

In the opinion of the writer, there has been too much emphasis placed on finding formulas which would be universally applicable and not enough thought given to the accuracy of fundamental assumptions for different stress conditions. The majority of text books show how formulas are developed without pointing out the limitations of the fundamental assumptions on which the formulas are based. It may be realized, however, that reinforced concrete is not a homogeneous material and cannot be expected to be as fitted for generalization as other homogeneous materials like steel.

The object of this Chapter is to attempt a better understanding of shearing stresses and related problems in reinforced concrete leading to a diagonal tension failure. In some cases, it seems possible to obtain such an understanding on the basis of theoretical considerations whereas this does not appear possible without additional tests in some other cases.

4.2 Proper Shear

Concrete possesses a relatively high shearing strength and "proper shear" is in most cases not an initial cause of failure. It is nearly impossible to determine the shearing strength accurately because of the difficulty of eliminating the accompanying tensile stresses during tests. There is a wide variation of the test results depending on how successful the investigator has been in reducing the tensile stresses accompanying the shear. Shear strength will depend, to a very large extent, on the restraint against expansion transversely to the direction

of shear. It is also probable that direct compression or tension perpendicular to the plane of shear will have a considerable influence on the actual shear strength.

Many tests have been conducted to find out the proper shearing strength of concrete. Shearing strength to compressive strength ratios showed a variation depending on the conditions of lateral confinement, concrete quality and the form of test piece. These ratios varied from 0.37 to 1.04 (15-H, 32). It is clear that these test results cannot give a true picture as there usually is little or no lateral restraint acting on reinforced concrete beams and the tensile stresses are not effectively eliminated. It can be deduced that under such conditions, shearing strength to compressive strength ratios will generally be lower.

Tests by Mörsch (45-H) on rectangular specimens without lateral confinement indicated a shearing strength equal to  $\sqrt{k_1 k_2}$  where  $k_1$  is the tensile strength and  $k_2$  the cube compressive strength of the concrete. Since the tensile stress will always be less than the compressive strength, this equation will give a shearing strength less than the compressive strength. Further tests on shearing strength are reported by Suenson (72-H) and Rausch (74-H); Laupa, Siess and Newmark (7).

It seems reasonable to assume that concrete specimens having no special lateral confinement will be influenced to such a large degree by unavoidable tensile stresses that a direct linear relation to compressive strength does not exist. For diagonal tension it is even more true. A more logical approach, thus, would involve stipulating a certain allowable shear for each of the standard concrete strengths instead of specifying the shearing strength to be proportional to compressive

strength.

Since the shearing strength is much higher than the tensile strength, there is very little possibility of initial failure by shear. One important case may be where a crack caused by diagonal tension has extended so far towards the compression side of the beam that the shearing resistance of the uncracked concrete is insufficient.

#### 4.3 Principal Stresses

As already pointed out, the shear combined with bending moment tension may form a resultant diagonal tensile stress which in most cases results in initial cracking. Thus the carrying capacity of the beam is limited by this principal stress unless some reinforcement is used. Although shear is only a component of the principal stress, it is generally employed as a measure of the beam's ability to resist principal tensile stresses. The magnitude of the resulting principal stresses will also be influenced by the bending stresses, torsional stresses and the axial stresses.

Considerations of shear alone as a measure of the principal stress intensity may be grossly misleading. This fact has often been disregarded and in certain cases it may lead to an unsafe design. The adoption of the transverse shear force alone and its corresponding shearing stresses as a basis for the design of web reinforcement has caused a great amount of misunderstanding of the actual conditions of internal stresses in reinforced concrete members.

Certain mathematical theories have employed the assumption that concrete has no tensile strength. The result, in such cases, would be unrealistic as, if the concrete did not have any tensile or shear strength, it would be entirely unsuited as a construction material.

When bending is considered, the assumption of no tensile strength in bending is almost universally acknowledged but for diagonal tension this assumption is not as widely used. If, on a portion of a beam, there is any danger of the formation of diagonal tension cracks, there should then be a sufficient number of stirrups at a reasonable spacing to hinder objectionable widening of the cracks.

It has been established experimentally that the uncracked concrete of a section will carry some external shear (32). If it can be determined as to how much shear the concrete itself would carry, only that amount of web reinforcement can be put in which is necessary to carry the excess shear. This would prevent failure by diagonal tension cracking.

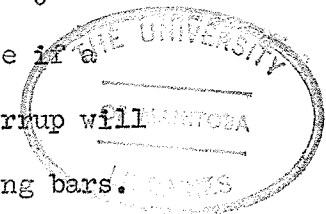
The determination of the portion of strength carried by concrete is not very easy. The strength of a beam in shear is a function of many variables. It depends not only on the compressive, shear and tensile strengths of concrete, but also on the amount and distribution of the web reinforcement as well as on its pattern and to a certain degree on the horizontal reinforcement. The influence of shrinkage stresses and temperature stresses may also be large. Empirical formulas have been attempted to include the effect of different variables. Reference has already been made to the formula developed by Moretto (1). This formula was a result of tests on simple beams and does not apply for restrained beams. Most of the tests in the laboratories have neglected the influence of the longitudinal forces set up by deformations arising out of temperature or shrinkage stresses, such as would most often be encountered in a structure. The empirical equations would thus be unsuited for accurate values.

Most of the tests for shear have been made on simple beams and the

above argument also applies to such beams. The present program of testing is also based on simple beams and it will be suitable to consider the stresses set up in such a beam. The principal stresses under this condition will be a combination of the shearing stresses and bending stresses. At midspan, the shearing stresses will be lowest and near the free ends, they will form the largest component. Generally, there will exist some residual axial tension from shrinkage and temperature deformation at the freely supported end. Under such conditions, a crack will start from the underside of the beam and move upwards towards the neutral axis. If there is no web reinforcement, the external shear will be carried partly by the uncracked compression zone above the neutral axis and partly by the dowel action of the bottom reinforcement.

#### 4.4 Dowel Action

If a bar is embedded in a concrete body of sufficient cross-section, considerable transverse load will be carried by the bar after the concrete has cracked. The amount of additional resistance by the bar to concrete before the cracking is probably small. Most generally, for horizontal reinforcement, the bars are placed quite close to each other near the surface. There will thus be a small concrete section left outside and between the bars to resist a transverse pressure from the bars. In addition, there will be appreciable amount of tension from transverse shrinkage which the relatively incompressible steel bars will set up. Under such a situation, there will be a small concrete section which cannot have much dowel resistance and there is no justification in depending on it. Dowel action may also take place if a horizontal split along the bars crosses a stirrup. Such a stirrup will then take up the transverse load from the horizontal reinforcing bars. Hence, if a diagonal tension crack starts close to a stirrup, there may still be substantial help from the stirrup after a split has formed even



if the diagonal crack does not cross the stirrup. The uncracked concrete section may thus be relieved of some shear load due to this action. When the stirrup gets overstressed, or the horizontal bars are deformed in bending, there is again a transfer of the load to the uncracked section of concrete. The portion of the shear load carried by the combination of bar-stirrup dowel action depends on the slope of the diagonal crack since this has an influence on the vertical effect of the adjacent sections at a crack.

It seems safest to disregard the dowel action completely as it is difficult to determine exactly the proportion of the shear load carried by it. Thus, if there is no web reinforcement crossing the crack, the uncracked compression concrete takes all the shear. The interlocking of the aggregate across the crack may carry a very small amount of shear which can be disregarded for all practical purposes. Thus for an uncracked portion, the concrete can carry some shear. As the crack moves upwards, a smaller section of concrete has to carry both shear and compression and ultimately a failure may result by combined shear and compression due to high unit principal stress. Hence, a well designed web reinforcement serves the purpose of carrying some of the shear directly and at the same time to retard the upward movement of a diagonal crack, thereby reducing the possibility of the uncracked portion becoming very small. Even if the web reinforcement is not designed for the full amount of shear, it will have an effect on the shear strength of the beam. The actual distribution of shear load between the concrete and web reinforcement is very difficult to establish. If a structure is composed of different materials, a possibility always exists that one kind of material may fail even before the other goes into action. For reinforced concrete structures, steel, being ductile,



will stand sufficient amount of overstressing and the web reinforcement in most cases can act with concrete until the complete failure of the member. If the concrete fails completely before the steel gets into action, the steel would have to carry all the load. The compression from a support has been found to increase the principal compressive stress and decrease the principal tensile stress near the support.

#### 4.5 Shear Failure

As it has been described in the preceding paragraphs, the shear failure in a beam is always initiated by a critical inclined crack. After the appearance of that crack, failure may develop in various ways. Thus the inclined crack may extend towards the center of the beam dividing the compression zone in two parts (69), the upper of which will crush. The inclined crack is usually accompanied by a horizontal crack along the reinforcement caused by the dowel action of the reinforcement at the lower end of the inclined crack. A collapse may be caused by an extension of the splitting towards the support. Such failures are referred to as shear-tension failures. It is difficult to establish the criteria for any one type of this failure but in general shear-compression failure is more predominant in short beams and shear-tension failure usually occurs with larger ratios of span-depth.

It seems important to have a theory to predict the critical shear corresponding to the appearance of the critical inclined crack in beams without web reinforcement. Web-shear cracking is noticed in the portion of the beam which is not affected by bending cracks, and is the result of the main tensile stress reaching the tensile strength of concrete.

On the other hand a flexural-shear crack results from a flexural crack when the concrete lamella between two consecutive flexural cracks is broken.

#### 4.6 Mechanism of Failure

The mechanism of failure due to the shearing forces is conceived as a modification of the internal force system (70) conventionally attributed to reinforced concrete members, accompanied by associated deformations consistent with the altered geometry of the member after diagonal cracks form in the beam web and horizontal cracking occurs at the level of the longitudinal reinforcing. While any diagonal crack in a beam web is potentially dangerous, since a separation produces a condition conducive to further re-distribution of internal forces, it is not, of itself, necessarily the cause of ultimate failure. This fact is well supported by the final flexural failures in the beams after extensive multiple diagonal cracking.

The critical extension of the diagonal crack at its upper end occurs when the dowel force transmitted by the reinforcement produces horizontal cracking of the concrete along the bars. Thereafter further redistribution of the internal forces results, with continued local bending of the bars, rotational deformations of the concrete, and increasing propagation of the crack in the compression zone.

#### 4.7 Cracking Phenomena

From the observations of crack formations in reinforced concrete beams, which are supplemented by strain measurements, it can be inferred that when the load is applied there are progressive changes in the magnitude and distribution of the stress. (70). Before the formation

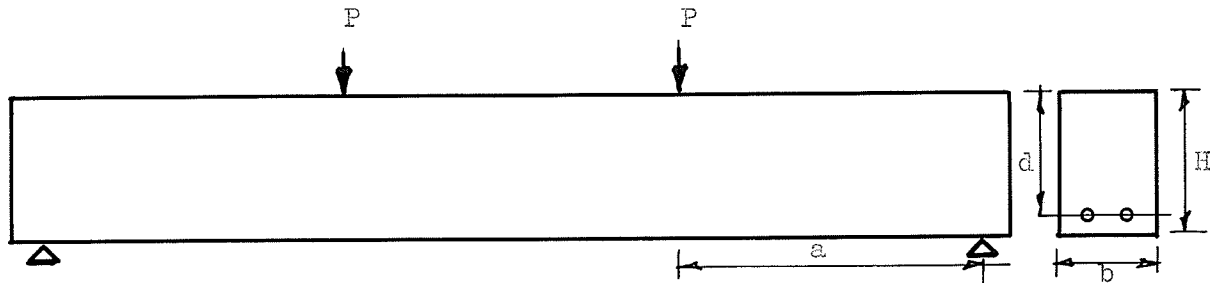


FIG. 3 - IMPORTANT PARAMETERS IN SHEAR

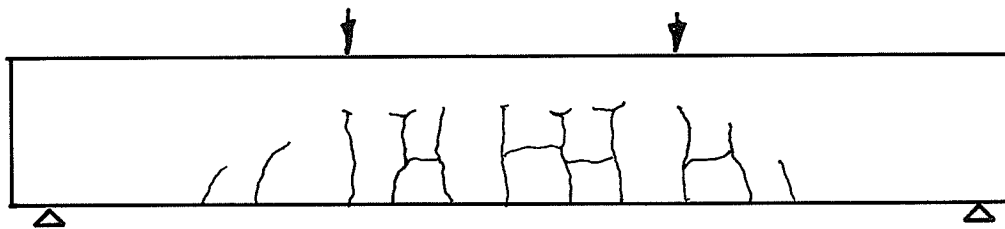


FIG. 4 (a) Flexure Failure

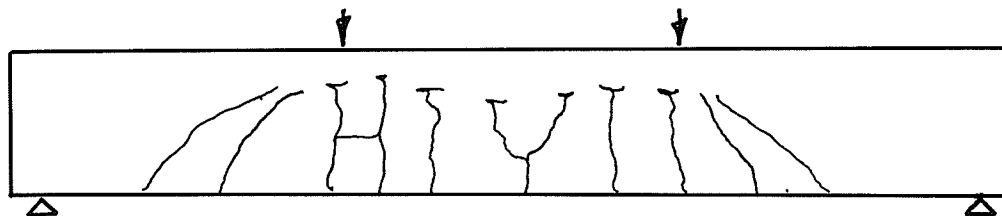


FIG. 4 (b) Balanced Flexure & Shear Failure

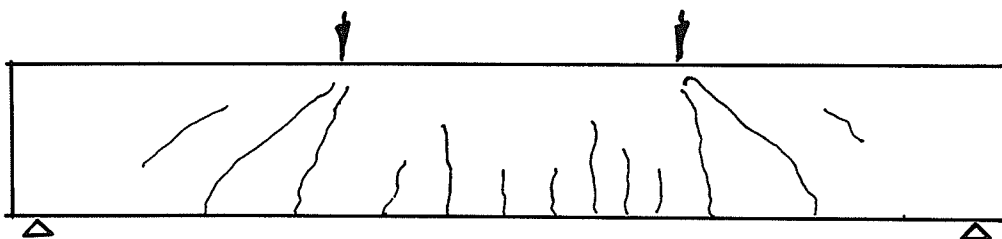


FIG. 4 (c) Shear Failure

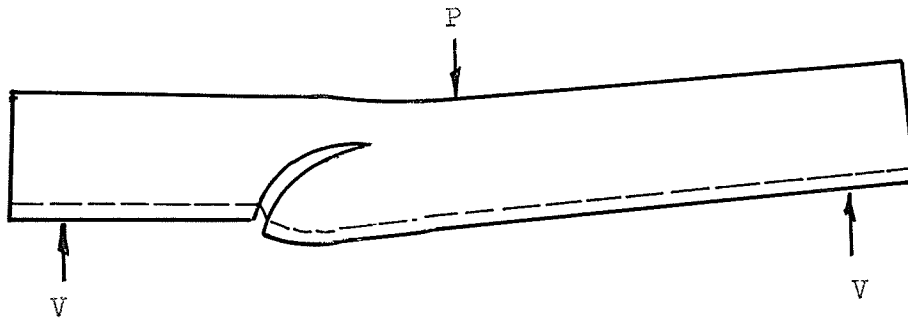
FIG. 4 - TYPICAL PATTERNS OF CRACKING

of any cracks, the concrete beam behaves as a composite member composed of concrete and steel. When the load is increased, and the limiting tensile strain in the concrete is reached, vertical flexural cracks form from the tension surface of the beam at intervals along the span according to the magnitude of the bending moment. There is a modification in the composite action with a resulting redistribution of stress at the cracked sections. Stresses in concrete and steel are increased and an increase in the rate of deflection of the beam is noticeable.

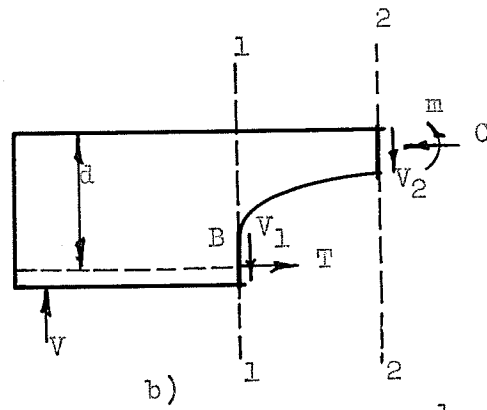
When the flexural cracks have moved above the longitudinal reinforcement, they start to become inclined at sections subjected to shear as well as bending. It may be kept in mind that the inclined extensions are mainly a result of the influence of shear in producing diagonal tension. This is true as the tensile stresses in the concrete due to bending moments decrease when approaching the neutral axis. A second redistribution of stresses occurs at this stage due to the progressive changes in the concrete and steel stresses as the inclined crack extends upwards.

Fig. 5 (a) shows an idealized diagonal tension crack. The continuity of the beam, after the separation of the beam web occurs, is provided by the tensile reinforcement on the bottom and the uncracked concrete in the compression zone above the end of the crack.

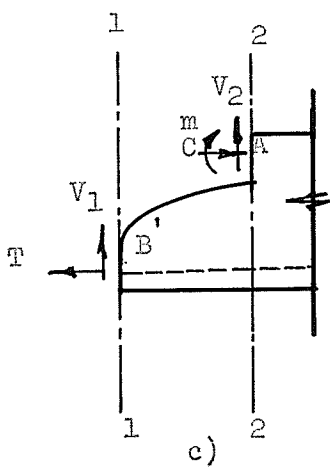
Figs. 5 (b) and (c) show the forces involved: the dowel shear  $V_1$ , and the tensile force  $T$  in the steel at  $B$  and  $B'$ , and a shear force  $V_2$  and a compressive force  $C$  at the intact section at  $A$ . A relative displacement of the segments adjacent to the crack occurs at  $B$  and  $B'$  which is accompanied by rotation at the reduced depth at  $A$ . This causes a secondary moment  $m$ . Longitudinal compressive strains below the upper



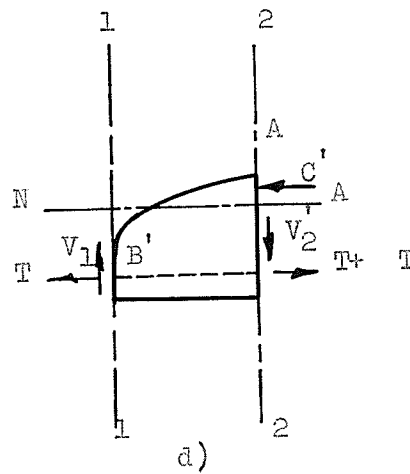
a) DEFORMATION DIAGRAM



b)



c)



d)

INTERNAL FORCE DIAGRAMSFIG. 5

end of the diagonal crack are noticed showing the presence of compressive force  $C'$  as shown in Fig. 5 (d). From considerations of statics, the dowel shear  $V_1$  must be present to be consistent with  $C'$  and  $V_2'$ . (32)

When the inclined crack reaches a point somewhat above the neutral axis in the compression zone, a horizontal crack, directed towards the adjacent support, forms at or above the level of the longitudinal bars at B where the inclined crack crosses the bars, marking the end of the second stage of stress re-distribution. Before this crack is formed, a portion of the vertical component of the displacement at B is due to the tensile strain in the concrete above the reinforcing bars accompanying the stresses resisting the dowel shear,  $V_1$ . After horizontal cracking along the bars, the deflection of  $B'$  with respect to B increases, and correspondingly, the relative rotation of the two segments at A also increases. The diagonal crack propagates more rapidly in the compression zone but with a flatter trajectory. Mostly it is noticed that this change is sudden. This third stage of stress redistribution continues until a complete "split" of the concrete occurs along the steel or until the compression zone disintegrates.

The ultimate failure of the compression zone may take different forms. In case there is an extensive and sudden split along the steel, the diagonal tension crack extends directly to the top surface of the beam with little change in direction, and the fracture resembles a shear surface after failure in compression. On the other hand, if the horizontal cracking above the bars progresses slowly, the diagonal crack propagates gradually towards the load point and, eventually, crushing occurs somewhat above the crack in the greatly reduced section. Another possibility of the beam failing without any crushing above the crack exists. This may, however, be due to the rotation about the section at the end of the crack, with the crack simply bending upward to the top

surface. In some cases the thin segment remaining above the crack may fail by buckling without any evidence of the crushing. (32)

The propagation of the crack in the compression zone, leading ultimately to separation and disintegration, results from the cracking or splitting above the bars at the bottom of a critical diagonal crack. Generally the diagonal crack has extended just above the neutral axis into the compression zone at the time this splitting occurs. The diagonal crack propagation after dowel action cracking is secondary though it is the ultimate cause to final failure. The amount of load which can be carried after the dowel action cracking is a variable and tends to be small for long span beams. Conservatively, the load producing dowel action cracking has been taken as the critical load. An increase of the resistance to the shear force by the bars <sup>by means of</sup> / stirrups etc., will delay the dowel action cracking and increase the critical load and ultimate load-carrying capacity of the beam. Reference to the dowel action, however, does not signify a dowel bar embedded in concrete, as the crushing strength of concrete is largely controlling the resistance. When applied to the beams where the bars are relatively close to the concrete surface, this resistance depends on other factors such as bar size, amount of cover below the bars, beam depth and the tensile strength of concrete.

## CHAPTER V

### SHEAR REINFORCEMENT

#### 5.1 General

It is clear from the discussion in the preceding chapters that due to the sudden and relatively brittle nature of shear failures, this type of failure should be avoided. The beams which are only reinforced by the longitudinal steel have a potential weakness to diagonal tension failure as the contribution by longitudinal steel to the shear strength is not sufficient to guarantee a flexural failure. The shear failures are guarded against usually by the provision of web reinforcement or the vertical steel. Although the shear strength is a function of many variables, the contribution to shear strength by web reinforcement is appreciable. The following section deals with the different forms of web reinforcement.

#### 5.2 Web Reinforcement

For very small values of the shear stress, the web reinforcement may not be needed. The problem of defining the limiting value of shear upto which no web reinforcement is required has been under discussion for a very long time. The ACI code and National Building Code of Canada have undergone a change in prescribing the permissible shear without web reinforcement. The present ACI code permits a shear of  $1.1\sqrt{f'_c}$  at a distance  $d$  from the face of the support unless a more detailed analysis is made. Whenever the shear stress exceeds the permissible values, web reinforcement is needed. The ACI code requires the web reinforcement to carry the excess shear stress over that permitted for the concrete of an unreinforced web. Upon thorough re-evaluation of all the work done in an effort to solve the problem of diagonal tension, ACI-ASCE committee 426 decided to carry on the practice of providing



only for excess shear. While shear in a member without web reinforcement is carried by the concrete web, the contribution of the concrete to the shear strength of a member with web reinforcement results from shear carried by the compression zone. Nevertheless, the approximation of assuming that the two contributions, i.e. by concrete web in unreinforced beam and concrete in compression in reinforced case, are numerically equal is not unreasonable. During the preceding discussion it was shown that the critical section is some distance away from the support. Usually the shear is calculated at a distance  $d$  from the support and the same reinforcement is continued upto the support.

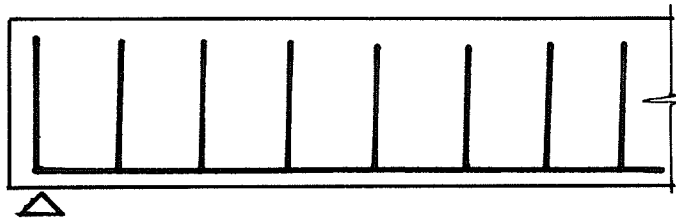
### 5.3 Types of Web Reinforcement

Following are some of the common types of web reinforcements:

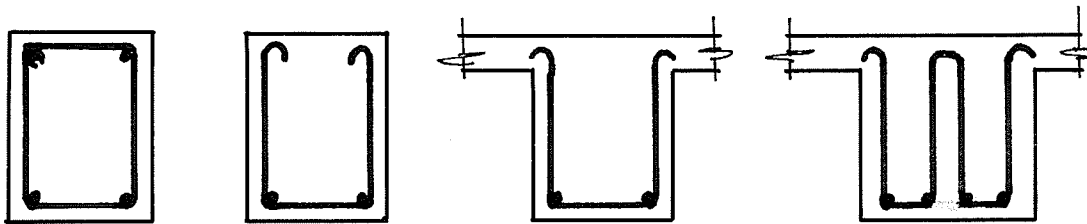
- 1) stirrups perpendicular to the longitudinal reinforcement.
- 2) stirrups making an angle of  $45^\circ$  or more with the longitudinal tension reinforcement.
- 3) longitudinal bars bent so that the axis of the bent bar makes an angle of  $30^\circ$  or more with the axis of the longitudinal portion of the bar.
- 4) combination of stirrups and bent-up bars.

The most common web reinforcement consists of vertical stirrups, usually in U-shape, but sometimes in W-shape. It may be noted that the vertical stirrup carries no significant stress until after the formation of a diagonal tension crack. Once a diagonal crack opens, the vertical stirrups act in tension to carry the load from one side of the crack to the other. A common analogy considers the stirrups acting as tension verticals in a truss, with the concrete acting as compression diagonals. This analogy is shown in Fig. 6 (c). It has been shown by tests that the beam cannot fail by further opening of the diagonal crack until the stirrup stress passes the yield point value.

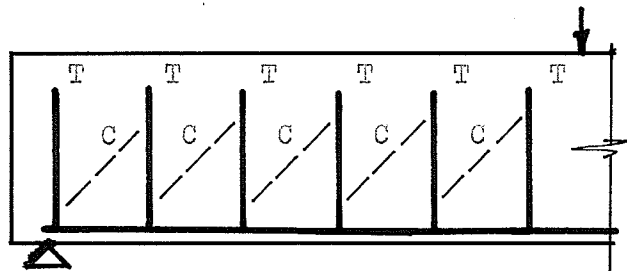
Fig. 6 (d) shows the diagonal or inclined stirrups. It can be



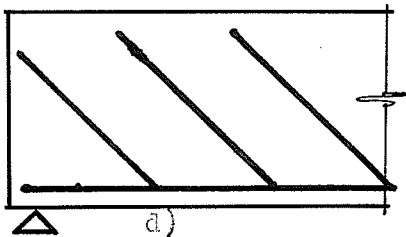
a) BEAM ELEVATION



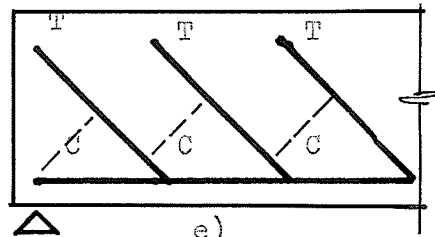
b) TYPES OF STIRRUPS



c) TRUSS ANALOGY



d)



e)

INCLINED STIRRUPS

FIG. 6

seen that these stirrups are more nearly aligned with the principal tension stresses in the beam. They share in carrying this tension, and also retard the formation of the diagonal tension cracks slightly. One practical problem with inclined stirrups is to tie them with the longitudinal steel. Welded stirrups are recommended in such cases. (2)

The use of welded stirrups as web reinforcement for concrete beams, instead of the conventional loose U-stirrup, should provide improved construction, since it gives perfect continuity to the reinforcing steel in the concrete and facilitates accurate steel-setting. Its practical use in units formed by longitudinal bars and stirrups welded together has been suggested as economically possible. Probable advantages in strength due to the welded stirrups lie in the improved anchorage of the stirrups, prevention of the slipping of the inclined stirrups along the main bars and improved anchorage of the main bars.

The stirrups inclined at  $67.5^\circ$  would give the maximum strength from theoretical considerations. If the strength of the beam with vertical stirrups or stirrups inclined at  $45^\circ$  is taken as unity, the beam strength works out to be about 20% greater for stirrups inclined at  $67.5^\circ$ . If the stirrups are to be welded, the inclination to be given should be the one that gives the maximum strength. (2)

The longitudinal bars are often bent up where no longer needed for the moment. These bent bars behave as inclined stirrups and carry some shear. Usually only a few bars are bent, and they may not be conveniently spaced for use as web reinforcement.

#### 5.4 Which Type of Web Reinforcement is Most Effective?

To determine what kind of web reinforcement is most effective, it is useful to study the stress trajectories of a homogeneous beam. The principal stresses will all have slopes of  $45^\circ$  at the neutral axis

where there is no bending stress. There is, however, a considerable difference in these trajectories along the beam from the neutral axis to the tension side. From the considerations of truss analogy (Warren truss or Pratt truss), the bent-up bars should not be spaced farther apart in the longitudinal direction than a distance corresponding to the effective depth, and the stirrups not more than 75% of this dimension if the web reinforcement is to be sufficiently effective. Web reinforcement is supposed to be effective only when crossing a crack. If the crack is very steep, it may not cross any stirrup. It would appear that the bent-up bars may generally provide much greater safety against diagonal cracking than can be provided by the vertical stirrups (32). This argument has been open to debate, and opinions differ in this connection (35, 36, 37, 38, 40, 41). The stirrups can perform the useful additional function of reinforcing a beam transversely against splitting caused by bond or anchorage stresses in the main horizontal reinforcement. The stirrups may be placed at a suitable slope as an alternative to the bent-up bars. If they are properly anchored to the main reinforcement, they can be very useful due to the generally smaller cross-section, and hence better anchorage at the compression side.

The vertical stirrups carry only the vertical component of the diagonal stress. A corresponding horizontal reinforcement is required to take up the horizontal stress component of the diagonal stress (74-H). If the horizontal steel is lacking, additional tension is put on the main horizontal reinforcement. For a relatively deep beam some special longitudinal reinforcement may be placed in the middle portion of the beam stress as a precautionary measure.

The combined use of stirrups and straight longitudinal bars has

been established as an entirely effective reinforcing system. This has been confirmed by tests and practical experiences (32).

CHAPTER VITHE TEST PROGRAM6.1 General

Kani (71) in his paper "Facts Concerning Shear Failure" writes:

"To study the influence of a single parameter on the shear stress at failure, a beam series of at least a dozen test specimens, in which all other parameters remain unchanged, is necessary. Therefore, in a test project designated to study the influence of three different grades of concrete, in conjunction with four different percentages of main reinforcement, the number of test specimens needed would be  $12 \times 3 \times 4 = 144$ . Such a project would present the behavior of only one type of cross-section, only one type of loading, and no web reinforcement. If the influences of these other parameters are to be established with a similar degree of reliability, it is obvious that several thousand specimens are required."

The above extract has been reproduced to emphasize the need for a systematic investigation. It is obvious that a test program has to be highly selective. It is better to assess the effect of one variable on the shear strength of concrete by the appropriate number of tests rather than attempting to investigate a number of variables with insufficient tests. The primary variable selected for the present study was the effect of size, pattern and the distribution of shear reinforcement on the shear strength. For this purpose various combinations were attempted, keeping the amount of steel used as shear reinforcement approximately constant. The study was, however, not confined to this aspect only. An important part of this study involved the initiation of the cracks and crack propagation, observations regarding the nature of crack formation and the ductility of concrete. The last named factor

being very important from the point of view of behavior was considered in relation with the shear strength, i.e. the reserve energy shown by the beams after ultimate shear failure at maximum load had occurred.

## 6.2 The Experimental Beam

It was decided to cast and test one beam before embarking upon the detailed test program. This was done with a view to ensure a failure of all beams due to shear, so that modifications can be made in the subsequent testing and adjustments, if any, can be made in the equipment or test procedures. A very brief description of this beam is given in this section. Details of actual test specimens, equipment and test procedures are given in the following sections.

The size of the experimental beam was chosen as 8 ft. long and 8 in. x 18 in. in cross-section. This size was suitable for the testing machine and the length of the testing bed of the machine. 4 No. 7 bars were used as longitudinal reinforcement at the bottom of the beam, using a concrete cover of about 2 in. with the effective depth of the beam as 16 in. These bars were 7 ft. 6 in. long giving a cover of 3 in. on either side. The shear reinforcement was provided by stirrups made out of No. 4 bars. The stirrups were spaced at 8 inch intervals. In order to achieve a good bond, enclosed stirrups having four sides were made. These stirrups <sup>were</sup> made in the laboratory and a small steel plate with special projections was made in order to facilitate making these stirrups. The diameter of the bends and projections at the ends of the stirrups were according to the ACI code. Two hanger bars were used at the top to keep the stirrups truly vertical.

All these bars for longitudinal and web reinforcement were deformed bars of intermediate grade steel. No tests were carried out to actually determine the yield point and ultimate tensile strength of the steel used for this beam.

As it was desirable to test this beam as early as possible, it was cast with High Early Strength Cement. The maximum size of the aggregate was  $3/4$  inches.

A special plywood form was made for casting this beam. In order to guard against the possible bulging out of the beam at the center, plywood strips were made and screwed on top of the form at three equal distances along the length. The mix ratio by weight was 1:3.5:5 and the cement/water ratio was 1.43.

Control cylinders were made and the average compressive strength was found out at the time of testing of the beam.

The beam was tested after 7 days. The sketch of the testing arrangement and a picture are shown in Figs. 24 to 27.

The load was applied at the third points of the beam and the deflections were recorded only at the mid point by a dial gauge.

Although the beam was designed to fail in shear (the calculated ultimate load in shear being 85 kips and that in flexure being 96 kips), it showed a final flexural failure.

The initial diagonal tension cracks appeared at a load of 50 kips and with the increase in load, they started extending upwards. At a load of 80 kips, small moment cracks in the middle span were noticed. However, these moment cracks did not extend upwards with a further increase in the load. The diagonal tension cracks were extending in the compressive zone when one moment crack in the middle span suddenly widened and extended upwards to cause flexural failure at a load of 119.3 kips. It was thus clear that the combination used for longitudinal and web reinforcement used in the beam could not ensure a shear failure. The design methods for web reinforcement are quite conservative due to the anxiety on the part of designers to avoid sudden shear failure. For the purposes of present study, the ratio of web/longitudinal reinforce-



ment had to be decreased. This could either be done by increasing the longitudinal reinforcement and/or increasing the spacing of the stirrups, thereby reducing the web reinforcement, or reducing the shear span.

The following conclusions were made from the testing of this beam.

1) Two brackets used on the outside of the plywood form to keep the beam at a constant cross-section and preventing from bulging out were insufficient. The maximum width of the beam (at the center) was  $8 \frac{3}{16}$  inch. In order to keep a constant width of 8 in. throughout, the number of supports had to be increased and they had to be firmly tied at the outside by a good number of screws, at a width preferably  $\frac{1}{16}$  in. less than 8 in. so that even if there were some expansion of forms, the final width of the beam would not be more than 8 in.

2) The effective depth of the beam was 16 in. and the shear span was 28 in., giving a ratio  $a/d = 1.75$ . This is already a low value for a normal beam and it was not considered suitable to further reduce the shear span in order to ensure a shear failure due to heavy shears.

3) An increase in the longitudinal reinforcement was the best way to make the beam comparatively weak in shear. A 50% increase in the longitudinal reinforcement seemed enough to guarantee a good shear failure. It was decided to use six No.7 bars (instead of four) as longitudinal reinforcement, considerably increasing the moment capacity of the beam.

4) The spacing of No. 4 stirrups could be increased from 8 in. to 9 in. Further increase in spacing was unsuitable as the effective depth was only 16 in.

5) The average concrete strength worked out to be 3014 psi. A reduction in concrete strength for mix design could be another way to cause shear failure. This was not considered very effective in view of an increase already made in the longitudinal reinforcement.

6) A water-cement ratio of 0.75 could give a better slump and added workability in mixing concrete. This would give a cement-water ratio of 1.33 instead of 1.42 used for this beam. It was decided to use the former value as cement-water ratio for all the beams to be tested.

7) Deflection readings were recorded only at the center of the beam. It was decided to take the deflection readings at more points along the length of the beam to gain a better insight into the crack formation. The following points were selected for deflection readings:

- (1) center of the beam
- (2) under load points
- (3) mid points of the end spans

8) The wires used for tying the web reinforcement with the longitudinal steel took a long time to make a rigid cage of the reinforcement. It was decided to use special ties for this purpose, which were especially suitable for smaller size stirrups and the wire-mesh used as web reinforcement.

9) Three wooden forms of the size 8 in. x 18 in. were made so that three beams could be cast at the same time. This was considered advisable as the conditions at the time of casting would be similar and the results could be more comparable. The work in mixing concrete could, thus, also be reduced.

### 6.3 Test Specimens

Six beams of rectangular cross-section were cast for the purpose of present investigation. They were all of the same size with a cross section 8 in. x 18 in. and a length of 8 ft. The nominal concrete strength was chosen as 3500 psi. The concrete cover on the longitudinal reinforcement was 2 in., giving an effective depth of 16 in. for all the beams. The longitudinal reinforcement was kept constant at six No.7

deformed bars, spaced in two layers at the bottom of the beams, Consequently, the ratio of the area of the tensile steel bars to the effective cross-sectional area of the concrete was 0.0281. 2 No.3 bars were used at top as hanger bars. All the beams were designed to fail in diagonal tension.

#### Beam 1

This beam employed No. 4 stirrups spaced at 9 in. centres. Details of reinforcement are shown in Figs. 7, 13, and 14. This was the maximum size used for the stirrups. The ratio of web reinforcement was 0.555%.

#### Beam 2

No. 3 size stirrups were used for this beam @ 5 in. centres giving a ratio of web reinforcement approximately the same as in Beam 1. A sketch of the reinforcement is shown in Fig. 8 and a picture of the reinforcement is shown in Fig. 15.

#### Beam 3

This beam was made with No. 2 size stirrups spaced @ 3 in. centres. This gave the ratio of web reinforcement lesser than the first two beams, i.e. 0.417%. If the same ratio of web reinforcement was to be used, then these stirrups had to be spaced at 2 1/4 in. centres. However, this spacing of 3 in. was used as the yield point strength of No. 2 bars was considerably higher than the No. 3 and No. 4 bars. As the basic factor to be kept constant was  $A_s f_{s_y}$  (Area of steel x yield stress), this spacing of 3 in. was found to be more appropriate. Details of tests on the No. 2 and other sizes of reinforcement are presented in Tables 6 to 16 in the next Chapter.

A sketch of the reinforcement is given in Fig. 9 and two pictures of reinforcement cage used for this beam are given in Figs. 16, and 17.

#### Beam 4

This beam employed the combination of No. 2, No. 3 and No. 4 size

stirrups. For details see Figs. 10, 18, and 19.

#### Beam 5

The shear reinforcement for this beam consisted of No. 3 stirrups inclined at  $45^{\circ}$  to the horizontal. The spacing worked out to be 7 in. for the same ratio of web reinforcement. As some other bars had to be used to give a good bond and a reasonable cage, the actual pattern of reinforcement used is shown in Fig. 11. Fig. 20 shows a picture of this reinforcement cage.

#### Beam 6

Three wire meshes each 2 in. square were used as shear reinforcement for this beam. These meshes were spaced in the width of the beam, i.e. one mesh each at the ends and the third at the center of the width. The verticals consisted of 0.156 in. dia wires and the horizontal wires were 0.115 in. dia.

If the ratio of the web reinforcement had to be kept constant, four such wire meshes were required. However, it was not possible to accommodate these meshes within a width of 8 in. It would not have been possible to pour concrete and to vibrate it within the meshes.

At it turned out, even three meshes presented some difficulty in vibrating the concrete during casting. It was decided to use three meshes and the results could then be compared on this basis.

A sketch of the beam is given in Fig. 12 and Figs. 21 and 22 show photographs of the reinforcement cage. As it can be seen four stirrups of No. 2 size were used to put the meshes in perfectly vertical condition and to facilitate tying them. Special ties were used for making the wire meshes, i.e. tying the verticals with the horizontals at all the corners.

TABLE 1 - Details of the Test Beams

Beam No.	a/d ratio	Nominal $f_c$ (psi)	Tensile Steel		Shear Reinforcement (stirrups etc)		
			Pieces & Bar No.	$p=A_s/bd$	Size	Spacing S(in)	$r=A_v/b_s$
1	1.75	3500	6-No. 7	0.0281	No. 4	9	0.00555
2	"	"	"	"	No. 3	5	0.0055
3	"	"	"	"	No. 2	3	0.00417
4	"	"	"	"	Nos. 2, 3 & 4	3 5 9	0.00417 0.0055 0.00555
5	"	"	"	"	No. 3 inclined	7	0.00555
6	"	"	"	"	3 wire * meshes 0.156" dia vert. 0.115" dia hor.	2	0.00358

$$*A_v = 3 \times 0.0191 = 0.0573 \text{ in}^2$$

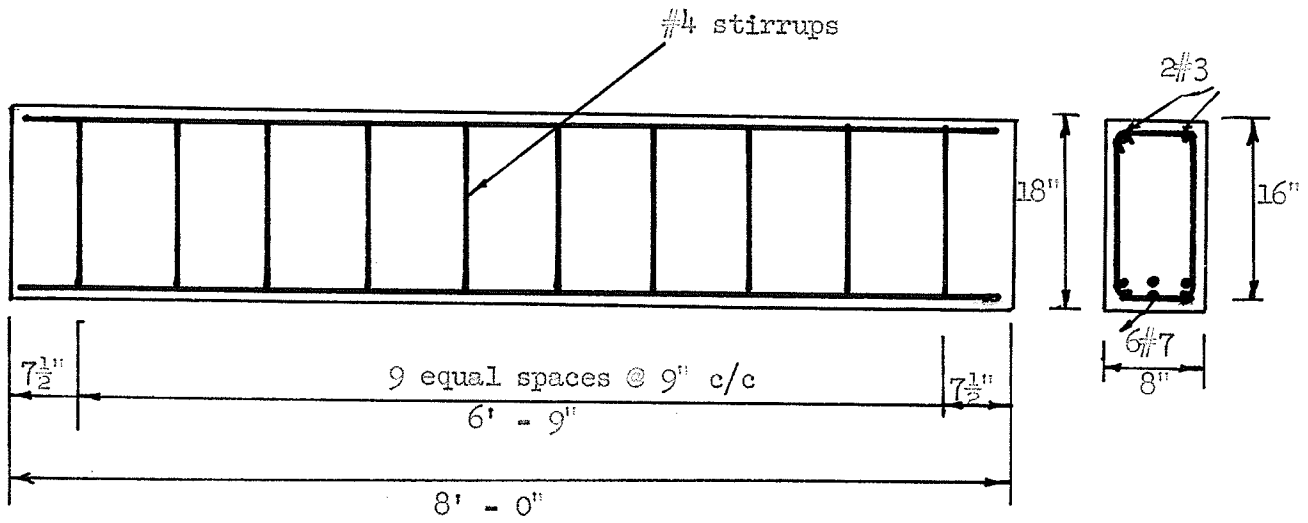


FIG. 7 SKETCH OF REINFORCEMENT BEAM 1

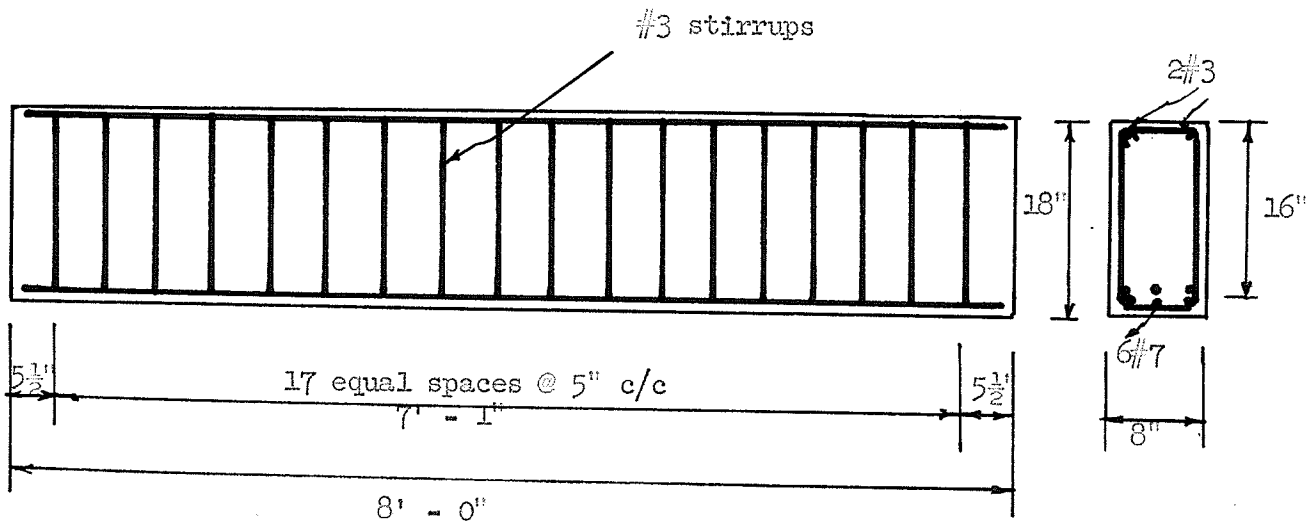


FIG. 8 SKETCH OF REINFORCEMENT BEAM 2

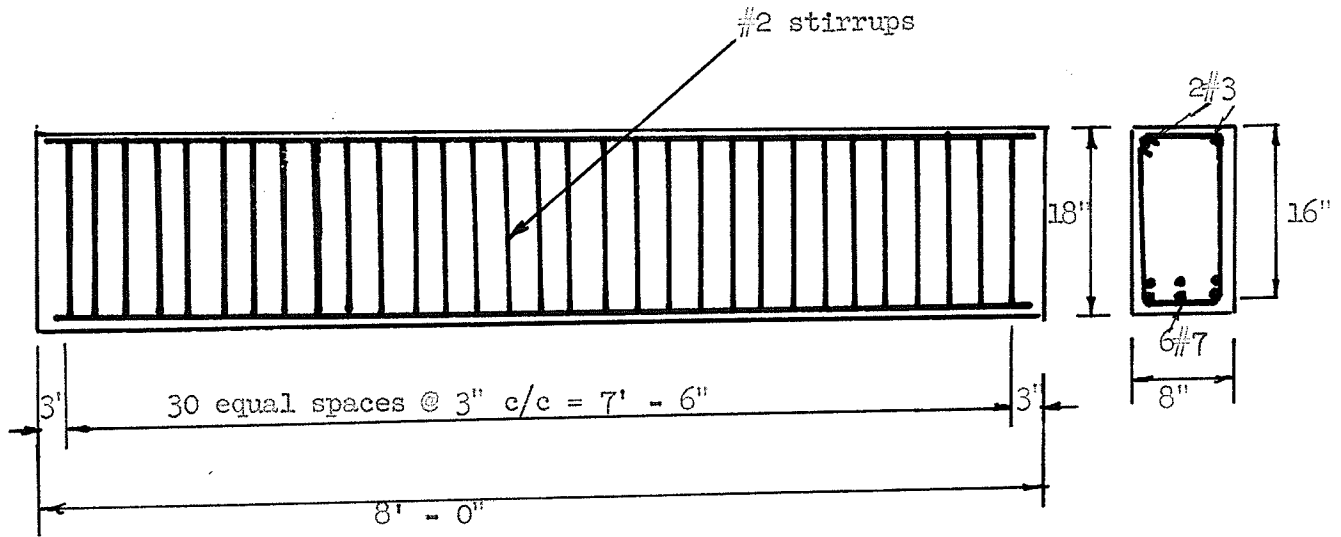


FIG. SKETCH OF REINFORCEMENT BEAM #3

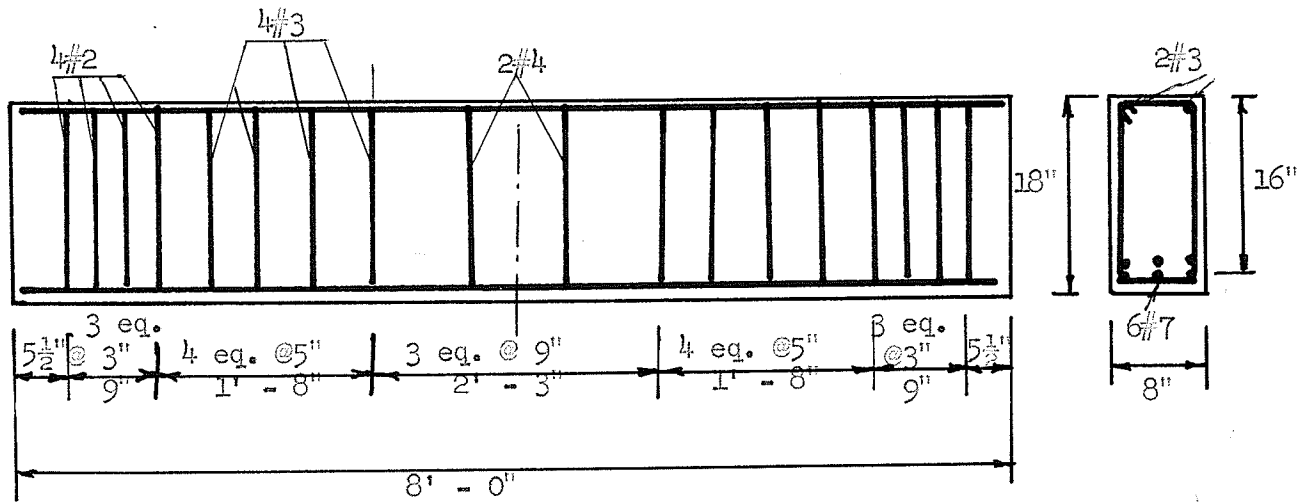


FIG 10 SKETCH OF REINFORCEMENT BEAM 4

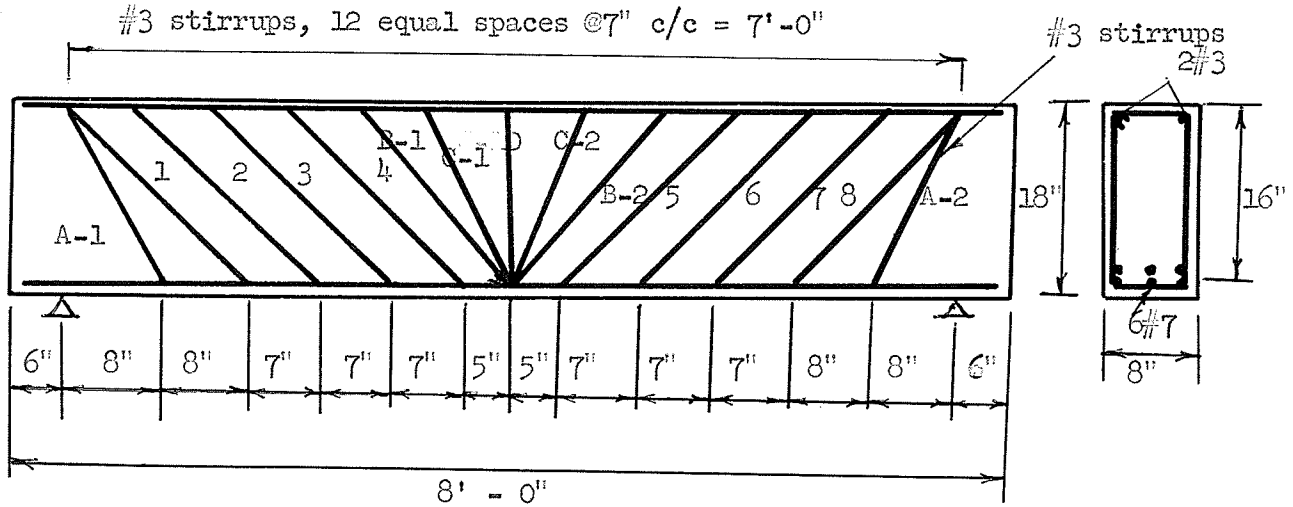


FIG. 11 SKETCH OF REINFORCEMENT BEAM 5

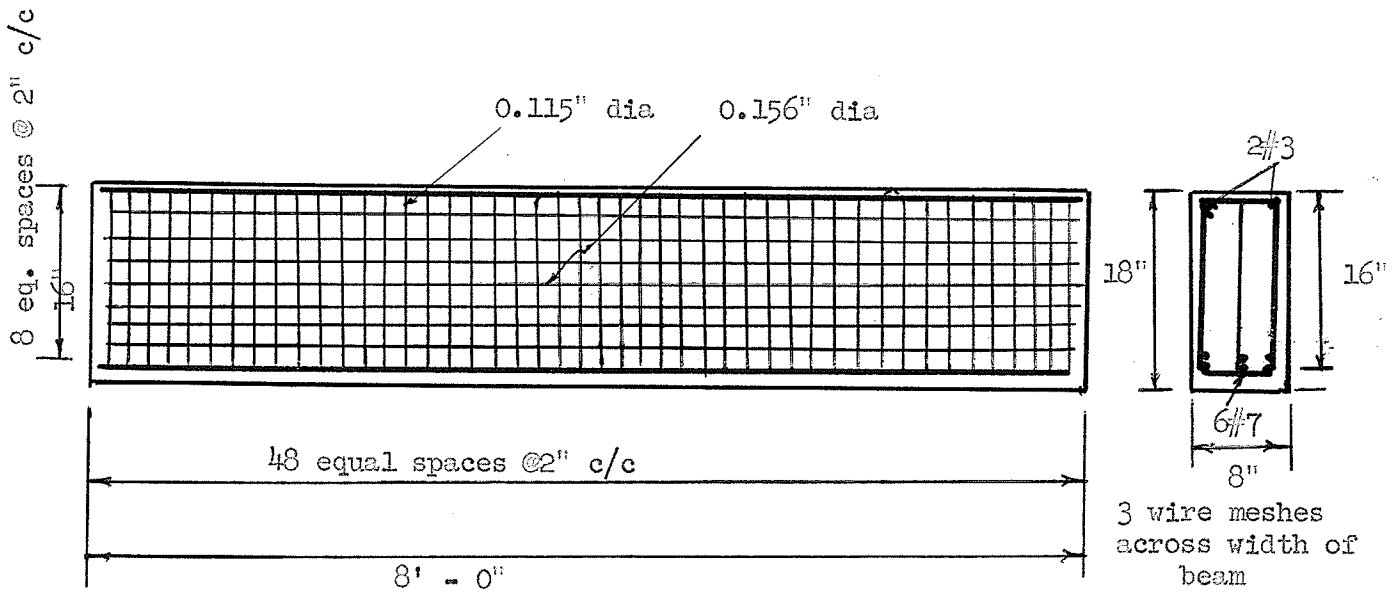


FIG. 12 SKETCH OF REINFORCEMENT BEAM 6



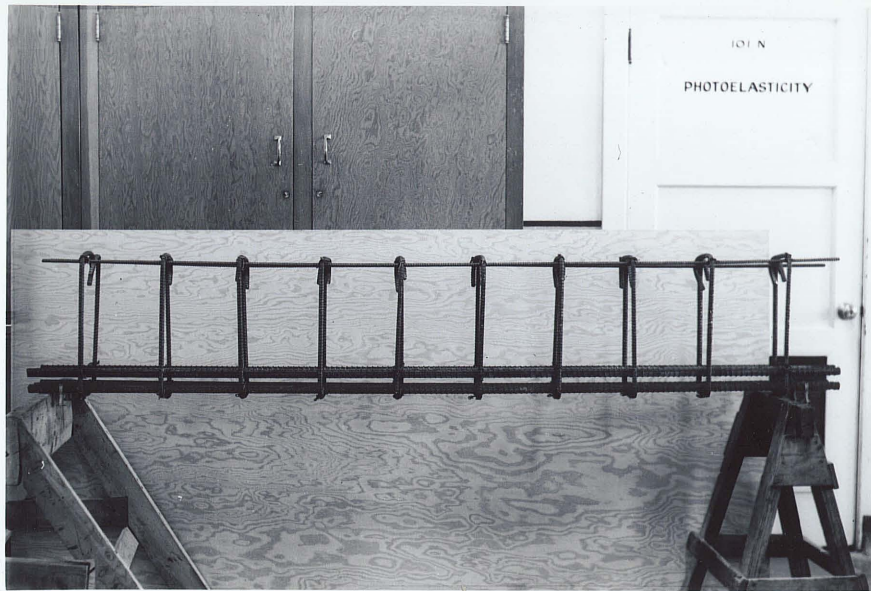


FIG. 13 - Reinforcement cage for Beam 1,  
#4 stirrups @ 9" c/c

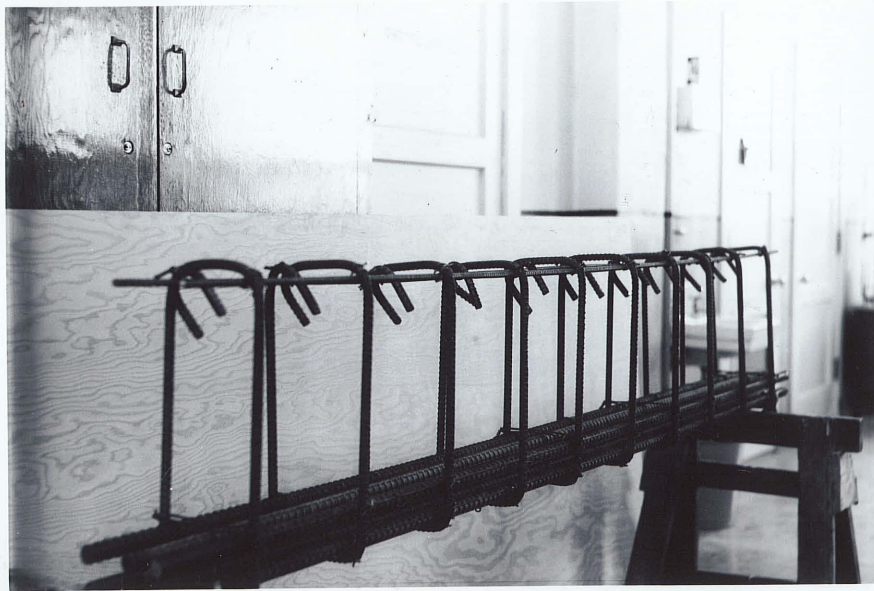


FIG. 14 - Side View of Reinforcement cage for Beam 1

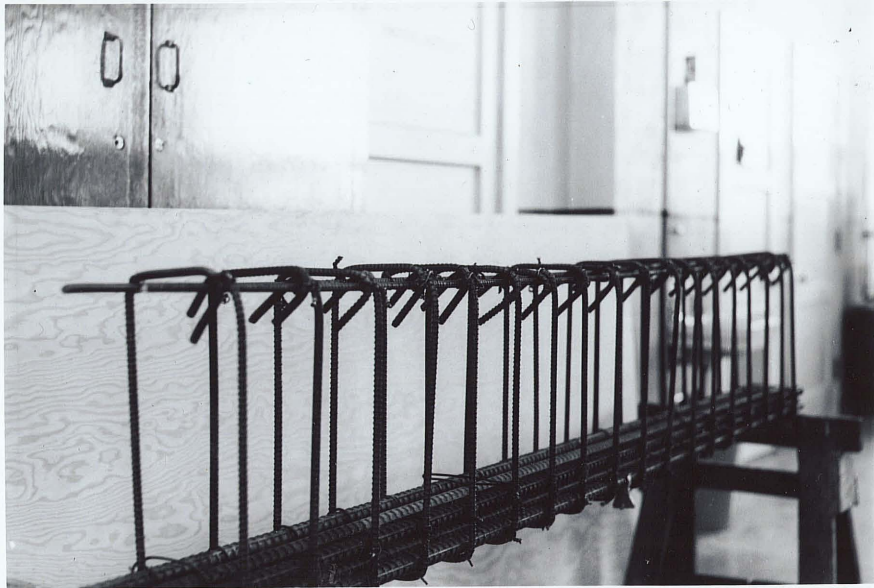


FIG. 15 - Reinforcement cage for Beam 2  
#3 stirrups @ 5" c/c

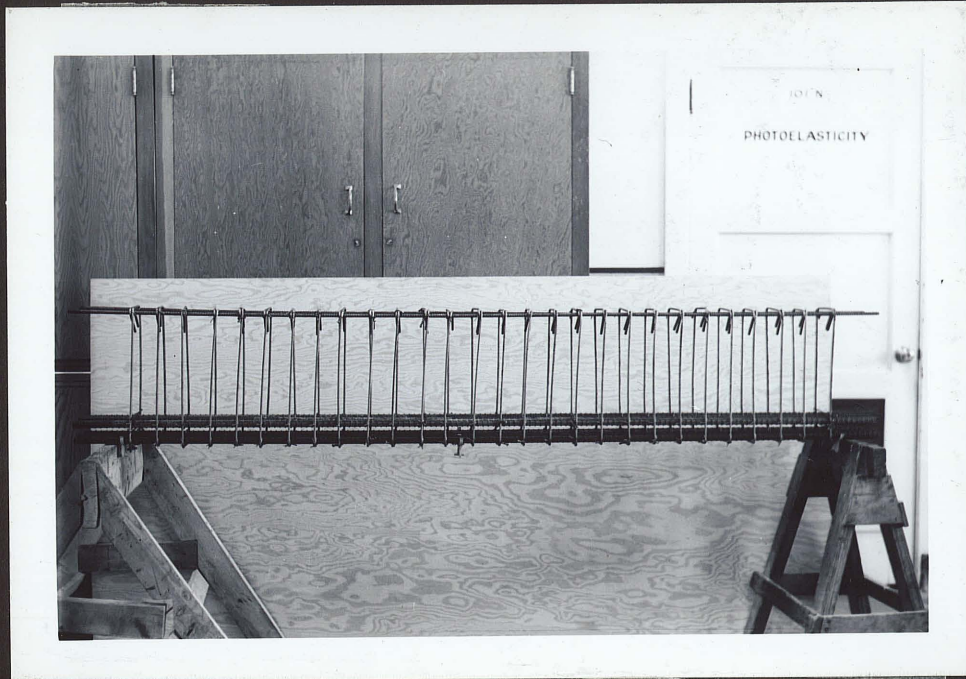


FIG. 16 - Reinforcement Cage for Beam 3

No. 2 stirrups @ 3" c/c



FIG. 17 - Reinforcement Cage for Beam 3

(Side View)

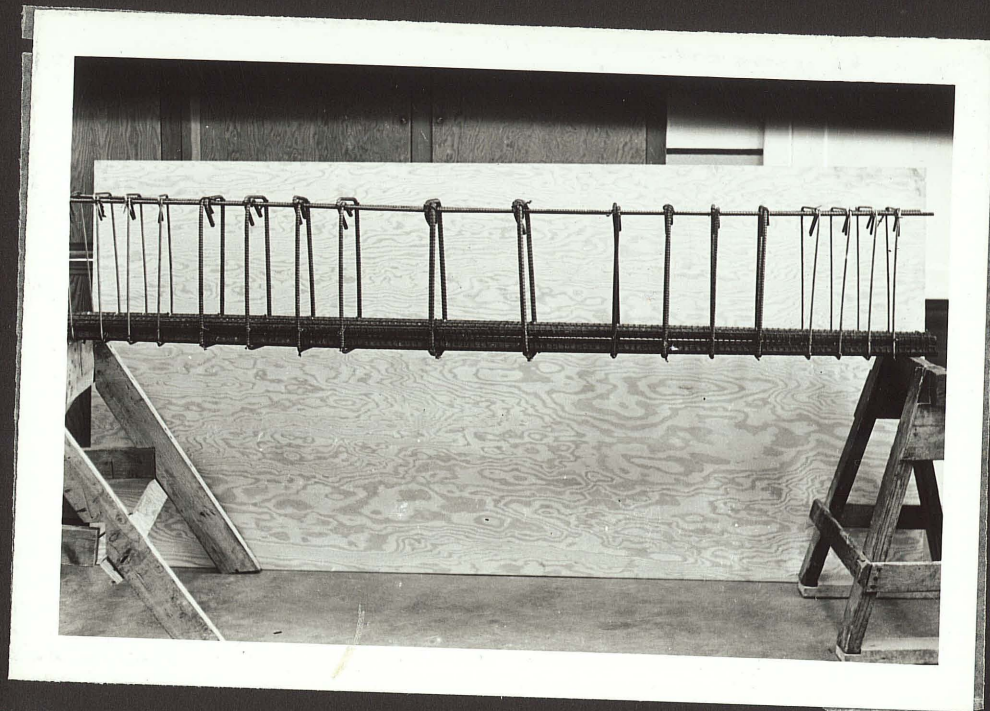


FIG. 18 - Reinforcement cage for Beam 4.  
#2, #3, and #4 stirrups

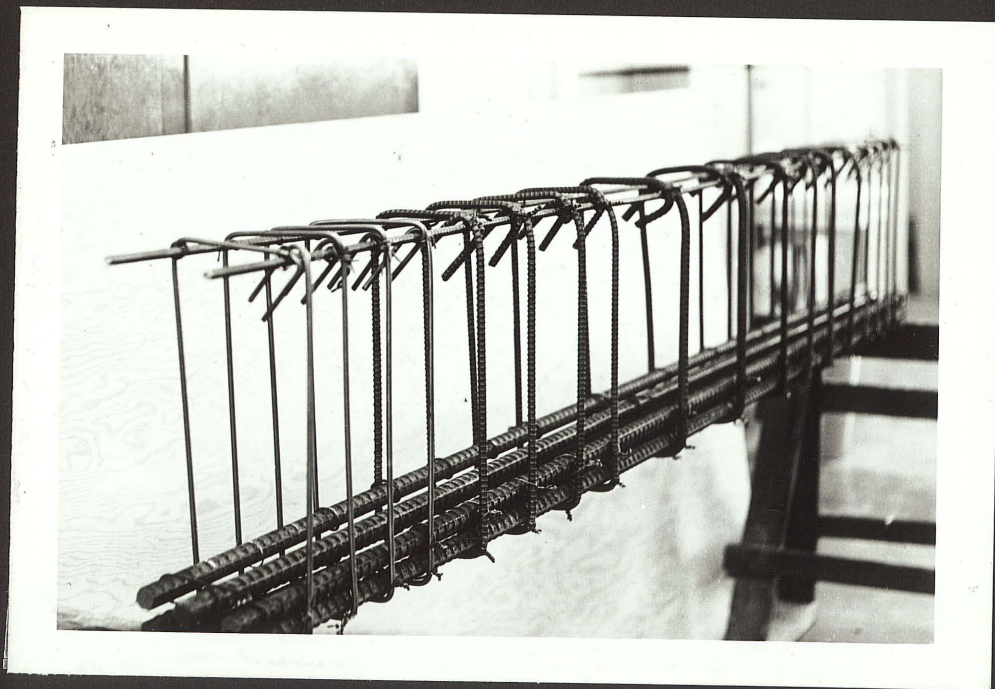


FIG. 19 - Side View of Reinforcement cage for Beam 4.

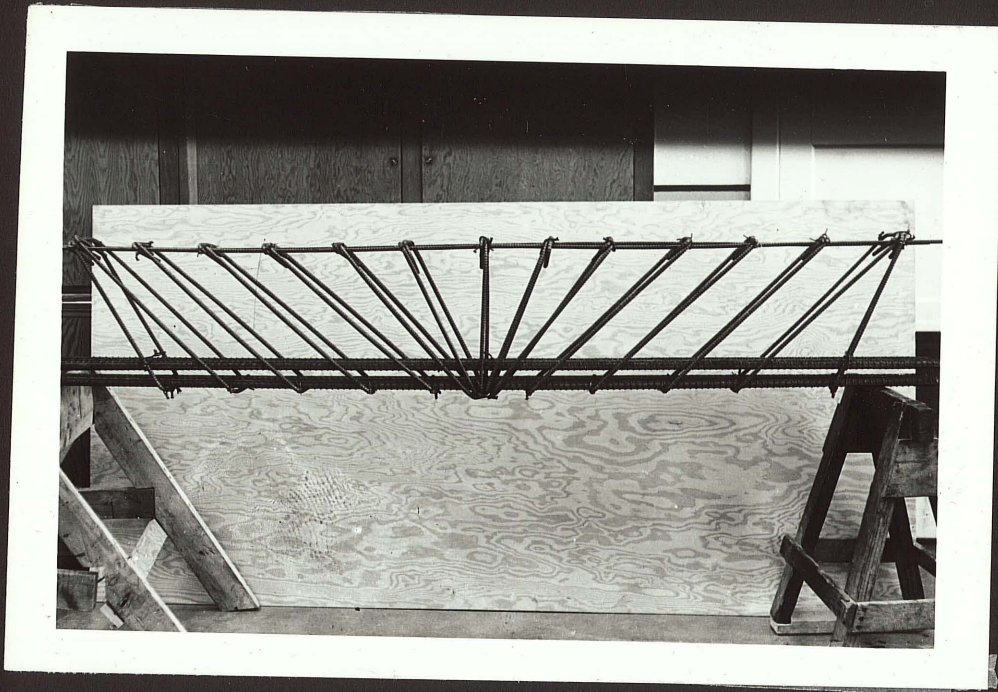


FIG. 20 - Reinforcement Cage for Beam 5

No. 3 stirrups

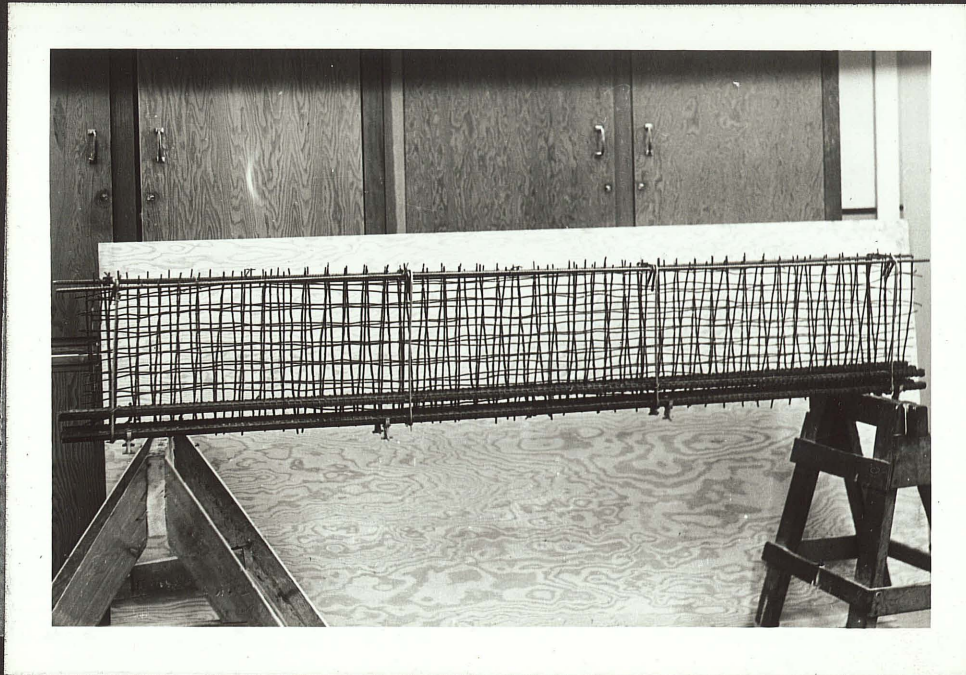


FIG. 21 - Reinforcement cage for Beam 6 (wire mesh)



FIG. 22 - Side View of Reinforcement cage for Beam 6

### 6.31 Design of the Beams

Since the failure of the specimens by other than diagonal tension was not desirable, all the beams were designed to fail in diagonal tension. Each beam had the theoretical flexural capacity more than 50% greater than its shear capacity. The estimated ultimate moments due to flexural failure were based on theoretical methods of ultimate strength design and were also checked by the ACI design methods. The estimated loads for shear failure were also checked by Clark's formula (2).

### 6.32 Materials

All the beams were made with ordinary portland cement. The specific gravity of the cement was 3.13. The specific gravity of coarse and fine aggregates was 2.70 and the specific gravity of sand was 2.66. The average fineness modulus of sand was 3.28, and the maximum size of aggregate was 1 in. Both aggregates passed the usual ASTM tests. Average properties of the concrete mixtures are given in Table 2. Actual concrete strengths are given in Table (4) Chapter 7.

The concrete mixtures were designed to attain their nominal strength of 3500 psi at the time of test after 28 days. Trial mixes were also used to keep the slump upto 2 inches. A greater slump was not considered desirable due to the use of vibrators.

TABLE 2 - Properties of Concrete

Beams	Av. mix ratio by wt. cement : sand : gravel	c/w ratio by wt.	Average slump	Nominal $f'_c$ (psi)
All	1 : 3.61 : 5.8	1.33	2 in.	3500

One practical problem encountered during casting concerned the sand and coarse aggregate which was used. The sand and coarse aggregate already stored in the lab was quite dry but as it was not enough, some

more materials were ordered. It had rained a few days before and the materials which arrived were wet. In order to keep the  $c/w$  ratio constant and hence the slump to be consistent, a weighed amount of sand and aggregate was dried in the oven at  $105^{\circ}\text{C}$  for three hours to determine the amount of moisture. Correspondingly the amount of water used for mixing was reduced for mixes using wet sand and aggregate.

Deformed bars of intermediate grade steel meeting ASTM Designation A305-50T were used for all longitudinal and web reinforcement except for No. 2 bars which were intermediate grade plain steel. These No. 2 bars seemed to be cold rolled and the subsequent tests confirmed this, but as no other bars of this size were available at the time of casting, these bars were used as shear reinforcement. The wires used as web reinforcement were also of plain steel.

As it was important to keep the effective steel ( $A_s f_y$ ) used for shear reinforcement as constant, the yield point of the reinforcement had to be obtained carefully. The physical properties of the bars were determined by conducting three tests on each size of the reinforcement. Physical properties of the reinforcement are given in Table 3. Graphs were also obtained during these tests.



TABLE - 3

PHYSICAL PROPERTIES OF THE REINFORCEMENT

Reinf. Size	Test No.	Yield Point		Ultimate			Elongation in 8", %	Average Elongation %	Remarks
		Load lbs	Stress psi	Av. Stress psi	Load lbs	Stress psi			
No. 7 Deformed	1	28,750	47,917		47,420	79,033	22.2		
	2	28,550	47,583	47,356	47,050	78,417	7.2*	22.6	* Fall outside the fracture zone and this reading is not considered for finding the average elongation
	3	28,000	46,667		46,670	77,783	23.0		
No. 4 Deformed	1	10,200	51,000		16,550	82,750	13.0*		
	2	10,000	50,000	50,500	15,350	76,750	18.5	19.7	
	3	10,100	50,500		15,650	78,250	21.0		
No. 3 Plain	1	6,380	58,000		9,020	82,000	17.8		
	2	6,220	56,545	57,151	8,900	80,909	14.0*	18.1	
	3	6,260	56,909		8,940	81,273	18.5		

TABLE - 3 (continued)

PHYSICAL PROPERTIES OF THE REINFORCEMENT

Reinf. Size	Test No.	Yield Point		Ultimate			Elongation in 8", %	Average Elongation %	Remarks
		Load lbs	Stress psi	Av. Stress psi	Load lbs	Stress psi			
No. 2 Plain	1	3,700	74,000		3,870	77,400			
	2	3,680	73,600	73,733	3,840	76,800	77,200	6.3	
	3	3,680	73,600		3,870	77,400			
0.156 in. dia. wire plain	1	480	25,200		805	42,100			
	2	505	26,400	25,700	800	41,900	42,033	22.3	
	3	485	25,500		805	42,100			
0.115 in dia. wire plain	1	410	39,400		630	60,500			
	2	385	37,000	38,130	585	56,200	57,970	24.0	
	3	395	38,000		595	57,210			

### 6.33 Fabrication of Specimens

The reinforcement for each beam was tied into a rigid cage as shown in Figs. 13 to 22. It was then placed in the plywood forms as shown in Fig. 23. The bottom bars were supported from the base of the form on plastic chairs at a height of 1 1/2 inches. All the forms were well oiled before casting the beams. The outer brackets shown in the plywood forms were tightly screwed at the top of the forms 1/16 in. less than 8 in. after the concrete had been poured.

The concrete was mixed in a tilting rotary mixer of 3 1/2 cu. ft. capacity by the side of the forms. External vibrators were used for vibrating the concrete in the forms. The vibrators were driven by an external motor. All the batching was done by weight. The concrete was chuted into the forms. After each batch had been placed in the form, the concrete mix was vibrated for about 1 1/2 minutes. The first three beams were cast on the same day. Eight mixes were required for these three beams. Beams 4 and 5 were cast on one day and Beam 6 was cast two days afterwards.

About two and one-half batches were required to fill up one form. Total time required for vibration for one beam was about 4 minutes. There was some difficulty in vibrating the mix for beam No. 6 due to wire-meshes. It required a little longer time for vibration, but it was ensured that the concrete had been fully vibrated within the cages. In most cases, the time of vibration was judged by the appearance of the concrete. Steel hangers were inserted on the two sides in the beam to facilitate transporting the beams to the testing machine. Care was exercised in choosing their locations so that they did not disturb the stirrups.

Forms were removed two days after casting. They were then sub-

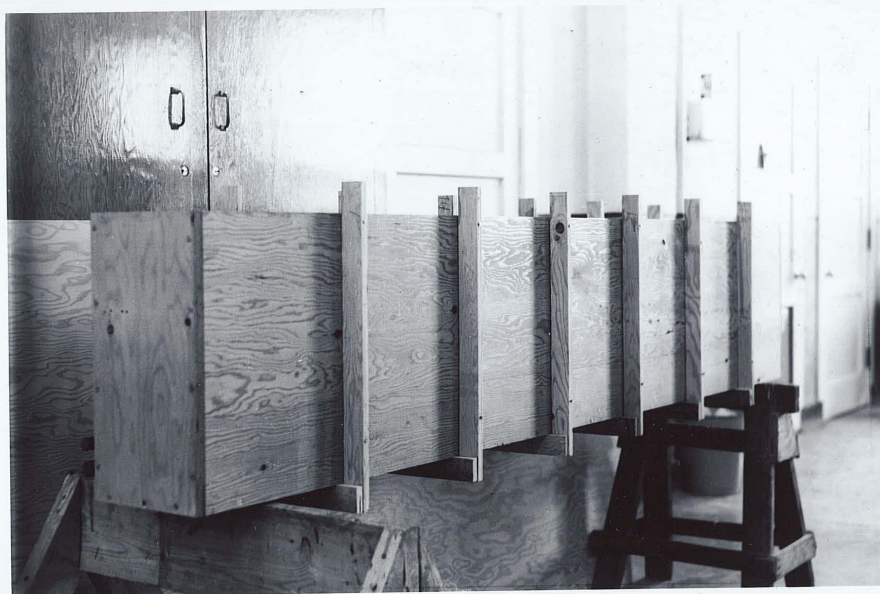


FIG. 23 - Plywood Forms 8" x 18", 8' long with 6 outer brackets equally spaced

jected to curing by placing them under wet burlap. The burlap was covered with building paper to prevent rapid evaporation of moisture. The burlap was moistened each 24 hours to prevent the beams from rapid drying. They were stored in the laboratory for 28 days prior to the testing.

Two standard 6 x 12 in. control cylinders were made from each batch of concrete. Most of the cylinders were cast in the steel forms, but some were also cast in the thick paper forms. The cylinders were cured and stored in an identical manner and tested on the same day as the corresponding beams, to determine the actual concrete strengths. The cylinders were capped with vitro bond which is a material made with tar and sulphur.

#### 6.34 Type of Loading

Third point loading was used for all the specimens. The supports were 7 ft. apart and the three spans were each 28 inches. The loading pattern is shown in Fig. 24.

#### 6.35 Designation of the Beams

The beams were marked at the top to show the beam No. All the four sides were marked as LS1, LS2, RS1 and RS2, the symbols LS and RS standing for left side and right side respectively. Date of casting and testing of the beams was also shown on the top right hand corners of the beams marked as RS1.

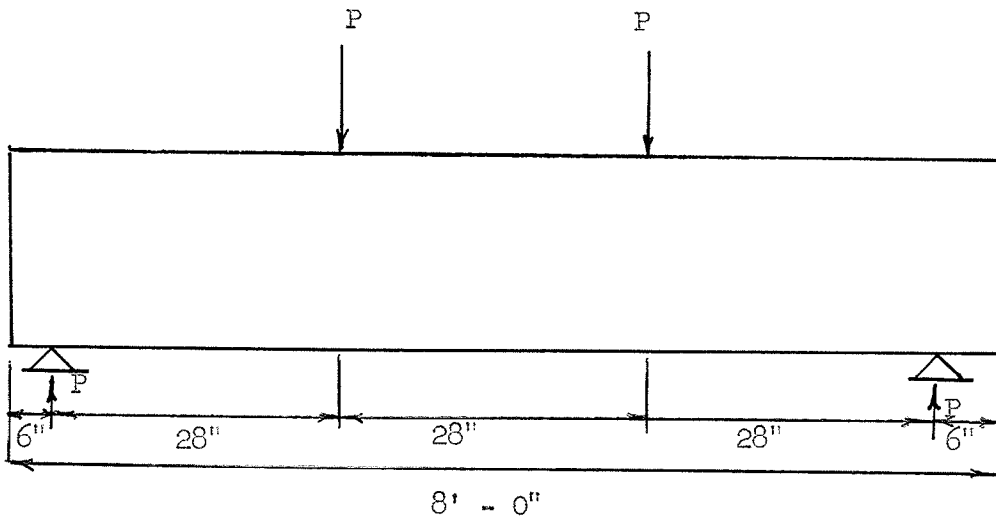


FIG. 24 LOADING PATTERN

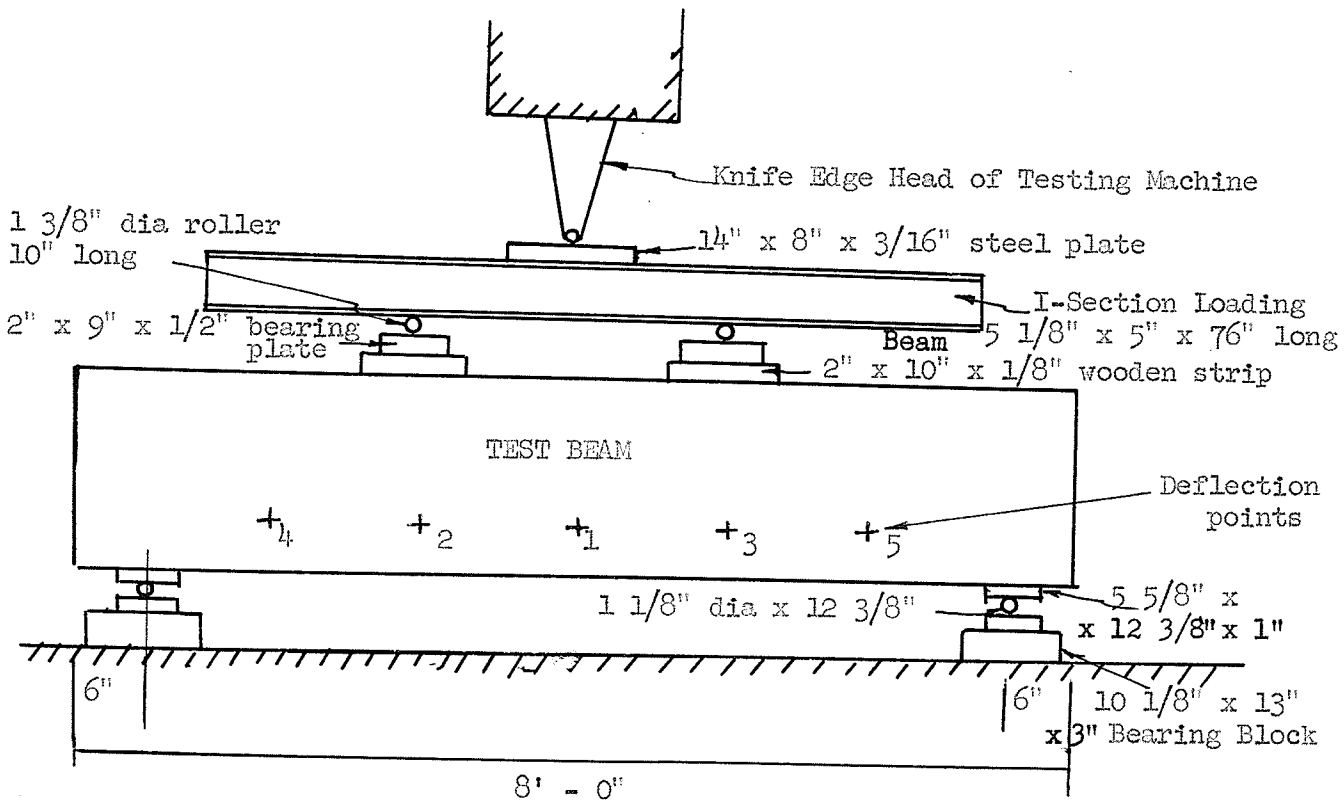


FIG. 25 SKETCH SHOWING TESTING ARRANGEMENT

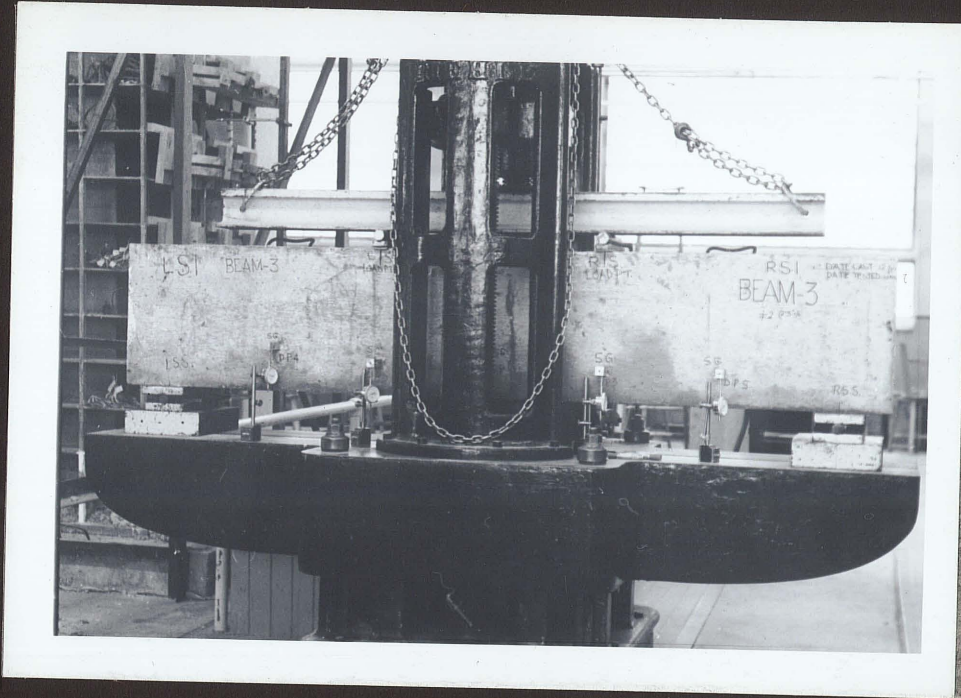


FIG. 26 - General Arrangement for Testing  
(North Side)

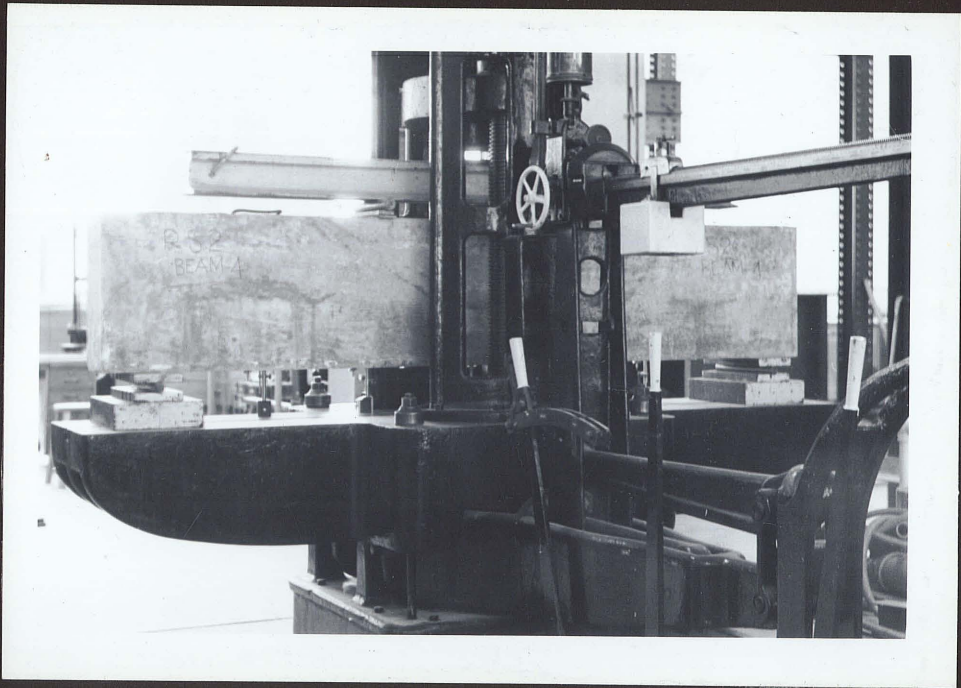


FIG. 27 - General Arrangement for Testing  
(South Side)

#### 6.4 Test Equipment and Procedure

All the beams were tested in a 200,000 pound capacity Riehle testing machine of the beam type. Details of the testing arrangement are shown in Figs. 25, 26, and 27. The load was applied at the third points of a 7 ft. span.

The load was usually applied in 5000-pound increments from zero load up to failure. After each increment the machine was operated intermittently to maintain the load constant while cracking and creep of concrete continued. After the load was stabilized, the deflections were measured, crack patterns noted and general observations were made. Deflections were measured at the midspan, at the load points and at the centers of end spans as shown by the dial gauges in Figs. 25 and 26. The dial gauges were graduated in 0.001" intervals.

The progress of cracking and phenomenon of failure was observed visually and photographs were taken during the time cracks were progressing or failure was taking place.

The dial gauges were marked as DPI, DP2, DP3, DP4, and DP5 as shown in Fig 26. The symbol DP signified deflection point. Point one was at the center, Points 2 and 3 were the left side and right side load points respectively and Points 4 and 5 were the mid points of left and right hand side spans respectively.

After the final failure of the beam, the load was sustained and the deflections at the mid point were recorded as the load dropped. As the deflections were very rapid, an exact reading was very difficult to obtain. During this time blocks of concrete chipped off and the reinforcement was visible. Further photographs were taken at this stage to record closely the effect of failure on the reinforcement.



## CHAPTER VII

### TEST RESULTS

#### 7.1 General

The results of the tests are summarized in this Chapter. This has been done with the help of tables, graphs, and photographs. The behavior of the beams before and after the formation of diagonal cracks is also described as witnessed during loading of the beams. Detailed description of the behavior of the beams as well as comparative results of tests with analysis are given in the next Chapter.

#### 7.2 Tests of the Reinforcement

Curves showing Load-Elongation relationships for the reinforcement are shown in Figs. 28 to 34. This includes No. 7, No. 4, No. 3, No. 2 bars; and 0.156" dia. and 0.115" dia. wires used as reinforcement. Fig. 32 shows the Load-Elongation curves for the new and old No. 2 plain reinforcement over an 8 inch length. The new reinforcement was used in the Beams 3 and 4 as the old reinforcement was not in sufficient quantity. As it can be seen that the No. 2 bars used in the Beams 3 and 4 were relatively brittle and probably cold rolled. As already mentioned in Chapter VI, bars of the 'old' type (see Fig. 32) were not available and hence relatively brittle No. 2 bars had to be used. A general idea of elongation and ductility can be obtained from these curves.

#### 7.3 Tests for Compressive Strength of Concrete

Tests were run on standard 6" x 12" cylinders and the results are given in Table 4. An average of three tests was considered necessary for each beam since there is always considerable variation in the results. Factors influencing the strength include the capping for the cylinders, exact diameter for the cylinders and the application of the load during

the compression test. The loading rate was 1/8 inches / minute.

Fig. 35 shows a test cylinder in the loading machine, whereas Fig. 36 shows the cracked cylinder after failure had occurred.

#### 7.4 Tensile Splitting Tests

Two tests were run for each beam to determine the tensile splitting strength of concrete. Results of these tests are given in Table 5.

The loading arrangement for this test is shown in Fig. 37, and a broken specimen is shown in Fig. 38. The results of these tests show a fairly close range except for the first beam and it may be concluded that tensile splitting tests are a better guide for the strength of concrete, there being a greater variation in the compressive strength test results as well as there being a greater possibility of the specimen being sensitive to capping arrangement and exact dimensions. Also, no capping is required for tensile splitting tests. The average tensile splitting strength obtained varies from 390 to 480 psi.

#### 7.5 Results of Beam Tests

The results of the six beams are given in Tables 6 to 16.

Both the first cracking load ( $P_c$ ) and the ultimate load ( $P_u$ ) at failure were measured by the testing machine. The first cracking loads were the loads at the formation of initial diagonal tension cracks, and were based on the visual observations in the tests.

The deflections were recorded at five points along the beam until failure occurred. However, it was not possible to record the deflections at all the points after failure had occurred, due to the continued rapid deflections and difficulty in obtaining a correct reading. Readings for deflection were thus only taken at mid-span after failure except for beam 6 where deflections were not so rapid and readings could be taken at all five points for some time.

TYPICAL LOAD-ELONGATION CURVE FOR NO. 7 BARS

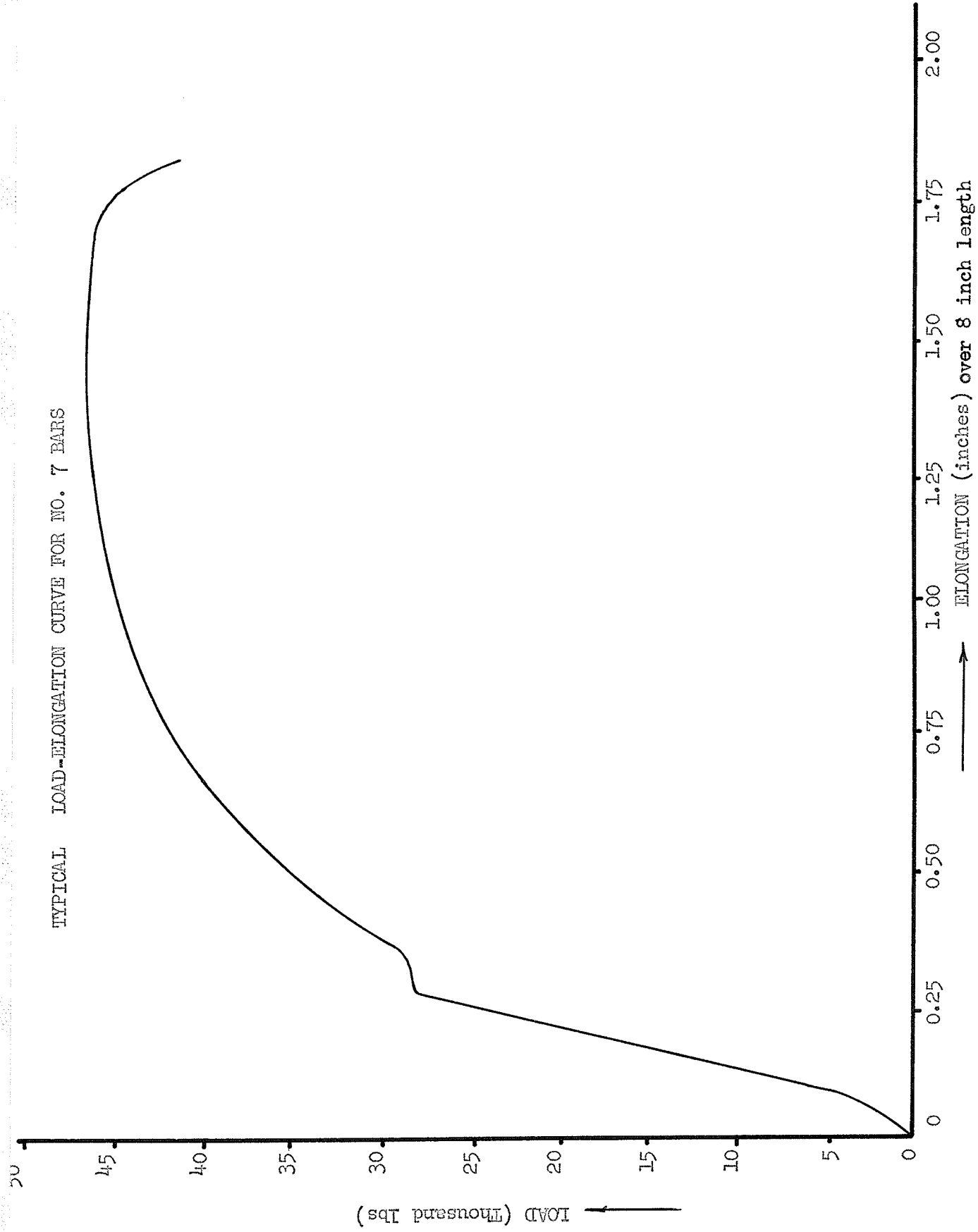


FIG. 28

TYPICAL  
LOAD-ELONGATION CURVE FOR NO. 4 BARS

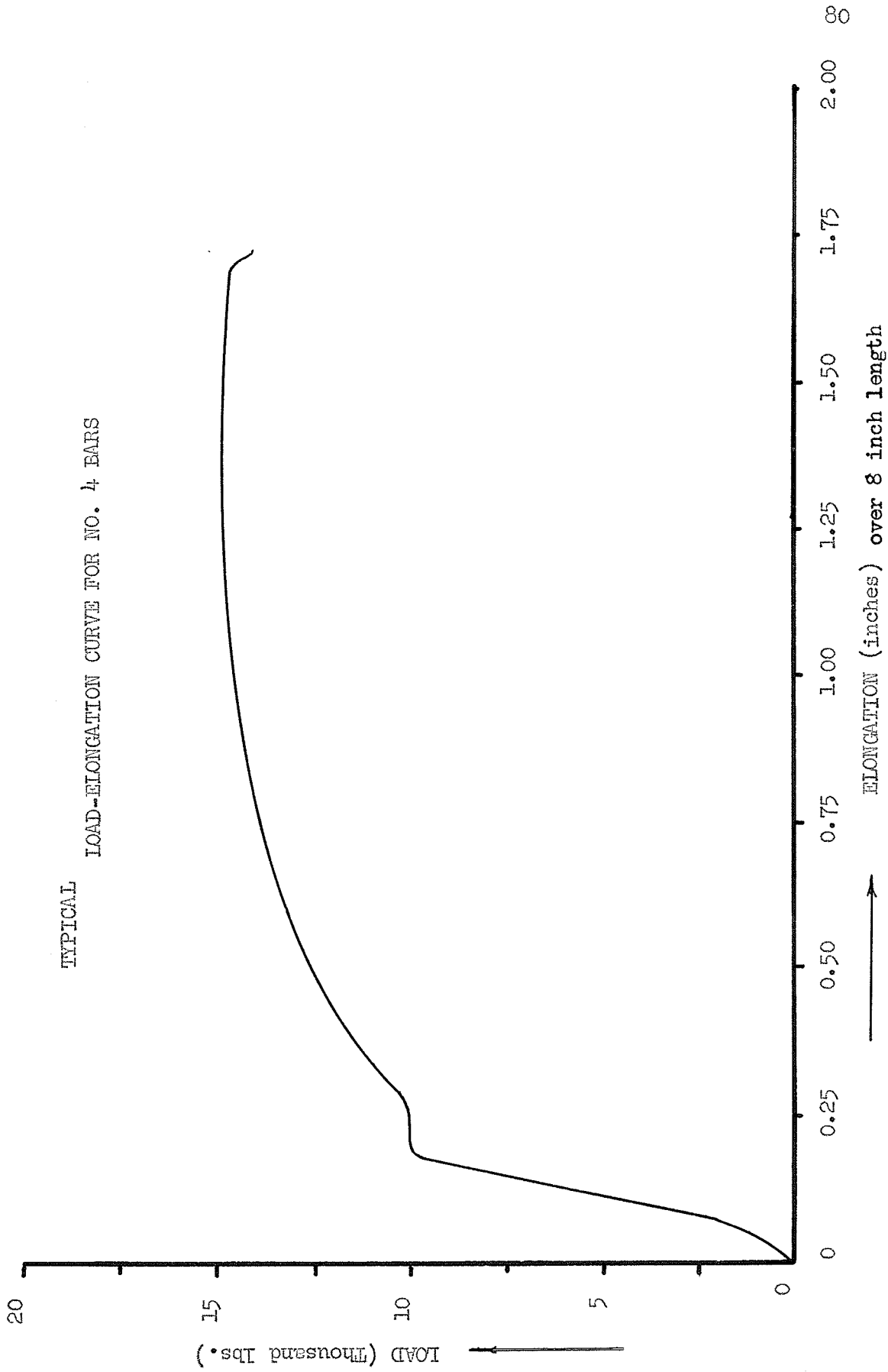


FIG. 29

TYPICAL

LOAD-ELONGATION CURVE FOR NO. 3 BARS

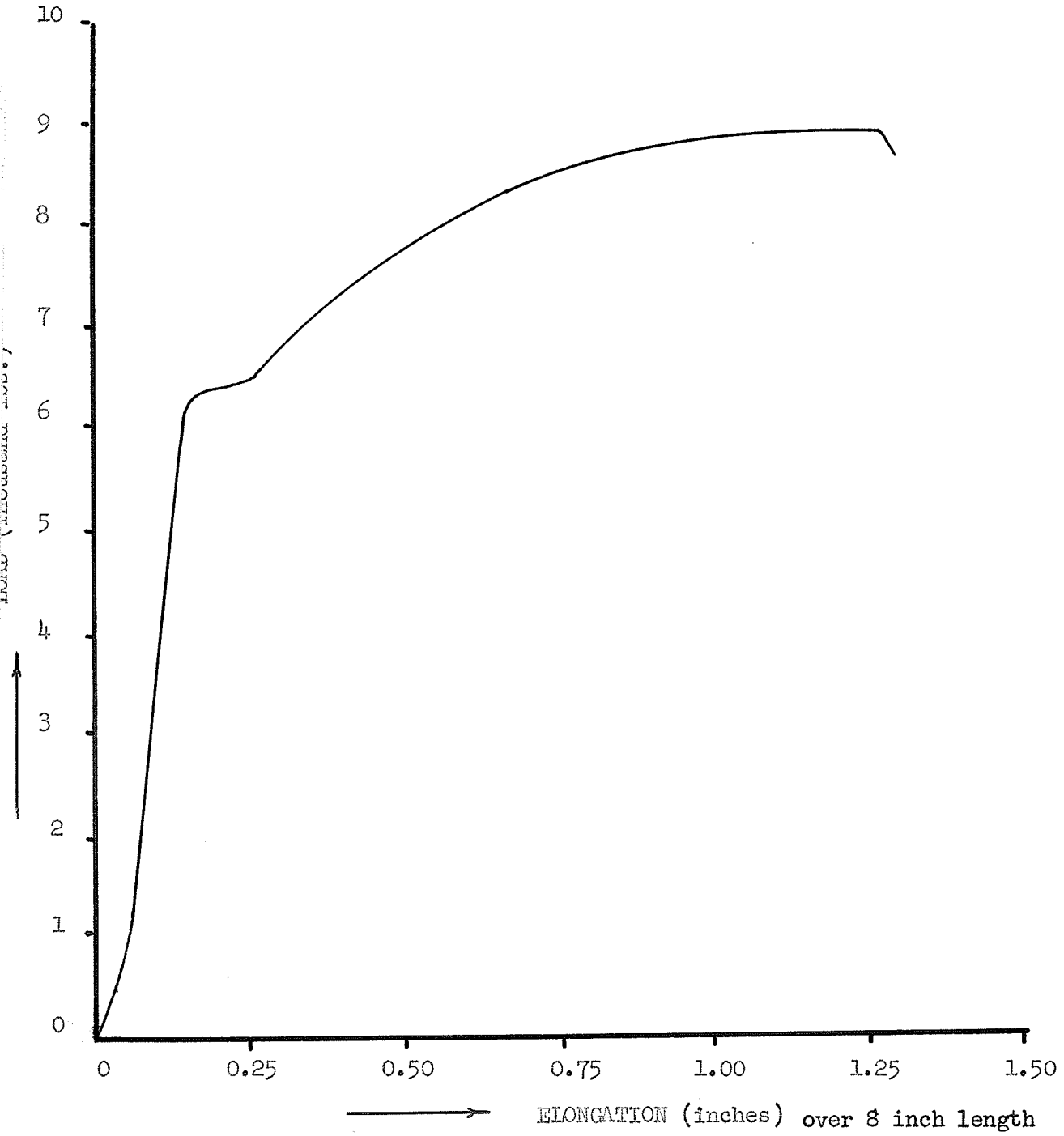


FIG. 30

TYPICAL

LOAD-ELONGATION CURVE FOR NO. 2 PLAIN BARS

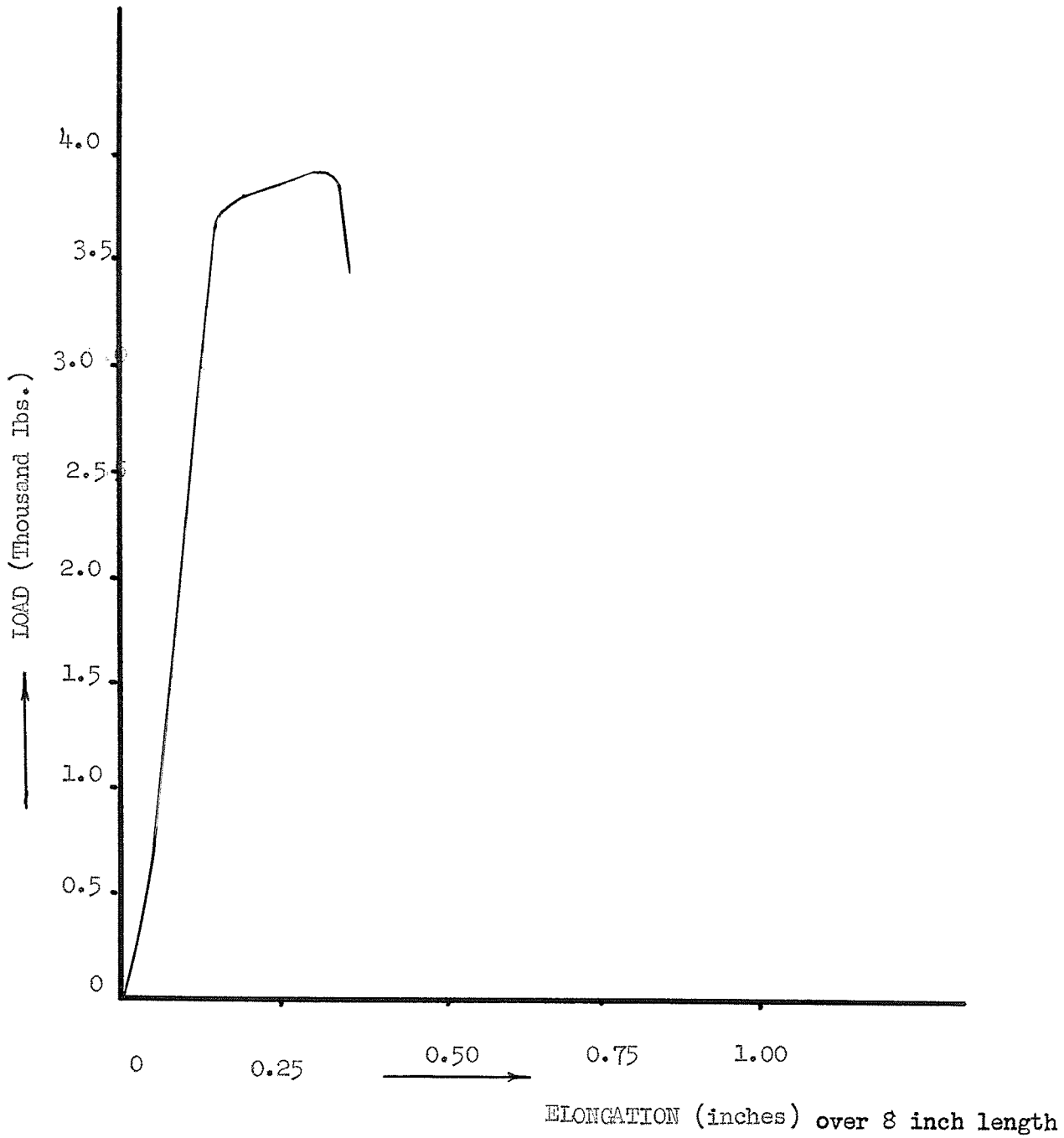


FIG. 31

COMPARITIVE TEST OF #2 PLAIN REINFORCEMENT - OLD & NEW

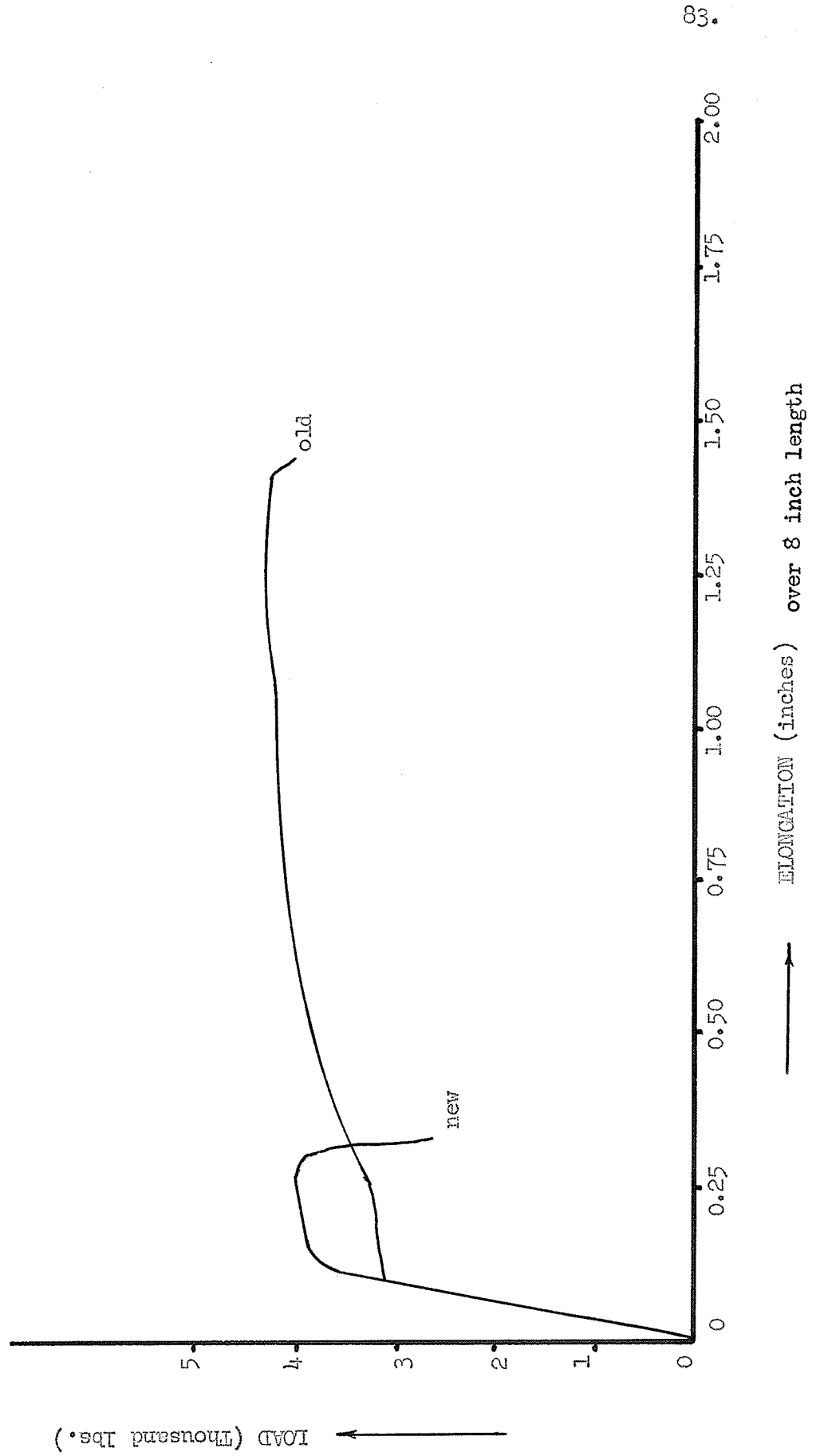


FIG. 32

TYPICAL

LOAD-ELONGATION CURVE FOR 0.156" DIA. WIRE

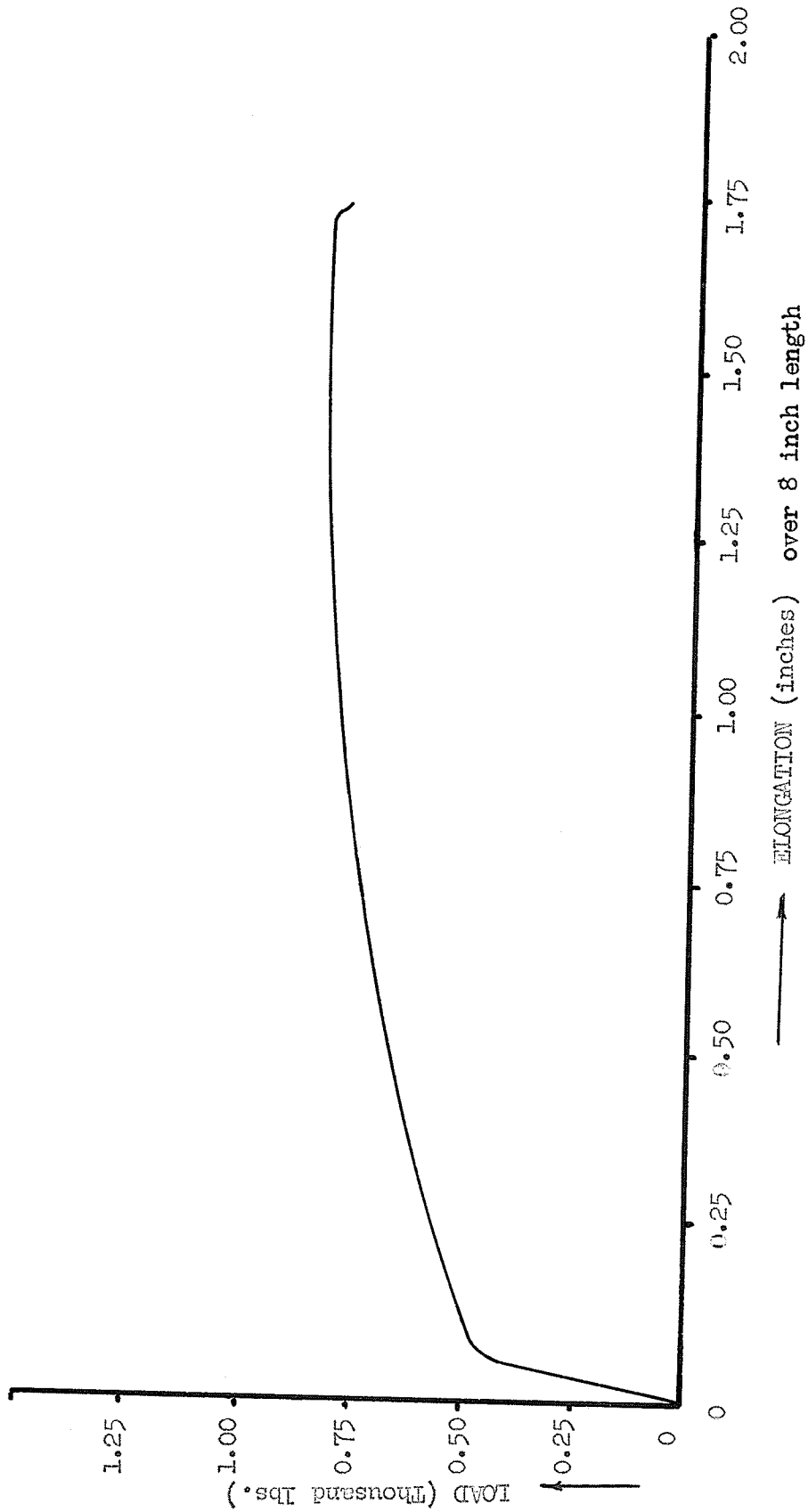


FIG. 33



TYPICAL

LOAD-ELONGATION CURVE FOR 0.116" DIA. WIRE

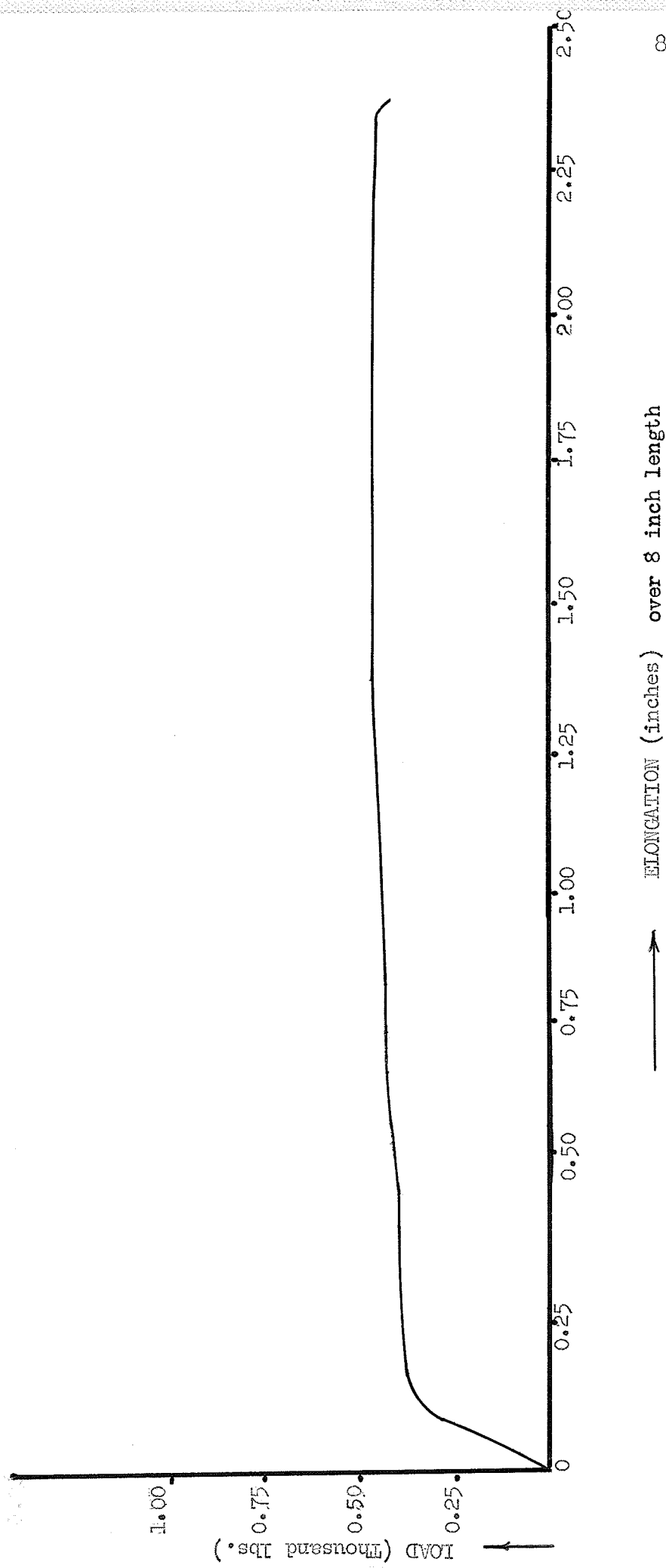


FIG. 34



FIG. 35 - Test Cylinder before Application of the Load



FIG. 36 - Compression Failure of the Cylinder

TABLE - 4

CYLINDER TESTS (Compressive Strength)  
(6" x 12" cylinders)

Beam No.	Test No.	Total load (pounds)	Comp. stress (psi)	Av. $f'_c$ (psi) <sup>c</sup>
1	1	95,500	3,340	3,430
	2	91,500	3,200	
	3	107,500	3,760	
2	1	108,000	3,780	4,110
	2	116,000	4,050	
	3	129,000	4,500	
3	1	109,500	3,830	4,230
	2	121,000	4,240	
	3	132,000	4,610	
4	1	99,000	3,460	3,680
	2	105,000	3,670	
	3	112,000	3,920	
5	1	117,500	4,100	4,000
	2	117,000	4,090	
	3	112,500	3,920	
6	1	111,000	3,850	3,680
	2	101,500	3,550	
	3	104,000	3,640	



FIG. 37 - Arrangement for Tensile Splitting Test

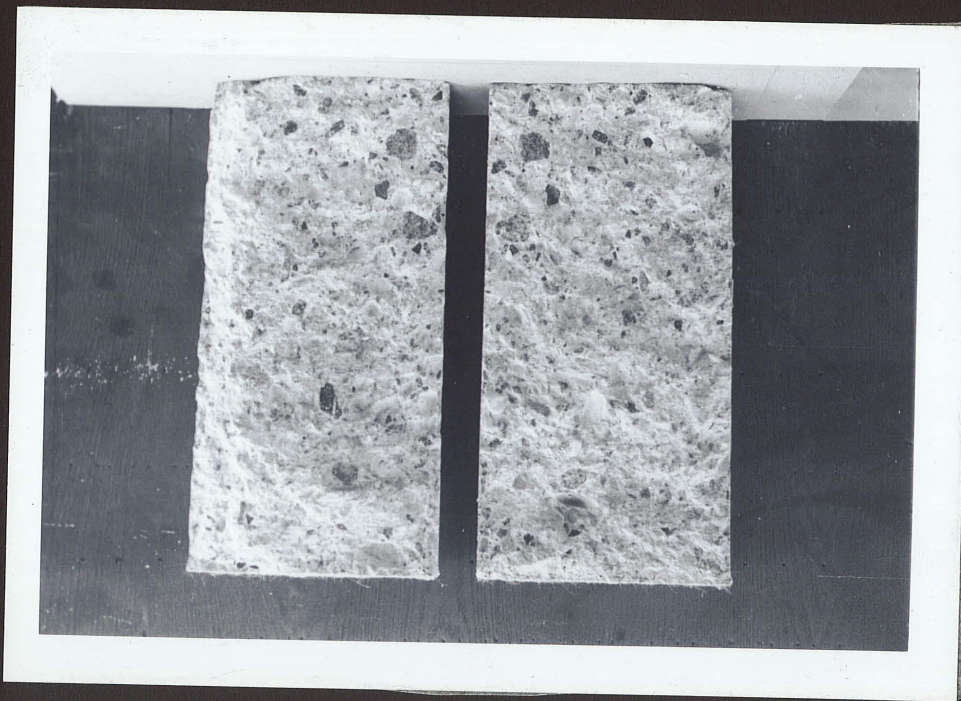


FIG. 38 - Broken Cylinder after Tensile Splitting Test

TABLE - 5

## CYLINDER TESTS (Tensile Splitting Tests)

Beam No.	Test No.	Total Load (Pounds)	$f_t' = \frac{2P}{\pi dl}$ (psi)	Av. $f_t'$ (psi)
1	1	43,700	387	390
	2	44,500	394	
2	1	54,500	482	480
	2	54,100	479	
3	1	50,900	450	447
	2	50,300	445	
4	1	52,900	468	467
	2	52,700	466	
5	1	51,750	457	471
	2	54,900	485	
6	1	50,600	449	471
	2	55,600	493	

\*  $d = 6''$ ,  $l = 12''$

$$\therefore \frac{\pi dl}{2} = 113 \text{ \& } f_t' = \frac{P}{113}$$

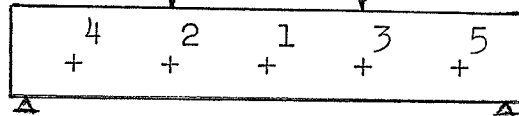
Beam 1 was cured in the laboratory where there was possibly some small irregularity on the floor. Though the beams were supported by steel weights from the floor, there appeared a very small twist in the first beam. Two small wooden strips were placed on each support under the beams to make the beam perfectly horizontal under the loading. As the wood was crushed during the loading, the deflections obtained for this beam were incorrect. Subsequently, a similar piece of wood was tested in the machine and deflections recorded. These deflections were subtracted from the original readings to obtain the values given in Table 6. However, these values vary considerably from other beam tests and these values cannot be taken as absolutely correct due to the method of approach involved and due to the possibility of non-uniform compression of wood during loading of the beam. In fact possibilities exist that a considerable deflection of the wood occurred in the initial stages due to the hard edge of the beam and due to the probable twist of the beam being counterbalanced. Deflections of beam 1 should thus be accepted with some reservation.

The load-deflection curves for these beams have been drawn only for the mid-span deflections in Figs. 39 to 44. The curves for other deflection points were not drawn because these curves followed very closely the curves for mid-span and it was difficult to differentiate the different curves within the scale used. However, some comments will be made, whenever required, during the analysis of the results and a direct reference will be made to Tables 6 - 16 giving deflections.

The progress of the cracking, crack pattern of the beams and close views of the beams after failure occurred are given in photographs shown in Figs. 45 to 74.

TABLE - 6

## RESULTS OF TESTS - BEAM 1



Date Cast - July 8, 1967

Date Tested - Aug. 9, 1967

Zero Set at 2,000 lbs.

LOAD Thousand lbs	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
5	8	7	8	6	7	
10	28	25	27	18	19	
15	50	45	48	33	35	
20	69	62	65	44	46	
25	88	79	84	55	58	
30	107	96	103	71	75	
35	133	123	128	90	96	
40	163	154	157	114	115	
45	197	189	187	146	145	
50	231	223	218	172	170	
55	268	260	252	199	197	
60	301	293	285	223	220	Appearance of first crack on L.S.1-L.S.2
65	331	322	314	237	234	
70	347	334	327	246	247	
75	358	345	338	253	247	
80	368	355	347	260	254	

TABLE - 6 (continued)

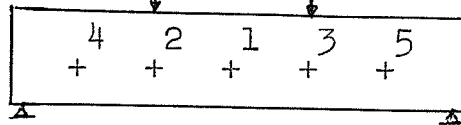
## RESULTS OF TESTS - BEAM 1

LOAD Thousand lbs	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
85	377	364	358	267	260	
90	387	374	369	272	268	
95	396	383	379	279	275	
100	406	392	388	286	282	
105	416	402	398	295	291	
110	426	412	408	304	300	
115	436	421	417	313	308	
120	447	436	430	330	325	
125	461	448	441	343	339	Cracks widen
130	475	462	456	356	352	
135	491	477	470	370	366	
140	514	499	489	387	384	
145	546	524	516	402	398	
150	583	557	550	423	419	
151.2	592	570	561	433	428	Failure occurs on L.S.1-L.S.2



TABLE - 7

## RESULTS OF TESTS - BEAM 2



Date Cast - July 8, 1967

Date Tested - Aug. 9, 1967

Zero Set at 2,000 lbs.

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
5	8	8	7	6	5	
10	21	21	20	19	15	
15	31	31	29	25	21	
20	41	39	38	31	26	
25	52	50	47	38	32	
30	62	58	55	44	38	
35	73	69	66	50	44	
40	83	78	74	55	50	
45	95	88	84	62	56	
50	108	100	96	68	63	
55	120	111	107	75	70	Initial cracks appear on R.S.2
60	131	122	118	81	76	
65	147	137	132	91	84	Small cracks on side L.S.1
70	161	150	144	100	91	Small cracks on L.S.2
75	176	163	157	108	100	Initial cracks on R.S.1

TABLE - 7 (continued)

## RESULTS OF TESTS - BEAM 2

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
80	191	176	169	116	109	
85	205	189	182	125	118	First crack at center
90	218	201	194	132	126	
95	232	214	206	140	133	
100	245	226	218	147	141	
105	259	239	231	155	149	
110	274	253	245	163	156	
115	290	267	259	172	165	
120	305	281	272	181	174	
125	321	296	286	191	183	
130	338	311	301	201	193	
135	356	330	317	210	203	
140	374	344	334	222	213	
145	394	362	352	235	225	
150	418	385	373	250	240	
155	456	417	406	273	263	
160	504	460	453	298	295	Deflection continues
161.17	629	580	571	355	349	Failure (Max. load) Final Failure on R.S.1-R.S.2

TABLE - 8

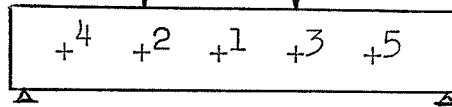
## BEAM - 2

MIDSPAN DEFLECTION AFTER FAILURE

Load (thousand lbs)	Midspan Deflection ( $10^{-3}$ inches)	Remarks
161.17	629	Max. failure load, deflection continues.
130	676	
120	801	
115	844	
110	914	
105	1014	
100	1079	
95	1244	Further readings not possible due to rapid deflections.

TABLE - 9

## RESULTS OF TESTS - BEAM 3



Date Cast - July 8, 1967

Date Tested - August 10, 1967

Zero Set at 2,000 lbs.

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
5	8	8	8	7	7	
10	22	21	20	18	16	
15	32	31	28	25	22	
20	42	40	37	32	27	
25	52	49	45	39	32	
30	62	59	54	45	38	
35	72	68	63	51	44	
40	85	79	73	58	50	
45	96	89	83	65	57	
50	109	100	94	71	63	
55	122	112	107	79	72	First crack on R.S.1
60	136	125	120	87	79	Cracks appear on all sides.
65	149	138	132	96	84	
70	163	151	144	105	90	
75	180	165	158	122	96	

TABLE - 9 (Continued)RESULTS OF TESTS - BEAM 3

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
80	194	178	170	135	103	
85	207	190	182	145	110	
90	222	203	196	157	119	
95	237	217	209	170	128	
100	251	231	223	182	137	
105	265	243	235	194	144	
110	281	257	250	209	153	
115	296	270	262	221	161	
120	311	283	276	233	170	
125	329	299	292	245	180	
130	345	314	307	256	189	
135	361	329	323	269	198	
140	378	344	338	281	208	
145	397	361	355	294	219	
150	418	379	375	310	231	
155	443	401	400	327	246	
160	480	433	438	349	279	Failure on R.S. 1 and R.S. x-section

TABLE - 10

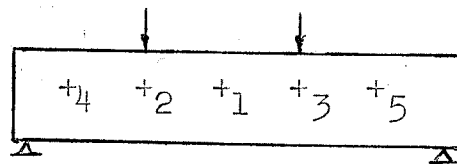
## BEAM - 3

MIDSPAN DEFLECTION AFTER FAILURE

Load (thousand lbs)	Midspan Deflection ( $10^{-3}$ inches)	Remarks
160	480	Failure load
64	524	
63	610	
61	700	
60	705	
59	725	
		Further readings not possible as I-beam edges touch the top of the beam

TABLE - 11

## RESULTS OF TESTS - BEAM 4



Date Cast - July 12, 1967

Date Tested - August 11, 1967

Zero set at 2,000 lbs.

LOAD: Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
5	5	5	5	4	4	
10	16	15	15	10	12	
15	28	25	26	17	21	
20	39	34	37	23	30	
25	51	45	49	30	39	
30	63	55	59	36	49	
35	74	65	71	42	57	
40	87	77	84	49	66	
45	100	88	96	55	75	
50	116	103	110	61	85	Initial cracks on L.S.1 and R.S.2
55	131	115	124	66	95	L.S.2 shows cracks
60	145	128	137	73	103	
65	160	142	151	84	103	First crack on R.S.1
70	174	155	164	92	122	

TABLE - 11 (continued)

## RESULTS OF TESTS - BEAM 4

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
75	189	168	177	101	131	
80	203	180	190	108	140	
85	218	194	204	116	150	
90	232	206	216	122	160	
95	246	218	229	130	169	
100	259	230	241	137	178	
105	274	244	255	144	187	
110	288	256	267	151	194	
115	304	271	282	159	204	
120	319	284	295	167	213	
125	335	298	309	175	222	
130	350	313	324	184	231	
135	367	328	339	194	241	
140	382	341	351	202	249	
145	401	359	368	214	261	
150	422	380	387	228	274	
155	451	408	412	249	290	
160	481	439	441	270	310	
165	524	487	478	302	335	Deflection continues to increase and final failure results



TABLE - 12

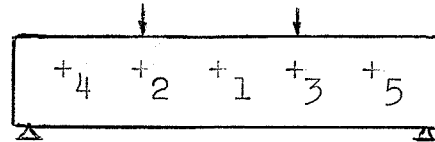
BEAM - 4MIDSPAN DEFLECTION AFTER FAILURE

Load (thousand lbs)	Midspan deflection ( $10^{-3}$ inches)	Remarks
165	524	Failure load
128	660	
122	702	
115.5	777	
116.3	850	
110	927	
111	970	
110.6	1025	Deflection continues at about the same load
109.9	1105	
108.5	1170	
107.5	1230	
106.3	1275	
104	1345	
103	1372	
101	1435	
101	1460	
101	1490	*

\* Further readings not possible as loading beam starts touching top of the beam.

TABLE - 13

## RESULTS OF TESTS - BEAM 5



Date Cast - July 12, 1967

Date Tested - August 11, 1967

Zero Set at 2,000 lbs.

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
5	12	11	12	9	9	
10	29	28	28	21	22	
15	46	44	43	33	35	
20	57	55	54	42	43	
25	69	66	65	51	50	
30	78	76	74	58	56	
35	87	86	84	65	63	
40	99	95	94	71	68	
45	112	107	106	79	76	
50	122	117	115	86	82	
55	134	128	126	93	89	
60	145	139	136	100	95	
65	157	150	147	100	101	First crack on all sides at bottom

TABLE - 13 (Continued)

## RESULTS OF TESTS - BEAM 5

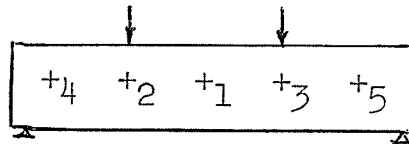
LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
70	168	161	157	118	108	
75	180	173	168	127	115	
80	192	184	180	135	122	First diagonal crack on R.S.1, following stirrup pattern
85	205	196	191	143	129	
90	217	207	203	151	137	
95	229	218	213	158	143	
100	243	231	226	167	151	
105	255	242	236	174	158	
110	266	253	247	181	165	
115	280	265	259	190	173	
120	293	278	271	198	180	Still good behavior
125	307	291	284	206	188	
130	320	303	296	214	195	
135	335	317	310	224	204	Cracks open on L.S.1
140	353	333	327	235	214	
145	368	348	340	245	223	
150	387	365	357	258	234	beam in fairly good shape
155	409	386	377	270	247	
160	436	411	401	285	262	
165	505	470	459	310	294	Final failure by flexure on sides L.S. 1 - and L.S.2

TABLE - 14BEAM - 5MIDSPAN DEFLECTION AFTER FAILURE

Load (thousand lbs)	Midspan deflection ( $10^{-3}$ inches)	Remarks
165	505	Failure load
154	511	
162.5	526	
157	566	
156.2	614	
155.6	641	
146	696	
143	736	
137	786	
133	836	
127	903	
127	942	Further readings not possible

TABLE - 15

## RESULTS OF TESTS - BEAM 6



Date Cast - July 18, 1967

Date Tested - August 14, 1967

Zero Set at 2,000 lbs.

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
5	13	13	13	12	12	
10	31	31	30	28	26	
15	47	47	46	43	38	
20	62	62	60	55	49	
25	74	74	71	64	57	
30	88	87	83	73	65	
35	100	98	93	82	72	
40	112	110	104	90	79	
45	125	122	116	98	87	
50	139	135	128	107	96	Diagonal crack on L.S.1
55	155	150	143	115	108	Cracks appear on other sides
60	169	163	156	122	117	

TABLE - 15 (Continued)RESULTS OF TESTS - BEAM 6

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
65	184	176	170	129	128	
70	198	190	181	135	138	Cracks appear at center
75	213	205	195	144	148	
80	228	219	208	153	158	
85	242	232	220	161	167	
90	256	245	233	170	176	
95	271	259	247	180	186	Main crack widens
100	289	275	264	191	199	
105	307	292	281	202	212	
110	325	309	298	213	225	
115	343	325	316	224	239	
120	367	345	342	237	264	Max. load

TABLE - 16BEAM - 6BEHAVIOR AFTER FAILURE

LOAD Thousand pounds	DEFLECTION ( $10^{-3}$ inches) AT POINTS					REMARKS
	1	2	3	4	5	
120	367	345	342	237	264	Deflection at failure
69	468	380	409	260	350	
68	510	415	465	263	420	
66	535	435	500	270	454	
65.5	575	460	565	280	(excessive deflections)	Main shear crack widens
61.6	613	485	620	290		
58.8	635	510	625	300		
59.3	704	540	73.4	315		
59.7	754	570	800	330		
59.7	790	610	850	450		
59.5	830	630	895	460		
56.6	850	680	(excessive deflections)	490		Beam deflects considerably on R.S.1 - R.S. 2
57.2	897	715		510		
57.2	940	745		525		
58.1	980	770		535		
57.9	1025	800		550		Very large deflection of reinf. cage

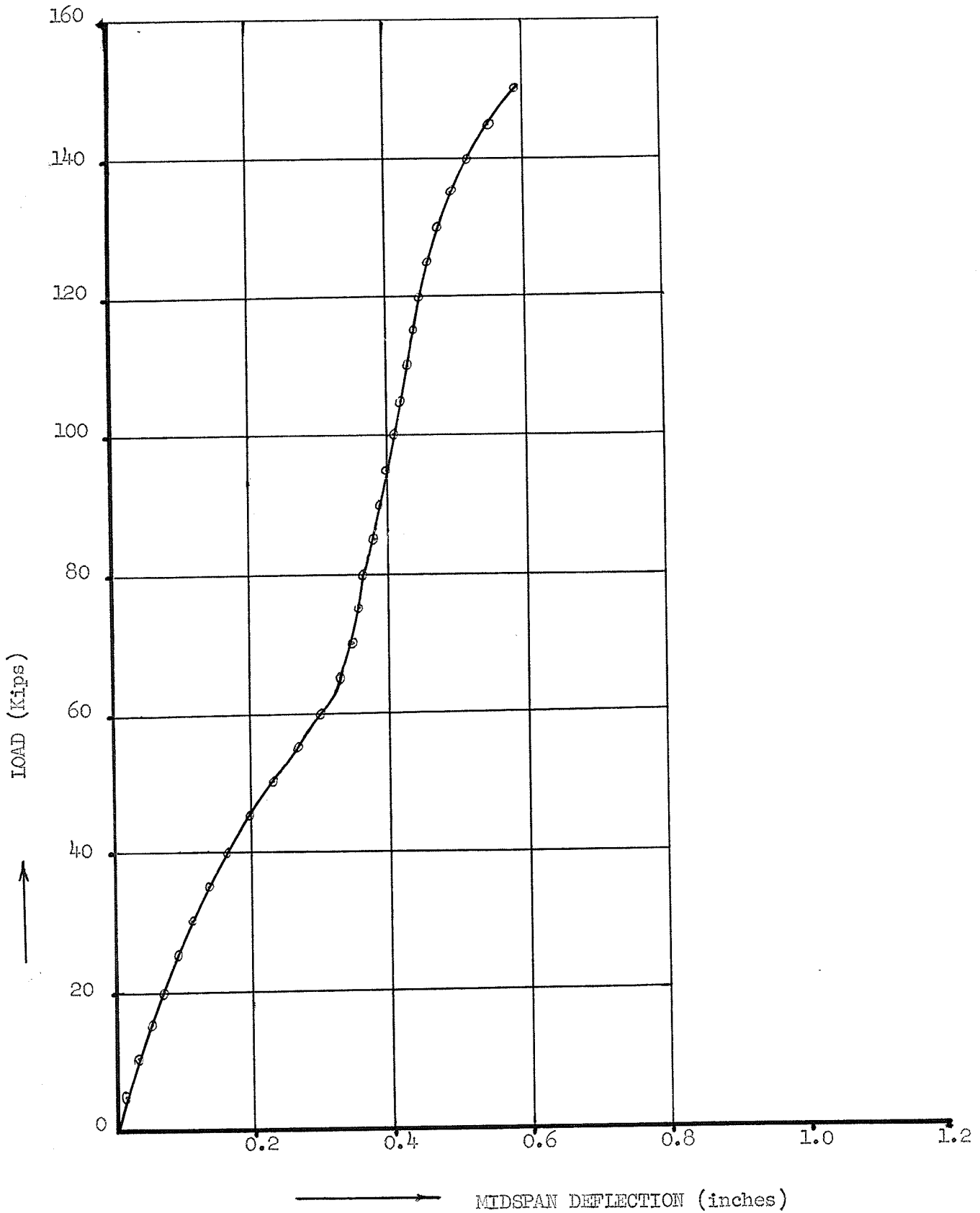


FIG 39 LOAD-DEFLECTION CURVE FOR BEAM 1



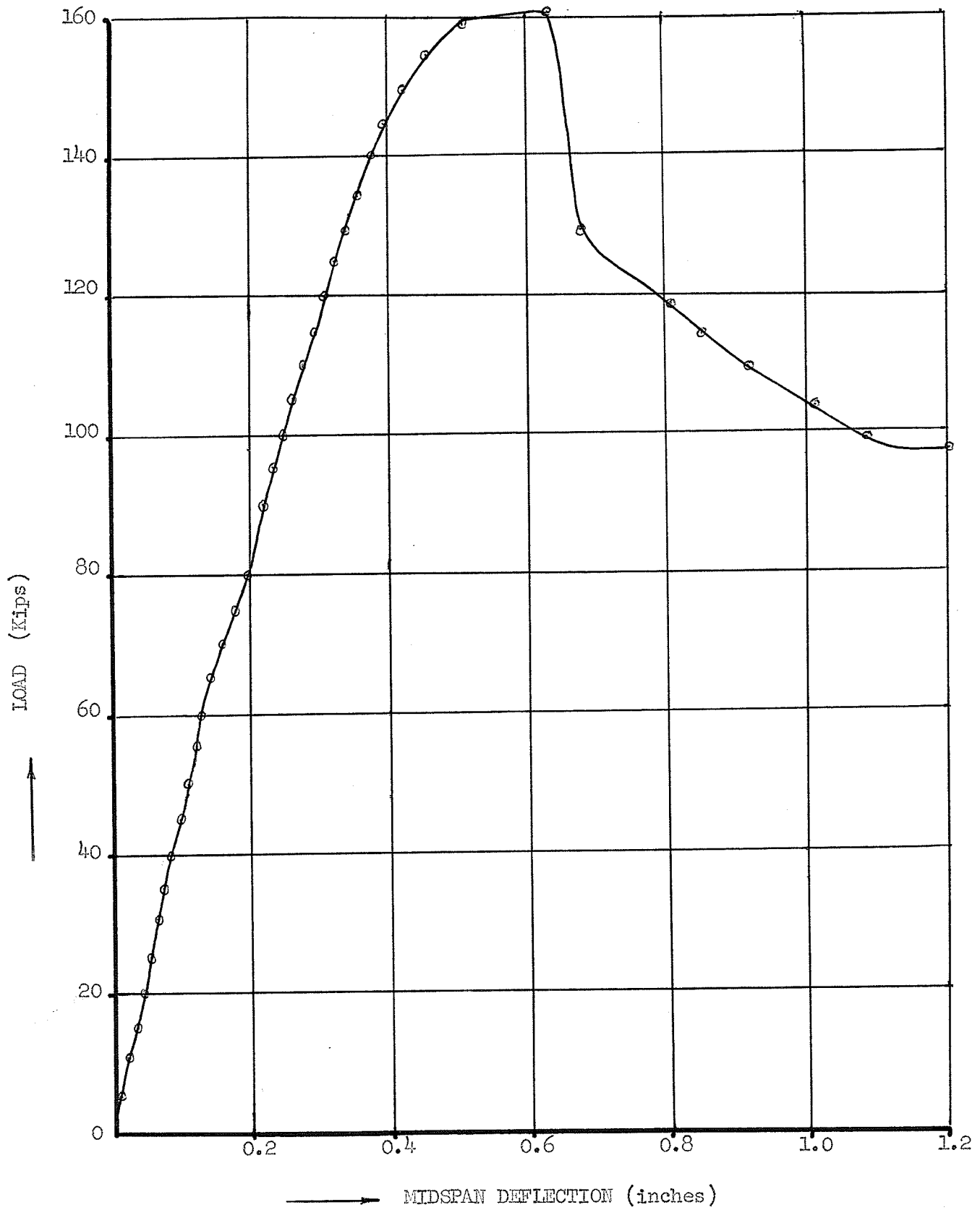


FIG 40 LOAD-DEFLECTION CURVE FOR BEAM 2

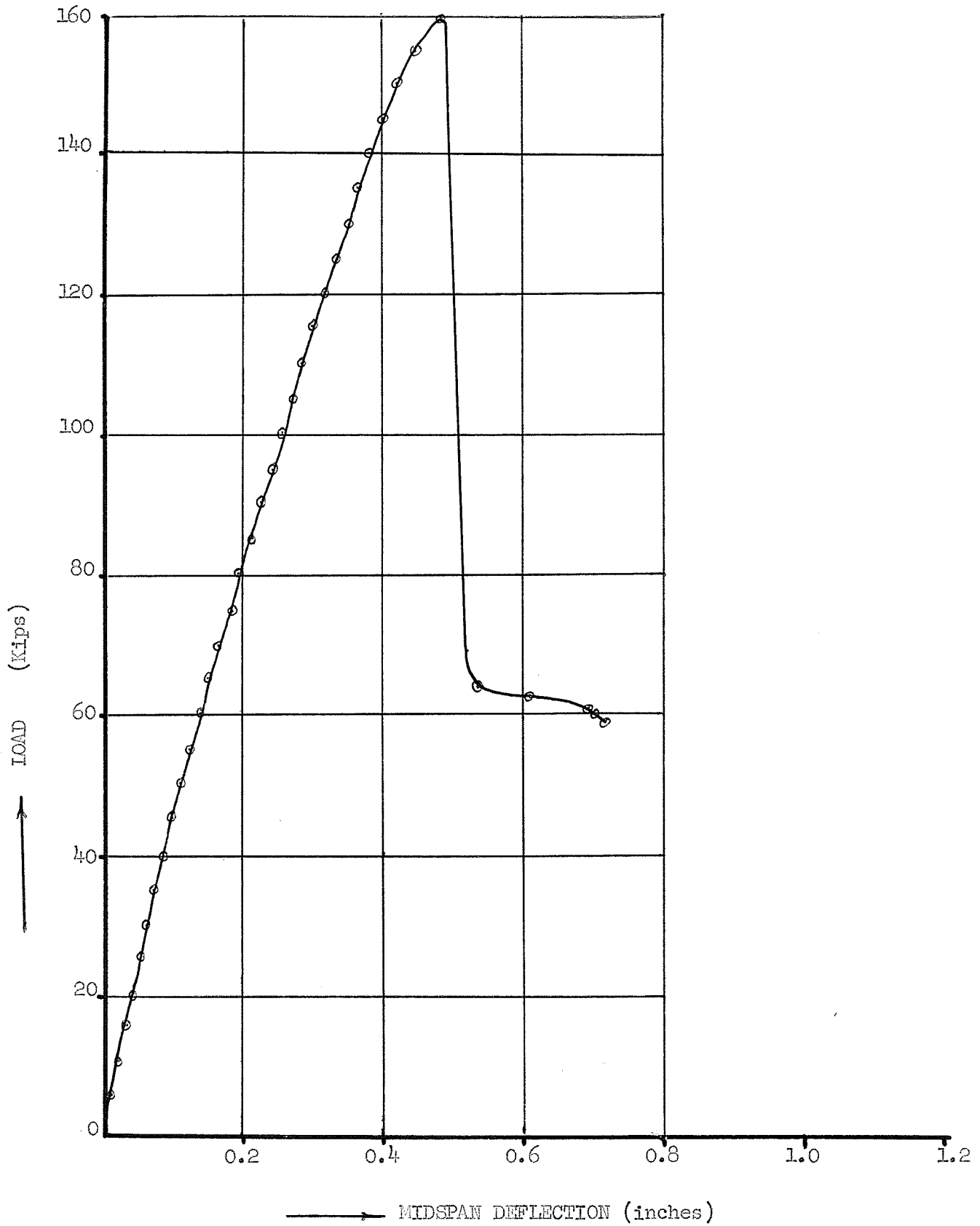


FIG. 41 LOAD-DEFLECTION CURVE FOR BEAM 3

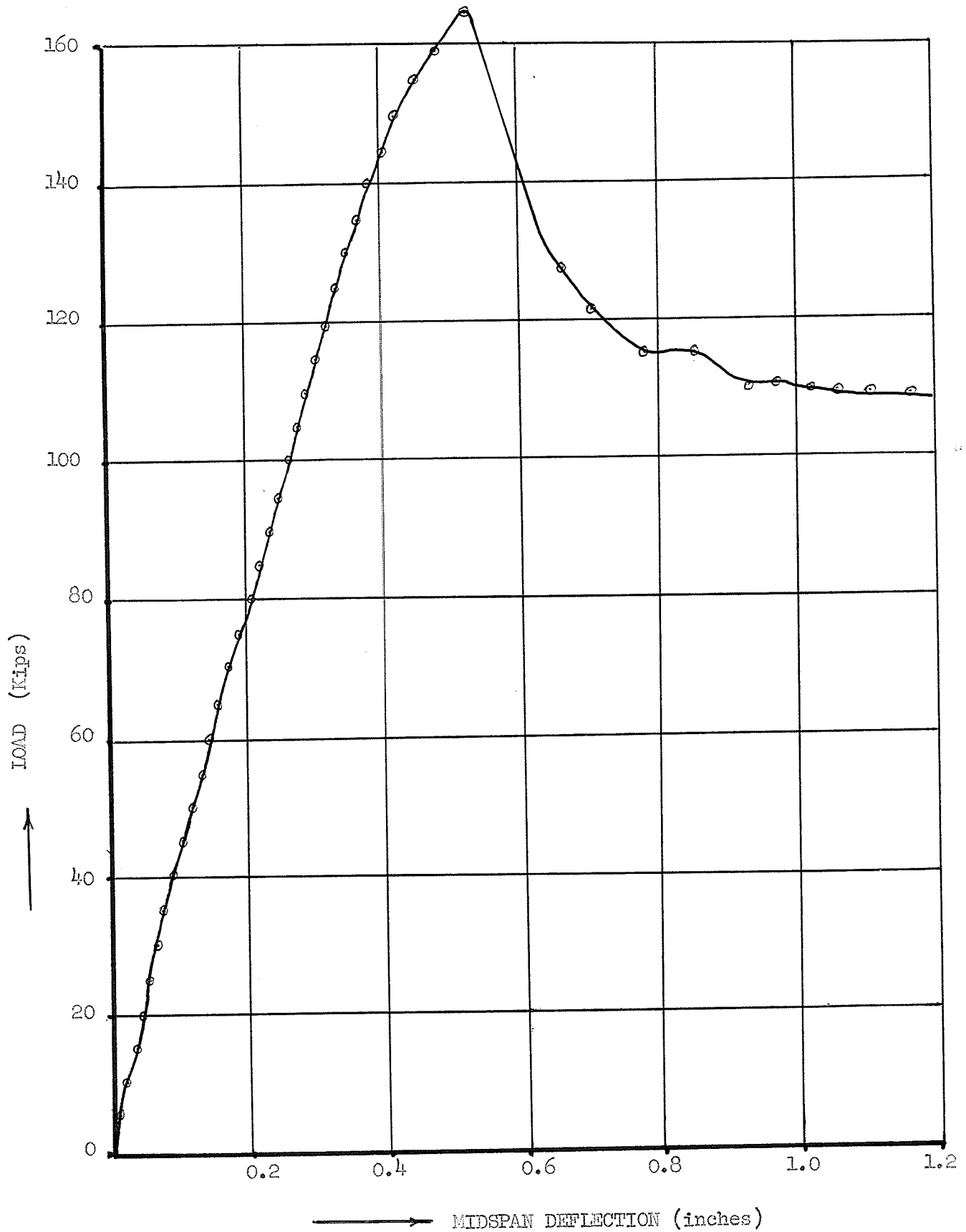


FIG. 42 LOAD-DEFLECTION CURVE FOR BEAM 4

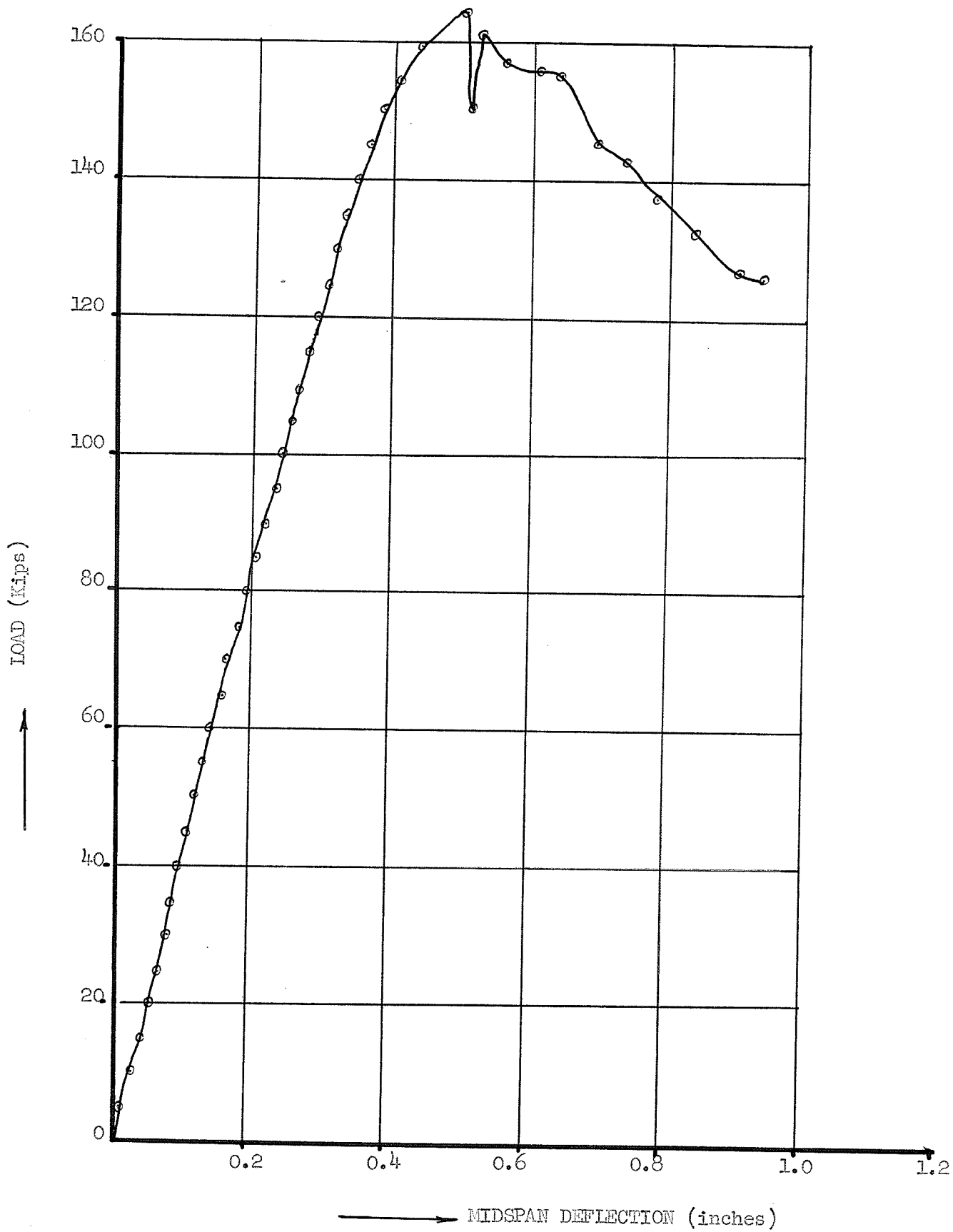


FIG. 43 LOAD-DEFLECTION CURVE FOR BEAM 5

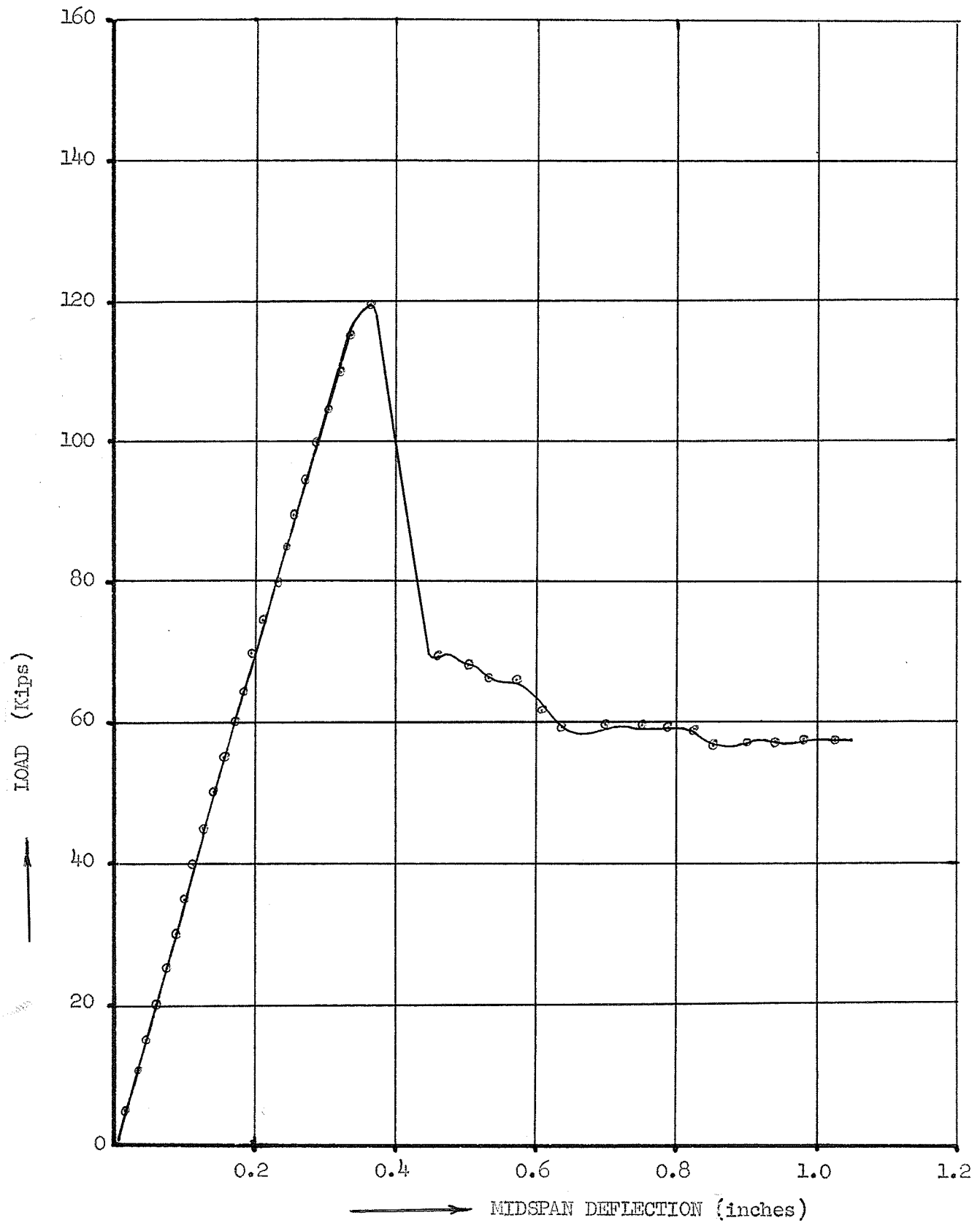


FIG. 44 LOAD-DEFLECTION CURVE FOR BEAM 6

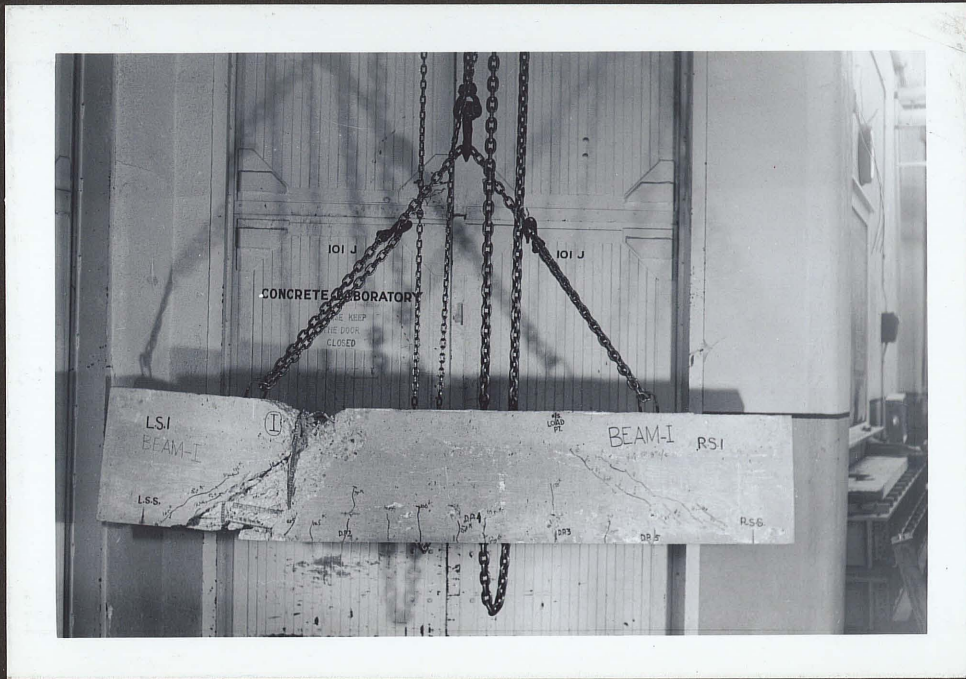


FIG. 45 - Beam 1 after Failure (side L.S.1-R.S.1)

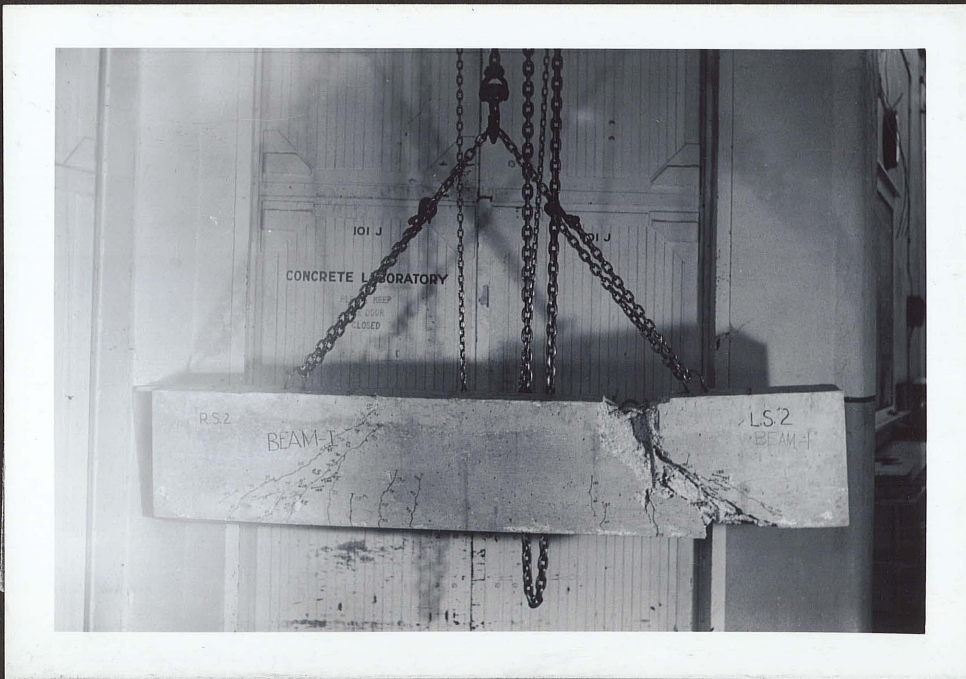


FIG. 46 - Beam 1 after Failure (side R.S.2-L.S.2)



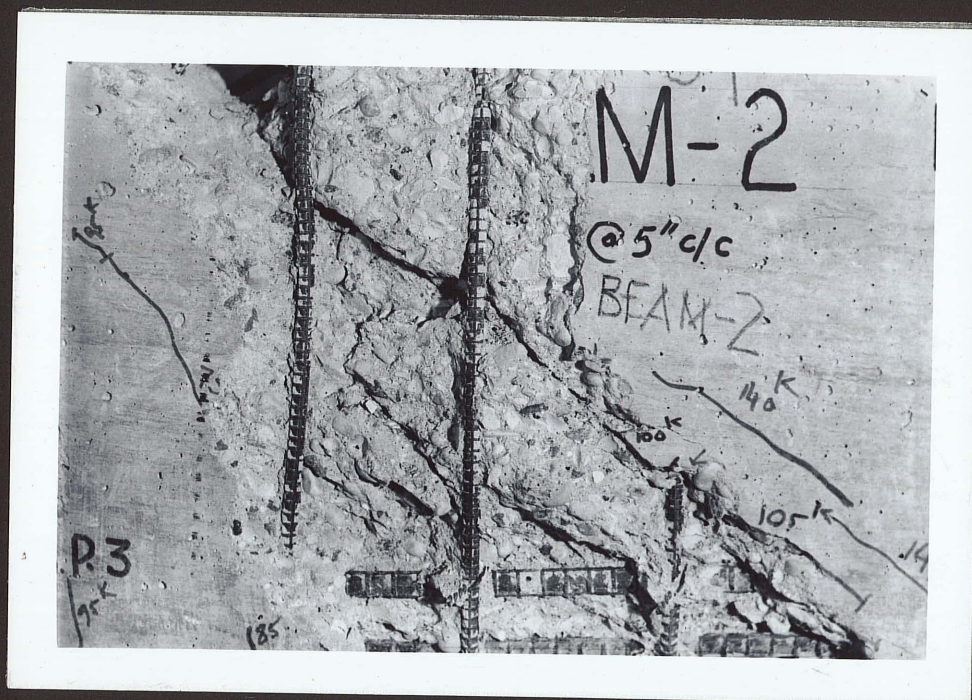


FIG. 48 - Close View of Beam after Cracking



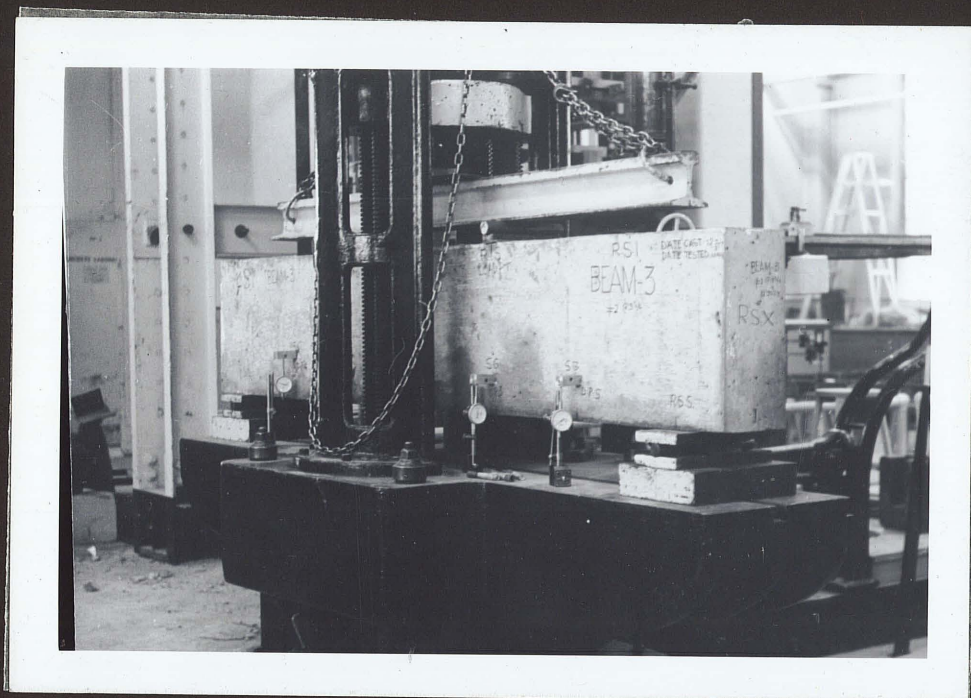


FIG. 49 - Test Arrangement for Beam 3

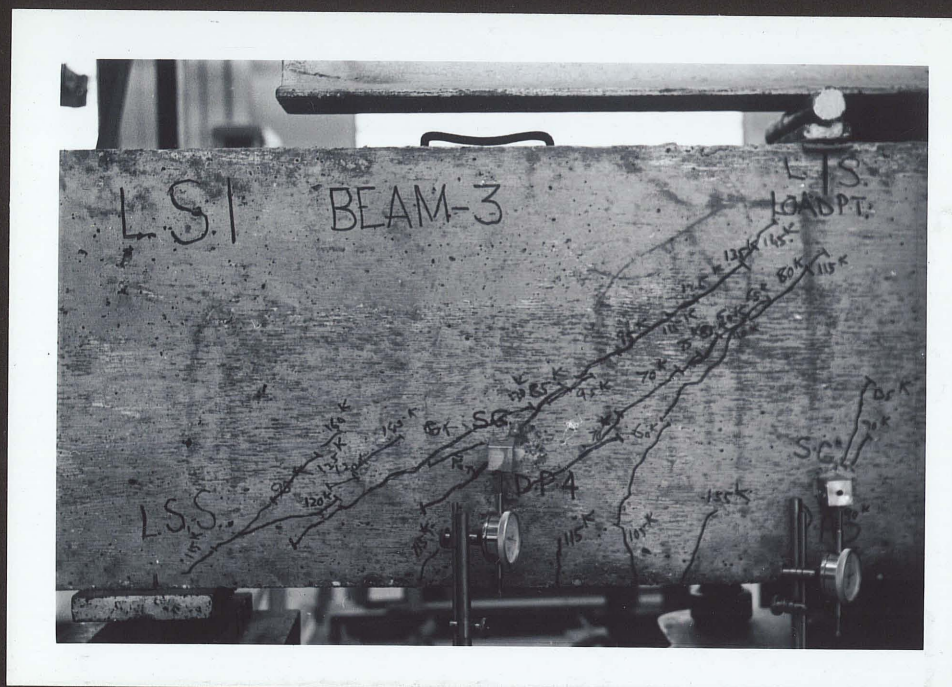


FIG. 50 - Beam 3: Detail & Crack Pattern (side L.S.1)

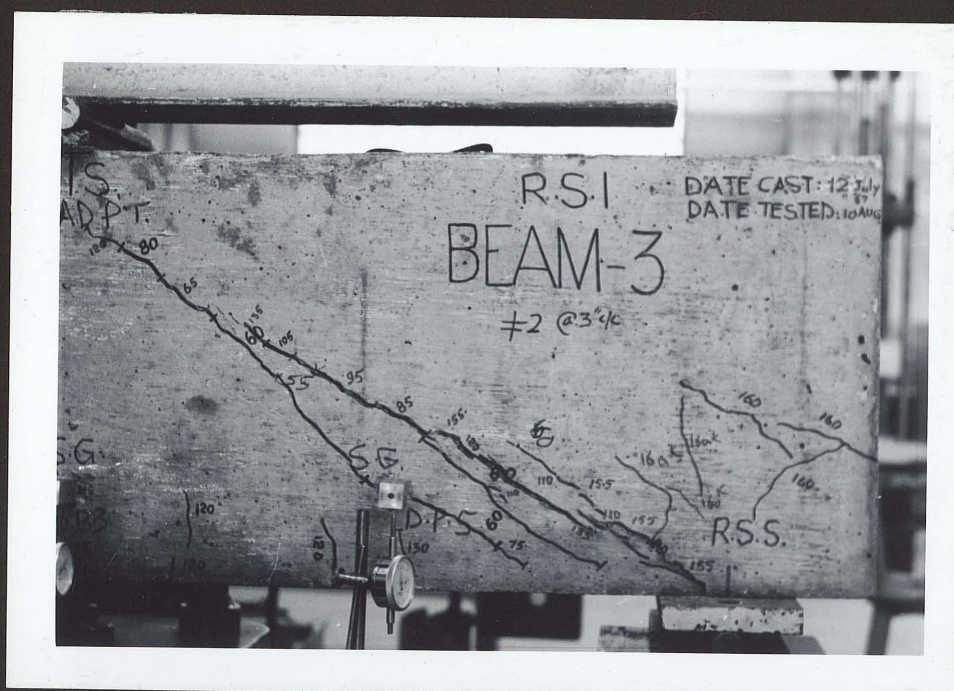


FIG. 51 - Beam 3: Detail and Crack Pattern (side R.S.1)



FIG. 52 - Beam 3 after Failure (side R.S.1)



FIG. 53 - Beam 3 on Further Loading after Failure



FIG. 54 - Beam 3: View on side R.S.2 after Complete Failure

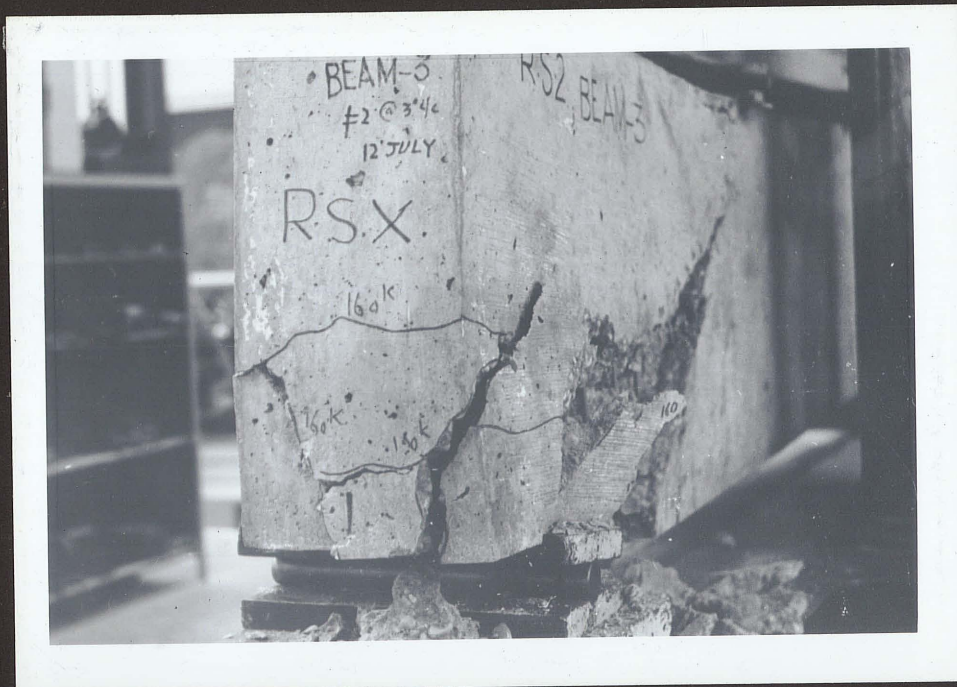


FIG. 55 - Side View of Beam 3 after Failure

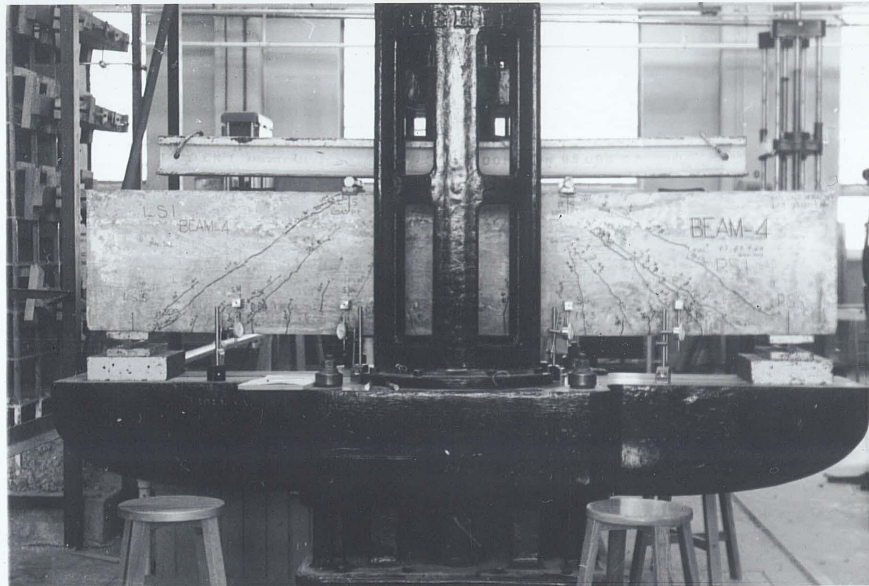


FIG. 56 - Beam 4 with Crack Pattern During Loading

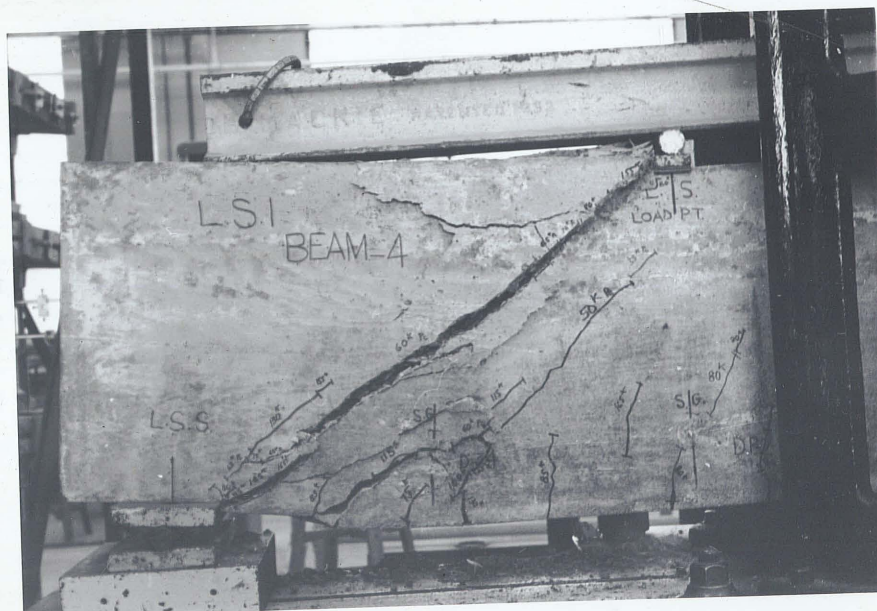


FIG. 57 - Beam 4 (side L.S.1) after Failure



FIG. 58 - Beam 4 (side L.S.1) on Further Loading after Failure



FIG. 59 - Beam 4 (side L.S.2) after Complete Failure

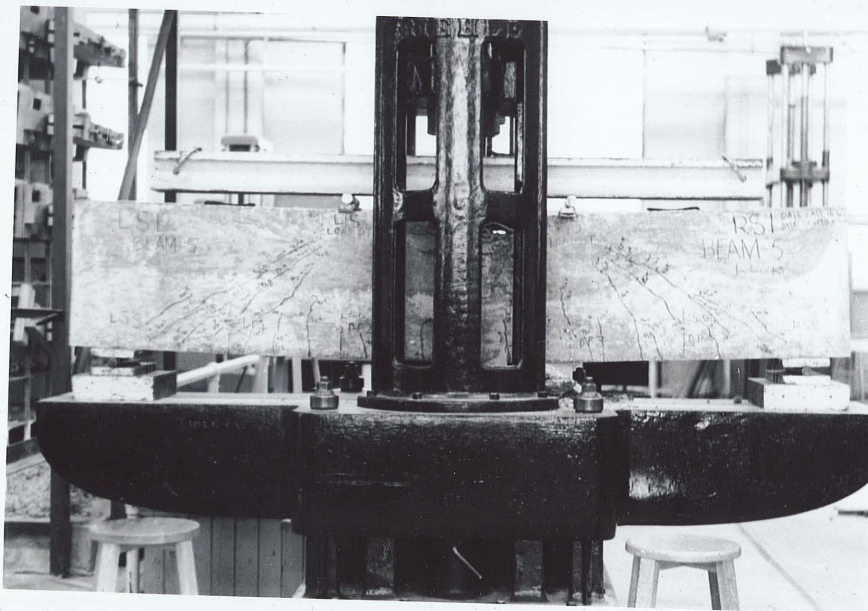


FIG. 60 - Beam 5: General Pattern of Crack  
Formation During Loading

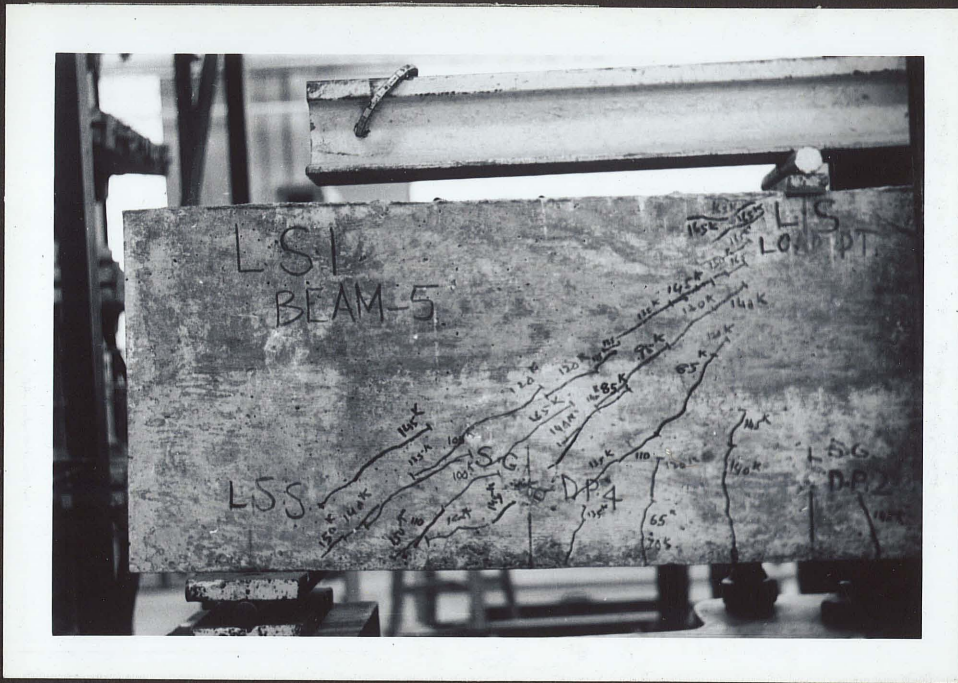


FIG. 61 - Beam 5: Close View of Side L.S.1 at Failure

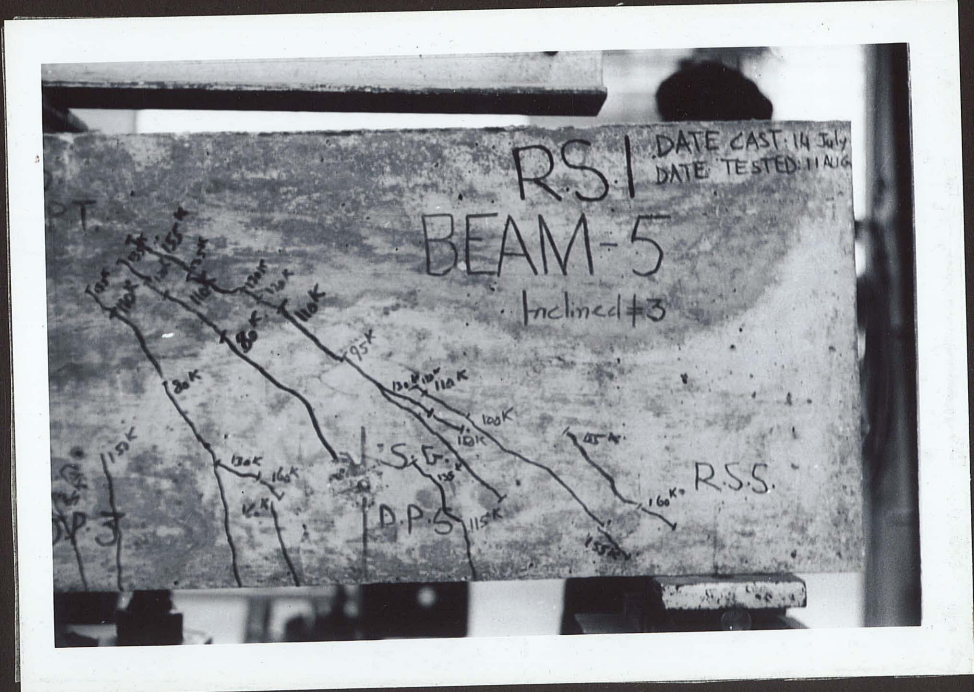


FIG. 62 - Beam 5: Close View of Side R.S.1 at Failure



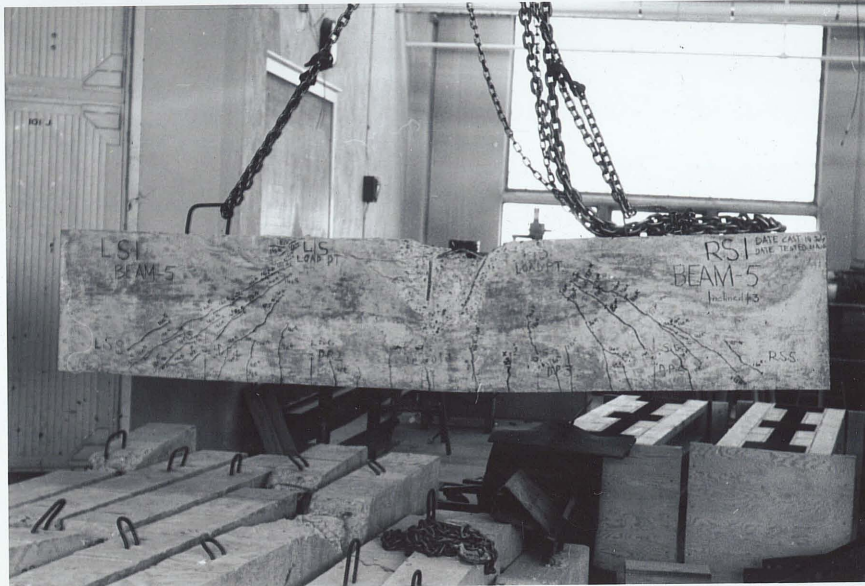


FIG. 63 - Flexural Failure of Beam 5  
(sides L.S.1 - R.S.1)

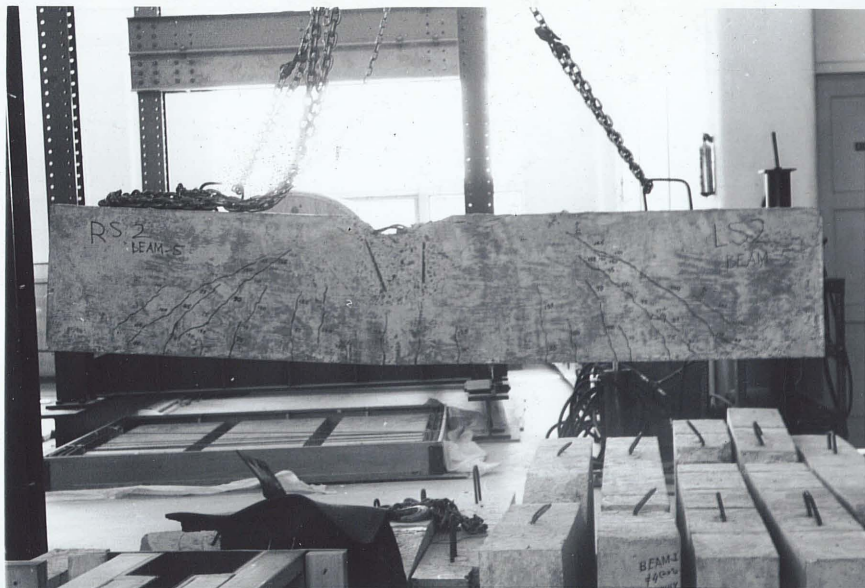


FIG. 64 - Failure of Beam 5  
(sides R.S.2 - L.S.2)



FIG. 65 - Close View of Cracked Portion in Beam 5  
(side L.S.1 - R.S.1)



FIG. 66 - Close View of Cracked Portion in Beam 5  
(side R.S.2 - L.S.2)

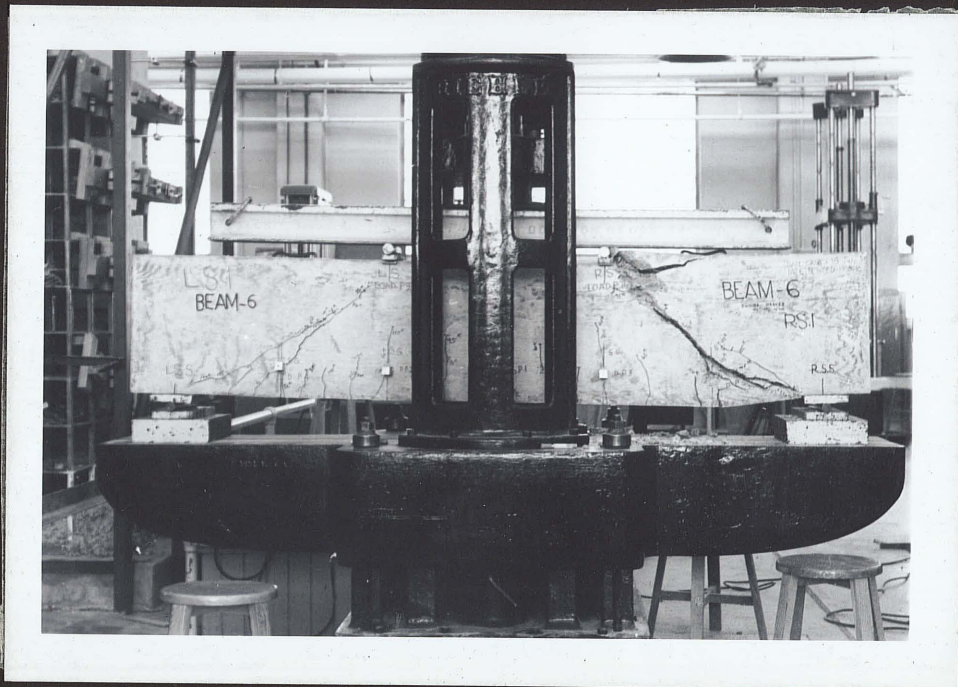


FIG. 67 - Failure of Beam 6: Details and Crack Pattern



FIG. 68 - Beam 6 after Failure  
(side L.S.1 - R.S.1)

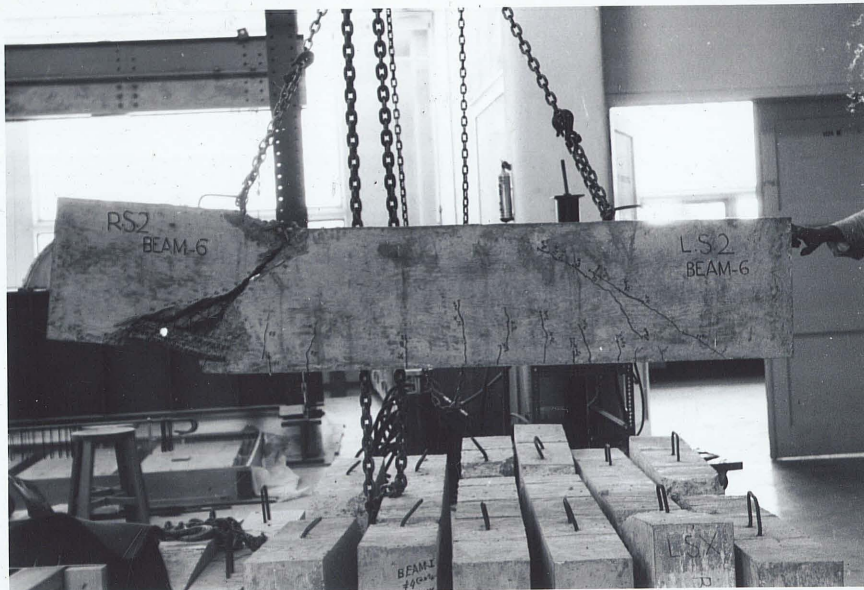


FIG. 69 - Beam 6 After Failure  
(side R.S.2 - L.S.2)



FIG. 70 - Beam 6: Close view of side R.S.1 after Failure



FIG. 71 - Beam 6: Side R.S.1 on Further Loading After Failure

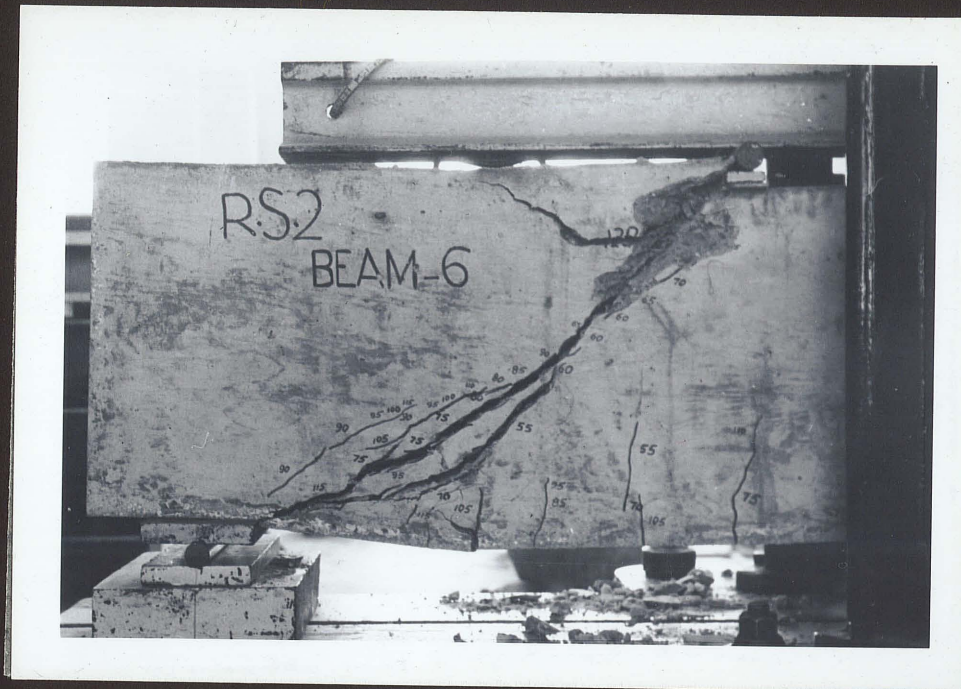


FIG. 72 - Beam 6: side R.S.2 after Failure



FIG. 73 - Beam 6: side R.S.2 on Further Loading after Failure



FIG. 74 - Beam 6: Very Close View of Wire-mesh after Failure

In testing reinforced concrete beams, the following three different stages may be distinguished.

- 1) Behavior prior to the formation of diagonal tension cracks.
- 2) Initial diagonal tension cracking.
- 3) Behavior after the formation of diagonal cracks.

These are discussed in the following paragraphs.

#### 7.51 Behavior Prior to the Formation of Diagonal Cracks

In Figs. 45 to 74 it will be noted that the portions of the load deflection curves below the cracking load are rather straight and they are more or less parallel for the different beams. This indicates that prior to the formation of diagonal cracks, the behavior of all the beams was essentially elastic and the presence of the web reinforcement had practically no effect on the behavior of the beams. No tension cracks were observed during this stage. This stage of behavior was ended when inclined cracks, hereafter called initial diagonal tension cracks, formed.

The loads at which initial diagonal tension cracks formed varied from 50K to 65K in these beams and practically there was not much difference in the initial diagonal tension cracking load due to the type of reinforcement used.

#### 7.52 Initial Diagonal Tension Cracking

For the present purposes, a diagonal tension crack is defined as a major inclined crack; this crack causes a significant redistribution of internal stresses. Several diagonal tension cracks may form in a beam before the ultimate load is reached. The one which forms first is called the initial diagonal tension crack and the corresponding load is called the initial diagonal tension cracking load.

Diagonal tension cracks observed in these tests had certain



common characteristics by which they could be distinguished from the ordinary tension cracks. In all the beams, the diagonal tension cracks were initiated at or above the tension reinforcement and were inclined from their inception. As the load increased, the diagonal tension cracks developed further at both the ends. These initial cracks were very thin and a very careful visual observation was required to trace them. The increments of load were 5,000 pounds and though care was exercised in locating the diagonal cracks as the load increased, an exact value of load at initial diagonal cracking could not be obtained. In fact once the approximate value of the initial diagonal tension load was obtained from tests on the first beam, it was observed that if the load was kept constant around that value or with a further increment, initial diagonal cracks usually appeared. Another point which was noted during this period was that the initial cracks did not appear on all four sides of the beam at the same load. Usually first cracks appeared on one or two faces and with a further increase of load, cracks subsequently formed on the other faces as well.

It was observed that for most of the beams, the initial cracks started from somewhere in the middle of the supports and the load points. Once a crack appeared, it extended immediately in a diagonal direction on either side. In some cases, two or three cracks started at the same load but one of these cracks was distinct in character and comparatively longer than the others. All these cracks started from above the tension reinforcement.

It may be seen that there is a slight difference in the initial diagonal cracking load of the beams. As concrete is a heterogeneous material and even the strength tests of concrete (Tables 4 and 5) show some variation in the range of about 15% in the strength of concrete, this may well be explained by the difference in strength of concrete.

The most common type of initial diagonal tension cracking was the starting of cracks at the midpoint of the shear span and the mid-depth of the beam and pointing towards both the support and the load.

### 7.53 Behavior After the Formation of Diagonal Cracks

A comparison of the results of the beam specimen is given in Table 17. The ratio of the ultimate load to the cracking load is shown in column  $P_u/P_c$  of this table. It can be seen that the ultimate load was considerably higher than the first cracking load. The ratios of the ultimate to the cracking load varied from 2.40 for beam 6 to 3.30 for beam 4. The nominal shearing stresses ranged from 470 psi to 645 psi.

The behavior of all the beams is described separately.

#### Beam-1

Figs. 45 and 46 show the beam after failure. As it can be seen, failure occurred on the sides L.S.1 - L.S.2 by diagonal tension. Loadings in Kips and crack pattern are marked on the beam. Failure occurred by the extension of the diagonal cracks from both ends of the initial cracks. The maximum load on this beam was 151.2 Kips.

As the load was increased from the initial cracking load, several cracks formed. These cracks were closely spaced and approximately parallel to the direction of the initial diagonal cracks.

With further increase of the load, the upper ends of the cracks extended towards the load. At this stage some flexural cracks were observed in the middle span. The first of such cracks appeared at 75 Kips. Further such cracks appeared with an increase in the load but they did not extend upwards and with a closer view it could be concluded that the stirrups were the starting points for these cracks. When the load reached about 100 Kips, the upward movement of the diagonal cracks became slower and further small cracks appeared. One of the cracks on side L.S.1 became wider and appeared likely to cause failure. The

TABLE - 17

COMPARISON OF RESULTS OF BEAM SPECIMENNominal  $f'_c = 3,500$  psi

Beam No.	Actual $f_c$ (psi)	Actual $f_t$ (psi)	First crack- ing Load $P_c$ (Kips)	Ultimate Load $P_u$ (Kips)	$v_u = \frac{V}{bd}$ (psi)	$P_u/P_c$	Mode of failure
1	3,430	390	60	151.20	590	2.52	D. T.
2	4,110	480	55	161.17	630	2.94	D. T.
3	4,230	447	55	160.00	625	2.91	B.
4	3,680	467	50	165.00	645	3.30	D. T.
5	4,000	471	65	165.00	645	2.54	F.
6	3,680	471	50	120.00	470	2.40	D. T.

Note: D. T. - Diagonal Tension

B. - Bond

F. - Flexure

deflections further increased and the diagonal cracks approached within a few inches of the load points.

Final failure was caused by a diagonal crack. At failure, the upper ends of the diagonal cracks were closer to the top surface of the beam than the upper ends of the tension cracks, indicating that the beam failed at a load lower than the flexural capacity.

The deflection at the center of the beam at failure was about 0.6 inches.  $P_u/P_c$  ratio was 2.52. <sup>The</sup> Load-deflection curve is shown in Fig. 39.

#### Beam-2

This beam showed a behavior very similar to Beam 1. After the initial diagonal tension cracks appeared at a load of 55 Kips on the side R.S.2, load was further increased and first diagonal cracks appeared on other sides up to 75 Kips. First flexural cracks appeared at a load of 85 Kips. However, by this time several other diagonal cracks had formed. As the load increased, the diagonal tension cracks widened and finally at a load of 161.17 Kips, the same crack which caused initial cracking resulted in a diagonal tension failure on the side R.S.1 - R.S.2. The final deflection at midspan was about 0.62 inches and  $R_u/P_c$  ratio was 2.94. After failure had occurred, the load was sustained to expose the reinforcement. A close view of Beam 2 is shown in Fig. 48. The deflection of the vertical No. 3 stirrups can easily be seen with the main diagonal crack passing through about three stirrups. Fig. 40 shows the load-deflection curve.

#### Beam-3

The final failure of this beam was different from the other beams. Initial diagonal cracks appeared at 55 Kips on side R.S.1. Further cracks appeared with an increase in the load and the behavior was quite similar to the first two beams until about 120 Kips. Afterwards,

the diagonal cracks did not move up considerably with further increase of load. Diagonal cracks were closer to the load on the side L.S.1. As the load increased to 145 Kips, the cracks moved closer to the load on the side L.S.1 but some cracks appeared in the cross-section as well. These cracks widened and a sudden failure occurred due to these cracks at 160 Kips at the right hand cross-section. Figs. 50 and 51 show the failure of the beam. The beam was continued to be loaded and the concrete on top of the reinforcement was shattered. This state of loading is shown by Fig. 52. The exposed steel bars are visible at this stage. A closer view revealed that some of these bars were broken. This is clear from Fig. 54. The concrete was completely broken from one side as well as from the cross-section and the longitudinal reinforcement was fully exposed. This is evident from Figs. 53 and 54. A closer look at the redistribution of internal forces shows that the formation of the diagonal tension crack is accompanied by a considerable increase of the tension force and this may lead to secondary bond failure. An anchorage of the reinforcement, such as hooks, may have prevented such a failure. From the behavior of the beam before failure, it appeared that the beam may have taken more load had this failure not occurred. Right hand side cross sectional view is shown in Fig. 55 where cracks causing ultimate failure may be observed. The ultimate nominal shear was 625 psi and  $P_u/P_c$  worked out as 2.91.

#### Beam-4

Crack pattern during loading of this beam is shown in Fig. 56 and the side L.S.1 at failure is shown in Fig. 57. Maximum load for this beam was 165 Kips and it failed by diagonal tension on the side L.S.1 - L.S.2. The beam showed a very good behavior upto 140 Kips

but afterwards the deflections increased considerably. The beam was continued to be loaded after failure and Figs. 58 and 59 represent the condition after the steel had been exposed. The main diagonal failure crack crossed No. 3 stirrups and a few No. 2 stirrups which were broken. No. 3 stirrups closer to the load deflected considerably.

#### Beam-5

Crack pattern during loading of this beam is shown in Fig. 60. This beam failed in flexure. After initial diagonal cracking at 65 Kips, the beam showed very good behavior with only few cracks appearing. Flexural cracks were very small. It appeared that the beam would fail by diagonal tension on the side L.S.1. At about 135 Kips, the cracks on L.S.1 opened slightly, but the beam appeared to be fairly strong even at 145 Kips with the deflection at midspan being only 0.25 inches. Diagonal cracks advanced very slowly towards the load. At 165 Kips the diagonal cracks had almost caused failure as side L.S.1 was fully lined up with cracks. However, a crack suddenly jumped up in the middle portion and caused failure by flexure at the same load. Figs. 63 to 66 represent this failure. This seems to be very nearly a balanced failure. Some of the concrete at top in the central portion was exposed and two stirrups could also be seen. This beam exhibited very good behavior under a high value of loads and showed smaller deflections. The ultimate nominal shear for this beam was 645 psi and  $P_u/P_c$  was 2.54.

#### Beam-6

Failure of this beam is shown in Fig. 67. Failure was caused by a diagonal crack on the side R.S.1 at 120 Kips.

Initial diagonal cracks appeared at 50 Kips. Some tension cracks were visible at a load of 70 Kips. The beam continued taking load

while the cracks widened at 95 Kips. The moment cracks in the center were very small and did not appear likely to cause failure. The diagonal cracks advanced on either side towards the load points and the supports until failure occurred at 120 Kips. The shear reinforcement in this case was considerably less than the other beams and hence failure occurred at a comparatively lower value of the ultimate load. Readings for deflection were taken even after failure and the beam continued taking a load of about 59 Kips for quite a long time while the deflections were increasing.

Details of failure of the beam are shown in Figs. 68 and 69. Fig. 70 shows a closer view of the beam after failure.

As the load was sustained after failure, concrete blocks chipped off from the beam and a larger portion of steel was uncovered. Such a condition is shown in Fig. 71. Figs. 72 and 73 show the side R.S.2 at failure and after failure as the load was sustained. As it can be seen from Fig. 73, there was a considerable deflection of the longitudinal reinforcement as failure occurred.

A closer view of the wire-meshes after failure can be seen in Fig. 74. Although the wires had an elongation of about 26%, some of these wires were found broken at ultimate load. There was a great deflection in the vertical wires and the 2 inch mesh was completely lost with the spacing differing at different levels. The concrete within the meshes remained intact but the failure occurred right across the section. The compression zone of concrete was destroyed above the diagonal tension crack and adjacent to one of the loading blocks. The ultimate nominal shear worked out as 470 psi and the  $P_u/P_c$  ratio as 2.4.

There were large deflections in the wire cages and the failure of concrete had resulted in forcing the cage outwards.

Thus out of the six beams tested four failed in diagonal tension, one resulted in secondary bond failure and one showed a flexural failure with the diagonal tension failure probably about to occur.



CHAPTER VIIIANALYSIS OF TEST RESULTS8.1 General

The purpose of this Chapter is to discuss the probable reasons for the failure of the beams tested in this program, and to present an analysis of the test results. It was mentioned Chapter I that the purpose of the present study was to determine the effect of size, distribution and pattern of shear reinforcement on the shear strength of concrete. In this connection, the behavior of the beams prior to ultimate failure and the ability of the beam to carry a portion of the ultimate load after failure was also an important study. Test results are analysed to draw conclusions on these important aspects of the study.

8.2 Comparison of Web Reinforcement

One of the important factors in the present study was the necessity of maintaining the amount of web reinforcement practically constant for all the cases. Considering the reinforcement used for different beams, an analysis has been made to see how far this attempt was successful. Table 18 shows a comparison of the web reinforcement in terms of the total steel used as web reinforcement. As only the vertical steel is effective against diagonal cracks, this table takes into account only the vertical or the inclined length of the web reinforcement. In order to correctly identify the steel effective in case of wire meshes, an attempt has been made to include the effect of horizontal wires also. Considering a small 2 in. sq. mesh consisting of 0.156 in. verticals and 0.115 in. horizontals, the effective inclined length in terms of the 0.156 in. vertical wire may be assumed as the square root of the sum

of the square of the vertical length of 0.156 in. wire (i.e. 2 in.) and the square of the horizontal length multiplied by the ratio of the diameters of the wires. If this assumption is not made the size of the mesh (2 in. square in this case) does not enter into calculations. It can be visualized, however, that the smaller the mesh size, the better the enclosed cage.

$$\begin{aligned} \text{i.e. inclined effective length} &= \sqrt{(2)^2 + (2)^2 \left(\frac{0.115}{0.156}\right)^2} \\ &= 2.46 \text{ inches.} \end{aligned}$$

Hence, total effective length in 16 in. height  
 $= 8 \times 2.46 = 19.52 \text{ inches} = 1.63 \text{ ft.}$

(There are 8 small meshes in a depth of 16 in.)

Total length is calculated by adding the 0.156" diameter verticals for the reinforcement, % wt. of reinforcement is calculated for each beam relative to Beam 1.

It should be realized, however, that the yield strength of each bar is different and the effective resistance against cracking is proportional to the weight of the reinforcement multiplied by the yield strength. This is shown in Table 19. This resistance is also calculated for each of the beams in terms of Beam 1.

It will be noticed from Table 18 and 19 that the reinforcement is fairly constant except for the sixth Beam with wire meshes. The first five beams show relative effective strength (Table 19) varying from +20.5% to -3.5% of Beam 1.

As the strength of small wires is considerably low and pouring of concrete as well as compaction limits the use of meshes to only three in a width of 8 inches, the effective wt. of steel used is only 77.9% of Beam 1 and the effective resistance against diagonal cracking is only 39.6% of Beam 1. On the other hand the percentage of effective weight for

TABLE - 18

COMPARISON OF WEB REINFORCEMENT

CASE 1 (a) STEEL EFFECTIVE AGAINST DIAGONAL TENSION

BEAM NO.	Stirrup Size	*Length of each Stirrup (ft)	No. of Stirrups	Total length (ft)	Wt/ft. of reinf. (lbs.)	Wt. of reinf- orcement (lbs)	Total eff. wt. of reinf. in each beam (lbs)	% wt. of beam 1
1	No. 4	2.67	10	26.70	0.668	17.8	17.8	100.0
2	No. 3	2.67	18	48.05	9.376	18.1	18.1	101.8
3	No. 2	2.67	31	82.80	0.167	13.8	13.8	77.5
4	No. 4	2.67	2	5.34	0.668	3.56		
	No. 3	2.67	8	20.36	0.376	7.65	14.6	82.0
	No. 2	2.67	8	20.36	0.167	3.39		
	No. 3 (1 to 8)	3.61	8	28.88	Total length = 50.55			
** 5	No. 3 (A1, A2)	3.00	2	6.00	0.376	19.0	19.0	106.8
	No. 3 (B1, B2)	3.58	2	7.16				
	No. 3 (C1, C2)	2.92	2	5.84				
	No. 3 (D)	2.67	1	2.67				

TABLE 18 (Continued)

COMPARISON OF WEB REINFORCEMENT

CASE 1 (a) STEEL EFFECTIVE AGAINST DIAGONAL TENSION

BEAM NO.	Stirrup Size	*Length of each Stirrup (ft)	No. of Stirrups	Total length (ft)	Wt./ft. of reinf. (lbs.)	Wt. of reinf. (lbs)	Total eff. wt. of reinf. in each beam (lbs)	% wt. of beam 1
6	0.156"	*** 1.63	47	*** 230.0	0.063	13.89	13.89	77.9

\* Total length in this case is the length of vertical or inclined web reinforcement

\*\* For symbols in cols. of stirrup size, refer to FIG. 11

\*\*\* The stirrup length is taken as the resultant of verticals (0.156" dia.), and horizontals (0.115" dia.) making 2 in. sq. small meshes. Total length given for three meshes of reinf.

TABLE - 19CASE 1 (b) EFFECTIVE RESISTANCE AGAINST DIAGONAL TENSION(Wt. of effective web reinf. x  $f_y$ )

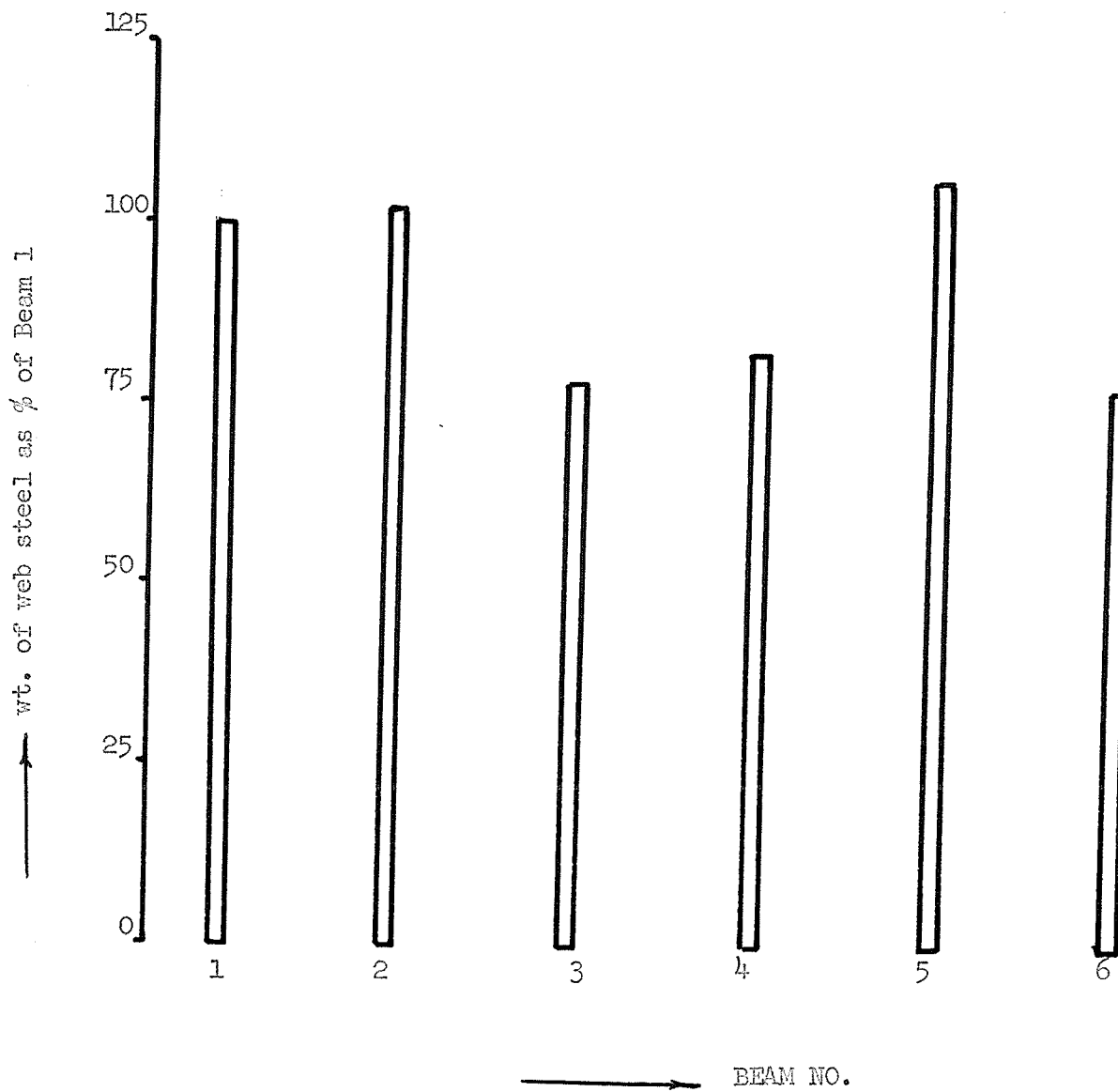
Beam No.	Wt. of effective web reinf. (lbs)	$f_y$ (Kips)	Effective Steel Resistance Wt x $f_y$ (lb-K)	% of Effective Resistance for Beam 1
1	17.8	50.50	900	100.00
2	18.1	57.15	1035	115.00
3	13.8	73.73	1018	113.00
4	3.56(No.4)	50.50	867	96.50
	7.65(No.3)	57.15		
	3.39(No.2)	73.73		
5	19.0	57.15	1088	120.50
6	13.89	25.70	356	39.60

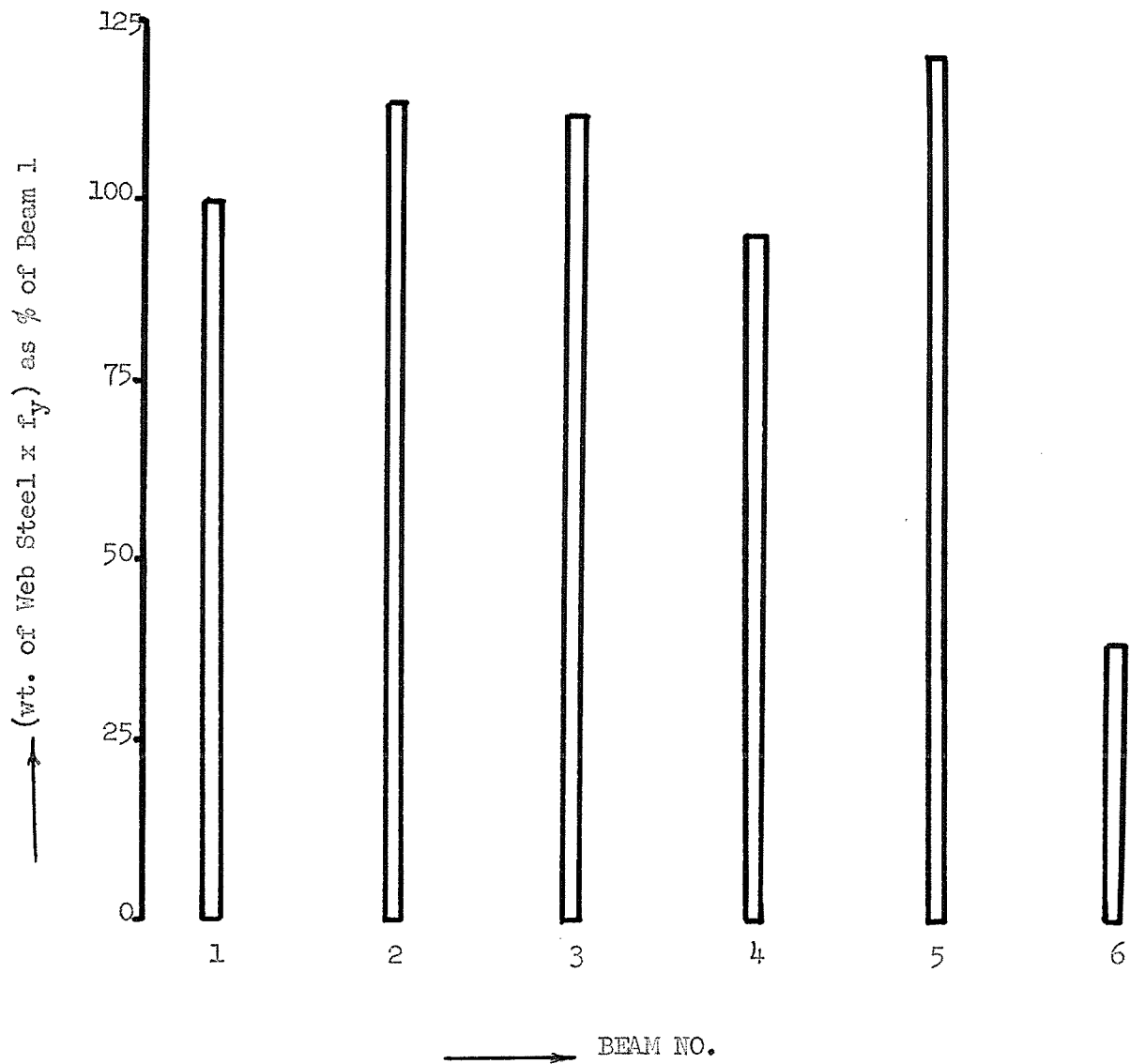
Beam 3 was deliberately chosen as less than Beam 1 (77.5%) as the yield point of No. 2 bars was considerably higher. This is evident from comparison with Table 19 which shows that the effective resistance for these bars is 113% of Beam 1. Similar arguments apply for beam 4 where the weight of effective reinforcement was 82.0% and the effective resistance increased to 96.5% of Beam 1 in Table 19. It seems that slightly larger amount of steel was used for Beam 5 where the effective resistance worked out to be as 20.5% greater than Beam 1.

Before the results are interpreted with these considerations, a comparison of total steel used is useful from the point of view of costs involved. Total web steel has been worked out in Table 20 using the actual quantities of steel used in making stirrups. In order to make use of the yield strength properties, which may influence cost, the weight steel has been multiplied by yield strength for each beam to find the steel resistance in Table 21. Percentage has been worked out in terms of Beam 1 for each case in Tables 20 and 21.

We shall consider first the Tables 18 and 19 to explain the behavior of the beams from the phenomenological point of view. Graphical representation of Table 18 is shown in Fig. 75 and that for Table 19 is given in Fig. 76.

The ultimate load carried by each beam has to be seen as affected by the strength of concrete (Tables 4 and 5) and the web reinforcement (Tables 18 and 19). It can be seen that the concrete used for Beam 1 is relatively weaker than the other beams, the average compressive strength being slightly lower than the nominal 3500 psi whereas the strength for all other beams is slightly higher than the nominal. This may well explain the slightly lower value of ultimate load for the first beam. Also Beams 2, 4, and 5 have higher web resistance. Beam 3 is

GRAPHICAL REPRESENTATION OF TABLE 18FIG. 75

GRAPHICAL REPRESENTATION OF TABLE 19FIG. 76



very nearly of the same strength as Beam 1. It can be seen that for Beams 1 to 5 there is only a small difference in the value of ultimate load carried, the variation being from 151 Kips to 165 Kips, which is only about 8-9%. These results tend to show that the pattern of reinforcement does not have much effect on the ultimate load carried by a simple beam. The analysis of results from the point of view of behavior will be discussed a little later. However, it can be said that closer spacing tends to give slightly higher values for ultimate load.

Considering Beam 6, the effective web resistance is only about 40% of the first beam. Though the amount of steel used in making up the wire meshes was 77.9% of Beam 1, the effective resistance against cracking dropped to 39.6% of Beam 1. In fact four meshes were required to get the same weight of steel for Beam 6 but due to the limitations already explained, only three were used. However, the yield point of these wires was considerably lower than other bars, being only 25.7 Ksi. This further reduced the effective resistance. The ultimate load at which this beam failed was 120 Kips. This load is 79.5% of the ultimate load carried by Beam 1. This analysis clearly shows the advantage of using the wire meshes. As the effective web resistance was only 39.6% of Beam 1 and the ultimate load was 79.5% of the first beam, the wire mesh can be considerably more effective than the normal bars. However, it cannot be said that the wire mesh was twice as effective as the stirrups seem only to become effective after the appearance of the initial diagonal cracks.

The total weight of steel used as web reinforcement (Table 20) is almost constant for Beam 1, 2, and 5, the variation being only of the order of 0.9%. Beam 3 employed only 76.2% of Beam 1 and Beam 4, 84.2%. Due to the higher yield strengths, the <sup>percentage</sup> of total reinforcement resistance in web for Beams 3 and 4 works out as 111.0% and 99.5%

TABLE - 20

CASE 2 (a) COMPARISON OF TOTAL STEEL USED AS WEB REINFORCEMENT

BEAM NO.	Stirrup Size	*Length of each Stirrup (ft)	No. of Stirrups	Total length (ft)	Wt./ft of reinf. (lbs)	Wt. of reinf. (lbs.)	Total eff. wt. of reinf. in each beam (lbs)	% wt. of beam l.
1	No. 4	4.04	10	40.40	0.668	26.95	26.95	100.00
2	No. 3	4.00	18	72.00	0.376	27.05	27.05	100.50
3	No. 2	3.96	31	122.76	0.167	20.50	20.50	76.10
4	No. 4	4.04	2	8.08	0.668	5.39	22.70	84.20
	No. 3	4.00	8	32.00	0.376	12.01		
	No. 2	3.96	8	31.68	0.167	5.30		
** 5	No. 3 (1 to 8)	5.17	8	41.36	0.376	27.15	27.15	100.90
	No. 3 (A1,A2)	4.33	2	8.66				
	No. 3 (B1,B2)	4.82	2	9.64				
	No. 3 (C1,C2)	4.25	2	8.50				
	No. 3 (D)	4.00	1	4.00				

Total length = 72.16

TABLE - 20 (Continued)

## Case 2 (a) COMPARISON OF TOTAL STEEL USED AS WEB REINFORCEMENT

BEAM NO.	Stirrup Size	*Length of each Stirrup (ft)	No. of Stirrups	Total Length (ft)	Wt./ft of reinf. (lbs)	Wt. of reinforcement (lbs)	Total eff. wt. of reinf. in each beam (lbs.)	% wt. of beam l.
6	0.156" (vert.)	1.33	47	*** 188.00	0.063	11.76	19.98	74.20
	0.115" (hor.)	7.50	8	** 240.00	0.0343	8.22		

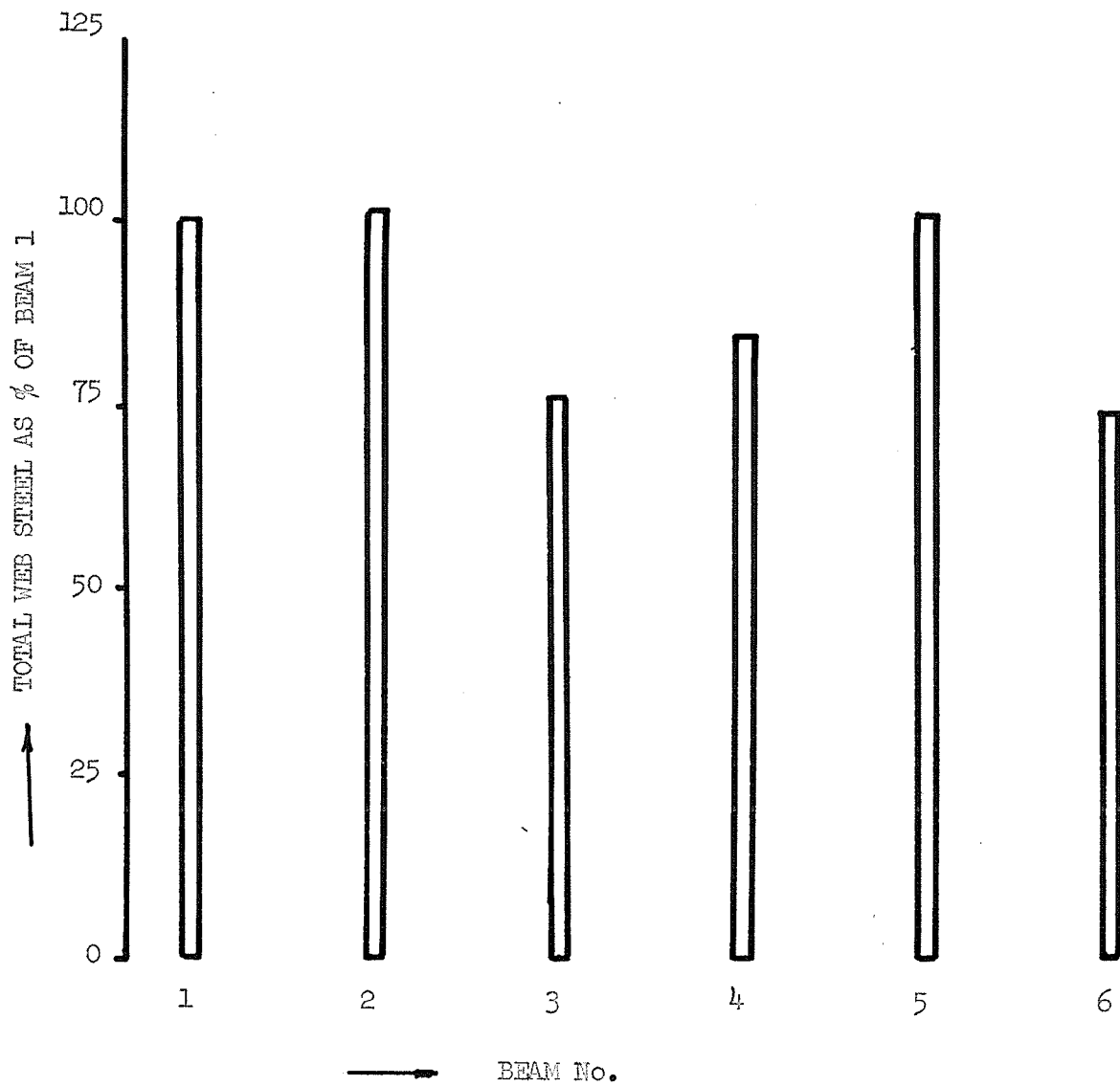
\* Total length including horizontals, hooks bends etc.

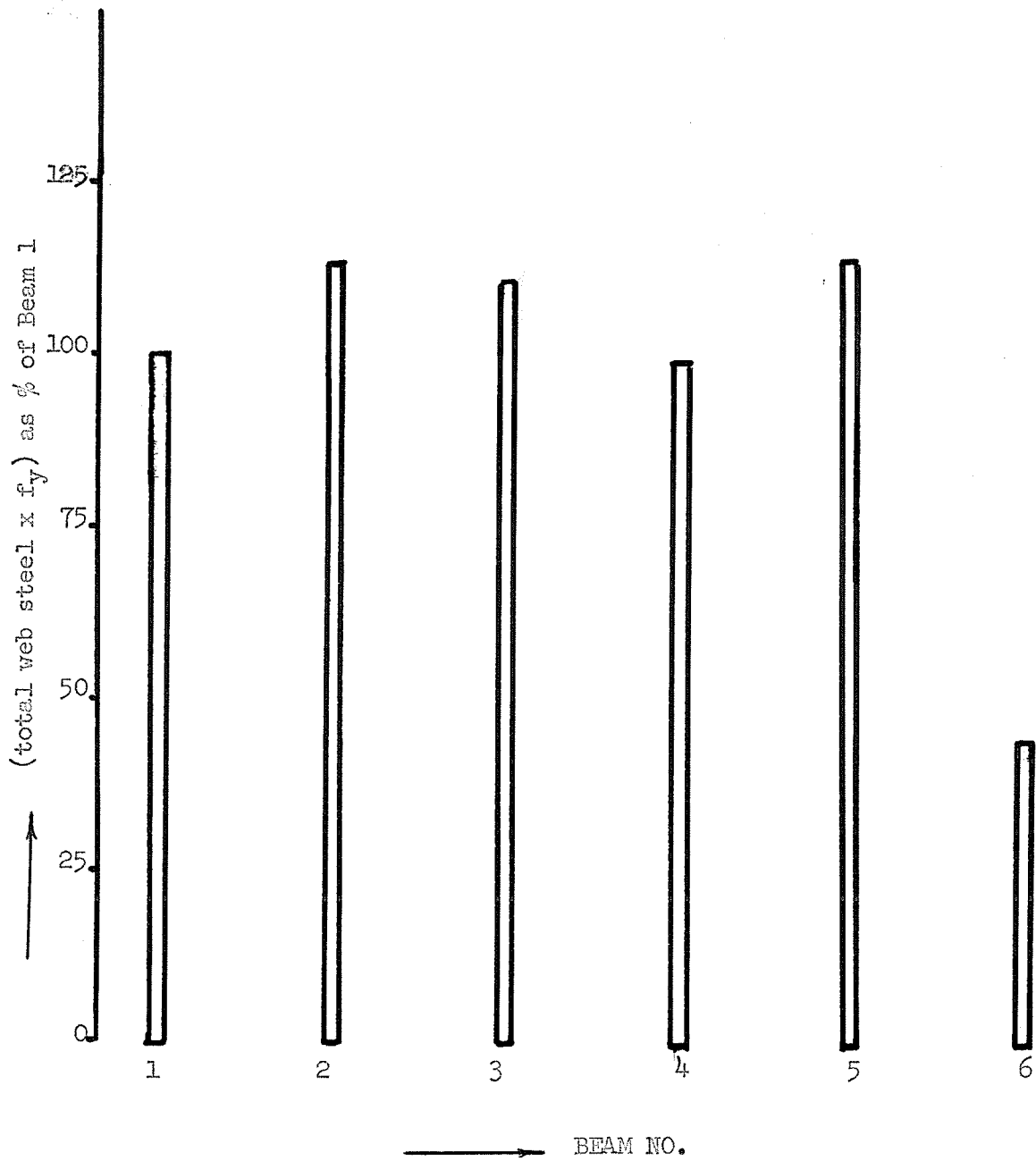
\*\* For symbols in Cols. of stirrup size, refer to FIG. 11

\*\*\* Total length given for three wire meshes

TABLE - 21CASE 2 (b) - COMPARISON OF TOTAL WEB STEEL RESISTANCE(Wt. of web reinf. x  $f_y$ )

BEAM NO.	Wt. of web reinf. (lbs)	$f_y$ (Kips)	Effective Reinforcement Wt x $f_y$ (lb-K)	% of Effective Web Reinforcement for Beam 1
1	26.95	50.50	1360.0	100.00
2	27.05	57.15	1549.0	113.80
3	20.50	73.73	1511.0	111.00
	5.39(No.4)	50.50		
4	12.01(No.3)	57.15	1351.0	99.50
	5.30(No.2)	73.73		
5	27.15	57.15	1550.0	113.80
6	11.76	25.70	616.0	45.40
	8.22	38.13		

GRAPHICAL REPRESENTATION OF TABLE 20FIG. 77

GRAPHICAL REPRESENTATION OF TABLE 21FIG. 78

respectively. Although both of these Tables 20 and 21 are essential as a guide-line for the costs involved, they cannot be considered individually to be representative of the costs involved. The exact value of yield strength is not important for cost as it is broadly based on the grades of steel. A better evaluation of the costs involved can be obtained by considering the total amount of steel used for each beam and comparing with the specified yield strengths of the steel. For beams 1, 2, and 5 the weights of web reinforcement are constant and specified yield is 40 Ksi, the steel being structural grade. Hence the approximate cost of the reinforcement will be the same. Beams 3 and 4 involved the use of No. 2 plain bars. They had a considerably higher yield point and their cost may be higher than the other bars, so that a lower percentage of weight is almost counterbalanced and in the opinion of the writer, approximate cost for web reinforcing for all the five beams would be constant.

Beam 6 consisting of wire meshes had 74.2% wt. of Beam 1 as web reinforcement and the yield was also lower. This would probably involve lower cost. An added factor involving labour will be discussed in the next Chapter.

A graphical representation of Tables 20 and 21 is given in Figs. 77 and 78 to get an idea of the steels involved at first glance.

### 8.3 Analysis of Beam Tests

The behavior aspect has not been considered with some detail as yet. The general behavior of the beams was described in the last Chapter (Chapter VII) prior and after the formation of the cracks. As it has already been pointed out, the ultimate load at which failure occurred did not show a wide variation for approximately the same amount of steel. A major criteria of selection of the type of stirrups as well as their

size would seem to be their behavior during loading and after failure had occurred. Reference is made to Tables 6 to 16 (Chapter VII), giving the load-deflection readings for the beams during the test. An analysis of the behavior will have to be made with due regard to the visual observations during the tests as well as the general performance shown by the Load-Deflection curves. A comparison of the mid-span deflections for these beams is given in Fig. 79 (Figures 39 to 44 superimposed).

The deflection readings for Beam 1 (Table 6) are not very correct due to the reasons already listed in Article 7.5, Chapter VII. This is indicated in Fig. 79 where this beam shows larger deflections prior to final failure than all other beams. There is not much difference in the deflection readings of Beams 2, 3, 4, and 5 prior to the final failure.

It must be realized at this point that the level of ultimate moment has also an effect on the shear resistance of the beams. Though the longitudinal reinforcement for these beams was kept constant, the ultimate flexural capacity was only slightly greater than the load at which Beams 2, 3, 4, and 5 failed. In fact Beam 5 failure in flexure. The designed load for shear failures was less than that at which failure actually occurred by diagonal tension. This reduced the ratio of the ultimate moment and shear capacities of the beams. This point is dealt with at some length in the discussion in the next Chapter.

The failure in Beam 3 (No. 2 plain web reinforcement) occurred as a secondary bond failure at 165 Kips. However, there was a very sharp



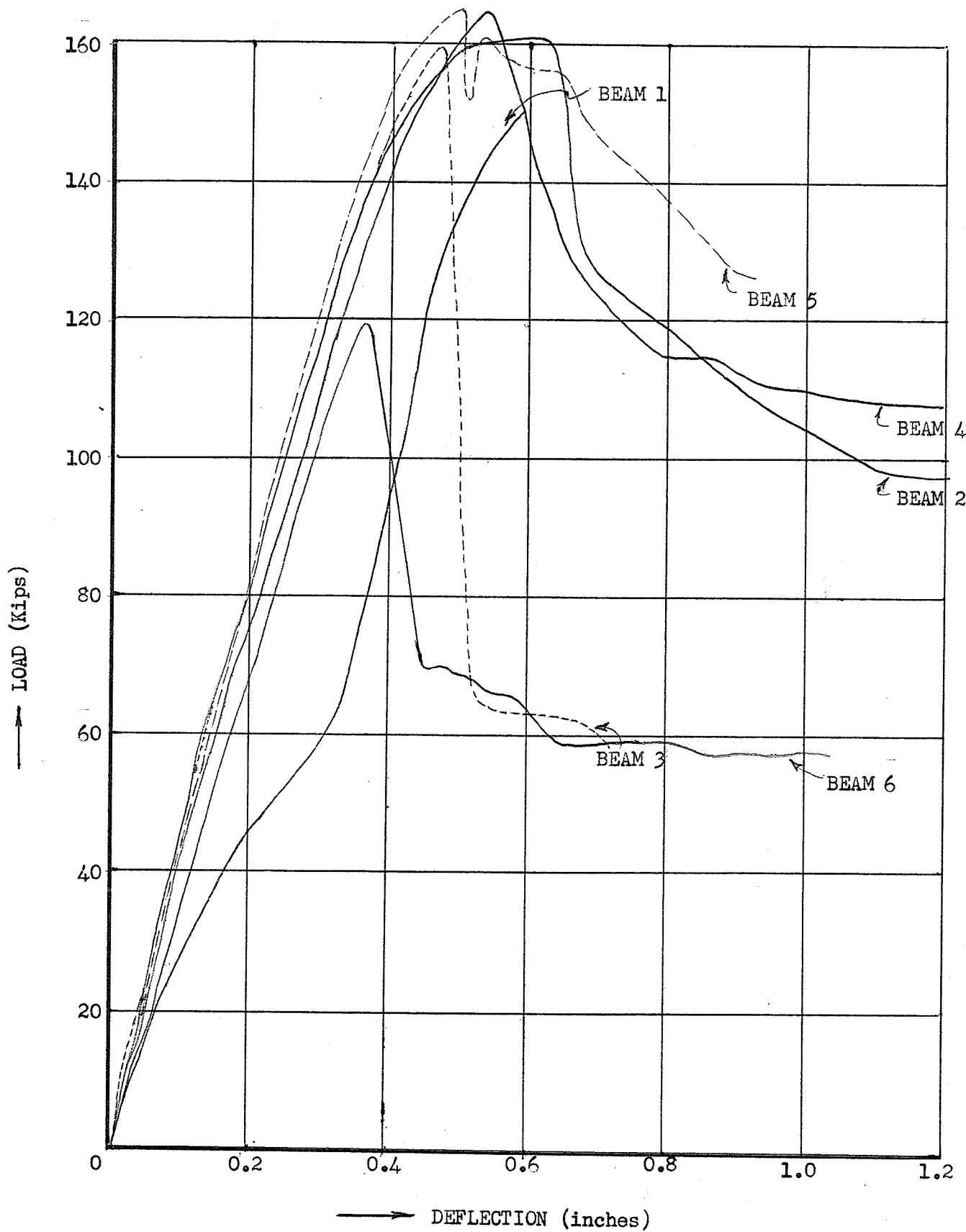


FIG. 79 COMPARISON OF MIDSPAN DEFLECTIONS

(FIGS. 39 to 44  
superimposed)

drop in the load carried by the beam after failure. This was due to the fact that the stirrups were relatively brittle and failure had resulted in a complete break of all the stirrups. The load dropped from 160 Kips to about 63 Kips at almost the same deflection. There seems a possibility that this beam may have carried some more load if premature bond failure had not occurred. Bars with end hooks for the longitudinal reinforcement may have served the purpose.

Out of the first five beams, the best performance was shown by Beam 5. In this case even the initial diagonal cracks appeared at a higher value of 65 Kips. The deflections were comparatively smaller and after the initial cracks had formed, the deflections further reduced with the same increments of the load. First diagonal crack on one of the sides appeared at 80 Kips whereas some of the other beams had shown considerable cracking at this load. During the time the beam was being loaded, it could easily be observed that this beam was showing a much better behavior than the other beams. The appearance of the cracks was slow and the movement of the cracks in the compression zone of concrete was also very slow. Even at a load of 150 Kips, the beam seemed to be fairly intact. This was a load at which other beams had shown considerable signs of failure, with corresponding increase in deflections. First visible signs of failure were observed at 155 Kips. The diagonal cracks started penetrating deeper into the compression zone and the beam was close to diagonal failure at 165 Kips when flexural failure suddenly occurred by shattering of concrete in the midspan. Had the flexural capacity been higher, the beam seemed to be capable of taking further load in diagonal tension.

Fig. 79 shows a good representation of the behavior of this beam after failure in relation to the other beams. The load dropped from

165 Kips to 154 Kips after failure, but it again jumped to 162 Kips. This may be due to added resistance of some stirrups as the load was transferred at this stage. The beam continued to take a large value of the load as the deflections increased and even at a deflection of about one inch at midspan, the beam was still carrying 125 Kips. This was the maximum load carried by any of the beams at this deflection.

When the deflections of this beam after initial diagonal cracking are analysed, it can be seen that the deflections at this stage were slower than any of the other beams. This shows a better crack arrest by the inclined stirrups.

Beams 2 and 4 showed behavior similar to Beam 5 before failure. After failure the load dropped off. This drop was not as steep as in the case of Beam 3 and not as slow as in the case of Beam 5. Beam 2 showed quite good behavior after failure for some time before the load dropped to 130 Kips. Afterwards, the drop in the load was more rapid than in Beam 4. Beam 4 showed a slightly quicker drop in the load after failure before the load almost stabilized at about 110 Kips. The drop in load after failure may be due to the fact that the main crack causing failure passed through two of the No. 2 plain stirrups which broke down due to their relatively brittle nature. The fact that the beam continued to carry 110 Kips may indicate that the No. 3 stirrups were carrying the load while probably going through a stage of strain hardening. It seems that the overall behavior of Beam 4 was better than Beam 3, particularly keeping in view the relatively brittle nature of the No. 2 plain stirrups. It could well be expected that this beam may well have shown better results with more ductile No. 2 stirrups. Beam 3 consisting of No. 2 plain stirrups, on similar grounds, could have shown better behavior after failure.

Beam 6 was, in the opinion of the writer, the most important for

this study. It has always been considered advisable to increase the ductibility of concrete to ensure a better performance. The wire meshes served the purpose of distributing the steel right through the beams. This method was considered the most suitable to make the beams more ductile. This has been very well supplemented by the test results. Table 16 gives the load-deflection readings of this beam after final failure. Fig. 79 also shows the mid-span deflections for this beam. This beam failed at a load of 120 Kips. The behavior before the diagonal tension failure was typical of the other beams, except that this beam showed slightly greater deflections at corresponding loads. This may be due to the fact that the maximum load was also lower than the other beams. The load dropped to about 70 Kips after failure at deflections close to 0.45 inches. However, the beam continued to carry about 59 Kips to about 1 inch deflection. The behavior at this stage was very ductile and the beam seemed to be capable of taking this load for fairly large deflections. The load was also sustained for a considerable times and the deflections were not as rapid so that readings of deflections could be taken at all five points. This shows that the beam could carry about 60 Kips of load for quite some time.

The preceding analysis shows that Beam 5 and 6 showed perhaps the best behavior out of all the beams. Table 22 gives a comparison of deflections at mid-span for all the beams in terms of the deflections at initial cracking and final failure. The values of Beam 1 cannot be considered to be very accurate. <sup>The</sup> maximum ratio of deflections at maximum load to deflection at initial cracking was obtained for the ~~second~~ beam (No.3 stirrups). This ratio cannot be taken as having direct significance in the behavior of the beams as the behavior after maximum load is more

TABLE - 22COMPARISON OF DEFLECTIONS AT MIDSPAN

BEAM NO.	Deflection at Initial cracking $\Delta_i$ (inches)	Deflection at failure $\Delta_f$ (inches)	$\frac{\Delta_f}{\Delta_i}$
1	0.301	0.592	1.97
2	0.120	0.629	5.24
3	0.122	0.480	3.94
4	0.116	0.524	4.50
5	0.157	0.505	3.22
6	0.139	0.367	2.64

important.

In the pictures shown in Chapter VII, crack formation and cracked beams could be seen. It will be observed that there was one major crack which caused failure in most of the cases (except for Beams 3 and 5). This crack usually appeared as a diagonal crack extending from one support to the load point. The photographs can be used as a guide to estimate roughly the number of stirrups across the major cracks of the beams. The participation of some stirrups in the free body to evaluate the shear force taken by the web reinforcement becomes doubtful when a stirrup is cut by the major crack in the vicinity of the tension reinforcement. This aspect, however, was not important for the present study.

CHAPTER IXDISCUSSION

This Chapter is mainly devoted to a critical study of the tests and test results. Factors which could have improved the performance of the tests are also discussed and reference is also made of the diagonal tension studies which have been conducted to work out the performance of different types of shear reinforcement.

A fairly detailed account of the diagonal tension failures has been given in Chapter IV. Clark (2) realized that the distribution of web reinforcement is also a factor in the ability of the beam to resist failure by diagonal tension. During these tests, it was noticed that initial diagonal cracks appeared at about 30% of the maximum load for the beam. Clark reported cracks at 40% of the maximum load which developed into diagonal cracks. However, during his tests on the beams with web reinforcement, he found small cracks near the center of the span when the applied load was approximately 20% of the maximum load. No such cracks were found in the present study. The vertical cracks in the central portion always initiated after initial diagonal cracks had appeared.

It could be seen from the propagation of cracks and especially after the reinforcement had been exposed after failure that there was considerable distortion of the web reinforcement, particularly in the region where the main inclined crack crossed the stirrups. There was visible shift of stirrups from concrete in this zone. It could be concluded that after the yield point was reached in one stirrup leg, the stress increased in adjacent stirrups indicating that the tension was being taken up by the other stirrups. Previous studies by Clark (2)

and others, have indicated that the loading condition is an important factor.

Previous studies have also indicated that the shear capacity of a beam increases to a certain extent with the strength of the concrete when the other factors are constant. Kani (71) has, concluded however, that for the specimens in his study, the concrete strength had little effect on the shear strength of concrete. The nominal concrete strength for present study was 3500 psi and the maximum variation in the actual values was about 16%. The actual values were normally on a higher side. The variation in compressive strength is not excessive and it may be assumed that this difference did not have any bearing on the ultimate diagonal tension failures of the beams. The tensile splitting tests also showed a comparatively close agreement.

Studies conducted by Moretto (1) on the strength of welded stirrups in reinforced concrete beams indicated that the shear strength of welded stirrups may be expected to be between 10 and 25 percent greater than that of the loose stirrups. Beam 5 had inclined stirrups, but the stirrups were not welded to the longitudinal reinforcement in order to keep the uniformity in all the beams. Connections of inclined stirrups with longitudinal stirrups are somewhat difficult, but the results did not show any slippage of the bars. Beam 5 already showed a better performance than vertical stirrups and welded stirrups could further increase the strength in shear.

It may be noted that until diagonal tension cracks form, the stresses in the tension steel and in the concrete are distributed along



the length of the beam in the same way as the external moments so that these stresses at any section are approximately proportional to the moment at that particular section. The formation of diagonal tension cracks changes these relationships. Such changes are called the redistribution of internal stresses (9). Thus in a beam with diagonal cracks, the distribution of stresses in the tensile reinforcement and in the concrete along the beams does not follow the distribution of external moments. This has been discussed in Chapter IV. Failure may result due to diagonal tension, destruction of the compression zone, bond or splitting. During the redistribution of internal forces, the formation of the diagonal tension crack is accompanied by a considerable increase of the tension force in the reinforcement. This may, sometimes, lead to a bond failure, such as in beam 3. An anchorage of the reinforcement, such as hooks, has been found to reduce the danger of this type of failure.

Generally, numerous short diagonal cracks formed since the stirrups distributed the cracks; more load was required for the cracks to develop and to penetrate into the compression zones, and thus to cause crushing of concrete. Inclined stirrups were more effective in distributing diagonal cracks than were vertical stirrups.

The phenomenon of diagonal tension cracking is one which involves a combination of flexural and shear stresses. Various attempts have been made to express this phenomenon in terms of a rational theory based on the ordinary theory of flexure, but they have not yielded any solution of general applicability.

It has been observed in tests of simple beams (9) that failure occurred by destruction of the compression zone over a diagonal crack, always near that end of the shear span at which the moment was greater.

These tests may suggest that the ultimate moment at this critical section reduces shear capacity. A semi-rational analysis was attempted by Moody (15) to express shear strength in terms of the ultimate moment.

Rensaa (32) commented on shear conditions where both bending and shear are maximum. The direction of cracks at such places will be much steeper than  $45^{\circ}$ , and it is not certain that cracks will be crossed by an ordinary vertical stirrup. The ordinary formula for shearing stresses does not give a true picture of the shearing conditions. Rensaa recommended more investigation and tests on this problem. This situation exists in the present series of beams under the load points.

One of the points noticed during all of these tests was that though the diagonal cracks started on both shear spans of the beams and extended upwards with an increase in the load, final failure occurred only on one side of the beam with the other side badly cracked, but still intact. It can be explained from the fact that concrete is a complex, multiphase material containing minute cracks and other flaws. Thus there are stress concentrations inherent in concrete. These are not homogeneously distributed and may result in small differences of strength. Also, there is always a possibility of the load not being applied at the exact third points. This could result in slightly heavier shears in one span.

Vertical stirrups have been commonly used as shear reinforcement for a long time, and there is available a considerable amount of experimental data on the strength of beams with a variety of stirrup types, sizes, and spacings. The generally accepted function of stirrups is to resist the opening or widening of the diagonal tension cracks,

and thus to prevent the failure of a reinforced concrete beam due to shear. It may be relevant to observe in this connection that as long as more than 40 years ago it was shown that beams differing only in web reinforcement exhibit first cracking at the same load (53-H). It is only after this cracking has started that the behavior of the beams depends on the type of web reinforcement, and the ultimate strength in shear is a function of this reinforcement. Failure may occur by flexure at midspan or in shear. The shear failure may be caused by crushing of the concrete at the top of the diagonal tension crack; by the destruction of the tension zone between the lower end of the diagonal crack and the beam support or by the opening of a flat-slope crack up to the top surface of the beam. Once the redistribution of moments has started, the shear strength of a beam depends on the strength of the compression zone above the diagonal tension crack and of the tension zone at the lower end of the crack. If both of these are strong enough the beam may fail in flexure at midspan, despite the apparently insufficient web reinforcement which resulted in the yield of the stirrups. This type of failure was observed in Beam 5 of present series of tests where failure occurred at a load of 165 Kips. Similar failures have been reported by Clark (2) in case of the beams C2-3, D1-2, D2-3, and D2-4.

It is known that the ratio of the shear span to the effective depth is important in determining the type of failure which can occur in a beam. This has been discussed by Clark (2) and Taub and Neville (56). Span depth ratio was constant in this study in order to eliminate its effect.

It has been observed that the higher the percentage area of web reinforcement, the higher the load carrying capacity of the beam. At the same time, it is important to note that the increase in capacity is small,

and, in particular, is not proportional to the total amount (weight) of the stirrups. This is so because the ultimate resistance of a beam to shear is only in part due to the stirrups, and the beam can take considerably more load after they have yielded. This action of the stirrups is shown, for instance, by a comparison of two beams tested by Moody, Viest, Elstner, and Hognestad (9, 14, 15), which differed only in the cross-sectional area of vertical stirrups. Beam III - 30 had  $r=0.52\%$ , while in Beam III - 31,  $r=0.95\%$ . The corresponding values of the ultimate load were 215 and 228 Kips, respectively - a negligible increase for almost a doubling of the weight of stirrups. For the purposes of the present study, the weight of reinforcement used was practically a constant (about 0.55%) with the variations being within a range of 20%. It may thus be conceived that this small variation in the ratio of web reinforcement (Table 1) can be neglected when the ultimate loads are compared.

Taub, and Neville (56) observed that the action of the stirrups in preventing the pressing down of steel depends more on their layout than on their size, or within limits, on their spacing.

They also argued that the greater the number of stirrups crossed by a crack, the less it opens and the later occurs the yield of the stirrups, and hence the higher the ultimate load on the beam. It is, however, difficult to analyze the results of present tests in this connection. The smallest size of stirrups used for this study was 1/4" and Beam 3 had all stirrups of this size. The spacing for this beam was also 3 inches, which was the least. The steel used for the stirrups was not very ductile and had an average elongation of 5.5% with a very high yield of more than 70 Ksi. This beam failed at 160 Kips but the final failure was not a shear-tension failure, but a secondary bond failure.

Beam 1 with No. 4 stirrups failed at 151 Kips, and Beam 2 with No. 3 stirrups failed at 161 Kips, whereas Beam 4 with a combination of stirrups failed at 165 Kips. Beam 3 would probably have carried more load if bond failure had not resulted. The failure load of 160 Kips is very close to other beams, and it is difficult to comment on the effectiveness of such close spacing. It could be said, however, that closer spacing would not give lower results. Also, it has been observed (25) that stirrups usually made of mild steel are preferable to high tensile steel. Cold drawn steel becomes brittle due to cold-hardening, and is not suitable for bending. Larsson (25) observed cracks, which could not be easily detected by a naked eye, in the corners of cold-drawn stirrups, and beams made with such stirrups failed prematurely and suddenly in shear. Considering that No. 2 stirrups of Beam 3 were high tensile, cold drawn steel, the failure load of 160 Kips and even that due to a secondary bond failure argues well for closer spacing.

Elstner, Moody, Viest, and Hognestad's tests (9, 14, 15) on rectangular restrained beams with vertical stirrups and with inclined stirrups, at the same horizontal spacing, show that, taking average values, the inclined stirrups give an approximately 35% higher ultimate load than the vertical stirrups. The spacing of No. 3 vertical stirrups in Beam 2 was 5 inches. This spacing was increased to 7 inches for 45° inclined stirrups of the same size in Beam 5. Beam 2 failed at a load of 161 Kips and Beam<sup>5</sup> failed at a load of 165 Kips. Also the failure of Beam 2 was due to diagonal tension, whereas Beam 5 failed due to flexure, though there was extensive diagonal cracking before failure.

Noting that the spacing of inclined stirrups was reduced and even then

they showed higher ultimate load provides evidence that inclined stirrups are better than the vertical stirrups. This shows that the vertical stirrups alone cannot always economically guarantee a full protection against shear failure because they do not lie in a direction normal to the diagonal tension crack.

Taub and Neville (52) have given some test results on orthogonal web reinforcement. This system of reinforcement consists of vertical stirrups combined with horizontal bars capable of resisting the horizontal component of the principal tension. This case of reinforcement is considered most suitable for comparison with Beam 6 composed of the wire-meshes.

Taub and Neville concluded that the destructive action of the force in the main steel at the lower end of the diagonal tension crack in a beam with orthogonal web reinforcement is smaller than in a beam with vertical stirrups only. The load carrying capacity of the former beam must, therefore, be higher than that of the latter. Experiments conducted by them showed a 25% increase in the load carrying capacity of the beam, the increase in the weight of steel used being barely 6%. The calculated value of the stress in the main steel of the beams with orthogonal reinforcement exceeded the yield stress of that steel, so that flexure-tension failure was unavoidable.

Rausch (74-H) also discussed the possibility of the use of orthogonal shear reinforcement in 1953 on the basis of multiple truss analogy. Walther (23) compared in 1956 beams with vertical stirrups and bent-up bars and beams with orthogonal web reinforcement, using three types of steel.

It should be noted that when there is more than one layer of

reinforcement present, the longitudinal tension in the bars below the neutral axis, arising from the applied bending moment, has to be taken into account.

Taub and Neville, and Rusch used standard size bars for the vertical and horizontal bars of the orthogonal reinforcement. On the other hand, the wire meshes employed in Beam 6 were of small-diameter wires and consequently the mesh size was only 2 in. square and three meshes were used across the width of the beam. This resulted in a fairly homogeneous spreading of the reinforcement across the entire beam. Analysis of the results in Chapter VIII already gives an indication of their superior performance. The effectiveness of the wire meshes may also be considered by taking into account the load carried by the beam after initial cracking. This can be compared with a corresponding value of the other beams. It can be seen that though the effective web resistance of this beam was only 39.6% of Beam 1, it carried about 76% of the load after initial cracking for Beam 1.

As the web steel in Beam 6 was less than the other beams, it is difficult to establish the exact effectiveness of the wire meshes, but the performance of wire meshes seems to be fairly good. This agrees with the findings of Taub and Neville (52) for the orthogonal reinforcement.

The uniform protection against shear throughout the length of the beam, provided by the wire meshes (as orthogonal reinforcement), is of considerable importance because, in beams subjected to uniform or unsymmetrical loading and reinforced against shear in the zone of a high shearing force only, failure in shear can occur where the shearing force acting is lower than the maximum on a beam. (56)

An attempt was made in the last Chapter (Chapter VIII) to consider

the costs involved. Wire meshes involved a lot of effort in joining the horizontal and vertical wires at a small interval of 2 inches. Labour involved in making wire meshes will definitely be greater. However, if this system of reinforcement becomes common, such meshes may be produced at a mass scale. They may then be rolled for ease in storage and a required size of mesh can easily be cut from them. The economic aspect of the comparison is difficult as so much depends on the relative costs of material and labour.

It may be observed that the use of inclined stirrups permits the main steel to continue up to the support, and this has shown to increase the load carrying capacity of a beam in shear. It is important that the inclined stirrups are well anchored, otherwise they will not be effective in taking over the horizontal force transmitted by the main steel.

The description of the mode of failure of Beam 3 has shown the importance of the end hooks when the shear reinforcement of a beam is otherwise inadequate, and the beam is so proportioned that incipient shear-tension failure arises. Several experiments have been conducted (56) to determine the influence of hooks on the strength of beams. The problem of bond is very closely related to the shear-tension failure of reinforced concrete beams. It has been shown that in normal beams, bond failure is never a primary cause of failure of a reinforced concrete member subjected to both flexure and shear, but merely a consequence of the redistribution of internal forces following the widening of the diagonal tension crack. (32)



CHAPTER XSUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH10.1 Summary

This investigation of diagonal tension embodies the tests of six simply supported concrete beams, subjected to third point loading. These beams were all of the same size, being eight inches wide, eighteen inches deep and eight feet long. The longitudinal reinforcement consisted of six No. 7 bars in all the beams. The amount of web reinforcement was also practically kept constant, except for beam 6, and the size, distribution and pattern of web reinforcement was varied. Beam 6 consisted of three wire meshes, homogeneously spread in the beam. This reinforcement was less than the other beams by about 25% by weight. The width of the beam limited further use of meshes because of difficulties in pouring the concrete.

The ratio of web reinforcement was 0.55% for most of the beams. An attempt was made to keep the effective resistance of web steel as constant. Physical properties of the reinforcement were determined before use in the beams. Beam 1 consisted of web reinforcement of No. 4 size stirrups @9"c/c. This spacing was changed to 5" in beam 2 where No. 3 size stirrups were used. Beam 3 consisted of No. 2 stirrups spaced at intervals of three inches. Beam 4 was a combination of stirrups used in first three beams. No. 3 stirrups inclined at 45° were used for beam 5, spaced at intervals of 7 inches, to keep the ratio of web reinforcement as constant. Beam 6 was composed of three wire meshes, 2 in. square, for use as web reinforcement. The verticals consisted of 0.156 in. dia. wire and the horizontals were 0.115 in. dia. wire. The yield strength of these wires was lower than the stirrups and the

effective web resistance of this beam was 39.6% of Beam 1.

The beams were loaded to failure in 5000 lb. increments. Deflection readings were taken at five points along the length of the beam, i.e. centers of end spans, under the load points, and at midspan. Load-deflection curves were plotted for midspan for all the beams. Crack pattern and the propagation of cracks were marked during the tests and loads were recorded on the cracks. Photographs were taken during loading and at failure of the beams. The beams were continued to be loaded after failure and deflections were recorded for the midspan. The results of the tests were analyzed and tables were made to compare the total steel used in the beams as well as to compare the total resistance of the beams. Midspan deflections at initial cracking and final failure were also compared.

Four of the beams, i.e. 1, 2, 4, and 6 failed in diagonal tension. Beam 3 failed due to a secondary bond failure with the diagonal cracks penetrating deep into the compression zone of concrete. Beam 5 failed due to flexure, though the diagonal cracks caused initial cracking and had almost completely penetrated the compression zone of concrete.

Diagonal tension failures were caused by one major crack, inclined and extending from a support to the load point.

The ultimate load at which failure occurred varied from 151 Kips to 165 Kips for Beams 1 to 5. Beam 6 failed at 120 Kips. The average shear stresses at failure,  $v_u$ , varied from 590 psi to 645 psi for Beams 1 to 5. The average shear at failure for Beam 6 was 470 psi. The ratio of ultimate load to the load causing initial cracking was also recorded.

This ratio varied from 2.40 to 3.30.

The behavior of different beams was analysed to determine which type of web reinforcement is most effective. It appeared that the inclined stirrups are more effective than the vertical stirrups and orthogonal reinforcement consisting of wire-meshes is also very effective against diagonal tension failure.

## 10.2 Conclusions

Shear failure occurs in beams in which the main tension reinforcement and the concrete in the compression zone are strong enough to resist the stresses induced by the moment applied while, as a result of the shearing force applied, severe diagonal tension cracking takes place.

The diagonal cracking may lead to failure in a variety of ways. The diagonal tension crack generally opens first in the mid-depth of the beam, and extends in an inclined direction. When the load applied increases, the diagonal tension crack increases in length and width. As the lower end of this crack reaches the main tension reinforcement, the balance of internal forces alters and redistribution of internal forces takes place. A further increase in load leads to shear failure in one of several ways.

In shear-compression failure, the extension of the upper end of the diagonal tension crack reduces the compression zone until crushing of concrete takes place. In shear-tension failure, it is the lower end of the diagonal tension crack that extends, leading to splitting of the concrete at the level of tension steel. In a beam with vertical stirrups horizontal splitting cannot take place as it is restrained by the stirrups. When stirrups are used as web reinforcement, the composite action of the concrete and the reinforcement is maintained in the part of the beam between the diagonal tension crack and the near support, and the

beam can resist an increase in the applied load beyond that which caused the yielding of the stirrups. As long ago as 1935, Evans (61-H) observed that "provision of web reinforcement preserves the homogeneity of a beam in flexure at higher loads".

When stirrups are provided, the opening of the initial diagonal tension crack is followed by formation of other diagonal cracks and by a considerable increase in the applied load before failure occurs. A sudden collapse of the beam does not take place. This fact is of tremendous importance and would warrant the provision of web reinforcement in all beams regardless of the value of the nominal shearing stress; should the beam be accidentally overloaded or subjected to a load distribution different from that assumed in design, the provision of stirrups would prevent failure without warning from taking place.

Wilby's (70-H) test data show that the load-carrying capacity of beams increases as the spacing of stirrups inclined  $45^{\circ}$  decreases, for a constant percentage of web reinforcement. In the present study, however, only one beam had inclined stirrups and so it is not possible to compare the spacings of different sizes of web reinforcement as inclined stirrups for a constant percentage of web reinforcement. A comparison is, however, made with the vertical stirrups. The spacing of the inclined No. 3 stirrups was reduced from 5 inches for the vertical stirrups to 7 inches for inclined stirrups. This kept the ratio of web reinforcement to be exactly constant in the end shear spans. In the midspan, the spacing was somewhat reduced to get a good cage of the reinforcement (see Fig. 11 for a sketch of the reinforcement). The overall total weight of the reinforcement was 100.90%, (Table 20) of the first beam. In terms of the effective wt. (inclined legs of stirrups), it was 106.8% of Beam 1 (Table 21). The ultimate failure occurred at the midspan due to flexure at 165 Kips. This load is 9.3% greater than

Beam 1, which failed at 151 Kips. The behavior prior to and after the final failure was also much better with relatively slow crack propagation. On the basis of these results it can be concluded that inclined stirrups are more effective than the vertical stirrups for the same amount of steel used as web reinforcement. The Beam 5 would have carried some more load before a diagonal tension failure if flexural failure had not occurred at 165 Kips. This conclusion is also in agreement with some previous studies, including that done by Elstner, Moody, Viest and Hognestad (9, 14, 15). The inclined stirrups gave 35% higher results for the same horizontal spacing. The reduction in spacing in the present case was 40% of the original 5" spacing for No. 3 stirrups used in beam 2. Even then the inclined stirrups appear to be more effective. Hence for the same horizontal spacing, the inclined stirrups are definitely more useful against diagonal tension failures. The inclined stirrups were not welded to the main reinforcement in order to observe the behavior under similar set of conditions as for the other beams. Moretto (1) concluded that welded inclined stirrups give more resistance to diagonal tension failures than loose stirrups.

It is difficult to draw any conclusions for the most suitable size of the stirrups from the present study. This is made difficult by the fact that No. 2 stirrups used in beam 3 were cold rolled. They were relatively brittle with a very high yield point. Larsson (25) observed that stirrups usually made of mild steel are preferable to high tensile steel. Larsson observed cracks in the corners of cold drawn stirrups which were not visible to the naked eye. Such stirrups failed prematurely in stirrups. Thus, according to his study there was an inherent weakness in beam 3 due to brittle stirrups.

The spacing for No. 2 stirrups was chosen as 3 inches. This gave

a lower ratio of web reinforcement, but this was done to keep the effective resistance as constant, keeping in view the high yield strength of No. 2 wires. The total steel used as web reinforcement for beam 3 was only 76.1% of Beam 1, but the effective resistance increased to 111.0% of Beam 1. This beam failed at 160 Kips due to a secondary bond failure. Cracks which caused ultimate failure appeared at 160 Kips over the supports and in the cross section of the beam. The beam deflected at the same load and failure occurred. Diagonal tension failure would probably have occurred at a higher load. The ultimate loads for Beam 1 and 2 were respectively 151 and 161 Kips. The load for No. 3 stirrups was thus 6.7% greater than for No. 4 stirrups. No. 2 stirrups showed a 6.0% higher value as the beam failed at 160 Kips. The possibility exists, however, that this load is not representative for No. 2 stirrups due to their brittle nature. All the stirrups along the crack were broken at failure. A ductile behavior would have taken the stirrups into strain hardening range with a possibility of higher loads. The failure load of No. 2 stirrups is only one Kip lower than No. 3 stirrups. The effective resistance of No. 3 stirrups was 113.8% of Beam 1 as against 111.0% for No. 2 stirrups in Beam 3. On the basis of these results it is difficult to say that closer spacing increases the resistance against diagonal cracking. Taub and Neville (56), however, concluded that the greater the number of stirrups crossed by a crack, the less it opens up and hence shows higher ultimate loads. The writer would recommend another series of tests to determine this factor, selecting the yield strength of different sizes of reinforcement very nearly the same and for the same percentage of effective web resistance. Even the present series of tests gives an indication that the closer spacing of stirrups may be more useful for resistance against diagonal cracking. Steel with lower percent elongation should be avoided as reinforcement where ductility is important due to the sudden nature of the failure.

Beam 4, which consisted of a combination of different sizes of stirrups failed at 165 Kips. Total web steel was 84.2% of beam 1, and total web resistance was 99.5% of beam 1. The results showed 9.3% higher ultimate load. This shows that a combination of stirrups of No. 2, No. 3 and No. 4 for the same amount of web reinforcement is more effective than any individual size. The cracks causing failure passed through No. 2 and No. 3 size stirrups.

The orthogonal reinforcement used in beam 6, consisting of three wire meshes, proved to be quite effective against diagonal cracking. An absolute comparison with other beams is not possible as the amount of steel used in this beam was less than the other beams. Total steel in this beam used as web reinforcement was 74.2% of beam 1 and the total web resistance of the reinforcement dropped to 45.4% (Table 21) of beam 1 due to lower yield strength of wires used in the meshes. The final failure due to diagonal tension occurred at 120 Kips as against 151 Kips for beam 1. This value is about 80% of beam 1. This shows that wire meshes can better resist all the forces in the direction of the principal stress. Taub and Neville (52) reached the same conclusion, though they used bigger bars for orthogonal reinforcement. The uniform protection against shear throughout the lengths of the beam, provided by the orthogonal reinforcement, especially by small meshes, is of considerable importance because there is a possibility of shear failure where shear is lesser than maximum on a beam in case of beams subjected to uniform or unsymmetrical loading (56).

The ratio of designed moment capacity to shear capacity for laboratory tests should be high to guard against any flexural failures. Most of the design methods for shear reinforcement seem to be conservative in approach.

The longitudinal reinforcement may also be provided with hooks to

avoid any possibility by secondary bond failure. Taub and Neville (56) showed that hooks at the ends of main bars, especially plain, can materially increase the load carrying capacity of the beam.

The use of orthogonal web reinforcement in continuous beams would also appear to be particularly advantageous: the difficulties of using bent-up bars in the part of the beam where the point of contraflexure may occur are avoided, and the tension reinforcement can be placed entirely to suit the bending moment.

### 10.3 Suggestions for Future Research

The present series of tests for determining the effect of size, pattern and distribution of web reinforcement have been carried out for the simple beams with third point loading. The limitations of time did not allow all the possible arrangements of web reinforcement to be compared. It could be better to conduct a more extensive program not only for the simple rectangular beams, but also for the restrained beams and beams of different sections. Bent-up bars were not used in the present study, but they can be very important for such studies, especially for continuous beams. Combinations of vertical stirrups and bent-up bars can also be considered for any further studies. It would also be useful to compare the orthogonal reinforcement of these wire meshes with bigger size orthogonal reinforcement for the same amount of reinforcement. Inclined stirrups at inclinations less than, or greater than  $45^{\circ}$ , to the horizontal can also be included in the study. It would also be more realistic to study the behavior of types of web reinforcement for uniform or unsymmetrical loading.

The published research on the suitability of orthogonal reinforcement, and especially wire meshes is insufficient to be conclusive and some further tests in this regard would be appropriate to develop a better understanding of the behavior of such reinforcement against



diagonal cracking.

A basic difference between vertical stirrups and wire mesh, during loading, after the cracks have formed was observed during the tests. It was noticed that the wire meshes seemed to slow down the cracks. There was considerable kinking in the wires at failure. This was not as noticeable in case of stirrups.

Future research in this field should be oriented towards developing a better understanding of the behaviour of the reinforcement and concrete after the cracks have developed. One of the points about which no conclusive evidence was observed during the present study concerned whether there was any change in the relative direction of displacement between elements on either side of the diagonal tension crack as the crack progressed. Future research on these lines is essential to develop a better understanding of the mechanism of shear failure.

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