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A STUDY OF STUDENT'S t-DISTRIBUTION UNDER NON-NORMALITY

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ABSTRACT

Student's t-distribution provides a well known test (called Student's t-test) for making exact probability statements about the mean of a population. While this distribution is only accurate for samples drawn from a normal population, it is frequently applied to populations whose distributions are either known to be non-normal, or are unknown and may or may not be normal. The purpose of this study is to examine how accurate this distribution is when applied to populations which are not normally distributed.

The validity of this distribution is first examined for populations which are moderately non-normal and then for certain populations which have an elementary standard non-normal distribution. From a more mathematical point of view, Student's distribution is compared to the t-distribution for distributions which can be specified by the first four terms of the Edgeworth series. Consideration is then given to a compound normal distribution.

Finally, some alternative tables are presented which may be used instead of Student's tables when the distribution is not normal but known to be similar to one of the distributions in the appendix.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

THE PROBLEM OF NON-NORMALITY IN SMALL SAMPLES

If $\bar{x}$ is the mean of a sample of size n drawn from a normal population whose mean is μ and if σ^2 is the variance of this population, then the quantity

$$\frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

is known to be normally distributed with mean zero and unit variance. This quantity serves as a suitable test criterion for making exact probability statements about the population mean.

One serious defect of this test is that in practical situations σ^2 is seldom known and although methods for obtaining precise estimates of the variance are available, they require large samples and thus are impractical to apply due to excessive costs or difficulties in repeating experiments. In a paper on the errors of small samples, Student (1908) noted:

> "The usual method of determining the probability that the mean of the population lies within a given distance of the mean of the sample, is to assume a normal distribution about the mean of the sample with a standard deviation equal to $s/\sqrt{n}$, where s is the standard deviation of the sample, and to use tables of the probability integral."

This method was unsatisfactory since the estimate of the standard error is subject to increasing error as n becomes small and

$$\frac{\bar{x} - \mu}{s/\sqrt{n}}$$

does not necessarily follow the standard unit normal distribution. The limiting form of the distribution of this quantity as n becomes large, is however, the normal distribution and

> "...although it is well known that the method of using the normal curve is only trustworthy when the sample is 'large', no one has yet told us very clearly where the limit between 'large' and 'small' samples is to be drawn." (Student, 1908)

In the same paper Student showed that in a sample drawn from a normal population the sample mean $\bar{x}$ and sample variance s^2 are independent. He also derived the probability density function (hereafter abbreviated p.d.f.) f(z) of the statistic

$$z = \frac{\bar{x} - \mu}{s} .$$

From this p.d.f. definite probability statements could be made about the deviation of a sample mean from the population mean, irrespective of the population variance.

Student's statistic z was modified by Fisher (1923) to

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

which is now known as "Student's t". The p.d.f. of t was found to be

$$f(t) = \frac{\Gamma(\frac{n}{2})}{\sqrt{\pi(n-1)}\,\Gamma(\frac{n-1}{2})} \; \frac{1}{\left[1 + t^2/(n-1)\right]^{n/2}}$$

where n-1 was referred to as the "number of degrees of freedom". This distribution is independent of the population variance and approaches the normal distribution in the limit as $n \to \infty$.

The integral

$$\int_{t_o}^{\infty} f(t)\, dt$$

is the probability of drawing from a population with mean μ a sample of size n for which

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} > t_o,$$

$\bar{x}$ and s^2 being the sample mean and variance respectively. Tabulated values of this integral have been calculated for normal parent distributions and can be found in most introductory text books on statistics and are usually referred to as "Student's t-tables".

To test the hypothesis

$$H_o : \mu = \mu_o$$

against the alternative hypothesis

$$H_1 : \mu \neq \mu_o$$

we must calculate the statistic

$$t = \frac{\bar{x} - \mu_o}{s/\sqrt{n}}$$

and compare this value with a tabulated value. The tabulated value t_o used for comparison in this test depends on n and on the level of significance desired. The level of significance, denoted as α, is the probability of obtaining a value of $|t| > t_o$ by chance when μ_o is the true mean. We define $t_{.025}$ and $t_{.05}$ by

$$\int_{t_{.025}}^{\infty} f(t)\,dt = 0.025$$

and

$$\int_{t_{.05}}^{\infty} f(t)\,dt = 0.05$$

and note that, due to the symmetry of f(t), we have

$$\int_{t_o}^{\infty} f(t)\,dt = \int_{-\infty}^{-t_o} f(t)\,dt.$$

Then if α is chosen as .05 for example, the test statistic t must be compared with the tabulated value of $t_{.025}$ with n-1 degrees of freedom. If $|t| > t_{.025}$ this means that the probability of obtaining a sample with an absolute value of t equal to or larger than that of the observed value from a population with mean μ_o is less than .05.

We then say that the observed value of t is significant and H_o is rejected. Is, on the other hand, $|t| \leq t_{.025}$, then we say that the observed value of t is non-significant and H_o is accepted. This test is called a two-tailed t-test, since $|t| > t_{.025}$ can be written as $t < -t_{.025}$ or $t_{.025} < t$, so that both "tails" of the graph of f(t) are involved.

If the alternative hypothesis is

$$H_1 : \mu > \mu_o$$

and α is chosen as .05, then H_o is rejected if $t > t_{.05}$ and accepted if $t < t_{.05}$. Similarily if the alternative hypothesis is

$$H_1 : \mu < \mu_o$$

H_o is rejected if $t < -t_{.05}$ and accepted if $t > -t_{.05}$. Such a test is called a one-tailed t-test, for obvious reasons.

Bringing this test into some degree of correspondence with practical situations has caused some difficulty, however, since the condition of normality of means for samples of size n, upon which the derivation of f(t) was based, is not always satisfied. Violation of this condition will be more serious in small samples, while as the sample size gets larger, the distribution of the means approaches the normal distribution.

The purpose of this study is to examine the confidence with which Student's distribution can be used when the population is not normal. It will be assumed that the p.d.f. of any parent population can be approximated with sufficient accuracy by the first four terms of the Edgeworth series. (Cramér (1928) proved rigorously that the Edgeworth series gives a valid asymptotic expansion for the p.d.f. f(x) of any universe.) In other words, we will assume that the effects of the fifth and higher moments are negligible and thus that the non-normality of the population can be expressed in terms of the first four moments. The standardized p.d.f. of the parent population can be represented as

$$f(x) = \phi(x) - \frac{\gamma_1}{3!}\,\phi^{(3)}(x) + \frac{\gamma_2}{4!}\,\phi^{(4)}(x) + \frac{10}{6!}\gamma_1^2\phi^{(6)}(x)$$

where

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2}$$

is the p.d.f. for the standardized normal distribution,

$$\phi^{(\nu)}(x) = \left(-\frac{d}{dx}\right)^{\nu}\phi(x)$$

is the ν^{th} derivative with respect to x, and γ_1

and γ_2 are measures of skewness and excess kurtosis. (see page 8 for definitions of these measures.)

Unless otherwise stated, we will assume, furthermore, that γ_1 and γ_2 fall inside the Barton and Dennis (1952) limits of values for γ_1 and γ_2 for which the Edgeworth series are no longer positive definite.

Consideration is given first to the effect of a moderate departure from normality, followed by a study of the effects of using Student's tables for the t-test in the case of certain standard non-normal distributions.

From a more mathematical point of view, Gayen's (1949) form of the corrective terms due to population skewness and excess kurtosis is used to compare the true distribution of t with Student's distribution. A compound normal distribution is then examined and it will be shown that Gayen's form can be applied to find the p.d.f. for populations of this type.

Finally, an appendix is included which contains some of the tables of the t-distribution that were calculated with the help of Gayen's corrective terms.

CHAPTER II

EFFECT OF A MODERATE DEPARTURE FROM NORMALITY

Departure from normality in the parent population is commonly evaluated by measures of skewness and kurtosis. Skewness, which is a measure of departure from symmetry, is given by

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}$$

and kurtosis, a measure of "peakedness" is given by

$$\frac{\mu_4}{\mu_2^2}.$$

Since for a normal distribution the kurtosis is 3, a better measure for non-normality is the excess of kurtosis, (hereafter called excess) given by

$$\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3.$$

In the above expressions μ_r represents the r^{th} moment about the mean. For a normal population γ_1 and γ_2 are both equal to zero. The form of γ_1 and γ_2 for the means of samples will be derived in this chapter.

The effect of skewness and excess on the following applications of the t-distribution will then be examined:

1. Testing the assumed mean of a population.

2. Testing the difference between the means of two samples.

3. Testing a correlation coefficient.

4. Power of the test.

Skewness and Excess For Means Of Samples

The first step in deriving γ_1 and γ_2 for the means of samples is to find the moments about zero for the means. Define m_r and $m_{r\bar{x}}$ *) as the r^{th} moment about zero for the original population and for the population of sample means respectively.

$$m_{1\bar{x}} = E\left(\frac{\sum x}{n}\right) = m_1$$

$$m_{2\bar{x}} = E\left(\frac{\sum x}{n}\right)^2 = \frac{m_2 - m_1^2}{n} + m_1^2$$

$$m_{3\bar{x}} = E\left(\frac{\sum x}{n}\right)^3$$

$$= \frac{1}{n^2}(m_3 + 3nm_2m_1 - 3m_2m_1 + n^2m_1^3 - 3nm_1^3 + 3m_1^3)$$

*) If a symbol such as γ_1 or m_r refers to a population other than the original population, an additional subscript will be included to indicate the distribution of the variable referred to, so that we will write γ_{1t} or $m_{r\bar{x}}$ etc., as the case may be.

$$m_{4\bar{x}} = E\left(\frac{\Sigma x}{n}\right)^4$$

$$= \frac{1}{n^3}(m_4 + 4nm_3m_1 - 4m_3m_1 + 3nm_2^2 - 3m_2^2$$

$$+ 6n^2m_2m_1^2 - 18nm_2m_1^2 + 12m_2m_1 + n^3m_1^4$$

$$- 6n^2m_1^4 + 4nm_1^4 - 6m_1^4)$$

From these non-central moments it is possible to obtain the moments about the mean, which are of prime importance in obtaining expressions for skewness and excess. The r^{th} moment about the mean (i.e. the central moment) is denoted by μ_r. The second, third and fourth central moments of the sample means in terms of the non-central and central moments of the population are derived next.

$$\mu_{2\bar{x}} = m_{2\bar{x}} - m_{1\bar{x}}^2 = \frac{m_2 - m_1^2}{n} = \frac{\mu_2}{n}$$

$$\mu_{3\bar{x}} = m_{3\bar{x}} - 3m_{2\bar{x}}m_{1\bar{x}} + 2m_{1\bar{x}}$$

$$= \frac{1}{n^2}(m_3 - 3m_2m_1 + 2m_1^3) = \frac{\mu_3}{n^2}$$

$$\mu_{4\bar{x}} = m_{4\bar{x}} - 4m_{3\bar{x}}m_{1\bar{x}} + 6m_{2\bar{x}}m_{1\bar{x}}^2 - 3m_{1\bar{x}}^4$$

$$= \frac{1}{n^3}(m_4 - 4m_3m_1 + 6m_2m_1^2 - 3m_1^4) + \frac{3(n-1)}{n^3}(m_2 - m_1^2)^2$$

$$= \frac{\mu_4}{3} + \frac{3(n-1)}{n^3}\mu_2^2$$

The coefficients of skewness and excess for the population of sample means can now be expressed in terms of those for the original populations and the sample size n:

$$\gamma_{1\bar{x}} = \frac{\mu_{3\bar{x}}}{\mu_{2\bar{x}}^{3/2}}$$

which is equal to $\gamma_1/\sqrt{n}$, and

$$\gamma_{2\bar{x}} = \frac{\mu_{4\bar{x}}}{\mu_{2\bar{x}}^{2}} - 3$$

which is equal to γ_2/n.

These results agree with those cited by Shewhart and Winters (1928). As n approaches infinity, $\gamma_{1\bar{x}}$ and $\gamma_{2\bar{x}}$ approach zero and the distribution of sample means approaches the normal distribution, irrespective of the distribution of the parent population.

Although both $\gamma_{1\bar{x}}$ and $\gamma_{2\bar{x}}$ approach zero, $\gamma_{1\bar{x}}$ approaches zero at a slower rate than does $\gamma_{2\bar{x}}$. Thus if skewness has a greater effect on the t-distribution than excess, then skewness will be the determining factor in choosing the sample size large enough for a good approximation to the distribution of t for a normal population. If excess has a greater effect (and it will be shown that it does not), then either factor could be the determining criterion.

Testing The Assumed Mean Of A Population

The t-distribution frequently provedes a useful tool for the testing of hypotheses. This test has been described on page 3.

Regardless of which hypothesis is accepted when doing a test, the nature of sampling, that is, using only a small part of a population to draw conclusions about the population, implies that one can never be certain that his conclusion will be the correct one. In the case of a normal parent population, testing a hypothesis H_o regarding the value of the population mean against an alternative hypothesis H_1 gives rise to two types of error:

1. Rejecting H_o when H_o is true, called the type I error, and

2. Accepting H_o when H_1 is true, called the type II error.

The probabilities of the type I and type II errors are denoted by α and β respectively. While it is desirable to minimize α and β , it will be shown later in this chapter that, ceteris paribus, a decrease in α will cause an increase in β and vice versa.

If α is chosen as .05, then the type I error will occur in 5% of the tests when μ_o is the true mean. In a two-tailed test (which will be implied unless otherwise stated) half of these errors would occur by rejecting H_o when $t < -t_{.025}$ and half when $t > t_{.025}$.

Student's t-test and the conclusions drawn from it are, however, based on the assumption of normality in the parent population. If the parent population is not normal serious errors could result as the tabulated values will then not be the correct values. The probability and control of the two types of errors would also be different from those which were intended when α was chosen.

Consider Figure 2.1, page 15. Curve A depicts the situation where the parent distribution is normal. Then, because of the symmetry of the distribution $P(t \geq t_o) = \alpha/2$ and $P(t \leq -t_o) = \alpha/2$. that is, $P(t \geq t_o) = P(t \leq -t_o)$ and $P(t \geq t_o) + P(t \leq -t_o) = \alpha$. Since the population has a normal distribution the t-test is trustworthy and the conclusions drawn from the test are correct.

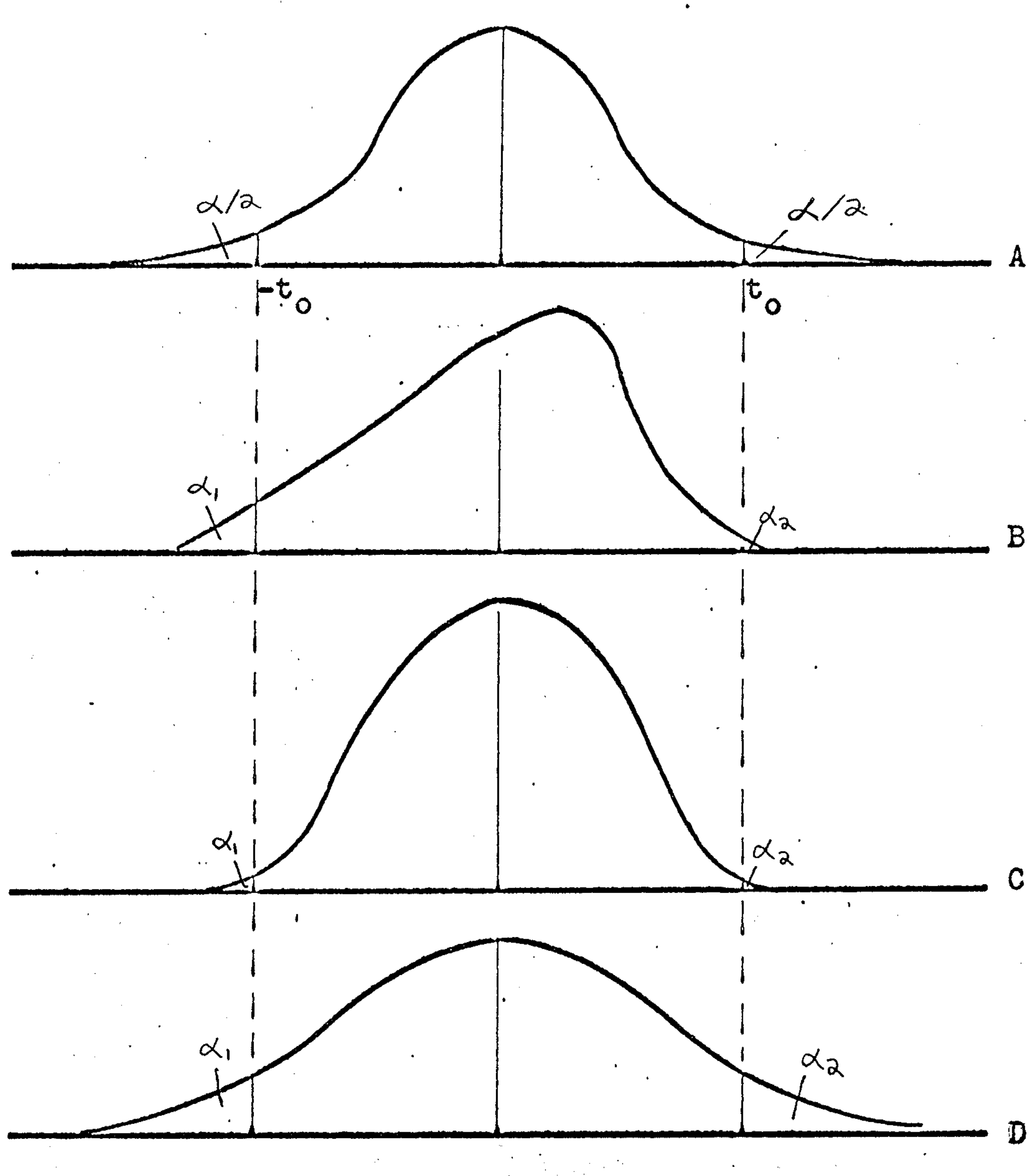
For a skew population as in curve B $P(t \geq t_o)$ and $P(t \leq -t_o)$ are not equal. (t_o is Student's tabular value for the chosen value of α and the proper number of degrees of freedom.) Pearson and Adanthya (1929) however, found in their empirical studies that control of the type I error would be almost as good as in the normal case if $P(t \geq t_o) + P(t \leq -t_o) \doteq \alpha$. Under these circumstances, however, H_o would be rejected too often when $t \leqslant -t_o$ and not often enough when $t \geqslant t_o$ or vice versa. For example, in the illustration considered, a value of t could be rejected as being too small when in fact it should not have been considered significant, whereas another value of t could be accepted although it should have been rejected. Unless the distribution is nearly symmetrical, however, the condition that $P(t > t_o) + P(t \leqslant -t_o) \doteq \alpha$ may not always be satisfied. In chapter IV it will be seen that the effect of γ_1 on the t-test is considerable and that a large deviation from symmetry will weaken conclusions that can be drawn from the test.

Curve C illustrates a leptokurtic population. If Student's t-tables are used to determine a critical value t_o for a given level of significance α, the true size of the type I error will actually be less than α. Since decreasing α will cause an increase in β as will be seen later in this chapter, having a smaller α than desired is not necessarily a desirable effect.

In a sample drawn from a platykurtic population, the true level of significance will be greater than the chosen α. It can be seen in diagram D that $P(t \geqslant t_o) + P(t \leqslant -t_o)$ is greater than α.

While it is obvious that departures from normality will affect the validity of Student's tables for the

Figure 2.1 An illustration of possible distributions.



t-test, both empirical and theoretical results referred to in this chapter have shown that for practical purposes these tables may be used confidently when only moderate departures from normality exist. Skewness has a more serious effect on the accuracy of the results than excess, but the results regarding $\gamma_{1\bar{x}}$ and $\gamma_{2\bar{x}}$ derived on page 11 indicate that the effect decreases as n increases.

Sampling investigations by Pearson et al. (1929) from distributions where $0 \leq \gamma_1 \leq 0.50$ and $-0.5 \leq \gamma_2 \leq 4.07$ suggest that no serious loss of control of the type I error occurs if Student's t-distribution is used for the test. In these investigations the control of the type I error is least satisfactory for the most leptokurtic population. He suggests, furthermore, that certain incomplete results indicate that for populations with greater skewness the distribution of t should be modified appreciably. For moderate departures from normality, however, use of Student's distribution for the t-test will be reasonably accurate.

Testing The Difference Between The Means of Two Samples

To test the hypothesis

$$H_o : \mu_1 - \mu_2 = \mu_d$$

against the alternative hypothesis

$$H_1 : \mu_1 - \mu_2 \neq \mu_d$$

when the sample sizes are equal, the test statistic t is given as

$$t = \frac{\bar{x}_1 - \bar{x}_2 - \mu_d}{\sqrt{(s_1^2 + s_2^2)/n}} .$$

Here n is the size of the two samples and s_1^2 and s_2^2 are the variances of the two samples.

If the sample sizes are unequal, say n_1 and n_2, this quantity is replaced by

$$t = \frac{\bar{x}_1 - \bar{x}_2 - \mu_d}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} .$$

A number of different situations can occur in testing the difference between two means. For example, one sample may come from a negatively skewed, platykurtic population while the other sample is drawn from a positively skewed, leptokurtic population. The populations could also both be normally distributed.

In most practical situations of testing the difference between two means, the populations could quite reasonably be assumed to have distributions of the same type, differing only in the values of their parameters.

Bartlett (1935), in a theoretical paper, states that if x_1 and x_2 come from populations with the same skewness, then the distribution of $x_1 - x_2$ is symmetrical and the effect disappears. This is an important point to notice since in Chapter IV we will see that the effect of γ_1 is likely to be more serious than that of γ_2. He adds, furthermore, that terms involving γ_1 and γ_2 in the p.d.f. of $\bar{x}_1 - \bar{x}_2$ are much reduced in testing the difference between two means, and in the important case where $n_1 = n_2$ terms involving γ_1 and γ_2 again disappear.

Boneau (1960) suggests that the t-test is functionally a distribution free test, provided that sample sizes are equal and not too small. If the two distributions are not the same, there is little difficulty if they are both symmetrical or have the same skewness. Only in the extreme case of opposite skewness would the t-distribution be markedly affected.

Pearson et al. (1929) in an empirical study concluded that no very serious error of judgement would be made by referring to Student's tables when examining the difference between means of pairs of small samples taken from moderately skew, leptokurtic or platykurtic populations if one of the samples contains at least five observations and the other at least ten.

Testing A Correlation Coefficient

The p.d.f. of a bivariate normal distribution is

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2} - \frac{2(x-\mu_x)(y-\mu_y)}{\sigma_x\,\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right]\right\}$$

where ρ is the correlation coefficient for the two populations. ρ is estimated by

$$r = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2\sum(y_i-\bar{y})^2}} .$$

r is the most suitable criterion for testing the hypothesis

$$H_o : \rho = 0$$

against the alternative

$$H_1 : \rho \neq 0$$

since, when $\rho = o$, the p.d.f. of r becomes

$$f(r) = \frac{\Gamma(\frac{n-1}{2})}{\sqrt{\pi}\,\Gamma(\frac{n-2}{2})} \quad (1-r^2)^{(\frac{n-4}{2})/2} \qquad \text{(a)}$$

With the substitution

$$t = \frac{r\sqrt{n-2}}{(1-r^2)}$$

this becomes

$$f(t) = \frac{\Gamma(\frac{n-1}{2})}{\sqrt{\pi}\,\Gamma(\frac{n-2}{2})\sqrt{n-2}} \qquad \frac{1}{[1+t^2/(n-2)]^{(n-1)/2}}$$

which is the p.d.f. of a t-distribution with n-2 degrees of freedom.

The statistic

$$t = \frac{r\sqrt{n-2}}{(1-r^2)}$$

is then the best criterion to test the hypothesis of zero correlation.

When x and y do not form a bivariate normal population, this test may be invalid because:

i) r may no longer be the appropriate criterion with which to test the hypothesis of no association, and

ii) the sampling distribution of r may no longer have the distribution (a). (E.S. Pearson, 1931)

Baker (1930) examined the validity of this test with some experimental sampling from an extremely skew population and found that r did not follow (a). Pearson (1931) found, however, that when the distributions of x and y are the same, the correlation coefficient, a criterion based on ratios, is remarkably insensitive to changes in population form.

When the null hypothesis is

$$H_0 : \rho = \rho_0 \; (\neq 0),$$

the distribution of r is more complicated and the t-test cannot be used to test this hypothesis.

Effect On The Power Of The Test

If β is defined as the probability of a type II error, then the power of the test is defined as $1 - \beta$, that is, the probability of rejecting H_0 when H_1 is true. Using a one-tailed test as an example we calculate the power of the test of the hypothesis

$$H_0 : \mu = \mu_0$$

against the alternative

$$H_1 : \mu = \mu_1, \mu_1 > \mu_0.$$

Putting

$$\bar{x}_\alpha = \mu_0 + \frac{t_\alpha s}{\sqrt{n}}$$

in which t_α is determined by $P(t > t_\alpha | H_o) = \alpha$ we have:

$$1 - \beta = \int_{\bar{x}_\alpha}^{\infty} f(t, H_1)\, dt$$

where $f(t | H_1)$ is the sampling distribution of t under H_1.

The size of $1 - \beta$ depends on a number of factors. Included among these are the value of α, the nature of the population and as will be explained later, the number of standard deviations that μ_o and μ_1 are apart. Define ψ as

$$\psi = \frac{|\mu_1 - \mu_o|}{s_{\bar{x}}}.$$

To illustrate the first and last of these, consider the following artificial probem:

Assume that a sample of size 9 is taken from a normal population and that $s^2 = 25$. Chose α to be .05 and test the hypothesis

$$H_o : \mu_o = 50$$

against the alternative

$$H_1 : \mu_1 = 55.$$

Then the estimated standard deviation of the mean $s_{\bar{x}}$ is

1.67 and $\Psi = 3$. This is a one-tailed test. The 5% critical value for rejecting H_o is

$$\bar{x} = 50 + (1.86)(5/3)s_{\bar{x}} = 53.1 .$$

The power of this test is

$$1-\beta = \int_{53.1}^{\infty} f(t/H_1)\, dt = 0.856$$

and $\beta = 0.144$. The relative size of α, β and $1-\beta$ are illustrated in Diagram A, Figure 2.2.

Now let $\alpha = .025$. The new critical value is

$$\bar{x}_{\alpha} = 50 + (2.306)(\frac{5/3}{3}) = 53.84$$

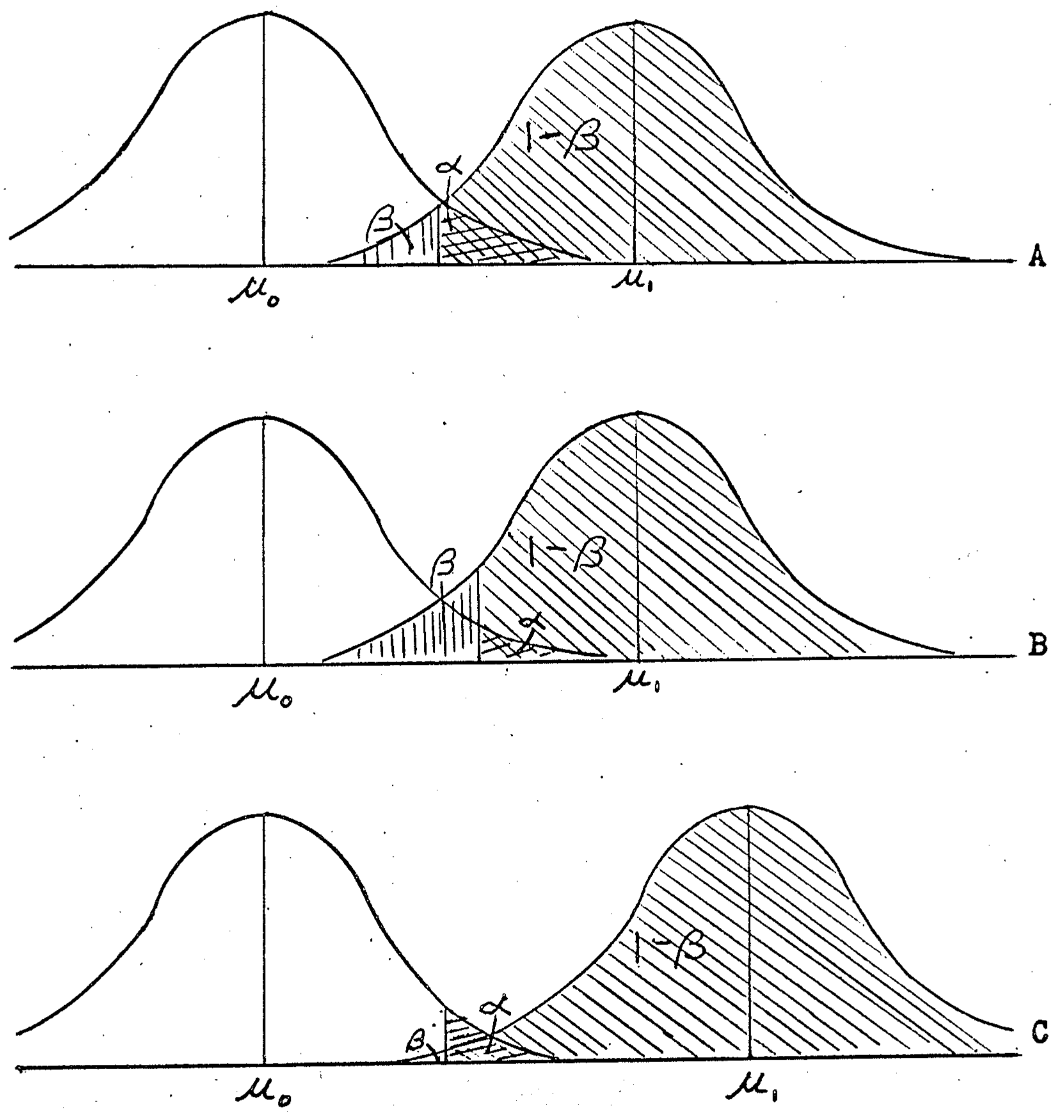
and the power is reduced to .746. This example illustrates that a decrease in α decreases the power of the test and increases β. Similarly an increase in α increases $1-\beta$ and decreases β. The former effect is shown in Diagram B.

To illustrate the effect of changing Ψ, let the alternative hypothesis be

$$H_1 : \mu_1 = 56.$$

Now $\Psi = 3.6$ and the new value of $1-\beta$ is 0.940. This increase can be seen in Diagram C. Thus the greater Ψ is, the greater $1-\beta$ will be while $1-\beta$ decreases as Ψ becomes smaller.

Figure 2.2 Comparison of α, β, and $1-\beta$.



It is perhaps important to note that increasing the sample size will have the effect of decreasing $s_{\bar{x}}$, thus increasing ψ and increasing $1-\beta$, ceteris paribus.

Another factor affecting the power of the t-test is the distribution of the population. In cases where the distribution of the population is not known it will be difficult to calculate the correct value of the power. In other words, it will be difficult to calculate exactly how much the value of $1-\beta$ for a non-normal population differs from that which is calculated when the population is assumed to be normal, ceteris paribus.

Ghurye (1949) made a study of the power of the t-test in an asymmetrical population with $\gamma_2 = 0$. In a sample of size n+1 he defined u as

$$u = \frac{(\bar{x} - \mu_1)\sqrt{n+1}}{\sigma}$$

and X^2 as

$$X^2 = \frac{ns^2}{\sigma^2}$$

and found the joint p.d.f. of $\bar{x}$ and s^2 to be

$$f(u,X) = \frac{\gamma_1}{2^{(n-2)/2}\,\Gamma(\frac{n}{2})\; 6\sqrt{2\pi(n+1)}}\left[u^3 + 3uX^2 - 3(n+1)u\right]^{n-1}$$

$$X \exp\left\{-\tfrac{1}{2}(X^2 + u^2)\right\} .$$

In a given t-test $1-\beta$ is a function of μ_1, called the power function. In a non-normal population, in addition to the true power function, a hypothetical power function can be calculated by treating the population as if it were normal (by using Student's t-distribution). The graphs of these functions intersect in a certain point such that for both curves to one side the ordinates are smaller than in this point (region of less power) and to the other side the ordinates are larger than in this point (region of higher power). Ghurye found that in the case of positive skewness the true power curve lies below the hypothetical one in the region of less power and above it in the region of higher power, and vice versa. He also found that subject to the additional restriction that the standardized γ_1 is sufficiently small, the error in the power of the test due to treating the population as normal would not materially affect the inferences drawn.

Srivastava (1958), using Gayen's p.d.f., found that even if the samples came from considerably non-normal populations, the power of the t-test is, for practical purposes, not seriously affected. When $|\gamma_2| < 1$ and $\gamma_1 = 0$ the effect on the power is quite small but when $\gamma_2 > 1$ there is a noticeable increase in power up to a certain point, then a subsequent decrease. If $\gamma_2 < -1$ the effect is just the reverse.

The effect of γ_1 on the power is relatively greater than the effect of γ_2.

Table 2.1, which has been taken from Srivastava's paper, gives a comparison of values of the power function, calculated for a sample of size 10 for a population where $\gamma_1 = 0.775$, $\gamma_2 = -2.6$. In this table:

i) The first column gives values of the power function for a normal distribution,

ii) in the second column the critical region is erroneously based on the upper (or lower) 5% point of Student's t-distribution, and

iii) in the third column the critical region is based on roughly the upper (or lower) 5% point of the t-distribution corresponding to the non-normal population considered.

ρ_n is defined as

$$\frac{(\mu_1 - \mu_0)\sqrt{n}}{\sigma},$$

that is, the difference of μ_1 and μ_0 measured in standard deviations of the mean.

For the case where the critical region is defined by $t_0 = 1.833$, power decreases in the region of less power as a consequence of α being .035 rather than .05,

Table 2.1

Comparison Of Values Of The Power Functions For Normal, Corrected, And Erroneously Assumed Normal Cases

ρ_n	$\gamma_1, \gamma_2 = 0$	$\gamma_1 = 0.775$	$\gamma_2 = -2.6$
	$t_o = 1.833$	$t_o = 1.833$	$t_o = 1.627$
0	0.050	0.035	0.051
1	0.236	0.198	0.259
2	0.580	0.579	0.663
3	0.868	0.910	0.940
4	0.979	0.997	0.9996
		$t_o = -1.833$	$t_o = -1.627$
0		0.070	0.051
-1		0.273	0.219
-2		0.585	0.530
-3		0.838	0.784
-4		0.957	0.933

while in the case of the corrected critical region the power is considerable greater than for a normal population.

We can see from the table that the power function, when μ_1 is in the immediate neighbourhood of μ_0, is as much influenced by the choice of an erroneous critical region based on the false assumption of normality as it is by non-normality.

The effect of γ_1 is more prominent when $|\gamma_2|$ is small. When γ_2 is quite large, however, the effects of γ_1 and γ_2 become equally prominent and could cancel each other out. For example, in the case where γ_1 and γ_2 are both positive the power of the test is likely to be quite close to that of a normal distribution in regions of low power.

Pearson's (1931) empirical findings are in agreement with these results. Here again the effect of non-normality on the power of the test decreases as sample size increases.

Findings mentioned in this chapter reveal that for moderate departures from normality, the t-test is quite reliable and no serious error will be made if the assumption of normality is made. While γ_1 has a greater effect on the t-test than γ_2, and the effect varies with the different applications of the test, the approximation to normality is improved in all cases by increasing the sample size.

CHAPTER III

STANDARD DISTRIBUTIONS

In order to execute a proper t-test it is necessary that the distribution for the t statistic corresponding to the distribution of the population from which the sample has been taken is available. For normally distributed populations this t-distribution is known as "Student's t-distribution" for which detailed tables are widely available. For populations with non-normal distributions it may be too laborious to calculate the proper t-distribution or even impossible (e.g. when the population being sampled has an unknown distribution). In such cases the population is often treated as if it were normal and a "t-test" is performed using Student's t tables, in the expectation that if the original population is not too far from having a normal distribution, the results arrived at by this "t-test" will not be too far from being correct. To investigate the reliability of this procedure we shall first derive the p.d.f. of t for an arbitrary parent population with continuous p.d.f. $f(x)$ for samples of size two and apply this result to two special cases, vis. the rectangular and exponential distributions, to see in how far this substitution of Student's t-distribution for the true t-distribution can be justified.

<u>Distribution Of The t-statistic In Samples of Size Two</u>

Laderman (1939) developed a method to find the p.d.f. of t for samples of size two taken from any parent population with an arbitrary continuous p.d.f. $f(x)$, $a < x < b$ and zero elsewhere. His results were for populations with mean zero and will be extended here for populations with an arbitrary mean, say m.

Suppose the two observations are x_1 and x_2. Then

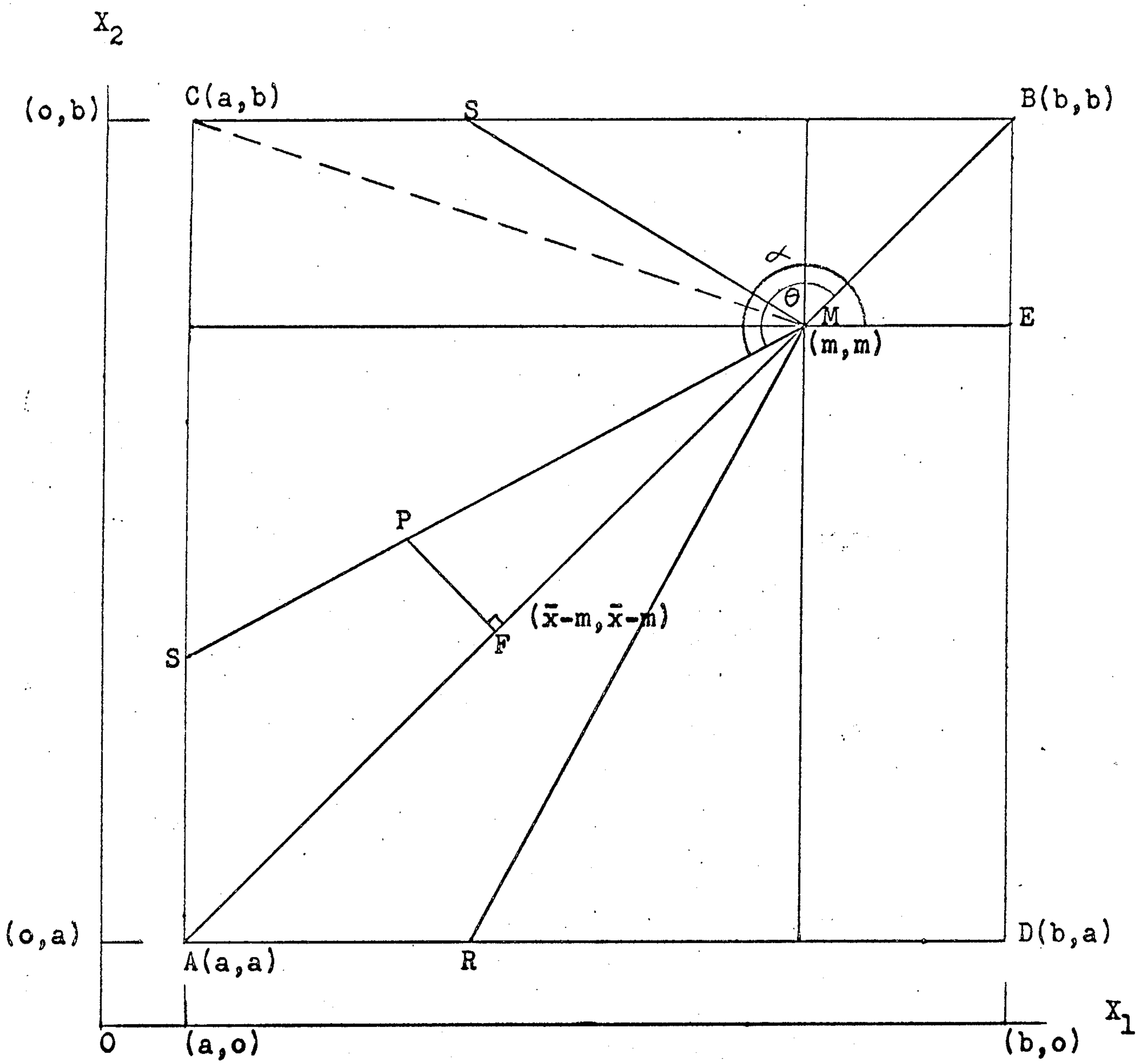
$$\bar{x} = \frac{x_1 + x_2}{2}$$

and

$$t = \frac{\sqrt{n}\ (\bar{x}-m)}{\sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}} = \frac{\sqrt{2}\ (\bar{x} - m)}{\sqrt{(x_1-\bar{x})^2 + (x_2-\bar{x})^2}} .$$

The sample (x_1, x_2) can be represented by a point in a square of side b-a, as point P in Figure 3.1. No generality is lost by assuming that $x_1 < x_2$ and thus considering only Δ ABC. The arguments presented for a point in Δ ABC will apply to the mirror image of that point in Δ ABD so the results derived for a point in Δ ABC must be multiplied by two. The coordinates of the point

Figure 3.1



P with respect to the mean are (x_1-m, x_2-m). For all points with the same mean as P, the sum of the coordinates is the same, so that they lie on a straight line with equation $x_1 + x_2 = 2\bar{x}$, which is perpendicular to AM (with equation $x_1 = x_2$) and intersects this line in the point F with coordinates $(\bar{x}-m, \bar{x}-m)$.

Then

$$MF = -\sqrt{2}\,(\bar{x} - m)$$

and

$$FP = \sqrt{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2}$$

and the t-statistic for samples of size two turns out to be

$$t = \frac{\bar{x} - m}{s/\sqrt{n}} = -\frac{MF}{FP} .$$

This is, furthermore, equivalent to the cotangent of $\angle$BMP (which we call θ) so that all points with the same t value lie on the half-line through M, which intersects the perimiter of the square in a point S.

If we let $\angle$ EMP $= \alpha$, we see that tan α is the slope of the line MP, and since $\alpha = \frac{1}{4}\pi + \theta$, we have

$$\tan\alpha = \frac{\tan\pi/4 + \tan\theta}{1 - \tan\pi/4\ \tan\theta} = \frac{t+1}{t-1} .$$

so that the equation of MP becomes:

$$x_2 - m = \frac{t+1}{t-1}(x_1 - m) .$$

From this it is again clear that all points on the half-line MP represent samples with the same t values.

For a point on MC we can show that the slope of the line MP is

$$t_{MC} = \cot \angle BMC = \frac{b + a - 2m}{b - a} ,$$

so that $\triangle$ ABC is divided into $\triangle$ AMC, for which

$$-\infty < t < \frac{b + a - 2m}{b - a} ,$$

and $\triangle$ BMC, for which

$$\frac{b + a - 2m}{b-a} < t < \infty.$$

The probability of getting a sample point in the element of area $dx_1 \, dx_2$ is $f(x_1) \, f(x_2) \, dx_1 dx_2$. Therefore the probability of $(\bar{x} - m)\sqrt{n}/s$ having a value less than that for any given point on the half-line MS, with S on AC is given by

$$\int_a^m \int_{x_1}^{\frac{(t+1)x_1 - 2m}{t - 1}} f(x_1) \; f(x_2) \; dx_1 \; dx_2,$$

and by

$$2\int_{a}^{m}\int_{x_1}^{\frac{(t+1)x_1 - 2m}{t-1}} f(x_1)\, f(x_2)\, dx_1\, dx_2$$

if we include points on MS with S on AD (i.e., for which $x_2 < x_1$).

Differentiating with respect to t and setting $x_2 = x$ we obtain the p.d.f. of t which is

$$g(t) = \frac{4}{(t-1)^2}\int_{a}^{m} (m-x)\, f(x)\, f\left[\frac{(t+1)x - 2m}{t-1}\right] dx, \quad t \leq \frac{b + a - 2m}{b - a}\ .$$

If $t \geq (b + a - 2m)/(b-a)$ the probability of t being greater than a value represented by a point P' on MS' (or its mirror image) can be shown to be

$$1 - 2\int_{m}^{b}\int_{\frac{(t-1)x_2 + 2m}{t+1}}^{x_2} f(x_1)\, f(x_2)\, dx_1\, dx_2\ .$$

In this case the p.d.f. of t is

$$g(t) = \frac{4}{(t+1)^2}\int_{m}^{b} (x-m)\, f(x)\, f\left[\frac{(t-1)x + 2m}{t+1}\right] dx.$$

Then for a sample of size two with p.d.f. f(x), the p.d.f. of the t-distribution is:

$$g(t) = \frac{4}{(t-1)^2}\int_a^m (m-x)\ f(x)\ f\left[\frac{(t+1)x-2m}{t-1}\right]dx, \quad t \leq \frac{b+a-2m}{b-a}$$

$$g(t) = \frac{4}{(t+1)^2}\int_m^b (x-m)\ f(x)\ f\left[\frac{(t-1)x+2m}{t+1}\right]dx, \quad t \geq \frac{b+a-2m}{b-a}.$$

These results will be used later to find the p.d.f. for the rectangular and exponential distributions. Definite probability statements about the mean of the population can be made with the help of these functions.

The Rectangular Distribution

The rectangular distribution is a uniform distribution over the entire range. This means that the probability that a value will fall in a certain interval in the range depends only on the size of the interval and not on its location.

If x has a rectangular distribution ranging from a to b, then the p.d.f. of x is

$$f(x) = \frac{1}{b-a}, \qquad a < x < b$$

$$f(x) = 0, \qquad \text{elsewhere.}$$

1) Samples of Size 2

The mean of a rectangular distribution is given by

$$E(x) = \int_a^b \frac{x}{b - a}\, dx = \frac{b + a}{2} .$$

Using the result of the previous section, the p.d.f. of t for a rectangular distribution can be shown to be

$$g(t) = \frac{1}{2(1-t)^2} \qquad \text{when } t \leq \frac{b + a - 2m}{b - a}$$

and

$$g(t) = \frac{1}{2(1+t)^2} \qquad \text{when } t \geq \frac{b + a - 2m}{b - a} .$$

In contrast, the p.d.f. of Student's t-distribution for a sample of size 2 is

$$f(t) = \frac{1}{\pi(1+t^2)} .$$

The true distribution of t and Student's distribution of t for samples of size two can lead to different conclusions about the mean of the population because of these differences in the p.d.f. which for some values of t can be considerable.

2) Sample Size > 2

Since the rectangular distribution is symmetrical, all odd order moments about the mean will be zero. This implies also that γ_1 will be zero and hence will not affect the t-distribution.

For the even moments we have

$$\mu_2 = \int_a^b \left[x - \tfrac{1}{2}(a+b)\right]^2 \cdot \frac{1}{b-a}\, dx = \frac{(b-a)^2}{12}$$

and similarly

$$\mu_4 = \frac{(b-a)^4}{80}$$

so that

$$\mu_2 = \frac{9}{5} - 3 = -1.2$$

and this shows that the rectangular distribution is platykurtic.

For the sample means of a rectangular distribution we then have

$$\gamma_{2\bar{x}} = \frac{-1.2}{n} \;.$$

which implies that, even for reasonably small samples the distribution of the sample means is approximately normal and hence the use of Student's t-tables will not cause any serious error.

For a rectangular distribution, the probability of obtaining a t value near the origin will be less than for a normal population, while the probability of a value of $|t|$ greater than a certain t_0 will be more for the rectangular population than for the normal.

For $|t| \leq 1$, "Student's theory based on the hypothesis of a normal universe is an excellent approximation of a rectangular universe" (Rider, 1929).

Neyman and Pearson (1928) point out that the assumption of a rectangular distribution will lead to more serious errors than the assumption of a normal distribution since normal or approximately normal distributions are more common in practise. They state, furthermore:

> "If we assume erroneously that the sampled population is normal when in fact it has a [γ_1 and γ_2] on or close to the stretch of the [γ_2] axis between [0 and -1.2] we shall not be led to any serious errors of judgement by using Student's test."

Since the distribution of the means of the samples is approximately normal, we can assume that the distribution of the actual t-statistic is sufficiently close to that of Student's distribution that Student's tables can be used without any significant loss of reliability.

The Exponential Distribution

The exponential distribution is frequently the probability model for waiting times; for instance, in life-testing, the waiting time until "death" is a random variable which frequently has an exponential distribution. A variable of this nature can take on only positive values.

If x has an exponential distribution with mean θ, then the p.d.f. of x is

$$f(x) = \frac{1}{\theta} e^{-x/\theta} \quad , 0 < x < \infty$$

$$f(x) = 0 \quad , \text{elsewhere.}$$

θ can also be called the "average waiting time".

1) Samples of Size 2

The mean of an exponential distribution is given by

$$E(x) = \int_0^\infty \frac{x}{\theta} e^{-x/\theta} \, dx$$

which is θ, as has already been stated.

By substitution into the equations derived earlier in this chapter, the p.d.f. of t for an exponential distribution of size two can be shown to be

$$g(t) = \frac{e^{-2}}{t^2}\left[1 - \frac{2t+1}{1-t}\, e^{\frac{-2t}{1-t}}\right], \quad \text{when} \quad t \leq 1$$

and

$$\hat{g}(t) = \frac{e^{-2}}{t^2}, \qquad \text{when } t \geq 1.$$

Neither one of these approximations appear to be a good approximation to Student's distribution for a sample of size two. This suggests that, for small samples at least, Student's distribution would not give reliable results when applying the t-test to an exponential distribution.

2) Sample Size > 2

For the moments of an exponential distribution we have

$$\mu_2 = \int_0^{\infty} \frac{(x-\Theta)^2}{} \quad e^{-x/\Theta}\, dx = \Theta^2$$

and similarly

$$\mu_3 = 2\theta^3$$

and

$$\mu_4 = 9\theta^4 \quad .$$

Hence

$$\gamma_1 = 2$$

and

$$\gamma_2 = 6.$$

These values of γ_1 and γ_2 suggest that the t-distribution corresponding to an exponential distribution would give a very poor approximation to Student's t-distribution, as the parent distribution is highly skewed and quite leptokurtic.

For the distribution of the means of samples drawn from the exponential distribution,

$$\gamma_{1\bar{x}} = 2/\sqrt{n}$$

and

$$\gamma_{2\bar{x}} = 6/n.$$

This value of $\gamma_{2\bar{x}}$ approaches zero much more slowly with an increase in n than does the corresponding value for a rectangular population. Furthermore, a rather large value of n is required for $\gamma_{1\bar{x}}$ to be nearly zero.

This suggests that results obtained by substituting Student's t-distribution for the actual t-distribution would probably not be very reliable.

Boneau (1960) points out that the probability of obtaining a t value in the negative tail will be greater than that of obtaining a value in the positive tail, but that, as the sample size gets larger, this difference becomes less and less.

If one takes only small samples from a population of which the distribution is unknown, it is generally impossible to be certain of the form of the distribution of the population from internal evidence, that is, from the samples. It has been shown that the reliability of results obtained by means of applying Student's t-distribution to an exponential parent distribution are far from that of the results obtained by applying it to symmetrical parent distributions.

The results of this chapter suggest that approximating the t-distribution corresponding to a rectangular distribution, or that corresponding to a similar symmetrical distribution, with Student's t distribution would not lead to serious errors. Approximating the t-distribution corresponding to an exponential distribution in this way, however, would not be valid and would lead to serious errors unless the sample size is very large.

CHAPTER IV

GAYEN'S DISTRIBUTION

For samples of size n from the parent population specified in Chapter I, Gayen (1949) has shown that the t-statistic has the p.d.f.

$$P(t) = \frac{\Gamma(n/2)}{\sqrt{\pi(n-1)}\,\Gamma(\frac{n-1}{2})} \cdot \frac{1}{\left[1 + t^2/(n-1)\right]^{n/2}}$$

$$\frac{\gamma_1 \left\{3(n-1)t - (2n-1)t^3\right\}}{6(n-1)\sqrt{2n\pi}\left[1 + t^2/(n-1)\right]^{(n+3)/2}}$$

$$-\gamma_2 \frac{\Gamma(\frac{n+2}{2})}{24n\sqrt{\pi(n-1)}\,\Gamma(\frac{n+3}{2})} \cdot \frac{3(n-1) - 6(n+1)t^2 + (n+1)t^4}{\left[1 + t^2/(n-1)\right]^{(n+4)/2}}$$

$$+\frac{\gamma_1^2\,\Gamma(\frac{n+2}{2}) \left\{\begin{array}{l} 3(n-1)^2(2n+11) - 9(n-1)(n+3)(2n-1)t^4 \\ -3(n+1)(n+3)(2n+13)t^4 + (n+1)(n+3)(2n+5)t^2 \end{array}\right\}}{144n(n-1)^{3/2}\sqrt{\pi}\,\Gamma(\frac{n+5}{2})\left[1 + t^2/(n-1)\right]^{(n+6)/2}}$$

$$= P_o(t) + \gamma_1 P_{\gamma_1}(t) - \gamma_2 P_{\gamma_2}(t) + \gamma_1^2 P_{\gamma_1^2}(t).$$

Here P(t) is the p.d.f. of the t-statistic for the population under consideration, $P_o(t)$ is the p.d.f. of this statistic for a normally distributed population, and $P_{\gamma_1}(t)$, $P_{\gamma_2}(t)$ and $P_{\gamma_1^2}(t)$ are the "correction" terms.

For purposes of statistical inference, only the tail areas are of interest and so we consider the integral

$$\int_{t_o}^{\infty} P(t)\, dt.$$

This integral may be calculated directly for values of n, t_o, γ_1, and γ_2. For computational purposes we make the transformation

$$\frac{1}{1 + t^2/(n+1)} = \sin^2 \Theta .$$

This changes the integral of P(t) into a linear combination of integrals of the form

$$\int_0^{\pi/2 - \arctan\sqrt{t_o^2/(n-1)}} \sin^{n+r} (\Theta)\, d\Theta$$

r is an integer.

To calculate the value of the integral on the computer, Simpson's extended 1/3 rule was used. The error involved in this procedure, E, is limited by

$$\left| E \right| \leq \frac{(x_N - x_o)^5 \quad f''''_{max}}{180\ N^4}$$

if differences of the 5[th] and higher orders are neglected. In this expression x_N and x_o are the upper and lower limits of the integration interval, N is the number of sub-intervals used and f''''_{max} is the maximum value of the fourth derivative of the function. Since the lower limit of the integration considered here is always zero, we have

$$\left| E \right| \leq \frac{x_N^5 \ f''''_{max}}{180\ N^4} .$$

The computer was programmed to calculate each integral for given values of t_o and n, for increasing values of N, until successive values of the integral differed by less than 10^{-7}.

The maximum possible error increases with x_N and with f''''_{max}. However, as x_N and f''''_{max} increased, N increased as well, thus limiting the maximum possible error. As a check this procedure was applied to the t-distribution for a normal population, and the results agreed perfectly with the published tables for Student's t-distribution.

Properties Of Gayen's t-distribution

For the p.d.f. P(t) as given above, Gayen gives the moments about t = 0, neglecting terms in 1/n of higher than the 2nd degree as

$$m_{1t} = \frac{1}{2\sqrt{n}} (1 + 3/4n)$$

$$m_{2t} = 1 + \frac{2}{n} (1 + \gamma_1^2) + \frac{2}{n^2} (3 - \gamma_2)$$

$$m_{3t} = \frac{-7}{2\sqrt{n}} (1 + \frac{15}{4n})$$

$$m_{4t} = 3 + \frac{2}{n} (9 - \gamma_2 + 14\gamma_1^2) + \frac{1}{n^2}(102 - 30\gamma_2 + 120\gamma_1^2).$$

Considering only terms involving γ_1, and ignoring expressions of higher order than those considered above, we get for the central moments

$$\mu_{2t} = m_{2t} - m_{1t}^2 = 1 + \frac{2}{n} + \frac{7\gamma_1^2}{4} + \frac{6}{n^2} + \frac{3\gamma_1^2}{8n^2}$$

and

$$\mu_{3t} = m_{3t} - 3m_{2t}m_{1t} + 2m_{1t}^2 = \frac{-2\gamma_1}{\sqrt{n}} - \frac{9\gamma_1}{n^{3/2}} .$$

The skewness of the t-distribution is

$$\gamma_{1t} = \frac{\mu_{3t}}{\mu_{2t}^{3/2}}$$

and we see that $\gamma_{1t} > 0$ if $\gamma_1 < 0$ and that $\gamma_{1t} < 0$ if $\gamma_1 > 0$. Hence for a positively skewed parent distribution, the resulting t-distribution is negatively skewed, and vice versa.

Similarly, considering the effect of γ_2 alone, and ignoring expressions of higher order than those considered above, we have

$$\gamma_{2t} \doteq (3 - \gamma_2)\left[\frac{2}{n} + \frac{10}{n^2}\right] .$$

Hence if $\gamma_2 < 3$, $\gamma_{2t} > 0$, that is, the t-distribution is leptokurtic. If $\gamma_2 > 3$, γ_2 falls outside the Barton and Dennis (1952) limit mentioned in Chapter I and Gayen's distribution is not valid.

Evaluation Of The t-distribution

As has been stated in the previous section, we are interested mainly in the tail area probabilities, that is, in

$$\int_{t_0}^{\infty} P(t)\, dt$$

or in

$$\int_{-\infty}^{-t_0} P(t)\, dt.$$

Values of the first of these integrals have been tabulated in Appendix A for $t_o = 0(0.1)3.9$ (i.e. for values of t_o from 0 to 3.9 in steps of 0.1); $n = 3, 4, 5, 6, 7, 9, 11, 13, 15, 17, 19, 21$; $\gamma_1 = 0(0.5)1$, and $\gamma_2 = 0(1)2$. While the range of these values is somewhat limited, more extensive tables can easily be constructed. For instance, for 6 degrees of freedom, or n = 7, values of the integral have been tabulated at smaller intervals, vis. for $t_o = 0(0.01)\ 3.79$.

In non-symmetrical populations

$$\int_{t_o}^{\infty} P(t)\, dt \neq \int_{-\infty}^{-t_o} P(t)\, dt.$$

The lower tails thus do not correspond to the upper tails, but have not been tabulated in order to conserve space.

If we assume that $\gamma_2 = 0$, that is, disregard the influence of γ_2, it will be noticed that the effect of γ_1 on the value of the integral is quite large but decreases as n increases. For example, in the normal case, the probability of $t > 1$ for 4 degrees of freedom is 0.1869. For $\gamma_1 = .5$ this probability is recuded to .1621 and for $\gamma_1 = 1$ it is further reduced to .1370. When n is increased to 19, the corresponding probabilities are .1653, .1517 and .1380 respectively. The approximation of a distribution with $\gamma_1 = 1$ to a normal distribution for either size of sample considered is poor, but when $\gamma_1 < 0.5$ the approximation is much better.

It is interesting to compare values of $P(t \geq t_o)$ for values of t_o between 3.5 and 4.0. In this range the differences between the probabilities for populations with $\gamma_1 = 0$ and $\gamma_1 = 1$ are less than they are for populations with $\gamma_1 = 0$ and $\gamma_1 = 0.5$. This is due to correction terms for γ_1 and γ_1^2 cancelling each other. A corresponding result would not be found for the lower tail where the correction terms would aggravate each other and the difference for populations with $\gamma_1 = 0$ and $\gamma_1 = 1$ would be much larger.

It is easily seen that where the true distribution is not known, no serious error of judgement would be made by assuming normality if, in fact, $\gamma_1 = .5$ or less. Serious errors could result, however, if normality was assumed for a population with values of γ_1 at or near 1, even for samples as large as 18.

If $\gamma_1 < 0$, the tails would be reversed, with the tables presented in the appendix now applying to the lower tail of the distribution.

As can be seen from earlier discussion in Chapter II, the effect of γ_2 is relatively much smaller than the effect of γ_1. To investigate the effect of γ_2, we now assume that γ_1 is zero. For a sample of size 5, $P(t > 1)$ is .1869 if $\gamma_2 = 0$, .1905 if $\gamma_2 = 1$ and .1941 if $\gamma_2 = 2$. The deviation of this latter value from the value for $\gamma_2 = 0$ is less than one-third of the deviation caused by having γ_1 equal to 0.5 instead of 0. For a sample of size 19 the corresponding probabilities are .1653, .1670 and .1688. This deviation is even less.

When $\gamma_2 > 0$, there is always a certain range about the origin, say $-t_1$ to t_1, such that the probability of t falling in this range is smaller than it is when $\gamma_2 = 0$. The value of t_1 varies with change in n and γ_2, but is usually near 0.7. Then there is a value t_2, usually near 2.5, such that the probability that $t_1 < |t| < t_2$ is greater for a population with $\gamma_2 > 0$ than it is for a population with $\gamma_2 = 0$. $P(t_2 < |t|)$ is again smaller for the population with $\gamma_2 > 0$.

If $\gamma_2 < 0$, an analogous effect occurs, but opposite to that just discussed.

In any case, γ_2 has very little effect on the validity of using Student's tables. This has been inferred by empirical and theoretical studies referred to in Chapter II, and is suggested again by the tables in the appendix.

Should both γ_1 and γ_2 be other than zero, the combined effect would be a linear combination of the effects just discussed. This could result in a very poor approximation to a population where both γ_1 and γ_2 are zero if the effects aggravate each other or it could result in a better approximation if the effects cancel each other. For the effects to cancel each other the absolute value of γ_2 must be much larger than that of γ_1 since the deviation caused by γ_1 is relatively much greater than that caused by γ_2.

Tail Area Probabilities

A common procedure in statistical analysis is to find the sample value of a test statistic and to compare this value with a tabulated value at a given level of significance. Values of t_o have been tabulated in Appendix B for levels of significance α equal to .25, .20, .15, .10, .05, .025, .010 and .005 for all combinations of n = 3, 4, 5, 6, 7, 9, 11, 13, 15, 17, 19, 21; $\gamma_1 = 0, 0.5, 1$; and $\gamma_2 = 0, 1, 2$.

The values of t_o were obtained by searching through a set of tables (such as that illustrated in Appendix A for six degrees of freedom) for a value of t_o, such that $P(t \geqslant t_o)$ equals the chosen α. Since probabilities were only calculated for values of t_o to two decimal places, only on rare occassions was a calculated t_o found for which $P(t \gtrsim t_o)$ was exactly equal to the chosen α. Linear interpolation was then used to obtain more accurate values of t_o. To illustrate, suppose $P(t > t_j) > \alpha$ and $P(t > t_{j+1}) < \alpha$. Then set

$$t_o = t_j + \frac{P(t > t_j) - \alpha}{P(t > t_j) - P(t > t_{j+1})} \cdot (t_{j+1} - t_j) .$$

To illustrate the use of these tables, consider the one-tailed test of the hypothesis

$$H_o : \mu \leq 70$$

against the alternative

$$H_1 : \mu > 70$$

Choose α as .05. The test statistic used is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

which we described in an earlier section. If a sample of size 11 is taken, there are 10 degrees of freedom and the tabulated value to be compared with the test statistic is 1.8125. Then H_o is rejected if $t > 1.8125$ and accepted if $t < 1.8125$.

In this example it was assumed, however, that the sample came from a normal population. If the population is not normal, then the wrong table was used to obtain the value $t_o = 1.8125$ and a value of t_o must be obtained from a table which has been corrected for the non-normality in the population. If, for example, the sample had been taken from a population with $\gamma_1 = .5$ and $\gamma_2 = 1$ instead of both being zero, the correct value for t_o would have been 1.6540 rather than 1.8125.

<u>True Level Of Significance If Normality Is Assumed</u>

In most practical situations, the true population values of γ_1 and γ_2 are seldom known and the population is treated as if it were normally distributed. If then a level of significance α^* is chosen and Student's tables are entered with this value to find the corresponding critical value of t_o, then, unless $\gamma_1 = \gamma_2 = 0$ this value of t_o will determine an actual level of significance $\alpha = \alpha^* + \delta$ which, except by accident, will be different from α^*. Here δ, which may be positive or negative, is the correction to be applied to α^* to obtain the actual level of significance α.

Table 4.1 gives the true significance levels $\alpha = P(t \geq t_o)$ and $\alpha = P(t \leq -t_o)$ for a one-tailed test where

t_o is the critical value for $\alpha^* = .05$. For $\gamma_1 = 0$, $\gamma_2 = 1$ and $n = 5$, α is .0470 (i.e., $\delta = -0.003$) and α approaches .0500 very rapidly as n increases. Even for $\gamma_2 = 2$ α does not deviate very far from .0500.

When $\gamma_2 = 0$ and $\gamma_1 \neq 0$, δ is much larger than for $\gamma_1 = 0$ and $\gamma_2 = 1$ or 2, and for smaller values of n and larger values of γ_1, δ is even very close to .05, our value of α^*. Here δ approaches zero much more slowly as n increases than it does when $\gamma_1 = 0$ and $\gamma_2 = 1$ or 2. For example, for 16 degrees of freedom with $\gamma_2 = 0$ and $\gamma_1 = .5$, $P(t \geq t_o) = .0390$ instead of .0500, making α too conservative and causing us to reject H_o more often than should be the case. However, with the same values of γ_1, γ_2 and n we find, in the negative tail, $\alpha = P(t \leq -t_o) = .0635$ and the results are less significant than assumed to be and H_o will not be rejected often enough.

Results for other values of α^* are similar to those for $\alpha^* = .05$. For more accurate results where $\gamma_1 \neq 0$, that t_o should be chosen which gives us the level of significance we desire.

In Chapter II we observed that, for a two-tailed test, the control of type I error achieved for a non-normal population by treating it as if it were normal was almost as good as if it actually were normal

Table 4.1 True level of significance when t_o is Student's critical value for $\alpha = .05$.

DEGREES OF FREEDOM		$\gamma_1 = 0$		$\gamma_2 = 0$		$\gamma_1 = .5$	$\gamma_1 = 1$
		$\gamma_2=1$	$\gamma_2=2$	$\gamma_1=.5$	$\gamma_1=1$	$\gamma_2=1$	$\gamma_2=2$
4	$P(t > t_o)$	.0470	.0440	.0364	.0298	.0334	.0238
	$P(t < -t_o)$			.0706	.0982	.0676	.0923
8	$P(t > t_o)$	.0489	.0478	.0368	.0280	.0357	.0258
	$P(t < -t_o)$			.0675	.0894	.0664	.0872
12	$P(t > t_o)$	.0495	.0490	.0380	.0292	.0375	.0281
	$P(t < -t_o)$			.0651	.0834	.0646	.0824
16	$P(t > t_o)$	.0497	.0495	.0390	.0305	.0387	.0300
	$P(t < -t_o)$			.0635	.0794	.0632	.0789
20	$P(t > t_o)$	.0498	.0497	.0399	.0318	.0397	.0315
	$P(t < -t_o)$			.0623	.0767	.0621	.0764

if $P(t \leq -t_o) + P(t \geq t_o) \doteq \alpha$. ($t_o$ is again the critical value corresponding to α^* in Student's tables.) Table 4.2 gives the sum of $P(t \leq -t_o)$ and $P(t \geq t_o)$ when α^* is chosen as .05.

The error in a two-tailed test is, ceteris paribus, much less than for a one-tailed test. If $\gamma_1 < .5$, $\gamma_2 = 0$, $P(t \leq -t_o) + P(t \geq t_o) \doteq .05$ and the control of type I error will be nearly as good as when γ_1 and γ_2 are both zero.

For small samples, say < 6, and large values of γ_2,

Table 4.2 True values of $P(t \leq -t_o) + P(t \geq t_o)$ when normality is erroneously assumed and $\alpha^* = .05$.

DEGREES OF FREEDOM	$\gamma_1 = 0$		$\gamma_2 = 0$		$\gamma_1 = .5$	$\gamma_1 = 1$
	$\gamma_2 = 1$	$\gamma_2 = 2$	$\gamma_1 = .5$	$\gamma_1 = 1$	$\gamma_2 = 1$	$\gamma_2 = 2$
4	.0442	.0384	.0565	.0759	.0507	.0653
8	.0468	.0434	.0549	.0697	.0517	.0631
12	.0478	.0458	.0538	.0651	.0517	.0609
16	.0484	.0470	.0530	.0621	.0515	.0591
20	.0488	.0478	.0525	.0602	.0514	.0579

$P(t \leq -t_o) + P(t \geq t_o)$ will not be acceptable. For example, if $n = 5$, $\gamma_2 = 2$ and $\gamma_1 = 0$, this sum is .0384 which is .0116 less than .05 and the control of type I error will not be very good.

In two-tailed tests for $\gamma_1 < .5$, $\gamma_2 < 2$ and $n > 6$, the control of type I error will be almost as good as for a normal population.

The observations made from Gayen's distribution agree with previous empirical and theoretical findings. Inferences drawn on the assumption of normality will be more accurate for relatively large values of γ_2 if $\gamma_1 = 0$, than for smaller values of γ_1 if $\gamma_2 = 0$, that is, the influence of γ_1 is the main source of errors in applying Student's t-test to a non-normal population. Furthermore, the control of type I error is better for a two-tailed test than it is for a one-tailed test.

CHAPTER V

THE COMPOUND NORMAL DISTRIBUTION

A compound normal distribution consists of a mixture of normal distributions with means and variances (a_1, λ_1), (a_2, λ_2), $\cdots$ (a_k, λ_k). p_i is the proportion of the mixture which comes from the i^{th} group and $p_1 + p_2 \cdots + p_k = 1$. In this study we consider only the case where $k = 2$ and $\lambda_1 = \lambda_2 = \lambda$, and set $p_1 = p$. This is a mixture of two normal populations with equal variances and the distribution of this population has the p.d.f.

$$f(x) = \frac{p}{\sqrt{2\pi}} \exp\left[-\frac{(x-a_1)^2}{2}\right] + \frac{1-p}{\sqrt{2\pi}} \exp\left[-\frac{(x-a_2)^2}{2}\right].$$

We can assume without loss of generality that $a_1 > a_2$, since these can be assigned to the populations arbitrarily.

With a suitable choice of parameters p, a_1 and a_2 this p.d.f. can be made to represent a variety of distributions showing considerable deviation from normality. Obviously if $p = 0$, $p = 1$ or $a_1 = a_2$ we will have a simple normal population. The numerical values of a_1 and a_2 themselves are not important, but the quantity

$$d = \frac{(a_1 - a_2)}{\sqrt{\lambda}}$$

is, since with a simple transformation one of the means can be moved to the origin and only as d changes does the compound normal distribution change.

In order to be able to give a better description of the compound normal distribution and how it deviates from normality as p and d vary, the moments and the expressions for γ_1 and γ_2 will be of great help. To obtain these, we first derive the moment generating function (that is, the expected value of e^{tx}) of this distribution. This is given by

$$E(e^{tx}) = M(t) = \frac{p}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} \exp\left[-\frac{(x-a_1)^2}{2\lambda}\right] dx + \frac{1-p}{\sqrt{2\pi}}$$

$$\times \int_{-\infty}^{\infty} e^{tx} \exp\left[-\frac{(x-a_2)^2}{2\lambda}\right] dx.$$

With suitable transformations and integration this is found to be

$$M(t) = p \exp(ta_1 + \lambda t^2/2) + (1 - p) \exp(ta_2 + \lambda t^2/2).$$

The moments about zero are found by differentiating

$M(t)$ r times for the r^{th} moment and setting $t = 0$.

$$M'(t) = p(a_1 + \lambda t) \exp (ta_1 + \lambda t^2/2) + (1-p)(a_2 + \lambda t) \exp(ta_2 + \lambda t^2/2)$$

$$m_1 = M'(0) = pa_1 + (1-\rho)\, a_2.$$

Similarly

$$m_2 = M''(0) = \lambda + pa_1^2 + (1-p)\, a_2^2$$

$$m_3 = M'''(0) = 3p\lambda a_1 + pa_1^3 + 3(1-p)\lambda a_2 + (1-p)a_2^3 .$$

and

$$m_4 = M''''(0) = 3p\lambda^2 + 6p\lambda a_1^2 + pa_1^4 + 3\lambda^2 - 3p\lambda^2 + 6\lambda a_2^2 - 6p\lambda a_2^2 + a_2^4 - pa_2^4 .$$

The moments about the mean can then be shown to be

$$\mu_2 = \lambda + p(1-p)(a_1 - a_2)^2$$

$$\mu_3 = p(1 - p)(1 - 2p)(a_1 - a_2)^3$$

and

$$\mu_4 = 3\lambda^2 + 6\lambda p(1-p)(a_1-a_2)^2 + (p-4p^2+6p^3-3p^4)(a_1\ a_2)^4.$$

These central moments confirm that d is the important factor along with p, rather than either a_1 or a_2 in themselves since each of the terms is a factor of $a_1 - a_2$.

We wish to find expressions for γ_1 and γ_2 to see how these values deviate from zero as the parameters vary. Two conditions have already been stated for which the distribution will be normal, so that $\gamma_1 = 0$ and $\gamma_2 = 0$.

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{p(1p)(1 - 2p)(a_1 - a_2)^3}{\left[\lambda + p(1 - p)(a_1 - a_2)^2\right]^{3/2}}.$$

A third condition for $\gamma_1 = 0$ is $p = .5$ since in this case $(1 - 2p) = 0$ and the numerator will vanish. (Since λ, the common variance is positive, and 1-p, p and $(a_1 - a_2)^2$ are never negative, the denominator is always positive.)

γ_1 can be positive or negative depending on p (since we assumed $a_1 > a_2$). If $p < .5$, then $\gamma_1 > 0$, and if $p > .5$, then $\gamma_1 < 0$.

Since, for all values of d, γ_1 becomes zero for

$p = 0$ and for $p = 0.5$, and $\gamma_1 > 0$ for $0 < p < .5$, there must, for any given d, be a value of p in the interval $0 < p < 0.5$ for which γ_1 attains a maximum. It can be observed from Table 5.1 and from Figures 5.1 to 5.4 that, for the values of d considered, γ_1 has a maximum for a value of p between .07 and .13. The exact value of p that makes γ_1 a maximum varies with d. Similarly it can be observed that γ_1 has a minimum for p between 1 - .13, or .87, and 1 - .07, or .93.

$$\gamma_2 = \frac{\mu_4}{\mu_2^{\,2}} - 3 = \frac{p(1 - \ p + 12p^2 - 6p^3)(a_1 - a_4)^4}{\left[\lambda + p(1-p)(a_1 - a_2)^2\right]^2} .$$

For the values of d considered, γ_2 has positive maxima for values of p that, depending on d, vary between .04 and .10 and between .90 and .96 and a negative minimum for $p = .5$.

Since $\gamma_2 < 0$ when $p = .5$, γ_2 will be zero between its minimum and maximum values. By setting $1 - 7p + 12p^2 - 6p^3$ equal to zero and solving the resulting equation for p, additional values of approximatly .2113 and 1 - .2113 or .7887 are found for which $\gamma_2 = 0$. It is of interest to note that these values of p also give the numerator of γ_1, μ_3, its maximum value, whereas γ_2 has its minimum value when $\gamma_1 = 0$.

Table 5.1 Values of γ_1 and γ_2 for a compound normal distribution $(p, 1 - p)$ with means 2, 3, and 3.5 standard deviations apart and a common variance.

p	d = 2.0		d = 3.0		d = 3.5	
	γ_1	γ_2	γ_1	γ_2	γ_1	γ_2
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.025	0.1611	0.2764	0.4643	1.1337	0.6709	1.8518
0.050	0.2635	0.3837	0.6768	1.3500	0.9213	2.0367
0.075	0.3267	0.3970	0.7691	1.2432	1.0049	1.7760
0.100	0.3632	0.3581	0.7983	1.0236	1.0126	1.4054
0.125	0.3808	0.2911	0.7923	0.7734	0.9827	1.0305
0.150	0.3848	0.2103	0.7657	0.5263	0.9332	0.6851
0.175	0.3789	0.1242	0.7267	0.2958	0.8734	0.3780
0.200	0.3657	0.0381	0.6801	0.0871	0.8082	0.1096
0.225	0.3649	-0.0488	0.6287	-0.0989	0.7404	-0.1230
0.250	0.3240	-0.1224	0.5745	-0.2628	0.6715	-0.3236
0.275	0.2978	-0.1938	0.5186	-0.4059	0.6023	-0.4955
0.300	0.2692	-0.2580	0.4616	-0.5295	0.5334	-0.6420
0.325	0.2388	-0.3149	0.4041	-0.6352	0.4649	-0.7657
0.350	0.2068	-0.3642	0.3464	-0.7242	0.3971	-0.8689
0.375	0.1738	-0.4058	0.2885	-0.7977	0.3298	-0.9535
0.400	0.1399	-0.4398	0.2307	-0.8566	0.2631	-1.0208
0.425	0.1055	-0.4662	0.1729	-0.9016	0.1969	-1.0721
0.450	0.0705	-0.4850	0.1152	-0.9334	0.1311	-1.1081
0.475	0.0353	-0.4962	0.0576	-0.9523	0.0655	-1.1295
0.500	0.0000	-0.5000	0.0000	-0.9586	0.0000	-1.1366

FIG. 5.1 DEPARTURE FROM NORMAL THEORY VALUES FOR A COMPOUND NORMAL DISTRIBUTION (P,1-P) WITH MEANS 1.0 STANDARD DEVIATION APART AND A COMMON VARIANCE.

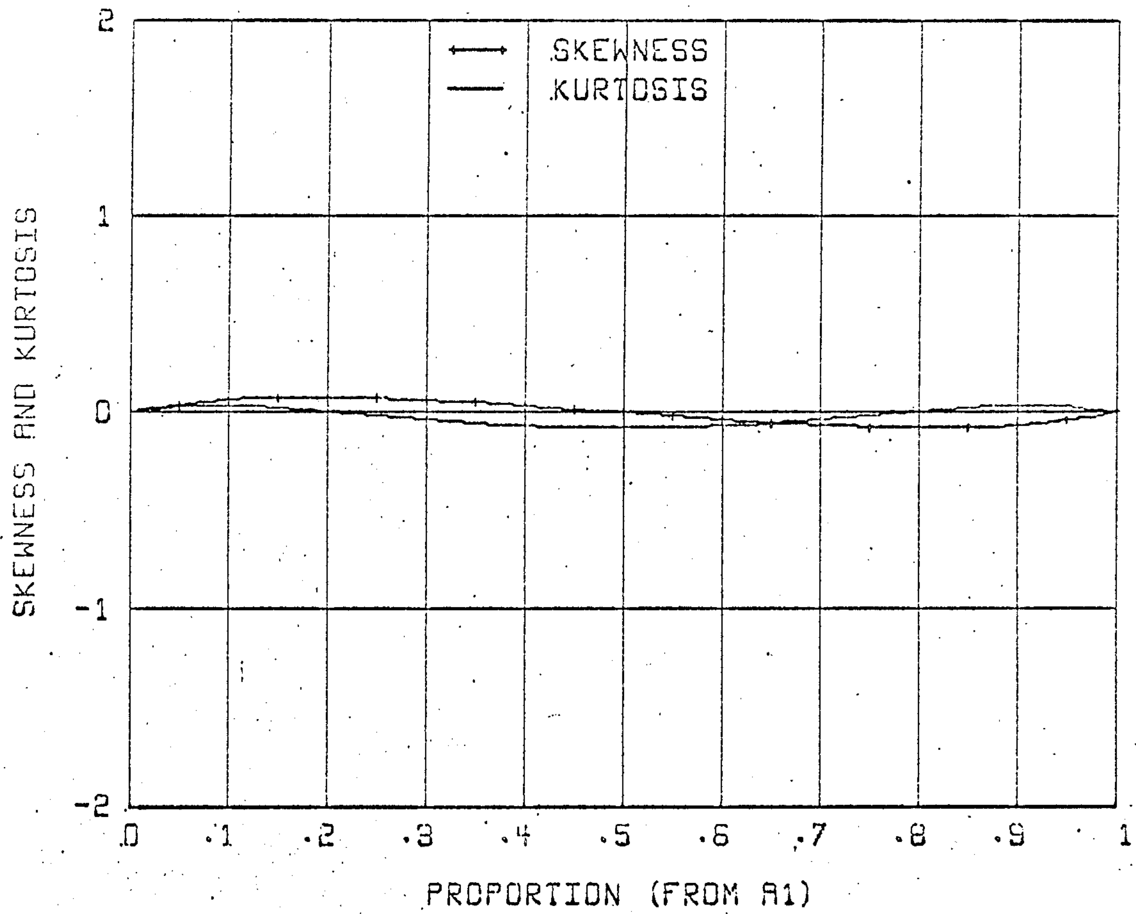


FIG. 5.2 DEPARTURE FROM NORMAL THEORY VALUES FOR A COMPOUND NORMAL DISTRIBUTION (P,1-P) WITH MEANS 2.0 STANDARD DEVIATIONS APART AND A COMMON VARIANCE.

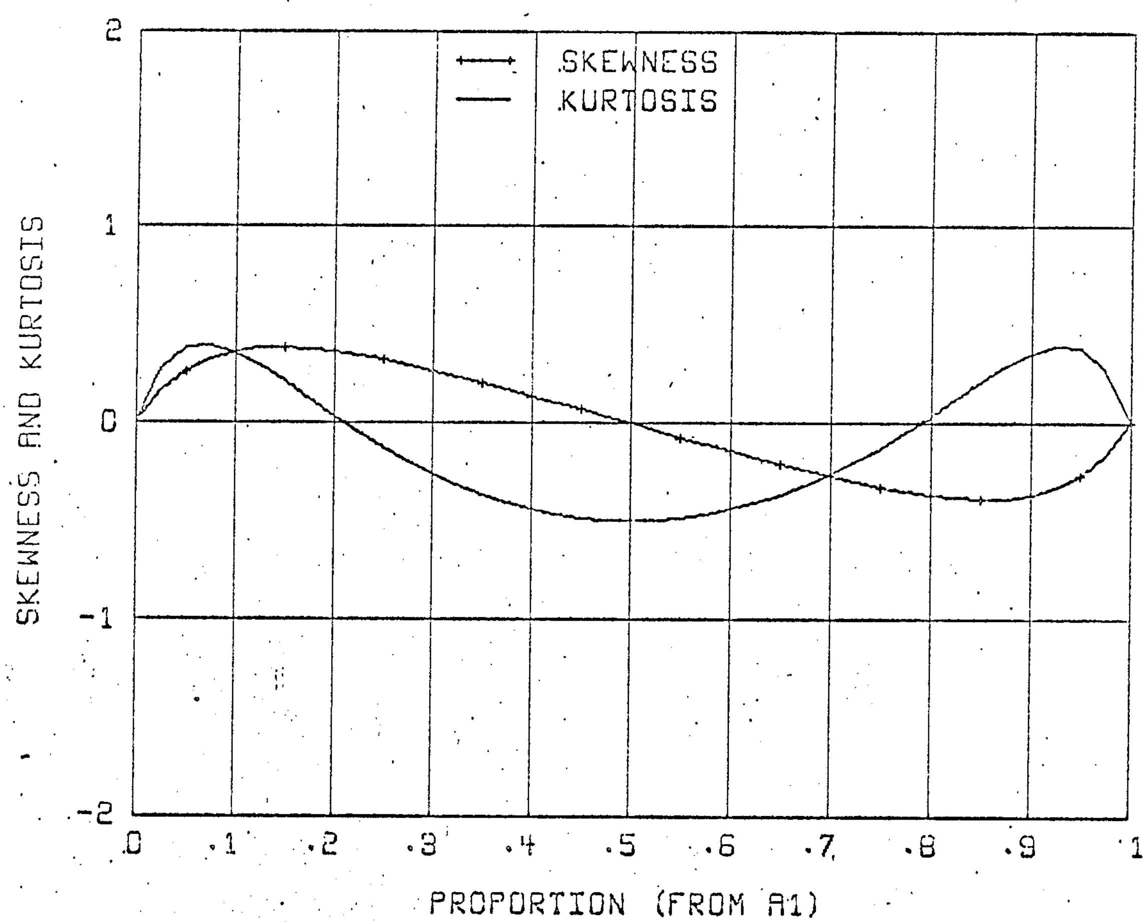


FIG. 5.3 DEPARTURE FROM NORMAL THEORY VALUES FOR A COMPOUND NORMAL DISTRIBUTION (P,1-P) WITH MEANS 3.0 STANDARD DEVIATIONS APART AND A COMMON VARIANCE.

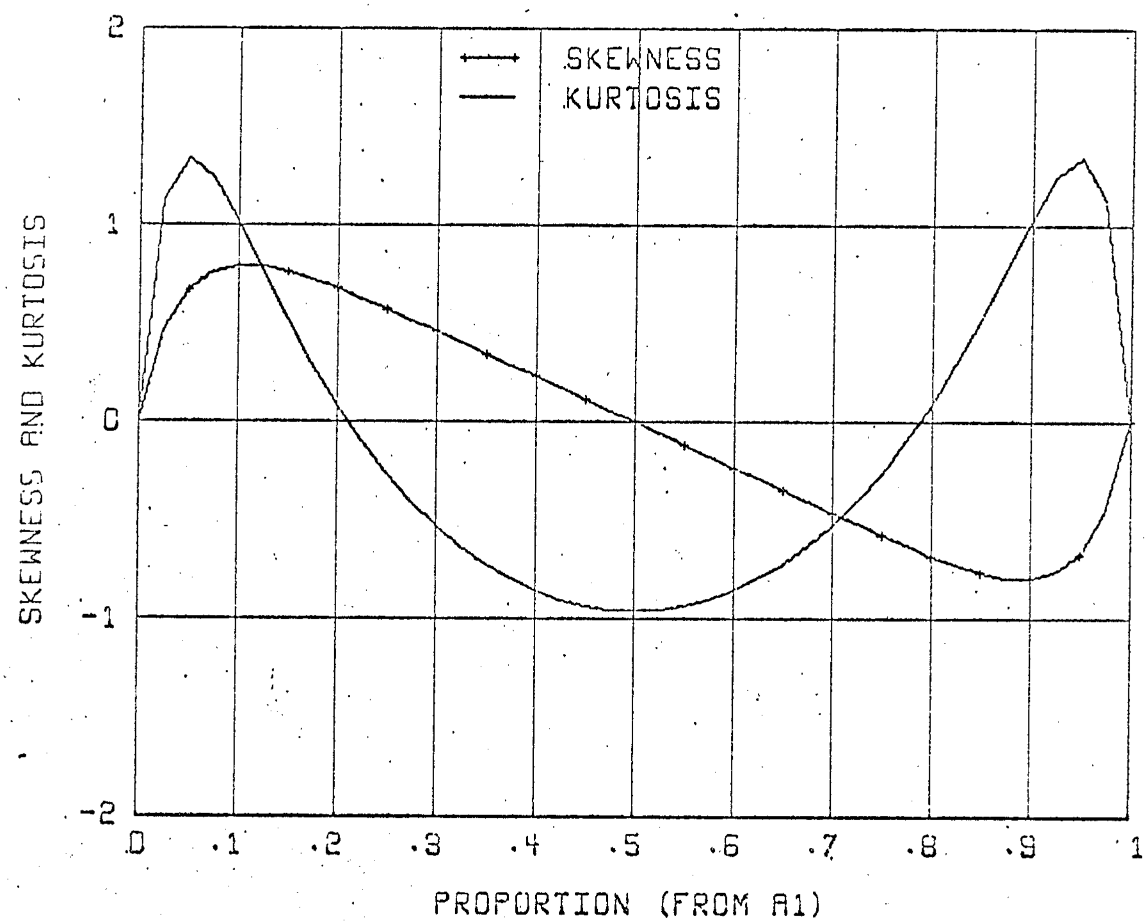
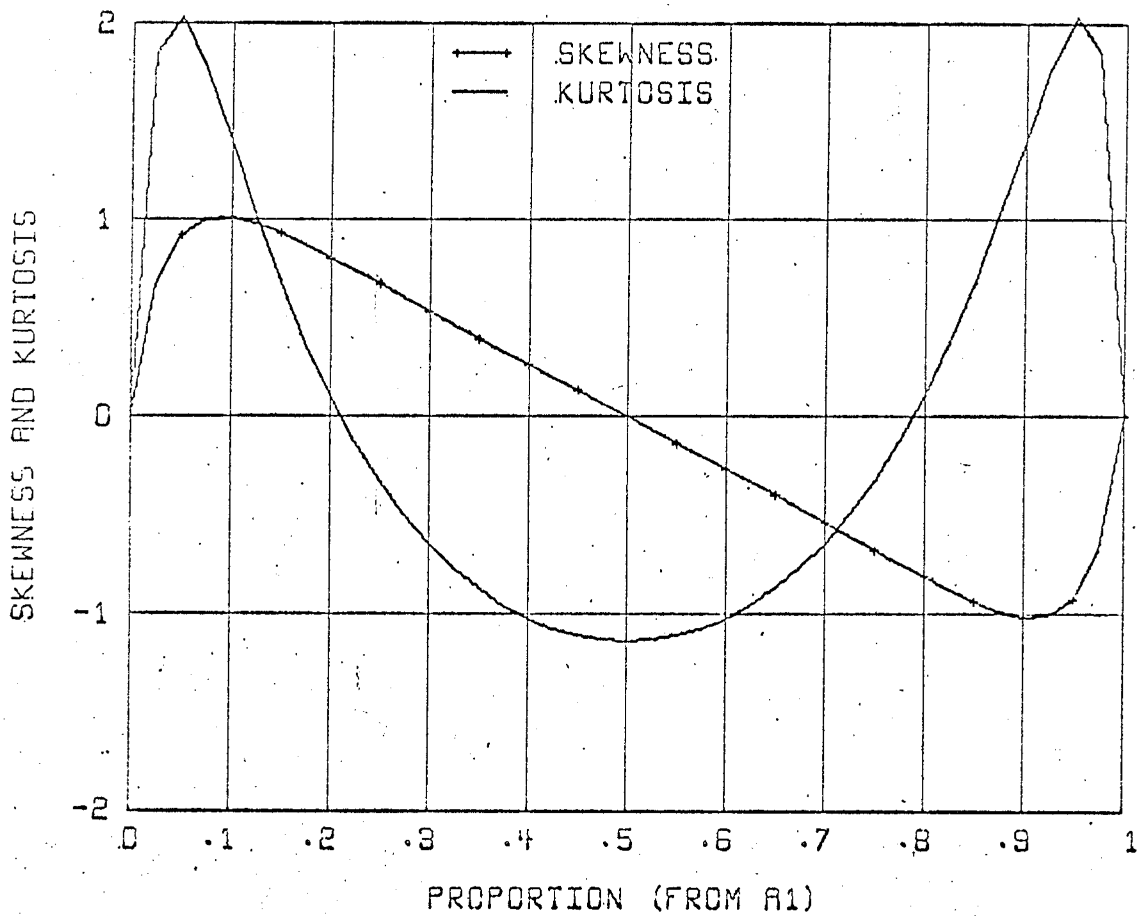


FIG. 5.4 DEPARTURE FROM NORMAL THEORY VALUES FOR A COMPOUND NORMAL DISTRIBUTION (P,1-P) WITH MEANS 3.5 STANDARD DEVIATIONS APART AND A COMMON VARIANCE.



Values have been calculated for γ_1 and γ_2 for $d = 1, 2, 3$ and 3.5 and for $p = 0(.025)1$ with the values graphed in Figures 5.1, 5.2, 5.3 and 5.4. The curves for γ_1 have radial symmetry with respect to the centre (0, 0.5) of the diagram; the curves for γ_2 have axial symmetry with respect to the axis (line) $p = 0.5$. In Table 5.1 the numerical values are given for $d = 2, 3$, and 3.5 and $p = 0(.025).5$. The values of γ_1 and γ_2 for $p = 0.5(.025)1$ can be found by the properties just described.

From Fig. 5.1 we can see that the distribution deviates very little from $\gamma_1 = 0$ and $\gamma_2 = 0$ which are the corresponding values for a normal distribution. In fact, $|\gamma_1| < .077$ and $|\gamma_2| < .080$. These values of γ_1 and γ_2 would have very little effect on the inferences drawn from treating the populations as normal in using the t-test.

As d increases, the distribution becomes more unlike a normal one. When p lies in the interval (.30, .75) and the population is treated as normal, no serious errors will be made, especially not on a two-tailed test, even where d is as large as 3.5. (It will be recalled from Chapter IV that if $\gamma_1 \leq .5$, use of Student's distribution would not lead to any serious error). When $d \geq 3$ and p lies in the interval (.025, .300) or in the interval (.700, .975), however, γ_1

is greater than .5 and the assumption of normality would lead to serious errors.

In cases where a compound normal population is known to exist the values of γ_1 and γ_2 can be calculated from values of a_1, a_2, p and λ, if the latter are known. Corrections can then be made for γ_1 and γ_2 by using Gayen's t-distribution as described in the preceeding chapter.

More complex distributions of the compound normal type can be obtained by increasing k to include more than two normal populations. Similar tables and graphs could be constructed for these distributions also to permit corrections for γ_1 and γ_2 and thus obtain the correct t-distribution. However, even for n = 3 the expressions of μ_2, μ_3, μ_4, γ_1, and γ_2 are awkward to handle and will not be considered here.

CHAPTER VI

CONCLUSIONS

Student's distribution was developed for testing the significance of an assumed mean when the variance of the population is unknown. The statistic

$$t = (\bar{x} - \mu)\sqrt{n/s}$$

is the ratio of a normal deviate $\bar{x} - \mu$ to the root of a χ^2 variate and its distribution is called Student's t-distribution. Tables for this distribution are widely available. Both one and two-tailed t-tests can be made, and treatment of these tests is similar. For example, consider the one-tailed test of the hypothesis

$$H_0 : \mu = m$$

against the alternative

$$H_1 : \mu > m.$$

We need merely calculate

$$t = (\bar{x} - m)\sqrt{n/s}$$

for a sample of size n drawn from the normal population considered. The value of the probability integral

$$P(t) = \frac{\Gamma(\frac{n}{2})}{\sqrt{\pi(n-1)}\,\Gamma\left[(n-1)/2\right]} \int_t^{\infty} \frac{1}{\left[1 + t^2/(n-1)\right]^{n/2}}\, dt$$

for this value of t can then be read from the appropriate tables and this is the probability that a value of t or greater would occur by chance if the true mean actually were m. If this value is too small, usually if it is less than 0.05, we will reject H_o.

Many tests such as this are made using Student's tables without any knowledge of the true distribution of the variable. Although the population will seldom be perfectly normally distributed, one knows from the Central Limit Theorem that the distribution of the means of samples of size n always approach normality as n becomes large. For smaller values of n, however, the normal distribution may not be a sufficiently accurate approximation of that of the means of samples. In such situations t will not have Student's t-distribution and the probability statements based on Student's t-integral will not be correct.

If the values of γ_1 and γ_2 for the population are known, Gayen's distribution gives the correct distribution of t and can be used to find out how large an error will be made if normality is erroneously assumed. Alternatively, we can use γ_1 and γ_2 to obtain the correct t-distribution and from this get the true probability of a value as large or larger than the t calculated from the given sample. If $|\gamma_1| < .5$ and $|\gamma_2| < 2$, the error made in assuming that the population is normal will not cause any serious misjudgements in making statistical inferences.

The control of type I error in a two-tailed test, where γ_1 or γ_2 or both are different from zero, will be nearly as good as in the case where both are zero, if the sum of the tail areas for the t-value used is equal to, or nearly equal to, the sum of the tail areas of Student's distribution for that t-value. In a one-tailed test the control will not be as good.

When testing for the difference of two means, the two populations will usually have their corresponding values of γ_1 and γ_2 nearly equal. The effects of γ_1 and of γ_2 will then be cancelled and the population of the differences of the means of the samples will have an approximately normal distribution.

γ_1 has relatively a much greater effect on the t-distribution than γ_2, and if $\gamma_1 = 0$ the error made in assuming normality erroneously will be negligible. Thus, in most practical work, the value of γ_2 can be ignored without danger of serious errors, but, unless $|\gamma_1|$ is quite small, say $< .5$, the effect of γ_1 should be taken into consideration in drawing inferences.

As an example, consider a population where $\gamma_1 = .5$, $\gamma_2 = 1$ and test the hypothesis

$$H_o : \mu = 50$$

against the alternative

$$H_1 : \mu \neq 50.$$

Assume that a sample of size 9 is taken from a population assumed to be normal, and that $\bar{x} = 56$ and $s = 7$. Choose α as .05. H_o will then be rejected if $|t| > |t_o|$ where t_o is the tabulated value for $\alpha = .05$ with 8 degrees of freedom, in our case 2.306. We find

$$t = \frac{56 - 50}{7/\sqrt{9}} = 2.57.$$

This is greater than 2.306 and H_o is rejected. However, since the population is not normal, the true level of significance actually turns out to be .0517 instead of .05. An error of .0017 in the level of significance is not considered serious and with an adjusted value for t the conclusions would still be the same, that is, we would reject H_o.

In general, if $|\gamma_1|$ and $|\gamma_2|$ are small, no serious error will be made in assuming normality when using Student's t-distribution. If they are somewhat larger, especially if γ_1 is large, corrections should be made for non-normality using Gayen's form of the t-distribution. If γ_1 or γ_2 or both fall outside the limits given by Barton and Dennis (1952) then the t-test is probably not the correct or best test to use. Either a transformation of the population into one with a more nearly normal distribution will have to be made or another test will have to be used.

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APPENDIX A

The tables in Appendix A give the probability of a random value of $t = (\bar{x} - \mu)\sqrt{n}/s$ being greater than the values tabulated in the margins. The number of degrees of freedom and the values of γ_1 (skewness) and γ_2 (kurtosis) are given at the top of each table.

D.F.= 2 SKEWNESS =0.0 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4647	.4300	.3962	.3639	.3333	.3047	.2782	.2538	.2316
1.	.2113	.1930	.1765	.1616	.1482	.1362	.1254	.1156	.1068	.0989
2.	.0918	.0853	.0794	.0741	.0692	.0648	.0608	.0571	.0537	.0506
3.	.0477	.0451	.0427	.0404	.0383	.0364	.0346	.0330	.0314	.0300

D.F.= 2 SKEWNESS =0.500 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4808	.4447	.4087	.3734	.3393	.3070	.2770	.2494	.2243	.2018
1.	.1817	.1639	.1481	.1341	.1219	.1110	.1014	.0929	.0853	.0786
2.	.0726	.0672	.0624	.0580	.0541	.0505	.0473	.0443	.0417	.0392
3.	.0369	.0349	.0330	.0312	.0296	.0281	.0267	.0254	.0242	.0231

D.F.= 2 SKEWNESS =1.000 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4616	.4237	.3855	.3478	.3116	.2775	.2462	.2180	.1931	.1713
1.	.1525	.1363	.1225	.1106	.1005	.0917	.0841	.0775	.0717	.0666
2.	.0620	.0579	.0543	.0509	.0479	.0452	.0426	.0403	.0382	.0362
3.	.0344	.0327	.0311	.0297	.0283	.0270	.0258	.0247	.0236	.0226

D.F.= 2 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4669	.4341	.4018	.3704	.3402	.3114	.2842	.2589	.2355
1.	.2140	.1945	.1768	.1608	.1465	.1337	.1222	.1119	.1027	.0944
2.	.0870	.0804	.0744	.0690	.0641	.0597	.0558	.0521	.0488	.0458
3.	.0431	.0405	.0382	.0361	.0341	.0323	.0306	.0291	.0277	.0263

D.F.= 2 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4808	.4469	.4128	.3789	.3458	.3139	.2837	.2554	.2294	.2057
1.	.1844	.1653	.1484	.1334	.1201	.1085	.0982	.0892	.0812	.0741
2.	.0679	.0623	.0574	.0530	.0490	.0455	.0423	.0394	.0368	.0344
3.	.0323	.0303	.0285	.0269	.0254	.0240	.0227	.0215	.0204	.0194

D.F.= 2 SKEWNESS =1.000 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4616	.4259	.3896	.3534	.3181	.2843	.2529	.2240	.1982	.1752
1.	.1552	.1378	.1228	.1099	.0988	.0892	.0809	.0738	.0675	.0621
2.	.0573	.0530	.0492	.0459	.0428	.0401	.0376	.0354	.0333	.0315
3.	.0297	.0281	.0267	.0253	.0241	.0229	.0218	.0208	.0199	.0190

D.F.= 2 SKEWNESS =0.0 KURTOSIS =2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4691	.4382	.4074	.3769	.3471	.3180	.2902	.2639	.2394
1.	.2167	.1959	.1770	.1600	.1448	.1312	.1190	.1082	.0985	.0899
2.	.0823	.0755	.0694	.0639	.0591	.0547	.0507	.0472	.0440	.0410
3.	.0384	.0360	.0338	.0318	.0299	.0282	.0267	.0252	.0239	.0227

D.F.= 2 SKEWNESS =0.500 KURTOSIS =2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4808	.4491	.4169	.3845	.3523	.3208	.2903	.2614	.2344	.2096
1.	.1870	.1667	.1486	.1326	.1184	.1060	.0950	.0854	.0770	.0696
2.	.0631	.0574	.0524	.0479	.0439	.0404	.0372	.0344	.0319	.0296
3.	.0276	.0258	.0241	.0226	.0212	.0199	.0187	.0177	.0167	.0158

D.F.= 2 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4616	.4281	.3937	.3590	.3246	.2912	.2595	.2301	.2032	.1791
1.	.1578	.1392	.1230	.1091	.0970	.0867	.0777	.0700	.0634	.0576
2.	.0526	.0481	.0442	.0408	.0378	.0350	.0326	.0304	.0285	.0267
3.	.0251	.0236	.0222	.0210	.0199	.0188	.0179	.0170	.0161	.0154

D.F.= 3 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4633	.4271	.3919	.3580	.3257	.2954	.2672	.2411	.2172
1.	.1955	.1758	.1581	.1422	.1280	.1153	.1040	.0938	.0848	.0768
2.	.0697	.0633	.0576	.0525	.0479	.0439	.0402	.0369	.0339	.0313
3.	.0288	.0266	.0247	.0229	.0212	.0197	.0184	.0171	.0160	.0150

D.F.= 3 SKEWNESS = 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4834	.4460	.4088	.3721	.3366	.3028	.2710	.2416	.2146	.1903
1.	.1685	.1491	.1319	.1169	.1036	.0921	.0820	.0732	.0655	.0587
2.	.0528	.0476	.0430	.0390	.0354	.0323	.0295	.0269	.0247	.0227
3.	.0209	.0193	.0178	.0165	.0153	.0142	.0132	.0123	.0115	.0107

D.F.= 3 SKEWNESS = 1.000 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4668	.4279	.3888	.3502	.3127	.2771	.2439	.2136	.1864	.1623
1.	.1414	.1232	.1077	.0945	.0833	.0738	.0657	.0588	.0529	.0479
2.	.0435	.0397	.0364	.0335	.0309	.0285	.0264	.0246	.0229	.0213
3.	.0199	.0186	.0175	.0164	.0154	.0145	.0136	.0128	.0121	.0114

D.F.= 3 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4651	.4306	.3967	.3637	.3319	.3017	.2731	.2464	.2217
1.	.1989	.1782	.1595	.1426	.1275	.1140	.1020	.0913	.0819	.0735
2.	.0661	.0596	.0538	.0486	.0440	.0400	.0364	.0331	.0303	.0277
3.	.0254	.0233	.0215	.0198	.0183	.0169	.0157	.0145	.0135	.0126

D.F.= 3 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4834	.4479	.4122	.3769	.3423	.3090	.2773	.2475	.2199	.1947
1.	.1719	.1515	.1333	.1172	.1031	.0908	.0800	.0707	.0625	.0554
2.	.0493	.0439	.0392	.0351	.0315	.0284	.0256	.0232	.0210	.0191
3.	.0174	.0159	.0146	.0134	.0123	.0114	.0105	.0097	.0090	.0083

D.F.= 3 SKEWNESS =1.000 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4668	.4298	.3923	.3549	.3184	.2833	.2502	.2195	.1917	.1668
1.	.1448	.1256	.1091	.0949	.0828	.0725	.0637	.0563	.0500	.0446
2.	.0400	.0360	.0326	.0296	.0270	.0247	.0226	.0208	.0192	.0178
3.	.0165	.0153	.0143	.0133	.0124	.0116	.0109	.0102	.0096	.0090

D.F.= 3 SKEWNESS =0.0 KURTOSIS =2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4669	.4340	.4015	.3694	.3382	.3080	.2791	.2517	.2261
1.	.2024	.1806	.1609	.1430	.1270	.1127	.1001	.0888	.0790	.0703
2.	.0626	.0559	.0499	.0447	.0401	.0361	.0325	.0294	.0266	.0241
3.	.0219	.0200	.0183	.0167	.0153	.0141	.0129	.0119	.0110	.0102

D.F. = 3 SKEWNESS = 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4834	.4497	.4157	.3817	.3481	.3152	.2836	.2535	.2253	.1992
1.	.1754	.1539	.1346	.1176	.1026	.0895	.0781	.0682	.0596	.0522
2.	.0457	.0402	.0354	.0321	.0276	.0245	.0218	.0194	.0174	.0156
3.	.0140	.0126	.0114	.0103	.0094	.0085	.0078	.0071	.0065	.0060

D.F.= 3 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4668	.4316	.3957	.3597	.3241	.2895	.2565	.2255	.1970	.1712
1.	.1482	.1280	.1104	.0953	.0823	.0712	.0618	.0538	.0471	.0413
2.	.0364	.0323	.0287	.0257	.0231	.0208	.0188	.0171	.0156	.0142
3.	.0130	.0120	.0110	.0102	.0095	.0088	.0082	.0076	.0071	.0066

D.F.= 4 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4626	.4256	.3896	.3548	.3217	.2904	.2612	.2343	.2095
1.	.1869	.1665	.1482	.1317	.1171	.1040	.0924	.0822	.0731	.0651
2.	.0581	.0518	.0463	.0415	.0372	.0334	.0300	.0270	.0244	.0221
3.	.0200	.0181	.0165	.0150	.0136	.0124	.0114	.0104	.0096	.0088

D.F.= 4 SKEWNESS = 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4851	.4472	.4092	.3719	.3357	.3011	.2685	.2381	.2102	.1849
1.	.1621	.1418	.1239	.1082	.0944	.0825	.0721	.0631	.0554	.0487
2.	.0429	.0378	.0335	.0297	.0264	.0236	.0211	.0189	.0169	.0152
3.	.0138	.0124	.0113	.0102	.0093	.0085	.0077	.0071	.0065	.0060

D.F.= 4 SKEWNESS =1.000 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4703	.4311	.3916	.3525	.3145	.2782	.2442	.2129	.1845	.1592
1.	.1370	.1177	.1012	.0871	.0752	.0651	.0567	.0496	.0437	.0387
2.	.0344	.0308	.0277	.0250	.0227	.0207	.0189	.0173	.0159	.0146
3.	.0135	.0125	.0116	.0107	.0099	.0092	.0086	.0080	.0075	.0070

D.F.= 4 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4641	.4286	.3937	.3598	.3272	.2961	.2668	.2393	.2139
1.	.1905	.1692	.1500	.1326	.1172	.1034	.0912	.0804	.0709	.0626
2.	.0553	.0489	.0433	.0384	.0341	.0303	.0270	.0241	.0215	.0193
3.	.0173	.0156	.0140	.0127	.0115	.0104	.0094	.0086	.0078	.0071

D.F.= 4 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4851	.4487	.4122	.3761	.3407	.3066	.2742	.2436	.2153	.1893
1.	.1657	.1445	.1257	.1091	.0945	.0819	.0709	.0614	.0532	.0461
2.	.0401	.0349	.0304	.0266	.0233	.0205	.0180	.0159	.0141	.0125
3.	.0111	.0099	.0088	.0079	.0071	.0064	.0058	.0052	.0047	.0043

D.F.= 4 SKEWNESS =1.000 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4703	.4326	.3945	.3566	.3195	.2837	.2499	.2184	.1895	.1636
1.	.1406	.1204	.1029	.0880	.0753	.0645	.0555	.0479	.0415	.0361
2.	.0316	.0279	.0247	.0219	.0196	.0176	.0159	.0144	.0130	.0119
3.	.0109	.0099	.0091	.0084	.0078	.0072	.0066	.0061	.0057	.0053

D.F.= 4 SKEWNESS = 0.0 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4657	.4315	.3978	.3648	.3327	.3018	.2723	.2444	.2183
1.	.1941	.1719	.1518	.1336	.1173	.1028	.0890	.0787	.0687	.0601
2.	.0525	.0460	.0402	.0353	.0310	.0272	.0240	.0211	.0187	.0165
3.	.0147	.0130	.0116	.0104	.0093	.0083	.0075	.0067	.0060	.0055

D.F.= 4 SKEWNESS = 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4851	.4502	.4152	.3802	.3458	.3122	.2798	.2491	.2203	.1936
1.	.1692	.1472	.1275	.1100	.0946	.0812	.0696	.0596	.0510	.0436
2.	.0373	.0320	.0274	.0235	.0202	.0174	.0150	.0130	.0112	.0097
3.	.0085	.0074	.0064	.0056	.0049	.0043	.0038	.0034	.0030	.0026

D.F.= 4 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4703	.4341	.3975	.3608	.3245	.2893	.2556	.2239	.1946	.1680
1.	.1441	.1231	.1047	.0889	.0754	.0639	.0542	.0461	.0393	.0336
2.	.0289	.0249	.0216	.0188	.0165	.0145	.0128	.0114	.0102	.0091
3.	.0082	.0074	.0067	.0061	.0056	.0051	.0047	.0043	.0039	.0036

D.F.= 5 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4621	.4247	.3881	.3528	.3192	.2873	.2576	.2300	.2047
1.	.1816	.1607	.1419	.1252	.1102	.0970	.0852	.0749	.0659	.0579
2.	.0510	.0449	.0395	.0349	.0308	.0272	.0241	.0214	.0190	.0169
3.	.0150	.0134	.0120	.0107	.0096	.0086	.0078	.0070	.0063	.0057

D.F.= 5 SKEWNESS =0.500 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4864	.4481	.4098	.3721	.3355	.3004	.2673	.2363	.2078	.1819
1.	.1585	.1377	.1193	.1031	.0890	.0768	.0663	.0572	.0494	.0427
2.	.0370	.0321	.0279	.0244	.0213	.0186	.0164	.0144	.0127	.0112
3.	.0100	.0089	.0079	.0070	.0063	.0056	.0051	.0045	.0041	.0037

D.F.= 5 SKEWNESS =1.000 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4729	.4334	.3938	.3545	.3163	.2796	.2452	.2132	.1842	.1582
1.	.1352	.1151	.0979	.0832	.0708	.0603	.0516	.0444	.0384	.0333
2.	.0292	.0257	.0227	.0202	.0181	.0163	.0147	.0133	.0121	.0110
3.	.0101	.0092	.0085	.0078	.0071	.0066	.0061	.0056	.0052	.0048

D.F.= 5 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4635	.4273	.3918	.3573	.3241	.2925	.2626	.2347	.2088
1.	.1851	.1635	.1439	.1263	.1106	.0967	.0844	.0736	.0642	.0559
2.	.0487	.0425	.0370	.0323	.0282	.0247	.0216	.0190	.0167	.0147
3.	.0129	.0114	.0101	.0089	.0079	.0071	.0063	.0056	.0050	.0045

D.F.= 5 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4864	.4494	.4124	.3757	.3399	.3053	.2724	.2414	.2125	.1861
1.	.1620	.1404	.1212	.1043	.0894	.0766	.0654	.0559	.0477	.0407
2.	.0348	.0297	.0254	.0218	.0187	.0161	.0139	.0120	.0104	.0090
3.	.0078	.0068	.0060	.0052	.0046	.0041	.0036	.0032	.0028	.0025

D.F.= 5 SKEWNESS = 1.000 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4729	.4348	.3963	.3581	.3207	.2846	.2503	.2183	.1889	.1623
1.	.1387	.1179	.0998	.0843	.0712	.0601	.0508	.0431	.0367	.0313
2.	.0269	.0233	.0202	.0177	.0155	.0137	.0122	.0109	.0098	.0088
3.	.0080	.0072	.0066	.0060	.0055	.0050	.0046	.0042	.0039	.0036

D.F.= 5 SKEWNESS = 0.0 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4648	.4298	.3954	.3617	.3290	.2976	.2676	.2394	.2130
1.	.1886	.1662	.1458	.1275	.1111	.0965	.0836	.0723	.0625	.0539
2.	.0465	.0401	.0345	.0298	.0257	.0222	.0192	.0166	.0144	.0125
3.	.0108	.0094	.0082	.0072	.0063	.0055	.0048	.0042	.0037	.0033

D.F.= 5 SKEWNESS = 0.500 KURTOSIS 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4864	.4507	.4149	.3793	.3443	.3103	.2775	.2464	.2172	.1902
1.	.1655	.1431	.1231	.1054	.0899	.0763	.0646	.0546	.0460	.0387
2.	.0325	.0273	.0229	.0193	.0162	.0136	.0114	.0096	.0081	.0068
3.	.0057	.0048	.0041	.0035	.0029	.0025	.0021	.0018	.0015	.0013

D.F. = 5 SKEWNESS 1.000 KURTOSIS 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4729	.4361	.3989	.3618	.3251	.2895	.2554	.2233	.1936	.1665
1.	.1421	.1206	.1018	.0855	.0716	.0599	.0500	.0418	.0350	.0293
2.	.0247	.0209	.0177	.0151	.0130	.0112	.0097	.0085	.0075	.0066
3.	.0058	.0052	.0047	.0042	.0038	.0034	.0031	.0028	.0026	.0024

D.F.= 6   SKEWNESS =0.0   KURTOSIS =0.0

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4962	.4924	.4885	.4847	.4809	.4771	.4732	.4694	.4656
0.1	.4618	.4580	.4542	.4504	.4466	.4428	.4391	.4353	.4315	.4278
0.2	.4240	.4203	.4166	.4129	.4092	.4055	.4018	.3981	.3944	.3908
0.3	.3872	.3835	.3799	.3763	.3727	.3691	.3656	.3620	.3585	.3550
0.4	.3515	.3480	.3446	.3411	.3377	.3343	.3309	.3275	.3241	.3208
0.5	.3174	.3141	.3108	.3076	.3043	.3011	.2979	.2947	.2915	.2884
0.6	.2852	.2821	.2790	.2760	.2729	.2699	.2669	.2639	.2609	.2580
0.7	.2551	.2522	.2493	.2464	.2436	.2408	.2380	.2353	.2325	.2298
0.8	.2271	.2244	.2218	.2192	.2165	.2140	.2114	.2089	.2064	.2039
0.9	.2014	.1990	.1965	.1941	.1918	.1894	.1871	.1848	.1825	.1802
1.0	.1780	.1757	.1735	.1714	.1692	.1671	.1650	.1629	.1608	.1588
1.1	.1567	.1547	.1528	.1508	.1489	.1470	.1451	.1432	.1413	.1395
1.2	.1377	.1359	.1341	.1324	.1306	.1289	.1272	.1256	.1239	.1223
1.3	.1207	.1191	.1175	.1159	.1144	.1129	.1114	.1099	.1084	.1070
1.4	.1055	.1041	.1027	.1013	.1000	.0986	.0973	.0960	.0947	.0934
1.5	.0921	.0909	.0897	.0884	.0872	.0861	.0849	.0837	.0826	.0815
1.6	.0804	.0793	.0782	.0771	.0761	.0750	.0740	.0730	.0720	.0710
1.7	.0700	.0691	.0681	.0672	.0663	.0653	.0644	.0636	.0627	.0618
1.8	.0610	.0601	.0593	.0585	.0577	.0569	.0561	.0553	.0546	.0538
1.9	.0531	.0524	.0516	.0509	.0502	.0495	.0488	.0482	.0475	.0469
2.0	.0462	.0456	.0450	.0443	.0437	.0431	.0425	.0419	.0414	.0408
2.1	.0402	.0397	.0391	.0386	.0381	.0376	.0370	.0365	.0360	.0355
2.2	.0351	.0346	.0341	.0336	.0332	.0327	.0323	.0318	.0314	.0310
2.3	.0306	.0301	.0297	.0293	.0289	.0285	.0281	.0278	.0274	.0270
2.4	.0266	.0263	.0259	.0256	.0252	.0249	.0246	.0242	.0239	.0236
2.5	.0233	.0230	.0226	.0223	.0220	.0217	.0215	.0212	.0209	.0206
2.6	.0203	.0201	.0198	.0195	.0193	.0190	.0188	.0185	.0183	.0180
2.7	.0178	.0176	.0173	.0171	.0169	.0166	.0164	.0162	.0160	.0158
2.8	.0156	.0154	.0152	.0150	.0148	.0146	.0144	.0142	.0140	.0138
2.9	.0137	.0135	.0133	.0131	.0130	.0128	.0126	.0125	.0123	.0122
3.0	.0120	.0119	.0117	.0115	.0114	.0113	.0111	.0110	.0108	.0107
3.1	.0106	.0104	.0103	.0102	.0100	.0099	.0098	.0097	.0095	.0094
3.2	.0093	.0092	.0091	.0090	.0088	.0087	.0086	.0085	.0084	.0083
3.3	.0082	.0081	.0080	.0079	.0078	.0077	.0076	.0075	.0074	.0073
3.4	.0072	.0072	.0071	.0070	.0069	.0068	.0067	.0067	.0066	.0065
3.5	.0064	.0063	.0063	.0062	.0061	.0060	.0060	.0059	.0058	.0058
3.6	.0057	.0056	.0055	.0055	.0054	.0054	.0053	.0052	.0052	.0051
3.7	.0050	.0050	.0049	.0049	.0048	.0048	.0047	.0046	.0046	.0045

D.F.= 6 SKEWNESS =0.500 KURTOSIS =0.0

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.4874	.4836	.4798	.4759	.4720	.4682	.4643	.4604	.4565	.4527
0.1	.4488	.4449	.4411	.4372	.4334	.4295	.4256	.4218	.4179	.4141
0.2	.4103	.4064	.4026	.3988	.3950	.3912	.3874	.3836	.3799	.3761
0.3	.3723	.3686	.3649	.3611	.3574	.3537	.3501	.3464	.3427	.3391
0.4	.3355	.3319	.3283	.3247	.3211	.3176	.3140	.3105	.3070	.3036
0.5	.3001	.2967	.2932	.2898	.2865	.2831	.2798	.2765	.2732	.2699
0.6	.2666	.2634	.2602	.2570	.2538	.2507	.2476	.2445	.2414	.2384
0.7	.2353	.2323	.2294	.2264	.2235	.2206	.2177	.2149	.2120	.2092
0.8	.2065	.2037	.2010	.1983	.1956	.1930	.1903	.1877	.1852	.1826
0.9	.1801	.1776	.1752	.1727	.1703	.1679	.1655	.1632	.1609	.1586
1.0	.1563	.1541	.1519	.1497	.1476	.1454	.1433	.1412	.1392	.1371
1.1	.1351	.1331	.1312	.1292	.1273	.1254	.1236	.1217	.1199	.1181
1.2	.1163	.1146	.1129	.1112	.1095	.1078	.1062	.1046	.1030	.1014
1.3	.0999	.0984	.0968	.0954	.0939	.0925	.0910	.0896	.0883	.0869
1.4	.0855	.0842	.0829	.0816	.0804	.0791	.0779	.0767	.0755	.0743
1.5	.0731	.0720	.0709	.0698	.0687	.0676	.0665	.0655	.0645	.0635
1.6	.0625	.0615	.0605	.0596	.0586	.0577	.0568	.0559	.0550	.0542
1.7	.0533	.0525	.0517	.0509	.0501	.0493	.0485	.0477	.0470	.0463
1.8	.0455	.0448	.0441	.0434	.0427	.0421	.0414	.0408	.0401	.0395
1.9	.0389	.0383	.0377	.0371	.0365	.0360	.0354	.0348	.0343	.0338
2.0	.0332	.0327	.0322	.0317	.0312	.0308	.0303	.0298	.0294	.0289
2.1	.0285	.0280	.0276	.0272	.0268	.0264	.0259	.0256	.0252	.0248
2.2	.0244	.0240	.0237	.0233	.0230	.0226	.0223	.0219	.0216	.0213
2.3	.0210	.0207	.0203	.0200	.0197	.0194	.0192	.0189	.0186	.0183
2.4	.0180	.0178	.0175	.0173	.0170	.0168	.0165	.0163	.0160	.0158
2.5	.0156	.0153	.0151	.0149	.0147	.0145	.0143	.0141	.0139	.0137
2.6	.0135	.0133	.0131	.0129	.0127	.0125	.0123	.0122	.0120	.0118
2.7	.0117	.0115	.0113	.0112	.0110	.0109	.0107	.0106	.0104	.0103
2.8	.0101	.0100	.0099	.0097	.0096	.0095	.0093	.0092	.0091	.0089
2.9	.0088	.0087	.0086	.0085	.0084	.0082	.0081	.0080	.0079	.0078
3.0	.0077	.0076	.0075	.0074	.0073	.0072	.0071	.0070	.0069	.0068
3.1	.0067	.0067	.0066	.0065	.0064	.0063	.0062	.0061	.0061	.0060
3.2	.0059	.0058	.0058	.0057	.0056	.0055	.0055	.0054	.0053	.0053
3.3	.0052	.0051	.0051	.0050	.0049	.0049	.0048	.0048	.0047	.0046
3.4	.0046	.0045	.0045	.0044	.0044	.0043	.0043	.0042	.0041	.0041
3.5	.0040	.0040	.0039	.0039	.0039	.0038	.0038	.0037	.0037	.0036
3.6	.0036	.0035	.0035	.0035	.0034	.0034	.0033	.0033	.0033	.0032
3.7	.0032	.0031	.0031	.0031	.0030	.0030	.0030	.0029	.0029	.0029

D.F.= 6 SKEWNESS =1.000 KURTOSIS =0.0

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.4749	.4710	.4671	.4631	.4591	.4552	.4512	.4472	.4433	.4393
0.1	.4353	.4313	.4274	.4234	.4194	.4154	.4114	.4075	.4035	.3995
0.2	.3956	.3916	.3876	.3837	.3797	.3758	.3719	.3679	.3640	.3601
0.3	.3562	.3523	.3484	.3445	.3407	.3368	.3330	.3292	.3254	.3216
0.4	.3178	.3141	.3103	.3066	.3029	.2992	.2955	.2919	.2882	.2846
0.5	.2810	.2774	.2739	.2703	.2668	.2633	.2599	.2564	.2530	.2496
0.6	.2463	.2429	.2396	.2363	.2330	.2298	.2266	.2234	.2202	.2171
0.7	.2140	.2109	.2079	.2048	.2019	.1989	.1960	.1931	.1902	.1873
0.8	.1845	.1817	.1790	.1762	.1735	.1709	.1682	.1656	.1631	.1605
0.9	.1580	.1555	.1530	.1506	.1482	.1458	.1435	.1412	.1389	.1367
1.0	.1345	.1323	.1301	.1280	.1259	.1238	.1218	.1198	.1178	.1158
1.1	.1139	.1120	.1101	.1083	.1065	.1047	.1029	.1012	.0995	.0978
1.2	.0961	.0945	.0929	.0913	.0898	.0883	.0868	.0853	.0838	.0824
1.3	.0810	.0796	.0782	.0769	.0756	.0743	.0730	.0718	.0706	.0694
1.4	.0682	.0670	.0659	.0648	.0637	.0626	.0615	.0605	.0594	.0584
1.5	.0574	.0565	.0555	.0546	.0537	.0528	.0519	.0510	.0502	.0493
1.6	.0485	.0477	.0469	.0462	.0454	.0446	.0439	.0432	.0425	.0418
1.7	.0411	.0405	.0398	.0392	.0385	.0379	.0373	.0367	.0361	.0356
1.8	.0350	.0345	.0339	.0334	.0329	.0324	.0319	.0314	.0309	.0304
1.9	.0300	.0295	.0291	.0287	.0282	.0278	.0274	.0270	.0266	.0262
2.0	.0258	.0255	.0251	.0247	.0244	.0240	.0237	.0234	.0230	.0227
2.1	.0224	.0221	.0218	.0215	.0212	.0209	.0206	.0204	.0201	.0198
2.2	.0196	.0193	.0191	.0188	.0186	.0183	.0181	.0179	.0176	.0174
2.3	.0172	.0170	.0168	.0166	.0163	.0161	.0160	.0158	.0156	.0154
2.4	.0152	.0150	.0148	.0147	.0145	.0143	.0141	.0140	.0138	.0137
2.5	.0135	.0134	.0132	.0131	.0129	.0128	.0126	.0125	.0123	.0122
2.6	.0121	.0119	.0118	.0117	.0116	.0114	.0113	.0112	.0111	.0109
2.7	.0108	.0107	.0106	.0105	.0104	.0103	.0102	.0101	.0100	.0099
2.8	.0098	.0097	.0096	.0095	.0094	.0093	.0092	.0091	.0090	.0089
2.9	.0088	.0087	.0086	.0086	.0085	.0084	.0083	.0082	.0082	.0081
3.0	.0080	.0079	.0078	.0078	.0077	.0076	.0075	.0075	.0074	.0073
3.1	.0073	.0072	.0071	.0071	.0070	.0069	.0069	.0068	.0067	.0067
3.2	.0066	.0066	.0065	.0064	.0064	.0063	.0063	.0062	.0061	.0061
3.3	.0060	.0060	.0059	.0059	.0058	.0058	.0057	.0057	.0056	.0056
3.4	.0055	.0055	.0054	.0054	.0053	.0053	.0052	.0052	.0051	.0051
3.5	.0050	.0050	.0049	.0049	.0049	.0048	.0048	.0047	.0047	.0046
3.6	.0046	.0046	.0045	.0045	.0044	.0044	.0044	.0043	.0043	.0043
3.7	.0042	.0042	.0041	.0041	.0041	.0040	.0040	.0040	.0039	.0039

D.F.= 6 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4963	.4926	.4889	.4852	.4815	.4778	.4741	.4704	.4667
0.1	.4630	.4593	.4556	.4519	.4483	.4446	.4409	.4373	.4336	.4300
0.2	.4263	.4227	.4191	.4155	.4118	.4082	.4046	.4011	.3975	.3939
0.3	.3904	.3868	.3833	.3798	.3763	.3728	.3693	.3658	.3623	.3589
0.4	.3555	.3520	.3486	.3452	.3418	.3385	.3351	.3318	.3285	.3252
0.5	.3219	.3186	.3153	.3121	.3089	.3057	.3025	.2993	.2961	.2930
0.6	.2899	.2868	.2837	.2806	.2776	.2745	.2715	.2685	.2656	.2626
0.7	.2597	.2568	.2539	.2510	.2481	.2453	.2425	.2397	.2369	.2342
0.8	.2314	.2287	.2260	.2234	.2207	.2181	.2155	.2129	.2104	.2078
0.9	.2053	.2028	.2003	.1979	.1954	.1930	.1906	.1883	.1859	.1836
1.0	.1813	.1790	.1767	.1745	.1723	.1701	.1679	.1657	.1636	.1615
1.1	.1594	.1573	.1553	.1533	.1513	.1493	.1473	.1454	.1434	.1415
1.2	.1396	.1378	.1359	.1341	.1323	.1305	.1288	.1270	.1253	.1236
1.3	.1219	.1203	.1186	.1170	.1154	.1138	.1122	.1107	.1091	.1076
1.4	.1061	.1046	.1032	.1017	.1003	.0989	.0975	.0962	.0948	.0935
1.5	.0921	.0908	.0895	.0883	.0870	.0858	.0846	.0834	.0822	.0810
1.6	.0798	.0787	.0775	.0764	.0753	.0742	.0732	.0721	.0711	.0700
1.7	.0690	.0680	.0670	.0661	.0651	.0642	.0632	.0623	.0614	.0605
1.8	.0596	.0587	.0579	.0570	.0562	.0554	.0546	.0538	.0530	.0522
1.9	.0514	.0507	.0499	.0492	.0485	.0478	.0471	.0464	.0457	.0450
2.0	.0443	.0437	.0430	.0424	.0418	.0412	.0406	.0400	.0394	.0388
2.1	.0382	.0377	.0371	.0366	.0360	.0355	.0350	.0344	.0339	.0334
2.2	.0329	.0324	.0320	.0315	.0310	.0306	.0301	.0297	.0292	.0288
2.3	.0284	.0280	.0276	.0272	.0268	.0264	.0260	.0256	.0252	.0248
2.4	.0245	.0241	.0238	.0234	.0231	.0227	.0224	.0221	.0218	.0214
2.5	.0211	.0208	.0205	.0202	.0199	.0196	.0193	.0191	.0188	.0185
2.6	.0182	.0180	.0177	.0175	.0172	.0170	.0167	.0165	.0162	.0160
2.7	.0158	.0155	.0153	.0151	.0149	.0147	.0145	.0142	.0140	.0138
2.8	.0136	.0135	.0133	.0131	.0129	.0127	.0125	.0123	.0122	.0120
2.9	.0118	.0117	.0115	.0113	.0112	.0110	.0109	.0107	.0106	.0104
3.0	.0103	.0101	.0100	.0098	.0097	.0096	.0094	.0093	.0092	.0090
3.1	.0089	.0088	.0087	.0086	.0084	.0083	.0082	.0081	.0080	.0079
3.2	.0078	.0077	.0075	.0074	.0073	.0072	.0071	.0070	.0070	.0069
3.3	.0068	.0067	.0066	.0065	.0064	.0063	.0062	.0062	.0061	.0060
3.4	.0059	.0058	.0058	.0057	.0056	.0055	.0055	.0054	.0053	.0052
3.5	.0052	.0051	.0050	.0050	.0049	.0048	.0048	.0047	.0046	.0046
3.6	.0045	.0045	.0044	.0044	.0043	.0042	.0042	.0041	.0041	.0040
3.7	.0040	.0039	.0039	.0038	.0038	.0037	.0037	.0036	.0036	.0035

D.F.= 6 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.4874	.4837	.4800	.4762	.4725	.4687	.4650	.4612	.4575	.4537
0.1	.4500	.4462	.4425	.4387	.4350	.4312	.4275	.4238	.4200	.4163
0.2	.4126	.4088	.4051	.4014	.3977	.3940	.3903	.3866	.3829	.3792
0.3	.3756	.3719	.3682	.3646	.3610	.3574	.3537	.3501	.3466	.3430
0.4	.3394	.3359	.3323	.3288	.3253	.3218	.3183	.3148	.3114	.3079
0.5	.3045	.3011	.2977	.2944	.2910	.2877	.2844	.2811	.2778	.2745
0.6	.2713	.2680	.2648	.2617	.2585	.2553	.2522	.2491	.2460	.2430
0.7	.2399	.2369	.2339	.2310	.2280	.2251	.2222	.2193	.2164	.2136
0.8	.2108	.2080	.2052	.2025	.1998	.1971	.1944	.1918	.1892	.1866
0.9	.1840	.1815	.1789	.1764	.1740	.1715	.1691	.1667	.1643	.1620
1.0	.1597	.1574	.1551	.1528	.1506	.1484	.1462	.1441	.1420	.1399
1.1	.1378	.1357	.1337	.1317	.1297	.1278	.1258	.1239	.1220	.1202
1.2	.1183	.1165	.1147	.1129	.1112	.1095	.1077	.1061	.1044	.1028
1.3	.1011	.0996	.0980	.0964	.0949	.0934	.0919	.0904	.0890	.0876
1.4	.0861	.0848	.0834	.0820	.0807	.0794	.0781	.0768	.0756	.0744
1.5	.0731	.0719	.0708	.0696	.0685	.0673	.0662	.0651	.0640	.0630
1.6	.0619	.0609	.0599	.0589	.0579	.0570	.0560	.0551	.0541	.0532
1.7	.0523	.0515	.0506	.0498	.0489	.0481	.0473	.0465	.0457	.0449
1.8	.0442	.0434	.0427	.0420	.0413	.0406	.0399	.0392	.0385	.0379
1.9	.0372	.0366	.0360	.0354	.0348	.0342	.0336	.0330	.0325	.0319
2.0	.0314	.0308	.0303	.0298	.0293	.0288	.0283	.0278	.0274	.0269
2.1	.0264	.0260	.0256	.0251	.0247	.0243	.0239	.0235	.0231	.0227
2.2	.0223	.0219	.0215	.0212	.0208	.0205	.0201	.0198	.0195	.0191
2.3	.0188	.0185	.0182	.0179	.0176	.0173	.0170	.0167	.0164	.0161
2.4	.0159	.0156	.0154	.0151	.0148	.0146	.0144	.0141	.0139	.0137
2.5	.0134	.0132	.0130	.0128	.0126	.0124	.0121	.0119	.0118	.0116
2.6	.0114	.0112	.0110	.0108	.0106	.0105	.0103	.0101	.0100	.0098
2.7	.0096	.0095	.0093	.0092	.0090	.0089	.0087	.0086	.0085	.0083
2.8	.0082	.0081	.0079	.0078	.0077	.0076	.0074	.0073	.0072	.0071
2.9	.0070	.0069	.0068	.0067	.0066	.0065	.0063	.0063	.0062	.0061
3.0	.0060	.0059	.0058	.0057	.0056	.0055	.0054	.0053	.0053	.0052
3.1	.0051	.0050	.0049	.0049	.0048	.0047	.0047	.0046	.0045	.0044
3.2	.0044	.0043	.0042	.0042	.0041	.0041	.0040	.0039	.0039	.0038
3.3	.0038	.0037	.0037	.0036	.0035	.0035	.0034	.0034	.0033	.0033
3.4	.0032	.0032	.0031	.0031	.0031	.0030	.0030	.0029	.0029	.0028
3.5	.0028	.0028	.0027	.0027	.0026	.0026	.0026	.0025	.0025	.0025
3.6	.0024	.0024	.0024	.0023	.0023	.0023	.0022	.0022	.0022	.0021
3.7	.0021	.0021	.0020	.0020	.0020	.0020	.0019	.0019	.0019	.0019

D.F.= 6 SKEWNESS =1.000 KURTOSIS =1.000

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.4749	.4711	.4673	.4634	.4596	.4558	.4519	.4481	.4442	.4404
0.1	.4365	.4326	.4288	.4249	.4210	.4172	.4133	.4094	.4056	.4017
0.2	.3978	.3940	.3901	.3863	.3824	.3786	.3747	.3709	.3671	.3632
0.3	.3594	.3556	.3518	.3480	.3442	.3405	.3367	.3329	.3292	.3255
0.4	.3218	.3181	.3144	.3107	.3070	.3034	.2998	.2962	.2926	.2890
0.5	.2854	.2819	.2784	.2749	.2714	.2679	.2645	.2610	.2576	.2543
0.6	.2509	.2476	.2442	.2409	.2377	.2344	.2312	.2280	.2249	.2217
0.7	.2186	.2155	.2124	.2094	.2064	.2034	.2004	.1975	.1946	.1917
0.8	.1888	.1860	.1832	.1805	.1777	.1750	.1723	.1697	.1670	.1644
0.9	.1619	.1593	.1568	.1543	.1519	.1495	.1471	.1447	.1424	.1401
1.0	.1378	.1355	.1333	.1311	.1290	.1268	.1247	.1226	.1206	.1186
1.1	.1166	.1146	.1127	.1107	.1089	.1070	.1052	.1034	.1016	.0998
1.2	.0981	.0964	.0947	.0931	.0915	.0899	.0883	.0867	.0852	.0837
1.3	.0823	.0808	.0794	.0780	.0766	.0752	.0739	.0726	.0713	.0700
1.4	.0688	.0676	.0664	.0652	.0640	.0629	.0617	.0606	.0596	.0585
1.5	.0574	.0564	.0554	.0544	.0535	.0525	.0516	.0506	.0497	.0489
1.6	.0480	.0471	.0463	.0455	.0447	.0439	.0431	.0423	.0416	.0409
1.7	.0401	.0394	.0387	.0381	.0374	.0367	.0361	.0355	.0349	.0343
1.8	.0337	.0331	.0325	.0320	.0314	.0309	.0303	.0298	.0293	.0288
1.9	.0283	.0279	.0274	.0269	.0265	.0260	.0256	.0252	.0248	.0244
2.0	.0240	.0236	.0232	.0228	.0225	.0221	.0217	.0214	.0210	.0207
2.1	.0204	.0201	.0198	.0194	.0191	.0188	.0186	.0183	.0180	.0177
2.2	.0174	.0172	.0169	.0167	.0164	.0162	.0159	.0157	.0155	.0152
2.3	.0150	.0148	.0146	.0144	.0142	.0140	.0138	.0136	.0134	.0132
2.4	.0130	.0128	.0127	.0125	.0123	.0122	.0120	.0118	.0117	.0115
2.5	.0114	.0112	.0111	.0109	.0108	.0106	.0105	.0104	.0102	.0101
2.6	.0100	.0099	.0097	.0096	.0095	.0094	.0093	.0091	.0090	.0089
2.7	.0088	.0087	.0086	.0085	.0084	.0083	.0082	.0081	.0080	.0079
2.8	.0078	.0077	.0076	.0076	.0075	.0074	.0073	.0072	.0071	.0071
2.9	.0070	.0069	.0068	.0067	.0067	.0066	.0065	.0065	.0064	.0063
3.0	.0063	.0062	.0061	.0061	.0060	.0059	.0059	.0058	.0057	.0057
3.1	.0056	.0056	.0055	.0054	.0054	.0053	.0053	.0052	.0052	.0051
3.2	.0051	.0050	.0050	.0049	.0049	.0048	.0048	.0047	.0047	.0046
3.3	.0046	.0045	.0045	.0045	.0044	.0044	.0043	.0043	.0042	.0042
3.4	.0042	.0041	.0041	.0040	.0040	.0040	.0039	.0039	.0039	.0038
3.5	.0038	.0038	.0037	.0037	.0036	.0036	.0036	.0035	.0035	.0035
3.6	.0034	.0034	.0034	.0034	.0033	.0033	.0033	.0032	.0032	.0032
3.7	.0031	.0031	.0031	.0031	.0030	.0030	.0030	.0030	.0029	.0029

D.F.= 6 SKEWNESS =0.0 KURTOSIS =2.000

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4965	.4929	.4892	.4857	.4821	.4785	.4749	.4713	.4677
0.1	.4642	.4606	.4570	.4535	.4499	.4463	.4428	.4392	.4357	.4321
0.2	.4286	.4251	.4216	.4180	.4145	.4110	.4075	.4040	.4005	.3971
0.3	.3936	.3901	.3867	.3832	.3798	.3764	.3730	.3696	.3662	.3628
0.4	.3594	.3560	.3527	.3493	.3460	.3427	.3394	.3361	.3328	.3295
0.5	.3263	.3230	.3198	.3166	.3134	.3102	.3070	.3039	.3007	.2976
0.6	.2945	.2914	.2883	.2853	.2822	.2792	.2762	.2732	.2702	.2672
0.7	.2643	.2613	.2584	.2555	.2527	.2498	.2470	.2441	.2413	.2385
0.8	.2358	.2330	.2303	.2276	.2249	.2222	.2196	.2170	.2144	.2118
0.9	.2092	.2066	.2041	.2016	.1991	.1966	.1942	.1918	.1894	.1870
1.0	.1846	.1823	.1799	.1776	.1753	.1731	.1708	.1686	.1664	.1642
1.1	.1621	.1599	.1578	.1557	.1536	.1516	.1495	.1475	.1455	.1436
1.2	.1416	.1397	.1378	.1359	.1340	.1321	.1303	.1285	.1267	.1249
1.3	.1232	.1215	.1197	.1180	.1164	.1147	.1131	.1115	.1099	.1083
1.4	.1067	.1052	.1037	.1022	.1007	.0992	.0978	.0963	.0949	.0935
1.5	.0921	.0908	.0894	.0881	.0868	.0855	.0842	.0830	.0817	.0805
1.6	.0793	.0781	.0769	.0758	.0746	.0735	.0724	.0713	.0702	.0691
1.7	.0680	.0670	.0660	.0650	.0640	.0630	.0620	.0611	.0601	.0592
1.8	.0583	.0574	.0565	.0556	.0547	.0539	.0530	.0522	.0514	.0506
1.9	.0498	.0490	.0482	.0475	.0467	.0460	.0453	.0446	.0439	.0432
2.0	.0425	.0418	.0411	.0405	.0398	.0392	.0386	.0380	.0374	.0368
2.1	.0362	.0356	.0351	.0345	.0339	.0334	.0329	.0323	.0318	.0313
2.2	.0308	.0303	.0298	.0294	.0289	.0284	.0280	.0275	.0271	.0267
2.3	.0262	.0258	.0254	.0250	.0246	.0242	.0238	.0234	.0230	.0227
2.4	.0223	.0220	.0216	.0213	.0209	.0206	.0202	.0199	.0196	.0193
2.5	.0190	.0187	.0184	.0181	.0178	.0175	.0172	.0170	.0167	.0164
2.6	.0162	.0159	.0156	.0154	.0151	.0149	.0147	.0144	.0142	.0140
2.7	.0137	.0135	.0133	.0131	.0129	.0127	.0125	.0123	.0121	.0119
2.8	.0117	.0115	.0113	.0112	.0110	.0108	.0106	.0105	.0103	.0101
2.9	.0100	.0098	.0097	.0095	.0094	.0092	.0091	.0089	.0088	.0087
3.0	.0085	.0084	.0083	.0081	.0080	.0079	.0077	.0076	.0075	.0074
3.1	.0073	.0072	.0071	.0069	.0068	.0067	.0066	.0065	.0064	.0063
3.2	.0062	.0061	.0060	.0059	.0058	.0058	.0057	.0056	.0055	.0054
3.3	.0053	.0052	.0052	.0051	.0050	.0049	.0049	.0048	.0047	.0046
3.4	.0046	.0045	.0044	.0044	.0043	.0042	.0042	.0041	.0040	.0040
3.5	.0039	.0039	.0038	.0037	.0037	.0036	.0036	.0035	.0035	.0034
3.6	.0034	.0033	.0033	.0032	.0032	.0031	.0031	.0030	.0030	.0029
3.7	.0029	.0029	.0028	.0028	.0027	.0027	.0027	.0026	.0026	.0025

D.F.= 6 SKEWNESS =0.500 KURTOSIS =2.000

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.4874	.4839	.4802	.4766	.4730	.4693	.4657	.4621	.4584	.4548
0.1	.4512	.4475	.4439	.4403	.4366	.4330	.4294	.4257	.4221	.4185
0.2	.4148	.4112	.4076	.4040	.4004	.3968	.3931	.3895	.3860	.3824
0.3	.3788	.3752	.3716	.3681	.3645	.3610	.3574	.3539	.3504	.3469
0.4	.3434	.3399	.3364	.3329	.3295	.3260	.3226	.3191	.3157	.3123
0.5	.3090	.3056	.3022	.2989	.2955	.2922	.2889	.2857	.2824	.2791
0.6	.2759	.2727	.2695	.2663	.2631	.2600	.2569	.2538	.2507	.2476
0.7	.2445	.2415	.2385	.2355	.2325	.2296	.2266	.2237	.2209	.2180
0.8	.2151	.2123	.2095	.2067	.2040	.2012	.1985	.1958	.1932	.1905
0.9	.1879	.1853	.1827	.1802	.1777	.1752	.1727	.1702	.1678	.1654
1.0	.1630	.1606	.1583	.1560	.1537	.1514	.1492	.1470	.1448	.1426
1.1	.1404	.1383	.1362	.1341	.1321	.1301	.1281	.1261	.1241	.1222
1.2	.1203	.1184	.1165	.1147	.1129	.1111	.1093	.1075	.1058	.1041
1.3	.1024	.1008	.0991	.0975	.0959	.0943	.0928	.0912	.0897	.0882
1.4	.0868	.0853	.0839	.0825	.0811	.0797	.0784	.0770	.0757	.0744
1.5	.0731	.0719	.0706	.0694	.0682	.0670	.0659	.0647	.0636	.0625
1.6	.0614	.0603	.0593	.0582	.0572	.0562	.0552	.0542	.0532	.0523
1.7	.0514	.0504	.0495	.0486	.0478	.0469	.0461	.0452	.0444	.0436
1.8	.0428	.0420	.0413	.0405	.0398	.0390	.0383	.0376	.0369	.0363
1.9	.0356	.0349	.0343	.0337	.0330	.0324	.0318	.0312	.0306	.0301
2.0	.0295	.0290	.0284	.0279	.0274	.0269	.0263	.0259	.0254	.0249
2.1	.0244	.0240	.0235	.0231	.0226	.0222	.0218	.0214	.0210	.0206
2.2	.0202	.0198	.0194	.0190	.0187	.0183	.0180	.0176	.0173	.0170
2.3	.0166	.0163	.0160	.0157	.0154	.0151	.0148	.0145	.0143	.0140
2.4	.0137	.0134	.0132	.0129	.0127	.0124	.0122	.0120	.0117	.0115
2.5	.0113	.0111	.0109	.0106	.0104	.0102	.0100	.0098	.0097	.0095
2.6	.0093	.0091	.0089	.0088	.0086	.0084	.0082	.0081	.0079	.0078
2.7	.0076	.0075	.0073	.0072	.0070	.0069	.0068	.0066	.0065	.0064
2.8	.0063	.0061	.0060	.0059	.0058	.0057	.0056	.0055	.0053	.0052
2.9	.0051	.0050	.0049	.0048	.0048	.0047	.0046	.0045	.0044	.0043
3.0	.0042	.0041	.0041	.0040	.0039	.0038	.0037	.0037	.0036	.0035
3.1	.0035	.0034	.0033	.0033	.0032	.0031	.0031	.0030	.0030	.0029
3.2	.0028	.0028	.0027	.0027	.0026	.0026	.0025	.0025	.0024	.0024
3.3	.0023	.0023	.0022	.0022	.0021	.0021	.0021	.0020	.0020	.0019
3.4	.0019	.0019	.0018	.0018	.0018	.0017	.0017	.0017	.0016	.0016
3.5	.0016	.0015	.0015	.0015	.0014	.0014	.0014	.0013	.0013	.0013
3.6	.0013	.0012	.0012	.0012	.0012	.0011	.0011	.0011	.0011	.0011
3.7	.0010	.0010	.0010	.0010	.0010	.0009	.0009	.0009	.0009	.0009

D.F. = 6 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.4749	.4712	.4675	.4638	.4601	.4564	.4526	.4489	.4452	.4414
0.1	.4377	.4339	.4302	.4264	.4227	.4189	.4152	.4114	.4076	.4039
0.2	.4001	.3964	.3926	.3888	.3851	.3813	.3776	.3738	.3701	.3664
0.3	.3626	.3589	.3552	.3515	.3478	.3441	.3404	.3367	.3330	.3294
0.4	.3257	.3221	.3184	.3148	.3112	.3076	.3040	.3005	.2969	.2934
0.5	.2898	.2863	.2829	.2794	.2759	.2725	.2690	.2656	.2623	.2589
0.6	.2555	.2522	.2489	.2456	.2423	.2391	.2359	.2327	.2295	.2263
0.7	.2232	.2201	.2170	.2139	.2109	.2079	.2049	.2019	.1990	.1961
0.8	.1932	.1903	.1875	.1847	.1819	.1791	.1764	.1737	.1710	.1684
0.9	.1658	.1632	.1606	.1581	.1556	.1531	.1506	.1482	.1458	.1434
1.0	.1411	.1388	.1365	.1342	.1320	.1298	.1276	.1255	.1234	.1213
1.1	.1192	.1172	.1152	.1132	.1112	.1093	.1074	.1055	.1037	.1019
1.2	.1001	.0983	.0966	.0948	.0931	.0915	.0898	.0882	.0866	.0851
1.3	.0835	.0820	.0805	.0790	.0776	.0762	.0748	.0734	.0720	.0707
1.4	.0694	.0681	.0668	.0656	.0644	.0632	.0620	.0608	.0597	.0586
1.5	.0574	.0564	.0553	.0543	.0532	.0522	.0512	.0503	.0493	.0484
1.6	.0475	.0466	.0457	.0448	.0439	.0431	.0423	.0415	.0407	.0399
1.7	.0392	.0384	.0377	.0370	.0363	.0356	.0349	.0342	.0336	.0329
1.8	.0323	.0317	.0311	.0305	.0299	.0294	.0288	.0283	.0277	.0272
1.9	.0267	.0262	.0257	.0252	.0247	.0243	.0238	.0234	.0229	.0225
2.0	.0221	.0217	.0213	.0209	.0205	.0201	.0198	.0194	.0191	.0187
2.1	.0184	.0180	.0177	.0174	.0171	.0168	.0165	.0162	.0159	.0156
2.2	.0153	.0151	.0148	.0145	.0143	.0140	.0138	.0136	.0133	.0131
2.3	.0129	.0126	.0124	.0122	.0120	.0118	.0116	.0114	.0112	.0110
2.4	.0109	.0107	.0105	.0103	.0102	.0100	.0098	.0097	.0095	.0094
2.5	.0092	.0091	.0089	.0088	.0087	.0085	.0084	.0083	.0081	.0080
2.6	.0079	.0078	.0077	.0075	.0074	.0073	.0072	.0071	.0070	.0069
2.7	.0068	.0067	.0066	.0065	.0064	.0063	.0062	.0061	.0061	.0060
2.8	.0059	.0058	.0057	.0056	.0056	.0055	.0054	.0053	.0053	.0052
2.9	.0051	.0051	.0050	.0049	.0049	.0048	.0047	.0047	.0046	.0046
3.0	.0045	.0045	.0044	.0043	.0043	.0042	.0042	.0041	.0041	.0040
3.1	.0040	.0039	.0039	.0038	.0038	.0037	.0037	.0037	.0036	.0036
3.2	.0035	.0035	.0035	.0034	.0034	.0033	.0033	.0033	.0032	.0032
3.3	.0032	.0031	.0031	.0030	.0030	.0030	.0029	.0029	.0029	.0029
3.4	.0028	.0028	.0028	.0027	.0027	.0027	.0026	.0026	.0026	.0026
3.5	.0025	.0025	.0025	.0025	.0024	.0024	.0024	.0024	.0023	.0023
3.6	.0023	.0023	.0022	.0022	.0022	.0022	.0022	.0021	.0021	.0021
3.7	.0021	.0021	.0020	.0020	.0020	.0020	.0020	.0019	.0019	.0019

D.F.= 8 SKEWNESS =0.0 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4614	.4232	.3859	.3498	.3153	.2826	.2519	.2234	.1972
1.	.1733	.1517	.1322	.1149	.0995	.0860	.0741	.0638	.0548	.0470
2.	.0403	.0345	.0295	.0252	.0216	.0185	.0158	.0135	.0116	.0099
3.	.0085	.0073	.0063	.0054	.0047	.0040	.0035	.0030	.0026	.0023

D.F.= 8 SKEWNESS =0.500 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4889	.4500	.4111	.3729	.3357	.3000	.2662	.2344	.2051	.1782
1.	.1540	.1323	.1130	.0962	.0815	.0688	.0580	.0487	.0409	.0343
2.	.0288	.0241	.0202	.0170	.0143	.0120	.0101	.0086	.0073	.0062
3.	.0052	.0045	.0038	.0033	.0028	.0024	.0021	.0018	.0016	.0014

D.F.= 8 SKEWNESS =1.000 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4778	.4381	.3983	.3589	.3204	.2834	.2484	.2157	.1857	.1586
1.	.1344	.1132	.0947	.0790	.0656	.0544	.0451	.0375	.0312	.0261
2.	.0220	.0186	.0159	.0136	.0118	.0103	.0090	.0080	.0071	.0063
3.	.0057	.0051	.0046	.0041	.0037	.0034	.0030	.0027	.0025	.0023

D.F.= 8 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4624	.4251	.3885	.3530	.3189	.2864	.2558	.2271	.2006
1.	.1762	.1541	.1341	.1162	.1003	.0862	.0739	.0632	.0538	.0458
2.	.0389	.0329	.0279	.0236	.0199	.0169	.0142	.0120	.0102	.0086
3.	.0073	.0062	.0052	.0044	.0038	.0032	.0027	.0023	.0020	.0017

D.F.= 8 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4889	.4509	.4130	.3755	.3390	.3037	.2700	.2383	.2088	.1816
1.	.1569	.1347	.1149	.0975	.0822	.0691	.0578	.0481	.0400	.0331
2.	.0274	.0226	.0186	.0153	.0126	.0104	.0086	.0071	.0058	.0048
3.	.0040	.0033	.0027	.0023	.0019	.0016	.0013	.0011	.0009	.0008

D.F.= 8 SKEWNESS =1.000 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4778	.4391	.4002	.3615	.3236	.2870	.2522	.2196	.1894	.1620
1.	.1374	.1156	.0966	.0802	.0663	.0546	.0449	.0369	.0303	.0249
2.	.0206	.0171	.0143	.0120	.0102	.0087	.0075	.0065	.0056	.0050
3.	.0044	.0039	.0035	.0031	.0028	.0025	.0023	.0020	.0019	.0017

D.F.= 8 SKEWNESS =0.0 KURTOSIS =2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4633	.4269	.3912	.3563	.3226	.2903	.2597	.2308	.2040
1.	.1792	.1565	.1360	.1175	.1010	.0865	.0737	.0625	.0529	.0446
2.	.0375	.0314	.0263	.0220	.0183	.0152	.0127	.0105	.0087	.0072
3.	.0060	.0050	.0041	.0034	.0028	.0024	.0020	.0016	.0014	.0011

D.F.= 8 SKEWNESS =0.500 KURTOSIS =2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4889	.4519	.4148	.3782	.3422	.3073	.2739	.2422	.2125	.1850
1.	.1599	.1371	.1168	.0987	.0830	.0693	.0575	.0475	.0390	.0319
2.	.0260	.0211	.0170	.0137	.0110	.0088	.0070	.0055	.0044	.0035
3.	.0027	.0021	.0017	.0013	.0010	.0007	.0006	.0004	.0003	.0002

D.F.= 8 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4778	.4401	.4020	.3641	.3269	.2907	.2561	.2235	.1931	.1654
1.	.1403	.1180	.0985	.0815	.0671	.0549	.0447	.0362	.0293	.0237
2.	.0192	.0156	.0127	.0103	.0085	.0070	.0059	.0050	.0042	.0036
3.	.0031	.0027	.0024	.0021	.0019	.0017	.0015	.0014	.0012	.0011

D.F.= 10 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4612	.4227	.3852	.3488	.3139	.2809	.2499	.2212	.1946
1.	.1704	.1486	.1289	.1114	.0959	.0823	.0703	.0600	.0510	.0433
2.	.0367	.0310	.0262	.0221	.0187	.0157	.0132	.0112	.0094	.0079
3.	.0067	.0056	.0047	.0040	.0034	.0029	.0024	.0021	.0017	.0015

D.F.= 10 SKEWNESS = 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4900	.4508	.4119	.3734	.3361	.3002	.2661	.2341	.0245	.1774
1.	.1528	.1308	.1113	.0942	.0793	.0665	.0555	.0462	.0383	.0317
2.	.0262	.0217	.0179	.0148	.0122	.0101	.0083	.0069	.0057	.0048
3.	.0040	.0033	.0028	.0023	.0020	.0017	.0014	.0012	.0010	.0009

D.F.= 10 SKEWNESS = 1.000 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4799	.4402	.4003	.3609	.3224	.2853	.2501	.2173	.1870	.1596
1.	.1350	.1133	.0945	.0783	.0645	.0530	.0435	.0356	.0292	.0240
2.	.0199	.0165	.0138	.0117	.0099	.0085	.0074	.0064	.0056	.0050
3.	.0044	.0039	.0035	.0031	.0028	.0025	.0022	.0020	.0018	.0016

D.F.= 10 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4620	.4243	.3874	.3515	.3171	.2842	.2533	.2244	.1976
1.	.1731	.1507	.1306	.1126	.0966	.0826	.0703	.0596	.0503	.0424
2.	.0356	.0298	.0249	.0208	.0173	.0144	.0120	.0100	.0083	.0069
3.	.0057	.0047	.0039	.0033	.0027	.0023	.0019	.0016	.0013	.0011

D.F.= 10 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4900	.4516	.4134	.3757	.3388	.3033	.2694	.2375	.2077	.1804
1.	.1554	.1330	.1130	.0954	.0801	.0668	.0554	.0458	.0376	.0308
2.	.0251	.0205	.0166	.0134	.0109	.0088	.0071	.0057	.0046	.0037
3.	.0030	.0024	.0020	.0016	.0013	.0011	.0009	.0007	.0006	.0005

D.F.= 10 SKEWNESS =1.000 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4799	.4410	.4019	.3631	.3251	.2884	.2535	.2206	.1903	.1625
1.	.1376	.1155	.0962	.0795	.0653	.0534	.0434	.0352	.0285	.0231
2.	.0188	.0153	.0125	.0103	.0086	.0072	.0061	.0052	.0045	.0039
3.	.0034	.0030	.0027	.0024	.0021	.0019	.0017	.0015	.0013	.0012

D.F.= 10 SKEWNESS =0.0 KURTOSIS =2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4628	.4259	.3896	.3543	.3202	.2875	.2566	.2276	.2006
1.	.1757	.1529	.1323	.1139	.0974	.0829	.0702	.0592	.0496	.0415
2.	.0345	.0286	.0237	.0195	.0160	.0131	.0107	.0088	.0071	.0058
3.	.0047	.0038	.0031	.0025	.0020	.0016	.0013	.0011	.0009	.0007

D.F.= 10 SKEWNESS = 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4900	.4524	.4149	.3779	.3416	.3064	.2727	.2408	.2110	.1833
1.	.1580	.1352	.1147	.0966	.0808	.0671	.0553	.0453	.0369	.0299
2.	.0240	.0192	.0153	.0121	.0096	.0075	.0058	.0045	.0035	.0027
3.	.0020	.0015	.0011	.0008	.0006	.0004	.0003	.0002	.0001	.0001

D.F.= 10 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4799	.4418	.4034	.3653	.3278	.2915	.2568	.2240	.1935	.1655
1.	.1402	.1177	.0979	.0808	.0661	.0537	.0433	.0348	.0278	.0222
2.	.0177	.0141	.0113	.0090	.0073	.0059	.0049	.0040	.0034	.0028
3.	.0024	.0021	.0018	.0016	.0014	.0013	.0011	.0010	.0009	.0008

D.F.= 12 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4610	.4224	.3847	.3481	.3131	.2798	.2486	.2196	.1929
1.	.1685	.1465	.1266	.1090	.0934	.0797	.0678	.0574	.0485	.0409
2.	.0343	.0288	.0241	.0201	.0168	.0140	.0116	.0097	.0080	.0067
3.	.0055	.0046	.0038	.0032	.0026	.0022	.0018	.0015	.0013	.0011

D.F.= 12 SKEWNESS = 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4908	.4515	.4124	.3739	.3364	.3004	.2662	.2341	.2043	.1770
1.	.1522	.1300	.1103	.0930	.0780	.0650	.0540	.0446	.0367	.0301
2.	.0247	.0201	.0164	.0134	.0109	.0089	.0072	.0059	.0048	.0039
3.	.0032	.0027	.0022	.0018	.0015	.0012	.0010	.0009	.0007	.0006

D.F.= 12 SKEWNESS =1.000 KURTOSIS =0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4816	.4418	.4019	.3624	.3239	.2868	.2516	.2187	.1882	.1606
1.	.1358	.1138	.0947	.0782	.0642	.0524	.0426	.0346	.0281	.0228
2.	.0186	.0152	.0126	.0104	.0087	.0074	.0063	.0054	.0047	.0041
3.	.0036	.0032	.0028	.0025	.0022	.0020	.0017	.0015	.0014	.0012

D.F.= 12 SKEWNESS =0.0 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4617	.4238	.3866	.3505	.3158	.2827	.2516	.2225	.1955
1.	.1709	.1484	.1282	.1102	.0942	.0801	.0678	.0571	.0480	.0401
2.	.0334	.0278	.0230	.0190	.0157	.0129	.0106	.0087	.0071	.0058
3.	.0047	.0039	.0032	.0026	.0021	.0017	.0014	.0011	.0009	.0008

D.F.= 12 SKEWNESS =0.500 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4908	.4522	.4137	.3758	.3388	.3031	.2691	.2370	.2071	.1796
1.	.1546	.1320	.1119	.0942	.0787	.0654	.0540	.0443	.0362	.0294
2.	.0238	.0191	.0154	.0123	.0098	.0078	.0062	.0049	.0039	.0031
3.	.0025	.0019	.0015	.0012	.0010	.0008	.0006	.0005	.0004	.0003

D.F.= 12 SKEWNESS =1.000 KURTOSIS =1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4816	.4424	.4032	.3643	.3263	.2895	.2545	.2216	.1911	.1632
1.	.1381	.1158	.0962	.0793	.0649	.0528	.0427	.0343	.0275	.0221
2.	.0177	.0142	.0115	.0093	.0076	.0063	.0053	.0044	.0038	.0032
3.	.0028	.0025	.0022	.0019	.0017	.0015	.0013	.0012	.0010	.0009

D.F.= 12 SKEWNESS = 0.0 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4624	.4251	.3885	.3528	.3185	.2856	.2545	.2253	.1982
1.	.1732	.1504	.1298	.1113	.0949	.0804	.0678	.0569	.0474	.0394
2.	.0325	.0268	.0219	.0179	.0146	.0118	.0095	.0077	.0062	.0050
3.	.0040	.0032	.0025	.0020	.0016	.0013	.0010	.0008	.0006	.0005

D.F.= 12 SKEWNESS = 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4908	.4529	.4151	.3777	.3412	.3058	.2720	.2400	.2100	.1823
1.	.1569	.1340	.1135	.0953	.0795	.0657	.0540	.0440	.0356	.0286
2.	.0229	.0181	.0143	.0112	.0087	.0067	.0052	.0039	.0030	.0022
3.	.0017	.0012	.0009	.0006	.0004	.0003	.0002	.0001	.0001	.0000

D.F.= 12 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4816	.4431	.4046	.3662	.3287	.2922	.2574	.2245	.1939	.1659
1.	.1404	.1178	.0978	.0805	.0657	.0531	.0427	.0340	.0270	.0213
2.	.0168	.0132	.0104	.0082	.0065	.0052	.0042	.0035	.0029	.0024
3.	.0020	.0017	.0015	.0013	.0012	.0010	.0009	.0008	.0007	.0006

D.F.= 14 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4609	.4222	.3843	.3476	.3124	.2790	.2477	.2185	.1917
1.	.1671	.1449	.1250	.1073	.0916	.0779	.0660	.0556	.0467	.0391
2.	.0326	.0272	.0226	.0187	.0154	.0127	.0105	.0086	.0071	.0058
3.	.0048	.0039	.0032	.0026	.0022	.0018	.0014	.0012	.0010	.0008

D.F.= 14 SKEWNESS = 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4914	.4521	.4129	.3743	.3368	.3007	.2664	.2342	.2043	.1768
1.	.1519	.1295	.1097	.0923	.0771	.0641	.0530	.0435	.0356	.0290
2.	.0236	.0191	.0154	.0125	.0100	.0081	.0065	.0052	.0042	.0034
3.	.0028	.0022	.0018	.0015	.0012	.0010	.0008	.0007	.0005	.0005

D.F.= 14 SKEWNESS = 1.000 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4829	.4430	.4031	.3637	.3252	.2881	.2529	.2199	.1893	.1615
1.	.1365	.1144	.0950	.0783	.0641	.0521	.0422	.0341	.0274	.0220
2.	.0178	.0144	.0117	.0096	.0079	.0066	.0056	.0048	.0041	.0035
3.	.0031	.0027	.0024	.0021	.0018	.0016	.0014	.0013	.0011	.0010

D.F.= 14 SKEWNESS = 0.0 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4615	.4234	.3860	.3497	.3148	.2816	.2503	.2211	.1940
1.	.1692	.1467	.1265	.1084	.0924	.0783	.0660	.0554	.0463	.0385
2.	.0319	.0263	.0216	.0177	.0145	.0118	.0096	.0078	.0063	.0051
3.	.0041	.0033	.0027	.0022	.0017	.0014	.0011	.0009	.0007	.0006

D.F.= 14 SKEWNESS = 0.500 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4914	.4527	.4141	.3760	.3389	.3031	.2690	.2368	.2068	.1792
1.	.1540	.1313	.1112	.0934	.0779	.0645	.0530	.0433	.0352	.0284
2.	.0228	.0183	.0145	.0115	.0091	.0072	.0056	.0044	.0034	.0027
3.	.0021	.0016	.0013	.0010	.0008	.0006	.0005	.0004	.0003	.0002

D.F.= 14 SKEWNESS 1.000 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4829	.4436	.4043	.3654	.3273	.2905	.2555	.2225	.1919	.1639
1.	.1387	.1162	.0965	.0794	.0648	.0525	.0423	.0339	.0270	.0214
2.	.0170	.0136	.0108	.0087	.0070	.0057	.0047	.0039	.0033	.0028
3.	.0024	.0021	.0018	.0016	.0014	.0012	.0011	.0010	.0008	.0008

D.F.= 14 SKEWNESS = 0.0 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4621	.4245	.3877	.3518	.3172	.2842	.2529	.2236	.1964
1.	.1714	.1485	.1279	.1095	.0931	.0787	.0661	.0552	.0458	.0379
2.	.0311	.0255	.0207	.0168	.0136	.0109	.0087	.0070	.0055	.0044
3.	.0035	.0027	.0021	.0017	.0013	.0010	.0008	.0006	.0005	.0004

D.F.= 14 SKEWNESS = 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4914	.4533	.4152	.3777	.3410	.3055	.2715	.2394	.2093	.1815
1.	.1561	.1331	.1126	.0944	.0786	.0648	.0531	.0431	.0348	.0278
2.	.0221	.0174	.0136	.0106	.0082	.0062	.0047	.0036	.0027	.0020
3.	.0014	.0010	.0007	.0005	.0004	.0002	.0001	.0001	.0000	.0000

D.F.= 14 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4829	.4442	.4055	.3671	.3294	.2929	.2580	.2251	.1944	.1662
1.	.1408	.1180	.0979	.0805	.0655	.0529	.0423	.0336	.0265	.0208
2.	.0163	.0127	.0099	.0077	.0061	.0048	.0038	.0031	.0025	.0021
3.	.0018	.0015	.0013	.0011	.0010	.0009	.0008	.0007	.0006	.0005

D.F.= 16 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4608	.4220	.3840	.3472	.3119	.2785	.2470	.2177	.1907
1.	.1661	.1438	.1238	.1060	.0903	.0765	.0646	.0542	.0454	.0378
2.	.0314	.0260	.0214	.0176	.0145	.0118	.0097	.0079	.0064	.0052
3.	.0042	.0034	.0028	.0023	.0018	.0015	.0012	.0010	.0008	.0006

D.F.= 16 SKEWNESS = 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4919	.4524	.4133	.3747	.3371	.3009	.2666	.2343	.2043	.1767
1.	.1517	.1293	.1093	.0918	.0766	.0635	.0523	.0428	.0349	.0283
2.	.0229	.0184	.0148	.0118	.0094	.0075	.0060	.0048	.0038	.0030
3.	.0024	.0019	.0016	.0013	.0010	.0008	.0007	.0005	.0004	.0004

D.F.= 16 SKEWNESS = 1.000 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4838	.4437	.4041	.3647	.3262	.2892	.2540	.2209	.1903	.1624
1.	.1373	.1150	.0954	.0786	.0642	.0521	.0420	.0337	.0270	.0216
2.	.0173	.0138	.0111	.0090	.0074	.0061	.0051	.0043	.0036	.0031
3.	.0027	.0023	.0020	.0018	.0016	.0014	.0012	.0010	.0009	.0008

D.F. = 16 SKEWNESS = 0.0 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4614	.4231	.3855	.3491	.3141	.2807	.2493	.2200	.1929
1.	.1680	.1454	.1251	.1070	.0910	.0769	.0647	.0541	.0450	.0373
2.	.0307	.0252	.0206	.0168	.0136	.0110	.0089	.0072	.0058	.0046
3.	.0037	.0029	.0023	.0019	.0015	.0012	.0009	.0007	.0006	.0005

D.F. = 16 SKEWNESS = 0.500 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4919	.4530	.4143	.3762	.3389	.3031	.2689	.2366	.2066	.1789
1.	.1537	.1309	.1107	.0928	.0773	.0638	.0524	.0427	.0345	.0278
2.	.0222	.0176	.0140	.0110	.0086	.0067	.0052	.0041	.0031	.0024
3.	.0019	.0014	.0011	.0009	.0007	.0005	.0004	.0003	.0002	.0002

D.F. = 16 SKEWNESS = 1.000 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4838	.4443	.4052	.3662	.3281	.2913	.2563	.2232	.1926	.1645
1.	.1392	.1166	.0968	.0796	.0649	.0524	.0421	.0336	.0266	.0210
2.	.0166	.0131	.0104	.0082	.0066	.0053	.0043	.0035	.0030	.0025
3.	.0021	.0018	.0016	.0014	.0012	.0011	.0009	.0008	.0007	.0006

D.F. = 16 SKEWNESS = 0.0 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4619	.4241	.3870	.3510	.3162	.2830	.2517	.2223	.1950
1.	.1699	.1471	.1265	.1080	.0917	.0773	.0647	.0539	.0446	.0367
2.	.0301	.0245	.0198	.0160	.0128	.0102	.0081	.0064	.0051	.0040
3.	.0031	.0024	.0019	.0015	.0011	.0009	.0007	.0005	.0004	.0003

D.F.= 16 SKEWNESS = 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4919	.4535	.4154	.3777	.3408	.3052	.2712	.2390	.2089	.1810
1.	.1556	.1326	.1120	.0938	.0779	.0642	.0524	.0425	.0342	.0272
2.	.0215	.0169	.0132	.0102	.0078	.0059	.0045	.0033	.0025	.0018
3.	.0013	.0009	.0007	.0005	.0003	.0002	.0001	.0001	.0000	.0000

D.F.= 16 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4838	.4448	.4062	.3677	.3300	.2935	.2586	.2255	.1949	.1667
1.	.1411	.1182	.0981	.0806	.0655	.0528	.0422	.0334	.0263	.0205
2.	.0160	.0124	.0096	.0074	.0057	.0045	.0035	.0028	.0023	.0019
3.	.0016	.0013	.0011	.0010	.0008	.0007	.0006	.0006	.0005	.0004

D.F.= 18 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4607	.4219	.3838	.3469	.3116	.2780	.2464	.2171	.1900
1.	.1653	.1429	.1228	.1050	.0893	.0755	.0635	.0532	.0443	.0368
2.	.0304	.0250	.0206	.0168	.0137	.0112	.0091	.0073	.0059	.0048
3.	.0038	.0031	.0025	.0020	.0016	.0013	.0010	.0008	.0007	.0005

D.F.= 18 SKEWNESS 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4924	.4529	.4136	.3750	.3374	.3012	.2668	.2344	.2044	.1768
1.	.1517	.1291	.1091	.0915	.0762	.0630	.0518	.0423	.0343	.0277
2.	.0223	.0179	.0142	.0113	.0090	.0071	.0056	.0044	.0035	.0028
3.	.0022	.0017	.0014	.0011	.0009	.0007	.0006	.0004	.0004	.0003

D.F.= 18 SKEWNESS = 1.000 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4848	.4450	.4050	.3657	.3272	.2901	.2549	.2218	.1912	.1632
1.	.1380	.1155	.0958	.0789	.0644	.0521	.0419	.0336	.0267	.0213
2.	.0169	.0134	.0107	.0086	.0070	.0057	.0047	.0039	.0033	.0028
3.	.0024	.0021	.0018	.0015	.0014	.0012	.0010	.0009	.0008	.0007

D.F.= 18 SKEWNESS = 0.0 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4612	.4228	.3852	.3486	.3135	.2801	.2486	.2192	.1920
1.	.1670	.1444	.1241	.1059	.0899	.0758	.0636	.0530	.0440	.0363
2.	.0298	.0244	.0199	.0161	.0130	.0104	.0084	.0067	.0053	.0042
3.	.0034	.0027	.0021	.0016	.0013	.0010	.0008	.0006	.0005	.0004

D.F.= 18 SKEWNESS 0.500 KURTOSIS 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4924	.4534	.4146	.3763	.3390	.3031	.2689	.2366	.2064	.1787
1.	.1534	.1306	.1103	.0924	.0768	.0634	.0519	.0422	.0340	.0273
2.	.0217	.0172	.0135	.0106	.0083	.0064	.0049	.0038	.0029	.0022
3.	.0017	.0013	.0010	.0008	.0006	.0004	.0003	.0003	.0002	.0002

D.F. = 18 SKEWNESS = 1.000 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4848	.4455	.4059	.3670	.3289	.2920	.2570	.2240	.1932	.1651
1.	.1397	.1171	.0970	.0798	.0650	.0525	.0420	.0335	.0264	.0208
2.	.0163	.0128	.0100	.0079	.0062	.0050	.0040	.0033	.0027	.0023
3.	.0019	.0016	.0014	.0012	.0011	.0009	.0008	.0007	.0006	.0005

D.F.= 18 SKEWNESS = 0.0 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4617	.4238	.3865	.3503	.3154	.2821	.2507	.2212	.1939
1.	.1688	.1459	.1253	.1069	.0905	.0762	.0637	.0529	.0437	.0359
2.	.0293	.0237	.0191	.0154	.0123	.0097	.0077	.0061	.0047	.0037
3.	.0029	.0022	.0017	.0013	.0010	.0008	.0006	.0004	.0003	.0002

D.F.= 18 SKEWNESS = 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4924	.4539	.4155	.3777	.3407	.3050	.2709	.2387	.2085	.1807
1.	.1552	.1321	.1116	.0934	.0775	.0637	.0520	.0421	.0337	.0268
2.	.0212	.0165	.0128	.0099	.0075	.0057	.0043	.0032	.0023	.0017
3.	.0012	.0009	.0006	.0004	.0003	.0002	.0001	.0001	.0000	.0000

D.F.= 18 SKEWNESS = 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4848	.4460	.4069	.3684	.3306	.2940	.2591	.2261	.1953	.1671
1.	.1415	.1186	.0983	.0807	.0656	.0528	.0421	.0333	.0261	.0203
2.	.0157	.0121	.0093	.0072	.0055	.0043	.0033	.0026	.0021	.0017
3.	.0014	.0012	.0010	.0009	.0008	.0006	.0006	.0005	.0004	.0004

D.F.= 20 SKEWNESS = 0.0 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4607	.4218	.3836	.3467	.3113	.2776	.2460	.2166	.1894
1.	.1646	.1422	.1221	.1042	.0884	.0746	.0626	.0523	.0435	.0360
2.	.0296	.0243	.0199	.0162	.0131	.0106	.0086	.0069	.0055	.0044
3.	.0035	.0028	.0022	.0018	.0014	.0011	.0009	.0007	.0006	.0004

D.F.= 20 SKEWNESS = 0.500 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4927	.4532	.4139	.3752	.3376	.3014	.2670	.2346	.2045	.1768
1.	.1517	.1291	.1090	.0913	.0759	.0627	.0514	.0419	.0339	.0273
2.	.0219	.0174	.0138	.0110	.0086	.0068	.0053	.0042	.0033	.0026
3.	.0020	.0016	.0012	.0010	.0008	.0006	.0005	.0004	.0003	.0002

D.F.= 20 SKEWNESS = 1.000 KURTOSIS = 0.0

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4855	.4456	.4058	.3664	.3280	.2910	.2557	.2226	.1919	.1639
1.	.1386	.1161	.0963	.0792	.0646	.0522	.0419	.0335	.0266	.0210
2.	.0166	.0131	.0104	.0083	.0067	.0054	.0044	.0036	.0030	.0025
3.	.0022	.0018	.0016	.0014	.0012	.0010	.0009	.0008	.0007	.0006

D.F.= 20 SKEWNESS = 0.0 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4611	.4226	.3849	.3482	.3130	.2795	.2479	.2185	.1912
1.	.1662	.1436	.1232	.1051	.0890	.0750	.0627	.0522	.0432	.0356
2.	.0291	.0237	.0192	.0155	.0125	.0100	.0080	.0063	.0050	.0039
3.	.0031	.0024	.0019	.0015	.0012	.0009	.0007	.0005	.0004	.0003

D.F.= 20 SKEWNESS = 0.500 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4927	.4537	.4148	.3765	.3391	.3032	.2689	.2365	.2064	.1786
1.	.1533	.1305	.1101	.0922	.0766	.0631	.0515	.0418	.0337	.0269
2.	.0214	.0169	.0132	.0103	.0080	.0062	.0047	.0036	.0027	.0021
3.	.0016	.0012	.0009	.0007	.0005	.0004	.0003	.0002	.0002	.0001

D.F. = 20 SKEWNESS = 1.000 KURTOSIS = 1.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4855	.4460	.4067	.3676	.3295	.2928	.2576	.2246	.1938	.1657
1.	.1402	.1175	.0975	.0801	.0652	.0526	.0421	.0334	.0263	.0206
2.	.0161	.0126	.0098	.0077	.0060	.0047	.0038	.0031	.0025	.0021
3.	.0017	.0015	.0013	.0011	.0009	.0008	.0007	.0006	.0005	.0005

D.F.= 20 SKEWNESS = 0.0 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.5000	.4616	.4235	.3861	.3498	.3148	.2814	.2499	.2204	.1930
1.	.1679	.1450	.1244	.1059	.0896	.0753	.0629	.0521	.0429	.0352
2.	.0286	.0231	.0186	.0149	.0118	.0093	.0074	.0058	.0045	.0035
3.	.0027	.0021	.0016	.0012	.0009	.0007	.0005	.0004	.0003	.0002

D.F.= 20 SKEWNESS 0.500 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4927	.4541	.4156	.3777	.3407	.3049	.2708	.2385	.2083	.1804
1.	.1549	.1318	.1113	.0931	.0772	.0634	.0517	.0417	.0334	.0265
2.	.0209	.0163	.0126	.0097	.0073	.0055	.0041	.0030	.0022	.0016
3.	.0011	.0008	.0006	.0004	.0003	.0002	.0001	.0001	.0000	.0000

D.F.= 20 SKEWNESS 1.000 KURTOSIS = 2.000

	.0	.10	.20	.30	.40	.50	.60	.70	.80	.90
0.	.4855	.4465	.4075	.3689	.3310	.2945	.2595	.2265	.1957	.1675
1.	.1418	.1189	.0986	.0809	.0658	.0529	.0422	.0333	.0260	.0202
2.	.0156	.0120	.0092	.0070	.0054	.0041	.0032	.0025	.0020	.0016
3.	.0013	.0011	.0009	.0008	.0007	.0006	.0005	.0005	.0004	.0003

APPENDIX B

The tables in Appendix B contain values of t_o for which the probability of having a larger value of t is that given in the margin at the top of the table. The left hand margin gives values of degrees of freedom, γ_1, and γ_2 for each value in that row.

D.F.	γ_1	γ_2	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
2	0.0	0.0	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	6.9646	9.9249
2	0.5	0.0	0.6976	0.9084	1.1871	1.6156	2.5154	3.7318	6.0770	8.6806
2	1.0	0.0	0.5873	0.7710	1.0145	1.4050	2.3299	3.6710	6.2137	8.9853
2	0.0	1.0	0.8370	1.0706	1.3747	1.8313	2.7634	4.0066	6.4122	9.0999
2	0.5	1.0	0.7200	0.9257	1.1897	1.5816	2.3737	3.4265	5.4653	7.7462
2	1.0	1.0	0.6095	0.7925	1.0283	1.3880	2.1791	3.3257	5.5743	8.0510
2	0.0	2.0	0.8558	1.0795	1.3646	1.7840	2.6198	3.7179	5.8383	8.2178
2	0.5	2.0	0.7414	0.9413	1.1920	1.5529	2.2513	3.1437	4.8407	6.7418
2	1.0	2.0	0.6314	0.8127	1.0405	1.3739	2.0561	3.0036	4.8993	7.0266

			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
3	0.0	0.0	0.7649	0.9785	1.2498	1.6378	2.3534	3.1825	4.5407	5.8409
3	0.5	0.0	0.6705	0.8589	1.0949	1.4301	2.0523	2.7861	4.0063	5.1804
3	1.0	0.0	0.5810	0.7486	0.9571	1.2564	1.8565	2.6762	4.1340	5.5100
3	0.0	1.0	0.7861	0.9952	1.2549	1.6179	2.2718	3.0180	4.2343	5.3999
3	0.5	1.0	0.6914	0.8783	1.1076	1.4242	1.9873	2.6246	3.6607	4.6544
3	1.0	1.0	0.6006	0.7691	0.9751	1.2621	1.8001	2.4842	3.7332	4.9651
3	0.0	2.0	0.8065	1.0106	1.2595	1.6004	2.1988	2.8637	3.9240	4.9282
3	0.5	2.0	0.7119	0.8967	1.1192	1.4190	1.9319	2.4835	3.3317	4.1107
3	1.0	2.0	0.6203	0.7890	0.9919	1.2671	1.7545	2.3247	3.3265	4.3473

			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
4	0.0	0.0	0.7407	0.9410	1.1896	1.5332	2.1319	2.7765	3.7470	4.6041
4	0.5	0.0	0.6599	0.8390	1.0582	1.3577	1.8788	2.4478	3.3236	4.1094
4	1.0	0.0	0.5824	0.7440	0.9398	1.2077	1.6942	2.3010	3.3923	4.3938
4	0.0	1.0	0.7602	0.9584	1.1998	1.5266	2.0817	2.6674	3.5377	4.3020
4	0.5	1.0	0.6785	0.8575	1.0729	1.3608	1.8433	2.3469	3.0910	3.7440
4	1.0	1.0	0.5996	0.7626	0.9575	1.2185	1.6700	2.1884	3.0940	3.9813
4	0.0	2.0	0.7793	0.9749	1.2092	1.5206	2.0369	2.5667	3.3315	3.9868
4	0.5	2.0	0.6971	0.8754	1.0867	1.3635	1.8126	2.2597	2.8802	3.3891
4	1.0	2.0	0.6171	0.7809	0.9743	1.2284	1.6497	2.0980	2.8157	3.5206

			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
5	0.0	0.0	0.7267	0.9195	1.1558	1.4759	2.0151	2.5706	3.3649	4.0322
5	0.5	0.0	0.6549	0.8292	1.0395	1.3209	1.7917	2.2811	2.9969	3.6123
5	1.0	0.0	0.5855	0.7444	0.9341	1.1869	1.6210	2.1212	3.0083	3.8425
5	0.0	1.0	0.7443	0.9362	1.1676	1.4753	1.9813	2.4913	3.2080	3.8037
5	0.5	1.0	0.6715	0.8462	1.0542	1.3274	1.7703	2.2114	2.8267	3.3369
5	1.0	1.0	0.6008	0.7611	0.9505	1.1989	1.6098	2.0504	2.7789	3.5005
5	0.0	2.0	0.7617	0.9522	1.1787	1.4748	1.9510	2.4184	3.0561	3.5705
5	0.5	2.0	0.6881	0.8628	1.0682	1.3334	1.7515	2.1510	2.6757	3.0801
5	1.0	2.0	0.6164	0.7777	0.9665	1.2101	1.6002	1.9927	2.5799	3.1360

D.F.	γ_1	γ_2	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
6	0.0	0.0	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074
6	0.5	0.0	0.6522	0.8237	1.0287	1.2992	1.7407	2.1843	2.8096	3.3308
6	1.0	0.0	0.5889	0.7463	0.9326	1.1769	1.5821	2.0227	2.7761	3.5074
6	0.0	1.0	0.7335	0.9213	1.1463	1.4423	1.9191	2.3858	3.0183	3.5247
6	0.5	1.0	0.6672	0.8392	1.0428	1.3072	1.7271	2.1329	2.6780	3.1129
6	1.0	1.0	0.6027	0.7615	0.9478	1.1890	1.5771	1.9746	2.5983	3.2144
6	0.0	2.0	0.7493	0.9364	1.1578	1.4446	1.8973	2.3297	2.8990	3.3410
6	0.5	2.0	0.6822	0.8546	1.0563	1.3146	1.7149	2.0877	2.5619	2.9142
6	1.0	2.0	0.6167	0.7766	0.9626	1.2004	1.5728	1.9345	2.4500	2.9202

			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
8	0.0	0.0	0.7064	0.8889	1.1082	1.3968	1.8596	2.3060	2.8965	3.3554
8	0.5	0.0	0.6500	0.8182	1.0174	1.2760	1.6853	2.0793	2.6078	3.0295
8	1.0	0.0	0.5952	0.7512	0.9342	1.1699	1.5451	1.9247	2.5214	3.1136
8	0.0	1.0	0.7196	0.9023	1.1196	1.4019	1.8458	2.2655	2.8098	3.2262
8	0.5	1.0	0.6624	0.8314	1.0299	1.2846	1.6791	2.0472	2.5198	2.8795
8	1.0	1.0	0.6066	0.7640	0.9472	1.1810	1.5453	1.8982	2.4095	2.8943
8	0.0	2.0	0.7328	0.9155	1.1306	1.4067	1.8332	2.2280	2.7273	3.0984
8	0.5	2.0	0.6748	0.8445	1.0421	1.2926	1.6735	2.0185	2.4418	2.7445
8	1.0	2.0	0.6182	0.7767	0.9600	1.1916	1.5454	1.8753	2.3171	2.6951

			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
10	0.0	0.0	0.6998	0.8791	1.0931	1.3722	1.8125	2.2282	2.7638	3.1693
10	0.5	0.0	0.6495	0.8161	1.0122	1.2644	1.6569	2.0252	2.5039	2.8747
10	1.0	0.0	0.6004	0.7560	0.9376	1.1691	1.5297	1.8798	2.3948	2.8919
10	0.0	1.0	0.7110	0.8907	1.1035	1.3780	1.8039	2.1986	2.6980	3.0700
10	0.5	1.0	0.6600	0.8274	1.0232	1.2726	1.6540	2.0027	2.4391	2.7623
10	1.0	1.0	0.6102	0.7670	0.9489	1.1791	1.5317	1.8627	2.3178	2.7273
10	0.0	2.0	0.7223	0.9023	1.1136	1.3835	1.7959	2.1711	2.6354	2.9725
10	0.5	2.0	0.6706	0.8387	1.0340	1.2805	1.6514	1.9823	2.3809	2.6608
10	1.0	2.0	0.6201	0.7779	0.9601	1.1888	1.5335	1.8475	2.2532	2.5845

			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
12	0.0	0.0	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.6810	3.0546
12	0.5	0.0	0.6497	0.8152	1.0095	1.2580	1.6402	1.9931	2.4421	2.7823
12	1.0	0.0	0.6048	0.7603	0.9412	1.1706	1.5230	1.8561	2.3236	2.7560
12	0.0	1.0	0.7052	0.8829	1.0927	1.3621	1.7766	2.1560	2.6283	2.9743
12	0.5	1.0	0.6588	0.8252	1.0193	1.2656	1.6390	1.9762	2.3913	2.6933
12	1.0	1.0	0.6133	0.7699	0.9512	1.1796	1.5256	1.8440	2.2666	2.6297
12	0.0	2.0	0.7150	0.8931	1.1019	1.3677	1.7712	2.1346	2.5781	2.8957
12	0.5	2.0	0.6680	0.8351	1.0290	1.2730	1.6379	1.9606	2.3452	2.6123
12	1.0	2.0	0.6220	0.7795	0.9611	1.1883	1.5280	1.8330	2.2177	2.5214

D.F.	γ_1	γ_2	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
14	0.0	0.0	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.62[illegible]	2.9769
14	0.5	0.0	0.6500	0.8149	1.0081	1.2541	1.6296	1.9724	2.40[illegible]8	2.7220
14	1.0	0.0	0.6084	0.7641	0.9448	1.1729	1.5200	1.8424	2.28[illegible]	2.6677
14	0.0	1.0	0.7010	0.8772	1.0849	1.3507	1.7573	2.1264	2.5808	2.9096
14	0.5	1.0	0.6581	0.8238	1.0169	1.2612	1.6294	1.9589	2.3602	2.6487
14	1.0	1.0	0.6161	0.7727	0.9537	1.1810	1.5228	1.8332	2.2351	2.5680
14	0.0	2.0	0.7096	0.8863	1.0933	1.3562	1.7535	2.1090	2.5389	2.8439
14	0.5	2.0	0.6663	0.8326	1.0256	1.2680	1.6291	1.9465	2.3221	2.5819
14	1.0	2.0	0.6238	0.7812	0.9625	1.1889	1.5255	1.8215	2.1960	2.4821
			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
16	0.0	0.0	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.5825	2.9208
16	0.5	0.0	0.6505	0.8150	1.0074	1.2517	1.6225	1.9582	2.3739	2.6800
16	1.0	0.0	0.6118	0.7674	0.9480	1.1754	1.5191	1.8341	2.2511	2.6081
16	0.0	1.0	0.6978	0.8729	1.0790	1.3422	1.7430	2.1046	2.5462	2.8631
16	0.5	1.0	0.6577	0.8230	1.0154	1.2582	1.6228	1.9472	2.3399	2.6178
16	1.0	1.0	0.6185	0.7751	0.9561	1.1828	1.5219	1.8270	2.2146	2.5269
16	0.0	2.0	0.7055	0.8811	1.0867	1.3474	1.7403	2.0900	2.5104	2.8066
16	0.5	2.0	0.6650	0.8309	1.0233	1.2646	1.6230	1.9368	2.3064	2.5603
16	1.0	2.0	0.6253	0.7828	0.9641	1.1900	1.5246	1.8202	2.1826	2.4562
			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
18	0.0	0.0	0.6884	0.8621	1.0672	1.3304	1.7341	2.1009	2.5524	2.6785
18	0.5	0.0	0.6510	0.8153	1.0070	1.2501	1.6175	1.9480	2.3537	2.6494
18	1.0	0.0	0.6143	0.7704	0.9507	1.1779	1.5193	1.8292	2.2313	2.5661
18	0.0	1.0	0.6953	0.8695	1.0744	1.3355	1.7320	2.0879	2.5199	2.8279
18	0.5	1.0	0.6576	0.8225	1.0144	1.2562	1.6181	1.9387	2.3233	2.5956
18	1.0	1.0	0.6205	0.7772	0.9582	1.1847	1.5221	1.8231	2.2013	2.4983
18	0.0	2.0	0.7023	0.8770	1.0815	1.3405	1.7300	2.0754	2.4887	2.7785
18	0.5	2.0	0.6642	0.8297	1.0216	1.2621	1.6186	1.9299	2.2952	2.5454
18	1.0	2.0	0.6267	0.7843	0.9656	1.1913	1.5247	1.8175	2.1733	2.4381
			0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050
20	0.0	0.0	0.6870	0.8600	1.0640	1.3254	1.7247	2.0860	2.5280	2.8454
20	0.5	0.0	0.6515	0.8156	1.0070	1.2491	1.6204	1.9405	2.3366	2.6263
20	1.0	0.0	0.6169	0.7728	0.9537	1.1805	1.5202	1.8262	2.2192	2.5355
20	0.0	1.0	0.6933	0.8668	1.0707	1.3302	1.7232	2.0747	2.4993	2.8004
20	0.5	1.0	0.6575	0.8222	1.0137	1.2548	1.6200	1.9324	2.3118	2.5789
20	1.0	1.0	0.6225	0.7793	0.9606	1.1867	1.5228	1.8210	2.1916	2.4776
20	0.0	2.0	0.6996	0.8737	1.0772	1.3349	1.7217	2.0638	2.4716	2.7565
20	0.5	2.0	0.6635	0.8289	1.0203	1.2603	1.6192	1.9248	2.2869	2.5344
20	1.0	2.0	0.6281	0.7857	0.9673	1.1928	1.5254	1.8163	2.1672	2.4253