WIND SETUP PARAMETERS

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by

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SYNOPSIS

A laboratory investigation was undertaken of the factors determining the rise in water level encountered when wind blows towards the shore of a shallow body of water. The factors were evaluated for bodies of water of uniform dimensions and for bodies of water of irregular surface areas and bottom configurations. On the basis of the laboratory observations and a theoretical analysis of the phenomenon, methods of predicting the rise in water level for a specific wind velocity and body of water are proposed in Chapter V.

ACKNOWLEDGEMENTS

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LIST OF SYMBOLS AND DEFINITIONS

A - the angle between the wind direction and tidal axis.

B - a coefficient in Boussinesq's formula indicating the roughness of a boundary.

F - fetch in feet, the distance from the windward shore.

K - a coefficient in Boussinesq's formula indicating the characteristics of a fluid.

M - a coefficient dependent on flow velocity.

N - a plan shape factor dependent on the shape of the water surface area (U.S. Corps of Engineers).

R - hydraulic radius in feet.

S - wind setup value in feet.

S_1 - wind setup due to skin friction between wind and water surface.

S_2 - wind setup due to the form resistance of waves.

U - the wind velocity in feet per second .

U_o - Keulegan's "characteristic formula velocity".

V - the wind velocity in feet per second.

a - a coefficient dependent on the ratio of fetch to depth of a body of water.

d - still water depth in feet.

g - acceleration of gravity in feet/sec.^{2}

h - wind setup in feet (bottom at windward shore not exposed).

h' - wind setup in feet (bottom at windward shore exposed).

m - distance along "m" axis.

n - distance along "n" axis.

p - pressure in pounds per square foot.

Z_s - distance from the bottom to the mean water level.

γ - unit weight of water.

ϵ - a coefficient depending on the eddy viscosity.

λ - a coefficient depending on the turbulence of flow.

μ - a coefficient of viscosity.

ρ - density of a fluid in pounds/cubic foot.

τ_b - shear stress on the bottom.

τ_s - shear stress on the water surface.

CHAPTER I

INTRODUCTION

A significant factor in the safe design of dams and dykes is the selection of freeboard (i.e) the vertical distance between the maximum still water level and the top of the structure. With many of these structures overtopping would mean failure, and failure, major disaster. It is desirable to select the freeboard with as much accuracy as possible, firstly for safety and secondly for economy. One of the components of freeboard that must be evaluated is the rise in water level encountered when a wind blows towards a structure or shore. This phenomenon is defined as wind setup at the leeward shore. In actual cases the setup may vary from a few inches in short deep lakes to several feet in long shallow lakes. Values of over six feet have been observed on Lake Erie, a typical example of the latter case.

Investigations into predicting wind setup have been undertaken from both the theoretical and empirical approach. Investigators have included Hellstrom (1941), U.S. Corps of Engineers (1945), Langhaar (1951), Keulegan (1951), and Sibul (1954). In nearly all investigations the theoretical analysis deals with an idealized body of water seldom encountered in nature. This presents the problem of modifying the theoretical predictions to apply to natural bodies of water. The fundamental difficulties are determining allowances for non-uniform depths of water, irregular plan shapes of some natural bodies of water, and increased obstruction on the bottom of shallow lakes. Surges that occur after the wind has begun to blow are also of significance in predicting the maximum water level.

The purpose of this thesis is to investigate briefly as many determining factors of wind setup as possible. To accomplish this a theoretical analysis of the wind setup phenomena has been made followed by a summary of the present methods of relating the theoretical predictions to natural bodies of water. From this point a laboratory investigation was undertaken that encompassed idealized conditions as well as some of the irregularities encountered under natural conditions.

CHAPTER II

THEORETICAL WIND SETUP ANALYSIS

When wind blows over a water surface a tangential stress develops between the wind and water. The stress generates waves and causes a surface current in the general direction of the wind. In addition, a pressure head is built up at the leeward end of the body which generates and sustains a return current in the opposite direction of the surface current (see Figure I). The pressure head is formed by the water surface assuming a slight slope upwards towards the leeward shore. The amount the water surface changes is defined as the wind "setup". The phenomenon is synonomous with wind "piling" at the leeward shore.

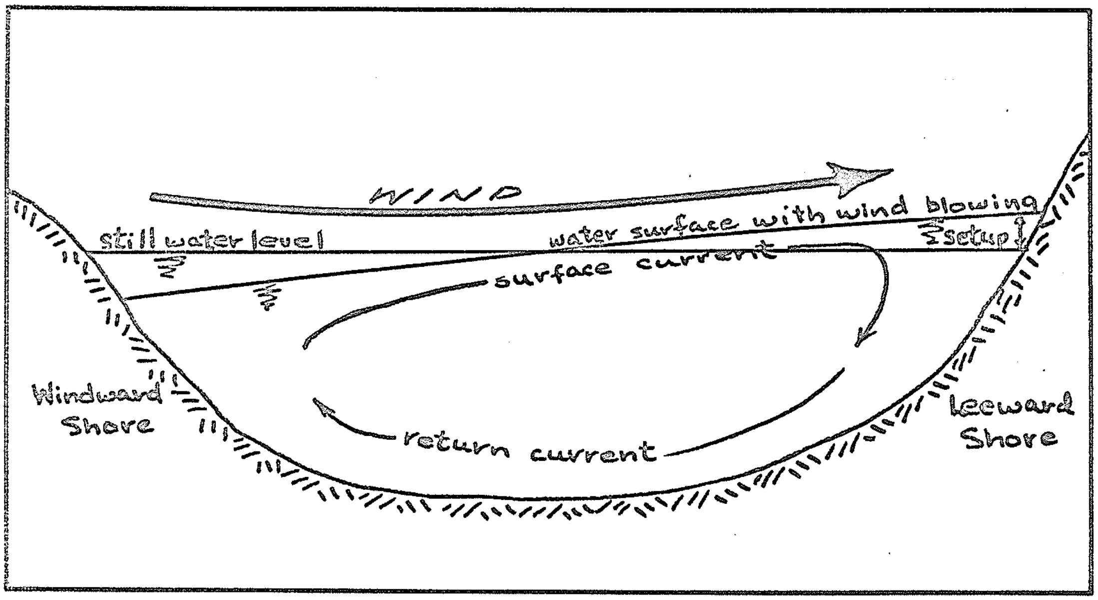

FIGURE I. WIND SETUP PHENOMENON

Wind Setup Formula

The development of a wind setup formula involves a number of factors. Some of the more significant include wind velocity, the transfer of energy from wind to water, length or fetch of water on which the wind acts, depth of water, plan shape of the body of water, bottom shape and roughness, wave height and period, currents, variations in atmospheric pressure, and the rotation of the earth.

One of the most comprehensive theoretical treatments of the phenomenon was proposed by Hellstrom in 1941[1]. He applied the basic Euler-Navier equation for the three dimensional motion of a viscous incompressible fluid, (i.e)

$$\begin{aligned}
\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right) &= \rho X-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)\\
\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right) &= \rho Y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \qquad (1)\\
\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right) &= \rho Z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}$$

To simplify equation (1) Hellstrom made the following assumptions:

(a) the flow is laminar,

(b) the depth is constant and small,

(c) the water surface slope is small,

(d) the pressure distribution is hydrostatic,

(e) the wind velocity and direction is constant,

(f) all motion is steady and equilibrium is established.

He then solved the equation to obtain the differential equation for the free water surface as:

$$\frac{\partial z_s}{\partial x} = \lambda \frac{\tau_s}{\gamma z_s} \qquad (2)$$

Equation (2) is the basic equation presented by most investigators regardless of the method of derivation. A derivation that is more readily visualized based on the same assumptions as Hellstrom has made in his analysis is as follows:

If equilibrium conditions prevail, a small portion of unit width and length "dx" of the lake shown below may be isolated.

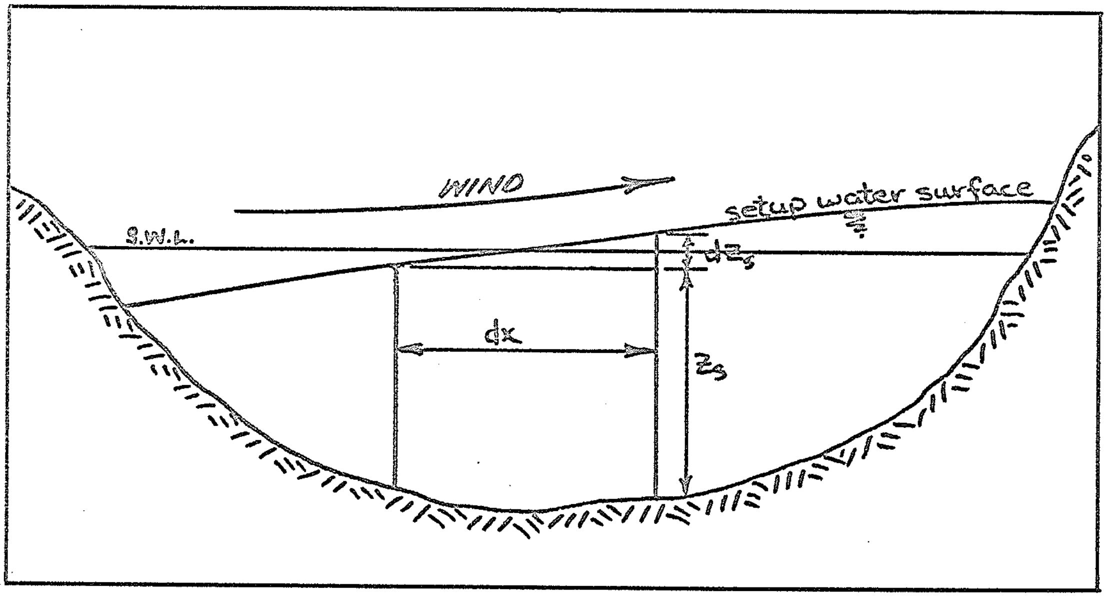

FIGURE II WIND SETUP ANALYSIS

The forces acting on this portion are shown in Figure III:

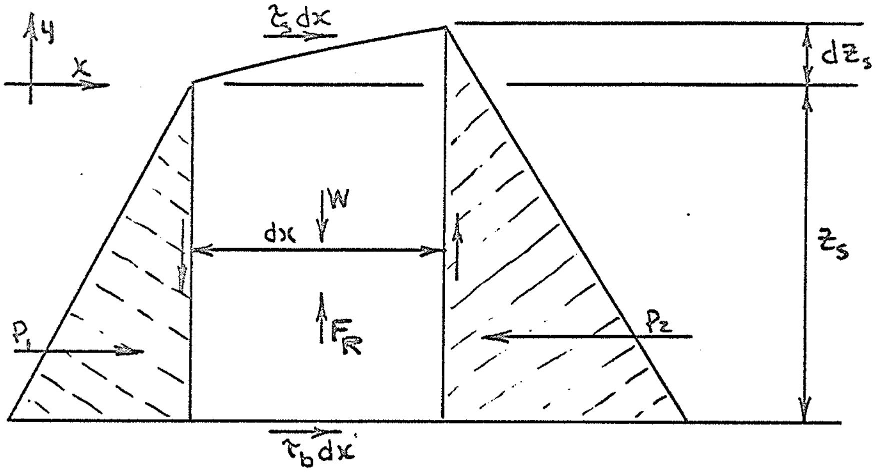

FIGURE III WIND SETUP FORMULA

For equilibrium: $\Sigma F_x = 0$

Writing $\Sigma F_x = 0$

$$P_1 + \tau_s dx + \tau_b dx - P_2 = 0$$

OR $(\tau_s + \tau_b) dx = \frac{1}{2}\gamma (z_s + dz_s)^2 - \frac{1}{2}\gamma (z_s)^2$

$$= \frac{1}{2}\gamma (2 z_s dz_s + dz_s^2)$$

IF dz_s is very small, $dz_s^2 \doteq 0$

$\therefore (\tau_s + \tau_b) dx = \gamma z_s dz_s$, OR

$$\frac{dz_s}{dx} = \frac{\tau_s + \tau_b}{\gamma z_s} \qquad (3)$$

The bottom stress, τ_b, is a function of the wind stress expressed by the following relationship:

$$\frac{\tau_b}{\tau_s} + 1 = \lambda \tag{4}$$

Equation (3) can now be written as:

$$\frac{dz_s}{dx} = \lambda \frac{\tau_s}{\gamma z_s} \tag{5}$$

Equation (5) for the free water surface is now in the same form as derived by Hellstrom (Equation (2)).

Turbulent Flow

Equations (2) and (5) were derived on the assumption that all flow is laminar. This is seldom the case under natural conditions. Investigations by Boussinesq[2,3,4] into turbulent flow indicate that the factor λ in equations (2) and (5) varies with the degree of turbulence of flow. When the flow is laminar, λ has a value of 1.5 and when completely turbulent 1.0, (i.e.) $1.0 \leq \lambda \leq 1.5$ Hellstrom gives

$$\lambda = 3/2 \, \frac{(K\sqrt{B} + 2)}{(K\sqrt{B} + 3)} \tag{6}$$

where $K = 45 \ m^{1/2}/sec.$

and $$B = \frac{g}{(3MR^{1/6} - K)^2} \tag{7}$$

where $M \doteq 25$ for lakes and reservoirs of low velocity and R = the hydraulic radius.

Summarizing the above, it is evident that the factor in equations (2) and (5) must be evaluated by equations (6) and (7) derived empirically by Boussinesq and Hellstrom to conform the water surface equation with the existing turbulence of flow.

Wind Shear Stress

The wind shear stress factor in equations (2) and (5) was investigated by Sibul in 1954[8]. On the basis of a theoretical analysis and laboratory observations he proposed the wind shear stress relationship to be:

$$\tau_s = 5.65 \times 10^{-6} U^{2.15} \tag{8}$$

where'U' is the average wind velocity. Equation (8) will be used to evaluate the results of this investigation.

CHAPTER III

APPLYING THE THEORETICAL WIND SETUP ANALYSIS

Equation (5), the differential equation for a free water surface affected by a constant wind force, must be integrated to obtain numerical wind setup values. There have been several approaches to this problem, three of which will be considered for a theoretical body of water of unit width and uniform depth.

The Ideal Case

1. Hellstrom integrates equation (5) to obtain:

$$z_s^2 = \frac{2\lambda\tau_s}{\gamma}(x+c_1) \tag{9}$$

Equation (9) indicates the water surface is parabolic in form and may be written in coordinates of m and n as:

$$n^2 = \frac{2\lambda\tau_s}{\gamma}(m) \tag{10}$$

Equation (10) is plotted in Figure IV illustrating Hellstrom's "Characteristic Water Surface Parabola".[1] To locate a particular portion of the water surface parabola that represents a specific case, two characteristics of the case are utilized, the fetch and "still" water depth. As shown in Figure IV the area between the curve and x-axis must equal the product of the fetch and still water depth since the volume of water does not increase or decrease due to the wind force. The distance between the m-axis and the Z_s - axis is represented by C_1.

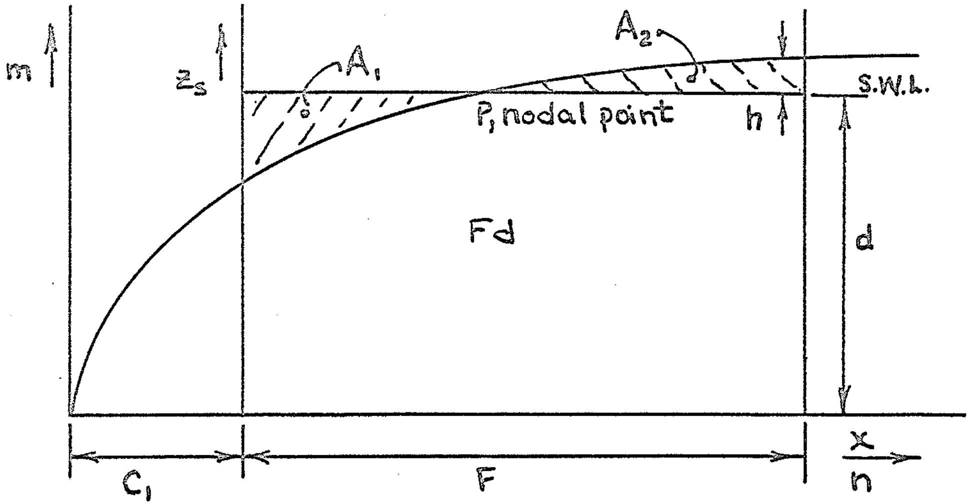

FIGURE IV CHARACTERISTIC WATER SURFACE PARABOLA-HELLSTROM

Figure IV represents a particular case where the bottom of a body of water is not exposed, giving C_1 a positive value. Figure V shown below represents the case where C_1 equals zero, (i.e) the water surface at the beginning of the lake has the same elevation as the bottom. The third case that may be encountered is illustrated in Figure VI where the bottom of the lake is exposed giving C_1 a negative value. The setup "h" can be isolated by forming equation (11)

$$h = \sqrt{\frac{2\lambda \tau_s}{\delta}(x + c_1)} - d \qquad (11)$$

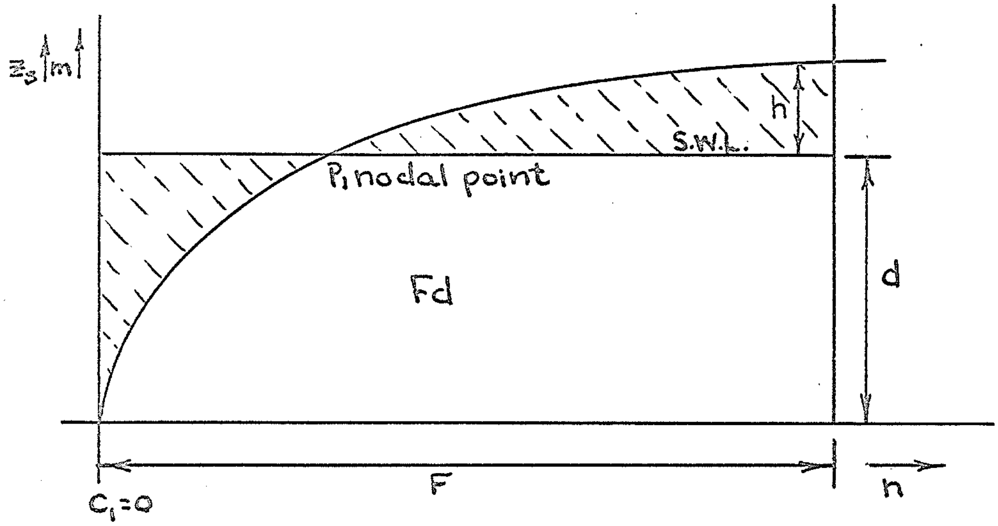

FIGURE V CHARACTERISTIC WATER SURFACE PARABOLA-HELLSTROM

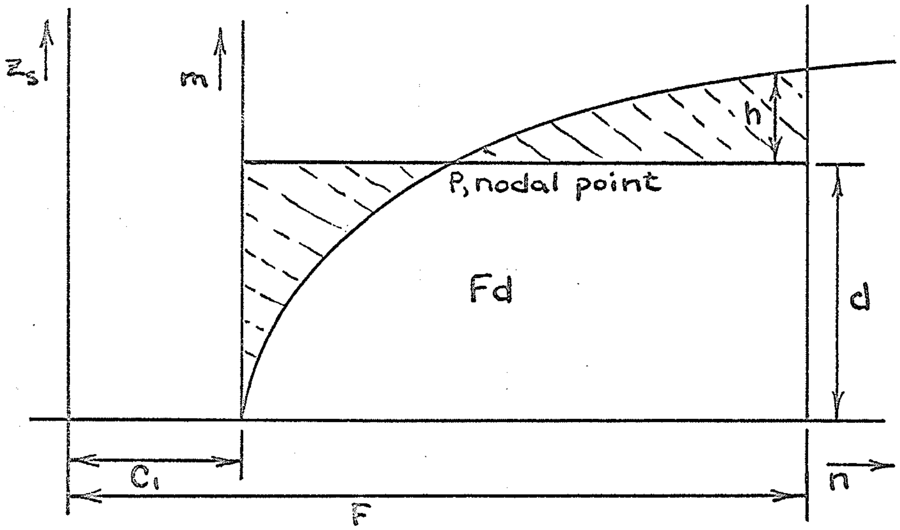

FIGURE VI CHARACTERISTIC WATER SURFACE PARABOLA-HELLSTROM

The nodal point "p" may be obtained from equation (11) by setting $h = 0$.

When the depth is large compared to the setup, Hellstrom proposes that the nodal point occurs at F/2. The setup equation for the windward shore then becomes:

$$h_{x=o} = \frac{\lambda \tau_s}{\gamma d}\left(x - F/2\right) \tag{12}$$

At the leeward shore where $x = F$

$$h_{x=F} = \frac{\lambda \tau_s}{2\gamma d} F \tag{13}$$

Equation (13) coincides with the setup formula proposed by Langhaar[5].

$$h_{x=F} = \frac{\tau_s F}{2\gamma d} \tag{14}$$

where he has assumed the factor λ equals one if the setup is small and the depth relatively great.

2. Keulegan conducted a laboratory investigation of the wind setup phenomenon in 1951[6]. He also derived the basic differential equation for the water surface in the form of equations (2) and (5). Keulegan proposed the factor λ in the equation has a value of 1.5 for laminar flow and 1.25 for turbulent flow. In his investigation he separated the total setup "S" into two parts:

1. S_1, the setup due to skin friction between the wind and water surface and,
2. S_2, the setup due to the form resistance of the waves.

Keulegan's setup "S" is defined as the difference between the water surface elevations at the windward and leeward shores.

He proposed the setup without wave action as:

$$S_1 = C_2 \frac{U^2F}{gd} \tag{15}$$

and the setup due to waves as:

$$S_2 = C_3 \frac{(U-U_o)^2}{gd} \left(\frac{d}{F}\right)^{1/2} \tag{16}$$

The constants C_2 and C_3 were given as $C_2 = 3.3 \times 10^{-6}$ and $C_3 = 2.08 \times 10^{-4}$. The total setup is then the sum of S_1 and S_2 or:

$$S = F\left[3.3\times10^{-6}\frac{U^2}{gd} + 2.08\times10^{-4}\frac{(U-U_o)^2}{gd}\left(\frac{d}{F}\right)^{1/2}\right] \tag{17}$$

The factor "U_o" is referred to by Keulegan as the "formula characteristic velocity" of the wind. It is approximately 1.3 times the lowest wind velocity needed to start waves. The value of "U_o" was established by his experiments as varying with depth. This is illustrated in Figure VII.

For larger bodies of water under actual conditions Keulegan proposed the factor U_o be ignored. The formula for larger bodies of water then becomes

$$S = 3.3\times10^{-6}\left[1+63\left(\frac{d}{F}\right)^{1/2}\right]\frac{U^2F}{gd} \tag{18}$$

3. The Zuider Zee[7] and Beach Erosion Board[9] formulae for wind setup are similar. They may be derived from the basic equation for

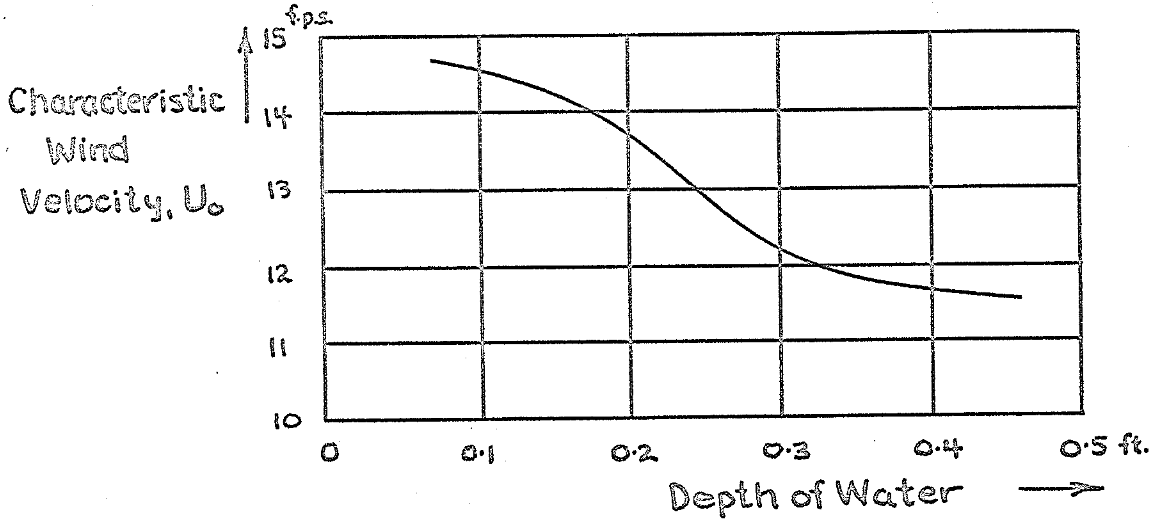

FIGURE VII CHARACTERISTIC WIND VELOCITIES-KEULEGAN

the water surface profile by applying several empirical constants. The Zuider Zee formula is

$$h = \frac{V^2 F}{C\,d} \cos A \qquad (19)$$

where h is the setup in feet above the still water elevation at the leeward shore, U is in miles per hour, and F is in miles. In deriving equation (19) the following assumptions were made:

(a) the bottom stress is only a small fraction of the wind stress and may be ignored and,

(b) the wind stress is proportional to the square of the velocity of the wind.

The value of "C" has been observed to be 1400 in field investigations.

The Beach Erosion Board[13] proposes a similar equation with slightly different constants and factors:

$$S = \frac{K \lambda \rho_a U^2 F}{\rho g d} \cos A \qquad (20)$$

where ρ_a is the air density and k is a numerical constant approximately equal to .003.

Modifications of the Ideal Case

The previous wind setup formulae were based on theoretical bodies of water of uniform dimensions and characteristics. Theoretical setup predictions must be modified when applied to the irregular conditions encountered in most natural bodies of water. Information presently available on this aspect may be briefly summarized as follows:

1. Irregular Dimensions

Allowances for irregular plan shapes and irregular depths have been proposed by several investigators, notably the Beach Erosion Board of the U.S. Corps of Engineers[9]. The suggested method for predicting wind setup values where non-uniform dimensions are encountered could best be described as mechanical integration. In essence, it is recommended that the body of water be broken up into sections of uniform width and depth, the setup values computed for each section using the general setup formula, and the water surface elevations adjusted to form a continuous water surface. Some adjustments in elevations may then be required to ensure the volume of water raised above the still water depth is equal to the volume of water depressed below the still water depth.

Other investigators have proposed factors by which the setup values from theoretical formulae may be multiplied to allow for irregular dimensions. Usually these factors apply to a specific body of water under observation for a period of years. From the observations, actual setup values are compared with the theoretical predictions and a factor is evaluated. The success of this method hinges on the period of observation and the specific case considered. Where new bodies of water are to be impounded, setup predictions must be based on a theoretical analysis only.

2. Bottom Roughness and Irregularities

A laboratory study by Sibul in 1954[10] has indicated the effects of bottom roughness and weeds on wind setup in shallow water. His experiments were conducted in a small wind tunnel with smooth and rough bottom conditions. Strips of cloth placed in the channel were used to simulate vegetation. Sibul summarized his findings as follows:

> "The results indicate a rapidly increasing setup when the still water depth decreases below a certain limit. There were no indications that the bottom roughness affects the setup for relatively deep water. In very shallow water, however, the rougher bottom conditions result in higher setups. The trend is especially pronounced for higher wind velocities. For the shallowest still water depth (0.05 ft) used in the experiments, the setup was approximately 10 percent higher for the rough bottom and approximately 20 percent higher when strips of cheese cloth were used in the

> channel to simulate the roughness effect of vegetation, than the setup observed with a smooth bottom".[10]

The preceding sections have briefly outlined the theoretical and empirical information available on the analysis of the wind setup phenomenon. No attempt has been made to present a complete and detailed review of the findings and recommendations of previous investigators. It is hoped that the present state of the wind tide phenomenon knowledge has been indicated. For detailed accounts of a specific investigation the reader is referred to the bibliography.

CHAPTER IV

LABORATORY INVESTIGATION

A series of laboratory experiments were performed to investigate the wind setup phenomenon with actual observations. The tests, thirty-five in all, were primarily concerned with measuring the water surface profiles of a small body of water under various setup conditions. They may be roughly divided into two categories; investigation of the general or theoretical case and investigation of the natural or irregular case. The testing apparatus was constructed and the tests run over a period of seven months in 1963. Information obtained from the tests is tabulated in Appendices A and B. The laboratory apparatus and testing procedure is briefly outlined in this chapter and considered in detail in Appendix C.

Laboratory Apparatus:

Essentially the apparatus consisted of a wind tunnel, 46 feet long, 3 feet wide, and 2 feet deep. The tunnel, actually a converted hydraulic flume, was dammed off at either end allowing a depth of water of 0.5 feet to be impounded. Air was drawn through the tunnel over the water surface with a variable centrifugal fan. Manometers, reading to the thousandth of a foot, were spaced along the glass-walled tunnel to record the water surface elevations at various fetches. An adjustable passage between the manometers and tunnel was used to dampen out the wave motion of the water. A carriage mounted on rails