

A COMPUTER STUDY OF SOME ASPECTS OF  
POWER SYSTEM STABILITY

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by  
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## ABSTRACT

The purpose of this thesis is to study transient stability, to devise a method of solving the swing equation that will make full use of the potentials of the IBM 1620 (20K) Digital Computer and to investigate three methods of improving transient stability, that is, the effects of exciter response, governor response and rapid reclosing on a two machine system.

## PREFACE

This thesis is a computer study of some of the aspects of transient stability in power systems. Stability became a problem when machines were first paralleled and today power systems are more complex and expensive and the results of stability studies continue to be an important consideration in power system design.

Although a computer program has been written to handle up to thirteen machines (30 buses), a two machine system was used here. The swing equation is solved by the Kutta-Runge method which provides an accurate way of solving a large number of first order differential equations. In succeeding chapters the effects of exciter response, governor response and rapid reclosing are considered. The final chapter takes into account the combined effect on the two machine system when all improvement methods are used at once.

I wish to acknowledge my indebtedness to Professor G. W. Swift for his never failing guidance and assistance, to R. W. Haywood of Manitoba Hydro for reviewing the thesis and making many suggestions for its improvement, and to my wife for all that she has done.

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## CHAPTER I

### TRANSIENT STABILITY

The purpose of this thesis is to study transient stability, to devise a method of solving the swing equation that will make the maximum use of the potentials of the IBM 1620 (20K) Digital Computer, and to investigate three methods of improving transient stability.

Transient stability studies are by no means rare, in fact, the opposite is true. Every power company must investigate the transient stability characteristics of its system each time a major generator addition is proposed. This thesis will not attempt to carry out a detailed study of a particular power system but it will however, endeavor to investigate the general over-all effect of governor response, exciter response and rapid reclosing on a system. The point is again made that the purpose is to observe the general effects of these improvement methods in order to gain insight into the benefits available from using them.

#### 1.1 Background

Stability, when referring to a power system, may be defined as that condition wherein all the machines in that system remain in synchronism with each other. Conversely then, instability denotes that condition when one or more machines within the system lose synchronism with the others,



or, in other words, fall out of step.

Instability became a problem when machines were first paralleled. These original machines were operated by directly coupling the generator to a steam engine, which delivered a pulsating torque causing hunting of the generator. Hunting, however, was reduced by the introduction of a damper winding in the generator, which in turn, developed a damping torque due to the rate of change of the rotor position with respect to the armature m.m.f. It is noted that today most generators are run by coupling them to a waterwheel or to steam turbines, both of which develop a relatively non-pulsating torque. <sup>1</sup>

Before the advent of automatic regulating systems, such as governors and exciters, all systems had to be designed with a good inherent voltage regulation, that is, circuits, machines and transformers that had a low reactance. This was possible due to the fact that the lines between the load and the power sources were comparatively short and the voltage comparatively low. Stability, or for that matter, instability, became a problem when it was necessary to go farther and farther away to reach sources of power. The development of automatic voltage regulating devices made it possible to increase the reactance in order to obtain a more economical design and to limit short circuit currents.

<sup>1</sup>. See Bibliography, Ref. #1. Superscripts, henceforth, will refer to numbered references in Bibliography.

Today, the problem has not vanished. The engineer, because of economic considerations, cannot arbitrarily apply methods such as rapid reclosing; it is necessary that he realize the effect of what each method can do for his specific problem. This thesis intends to show generally the ramifications of each of the three methods studied-- with respect to transient stability for the particular system studied. The system used was a two machine system, i.e. a finite machine connected to an infinite bus. (See Figure 2.3)

With the era of long lines and rising costs, the engineer has tried to transmit more and more power through the line, approaching the theoretical power limit of that line. With these increased loadings, transient stability has thus become the focal point of many investigations.

## 1.2 Procedure

In order to have a common base on which to compare the various improvement methods, a sample power system was selected. It was also decided that a three phase fault lasting a specified time would be applied for each series of tests. (See Chapter II)

The first problem encountered in transient stability studies is how to solve the swing equation. Because a small computer was being used and because governor and exciter response were to be considered, it was felt that a method

other than the classic step-by-step solution had to be found. Gills' variation of the Kutta-Runge process was employed because of the accuracy and flexibility it offered in solving first order differential equations<sup>2</sup>.

This thesis is grouped into five sections. The first will deal with the solving of the swing equation and its application to the "raw" or uncompensated system<sup>a</sup>. The second will consider governor response and its abilities to compensate the "raw" system. The third and fourth sections will consider exciter response and rapid reclosing respectively and their effect on improving stability. The last section will deal with the combined effects of governor response, exciter response and reclosing on the system.

In order to supply the transient stability program with all the data needed it was necessary to prepare several programs. For each loading condition, the IBM Load Flow program was used in order to obtain the output power,

<sup>a</sup> By "raw" system it is meant a system based on the following assumptions:

1. Synchronous-machine transient reactances in the direct and quadrature axes are assumed to be alike.
2. Voltages behind transient reactances of the synchronous machines are assumed to remain constant.
3. Damping torques are neglected.
4. The influence of saturation may be either entirely neglected, or taken into account in an approximate manner by modifying the value of transient reactance.
5. Constant shaft torques are assumed for all of the machine groups, and governor-action and load-speed characteristics are neglected.

See Bibliography, Ref. #19.

voltage and angle at the generator internal buses. Secondly, it was necessary to obtain the values of the admittance (self and mutual) between each generator and every other generator. Two programs were prepared and for each loading condition, <sup>and</sup> a set of admittances was procured<sup>b</sup>.

The transient stability program was written so as to accept data directly from the previous program. The first program solved the swing equation by the Kutta-Runge method and considered only reclosing. This program has the capacity of handling up to thirteen machines (30 buses)<sup>c</sup>. The second program written again solved the swing equation by the Kutta-Runge method and considered reclosing. However, this time logic for both governor response and exciter response was added. This program handles five machines (30 buses)<sup>d</sup>.

The transient stability programs were then used to compute the resulting swing curve in the test runs<sup>e</sup>. The results of each test will be given in the appropriate chapters.

<sup>b</sup> New England Electric ("Transient Stability" package).

<sup>c</sup> See Appendix I.

<sup>d</sup> See Appendix II.

<sup>e</sup> See Appendix III for typical outputs of various programs.

## CHAPTER II

### SOLUTION OF THE SWING EQUATION

The purpose of this chapter is to introduce the method used to solve the swing equation and to present the sample system studied, along with the criteria applied to the faulting and fault clearing of the system. Furthermore, the results of a three phase fault applied under various loading conditions to the sample system will also be given.

#### 2.1 Swing Equation

Consider the swing equation:

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M} \quad 2.1$$

where:  $\delta$  = the displacement angle of the rotor of a synchronous machine, with respect to a reference axis rotating at normal speed in radians. In this thesis a two machine system was studied, with one of the machines used as reference.

$M$  = the inertia constant of a machine in per unit megawatts per radian per second squared.

$M = \frac{GH}{B\pi f}$  where:  $G$  = the machine rating in Megavolt-amperes.

$H$  = the stored energy in magawatts.

$B$  = the system base (MVA).

$f$  = the frequency in cycles per second.

$p_m$  = the shaft power input corrected for rotational losses in per unit megawatts.

$p_e$  = the electrical power output corrected for electrical losses in per unit megawatts. Note that the difference between  $p_m$  and  $p_e$  is the accelerating power of the machine.

By examining the swing equation we see that the acceleration of the machine, given by the second derivative of delta, varies directly with the accelerating power and, inversely with the inertia constant.

## 2.2 Classical Solution

The classical approach in solving the swing equation has been a formal solution, which proves in fact, to be impracticable. As an example, consider a three machine system: by examining the equations given below we see that the output of a machine and therefore its accelerating power, depends on its angular position and angular speed with respect to every other machine in the system.

$$M_1 \frac{d^2 \delta_1}{dt^2} = P_{m1} - P_{e1}(\delta_1, \delta_2, \delta_3, \frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}) \quad 2.2$$

$$M_2 \frac{d^2 \delta_2}{dt^2} = P_{m2} - P_{e2}(\delta_1, \delta_2, \delta_3, \frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}) \quad 2.3$$

$$M_3 \frac{d^2 \delta_3}{dt^2} = P_{m3} - P_{e3}(\delta_1, \delta_2, \delta_3, \frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}) \quad 2.4$$

The simplest system of one finite machine connected through a reactance to an infinite bus, with damping neglected, yields the equation

$$M \frac{d^2 \delta}{dt^2} = P_m - P_e \sin \delta \quad 2.5$$

for which the formal method (with input power equal to zero) gives a solution involving elliptic integrals.

Another method used to solve the problem of stability of a very simple system is the equal area criterion. A solution of a swing equation, with the usual assumptions of constant input power, and constant voltage behind the transient reactance, shows that  $\delta$  oscillates about some equilibrium point with a constant amplitude, providing the system is stable. The method used to indicate stability without solving the swing equation is called the equal area criterion of stability. This criterion applies only to a two machine system, but because exciter and governor response are to be considered this method proves to be inadequate for the purpose of this thesis.

However, the method which is usually used to solve the swing equation is the step-by-step solution. In this solution one or more of the variables are assumed to be constant and another is varied according to the assumed laws over a small interval  $\Delta t$ . It is usual practice to assume accelerating power and hence the acceleration to be constant over the time interval  $\Delta t$ . The mechanical input power is also assumed constant. Good accuracy can be

obtained by the step-by-step method and the computations are fairly simple.

The step-by-step method of solving the swing equation is familiar to most persons in the field of power systems. The author felt that in view of the fact that regulators and exciters were to be taken into account in this study, another method more conducive to digital computation had to be used. The method selected was Gills' variation of the Kutta-Runge solution for simple first order differential equations<sup>2</sup>.

### 2.3 Kutta-Runge Method

The Kutta-Runge method has been used recently in studies involving large systems using large computers<sup>3</sup>. Most methods used in computing the step-by-step integration of differential equations have one essential in common with each other; that is, at each step of the integration one must use values previously calculated in order to proceed.

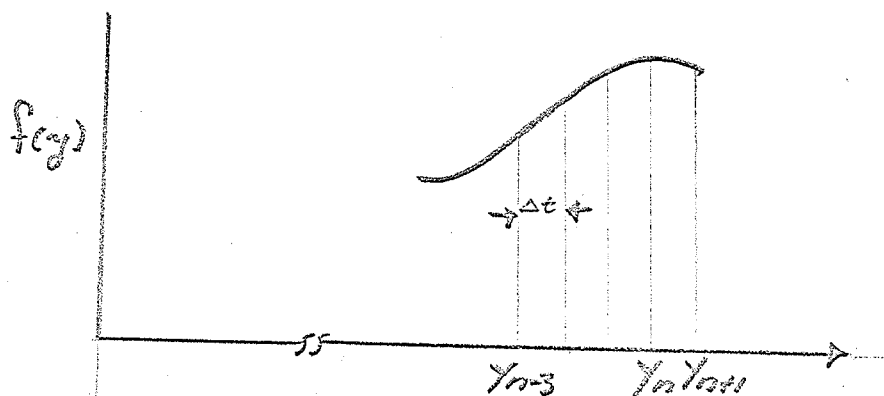


Figure 2.1



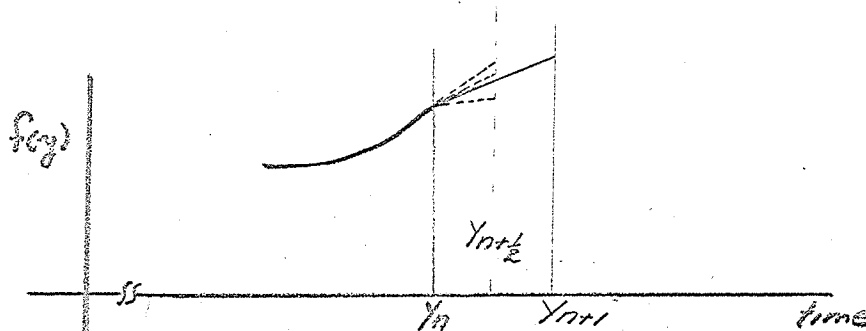
In other words, if one has arrived at  $y_n$  and wishes to calculate  $y_{n+1}$  (see figure 2.1) one must have a knowledge of  $y_{n-1}$ ,  $y_{n-2}$  etc.--the number of the previous values required depending on the accuracy needed and the method used. The previous values are indications of how the function will act in the region of  $y_n$  to  $y_{n+1}$  and it would indeed be wasteful to disregard these values. In hand calculations most formulae are of the difference type, are simple and are easily remembered. This is a definite advantage, wherein mental labour is to be kept at a minimum. There are however, disadvantages with this method. The first is that it cannot be used at the beginning of the integration because two or more values of  $y_n$  are needed and therefore some auxiliary method must be used to begin the calculations. The second disadvantage is that it is difficult to change the size of the time interval in the middle of a run. Halving or doubling the size is relatively simple but changing by other factors proves to be cumbersome.

When using digital computers as opposed to hand calculations, these considerations assume different proportions. It is a serious drawback now to supply the computer with special methods and instructions to begin the calculation. A simple operation when solving the swing equation by hand, that of changing the variable  $y_{ni}$  in order to carry on to the next interval, can assume serious proportions in time and instructions in a digital computer.

As an example, having calculated  $y_{n+1}$  and wishing to calculate  $y_{n+2}$ , one must first write the program to take  $y_n$ , shift it to  $y_{n+1}$  (used to calculate  $y_n$ ) and shift this in the memory to position  $y_n$ , whereas for hand calculation the operator would merely shift his eyes down the page.

There is another consideration when using computers, and this is storage space. Storage becomes critical as the number of machines in a study increases. Therefore, one is led to look for processes which do not make use of preceding values. A large general classification of such processes that do not require previous values is given by Kutta<sup>4, 5</sup>. Kutta investigated a large number of these processes dealing with various orders of accuracy. A most attractive method is given by the fourth order Kutta-Runge process. By fourth order it is meant that the error in each step is of the order of  $h^5$ , where  $h$  is the interval size between calculations.

The Kutta-Runge process was used in this thesis in the form of Gills' variation<sup>2</sup>. This method consists of four approximations (see figure 2.2).



The first approximation calculates the slope of the function at half the interval length between  $y_n$  and  $y_{n+1}$ , the second approximation again calculates the slope at the half-way point, this time using the value of the first approximation as a weighted term; the third approximation uses the first and second approximations as weighted values and calculates another value of slope at half the distance between this interval; the fourth approximation then calculates the value of the slope at the end of the interval using the weighted values of the first, second and third approximations.

The weighting of the previous value of the slope has been derived by Kutta, extended by Runge, and amplified by Gill. Kutta suggested five special cases or solutions which Runge soon developed into the simplest particular solution. Briefly, these weighted terms allow us to obtain a high degree of accuracy without any knowledge of previous terms.

#### 2.4 Application of the Kutta-Runge Method

The swing equation is a second order differential equation. In order to use the Kutta-Runge process the swing equation is simplified into the following two first order differential equations.

$$\frac{dy}{dt} = \frac{p_a}{M} \quad 2.6$$

$$\frac{d\delta}{dt} = \omega \quad 2.7$$

where:  $w$  = the speed in radians per second.

$p_a$  = the accelerating power (i.e.  $p_m - p_e$ ).

$M$  = the inertia constant.

One can now begin to understand the reason for the choice of the Kutta-Runge process. The method is capable of handling as many single order differential equations as the memory will allow. It handles each differential equation separately, thus a concise subroutine within the computer program can be written and used for each differential equation in turn. Since part of the purpose of this thesis is to consider exciter and governor response, both of which can be approximated by first and second order differential equations, the simplicity of the method becomes apparent. The exact formulae used to consider regulation will be dealt with in separate chapters.

The accuracy of the Kutta-Runge process as compared to the step-by-step method of solution has been investigated and shown to be superior<sup>15</sup>. In order to check the accuracy of the Kutta-Runge process two problems with known solutions were solved and both times results obtained proved more accurate than those obtained by the step-by-step solution.

## 2.5 Power Equation

Consider the equation 2.6; the accelerating power, is calculated from the following equation