

The Development of a Fast Sum-Coincidence
Angular Correlation Gamma Ray Spectrometer and its
Application to Decay Scheme Studies

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ABSTRACT

The construction of a fast sum-coincidence angular correlation gamma ray spectrometer is described. With the exception of the high voltage power supply the apparatus is completely transistorized.

A method has been developed for removing fictitious cascades, caused by Compton interference, in sum-coincidence spectra, by using in turn, two small crystals ($1\frac{3}{4}$ " diameter x 2") and one small and one large crystal (3" x 3").

The method was tested using the decay of Co^{60} and then applied to the decay of Cs^{134} with the object of elucidating the decay scheme.

Section I

INTRODUCTION

In 1911, Rutherford proposed that the atom consisted of a very small positively charged core, which he called the nucleus, surrounded by moving electrons. Since that time physicists have devoted much effort to learn more about the detailed structure of the nucleus. To this date several models have been proposed (for example, The Liquid Drop Model, The Shell Model, The Optical Model and The Collective Model are some of them.) but no one model has yet been devised which will explain all the phenomena related to the nucleus.

One method of studying a nucleus is to examine the emissions from it. These are generally of three types; alpha (α) particle, beta (β) particle and gamma (γ) radiation. Gamma rays, which are electromagnetic in nature, are believed to come from the nucleus during transitions between the various energy states in which it can exist. Therefore an examination of the gamma rays emitted from a nucleus may give some insight into the structural levels of that nucleus. The study of these nuclear levels is called "Nuclear Spectroscopy".

When a nucleus decays by alpha or beta emission, or by any other nuclear process the daughter nucleus is usually left in an excited state. This new nucleus may then de-excite by several methods, one of the more important of which is by gamma ray emission. When it de-excites by gamma ray emission it can do so by going directly to the ground state or by going to another excited state and then to ground. Thus it is possible for a level to de-excite by several different paths so that a transition between a highly excited state and the ground state may be represented by several gamma rays of different energies. The energy of each of these gamma rays is equal to the difference in energy between the two levels involved.

It is the job of the nuclear spectroscopist to examine these gamma rays and by means of energy considerations, spin consideration and relative intensities, decide upon the structure of the particular nucleus under investigation. It is to be hoped that when enough information has been gathered about the many radioactive nuclei, a suitable nuclear theory may be developed which can explain all the phenomena related to the nucleus.

Spectroscopic investigations of the nucleus began as early as 1911 and for the next thirty years physicists devoted themselves to the laborious task of

gathering data. When isotopes were produced artificially the field of nuclear spectroscopy grew rapidly. During the "forties" a tremendous effort was put into the development of better instruments and techniques in order that data of higher quality might be obtained.

With the advent of the scintillation counter about 1944, nuclear spectroscopy was given one of its most powerful tools - the scintillation spectrometer.

The Scintillation Spectrometer .

A complete description of the scintillation spectrometer can be found in the literature (Roulston 1952, Bell 1955). It is not intended that the following description of a scintillation spectrometer be a thorough one. It is given merely as an outline of how the instrument works.

When a gamma ray passes through matter it loses some or all of its energy. If the gamma ray passes through certain crystals (for example NaI(Tl)) then a light pulse is given off which is proportional to the energy deposited. This light pulse is detected and converted into an electrical pulse by means of a photomultiplier. These pulses are then fed into a linear amplifier to produce pulses of usable size (up to 10v for transistorized equipment and 100v for tube equipment). The pulses are then fed into an analyser which is capable of determining the pulse amplitude distribution or the "scintillation spectrum" of the gamma rays detected.

The scintillation spectrum of a gamma ray source contains distinctive features corresponding with the different interactions between the radiation and the crystal material. These features can generally be identified and permit measurement of gamma ray intensities and energies.

The main features in a spectrum are the Compton distribution, corresponding to Compton interactions, the Gaussian shaped photo-electric "peak" which corresponds with photo-electric interactions, and the pair production "peak" which corresponds with the production of a positron-electron pair. In the pair production process 1.022 Mev of energy is required to create the pair resulting in a peak corresponding with energy absorption by the crystal equal to $E - 1.022$ Mev. When the positron annihilates two 511 Kev gamma rays are emitted resulting in further peaks in the spectrum at $(E - .511)$ Mev and E corresponding to the absorption of one or both annihilation quanta in the crystal.

Other processes also occur with corresponding features in the spectrum. These include peaks at 511 Kev and 1.022 Mev corresponding to the capture of one or both annihilation gamma rays if the source is a positron emitter; a peak at $E(1 + \frac{2E}{.511})^{-1}$ Mev corresponding to the energy of gamma rays back scattered from the materials surrounding the source; escape peaks at $(E - E_K)$ and $(E - E_L)$ provided the energy E is close to E_K and E_L , the binding energy of K and L electrons in the scintillator; a peak corresponding to the x-ray emitted by the daughter nucleus (usually only the K x-ray). While these effects are generally small their existence should be recognised for the accurate interpretation of spectra.

The photopeak of a weak intensity, low energy gamma ray can become completely obscured by the Compton distribution of a stronger, higher energy gamma ray. For this reason, many attempts have been made to construct a spectrometer which will reduce or eliminate the Compton portion of the spectrum (Albert 1953, Roulston 1956, Bell 1954, 1955 and Hoogenboom 1958). A summary of these spectrometers can be found in the literature (Naqvi 1961).

Sum Coincidence Spectrometer

Hoogenboom, in 1958, developed a spectrometer which was particularly suited to measure gamma-gamma coincidence spectra and gamma-gamma angular correlation functions. The main features of this new spectrometer are the following:

- 1) The pulse distribution shows only the "full energy" peak for one specific gamma ray transition.
- 2) The absolute half-widths of the peaks due to coincident gamma rays are, to a first approximation, equal and smaller than the half-widths of the two corresponding full energy peaks in the singles spectrum.
- 3) The detection efficiencies for coincident gamma rays are equal.

The spectrometer works on the principle that the energy involved in a transition from one particular level to the ground state is a constant. Pulses from two crystals are added and sent through a differential discriminator which has a "window" set so that only those pulses from gamma rays which have been totally

absorbed get through. The output from this differential discriminator is used to gate a multi-channel analyser which analyses the pulses from one of the crystals. Thus only full energy peaks of the gamma rays involved in the cascade and a peak due to the absorption of both gamma rays in the one crystal appear in the spectrum.

The resolution of the spectrometer can be calculated if it is assumed that the full energy peaks of the two gamma rays (γ_a and γ_b with energies E_a and E_b) have a Gaussian shape with half-widths Γ_a and Γ_b and that the sum peak is also Gaussian with a half-width Γ_s at energy $E_s = E_a + E_b$. Under these conditions it can be shown* that the half-width of the peak in the sum spectrum corresponding to γ_a is

$$\Gamma_{sa} = \frac{\Gamma_a \sqrt{\Gamma_b^2 + \Gamma_s^2}}{\Gamma} \quad \text{--- (1)}$$

where $\Gamma^2 = \Gamma_a^2 + \Gamma_b^2 + \Gamma_s^2$.

Inspecting this formula it can be seen that the peak in the sum spectrum is narrower than the corresponding peak in the single spectrum. If the sum peak is made much narrower than Γ_a or Γ_b then

$$\Gamma_{sa} = \frac{\Gamma_a \Gamma_b}{\Gamma} \quad \text{--- (2)}$$

*

Naqvi (1961)

This means that the peaks in a sum coincidence spectrum have the same half-widths to a first approximation.

The efficiency ϵ_{sa} of detecting γ_a in the sum spectrum can be expressed in terms of the full energy efficiencies of detecting γ_a and γ_b in the singles spectrum. The efficiency ϵ_{sa} is given as

$$\epsilon_{sa} = 2 \sqrt{\frac{\ln 2}{\pi}} \epsilon_a \epsilon_b \frac{\sqrt{s}}{\Gamma} \quad (3)$$

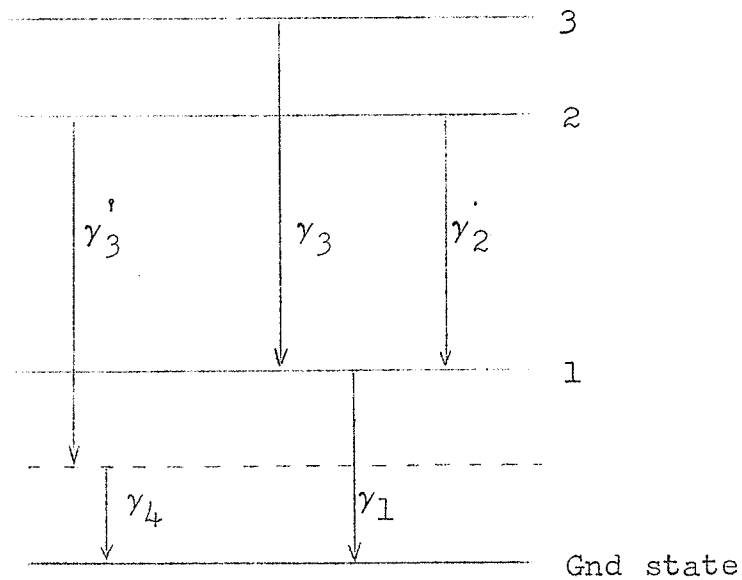
Provided the crystals being used in the experiment are of the same size, since the above expression for ϵ_{sa} is symmetrical with respect to the indices a and b, then the area under the peaks in the sum spectrum corresponding to γ_a and γ_b have to be equal.

In comparing equations (2) and (3) it should be noted that a compromise between good resolution and good efficiency has to be effected in regard to the setting of the sum "window" width. Hoogenboom has suggested that a good compromise is to make Γ_s of the order of the smaller of Γ_a and Γ_b .

The efficiency of detection of the cross-over equals $\epsilon_{sab} = \frac{\sqrt{s}}{\Gamma_{ab}} \epsilon_{ab}$, where Γ_{ab} and ϵ_{ab} are the half-width and efficiency of detecting the cross-over transition in the singles spectrum. In order to calculate the intensity of this cross-over transition a correction

must be made for background and for the contribution from both gamma rays depositing their energy in the same crystal. If the geometry is symmetric, then this correction amounts to one half the intensity of all other peaks in the spectrum.

In the foregoing, it has been assumed that only full energy peaks give rise to peaks in the sum spectrum. This is the case only when the sum peak corresponds to the highest energy level in the nucleus. If the sum window is set at any intermediate level then the coincident gamma rays from that level will appear in the spectrum, but there can also be a contribution from Compton events of higher energy transitions.



For example, consider a scheme such as that shown in the accompanying figure with the sum window set at the energy corresponding to level two. The sum spectrum will show peaks corresponding to γ_1 and γ_2 . However peaks could also appear in the spectrum from γ_3 adding with Compton events from γ_1 giving the impression that there should be another level in the scheme (dotted in the figure) with a gamma ray γ_4 between it and the ground state.

Recently in a paper by Schriber and Hogg (Schriber 1963), a method has been developed for calculating the amount of Compton interference in the spectrum and subtracting it. It is the object of this thesis to construct a sum coincidence spectrometer and to develop an experimental method of eliminating, or reducing the Compton interference.

Angular Correlations

The study of angular correlations of successive gamma rays in a cascade began in 1940. The first theory on directional correlations was proposed by Hamilton (Hamilton 1940) in that year and for the next few years many people tried to obtain results which would agree with this theory. They were however unsuccessful, and even the attempts of Goertzel (Goertzel 1946) in a theoretical paper on the effects of the influence of extra nuclear fields on the directional correlation could not bring these results into line. The first successful experiments on directional correlation were performed by Brady and Deutsch in 1947 (Brady 1947) using Geiger counters. They showed that it was not the theory that was at fault but the experimental techniques.

In 1948 Brady and Deutsch (Brady 1948) introduced scintillation counters into directional correlation work. These greatly reduced the time required for a correlation and since that date the literature has contained several hundred papers on the subject of directional correlations. There have been many theoretical papers written on the subject, with different degrees of sophistication ranging from Hamilton's treatment of the "Naive Theory" (see Siegbahn 1955) to treatments involving Racah algebra.

The reader is referred to the literature for a detailed description of the theory. It will suffice to give here a brief description of Hamilton's treatment with some comments regarding its limitations and subsequent improvements.

The "Naive" Theory of Angular Correlation

The probability of emission of a gamma ray depends upon the angle between the direction of the nuclear spin axis and the direction of emission. Since in a radioactive source the nuclei are randomly orientated, an isotropic distribution is observed. An anisotropic distribution will be observed only from a source which does not have randomly orientated nuclei. This can be detected by placing the source in a magnetic field or by measuring the distribution of gamma rays with respect to other gamma rays emitted in a known direction. The nuclei emitting gamma rays in a fixed direction have a non isotropic spin distribution. The directional distributions of the other gamma rays from these nuclei can be measured relative to this orientation.

Consider first of all a single gamma ray transition with angular momentum \underline{L} between the nuclear levels B and C with spins I_b and I_c . Let the gamma ray

be characterized by the quantum numbers L and M and the levels by $I_b m_b$ and $I_c m_c$. Then the following relations hold: $\underline{I}_b = \underline{I}_c + \underline{L}$ $m_b = m_c + M$ where $\underline{L}^2 = L(L+1)\hbar^2$, $L_z = M\hbar$.

Each component $m_b \rightarrow m_c$ between specified magnetic sub levels has a directional distribution function $F_L^M(\theta)$ where θ is the angle between the gamma ray and the z axis. In atomic spectroscopy the individual components of the line $B \rightarrow C$ can be resolved but in nuclear spectroscopy they cannot be. In order to calculate the directional distribution of the line it is necessary to know the relative population $P(m_b)$ of each sub level m_b and the transition probability $G(m_b m_c)$ for each component $m_b \rightarrow m_c$. Then

$$F_L^M(\theta) \sim \sum_{m_b m_c} P(m_b) G(m_b m_c) F_L^M(\theta)$$

is the directional distribution for the unresolved line.

The absolute transition probability for a component $m_b \rightarrow m_c$ can be written as a product of a "nuclear" factor and a "geometrical" factor. It can be shown that the relative transition probability for a transition from $m_b \rightarrow m_c$ is given by

$$G(m_b m_c) = (I_c L m_c M | I_b m_b)^2$$

where the right hand side is a Clebsch-Gordan coefficient.

In order to calculate the relative populations it is necessary to know how the level B was created. Consider another level A decaying to B and then to C. If the direction of the gamma ray emitted from level A is taken as the Z axis the calculations will be greatly simplified and a directional correlation between the two successive gamma rays will be obtained. If all the m_a states are equally populated then

$$P(m_b) \sim \sum_{m_a} G(m_a m_b) F_{L_1}^{M_1} (\theta = 0) \text{ where}$$

$M_1 = m_a + m_b$. (for short-lived intermediate level)

This special choice of Z axis has also restricted the possible values of $F_{L_1}^{M_1}$ to $F_{L_1}^{\pm 1}$ since a gamma ray emitted in a specific direction can carry only the angular momentum $+\hbar$ or $-\hbar$.

Thus the directional correlation function $W(\theta)$ becomes

$$W(\theta) \sim \sum_{m_b m_c m_a} (I_b L_1 m_b \pm 1 | I_a m_a)^2 F_{L_1}^{\pm 1}(0) (I_c L_2 m_c M_2 | I_b m_b)^2 F_{L_2}^{M_2}(\theta)$$

Hamilton derived this equation using second order damping theory and evaluated it for possible cascades involving pure dipole and quadrupole gamma rays. He used an expansion

$$W(\theta) = 1 + a_2 \cos^2 \theta + a_4 \cos^4 \theta$$

and gave explicit forms for a_2 and a_4 . These calculations were, however, very tedious and extremely difficult to transform to higher multipolarities.

The next improvement in the theory of angular correlations came from Yang (Yang 1948) and Fierz (Fierz 1949) using group theory methods. Yang proved some general statements about $W(\theta)$ and Fierz calculated the expressions for the angular correlation functions of gamma rays and conversion electrons but could not give closed formulae.

The next improvement came from Gardner (Gardner 1949) who expressed the function $W(\theta)$ as

$$W(\theta) = 1 + \sum_{k=1}^{k_m} A_{2k} P_{2k}(\cos \theta)$$

where $P_{2k}(\cos \theta)$ are Legendre Polynomials. The coefficients A_{2k} were now in closed form and could be calculated since they were the product of five Clebsch-Gordan coefficients. The calculations of Gardner were restricted to conversion electrons, but Racah (Racah 1951), Lloyd (Lloyd 1950, 1951) and Alder (Alder 1951, 1952) extended them to cover different types of radiation, i.e., alpha-gamma and beta-gamma correlations as well as gamma-gamma correlations involving different multipolarity gamma rays.

The correlation function $W(\theta)$ can be written as

$$W(\theta) = 1 + A_2 P_2(\cos \theta) + \dots + A_{\gamma_{\max}} P_{\gamma_{\max}}(\cos \theta)$$

where the highest order in the expansion is determined by the selection rule

$$\gamma_{\max} = \text{Min} (2I_b, 2L_1, 2L_2).$$

The calculation of the coefficients is relatively simple since they are a product of two functions, each of which depends only on one transition of the cascade.

$$A_{\gamma} = F_{\gamma}(L_1 I_a I_b) F_{\gamma}(L_2 I_c I_b)$$

These coefficients have been calculated by Biedenharn and Rose (Biedenharn 1953) and can be found in a report by Ferentz and Rosenzweig (Ferentz 1954).

Section II

APPARATUS

Before embarking upon a description of the individual units in the apparatus, a brief description of the operation of a sum coincidence spectrometer will be given. The apparatus described here (see Fig. 1) differs from that described by Hoogenboom (Hoogenboom 1958) in that it employs the fast coincidence technique in addition to the sum coincidence.

The gamma rays from the source under investigation are detected by means of two scintillation counters. The base unit (the header) of each photomultiplier contains two circuits. One of these, a timing pulse shaper, produces a short fast rising pulse which is fed to the fast coincidence unit, the other, a slower rising linear pulse used for analysis. Thus if two gamma rays are detected by the detectors within the resolving time of the fast coincidence unit, a pulse is generated by this circuit. The preamplifier outputs are fed through linear amplifiers, with equal energy calibration, to an adding circuit. The output from the adder is connected to a single channel analyser so adjusted that only pulses of a certain height

FIGURE 1

SCHEMATIC DIAGRAM OF THE SUM
COINCIDENCE SPECTROMETER