INFLUENCE VALUES FOR COMPUTATION OF STRESSES IN A SEMI-INFINITE ELASTIC SOLID

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ABSTRACT

The purpose of this thesis was to evaluate the influence values for vertical stress due to concentrated or uniform load on the surface of a semi-infinite, elastic, isotropic and homogeneous solid by using the Froelich Equation with the concentration indexes from three to six. None of these tables have been known to be previously published except for the case of the concentration index equal to three, commonly known as the Boussinesq solution.

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TABLE OF CONTENTS

CHAPTER

CHAPTER

LIST OF TABLES

TABLE

CHAPTER I

INTRODUCTION

The determination of the distribution of vertical stresses in the ground under a foundation constitutes a major part of settlement investigations. The problem has been studied theoretically, and it is the purpose of this thesis to use the Froelich Equation[1]; to determine the stresses in an isotropic, elastic and homogeneous mass of semi-infinite extent due to a vertical load, concentrated or uniform, applied on the surface.

This problem for a single concentrated load was first solved by Lamé and Clapeyron[2] in the nineteenth century. Later the solution was developed in considerable detail by Boussinesq[3] in 1885. His solutions are much more usable than previous ones.

In 1897 August Föppl[4] pointed out that the behavior of soils under load was not exactly in accordance with the theory of elasticity. In other words, the stresses and strains in soils are not proportional in the manner required by the assumptions made by

Boussinesq. In 1929 J. H. Griffith[A] and in 1932 Dr. Ing. O. K. Froelich[1] published stress equations involving a parameter, which may be adjusted to suit materials other than elastic isotropic solids. Though, the two derivations differ in several important respects, each makes use of the necessary condition of equilibrium between the summation of the vertical forces within the solid and the load applied at the surface. The parameter is sometimes called the concentration index. It is of considerable importance in problems relating to soils and should receive much more consideration than it has received.

CHAPTER II

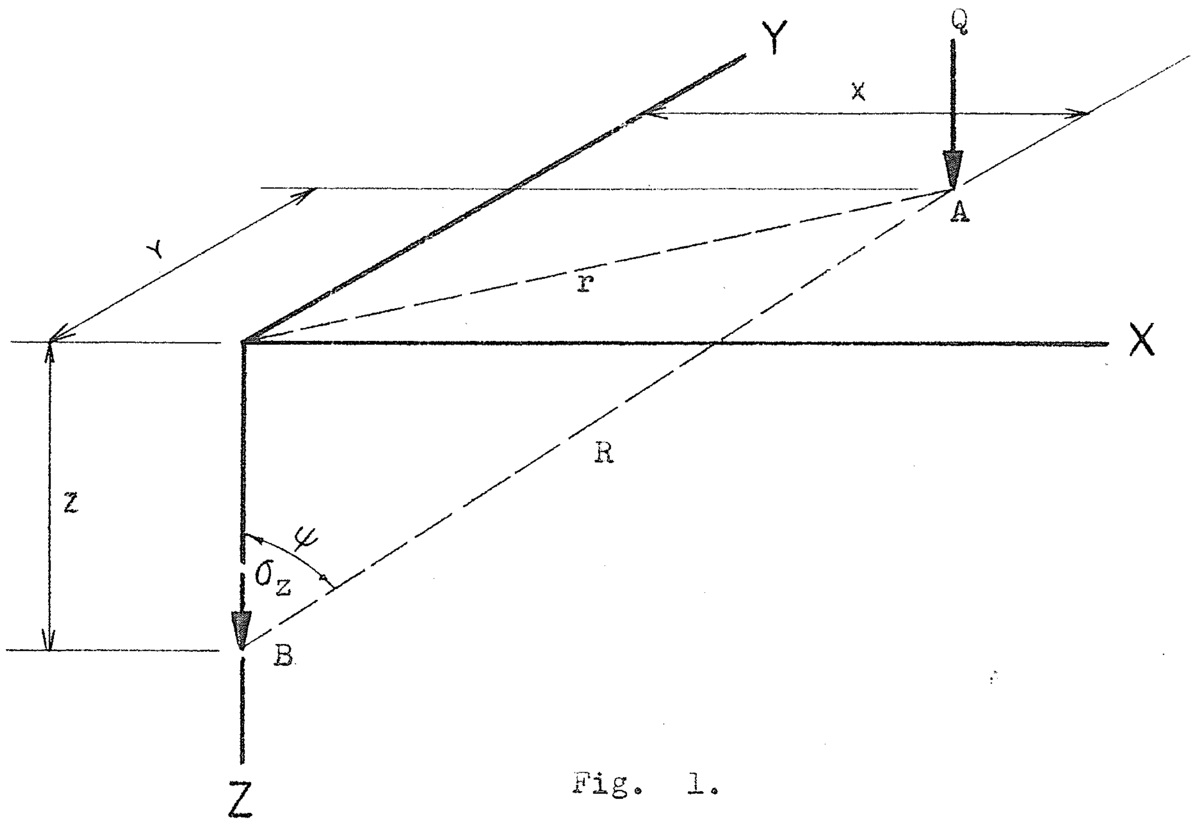

Fig. 1.

VERTICAL STRESSES UNDER VERTICAL LOADS APPLIED TO THE SURFACE OF A SEMI-INFINITE, ELASTIC, ISOTROPIC AND HOMOGENEOUS SOLID

The equation given by Dr. Ing. O. K. Froelich for the stress at point "B" in a vertical direction shown in Fig. 1 is as follows:

$$\sigma_z = \frac{\nu Q}{2\pi z^2} \cos^{(\nu + 2)} \psi \qquad \ldots\ldots\ldots\ldots(1)$$

in which:

Q = the vertical load,

r = the horizontal radial distance between an arbitrary point "B", below the surface and a vertical axis through the point "A" of application of "Q",

ψ = the angle between the vector "AB" and the vertical axis through the point of application,

ν = the concentration index, that may have different values for different soil structures and loading conditions.

Having an equation for the stress due to a load applied at a point, the equations for stresses due to distributed loads can be found by integration.

In order to get the formulas for vertical stresses under uniform loads, circular and rectangular, a double integral of the form:

$$\sigma_z = \iint F(x,y)\,dx\,dy \qquad \text{is involved.}$$

As the integrations are so tedious, in general it was only possible to reduce mathematically the above form to a single integral of the form:

$$\sigma_z = \int f(y)\,dy \qquad \text{and then,}$$

simply to find the area under the curve " Y = f(y) " with the aid of computer to obtain the influence values.

The loads which are transmitted by a building foundation can in general be assumed to act at a point, uniformly along a line or distributed uniformly over a rectangular area. Isolated column loads are approximately representative of the first type, wall loads of the second, spread footings and weight removed by excavation of the third.

In order to simplify a stress analysis, it is desirable to express the equations for the stresses in terms of dimensionless ratios. The quantities entering into the ratios being dimensions, which can be easily measured from a plan of the foundation.

Tables of influence values for the vertical stresses produced by loads applied at a point; applied uniformly along a line of infinite length; uniformly over a rectangular and over a circular area with concentration indexes of three, four, five and six were calculated and are presented herewith.

CHAPTER III

VERTICAL

STRESS DUE TO A POINT LOAD

The vertical stress at "B" due to a concentrated load "Q" at "A" on the surface of a semi-infinite, elastic, isotropic and homogeneous solid is determined by the equation:

$$\sigma_z = \frac{\nu Q}{2\pi z^2} \text{Cos}^{(\nu+2)} \psi$$

From Fig. 1 , $\text{Cos}\ \psi = \dfrac{z}{(r^2 + z^2)^{1/2}}$

Then, by substitution:

$$\sigma_z = \frac{\nu Q}{2\pi z^2} \left[\frac{z}{(r^2 + z^2)^{1/2}}\right]^{(\nu+2)}$$

or

$$\sigma_z = \frac{\nu Q}{2\pi z^2} \left[\frac{1}{1 + (r/z)^2}\right]^{(\nu+2)/2}$$

Let $m = r/z$

and $\sigma_z = (Q/z^2)\ I_\nu$ ....(2)

wherein , $I_\nu = \dfrac{\nu}{2\pi} \left[\dfrac{1}{1 + m^2}\right]^{(\nu+2)/2}$ .....(3)

in which, I_ν = influence value .

For $\nu = 3$, $I_3 = (3/2\pi)\left[1/(1 + m^2)\right]^{5/2}$ ......(4)

For $\nu = 4$, $I_4 = (2/\pi)\left[1/(1 + m^2)\right]^{3}$ ......(5)

For $\nu = 5$, $I_5 = (5/2\pi)\left[1/(1 + m^2)\right]^{7/2}$ ......(6)

For $\nu = 6$, $I_6 = (3/\pi)\left[1/(1 + m^2)\right]^{4}$ ......(7)

In Tables I to IV are given influence values for various values of the ratio, $m = r/z$.

To find the vertical stress, it is necessary only to measure the distance "r", from the point of load application to the point on the surface immediately above the point at which the stress is desired; divide this by the vertical distance "z"; select the proper influence value corresponding to this ratio "m", and compute the stress from Eq. (2).

TABLE I - INFLUENCE VALUES FOR CASE OF POINT LOAD $\nu = 3$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.4775	0.4774	0.4770	0.4764	0.4756	0.4745	0.4732	0.4717	0.4699	0.4679
.1	0.4657	0.4633	0.4607	0.4579	0.4549	0.4516	0.4482	0.4446	0.4409	0.4370
.2	0.4329	0.4286	0.4243	0.4197	0.4151	0.4103	0.4054	0.4005	0.3954	0.3902
.3	0.3849	0.3796	0.3742	0.3687	0.3632	0.3577	0.3521	0.3465	0.3408	0.3351
.4	0.3295	0.3238	0.3181	0.3124	0.3068	0.3011	0.2955	0.2899	0.2843	0.2788
.5	0.2733	0.2679	0.2625	0.2571	0.2518	0.2466	0.2414	0.2363	0.2312	0.2263
.6	0.2214	0.2165	0.2117	0.2070	0.2024	0.1978	0.1934	0.1890	0.1846	0.1804
.7	0.1762	0.1721	0.1681	0.1641	0.1603	0.1565	0.1527	0.1491	0.1455	0.1420
.8	0.1386	0.1353	0.1320	0.1288	0.1257	0.1226	0.1196	0.1167	0.1138	0.1111
.9	0.1083	0.1057	0.1031	0.1005	0.0981	0.0956	0.0933	0.0910	0.0887	0.0865
1.0	0.0843	0.0823	0.0803	0.0783	0.0763	0.0743	0.0727	0.0709	0.0691	0.0673
.1	0.0658	0.0641	0.0626	0.0609	0.0595	0.0581	0.0567	0.0553	0.0539	0.0526
.2	0.0513	0.0501	0.0489	0.0477	0.0465	0.0453	0.0443	0.0432	0.0422	0.0412
.3	0.0402	0.0393	0.0383	0.0374	0.0365	0.0357	0.0348	0.0340	0.0332	0.0324
.4	0.0317	0.0309	0.0302	0.0295	0.0288	0.0282	0.0275	0.0268	0.0263	0.0257
.5	0.0251	0.0245	0.0239	0.0234	0.0229	0.0223	0.0219	0.0214	0.0209	0.0204
.6	0.0200	0.0195	0.0191	0.0187	0.0183	0.0179	0.0175	0.0171	0.0166	0.0164
.7	0.0160	0.0157	0.0153	0.0150	0.0147	0.0144	0.0140	0.0137	0.0135	0.0132
.8	0.0129	0.0126	0.0124	0.0121	0.0119	0.0116	0.0114	0.0111	0.0109	0.0107
.9	0.0105	0.0103	0.0100	0.0098	0.0096	0.0094	0.0093	0.0090	0.0089	0.0087
2.0	0.0085	0.0084	0.0082	0.0080	0.0079	0.0077	0.0076	0.0074	0.0073	0.0072
.1	0.0070	0.0069	0.0067	0.0066	0.0065	0.0064	0.0062	0.0061	0.0060	0.0059
.2	0.0058	0.0057	0.0056	0.0055	0.0054	0.0053	0.0052	0.0051	0.0050	0.0049
.3	0.0048	0.0047	0.0046	0.0045	0.0044	0.0044	0.0043	0.0042	0.0041	0.0041
.4	0.0040	0.0039	0.0039	0.0038	0.0037	0.0037	0.0036	0.0035	0.0035	0.0034
.5	0.0034	0.0033	0.0032	0.0032	0.0031	0.0031	0.0031	0.0030	0.0030	0.0029
.6	0.0029	0.0029	0.0028	0.0028	0.0027	0.0026	0.0026	0.0025	0.0025	0.0025
.7	0.0024	0.0024	0.0023	0.0023	0.0023	0.0022	0.0022	0.0022	0.0021	0.0021
.8	0.0021	0.0020	0.0020	0.0020	0.0019	0.0019	0.0019	0.0018	0.0018	0.0018
.9	0.0018	0.0017	0.0017	0.0017	0.0017	0.0016	0.0016	0.0016	0.0016	0.0015

m		m		m		m	
3.0	0.0015	4.0	0.0004	5.0	0.0001	6.0	0.0001
.1	0.0013	.1	0.0004	.1	0.0001	.1	0.0001
.2	0.0011	.2	0.0003	.2	0.0001	.2	0.0000
.3	0.0010	.3	0.0003	.3	0.0001		
.4	0.0009	.4	0.0003	.4	0.0001		
.5	0.0007	.5	0.0002	.5	0.0001		
.6	0.0007	.6	0.0002	.6	0.0001		
.7	0.0006	.7	0.0002	.7	0.0001		
.8	0.0005	.8	0.0002	.8	0.0001		
.9	0.0005	.9	0.0002	.9	0.0001		

TABLE II - INFLUENCE VALUES FOR CASE OF POINT LOAD $\nu = 4$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.6366	0.6364	0.6359	0.6349	0.6336	0.6319	0.6298	0.6274	0.6246	0.6214
.1	0.6179	0.6141	0.6099	0.6054	0.6006	0.5955	0.5901	0.5845	0.5785	0.5724
.2	0.5660	0.5593	0.5525	0.5454	0.5382	0.5308	0.5232	0.5155	0.5076	0.5000
.3	0.4916	0.4834	0.4752	0.4669	0.4585	0.4501	0.4417	0.4332	0.4248	0.4163
.4	0.4079	0.3994	0.3910	0.3827	0.3744	0.3661	0.3579	0.3498	0.3418	0.3338
.5	0.3259	0.3182	0.3105	0.3029	0.2955	0.2881	0.2809	0.2737	0.2667	0.2598
.6	0.2531	0.2464	0.2399	0.2336	0.2273	0.2212	0.2152	0.2093	0.2036	0.1979
.7	0.1925	0.1871	0.1819	0.1767	0.1718	0.1669	0.1621	0.1575	0.1530	0.1486
.8	0.1443	0.1402	0.1361	0.1321	0.1283	0.1246	0.1209	0.1174	0.1140	0.1106
.9	0.1074	0.1042	0.1011	0.0982	0.0953	0.0924	0.0897	0.0871	0.0845	0.0820
1.0	0.0794	0.0772	0.0749	0.0727	0.0705	0.0685	0.0665	0.0645	0.0626	0.0605
.1	0.0590	0.0572	0.0556	0.0539	0.0523	0.0518	0.0493	0.0479	0.0465	0.0451
.2	0.0438	0.0425	0.0413	0.0401	0.0390	0.0377	0.0367	0.0355	0.0347	0.0337
.3	0.0327	0.0318	0.0309	0.0300	0.0291	0.0283	0.0275	0.0267	0.0260	0.0253
.4	0.0245	0.0239	0.0232	0.0225	0.0219	0.0213	0.0207	0.0201	0.0196	0.0191
.5	0.0185	0.0180	0.0175	0.0171	0.0166	0.0162	0.0157	0.0153	0.0149	0.0145
.6	0.0141	0.0137	0.0134	0.0130	0.0127	0.0123	0.0120	0.0117	0.0113	0.0111
.7	0.0108	0.0105	0.0103	0.0100	0.0097	0.0095	0.0093	0.0090	0.0088	0.0086
.8	0.0084	0.0081	0.0079	0.0077	0.0075	0.0074	0.0072	0.0070	0.0068	0.0067
.9	0.0065	0.0063	0.0062	0.0060	0.0059	0.0057	0.0056	0.0054	0.0053	0.0052
2.0	0.0051	0.0050	0.0049	0.0047	0.0046	0.0045	0.0044	0.0043	0.0042	0.0041
.1	0.0040	0.0039	0.0038	0.0038	0.0037	0.0036	0.0035	0.0034	0.0033	0.0033
.2	0.0032	0.0031	0.0031	0.0030	0.0029	0.0028	0.0028	0.0027	0.0027	0.0026
.3	0.0025	0.0025	0.0024	0.0024	0.0023	0.0023	0.0022	0.0022	0.0021	0.0021
.4	0.0021	0.0020	0.0020	0.0019	0.0019	0.0018	0.0018	0.0018	0.0017	0.0017
.5	0.0017	0.0016	0.0016	0.0016	0.0015	0.0015	0.0015	0.0015	0.0014	0.0014
.6	0.0014	0.0014	0.0013	0.0013	0.0013	0.0012	0.0012	0.0012	0.0012	0.0011
.7	0.0011	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
.8	0.0009	0.0009	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0008
.9	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0006

m		m	
3.0	00.0006	4.0	0.0001
.1	0.0005	.1	0.0001
.2	0.0004	.2	0.0001
.3	0.0004	.3	0.0001
.4	0.0003	.4	0.0001
.5	0.0003	.5	0.0001
.6	0.0002	.6	0.0001
.7	0.0002	.7	0.0001
.8	0.0002	.8	0.0000
.9	0.0001		

TABLE III - INFLUENCE VALUES FOR CASE OF POINT LOAD $\nu = 5$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.7958	0.7955	0.7947	0.7933	0.7913	0.7889	0.7858	0.7823	0.7782	0.7736
.1	0.7685	0.7630	0.7569	0.7505	0.7435	0.7362	0.7284	0.7203	0.7117	0.7029
.2	0.6937	0.6842	0.6745	0.6644	0.6541	0.6436	0.6329	0.6221	0.6110	0.5999
.3	0.5888	0.5772	0.5657	0.5542	0.5426	0.5311	0.5195	0.5079	0.4963	0.4848
.4	0.4734	0.4620	0.4507	0.4394	0.4283	0.4173	0.4065	0.3957	0.3852	0.3747
.5	0.3644	0.3543	0.3443	0.3346	0.3250	0.3156	0.3063	0.2973	0.2884	0.2797
.6	0.2713	0.2630	0.2549	0.2470	0.2393	0.2318	0.2245	0.2173	0.2104	0.2036
.7	0.1971	0.1907	0.1845	0.1784	0.1726	0.1669	0.1614	0.1560	0.1508	0.1458
.8	0.1409	0.1361	0.1316	0.1271	0.1228	0.1186	0.1146	0.1107	0.1069	0.1033
.9	0.0997	0.0963	0.0930	0.0898	0.0868	0.0838	0.0809	0.0781	0.0754	0.0728
1.0	0.0703	0.0679	0.0656	0.0633	0.0611	0.0589	0.0570	0.0549	0.0532	0.0512
.1	0.0496	0.0479	0.0463	0.0446	0.0432	0.0417	0.0403	0.0389	0.0376	0.0363
.2	0.0351	0.0339	0.0327	0.0316	0.0306	0.0295	0.0286	0.0275	0.0267	0.0258
.3	0.0249	0.0241	0.0233	0.0225	0.0218	0.0211	0.0204	0.0197	0.0191	0.0184
.4	0.0178	0.0173	0.0167	0.0162	0.0156	0.0151	0.0146	0.0142	0.0137	0.0133
.5	0.0129	0.0124	0.0121	0.0117	0.0113	0.0109	0.0106	0.0103	0.0100	0.0096
.6	0.0093	0.0091	0.0088	0.0085	0.0082	0.0080	0.0078	0.0075	0.0072	0.0071
.7	0.0069	0.0066	0.0064	0.0063	0.0061	0.0059	0.0057	0.0055	0.0054	0.0052
.8	0.0051	0.0049	0.0048	0.0046	0.0045	0.0044	0.0042	0.0041	0.0040	0.0039
.9	0.0038	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0030
2.0	0.0028	0.0028	0.0027	0.0026	0.0025	0.0025	0.0024	0.0023	0.0023	0.0022
.1	0.0022	0.0021	0.0020	0.0020	0.0019	0.0019	0.0018	0.0018	0.0017	0.0017
.2	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0014	0.0013	0.0013
.3	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0011	0.0010	0.0010
.4	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008
.5	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006
.6	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005
.7	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
.8	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003
.9	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003

m	
3.0	0.0003
.1	0.0002
.2	0.0002
.3	0.0001
.4	0.0001
.5	0.0001
.6	0.0001
.7	0.0001
.8	0.0001
.9	0.0000

TABLE IV - INFLUENCE VALUES FOR CASE OF POINT LOAD $\nu = 6$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.9549	0.9546	0.9534	0.9515	0.9488	0.9454	0.9413	0.9364	0.9309	0.9246
.1	0.9177	0.9101	0.9019	0.8930	0.8836	0.8736	0.8631	0.8521	0.8406	0.8286
.2	0.8163	0.8035	0.7904	0.7770	0.7633	0.7493	0.7351	0.7207	0.7661	0.6913
.3	0.6765	0.6616	0.6466	0.6315	0.6165	0.6015	0.5865	0.5716	0.5568	0.5420
.4	0.5274	0.5129	0.4986	0.4844	0.4704	0.4567	0.4431	0.4298	0.4167	0.4038
.5	0.3911	0.3787	0.3666	0.3547	0.3422	0.3318	0.3207	0.3099	0.2993	0.2891
.6	0.2791	0.2674	0.2600	0.2508	0.2409	0.2332	0.2248	0.2167	0.2088	0.2011
.7	0.1937	0.1866	0.1796	0.1729	0.1665	0.1602	0.1542	0.1483	0.1427	0.1373
.8	0.1320	0.1265	0.1221	0.1174	0.1128	0.1085	0.1043	0.1002	0.0963	0.0926
.9	0.0890	0.0855	0.0822	0.0789	0.0759	0.0729	0.0700	0.0673	0.0646	0.0621
1.0	0.0596	0.0573	0.0551	0.0529	0.0508	0.0489	0.0470	0.0451	0.0432	0.0413
.1	0.0400	0.0385	0.0370	0.0355	0.0341	0.0328	0.0315	0.0303	0.0291	0.0280
.2	0.0268	0.0259	0.0249	0.0239	0.0230	0.0221	0.0213	0.0202	0.0197	0.0190
.3	0.0182	0.0175	0.0169	0.0162	0.0156	0.0150	0.0145	0.0139	0.0134	0.0129
.4	0.0124	0.0120	0.0115	0.0111	0.0107	0.0103	0.0099	0.0095	0.0092	0.0089
.5	0.0086	0.0082	0.0080	0.0077	0.0074	0.0071	0.0069	0.0066	0.0064	0.0062
.6	0.0059	0.0057	0.0055	0.0053	0.0052	0.0050	0.0048	0.0046	0.0044	0.0043
.7	0.0042	0.0040	0.0039	0.0038	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031
.8	0.0030	0.0029	0.0028	0.0027	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022
.9	0.0021	0.0020	0.0020	0.0019	0.0019	0.0018	0.0017	0.0016	0.0016	0.0016
2.0	0.0015	0.0015	0.0014	0.0014	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011
.1	0.0011	0.0011	0.0010	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008
.2	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006
.3	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
.4	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
.5	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
.6	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
.7	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
.8	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
.9	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

m	
3.0	0.0001
.1	0.0001
.2	0.0001
.3	0.0000

CHAPTER IV

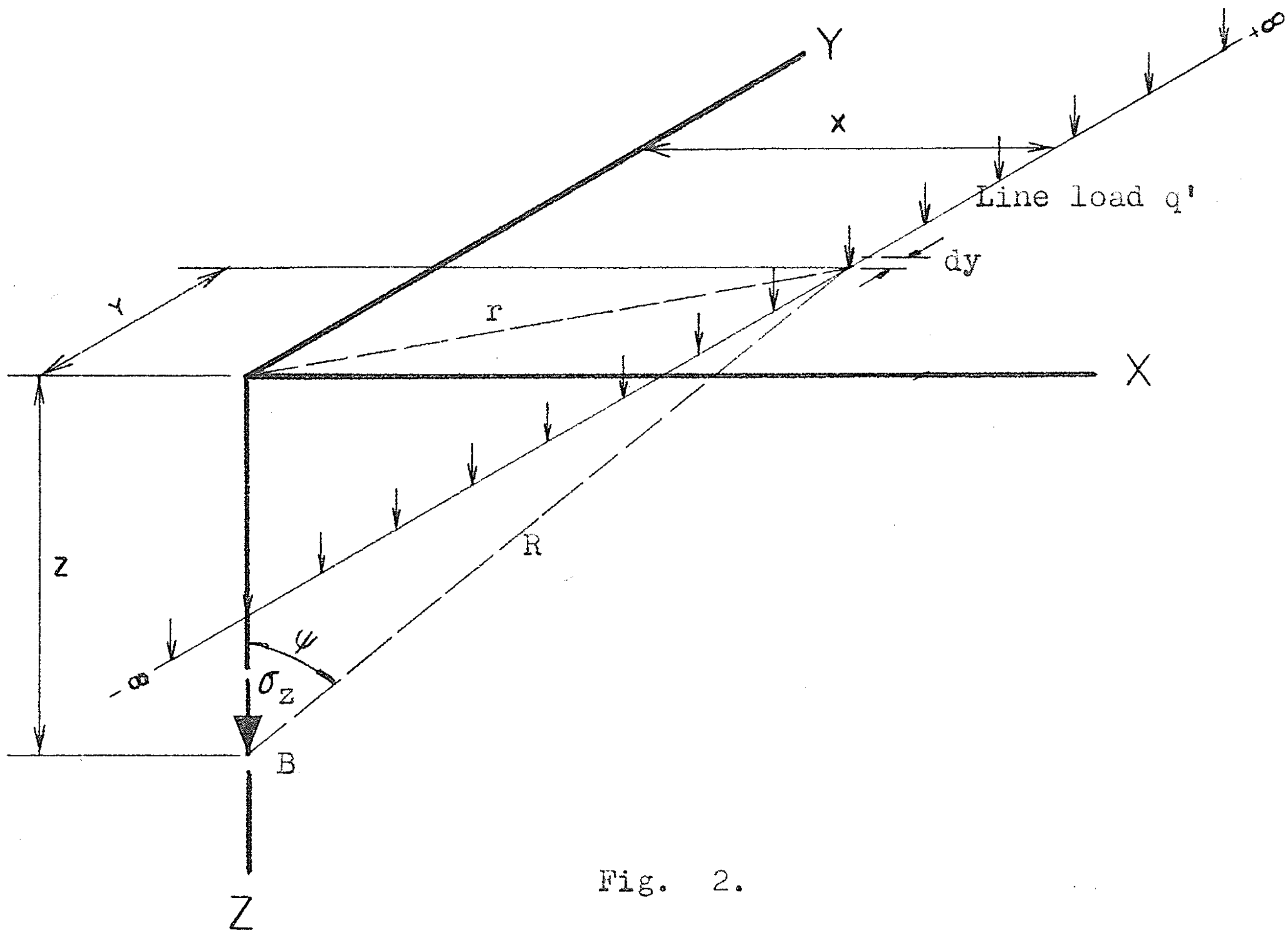

Fig. 2.

VERTICAL
STRESS DUE TO A LINE LOAD OF INFINITE LENGTH

The vertical stress at "B" due to a strip of infinite length from $-\infty$ to $+\infty$, loaded uniformly with a load of "q'" per unit of length on the surface of a semi-infinite, elastic, isotropic and homogeneous solid may be found by considering the relationship:

$$d\,\sigma_z = \frac{\nu\,Q}{2\pi z^2}\,\text{Cos}^{(\nu+2)}\psi$$

wherein , $Q = q'\,dy$

and $\text{Cos}\,\psi = z/(x^2 + y^2 + z^2)^{1/2}$.

Therefore , $$\sigma_z = \int_{-\infty}^{+\infty} \frac{\nu\,q'}{2\pi z^2}\left[\frac{z^2}{x^2 + y^2 + z^2}\right]^{(\nu+2)/2} dy \quad \text{.......(8)}$$

Let $m = x/z$

Then, by substitution and integration:

$$\sigma_z = (q'/z)\,I_\nu \quad \text{......................(9)}$$

For $\nu = 3$, $I_3 = (2/\pi)\left[1/(1 + m^2)\right]^2$ ....(10)

For $\nu = 4$, $I_4 = (3/4)\left[1/(1 + m^2)\right]^{5/2}$..............(11)

For $\nu = 5$, $I_5 = (8/3\pi)\left[1/(1 + m^2)\right]^3$ ....(12)

For $\nu = 6$, $I_6 = (15/16)\left[1/(1 + m^2)\right]^{7/2}$..............(13)

In Tables V to VIII are given influence values for various values of the ratio, $m = x/z$.

To find the vertical stress at any point "B", it is necessary to find the distance "x", and to divide this by the vertical distance "z" to obtain the ratio "m". Then, the proper influence value is selected, and the stress computed from Eq. (9).

TABLE V - INFLUENCE VALUES FOR CASE OF LINE LOAD OF INFINITE LENGTH $\nu = 3$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.6366	0.6365	0.6361	0.9355	0.6346	0.6335	0.6321	0.6304	0.6286	0.6264
.1	0.6241	0,6215	0.5187	0.6156	0.6124	0.6089	0.6052	0.6014	0.5973	0.5930
.2	0.5886	0.5840	0.5792	0.5743	0.5692	0.5639	0.5586	0.5531	0.5474	0.5417
.3	0.5385	0.5299	0.5238	0.5177	0.5115	0.5053	0.4989	0.4925	0.4861	0.4796
.4	0.4731	0.4666	0.4600	0.4534	0.4469	0.4403	0.4337	0.4271	0.4205	0.4140
.5	0.4074	0.4009	0.3945	0.3880	0.3816	0.3753	0.3689	0.3627	0.3564	0.3503
.6	0.3442	0.3381	0.3322	0.3262	0.3204	0.3146	0.3089	0.3033	0.2977	0.2922
.7	0.2868	0.2814	0.2761	0.2709	0.2658	0.2608	0.2558	0.2509	0.2461	0.2414
.8	0.2367	0.2321	0.2276	0.2232	0.2188	0.2146	0.2104	0.2062	0.2022	0.1982
.9	0.1943	0.1905	0.1867	0.1830	0.1794	0.1759	0.1724	0.1690	0.1656	0.1624
1.0	0.1588	0.1560	0.1529	0.1499	0.1469	0.1440	0.1412	0.1384	0.1356	0.1324
.1	0.1303	0.1278	0.1253	0.1228	0.1204	0.1180	0.1157	0.1134	0.1112	0.1091
.2	0.1066	0.1048	0.1028	0.1008	0.0989	0.0969	0.0951	0.0928	0.0914	0.0897
.3	0.0880	0.0863	0.0846	0.0830	0.0815	0.0799	0.0784	0.0769	0.0755	0.0740
.4	0.0727	0.0713	0.0700	0.0687	0.0674	0.0661	0.0649	0.0635	0.0625	0.0614
.5	0.0603	0.0592	0.0581	0.0570	0.0560	0.0550	0.0540	0.0530	0.0521	0.0511
.6	0.0502	0.0493	0.0485	0.0476	0.0468	0.0459	0.0451	0.0443	0.0432	0.0428
.7	0.0421	0.0413	0.0406	0.0399	0.0392	0.0386	0.0379	0.0373	0.0366	0.0360
.8	0.0354	0.0347	0.0342	0.0337	0.0331	0.0325	0.0319	0.0315	0.0310	0.0305
.9	0.0300	0.0295	0.0290	0.0285	0.0281	0.0276	0.0272	0.0267	0.0263	0.0259
2.0	0.0255	0.0251	0.0247	0.0242	0.0239	0.0235	0.0232	0.0228	0.0224	0.0221
.1	0.0218	0.0214	0.0210	0.0208	0.0204	0.0201	0.0198	0.0195	0.0192	0.0190
.2	0.0187	0.0184	0.0181	0.0178	0.0176	0.0173	0.0170	0.0168	0.0165	0.0163
.3	0.0160	0.0160	0.0156	0.0154	0.0151	0.0149	0.0147	0.0145	0.0143	0.0141
.4	0.0139	0.0137	0.0135	0.0133	0.0131	0.0129	0.0128	0.0125	0.0124	0.0122
.5	0.0119	0.0119	0.0117	0.0116	0.0114	0.0113	0.0112	0.0111	0.0109	0.0108
.6	0.0107	0.0105	0.0103	0.0102	0.0101	0.0096	0.0096	0.0096	0.0096	0.0094
.7	0.0093	0.0092	0.0091	0.0090	0.0088	0.0087	0.0086	0.0085	0.0084	0.0083
.8	0.0082	0.0081	0.0080	0.0078	0.0077	0.0076	0.0076	0.0075	0.0074	0.0073
.9	0.0072	0.0071	0.0070	0.0069	0.0068	0.0068	0.0067	0.0066	0.0065	0.0064

m		m		m		m		m	
3.0	0.0064	4.5	0.0014	6.0	0.0005	7.5	0.0002	9.0	0.0001
.1	0.0057	.6	0.0013	.1	0.0004	.6	0.0002	.1	0.0000
.2	0.0050	.7	0.0012	.2	0.0004	.7	0.0002		
.3	0.0045	.8	0.0011	.3	0.0004	.8	0.0002		
.4	0.0040	.9	0.0010	.4	0.0004	.9	0.0002		
.5	0.0036			.5	0.0003				
.6	0.0033	5.0	0.0009	.6	0.0003	8.0	0.0002		
.7	0.0030	.1	0.0009	.7	0.0003	.1	0.0001		
.8	0.0027	.2	0.0008	.8	0.0003	.2	0.0001		
.9	0.0024	.3	0.0008	.9	0.0003	.3	0.0001		
		.4	0.0007			.4	0.0001		
4.0	0.0022	.5	0.0007	7.0	0.0003	.5	0.0001		
.1	0.0020	.6	0.0006	.1	0.0002	.6	0.0001		
.2	0.0018	.7	0.0006	.2	0.0002	.7	0.0001		
.3	0.0017	.8	0.0005	.3	0.0002	.8	0.0001		
.4	0.0015	.9	0.0005	.4	0.0002	.9	0.0001		

TABLE VI - INFLUENCE VALUES FOR CASE OF LINE LOAD OF INFINITE LENGTH $\nu = 4$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.7500	0.7498	0.7493	0.7483	0.7470	0.7453	0.7433	0.7409	0.7381	0.7350
.1	0.7316	0.7278	0.7237	0.7192	0.7145	0.7094	0.7041	0.6984	0.6925	0.6864
.2	0.6800	0.6733	0.6664	0.6593	0.6520	0.6445	0.6369	0.6290	0.6210	0.6129
.3	0.6046	0.5963	0.5879	0.5792	0.5705	0.5618	0.5530	0.5442	0.5353	0.5264
.4	0.5175	0.5086	0.4997	0.4907	0.4819	0.4730	0.4642	0.4554	0.4466	0.4379
.5	0.4293	0.4208	0.4123	0.4039	0.3956	0.3874	0.3792	0.3712	0.3632	0.3554
.6	0.3477	0.3401	0.3326	0.3252	0.3179	0.3108	0.3037	0.2968	0.2900	0.2833
.7	0.2768	0.2703	0.2640	0.2578	0.2517	0.2458	0.2399	0.2342	0.2286	0.2231
.8	0.2177	0.2125	0.2074	0.2023	0.1974	0.1926	0.1879	0.1833	0.1788	0.1744
.9	0.1702	0.1660	0.1619	0.1579	0.1540	0.1502	0.1465	0.1429	0.1394	0.1359
1.0	0.1325	0.1293	0.1261	0.1230	0.1199	0.1167	0.1141	0.1109	0.1086	0.1057
.1	0.1033	0.1008	0.0983	0.0959	0.0935	0.0912	0.0890	0.0868	0.0847	0.0827
.2	0.0806	0.0787	0.0768	0.0749	0.0731	0.0712	0.0696	0.0678	0.0663	0.0647
.3	0.0632	0.0617	0.0602	0.0588	0.0574	0.0560	0.0547	0.0534	0.0522	0.0508
.4	0.0498	0.0486	0.0475	0.0464	0.0453	0.0442	0.0432	0.0422	0.0413	0.0403
.5	0.0394	0.0385	0.0376	0.0368	0.0359	0.0351	0.0343	0.0336	0.0328	0.0321
.6	0.0314	0.0307	0.0300	0.0293	0.0287	0.0281	0.0274	0.0268	0.0260	0.0257
.7	0.0251	0.0246	0.0241	0.0235	0.0230	0.0225	0.0221	0.0216	0.0211	0.0207
.8	0.0203	0.0198	0.0194	0.0190	0.0186	0.0182	0.0179	0.0175	0.0171	0.0168
.9	0.0164	0.0161	0.0158	0.0155	0.0151	0.0148	0.0145	0.0142	0.0140	0.0137
2.0	0.0134	0.0132	0.0129	0.0126	0.0124	0.0121	0.0119	0.0117	0.0115	0.0112
.1	0.0110	0.0108	0.0106	0.0104	0.0102	0.0100	0.0098	0.0096	0.0095	0.0093
.2	0.0091	0.0089	0.0088	0.0086	0.0084	0.0083	0.0081	0.0079	0.0078	0.0077
.3	0.0075	0.0074	0.0073	0.0071	0.0070	0.0069	0.0068	0.0066	0.0065	0.0064
.4	0.0063	0.0062	0.0061	0.0060	0.0059	0.0058	0.0057	0.0056	0.0055	0.0054
.5	0.0053	0.0052	0.0051	0.0050	0.0049	0.0048	0.0048	0.0047	0.0047	0.0046
.6	0.0045	0.0044	0.0043	0.0042	0.0041	0.0041	0.0040	0.0039	0.0039	0.0039
.7	0.0038	0.0038	0.0037	0.0036	0.0036	0.0035	0.0035	0.0034	0.0034	0.0033
.8	0.0032	0.0032	0.0031	0.0031	0.0030	0.0030	0.0029	0.0029	0.0028	0.0028
.9	0.0028	0.0027	0.0027	0.0026	0.0026	0.0026	0.0025	0.0025	0.0024	0.0024

m		m		m		m	
3.0	0.0024	4.0	0.0006	5.0	0.0002	6.0	0.0001
.1	0.0020	.1	0.0006	.1	0.0002	.1	0.0001
.2	0.0018	.2	0.0005	.2	0.0002	.2	0.0001
.3	0.0015	.3	0.0004	.3	0.0002	.3	0.0001
.4	0.0013	.4	0.0004	.4	0.0002	.4	0.0001
.5	0.0012	.5	0.0004	.5	0.0001	.5	0.0001
.6	0.0010	.6	0.0003	.6	0.0001	.6	0.0001
.7	0.0009	.7	0.0003	.7	0.0001	.7	0.0001
.8	0.0008	.8	0.0003	.8	0.0001	.8	0.0001
.9	0.0007	.9	0.0002	.9	0.0001	.9	0.0000

TABLE VII - INFLUENCE VALUES FOR CASE OF LINE LOAD OF INFINITE LENGTH $\nu = 5$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.8488	0.4868	0.8478	0.8465	0.8448	0.8425	0.8397	0.8365	0.8327	0.8285
.1	0.8239	0.8188	0.8132	0.8072	0.8008	0.7940	0.7868	0.7793	0.7714	0.7632
.2	0.7546	0.7458	0.7366	0.7272	0.7176	0.7077	0.6976	0.6873	0.6768	0.6662
.3	0.6555	0.6446	0.6336	0.6225	0.6114	0.6006	0.5889	0.5776	0.5664	0.5551
.4	0.5438	0.5326	0.5214	0.5102	0.4992	0.4882	0.4772	0.4664	0.4557	0.4451
.5	0.4346	0.4242	0.4140	0.4039	0.3939	0.3841	0.3745	0.3650	0.3556	0.3465
.6	0.3374	0.3286	0.3199	0.3114	0.3031	0.2949	0.2869	0.2791	0.2714	0.2639
.7	0.2566	0.2495	0.2425	0.2357	0.2290	0.2225	0.2162	0.2100	0.2040	0.1981
.8	0.1924	0.1869	0.1815	0.1762	0.1711	0.1661	0.1612	0.1565	0.1519	0.1475
.9	0.1431	0.1389	0.1348	0.1309	0.1270	0.1233	0.1196	0.1161	0.1127	0.1093
1.0	0.1059	0.1030	0.0999	0.0968	0.0941	0.0913	0.0886	0.0860	0.0835	0.0807
.1	0.0786	0.0763	0.0741	0.0719	0.0698	0.0678	0.0658	0.0638	0.0620	0.0602
.2	0.0584	0.0567	0.0551	0.0535	0.0519	0.0503	0.0490	0.0473	0.0462	0.0449
.3	0.0436	0.0424	0.0412	0.0400	0.0388	0.0377	0.0367	0.0356	0.0346	0.0337
.4	0.0327	0.0318	0.0309	0.0301	0.0292	0.0284	0.0276	0.0268	0.0261	0.0254
.5	0.0247	0.0241	0.0234	0.0228	0.0221	0.0215	0.0210	0.0204	0.0199	0.0193
.6	0.0188	0.0183	0.0178	0.0174	0.0169	0.0165	0.0160	0.0156	0.0150	0.0148
.7	0.0144	0.0140	0.0137	0.0133	0.0130	0.0127	0.0123	0.0120	0.0117	0.0114
.8	0.0111	0.0108	0.0106	0.0103	0.0101	0.0098	0.0096	0.0093	0.0091	0.0089
.9	0.0087	0.0085	0.0082	0.0080	0.0079	0.0077	0.0075	0.0073	0.0071	0.0070
2.0	0.0068	0.0066	0.0065	0.0063	0.0062	0.0060	0.0059	0.0057	0.0056	0.0055
.1	0.0053	0.0052	0.0051	0.0050	0.0049	0.0048	0.0047	0.0046	0.0045	0.0044
.2	0.0043	0.0042	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035	0.0035
.3	0.0034	0.0034	0.0032	0.0032	0.0031	0.0030	0.0030	0.0029	0.0029	0.0028
.4	0.0027	0.0027	0.0026	0.0026	0.0025	0.0025	0.0024	0.0023	0.0023	0.0023
.5	0.0022	0.0022	0.0021	0.0021	0.0020	0.0020	0.0020	0.0020	0.0019	0.0019
.6	0.0018	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0016	0.0016	0.0015
.7	0.0015	0.0015	0.0014	0.0014	0.0014	0.0014	0.0013	0.0013	0.0013	0.0013
.8	0.0012	0.0012	0.0012	0.0012	0.0011	0.0011	0.0011	0.0011	0.0011	0.0010
.9	0.0010	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009

m		m		m	
3.0	0.0008	4.0	0.0002	5.0	0.0000
.1	0.0007	.1	0.0002		
.2	0.0006	.2	0.0001		
.3	0.0005	.3	0.0001		
.4	0.0004	.4	0.0001		
.5	0.0004	.5	0.0001		
.6	0.0003	.6	0.0001		
.7	0.0003	.7	0.0001		
.8	0.0002	.8	0.0001		
.9	0.0002	.9	0.0001		

TABLE VIII - INFLUENCE VALUES FOR CASE OF LINE LOAD OF INFINITE LENGTH $\nu = 6$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.9375	0.9372	0.9362	0.9346	0.9323	0.9293	0.9258	0.9216	0.9168	0.9114
.1	0.9054	0.8989	0.8917	0.8841	0.8759	0.8673	0.8581	0.8485	0.8385	0.8281
.2	0.8173	0.8661	0.7946	0.7827	0.7706	0.7583	0.7457	0.7329	0.7199	0.7067
.3	0.6934	0.6800	0.6665	0.6529	0.6393	0.6256	0.6120	0.5983	0.5847	0.5712
.4	0.5577	0.5442	0.5309	0.5177	0.5046	0.4917	0.4789	0.4662	0.4537	0.4414
.5	0.4293	0.4174	0.4057	0.3942	0.3828	0.3717	0.3609	0.3502	0.3397	0.3286
.6	0.3196	0.3098	0.3003	0.2910	0.2810	0.2731	0.2645	0.2561	0.2479	0.2399
.7	0.2322	0.2246	0.2173	0.2102	0.2033	0.1966	0.1901	0.1838	0.1777	0.1717
.8	0.1660	0.1604	0.1550	0.1497	0.1447	0.1398	0.1350	0.1304	0.1260	0.1217
.9	0.1175	0.1135	0.1096	0.1058	0.1022	0.0987	0.0953	0.0920	0.0889	0.0858
1.0	0.0828	0.0800	0.0773	0.0746	0.0720	0.0964	0.0672	0.0647	0.0626	0.0604
.1	0.0584	0.0564	0.0545	0.0525	0.0518	0.0491	0.0474	0.0458	0.0443	0.0428
.2	0.0413	0.0399	0.0384	0.0373	0.0360	0.0347	0.0336	0.0324	0.0314	0.0304
.3	0.0294	0.0284	0.0274	0.0265	0.0257	0.0248	0.0240	0.0232	0.0225	0.0217
.4	0.0210	0.0203	0.0197	0.0190	0.0184	0.0178	0.0172	0.0167	0.0162	0.0156
.5	0.0151	0.0147	0.0142	0.0138	0.0133	0.0129	0.0125	0.0121	0.0117	0.0114
.6	0.0110	0.0107	0.0103	0.0100	0.0097	0.0094	0.0091	0.0089	0.0085	0.0083
.7	0.0081	0.0078	0.0076	0.0074	0.0071	0.0069	0.0067	0.0065	0.0063	0.0062
.8	0.0060	0.0058	0.0056	0.0055	0.0053	0.0052	0.0050	0.0049	0.0047	0.0046
.9	0.0045	0.0043	0.0042	0.0041	0.0040	0.0039	0.0038	0.0036	0.0035	0.0034
2.0	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0028	0.0027	0.0026
.1	0.0025	0.0025	0.0024	0.0023	0.0023	0.0022	0.0022	0.0021	0.0021	0.0020
.2	0.0019	0.0019	0.0018	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015
.3	0.0015	0.0015	0.0014	0.0014	0.0013	0.0013	0.0013	0.0012	0.0012	0.0012
.4	0.0012	0.0011	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010	0.0010	0.0009
.5	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0007
.6	0.0007	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
.7	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
.8	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
.9	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003

m		m	
3.0	0.0003	4.0	0.0000
.1	0.0002		
.2	0.0002		
.3	0.0002		
.4	0.0001		
.5	0.0001		
.6	0.0001		
.7	0.0001		
.8	0.0001		
.9	0.0001		

CHAPTER V

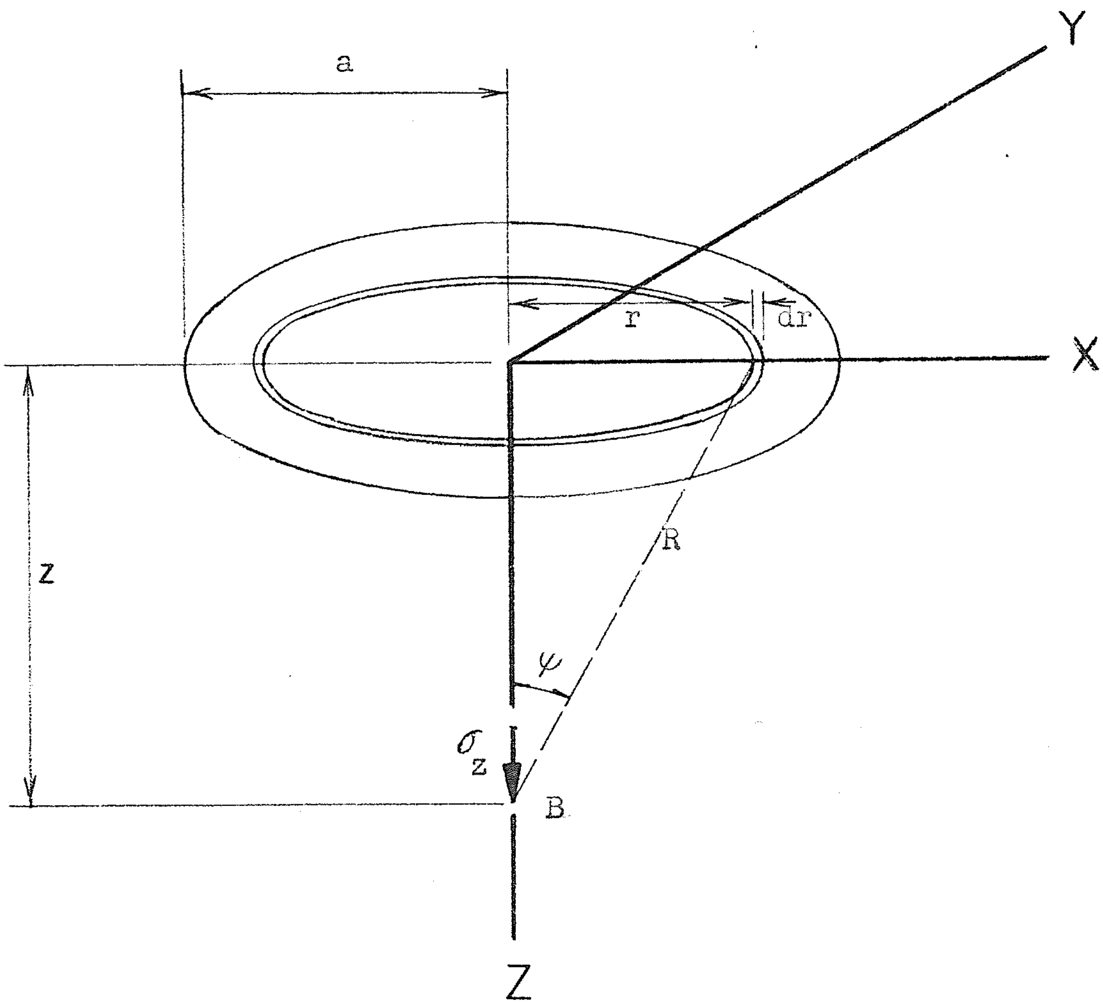

Fig. 3

VERTICAL

STRESS BENEATH THE CENTER OF A CIRCULAR AREA

UNIFORMLY LOADED

The vertical stress at "B", beneath the center of a circular area of radius "a", loaded with a load of "q" per unit area on the surface of a semi-infinite, isotropic, elastic and homogeneous solid may be found as follows:

$$d\sigma_z = \frac{\nu Q}{2\pi z^2} \cos^{(\nu+2)}\psi$$

wherein, $Q = 2\pi r \, dr \, q$

and $\cos\psi = \frac{z}{(z^2 + r^2)^{1/2}}$

Therefore, $$\sigma_z = \int_0^a \nu q z^\nu \frac{r dr}{(z^2 + r^2)^{(\nu+2)/2}} \quad \ldots.(14)$$

Let $m = a/z$

Then, by integration and substitution: $\sigma_z = q \, I_\nu$ ......(15)

For $\nu = 3$, $I_3 = 1 - [1/(1 + m^2)]^{3/2}$ ....(16)

For $\nu = 4$, $I_4 = 1 - [1/(1 + m^2)]^2$ ....(17)

For $\nu = 5$, $I_5 = 1 - [1/(1 + m^2)]^{5/2}$ ....(18)

For $\nu = 6$, $I_6 = 1 - [1/(1 + m^2)]^3$ ....(19)

In Tables IX to XII are given influence values for various values of the ratio, m = a/z.

In order to find the value of a stress with the aid of these tables, it is necessary to know the radius "a" of the circle which is uniformly loaded, and to obtain the value of the ratio "a/z". Then, select the proper value of "I_ν" corresponding to this ratio. The stress can then be computed from Eq. (15).

TABLE IX - INFLUENCE VALUES FOR POINT BENEATH THE CENTER OF A

UNIFORMLY LOADED CIRCULAR AREA $\nu = 3$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0001	0.0006	0.0013	0.0024	0.0037	0.0054	0.0073	0.0095	0.0120
.1	0.0148	0.0179	0.0212	0.0248	0.0287	0.0328	0.0372	0.0418	0.0467	0.0518
.2	0.0571	0.0627	0.0684	0.0744	0.0806	0.0869	0.0935	0.1002	0.1070	0.1141
.3	0.1213	0.1286	0.1361	0.1436	0.1513	0.1592	0.1671	0.1751	0.1832	0.1914
.4	0.1996	0.2079	0.2163	0.2247	0.2332	0.2417	0.2502	0.2588	0.2673	0.2759
.5	0.2845	0.2931	0.3016	0.3102	0.3188	0.3273	0.3358	0.3443	0.3527	0.3611
.6	0.3695	0.3778	0.3861	0.3943	0.4025	0.4106	0.4186	0.4266	0.4346	0.4424
.7	0.4502	0.4579	0.4655	0.4731	0.4806	0.4880	0.4953	0.5026	0.5098	0.5169
.8	0.5239	0.5308	0.5376	0.5444	0.5511	0.5577	0.5642	0.5706	0.5769	0.5832
.9	0.5894	0.5954	0.6014	0.6074	0.6132	0.6189	0.6246	0.6302	0.6357	0.6411
1.0	0.6468	0.6517	0.6569	0.6620	0.6671	0.6720	0.6769	0.6817	0.6864	0.6917
.1	0.6956	0.7001	0.7046	0.7090	0.7133	0.7175	0.7216	0.7257	0.7298	0.7337
.2	0.7376	0.7415	0.7453	0.7490	0.7526	0.7566	0.7598	0.7639	0.7667	0.7700
.3	0.7734	0.7766	0.7798	0.7830	0.7861	0.7891	0.7921	0.7951	0.7980	0.8008
.4	0.8036	0.8064	0.8091	0.8118	0.8144	0.8170	0.8196	0.8223	0.8245	0.8270
.5	0.8293	0.8317	0.8340	0.8363	0.8385	0.8409	0.8428	0.8450	0.8471	0.8491
.6	0.8511	0.8531	0.8551	0.8570	0.8589	0.8608	0.8626	0.8644	0.8669	0.8679
.7	0.8698	0.9714	0.8730	0.8747	0.8763	0.8779	0.8795	0.8810	0.8825	0.8840
.8	0.8855	0.8870	0.8883	0.8897	0.8911	0.8925	0.8938	0.8951	0.8964	0.8977
.9	0.8990	0.9002	0.9014	0.9026	0.9038	0.9050	0.9061	0.9073	0.9084	0.9095
2.0	0.9106	0.9116	0.9127	0.9221	0.9147	0.9157	0.9167	0.9178	0.9187	0.9196
.1	0.9207	0.9215	0.9223	0.9233	0.9241	0.9250	0.9259	0.9267	0.9275	0.9283
.2	0.9291	0.9299	0.9307	0.9315	0.9323	0.9332	0.9339	0.9348	0.9354	0.9362
.3	0.9368	0.9369	0.9381	0.9388	0.9396	0.9402	0.9408	0.9414	0.9420	0.9426
.4	0.9432	0.9438	0.9444	0.9459	0.9456	0.9462	0.9465	0.9475	0.9478	0.9484
.5	0.9489	0.9493	0.9499	0.9504	0.9510	0.9514	0.9516	0.9520	0.9525	0.9530
.6	0.9535	0.9539	0.9545	0.9550	0.9554	0.9560	0.9564	0.9569	0.9571	0.9576
.7	0.9580	0.9584	0.9588	0.9592	0.9596	0.9600	0.9603	0.9588	0.9611	0.9615
.8	0.9618	0.9622	0.9625	0.9630	0.9634	0.9637	0.9640	0.9644	0.9647	0.9650
.9	0.9654	0.9657	0.9660	0.9663	0.9666	0.9669	0.9672	0.9675	0.9678	0.9681

m		m		m		m	
3.0	0.9684	4.5	0.9898	6.0	0.9956	8.8	0.9986
.1	0.9711	.6	0.9904	.2	0.9960		
.2	0.9735	.7	0.9910	.4	0.9963	9.0	0.9987
.3	0.9756	.8	0.9915	.6	0.9966	10.0	0.9990
.4	0.9775	.9	0.9920	.8	0.9969	11.0	0.9993
.5	0.9793					12.0	0.9994
.6	0.9808	5.0	0.9925	7.0	0.9972	13.0	0.9996
.7	0.9822	.1	0.9929	.2	0.9974	14.0	0.9996
.8	0.9835	.2	0.9933	.4	0.9976	15.0	0.9997
.9	0.9847	.3	0.9936	.6	0.9978	16.0	0.9998
		.4	0.9940	.8	0.9979	17.0	0.9998
4.0	0.9857	.5	0.9943			18.0	0.9998
.1	0.9867	.6	0.9946	8.0	0.9981	19.0	0.9999
.2	0.9876	.7	0.9948	.2	0.9982	20.0	0.9999
.3	0.9884	.8	0.9951	.4	0.9984	25.0	0.9999
.4	0.9891	.9	0.9953	.6	0.9985	30.0	1.0000

TABLE X - INFLUENCE VALUES FOR POINT BENEATH THE CENTER OF A

UNIFORMLY LOADED CIRCULAR AREA $\nu = 4$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0002	0.0008	0.0018	0.0032	0.0050	0.0072	0.0097	0.0127	0.0160
.1	0.0197	0.9238	0.0282	0.0330	0.0381	0.0435	0.0493	0.0554	0.0618	0.0685
.2	0.0754	0.9827	0.0902	0.0980	0.1060	0.1141	0.1226	0.1313	0.1401	0.1491
.3	0.1583	0.1677	0.1772	0.1868	0.1965	0.2064	0.2163	0.2263	0.2364	0.2466
.4	0.2568	0.2671	0.2774	0.2878	0.2981	0.3085	0.3188	0.3291	0.3395	0.3498
.5	0.3600	0.3702	0.3804	0.3905	0.4006	0.4106	0.4205	0.4303	0.4400	0.4498
.6	0.4594	0.4688	0.4782	0.4875	0.4967	0.5058	0.5148	0.5237	0.5324	0.5411
.7	0.5496	0.5580	0.5663	0.5744	0.5825	0.5904	0.5982	0.6059	0.6135	0.6209
.8	0.6282	0.6354	0.6425	0.6494	0.6563	0.6630	0.6696	0.6760	0.6824	0.6886
.9	0.6948	0.7008	0.7067	0.7125	0.7182	0.7237	0.7292	0.7346	0.7398	0.7450
1.0	0.7500	0.7550	0.7598	0.7646	0.7693	0.7738	0.7783	0.7827	0.7869	0.7920
.1	0.7953	0.7993	0.8033	0.8071	0.8109	0.8146	0.8183	0.8218	0.8253	0.8287
.2	0.8320	0.8353	0.8385	0.8417	0.8447	0.8477	0.8507	0.8543	0.8564	0.8191
.3	0.8618	0.8645	0.8670	0.8696	0.8721	0.8745	0.8769	0.8792	0.8815	0.8837
.4	0.8859	0.8880	0.8901	0.8922	0.8942	0.8961	0.8980	0.9002	0.9018	0.9036
.5	0.9053	0.9071	0.9088	0.9104	0.9120	0.9136	0.9150	0.9167	0.9182	0.9197
.6	0.9211	0.9225	0.9239	0.9252	0.9266	0.9278	0.9291	0.9304	0.9321	0.9328
.7	0.9339	0.9351	0.9362	0.9373	0.9384	0.9394	0.9405	0.9415	0.9425	0.9434
.8	0.9444	0.9453	0.9462	0.9471	0.9480	0.9489	0.9497	0.9506	0.9514	0.9522
.9	0.9510	0.9537	0.9545	0.9552	0.9559	0.9567	0.9573	0.9580	0.9587	0.9594
2.0	0.9600	0.9606	0.9613	0.9620	0.9625	0.9631	0.9636	0.9642	0.9648	0.9653
.1	0.9658	0.9564	0.9669	0.9674	0.9679	0.9684	0.9689	0.9693	0.9698	0.9702
.2	0.9707	0.9711	0.9716	0.9720	0.9724	0.9729	0.9733	0.9737	0.9741	0.9744
.3	0.9748	0.9749	0.9755	0.9759	0.9762	0.9766	0.9769	0.9772	0.9776	0.9779
.4	0.9782	0.9785	0.9788	0.9791	0.9794	0.9797	0.9799	0.9804	0.9805	0.9808
.5	0.9810	0.9812	0.9816	0.9818	0.9820	0.9823	0.9825	0.9826	0.9828	0.9830
.6	0.9833	0.9835	0.9838	0.9840	0.9843	0.9845	0.9846	0.9847	0.9850	0.9852
.7	0.9854	0.9856	0.9858	0.9859	0.9861	0.9863	0.9865	0.9867	0.9868	0.9870
.8	0.9872	0.9874	0.9875	0.9877	0.9878	0.9880	0.9881	0.9883	0.9884	0.9886
.9	0.9887	0.9888	0.9890	0.9891	0.9893	0.9894	0.9895	0.9896	0.9898	0.9899

m		m		m		m		m	
3.0	0.9900	4.0	0.9965	5.0	0.9985	6.0	0.9993	8.0	0.9998
.1	0.9911	.1	0.9969	.1	0.9986	.2	0.9994	.2	0.9998
.2	0.9921	.2	0.9971	.2	0.9987	.4	0.9994	.4	0.9998
.3	0.9929	.3	0.9974	.3	0.9988	.6	0.9995	.6	0.9998
.4	0.9937	.4	0.9976	.4	0.9989	.8	0.9996	.8	0.9998
.5	0.9943	.5	0.9978	.5	0.9990				
.6	0.9949	.6	0.9980	.6	0.9991	7.0	0.9996	9.0	0.9999
.7	0.9954	.7	0.9981	.7	0.9991	.2	0.9996	10.0	0.9999
.8	0.9958	.8	0.9983	.8	0.9992	.4	0.9997	11.0	1.0000
.9	0.9962	.9	0.9984	.9	0.9992	.6	0.9997		
						.8	0.9997		

TABLE XI - INFLUENCE VALUES FOR POINT BENEATH THE CENTER OF A

UNIFORMLY LOADED CIRCULAR AREA $\nu = 5$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0002	0.0010	0.0022	0.0040	0.0062	0.0089	0.0121	0.0158	0.0200
.1	0.0246	0.0296	0.0351	0.0410	0.0474	0.0541	0.0612	0.0687	0.0766	0.0848
.2	0.0934	0.1023	0.1115	0.1209	0.1307	0.1406	0.1509	0.1613	0.1720	0.1828
.3	0.1938	0.2050	0.2163	0.2277	0.2393	0.2509	0.2626	0.2744	0.2862	0.2981
.4	0.3100	0.3219	0.3338	0.3457	0.3575	0.3694	0.3811	0.3929	0.4045	0.4161
.5	0.4276	0.4390	0.4503	0.4615	0.4726	0.4835	0.4944	0.5051	0.5156	0.5261
.6	0.5364	0.5466	0.5566	0.5664	0.5761	0.5857	0.5950	0.6043	0.6134	0.6223
.7	0.6310	0.6396	0.6480	0.6563	0.6644	0.6723	0.6801	0.6877	0.6952	0.7025
.8	0.7097	0.7167	0.7235	0.7302	0.7368	0.7432	0.7495	0.7556	0.7616	0.7674
.9	0.7731	0.7787	0.7841	0.7895	0.7946	0.7997	0.8046	0.8095	0.8142	0.8188
1.0	0.8234	0.8276	0.8319	0.9360	0.8401	0.8445	0.8478	0.8516	0.8553	0.8591
.1	0.8623	0.8657	0.8690	0.8722	0.8753	0.8784	0.8813	0.8842	0.8871	0.8898
.2	0.8925	0.8951	0.8976	0.8001	0.9025	0.9050	0.9072	0.9096	0.9116	0.9137
.3	0.9158	0.9178	0.9197	0.9216	0.9235	0.9253	0.9271	0.9287	0.9304	0.9322
.4	0.9337	0.9352	0.9367	0.9382	0.9396	0.9410	0.9424	0.9438	0.9450	0.9463
.5	0.9475	0.9487	0.9499	0.9510	0.9521	0.9532	0.9542	0.9553	0.9563	0.9572
.6	0.9582	0.9591	0.9600	0.9609	0.9618	0.9626	0.9634	0.9642	0.9653	0.9658
.7	0.9665	0.9672	0.9679	0.9686	0.9693	0.9699	0.9706	0.9712	0.9718	0.9724
.8	0.9730	0.9736	0.9741	0.9747	0.9752	0.9757	0.9762	0.9767	0.9772	0.9776
.9	0.9781	0.9785	0.9790	0.9794	0.9798	0.9802	0.9806	0.9810	0.9814	0.9818
2.0	0.9821	0.9825	0.9828	0.9833	0.9835	0.9838	0.9841	0.9844	0.9847	0.9850
.1	0.9853	0.9856	0.9858	0.9861	0.9864	0.9867	0.9869	0.9872	0.9874	0.9876
.2	0.9879	0.9881	0.9883	0.9885	0.9887	0.9889	0.9892	0.9894	0.9896	0.9898
.3	0.9899	0.9901	0.9903	0.9905	0.9907	0.9908	0.9910	0.9912	0.9913	0.9915
.4	0.9916	0.9918	0.9919	0.9921	0.9922	0.9923	0.9924	0.9927	0.9927	0.9928
.5	0.9930	0.9930	0.9932	0.9933	0.9934	0.9935	0.9936	0.9937	0.9938	0.9939
.6	0.9940	0.9941	0.9942	0.9943	0.9944	0.9945	0.9946	0.9947	0.9948	0.9948
.7	0.9949	0.9950	0.9951	0.9952	0.9952	0.9953	0.9954	0.9955	0.9955	0.9956
.8	0.9957	0.9957	0.9958	0.9959	0.9960	0.9960	0.9961	0.9961	0.9962	0.9963
.9	0.9963	0.9964	0.9964	0.9965	0.9965	0.9966	0.9966	0.9967	0.9967	0.9968

m		m		m		m	
3.0	0.9968	4.0	0.9992	5.0	0.9997	6.0	0.9999
.1	0.9973	.1	0.9993	.1	0.9997	.2	0.9999
.2	0.9976	.2	0.9993	.2	0.9998	.4	0.9999
.3	0.9980	.3	0.9994	.3	0.9998	.6	0.9999
.4	0.9982	.4	0.9995	.4	0.9998	.8	0.9999
.5	0.9984	.5	0.9995	.5	0.9998		
.6	0.9986	.6	0.9996	.6	0.9998	7.0	0.9999
.7	0.9988	.7	0.9996	.7	0.9999	.1	1.0000
.8	0.9989	.8	0.9997	.8	0.9999		
.9	0.9991	.9	0.9997	.9	0.9999		

TABLE XII - INFLUENCE VALUES FOR POINT BENEATH THE CENTER OF A UNIFORMLY LOADED CIRCULAR AREA $\nu = 6$

m	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.0000	0.0003	0.0010	0.0027	0.0048	0.0075	0.0107	0.0146	0.0190	0.0239
.1	0.0294	0.0354	0.0420	0.0490	0.0566	0.0646	0.0730	0.0819	0.0912	0.0109
.2	0.1110	0.1214	0.1322	0.1433	0.1547	0.1663	0.1782	0.1903	0.2026	0.2151
.3	0.2278	0.2406	0.2536	0.2666	0.2798	0.2930	0.3062	0.3195	0.3328	0.3461
.4	0.3594	0.3726	0.3858	0.3989	0.4119	0.4249	0.4378	0.4505	0.4632	0.4757
.5	0.4880	0.5002	0.5123	0.5242	0.5359	0.5475	0.5588	0.5700	0.5810	0.5919
.6	0.6025	0.6129	0.6231	0.6332	0.6430	0.6526	0.6620	0.6713	0.6803	0.6891
.7	0.6977	0.7061	0.7144	0.7224	0.7302	0.7379	0.7453	0.7526	0.7597	0.7666
.8	0.7733	0.7799	0.7862	0.7924	0.7985	0.8043	0.8101	0.8156	0.8210	0.8263
.9	0.8314	0.8363	0.8411	0.8458	0.8504	0.8548	0.8591	0.8632	0.8673	0.8712
1.0	0.8753	0.8787	0.8823	0.8858	0.8891	0.8924	0.8956	0.8987	0.9017	0.9050
.1	0.9074	0.9101	0.9127	0.9153	0.9178	0.9202	0.9225	0.9248	0.9270	0.9291
.2	0.9312	0.9332	0.9351	0.9370	0.9388	0.9408	0.9423	0.9442	0.9456	0.9471
.3	0.9486	0.9501	0.9515	0.9529	0.9542	0.9555	0.9568	0.9580	0.9592	0.9603
.4	0.9614	0.9625	0.9636	0.9646	0.9656	0.9665	0.9674	0.9684	0.9692	0.9701
.5	0.9709	0.9717	0.9724	0.9732	0.9739	0.9746	0.9753	0.9760	0.9766	0.9772
.6	0.9778	0.9784	0.9790	0.9780	0.9801	0.9806	0.9811	0.9816	0.9823	0.9826
.7	0.9830	0.9835	0.9839	0.9843	0.9847	0.9851	0.9855	0.9858	0.9862	0.9865
.8	0.9869	0.9872	0.9875	0.9878	0.9882	0.9884	0.9887	0.9890	0.9893	0.9895
.9	0.9897	0.9900	0.9903	0.9905	0.9908	0.9910	0.9912	0.9914	0.9916	0.9918
2.0	0.9920	0.9922	0.9924	0.9926	0.9927	0.9929	0.9931	0.9932	0.9933	0.9935
.1	0.9937	0.9938	0.9940	0.9941	0.9942	0.9944	0.9945	0.9946	0.9948	0.9949
.2	0.9950	0.9951	0.9952	0.9953	0.9954	0.9955	0.9956	0.9957	0.9958	0.9959
.3	0.9960	0.9960	0.9962	0.9963	0.9963	0.9964	0.9965	0.9966	0.9966	0.9967
.4	0.9968	0.9968	0.9969	0.9970	0.9970	0.9971	0.9972	0.9972	0.9973	0.9973
.5	0.9974	0.9974	0.9975	0.9975	0.9975	0.9976	0.9977	0.9977	0.9978	0.9978
.6	0.9978	0.9979	0.9979	0.9980	0.9980	0.9981	0.9981	0.9981	0.9982	0.9982
.7	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984	0.9985	0.9985	0.9985
.8	0.9986	0.9986	0.9986	0.9986	0.9987	0.9987	0.9987	0.9987	0.9988	0.9988
.9	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990

m		m		m	
3.0	0.9990	4.0	0.9998	5.0	0.9999
.1	0.9992	.1	0.9998	.1	1.0000
.2	0.9993	.2	0.9999		
.3	0.9994	.3	0.9999		
.4	0.9995	.4	0.9999		
.5	0.9996	.5	0.9999		
.6	0.9996	.6	0.9999		
.7	0.9997	.7	0.9999		
.8	0.9997	.8	0.9999		
.9	0.9998	.9	0.9999		

CHAPTER VI

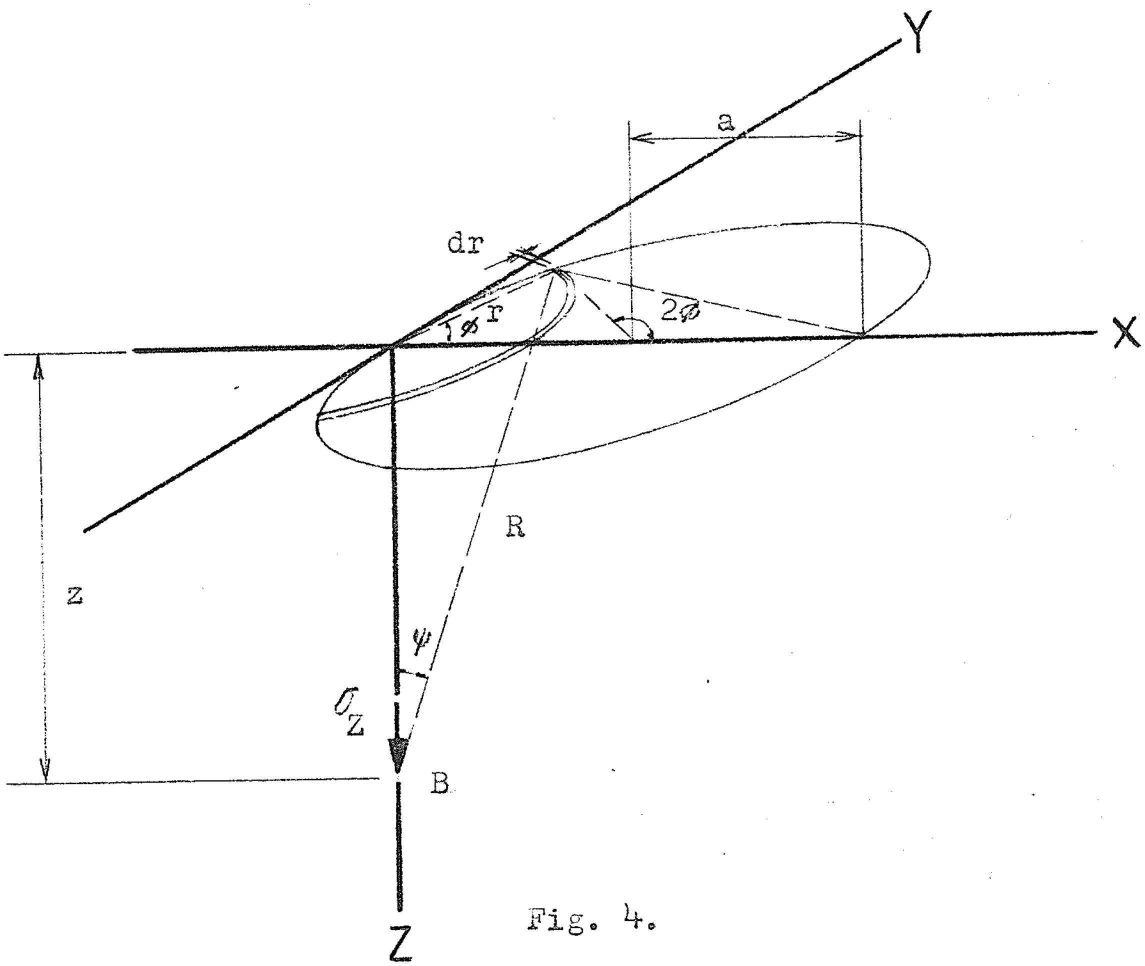

Fig. 4.

VERTICAL STRESS BENEATH THE CIRCUMFERENCE OF A CIRCULAR AREA UNIFORMLY LOADED

The vertical stress at "B" beneath the circumference of a circular area of radius "a", loaded uniformly with a load "q" per unit of area on the surface of a semi-infinite, isotropic, elastic and homogeneous solid may be found as follows:

$$d\sigma_z = \frac{\nu Q}{2\pi z^2} \cos^{(\nu+2)} \psi$$

in which, $Q = q\ dA$

From Fig. 4, $dA = 2r\phi\ dr$

and $r = 2a \cos \phi$

By differentiation: $dr = -2a \sin \phi\ d\phi$

Hence, $dA = -4ra\phi \sin \phi\ d\phi$

From Fig. 4, $\cos \psi = z/(z^2+r^2)^{1/2} \equiv z/(z^2 + 4a^2\cos^2\phi)^{1/2}$

$$\text{Thus, } \sigma_z = \int_0^{\pi/2} \frac{\nu q}{2\pi z^2} \frac{z^{(\nu+2)}}{(z^2 + 4a^2 \cos^2\phi)^{(\nu+2)/2}} (-4ar\phi \sin\phi\ d\phi)$$

Let $z/a = 2m_1 = m$

By integration and substitution:

$$\sigma_z = -\int_0^{\pi/2} \frac{\nu q m_1^{\nu}}{\pi} \frac{\phi \sin\phi \cos\phi}{(m_1^2 + \cos^2\phi)^{(\nu+2)/2}} d\phi$$

or $\sigma_z = qI_\nu$ ....(20)

in which, $$I_\nu = \frac{\nu\, m_1^\nu}{\pi} \int_0^{\pi/2} \frac{\phi \sin\phi \cos\phi}{(m_1^2 + \cos^2\phi)^{(\nu+2)/2}}\, d\phi \quad ..(21)$$

I_ν = influence value

For ν = 3, $$I_3 = \frac{3m_1^3}{\pi} \int_0^{\pi/2} \frac{\phi \sin\phi \cos\phi}{(m_1^2 + \cos^2\phi)^{5/2}}\, d\phi \quad ......(22)$$

For ν = 4, $$I_4 = \frac{4m_1^4}{\pi} \int_0^{\pi/2} \frac{\phi \sin\phi \cos\phi}{(m_1^2 + \cos^2\phi)^{3}}\, d\phi \quad ......(23)$$

For ν = 5, $$I_5 = \frac{5m_1^5}{\pi} \int_0^{\pi/2} \frac{\phi \sin\phi \cos\phi}{(m_1^2 + \cos^2\phi)^{7/2}}\, d\phi \quad ......(24)$$

For ν = 6, $$I_6 = \frac{6m_1^6}{\pi} \int_0^{\pi/2} \frac{\phi \sin\phi \cos\phi}{(m_1^2 + \cos^2\phi)^{4}}\, d\phi \quad ......(25)$$

The above integrations were evaluated by taking the area under the curve "$Y = F(\phi)$" between the limits of $\phi = 0$, to $\pi/2$ with the aid of computer. "$F(\phi)$" was taken as : $$\frac{\nu\, m_1^\nu}{\pi}\left[\frac{\phi \sin\phi \cos\phi}{(m_1^2 + \cos^2\phi)^{(\nu+2)/2}}\right]$$, for given values of "m_1".

In Tables XIII to XVI are given influence values for various values of the ratio, m = z/a.

To check the accuracy of the results, the number of strips used in evaluating the area under the curve " Y = F(Ø) " was varied from 18 to 180. The number of strips was increased until the influence value showed no change to four decimal places, when the number of strips was further increased. A further check on the accuracy was made by employing graphical integration for occasional verifications of computer results. It should be noted that 180 strips were required for each value of "m" from zero to 2.0; 36 strips for each value of "m" from 2.1 to 6.0 and 18 strips for each value of "m" from 6.1 to 250.0.

To find the value of a stress with the aid of the following tables, it is necessary to know the depth "z", and to divide this by the radius "a" to obtain the ratio "m". Then, the proper influence value is selected, and the stress computed from Eq. (20).

TABLE XIII - INFLUENCE VALUES FOR POINTS BENEATH THE CIRCUMFERENCE OF A CIRCULAR AREA UNIFORMLY LOADED $\sqrt{} = 3$

m	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	0.5000	0.4860	0.4683	0.4516	0.4347	0.4176	0.4004	0.3831	0.3659	0.3490
1.0	0.3324	0.3161	0.3002	0.2850	0.2703	0.2563	0.2429	0.2302	0.2182	0.2068
2.0	0.1960	0.1859	0.1764	0.1674	0.1590	0.1510	0.1436	0.1367	0.1301	0.1240
3.0	0.1182	0.1127	0.1079	0.1028	0.0983	0.0940	0.0900	0.0862	0.0827	0.0793
4.0	0.0761	0.0731	0.0703	0.0676	0.0650	0.0626	0.0603	0.0581	0.0561	0.0541
5.0	0.0522	0.0505	0.0488	0.0472	0.0756	0.0442	0.0428	0.0414	0.0402	0.0389
6.0	0.0378	0.0367	0.0356	0.0346	0.0336	0.0327	0.0318	0.0309	0.0301	0.0293
7.0	0.0285	0.0277	0.0270	0.0263	0.0257	0.0250	0.0244	0.0238	0.0233	0.0227
8.0	0.0222	0.0217	0.0212	0.0207	0.0202	0.0198	0.0193	0.0189	0.0185	0.0181
9.0	0.0177	0.0174	0.0170	0.0166	0.0163	0.0160	0.0157	0.0154	0.0151	0.0148
10.0	0.0145	0.0142	0.0139	0.0137	0.0134	0.0132	0.0129	0.0127	0.0125	0.0123
11.0	0.0120	0.0118	0.0116	0.0114	0.0112	0.0110	0.0109	0.0107	0.0105	0.0103
12.0	0.0102	0.0100	0.0098	0.0097	0.0095	0.0094	0.0092	0.0091	0.0090	0.0088
13.0	0.0087	0.0086	0.0084	0.0083	0.0082	0.0081	0.0080	0.0078	0.0077	0.0076
14.0	0.0075	0.0074	0.0073	0.0072	0.0071	0.0070	0.0069	0.0068	0.0067	0.0067

m		m		m		m	
15.0	0.0067	25.0	0.0024	65.0	0.0004	140.0	0.0001
15.5	0.0062	26.0	0.0022	70.0	0.0003	145.0	0.0001
16.0	0.0059	27.0	0.0020	75.0	0.0003	150.0	0.0001
16.5	0.0054	28.0	0.0019	80.0	0.0002	160.0	0.0001
17.0	0.0051	29.0	0.0018	85.0	0.0002	170.0	0.0001
17.5	0.0048	30.0	0.0017	90.0	0.0002	180.0	0.0000
18.0	0.0046	32.0	0.0015	95.0	0.0002		
18.5	0.0043	34.0	0.0013	100.0	0.0002		
19.0	0.0041	36.0	0.0012	105.0	0.0001		
19.5	0.0039	38.0	0.0010	110.0	0.0001		
20.0	0.0037	40.0	0.0009	115.0	0.0001		
21.0	0.0034	45.0	0.0008	120.0	0.0001		
22.0	0.0031	50.0	0.0006	125.0	0.0001		
23.0	0.0028	55.0	0.0005	130.0	0.0001		
24.0	0.0026	60.0	0.0004	135.0	0.0001		

TABLE XIV - INFLUENCE VALUES FOR POINTS BENEATH THE CIRCUMFERENCE OF A CIRCULAR AREA UNIFORMLY LOADED $\nu = 4$

m	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	0.5000	0.4901	0.4755	0.4624	0.4493	0.4359	0.4223	0.4085	0.3944	0.3802
1.0	0.3660	0.3517	0.3374	0.3234	0.3096	0.2961	0.2830	0.2702	0.2580	0.2462
2.0	0.2349	0.2241	0.2138	0.2040	0.1946	0.1858	0.1774	0.1695	0.1619	0.1548
3.0	0.1481	0.1417	0.1359	0.1299	0.1245	0.1194	0.1145	0.1099	0.1056	0.1015
4.0	0.0976	0.0939	0.0903	0.0870	0.0838	0.0808	0.0780	0.0752	0.0726	0.0702
5.0	0.0678	0.0656	0.0634	0.0614	0.0594	0.0576	0.0558	0.0541	0.0525	0.0509
6.0	0.0494	0.0480	0.0466	0.0453	0.0441	0.0428	0.0417	0.0405	0.0395	0.0384
7.0	0.0374	0.0365	0.0355	0.0347	0.0338	0.0330	0.0322	0.0314	0.0306	0.0299
8.0	0.0292	0.0286	0.0279	0.0273	0.0267	0.0261	0.0255	0.0250	0.0244	0.0239
9.0	0.0234	0.0229	0.0225	0.0220	0.0216	0.0211	0.0207	0.0203	0.0199	0.0195
10.0	0.0192	0.0188	0.0185	0.0181	0.0178	0.0175	0.0171	0.0168	0.0165	0.0162
11.0	0.0160	0.0157	0.0154	0.0151	0.0149	0.0146	0.0144	0.0142	0.0139	0.0137
12.0	0.0135	0.0133	0.0131	0.0129	0.0127	0.0125	0.0123	0.0121	0.0119	0.0117
13.0	0.0115	0.0114	0.0112	0.0110	0.0109	0.0107	0.0106	0.0104	0.0103	0.0101
14.0	0.0100	0.0098	0.0097	0.0096	0.0095	0.0093	0.0092	0.0091	0.0090	0.0088

m		m		m		m	
15.0	0.0087	25.0	0.0032	65.0	0.0005	140.0	0.0001
15.5	0.0082	26.0	0.0029	70.0	0.0004	145.0	0.0001
16.0	0.0077	27.0	0.0027	75.0	0.0004	150.0	0.0001
16.5	0.0072	28.0	0.0025	80.0	0.0003	160.0	0.0001
17.0	0.0068	29.0	0.0024	85.0	0.0003	170.0	0.0001
17.5	0.0064	30.0	0.0022	90.0	0.0002	180.0	0.0001
18.0	0.0061	32.0	0.0019	95.0	0.0002	190.0	0.0001
18.5	0.0058	34.0	0.0017	100.0	0.0002	200.0	0.0001
19.0	0.0055	36.0	0.0015	105.0	0.0002	210.0	0.0000
19.5	0.0052	38.0	0.0014	110.0	0.0002		
20.0	0.0050	40.0	0.0012	115.0	0.0002		
21.0	0.0045	45.0	0.0010	120.0	0.0001		
22.0	0.0041	50.0	0.0008	125.0	0.0001		
23.0	0.0038	55.0	0.0007	130.0	0.0001		
24.0	0.0034	60.0	0.0006	135.0	0.0001		

TABLE XV - INFLUENCE VALUES FOR POINTS BENEATH THE CIRCUMFERENCE OF A CIRCULAR AREA UNIFORMLY LOADED $\nu = 5$

m	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	0.5000	0.4927	0.4800	0.4684	0.4573	0.4462	0.4349	0.4233	0.4115	0.3995
1.0	0.3874	0.3750	0.3624	0.3499	0.3373	0.3248	0.3125	0.3003	0.2884	0.2769
2.0	0.2656	0.2547	0.2442	0.2341	0.2243	0.2150	0.2061	0.1976	0.1895	0.1817
3.0	0.1743	0.1672	0.1606	0.1541	0.1480	0.1423	0.1368	0.1315	0.1266	0.1218
4.0	0.1173	0.1131	0.1090	0.1051	0.1014	0.0979	0.0945	0.0913	0.0883	0.0853
5.0	0.0826	0.0799	0.0774	0.0749	0.0726	0.0704	0.0683	0.0662	0.0643	0.0624
6.0	0.0606	0.0589	0.0573	0.0557	0.0541	0.0527	0.0513	0.0499	0.0486	0.0473
7.0	0.0461	0.0450	0.0438	0.0427	0.0417	0.0407	0.0397	0.0388	0.0379	0.0370
8.0	0.0361	0.0353	0.0345	0.0338	0.0330	0.0323	0.0316	0.0309	0.0303	0.0296
9.0	0.0290	0.0284	0.0278	0.0273	0.0267	0.0262	0.0257	0.0252	0.0247	0.0242
10.0	0.0238	0.0233	0.0230	0.0225	0.0221	0.0217	0.0213	0.0209	0.0205	0.0202
11.0	0.0198	0.0195	0.0192	0.0188	0.0185	0.0182	0.0179	0.0176	0.0173	0.0170
12.0	0.0168	0.0165	0.0162	0.0160	0.0157	0.0155	0.0153	0.0150	0.0148	0.0146
13.0	0.0144	0.0142	0.0139	0.0137	0.0135	0.0134	0.0132	0.0130	0.0128	0.0126
14.0	0.0124	0.0123	0.0121	0.0119	0.0118	0.0116	0.0115	0.0113	0.0112	0.0110

m		m		m		m	
15.0	0.0109	25.0	0.0040	65.0	0.0006	140.0	0.0001
15.5	0.0102	26.0	0.0037	70.0	0.0005	145.0	0.0001
16.0	0.0096	27.0	0.0034	75.0	0.0004	150.0	0.0001
16.5	0.0090	28.0	0.0032	80.0	0.0004	160.0	0.0001
17.0	0.0085	29.0	0.0030	85.0	0.0003	170.0	0.0001
17.5	0.0080	30.0	0.0028	90.0	0.0003	180.0	0.0001
18.0	0.0076	32.0	0.0024	95.0	0.0003	190.0	0.0001
18.5	0.0072	34.0	0.0022	100.0	0.0003	200.0	0.0001
19.0	0.0068	36.0	0.0019	105.0	0.0002	210.0	0.0001
19.5	0.0065	38.0	0.0017	110.0	0.0002	220.0	0.0001
20.0	0.0062	40.0	0.0016	115.0	0.0002	230.0	0.0000
21.0	0.0056	45.0	0.0013	120.0	0.0002		
22.0	0.0051	50.0	0.0010	125.0	0.0002		
23.0	0.0047	55.0	0.0008	130.0	0.0001		
24.0	0.0043	60.0	0.0007	135.0	0.0001		

TABLE XVI - INFLUENCE VALUES FOR POINTS BENEATH THE CIRCUMFERENCE OF A CIRCULAR AREA UNIFORMLY LOADED $\nu = 6$

m	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	0.5000	0.4946	0.4822	0.4722	0.4625	0.4528	0.4429	0.4329	0.4227	0.4123
1.0	0.4019	0.3910	0.3799	0.3686	0.3574	0.3460	0.3347	0.3233	0.3121	0.3010
2.0	0.2901	0.2795	0.2691	0.2590	0.2492	0.2397	0.2305	0.2218	0.2133	0.2051
3.0	0.1973	0.1898	0.1828	0.1758	0.1692	0.1630	0.1570	0.1512	0.1458	0.1406
4.0	0.1356	0.1308	0.1263	0.1220	0.1178	0.1139	0.1101	0.1065	0.1030	0.0997
5.0	0.0965	0.0935	0.0906	0.0878	0.0852	0.0826	0.0802	0.0778	0.0756	0.0734
6.0	0.0713	0.0694	0.0675	0.0657	0.0639	0.0622	0.0606	0.0590	0.0575	0.0560
7.0	0.0546	0.0532	0.0519	0.0506	0.0494	0.0482	0.0471	0.0460	0.0450	0.0439
8.0	0.0429	0.0419	0.0410	0.0401	0.0392	0.0384	0.0376	0.0368	0.0360	0.0352
9.0	0.0345	0.0338	0.0331	0.0325	0.0318	0.0312	0.0306	0.0300	0.0294	0.0289
10.0	0.0283	0.0278	0.0273	0.0268	0.0263	0.0258	0.0254	0.0249	0.0245	0.0241
11.0	0.0236	0.0232	0.0229	0.0225	0.0221	0.0217	0.0214	0.0210	0.0207	0.0203
12.0	0.0200	0.0197	0.0194	0.0191	0.0188	0.0185	0.0182	0.0180	0.0177	0.0174
13.0	0.0172	0.0169	0.0167	0.0164	0.0162	0.0160	0.0157	0.0155	0.0153	0.0151
14.0	0.0149	0.0147	0.0145	0.0143	0.0141	0.0139	0.0137	0.0135	0.0133	0.0132

m		m		m		m	
15.0	0.0130	25.0	0.0048	65.0	0.0007	140.0	0.0002
15.5	0.0122	26.0	0.0044	70.0	0.0006	145.0	0.0001
16.0	0.0115	27.0	0.0041	75.0	0.0005	150.0	0.0001
16.5	0.0108	28.0	0.0038	80.0	0.0005	160.0	0.0001
17.0	0.0102	29.0	0.0035	85.0	0.0004	170.0	0.0001
17.5	0.0096	30.0	0.0033	90.0	0.0004	180.0	0.0001
18.0	0.0091	32.0	0.0029	95.0	0.0003	190.0	0.0001
18.5	0.0086	34.0	0.0026	100.0	0.0003	200.0	0.0001
19.0	0.0082	36.0	0.0023	105.0	0.0003	210.0	0.0001
19.5	0.0078	38.0	0.0021	110.0	0.0002	220.0	0.0001
20.0	0.0074	40.0	0.0019	115.0	0.0002	230.0	0.0001
21.0	0.0067	45.0	0.0015	120.0	0.0002	240.0	0.0001
22.0	0.0061	50.0	0.0012	125.0	0.0002	250.0	0.0000
23.0	0.0056	55.0	0.0010	130.0	0.0002		
24.0	0.0052	60.0	0.0008	135.0	0.0002		

CHAPTER VII

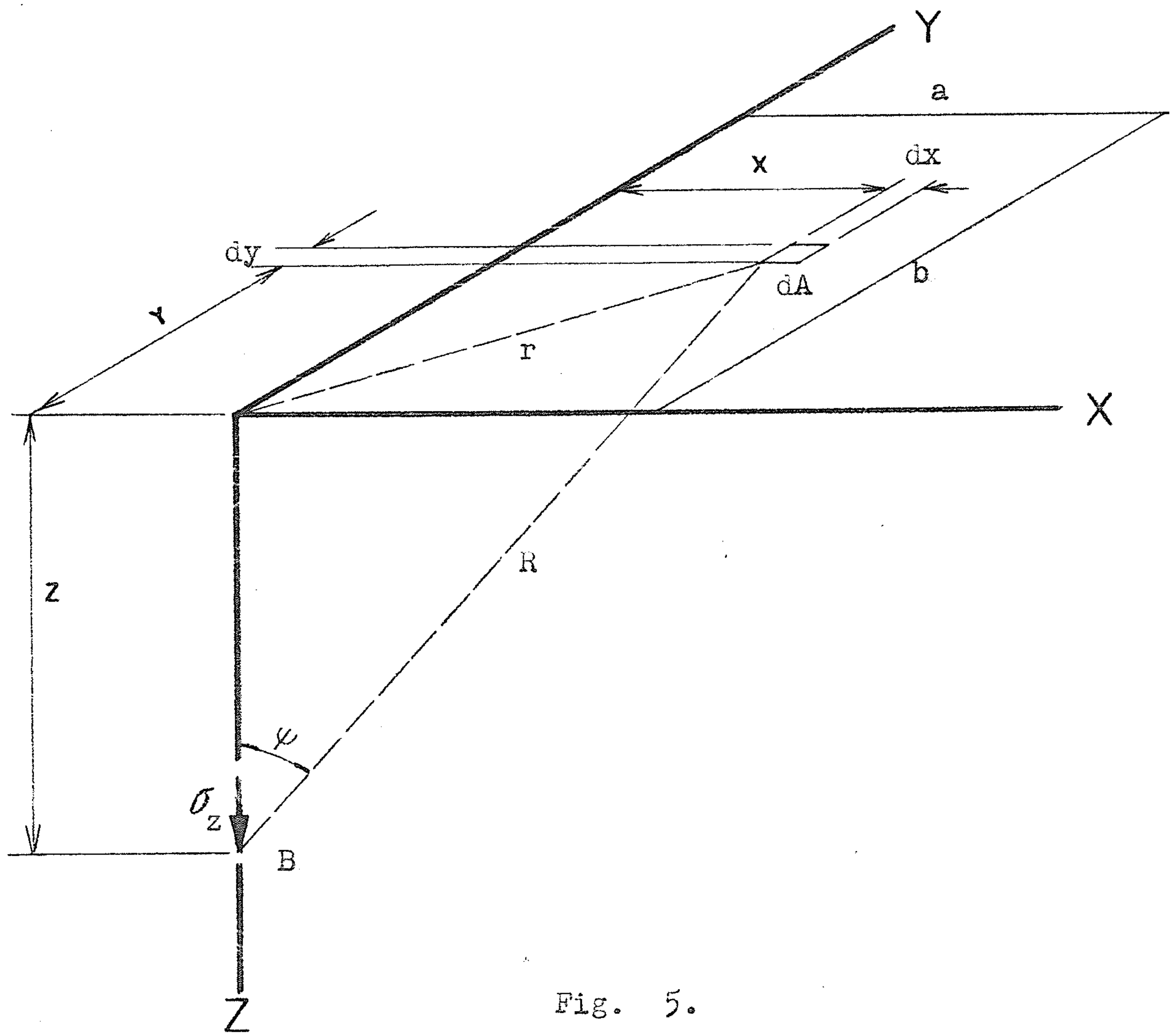

Fig. 5.

VERTICAL

STRESS BENEATH ONE OF THE CORNERS OF A

RECTANGULAR AREA UNIFORMLY LOADED

The vertical stress at "B" beneath a corner of a rectangular area, which carries a load "q" per unit of area on the surface of a semi-infinite, elastic, isotropic and homogeneous solid may be found by considering the relationship:

$$d\sigma_z = \frac{\nu Q}{2\pi z^2} \cos^{(\nu+2)}\psi$$

wherein, $Q = q\ dA$

and $dA = dy\ dx$

From Fig. 5, $\cos\psi = \frac{z}{(x^2 + y^2 + z^2)^{1/2}}$

Hence, $$\sigma_z = \int_0^{x=a}\int_0^{y=b} \frac{\nu q}{2\pi z^2} \frac{z^{(\nu+2)}}{(x^2 + y^2 + z^2)^{(\nu+2)/2}}\, dy\ dx$$

............(26)

For $\nu = 3$,
Eq. (26) becomes $$\sigma_z = \int_0^{x=a}\int_0^{y=b} \frac{3qz^3}{2\pi} \frac{dy\ dx}{(x^2 + y^2 + z^2)^{5/2}}$$

Integrating with respect to x and simplifying,

$$\sigma_z = \frac{qz^3}{2\pi}\int_0^{y=b} \frac{a(2a^2 + 3y^2 + 3z^2)}{(y^2 + z^2)^2(a^2 + y^2 + z^2)^{3/2}}\, dy \quad ..(27)$$

Let $a = mz$ and $y = nz$

Differentiating, $dy = z\,dn$

Then,

Eq. (27) becomes $$\sigma_z = \frac{q}{2\pi}\int_0^{b/z} \frac{m(2m^2+3n^2+3)}{(n^2+1)^2(m^2+n^2+1)^{3/2}}\,dn$$

$$= q\,I_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(28)$$

wherein, $$I_3 = \frac{1}{2\pi}\int_0^{b/z} \frac{m(2m^2+3n^2+3)\,dn}{(n^2+1)^2(m^2+n^2+1)^{3/2}} \quad \ldots(29)$$

The influence values,"I_3" , for given values of "m"and"n"can be determined from Eq. (29) by using the computer to find the area under the curve:

$Y = F(m,n) = m(2m^2+3n^2+3)/(n^2+1)^2(m^2+n^2+1)^{3/2}$ and divide this by 2π. In Table XVII are given influence values for various values of the ratios, $m = a/z$ and $n = b/z$.

Similarily, expressions were derived for

$$\nu = 4, \qquad \sigma_z = q\, I_4 \qquad \ldots\ldots\ldots\ldots(30)$$

wherein,

$$I_4 = \frac{1}{4\pi}\int_0^{b/z}\left[\frac{m(3m^2+5n^2+5)}{(n^2+1)^2(m^2+n^2+1)^2} + \frac{3}{(n^2+1)^{5/2}}\,\mathrm{Tan}^{-1}\frac{m}{(n^2+1)^{1/2}}\right] dn$$

$$\ldots\ldots\ldots\ldots(31)$$

for

$$\nu = 5; \qquad \sigma_z = q\, I_5 \qquad \ldots\ldots\ldots\ldots(32)$$

wherein,

$$I_5 = \frac{1}{6\pi}\int_0^{b/z}\frac{3m^5 + 5m(m^2+n^2+1)(m^2+3n^2+3)}{(n^2+1)^3(m^2+n^2+1)^{5/2}}\, dn$$

$$\ldots\ldots\ldots\ldots(33)$$

and for

$$\nu = 6; \qquad \sigma_z = q\, I_6 \qquad \ldots\ldots\ldots\ldots(34)$$

wherein,

$$I_6 = \frac{1}{16\pi}\int_0^{b/z}\left[\frac{8m(n^2+1)^2 + 5m(m^2+n^2+1)(3m^2+5n^2+5)}{(n^2+1)^3(m^2+n^2+1)^3} + \frac{15}{(n^2+1)^{7/2}}\,\mathrm{Tan}^{-1}\frac{m}{(n^2+1)^{1/2}}\right] dn$$

$$\ldots\ldots\ldots\ldots(35)$$

The influence values "I_4", "I_5" and "I_6" were also computed with the aid of computer and tabulated in Tables XVIII, XIX and XX respectively.

To find the value of the stress at the point "B" (Fig. 5) beneath the corner of the uniformly loaded rectangular area "a" by "b", simply determine the ratios, "m =a/z" and "n=b/z" and select the proper influence value corresponding to these ratios. The stress is equal to "I_v" times "q".

It will be noted that the co-ordinate system in Fig. 5 is so chosen that the origin coincides with the corner of the uniformly loaded rectangular area. If, it is desired to determine the stress at some point, which is not beneath a corner, as for example, point "B" in Fig. 6, the computation procedure is as follows:

(1) Select the influence values I_I, I_{II}, I_{III} and I_{IV} for the stress due to a uniformly distributed load applied to the rectangular areas I, II, III and IV respectively.

(2) Assuming the theory of superposition applies, the vertical stress at "B" can be computed from the following equation:

$$\sigma_z = q(I_I + I_{II} + I_{III} + I_{IV})$$

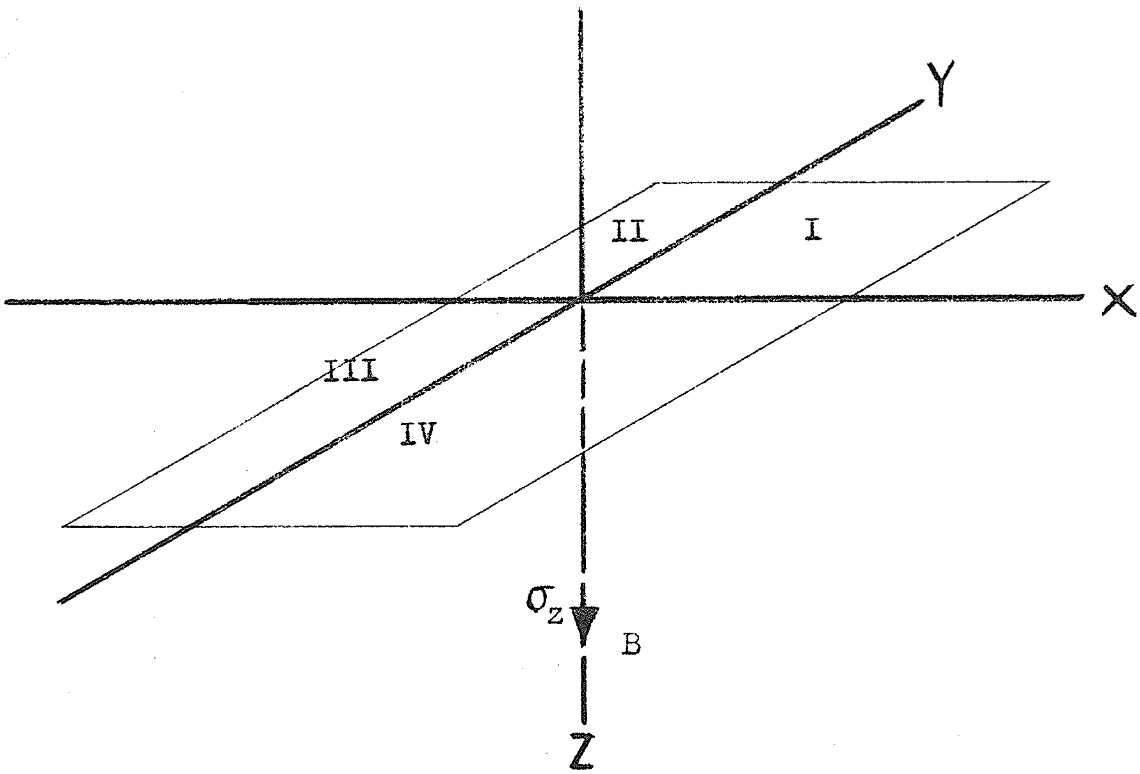

Fig. 6.

TABLE XVII - INFLUENCE VALUES FOR RECTANGULAR AREA UNIFORMLY LOADED $\nu = 3$

m \ n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00469	0.00916	0.01323	0.01678	0.01977	0.02223	0.02420	0.02575	0.02698	0.02793	0.02926	0.03007
0.2	0.00916	0.01790	0.02585	0.03280	0.03866	0.04347	0.04735	0.05042	0.05283	0.05471	0.05733	0.05894
0.3	0.01323	0.02585	0.03735	0.04741	0.05593	0.06293	0.06859	0.07308	0.07661	0.07938	0.08324	0.08561
0.4	0.01678	0.03280	0.04741	0.06024	0.07111	0.08009	0.08735	0.09314	0.09771	0.10130	0.10631	0.10941
0.5	0.01977	0.03866	0.05593	0.07111	0.08403	0.09472	0.10341	0.11035	0.11585	0.12018	0.12626	0.13004
0.6	0.02223	0.04347	0.06293	0.08009	0.09472	0.10689	0.11679	0.12474	0.13106	0.13605	0.14310	0.14749
0.7	0.02420	0.04735	0.06859	0.08735	0.10341	0.11679	0.12773	0.13654	0.14357	0.14914	0.15704	0.16200
0.8	0.02575	0.05042	0.07308	0.09314	0.11035	0.12474	0.13654	0.14607	0.15371	0.15978	0.16843	0.17390
0.9	0.02698	0.05283	0.07661	0.09771	0.11585	0.13106	0.14357	0.15371	0.16185	0.16836	0.17766	0.18357
1.0	0.02793	0.05471	0.07938	0.10130	0.12018	0.13605	0.14914	0.15978	0.16836	0.17522	0.18509	0.19139
1.2	0.02926	0.05733	0.08324	0.10631	0.12626	0.14310	0.15704	0.16843	0.17766	0.18509	0.19584	0.20278
1.4	0.03007	0.05894	0.08561	0.10941	0.13004	0.14749	0.16200	0.17390	0.18357	0.19139	0.20278	0.21020
1.6	0.03057	0.05994	0.08709	0.11136	0.13242	0.15028	0.16516	0.17739	0.18737	0.19547	0.20732	0.21509
1.8	0.03090	0.06059	0.08805	0.11261	0.13395	0.15208	0.16721	0.17967	0.18986	0.19815	0.21032	0.21836
2.0	0.03111	0.06101	0.08867	0.11343	0.13496	0.15327	0.16856	0.18119	0.19152	0.19994	0.21235	0.22058
2.5	0.03139	0.06155	0.08949	0.11450	0.13629	0.15484	0.17037	0.18321	0.19375	0.20236	0.21512	0.22364
3.0	0.03150	0.06178	0.08983	0.11496	0.13685	0.15550	0.17113	0.18407	0.19471	0.20341	0.21633	0.22499
4.0	0.03158	0.06194	0.09007	0.11528	0.13725	0.15598	0.17169	0.18470	0.19540	0.20417	0.21722	0.22601
5.0	0.03161	0.06199	0.09014	0.11537	0.13737	0.15612	0.17185	0.18489	0.19561	0.20440	0.21749	0.22632
6.0	0.03161	0.06201	0.09017	0.11541	0.13742	0.15618	0.17191	0.18496	0.19569	0.20449	0.21760	0.22644
8.0	0.03162	0.06202	0.09019	0.11543	0.13744	0.15621	0.17196	0.18501	0.19575	0.20455	0.21767	0.22652
10.0	0.03162	0.06203	0.09019	0.11544	0.13745	0.15622	0.17197	0.18502	0.19576	0.20457	0.21769	0.22654

TABLE XVII - CONTINUED

m \ n	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0
0.1	0.03057	0.03090	0.03111	0.03139	0.03150	0.03158	0.03161	0.03161	0.03162	0.03162
0.2	0.05994	0.06059	0.06101	0.06155	0.06178	0.06194	0.06199	0.06201	0.06202	0.06203
0.3	0.08709	0.08805	0.08867	0.08949	0.08983	0.09007	0.09014	0.09017	0.09019	0.09019
0.4	0.11136	0.11261	0.11343	0.11450	0.11496	0.11528	0.11537	0.11541	0.11543	0.11544
0.5	0.13242	0.13395	0.13496	0.13629	0.13685	0.13725	0.13737	0.13742	0.13744	0.13745
0.6	0.15028	0.15208	0.15327	0.15484	0.15550	0.15598	0.15612	0.15618	0.15621	0.15622
0.7	0.16516	0.16721	0.16856	0.17037	0.17113	0.17169	0.17185	0.17191	0.17196	0.17197
0.8	0.17739	0.17967	0.18119	0.18323	0.18407	0.18470	0.18489	0.18496	0.18508	0.18502
0.9	0.18737	0.18986	0.19152	0.19375	0.19471	0.19540	0.19561	0.19569	0.19575	0.19576
1.0	0.19547	0.19815	0.19994	0.20236	0.20341	0.20417	0.20440	0.20449	0.20455	0.20457
1.2	0.20732	0.21032	0.21235	0.21512	0.21633	0.21722	0.21749	0.21760	0.21767	0.21769
1.4	0.21509	0.21836	0.22058	0.22364	0.22499	0.22601	0.22632	0.22644	0.22652	0.22654
1.6	0.22025	0.22372	0.22610	0.22940	0.23088	0.23200	0.23235	0.23249	0.23258	0.23261
1.8	0.22372	0.22736	0.22986	0.23336	0.23495	0.23617	0.23656	0.23671	0.23681	0.23684
2.0	0.22610	0.22986	0.23247	0.23614	0.23782	0.23912	0.23954	0.23970	0.23981	0.23985
2.5	0.22940	0.23336	0.23614	0.24010	0.24196	0.24343	0.24392	0.24412	0.24425	0.24429
3.0	0.23088	0.23495	0.23782	0.24196	0.24394	0.24554	0.24608	0.24630	0.24646	0.24650
4.0	0.23200	0.23617	0.23912	0.24343	0.24554	0.24729	0.24791	0.24817	0.24836	0.24841
5.0	0.23235	0.23656	0.23954	0.24392	0.24608	0.24791	0.24857	0.24886	0.24907	0.24914
6.0	0.23249	0.23671	0.23970	0.24412	0.24630	0.24817	0.24886	0.24916	0.24939	0.24946
8.0	0.23258	0.23681	0.23981	0.24425	0.24646	0.24836	0.24907	0.24939	0.24964	0.24972
10.0	0.23261	0.23684	0.23985	0.24429	0.24650	0.24841	0.24914	0.24946	0.24972	0.24981

TABLE XVIII - INFLUENCE VALUES FOR RECTANGULAR AREA UNIFORMLY LOADED $\nu = 4$

m \ n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00624	0.01212	0.01738	0.02184	0.02547	0.02834	0.03054	0.03220	0.03344	0.03437	0.03556	0.03622
0.2	0.01212	0.02357	0.03379	0.04248	0.04958	0.05518	0.05949	0.06275	0.06520	0.06702	0.06937	0.07068
0.3	0.01738	0.03379	0.04847	0.06098	0.07122	0.07932	0.08558	0.09034	0.09390	0.09656	0.10002	0.10194
0.4	0.02184	0.04248	0.06098	0.07679	0.08977	0.10008	0.10806	0.11415	0.11873	0.12216	0.12663	0.12913
0.5	0.02547	0.04958	0.07122	0.08977	0.10504	0.11722	0.12669	0.13393	0.13941	0.14352	0.14891	0.15195
0.6	0.02834	0.05518	0.07932	0.10008	0.11722	0.13093	0.14165	0.14988	0.15610	0.16083	0.16703	0.17055
0.7	0.03054	0.05949	0.08558	0.10806	0.12669	0.14165	0.15338	0.16243	0.16932	0.17453	0.18144	0.18538
0.8	0.03220	0.06275	0.09034	0.11415	0.13393	0.14988	0.16243	0.17213	0.17956	0.18520	0.19271	0.19703
0.9	0.03344	0.06520	0.09390	0.11873	0.13941	0.15612	0.16932	0.17956	0.18742	0.19342	0.20143	0.20608
1.0	0.03437	0.06702	0.09656	0.12216	0.14352	0.16083	0.17453	0.18520	0.19342	0.19970	0.20814	0.21306
1.2	0.03556	0.06937	0.10002	0.12663	0.14891	0.16703	0.18144	0.19271	0.20143	0.20814	0.21722	0.22258
1.4	0.03622	0.07068	0.10194	0.12913	0.15195	0.17055	0.18538	0.19703	0.20608	0.21306	0.22258	0.22824
1.6	0.03659	0.07642	0.10304	0.13057	0.15369	0.17258	0.18768	0.19955	0.20881	0.21597	0.22578	0.23165
1.8	0.03682	0.07186	0.10368	0.13141	0.15472	0.17379	0.18904	0.20107	0.21045	0.21773	0.22773	0.23375
2.0	0.03695	0.07212	0.10707	0.13192	0.15535	0.17452	0.18987	0.20199	0.21146	0.21882	0.22895	0.23507
2.5	0.03710	0.07242	0.10452	0.13251	0.15607	0.17537	0.19085	0.20308	0.21266	0.22011	0.23042	0.23668
3.0	0.03715	0.07253	0.10468	0.13271	0.15633	0.17568	0.19120	0.20348	0.21309	0.22059	0.23096	0.23729
4.0	0.03718	0.07259	0.10477	0.13284	0.15648	0.17586	0.19141	0.20371	0.21335	0.22088	0.23130	0.23766
5.0	0.03719	0.07260	0.10479	0.13287	0.15652	0.17590	0.19146	0.20377	0.21342	0.22094	0.23138	0.23776
6.0	0.03719	0.07261	0.10480	0.13287	0.15653	0.17592	0.19148	0.20379	0.21344	0.22097	0.23140	0.23778
8.0	0.03720	0.07261	0.10480	0.13288	0.15654	0.17592	0.19148	0.20380	0.21345	0.22098	0.23142	0.23780
10.0	0.03720	0.07261	0.10480	0.13288	0.15654	0.17592	0.19149	0.20380	0.21345	0.22098	0.23142	0.23780

TABLE XVIII - CONTINUED

m \ n	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0
0.1	0.03659	0.03682	0.03695	0.03710	0.03715	0.03718	0.03719	0.03719	0.03720	0.03720
0.2	0.07642	0.07186	0.07212	0.07242	0.07253	0.07259	0.07260	0.07261	0.07261	0.07261
0.3	0.10304	0.10368	0.10707	0.10452	0.10468	0.10477	0.10479	0.10480	0.10480	0.10480
0.4	0.13057	0.13141	0.13192	0.13251	0.13271	0.13284	0.13287	0.13287	0.13288	0.13288
0.5	0.15369	0.15472	0.15535	0.15607	0.15633	0.15648	0.15652	0.15653	0.15654	0.15654
0.6	0.17258	0.17379	0.17452	0.17537	0.17568	0.17586	0.17590	0.17592	0.17592	0.17592
0.7	0.18768	0.18904	0.18987	0.19085	0.19120	0.19141	0.19146	0.19148	0.19148	0.19149
0.8	0.19955	0.20107	0.20199	0.20308	0.20348	0.20371	0.20377	0.20379	0.20380	0.20380
0.9	0.20881	0.21045	0.21146	0.21266	0.21309	0.21335	0.21342	0.21344	0.21345	0.21345
1.0	0.21597	0.21773	0.21882	0.22011	0.22059	0.22088	0.22094	0.22097	0.22098	0.22098
1.2	0.22578	0.22773	0.22895	0.23042	0.23096	0.23130	0.23138	0.23140	0.23142	0.23142
1.4	0.23165	0.23375	0.23507	0.23668	0.23729	0.23766	0.23776	0.23778	0.23780	0.23780
1.6	0.23522	0.23743	0.23883	0.24055	0.24121	0.24162	0.24173	0.24176	0.24178	0.24178
1.8	0.23743	0.23972	0.24118	0.24300	0.24370	0.24414	0.24426	0.24429	0.24431	0.24432
2.0	0.23883	0.24118	0.24269	0.24458	0.24531	0.24579	0.24591	0.24595	0.24597	0.24597
2.5	0.24055	0.24300	0.24458	0.24658	0.24839	0.24791	0.24805	0.24810	0.24813	0.24813
3.0	0.24121	0.24370	0.24531	0.24739	0.24833	0.24879	0.24894	0.24900	0.24903	0.24903
4.0	0.24162	0.24414	0.24579	0.24791	0.24879	0.24940	0.24957	0.24963	0.24966	0.24967
5.0	0.24173	0.24426	0.24591	0.24805	0.24894	0.24957	0.24975	0.24981	0.24985	0.24986
6.0	0.24176	0.24429	0.24595	0.24810	0.24900	0.24963	0.24981	0.24988	0.24992	0.24993
8.0	0.24178	0.24431	0.24597	0.24813	0.24903	0.24966	0.24985	0.24992	0.24996	0.24998
10.0	0.24178	0.24432	0.24597	0.24813	0.24903	0.24967	0.24986	0.24993	0.24998	0.24999

TABLE XIX - INFLUENCE VALUES FOR RECTANGULAR AREA UNIFORMLY LOADED $\nu = 5$

m \ n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00777	0.01503	0.02139	0.02665	0.03079	0.03392	0.03623	0.03789	0.03908	0.03991	0.04092	0.04143
0.2	0.01503	0.02909	0.04141	0.05161	0.05965	0.06576	0.07027	0.07352	0.07584	0.07749	0.07947	0.08057
0.3	0.02139	0.04141	0.05898	0.07357	0.08511	0.09390	0.10041	0.10512	0.10849	0.11089	0.11379	0.11525
0.4	0.02665	0.05161	0.07357	0.09185	0.10637	0.11746	0.12571	0.13170	0.13601	0.13908	0.14282	0.14472
0.5	0.03079	0.05965	0.08511	0.10637	0.12330	0.13631	0.14601	0.15310	0.15821	0.16187	0.16635	0.16865
0.6	0.03392	0.06576	0.09390	0.11746	0.13631	0.15083	0.16171	0.16971	0.17550	0.17966	0.18478	0.18742
0.7	0.03623	0.07027	0.10041	0.12571	0.14601	0.16171	0.17353	0.18225	0.18859	0.19317	0.19884	0.20179
0.8	0.03789	0.07352	0.10512	0.13170	0.15310	0.16971	0.18225	0.19154	0.19832	0.20324	0.20936	0.21257
0.9	0.03908	0.07584	0.10849	0.13601	0.15821	0.17550	0.18859	0.19832	0.20546	0.21065	0.21714	0.22057
1.0	0.03991	0.07749	0.11089	0.13908	0.16187	0.17966	0.19317	0.20324	0.21065	0.21606	0.22285	0.22646
1.2	0.04092	0.07947	0.11379	0.14282	0.16635	0.18478	0.19884	0.20936	0.21714	0.22285	0.23008	0.23397
1.4	0.04143	0.08047	0.11525	0.14472	0.16865	0.18742	0.20179	0.21257	0.22057	0.22646	0.23397	0.23804
1.6	0.04169	0.08099	0.11602	0.14571	0.16985	0.18882	0.20335	0.21428	0.22241	0.22841	0.23609	0.24028
1.8	0.04183	0.08127	0.11642	0.14624	0.17050	0.18957	0.20420	0.21522	0.22343	0.22950	0.23728	0.24155
2.0	0.04191	0.08142	0.11665	0.14654	0.17086	0.19000	0.20468	0.21575	0.22400	0.23011	0.23797	0.24228
2.5	0.04199	0.08158	0.11688	0.14684	0.17123	0.19044	0.20518	0.21631	0.22461	0.23077	0.23870	0.24309
3.0	0.04201	0.08162	0.11695	0.14693	0.17134	0.19057	0.20533	0.21648	0.22480	0.23097	0.23893	0.24334
4.0	0.04202	0.08164	0.11698	0.14698	0.17140	0.19063	0.20541	0.21656	0.22489	0.23107	0.23905	0.24347
5.0	0.04202	0.08165	0.11699	0.14698	0.17141	0.19064	0.20542	0.21658	0.22491	0.23109	0.23907	0.24350
6.0	0.04202	0.08165	0.11699	0.14699	0.17141	0.19065	0.20542	0.21658	0.22491	0.23110	0.23908	0.24350
8.0	0.04203	0.08165	0.11699	0.14699	0.17141	0.19065	0.20542	0.21658	0.22491	0.23110	0.23908	0.24351
10.0	0.04203	0.08165	0.11699	0.14699	0.17141	0.19065	0.20542	0.21658	0.22491	0.23110	0.23908	0.24351

TABLE XIX - CONTINUED

m \ n	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0
0.1	0.04169	0.04183	0.04191	0.04199	0.04201	0.04202	0.04202	0.04202	0.04203	0.04203
0.2	0.08099	0.08127	0.08142	0.08158	0.08162	0.08164	0.08165	0.08165	0.08165	0.08165
0.3	0.11602	0.11642	0.11665	0.11688	0.11695	0.11698	0.11699	0.11699	0.11699	0.11699
0.4	0.14571	0.14624	0.14654	0.14684	0.14693	0.14698	0.14698	0.14699	0.14699	0.14699
0.5	0.16985	0.17050	0.17086	0.17123	0.17134	0.17140	0.17141	0.17141	0.17141	0.17141
0.6	0.18882	0.18957	0.19000	0.19044	0.19057	0.19063	0.19064	0.19065	0.19065	0.19065
0.7	0.20335	0.20420	0.20468	0.20518	0.20533	0.20541	0.20542	0.20542	0.20542	0.20542
0.8	0.21428	0.21522	0.21575	0.21631	0.21648	0.21656	0.21658	0.21658	0.21658	0.21658
0.9	0.22271	0.22343	0.22400	0.22461	0.22480	0.22489	0.22491	0.22491	0.22491	0.22491
1.0	0.22841	0.22950	0.23011	0.23077	0.23097	0.23107	0.23109	0.23110	0.23110	0.23110
1.2	0.23609	0.23728	0.23797	0.23870	0.23893	0.23905	0.23907	0.23908	0.23908	0.23908
1.4	0.24028	0.24155	0.24228	0.24309	0.24334	0.24347	0.24350	0.24350	0.24351	0.24351
1.6	0.24261	0.24393	0.24471	0.24556	0.24583	0.24597	0.24600	0.24601	0.24601	0.24601
1.8	0.24393	0.24529	0.24610	0.24699	0.24728	0.24743	0.24746	0.24747	0.24747	0.24747
2.0	0.24471	0.24610	0.24692	0.24784	0.24814	0.24830	0.24834	0.24834	0.24835	0.24835
2.5	0.24556	0.24699	0.24784	0.24880	0.24913	0.24931	0.24934	0.24935	0.24936	0.24936
3.0	0.24583	0.24728	0.24814	0.24913	0.24946	0.24965	0.24969	0.24971	0.24971	0.24971
4.0	0.24597	0.24743	0.24830	0.24931	0.24965	0.24985	0.24990	0.24991	0.24992	0.24992
5.0	0.24600	0.24746	0.24834	0.24934	0.24969	0.24990	0.24994	0.24996	0.24996	0.24996
6.0	0.24601	0.24747	0.24834	0.24935	0.24971	0.24991	0.24996	0.24997	0.24998	0.24998
8.0	0.24601	0.24747	0.24835	0.24936	0.24971	0.24992	0.24996	0.24998	0.24998	0.24998
10.0	0.24601	0.24747	0.24835	0.24936	0.24971	0.24992	0.24996	0.24998	0.24998	0.24998

TABLE XX - INFLUENCE VALUES FOR RECTANGULAR AREA UNIFORMLY LOADED $\nu = 6$

m \ n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00930	0.01790	0.02529	0.03124	0.03576	0.03906	0.04138	0.04298	0.04406	0.04479	0.04561	0.04598
0.2	0.01790	0.03447	0.04873	0.06022	0.06899	0.07538	0.07990	0.08302	0.08513	0.08656	0.08817	0.08890
0.3	0.02529	0.04873	0.06893	0.08526	0.09775	0.10691	0.11340	0.11789	0.12095	0.12302	0.12536	0.12644
0.4	0.03124	0.06022	0.08526	0.10557	0.12116	0.13264	0.14080	0.14648	0.15037	0.15302	0.15602	0.15740
0.5	0.03576	0.06899	0.09775	0.12116	0.13921	0.15255	0.16210	0.16877	0.17335	0.17649	0.18006	0.18173
0.6	0.03906	0.07538	0.10691	0.13264	0.15255	0.16734	0.17796	0.18542	0.19058	0.19412	0.19819	0.20010
0.7	0.04138	0.07990	0.11340	0.14080	0.16210	0.17796	0.18941	0.19749	0.20311	0.20697	0.21144	0.21356
0.8	0.04298	0.08302	0.11789	0.14648	0.16877	0.18542	0.19749	0.20604	0.21201	0.21613	0.22093	0.22322
0.9	0.04406	0.08513	0.12095	0.15037	0.17335	0.19058	0.20311	0.21202	0.21824	0.22257	0.22763	0.23006
1.0	0.04479	0.08656	0.12302	0.15302	0.17649	0.19412	0.20697	0.21613	0.22257	0.22705	0.23232	0.24059
1.2	0.04561	0.08817	0.12536	0.15602	0.18006	0.19819	0.21144	0.22093	0.22763	0.23232	0.23787	0.24059
1.4	0.04598	0.08890	0.12644	0.15740	0.18173	0.20010	0.21356	0.22322	0.23006	0.23487	0.24059	0.24341
1.6	0.04616	0.08925	0.12695	0.15806	0.18252	0.20101	0.21458	0.22434	0.23126	0.23613	0.24194	0.24483
1.8	0.04625	0.08942	0.12719	0.15839	0.18292	0.20147	0.21510	0.22490	0.23186	0.23677	0.24264	0.24557
2.0	0.04629	0.08950	0.12732	0.15855	0.18312	0.20170	0.21536	0.22519	0.23218	0.23710	0.24301	0.24597
2.5	0.04633	0.08958	0.12743	0.15870	0.18330	0.20192	0.21561	0.22547	0.23247	0.23742	0.24337	0.24635
3.0	0.04634	0.08960	0.12746	0.15874	0.18335	0.20197	0.21567	0.22553	0.23255	0.23751	0.24346	0.24645
4.0	0.04634	0.08961	0.12747	0.15875	0.18337	0.20200	0.21569	0.22556	0.23258	0.23754	0.24350	0.24650
5.0	0.04634	0.08961	0.12748	0.15876	0.18337	0.20200	0.21570	0.22557	0.23259	0.23755	0.24351	0.24650
6.0	0.04634	0.08961	0.12748	0.15876	0.18337	0.20200	0.21570	0.22557	0.23259	0.23755	0.24351	0.24650
8.0	0.04634	0.08961	0.12748	0.15876	0.18337	0.20200	0.21570	0.22557	0.23259	0.23755	0.24351	0.24651
10.0	0.04634	0.08961	0.12748	0.15876	0.18337	0.20200	0.21570	0.22557	0.23259	0.23755	0.24351	0.24651

TABLE XX - CONTINUED

m \ n	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0	8.0	10.0
0.1	0.04616	0.04625	0.04629	0.04633	0.04634	0.04634	0.04634	0.04634	0.04634	0.04634
0.2	0.08925	0.08942	0.08950	0.08958	0.08960	0.08961	0.08961	0.08961	0.08961	0.08961
0.3	0.12695	0.12719	0.12732	0.12743	0.12746	0.12747	0.12748	0.12748	0.12748	0.12748
0.4	0.15806	0.15839	0.15855	0.15870	0.15874	0.15875	0.15876	0.15876	0.15876	0.15876
0.5	0.18252	0.18292	0.18312	0.18330	0.18335	0.18337	0.18337	0.18337	0.18337	0.18337
0.6	0.20101	0.20147	0.20170	0.20192	0.20197	0.20200	0.20200	0.20200	0.20200	0.20200
0.7	0.21458	0.21510	0.21536	0.21561	0.21567	0.21569	0.21570	0.21570	0.21570	0.21570
0.8	0.22434	0.22490	0.22519	0.22547	0.22553	0.22556	0.22557	0.22557	0.22557	0.22557
0.9	0.23126	0.23186	0.23218	0.23247	0.23255	0.23258	0.23259	0.23259	0.23259	0.23259
1.0	0.23613	0.23677	0.23710	0.23742	0.23751	0.23754	0.23755	0.23755	0.23755	0.23755
1.2	0.24194	0.24264	0.24301	0.24337	0.24346	0.24350	0.24351	0.24351	0.24351	0.24351
1.4	0.24483	0.24557	0.24697	0.24635	0.24645	0.24650	0.24650	0.24650	0.24651	0.24651
1.6	0.24630	0.24706	0.24748	0.24788	0.24734	0.24804	0.24805	0.24805	0.24805	0.24805
1.8	0.24706	0.24785	0.24827	0.24869	0.24881	0.24886	0.24887	0.24887	0.24887	0.24887
2.0	0.24748	0.24827	0.24871	0.24914	0.24926	0.24931	0.24932	0.24932	0.24932	0.24932
2.5	0.24788	0.24869	0.24914	0.24958	0.24971	0.24977	0.24978	0.24978	0.24978	0.24978
3.0	0.24799	0.24881	0.24926	0.24971	0.24986	0.24991	0.24991	0.24992	0.24992	0.24992
4.0	0.24804	0.24886	0.24931	0.24977	0.24991	0.24997	0.24998	0.24998	0.24998	0.24998
5.0	0.24805	0.24887	0.24932	0.24978	0.24991	0.24998	0.24999	0.24999	0.25000	0.25000
6.0	0.24805	0.24887	0.24932	0.24978	0.24992	0.24998	0.24999	0.24999	0.25000	0.25000
8.0	0.24805	0.24887	0.24932	0.24978	0.24992	0.24998	0.25000	0.25000	0.25000	0.25000
10.0	0.24805	0.24887	0.24932	0.24978	0.24992	0.24998	0.25000	0.25000	0.25000	0.25000

CHAPTER VIII

DISCUSSION

It will be noted for a value of $\nu = 3$, the Froelich equation becomes identical with the Boussinesq equation. It is possible to integrate Eq. (29)* by conventional methods as has been done by Newmark[E]. Instead, Eq. (29) was evaluated with the aid of computer to independently check the Newmark Table[E]. The values agreed to Newmark's values to within 0.00001. The difference is attributed to truncation at the last figure in using the computer. For the other concentration factors, plots were made of the curves " $Y = F(m,n)$ " and these were so similar in nature, that it was obvious the computer gave satisfactory results to five decimal places.

At each point of the earth mass, loaded with its own weight there is a vertical pressure "p_z" and a horizontal pressure "$p_x = p_y$". The ratio "$p_x/p_z = p_y/p_z = K$", which is the coefficient of earth pressure at rest[C]. For practically incompressible materials, the value of Poisson's ratio "μ" is 0.5 and the corresponding value of "K" is unity.

The "K"value decreases as isotropy of earth mass ceases to be monotonous. Simultaneously, the concentration index "ν" increases. Professor D. P. Krynine[B]

*Note: See p.36

has given the relationship between "ν" and "K" mathematically as:

$$\nu = 2 + \frac{1}{K}$$

for an incompressible isotropic elastic body, $\frac{p_x}{p_y} = 1$ and hence, $\nu = 3$

If the horizontal pressure exerted by a clay is three fourths of the vertical, the value of "ν" in this case is about 3.3. For a sand having an angle of internal friction of $\phi = 35^\circ$, test results[c] indicate that "K" is about 0.25. Hence, "ν" is 6.

It will be noted that the Boussinesq formula should only be applied to clays and not to sands. It does not mean however, that clays are elastic, and sands inelastic. For each sand particle is an elastic body, as demonstrated by the rebound after releasing a load from a sand mass. But the reason why the Boussinesq formula can not be applied unconditionally to sands, lies in their lack of monotonous isotropy. It should be noted bodies termed isotropic in reality may have quite different properties, depending on their degree of isotropy.

It is the author's belief, the correct concentration index to be used must be based upon observations of actual cases. At present there are little available data on this aspect.

For comparison, Figures 7, 8, 9 and 10 show the relative forms of stress distribution below points along line "A' - A" with different values of concentration index and at various depths. It will be noted that the higher concentration index values result in higher stresses, below the point or area of load application. Consequently, higher settlements should be anticipated with higher concentration factors.

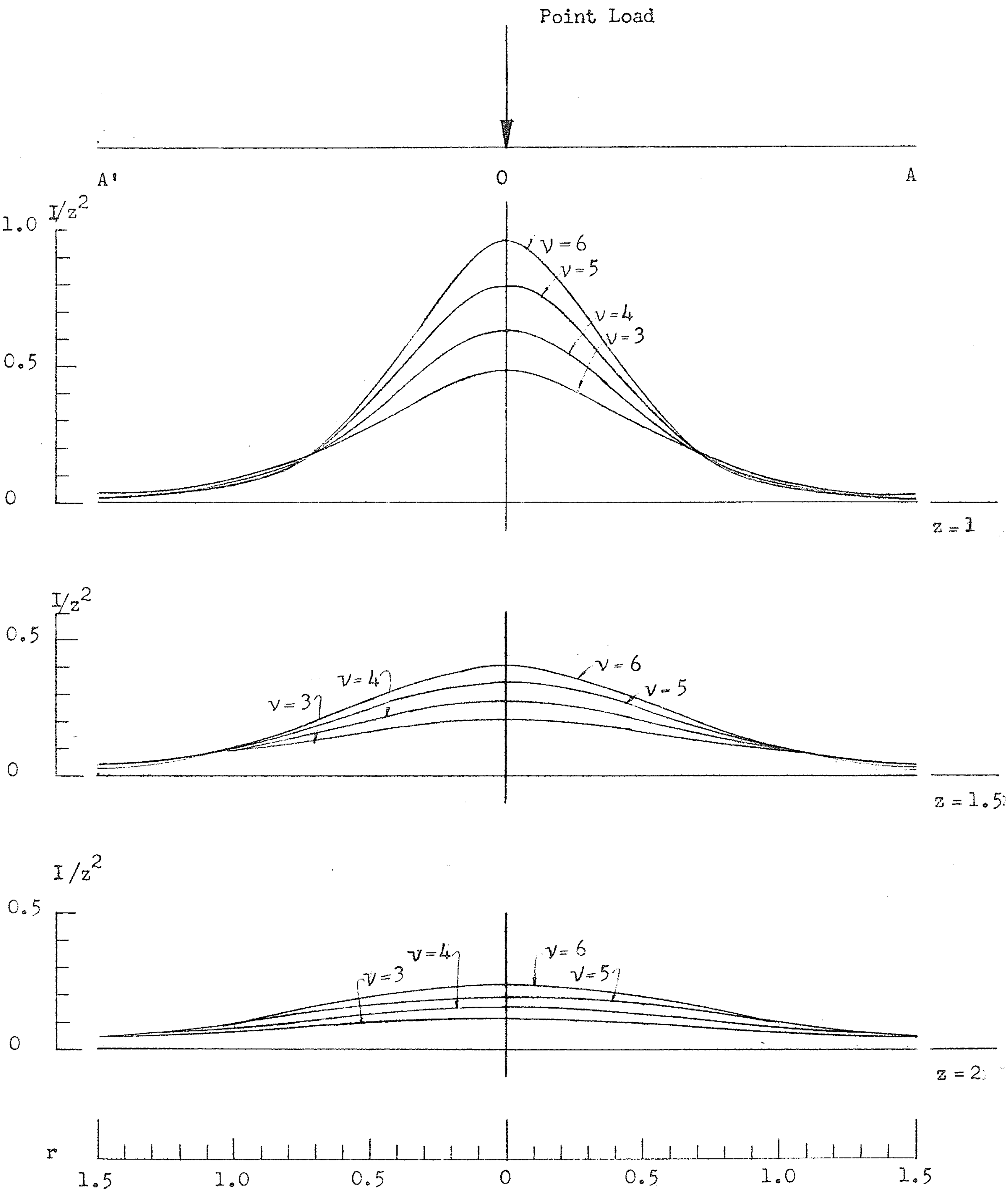

Fig. 7. Relative Forms of Stress Surface for Different Values of ν at Various Depths

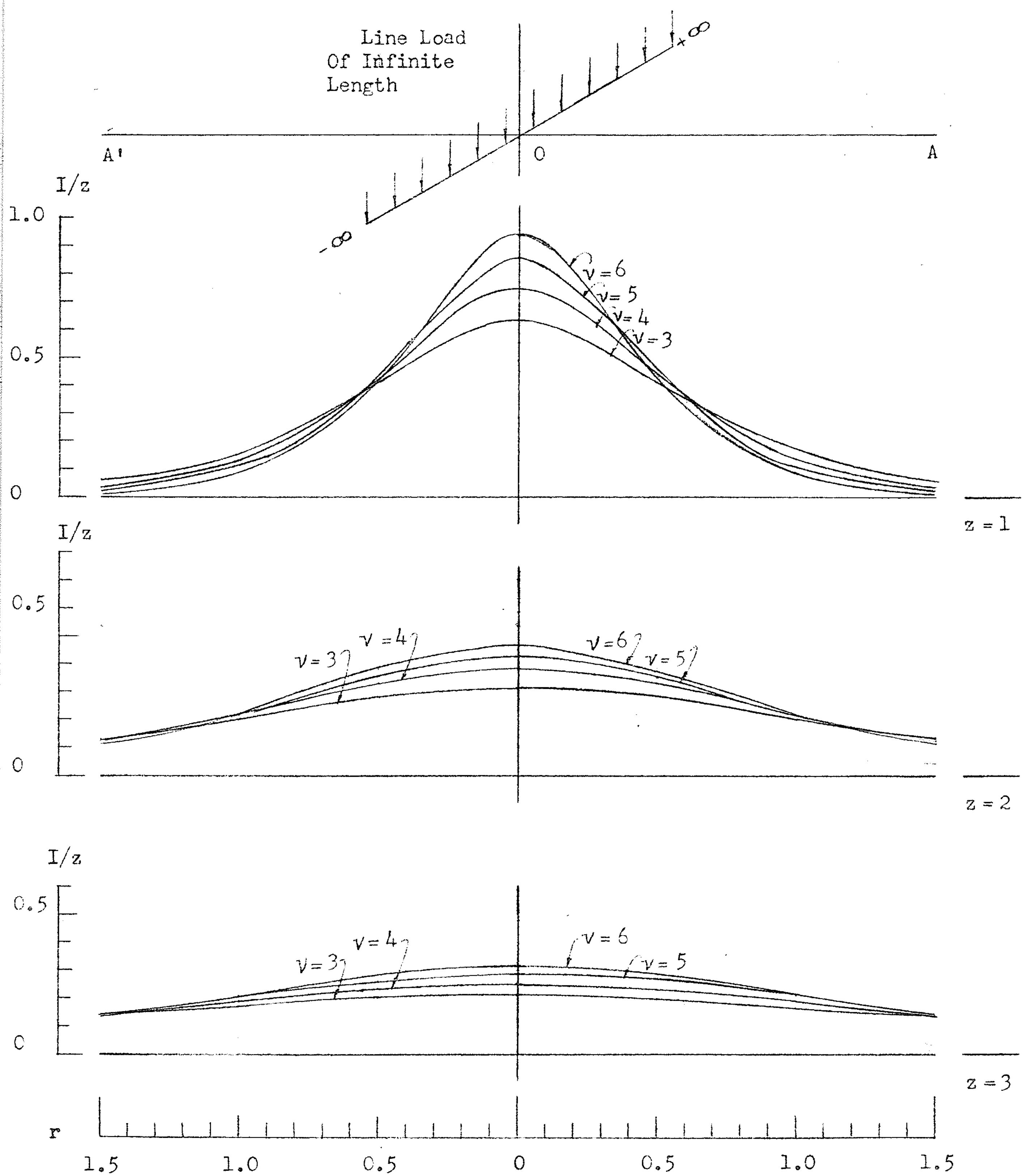

Fig. 8. Relative Forms of Stress Surface for Different Values of ν at Various Depths

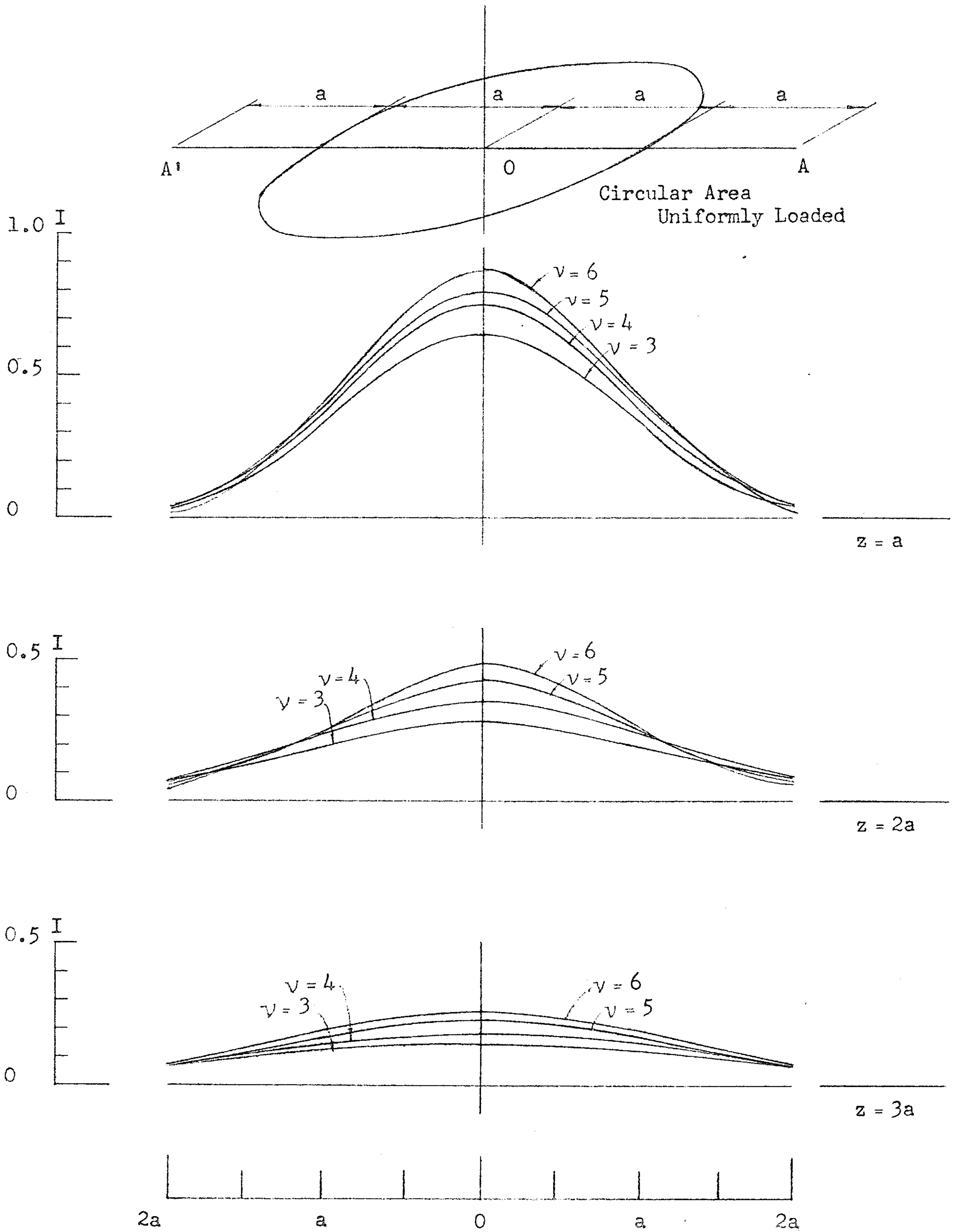

Fig. 9. Relative Forms of Stress Surface for Different Values of ν at Various Depths

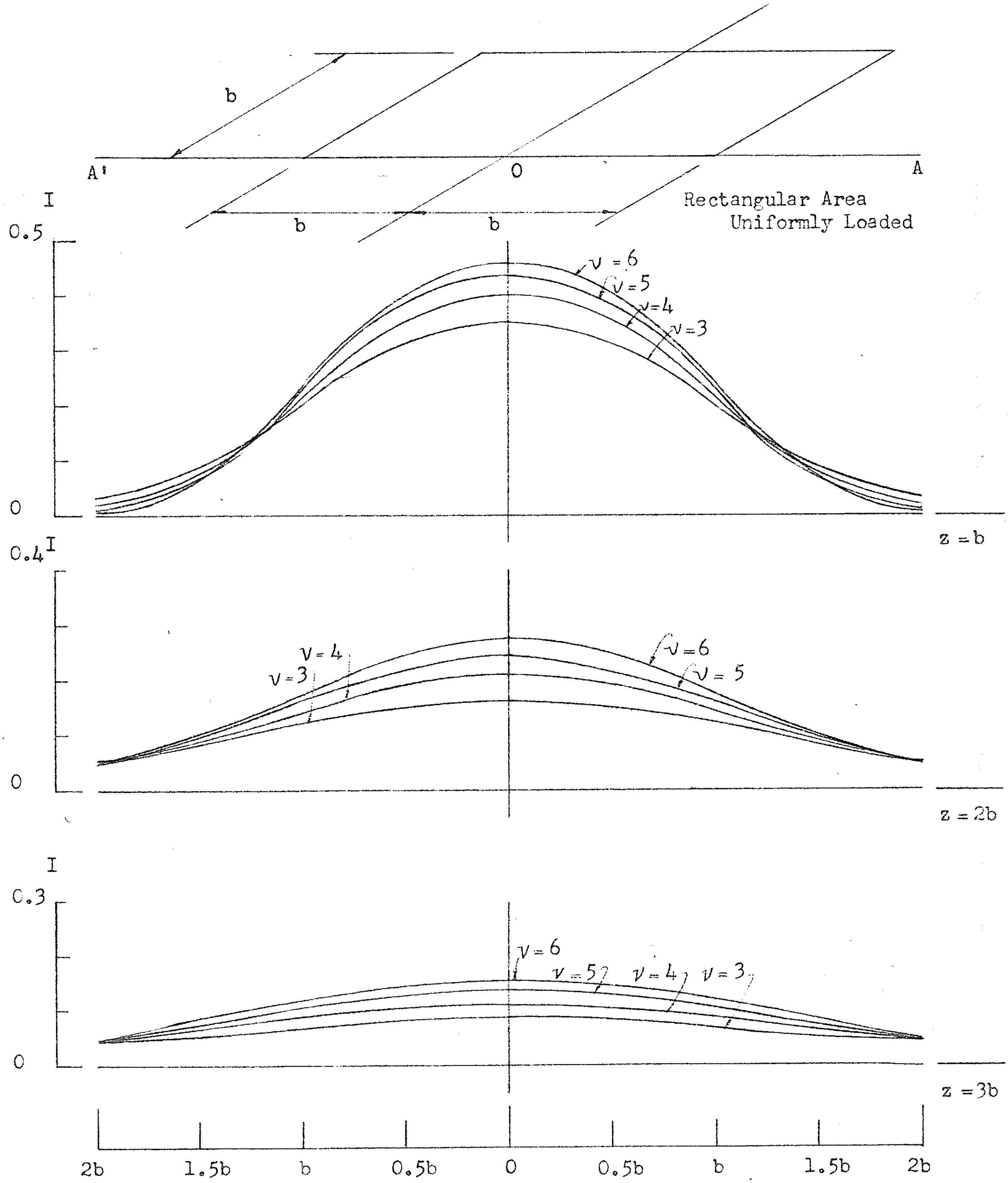

Fig. 10. Relative Forms of Stress Surface for Different Values of ν at Various Depths

APPENDIX A

Sample Programme for the Bendix G-15 D Computer

This appendix shows a sample programme by using the intercom 1000 programming system for the BENDIX G-15 digital computer for computation of influence values for the vertical stress due to a rectangular area uniformly loaded.

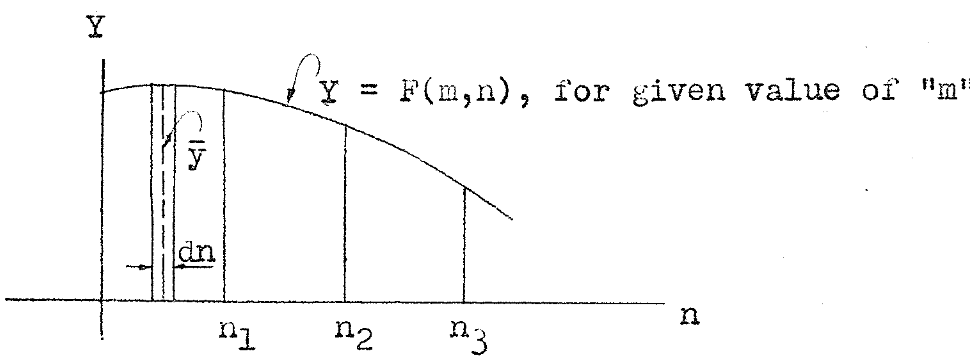

Fig. 11

Equations (29), (31), (33)and (35)* can be rewritten in a general form as follows:

$$I_v = \int_0^n F(m,n)\ dn$$

Fig. 11 shows the curve " Y= F(m,n)", for given value of "m". It is possible to find the area under the curve for obtaining influence values instead of integrating the above expression.

*Note: See pp. 36-37

In Fig. 11, $dA = \bar{y}\, dn$

in which, dA = the area of the strip,

$\bar{y}$ = the mid-ordinate of the strip,

dn = the width of the strip.

It will be noted that: $I_v = \int_0^{n_1} F(m,n)\, dn \doteq \sum_0^{n_1} \bar{y}\, dn$

The computer was used to calculate the above summation. The width of the element of area used for the case of the rectangular area uniformly loaded was 0.02. In other words, there were five elements of area for each type-out of influence value for the increment of "n" of 0.1 ; or fifty elements of area for the increment of "n" of 1.0 . These calculations gave answers that agreed to within 0.00001 of those given by Newmark[E] for the case of concentration index equal to three.

It may be also noted that computations were simultaneously made for concentration indexes of three, four, five and six, which greatly reduced the time of computer operation for each set of calculations.

PROBLEM : INFLUENCE VALUES FOR RECTANGULAR AREA UNIFORMLY LOADED	Prepared by :	
	Date :	Page

$$I_3 = \frac{1}{2\pi}\int_0^n \frac{m(2m^2 + 3n^2 + 3)}{(n^2+1)^2(m^2+n^2+1)^{3/2}}\,dn$$

$$I_4 = \frac{1}{4\pi}\int_0^n \left[\frac{m(3m^2 + 5n^2 + 5)}{(n^2+1)^2(m^2+n^2+1)^2} + \frac{3}{(n^2+1)^{5/2}}\,\mathrm{Tan}^{-1}\frac{m}{(n^2+1)^{1/2}}\right]dn$$

$$I_5 = \frac{1}{6\pi}\int_0^n \frac{3m^5+5m(m^2+n^2+1)(m^2+3n^2+3)}{(n^2+1)^3(m^2+n^2+1)^{5/2}}\,dn$$

$$I_6 = \frac{1}{16\pi}\int_0^n \left[\frac{8m(n^2+1)^2 + 5m(m^2+n^2+1)(3m^2+5n^2+5)}{(n^2+1)^3(m^2+n^2+1)^3} + \frac{15}{(n^2+1)^{7/2}}\,\mathrm{Tan}^{-1}\frac{m}{(n^2+1)^{1/2}}\right]dn$$

LOC	K	CODE	AD	DR	(A)	NOTES
997		42	09	18	Clear & add zero	0900 $(1/2\pi)dn$ 0.0031831
998		49	09	45	Store zero in I_3	0901 $(1/4\pi)dn$ 0.0015916
999		49	09	46	Store zero in I_4	0902 $(1/6\pi)dn$ 0.0010610
1000		49	09	47	Store zero in I_5	0903 $(1/16\pi)dn$ 0.00039789
1001		49	09	48	Store zero in I_6	0904 2.0
1002		42	09	10	Clear & add m	0905 3.0
1003		49	09	19	Store m_{start} in m	0906 1.0
1004		42	09	19	Clear & add m	0907 5.0
1005		43	09	49	m + 0.1	0908 8.0
1006		49	09	15	Store ditto in $n_{lim-lim}$	0909 15.0
1007		42	09	19	Clear & add m	0910 m_{start} 0.1
1008		44	21	01	m^2	0911 m_{inc} 0.1
1009		49	09	22	Store ditto	0912 m_{limit} 10.0

INTERCOM 1000

PROBLEM : Continued	Prepared by :	
	Date :	Page

LOC	K	CODE	AD	DR	(A)	NOTES		
1010		42	09	13	$n_{lim-start}$	0913	$n_{lim-start}$	0.1
1011		49	09	20	Store ditto in n_{lim}	0914	$n_{lim-inc}$	0.1
1012		42	09	16	n_{start}	0915	$n_{lim-lim}$	10.0
1013		49	09	21	Store ditto in n	0916	n_{start}	0.01
1014		42	09	21	Clear & add n	0917	n_{inc}	0.02
1015		44	21	01	n	0918		0.0
1016		49	09	23	Store ditto	0919	m	
1017		43	09	22	$m^2 + n^2$	0920	n_{lim}	
1018		43	09	06	$m^2 + n^2 + 1$	0921	n	
1019		49	09	24	Store ditto	0922	m^2	
1020		47	09	06	$1/(m^2 + n^2 + 1)$	0923	n^2	
1021		49	09	25	Store ditto	0924	$m^2 + n^2 + 1$	
1022		08	11	97	$1/(m^2 + n^2 + 1)$	0925	$1/(m^2 + n^2 + 1)$	
1023		49	09	26	Store ditto	0926	$1/(m^2 + n^2 + 1)^{1/2}$	
1024		44	09	25	$1/(m^2 + n^2 + 1)$	0927	$1/(m^2 + n^2 + 1)^{3/2}$	
1025		49	09	27	Store ditto	0928	$1/(m^2 + n^2 + 1)^2$	
1026		44	09	26	$1/(m^2 + n^2 + 1)$	0929	$1/(m^2 + n^2 + 1)^{5/2}$	
1027		49	09	28	Store ditto	0930	$1/(m^2 + n^2 + 1)^3$	
1028		44	09	26	$1/(m^2 + n^2 + 1)$	0931	$1/(n^2 + 1)$	
1029		49	09	29	Store ditto	0932	$1/(n^2 + 1)^{1/2}$	
1030		44	09	26	$1/(m^2 + n^2 + 1)$	0933	$1/(n^2 + 1)^2$	
1031		49	09	30	Store ditto	0934	$1/(n^2 + 1)^{5/2}$	
1032		42	09	23	n^2	0935	$1/(n^2 + 1)^3$	

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PROBLEM : Continued	Prepared by :	
	Date :	Page

LOC	K	CODE	AD	DR	(A)	NOTES
1033		43	09	06	$n^2 + 1$	0936 $1/(n^2 + 1)^{7/2}$
1034		47	09	06	$1/(n^2 + 1)$	0937 $2m^2$
1035		49	09	31	Store ditto	0938 $3m^2$
1036		08	11	97	$1/(n^2 + 1)^{1/2}$	0939 $3m^2 + 5n^2 \quad 5$
					0940 $m(3m^2 + 5n^2 + 5)/(n^2+1)^2(m^2 + n^2 + 1)^2$	
1037		49	09	32	Store ditto	0941 $\text{Tan}^{-1} m/(n^2 + 1)^{1/2}$
1038		42	09	31	$1/(n^2 + 1)$	0942 $3m^5$
1039		44	21	01	$1/(n^2 + 1)^2$	0943 $8m(n^2 + 1)^2$
		0944 $[8m(n^2 + 1)^2 + 5m(m^2+n^2+1)(3m^2+5n^2+5)]/(n^2+1)^3 (m^2 + n^2 + 1)^3$				
1040		49	09	33	Store ditto	0945 I_3
1041		44	09	32	$1/(n^2 + 1)^{5/2}$	0946 I_4
1042		49	09	34	Store ditto	0947 I_5
1043		44	09	32	$1/(n^2 + 1)^3$	0948 I_6
1044		49	09	35	Store ditto	0949 0.1
1045		44	09	32	$1/(n^2 + 1)^{7/2}$	
1046		49	09	36	Store ditto	
1047		42	09	22	m^2	
1048		44	09	04	$2m^2$	
1049		49	09	37	Store ditto	
1050		42	09	23	n^2	
1051		44	09	05	$3n^2$	
1052		43	09	37	$2m^2 + 3n^2$	
1053		43	09	05	$2m^2 + 3n^2 + 3$	

PROBLEM : Continued	Prepared by :	
	Date :	Page

LOC	K	CODE	AD	DR	(A)	NOTES
1054		44	09	19	$m(2m^2+3n^2+3)$	
1055		44	09	33	$m(2m^2+3n^2+3)/(n^2+1)$	
1056		44	09	27	$m(2m^2+3n^2+3)/(n^2+1)(m^2+n^2+1)$	
1057		44	09	00	Multiply ditto by $(1/2\pi)dn$	
1058		43	09	45	Add previous I value	
1059		49	09	45	Store in I_3	
1060		42	09	22	m^2	
1061		44	09	05	$3m^2$	
1062		49	09	38	Store ditto	
1063		42	09	23	n^2	
1064		44	09	07	$5n^2$	
1065		43	09	38	$3m^2 + 5n^2$	
1066		43	09	07	$3m^2 + 5n^2 + 5$	
1067		49	09	39	Store ditto	
1068		44	09	19	$m(3m^2 + 5n^2 + 5)$	
1069		44	09	33	Multiply by $1/(n^2 + 1)$	
1070		44	09	28	Multiply ditto by $1/(m^2 + n^2 + 1)$	
1071		49	09	40	Store ditto	
1072		42	09	19	m	
1073		44	09	32	$m/(n^2 + 1)^{1/2}$	
1074		08	12	24	$\mathrm{Tan}^{-1} m/(n^2 + 1)^{1/2}$	
1075		49	09	41	Store ditto	
1076		44	09	34	Mutiply ditto by $1/(n^2+1)^{5/2}$	

PROBLEM : Continued	Prepared :	
	Date :	Page

LOC	K	CODE	AD	DR	(A)	NOTES
1077		44	09	05	Multiply ditto by 3	
1078		43	09	40	Add the content in 0940 to ditto	
1079		44	09	01	Multiply ditto by $(1/4\pi)$dn	
1080		43	09	46	Add previous I value	
1081		49	09	46	Store ditto in I_4	
1082		42	09	22	m^2	
1083		44	21	01	m^4	
1084		44	09	19	m^5	
1085		44	09	05	$3m^5$	
1086		49	09	42	Store ditto	
1087		42	09	23	n^2	
1088		44	09	05	$3n^2$	
1089		43	09	22	$m^2 + 3n^2$	
1090		43	09	05	$m^2 + 3n^2 + 3$	
1091		44	09	24	Multiply ditto by $(m^2 + n^2 + 1)$	
1092		44	09	19	Multiply ditto by m	
1093		44	09	07	Multiply ditto by 5	
1094		43	09	42	Add 3m5 to ditto	
1095		44	09	35	Multiply ditto by $(n^2 + 1)^3$	
1096		44	09	29	Multiply ditto by $1/(m^2 + n^2 + 1)^{5/2}$	
1097		44	09	02	Multiply ditto by $(1/6\pi)$dn	
1098		43	09	47	Add previous I value	
1099		49	09	47	Store in I_5	

PROBLEM : Continued	Prepared :	
	Date :	Page

LOC	K	CODE	AD	DR	(A)	NOTES
1100		42	09	23	n^2	
1101		43	09	06	$n^2 + 1$	
1102		44	21	01	$(n^2 + 1)^2$	
1103		44	09	19	$m(n^2 + 1)^2$	
1104		44	09	08	$8m(n^2 + 1)^2$	
1105		49	09	43	Store ditto	
1106		42	09	39	$(3m^2 + 5n^2 + 5)$	
1107		44	09	24	Multiply ditto by $(m^2 + n^2 + 1)$	
1108		44	09	19	Multiply ditto by m	
1109		44	09	07	Multiply ditto by 5	
1110		43	09	43	Add $8m(n^2 + 1)^2$ to ditto	
1111		44	09	35	Multiply ditto by $1/(n^2 + 1)^3$	
1112		44	09	30	Multiply ditto by $1/(m^2 + n^2 + 1)^3$	
1113		49	09	44	Store ditto	
1114		42	09	41	Clear & add $\mathrm{Tan}^{-1} m/(n^2 + 1)^{1/2}$	
1115		44	09	09	Multiply ditto by 15	
1116		44	09	36	Multiply ditto by $1/(n^2 + 1)^{7/2}$	
1117		43	09	44	Add the content in 0944 to ditto	
1118		44	09	03	Multiply ditto by $(1/16\pi)dn$	
1119		43	09	48	Add previous I value	
1120		49	09	48	Store ditto in I_6	
1121		42	09	21	Clear & add n	
1122		43	09	17	$n + n_{inc}$	

PROBLEM : Continued	Prepared :	
	Date :	Page

LOC	K	CODE	AD	DR	(A)	NOTES
1123		49	09	21	Store n + n_{inc} in n	
1124		41	09	20	Sub. n_{lim}	
1125		22	10	14	If (-), trans. to 1014	
1126		33	09	19	If (+), type m	
1127		38	09	20	Type n_{lim} & C.R.	
1128		33	09	45	Type I_3	
1129		33	09	46	Type I_4	
1130		33	09	47	Type I_5	
1131		38	09	48	Type I_6	
1132		42	09	20	Clear & add n	
1133		43	09	14	n + n_{lim}	
1134		49	09	20	Store ditto in n_{lim}	
1135		41	09	15	Sub. m + 0.1	
1136		22	10	14	If (-), trans. to 1014	
1137		42	09	19	If (+), clear & add m	
1138		43	09	11	m m_{inc}	
1139		49	09	19	Store ditto in m	
1140		41	09	12	Sub. m_{lim}	
1141		22	11	44	If (-), trans. to 1144	
1142		63	00	00	If (+), ring bell	
1143		67	00	00	Halt	
1144		42	09	18	Clear & add zero	
1145		49	09	45	Store zero in I_3	

PROBLEM : Continued	Prepared :	
	Date :	Page

LOC	K	CODE	AD	DR	(A)	NOTES
1146		49	09	46	Store zero in I_4	
1147		49	09	47	Store zero in I_5	
1148		49	09	48	Store zero in I_6	
1149		29	10	04	Trans.to 1004	

APPENDIX B

Derivation of Formulas for Vertical Stress beneath One of the Corners of a Rectangular Area Uniformly Loaded

With reference to Fig. 5 on page 34,

$$d\sigma_z = \frac{\nu Q}{2\pi z^2} \mathrm{Cos}^{(\nu+2)}\psi$$

wherein, $Q = q\ dA$

and $dA = dy\ dx$

Also, $\mathrm{Cos}\ \psi = \dfrac{z}{(x^2 + y^2 + z^2)^{1/2}}$

Hence, $$\sigma_z = \int_0^{x=a}\int_0^{y=b} \frac{\nu q}{2\pi z^2} \frac{z^{(\nu+2)}}{(x^2 + y^2 + z^2)^{(\nu+2)/2}}\ dy\ dx$$

............(26)

For $\nu = 3$,
Eq. (26) becomes $$\sigma_z = \int_0^{x=a}\int_0^{y=b} \frac{3qz^3}{2\pi} \frac{dy\ dx}{(x^2 + y^2 + z^2)^{5/2}}$$

Let $x = (y^2 + z^2)^{1/2}\ \mathrm{Tan}\theta$

Then, $dx = (y^2 + z^2)^{1/2}\ \mathrm{Sec}^2\theta\ d\theta$

Substituting,

$$\sigma_z = \frac{3qz^3}{2\pi}\int_0^{y=b} dy \int_{x=0}^{x=a} \frac{(y^2 + z^2)^{1/2} \mathrm{Sec}^2\theta \, d\theta}{\left[(y^2 + z^2)\mathrm{Tan}^2\theta + (y^2 + z^2)\right]^{5/2}}$$

$$= \frac{3qz^3}{2\pi}\int_0^{y=b} dy \int_{x=0}^{x=a} \frac{\mathrm{Cos}^3\theta \, d\theta}{(y^2 + z^2)^2}$$

$$= \frac{3qz^3}{2\pi}\int_0^{y=b} \frac{dy}{(y^2 + z^2)^2}\left[\mathrm{Sin}\,\theta - \frac{1}{3}\mathrm{Sin}^3\theta\right]_{x=0}^{x=a}$$

$$= \frac{3qz^3}{2\pi}\int_0^{y=b} \frac{dy}{(y^2 + z^2)^2}\left[\frac{x}{(x^2 + y^2 + z^2)^{1/2}} - \frac{1}{3}\frac{x^3}{(x^2 + y^2 + z^2)^{3/2}}\right]_0^{x=a}$$

$$= \frac{qz^3}{2\pi}\int_0^{y=b} \frac{a\,(2a^2 + 3y^2 + 3z^2)}{(y^2 + z^2)^2 (a^2 + y^2 + z^2)^{3/2}} dy \quad ....(27)$$

Let $a = mz$ and $y = nz$

Then, $dy = z\,dn$

Substituting and simplifying,

$$\sigma_z = \frac{q}{2\pi}\int_0^{b/z} \frac{m(2m^2 + 3n^2 + 3)}{(n^2+1)^2(m^2+ n^2+ 1)^{3/2}} \, dn$$

$$= q \, I_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(28)$$

wherein, $$I_3 = \frac{1}{2\pi}\int_0^{b/z} \frac{m(2m^2 + 3n^2 + 3)}{(n^2 + 1)^2(m^2 + n^2 + 1)^{3/2}} \, dn$$

$$\ldots\ldots\ldots\ldots\ldots(29)$$

Similarily, expressions were derived for

$$\nu = 4, \qquad \sigma_z = q \, I_4 \qquad \ldots\ldots\ldots\ldots(30)$$

wherein, $$I_4 = \frac{1}{4\pi}\int_0^{b/z} \left[\frac{m(3m^2 + 5n^2 + 5)}{(n^2 + 1)^2(m^2 + n^2 1)^2} + \frac{3}{(n^2 + 1)^{5/2}} \mathrm{Tan}^{-1} \frac{m}{(n^2 + 1)^{1/2}}\right] dn$$

$$\ldots\ldots\ldots\ldots(31)$$

for $\nu = 5$; $\sigma_z = q\, I_5$ ....(32)

wherein, $$I_5 = \frac{1}{6\pi}\int_0^{b/z} \frac{3m^5 + 5m(m^2 + n^2 + 1)(m^2 + 3n^2 + 3)}{(n^2 + 1)^3(m^2 + n^2 + 1)^{5/2}}\, dn$$

..............(33)

and for $\nu = 6$; $\sigma_z = q\, I_6$ ....(34)

wherein, $$I_6 = \frac{1}{16\pi}\int_0^{b/z}\left[\frac{8m(n^2+1)^2 + 5m(m^2+n^2+1)(3m^2+5n^2+5)}{(n^2+1)^3(m^2+n^2+1)^3} + \frac{15}{(n^2+1)^{7/2}}\, \mathrm{Tan}^{-1}\frac{m}{(n^2+1)^{1/2}}\right] dn$$

..............(35)

BIBLIOGRAPHY

1. "Drukverdeeling in Bouwgrond" De Ingenieur, April 15, 1932, p. B-52.

2. "Sur l'équilibre intérieur des solides homogènes" Savants étrangers de l'Académie des Sciences de Paris, 1833, Tome IV, p. 541.

3. "Application des Potentiels a l'Étude de l'Équilibre et du Mouvement des Solides Élastiques" Paris, 1885.

4. "Versuche uber die Elastizität des Erdbodens" Zentralblatt der Bauverwaltung, 1897.

REFERENCES

A. "Pressure under Substructures" Engineering and Contracting, Mar. 1929, pp. 113-119.

B. "Distribution of Stresses under a Foundation" A. E. Cummings, Transactions of ASCE Vol. 101, 1936, pp. 1072-1134.

C. "Theoretical Soil Mechanics" Terzaghi.

D. "Soil Engineering" M. G. Spangler.

E. "Simplified Computation of Vertical Pressures in Elastic Foundations" Univ. Illinois Eng. Exper. Sta. Circular 24.