SCATTERING OF Co^{60} GAMMA RAYS AT 90^{o}

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ABSTRACT

The elastic scattering of gammas of 1.17 MeV and 1.33 Mev from a Co^{60} source was measured for the scattering angle of 90° and four different materials (Pb, Cu, Al and H_2O). Intensity of the source used was about 1000 curies and the scattered γ-ray spectrum was obtained by using a NaI(Tl) crystal ($1\frac{1}{2}$" thick and $1\frac{1}{4}$" diameter) and a single channel pulse height analyzer. The cross sections obtained are tabulated on page 59. These results agree fairly well with Bethe's form factor calculations for Rayleigh scattering and with the classical Thomson formula for elastic scattering by the nucleus.

The hard continuous gamma spectrum, usually ascribed to bremsstrahlung from photo- and Compton electrons, was examined for different thicknesses of Sn, Cu and Pb scatterers. The intensity of the spectra and the fact that it is almost independent of the scatterer thickness unambiguously proves that only a minor part of the hard component is caused by bremsstrahlung. The assumption that its origin is in the Compton effect on bound electrons seems consistent with the measurements.

Contents

List of figures

I INTRODUCTION

The scattering of a gamma ray by an atom can take place in several ways. The most probable process is Compton scattering from the atomic electrons, very well explained theoretically [1] and checked experimentally [2, 3] for the usual case where the electrons may be considered free and at rest. Here a gamma ray scattered at any angle has a definite energy given by the well-known Compton formula. The effect of electron binding is to give a distribution of energies to the scattered gamma rays as a result of the distribution of electron velocities within the atom [4]. This is well known in the X-ray region, but for higher energies the Compton line broadening has not been well treated theoretically or observed experimentally.

A process with much smaller probability is the elastic scattering which can occur in several different manners. In this case theoretical treatments have preceded experiments and four different elastic processes have been predicted. These are Rayleigh scattering by bound electrons [5, 6], Thomson scattering by the nucleus [7], Delbruck (potential) scattering from the electric field of the nucleus [8] and nuclear resonant scattering [9, 10]. All these four processes are coherent meaning that their scattering amplitudes must be added rather than their intensities. Since scintillation counter techniques have been developed, several successful experiments have been done on resonance scattering from the nucleus as well as a number of experiments on Rayleigh and Thomson scattering [11-20]. Delbruck scattering has not yet been observed experimentally.

In all scattering experiments we must use scatterers of finite thickness. This has the effect that some secondary processes take place. Among these processes the annihilation of positrons created by gammas is the best known one [c.f. sec. 26, 27 in ref. 1, and for instance ref. 14]. Another secondary process is bremsstrahlung due to electrons produced by the photo- and Compton effects. The energy spectrum of the gammas created in this manner is evidently dependent on the scatterer thickness and covers continuously the range from zero to the maximum energy of the ejected photoelectrons. In almost all experiments on elastic scattering a hard+ continuous component of gammas was observed [14, 15, 16, 19, 20] and has been ascribed to bremsstrahlung*.

The aim of this experiment was to get some more data on the elastic cross section for different atomic numbers (particularly for low Z) and to investigate more closely the origin of the hard continuous component. In section I a short theoretical discussion of scattering processes is presented in the scope that will be necessary for the discussion developed in section V. In section II and III the experimental set up and method are explained and the results obtained are presented in section IV.

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* See, however, Mann [19] who suggests Compton effect on bound electrons for part of it.

\+ By hard we mean γ-rays of energies higher than the energy of gammas scattered by the Compton effect.

II. THEORETICAL CONSIDERATIONS.

(A) Main primary processes.

(a) Photo effect.

For the discussion that follows the experiment we need to know roughly the angular distribution of ejected photo-electrons. We are not particularly interested in the total cross section for photo effect, so we shall present only the angular dependence of the differential cross section. Also, because the experimental results show that about 80 per cent of the photo electric absorption occurs in the K-shell for γ-ray energies of .5 MeV or higher we shall not consider here effects in the L-shell.

Several formulas for the differential cross section have been derived all having a limited range of validity. Sauter [21] has derived one that is most applicable in our case. It is valid for relativistic energies of the ejected photoelectrons and for $Z \ll 137$. Its angular dependent part for the case of an unpolarized primary beam is

$$\sigma(\theta) \sim \sin^2\theta \left[\frac{\pi\sqrt{1-\beta^2}}{(1-\beta\cos\theta)^4} - \frac{\pi(1-\sqrt{1-\beta^2})}{2\sqrt{1-\beta^2}(1-\beta\cos\theta)^3} + \frac{(1-\sqrt{1-\beta^2})}{4(1-\beta^2)(1-\beta\cos\theta)^3} \right] \tag{2.1}$$

where (θ) is the angle between the direction of the ejected electron and the direction of the primary photon, and β is the ratio of velocity of photoelectrons to the velocity of light.

It can be easily shown from the above formula that in the case when the energy of the ejected photoelectrons is 1.2 MeV (this is applicable to the radiation from Co^{60}) almost all of the electrons are emitted within an angle of 25° with the primary beam.

(b) Compton effect

From the relativistic relations for the energy and momentum it is easy to obtain the familiar equation that expresses the energy E of the scattered photon in terms of its primary energy E_0 and the scattering angle Θ i.e.

$$E = \frac{E_0}{1 + \gamma(1 - \cos\Theta)} . \qquad (2.2)$$

Here $\gamma = E_0/mc^2$, where mc^2 is the rest energy of the electron.

The differential cross section $\frac{d\sigma(\Theta)}{d\Omega}$ + for the process is given by the well-known Klein-Nishina formula (ref. [1] p. 219, eq. (40)). In the nonrelativistic case ($\gamma \ll 1$) it tends to a limit

$$\frac{d\sigma(\Theta)}{d\Omega} = r_0^2 \frac{1 + \cos^2\Theta}{2} \qquad (2.3)$$

where $r_0 = e^2/mc^2 = 2.82 \times 10^{-13}$ cm is the classical electron radius. This expression was initially derived by J.J. Thomson on purely classical grounds.

From the Klein-Nishina formula for the recoiling electrons (ref. [1], p. 220, eq. (43)) or from its graphs [22] it can be deduced that most of the recoil electrons go in

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+ $\frac{d\sigma(\Theta)}{d\Omega}$ is the cross section for scattering per unit solid angle.

the forward direction, and from eq. (2.2) it is clear that these electrons have the largest energies.

(c) Pair production

The differential cross section for the creation of a pair (ref [1], p. 257, eqs. (6) and (7)) shows that the angular distribution of positrons is strongly peaked in the forward direction and that the cross section for pair creation is proportional to Z^2. For heavy elements and relativistic energies and in the case of screening there is a small correction factor that slightly changes the Z^2-dependence.

(B) Secondary Processes

(a) Bremsstrahlung due to photo and Compton electrons.

The differential cross-section for bremsstrahlung has been derived theoretically by several authors (section 25 in ref. [1]) but recent experiments of Motz [23] have shown that the theory does not give satisfactory results. We reproduce in fig. (1), in a convenient and simplified form, his results for the cross section for 1 MeV electrons at 90°. Cross sections for Cu and Su have been interpolated using theoretically obtained Z^2-dependence that is valid for low Z (ref. [1], sec. 25).

(b) Annihilation of positrons at rest and in flight.

When positrons are annihilated at rest two γ-quanta of energy 511 KeV are produced and their angular distribution is isotropic.

The probability for the annihilation of a positron when it collides with a negative electron is treated in ref. [1] (sec. 27, subsection 1) for the case when the centre of mass of both electrons is at rest. In order to get the differential cross section for the annihilation in flight it is necessary

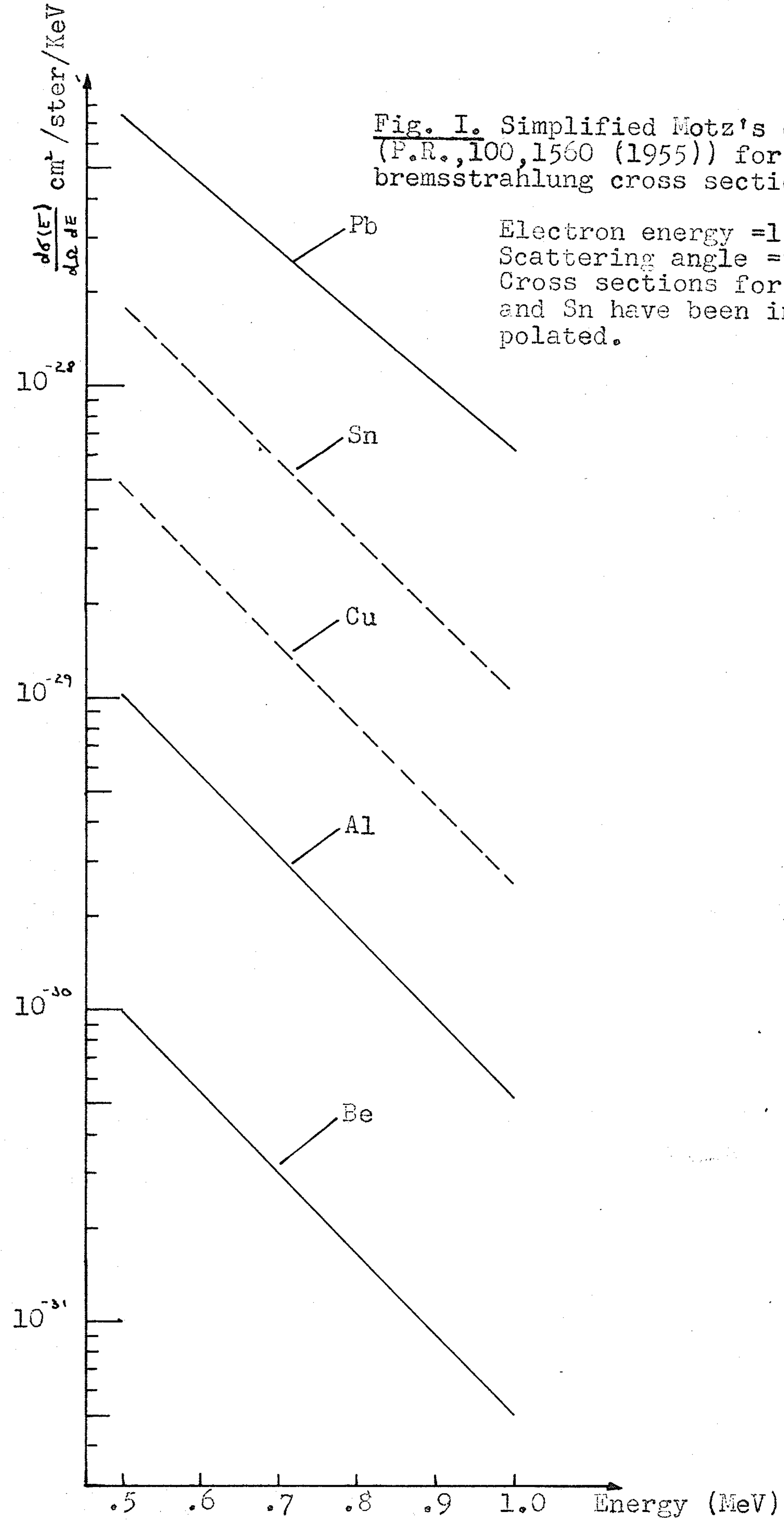

Fig. I. Simplified Motz's data (P.R., 100, 1560 (1955)) for the bremsstrahlung cross section.

Electron energy = 1.0 MeV
Scattering angle = 90°
Cross sections for Cu and Sn have been interpolated.

to make a simple Lorentz transformation [24]. Energies of γ-rays created $(h\nu)_1$ and $(h\nu)_2$ are directly related to the angle θ one of them make with the original direction of positron. So, it can be shown (ref. [24] eq. 2) that

$$(h\nu)_1 = \frac{mc^2}{1 - \frac{E - mc^2}{E + mc^2}\cos\theta} \tag{2.4}$$

and of course

$$(h\nu)_2 = E + mc^2 - (h\nu)_1 \tag{2.5}$$

Here, E is the total energy and mc^2 is the rest energy of a positron.

Besides this two quanta annihilation a positron can combine with an electron and produce three quanta, or only one. For small positron energies these processes are very improbable with respect to two quanta annihilation and usually their effects can be neglected [25].

(C) Less probable primary processes.

(a) Thomson scattering

The Compton effect on the nucleus exists as well as on electrons and the discussion reviewed in section (A-b) is applicable in this case, too. But because the mass M of any nucleus is much greater than the mass of the electron we have that

$$\gamma = \frac{E_0}{Mc^2} \ll 1$$

Consequences are that the scattering of photons is elastic (see eq. (2.2)) and that the scattering cross section is given simply by the classical Thomson formula (2.3) where r_0 is replaced now by $(Ze^2)/Mc^2$.

That is

$$\frac{d\sigma_T}{d\Omega} = \frac{(Ze)^4}{(Mc^2)^2} \frac{1 + \cos^2\theta}{2} \tag{2.6}$$

or introducing the atomic weight A and substituting numerical values for the universal constants we have

$$\frac{d\sigma_T}{d\Omega} = 2.39 \times 10^{-32} \frac{Z^4}{A^2} \frac{1 + \cos^2\theta}{2} \tag{2.7}$$

(b) Rayleigh scattering

Elastic scattering of γ-rays by bound electrons, usually called Rayleigh scattering, or electron resonance scattering, may be treated in the following way: a primary photon is absorbed by an atomic electron that is excited to an energy state in the discrete or continuous spectrum.

It has been shown that in the non-relativistic limit the amplitude for Rayleigh scattering a_R can be expressed as the ordinary Thomson scattering amplitude a_T corrected by an atomic form factor, i.e.

$$a_R = a_T \cdot F(q) = \frac{e^2}{mc^2}\sqrt{\frac{1}{2}(1+\cos^2\Theta)} \cdot F(q) \qquad (2.8)$$

where q is the change of momentum of the photon that is

$$q = 2\frac{h\nu}{c}\sin\frac{\Theta}{2} \qquad (2.9)$$

and, as shown by Franz [5], $F(q)$ is the form factor

$$F(q) = \int \bar{u}\, e^{i\vec{q}\cdot\vec{r}}\, u\, d^3\vec{r}$$

where $\bar{u}u$ is the charge density. Franz used the following simplifying assumptions: (a) the Thomas-Fermi electronic charge distribution; (b) the assumption that the change of the momentum of the photon is much smaller than mc i.e. $q \ll mc$; (c) that the velocity v of the electron that scatters the quantum is non-relativistic, i.e. $(v/c)^2 \sim (\alpha Z/n)^2 \ll 1$ where α is the fine structure constant and n is the principal quantum number; and (d) he neglected binding in the intermediate state, i.e. he used plane wave functions. By this method he obtained the following result for the scattering amplitude a_F

$$a_F = \frac{e^2}{mc^2}\sqrt{\tfrac{1}{2}(1+\cos^2\theta)}\,\frac{Z}{u}\sqrt{\frac{\pi}{2u}} \tag{2.10}$$

where

$$u = 1.21 \times 10^{-3}\,\frac{Z^{1/3}}{mc}\cdot q \tag{2.10'}$$

The corresponding cross section, providing that other forms of coherent scattering are negligible, is

$$\frac{d\sigma_F(\theta)}{d\Omega} = \frac{e^4 Z^2}{(mc^2)^2}\,\frac{\pi}{2u^3}\,\frac{1+\cos^2\theta}{2} \tag{2.11}$$

or, by putting in numerical values

$$\frac{d\sigma_F(\theta)}{d\Omega} = \frac{8.67\times 10^{-33}}{\sin^3\frac{\theta}{2}}\left(\frac{Z\cdot mc^2}{h\nu}\right)^3\frac{1+\cos^2\theta}{2}\ \text{cm}^2/\text{sterad.} \tag{2.11'}$$

The scattering amplitude a_F is in phase with a_T due to Thomson scattering so that the differential cross section σ_{TF} in the case where neither of these two processes is negligible is

$$\frac{d\sigma_{TF}(\theta)}{d\Omega} = a_T^2 + a_F^2 + 2a_T a_F \tag{2.12}$$

Theoretical calculations of the scattering amplitude for this process applicable to a general case have not yet been made. Bethe [26] has calculated the contribution (of K electrons only) to the scattering amplitude using Dirac wave functions, neglecting binding energy in the intermediate states and assuming that changes of the photon momentum are larger than the characteristic momentum of K electrons. i.e.

$$q \gg \frac{mcZ}{137}$$

He has shown that, in this case, the scattering amplitude a_B is still given by eq. (2.9) where the form factor is

$$F(q') = \frac{\sin(2\gamma \tan^{-1} q')}{\gamma q' (1+q'^2)^\gamma} \tag{2.13}$$

here

$$q' = \frac{137}{Z}\frac{h\nu}{mc^2} \sin\tfrac{1}{2}\theta \tag{2.13'}$$

and

$$\gamma = \sqrt{1-(Z/137)^2} \tag{2.13''}$$

This, Bethe's expression for the scattering amplitude a_B

$$a_B = \frac{e^2}{mc^2}\sqrt{\tfrac{1}{2}(1+\cos^2\theta)}\,\frac{\sin(2\gamma \tan^{-1} q')}{\gamma q' (1+q'^2)^\gamma} \tag{2.14}$$

is again in phase with the scattering amplitude a_T due to Thomson scattering so that the differential cross section, provided that the contribution due to other elastic processes is negligible, is

$$\frac{d\sigma_{TB}(\theta)}{d\Omega} = a_T^2 + a_B^2 + 2a_T a_B \tag{2.15}$$

Rohrlich and Rosenzweig [27] have extended Bethe's calculation to the case of L-electrons. They have shown (p. 388 in [8]) that at intermediate energies (momentum transfer $\sim \alpha Z m$) the contribution of the L-electrons to the differential cross section for the elastic scattering is about 20% of that due to K-electrons. With this modification the eq. (2.15) becomes

$$\frac{d\sigma_{TB}(\theta)}{d\Omega} = a_T^2 + 1.44\, a_B^2 + 2.4\, a_T a_B \tag{2.16}$$

Levinger [6] has calculated some corrections to Bethe's form factor formula mainly applicable to small angles. We shall not discuss them here because our scattering angle is 90°.

Brown et al. [28] have shown that taking into account the binding energies of the intermediate states may change the expression for the scattering amplitude significantly. Unfortunately these calculations are also inapplicable to the elastic scattering of Co^{60} γ-rays at 90° although they are now being extended.

(c) Delbrück scattering

The interaction of strong electric fields with a photon may cause a change in its momentum. Because only atomic nuclei can produce fields strong enough and the mass of the nucleus is large, the energy of nuclear recoil is negligible and the scattering is elastic. This scattering is frequently refered to as potential scattering of light. The process takes place through an intermediate state. Two intermediate states are possible, one where an electron pair is actually produced and immediately again annihilated, and the second where the pair is only virtually produced.

Theoretically, the problem is very complicated. It has been possible to carry out calculations analytically only for the scattering angle of 0°, and this has been done by Rohrlich and Gluckstern [29] and Toll [30]. Some approximate