THE ALPHA-PARTICLE COMPONENT OF THE PRIMARY COSMIC RADIATION

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ABSTRACT

Nuclear Emulsion techniques have been used to obtain a measure of the absolute flux and energy spectrum of the alpha-particle component of the primary cosmic radiation at geomagnetic latitude 55° N. A discussion of the technique with particular reference to the limitations of the method of energy estimation through scattering measurements is presented.

A flux of 340 ± 28 particles/ m^2 / sec./ sterad., in good agreement with that reported by Waddington(1954) , has been found. An integral energy distribution which conflicts strongly with distributions given by other workers has been observed.

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INTRODUCTION

It is now well-known that the primary cosmic radiation consists of energetic nuclei of elements with atomic numbers from 1 to at least 26. Hydrogen nuclei make up 79% of the primary beam, helium nuclei 20% and nuclei of $Z \geqslant 2$ account for the remaining 1%. The origin of these particles has not yet been determined with any degree of certainty although many theories, most notable amongst which is that of Fermi (1949), have been advanced. It is recognized that an adequate origin theory must be consistent with the observed properties of the primary beam so that a knowledge of these properties is basic to the construction of such a theory. Consequently, the past five or six years have seen the attention of investigators directed toward the measurement of relative and absolute flux, mean free paths, and energy spectra of the various primary particles. Results of different workers have not, however, always been in agreement. In particular there exists some question about the true form of the energy spectrum of the alpha-particle component of the primary beam.

As the result of an investigation carried out in 1951, Kaplon et al. have reported that the integral spectrum of any given component has the general form

$$N(E) = \frac{C}{(1 + E)^n}$$

where $N(E)$ is the number of particles whose kinetic energies are in excess of E BeV., C and n are constant for a given component.

Because n enters as an exponent, it is rather insensitive to changes in N(E) and E and it might be expected that a reasonable degree of agreement might be achieved amongst workers on the value of n. This has not in fact been the case.

Kaplon gives n = 1.35 for all particles for which $Z \geqslant 2$. Pomerantz (1954) gives n = 1.2 for He nuclei, and more recently, Waddington (1954) has made a study of the alpha-particle component and has found n = 1.9 which, though in conflict with Kaplon and Pomerantz, tends to support the findings of Dainton et al. who give n = 1.9 for particles of $Z \geqslant 3$. It was felt that further investigation into the form of the energy distribution of the alpha-particle beam at low energies (0.5 - 4.0 BeV.) was called for, and it was for this purpose that the present study was undertaken.

Like many of the previous experiments in the field, this one was carried out with nuclear emulsions, but several distinct advantages over earlier emulsion work were gained through

1) carrying the emulsions to a greater height than previously done (Waddington's were flown at a depth 12 g cm^{-2}, these at 10.5 g cm^{-2}),

2) the use of very thick (1 mm.) emulsions which made it possible to obtain more long tracks than other workers have done. (The accuracy with which energy measurements can be made depends upon the track length.)

3) an extended time of flight (Eight hours as compared to Waddington's four. Thus more tracks were available; they were more readily found and hence statistics were improved.)

In this way it was hoped that some light might be thrown upon the discrepancies reported.

METHOD OF INVESTIGATION

The techniques associated with the use of nuclear emulsions were employed in this study and because these techniques are not, perhaps, so familiar as other ones, they will be described briefly here with a view to justifying them and their limitations will be discussed.

In general, the procedure is the following one. A nuclear emulsion[(1)], that is, a photographic emulsion in which the silver halide content has been increased by a factor of eight or more over that normal to photo film and whose thickness may be as great as 1 mm., is flown by means of a balloon to a height so great that it may be said that the emulsion is within a few g cm^{-2} 's of the 'top' of the atmosphere. The purpose of so doing, of course, is to minimize the probability that a given track produced in the emulsion is that of a secondary particle, that is, one which has been produced by an interaction between a cosmic ray particle coming from outside the atmosphere, i.e. a primary particle, and an atmospheric nucleus. It is maintained at this height for a period of a few hours, then brought to the ground, developped, and examined by means of a microscope[(2)] which is capable optically of both a high degree of resolution and of very high magnification and mechanically, of very precise stage movements. The well-known particle tracks are then observed.

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(1) Yagoda; see the chapter Characteristics of Nuclear Emulsions.
(2) ibid. Microscopy of Radioactive Patterns

In the present work, it was desired to measure the flux and energy spectrum of the primary alpha-particles and therefore three steps were indicated:

1) the identification of those tracks produced by alpha-particles,
2) a simple count of the number of such tracks to determine the flux,
3) the measurement of the energy of the alpha-particle as it passed through the emulsion.

Now associated with a track there are only a certain few parameters available for measurement. The most important of these are range, grain density, and scattering. Because this work was concerned only with very fast particles, it could be expected that their tracks would exhibit sensibly constant characteristics along the whole length and that they would not be stopped, except perhaps catastrophically, in the emulsion. Range measurements were therefore of no value and use was made exclusively of grain density and scattering.

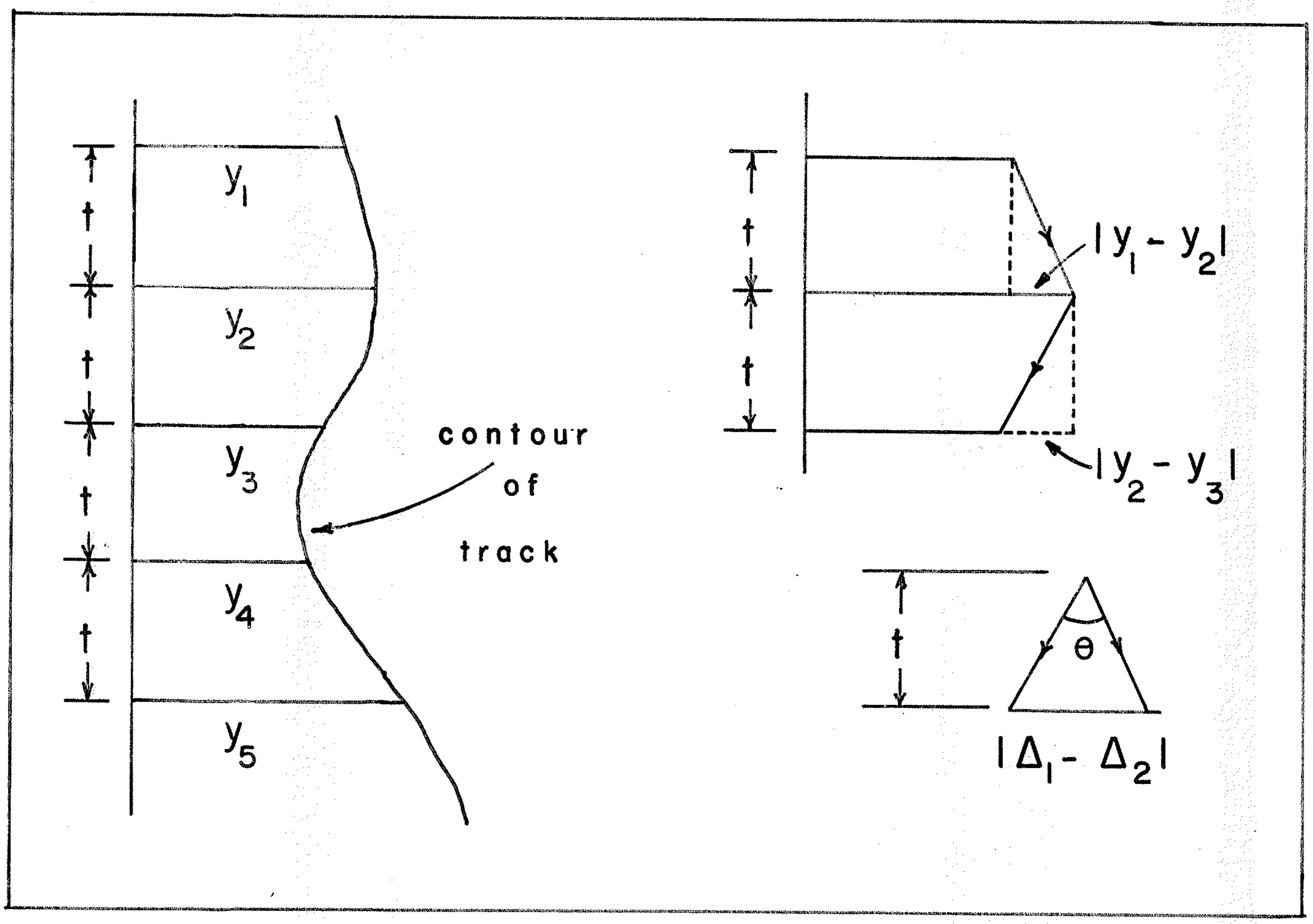
(a) Grain Density (g) is the number of developped grains per unit track length and is a measure of the rate of energy loss of the particle due to ionization. The absolute value (g) is dependent to a large extent upon the composition of the emulsion and therefore one introduces a unit grain density for a particular emulsion. The unit customarily chosen is (g_{min}), the grain density of the least dense track in the plate. This will be due to singly charged relativistic particles. In cosmic ray plates, (g_{min}) must be thought of as associated with high energy primary protons. It is then possible to express grain density by means of the ratio (g/g_{min}). The theory of ionization loss (Rossi, Sect. 2.5) indicates that for particles having the same velocity, (g/g_{min}) varies as Z^2.

(b) Scattering The theory of the multiple Coulomb scattering of a charged particle passing through matter has been treated by E.J. Williams (1939 and 1940) and by G. Moliere (1948) and their treatment has been adapted to photographic emulsion techniques by L. Voyvodic and E. Pickup (1949 and 1950) .

When a charged particle passes in the neighbourhood of a nucleus, a change in the direction in which the particle is travelling takes place due to the Coulomb interaction and this change is called the Coulomb scattering. Williams and others have attempted to calculate the differential scattering probability as a function of the scattering angle θ and have obtained expressions which reduce, in small angle approximation, to the well-known Rutherford formula

$$\Xi(\theta)\, d\omega = 4 N \frac{Z^2}{A} r_e^2 \left(\frac{m_e c}{p\beta} \right)^2 \frac{d\omega}{\theta^4}$$

It can be seen that the differential scattering cross-section is inversely proportional to the square of (βcp), where β is the velocity of the particle in units of 'c' and where 'p' is the momentum. (pβ) has the dimensions of momentum, but the momentum, expressed in e.v./c is numerically equal to the energy of the particle expressed in e.v.



y_1
y_2
y_3
y_4
y_5
t
t
t
t
contour of track
$|y_1 - y_2|$
$|y_2 - y_3|$
t
t
t
θ
$|\Delta_1 - \Delta_2|$

FIGURE I

In applying this to use in nuclear emulsion techniques, it is customary to estimate the mean deflection, $\bar{a}(t)$, for a track. This is done by dividing the track into cells of equal length (t) and measuring the changes in direction of the track from cell to cell. $\bar{a}(t)$ is the mean of such a set of deflections. The angles may be measured directly through the use of a goniometer, but a preferred method is that described by Fowler and called the 'sagitta' method. Because it was used in the present study, a brief description of it will be given here.

The track is aligned as closely parallel as possible to the y-motion of the microscope stage. Lateral track co-ordinates are read at successive displacements (t) along the y-direction. Differences of successive co-ordinate readings, $\Delta_1 = y_1 - y_2$ etc., give a measure of the slope of adjacent imaginary chords. The absolute values of the second differences, $|\Delta_2|$, give a measure of the angles between successive chords. This is made clear in Figure 1.

The arithmetic mean, $\overline{|\Delta_2|}$, is next formed. It is associated with $\bar{a}(t)$ through

$$\bar{a}(t) = \frac{\overline{|\Delta_2|}}{t} \times \frac{180}{\pi} \qquad \text{degrees.}$$

The textbooks introduce the variation, $\bar{\bar{a}}$, the mean deflection per 100 microns (μ), which is related to $\bar{a}(t)$ by

$$\bar{\bar{a}} = \frac{\bar{a}(t)}{t^{\frac{1}{2}}} \qquad \text{if (t) is in units of 100} \quad .$$

Here, however, (t) is expressed in units of 1 micron and so a factor of 100 must be introduced into the foregoing equation. Thus, where the sagitta technique has been used, the mean deflection per 100 microns is simply

$$\bar{a} = \frac{\overline{|\Delta_2|}}{t^{3/2}} \times \frac{180}{\pi} \times \frac{1}{100} \qquad \text{degrees.}$$

Now from the scattering formula of Williams, we have the relation

$$\bar{a} = \frac{K\,Z}{p\,\beta}$$

called the scattering-energy relationship. When $\bar{a}$ is expressed in degrees, $p\beta$ in MeV., K lies between 20 and 40[3]. K is a function of cell length (t) and the function $K = K(t)$ appears in Voyvodic's article[4]. Substituting this for $\bar{a}$ above gives

$$\overline{|\Delta_2|} = \frac{\pi}{180}\; t^{3/2}\; \frac{K\,Z}{p\,\beta} \times 100$$

whence $$\overline{|\Delta_2|}^{-1} \propto (p\,\beta)$$

and it is this proportionality of which use is made in estimating the energy associated with the tracks being examined.

If the scattering of a track were due entirely to the Coulomb effect, the reciprocal of the scattering as measured would give the particle energy directly, as has been indicated. It turns out, unfortunately, that the measured scattering $\overline{|\Delta_2|}$ depends upon several factors other than the one already treated. These new factors can be

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(3) Voyvodic, Prog. Cos. Ray Phys. Vol.11 P. 234

(4) ibid. P.273, Fig. 17

divided into two classes, namely 1) those arising from measuring errors for which correction can be made, and 2) those factors arising from the shrinking of the emulsion and the loss of perhaps as much as 2/3 of its original volume, suffered in the process of developping. As has been pointed out by Peters and recently confirmed by Lohrmann and Teucher (1956), these factors produce an error for which it appears at present that no correction is available. These factors do not appreciably affect energy measurements up to perhaps 0.5 BeV.(5) but energies much beyond that figure must be greatly underestimated where no allowance is made for what will be called hereafter, the 'Peters' effect'.

While it is now abundantly clear that scattering measurements on tracks made by high energy particles are going to yield results which cannot be regarded as systematically related to energy, still, such measurements certainly do provide a lower limit for the energy, and as long as such an interpretation is taken, scattering measurements will retain their validity in this limited sense.

Identification The grain density and the scatter of a track together determine the identity of the particle that produced it. It is found that a plot of the relative grain density against the reciprocal of the scattering for a group of tracks

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(5) Peters, as well as a private communication from E. Pickup.

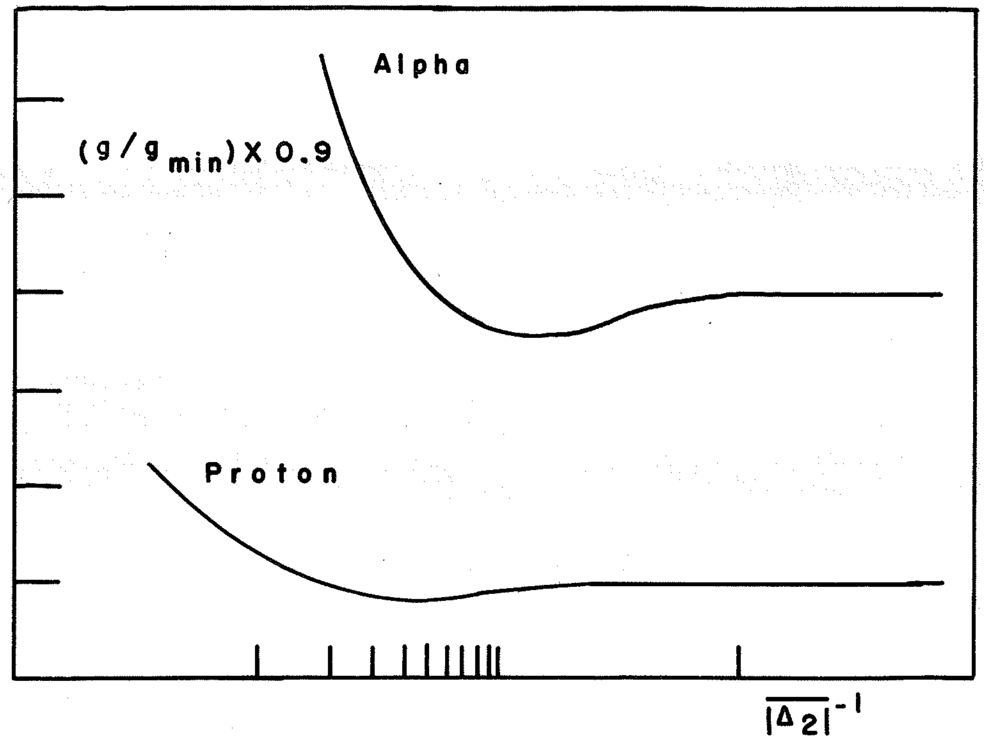

FIGURE II

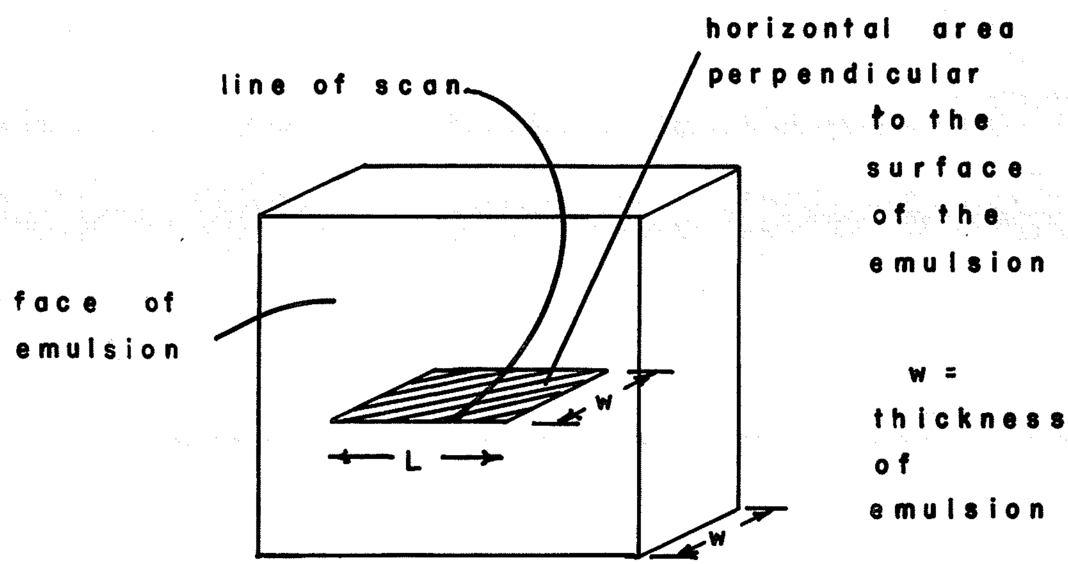

FIGURE III

made by the same sort of particle determines a curve which is characteristic of the particle. This is fully discussed in Voyvodic's article. Such curves have the form indicated in Figure 11. The horizontal scale upon which $\overline{|\Delta_2|}^{-1}$ is plotted is logarithmic, the vertical is linear. It is to be noticed that as energy increases, the curve has a steep decline in grain density, it goes through a trough and then flattens out into a plateau. The results of various workers(7) indicate that the minimum for a curve lies about 10% below the plateau value. In the present case, since g_{min} belonged to protons, the minimum value of the proton curve is called g_{min} and, following the custom, the plateau of this curve is taken as unit relative grain density. Therefore $g_{min} = 0.9$ units and hence the relative grain density on the graph is given by

$$(g/g_{min}) \times 0.9.$$

The position of the alpha-particle curve relative to the proton curve is determined by the consideration that, for particles 1 and 2 having the same velocity, having atomic numbers Z_1 and Z_2 and grain densities g_1 and g_2,

$$g_1/g_2 = (Z_1/Z_2)^2$$

which, where 1 and 2 are proton and alpha-particle, becomes

$$g_{alpha} = 4\, g_{proton} \quad .$$

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(7) Voyvodic, Prog. Cos. Ray Phys. P. 255

If E_{alpha} and E_{proton} are the energies of the two, then where they have the same velocity, β ,

$$E_{alpha} = 4\, E_{proton}$$

because

$$E_{alpha} = M_{alpha}\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$$

$$E_{proton} = M_{proton}\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$$

Hence
$$\frac{E_{alpha}}{E_{proton}} = \frac{M_{alpha}}{M_{proton}} = 4 .$$

With the help of these curves, a particle may be identified by simply locating the point ($g/g_{min} \times 0.9$) and $\overline{|\Delta_2|}^{-1}$ on a diagram such as that in Figure 11. This was the procedure followed in the present investigation.

Corrections for Scattering Errors It has already been observed that the scattering as measured is not directly proportional to the energy. Instead, it is distorted by various effects which must be accounted for. It was pointed out that nothing could be done about the error introduced by the spurious scattering described by Peters, but that there were other disturbances which produce error and for which measurements could not be corrected. The predominence of these effects over the Peters' effect varies inversely with energy, so that at low energies it is expected that they are responsible for the greater part of the error. A description and a detailed analysis of the systematic errors are given here.

The errors are these:

(a) Temperature Noise, which is caused by the differential thermal expansion of the microscope parts during the measuring period.

(b) Setting and grain noise, which arises partly from the failure of the observer to place the hair-line of the microscope eyepiece accurately, and partly from the fact that the grains of developped silver have a finite size and the particle trajectory does not necessarily pass through their centres of gravity.

(c) Stage Noise, arising from the imperfection of the linearity of motion of the microscope stage.

(a) <u>Temperature Noise</u> This is observed as a uniform motion of the optical axis relative to the stage.

After some experimenting, temperature noise was rendered practically negligible in the present investigation by the observance of the following precautions.

1) All heat sources, such as desk lamps, were removed from the neighbourhood of the microscope.

2) The microscope was permitted to reach thermal equilibrium after the illumination was switched on before any measurements were attempted. If at any time during the day temperature noise was observed to increase due to changes in the temperature of the room, measurements were discontinued until a new equilibrium had been attained.

3) Successive readings were taken at as close to regular intervals as possible . In this way, even if thermal shift were taking place, its linear character would prevent it from interfering with the second differences.

It was not possible to prevent factors (b) and (c) from altering the readings, and so it was necessary to carry out an analysis of their effect upon the scattering.

Recalling that the sagitta method involves measuring the displacement of the track from some arbitrary ideal straight line, one can regard an observed displacement x_i as the sum of two parts:

1) a_i , the 'true' displacement of the track, and

2) λ_i , the displacement due to the grain and stage noise.

$$\text{i.e.} \quad x_i = a_i + \lambda_i \quad .$$

Suppose the set of such displacements x_i to have been noted. If now the track is shifted vertically (in the y-direction) along the stage, and if measurements are repeated along the same portion of the track as before, at precisely the same points as before (and in fact this can be done with a considerable degree of success), then one may assume a_i to be unchanged, but because the measurements are carried out on a new part of the stage motion a change may be looked for in the stage and setting noise. Let the new noise be represented by Λ_i. The observed displacement will then be given by X_i where

$$X_i = a_i + \Lambda_i \quad .$$

Denoting second differences by the prefix Δ , one has

$$\Delta x_i = \Delta a_i + \Delta \lambda_i$$

$$\sum (\Delta x_i)^2 = \sum (\Delta a_i)^2 + \sum (\Delta \lambda_i)^2 \qquad \ldots(1)$$

Similarly $$\sum (\Delta X_i)^2 = \sum (\Delta a_i)^2 + \sum (\Delta \Lambda_i)^2$$

since the λ_i's and the a_i's are surely independent.

Taking the difference above, one has

$$\sum (\Delta x_i)^2 - \sum (\Delta X_i)^2 = \sum (\Delta \lambda_i)^2 - \sum (\Delta \Lambda_i)^2 .$$

Now it is reasonable to suppose that

$$\sum (\Delta x_i)^2 = \sum (\Delta X_i)^2$$

where i is summed over a sufficiently large range. Hence the right hand of the foregoing equation is identically zero as has been indicated above, and therefore

$$\sum (\Delta \lambda_i)^2 = \sum (\Delta \Lambda_i)^2 \qquad \ldots(2)$$

One can next introduce the quantity

$$s_i = X_i - x_i = \Lambda_i - \lambda_i \qquad \ldots(3)$$

Forming the second differences, $\Delta s_i = \Delta \Lambda_i - \Delta \lambda_i$,

one has finally

$$\sum (\Delta s_i)^2 = \sum (\Delta \Lambda_i)^2 + \sum (\Delta \lambda_i)^2 .$$

Again, this is true only if $\Delta \lambda_i$ and $\Delta \Lambda_i$ are independent.

The contributions to λ and Λ from grain and setting noise are certainly independent. Those that arise from imperfect linearity of stage motion may be systematically connected if there is, for example, a periodicity in the motion. Although a rigorous test for such a periodicity has not been carried out, there was no indication during the course of the measurements that it existed. The independence of stage noise readings has been accepted here as a fairly plausible assumption.

One has, then, that

$$\sum (\Delta s_i)^2 = 2 \sum (\Delta \lambda_i)^2 \qquad \ldots(4)$$

Therefore,

$$\sum (\Delta a_i)^2 = \sum (\Delta x_i)^2 - \tfrac{1}{2} \sum (\Delta s_i)^2 \qquad \ldots(5)$$

That is,

$$\overline{|\Delta a_i|}^2 = \overline{|\Delta x_i|}^2 - \tfrac{1}{2} \overline{|\Delta s_i|}^2 \qquad \ldots(6)$$

This last step is based upon the fact that for a given distribution for the Δ's, there is the simple relation

$$\overline{|\Delta|}^2 = k \cdot \frac{\sum \Delta^2}{n} = k\,(\overline{\Delta^2})$$

where the sum on i is up to n and where k is a numerical factor depending for its value upon the form of the distribution. As long as all the statistical quantities involved do in fact follow the same distribution function - they are usually approximately Gaussian - k will be the same for them all and the last step is valid.

The following is an example of the way in which equation (6) has been used in this investigation. It has been found by following the procedure just outlined, that $\overline{|\Delta s_i|} = 0.08 / 500$, for the present microscope and observer. Consider a track whose observed scattering is 0.09 / 500 . This, when referred directly to the energy scale, indicates a particle energy of about 11 Bev. Applying equation (6), one has

$$\overline{|\Delta a_i|^2} = 0.0081 - 0.0032 = 0.0049$$

whence

$$\overline{|\Delta a_i|} = 0.07\mu / 500\mu \, .$$

This corresponds to an energy of about 15.7 BeV. per particle.

FLUX AND ENERGY SPECTRUM THEORY

1 Flux The primary flux of alpha-particles, that is, the number of primary alpha-particles incident upon the top of the atmosphere per square metre, per second, per steradian has been measured with nuclear emulsions (Goldfarb et al.), with proportional counters (Perlow and Davis), and with scintillation counters (Ney and Thon) and these measurements have been made at various geomagnetic latitudes. It has of course so far been impossible to carry the measuring apparatus entirely beyond the atmosphere, but with the advent of the 'skyhook' balloon it has become feasible to post equipment for several hours at altitudes such that only 10 to 15 g cm^{-2} of residual atmosphere remain above the detector to interfere with the primary beam. It is necessary, even at these altitudes, to apply corrections to the observed flux values. Two steps are required, namely

1) from a count of the number of alpha-particles present, to obtain a measure of the flux at the plate, and
2) to extrapolate this flux value back to the top of the atmosphere to obtain a value for the primary flux.

1) **Flux at the Plate** It is presumed that the plate lies in a vertical plane. The tracks which are counted pass through a horizontal cross-section of the emulsion, bounded upon the surface by the line of scan. Figure 111 serves to clarify this.

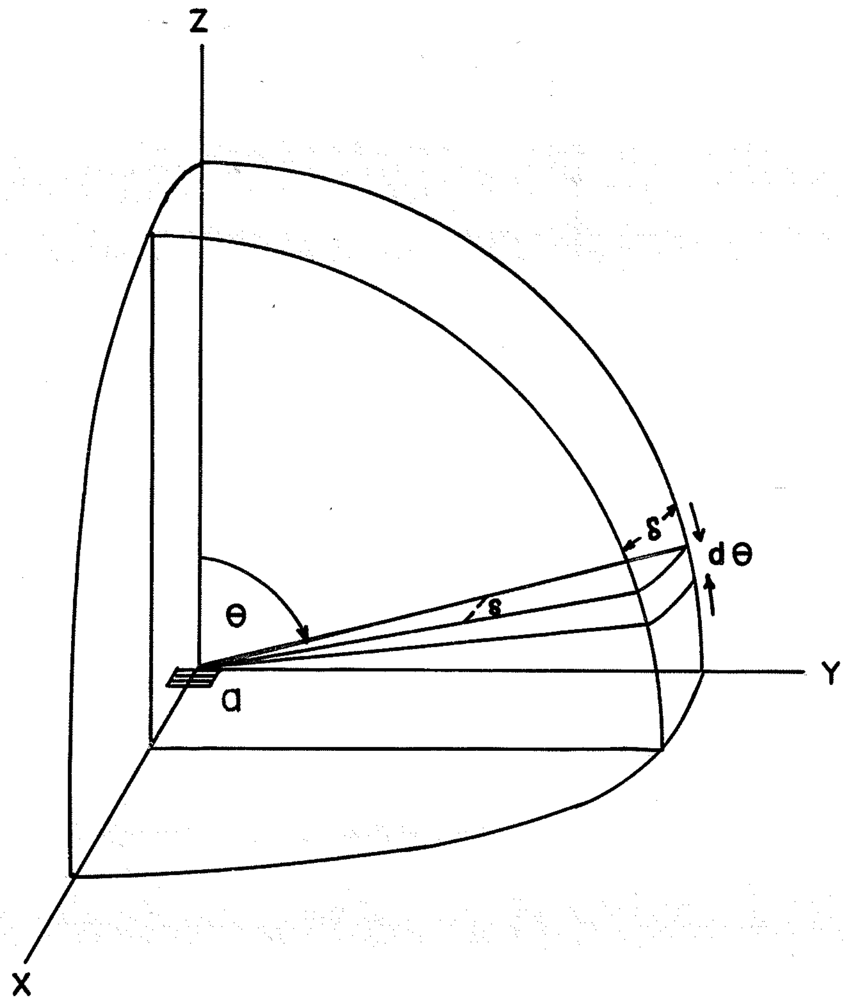

FIGURE IV

Reference will now be made to the diagram in Figure 1V. An element α of the area just mentioned is located at the centre of unit sphere; its normal is in the $+z$ direction. Two planes parallel to the yz plane, namely $x = a$ and $x = -a$ cut off an area on the sphere's surface whose width subtends the angle (δ) at the origin. The angle θ measured as shown defines the position of an element of this spherical surface area. Such an element itself has the area $\delta \, d\theta$. δ is defined as $= w/\ell$ where (w) is the thickness of the emulsion and (ℓ) is the length of the shortest acceptable track. Introducing the flux at the plate as n particles/ m^2/sec./sterad., one can write that the flux reaching from $\delta \, d\theta$ is dN, given by

$$dN = (\alpha \cos \theta) \, n \, (\delta \, d\theta) \, \tau$$

where τ is the time taken.

We have thus

$$N = n \alpha \delta \tau \int_{-\theta_1}^{\theta_1} \cos \theta \, d\theta$$

$$= 2 \, n \alpha \delta \tau \sin \theta_1 .$$

Further, integration is carried over all elements α , which results simply in the replacing of α in the foregoing equation by (L w). Finally, in the present instance, θ_1 was taken as 60°. Hence $\sin \theta_1 = \sqrt{3}/2$ so that the expression for N is

$$N = 2 \, n \, L \, w \, (w/\ell) \, \tau \sqrt{3}/2 .$$

Upon solving for n, the flux at the plate, one has

$$n = (N/L) \cdot \frac{\ell}{w^2 \tau \sqrt{3}} \quad \text{particles/ m}^2\text{/sec/ sterad.}$$

The bracketted quantity is a constant for the investigation ; (N/L) is the number of particle tracks per unit length of scan.

It is next necessary to obtain from n a measure of the primary flux I.

2) <u>Flux at the Top of the Atmosphere</u> The few g cm^{-2} of atmosphere above the detecting emulsion affect the observed flux in a way which must be taken into account. Three effects are anticipated. They are the following ones.

(a) Some primary alpha-particles will be removed from the beam through collisions with atmospheric nuclei.

(b) Some secondary alpha-particles will be introduced into the beam through the collision of heavy primaries with atmospheric nuclei.

(c) The beam of secondary alpha-particles so produced will itself be attenuated somewhat by the atmosphere through which it passes before it reaches the emulsion.

(a) The correction for the first effect mentioned above is readily determined from a knowledge of the mean free path of alpha-particles in air, glass, and emulsion. It has been shown by Bradt and Peters (1950) that the effective collision radius for the collision of a heavy primary with the particles of the medium through which it passes is approximately the sum of the geometric radii ($R_1 + R_2$) where $R_{geom} = r_o A^{1/3}$ and where $r_o = 1.45 \times 10^{-13}$ cm., less a certain decrement, ΔR, which may be expected to be of the order of the range of nuclear forces. If R is the geometric radius of the incident nucleus and R_i is the geometric radius of the nuclei of kind (i) present in the absorbing medium, then, according to Bradt and Peters, the collision cross-section is

$$\sigma_i = \pi (R + R_i - 2 \Delta R)^2 \qquad \text{...(1)}$$

where ΔR, determined empirically, is $= 0.85 \times 10^{-13}$ cm.

The mean free path, λ, to be associated with the σ_i's is given by

$$\lambda = \frac{1}{\sum_i n_i \sigma_i} \text{ cm.} \qquad \text{...(2)}$$

where n_i is the number of nuclei of kind (i) per unit volume in the medium. In at least one of the cases with which one has to deal here, namely that where the medium is the air of the upper atmosphere, n_i is not constant but a function of position. A small modification in equation (2) above is necessary to handle this.

Suppose one has a medium composed of (s) elements. Let $n_i(\underline{r})$ represent the number of nuclei of type (i) per unit volume at some location $\underline{r}$ in the medium. Then the density at $\underline{r}$ will be given by

$$\rho (\underline{r}) = \sum_{i=1}^{s} n_i m_i$$

where m_i is the mass of the nuclei of the ith kind. Define $q_i \equiv n_i/n_1$ and assume that the relative composition of the medium is constant throughout. One then has an expression for the density at any place $\underline{r}$ in the medium in terms of the number of nuclei of type $i = 1$ at that place, and the q_i's which are not functions of position.

One has

$$\rho(\underline{r}) = n_1(\underline{r}) \sum_{i=1}^{s} q_i m_i \quad .$$

One has thus an expression for $n_1(\underline{r})$ in terms of $\rho(\underline{r})$, namely

$$n_1(\underline{r}) = \frac{\rho(\underline{r})}{\sum_i q_i m_i}$$

and for $n_j(\underline{r})$ in terms of $\rho(\underline{r})$, namely

$$n_j(\underline{r}) = n_1(\underline{r})\, q_j = \frac{q_j \, \rho(\underline{r})}{\sum_i q_i m_i} \qquad \ldots(3)$$

Now $\lambda = \dfrac{1}{\sum_i n_i(\underline{r})\, \sigma_i}$ cm. or $\lambda = \dfrac{\rho(\underline{r})}{\sum_i n_i(\underline{r})\, \sigma_i}$ g cm^{-2}

Substituting expression (3) into the above, one gets

$$\lambda = \frac{\rho(\underline{r})}{\sum_i \sigma_i \left\{ \dfrac{q_i \; (\underline{r})}{\sum_j q_j m_j} \right\}} = \frac{\sum_j q_j m_j}{\sum_i \sigma_i q_i}$$

i.e. $$\lambda = \frac{\sum_i q_i m_i}{\sum_j q_j \sigma_j} \quad g\ cm^{-2} \qquad \ldots(4)$$

This expression has no positional dependence and is therefore suited to the present purpose. By means of it, one can calculate the mean free path of alphas in air, glass, and emulsion , (λ_{air}, λ_{gl}, λ_{em}.). These appear in the following provisional relation between observed flux (n) and primary flux (I).

$$n = I \exp -\left(\frac{x_{air}}{\lambda_{air}} + \frac{x_{gl}}{\lambda_{gl}} + \frac{x_{em}}{\lambda_{em}} \right)$$

where x_{air} , x_{gl}, and x_{em} are the average distances in g cm^{-2} of air, glass, and emulsion traversed by the alpha-particles before crossing the line of scan.

(b) Secondary particles may be produced above the emulsion by one of two possible processes, 1) by the collision of primary protons and alpha-particles with the oxygen and nitrogen nuclei of the atmosphere, and 2) by the collisions between heavy primaries ($Z \geqslant 3$) and the atmospheric nuclei.

In the first case, the collision of the primary with the relatively heavy nucleus produces a shower of high energy singly charged particles as well as some multiply charged 'evaporation' particles, most of which seem to be alphas. Perkins (1950), however, has shown that the energy of such alphas seldom exceeds a few MeV/ nucleon in the rest system. Because the mass of the primary is much smaller than that of the target nucleus the energy of the alphas produced in this way never approach the relativistic range in the laboratory system. Secondary alpha-particles originating in this way will not, therefore, interfere with the primary beam.

The collision of heavy primaries with atmospheric nuclei will also give rise to evaporation alphas which again will have an energy of only a few MeV/ nucleon in the rest system, but whose observed energy/ nucleon in the laboratory system will be essentially the energy/ nucleon of the primary. Consequently this may be regarded as a source of relativistic secondary alpha-particles. In this connection, Gottstein (1954) has reported an average of 1.5 secondary alphas per collision.

In order to calculate the number of relativistic secondary alpha-particles, one needs to know the number of collisions between heavy primaries and atmospheric nuclei that have occurred above the plate. A somewhat over-simplified picture leads one to say that where is the average mean free path for heavy primaries, the number of such events in a depth (h) is given by

$$N(0)\ (1 - e^{-h/\lambda})$$

where N(0) is the total primary flux of heavies. When this number is multiplied by 1.5, the result is the total alpha-particle flux produced in this way.

A more rigorous analysis has been suggested by Gottstein. Assigning all nuclei for which $3 \geqslant Z \geqslant 5$ to class A, average m.f.p. λ_A

$6 \geqslant Z \geqslant 9$ B, λ_B

$10 \geqslant Z$ C, λ_C

he considers the 'step-decays' that may occur. Thus, C nuclei decay to B and A types, B nuclei to A types and A types vanish by decaying into alphas and nucleons. In order to calculate the total number of decays and therefore of collisions giving rise to secondary alphas that have occurred above the emulsion, it is necessary to solve three differential equations of the first degree. These, though complicated by a multitude of terms of the zeroth degree, are really perfectly straightforward and were solved with no difficulty. The resulting total number of events, taking $h = 10.5$ g cm^{-2}, was 9. The number of relativistic alphas to be expected is therefore about 14 particles/ m^2/ sec./ sterad. If the simple picture had been used, taking

$$\lambda = \frac{N_A(0)\ \lambda_A + N_B(0)\,\lambda_B + N_C(0)\,\lambda_C}{N_A(0) - N_B(0) - N_C(0)}$$

$$\lambda = \frac{(0.4 \times 33) + (5 \times 26) + (2 \times 20)}{7.4}$$

$\lambda =$ 25 g cm^{-2}, where the various values are Gottstein's, the number of secondary alphas would have appeared to be about 4.

(c) The attenuation of the beam of secondary alphas by the atmosphere can be most readily obtained by making the simplifying assumption that they were all produced at a level half-way between the plate and the top of the atmosphere. This simplification has been discussed in Waddington's paper. He states there that practically nothing is gained by a more rigorous treatment.

One has that the number, Z , of secondary alphas/m^2/ sec./sterad. observed at the plate is

$$Z = 14 \exp\left(- \frac{h/2}{h} \right)$$

$$= 8.5 \,/\, m^2/ \text{ sec/ sterad.}$$

The number of primaries in the flux observed at the plate can therefore be written (n - Z) and hence the primary flux at the top of the atmosphere is given by

$$I = (n - Z) \exp\left(\frac{x_{air}}{\lambda_{air}} + \frac{x_{gl}}{\lambda_{gl}} + \frac{x_{em}}{\lambda_{em}} \right)$$

2 Energy Spectrum Before a primary alpha-particle reaches the detecting emulsion, it passes through some length of residual atmosphere, through the glass backing of the emulsion, and some distance through the emulsion itself before reaching the line of scan. In its passage it loses energy so that the energy observed in the plate is not the primary energy of the particle.

If residual atmosphere were the only medium which interfered with the progress of the particle, the correction might be made in this way. After having traversed x g cm^{-2} of atmosphere, the particle is known to have an energy E. There can be associated with E a residual range r g cm^{-2} in air which is the distance which the particle can yet be expected to travel before being brought to a stop by ionization loss. The number r can readily be obtained from a range-energy diagram such as that in the appendix. One can think of a primary particle about to enter the atmosphere with an energy E' and a range (x + r) g cm^{-2}. Since this range is known, then the same range-energy diagram as before yields E', the primary energy being sought.

To account for the x_1 g cm^{-2} of glass and the x_2 g cm^{-2} of emulsion through which the particle passes, one need only express x_1 and x_2 in terms of g cm^{-2} of atmosphere, and proceed as before.

An additional factor will take account of the different angles of approach of the various particles. In order to write down x, the number of g cm^{-2} of atmosphere traversed by the alpha-particle before reaching the scan, one must multiply an element of path length, ds, by the density at that element, then integrate along the path. Thus, if the particle had come vertically downwards (azimuth $\theta = 0$) we should have simply

$$x = \int_{h_o}^{h_1} \rho \, dh \quad \text{g cm}^{-2}$$

where h_o is the altitude of the top of the atmosphere, h_1 the altitude of the emulsion. Atmospheric density is , of course, a function of depth only, i.e. $\rho = \rho(h)$. A particle moving in an azimuthal direction θ however, has $ds = d(h \sec\theta) = \sec\theta\, dh$, instead of simply dh for its element of path length.

Thus

$$x(\theta) = \sec\theta \int_{h_\theta}^{h_1} \rho(h)\, dh \quad g\ cm^{-2} \quad .$$

But the value of the integral is just the depth of the emulsion expressed in $g\ cm^{-2}$. That is, if h_1 is expressed as $h_1\ g\ cm^{-2}$, then the distance which has been traversed by the alpha-particle moving along a direction θ is given by

$$x(\theta) = h_1 \sec\theta \quad g\ cm.^{-2} \quad .$$

A stack of glass-backed Ilford G5 emulsions was flown for eight hours at an altitude of 103,000 feet (10.5 g cm^{-2}) at geomagnetic latitude 55° N [(6)]. The dimensions of the emulsions were 4 inches by 4 inches by 1 m.m. The glass plates upon which they were supported were 2 m.m.'s thick. One of these emulsions was examined by means of a Leitz Ortholux Nuclear Research microscope which was equipped with a Leitz 40X objective for scanning purposes and a Koristka 100X oil immersion objective for track measurement. The stage of the microscope was slightly modified in that a micrometer screw was installed to provide a more precise y-motion, so that in scattering measurements, cell lengths could be made as nearly equal to each other as possible. During actual measurements, the plate was kept fixed to the stage by means of blobs of plasticene.

Track Selection Scanning was done along horizontal lines parallel to the top edge of the plate. Three such lines, separated each from its neighbour by 20 m.m.'s, were scanned and tracks were selected which satisfied the following criteria.

(1) The track must cross the line of scan.

(2) The track's length in emulsion must have been greater than or at least equal to 15 m.m. This automatically eliminated all slow singly charged particles and ensured a sufficient length of track for an adequate determination of the scattering parameter.

---

(6) South of Minnesota . The geomagnetic latitude comes from a diagram given on page 569 of Heisenberg's Kosmische Strahlung .

(3) The angle between the track and the y-axis (which was vertical during the flight) must have been 60^{o} or less. This kept the correction for direction of approach within reasonable limits, because with $\theta_{max} = 60^{o}$, sec 0 lies between 1 and 2.

(4) The grain density must have been about four times minimum grain density. This ensured the elimination of all fast singly charged particles.

Tracks which crossed two lines of scan were, of course, counted only once. Because of the wide separation of the scans , there were not a great many of these, but those that there were were readily recognized. Recognition was accomplished in two ways . 1) The position of the upper end of the track was recorded. The positional co-ordinates of the tracks in the second scan were compared with those tabulated for the first scan and the coincidences were eliminated from the list of tracks in the second scan. 2) Any track which crossed the second line of scan which was long enough to cross the first line of scan should have been observed and measured on the first scan. That this was in fact so was checked by comparing positional co-ordinates. It is presumed that because a close check of this sort was kept, few if any tracks were counted more than once.

In every case, a preliminary scattering measurement and grain count was made to determine the identity of the particle. Only those tracks which proved to be certainly due to alpha-particles were finally recorded.

<u>Track Measurement</u> In order to make measurements on grain density, it was necessary to determine g_{min} for the plate. An examination of various tracks of singly charged particles led the observer to conclude that there were no tracks with grain density less than 4.5 grains per unit track length. The unit of track length was arbitrarily chosen as the length of the microscope eyepiece scale. Grain densities were measured by counting the number of grains per unit length for many such units and then averaging. The total number of grains counted usually varied between 500 and 1000. This ensured that the statistical error in the grain density was not greater than 3%. Grain densities were finally expressed in the form

$$g/g_{min} \times 0.9$$

where g was the average number of grains per unit track length.

Scattering measurements were made by means of the sagitta technique already described. The basic cell-length adopted for this purpose was 0.5 m.m. and the identification curve was constructed on this basis. It was found, as scanning progressed, however, that a 1 m.m. cell-length was to be preferred and this was used in practice and the results so obtained were converted to a scale modulo 0.5 m.m. by dividing by $2^{3/2}$ (7).

---

(7) Where Δ represents the scattering and t the cell-length,

$$\Delta_1 \propto t_1^{3/2}$$

$$\Delta_2 \propto t_2^{3/2}$$

If $t_1 = 2t_2$, $\quad \dfrac{\Delta_1}{\Delta_2} = \left(\dfrac{t_1}{t_2}\right)^{3/2} = 2^{3/2}$

$$\therefore \quad \Delta_2 = \frac{\Delta_1}{2^{3/2}}$$

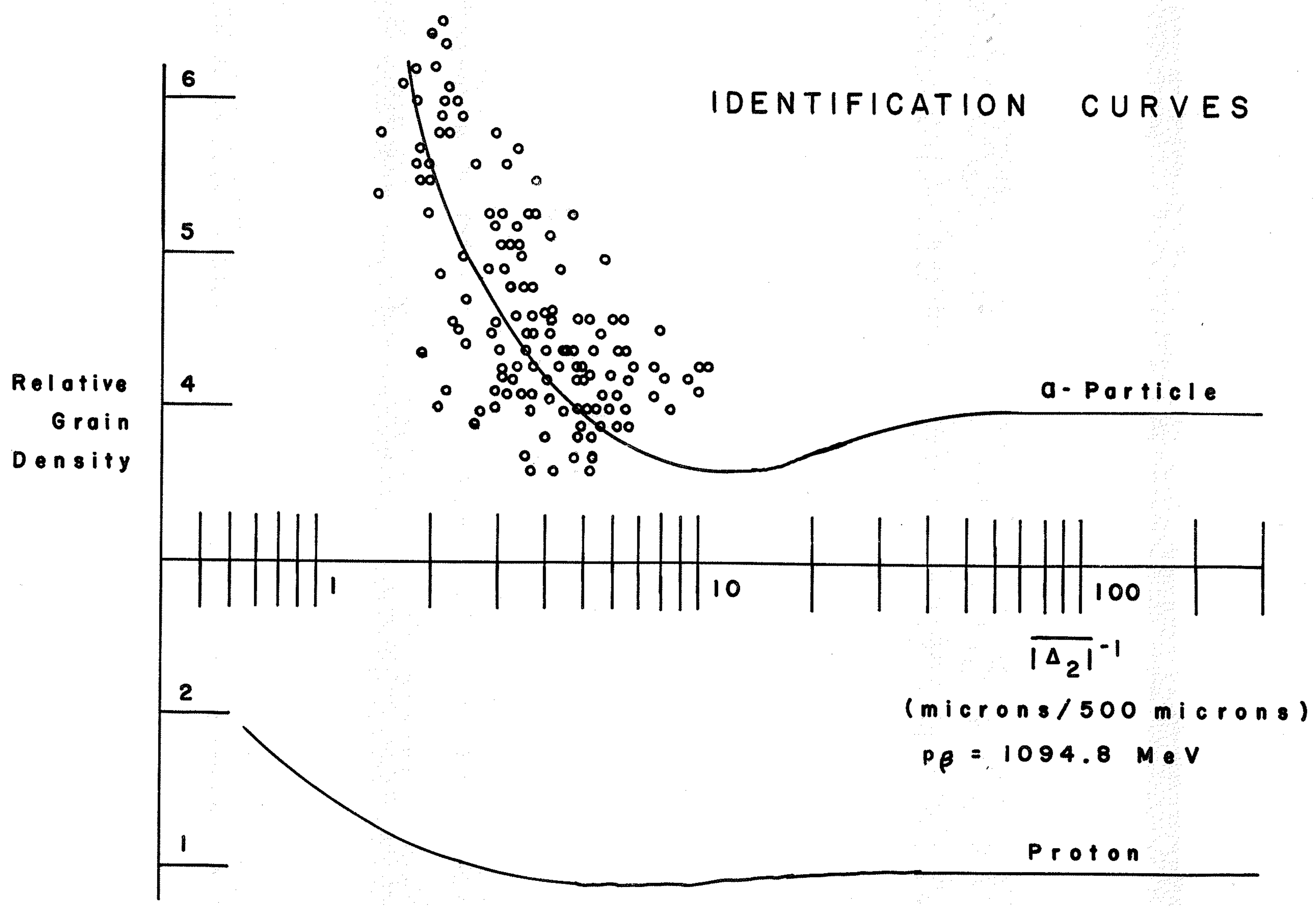
IDENTIFICATION CURVES
Relative Grain Density
6
5
4
2
1
1
10
100
$\overline{|\Delta_2|}^{-1}$
(microns/500 microns)
$p\beta$ = 1094.8 MeV
α- Particle
Proton

FIGURE V

RESULTS

155 tracks assumed to be due to primary alpha-particles were obtained and they are exhibited as points on the $g = g\ (\overline{|\Delta_2|}^{-1})$ curve in Figure V. No particles were found whose scatter was greater than $\overline{|\Delta_2|}^{-1} = 1.2$ which corresponds to about 1.3 BeV. This is consistent with the geomagnetic cut-off for 55° N. (0.33 BeV./nucleon, quoted in Waddington's paper), and appears to indicate that at this energy the Peters' spurious scattering is not too important. The departure of the points from the curve in the region of the trough , due to a grain density greater than that which was expected, has not been explained. The smallest scatter measured was $\overline{|\Delta_2|}^{-1} = 11$ units, this being of the order of the stage noise.

<u>Flux</u> Using the formula hitherto developped, the mean free paths of the alphas in air, glass, and emulsion were calculated on the basis of the following compositions.

<u>1</u> air ... N,O ... 5:1

<u>2</u> glass ... SiO_2 ... 1:2

<u>3</u> emulsion is assumed to consist in the main of Ag Br and gelatin in the proportions 3:1 by weight. The composition of the gelatin is given by Yagoda(8) as

Carbon	...	50%
Hydrogen	...	7%
Nitrogen	...	18%
Oxygen	...	25%

---

(8) Yagoda, P.87, Table 7

The resulting values for the mean free paths were

$$\lambda_{air} = 45 \text{ g cm}^{-2}$$

$$\lambda_{gl} = 54 \text{ g cm}^{-2}$$

$$\lambda_{em} = 72 \text{ g cm}^{-2}$$

The average grammage of atmosphere traversed, x_{air}, was calculated on the assumption that the average azimuthal angle was 30°. This resulted in

$$x_{air} = 10.5 \times \sec 30^\circ$$

$$= 12.2 \text{ g cm}^{-2}.$$

The average grammage of glass traversed was taken as

$$x_{gl} = 2.2 \text{ g cm}^{-2}$$

and that for emulsion, as

$$x_{em} = 4.1 \text{ g cm}^{-2}.$$

Now n, the flux observed at the plate, is given by

$$n = N/L \cdot \frac{\ell}{w^2 \tau \sqrt{3}}$$

where ℓ, the minimum acceptable track length was 15 m.m.'s

w, the thickness of the emulsion, was 1 m.m.

τ, the time of exposure, in seconds, was 8 x 3600 seconds

N, the total number of alpha-particle tracks, 155

L, the length of the scan, 19.6 cm.

Hence $n = 242$ particles/m^2/sec./sterad.

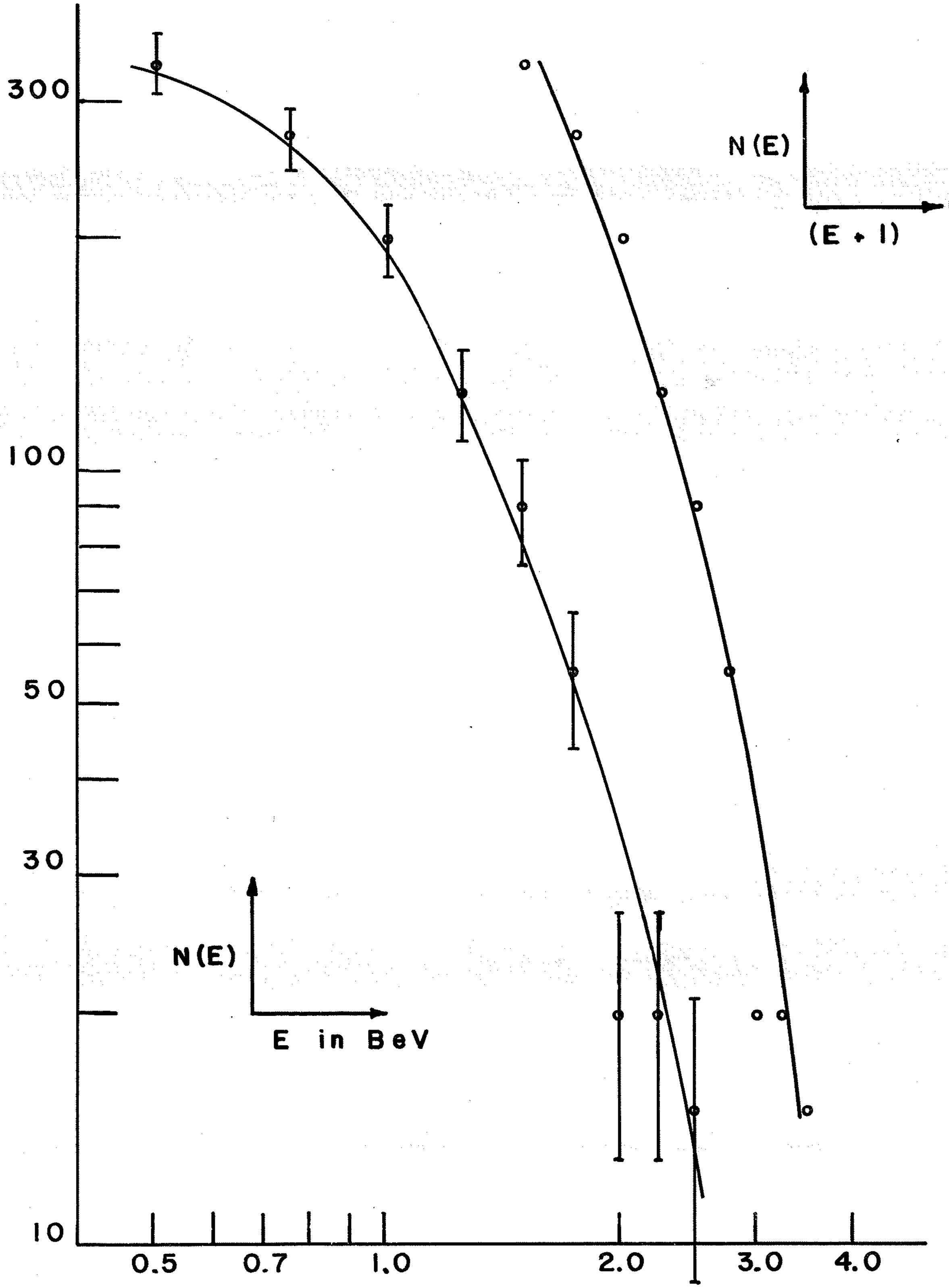

FIGURE VI

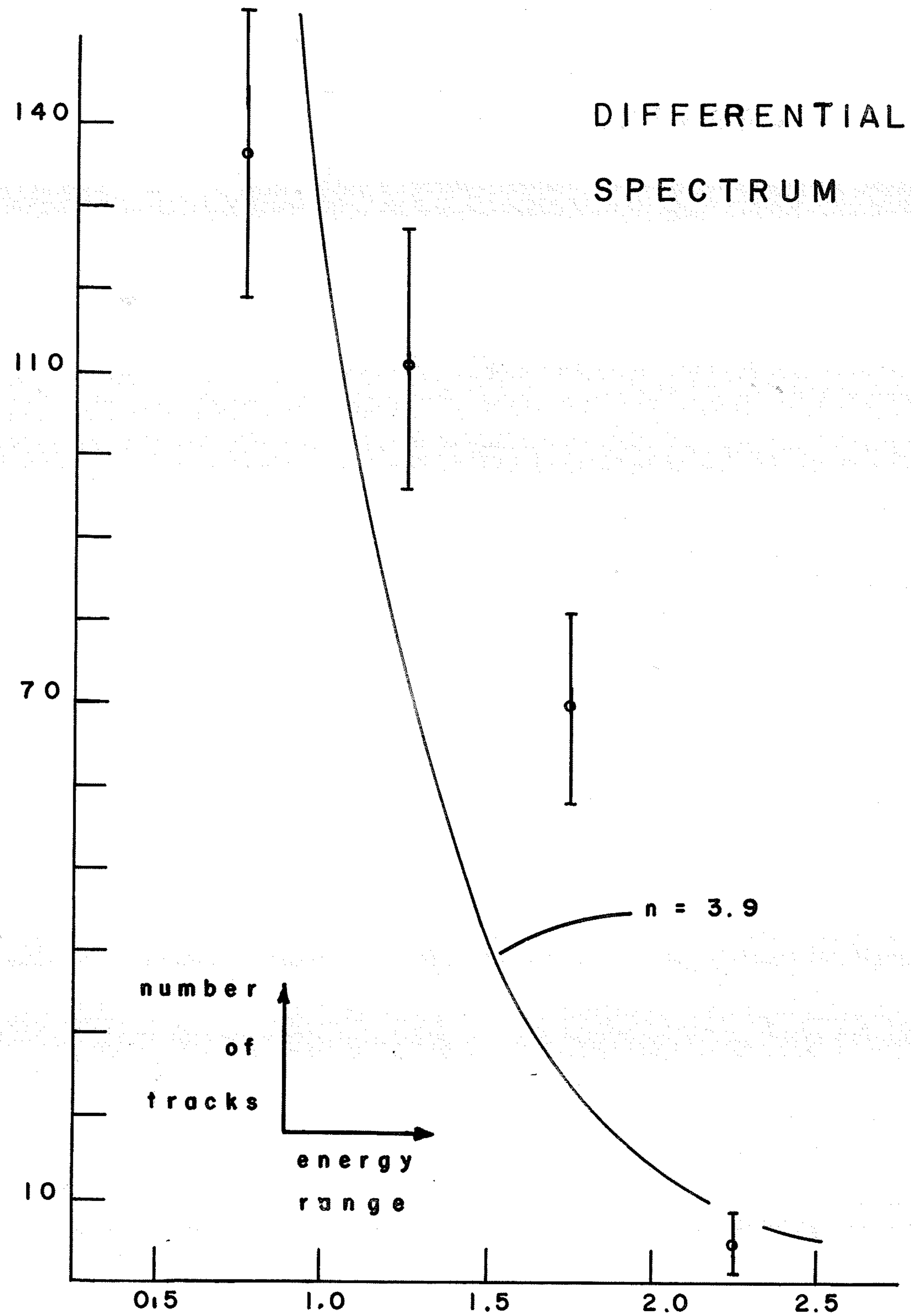
DIFFERENTIAL
SPECTRUM
140
110
70
10
n = 3.9
number
of
tracks
energy
range
0.5
1.0
1.5
2.0
2.5

FIGURE VII

The primary flux is given by

$$I = (n - Z) \exp \left(\frac{x_{air}}{\lambda_{air}} + \frac{x_{gl}}{\lambda_{gl}} + \frac{x_{em}}{\lambda_{em}} \right)$$

$$= (242 - 9) \exp (12.2/45 + 2.2/54 + 4.1/72)$$

$$= 340 \text{ particles/m}^2\text{/sec/ sterad.}$$

The error in I is calculated purely statistically on the basis of 155 tracks . It is given in terms of % error by means of ($100/\sqrt{n}$)%. The result is ± 28 particles. The uncertainties introduced by mean free path corrections have not been evaluated, but they may be expected to be small since I is quite insensitive to small changes in the λ's and the x's.

Energy Spectrum The values of $\overline{|\Delta_2|}^{-1}$, corrected for stage and setting noise, have been regarded as a measure of the energy of the particle in the emulsion, in the sense that they represent the lowest energy that each particle could have had. Extrapolation to the top of the atmosphere has been carried out in the manner described in an earlier section and an energy distribution has been obtained. This appears in the integral form in Figure Vl and in the differential in Figure Vll.

Kaplon's formula for the integral energy spectrum has already appeared in the introduction to this thesis. It can be written in the form

$$\ln N(E) = \ln C - n \ln (1 + E)$$

which, on a log-log plot is simply a straight line with slope (-n). Figure Vl shows the curve N(E) versus E as well as the curve N(E) versus 1 + E , which ought, according to Kaplon, to define a straight line. It will be seen that in fact the points lie along a definite curve which is nearly straight but not quite. A least squares fit has been applied to these points to determine the slope of the best straight line through them and it has been found to be n = 3.9 .

The factor C has been calculated using this value for n. At some energy E_o corresponding to $N(E_o)$ one has

$$N(E_o) = \frac{C}{(1 + E_o)^n}$$

So

$$C = N(E_o)\,(1 + E_o)^n = 336\,(1 + 0.50)^{3.9}$$

$$= 1640 \text{ flux units.}$$

C, of course, is the number of particles having energy greater than 0 BeV/nucleon. C is not, as might appear at first glance, the total flux. The range of energies for the primary particles does not extend to zero but stops at the geomagnetic cut-off which is the minimum energy that a primary of a given sort must possess in order to avoid being repelled from the earth by the interaction of the particle's charge with the earth's magnetic field.

The differential distribution is exhibited in Figure Vll. The four points have been obtained experimentally and the error on each as been calculated in the usual statistical way. The accompanying curve has been calculated from Kaplon's formula using $n = 3.9$ and indicates where the experimental points might have been expected to fall.

Table 1 is a break-down of the statistics. The numbers that appear in the row labelled 'Integral' are N(E) where E is the number at the head of each column. The numbers in the row 'Differential' are the number of particles whose energies were greater than E and less than or equal to E + 0.25 BeV/ nucleon.

Errors were calculated on the basis of the number of tracks observed, i.e. 155, not on the total flux value. The error was taken as $\sqrt{\nu}$ where ν was the number of tracks involved in each case. This number was then converted to modulus 340 by multiplying by 340/155.

The numbers in the third row are those which actually appear in Figure V11. The larger energy interval was necessary in order that some resemblance to Kaplon's curve appear.

DISCUSSION AND CONCLUSIONS

The flux measurement obtained here is in agreement, within experimental error, with that usually quoted, for example, Waddington's is 320 $\pm$ 36 particles/ m^2/sec./sterad. On the other hand, the energy distribution, characterized by the parameter $n = 3.9$, differs from the accepted distribution ($n = 1.35$, Kaplon) by a factor of 2.9 and from the distribution obtained by Waddington ($n = 1.9$) by a factor of 2.

Various reasons for such a discrepancy suggest themselves.

1) Higher energy tracks have been systematically missed.
2) Noise has been considerably under-estimated.
3) Statistical variations in the actual energy measurements are so large as to account for the discrepance.
4) Few tracks have been missed, the energies of high energy particles have been under-estimated in a majority of cases.

If any of the forgoing proves to be the correct explanation, then the discrepancy is not a real one. If it is in fact a real one, it is more difficult to suggest a reason for it. The possibility that n is a function of geomagnetic latitude cannot be discounted altogether, but in view of the fact that Waddington's work was done at the same latitude as this, very little should be looked for here.

It is unlikely that tracks at the high energy end of the spectrum have been systematically missed. That the absolute flux value is in good agreement with the values obtained by other investigators argues that few tracks have in fact been missed. It is to be expected that if a number of tracks sufficiently large to alter n (whose dependence upon N(E) is weak) by a factor of two or more had been left out, the flux calculated would have been a good deal smaller and not as large or larger than previous measurements.

It is possible that the noise level $\overline{\Delta s_i}$ has been underestimated. Correcting for stage and setting noise has the effect of spreading the points towards higher energies; the amount of spreading increases with the energy. This increases the values of N(E) for the high energies but leaves N(E) virtually unchanged at the low energy end. This means that a sufficiently large noise correction could account for the large value of n. However, the existence of an upper limit for $\overline{\Delta s_i}$ can be argued from the consideration that while it is possible, it is very unlikely that many tracks will have measured scatter $\overline{\Delta x} > \overline{\Delta s}$. Indeed, since

$$\overline{\Delta a}^{\,2} = \overline{\Delta x}^{\,2} - \tfrac{1}{2}\,\overline{\Delta s}^{\,2},$$

one should expect the maximum observable scatter to be

$$\overline{\Delta x}^{\,2} = \tfrac{1}{2}\,\overline{\Delta s}^{\,2}$$

$$\text{i.e.} \qquad \overline{\Delta x} = \frac{\overline{\Delta s}}{1.416}$$

The smallest scatter observed was $\overline{|\Delta_2|}^{1}$ = 11 units which corresponds to $0.09\mu/500\mu$. If this represented the noise level, then $\overline{\Delta s} = 0.13\mu/500\mu$, which is not quite twice as great as the value previously determined. ($0.08\mu/500\mu$) This value of $\overline{\Delta s}$ has been used in a recalculation of n which turns out to be n = 3.3 . Since the largest value that $\overline{\Delta s}$ could be imagined to have has been used, the difference n = 3.9 to n = 3.3 represents the largest error in n that under-estimation of stage and setting noise could produce. Clearly, it is not enough to account for the difference between n = 3.9 and n = 1.9 .

There is practically no chance that the statistical error in the scattering measurements is responsible for the discrepancy. Since the tracks were all at least 15 m.m.'s in length and since readings were taken at 1 m.m. intervals, there will have been at least 16 readings per track, usually more. The scattering is the average of the second differences, which in the worst case, number 14. A liberal estimate of the error due to statistics is, at the most, $\frac{100}{\sqrt{n'}}\% = \frac{100}{\sqrt{14}}\% = 27\%$. Again, few tracks longer than 30 m.m.'s were found so that $\frac{100}{\sqrt{29}}\% = 19\%$ is probably a good estimate of the lower limit for the statistical error. Suppose on the average, a statistical error of 25% is allowed in all scattering measurements. This implies an error of about the same amount in the final determination of energy at the top of the atmosphere. Use has been made of this to extend the energy intervals in Table 1.

TABLE 11

Energy	0.25	0.50	0.75	1.00	1.25	1.50	1.75
Integral	340	328	286	221	154	108	71
Diff'l	12	42	65	67	46	37	23

Energy in BeV/ nucleon

Energy	2.00	2.25	2.50	2.75	3.00	3.25	3.50
Integral	48	31	21	15	13	10	
Diff'l	17	10	6	2	3	2	8 (greater than 3.50)

Thus, for example, the tracks which formerly occupied the energy interval 0.75 to 1.0 BeV/ nucleon must now be thought of as occupying the extended interval 0.6 to 1.25 BeV/nucleon. If one assumes a uniform density of tracks throughout the interval (in the foregoing, that density will be 280/2.6 = 107.5 tracks per unit (0.25 BeV) interval) one can affect a redistribution of the tracks and thereby obtain a new differential spectrum. Table 11 gives such a spectrum together with the integral one which is derived from it.

The new value for n obtained from these figures is n = 3.3 . Because this new value has been corrected for the possible statistical fluctuations in the energy measurements it is probably preferable to the value quoted earlier.

Concluding Remarks A discrepancy between this author's value of n and the value obtained by other workers has been remarked and a number of suggestions of possible sources of error in the present work which might account for this have been discussed.

That n is large indicates that a good deal fewer than usual tracks of high energies have been observed, and reasons for this have been put forward. It has been suggested first of all that high energy tracks have somehow been missed in the process of scanning, but this has been disposed of by means of an argument based upon the goodness of the absolute flux value obtained. Further possibilites of error included under-estimation of noise level and the chance that the statistical fluctuations in the actual scattering measurements had been so great as to account for the difference observed.

In the first case, it has been shown that if the worst situation prevailed, the value of n would be reduced only to n = 3.3 .

A redistribution of the tracks has been carried out to take account of a maximum possible error of 25% in the energy determinations and the integral spectrum resulting from this is found to be characterized by n = 3.2 .

There appear to be three possibilites left. 1) The energies have been greatly under-estimated in quite a random way (due to the Peters' effect), something for which no correction can be applied . 2) Some effect which has not been considered is responsible for the discrepance. 3) The discrepancy is a real one and n = 3.9 or 3.3 is in fact the true integral energy spectrum parameter.

It must be noted in referring to the first of these that the emulsion used here lost about 2/3 of its original volume in the developping process. The rearrangement of the developped grains might indeed be enough to introduce a relatively considerable but entirely random spurious scattering effect. It is reasonable to assume that this effect would not be so great in Waddington's emulsions because his were 0.4 m.m.'s in thickness as compared with the present writer's 1 m.m. emulsion, and it might easily be that Waddington's emulsions lost a good deal less volume in processing than that used here as a consequence.

It is a distinct possibility that some effect which has not been considered here has caused the difficulty, but this seems rather unlikely due to the comparative simplicity of the technique.

APPENDIX

RANGE-ENERGY RELATIONS FOR HIGH ENERGY ALPHA-PARTICLES IN AIR

Range-energy relations for alpha-particles of total energy in excess of 1 BeV. in air are not readily available so that it became necessary in connection with the present investigation to derive an alpha-particle relation from the range-energy curve for high energy protons which appears on page 573 of Heisenberg's *Kosmische Strahlung* .

The relation between the curve for alpha-particles and that for protons is obtained in the following way. On the basis of the theory outlined by Rossi (page 388), one has, where R signifies range, M mass, Z charge number, and E energy, that

$$R = \frac{M}{Z^2} f(E/M) .$$

Experiments have shown that range-energy relations have the general form

$$E = AR^n$$

to a close approximation.

The result

$$E = M^{1-n} Z^{2n} R^n$$

is readily obtained.

Bradner et al., Gottstein and others have been quoted by Voyvodic(9) as agreeing that $n = 0.581$ for all particles.

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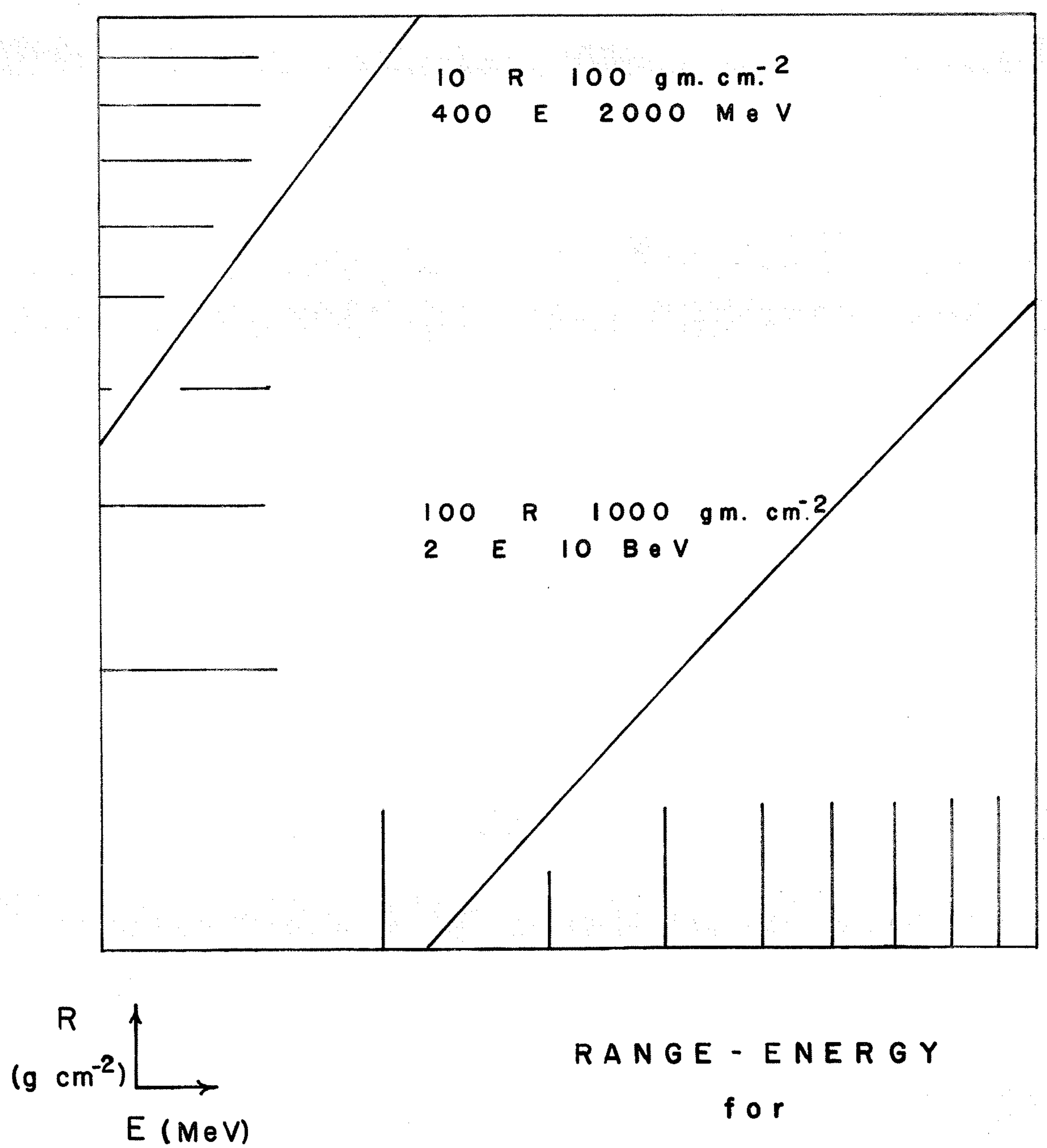
(9) Progress in Cos. Ray Phys. Vol.11 P.237



RANGE - ENERGY
for
α - PARTICLES in AIR

FIGURE VIII

Consider an alpha-particle and a proton having the same energy.

i.e. $E_p = E_a$

One has $E_a = M_a^{1-n} Z_a^{2n} R_a^n$

and $E_p = M_p^{1-n} Z_p^{2n} R_p^n$

Dividing
$$1 = (M_a/M_p)^{1-n} (Z_a/Z_p)^{2n} (R_a/R_p)^n$$
$$= 4^{1-n} 2^{2n} (R_a/R_p)^n$$
$$= 4 (R_a/R_p)^n .$$

Therefore, $\underline{R_a = 0.092 R_p}$

The curve that was subsequently obtained is appended. See Figure V111.

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