The Application of the Principle Of Finite Increments of Sensation applied to determine a Psychophysical Law for the Tactile Sense.

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A Thesis Submitted to the
University of Manitoba
As a Partial Requirement for the
Master of Arts Degree
April, 1929.

I wish to express my thanks to Mr. P. A. Macdonald, at whose suggestion this investigation was undertaken.

I also wish to thank Professor Allen for the apparatus placed at my disposal.

Physics Department,
University of Manitoba,
April, 1929.

The Application of the Principle of Finite Increments of Sensation, Applied to determine a Psycho-physical Law for the Tactile Sense.

In all sensory reactions, the materials involved, forces set free and stimuli employed, appear to be closely allied to physics and chemistry. From this it appears that the problems of psychology and physiology might well be attacked by the methods of the physicist and chemist.

If the senses of touch, taste, sound and vision are studied, it is found that the stimuli employed to excite these sensations are mainly electrical, mechanical, chemical, and thermal. In some sensory receptors, the reactions produced are physical and in others chemical. It can be seen that ultimate explanations will be difficult due to the limitations of these sciences.

Psychologists have been trying for a great many years to find some law relating intensity of stimulus and sensation. That is, they wanted to express sensation in physical or chemical units.

In 1760 Bouguer endeavoured to ascertain if it were possible to increase a sensory stimulus without being able to detect the increase in sensation. He found this condition to exist, and that the least increase in intensity of stimulus that could be distinguished by the visual receptors was 1 in 64.

This idea was extended by Weber in 1834. He discovered by using the sense of touch that it was possible to discriminate between two weights if they differed by 1 or 2 parts in 30. He also found that it was possible to discriminate visually between the length of two lines if they differed by 1 part in 100, regardless of the length of the lines.

Masson repeated the experiments in 1845 with a new method. He found that the ratio of the just distinguishable increase in intensity of stimulus to the original intensity was constant for a given person. This constant varied with different people.

It was not until 1860 that Fechner realized the full significance of what he called Weber's law. It was apparent to Fechner that Weber's law only held between certain limits of intensity but he set these limits at the two extremes of the intensity scale.

The accepted definition of Weber's law, according to Myers (1) is: "If I be the magnitude of a given stimulus and ΔI be its differential threshold, it is found that $\frac{\Delta I}{I}$ has an approximately constant value except for extremely large or extremely small values of I." According to this law $\frac{\Delta I}{I}$ is constant within wide limits of intensity not only for vision but also for the other senses.

Fechner assumed that just noticeable differences of sensation contain an equal number of sensation units. That is, he said that $\frac{\Delta I}{I}$ was a measure of sensation. If this is true, then it may be stated algebraically that:

$$K \frac{\Delta I}{I} = \Delta S$$

where S is a measure of sensation, ΔS a small increment of sensation, and K some constant.

On the very questionable assumption that it is permissible to integrate small finite quantities

$$S = K \int \frac{\Delta I}{I}$$

$$= K \log I + C$$

where C is another constant. Thus Fechner's law states that sensation varies as the logarithm of the stimulus. Fechner called this the psychophysical law.

Helmholtz in 1866 made the first criticism of the Weber-Fechner law. When he repeated the work of Masson using the same method he found the ratio $\frac{\Delta I}{I}$ to vary at different intensities. Since then the work has been repeated by other writers and a variation in the constant has been found.

Since Fechner's law is based on the assumption that Weber's law is correct, it is evident that it is not on a very firm basis. Even if we assume that $\frac{\Delta I}{I}$ is equal to a constant it is very questionable whether it is correct to integrate small finite quantities. If the increment of intensity is very small compared to the initial intensity then it may possibly be permitted.

During the past year it has been found in this laboratory that $\frac{\Delta I}{I}$ = constant for light. Helmholtz found that this constant varies from 1/100 to 1/165. In this case with such a small increment of intensity as compared to the original intensity, integration might be permitted by the mathematical physicist. However, in the case of sound, it has been found that $\frac{\Delta I}{I}$ is not equal to a constant. Even if it were equal to a constant the increment would be very large as compared to the original intensity and it would not be permissible to integrate.

Although the foundation of Fechner's law is weak, it seems to have many successful applications. Intermittent sensory stimuli for the visual, auditory, gustatory and tactile senses have been shown to obey this generalization.

$$\frac{I}{D} = K^2 \log I + C \quad \text{(visual)}$$

$$D = K^2 \log I + C \quad \text{(auditory)}$$

$$\frac{I}{D} = -K^2 \log I + C \quad \text{(gustatory)}$$

$$D = -K^2 \log I + C \quad \text{(tactile)}$$

Where D is the critical frequency of flicker, I the intensity of stimulus, and C and K are constants.

The fact that Weber's law only holds over a certain range of intensity might suggest that the correct law had not been found. In view of recent research conducted by Allen (2) and his associates with regard to reflex actions this investigation was undertaken to verify the law for the tactile sense.

Weber (3) in his experimental work applied the comparable stimuli successively to the same surface of the skin. He used the two distal phalanges of two adjacent fingers which with the whole arm lay fully supported on a rigid surface. The time interval between two successive stimuli was kept brief and regular in comparing pairs successively. The duration of the stimuli was kept equal. He stimulated the receptors by the addition of weights. A weight was added and left for a short interval. It was then removed and a slightly heavier weight was added. The second weight to be added was then increased, and the process repeated. This was continued until the increase in weight could be noticed.

If this method is considered it is easy to see where errors might enter. He was not making a quantitative measurement, and it would be difficult to compare the weights after a short interval of time.

Wienberg[2] and Hollenberg[4] in their work on touch have shown that if we stimulate a receptor there are reflex actions which have been called enhancement and inhibition. These reflexes would affect Weber's results to a large extent. He stimulated the receptors by the addition of a weight, then he removed the weight. Reflex actions would be set up and these would affect his estimation of the next weight.

Other experimental work has been done on touch by means of a stiff hair. The hair was applied to a touch receptor with a certain pressure. The pressure was then gradually increased until the increase was noticeable.

In this method the pressure applied was judged by the bending of the hair. This was a crude method of measurement and errors would be introduced. Enhancement and inhibition would also cause errors in this method. The touch receptor would be stimulated for a short length of time before the correct increase in pressure was found. The receptor would become adapted to this stimulus and this would affect the results.

During previous work on the tactile sense, Allen and his associates have used a jet of air as stimulus. This method was found to be very satisfactory and has been used as a source of stimulation during this investigation.

Several different methods were tried before a satisfactory method of adding the increment of stimulus was found. The most satisfactory method was by the use of a large constant pressure tank 48 cms. in diameter and 80 cms. high. It was found by experiment that when weights were added to the top of the tank the pressure increased directly as the weight added. This principle was used to produce the increase in pressure.

The apparatus was very simple in design. A chin rest was made since the receptors of the lip were used. The nozzle was fixed in position in front of this chin rest. By means of a water manometer in the circuit the pressure could be read. A lever and pulley

PHOTOGRAPH OF APPARATUS

1. Parts Used for Movement Work:

A. Constant Pressure Tank.

B. Weights added to top of tank to produce a definite pressure.

C. Lever used to add weights to the top of the tank in order to increase intensity of stimulus.

D. Adjustable chin rest.

E. Hole through which air from the nozzle came and struck the lip.

F. Water Manometer.

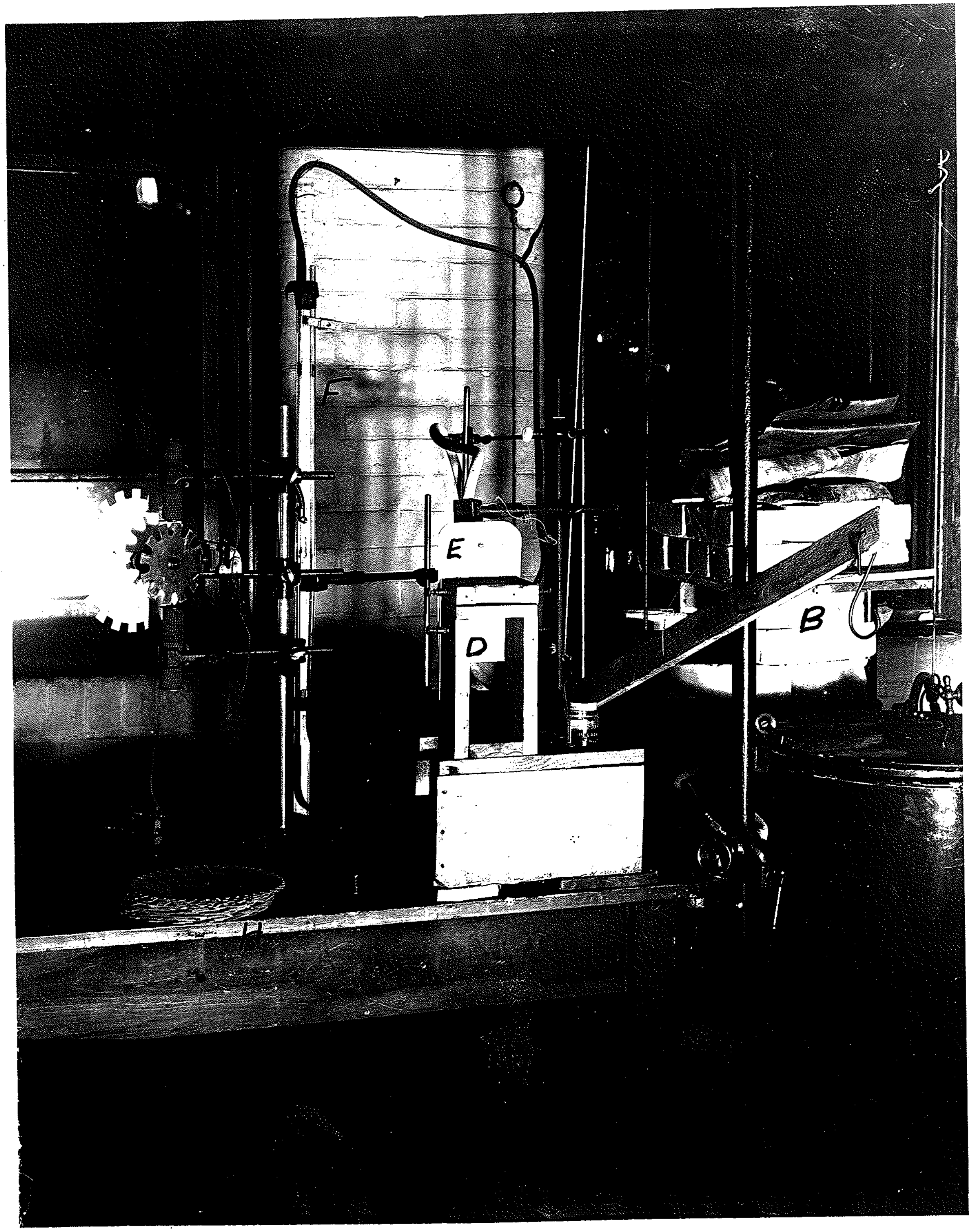
2. Parts Substituted for Flicker Work:

G. Synchronous Motor.

H. Cardboard discs which could be put on the motor.

I. Needle valve to adjust pressure.

Plate 1



were used to add the weights to the top of the tank in order to increase the intensity of the stimulus.

It has been found by previous experimenters that hairs when touched act as levers and affect the results. For this reason the receptors of the lip have been used during this investigation. The receptors on the centre of the left side of the lower lip were stimulated. The lips were kept soft and moist by the application of a thin layer of vaseline.

A constant routine was followed for each reading. Weights were first added to the top of the tank until the desired pressure was produced. This pressure was read on the manometer to within a millimetre of water.

The chin was next placed on the rest with the lips directly in front of the nozzle. By means of the lever a weight was added to the top of the tank. If the increase in pressure could be noticed the weight was decreased and the process was repeated. The weight added was gradually decreased until the increase in pressure could not be distinguished. This weight was taken as the increment of intensity.

Several precautions had to be taken in order to obtain accurate results. The tank was always raised to a constant height since there was a slight variation in the pressure if this was not done. Sometimes when adding the weights to the tank the body was moved. This gave the sensation of a change in pressure and the reading was neglected. Care was also taken to add the weights gently in order not to jar the tank. A pad was placed on the tank to prevent any jar. This also prevented any noise.

Every possible care was taken to eliminate effects due to adaptation and reflex actions. Errors due to adaptation were eliminated by adding the weight to the top of the tank as soon as the lip was in place. To eliminate the reflex actions a rest of ten minutes was taken between each reading. This period was found to be long enough for the receptors to return to what appeared to be a normal state.

It was found that the increment of intensity for a definite pressure was the same from day to day unless the writer was tired or not feeling well. Due to the fact that the writer was working most of the day time, an attempt was made to take readings at night. It was soon found that these readings could not all be repeated, with the result that this practice was discontinued. This condition is probably due to neural fatigue.

For the first curve taken the points were plotted as soon as they were ascertained. This indicated where the next point would be and made it easier to find the increment for the next pressure. It was found that the readings could not be accurately repeated so this was stopped.

Practice plays an important part in finding the increment of intensity. It takes considerable practice before the change in pressure can be easily distinguished. Practice is necessary before the weight can be added so as not to jar the tank.

It was found easier to appreciate the difference in pressure when the weight was suddenly added to the tank. When the weight was added quickly the increase was immediate. If this was done it lowered the threshold value.

Several nozzles of different sizes were tried, but the one used was a .7 mms. in diameter. It was found that the threshold value could be more easily distinguished with it. A complete curve was taken with this nozzle. The results are given in Table 1, and the corresponding curve is plotted in Figure 1.

For reasons which will appear later, the work was repeated with a nozzle 1.5 mms. in diameter. Although the points could not be obtained so accurately with this nozzle, the results were fairly consistent. These results are given in Table 2, and are plotted in Figure 2.

In both sets of data the increment of intensity has been left in terms of the kilograms added to the top of the tank. This was thought to be more accurate. For the first set of data 1 kilogram pro-

TABLE 1.

I	$\log \frac{1}{I}$	ΔI	$\frac{1}{\Delta I}$
7	$\bar{1}.1523$	.760	1.31
14	$\bar{2}.8537$	1.400	.714
16	$\bar{2}.7958$	1.560	.641
45	$\bar{2}.3463$	3.110	.321
56	$\bar{2}.2504$	3.660	.273
59	$\bar{2}.2279$	3.660	.273
67	$\bar{2}.1732$	4.010	.249
78	$\bar{2}.1072$	3.360	.229
88	$\bar{2}.0531$	4.610	.217
100	$\bar{2}.000$	5.160	.194
108	$\bar{3}.9661$	5.510	.181
109	$\bar{3}.9624$	5.560	.180
110	$\bar{3}.9585$	5.610	.178
117	$\bar{3}.9314$	5.710	.175
119	$\bar{3}.9243$	6.160	.162
129	$\bar{3}.8893$	6.510	.154
132	$\bar{3}.8791$	5.860	.171
138	$\bar{3}.8597$	6.710	.149
143	$\bar{3}.8445$	5.960	.168
149	$\bar{3}.8267$	6.760	.148
158	$\bar{3}.8014$	6.810	.147
159	$\bar{3}.7986$	6.810	.147
161	$\bar{3}.7931$	6.860	.146
166	$\bar{3}.7796$	6.910	.145
168	$\bar{3}.7745$	7.060	.142
179	$\bar{3}.7474$	7.460	.134
180	$\bar{3}.7443$	7.510	.133
192	$\bar{3}.7168$	8.010	.125
199	$\bar{3}.7007$	8.360	.119
208	$\bar{3}.6821$	9.710	.115
214	$\bar{3}.6693$	9.360	.107
224	$\bar{3}.6493$	9.860	..101

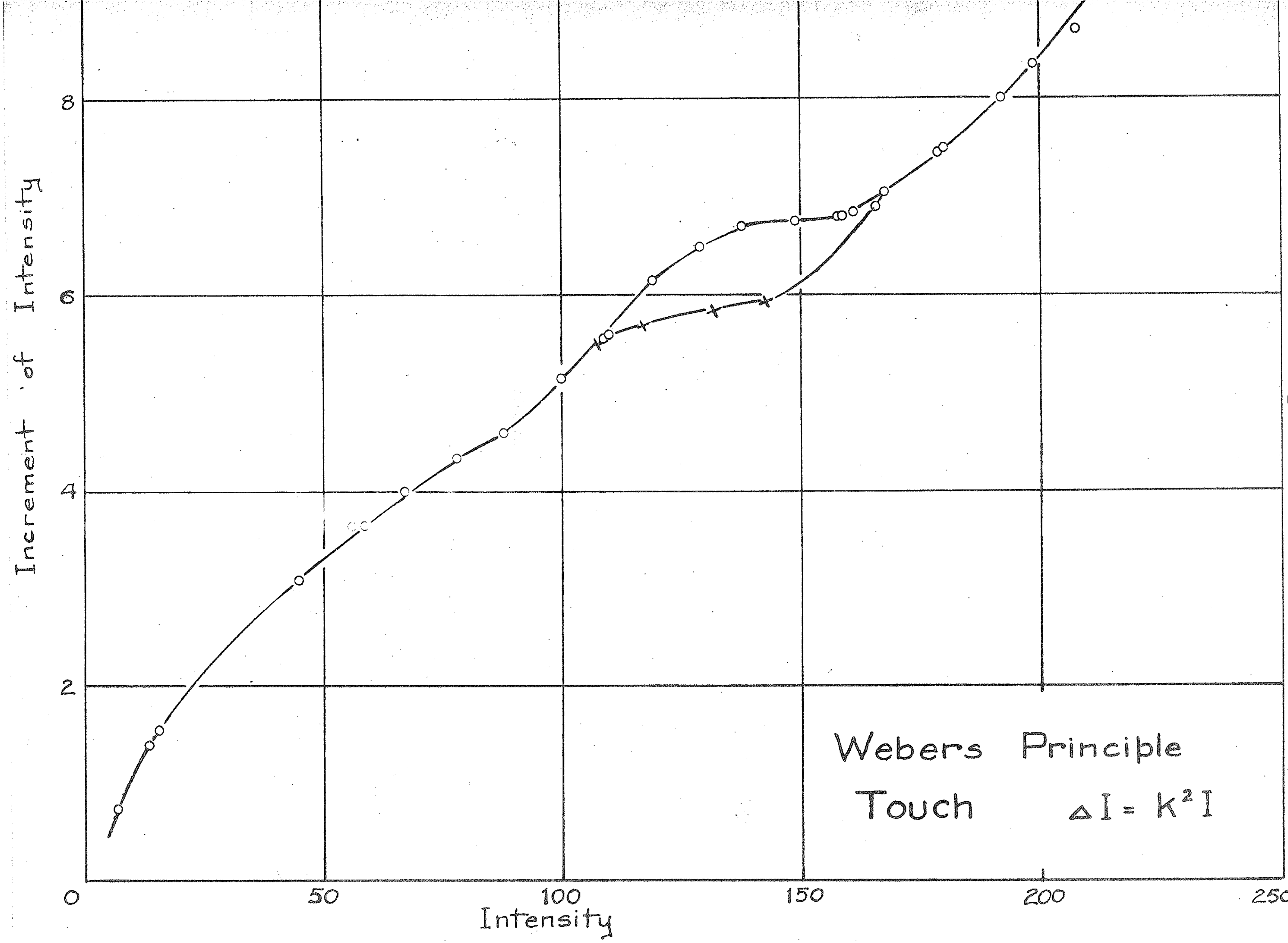
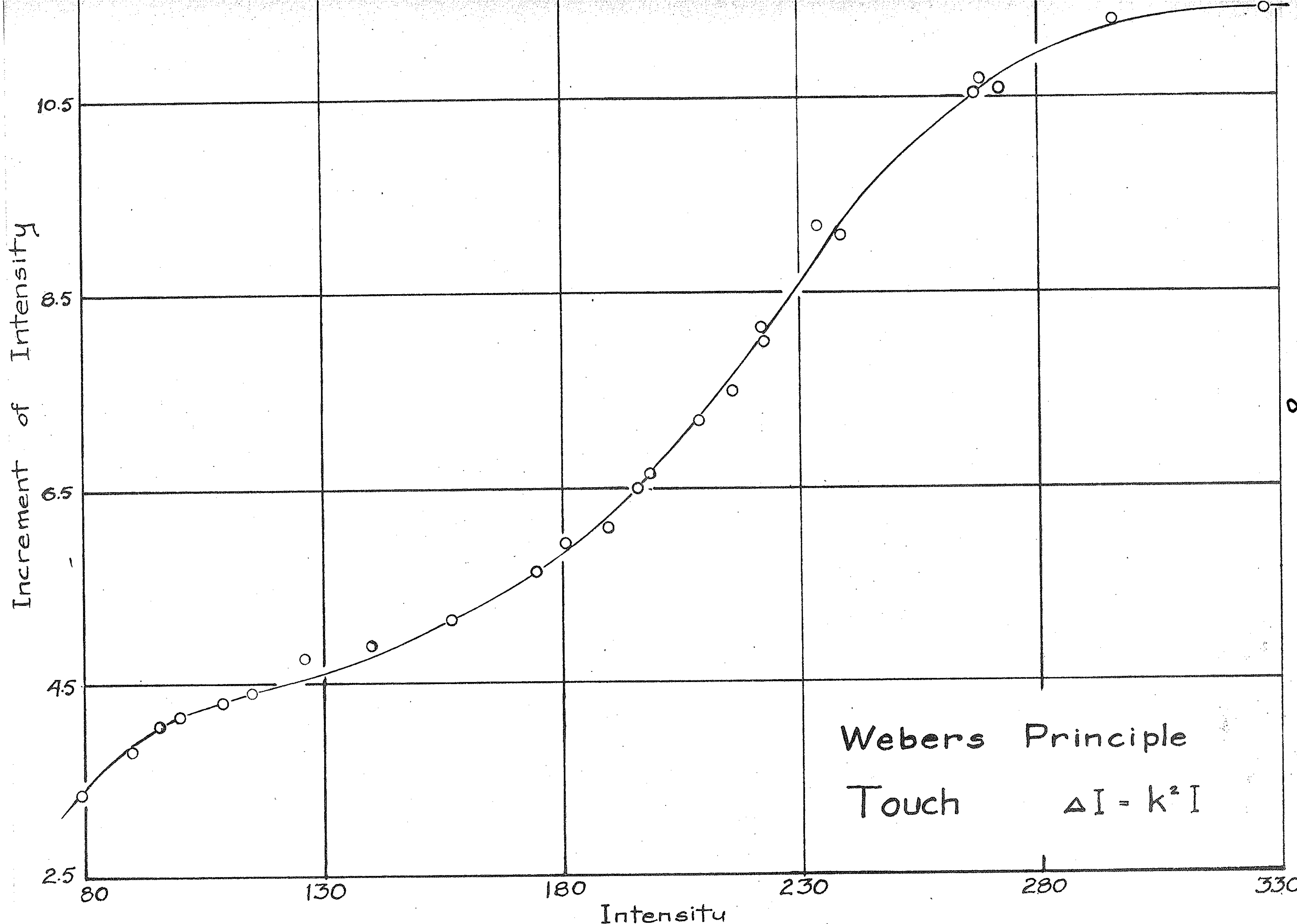

Increment of Intensity
Intensity
8
6
4
2
0
50
100
150
200
250
Webers Principle
Touch ΔI = k²I

Fig. 1

TABLE 2.

I	Log $\frac{1}{I}$	Δ I	$\frac{1}{\Delta I}$
79	$\bar{2}.079$	3.360	.297
90	$\bar{2}.041$	3.810	.262
96	$\bar{2}.017$	4.060	.246
100	$\bar{2}.000$	4.160	.240
109	$\bar{3}.954$	4.310	.232
115	$\bar{3}.939$	4.410	.226
126	$\bar{3}.897$	4.760	.210
140	$\bar{3}.854$	4.910	.203
157	$\bar{3}.799$	5.141	.194
175	$\bar{3}.755$	5.641	.177
181	$\bar{3}.740$	5.960	.167
190	$\bar{3}.716$	6.110	.163
196	$\bar{3}.707$	6.491	.154
199	$\bar{3}.702$	6.660	.150
209	$\bar{3}.672$	7.210	.138
216	$\bar{3}.662$	7.491	.133
222	$\bar{3}.653$	8.160	.122
223	$\bar{3}.651$	8.010	.124
234	$\bar{3}.623$	9.210	.108
239	$\bar{3}.620$	9.110	.109
267	$\bar{3}.572$	10.541	.095
268	$\bar{3}.570$	10.710	.093
272	$\bar{3}.566$	10.610	.094
296	$\bar{3}.529$	11.291	.089
328	$\bar{3}.484$	11.391	.088



Increment of Intensity
10.5
8.5
6.5
4.5
2.5
80
130
180
230
280
330
Intensity
Webers Principle
Touch $\Delta I = k^2 I$

Fig. 2

duced an increase of approximately 9 mms. Before the second curve was taken the apparatus was changed and it was found that 1 kilogram added to the top of the tank produced an increase in pressure of approximately 10 mms. These measurements were taken in both cases on the manometer in the circuit.

Some of the points do not fall on the curve but it will be noticed that the scale used for both graphs is very large. However,, if allowance is made for the fact that the smallest weight by which the increment could be varied was a 50 gram weight, the results do not appear so inaccurate.

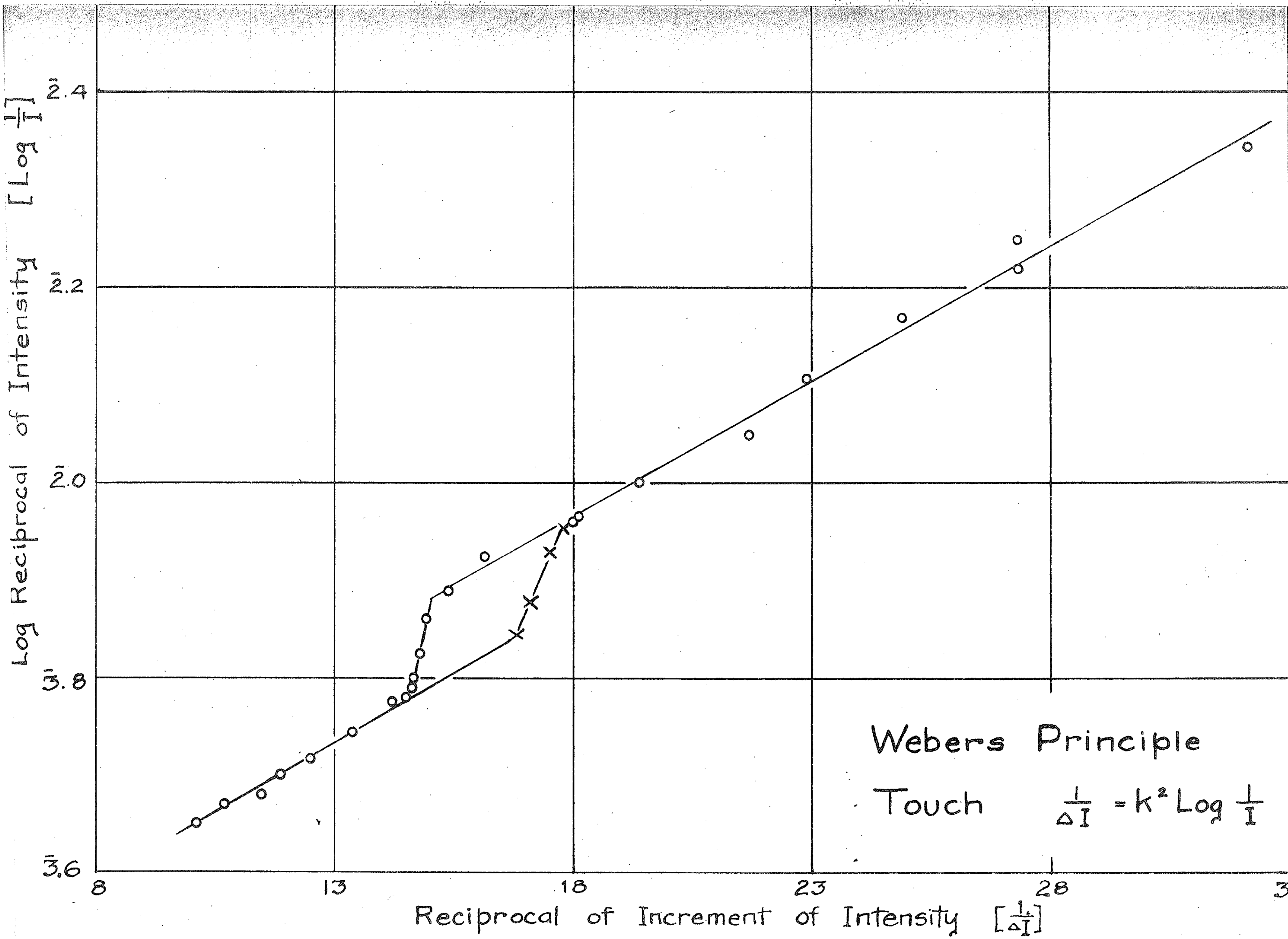
In Figure 1 there is a loop which does not appear in Figure 2. The points marked by crosses were taken in one day. This loop is probably accounted for by the fact that the writer was sick the next day.

From Figures 1 and 2 it can be seen that Weber's law does not hold for the tactile receptors over the range of intensities used. A large number of points have been taken for both curves, so there is no doubt.

In an attempt to find some law relating the increment of intensity and original intensity a large number of combinations were tried. The equation

$$\frac{I}{\Delta I} = K^2 \log \frac{1}{I} + C$$

where ΔI is increment of intensity, I original intensity and K and C are constants, was the only one which held



Log Reciprocal of Intensity [Log 1/I]
Reciprocal of Increment of Intensity [1/ΔI]
2.4
2.2
2.0
3.8
3.6
8
13
18
23
28
33
Webers Principle
Touch 1/ΔI = k² Log 1/I

Fig. 3

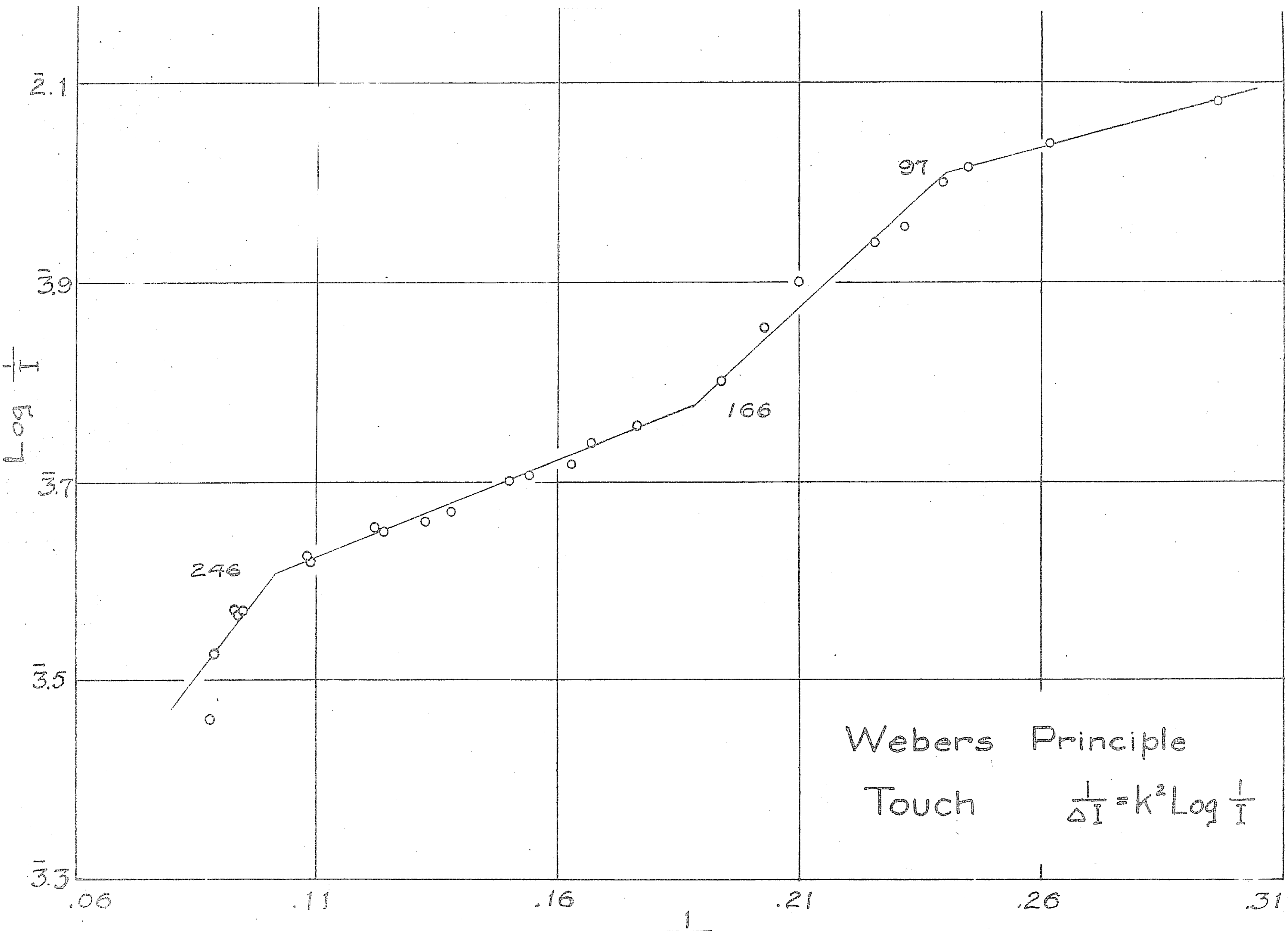

2.1
3.9
3.7
3.5
3.3
Log 1/I
.06
.11
.16
.21
.26
.31
1
246
166
97
Webers Principle
Touch $\frac{1}{\Delta I} = K^2 \text{Log} \frac{1}{I}$

Fig. 4

for both sets of data. This relation for the two sets of data is shown in Figures 3 and 4.

In order to show the relation clearly an excessively large scale has been used to plot these results. The first three readings taken for the .7 mm nozzle have not been plotted, the scale would need to be so small that all the points would be grouped together. However, if these points are plotted they are found to fall on a straight line.

Due to the limitations of the apparatus the law could not be tested for higher intensities of stimulus. It might be expected, however, that as I was increased, the same law would be found to hold.

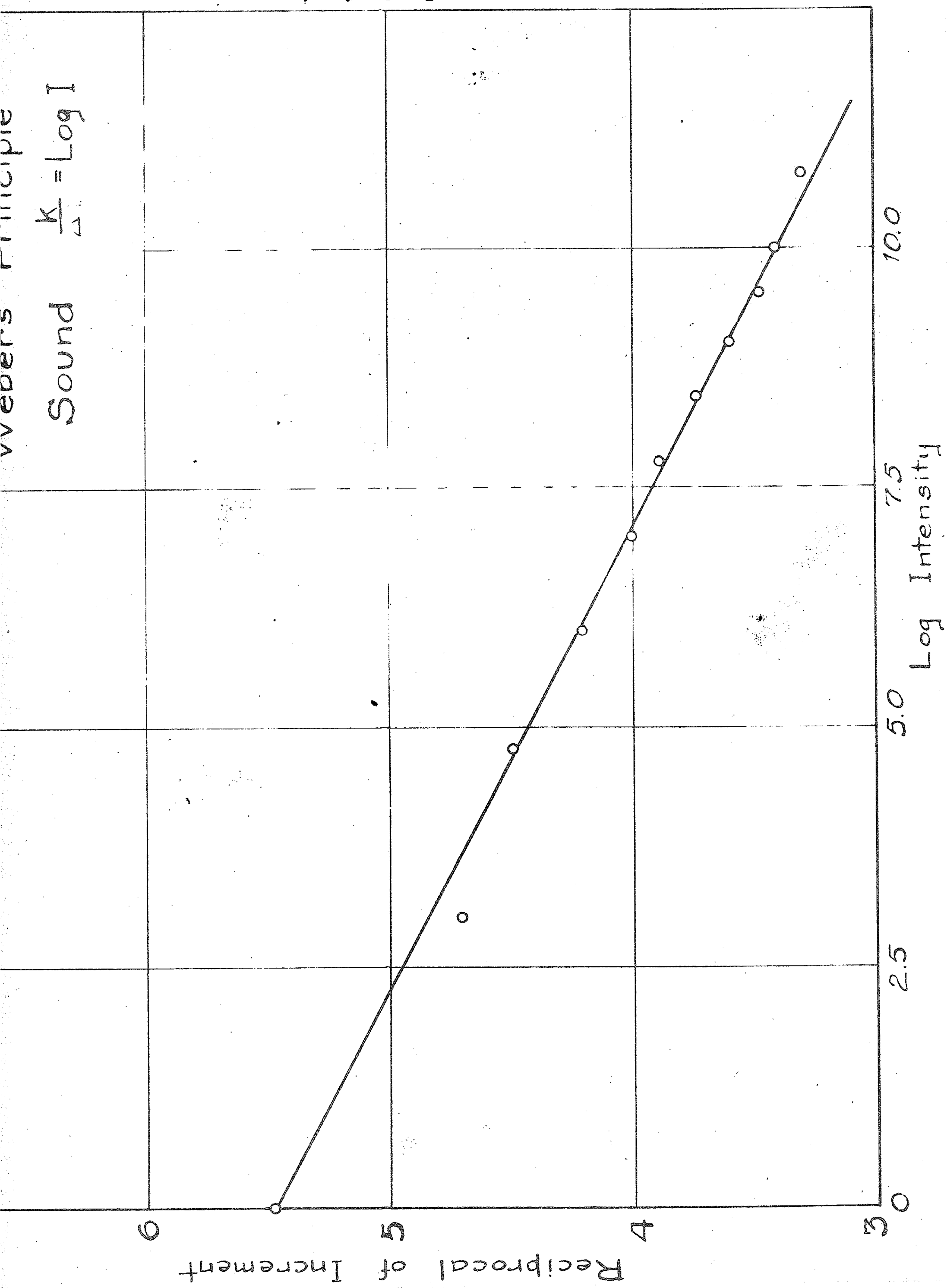
In Figure 3 it may be noticed that the four points marked by crosses are on a straight line. The effect has been to shift the breaks in the curve. A break in the curve indicates a distinct change in the sensitivity of the receptor at a certain intensity. If the break is shifted this would indicate that the receptors have either become more or less sensitive.

During the past year it has been found for sound that

$$\Delta I = K^2 \log I + C$$

where ΔI is increment of intensity of stimulus, I original intensity, and K and C are constants. This equation is very similar in form to the one found for touch. For visual sensations it has been proved that

Plate 2



Sound
Log Intensity
Reciprocal of Increment
0
2.5
5.0
7.5
10.0
3
4
5
6

$$\Delta I = K^2 I + C$$

The similarity between the laws found for touch and sound may be partly explained by the fact that in both cases the stimulus is mechanical. For visual sensations the stimulus is probably electro-chemical. No increment law has been found for taste as yet, but since the stimulus is chemical it will very likely be similar to that which holds for vision.

In vision, no matter what intensity of stimulus is applied we use the same end organs. In touch and probably in sound the number of receptors stimulated is increased when the intensity is increased.

The adequate stimulus for touch receptors is either to push the surface of the skin inward or to pull it outward. As the intensity of stimulus is increased the skin is gradually depressed with the result that more receptors are brought into the area which is being stimulated. If one receptor could be isolated from the rest and stimulated, it is possible that $\frac{\Delta I}{I}$ might be equal to a constant. This is impossible, however, because as soon as the intensity of stimulus is increased more receptors are stimulated and the conditions are altered.

If the range of intensities between 78 mms and 224 mms is considered, there is found to be a decided difference for the two nozzles. For the .7 mm nozzle $\frac{\Delta I}{I}$ varies from 1/2.6 to 1/2.5. For the 1.5 mm nozzle it varies from 1/2.3 to 1/2.7. There is more variation in the constant when a 1.5 mm nozzle is used than when a .7 mm nozzle is used. With a 1.5 mm nozzle more receptors are stimulated and this may complicate the results.

Hollenberg and Wienberg have by their work in touch established the law

$$D = K^2 \log \frac{1}{I} + C$$

where D is duration of intermittent stimulus, I intensity of stimulus, and C and K are constants. This is a specific instance of Fechner's psycho-physical law.

It was thought that there might be some relation between this and the increment law. This work has been repeated under the same conditions as the increment law.

The apparatus used was the same as that used for the first part of the investigation with a few alterations. A needle valve was inserted between the manometer and the tank. A sectored disc was placed in front of the nozzle.

The method used was a decided improvement over that used by Hollenberg and Wienberg. For their work a direct current motor was used to rotate the sectored disc. The pressure of the air was kept constant and the speed of the motor was varied by means of a friction brake until the flicker could not be detected. The speed of the motor was recorded on a chronograph. The motor was noisy and was very difficult to adjust.

In this work the speed of the motor was kept constant and the pressure was varied by means of the needle valve. A synchronous motor which was found to have a constant speed was used to rotate the sectored disc.

The motor was very simple in design. An iron disc was cut with an even number of poles and mounted on a shaft which was set in bearings. Above and below the disc were two electro-magnets of approximately the same strength. These two magnets were connected in such a way that the poles next the disc were opposite. The magnets were connected to the alternating current.

On the other end of the shaft there was a clamp by means of which sectored cardboard discs were put on. These sectored discs were cut accurately so that the opening was equal to the solid part. By means of changing these discs which were cut with various numbers of openings, the duration of the stimulus could be varied. The duration could also be changed by using discs with different numbers of poles.

A disc with 16 poles placed at equal distances from each other was used throughout this investigation. It was found to be the easiest to start and was about the correct speed. The motor had to be started with a jet of air. The speed of the motor had to be such that a point on the circumference of the disc would travel twice the distance between two poles while the electro-magnets were changing. When this speed was reached the air was shut off and the motor would run with a constant velocity.

The alternating current was found to have a frequency of 60 cycles per second. A 16 pole motor was used so the motor would make

$$\frac{60 \times 2}{16} = 7\tfrac{1}{2} \text{ revolutions per second}$$

If we let X be the number of openings in the sectored disc the duration of the stimulus would be

$$\frac{8}{60} \times \frac{1}{24} = \frac{8}{120X} = \frac{1}{15X}$$

The same part of the lip was used as in the first part of the investigation. The same precautions were taken to prevent reflex actions affecting the results.

In making measurements the following procedure was adopted. The motor was started and the tank filled. The chin was then placed on the rest with the lip in front of the nozzle. When all was in readiness the pressure was turned on by means of the needle valve and quickly adjusted so that the flickering could not be detected. The lip was then rested for 10 minutes before another reading was taken. When the correct pressure had been ascertained by means of repeating this process several times the sectored disc was changed and the work repeated. With very little practice the readings could be repeated to within 4 or 5 mms of water.

An attempt was made to obtain a curve using a nozzle .7 mms in diameter, but it was found that the results were not consistent. When a 1.5 mm nozzle was used, the readings were consistent and could easily be repeated from day to day.

The flicker law has been well established so only enough points were taken to determine the curve. The results obtained are given in Table 3. Figure 5 shows the results plotted according to the law

$$D = K^2 \log \frac{1}{I} + C$$

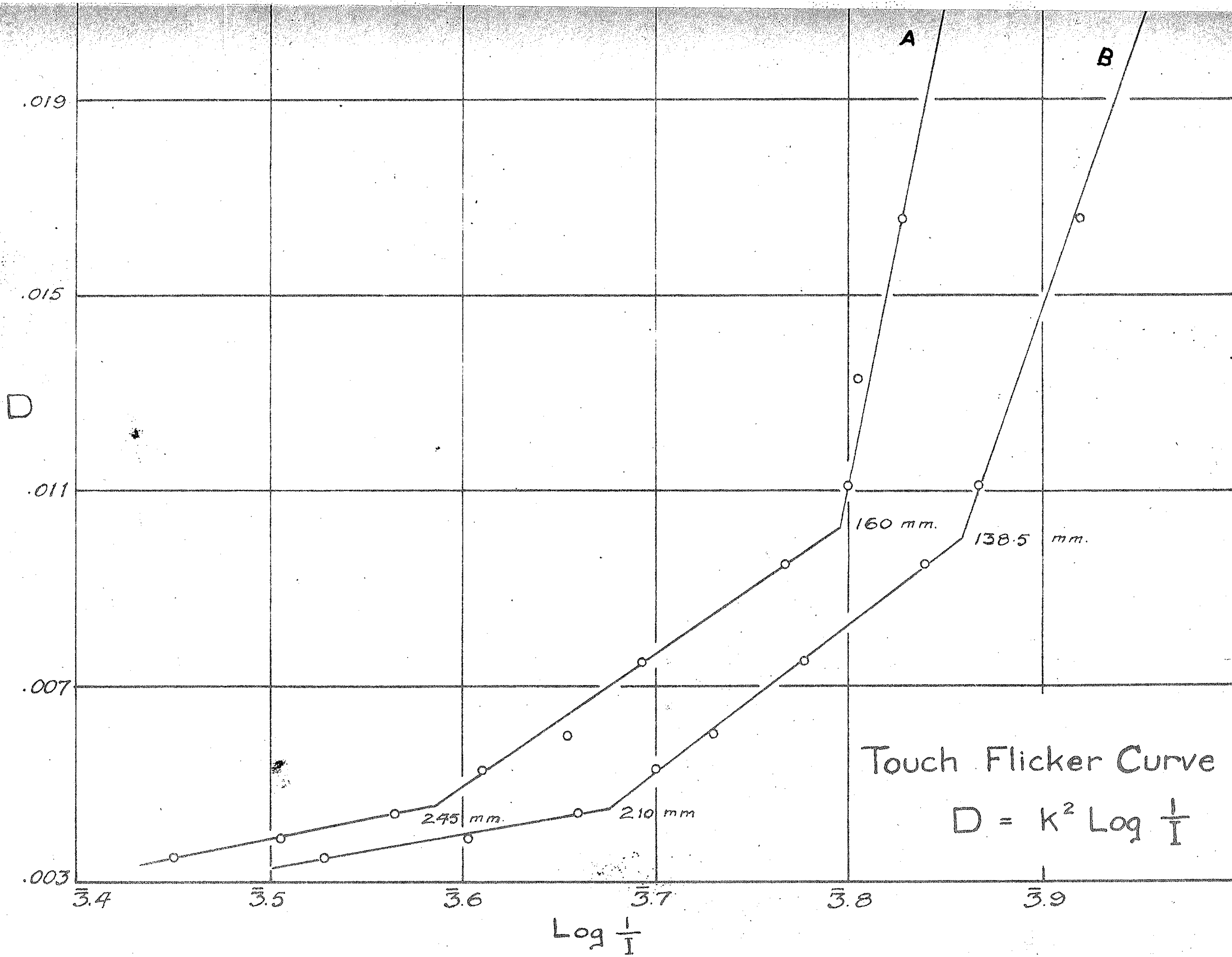
where D is duration, I intensity of stimulus, and C and K are constants.

Two distinct curves were found when the results were plotted. Since two sets of receptors are known to be present, this result would be expected. The one set of receptors has been called the superficial, the others the deep.

If the duration is kept constant it is found that it requires a greater intensity of stimulus to stimulate the one set of receptors than the other. Wienberg has made what appears to be an arbitrary statement by saying that the set which it is most difficult

TABLE 3.

Openings	Duration	Pressure		Log $\frac{1}{P}$	
		A	B	A	B
3	.0222	140	108	$\bar{3}.854$	$\bar{3}.966$
4	.0166	149	120	$\bar{3}.827$	$\bar{3}.921$
5	.0133	156	-	$\bar{3}.807$	-
6	.0111	158	136	$\bar{3}.801$	$\bar{3}.866$
7	.0095	171	143	$\bar{3}.767$	$\bar{3}.841$
9	.0075	203	167	$\bar{3}.693$	$\bar{3}.777$
11	.0060	221	186	$\bar{3}.655$	$\bar{3}.731$
13	.0053	245	200	$\bar{3}.611$	$\bar{3}.699$
15	.0044	260	219	$\bar{3}.584$	$\bar{3}.660$
17	.0039	313	250	$\bar{3}.504$	$\bar{3}.602$
19	.0035	354	297	$\bar{3}.450$	$\bar{3}.528$



A
B
.019
.015
.011
.007
.003
D
160 mm.
138.5 mm.
245 mm.
210 mm
Touch Flicker Curve
D = K² Log 1/I
3̄.4
3̄.5
3̄.6
3̄.7
3̄.8
3̄.9
Log 1/I

Fig. 5

to stimulate is the superficial and the set which can be stimulated the more easily has been called the deep.

From the fact that one set is easier to stimulate than the other it appears that the logical thing would be to assume that the receptors were nearer the surface of the skin. These would be the superficial.

If cocaine was applied to the lip and measurements made it would be possible to determine which was the deep and which the superficial receptors. This was not done on account of the fact that it would take considerable time before the end-organs returned to their normal state.

If the flicker curves are examined it is found that there are two distinct breaks in both curves. In the "A" curve there is a break at 160 mms intensity and another at 245 mms. For the "B" curve the breaks are at 138.5 mms and 210 mms pressure.

On examining the increment curve which was taken with the same nozzle and same apparatus it is found that there are three breaks in the curve. One break is at an initial pressure of 96 mms, another at 160 mms and the third at 245 mms pressure.

The breaks in the increment curve and those in the "A" curve are at almost the same intensity of stimulus. The one set correspond to with 6 mms intensity of stimulation and the other set to within 1 mm.

difference of intensity.

It may be inferred from this that the same set of receptors has been used to obtain the "A" flicker curve and the increment curve. That is to say, if Wienberg's statement is disregarded, that the deep receptors have been used to obtain the law,

$$\frac{I}{\Delta I} = K^2 \log \frac{1}{I} + C$$

Due to the correspondence of these breaks it may also be concluded that at a definite intensity of stimulus the receptors change their sensitivity.

In the case of efferent nerves the end reaction is a contraction or secretion which can be measured quantitatively in terms of physical and chemical units of sensation. The end reaction of sensory nerves is a state of consciousness for which there is no standard of measurement.

During this experimental work an attempt has been made to keep all conditions constant. Knowing what stimulus is applied and what results are obtained our problem is to find out what happens in the receptor when it is stimulated.

Two laws have been found to hold for the tactile receptors:

$$D = K_1^2 \log \frac{1}{I_1} + C_1 \quad \ldots\ldots (1)$$

$$\frac{1}{\Delta I_2} = K_2^2 \log \frac{1}{I_2} + C_2 \quad \ldots\ldots (2)$$

On account of the correspondense in the

breaks of the two curves

$$I_1 = I_2 = I$$

On substituting in (1) and (2)

$$D = K_1^2 \log \frac{1}{I} + C_1 \quad \ldots . (3)$$

$$\frac{1}{\Delta I} = K_2^2 \log \frac{1}{I} + C_2 \quad \ldots . (4)$$

Solve (3) and (4) by eliminating I. Then

$$\Delta I = \frac{K_1^2}{K_2^2 D} - \frac{K_1^2}{K_2^2 C_1} + \frac{1}{C_2}$$

$$= \frac{K}{D} + \text{constant}$$

$$= K \left(\frac{1}{D}\right) + M \quad \ldots \ldots (5)$$

Where ΔI is increment of intensity, D is duration of stimulus and K and M are constants.

Two stimuli must differ by a minimal amount in order that the difference in sensation produced may be distinguished. Let us take this minimal amount to be ΔI. ΔI will produce a certain number of sensation units.

Therefore for a certain intensity sensation produced by ΔI = constant $\times$ unit of sensation.

Therefore for a certain intensity of stimulus the sensation produced by ΔI is equal to a constant $\times$ unit of sensation.

It has been shown mathematically that

$$\Delta I = K\left(\frac{1}{D}\right) + M$$

Hence $\frac{1}{D}$ is unit of sensation.

Burns [5] says: "Each receptor has a certain functional inertia and will not respond to stimulation until the energy of the stimulus has reached a minimal value which is specific for each receptor and for each form of stimulus." This is probably represented by "M" in equation (5).

It has been shown that sensation varies as $\frac{1}{D}$

but $D \propto \log \frac{1}{I}$

since $D = K^2 \log \frac{1}{I} + C$

Therefore $\frac{1}{D} \propto \frac{1}{\log \frac{1}{I}}$

Hence $S \propto \frac{1}{\log \frac{1}{I}}$

That is sensation varies as the reciprocal of $\log \frac{1}{I}$.

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