

ENERGY EFFICIENT STABILITY CONTROL OF A BIPED BASED ON THE CONCEPT OF LYAPUNOV EXPONENTS

by

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Abstract

Balance control is important for biped standing. Due to the time-varying control bounds induced by the foot constraints, and the lack of tools for analyzing stability of highly nonlinear systems, it is extremely difficult to design balance control strategies for a standing biped with a rigorous stability analysis in spite of large efforts. In this thesis, three important issues are fully considered for a standing biped: maintaining the postural stability, minimizing the energy consumption and satisfying the constraints between the biped feet and the ground. Both the theoretical and the experimental studies on the constrained and energy-efficient control are carried out systematically using the genetic algorithm (GA). The stability for the proposed balancing system is thoroughly investigated using the concept of Lyapunov exponents.

On the other hand, the controlled standing biped is characterized by high nonlinearity and great complexity. For systems with such features, in general the Lyapunov exponents are hard to be estimated using the model-based method. Meanwhile the biped is supposed to be stabilized at the upright posture, indicating that the system should possess negative Lyapunov exponents only. However the accuracy of negative exponents is usually poor if following the traditional time-series-based methods. As it is nontrivial to examine the system stability for bipedal robots, the numerical accuracy of the estimated Lyapunov exponents is extremely demanding. In this research, two novel approaches are proposed based upon system approximation using different types of Radial-Basis-Function (RBF) networks. Both the proposed methods can estimate the exponents reliably with straightforward algorithms, yet no mathematical model is required in any newly developed method. The efficacies of both methods are demonstrated through a linear quadratic regulator (LQR) balancing system for a standing biped, as well as several other dynamical systems.

The thesis as a whole, has set up a framework for developing more sophisticated controllers in more complex movement for robot models with less conservative assumptions. The systematic stability analysis shown in this thesis has a great potential for many other engineering systems.

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Table of Contents

Abstract	i
Acknowledgments	iii
Dedication	iv
Table of Contents	v
List of Tables	viii
List of Figures	x
List of Notations	xiii
1 Introduction	1
1.1 Motivations	1
1.2 Literature Review	3
1.2.1 Modeling for Biped Quiet Standing	4
1.2.2 Balance Control for Biped Standing	6
1.2.3 Stability Analysis	10
1.2.4 Experimentation on Balance Control	19
1.3 Objectives	20
1.4 Thesis Organization	22
2 Mathematical Preliminaries	24
2.1 Introduction	24
2.2 Model Description	25
2.3 The Concept of Lyapunov Exponents	29
2.4 Methods of Calculation of Lyapunov Exponents	33
2.4.1 Calculation of Lyapunov Exponents from Mathematical Models	33
2.4.2 Estimation of Lyapunov Exponents from Time Series	36
2.5 Discussion	38
3 Estimation of Lyapunov Exponents from Time Series Using Radial-Basis-Function (RBF) Networks	41
3.1 Introduction	41
3.2 Radial-Basis-Function (RBF) Network	42

3.3	Estimation of Lyapunov Exponents via MIMO RBF Networks	46
3.3.1	Lyapunov exponents of bipedal standing with an LQR control	48
3.3.2	Lyapunov exponents of a hydraulic actuator system	55
3.4	Estimation of Lyapunov Exponents via MISO RBF Networks	62
3.4.1	Reconstruction of system attractors	62
3.4.2	Generation of Jacobian matrices	64
3.4.3	Lyapunov exponents of the LQR balance control system	65
3.4.4	Estimating Lyapunov exponents of the LQR balance control system using noisy data	70
3.4.5	Lyapunov exponents of the Hénon map	74
3.4.6	Lyapunov exponents of the Lorenz system	79
3.5	Discussion	84
4	Constrained and Energy Efficient Control of Biped Standing	88
4.1	Introduction	88
4.2	Development of the PD Controller Using Genetic Algorithm	89
4.3	Simulation Results	92
4.4	Stability Analysis	96
4.4.1	Stability of the biped balancing system	97
4.4.2	Structural stability analysis	100
4.4.3	Discussion	103
4.5	Summary	107
5	An Experimental Study on Constrained Balance Control of a Standing Biped	109
5.1	Introduction	109
5.2	Method	111
5.2.1	Model Description	111
5.2.2	GA-based Joint Control	116
5.3	Results	117
5.3.1	Experimental Setup	117
5.3.2	Experiment with the GA-based PID control	120
5.3.3	Comparative experiment with different PID gains	127
5.4	Stability Analysis	138
5.5	Summary	140
6	Conclusions and Future Works	143
6.1	Conclusions	143
6.1.1	Balance control and stability analysis of standing bipeds	143
6.1.2	Stability analysis using time-series-based Lyapunov exponents	145

6.2 Future Works	147
References	148

List of Tables

Table 2.1	Biped model parameters.	27
Table 3.1	Nominal values of the biped model parameters.	49
Table 3.2	Structure of the RBF network for the biped balancing system.	52
Table 3.3	Lyapunov exponents of the biped balancing system and their relative errors after 100 seconds.	55
Table 3.4	Hydraulic actuator system parameters.	57
Table 3.5	Structure of the RBF network for the hydraulic actuator system.	59
Table 3.6	Lyapunov exponents of the hydraulic actuator system and their relative errors after 100 seconds	61
Table 3.7	Structure of the RBF network for estimating the regression Equation (3.12) for the biped balancing system.	66
Table 3.8	Lyapunov exponents (LEs) and their relative errors for the biped balancing system.	67
Table 3.9	Lyapunov exponents (LEs) and their percentage errors for the biped balancing system in a 4-dimensional embedding space with different measurement noise levels.	72
Table 3.10	Structure of the RBF network for estimating the regression Equation (3.12) for the Hénon map.	76
Table 3.11	Lyapunov exponents (LEs) and their percentage errors for the Hénon map in a 2-dimensional embedding space.	77
Table 3.12	Lyapunov exponents (LEs) and their percentage errors for the Hénon map in a 2-dimensional embedding space with different measurement noise levels.	78
Table 3.13	Structure of the RBF network for estimating the regression Equation (3.12) for the Lorenz system.	81
Table 3.14	Lyapunov exponents (LEs) and their percentage errors for the Lorenz system in 3-dimensional embedding space.	82

Table 3.15	Lyapunov exponents (LEs) and their percentage errors for the Lorenz system in a 3-dimensional embedding space with different measurement noise levels.	83
Table 4.1	Lyapunov exponents after 100 seconds.	98
Table 4.2	Values of the biped balancing system subject to different control gains.	103
Table 5.1	Nominal values of the biped model parameters.	112
Table 5.2	Parameters of the actuators.	119
Table 5.3	Numerical values of the index J under different cases. . . .	137

List of Figures

Figure 2.1	(A) simplified biped model, (B) free body diagram of the two link inverted pendulum, and (C) free body diagram of the foot-link.	26
Figure 2.2	Evolution of an initially infinitesimal 2-dimensional sphere. .	31
Figure 2.3	The geometrical interpretation of GSR for $\delta x_1^{(j)}$ and $\delta x_2^{(j)}$ ($j = 1, \dots, k$. j is the number of integration step). $\delta x_1^{(j)}$ and $\delta x_2^{(j)}$ are orthogonalized into $v_1^{(j)}$ and $v_2^{(j)}$, then normalized into $u_1^{(j)}$ and $u_2^{(j)}$. Here $proj_{v_1^{(j)}}(\delta x_2^{(j)})$ denotes the projector of the vector $\delta x_2^{(j)}$ on the vector $v_1^{(j)}$. $v_2^{(j)}$ is exactly equal to $\delta x_2^{(j)} - proj_{v_1^{(j)}}(\delta x_2^{(j)})$	35
Figure 3.1	Typical schematics of (A) a multiple-input multiple-output (MIMO) RBF network, and (B) a multiple-input single-output (MISO) RBF network.	43
Figure 3.2	Evolution of (A) the system states, and (B) the control torques. .	50
Figure 3.3	Evolution of (A) the vertical ground reaction force, (B) the horizontal ground reaction force and (C) location of COP. .	51
Figure 3.4	Absolute errors of Jacobians (AEJ).	53
Figure 3.5	Relative errors of Jacobians (REJ).	54
Figure 3.6	(A) Evolution of the Lyapunov exponents of the biped balancing system; (B) Close-up views of the exponents.	54
Figure 3.7	Schematic of a typical valve-controlled hydraulic actuator. .	56
Figure 3.8	Evolution of (A) the states of the hydraulic actuator system, where the left y -axis is for the states $\tilde{x}$ and $\tilde{v}$ while the right y -axis is for the state $\tilde{P}$; (B) the control signal x_v	58
Figure 3.9	Comparison between the mathematic model and the neural model of (A) $\tilde{x}$, (B) $\tilde{v}$ and (C) $\tilde{P}$	60
Figure 3.10	(A) Evolution of the Lyapunov exponents of the hydraulic actuator system; (B) Close-up views of the exponents.	61

Figure 3.11	Evolution of Lyapunov exponents (LEs) in the 4-dimensional embedding state space of the biped balancing system. . . .	66
Figure 3.12	The trajectory of the Hénon map in (A) the original phase space; (B) a 2-dimensional embedding space.	75
Figure 3.13	Evolution of Lyapunov exponents in the 2-dimensional embedding phase space.	76
Figure 3.14	The trajectory of the Lorenz system in (A) the original phase space; (B) a 3-dimensional embedding space.	80
Figure 3.15	Evolution of Lyapunov exponents in the 3-dimensional embedding phase space.	82
Figure 4.1	Block diagram of GA-based PD control.	92
Figure 4.2	The variation of the cost function.	93
Figure 4.3	Evolution of (A) the biped states, and (B) the control torques.	94
Figure 4.4	Evolution of (A) the vertical ground reaction force, (B) the horizontal ground reaction force and (C) location of COP.	95
Figure 4.5	Evolution of the Lyapunov exponents.	98
Figure 4.6	A sample view of the stability region of the balancing biped determined by the LLE, which are slices with depth. The colorful points shown in the figure represent the initial states from which the system trajectory can be regulated to the equilibrium point (the upright posture). The grey points represent the initial states from which certain constraint between the biped foot-link and the ground will be violated during the regulation.	99
Figure 4.7	A sample view of the structural stability region of the balancing biped determined by the LLE, which are slices with depth. The colorful points shown in the figure represent the stable PD gains combinations, and those marked by the grey points represent the PD gains combinations with which certain constraint between the biped foot-link and the ground will be violated during the regulation.	102
Figure 4.8	General trend of the system largest Lyapunov exponent (LLE).	104
Figure 4.9	General trend of the system performance (i.e., the value of the cost function).	105
Figure 5.1	Block diagram of GA-based PID control.	117

Figure 5.2	Experimental setup: (A) the prototype of a one legged robot to be balanced via two electric actuators attached at the ankle and hip joints with (B) the hardware facilities: PC, DAQ, servo amplifiers, and power supply.	118
Figure 5.3	Performance of the convergence test.	121
Figure 5.4	Evolution of (A)the angle and the angular speed of the lower link; (B)the angle and the angular speed of the upper link; (C)two control torques applied at the ankle joint and the hip joint respectively; and (D)two voltage signals generated by the PID controllers.	123
Figure 5.5	Evolution of (A) the vertical ground reaction force, (B1)&(B2) the horizontal ground reaction force and (C) the location of COP.	126
Figure 5.6	Evolution of (A)the angle and the angular speed of the lower link; (B)the angle and the angular speed of the upper link; (C)two control torques applied at the ankle joint and the hip joint respectively; and (D)two voltage signals generated by the PID controllers.	129
Figure 5.7	Evolution of (A) the vertical ground reaction force, (B1)&(B2) the horizontal ground reaction force and (C) the location of COP.	131
Figure 5.8	Evolution of (A)the angle and the angular speed of the lower link; (B)the angle and the angular speed of the upper link; (C)two control torques applied at the ankle joint and the hip joint respectively; and (D)two voltage signals generated by the PID controllers.	133
Figure 5.9	Evolution of (A) the vertical ground reaction force, (B1)&(B2) the horizontal ground reaction force and (C) the location of COP.	135
Figure 5.10	J values under different cases.	137
Figure 5.11	Evolution of the system Lyapunov exponents. The converged value of each exponent has been shown as the Y reading of the datatip in the corresponding subplot, while the X reading reflects the time the system has progressed over.	138
Figure 5.12	A projection of the stability region on the $\theta_1 - \theta_2$ plane of the controlled robot. The black points shown in the figure represent the initial states from which the system trajectory can be regulated to the equilibrium state (the upright posture) without foot motion.	140

List of Notations

$\mathbb{R}^n$	n -dimensional Euclidean space
$\in$	Belong to
$\mathbf{A}_{n \times m}$	Matrix $\mathbf{A}$ of size $n \times m$
$\mathbf{A}^T$	Transpose of the matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	Inverse of the matrix $\mathbf{A}$
$diag(\mathbf{A}_1, \dots, \mathbf{A}_n)$	A diagonal matrix with $\mathbf{A}_i$ as its i^{th} diagonal element
$\dot{x}$	First derivative of a variable x with respect to time t
$\ddot{x}$	Second derivative of a variable x with respect to time t
∂	Partial derivative
$D\mathbf{f}_{\mathbf{x}}$	First derivative of a vector function $\mathbf{f}$ with respect to the vector $\mathbf{x}$ such that $D\mathbf{f}_{\mathbf{x}}$ is a matrix with its entries as $\frac{\partial f_i}{\partial x_j}$
$\sum$	Sumation
$\ \cdot\ $	Norm function

Chapter 1

Introduction

1.1 Motivations

Bipedal robots have significant advantages over conventional wheeled robots, as the mechanical design of legged robots endows them with a stronger ability to move over rough terrain and negotiate obstacles than wheeled robots. However, along with the advantage of increased mobility, come challenges associated with balance control and stability, which need to be addressed through advanced design and analysis.

A standing biped in an erect state, is inherently prone to fall during quiet standing since only the biped feet are in contact with the ground. Additionally, its center of mass (COM) is usually maintained above a relatively small base of support (Masani, Vette, and Popovic, 2006). The biped body thus features a high potential energy, leading to the priority of equilibrium control, which is essential for safe and successful co-existence of bipedal robots within normal human environments among almost all motor tasks including quiet standing (Gatev, Thomas, Kepple, and Hallett, 1999).

Due to the fact that during standing only the biped feet are in contact with the ground but not rigidly attached to the ground, a standing biped is always subject to constraints: there should be no lifting, no slipping, no tipping over about either the toe or the heel (Pai and Patton, 1997). It has been documented by Yang and Wu (2006a) that such constraints jointly determine the upper and the lower bounds of the control torque, which turn out to be two time-varying functions of the system states. Many published works on balance control have been carried out under the assumption that all these constraints are satisfied automatically. This assumption although simplifies the problem, can be misleading.

There is limited previous research on biped balance control, which considers the constraints between the foot-link and the ground. The balancing systems that do consider constraints between the foot-link and the ground, however, are not energy efficient. Energy efficiency is an important issue to be resolved before the use of bipeds is viable. Since bipedal robots need to carry their energy sources, a lower rate of energy consumption would directly contribute to a longer working cycle. Usually when a balance control algorithm is developed based on Lyapunov's stability theory, it has little flexibility to include optimization criteria associated with certain constraints. Thus the design of a constrained and energy-efficient balance control for the standing biped is a demanding task.

Another challenge for balancing a standing biped is the lack of effective tools for stability analysis. Considering that a standing biped is always prone to fall over, control methods have to be very effective and safety aspects are mandatory since any collapsing may result in a failure. The stability region of a controlled biped needs to be

determined for measuring the biped's ability to recover from disturbances. However, due to the complexity of the biped's dynamics, especially the existing constraints, the use of classical stability analysis tools is extremely difficult. For example, Lyapunov's second method has been widely used for stability analysis, but as far as the complex standing biped models are concerned, derivation of appropriate Lyapunov functions is not simple as no constructive rule is available. Alternatively, Lyapunov exponents, defined as the average exponential rates of divergence or convergence of nearby trajectories in the state space, can characterize the system stability. Generally, the signs of Lyapunov exponents indicate the asymptotic property of the dynamical system.

Motivated by the above discussion, the primary purpose of this research is to (1) design an effective control strategy, with which a standing biped can achieve the erect posture at a minimal level of energy consumption and without foot motion, i.e., always satisfying the constraints between the biped feet and the ground during the regulation phase; (2) develop a constructive and reliable method for stability analysis of such a complicated nonlinear system. The research, in summary, can have significant impact not only on the stable control of locomotion of the bipedal robots, but also on the wider subjects of stability analysis of nonlinear dynamical systems.

1.2 Literature Review

The complexity of the biped balancing system and its ability to maintain stable standing without falling or resort to taking a step despite various perturbations, has been paid considerable attention in both the field of biomechanics and robotics. A variety of theories have been developed, trying to reveal the control mechanism of

biped quiet standing. The following sections provide a detailed literature review of studies on bipedal robots, which mainly cover topics on dynamic modeling, control design and stability analysis.

1.2.1 Modeling for Biped Quiet Standing

Bipedal robots are thought of as a natural form of machine to interact with humans. Selecting a mechanical model with proper degrees of freedom to keep the equations of motion of a standing biped at a manageable level and yet reasonably describe the motion of interest requires advanced knowledge of classic mechanics, nonlinear dynamics and advanced mathematics.

It has been reported from the experimental observation that the oscillation of the COM on the contact surface is in phase with the center of pressure (COP). Such a phase lock is a consequence of the dynamics of the plant and is independent of the stabilization mechanism (Winter, Patla, Riedtyk, and Ishac, 1998; Morasso and Schieppati, 1999). As a result, studying the sway of a body in the horizontal plane as a time function has become the basic method of judging the correctness of stability system functioning. This can usually be done in two ways: by measuring the movement of the whole-body COM, which is capable of representing individual sway satisfactorily during quiet standing, or by measuring the movement of the COP. Many studies choose to measure the COP directly to avoid the experimental difficulty of determining and tracking the COM.

There are many approaches available to estimate COM movement. One of them is the measurement of body segment movement, which is described in (Gage, Winter,

Frank, and Adkin, 2003). The total body COM is a weighted average of the COM of all individual body segments. By comparing the whole-body COM movement with the one predicted from a simple inverted pendulum, the researchers examined the validity of how well the individual segments and lower limb angles temporally and spatially synchronize with the total body COM. The collected kinematic and kinetic data was found to support the inverted pendulum model of quiet standing.

One can find a lot of examples of investigating biped quiet standing using an inverted pendulum model (Morasso and Schieppati, 1999; Winter, Patla, Riedtyk, and Ishac, 2001; Kiemel, Kelvin, and Jeka, 2002; Micheau, Kron, and Bourassa, 2003; Peterka, 2002). Ito, Takishita, and Sasaki (2006) modeled the biped as an inverted pendulum with a supporting foot segment, connected at the ankle joint. The body segment moves only within the sagittal plane. The foot segment contacts the ground with only two points: the heel and the toe, which does not slip on the ground and its shape is symmetrical in the anterior-posterior direction. The ankle joint is located in the middle of the foot segment with zero height. Pai and Patton (1997) also simplified the biped as an inverted pendulum, but with a triangular foot segment. In their work, they identified the roles of constraints in developing foot motion and numerically determined the movement region in terms of sets of feasible COM velocity-position combinations, within which the biped's balance can be maintained. Based on this model, Yang and Wu (2006a) thoroughly investigated the effects of the three constraints on biped balance control. The COP constraint in their paper was reported to be the dominant one in limiting the control torque when the angular velocity is kept below the critical value, which suggests the importance of foot design

in balance control of a standing biped.

Modeling postural sway as an inverted pendulum assumes a rigid structure above the ankles. Although this simplifies analysis, it also induces resultant performance limitations. In general cases, the biped body is a multi-linked segmented structure capable of moving at all joints above the ankle (Gage, Winter, Frank, and Adkin, 2003; Alexandrov, Frolov, and Horak, 2005). Movement at joints other than the ankle, especially the hip, has been demonstrated in response to external perturbations (Horak and Nashner, 1986; Kuo and Zajac, 1993a,b), as well as during unperturbed quiet stance (Day, Steiger, Thompson, and Marsden, 1993). The assumption that the model need only consider the ankle joint to balance the biped is valid only if the movements in all the joints, are artificially blocked (Peterka, 2002). Barin (1989) found that a two-link model with feedback control gains computed using multiple regression, is sufficient to accurately model and predict the COM excursion. Additionally, from experiment data, Jiang, Sachio, and Hidenori (2006) found the ankle and the lumbosacral joint swayed with approximately the same amplitude during the static upright stance of humans. They modeled the human body as a two-link inverted pendulum system, providing that this model is reasonable and useful for studying the balance of biped standing.

1.2.2 Balance Control for Biped Standing

Balance maintenance during biped standing is essential for biped balance control. Various control strategies have been developed for balancing biped standing. For example, Gatev, Thomas, Kepple, and Hallett (1999) found preceding muscle activities

in plantar flexors during human standing and suggested that a feed-forward control mechanism is responsible for ensuring balance during quiet standing. This claim was supported by Morasso and Schieppati (1999), who also reported that the feed-forward control mechanism is required to generate the preceding motor command. However, Masani, Vette, and Popovic (2006) demonstrated that the preceding motor command could be accomplished by applying an appropriate proportional-derivative (PD) feed-back control strategy, which, in fact, has been popularly studied for biped balance control (Kuo, 1995). Matjačić, Hunt, Gollee, and Sinkjaer (2003) documented that a PD controller can theoretically stabilize an inverted pendulum of human body size when the system's closed-loop time delay is not too long. In an extreme case where the closed-loop time delay is zero, the PD controller acts as a regulator of mechanical stiffness and viscosity. Morasso and Sanguineti (2001) demonstrated that derivative control action is necessary as ankle muscle stiffness alone cannot stabilize balance during quiet standing.

Moreover, Ito, Takishita, and Sasaki (2006) proposed a biped stance maintenance method that contains the integral feedback of the ground reaction forces, which is used to improve the convergence of the posture to the equilibrium upright position. As a result, the biped posture can adaptively change with respect to external forces. Abdallah and Goswami (2005) presented a two-phase control strategy for robust balance maintenance under a force disturbance. The first phase, called the Reflex Phase, is designed to withstand the immediate effect of the force. The second phase is the Recovery Phase where the system is steered back to a statically stable "home" posture. A control model of individual stance using fuzzy logic was first published by

Jacobs (1997). The proposed control model was not based on purely inverted pendulum body mechanics. However, due to its reliable use of physiological knowledge, the control model was found to be useful in studying the states going beyond the quiet standing. Lower (2008) presented a simulation model of a human in quiet standing that was based on a stiff inverted pendulum model with a Mamdani fuzzy controller. The simulation experiments showed that the presented model is helpful in estimation of empirically observed phenomena.

A standing biped is always subject to three constraints between the feet and the ground, which include the vertical ground reaction force being upward (gravity constraint), the horizontal ground reaction force being lower than the maximum static friction (friction constraint), no tipping-over about either the toe or the heel and the COP staying within the boundary of contact surface (COP constraint). However, research on the effects of constraints on biped locomotion are sparse. One distinguished work is from Pai and Patton (1997), where all three constraints during biped standing have been taken into consideration. Although the movement region within which the biped's balance can be achieved was determined in their work, the extreme values of the torques in their paper were directly taken from a musculoskeletal model (Delp, Loan, Hoy, Zajac, Topp, and Rosen, 1990), and the effects of constraints on the balance control of the standing biped were excluded from their report. This limitation was improved in the work conducted by Yang and Wu (2006a), where a solid study on effects of the three foot constraints on balance control was presented. They also showed that such control bounds have significant effects on fall prediction and prevention during biped standing. Furthermore, Yang and Wu (2006b) proposed a

switching state feedback control scheme to stabilize the biped at the upright position while satisfying the constraints between the feet and the ground. The controller is a simple PD controller, whose output torque takes the value of the control bounds when it approaches them.

The design of biped balance control should consider reducing energy consumption of the control system since bipedal robots with a lower rate of energy consumption would directly contribute to a longer working cycle. In this sense, the design of control strategies with low energy consumption is essential. The linear quadratic regulator (LQR) algorithm is one of the tools available to improve control performance. Via the LQR design, a set of feedback gains may be found that can minimize a specific quadratic index and make a closed-loop linear system stable (Lewis and Syrmos, 1995; Bryson and Ho, 1975). As another optimization tool, Genetic Algorithms (GAs) are more frequently employed for nonlinear problems and multi-objective optimizations. Arakawa and Fukuda (1996) used a GA to generate the natural motion of biped locomotion with energy optimization. Capi et al. (2002), Park and Choi (2004) applied GAs to generate the optimal trajectory for biped walking. Garder and Høvin (2006) combined an incremental approach with a GA to generate precise walking patterns. Almost all the above works focused on energy consumption of walking robots. For balancing biped standing, Ghorbani, Wu, and Wang (2007) developed a general regression neural network (GRNN) feedback controller. Apart from stabilizing standing, the GRNN controller was also designed to minimize an energy-related cost function. The optimization was carried out using a GA. During each optimization iteration process, the GRNN was directly trained to provide the feedback optimal

control gains.

1.2.3 Stability Analysis

Stability is a basis for the successful bipedal locomotion including standing. There are two different stability concepts that have been widely utilized in bipedal locomotion. One is termed as transient stability, which can be judged by following the COP criterion (Murray, Seireg, and Scholz, 1967), zero-moment-point (ZMP) criterion (Vukobratović and Stepanenko, 1972) or the feet rotation indicator (FRI) criterion (Goswami, 1999). The COP is a projection of the centroid of the vertical force distribution on the ground plane (Benda, Riley, and Krebs, 1994), which in effect, is the location where the net ground reaction force vector would act if it is considered to have a single point of application (Cavanagh, 1978). The FRI is a point inside or outside the support polygon on the feet/ground surface, where the net ground reaction force would have to act to keep the feet stationary (Goswami, 1999). The support polygon of the biped is defined as the area of physical interaction between the biped and the ground surface, which may be the area of the supporting foot during the single-support phase, or the polygon created by the boundary of the two feet during the double-support phase. The concept of ZMP is frequently used in the study of biped gait synthesis. It specifies the point on the feet/ground contact surface with respect to which the dynamic reaction force does not produce any moment in the horizontal direction, i.e. the point where the total of vertical inertia and gravity forces equals zero (Vukobratović and Borovac, 2004). If ideal conditions are considered whereby neither the feet nor the ground can deform under the load and the

ground is level, the COP and ZMP locations will always coincide.

There is another stability concept, which is fundamentally based on the Lyapunov's stability theory and investigates the long-term behavior of motion under the influence of disturbances. Generally the effects of a disturbance are insignificant for a stable motion, which can be perceived from the observation that the disturbed motion always stays close to the undisturbed one. On the contrary, in an unstable case any infinitesimal disturbance will lead to a considerable change in the motion.

The classical method of conducting this long-term stability analysis for nonlinear control systems is using Lyapunov's stability theory. The key requirement for evaluating the ability of a system to maintain stability based on this theory is to construct a Lyapunov function. Since no constructive and general rules are provided for the derivation of Lyapunov functions, current understanding of Lyapunov's stability theory leaves stability analysis for most complicated nonlinear systems challenging, which restricts the application of this theory. Over the last few decades, numerous techniques have been proposed to construct Lyapunov functions for some special nonlinear systems. These techniques among others, include the method of analogy with linear systems by Barbasin (1960), the method of partial integration by Ponzo (1965) and Huaux (1965), the method of system energy by Marino and Nicosia (1983), the integral method, the scalar-Lyapunov-function method and the intrinsic method all by Chin (1986, 1987a,b, 1989), the extended integral method by Wu (1996) and the references cited in her work.

As an important point, it should be noticed that Lyapunov's stability theory is based on conventional solution theory, i.e., the dynamical systems must be smooth.

For the stability analysis of non-smooth systems, one has to extend Lyapunov's second method. Paden and Sastry (1986) first generalized Lyapunov's second method. By imposing a non-zero upper bound of the derivative of the Lyapunov function with respect to time, they proved that the states of the system (solution in the sense of Filippov) converge to the equilibrium point in finite time. Southwood, Radcliffe, and MacCluer (1990) extended Lyapunov's second method again based on Filippov's solution theory. The derivative of Lyapunov functions on the discontinuity surfaces in their work were replaced with Dini-derivate. Shevitz and Paden (1994) also made a systematic extension of Lyapunov's second method to non-smooth dynamical systems by means of developing non-smooth Lyapunov functions. Their result turned out to be applicable to switching systems, for which the established Lyapunov functions are only piece-wise smooth. All these extensions of Lyapunov's stability theory to non-smooth systems were based on the belief that non-smooth Lyapunov functions are natural for non-smooth dynamical systems. However, to apply the Lyapunov's stability theory with non-smooth Lyapunov functions, handling derivatives of Lyapunov functions is a big challenge when the solution trajectories approach the discontinuity surfaces. Stimulated by this, Wu (1996) proved that Lyapunov's second method can be directly applied to non-smooth dynamical systems in cases when the existence and uniqueness of Filippov's solution are guaranteed. Later in (Wu, Onyshko, Sepehri, and Thornton-Trump, 1998), a method of constructing smooth Lyapunov functions for non-smooth systems was developed, which was demonstrated to be much easier for some engineering systems as compared to its non-smooth counterpart. The results provided a solid framework in the study of posture stability and control of biped

locomotion.

Lyapunov's stability theory has been used to analyze biped posture stability. Early works on the stability of biped models (Vukobratović, Frank, and Juričić, 1970; Jr.Golliday and Hemami, 1976; Hemami and Jr.Golliday, 1977; Hemami and Cvetković, 1977; Hemami and Katbab, 1982) were restricted to small deviations about a vertical stance. Hemami and Wyman (1979) proposed a three-link biped model and the pole-assignment techniques based on Lyapunov's linearization method were employed to control the dynamical system. Iqbal, Hemami, and Simon (1993) studied human postural and movement stability for simple voluntary movements through a frontal four-link analytical biped model. Hemami and Utkin (2002) studied Lyapunov stability of embedded rigid bodies with holonomic constraints. They presented a systematic method of stabilizing the systems and a procedure for constructing Lyapunov functions, but the effects of the foot constraints on the stabilization mechanism were excluded from their consideration.

Although Lyapunov's second method is efficient for certain systems, due to the lack of construction methods, it is difficult to derive a Lyapunov function for highly nonlinear systems. An alternative method for stability analysis is therefore desirable, especially for the biped balancing system. Pai and Patton (1997) carried out stability analysis on the balance control of human subjects with an inverted pendulum biped model. Based on clinical observations, they numerically determined the stable movement region in terms of sets of feasible COM velocity-position combinations. In detail, if the ankle torque can drive the COM projection of the biped within a short time period (1s) into a region on the ground between the heel and the toe and with a

zero angular velocity, the corresponding initial state is then included in the stability region. All initial states satisfying the above condition then constitute the entire stability region associated with the upright posture. The limitation, from a viewpoint of stability, comes from their definition of the stability, which is more rooted in the concept of the transient stability. Such a definition only concerns the system performance within a short time period. More recently, Verdaasdonk, Koopman, van Gils, and van der Helm (2004) published a paper on bifurcation and stability analysis in musculoskeletal systems, where the influence of stretch reflexes on the behavior and stability of musculoskeletal systems was examined using a linearized model of human stance. Their work successfully combined bifurcation analysis with more common biomechanical concepts and tools, establishing that how reflexive feedback terms of muscle can lead to stable equilibria (i.e., posture) and limit cycles (i.e., ankle clonus and gait).

On the other hand, Sekhavat, Sepehri, and Wu (2004) employed the concept of Lyapunov exponents to analyze the stability of a nonlinear dynamical system with success, showing the efficacy of this exponent-based method. A Lyapunov exponent is a number that reflects the rate of divergence or convergence, averaged over the entire attractor, of two neighboring state space trajectories. The concept was fully developed by a Russian mathematician, Alexandr Mikhailovich Lyapunov. Oseledec (1968) gave the theory of Lyapunov exponents in a form adapted to the needs of both the theory of dynamical systems and ergodic theory. It is believed that Lyapunov exponents have many important characteristics, which can be estimated directly from experimental data, without solving the differential or difference equations describing

the corresponding dynamical systems (Kinsner, 2006). As compared to its counterpart, say, Lyapunov's second method, the most attractive point of the concept of Lyapunov exponents is that the methods of calculating the exponents are constructive, making the stability analysis of complex nonlinear systems possible.

Methods of computing/estimating Lyapunov exponents can be categorized into two streams. For systems with explicit mathematical models, Wolf and his collaborators (1985) presented an algorithm for calculating the spectrum of Lyapunov exponents. This model-based algorithm has been successfully applied to many smooth dynamical systems. Müller (1995) extended Wolf's method into non-smooth dynamical systems, i.e., cases when the ordinary differential equations contain non-differentiable terms. He pointed out that the required linearized equations have to be supplemented by certain transition conditions at the instances of discontinuities. Since Lyapunov exponents are calculated numerically over a long period of time, Mickens and his colleagues (2002a; 2002b) developed nonstandard finite difference techniques to improve numerical stability and computing efficiency.

The other stream of methods for calculating Lyapunov exponents are based on time series. The main advantage of using time series is that the data can often be measured experimentally without the knowledge of the system dynamics. This attractiveness makes calculation of Lyapunov exponents feasible for systems with complexities and uncertainties, which are hard to be modeled. The basic idea behind these methods is following sets of trajectories over short time-spans and computing their rates of separation repeatedly, then averaging those rates over the attractor. Wolf, Swift, Swinney, and Vastano (1985) developed a computational approach for

approximating the largest Lyapunov exponent directly from the rate of separation of neighboring points. Sano and Sawada (1985) proposed a method to determine the spectrum of system Lyapunov exponents from the observed time series of a single variable. Abarbanel (1997) reviewed research works on the computation of Lyapunov exponents based on time series for chaotic systems.

Regarding biped balancing systems, Yang and Wu (2006b) successfully applied the concept of Lyapunov exponents to analyze the stability of a PD switching feedback control for biped standing on the basis of a time series. Similarly, through estimating the system Lyapunov exponents, Ghorbani, Wu, and Wang (2007) conducted stability analysis for the proposed GRNN feedback balancing system, where the exponents were derived from a time series as well. The upright posture considered in their works is an example of a disturbed equilibrium; the system stability is assessed by the signs of system Lyapunov exponents. Theoretically, local asymptotic stability of the posture guarantees convergence back to the equilibrium provided the fluctuations around the equilibrium caused by environmental perturbations (such as a push) are small (Alexandrov, Frolov, and Horak, 2005). Dingwell, Cusumano, Cavanagh, and Sternad (2001) reported that traditional measures based on kinematic variability are poor predictors of local stability. They also proposed a method by estimating the largest finite-time Lyapunov exponent based on collected experimental data, to assess local stability for human walking (Dingwell and Cusumano, 2000; Dingwell, Cusumano, Sternad, and Cavanagh, 2000).

Traditional time-series-based method of calculating Lyapunov exponents were developed primarily for diagnosing chaotic systems, of which at least one Lyapunov

exponent is positive. The procedures are claimed to be unreliable for potentially stable engineering systems with only zero and negative exponents, due to the inaccuracy induced by the local linear mapping. To overcome this drawback, Brown, Bryant, and Abarbanel (1991) introduced higher-order Taylor expansion for the local neighborhood-to-neighborhood mapping, which separated the problem of finding the analogue Jacobian matrices from the determination of mapping neighborhoods to neighborhoods and improved the accuracy of the full set of Lyapunov exponents significantly. Later, Yang and Wu (2010) applied this method to a simplified 2-dimensional biped control system, and found that both negative Lyapunov exponents can be estimated accurately by using a quadratic polynomial mapping. However, compared with linear mapping, nonlinear mapping involves a large amount of mathematical derivation. Both the size of the neighborhood-to-neighborhood matrices associated with nonlinear mapping and the number of unknown parameters associated with such matrices, grow rapidly with the increment of the embedding dimension and the order used in the Taylor expansion. In addition, Brown's work shows that the numerical precision of the estimated Lyapunov exponents does not always get improved along with increasing the order of the Taylor expansion. Properly selecting the order for Taylor expansion still remains problematic. Recently, however, a more general form developed for deriving the neighborhood-to-neighborhood matrix of arbitrary dimensionality was developed by Yang and Wu (2011), the order of the Taylor expansion in their derivation was restricted to 2. Gencay and Dechert (1992) proposed another advanced approach based on the neural model, which is also under the stream of time-series-based methods. They employed multiple layer feedforward

(MLF) networks to discover the embedded mapping and extract the Jacobian matrices of the reconstructed dynamics. However, details of the networks employed for the corresponding systems were not provided in their paper.

The accuracy of Lyapunov exponents derived from a time series is sensitive to many factors, including the magnitude of measurement noise, the length of the time series, the choice of time lag and the evolution time for state space reconstruction. Moreover, spurious Lyapunov exponents are often generated when a time series is used for reconstruction of the state space. Identifying true exponents from those spurious ones remains challenging. To sum up, although the time-series-based methods for estimating Lyapunov exponents have the advantage of not requiring mathematical model, more research is still needed to develop methods for accurately estimating Lyapunov exponents from time series of large systems.

An interesting measure of stability is the basin of attraction. The basin of attraction of an attractor, such as a limit cycle, is the set of all the initial conditions in state space that lead to an orbit which approaches the attractor (Seydel, 1988). Schwab and Wisse (2001) calculated the basin of attraction for the gait cycle of the simplest walking model for different slopes (i.e., different gravitational energy inputs). The interpretation of the basin of attraction is easy in their case study because the basin of attraction equals all combinations of initial stance leg angles and angular velocities that lead to stable walking. If the basin of attraction is large, the biped is easier to start up, which is desirable. Thus, they concluded that the size of the basin of attraction is the most important stability measure in designing bipeds. Similar works can be seen in (Yang and Wu, 2006b) and (Ghorbani, Wu, and Wang, 2007), where

the stability region is partially identified in the space of system states based on the largest Lyapunov exponent.

In order to deal with stability analysis using the concept of Lyapunov exponents for systems with implicit mathematical models or great complexities, such as a constrained biped balancing system, two new approaches based on neural networks are proposed in this research work for estimating Lyapunov exponents with high accuracy. In both methods, radial-basis-function (RBF) networks for system approximation should be developed first by using the available time series. Once the networks are trained properly, the spectrum of Lyapunov exponents can be estimated by following the model-based algorithm with the structural information of the neural model.

1.2.4 Experimentation on Balance Control

Although balance control for a standing biped has inspired many research efforts, it is worthy to point out that most of the works was carried out from the perspective of biomechanics. In this category, maintenance of human bipedal individual in quiet standing is assumed to be mainly a sensory task, thus it is dependent on central nervous system (CNS) and its cooperation with the executive system. The common goal of this type of works is to identify the contribution of the intrinsic mechanical properties of the muscular-skeletal system and the reflex gains in controlling balance during standing (van der Kooij, Donker, de Vrijer, and van der Helm, 2004). Resultantly, the motor command data (i.e., joint torques) are only acquired from simulation experiments. The comparative data could be physiologically measured. Within such a framework, conducting real experiments on motor joint control of a standing biped

represented by the inverted pendulum model is unnecessary.

On the other hand, global regulation of rigid robots to a desired configuration has been systematically studied, especially for robot manipulators (Colbaugh, Barany, and Glass, 1997; Luca, Siciliano, and Zollo, 2005; Alvarez-Ramirez, Jose, and Cervantes, 2003). However, for the regulation of biped standing, noting that it is very difficult to synthesize a stable control design in the presence of time-varying bounds on the control output induced by the existing constraints between the biped feet and the ground, experimental studies on constrained balance control are less common. To practically implement a biped balance control, actuators are needed to provide mechanical torques to the biped links. Thus, taking into account the dynamics of the actuators is required, which complicates balance stabilization in the sense that it not only impacts the foot constraints, making the control design more challenging, but also invites problems with system modeling and stability analysis.

1.3 Objectives

There are two objectives for this research. The first objective is to study the balance control strategy of a standing biped. This objective includes two phases. One is designing an energy-efficient balance control to stabilize a standing biped at the upright posture without foot motion (i.e., with the full consideration of the constraints between the biped feet and the ground). The concept of Lyapunov exponents will be used to analyze the stability of the controlled biped subject to the constraints. The stability regions in the system state space and in the parametric manifold will then be determined based on the largest Lyapunov exponent calculated from the

mathematical model. The second phase is to extend the designed constrained balance control strategy to a standing bipedal robot, making physical implementation feasible. For this scenario, inclusion of the actuators' dynamics is necessary, which complicates the overall system, obstructs the control design, and makes the stability analysis more difficult. Experiments will be carried out to demonstrate the efficacy of the proposed controller. The systematic work presented in this research including system modeling, control design, and stability analysis, is believed to be one of the first studies on constrained and energy-efficient balance control of a standing biped with a high level of rigorous stability analysis.

The second objective of this research is to develop new approaches of estimating Lyapunov exponents reliably from time series. The rational of this objective comes from the two concerns. One is the fact that the controlled biped model is of high nonlinearity and subject to time-varying control bounds induced by the constraints. These features in terms of system complexities and uncertainties, generally restrict the application of the model-based method to extract Lyapunov exponents. Meanwhile the biped is supposed to maintain an upright posture (i.e., be stable about the equilibrium point) with the developed controller. It indicates that the system should possess only negative exponents. In this case, the accuracy of the estimated exponents usually cannot be guaranteed using the traditional time-series-based method. However, since it is nontrivial to examine the system stability and structural stability of a standing biped quantitatively, the numerical accuracy of Lyapunov exponents is extremely demanding. The new developed approaches, therefore, are intended to take the benefits of both the model-based method and the existing time-series-based meth-

ods. Two different types of RBF networks are needed in the developed approaches to assist in estimation of the Jacobian matrices for the complex or unknown systems. The Jacobian matrix is the first-order partial derivatives of the right-hand state space model, which plays a crucial role in the model-based method. The generated results show that both the newly developed methods are constructive and effective, allowing for stability analysis of more general and complicated robotic systems which were previously difficult to be carried out.

1.4 Thesis Organization

The remainder of this thesis is divided into five chapters. A description of each is outlined below:

Chapter 2 describes the dynamics of the standing biped model with discussion on the feet constraints, then an introduction of the concept of Lyapunov exponents is provided. The chapter also devotes to descriptions of the model-based method of calculating Lyapunov exponents, as well as the traditional time-series-based method via linear mapping operation to estimate Lyapunov exponents.

Chapter 3 begins by presenting two new methods for estimating Lyapunov exponents reliably from time series, which were developed based on different types of RBF neural models. Following that, balance control of a standing biped subject to three foot constraints in the form of a classical linear quadratic regulator (LQR) is taken as a common case study to validate the efficacy of both methods. Several other dynamical systems are also adopted as examples for each individual method. In all case studies, the numerical values of the estimated Lyapunov exponents from the

RBF networks are compared with those derived from the mathematical models.

Chapter 4 develops a constrained and energy efficient controller for a standing biped. The controller is in the form of a feedback PD control, of which the control gains are obtained via GA. The stability analysis of the constrained balancing system is carried out using the concept of Lyapunov exponents. Meanwhile, the ability of the balancing system to reject disturbances is partially quantified in terms of identifying the system stability region in state space and the structural stability region in parametric space, by estimating the system largest Lyapunov exponent. As part of the structural stability analysis, the effects of various gains on balancing stability and energy efficiency are investigated. The analysis reveals the balance control mechanisms for biped standing, to a certain extent.

Chapter 5 establishes an experimental study on the constrained and energy-efficient balance control of a standing bipedal robot based on the work presented in Chapter 4. To realize the physical implementation, the dynamics of the selected actuators is taken into account for developing the motion equations of the overall control system. Both the control design and the stability of the overall control system are revisited accordingly.

Chapter 6 gives conclusive remarks, sketches some recommended future developments.

Chapter 2

Mathematical Preliminaries

2.1 Introduction

Modeling biped quiet standing serves as a primary starting point for designing a balance control strategy. Investigators have reported that standing human subjects, when perturbed and instructed not to move their feet, typically respond by moving in the sagittal plane, using one or a combination of two strategies (Nashner and McCollum, 1985). For small disturbances, they tend to keep the knees, hips and neck fairly straight, moving predominantly about the ankles (the “ankle strategy”). For disturbances that place their COM near the perimeter of foot support, they tend to use a motion coordinating flexion or extension of the hips with smaller concurrent extension or flexion of the ankles (the “hip strategy”), keeping the other joints fairly straight. Hemami and his colleagues (1982) developed a two-link inverted pendulum model to study the problem of postural stabilization and found that reasonable predictions of behavior can be made using linearized dynamics. Later, from experimental data,

Barin (1989) used multiple regression to compute state feedback gains, with which he found that a two-link model is sufficient to model and predict the trajectory of the COM accurately.

There remains a gap, however, between the stabilized biped model and the rigorous theoretical proof of system stability. Due to the high nonlinearity and the existing constraints between the biped feet and the ground, (i.e., there is no lifting, no slipping, no rolling about either the toe or the heel), using Lyapunov's second method is extremely difficult, not to mention examining the system's ability to reject disturbances quantitatively. The concept of Lyapunov exponents is an important tool for categorizing the steady-state behavior of dynamical systems, determining the instability of systems, classifying invariant sets and approximating the dimension of strange attractors or other nontrivial invariant sets.

This chapter is organized as follows. Section 2.2 provides a description of the standing biped model, which includes mathematical expressions of the three constraints. Then the concept of Lyapunov exponents is introduced in Section 2.3, which is followed by Section 2.4 that describes the procedures of calculating the Lyapunov exponents based on both a mathematical model and a time series.

2.2 Model Description

To model a standing biped without foot motion appropriately, two assumptions are made. First of all, the biped body and feet are both bilaterally symmetrical in the frontal plane. Secondly, rather than a three-dimensional space, the biped is assumed to move only in the sagittal plane and the foot-link is stationary, but

not rigidly attached to the ground. Figure 2.1(A) shows a standing bipedal robot, which is simplified as an inverted double pendulum representing the leg and the torso respectively, with one additional rigid foot-link. The foot-link provides a base of support (BOS) on the ground, which is the roughly trapezoidal contact area between the robot foot and the ground. Based on the above two assumptions, the BOS in this work is actually a region within the anteroposterior length of the biped foot, where the COP may travel (Pai and Patton, 1997). Joints 1 and 2 can be considered as the ankle and hip, at which the control torques are supposed to be applied to maintain the biped at the upright posture without falling or resort to taking a step. The free body diagram of the inverted double pendulum and the free body diagram of the foot-link are shown in Figure 2.1(B) and (C) respectively. From the figure it can be seen that $\boldsymbol{\theta} = [\theta_1; \theta_2]$ are the two joint angles (clockwise as “+”); $\boldsymbol{\tau} = [\tau_1; \tau_2]$ are the corresponding control torques. Table 2.1 lists the mass and geometric parameters of the model along with their physical meanings as shown in Figure 2.1.

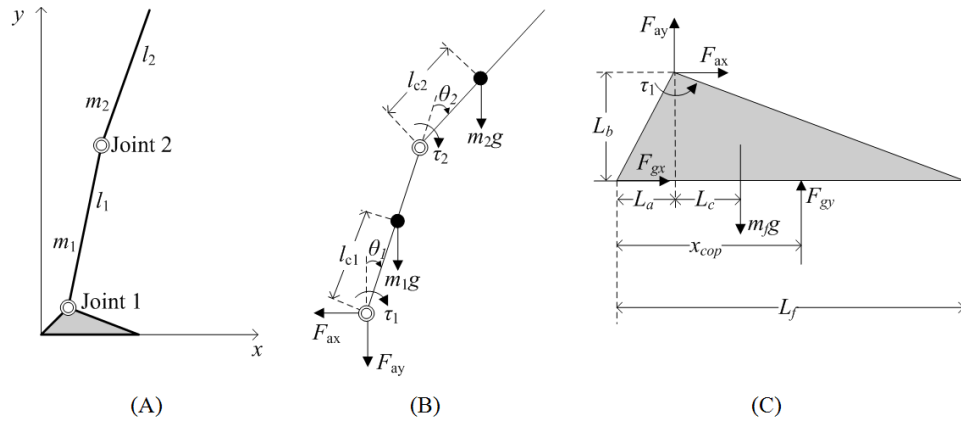


Figure 2.1: (A) simplified biped model, (B) free body diagram of the two link inverted pendulum, and (C) free body diagram of the foot-link.

Table 2.1: Biped model parameters.

Parameters	Physical Meanings
m_1	mass of the link 1 ^a [kg]
m_2	mass of the link 2 ^b [kg]
m_f	mass of the foot-link [kg]
l_1	length of the link 1 [m]
l_2	length of the link 2 [m]
l_{c1}	distance between the mass center of the link 1 and the ankle joint [m]
l_{c2}	distance between the mass center of the link 2 and the hip joint [m]
I_1	inertia of the link 1 with respect to the mass center of the link 1 [kg·m ²]
I_2	inertia of the link 2 with respect to the mass center of the link 2 [kg·m ²]
L_f	length of the foot-link [m]
L_a	horizontal distance between the ankle and the heel [m]
L_b	ankle height [m]
L_c	horizontal distance between the mass center of the foot and the ankle [m]

^a m_1 denotes the sum of the shank mass and the thigh mass in both legs.

^b m_2 is the torso mass, which is excluding the arm mass and the head mass.

Specifying $\dot{\boldsymbol{\theta}} = [\dot{\theta}_1; \dot{\theta}_2]$ and $\ddot{\boldsymbol{\theta}} = [\ddot{\theta}_1; \ddot{\theta}_2]$, the equations of motion of the biped shown in Figure 2.1 can be written in the following form according to the Euler-Lagrange formulation:

$$\boldsymbol{\tau} = \mathbf{D}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{G}(\boldsymbol{\theta}), \quad (2.1)$$

By defining $\alpha = m_1 l_{c1}^2 + m_2 l_1^2 + m_2 l_{c2}^2 + I_1 + I_2$, $\beta = 2m_2 l_1 l_{c2}$ and $\delta = m_2 l_{c2}^2 + I_2$, the matrices $\mathbf{D}(\boldsymbol{\theta})$, $\mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$, $\mathbf{G}(\boldsymbol{\theta})$ can be expressed as:

$$\mathbf{D}(\boldsymbol{\theta}) = \begin{bmatrix} \alpha + \beta \cos \theta_2 & \delta + \frac{\beta}{2} \cos \theta_2 \\ \delta + \frac{\beta}{2} \cos \theta_2 & \delta \end{bmatrix};$$

$$\mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} -\frac{\beta}{2} \dot{\theta}_2 \sin \theta_2 & -\frac{\beta}{2} (\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2 \\ \frac{\beta}{2} \dot{\theta}_1 \sin \theta_2 & 0 \end{bmatrix};$$

$$\mathbf{G}(\boldsymbol{\theta}) = \begin{bmatrix} -(m_1 l_{c1} + m_2 l_1)g \sin \theta_1 - m_2 l_{c2}g \sin(\theta_1 + \theta_2) \\ -m_2 l_{c2}g \sin(\theta_1 + \theta_2) \end{bmatrix}.$$

Additionally, the horizontal and vertical ground reaction forces F_{gx} , F_{gy} and the location of COP x_{COP} as displayed in Figure 2.1(C) can be obtained in the following form:

$$F_{gx} = m_1 a_{x1} + m_2 a_{x2}; \quad (2.2)$$

$$F_{gy} = m_1 a_{y1} + m_2 a_{y2} + (m_1 + m_2 + m_f)g; \quad (2.3)$$

$$x_{\text{COP}} = L_a - \frac{L_b F_{gx} + \tau_1 - L_c m_f g}{F_{gy}}. \quad (2.4)$$

The variables a_{x1} , a_{x2} , a_{y1} , a_{y2} are the resultant accelerations of two links along the horizontal and vertical directions respectively, which can be deduced as:

$$a_{x1} = -l_{c1} \sin \theta_1 \dot{\theta}_1^2 + l_{c1} \cos \theta_1 \ddot{\theta}_1;$$

$$a_{x2} = -l_1 \sin \theta_1 \dot{\theta}_1^2 + l_1 \cos \theta_1 \ddot{\theta}_1 - l_{c2} \sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)^2 + l_{c2} \cos(\theta_1 + \theta_2)(\ddot{\theta}_1 + \ddot{\theta}_2);$$

$$a_{y1} = -l_{c1} \cos \theta_1 \dot{\theta}_1^2 - l_{c1} \sin \theta_1 \ddot{\theta}_1;$$

$$a_{y2} = -l_1 \cos \theta_1 \dot{\theta}_1^2 - l_1 \sin \theta_1 \ddot{\theta}_1 - l_{c2} \cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)^2 - l_{c2} \sin(\theta_1 + \theta_2)(\ddot{\theta}_1 + \ddot{\theta}_2).$$

Since the foot-link is assumed to be still, but not fixed on the ground, there is a set of constraints imposed on the system. The gravity constraint in terms of that the resultant vertical ground reaction force F_{gy} should always be non-negative, guarantees the robot's foot will not lift off the ground. The friction constraint addressed in the way that the resultant horizontal reaction force F_{gx} should be bounded by two

threshold values, ensures the biped foot will never slide on the ground. The two boundary values are dedicated by the peak static friction $\pm\mu F_{gy}$ (μ is known as the static friction coefficient). The COP constraint restrains the location of the COP (x_{COP}) from wandering out of the BOS, such that there is no rolling of the foot-link about either the toe or the heel. All these constraints jointly determine the bounds on the control torques, which have been found to vary with the states of the system (Yang and Wu, 2006a). However, owing to the high nonlinearity of this three-link biped model, it is not feasible to derive analytical expressions of the control bounds in terms of θ and $\dot{\theta}$. Satisfaction of such constraints are assumed to be natural in many other works on control design for biped locomotion (Liu and Li, 2003; Mu and Wu, 2004). Although this assumption simplifies the problem, it can be misleading.

2.3 The Concept of Lyapunov Exponents

Stability analysis of a biped during standing subject to the constraints is a challenging task. Since the presence of the constraints causes time-varying control bounds, it is hard to apply Lyapunov's second method to synthesize a control scheme and to analyze stability. Alternatively, as described in the work of Oseledec (1968), the concept of Lyapunov exponents provides a meaningful way to characterize the asymptotic behavior of a nonlinear dynamical system. This concept was first introduced by Lyapunov in order to investigate the stability of non-stationary solutions of ordinary differential equations (ODEs) ^{*}, but has since been extensively studied and

^{*}According to the traditional definition, a solution $X(t)$ is strictly stationary, if the joint distribution of $[X(t_1), X(t_2), \dots, X(t_n)]$ and $[X(t_1 + \tau), X(t_2 + \tau), \dots, X(t_n + \tau)]$ are the same for all t_i and τ . Practically one cannot have data to cover all possible points in the phase plane, therefore most of the cases facing us are transient in nature, which are non-stationary.

frequently applied in the literature (Kaplan and Yorke, 1979; Hilborn, 2000; Stefanski, 2000; Sekhavat, Sepehri, and Wu, 2004; Zribi, Oteafy, and Smaoui, 2009). Consider a dynamical system in an n -dimensional state space expressed in the following form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t), \quad (2.5)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{x}(0) = \mathbf{x}_0$. $\mathbf{f}(\mathbf{x}, t)$ is a continuously differentiable vector function. Monitoring the long-term evolution of an infinitesimal n -sphere of initial conditions, the sphere becomes an n -ellipsoid due to the local deforming nature of the flow. The i^{th} dimensional Lyapunov exponent λ_i , is then defined in terms of the length of the ellipsoidal i^{th} principal axis $\|\delta x_i(t)\|$:

$$\lambda_i = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \frac{\|\delta x_i(t)\|}{\|\delta x_i(t_0)\|}, \quad i = 1, \dots, n, \quad (2.6)$$

where $\|\delta x_i(t_0)\|$ and $\|\delta x_i(t)\|$ represent the lengths of the i^{th} principal axis of the infinitesimal n -dimensional hyper-ellipsoid[†] at initial and current time instances, t_0 and t respectively. This definition indicates that Lyapunov exponents are related to the expanding or contracting nature of different directions in the state space, and the spectrum of Lyapunov exponents is equal in number to the dimensionality of the state space. To facilitate understanding, Figure 2.2 shows the evolution of a 2-dimensional sphere that is initially infinitesimal. One can imagine the two end-points of each principal axis $\delta x_i(t)$ as neighboring points in the state space. The growth or

[†]A general n -dimensional hyper-ellipsoid is represented by a center point $\mathbf{C}$, an orthonormal set of axis-direction vectors $\{\mathbf{U}_i\}_0^{n-1}$, and associated extents e_i with $e_{j+1} > e_j > 0$ for all j . The hyper-ellipsoid points are $\mathbf{P} = \mathbf{C} + \sum_{i=0}^{n-1} x_i \mathbf{U}_i$, where $\sum_{i=0}^{n-1} \left(\frac{x_i}{e_i}\right)^2 = 1$.

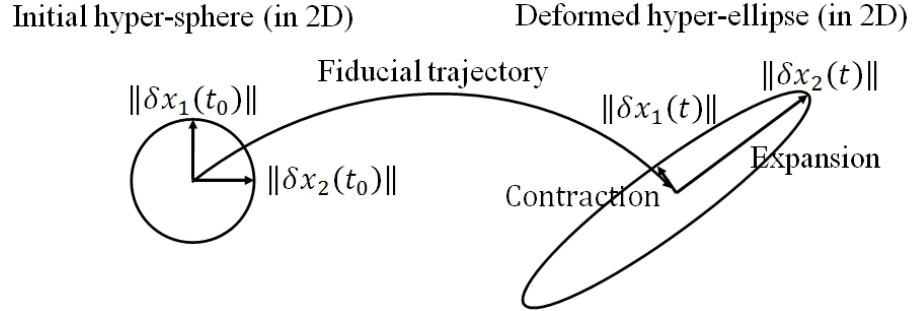


Figure 2.2: Evolution of an initially infinitesimal 2-dimensional sphere.

shrinkage of $\delta x_i(t)$ for $i = 1, \dots, n$ over the entire attractor[‡] is then measured according to whether the corresponding end-points get closer or further apart. With such a scenario, the largest Lyapunov exponent in the spectrum quantifies the averaged exponential rate of expansion of the first principal axis, i.e., the one which shows the largest amount of growth (or the slowest rate of shrinkage) over the attractor (Williams, 1997). Meanwhile, since the orientation of the ellipsoid varies continuously as it evolves, the growing direction associated with a given exponent changes in a complicated way through the attractor. As a result, one cannot speak of a well-defined[§] direction associated with a specific exponent.

Lyapunov exponents are global properties and are independent of the fiducial trajectory selected to estimate them. This feature was presented in Oseledec's work (1968), which has been applied in the limit of infinite time. In practical application, the finite-time Lyapunov exponents are frequently examined, which are defined in the

[‡]The attractor in more general terms, is that set of points to which trajectories approach as the number of iterations or the evolving time goes to infinity.

[§]In mathematics, an expression is well-defined if it is unambiguous and its objects are independent of their representative.

following form:

$$\lambda_i = \frac{1}{t} \ln \frac{\|\delta x_i(t)\|}{\|\delta x_i(t_0)\|}, \quad i = 1, \dots, n. \quad (2.7)$$

In the limit as $t \rightarrow \infty$, the finite-time Lyapunov exponents converge to the true Lyapunov exponents (Zeng, Pielke, and Eykholt, 1992).

Signs of the spectrum of Lyapunov exponents $(+, 0, -)$ are believed to be capable of indicating the sort of steady-state behavior the system will ultimately show, e.g., equilibrium point, limit cycle, quasi-periodic or chaotic, as well as judging the stability of such behaviors if certain non-generic cases with non-hyperbolic attractors[¶] are ignored (Bockman, 1991; Parker and Chua, 1989). Generally the sum of all the Lyapunov exponents represents the average volume changing rate in a phase space. Thus any attractor of a dissipative system^{||} will have at least one negative exponent and the sum of all exponents will be negative. Furthermore, in a dissipative system, an attractor is defined as strange or chaotic if the spectrum of Lyapunov exponents contains at least one positive exponent. For non-chaotic attractors such as an exponentially stable limit cycle, or an exponentially stable k -torus, the spectrum features only zero and negative exponents, while those exponentially stable equilibrium points are characterized by exclusively negative Lyapunov exponents (Williams, 1997).

It should be noted that while Lyapunov exponents quantify the time-averaged behaviors of nearby orbits in the state space, they are not local quantities in either the spatial or the temporal sense (Wolf, Swift, Swinney, and Vastano, 1985). Thus to

[¶]Non-hyperbolic attractors are not robust. Systems featuring such attractors are usually not structurally stable: small perturbations on system itself can result in a local bifurcation of a non-hyperbolic equilibrium.

^{||}Roughly speaking, a dissipative system is characterized by the property that at any time the amount of energy that the system has absorbed and conceivably transformed into other kinds of energy losses can not exceed the amount of energy that has been supplied to it.

the best of my knowledge, there is no quantitative relationship between the transient period for the system states and the convergent period for the exponents. Indeed, these two time periods are different in concept, although both can be intentionally defined by researchers based on specific design principles. The former is determined by the system dynamics and the initial conditions, while the latter is determined by the system dynamics in terms of the right-hand side of the state space model and its derivatives with respect to the states in the tangent space of the states.

2.4 Methods of Calculation of Lyapunov Exponents

Generally it is almost impossible to determine the Lyapunov exponents analytically. These characteristic numbers are hence often calculated numerically. Methods of calculating Lyapunov exponents basically can be categorized into two main streams. One is based on the system's mathematical model; the other is based on time series data, which can be measured experimentally. Application examples can be referred to works by Wolf, Swift, Swinney, and Vastano (1985); Sandri (1996); Hilborn (2000). The following subsections describe both approaches in detail.

2.4.1 Calculation of Lyapunov Exponents from Mathematical Models

Wolf and his colleagues (1985) developed an algorithm for calculating the spectrum of Lyapunov exponents from an explicit mathematical model of a system. In their work, a “fiducial” trajectory (the center of the sphere) is defined by the action of

the nonlinear motion equations on some initial conditions. The principal axes are determined by the evolution (via the linearized equations) of an initially orthonormal vector frame anchored to the fiducial trajectory. This leads to the following set of equations:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)), \quad (2.8a)$$

$$\dot{\mathbf{\Psi}}_t = \mathbf{J}(\mathbf{x}(t))\mathbf{\Psi}_t, \quad (2.8b)$$

where $\mathbf{\Psi}_t$ is the state transition matrix of the linearized system $\delta\mathbf{x}(t) = \mathbf{\Psi}_t\delta\mathbf{x}(0)$. The Jacobian matrix $\mathbf{J}(\mathbf{x}(t))$ is defined as

$$\mathbf{J}(\mathbf{x}(t)) = \left. \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}^T} \right|_{\mathbf{x}=\mathbf{x}(t)}, \quad (2.9)$$

and the initial conditions for numerical integrations are $\begin{Bmatrix} \mathbf{x}(t_0) \\ \mathbf{\Psi}_t(t_0) \end{Bmatrix} = \begin{Bmatrix} \mathbf{x}_0 \\ \mathbf{I} \end{Bmatrix}$, where $\mathbf{I}$ is the identity matrix of proper dimension.

To avoid misalignment of the vectors δx_i along the direction of maximal expansion, they are reorthonormalized at each integration step by involving the Gram-Schmidt Reorthonormalization (GSR) scheme, which generates an orthonormal set $\{u_1, \dots, u_n\}$ of n vectors with the property that $\{u_1, \dots, u_n\}$ spans the same subspace as $\{\delta x_1, \dots, \delta x_n\}$. This orientation-preserving property of GSR suggests that the initial labeling of the vectors may be done arbitrarily. Figure 2.3 shows the geometrical interpretation of the orthonormalization for two principal axes at the j^{th} step. Once the orthonormal vector frame $\{u_1, \dots, u_n\}$ is produced by GSR, for a

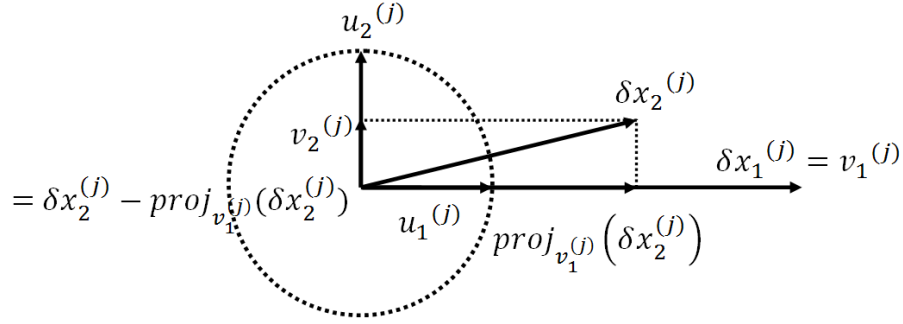


Figure 2.3: The geometrical interpretation of GSR for $\delta x_1^{(j)}$ and $\delta x_2^{(j)}$ ($j = 1, \dots, k$. j is the number of integration step). $\delta x_1^{(j)}$ and $\delta x_2^{(j)}$ are orthogonalized into $v_1^{(j)}$ and $v_2^{(j)}$, then normalized into $u_1^{(j)}$ and $u_2^{(j)}$. Here $proj_{v_1^{(j)}}(\delta x_2^{(j)})$ denotes the projector of the vector $\delta x_2^{(j)}$ on the vector $v_1^{(j)}$. $v_2^{(j)}$ is exactly equal to $\delta x_2^{(j)} - proj_{v_1^{(j)}}(\delta x_2^{(j)})$.

large enough integer k , one can obtain Lyapunov exponents as follows with time-step size h properly chosen:

$$\lambda_i \approx \frac{1}{kh} \sum_{j=1}^k \ln \|u_i^{(j)}\|, \quad i = 1, 2, \dots, n, \quad (2.10)$$

where j is the number of integration steps.

Since the model-based method essentially takes full advantage of the information brought by the system Jacobian matrix, one can always estimate the system Lyapunov exponents reliably with this method, provided that the numerical artifact** is under control. The significance of the Jacobian matrix lies in the fact that it is related to a linear approximation to a nonlinear function near a given point on the solution trajectory. In this sense the Jacobian matrix of a dynamical system can describe

**Usually some errors will invariably be present when one translates an analytical solution into numerical values for a given set of parameters with the use of a computer. This is due to the inherent limitations in the way numbers are stored in computers with finite precision. The errors will typically be small but they are an artifact about which one should be aware.

the amount of distortion of the flow at certain time instance, which is induced by a transformation in the neighborhood of a given point. The model-based method for computing Lyapunov exponents according to this viewpoint is constructive and reliable for determining all positive, zero and negative exponents, provided that Jacobian matrices are available.

2.4.2 Estimation of Lyapunov Exponents from Time Series

Explicit mathematical models are not always available in practice. Even if they are at hand, derivation of system Jacobian matrices can sometimes be troublesome because of a systems' inherent complexity. This limitation bolstered many efforts for determining the spectrum of Lyapunov exponents from a time series (Wolf, Swift, Swinney, and Vastano, 1985; Zeng, Pielke, and Eykholt, 1992). A time series is a sequence of data points sampled at ordered time instance with uniform intervals. In general the character of the overall dynamics of a nonlinear system must be reflected in the behavior of any one dynamical variable (Hilborn, 2000). Thus a one-dimensional time series is sufficient to capture the dynamical features of systems under study. This fact shows the very attractiveness of using time series: data can often be measured experimentally without awareness of the system dynamics. Basically the procedure of estimating Lyapunov exponents from a time series includes the following steps (Wolf, Swift, Swinney, and Vastano, 1985; Holzfuss and Lauterborn, 1989; Zeng, Pielke, and Eykholt, 1992):

Step 1: Choosing an embedding dimension d_E and constructing an orbit of d_E dimensions representing the time evolution of the system by the time-lag method, which

was suggested in 1980 by Packard, Crutchfield, Farmer, and Shaw (1980) and was put on a firm theoretical basis by Taken (1981). This means that defining

$$\vec{y}_i = (x_i, x_{i+T_{lag}}, \dots, x_{i+(d_E-1)T_{lag}}), \quad (2.11)$$

where the time lag T_{lag} is the time period between successive components of $\vec{y}_i$. Equation (2.11) provides the fiducial trajectory for the analysis of Lyapunov exponents.

Step 2: Determining the neighbor points $\vec{y}_j$ of the point $\vec{y}_i$, i.e., the points of the orbit which are contained in a shell of proper radius r , and centered at $\vec{y}_i$,

$$r_{min} \leq \|\vec{y}_j - \vec{y}_i\| \leq r. \quad (2.12)$$

Step 3: Determining the neighborhood-to-neighborhood matrix J_i of $d_E \times d_E$ dimensions using local linear mapping. The matrix functions analogously as the Jacobian matrix in the model-based method, which describes how the time evolution sends small vectors around $\vec{y}_i$ to small vectors around $\vec{y}_{i+1}$. J_i is obtained by looking for neighbors $\vec{y}_j$ of $\vec{y}_i$, and imposing

$$J_i(\vec{y}_j - \vec{y}_i) \approx (\vec{y}_{j+1} - \vec{y}_{i+1}). \quad (2.13)$$

The elements of J_i can be derived by a least-squares fit.

Step 4: Once the sequence of matrix J_i is obtained, using the QR decomposition, one can determine successively orthogonal matrices $Q_{(l)}$ and upper-triangular matrices $R_{(l)}$ with positive diagonal elements such that $Q_{(0)}$ is the unit matrix and

$$\begin{aligned}
J_1 Q_{(0)} &= Q_{(1)} R_{(1)}, \\
J_2 Q_{(1)} &= Q_{(2)} R_{(2)}, \\
&\dots \\
J_{l+1} Q_{(l)} &= Q_{(l+1)} R_{(l+1)}.
\end{aligned}$$

Then the Lyapunov exponents λ_K^i are given by

$$\lambda_K^i = \frac{1}{TK} \sum_{l=1}^{K-1} \ln R_{(l)ii}, \quad (2.14)$$

where K is the available number of matrices, T is the sampling time step, $i = 1, 2, \dots, d_E$.

Step 5: Repeating *Step 2* through *Step 4* along the fiducial trajectory, until the convergent Lyapunov exponents are obtained.

2.5 Discussion

Although conceptually Lyapunov exponents pertain to the eigenvalues of the linear system, it is fundamentally different to carry out stability analysis using the concept of Lyapunov exponents compared with the classical linear stability analysis. To apply linear stability analysis, the nonlinear system under study should first be linearized in a region around an operating point; then the eigenvalues of the linearized system are calculated. For the cases that the region of interest is not close to the operating point or the nonlinear system cannot be linearized, such as a non-smooth system, linear stability analysis is not applicable. From this viewpoint, the concept of Lyapunov

exponents is advanced compared to the classical linear stability analysis as it provides a generalization of linear stability analysis for perturbations of steady state solutions to time-dependent solutions, i.e., the involved linearization is around the points on the trajectory, rather than a certain operating point.

From Section 2.4 one may conclude that general approaches of calculating Lyapunov exponents are very constructive. In summary, the model-based method can estimate the spectrum of Lyapunov exponents accurately for systems whose mathematical models are well-developed, provided that the numerical artifact is under control. For cases when only experimental data are at hand, the traditional time-series-based method using linear mapping can be applied. However, this method arose primarily for diagnosing chaotic systems, of which the largest Lyapunov exponent is positive. For estimating exponents that potentially possess negative values or equal zero, due to the inaccuracy induced by the local linear mapping (Brown, Bryant, and Abarbanel, 1991), this time-series-based method using local linear mapping has been claimed to be unreliable. Replacing linear mapping with a higher-order Taylor expansion for the local neighborhood-to-neighborhood mapping, has been demonstrated to be capable of improving the numerical accuracy of negative exponents (Brown, Bryant, and Abarbanel, 1991; Yang and Wu, 2010). Nevertheless, the increased precision with such an operation always comes along with substantial extensions in the size of mapping matrices and the number of unknown coefficients associated with such matrices. Hence, for large systems with high dimensions, estimating Lyapunov exponents by employing the time-series-based nonlinear mapping method can be very knotty.

In Chapter 3, the numerical values of the Lyapunov exponents computed for a controlled standing biped subject to a set of constraints using the model-based method are listed as a reference. Comparative results obtained from two newly developed time-series-based methods are displayed as well.

Chapter 3

Estimation of Lyapunov Exponents from Time Series Using Radial-Basis-Function (RBF) Networks

3.1 Introduction

In this chapter, two newly developed methods for estimating the Lyapunov exponents with high accuracy are proposed. Both methods proposed can be applied to systems without explicit mathematical models, and the algorithms of both are quite straightforward. Specifically, as the common point in both methods, a radial-basis-function (RBF) network has to be tuned first using the available time series. The developed methods are applied to a biped balancing system in this chapter. The qui-

etly standing biped model can be referred to the one described in Section 2.2, which is subjected to a linear-quadratic regulator (LQR). The simulation results validate the efficacy of both methods. Apart from this example, different dynamical systems are taken as supplementary case studies for each developed method. All the data were generated by computer simulations under the MATLAB environment. As the reference, numerical values of the Lyapunov exponents of every individual system were first calculated from the mathematical models, provided that the numerical artifact was under control.

3.2 Radial-Basis-Function (RBF) Network

Modeling through RBF networks is a widely used method for nonlinear mapping approximation. It has been proven (Park and Sandberg, 1991; Funahashi, 1989) that any Borel-measurable function (a function for which all subsets of the type $E(x : f(x) \geq \alpha)$ in its domain of definition are Borel sets*, and such functions are also known as Borel-measurable functions (Halmos, 1950)) can be approximated to any desired degree of accuracy by carefully choosing parameters of the network, provided the network structure is sufficiently large.

The classical architecture of the RBF network is a three-layer feedforward network, which contains the input layer, the hidden layer and the output layer. A typical multiple-input multiple-output (MIMO) RBF network configuration with l hidden nodes is depicted in Figure 3.1(A). Such a network implements a mapping $f : \mathbb{R}^n \rightarrow$

*A Borel set in mathematics is any set in a topological space that can be formed from open sets (or, equivalently, from closed sets) through the operations of countable union, countable intersection, and relative complement.

$\mathbb{R}^m$ according to the overall outputs:

$$\hat{y}_i = w_{i0} \times bias + \sum_{j=1}^l w_{ij} \phi_j(\|\mathbf{x} - \mathbf{c}_j\|), \quad i = 1, 2, \dots, m, \quad (3.1)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the input vector; the output vector $\hat{\mathbf{y}} \in \mathbb{R}^m$ is constituted by $\hat{y}_i$ in the way that $\hat{\mathbf{y}} = [\hat{y}_1, \dots, \hat{y}_m]^T$. w_{ij} are the output weights, and the first index of the weight indicates the destination unit of the weight, and the second one indicates the input source for that weight. The bias neuron always emits 1. Its connection weight w_{i0} has an effect of increasing or lowering the net input of the summation junction in the next layer, which facilitates training. As regards a multiple-input single-output (MISO) RBF network shown in Figure 3.1(B), given the weights in the output layer w_j , the overall output becomes a scalar number, which has an analogous expression

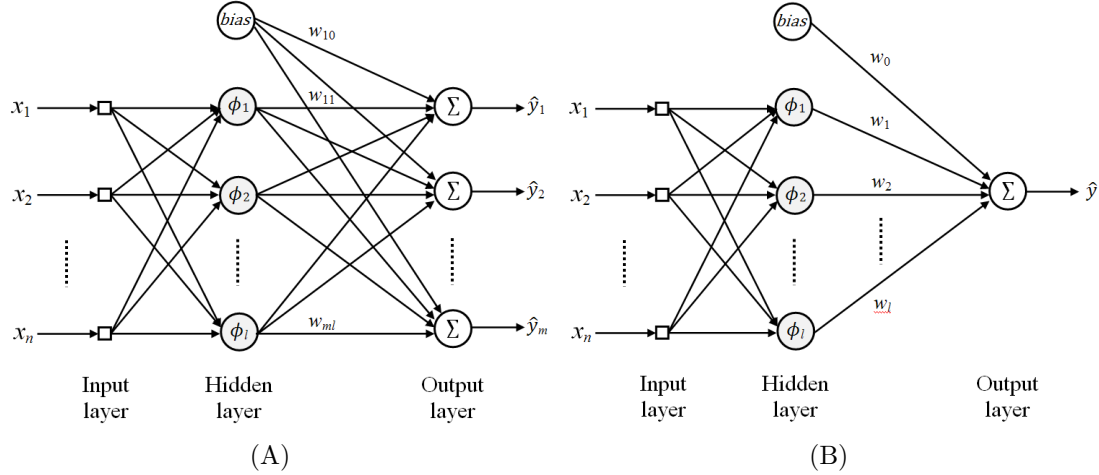


Figure 3.1: Typical schematics of (A) a multiple-input multiple-output (MIMO) RBF network, and (B) a multiple-input single-output (MISO) RBF network.

as Equation (3.1):

$$\hat{y} = w_0 \times bias + \sum_{j=1}^l w_j \phi_j(\|\mathbf{x} - \mathbf{c}_j\|). \quad (3.2)$$

Additionally, for $1 \leq j \leq l$, $\phi_j(\cdot)$ is a given function with its center vector defined as $\mathbf{c}_j = [c_{j1}, c_{j2}, \dots, c_{jn}]^T$. Here the Gaussian function is selected for ϕ_j , which has the following expression:

$$\phi_j = \exp\left(-\frac{\|\mathbf{x} - \mathbf{c}_j\|^2}{2\sigma_j^2}\right), \quad j = 1, \dots, l \quad (3.3)$$

where σ_j is the width of the j^{th} Gaussian function.

It is worthy to point out that the transfer function for the radial basis neurons are unnecessarily of Gaussian type as expressed in Equation (3.3). Functions of other types whose values depend only on the distance $r_j = \|\mathbf{x} - \mathbf{c}_j\|$ from certain centers $\mathbf{c}_j$, e.g., Multiquadratics ($\phi_j = \sqrt{\epsilon^2 + r_j^2}$ for some $\epsilon > 0$), Inverse multiquadratics ($\phi_j = (\epsilon^2 + r_j^2)^{-\frac{1}{2}}$ for some $\epsilon > 0$), Inverse quadratics ($\phi_j = (\epsilon^2 + r_j^2)^{-1}$ for some $\epsilon > 0$) etc., can also be implemented in principle for the approximation of any continuous function on a compact interval (Haykin, 1999). Among them, the inverse multiquadratic function, the inverse quadratic function and the Gaussian function share a common property: the nonlinearity $\phi_j \rightarrow 0$ as $r_j \rightarrow \infty$. By contrast, the multiquadratic function becomes unbounded along with $r_j \rightarrow \infty$. Theoretical investigation and practical results suggest that the choice of the radial basis function ϕ_j is not crucial to the performance of the RBF network (Chen, Cowan, and Grant, 1991).

To appropriately choose the parameters of the network, the centers of the RBF network $\mathbf{c}_j$ can be determined using the K -means clustering method, while the width

σ_j can be fixed by employing the K -nearest neighbors heuristic typically. Briefly speaking, the aim of the K -means algorithm is to partition a set of the observed n -dimensional vectors $\mathbf{x}$, into K groups, and find a cluster center in each group such that a cost function of dissimilarity measure is minimized (Moody and Darken, 1989; Kim, Park, Choi, and Chen, 2000). The K -nearest neighbors algorithm is used to vary the widths in order to achieve a certain amount of response overlap between the hidden nodes (Moody and Darken, 1989). The width parameter of the i^{th} hidden node is determined by averaging the sum of the distances from its center to the nearest K centers, i.e., $\sigma_i = \frac{1}{K} \sum_{j=1}^K \|\mathbf{c}_i - \mathbf{c}_j\|$. $\mathbf{c}_j$ are the nearest centers to $\mathbf{c}_i$. For the output layer, the linear weights (w_{ij} of a MIMO network, or w_j of a MISO network) can be trained by following the steps of gradient descent with momentum algorithm as addressed below:

Step 1: Find the error $E(k)$ at step k . For a MIMO network, $E(k) = \frac{1}{2}(\mathbf{y}(k) - \hat{\mathbf{y}}(k))^T(\mathbf{y}(k) - \hat{\mathbf{y}}(k))$, where $\mathbf{y}(k)$ and $\hat{\mathbf{y}}(k)$ respectively denote the actual system output vector and the RBF output vector at step k associated with the data sample $\mathbf{x}(k)$. For a MISO system, $E(k) = \frac{1}{2}(y(k) - \hat{y}(k))^2$, and again associated with the input data $\mathbf{x}(k)$ at step k known as the clustering sample, $y(k)$ represents the output of the actual system and $\hat{y}(k)$ is the RBF output.

Step 2: Change the connection weights in the following way:

For a MIMO network,

$$w_{ij}(k+1) = w_{ij}(k) + \mu(y_i(k) - \hat{y}_i(k))\phi_j(k) + \eta(w_{ij}(k) - w_{ij}(k-1)).$$

For a MISO network,

$$w_j(k+1) = w_j(k) + \mu(y(k) - \hat{y}(k))\phi_j(k) + \eta(w_j(k) - w_j(k-1)).$$

Here μ is the learning rate and η is the momentum constant introduced to prevent the network from being trapped in a local minimum. These two parameters are both positive constants within the range $(0, 1)$ (Haykin, 1999).

Step 3: If the error becomes lower than a predetermined value, stop training. Otherwise, replace k by $k + 1$ and go back to step 1.

In general, the best number of the hidden units depends in a complex way on the numbers of input and output data, the noise level of the data points, the complexity of the function to be approximated, the type of the hidden unit transfer function, the training algorithm (Geman, Bienenstock, and Doursat, 1992), etc.. In the remainder of this chapter, the numbers of hidden nodes when training the RBF models for all the examples were selected based on the trial and error following no specific rules. Furthermore, the numbers of clustering samples for tuning the neural models of all cases were selected respectively according to the length of the collected useful observations, which, to be specific, were proportional to the total number of the corresponding data series.

3.3 Estimation of Lyapunov Exponents via MIMO RBF Networks

Assume a dynamical system satisfies the following conditions: 1) the system has at least two state variables; 2) all state variables can be recorded practically. Once a

MIMO RBF network as shown in Figure 3.1(A) is well tuned for approximating such a system, the system Jacobian matrices, which are crucial in calculating Lyapunov exponents, can be obtained from the structural information of the developed network. Specifically, at step k , one entry of the Jacobian matrix, located at the m^{th} row and the n^{th} column can be written in the following form:

$$J_{mn}(k) = \frac{\partial y_m(k)}{\partial x_n(k)} \approx \frac{\partial \hat{y}_m(k)}{\partial x_n(k)} = \sum_{j=1}^l w_{mj} \phi_j \frac{c_{jn} - x_n(k)}{\sigma_j^2}. \quad (3.4)$$

For distinction, the Jacobian matrices derived through Equation (2.9) are referred as actual system Jacobians, and those obtained through Equation (3.4) are termed as neural model Jacobians in the balance of this section. Given the series of neural model Jacobians at different time instances, computing Lyapunov exponents based on time series reduces to the problem of calculating exponents from an explicit mathematical expression, which is a substitute for the system under study. The theoretical justification of this idea is that, since the RBF network can approximate the actual dynamical system with an arbitrary accuracy, its Jacobian matrices should be capable of reflecting the actual system Jacobians precisely. From this point of view, the amount of deformation at each time instance imposed by the actual transformation function can be drawn out by the neural model Jacobians with enough accuracy. The efficacy of this method is demonstrated through the following two case studies in this subsection.

3.3.1 Lyapunov exponents of bipedal standing with an LQR control

Recall the standing biped model as shown in Figure 2.1. Defining the system states as $\mathbf{q} = [q_1, q_2, q_3, q_4]^T = [\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2]^T$, the equation of motion of such a standing biped thus can be written as follows according to Equation (2.1):

$$\left\{ \begin{array}{l} \dot{q}_1 = q_3, \\ \dot{q}_2 = q_4, \\ \dot{q}_3 = \left(\delta\tau_1 - (\delta + \frac{\beta}{2} \cos q_2)\tau_2 + \frac{\beta}{2} \sin q_2 (\delta q_4 (2q_3 + q_4) + q_3^2 (\delta + \frac{\beta}{2} \cos q_2)) \right. \\ \quad \left. + (\delta(m_1 l_{c1} + m_2 l_1) \sin q_1 - \frac{\beta}{2} m_2 l_{c2} \cos q_2 \sin(q_1 + q_2))g \right) / D, \\ \dot{q}_4 = \left(-(\delta + \frac{\beta}{2} \cos q_2)\tau_1 + (\alpha + \beta \cos q_2)\tau_2 - \frac{\beta}{2} \sin q_2 ((\alpha + \beta \cos q_2)q_3^2 \right. \\ \quad \left. + q_4(\delta + \frac{\beta}{2} \cos q_2)(2q_3 + q_4)) + ((\alpha - \delta + \frac{\beta}{2} \cos q_2)m_2 l_{c2} \sin(q_1 + q_2) \right. \\ \quad \left. - (\delta + \frac{\beta}{2} \cos q_2)(m_1 l_{c1} + m_2 l_1) \sin q_1)g \right) / D. \end{array} \right. \quad (3.5)$$

where D stands for the determinant of the matrix $\mathbf{D}(\boldsymbol{\theta})$ displayed in Equation (2.1). Meantime, induced by the restriction that the biped foot is not rigidly fixed to the ground, the three constraints between the biped foot and the ground can be analytically expressed as the following set of inequalities:

$$\left\{ \begin{array}{l} F_{gy} \geq 0 \quad (\textbf{Gravity constraint}); \\ -\mu F_{gy} < F_{gx} < \mu F_{gy} \quad (\textbf{Friction constraint}); \\ 0 < x_{\text{COP}} < L_f \quad (\textbf{COP constraint}). \end{array} \right. \quad (3.6)$$

The nominal values of the system parameters are listed in Table 3.1, whose physical meanings can be referred to Table 2.1.

To stabilize the standing biped as well as minimize the control torques, a classical state feedback control law via LQR algorithm was adopted. It has been established that for a controllable linear time-invariant system, described by $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ with a quadratic cost function defined as $J = \frac{1}{2} \int_0^\infty (\mathbf{x}^T \mathbf{Q}\mathbf{x} + \mathbf{u}^T \mathbf{R}\mathbf{u})dt$, a feedback control law $\mathbf{u} = -\mathbf{F}\mathbf{x}$ may force the closed-loop system to be stable at the minimum cost, where the gain matrix $\mathbf{F}$ can be calculated by solving the algebraic Riccati equation (Kwakernaak and Sivan, 1972). To obtain the LQR parameters, the system as shown in Equation (3.5) was first linearized via Taylor's first-order expansion about the equilibrium point, which is specified as the upright position of the links 1 and 2 ($\mathbf{q} = [0, 0, 0, 0]^T$). Given the values of the model parameters listed in Table 3.1, one may obtain the system matrix $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{2 \times 2} \\ \mathbf{A}_{21} & \mathbf{0} \end{bmatrix}$ and the input matrix $\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{B}_{12} \end{bmatrix}^T$ after linearization, where $\mathbf{A}_{21} = \begin{bmatrix} 13.2155 & -9.0836 \\ -20.3555 & 48.8281 \end{bmatrix}$, $\mathbf{B}_{12} = \begin{bmatrix} 0.0428 & -0.1327 \\ -0.1327 & 0.6159 \end{bmatrix}$ and $\mathbf{0}$ denotes a squared zero matrix of dimension 2, $\mathbf{I}_{2 \times 2}$ stands for the unit ma-

Table 3.1: Nominal values of the biped model parameters.

Parameters	Values	Parameters	Values	Parameters	Values
m_1	48.72	l_{c1}	0.499	L_a	0.05
m_2	28.96	l_{c2}	0.356	L_b	0.07
m_f	2.32	I_1	4.044	L_c	0.085
l_1	0.998	I_2	1.223	μ^a	0.5
l_2	0.712	L_f	0.27	g^b	9.80

^a μ is the dimensionless static friction coefficient.

^b g is the gravitational acceleration in [m/s²].

trix also of dimension 2. The weight matrix $\mathbf{Q} = \text{diag}(\mathbf{Q}_1, \mathbf{Q}_2)$ with $\mathbf{Q}_1 = \mathbf{Q}_2 = \begin{bmatrix} 1000 & -500 \\ -500 & 1000 \end{bmatrix}$, $\mathbf{R} = \text{diag}(1000, 10000)$, both of which were obtained based on trial and error. The state feedback gain $\mathbf{F} = \begin{bmatrix} 1061.5 & 63.5 & 345.3 & 53.9 \\ 273.3 & 206.3 & 106.5 & 43.8 \end{bmatrix}$.

Provided that the system initial condition $\mathbf{q} = [-0.05, 0.03, 0.05, -0.03]^T$, the simulation results are displayed in Figures 3.2 and 3.3. One can observe the evolution of the system states and control torques from Figure 3.2(A) and (B) respectively, which indicate that the biped can be driven successfully to the equilibrium posture within 2.5 seconds associated with the proposed LQR controller. Meanwhile, the time history of F_{gy} and of F_{gx} are displayed in Figure 3.3(A) and (B) respectively. The positive vertical ground reaction force F_{gy} implies the support foot was always

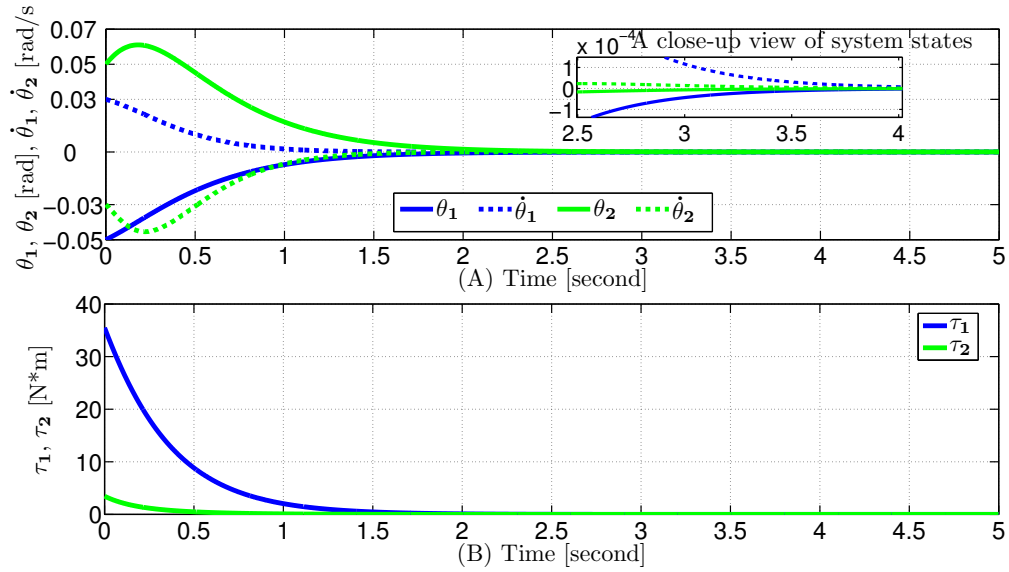


Figure 3.2: Evolution of (A) the system states, and (B) the control torques.

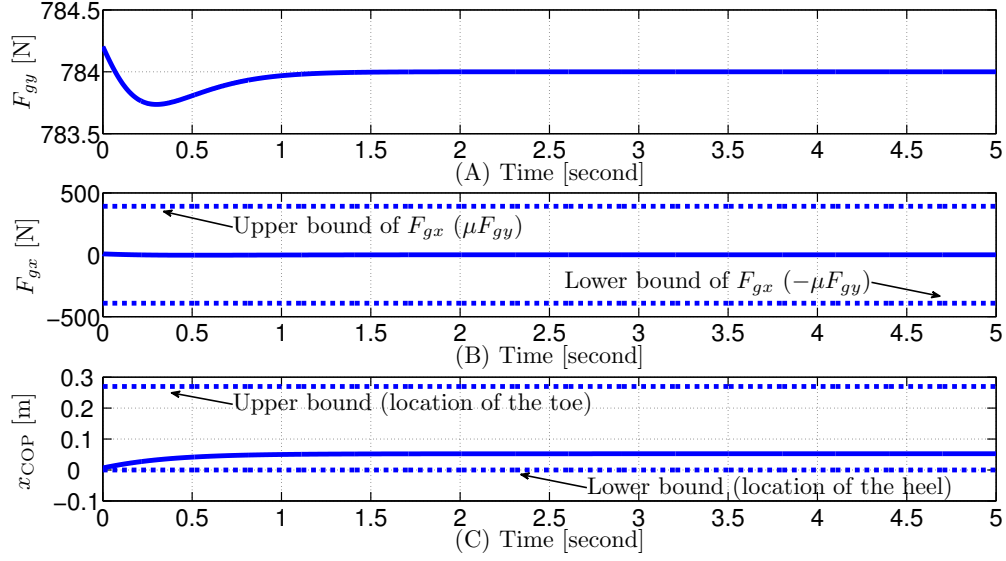


Figure 3.3: Evolution of (A) the vertical ground reaction force, (B) the horizontal ground reaction force and (C) location of COP.

in contact with the ground. The horizontal ground reaction force F_{gx} can be found residing within the bounds of the peak static friction $(-\mu F_{gy}, \mu F_{gy})$ represented by two dashed lines, which suggests the foot-link did not slip during the regulation. The numerical records show that after about 2.5 seconds, the friction magnitude between the foot-link and the ground is extremely low, indicating the relative potential movement between the biped foot and the contact ground was resisted. Additionally, the location of COP (x_{COP}) staying within the BOS between the foot-link and the ground is shown in Figure 3.3(C).

Care must be taken to the fact that the proposed LQR control strategy for the system as shown in Equation (3.5) was designed through the linearized system. There is no guarantee that it works well when implemented on the nonlinear system. This is particularly true because the biped's standing posture is not restricted close to

the upright position. Due to the complexity of the original nonlinear system, stability analysis of the system in Equation (3.5) combined with the proposed LQR controller was conducted by employing the concept of Lyapunov exponents, which were estimated using the collected time series via the newly developed MIMO RBF-network-based method.

For approximation of the above biped LQR balancing system displayed in Equation (3.1), a MIMO RBF network was constructed first. The training algorithm implemented with a digital computer invites the truncation in the form of input and output signal sampling. The problem treatment based upon truncation is acceptable since in practice only a restricted approximation accuracy is required (Gorinevsky, 1998). By defining a sampling time sequence $\{t_k\}_{k=0}^f$, the network was trained using $[\mathbf{q}(t_0), \dots, \mathbf{q}(t_f)]$ as the input of the neural model, and $[\dot{\mathbf{q}}(t_0), \dots, \dot{\mathbf{q}}(t_f)]$ as the desired output of the neural model. It is well studied that under appropriate observability conditions imposed on the sampling time sequence and on the controlled system, the desired outputs of the system can be evaluated by monitoring only the sampled outputs, and as a result the approximation problem here becomes mapping between finite-dimensional vector spaces. Table 3.2 summarizes the RBF network for the biped balancing system.

Table 3.2: Structure of the RBF network for the biped balancing system.

Number of hidden nodes	10
Number of inputs, outputs	4, 4
Number of clustering samples	3000
Mean-squared error after training	1.870e-12

Since the system is 4-dimensional, totally there are 16 entries in the Jacobian matrix. With respect to those zero entries in the actual system Jacobians derived from the mathematical model, Figure 3.4 shows the absolute error sequences between the actual system Jacobians and the neural model Jacobians (AEJ). All of them can be observed to be lower than 5×10^{-4} during system evolution. Figure 3.5 presents the relative error sequences for the remainder nonzero entries in the actual Jacobians (REJ), where the largest error during evolution (REJ₃₃) is found to be lower than 2.5%. It can be seen that in both figures the error signals stay nearly invariably after the biped was stabilized at the equilibrium posture. All these findings imply that the RBF model is accurate enough to approximate the controlled biped system. Moreover, the four Lyapunov exponents of the system in 100 seconds are displayed in Figure 3.6(A), where the solid lines represent the evolution of Lyapunov exponents

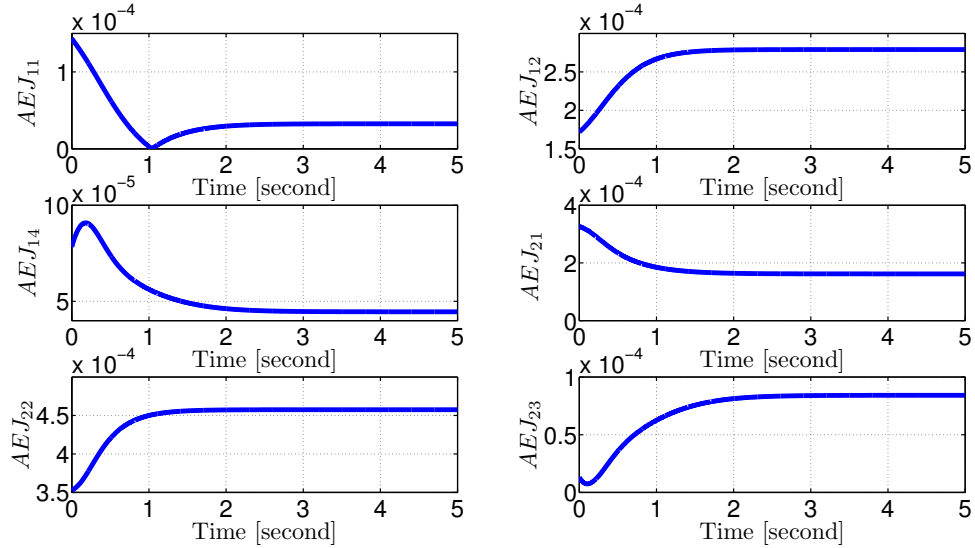


Figure 3.4: Absolute errors of Jacobians (AEJ).

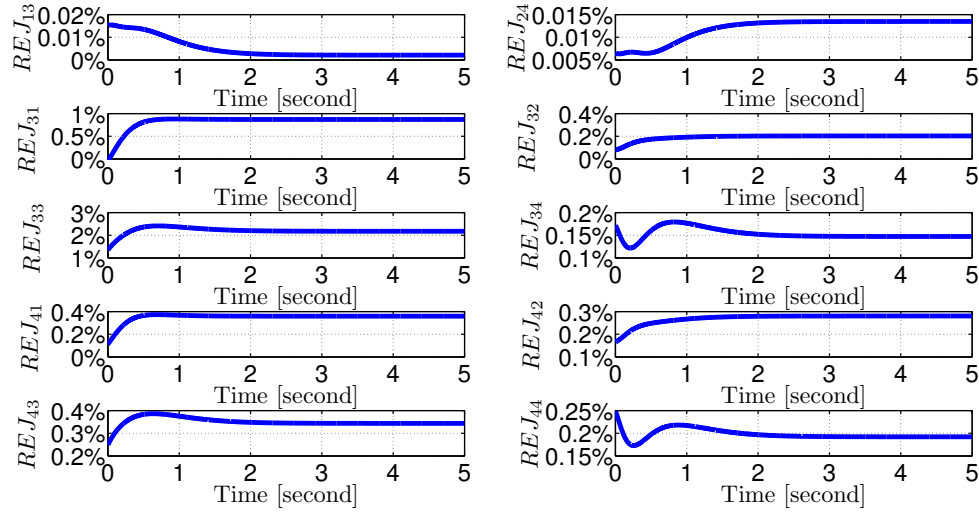


Figure 3.5: Relative errors of Jacobians (REJ).

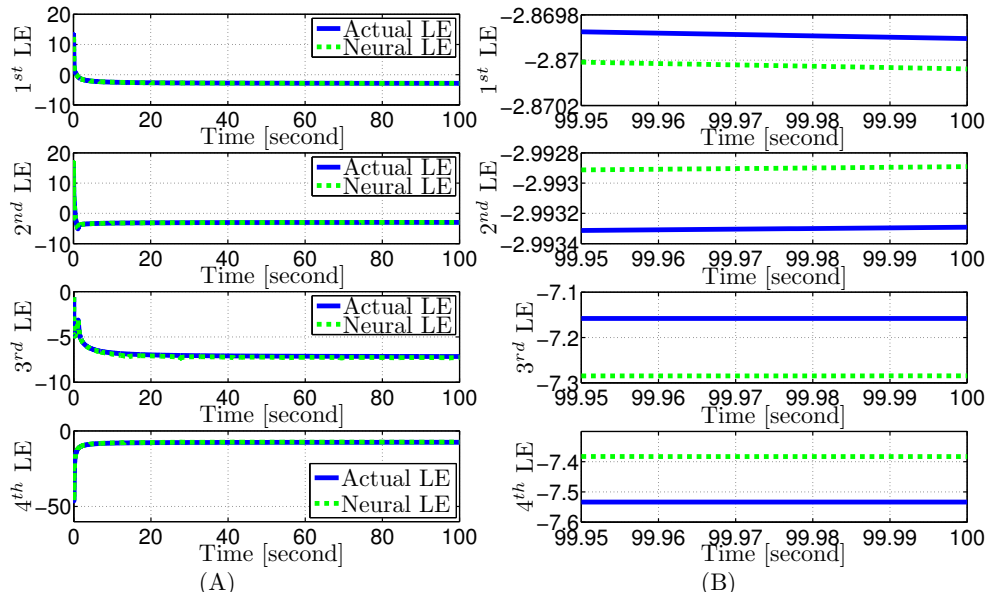


Figure 3.6: (A) Evolution of the Lyapunov exponents of the biped balancing system; (B) Close-up views of the exponents.

based on the actual system Jacobians, being compared with the dashed lines representing those from the neural model Jacobians. Close-up views of each exponent are provided in Figure 3.6(B). All Lyapunov exponents converge to negative constants, which are listed in Table 3.3, indicating that the biped balancing system subjected to the proposed LQR controller is exponentially stable about the equilibrium point. The low relative errors demonstrate that the proposed method of deriving Jacobians from the RBF network is effective for the calculation of Lyapunov exponents.

Table 3.3: Lyapunov exponents of the biped balancing system and their relative errors after 100 seconds.

Lyapunov Exponents (LEs)	Mathematic Model	Neural Model	Relative Error
1 st LE	-2.8699	-2.8700	0.003%
2 nd LE	-2.9933	-2.9929	0.01%
3 rd LE	-7.1580	-7.2845	1.77%
4 th LE	-7.5334	-7.3829	2.00%

3.3.2 Lyapunov exponents of a hydraulic actuator system

Attention should be paid to the fact that the rates of the system states in practice are rarely available or difficult to be measured. It is therefore desirable to monitor the system states rather than rates of the states under such conditions. A servovalve-controlled hydraulic actuator system is studied in this subsection based upon the above concern. Figure 3.7 shows the schematic of the system. The system is composed of a double-ended horizontal hydraulic actuator heading towards the desired position, x_d . The basic state equations describing the dynamics of the servovalve-actuator can

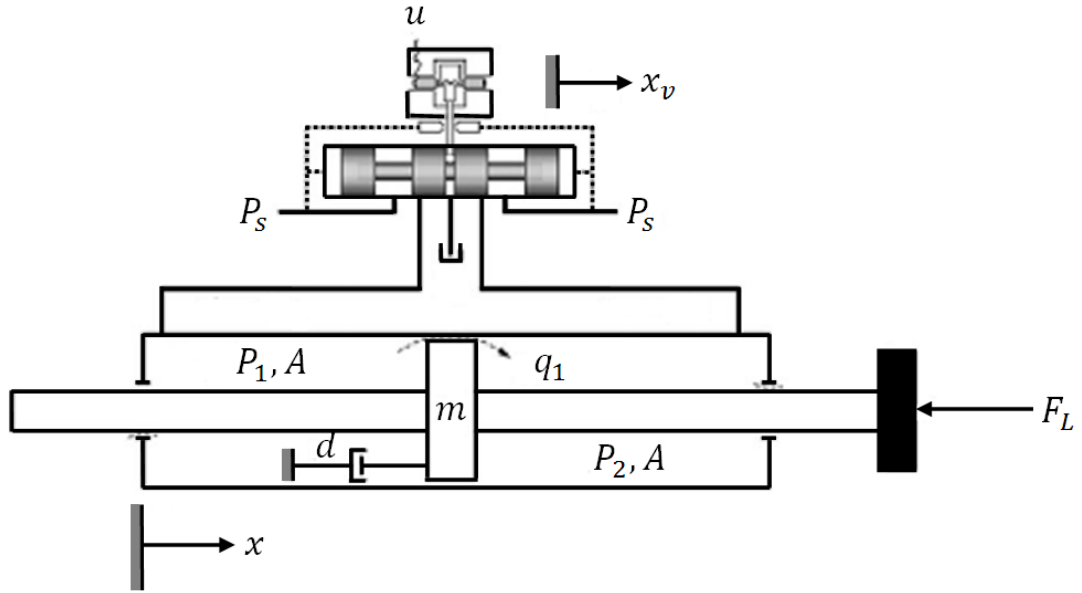


Figure 3.7: Schematic of a typical valve-controlled hydraulic actuator.

be formed as:

$$\begin{cases} \dot{x}_p = v_p, \\ \dot{v}_p = \frac{1}{m}(AP_L - bv_p - F_L), \\ \dot{P}_L = \frac{4\beta}{V}(C_v w x_v \sqrt{\frac{P_s - \text{sgn}(x_v)P_L}{\rho}} - Av_p). \end{cases} \quad (3.7)$$

Referring to Equation (3.7), the system states are selected as the position and velocity of the actuator x_p , v_p , and the load pressure $P_L = P_1 - P_2$. $F_L = kx_p$ denotes an external (environmental) force. $x_v = k_p(x_d - x_p)$, which is the valve spool displacement, serving as a control signal here. The term ‘ $\text{sgn}(x_v)$ ’ is used to account for the directionality of the valve spool and

$$\text{sgn}(x_v) = \begin{cases} 1, & \text{if } x_v > 0 \\ -1, & \text{if } x_v < 0 \\ 0, & \text{if } x_v = 0 \end{cases}$$

All the system parameters were taken from (Karpenko and Sepehri, 2005) and listed in Table 3.4.

One can get the equilibrium point of the system in Equation (3.7) as $[x_d \ 0 \ \frac{kx_d}{A}]^T$. Define a new state vector $\tilde{\mathbf{x}} = [\tilde{x} \ \tilde{v} \ \tilde{P}]^T = [x_p - x_d \ v_p \ P_L - \frac{kx_d}{A}]^T$. Then the system can be transferred into the following form with the new equilibrium point located at the origin:

$$\begin{cases} \dot{\tilde{x}} = \tilde{v}, \\ \dot{\tilde{v}} = \frac{1}{m}(A\tilde{P} - b\tilde{v} - k\tilde{x}), \\ \dot{\tilde{P}} = -\frac{4\beta}{V}(C_v w k_p \tilde{x} \sqrt{\frac{P_s + k_p \text{sgn}(\tilde{x})(\tilde{P} + \frac{kx_d}{A})}{\rho}} + A\tilde{v}). \end{cases} \quad (3.8)$$

Table 3.4: Hydraulic actuator system parameters.

Parameters	Physical Meanings	Values
m	total mass of piston, rods and load [kg]	12.00
A	piston annulus area [mm ²]	633.00
b	viscous damping coefficient [N·sec/m]	1000.00
k	spring constant [kN/m]	75.00
β	effective bulk modulus [MPa]	689.00
V	volume of oil at each side of cylinder when centered [cm ³]	468.00
w	servo valve orifice area gradient [mm ² /mm]	20.75
P_s	supply pressure [MPa]	17.20
ρ	hydraulic fluid density [kg/m ³]	847.00
x_d	desired actuator displacement [m]	0.01
C_v	dimensionless servo valve coefficient of discharge	0.60
k_p	proportional control gain	0.003

Given the initial state $\tilde{\mathbf{x}} = [-0.01\text{m} \ 0\text{m/s} \ -1.18\text{MPa}]^T$ (i.e., $[x_p \ v_p \ P_L]^T = [0\text{m} \ 0\text{m/s} \ 0\text{MPa}]^T$), the system shown in Equation (3.8) was regulated for guiding the states to the fixed point. Simulation results are displayed in Figure 3.8, where trajectories of the system states are shown in Figure 3.8(A), and the evolution of the control signal can be observed from Figure 3.8(B). It can be seen that all the states were quickly driven to zero, indicating the effectiveness of the proportional control law.

To investigate the system stability rigorously, an RBF network was established as well to approximate the dynamics of the discretized mathematical model. Different from the first case, here the desired output of the network was set to $[\tilde{\mathbf{x}}(t_1), \dots, \tilde{\mathbf{x}}(t_f)]$, whilst the input was $[\tilde{\mathbf{x}}(t_0), \dots, \tilde{\mathbf{x}}(t_f - 1)]$. The fourth-order Runge-Kutta method

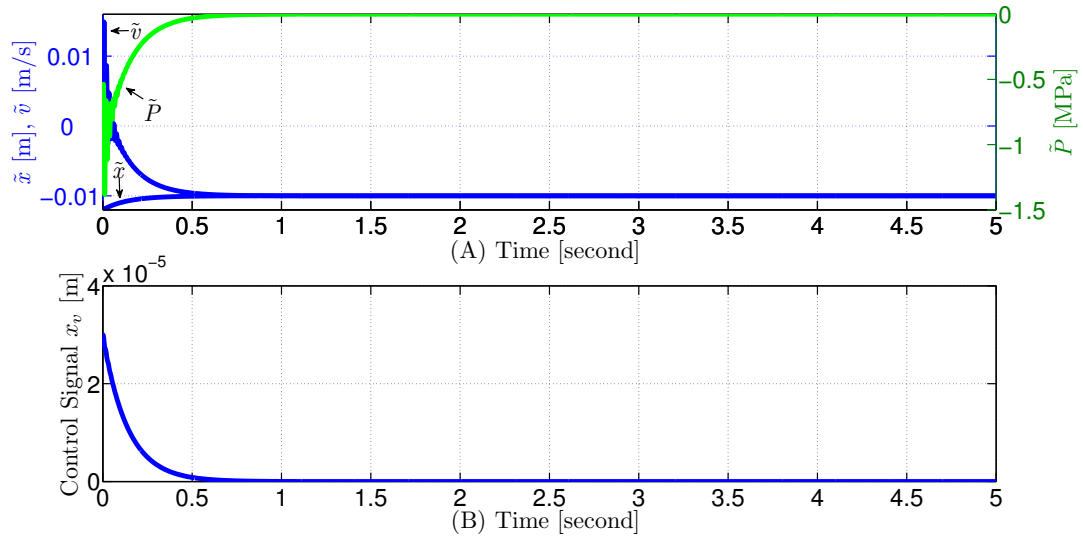


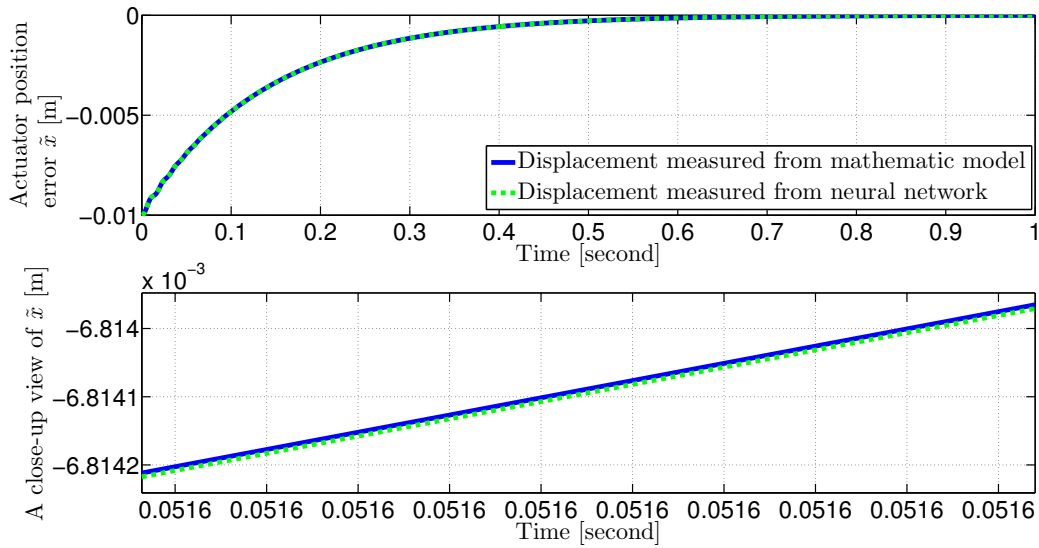
Figure 3.8: Evolution of (A) the states of the hydraulic actuator system, where the left y -axis is for the states $\tilde{\mathbf{x}}$ and $\tilde{v}$ while the right y -axis is for the state $\tilde{P}$; (B) the control signal x_v .

was adopted in this case for collecting the numerical values of the state variables at each time instant. Table 3.5 provides the summary of the trained RBF network. Comparisons were made between the outputs of the mathematical model as presented in Equation (3.8) and the RBF neural model during the first 1 second. The straight lines in Figure 3.9 represent the three states of the mathematical model, and the dashed lines represent the states of the neural model. Figure 3.9 also gives close-up views of the errors between two models of all three states, showing that the neural outputs match well to the mathematical states.

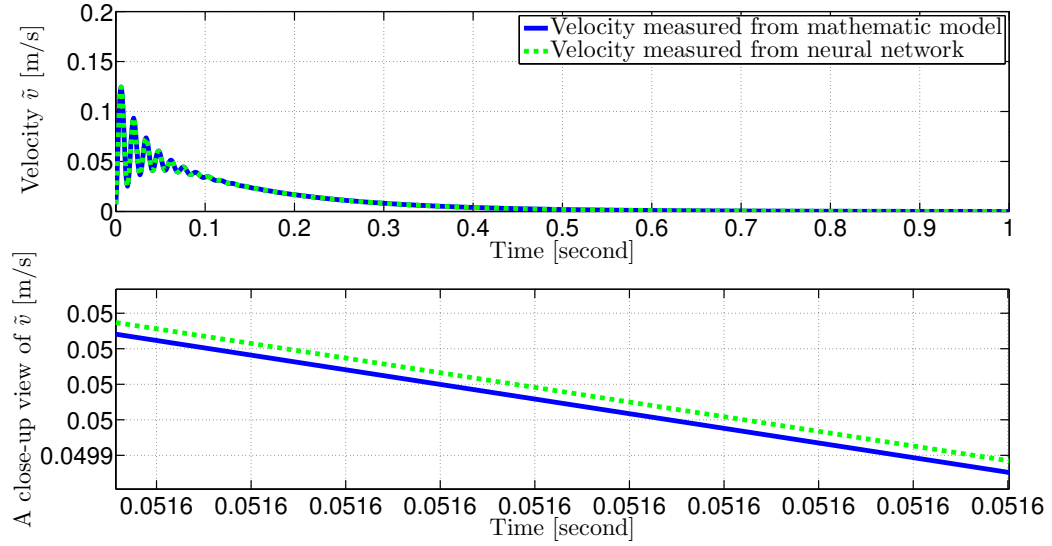
Since the established RBF network is an approximator of the discretized actuat-

Table 3.5: Structure of the RBF network for the hydraulic actuator system.

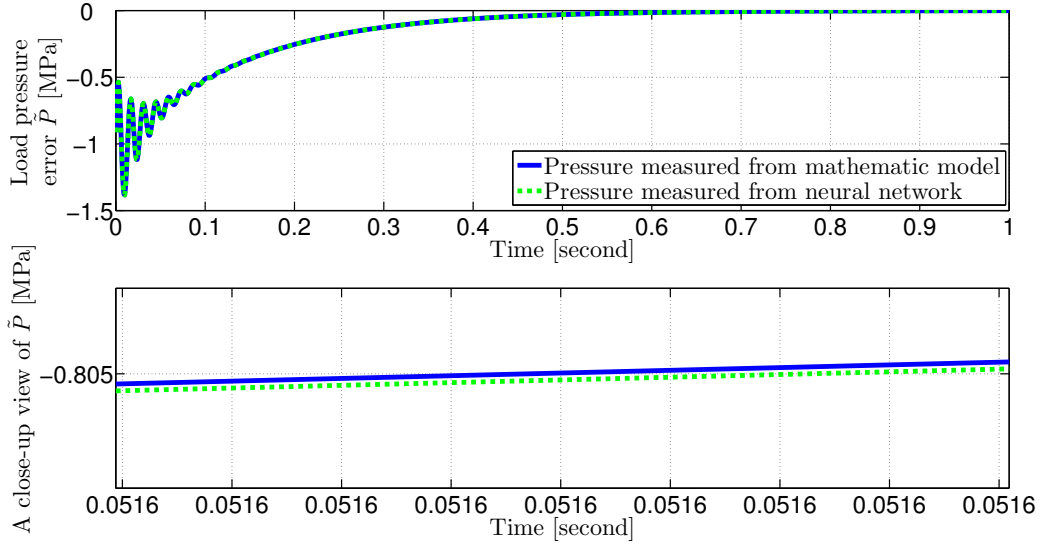
Number of hidden nodes	100
Number of inputs, outputs	3, 3
Number of clustering samples	2500
Mean-squared error after training	1.0310e-12



(A)



(B)



(C)

Figure 3.9: Comparison between the mathematic model and the neural model of (A) $\tilde{x}$, (B) $\tilde{v}$ and (C) $\tilde{P}$.

or system based upon the Runge-Kutta method, the numerical values of these neural model Jacobians are incomparable with ones of the original continuous system.

However, Lyapunov exponents calculated from the RBF network are very close to the actual exponents derived from mathematical model. Figure 3.10 illustrates the evolution of all three exponents of the actuator system in 100 seconds, where the solid lines stand for the exponents estimated from the actual system, and those dashed lines correspond to the ones obtained from the RBF model. All the negative constants to which the system exponents converge are listed in Table 3.6, suggesting the system is exponentially stable. The average relative error of all three exponents is 1.88%, demonstrating effectiveness of the proposed method as well.

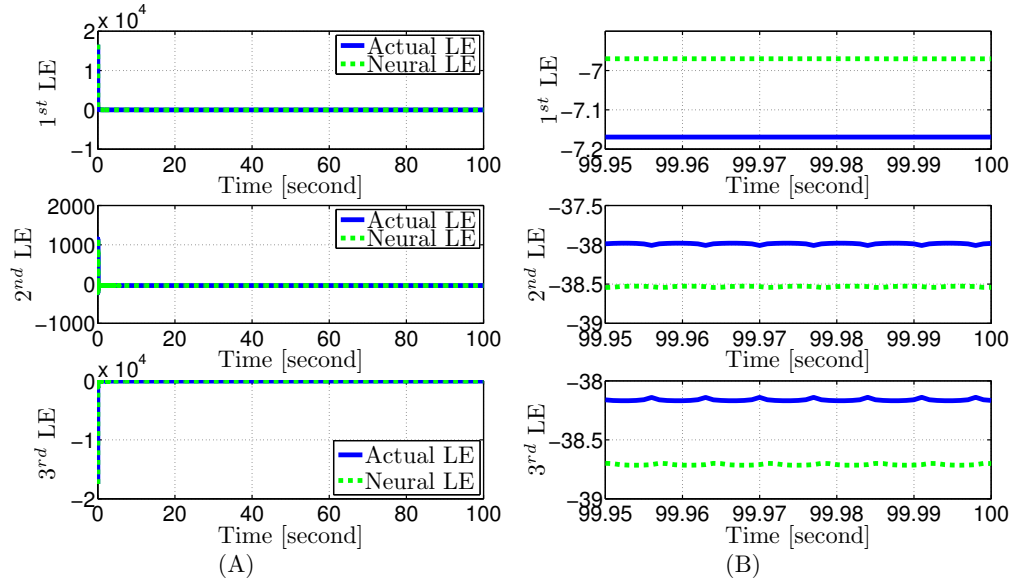


Figure 3.10: (A) Evolution of the Lyapunov exponents of the hydraulic actuator system; (B) Close-up views of the exponents.

Table 3.6: Lyapunov exponents of the hydraulic actuator system and their relative errors after 100 seconds

Lyapunov Exponents (LEs)	Mathematic Model	Neural Model	Relative Error
1 st LE	-7.1693	-6.9701	2.78%
2 nd LE	-37.9826	-38.5416	1.47%
3 rd LE	-38.1660	-38.7008	1.40%

3.4 Estimation of Lyapunov Exponents via MISO

RBF Networks

Developing a MIMO RBF network to approximate a certain nonlinear system without being aware of the system dynamics requires that all state profiles are at hand, which can be satisfied seldom in the real world. In this section, estimating Lyapunov exponents reliably via MISO RBF networks is discussed, which is capable of dealing with cases when only a scalar time series is available.

Briefly speaking, two important components constitute the main idea of this method. One is reconstruction of the system attractor in an embedding state space using a scalar time series, and another is an appropriate MISO RBF network tuned to reveal the embedded mapping, of which the structural information is provided for deriving Jacobian matrices of the reconstructed mapping. Once the approximated sequence of Jacobian matrices are at hand, Lyapunov exponents can be estimated following the model-based algorithm with ease.

3.4.1 Reconstruction of system attractors

Associated with the n -dimensional dynamical system as shown in Equation (2.5), there is a collected scalar time series $x_i = x(i\Delta t)$, where $i = 1, 2, \dots, N$. N is the number of observations and Δt is the time interval between measurements. According to the conventional time delay embedding technique (Taken, 1981), an orbit representing the time evolution of the system can be reconstructed using the vectors formed by the delay coordinates:

$$\mathbf{y}_k^{d_E} = [x_i, x_{i+T_{lag}}, \dots, x_{i+(d_E-1)T_{lag}}]^T, \quad k = 1, 2, \dots, f \quad (3.9)$$

where d_E is the embedding dimension, T_{lag} denotes the time lag, k labels the iteration step such that after one step iteration, $\mathbf{y}_k^{d_E}$ becomes $\mathbf{y}_{k+1}^{d_E}$. The whole iteration steps can be inferred as $f - 1$ since f is the total number of the reconstructed vectors $\mathbf{y}^{d_E}$. i may be arbitrarily selected from $[1, N - (f + d_E - 2) \times T_{lag}]$. It is different from the notation used in (Gencay and Dechert, 1992), where T_{lag} was defaulted to be 1 for all systems. This default value will cause problems in cases when the sampling rate Δt is very small, as the delayed coordinates at successive points in the state space represent almost the same information. To avoid this shortcoming, no fixed values are assigned to T_{lag} in this newly developed method, making it more applicable.

Rather than a single point in the reconstructed space, Equation (3.9) depicts a fiducial trajectory for estimation of the Lyapunov exponents. Generically, for $d_E \geq 2n + 1$ there exists a mapping $\mathbf{L} : \mathbb{R}^{d_E} \rightarrow \mathbb{R}^{d_E}$ such that:

$$\mathbf{y}_{k+1}^{d_E} = \mathbf{L}(\mathbf{y}_k^{d_E}) \quad (3.10)$$

where $\mathbf{y}_{k+1}^{d_E} = [x_{i+T_{lag}}, x_{i+2T_{lag}}, \dots, x_{i+d_ET_{lag}}]^T$. Considering that $\mathbf{L}$ may be taken as:

$$\mathbf{L} : \begin{bmatrix} x_i \\ x_{i+T_{lag}} \\ \vdots \\ x_{i+(d_E-1)T_{lag}} \end{bmatrix} \rightarrow \begin{bmatrix} x_{i+T_{lag}} \\ x_{i+2T_{lag}} \\ \vdots \\ x_{i+d_ET_{lag}} \end{bmatrix} = \begin{bmatrix} x_{i+T_{lag}} \\ x_{i+2T_{lag}} \\ \vdots \\ g(x_i, x_{i+T_{lag}}, \dots, x_{i+(d_E-1)T_{lag}}) \end{bmatrix}, \quad (3.11)$$

estimating $\mathbf{L}$ thus reduces to revealing the following regression map

$$x_{i+d_E T_{lag}} = g(x_i, x_{i+T_{lag}}, \dots, x_{i+(d_E-1)T_{lag}}), \quad (3.12)$$

which indeed can be estimated by resort to develop a MISO RBF network as described in Figure 3.3(B).

3.4.2 Generation of Jacobian matrices

Consider the reconstructed map expressed as Equation (3.11) in an embedding state space. Following that, the derivatives of $\mathbf{L}$ can be obtained as

$$\begin{aligned} \mathbf{J}(\mathbf{y}_k^{d_E}) = D\mathbf{L}_{\mathbf{y}_k^{d_E}} &= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ J_{d_E 1} & J_{d_E 2} & \cdots & J_{d_E d_E} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ \frac{\partial g}{\partial x_i} & \frac{\partial g}{\partial x_{i+T_{lag}}} & \cdots & \frac{\partial g}{\partial x_{i+(d_E-1)T_{lag}}} \end{bmatrix} \end{aligned} \quad (3.13)$$

Meanwhile, once a MISO RBF network is tuned well for the mapping $g : \mathbb{R}^{d_E} \rightarrow \mathbb{R}$, the system Jacobian matrices can be obtained again from the structural information of the network. Specifically, at step k , assume the input vector $\mathbf{x}(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T$ of the RBF network shown in Figure 3.3(B) has the same

expression as $\mathbf{y}_k^{d_E} = [x_i, x_{i+T_{lag}}, \dots, x_{i+(d_E-1)T_{lag}}]^T$. The p^{th} element of the input $\mathbf{x}(k)$ thus can be written as $x_p(k) = x_{i+(p-1)T_{lag}}$ ($1 \leq p \leq d_E$), and the output known as $\hat{y}(k)$ is an approximation of $x_{i+d_E T_{lag}}$. Consequently estimation of the entry $J_{d_E p}$ that located at the end row and the p^{th} column in the Jacobian matrix in Eq. (3.13), can be deduced to take the following form:

$$\begin{aligned} J_{d_E p}(k) &\approx \frac{\partial \hat{y}(k)}{\partial x_p(k)} = \sum_{j=1}^l w_j \phi_j \frac{c_{jp} - x_p(k)}{\sigma_j^2} \\ &= \sum_{j=1}^l w_j \phi_j \frac{c_{jp} - x_{i+(p-1)T_{lag}}}{\sigma_j^2}. \end{aligned} \quad (3.14)$$

Hence calculating Lyapunov exponents based on a scalar time series, reduces to the problem of estimating exponents from an explicit mathematical model, which is a substitute for the reconstructed mapping featuring the same dynamics as the system under study.

3.4.3 Lyapunov exponents of the LQR balance control system

Consider again the biped balancing system in Equation (3.5) subjected to an LQR control. With setting the numerical integration time step $h = 0.001s$, a time series including 100,000 data points along the coordinate of q_1 was generated first from the mathematical model, among which 1,426 observations were used to reconstruct the attractor in a 4-dimensional embedding state space with setting $T_{lag} = 70$, equivalently in terms of time, $T_{lag} = 70 \times 0.001 = 0.07s$.

To derive the Jacobian matrices described by Equation (3.13) at different time

instances, a MISO RBF network summarized in Table 3.7 was then constructed for the estimation of the regression mapping g presented in Equation (3.12), based on which the Lyapunov exponents of the reconstructed attractor can be easily estimated following the model-based algorithm. Figure 3.11 shows the evolution of Lyapunov exponents of the reconstructed attractor. It can be seen that all Lyapunov exponents remain negative and converge to constants after around 30s, indicating that the biped

Table 3.7: Structure of the RBF network for estimating the regression Equation (3.12) for the biped balancing system.

Number of hidden nodes	20
Number of inputs, outputs	4, 1
Number of clustering samples	855
Mean-squared error after training	2.32e-17

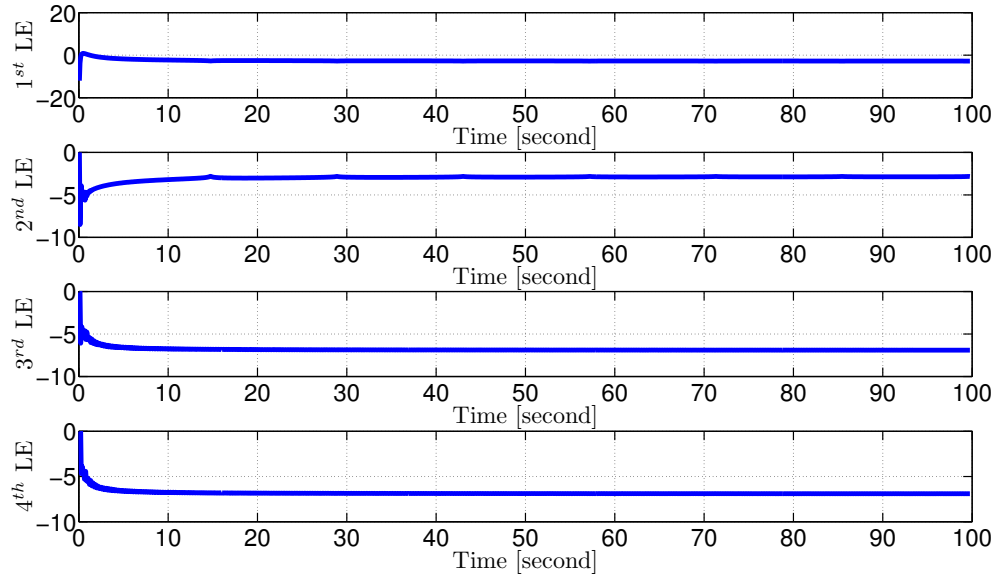


Figure 3.11: Evolution of Lyapunov exponents (LEs) in the 4-dimensional embedding state space of the biped balancing system.

balancing system is exponentially stable about its equilibrium (i.e., the upright posture). The property of the exponents remaining negative suggests that the nearby trajectories converge monotonically, which is an important condition for system stability. Otherwise the stability of the systems cannot be guaranteed even though the averaged exponents are negative (Slotine and Li, 1991).

Table 3.8 lists the numerical values of all four negative estimates and their relative errors with respect to the ones computed from the mathematical model, where the low percentage errors demonstrate that the proposed method is effective for the estimation of Lyapunov exponents. Moreover, a comparison between the results derived from the

Table 3.8: Lyapunov exponents (LEs) and their relative errors for the biped balancing system.

T_{lag}	1 st LE		2 nd LE		3 rd LE		4 th LE	
	$(\lambda_1^* = -2.8699)$		$(\lambda_2^* = -2.9933)$		$(\lambda_3^* = -7.1580)$		$(\lambda_4^* = -7.5334)$	
	LM ^a (Err.%) ^c	SW ^b (Err.%)	LM (Err.%)	SW (Err.%)	LM (Err.%)	SW (Err.%)	LM (Err.%)	SW (Err.%)
50	-1.1347 (60.46%)	-2.5386 (11.55%)	-1.2925 (56.82%)	-2.6523 (11.39%)	-1.8149 (74.65%)	-9.0130 (25.92%)	-3.5221 (53.25%)	-9.0146 (19.66%)
60	-1.1824 (58.80%)	-2.7681 (3.55%)	-1.3820 (53.83%)	-2.8366 (5.23%)	-2.0049 (71.99%)	-6.6833 (6.63%)	-3.6426 (51.65%)	-6.6880 (11.22%)
70	-1.2533 (56.33%)	-2.8040 (2.30%)	-1.4032 (53.12%)	-2.8885 (3.50%)	-2.7691 (61.32%)	-6.8241 (4.66%)	-3.9241 (47.91%)	-6.8301 (9.34%)
80	-1.0355 (63.92%)	-2.7335 (4.75%)	-1.3177 (55.98%)	-2.8181 (5.85%)	-2.9559 (58.70%)	-6.5946 (7.87%)	-4.2919 (43.03%)	-6.6003 (12.39%)
90	-1.2295 (57.16%)	-2.7501 (4.18%)	-1.4139 (52.76%)	-2.8204 (5.78%)	-2.0387 (71.52%)	-6.5976 (7.83%)	-4.6871 (37.78%)	-6.6055 (12.32%)
100	-1.1065 (61.44%)	-2.8106 (2.07%)	-1.2150 (59.41%)	-2.8923 (3.37%)	-1.9856 (72.26%)	-5.9517 (16.85%)	-3.4353 (54.40%)	-5.9546 (20.96%)

^aFor brevity, LM denotes the traditional time-series-based method using Linear Mapping, and the corresponding numerical results are derived from a 9-dimensional embedding space.

^bSW denotes the newly proposed method via the MISO RBF network, and the corresponding numerical results are estimated from an approximated 4-dimensional embedding space.

^cThe percentage error is calculated w.r.t. the results obtained through the mathematical model, which are listed as $\lambda_i^*, i = 1, \dots, 4$.

same time series but different methods, i.e., the traditional time-series-based method using local linear mapping and the newly proposed method via training a MISO RBF neural model, can be observed from Table 3.8, where the estimated Lyapunov exponents after 99.75s ($1425 \times T_{lag}$) and the corresponding percentage errors with different T_{lag} are provided. It can be found that using the proposed method, the numerical values of Lyapunov exponents estimated from an approximated 4-dimensional embedding space are much more accurate, compared with those derived directly from a reconstructed space with even a higher embedding dimension ($d_E = 2 \times 4 + 1$). To be specific, the smallest percentage error regarding the first Lyapunov exponent using the traditional linear mapping method turns out to be 56.33% when T_{lag} is set to 70, which is nearly 25 times the number as compared with the corresponding result derived from the new method (2.30%). The difference in the estimation accuracy between the two methods reaches the lowest level when T_{lag} equals 5. However the result based on the linear mapping method, 60.46%, is still more than 5 times the datum derived from the new method, which is 11.55% only. This difference becomes smaller although with respect to the remaining three exponents (e.g., for the forth exponent with T_{lag} is 50, the competition between the percentage errors from two methods is 53.25% vs. 19.66%), the gap is still striking. All these findings demonstrate that the newly proposed method can estimate Lyapunov exponents much more reliably compared with the traditional time-series-based method using linear mapping.

It has been documented that in estimating Lyapunov exponents based on a time series, some parameters for the state space reconstruction have significant effects on the accuracy of the estimated Lyapunov exponents. In this newly developed method

using MISO RBF model, such parameters include the value of the time lag (T_{lag}), and the embedding dimension (d_E). These two parameters determine the number of the data points to be used in the analysis. Regarding the time lag (T_{lag}), Taken's results (Taken, 1981) indicate that the choice of T_{lag} is arbitrary. However, in practice, if T_{lag} is quite small, the components of the successive vectors to be plotted are almost identical; while if T_{lag} is very large, then there is only very little correlation between the components of the vectors, and the trajectories on the attractor appear to wander all around the state space, leaving the structure hard to be detected (Peitgen, Jürgens, and Saupe, 2004). Methods have been developed for determining T_{lag} (Liebert and Schuster, 1989; Rosenstein, Collins, and DeLuca, 1993; Yang and Wu, 2011). In this case, it is found that $T_{lag} = 70$ is a good approximation of the reconstruction delay where the autocorrelation function drops to $1 - \frac{1}{3e}$ (e is the base of the natural logarithm) of its initial value. In addition, from Table 3.8 one can find that within the range of T_{lag} from 60 to 90, the accuracy of the estimated Lyapunov exponents is not sensitive to T_{lag} as the percentage errors of four different sets are below 15%. Such robustness with respect to T_{lag} provides the new method with more flexibilities for reconstruction of system dynamics, making it attractive especially when the length of the available time series is limited.

On the other hand, Taken's theorem stated that in order to preserve the dynamical properties of the original attractor, theoretically the embedding dimension should satisfy $d_E \geq 2n + 1$, where n is the dimensionality of the system to be investigated. In Yang and Wu's work (2010), it has been demonstrated that for the biped system, the embedding dimension of n is large enough to guarantee the accuracy of the estimated

Lyapunov exponents when the nonlinear mapping is introduced. For the cases of unknown system dimensions, various d_E have to be used, which leads to spurious exponents. Identifying true Lyapunov exponents from spurious ones is beyond the scope of this work. The related discussion can be referred to (Gencay and Dechert, 1992).

3.4.4 Estimating Lyapunov exponents of the LQR balance control system using noisy data

One important issue of estimating Lyapunov exponents using experimental data is the robustness of the estimates to the measurement noise, as in practice it is inevitable to have measurement noise in the observations. Several methods for estimating Lyapunov exponents using a noisy time series for chaotic systems have been developed (Wolf, Swift, Swinney, and Vastano, 1985; Brown, Bryant, and Abarbanel, 1991; Gencay and Dechert, 1992; Zeng, Pielke, and Eykholt, 1992). As regards the nonlinear mapping method, it was documented that the values of negative exponents in ‘thin’ directions of the data set start to become affected when the measurement noise level grows above about 10% of the thickness of the data set in the associated Lyapunov direction (Brown, Bryant, and Abarbanel, 1991). However, in Yang and Wu’s work (2011), it can be observed that the negative exponent of the selected system was apparently underestimated using nonlinear mapping even when the measurement noise was maintained at a relatively low level as 5%. Regarding the method based on multiple layer feedforward (MLF) networks, the extent to which the original system dynamics can be uncovered is reduced due to the presence of noise. The performance

of successive GSR operation may also be deteriorated (Gencay and Dechert, 1992). Although in (Gencay and Dechert, 1992) the method was successfully applied to the observed chaotic data with measurement noise as well as system noise, of which the largest exponents are all positive, the validity of the method has not been tested on exponentially stable systems featuring only negative exponents, leaving the power of the method less convincing.

In this research work, only additive Gaussian white noise is considered. Gaussian white noise is of special interest here since it is the type of measurement noise commonly encountered in experimental situations, especially for mechanical systems (Yang and Wu, 2011). The noise level has been frequently represented by the signal-to-noise ratio (SNR), which is defined as the power ratio between a signal and the background noise. It is usually expressed using the logarithmic decibel scale as follows:

$$\text{SNR(dB)} = 10 \log_{10}(P_{\text{signal}}/P_{\text{noise}}) \quad (3.15)$$

where P is the average power. Consequently, as an alternative measure of the noise level, the percentage of noise can be obtained as:

$$\text{Noise(\%)} = (P_{\text{noise}}/P_{\text{signal}})^{1/2} \times 100. \quad (3.16)$$

When the percentage of noise is greater than 5%, the measured values would be considered unreliable. Correspondingly, a time series with an SNR greater than 35 dB is believed to be contaminated with low noise, and those with an SNR less than 25 dB are regarded as having high noise.

To validate the noise robustness of the new method, Table 3.9 shows the estimated Lyapunov exponents and their percentage errors for the biped LQR balancing system in Equation (3.5) with different measurement noise levels using the proposed method. T_{lag} is set to 70 and the embedding dimension is 4. Each entry in Table 3.9 is an average of 100 simulations and σ_{λ_i} denotes the standard deviation of the corresponding λ_i . The results listed in this table were obtained under two different cases. In Case I, the noisy data were used only for calculating the exponents, based upon the approximated Jacobian matrices, which were derived from a MISO RBF model trained with a noise-free data sequence. In Case II, even the Jacobian matrices were derived from the RBF network tuned by noisy data. It can be observed that the results are very

Table 3.9: Lyapunov exponents (LEs) and their percentage errors for the biped balancing system in a 4-dimensional embedding space with different measurement noise levels.

Noise%	SNR (dB)	1 st LE		2 nd LE		3 rd LE		4 th LE		
		$(\lambda_1^* = -2.8699)$		$(\lambda_2^* = -2.9933)$		$(\lambda_3^* = -7.1580)$		$(\lambda_4^* = -7.5334)$		
		λ_1 (σ_{λ_1})	Err. %	λ_2 (σ_{λ_2})	Err. %	λ_3 (σ_{λ_3})	Err. %	λ_4 (σ_{λ_4})	Err. %	
I ^a	1.0	40	-2.8040 (9.0e-4)	2.30%	-2.8886 (0.4025)	3.50%	-6.8241 (0.9508)	4.66%	-6.8301 (0.9517)	9.34%
	5.0	26	-2.8042 (0.0054)	2.29%	-2.8884 (0.4025)	3.50%	-6.8241 (0.9508)	4.66%	-6.8301 (0.9517)	9.34%
	10.0	20	-2.8031 (0.0084)	2.33%	-2.8894 (0.4027)	3.47%	-6.8242 (0.9508)	4.66%	-6.8301 (0.9517)	9.34%
II ^b	0.01	80	-2.8075 (0.0503)	2.17%	-3.0093 (1.7388)	0.54%	-8.3617 (4.8282)	16.82%	-8.3598 (4.8272)	10.97%
	0.05	66	-2.6851 (0.0343)	6.44%	-2.7476 (1.5864)	8.21%	-5.8805 (3.3964)	17.85%	-5.8814 (3.3970)	21.93%
	0.1	60	-2.8412 (0.0572)	1.00%	-3.0021 (1.7343)	0.29%	-5.6196 (3.2516)	21.49%	-5.6240 (3.2541)	25.35%

^aThe reconstructed map in Case I are estimated using noise-free data, the noisy observations are only used for computing the exponents.

^bIn Case II, the noisy observations are used for estimating the embedded map first, then computing the exponents.

robust to the noise in Case I. Compared with the exponents derived directly from the system's equation of motion, the percentage errors of all estimated exponents are always kept below 10% (specifically, 9.34% is the largest error) even when the noise level is increased to 10%. Correspondingly, the standard deviations of all four estimates with different SNR are acceptable ($\sigma_{\lambda_4} = 0.9517$ is the largest value). From Case II, one can find that the percentage errors of the estimated Lyapunov exponents, especially those of the last two exponents, grow up obviously along with the increment of noise levels. To be specific, when the SNR is set to 60db (0.1% noise level), both the percentage errors for the third and fourth exponents exceed 20%, climbing to 21.49% and 25.35% respectively. The cause to this degraded accuracy may be inferred as the noisy data are employed for approximating the embedded attractor, leading to the sequence of the approximated Jacobian matrices being contaminated. In one word, the findings from Table 3.9 indicate again that the RBF model plays a crucial role for estimating Lyapunov exponents. If the structural information of the RBF model can be derived accurately, the sensitivity of the new method is very low to a noisy time series. To put it in another way, noise control is critical to the numerical accuracy of the estimated Lyapunov exponents even for the newly proposed method, especially in the procedure of approximating the reconstructed dynamics based on the RBF model.

One thing should be noticed that here values of the exponents estimated using the nonlinear mapping method are not provided since the controlled biped is a system of dimensionality higher than 3, for which using the polynomial expansion would introduce complicated mathematical derivation. Besides, accounting that one can

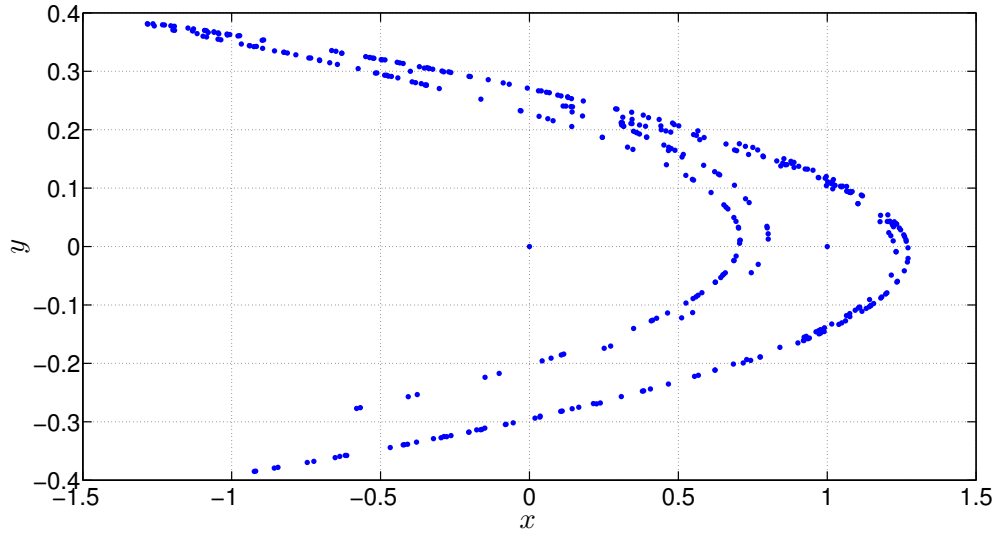
always find an existing RBF network capable of accurately mimicking a specified multiple layer feedforward (MLF) network, or vice versa (Haykin, 1999), the proposed time-series-based method of estimating Lyapunov exponents via MISO RBF networks actually is inspired to a great extent by Gencay and Dechert's work (1992), where MLF networks were employed to reveal the reconstructed system dynamics. Comparisons of the proposed method via MISO RBF network with the mentioned nonlinear mapping method and the MLF-network-based method are provided through the following two examples.

3.4.5 Lyapunov exponents of the Hénon map

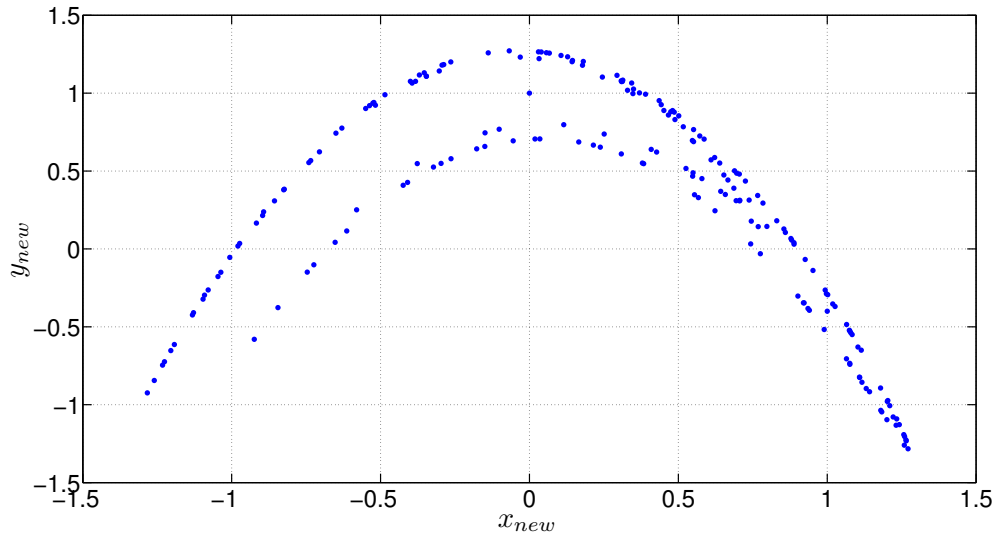
The classical Hénon map is a frequently studied dynamical system that exhibits chaotic behavior. The system has the following form as a 2-dimensional iterated map:

$$\begin{cases} x_{n+1} = 1 - 1.4x_n^2 + y_n \\ y_{n+1} = 0.3x_n \end{cases} \quad (3.17)$$

It is selected here as one of the examples since the full set of Lyapunov exponents of the map has been well investigated. A time series including 400 data along the x direction was generated first with the initial condition set to (0.0, 0.0). To avoid transients, the first 200 observations were discarded, thus the number of observations used in estimating the exponents was only 200 along with setting $T_{lag} = 1$, $d_E = 2$. Figure 3.12(A) displays the dynamics of the Hénon map in the original phase space and Figure 3.12(B) shows the one in a 2-dimensional embedding space, where x_{new} denotes the first component of the reconstructed vector and y_{new} is corresponding



(A)



(B)

Figure 3.12: The trajectory of the Hénon map in (A) the original phase space; (B) a 2-dimensional embedding space.

to the second component analogously. It can be seen from Figure 3.12(B) that the chaotic geometric structure of the Hénon map in Figure 3.12(A) is retrieved in the

embedding space.

To derive the sequence of Jacobian matrices of the reconstructed attractor shown in Figure 3.12(B), a MISO RBF network summarized in Table 3.10, was tuned for approximating the nonlinear regression mapping g in Equation (3.12) first. Lyapunov exponents were then estimated based upon the structural information of the tuned network. Figure 3.13 shows the evolution of the estimated Lyapunov exponents for the Hénon map. The constants that two exponents finally converged to are 0.409

Table 3.10: Structure of the RBF network for estimating the regression Equation (3.12) for the Hénon map.

Number of hidden nodes	20
Number of inputs, outputs	2, 1
Number of clustering samples	120
Mean-squared error after training	1.76e-8

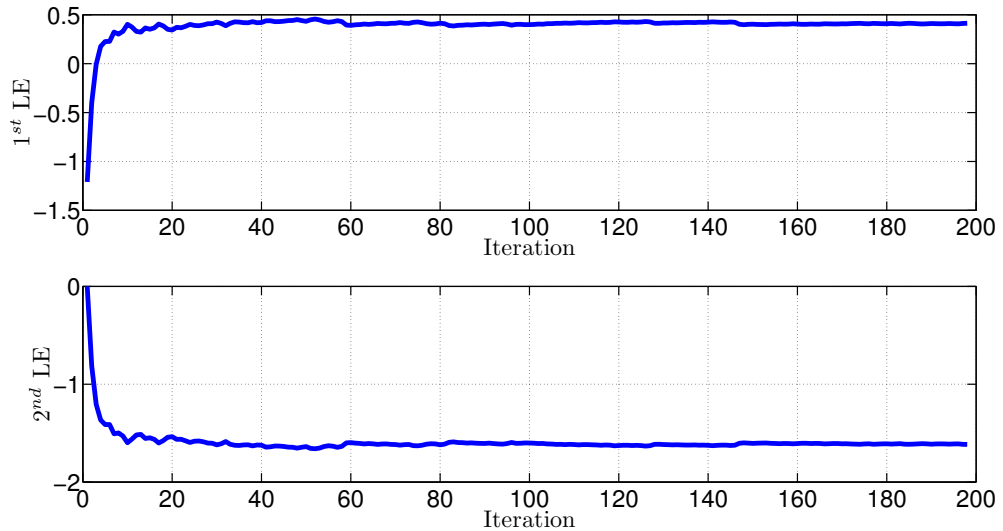


Figure 3.13: Evolution of Lyapunov exponents in the 2-dimensional embedding phase space.

and -1.612 respectively as listed in Table 3.11. To make a comparison, the numerical results estimated from the linear and nonlinear mapping methods, the Gencay and Dechert's method (Gencay and Dechert, 1992) are also provided in Table 3.11, where values derived from the original map (0.408 and -1.620) are taken as the standards. It can be seen from the percentage errors shown in the table that, for the positive exponent, the newly proposed method in this work can generate the most accurate estimate compared with other three methods, that is 0.24% vs. 116.18%, 9.58% and 0.74%. Regarding the negative exponent, although the accuracy in terms of percentage error using the proposed method (0.49%) is at the weak position compared with the Gencay and Dechert's method (0.31%), it is still within the acceptable error tolerance (far below 15%), let alone winning in the competition with the linear

Table 3.11: Lyapunov exponents (LEs) and their percentage errors for the Hénon map in a 2-dimensional embedding space.

Method	1 st LE		2 nd LE	
	$(\lambda_1^* = 0.408)$		$(\lambda_2^* = -1.620)$	
	λ_1	Err.%	λ_2	Err.%
LM ^a	0.882	116.18%	-0.865	46.60%
NM ^b	0.447	9.58%	-1.509	6.81%
GD ^c	0.405	0.74%	-1.625	0.31%
SW ^d	0.409	0.24%	-1.612	0.49%

^aFor brevity, LM denotes the traditional time-series-based method using Linear Mapping. Results are derived using 50,000 data with setting $T_{lag} = 3$. The embedding dimension $d_E = 3$, generating a spurious exponent $\lambda_{spurious} = -0.017$.

^bNM denotes the time-series-based method using Nonlinear Mapping proposed by Brown, Bryant, and Abarbanel (1991). The number of the observations is 11,000. $T_{lag} = 1, d_E = 2$, and the order of the Taylor expansion is selected as 2.

^cGD denotes the method proposed by Gencay and Dechert (1992).

^dSW denotes the newly proposed method via the MISO RBF network. Parameters used in GD and SW are the same: the number of the observations is only 200 with $T_{lag} = 1, d_E = 2$.

(46.60%) and the nonlinear mapping (6.81%) methods. Moreover, the newly proposed method overwhelms the linear and nonlinear mapping methods not only in terms of accuracy, but also in the way of the length of observations used for estimating (200 vs. 50,000 and 11,000).

To investigate the robustness of the new method to the measurement noise, Gaussian white noise was added to the data set and the noise level was increased gradually. Table 3.12 presents the estimated Lyapunov exponents under different noise levels using the newly proposed method, along with the corresponding results derived from Gencay and Dechert's method for comparison. Parameters used for estimating are the same as those in Table 3.11, i.e., $N = 200$, $T_{lag} = 1$, $d_E = 2$. Again each entry in Table 3.12 is an average of 100 simulations and σ_{λ_i} denotes the standard deviation of the corresponding λ_i . Compared with the results reported in (Gencay and Dechert, 1992), the estimated Lyapunov exponents using the newly proposed method achieve much higher accuracy. The smallest gap occurs when the noise level is set to 0.01%,

Table 3.12: Lyapunov exponents (LEs) and their percentage errors for the Hénon map in a 2-dimensional embedding space with different measurement noise levels.

Noise%	SNR (dB)	Method ^a	1 st LE ($\lambda_1^* = 0.408$)		2 nd LE ($\lambda_2^* = -1.620$)	
			λ_1 (σ_{λ_1})	Err.%	λ_2 (σ_{λ_2})	Err.%
0.01	80	GD	0.3899 (0.0573)	4.44%	-1.7251 (0.6114)	6.49%
		SW	0.4108 (0.0013)	0.69%	-1.6118 (0.2249)	0.51%
0.05	66	GD	0.3612 (0.0633)	11.47%	-1.7961 (0.6114)	10.87%
		SW	0.4111 (0.0035)	0.75%	-1.6155 (0.2258)	0.28%
0.1	60	GD	0.3591 (0.0909)	11.99%	-2.2514 (0.5369)	38.98%
		SW	0.4097 (0.0049)	0.42%	-1.6148 (0.2262)	0.32%

^aThe notations in this column are similar as those in Table 3.11. GD denotes the method proposed by Gencay and Dechert (1992), and SW denotes the new method proposed in this section.

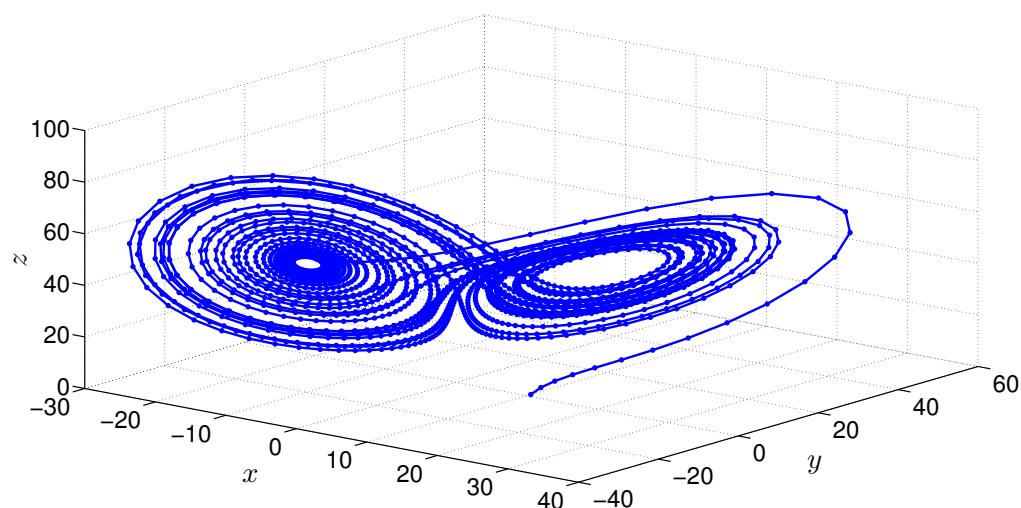
where the algorithm performance in terms of percentage error is 6.49% vs. 0.51% for the negative exponent; when the noise level is increased to 0.1%, the gap is enlarged to 38.98% vs. 0.32%. Since no details of the network approximating the Hénon map was provided in (Gencay and Dechert, 1992), the relatively large errors following Gencay and Dechert's method shown in Table 3.12 are inferred from over-fitting in the noisy environment.

3.4.6 Lyapunov exponents of the Lorenz system

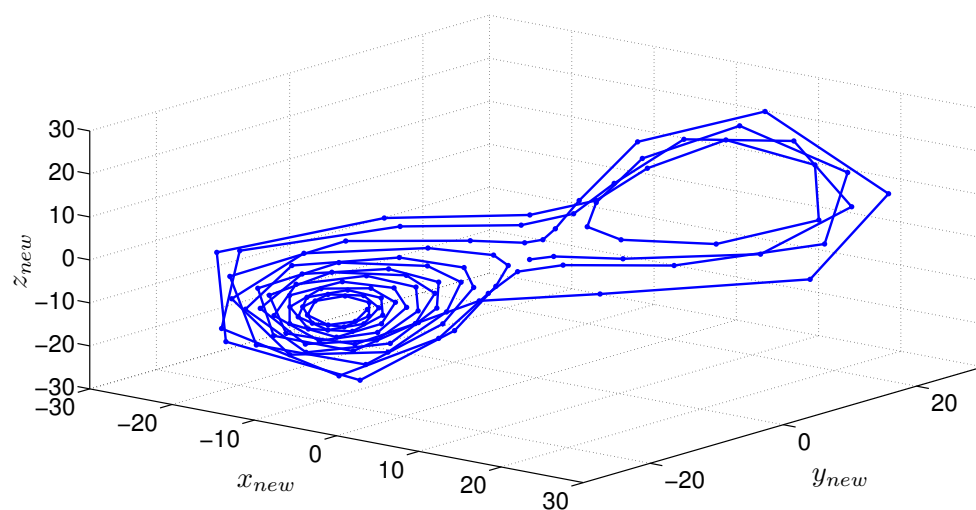
The Lorenz system is modeled by three ordinary differential equations as follows

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = rx - y - xz \\ \dot{z} = -bz + xy \end{cases} \quad (3.18)$$

It was first developed to describe the fluid convection problem (Lorenz, 1963). As an example, here the system parameters are taken from (Wolf, Swift, Swinney, and Vastano, 1985): $\sigma = 16$, $r = 45.92$, and $b = 4.0$. By employing the fourth-order Runge-Kutta method, a time series including 1000 data along the x direction was generated first with a sampling rate $\Delta t = 0.01s$, and the initial condition was $(10.0, 1.0, 0.0)$. The first 200 observations were discarded to avoid transients. Thus with setting $T_{lag} = 6\Delta t = 0.06s$, $d_E = 3$, the number of observations used in estimating the exponents is only 132, indicating that totally the useful length of the observations in terms of time is $131 \times T_{lag} = 7.86s$. Figure 3.14(A) shows the original attractor of the Lorenz system, of which the chaotic geometric structure is restored in a 3-



(A)



(B)

Figure 3.14: The trajectory of the Lorenz system in (A) the original phase space; (B) a 3-dimensional embedding space.

dimensional embedding space as shown in Figure 3.14(B). x_{new} in Figure 3.14(B) has the similarity with the legend labeled in the reconstructed Hénon map, which denotes

the first component of the reconstructed vector. y_{new} and z_{new} are corresponding to the second and third components of the reconstructed vector in the delay coordinates respectively.

Table 3.13 lists the summary of the RBF network tuned for approximating the nonlinear regression mapping g in Equation (3.12). As an approximation of the reconstructed attractor, the structural information of the trained network was extracted for deriving the sequence of Jacobian matrices. Evolution of the estimated Lyapunov exponents is shown in Figure 3.15, of which the numerical values and their percentage errors with respect to those exponents estimated from the mathematical model are listed in Table 3.14, where the results of the traditional time-series-based method using linear mapping and the nonlinear mapping method proposed by Brown, Bryant, and Abarbanel (1991) are also provided for comparison.

It can be seen from Table 3.14 that, for the positive exponent, although the nonlinear mapping method can achieve the highest level of accuracy in terms of percentage error (1.87%) among the listed three methods, the numerical accuracy of the newly proposed method via the MISO RBF network is still preferable in terms of percentage error (4.57%) compared with the one of the linear mapping method

Table 3.13: Structure of the RBF network for estimating the regression Equation (3.12) for the Lorenz system.

Number of hidden nodes	50
Number of inputs, outputs	3, 1
Number of clustering samples	79
Mean-squared error after training	1.366e-5

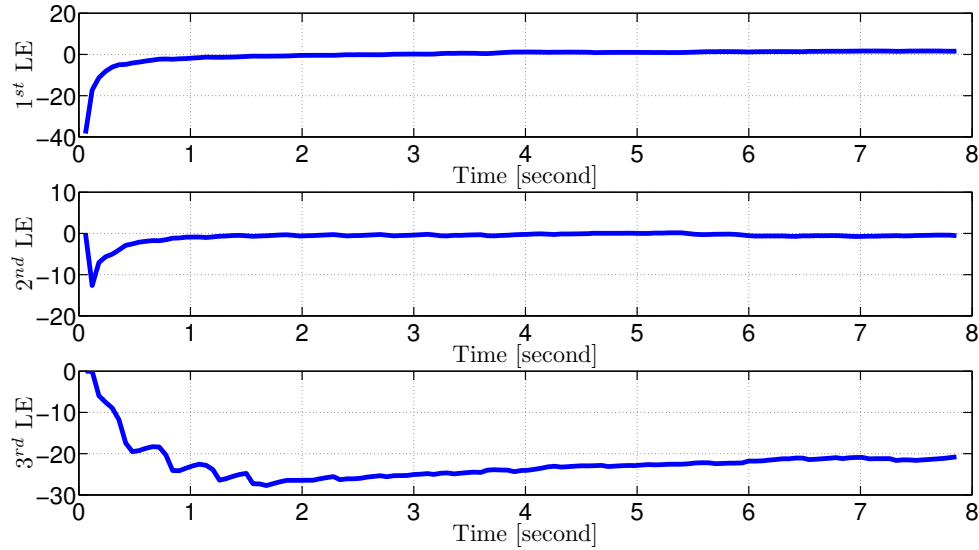


Figure 3.15: Evolution of Lyapunov exponents in the 3-dimensional embedding phase space.

Table 3.14: Lyapunov exponents (LEs) and their percentage errors for the Lorenz system in 3-dimensional embedding space.

Method	1 st LE		2 nd LE		3 rd LE	
	$(\lambda_1^* = 1.50)$		$(\lambda_2^* = 0.0)$		$(\lambda_3^* = -22.50)$	
	λ_1	Err.%	λ_2	Err.%	λ_3	Err.%
LM ^a	1.4151	5.67%	-0.1198	n/a	-8.2033	63.54%
NM ^b	1.4719	1.87%	-0.5244	n/a	-13.1957	41.35%
SW ^c	1.5686	4.57%	-0.5660	n/a	-20.7693	7.69%

^aLM denotes the traditional time-series-based method using Linear Mapping.

^bNM denotes the time-series-based method using second-order Nonlinear Mapping proposed by Brown et al. (1991). Parameters used in the LM and NM methods are same: both the numbers of the observations are 20,000 with $\Delta t = 0.001s$, $T_{lag} = 4$, and the results are reported in Yang and Wu's work (2010).

^cSW denotes the newly proposed method in this paper.

(5.67%). For the negative exponent, the numerical accuracy using the new method hits the highest level as the percentage errors of three methods turn out to be 7.69% (new method) vs. 63.54% (linear mapping) and 41.35% (nonlinear mapping), and

the sum of the exponents spectrum using three different methods are correspondingly -19.77(new method) vs. -6.91 (linear mapping), -12.23 (nonlinear mapping), showing that the newly proposed method can approximate the averaged convergence rate for the hyper volume in the state space best associated with the standard rate -21.00.

For counteracting the effect of the measurement noise on the numerical accuracy of estimated Lyapunov exponents, Table 3.15 provides the average values of the estimated Lyapunov exponents of 100 estimations, along with the percentage errors using the newly proposed method for the Lorenz system undergoing different noise levels. σ_{λ_i} denotes the standard deviation of the corresponding λ_i . Similar to Table 3.9, the numerical results were carried out under two cases as shown in Table 3.15: the noisy data only got involved for computing Lyapunov exponents in the first case, provided that the MISO RBF model was derived from noise-free data; while in the second case both the neural model and Lyapunov exponents were estimated using

Table 3.15: Lyapunov exponents (LEs) and their percentage errors for the Lorenz system in a 3-dimensional embedding space with different measurement noise levels.

	Noise%	SNR (dB)	1 st LE ($\lambda_1^* = 1.50$)		2 nd LE ($\lambda_2^* = 0.0$)		3 rd LE ($\lambda_3^* = -22.50$)	
			$\lambda_1(\sigma_{\lambda_1})$	Err. %	$\lambda_2(\sigma_{\lambda_2})$	Err. %	$\lambda_3(\sigma_{\lambda_3})$	Err. %
I ^a	1.0	40	1.4760(0.158)	1.60%	-0.4762(0.301)	n/a	-20.6343(2.912)	8.29%
	5.0	26	1.4030(0.292)	6.46%	-0.2886(0.459)	n/a	-20.5106(2.960)	8.84%
	10.0	20	1.6230(0.401)	8.20%	-0.4489(0.577)	n/a	-20.5218(3.156)	8.79%
II ^b	0.01	80	1.5453(0.306)	3.02%	-0.3316(0.607)	n/a	-22.3765(4.571)	0.55%
	0.05	66	1.6466(0.308)	9.77%	-0.3281(0.615)	n/a	-22.3997(4.712)	0.45%
	0.1	60	1.5538(0.296)	3.59%	-0.4189(0.581)	n/a	-22.4899(4.659)	0.04%
	1	40	1.6350(0.116)	9.00%	-0.5365(0.772)	n/a	-18.6447(2.581)	17.13%

^aThe reconstructed map in Case I are estimated using noise-free data, the noisy observations are only used for computing the exponents.

^bIn Case II, the noisy observations are used for estimating the embedded map first, then computing the exponents.

the noisy observations. As demonstrated in Case I, since the structural information of the embedded map estimated from the noise-free data is at hand, the percentage errors of all non-zero exponents are below 9% even when the noise level is increased to 10% (i.e., 8.20% for the positive exponent and 8.79% for the negative one). However, once the available structural information is also estimated from the contaminated observations, as shown in Case II, the numerical accuracy of the estimated Lyapunov exponents can only be maintained at a high level in terms of the percentage error lower than 10% within a very limited range of noise level (less than 0.1%). When the noise level is increased to 1%, the percentage error of the negative exponent exceeds 15%, achieving 17.13%. Thus the conclusion that the RBF model approximating the embedded attractors plays a dominant role for estimating Lyapunov exponents reliably could be driven from Table 3.15 once again.

3.5 Discussion

Two methods of estimating Lyapunov exponents using RBF networks are proposed in this chapter, both of which feature common advantages in three folds. First of all, no mathematical models are required, which inherits the attractiveness of the time-series-based methods; Secondly, the derivation of Jacobian matrices based on the RBF network is quite straightforward; Last but not least, Lyapunov exponents can be estimated reliably regardless of the signs, that is the very merit of the model-based method.

In particular, if all the state variables of the systems under study are practically measurable, the proposed method using MIMO RBF models can be applied. Since

there is no need to reconstruct the system dynamics in an embedding space, this method is more attractive compared with those approaches demanding reconstruction of the state space. The up side of the method is especially perceivable to systems of large dimensions (such as certain power systems). The reason not only lays in that the size of the neural Jacobian matrices can be maintained same as of the original system, but also in the sense that with this method the required length of the time series along each state is relatively short. A standing biped balance system subjected to an LQR control and a servovalve-controlled hydraulic actuator system have been employed in Section 3.4 to demonstrate the efficacy of this developed method. The accuracy of Lyapunov exponents calculated based upon the neural model Jacobians in both cases are high enough with respect to the ones derived directly based on the mathematical models.

As regards the method using MISO RBF networks, the fact that only a scalar time series is required lifts the prohibition on the widespread use of this method. The advantage also comes from the finding that the high numerical accuracy of the estimated Lyapunov exponents can be guaranteed even from noisy observations in most cases except when the structural information of the reconstructed attractor are corrupted seriously. For this case, reconstructing the attractors in the embedding space and revealing the original dynamics becomes challenging yet. The validity of this method has been demonstrated in Section 3.5 via again the standing biped balance system subjected to the LQR control, plus the Hénon mapping and the classical Lorenz system. Concerning these three case studies, several remarks are provided here.

Remark 1: The three dynamical systems studied in Section 3.5 are quite different. Both the Hénon map and the Lorenz system have strange attractors, while the actuated biped model features an exponentially stable equilibrium point. It has been demonstrated that in spite of the noise involved in the time series, for all the above dynamical systems, the proposed method works well in terms of achieving highly accurate estimates of the entire spectra of the system Lyapunov exponents. Noting that the standing biped was regulated by a state feedback controller, the biped balancing system should be considered as an autonomous system[†]. However, the proposed method can also be applied to those non-autonomous systems that are usually represented by a set of ordinary differential equations with finite dimensions. For such a system in the form of Equation (2.5), by treating t as an additional state with $\dot{t} = 1$, calculating Lyapunov exponents can be reduced to the problem of computing exponents for an autonomous system at the expense of increasing the dimension by only one (Sandri, 1996).

Remark 2: It should be clarified here that, the data segment representing the system transient section discarded in the Hénon and the Lorenz examples, were counted in the biped case. However, if one estimates the Lyapunov exponents for the controlled biped following the same procedure, i.e., discard the data collected from the transient period, the same convergent exponents would be obtained. Indeed due to that the solution is unique, rather than exactly achieving and resting at the equilibrium state, the biped was driven toward to the upright posture asymptotically, which can be seen from the close-up view of the state trajectories provided in Figure

[†]An autonomous system in mathematics is a system of ordinary differential equations whose output does not explicitly depend on time.

3.2(A). The fundamental invariance property of Lyapunov exponents to the initial conditions warrants that within the same basin of attraction, the numerical values of the exponents estimated from different initial conditions should be identical. This characteristic of Lyapunov exponents can be perceived from the computational viewpoint: the effects of the data points selected from the transient period are removed due to the averaging process.

Remark 3: From Figure 3.2 and Figure 3.11, one may notice that while the actuated biped moved to the position close to the upright one after around 2.5 seconds, the Lyapunov exponents didn't converge to their final values until about 20 seconds. Thus it is important to distinguish these two time scales. Basically, the transient period in terms of settling time of the system states, and the convergent period of the Lyapunov exponents are different in concept, and both can be selected by the researchers based on different design principles. In this work, an exponent is considered to achieve the converged constant if the change in the numerical values of the exponent between two subsequent steps is within 10^{-4} . As no solid proof is available to quantify the relationship between the transient period for the system states and the convergent period for the exponents, one cannot determine the adequate data length required for estimating Lyapunov exponents merely from the transient period of the system states; and although important, finding the determination rule for the data length is beyond the scope of this work.

Chapter 4

Constrained and Energy Efficient Control of Biped Standing

4.1 Introduction

In this chapter, based on the concept of Lyapunov exponents, a systematic review of the stability property for a standing biped balancing system, including the system stability and the structural stability analysis, is presented. The simplified standing biped model described in Figure 2.1 is used here with the same parameters listed in Chapter 3. However, rather than the LQR control employed in Chapter 3, a proportional-derivative (PD) control is developed with the control gains obtained from a genetic algorithm (GA), which is able to minimize the energy consumed during the regulation without foot motion. As the PD gains derived via the GA are optimal only associated with a certain initial condition in a local sense, the stability of the control system should be analyzed over the state space. Moreover, the system largest

Lyapunov exponent (LLE) and a specific cost function are calculated for the standing biped with different sets of control gains combinations. Comparisons and analysis on this basis demonstrate the effects of various control actions in different magnitudes on the balancing stability and on energy efficiency. The proposed method using the concept of Lyapunov exponents forms a framework for tuning the control gains to achieve the balance between the system stability and the energy consumption.

4.2 Development of the PD Controller Using Genetic Algorithm

It is well known the joint torques needed to balance the body during standing can be evoked passively by the viscoelasticity generated from muscles and surrounding tissues (Meyer, McCulloch, and Lieber, 2011). However, the elastic components alone of the ankle muscle cannot stabilize balance (Morasso and Sanguineti, 2001; Matjačić, Hunt, Gollee, and Sinkjaer, 2003; Loram and Lakie, 2003). Viscosity is reported to be a sizeable contributor to muscle stress and extensibility during passive stretch (Meyer, McCulloch, and Lieber, 2011). On account of this consideration, a PD controller shown in Equation (4.1) is proposed to stabilize the standing biped.

$$\begin{cases} \tau_1 = -K_{P1}q_1 - K_{D1}q_3, \\ \tau_2 = -K_{P2}q_2 - K_{D2}q_4. \end{cases} \quad (4.1)$$

The proportional control action serves as active elasticity and the derivative control action serves as active viscosity. For a given set of initial conditions, a GA is employed

to tune the PD gains of the controller, which are supposed to keep the biped at an upright posture, meanwhile satisfy the constraints between the biped feet and the ground, minimize the energy consumption. Following this target, an energy-related cost function is defined as

$$J = \frac{1}{2} \left(\int_0^{t_f} (C_1 \tau_1^2 + C_2 \tau_2^2 + C_3 q_1^2 + C_4 q_2^2 + C_5 q_3^2 + C_6 q_4^2) dt + \int_0^{t_f} C_{constraint} dt \right) \quad (4.2)$$

where $C_{constraint} = \begin{cases} 0 & \text{if the constraints are satisfied} \\ C_7 & \text{if the constraints are violated} \end{cases}$ and t_f is the final time instance. The first two terms of the cost function are the control effort. It has been pointed out that reducing the control effort indicates lowering the energy consumption (Chevallereau and Austin, 2001). The remaining four terms in the first integral of the cost function, are the squared errors of the actual angular displacements and angular velocities of the two links of inverted pendulums with respect to the desired states ($\mathbf{q}_{desired} = [0, 0, 0, 0]^T$). By choosing different values of the weighting coefficients C_j ($j = 1, \dots, 7$), it is possible to tune the balancing system to meet different criteria.

In this work, searching the optimal control gains is carried out by employing GA. A GA comprises a set of individuals (the population) and a set of biologically inspired operators (the genetic operators). The individuals have genes, which are the potential solutions for the problem. The following outline summarizes how GA works:

Step 1: The algorithm begins by creating a random initial population as a starting point for the search.

Step 2: The algorithm then creates a sequence of new populations. At each step, the algorithm uses the individuals in the current generation to create the next population.

To create the new population, the algorithm performs the following steps:

- a. Score each member of the current population by computing its fitness value based on the cost function.
- b. Select some individuals (called parents) in the population for reproduction based on the relative fitness of the individuals.
- c. Choose some of the individuals in the current population that have lower fitness as elite. These elite individuals are passed on to the next population.
- d. Produce children from the parents by means of a crossover and mutation operator. A crossover takes two parents and swaps parts of their genetic information to produce new chromosomes. A mutation operator produces new single parents in the population by randomly modifying some of the genes.
- e. Replace the current population with the children to form the next generation.

Step 3: These processes are repeated until a satisfactory individual is found or a certain stop condition is met.

Figure 4.1 demonstrates the block diagram of the optimal PD control using a GA. In this research a real-valued GA (representing each chromosome as a real-valued number) is employed because it has several advantages over a binary GA (representing each chromosome as a bit string). Programming is simple and the searching speed is improved since the encoding and decoding processes are not necessary. This is due to the one-to-one correspondence between a phenotype and a genotype. It is possible to define a very large domain and easy to deal with highly complex constraints. Many experiments comparing real-valued and binary GAs have proven that the real-valued generates better results in terms of the solution quality and computation time

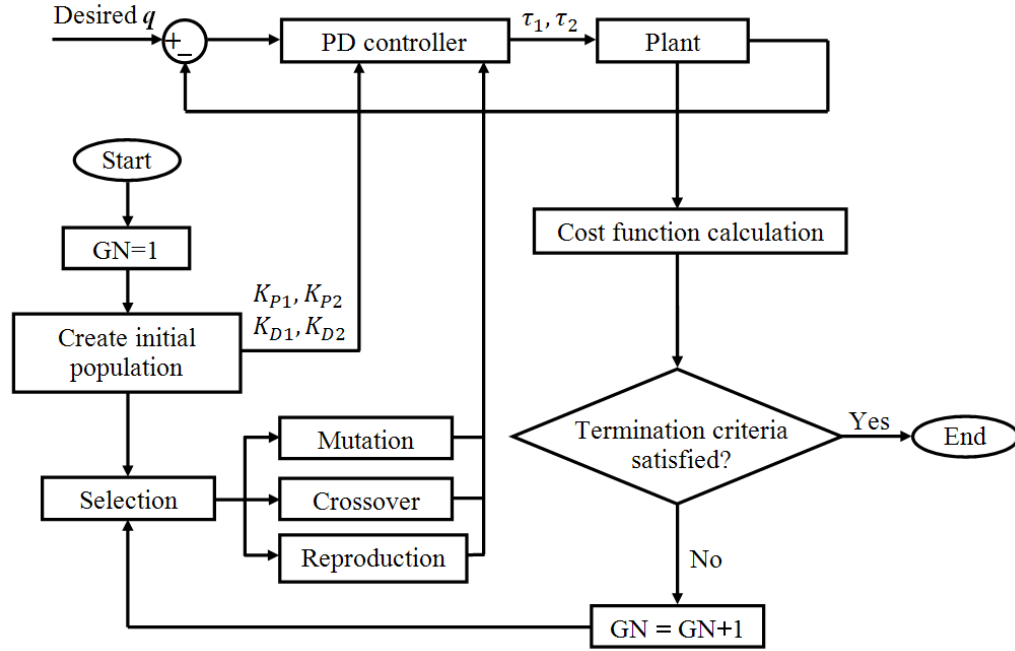


Figure 4.1: Block diagram of GA-based PD control.

(Michalewicz, 1996).

4.3 Simulation Results

The developed control strategy was implemented with MATLAB in the PC environment. The simulated motion of the biped, evolution of the control torques, ground reaction forces and COP migration are presented in this section, which are followed by a systematic stability analysis displayed in Section 4.4.

To obtain the optimal PD control gains, the convergence test was carried out first. The weighting coefficients were selected as $C_1 = C_2 = 0.001$, $C_3 = C_4 = 50$, $C_5 = C_6 = 10$, $C_7 = 1000$, indicating that the main target is driving the biped at

the upright posture. The population size for the GA was set to 200. The crossover probability was 0.9. For mutation probability, it varied from 0.09 to 0.1 according to different fitness values. The better the performance is, the lower the mutation probability would be taken. The maximum number of generations was selected as 120. The GA will terminate either a maximum number of generations has been produced, or when the value of the cost function does not improve for 30 consecutive generations. If in the final generation J values higher than C_7 or at $t_f = 5$ s the biped is outside the upright region, $\{|q_1| < 0.002, |q_2| < 0.002, |q_3| < 0.005, |q_4| < 0.005\}$, the GA is then considered as failed to find the solution of PD gains. The system initial condition was given by $\mathbf{q} = [-0.05, 0.03, 0.05, -0.03]^T$, and the step size $h = 0.001$. It can be seen from Figure 4.2 that the cost function converged to 0.1563 within 112 generations.

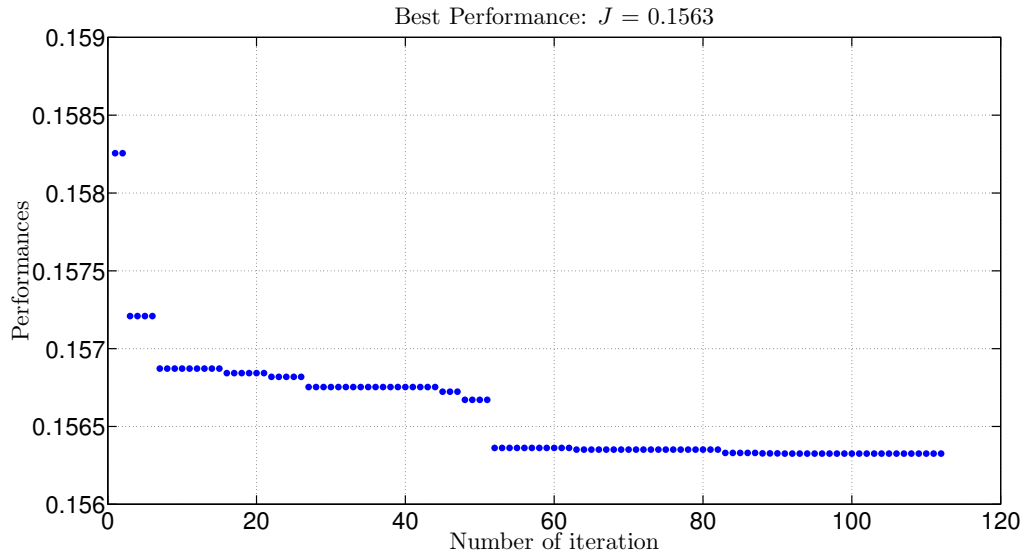


Figure 4.2: The variation of the cost function.

The PD gains obtained by the GA are $K_{P1} = 1144.10, K_{P2} = 271.42, K_{D1} = 361.95$ and $K_{D2} = 135.00$. Figure 4.3(A) and 4.3(B) show the evolution of the system states and control torques respectively, indicating that the biped can be driven successfully to the equilibrium posture within 2.5 seconds with the proposed controller. Meanwhile, time history versus F_{gy}, F_{gx} are displayed in Figure 4.4(A) and 4.4(B) respectively. Again the positive vertical ground reaction force F_{gy} implies the support foot was always in contact with the ground. The horizontal ground reaction force F_{gx} can be observed residing in the bounds of the peak static friction $(-\mu F_{gy}, \mu F_{gy})$ represented by two dashed lines, which suggests the foot-link did not slip. Migration of the COP which remained within the contact surface between the foot-link and the ground, is displayed in Figure 4.4(C).

Additionally, it can be found that the low control torques have insignificant effects

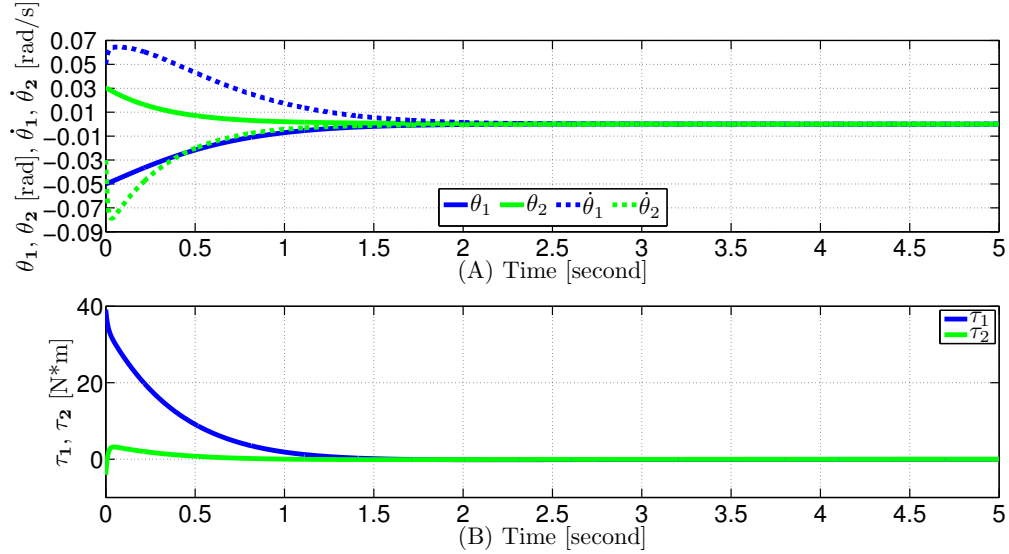


Figure 4.3: Evolution of (A) the biped states, and (B) the control torques.

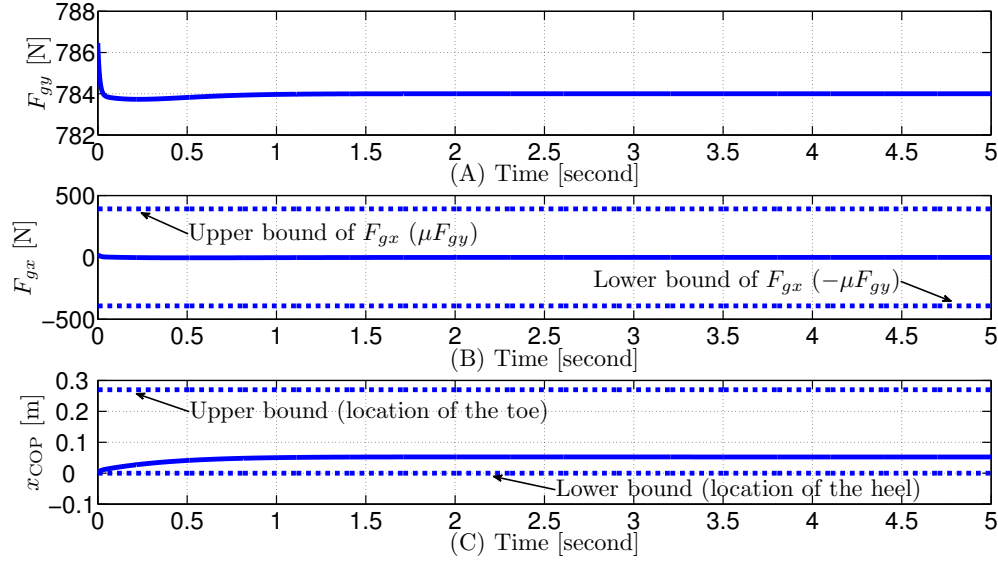


Figure 4.4: Evolution of (A) the vertical ground reaction force, (B) the horizontal ground reaction force and (C) location of COP.

on the ground reaction forces. The vertical ground reaction force is dominated by the gravity of the biped. As shown in Figure 4.4(A), the relative changes in F_{gy} with respect to the net weight of the biped are unapparent. The horizontal ground reaction force is always far lower than the maximum static friction, which indicates the friction constraint can be easily satisfied. It is also found that among the three constraints, the COP constraint is more likely to be violated especially during the transient period, which can be observed from Figure 4.4(C). This is due to the geometry of the biped (the foot length is much shorter compared with the total length of leg plus torso), and the finding is consistent with the previous research (Masani, Vette, and Popovic, 2006).

4.4 Stability Analysis

Basically adequate stability analysis is composed of four important issues: judging the asymptotic stability of the system that indicates the system capability of rejecting disturbances imposed on the system states; measuring the stability region in terms of shape and size in the state space in which the stability of the system is guaranteed; answering the question that how do the system parameters affect the system stability; and identifying the allowable variation range of the system parameters within which the system remains stable. In this section, all these four issues are explored using the concept of Lyapunov exponents for the standing biped associated with the proposed GA-based PD control. Although for such a system, the asymptotic stability of the system equilibrium point can be established by applying the linearization theorem as well, the concept of Lyapunov exponents is employed here based on the following considerations. First of all, the linearization theorem states that the behavior of a dynamical system near a hyperbolic equilibrium point is qualitatively the same as the behavior of its linearization near this equilibrium point. However, when tackling the system stability analysis, the mathematical term “neighborhood of the equilibrium point” is one of the obstacles restricting to determine system stability regions. For example, by applying the linearization theorem only, one cannot find the unstable initial states for the controlled biped. Secondly, the fact that numerical values of Lyapunov exponents do give an intuitive vision of system divergence or convergence rate can facilitate parametric analysis, which can be seen in the following section.

One point should be emphasized here is that the obtained GA-based PD gains are only optimal with respect to the specific initial condition as predefined; as regards a

different set of initial conditions, these previously obtained gains may be no longer optimal. Considering that stability has the higher priority compared with energy optimization for most dynamical systems, the stability region for the standing biped with the initially obtained control gains is determined in the state space at the cost of sacrificing the energy consumption. Re-calculation of the optimal gains via GA for those initial conditions deviated from the predefined one is out of the scenario of this work.

4.4.1 Stability of the biped balancing system

To investigate the stability of the standing biped subjected to the proposed controller, the algorithm developed by Wolf, Swift, Swinney, and Vastano (1985) to calculate the spectrum of Lyapunov exponents based on a mathematical model is used. Since Lyapunov exponents involve averaged long-term behaviors of the system, Figure 4.5 shows the evolution of all four Lyapunov exponents of the actuated system as shown in Equation (1), from which it can be observed that all the Lyapunov exponents converge to negative constants within 100 seconds. The numerical values of these four constants are listed in Table 4.1, indicating that the biped model combined with the proposed balance controller is exponentially stable about the equilibrium point.

To determine the stability region, the approach developed by Nusse and Yorke (1994) was adopted, where the region of interest is first divided into grid boxes. Next the size of neighboring grid boxes is chosen and LEs are calculated using the initial states from each neighboring box. If the same convergent and negative exponents are obtained, the neighboring grid box belongs to the stability region. In this research

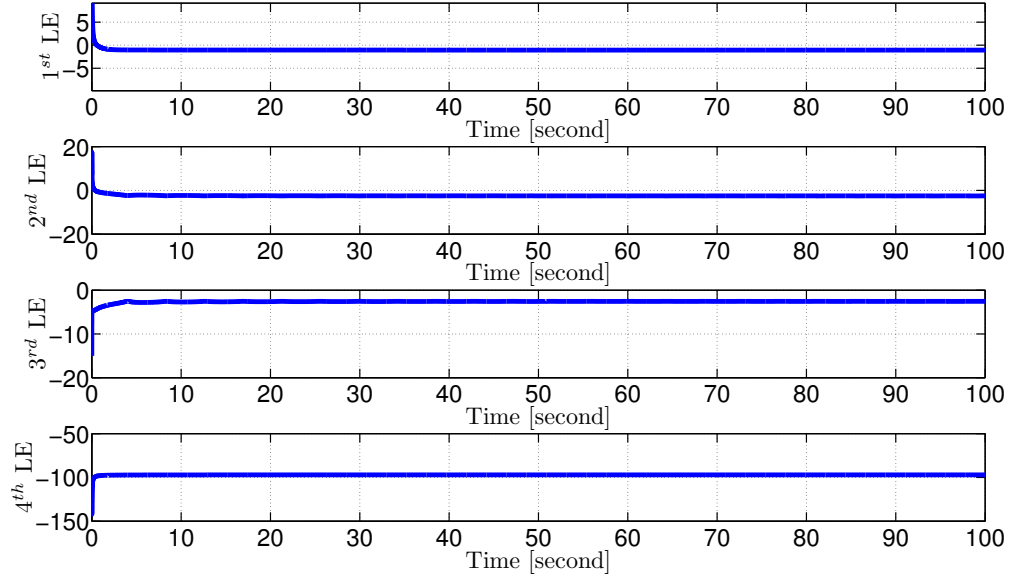


Figure 4.5: Evolution of the Lyapunov exponents.

Table 4.1: Lyapunov exponents after 100 seconds.

Lyapunov exponents (LEs)	1 st LE	2 nd LE	3 rd LE	4 th LE
Numerical value	-1.0616	-2.5259	-2.5594	-97.0509

four regions in the phase plane were tested:

$$\Gamma_1 = \{q_1 = 0\text{rad}, q_2 \in [-0.15, 0.15]\text{rad},$$

$$q_3 \in [-0.15, 0.15]\text{rad/s}, q_4 \in [-0.15, 0.15]\text{rad/s}\};$$

$$\Gamma_2 = \{q_1 \in [-0.15, 0.15]\text{rad}, q_2 = 0\text{rad},$$

$$q_3 \in [-0.15, 0.15]\text{rad/s}, q_4 \in [-0.15, 0.15]\text{rad/s}\};$$

$$\Gamma_3 = \{q_1 \in [-0.15, 0.15]\text{rad}, q_2 \in [-0.15, 0.15]\text{rad},$$

$$q_3 = 0\text{rad/s}, q_4 \in [-0.15, 0.15]\text{rad/s}\};$$

$$\Gamma_4 = \{q_1 \in [-0.15, 0.15]\text{rad}, q_2 \in [-0.15, 0.15]\text{rad}, \\ q_3 \in [-0.15, 0.15]\text{rad/s}, q_4 = 0\text{rad/s}\}.$$

And each region was divided into grid boxes with the size of 0.01rad, 0.01rad, 0.01rad/s and 0.01rad/s for q_1 , q_2 , q_3 and q_4 respectively. Figure 4.6 shows the stability regions determined by the largest Lyapunov exponent (LLE), which is a sample view with certain special fixed states, rather than an overall scenery. In other words, the fixed states in the above four regions are unnecessarily equal to 0. In Figure 4.6, the colorful points represent the initial states from which convergent and negative exponents

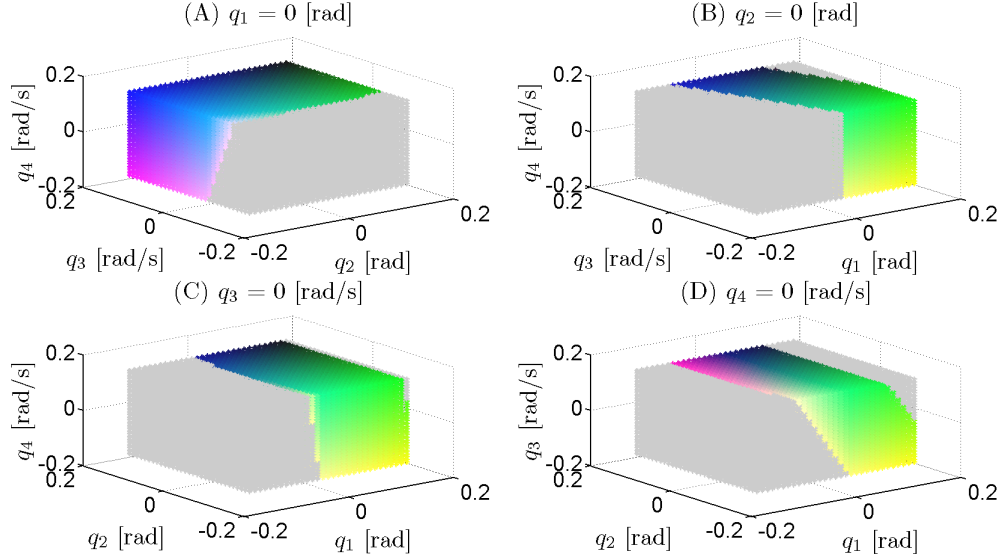


Figure 4.6: A sample view of the stability region of the balancing biped determined by the LLE, which are slices with depth. The colorful points shown in the figure represent the initial states from which the system trajectory can be regulated to the equilibrium point (the upright posture). The grey points represent the initial states from which certain constraint between the biped foot-link and the ground will be violated during the regulation.

can be obtained using the proposed PD controller. In these regions, all the LLEs are negative, of which the mean value is -1.0616 with a standard deviation of 1.37×10^{-14} , indicating that all the colorful points belong to the same stability region as the LLEs remain constant. The initial states marked by the grey points in Figure 4.6 constitute an unstable space, where convergent Lyapunov exponents cannot be obtained associated with the obtained PD controller. Failure of deriving exponents is caused by violation of the constraints between the foot-link and the ground, the calculation program terminated automatically since the biped collapsed, thus the model is no longer relevant to the standing biped.

4.4.2 Structural stability analysis

One cannot guarantee the proposed GA-based optimal controller is always at hand due to the limited experimental resolution, and in practice electronic noises of the controller usually introduce a certain degree of inaccuracy. It is worthy to point out that once the values of the control gains shift from the previously obtained ones even by a small amount, they may not be optimal gains any longer with respect to the predefined initial condition. For such a case, the objective is to ensure that in spite of the compromised energy consumption, the system asymptotic stability is still guaranteed since stability has the highest priority to most dynamical systems. From this viewpoint the structural stability analysis for the controlled biped is non-trivial. Structural stability is a fundamental property of a dynamical system. If the dynamics of the system in the phase space changes radically, for example by the appearance of a new equilibrium or a new periodic orbit, due to small external perturbations imposed

on the system itself, rather than the state variables, the system is then considered to be structurally unstable (Chen, 2005). To be more specific, a parametric analysis, which is a branch of the structural stability analysis, is provided here since the disturbance in this work is imposed on the PD control gains. As discussed in the work published by Tavazoei (2011), PD gains have significant effects on the structural stability. Here the system LLE is used to analyze the structural stability of the controlled biped and to determine the stability regions in the parametric space of PD gains, which is different from (Tavazoei, 2011), where the perturbation bound of the output feedback control gain for a chaotic system was investigated through following the eigenvalue justification of the linearized system.

For visual presentation, the analysis here is carried out by keeping one of the four gains at its optimal value, assigning the values for the remaining three ones from 70% to 130% (3% as step size) of their optimal values. Figure 4.7 shows parts of the structural stability regions in the parametric manifold, which are sets of $K_{P1} - K_{D1} - K_{P2} - K_{D2}$ combinations. The colorful points in Figure 4.7 constitute four different irregular manifolds which ensure the LLE of the biped balancing system is negative, and in these areas the biped can be stabilized at the upright position with the foot-link sustaining stationary. The areas marked by the grey points are sets of unstable gains combinations, where convergent Lyapunov exponents can not be obtained, again due to violation of the constraints between the foot-link and the ground.

Unlike the situation shown in Figure 4.6, the system LLEs are different from each other since every point in Figure 4.7 corresponds to a different control system.

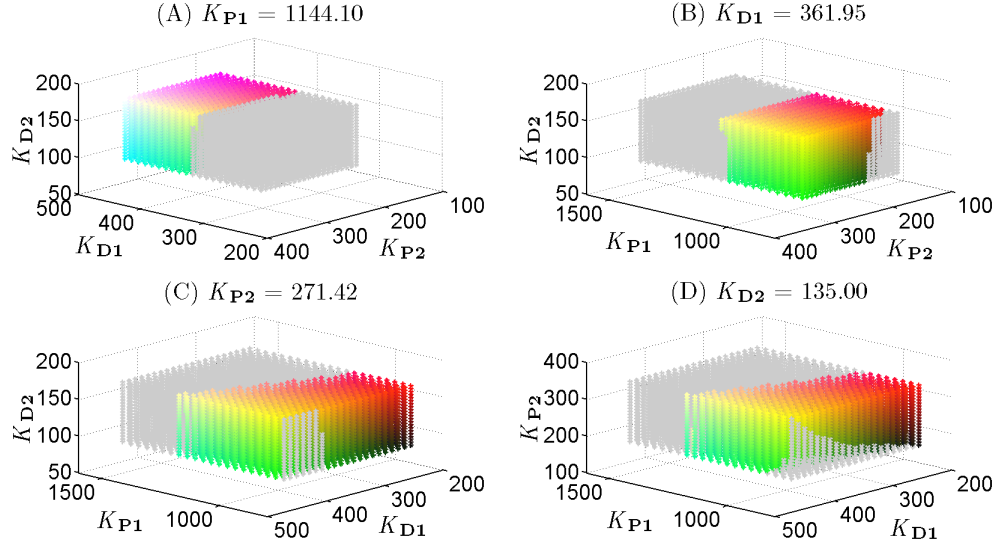


Figure 4.7: A sample view of the structural stability region of the balancing biped determined by the LLE, which are slices with depth. The colorful points shown in the figure represent the stable PD gains combinations, and those marked by the grey points represent the PD gains combinations with which certain constraint between the biped foot-link and the ground will be violated during the regulation.

However, certain systems may have very close LLEs and examples are provided in Table 4.2, Case I. The estimated LLEs of three biped balancing systems are all around -1.0328 with different control gains. Since the system LLE represents the convergence rate, Case I indicates that although the listed three controlled bipeds are characterized by different energy consumption, they have approximately the same asymptotic decay of the disturbances imposed on the initial conditions to the upright position. Contrary examples are also given in Table 4.2, Case II, where another three biped balancing systems are found to possess close values of the cost function (around 0.1838) but different LLEs. This indicates that three controlled bipeds listed in Case II consumes approximately the same amount of energy, whereas their asymptotic decays of the

Table 4.2: Values of the biped balancing system subject to different control gains.

	LLE	Cost value	K_{P1} (Incr. %)	K_{D1} (Incr. %)	K_{P2} (Incr. %)	K_{D2} (Incr. %)
Case I	-1.0328	0.1586	1178.42 (3)	416.24 (15)	271.42 (0)	135.00 (0)
	-1.0328	0.1714	1109.78 (-3)	427.10 (18)	295.85 (9)	135.00 (0)
	-1.0328	0.1845	1006.81 (-12)	361.95 (0)	287.71 (6)	102.60 (-24)
Case II	-0.2870	0.1838	903.84 (-21)	318.52 (-12)	189.99 (-30)	135.00 (0)
	-0.7164	0.1838	1144.10 (0)	470.54 (30)	206.28 (-24)	94.50 (-30)
	-1.2289	0.1838	1144.10 (0)	459.68 (27)	320.28 (18)	102.60 (-24)

disturbances imposed on the initial states vary from each other. Thus one may get a first sight that different control gains impact the system differently, and the value of each control gain should be tuned carefully to reach different control objectives and make tradeoffs among them. Discussion on effects of the different control gains is made in the following subsection.

4.4.3 Discussion

To investigate the effects of the control gains on the convergence rate and on the energy consumption of the biped balancing system, values of the system LLE and of the cost function (4.2) are estimated for all colorful points shown in Figure 4.7. Figures 4.8 and 4.9 show the general trends of the system LLE and the cost function respectively, and both figures contain 4 subplots, which correspond to the four cases presented in Fig. 4.7. In each subplot of these two figures, there are 21 hyper-planes dotted by different colors, which relate to the specific values of K_i . Here K_i represents the gain labeled in the vertical z -direction of the corresponding subplot of Figure 4.7. Thus, according to Figure 4.7, K_i means K_{D2} in Figure 4.8(A), (B) and (C), while

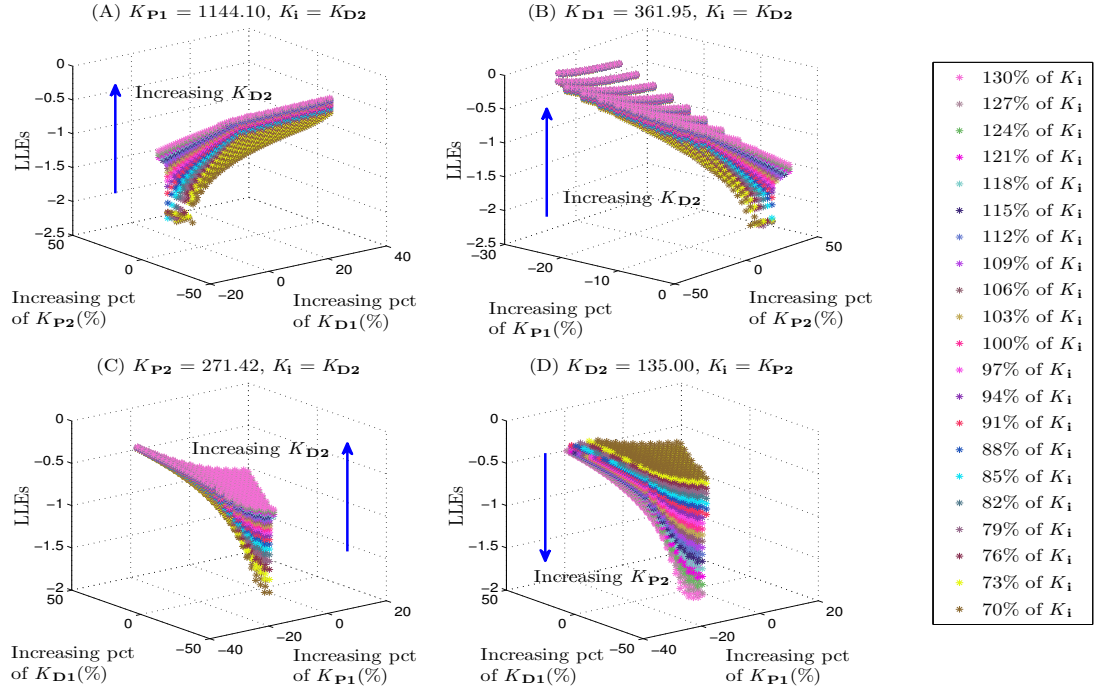


Figure 4.8: General trend of the system largest Lyapunov exponent (LLE).

in Figure 4.8(D) K_i denotes K_{P2} . Same manner can be seen from Figure 4.9. In addition, the increasing direction of K_i is marked by the arrow in every subplot, which assists in observation.

Figure 4.8(A) shows that, for the entire stable gains combinations, setting $K_{P1} = 1144.10$ and fixing the K_{P1} - K_{D1} - K_{P2} combination, the system LLE becomes less negative along with increasing the values of K_{D2} independently. Additionally, one may get a first impression of Figure 4.9(A) that for the same combinations, increasing K_{D2} lead to lower cost value. However, there are some exceptions. The numerical results show that increasing K_{D2} independently reduce the value of the cost function in the area where K_{D1} is relatively high; while in the area where K_{D1} is relatively low,

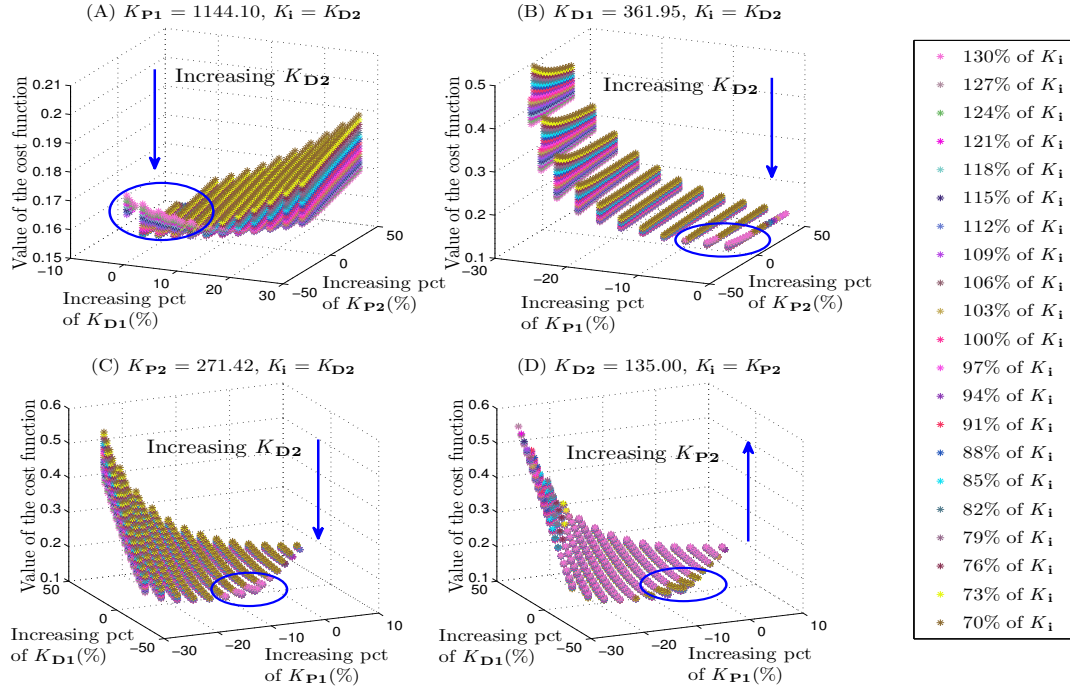


Figure 4.9: General trend of the system performance (i.e., the value of the cost function).

the trend is reversed. The circled area in Figure 4.9(A) reflects this finding, which in fact, is caused by the intersection of different hyper-planes. As a conclusion, one may say that when $K_{P1} = 1144.10$, for the same combinations, generally the convergence rate of the system becomes lower with increasing K_{D2} , but the system has a better performance (lower value of the cost function) especially with high K_{D1} .

Next, to investigate the effects of K_{D1} on the system LLE, the K_{P1} - K_{P2} - K_{D2} combination is pinned down, and the value of K_{D1} is tuned independently. From Figure 4.8(A), by viewing the K_{D1} -LLEs projection, it can be found that the negative LLE keeps increasing when the value of K_{D1} is increased independently, implying the convergence rate is decreasing. The K_{D1} -Performances projection in Figure 4.9(A)

shows that generally the higher K_{D1} is, the higher value does the cost function have. Exceptions occur in the area where K_{P2} is relatively low but K_{D2} is relatively high, within which the change of the value of the cost function is parabolic with respect to K_{D1} . Based on this observation, one may conclude that when $K_{P1} = 1144.10$, generally, increasing K_{D1} independently will slow down the system convergence rate, and degrade the system performance by causing higher value of the cost function especially when K_{P2} is high while K_{D2} is low.

Similarly, the K_{P2} -LLEs projection of Figure 4.8(A) illustrates that for a fixed K_{P1} - K_{D1} - K_{D2} combination, when increasing K_{P2} independently, the system LLE becomes more negative, meaning that the states converges faster to the attractor; And the K_{P2} -Performances projection of Figure 4.9(A) displays that in general higher K_{P2} causes higher value of the cost function, meaning that the system consumes more energy. Exceptions happen in the area where K_{D1} is relatively low but K_{D2} is relatively high. In a word, when $K_{P1} = 1144.10$, the convergence rate usually speeds up along with increasing K_{P2} , but the system performance becomes worse due to higher energy consumption especially with high K_{D1} and low K_{D2} .

Similar sceneries can be observed from other subplots in Figures 4.8 and 4.9 following the same manner. In summary, within the parametric stability region, if the PD controller is strongly position-based, i.e., with high proportional gains, the system LLE of the controlled biped has a low value, thus the biped can be stabilized at the balance state with a relatively high convergence speed; while if one increases derivative gains in the controller, value of the system LLE will also increase, the biped may be driven to the erect posture after a relatively long time. On the other

hand, the effects of the control gains on the cost function are more complex compared with those on the system LLE. Simulation results reveal that the change of the cost value is not monotone. This is reasonable because the control gains combinations are varied with respect to the optimal one. All the points except the one representing the optimal gains combination in Figure 4.9 are found to have values higher than the minimum 0.1563.

4.5 Summary

Two different types of stability have been rigorously analyzed in this chapter using the concept of Lyapunov exponents for a controlled standing biped: system stability with the disturbance imposed on the initial conditions, and structural stability with the disturbance imposed on the control gains. The stability regions for the above mentioned stability are identified in both the state space and the parametric manifold. Furthermore, based on the structural stability analysis, significant insights into the effects of different control gains on the system convergence rate and on energy consumption have been revealed. For the PD controller in the form of Equation (4.1), roughly speaking, within the parametric stability regions, high values of the proportional control gains can speed up the system convergence rate at the price of lowering the system energy efficiency. On the other hand, high values of the derivative control gains can slow down the system convergence rate, but consequences reflected from the value of the cost function are dependent. In specific, when the proportional control gains are fixed, a high value of the cost function may be generated by emphasizing the derivative control applied on the ankle joint, or by mitigating the derivative control

acted on the hip joint. However, the findings discussed here are for the selected range of control gains. Attentions are still supposed to be paid when tuning control gains to achieve desired balance between stability (convergence rate) and energy consumption.

The main contribution of the work presented in this chapter lies in the fact that, it is very challenging to synthesize a multi-objective Lyapunov control design (i.e., keeping the biped upright at the minimum level of energy consumption without foot motion) and a systematic stability analysis for a quietly standing biped using classical control theories. Since no constructive rule was provided for developing a candidate Lyapunov function for such a complicated control system, little success has been reported in the previous literatures on thoroughly tackling this problem. In general, when a balance control is designed based on Lyapunov's stability theory, it has little flexibility to include the optimization criterion owing to the system complexity. While when an optimal control is designed, using Lyapunov's second method for stability analysis is extremely difficult. The problem is solved in this chapter by applying the GA technique and the concept of Lyapunov exponents, both of which make up a balanced approach that provides not only the system stability, but also the optimized energy-related cost. Moreover, the method based on the concept of Lyapunov exponents shown here constitutes a general framework of systematic stability analysis for certain deterministic nonlinear dynamical systems. Application of this method is not restricted to the studied case of biped.

Chapter 5

An Experimental Study on Constrained Balance Control of a Standing Biped

5.1 Introduction

Regulation of biped standing is very challenging due to the existing constraints between the biped feet and the ground, which lead to time-varying bounds on the control output. Motivated by this, a GA-based PD control was proposed in the previous chapter, which is capable of stabilizing the standing biped at the upright position, satisfying the constraints between the foot-link and the ground, and minimizing the energy consumption. However, the efficacy is validated through the simulation only. To practically implement such a control, it is noticed that the dynamics of the actuators, which provide mechanical torques to the biped links must be considered. The

inclusion of the actuator dynamics not only makes control design more challenging, but also invites problems for the system modeling, which complicates the stability analysis. To be more specific, the dynamics of the overall control system, rather than a 4-dimensional nonlinear model in the simulation, has to be remodeled and extended to the one of the higher dimensionality owing to the presence of the selected actuators. As the induced effect, the constraints between the biped feet and the ground are not determined only by the state of the biped any more, which turn out to be influenced also by the actuators in a certain fashion. As a result, the control design faces more problems and the previously developed GA-based PD control in Chapter 4 has to be revisited. Stability of such a practical system (i.e., the biped is balanced by certain selected actuators, which are regulated by the developed compensators), is not trivial and thus has to be re-examined.

Based upon the above discussion, in this chapter, the constrained balance control proposed in Chapter 4 is extended to make the physical implementation feasible. The dynamics of the selected actuators is accounted for in the design of the balancing strategy. Experiments are carried out to demonstrate the efficacy of the controller. The systematic work including system modeling, control design, and stability analysis is presented in detail.

5.2 Method

5.2.1 Model Description

Let's first recall the standing biped model described in Figure 2.1, which is represented by an inverted double pendulum, plus a rigid foot link. The control torques are supposed to be applied at the joints 1 and 2 to maintain the biped at the upright posture without falling or resort to taking a step. According to this model, a real prototype has been designed and made, from which the estimated resultant model parameters are listed in Table 5.1.

A PD control strategy has been proposed to stabilize the standing biped in Chapter 4. The outputs of both PD compensators are two generalized torques satisfying the Euler-Lagrange principle as shown in Equation (2.1). To produce and implement real torques, one needs actuators, which may be electric, hydraulic or pneumatic. In this chapter, considering that the motor dynamics has been well understood, the control torques injected to the biped joints are supposed to be provided by two brushed DC motors anchored to the biped joints. Generally the differential equation for the armature circuit of a DC motor can be expressed as

$$L\dot{i}_a + Ri_a = V - V_b, \quad (5.1)$$

where i_a is the armature current, V is the armature voltage and V_b stands for the voltage generated by the back electro magnetic force. Given the motor speed constant

K_v and the rotor position θ_m , V_b usually can be estimated by following that

$$V_b = \frac{1}{K_v} \dot{\theta}_m. \quad (5.2)$$

It is worthy to note that in most applications, the DC motor output shaft is in series with a gearhead of a given gear reduction r , and the load is connected to the output shaft of the gearhead. Ideally, the load position θ_l is then related to θ_m by $\theta_m = r\theta_l$. Referring to the system with this scenario, the governing motion equation can be yielded from Newton's moment balance equation in the form that:

$$J_{eff}\ddot{\theta}_m + B_{eff}\dot{\theta}_m = \tau_m - \tau_l/r, \quad (5.3)$$

where J_{eff} is the resultant inertia reflected at the motor output shaft, which is the summation of the rotor inertia and the equivalent gearhead inertia at the motor side. B_{eff} is the resultant mechanical damping at the motor shaft. τ_m and τ_l denote the motor torque and the load torque respectively. Provided with the torque constant

Table 5.1: Nominal values of the biped model parameters.

Parameters	Values ^a	Parameters	Values	Parameters	Values
m_1	1.3154	l_{c1}	0.1527	L_a	0.0762
m_2	0.9752	l_{c2}	0.1165	L_b	0.1118
m_f	2.1818	I_1	0.0095	L_c	0.0198
l_1	0.2381	I_2	0.0034	μ	0.593
l_2	0.1873	L_f	0.2032	g	9.80

^aAll the values listed in this table are the ones measured without attached motors.

K_m , typically the torque developed by the motor can be derived via

$$\tau_m = K_m i_a. \quad (5.4)$$

Equivalently, if looking at the end of the gearhead, the equation of motion becomes

$$r^2 J_{eff} \ddot{\theta}_l + r^2 B_{eff} \dot{\theta}_l = r \tau_m - \tau_l. \quad (5.5)$$

Regarding the fact that the biped model has two rigid links apart from the foot link, here the subscripts 1 and 2 are added to the system states and parameters to distinguish the quantities related to the lower link from those associated with the upper link. Such a nomenclature coincides with the marks shown in Figure 2.1. For $i = 1, 2$, defining the error between the desired and the actual angular positions of each biped link at the time instance t as $e_i(t) = \theta_{di}(t) - \theta_i(t)$, the voltage signals V applied at the armature side as displayed in Equation (5.1) in both motors are correspondingly generated by two PID controllers following such an algorithm:

$$V_i(t) = K_{Pi} e_i(t) + K_{Ii} \int_0^t e_i(t) dt + K_{Di} \dot{e}_i(t), \quad i = 1, 2. \quad (5.6)$$

Here the integral term is intentionally incorporated not only to reduce the steady state error, but also to attenuate the oscillations caused by the differential action. Accordingly, for mathematical modeling, two artificial states specifying the integrals of position error signals, i.e., $a_i = \int_0^t e_i(t) dt$ with $i = 1, 2$ have been incorporated into the overall system states. Hence by defining the state vector as

$\mathbf{q} = [q_1; q_2; \dots; q_8] = [\theta_1; \theta_2; \dot{\theta}_1; \dot{\theta}_2; a_1; a_2; i_{a1}; i_{a2}]$, the overall system equations of motion can be derived in an 8-dimensional space via combining Equations (2.1), (5.1)-(5.4), and (5.6) in the form of

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_8 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_1 & \mathbf{0} & \mathbf{M}_2 \\ -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_3 & \mathbf{M}_4 & \mathbf{M}_5 & \mathbf{M}_6 \end{bmatrix}_{8 \times 8} \times \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_8 \end{bmatrix} + \mathbf{N}, \quad (5.7)$$

where $\mathbf{0}$ is a squared zero matrix of size 2, $\mathbf{I}$ is an unit matrix also of size 2. For other block matrices, they can be expressed as

$$\begin{aligned} \mathbf{M}_1 &= \begin{bmatrix} \alpha + \beta \cos q_2 + \eta_{g1} r_1^2 J_{eq1} & \delta + \frac{\beta}{2} \cos q_2 \\ \delta + \frac{\beta}{2} \cos q_2 & \delta + \eta_{g2} r_2^2 J_{eq2} \end{bmatrix}^{-1} \times \begin{bmatrix} -\eta_{g1} r_1^2 B_{eq1} + \frac{\beta}{2} \dot{q}_2 \sin q_2 & \frac{\beta}{2} (\dot{q}_1 + \dot{q}_2) \sin q_2 \\ \frac{\beta}{2} \dot{q}_2 \sin q_2 & -\eta_{g2} r_2^2 B_{eq2} \end{bmatrix}; \\ \mathbf{M}_2 &= \begin{bmatrix} \alpha + \beta \cos q_2 + \eta_{g1} r_1^2 J_{eq1} & \delta + \frac{\beta}{2} \cos q_2 \\ \delta + \frac{\beta}{2} \cos q_2 & \delta + \eta_{g2} r_2^2 J_{eq2} \end{bmatrix}^{-1} \times \begin{bmatrix} \eta_{g1} \eta_{m1} r_1 K_{m1} & 0 \\ 0 & \eta_{g2} \eta_{m2} r_2 K_{m2} \end{bmatrix}; \\ \mathbf{M}_3 &= \begin{bmatrix} -K_{P1}/L_1 & 0 \\ 0 & -K_{P2}/L_2 \end{bmatrix}; \\ \mathbf{M}_4 &= \begin{bmatrix} -(K_{D1} + r_1/K_{v1})/L_1 & 0 \\ 0 & -(K_{D2} + r_2/K_{v2})/L_2 \end{bmatrix}; \\ \mathbf{M}_5 &= \begin{bmatrix} K_{I1}/L_1 & 0 \\ 0 & K_{I2}/L_2 \end{bmatrix}; \\ \mathbf{M}_6 &= \begin{bmatrix} -R_1/L_1 & 0 \\ 0 & -R_2/L_2 \end{bmatrix}; \\ \mathbf{N} &= \begin{bmatrix} 0 & 0 & \mathbf{N}_1 & -\theta_{d1} & -\theta_{d2} & 0 & 0 \end{bmatrix}^T; \end{aligned}$$

$$\mathbf{N}_1 = \begin{bmatrix} \alpha + \beta \cos q_2 + \eta_{g1} r_1^2 J_{eq1} & \delta + \frac{\beta}{2} \cos q_2 \\ \delta + \frac{\beta}{2} \cos q_2 & \delta + \eta_{g2} r_2^2 J_{eq2} \end{bmatrix}^{-1} \times \begin{bmatrix} -(m_1 l_{c1} + m_2 l_1) g_1 - m_2 l_{c2} g \sin(q_1 + q_2) \\ -m_2 l_{c2} g \sin(q_1 + q_2) \end{bmatrix}.$$

For $i = 1$ and 2 , η_{gi} , η_{mi} signify the individual maximum gearhead efficiency and maximum motor efficiency.

Apparently, one can see from Equation (5.7) that the angular velocities of both the lower link and the upper link are coupled with the armature currents. As a consequence, it can be found that the three constraint variables: F_{gx} , F_{gy} and x_{COP} as shown in Equations (2.2)-(2.4), are affected as well by the armature currents, indicating that the presence of the motors not only complicates the overall system, but also obstructs the control design for the constrained system. Such a scenery can be further perceived from designing the prototype since the invited structural asymmetry in the frontal plane should be compensated by additional offset blocks. Moreover, the actuators' presence should be accounted for a new mass distribution of the biped model. Namely, instead of substituting the values listed in Table. 5.1 into Equation (5.7) directly, the nominal values of the corresponding parameters should be revisited combined with the characteristics of the selected actuators. Apart from all these concerns, in the experiment the selected two motors are both equipped with a built-in planetary gearhead. Due to the limited experimental resolution, selection of such actuators is a trade-off between geometric size, output power and overall budget, which also complicates the dynamics of the system by introducing more backlash and friction.

5.2.2 GA-based Joint Control

Since the design objective is developing a set of control gains, with which the selected actuators are capable of balancing the biped around the upright position without foot motion at a minimum level in terms of energy consumption, a cost function J was defined in the following form:

$$J = \frac{1}{2} \int_0^{t_f} C_1 \boldsymbol{\tau}^T \boldsymbol{\tau} dt + \frac{1}{2} \left(C_2 \mathbf{e}(t_f)^T \mathbf{e}(t_f) + C_3 \dot{\mathbf{e}}(t_f)^T \dot{\mathbf{e}}(t_f) \right) + \int_0^{t_f} C_{constraint} dt, \quad (5.8)$$

where $\mathbf{e}(t_f) = [e_1(t) \ e_2(t)]^T|_{t=t_f}$ is the error signal at the final time instance, $\dot{\mathbf{e}}(t_f) = [\dot{e}_1(t) \ \dot{e}_2(t)]^T|_{t=t_f}$ correspondingly stands for the changing rate of the error at t_f .

$C_{constraint}$ has the similar expression as the one shown in Equation (4.2) in the form

$$\text{that } C_{constraint} = \begin{cases} 0 & \text{if the constraints are satisfied} \\ C_4 & \text{if the constraints are violated} \end{cases}. \quad \text{The first integral of the}$$

control effort associated with the weight C_1 is the cost representation in the transitory regime. The weighted squared position errors and velocity errors of two biped links with respect to their desired states at final time instance represent the cost in the permanent regime. Figure 5.1 demonstrates the block diagram of the GA-based joint control for the standing bipedal robot. Although it is different from the case displayed in Figure 4.1, where the outputs of two PD controllers are directly taken as the control torques, the work flow of the GA can still be referred to Section 4.2.

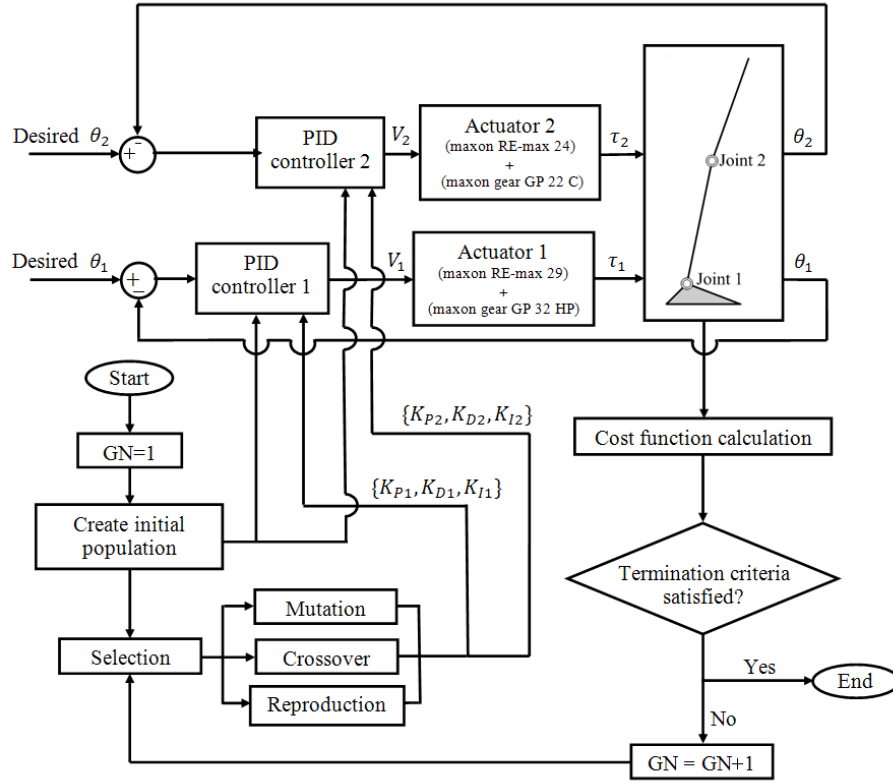


Figure 5.1: Block diagram of GA-based PID control.

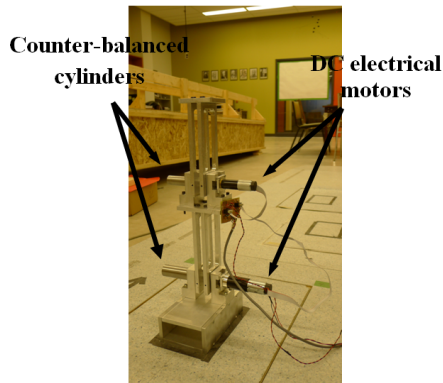
5.3 Results

5.3.1 Experimental Setup

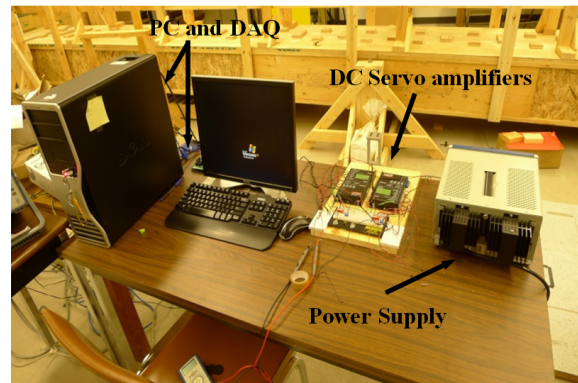
The experimental setup of balancing a standing biped is displayed by Figure 5.2. Being formed by a mechanism of two degrees of freedom (DOFs) as shown in Figure 2.1, which is in unilateral contact with the ground, the prototype of the bipedal robot has been designed to possess three links, i.e., the foot, the leg, and the torso, and they were connected by the ankle and hip joints in the sagittal plane as shown in Figure 5.2(A). Two geared-brushed DC electrical motors equipped with two encoders on its

back-side shafts were selected to actuate the ankle and hip joints. Having known that both motors are from Maxon motor Co., the one at the ankle joint is of RE-max 29, of which the output shaft is serialized by a planetary gearhead of GP 32 HP, and a quadrature encoder of 1000 count per revolution has been installed on the back-side shaft; while the motor at the hip joint is of RE-max 24 with a planetary gearhead of GP 22 C cascade connected on its output shaft, and similarly a quadrature encoder of 512 count per revolution has been planted on the back-side shaft of the motor. Table. 5.2 provides most of the characteristics of two selected actuators.

Furthermore, from Figure 5.2(B) one can see that both motors in the experiment communicated with a PC of Dell Precision Intel (R) Core (TM) 2 Quad CPU 2.40 GHz, 3.25 GB of RAM through a data acquisition board, Quanser DAQ, 16 chan-



(A)



(B)

Figure 5.2: Experimental setup: (A) the prototype of a one legged robot to be balanced via two electric actuators attached at the ankle and hip joints with (B) the hardware facilities: PC, DAQ, servo amplifiers, and power supply.

nels I/O, which served as the interface between the robot and the PC. Specifically speaking, at each time instance the actual joint angles θ_1 and θ_2 sensed by the encoders were first sent to the PC, then the differences between the desired joint angles θ_{d1} , θ_{d2} and the sensed ones were taken as the inputs of the controllers. Following that, two 4-Q-DC servo amplifiers were incorporated to send the control outputs back to the motors. All calculations of the control signals were performed in QuaRC 2, SIMULINK, MATLAB R2008b with a fixed sample time 0.002s. It is noteworthy that since the actuators were installed on one side of the robot, as mentioned in the previous section, the balance of the robot in the sagittal plane should be compromised. Therefore, two counter-balanced mild-steel cylinders were attached on the other side of the robot, which have been pinpointed in Figure 5.2(A). The mass and geometry

Table 5.2: Parameters of the actuators.

Symbols	Parameters	Nominal Values			
		maxon RE-max 29	maxon RE-max 24	maxon gear GP 32 HP	maxon gear GP 22 C
m_{motor}	weight of motor [g]	161	71		
R	terminal resistance [Ω]	5.29	20.2		
L	terminal inductance [mH]	0.454	0.952		
K_m	torque constant [mNm/A]	39.4	37.4		
K_v	speed constant [rpm/V]	242.0	255.0	N/A	N/A
J_m	rotor inertia [gcm ²]	13.2	4.11		
$V_{nominal}$	nominal voltage [V]	36.0	36.0		
P	assigned output power [W]	22.0	11.0		
η_m	max. efficiency at nominal voltage (%)	88	84		
m_{gear}	weight of gearhead [g]			249	81
r	gear reduction			318:1	370:1
J_g	inertia [gcm ²]	N/A	N/A	0.7	0.4
η_g	max. efficiency (%)			60	49
E_θ	average backlash no load [°]			1	2

parameters of the equipped prototype were re-identified based on the nominal values listed in Table 5.2.

5.3.2 Experiment with the GA-based PID control

To obtain the PID gains which can fulfill the control design objective, the convergence test was conducted first. Having in mind that stabilizing the biped at the equilibrium posture is the primary target of the proposed controllers, the weighting coefficients were then selected as $C_1 = 1$, $C_2 = 100$, $C_3 = 10$, $C_4 = 1000$. Here the parameters set for the GA have the same values as those used in Chapter 4, except that the maximum number of generations was 300. The MATLAB function `ode15s` was selected as the integral solver. GA will terminate when a maximum number of generations has been produced or the value of the cost function does not improve for 80 consecutive generations. If the value of the cost function is greater than C_4 or at $t_f=20s$ the biped is outside the upright region: $\{|\theta_1| < 0.0035, |\theta_2| < 0.0035, |\dot{\theta}_1| < 0.005, |\dot{\theta}_2| < 0.005\}$, or at any time instance the armature voltage is higher than the nominal value, then the algorithm will be considered as failed to find the solution of appropriate PID gains. The voltage constraint indeed is an unnecessary condition since usually a motor is allowed to work with a relatively high voltage at certain time instance. It is intentionally incorporated into the GA algorithm not only for the sake of safety, but more importantly, for the reason that the output power of any actuator in the real world is always limited, and once the voltage is greater than a certain extent, the current magnitude will stay at the saturation limit, as a result the output response of the actuator may lag behind the expectation. As the initial condition for

the system simulation, both angles of the robot links were set to 10 degree with zero angular speed. Selection of such an initial condition is mainly based on the concern that in the experiment a stationary robot in general can be launched with higher accuracy. In other words, during the experiment, at each initial time instance both the robot links could be manually rotated about their respective joints to a certain degree easily and precisely. While with the available facilities, calibrating initial angular speeds for the robot is extremely difficult. Figure 5.3 displays the performance of the convergence test, from which one can see that the value of the cost function J converged to 0.3660 within 300 generations.

Given the obtained PID control gains associated with the minimal J value, the evolution of two joint angles are shown in Figure 5.4(A) and 5.4(B) respectively, where the profiles of the corresponding angular speeds are displayed versus time as

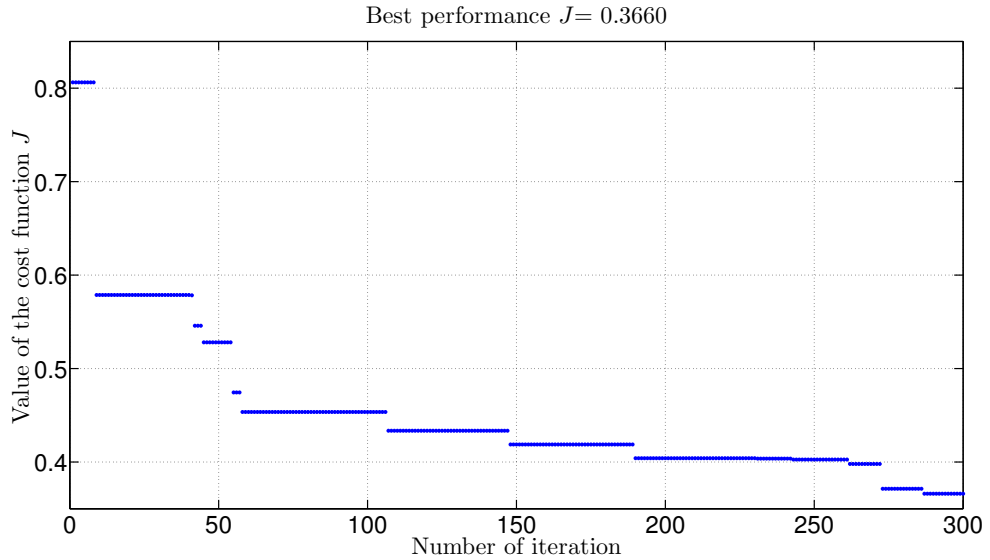
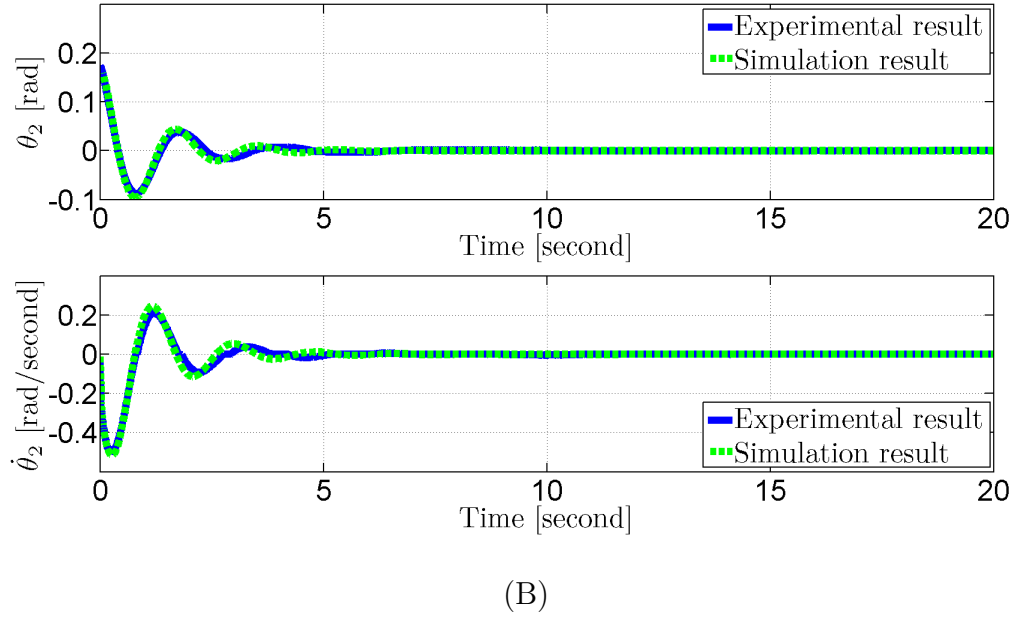
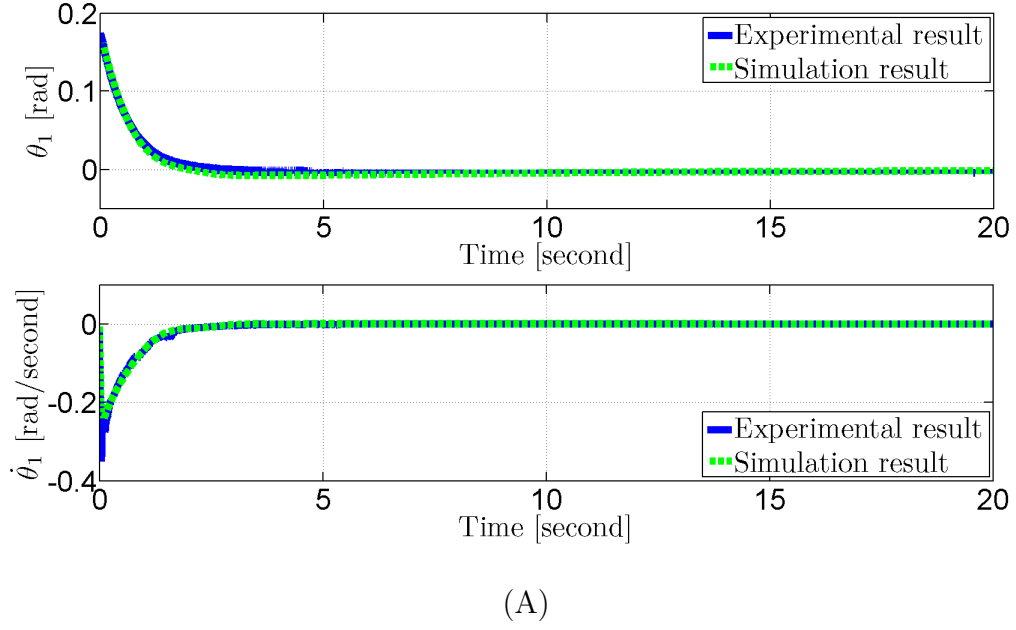
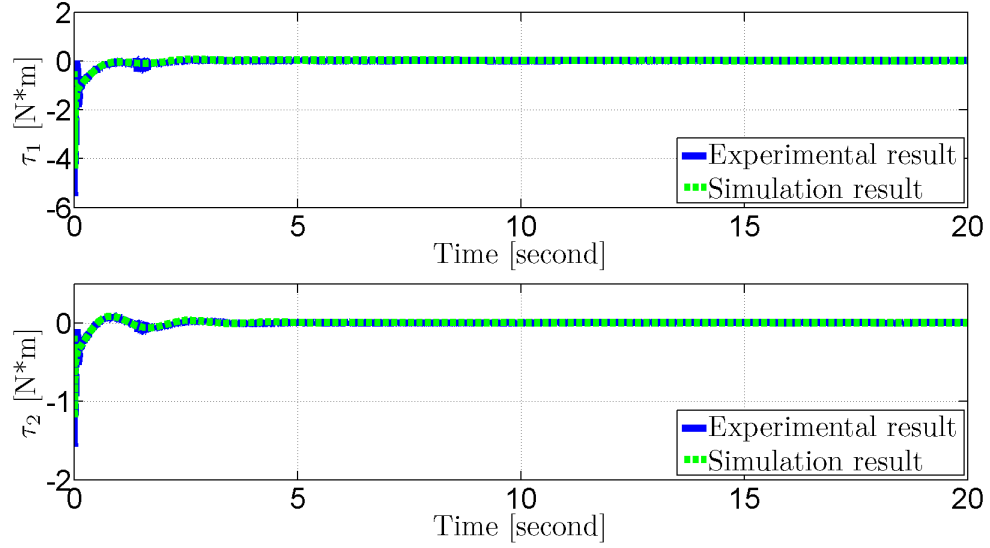


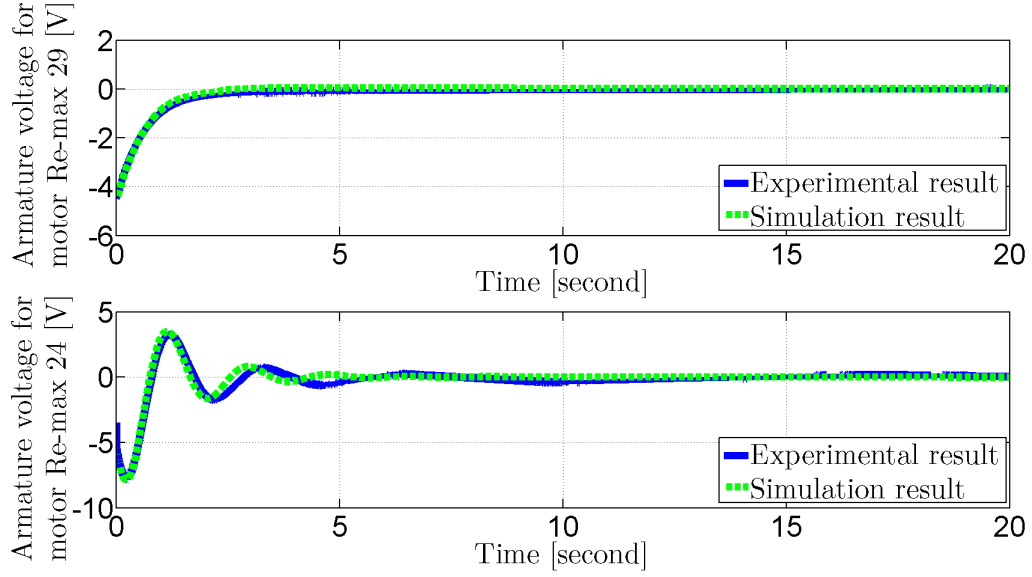
Figure 5.3: Performance of the convergence test.

well. Figure 5.4(C) shows the evolution of the control torques spent on regulating the robot, while the armature voltage records are displayed in Figure 5.4(D). To make comparisons, here the dashed lines in each subplot represent the results obtained from





(C)



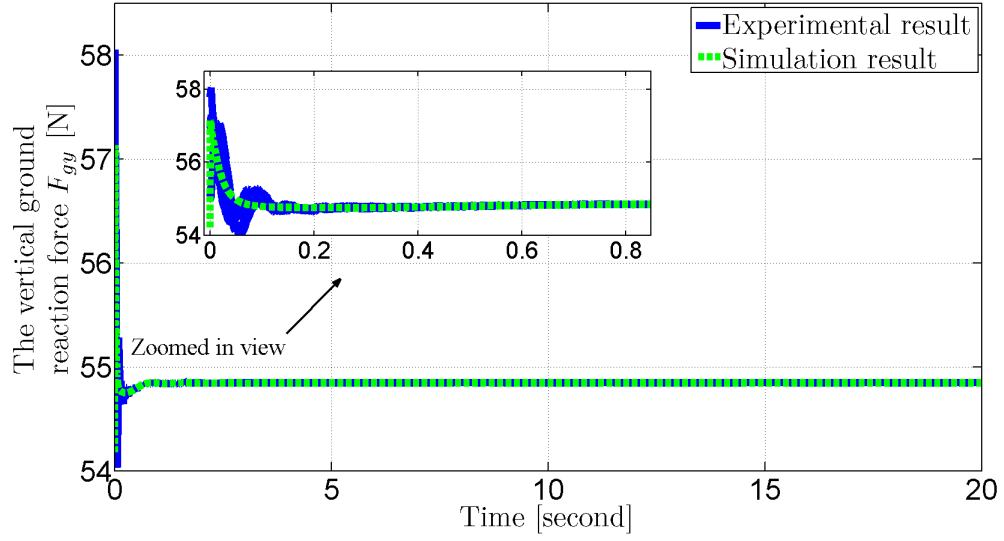
(D)

Figure 5.4: Evolution of (A)the angle and the angular speed of the lower link; (B)the angle and the angular speed of the upper link; (C)two control torques applied at the ankle joint and the hip joint respectively; and (D)two voltage signals generated by the PID controllers.

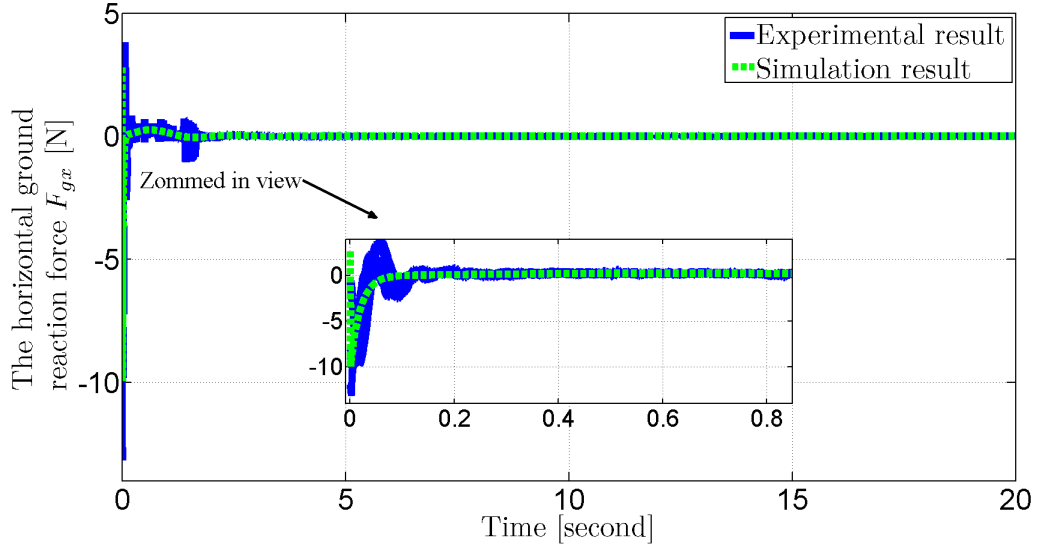
simulation, the solid lines exhibit the experimental results of ten trials, which were

randomly chosen from a large number of repeated trials with the fixed initial condition. The experimental results although were collected from different trials, it can be observed from the figure that they almost overlap each other, and to distinguish one from another is hard. Such small dissimilarities between the results of different trials imply that the experimental results are statistically reliable. It should be noticed that histories of two joint angles were plotted using the raw data, which were sensed by the corresponding encoders during the control phase. Differently, the evolution of two angular speeds were depicted using the filtered data, which were the remainder signals after noise reduction operated by a classical second-order Butterworth filter. As the inputs to the joints, the torques consumed by the robot shown in Figure 5.4(C) are two series composed by the computational results at successive sample time instances. To be specific, magnitudes of the consumed torques were derived by an off-line calculation according to Equation (5.3) based on the angle observations, plus the filtered angular speed and acceleration signals, and the filtered armature current signals. Experimental results in Figure 5.4(D) are faithful records of armature voltages in two motors, which are manifested by the raw data directly read from the output of the controller. Both the simulation and the experiment trials demonstrated that subjected to a PID controller with the gains developed from GA, the robot could be driven to the upright posture within approximately 5 seconds and then rest at the equilibrium states. Furthermore, it can be seen from all subplots that the experimental results matched the simulation results quite well, indicating the developed mathematical model with those re-identified parameters, is a good representative of the real system.

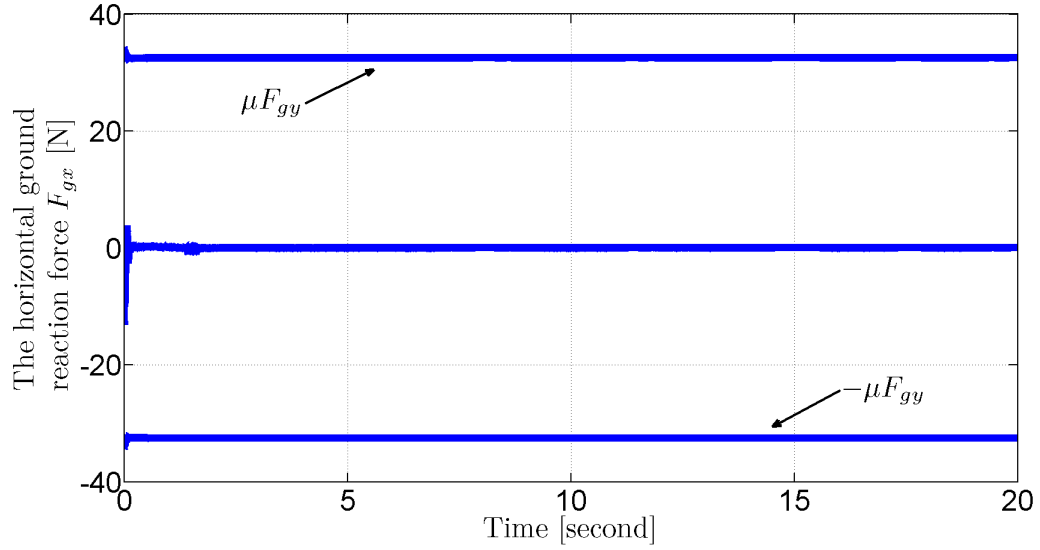
On the other hand, the trajectories from the simulation and the experiment trials of the vertical ground reaction force F_{gy} have been shown in Figure 5.5(A). The positive vertical ground reaction force F_{gy} implies the support foot was always in co-



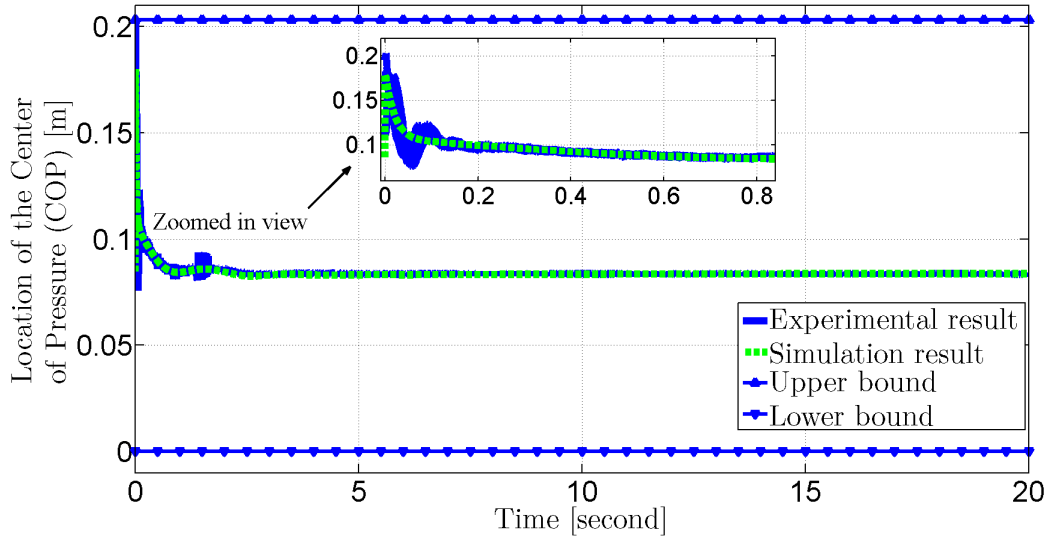
(A)



(B1)



(B2)



(C)

Figure 5.5: Evolution of (A) the vertical ground reaction force, (B1)&(B2) the horizontal ground reaction force and (C) the location of COP.

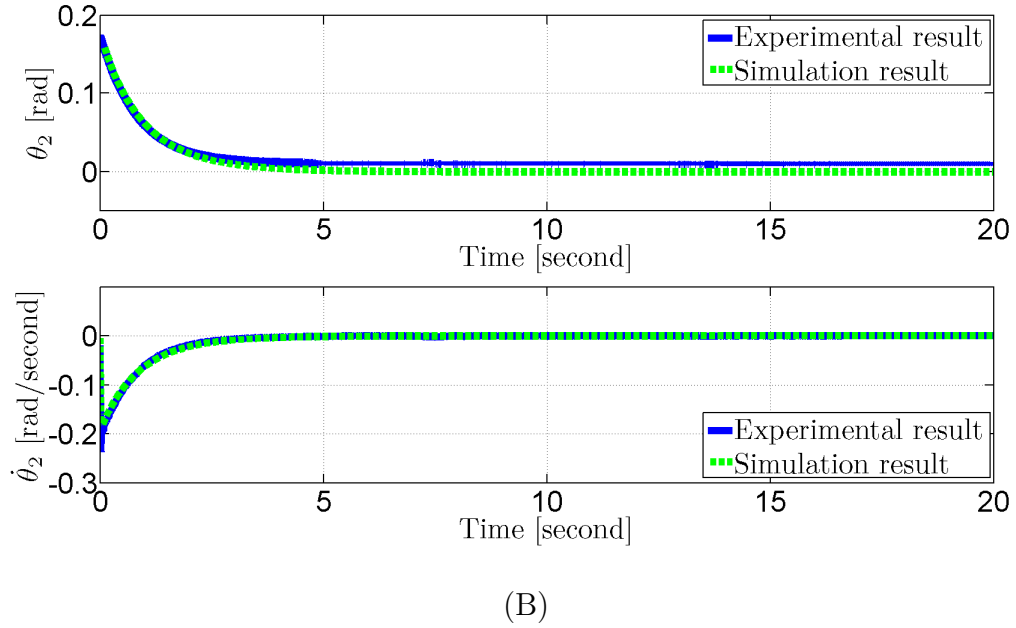
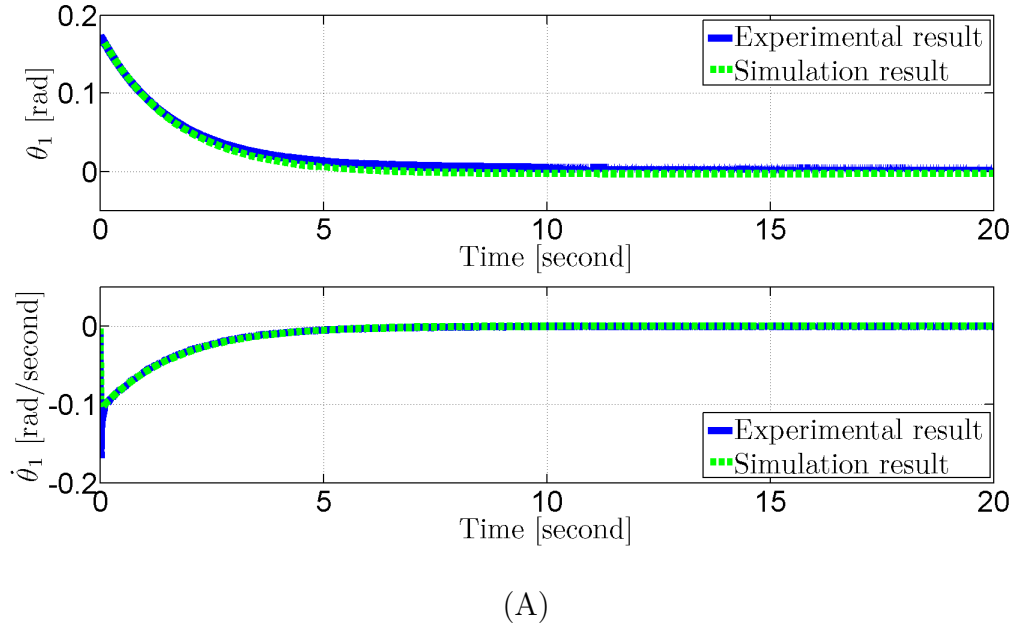
ntact with the ground. Figure 5.5(B1) is a display of the evolution of the horizontal ground reaction force F_{gx} , of which the actual lower and upper bounds according

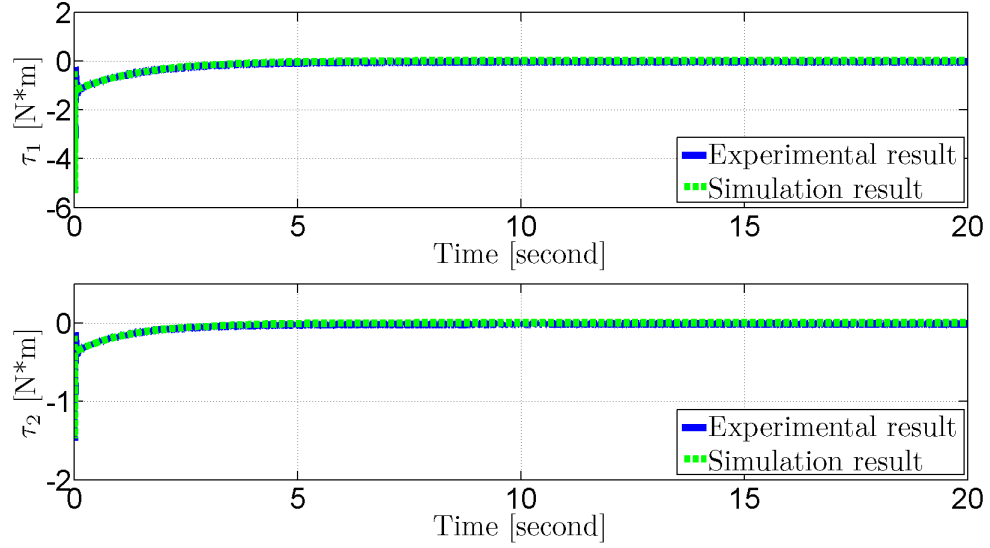
to the real F_{gy} have been shown in Figure 5.5(B2), labeled by $\pm\mu F_{gy}$ respectively. It can be found that F_{gx} always resided in the bounds of the peak static friction $(-\mu F_{gy}, \mu F_{gy})$, suggesting the foot-link did not slip during the control phase. Similarly from Figure 5.5(C) the emulated migration of COP together with the experimental COP migration during the control phase can be observed staying within its two fixed bounds, say, locations of the heel and the toe represented by the dashed lines in black. All the three constraints in the experiment were estimated based on the measurements of the joint angles and the denoised profiles of two angular speeds and accelerations, by following the Equations (2.2)-(2.4) of the system. The off-line computations were fulfilled in a way similar to the one by which the consumed torques were estimated.

5.3.3 Comparative experiment with different PID gains

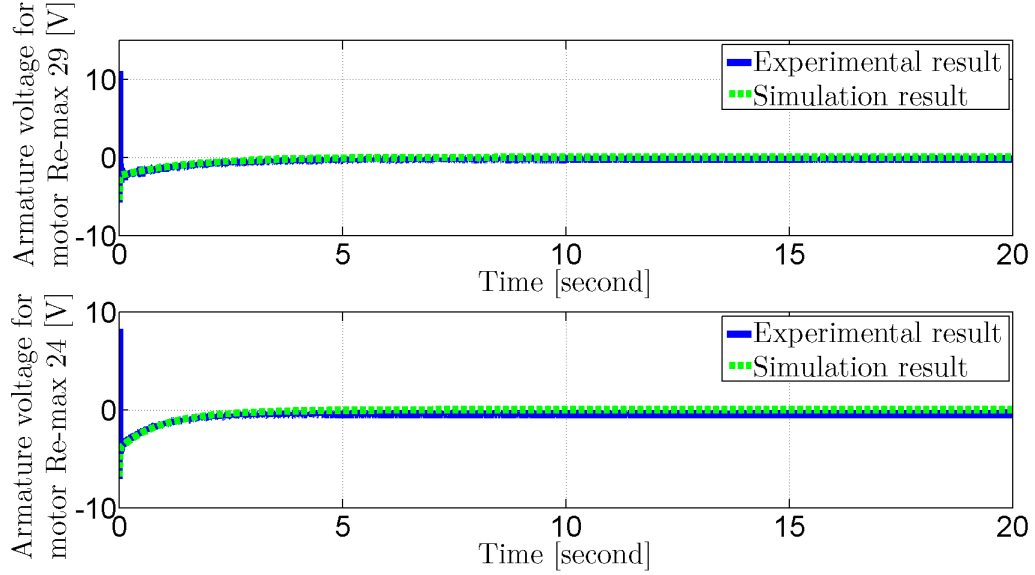
In this subsection, experimental results with different sets of PID gains will be displayed to make comparisons with those associated with the GA-based control gains. Instead of multivariable control design completed in one shot, at this stage the control gains were tuned for each joint independently, and the coupling dynamics between the lower and the upper links was regarded as disturbances to the individual systems. Specifically, to have an overdamped response, two subsystems were modeled independently based on the motor dynamics that is expressed by linear differential equations. Then, the PID gains for each subsystem were designed employing the classical pole placement method. The system response consequently can be observed via the simulation and experiment. Figure 5.6 displays the states trajectories for a typical overdamped response. Migration of the three constraints between the robot

foot and the ground is demonstrated in Figure 5.7. Again for visual presentation, the simulation results were plotted by dashed lines, while those solid lines in each subplot represent the corresponding trajectories obtained from ten repeated experimental tr-



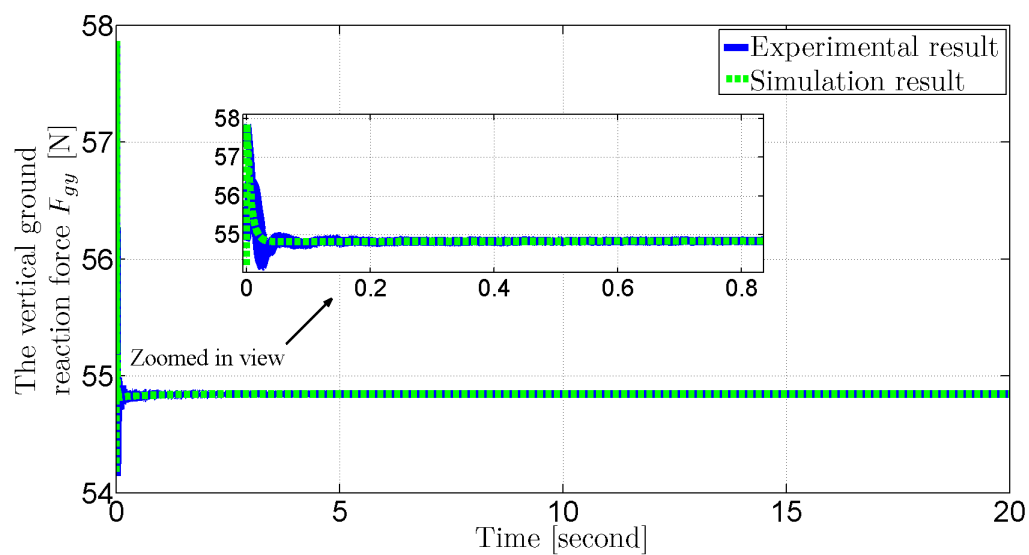


(C)

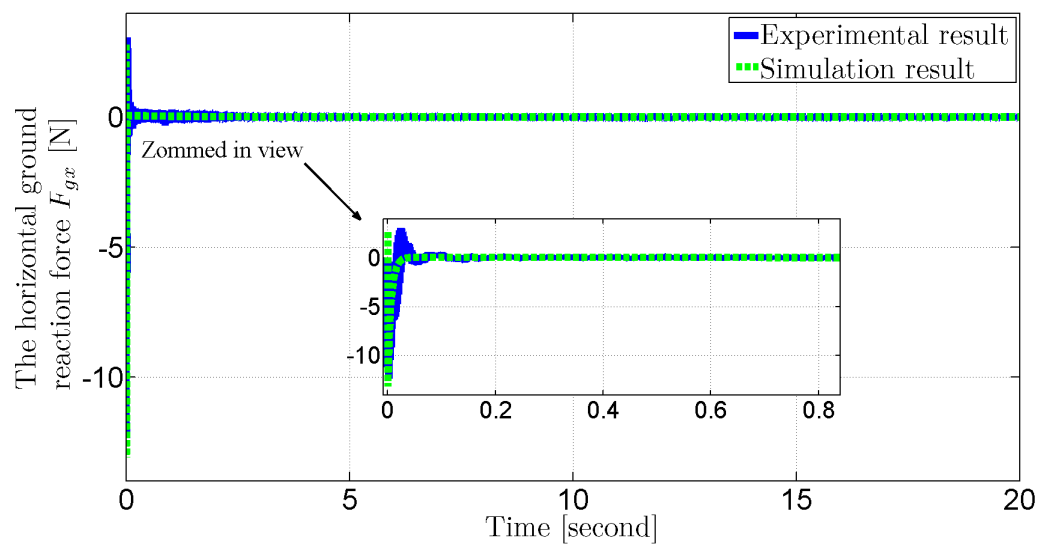


(D)

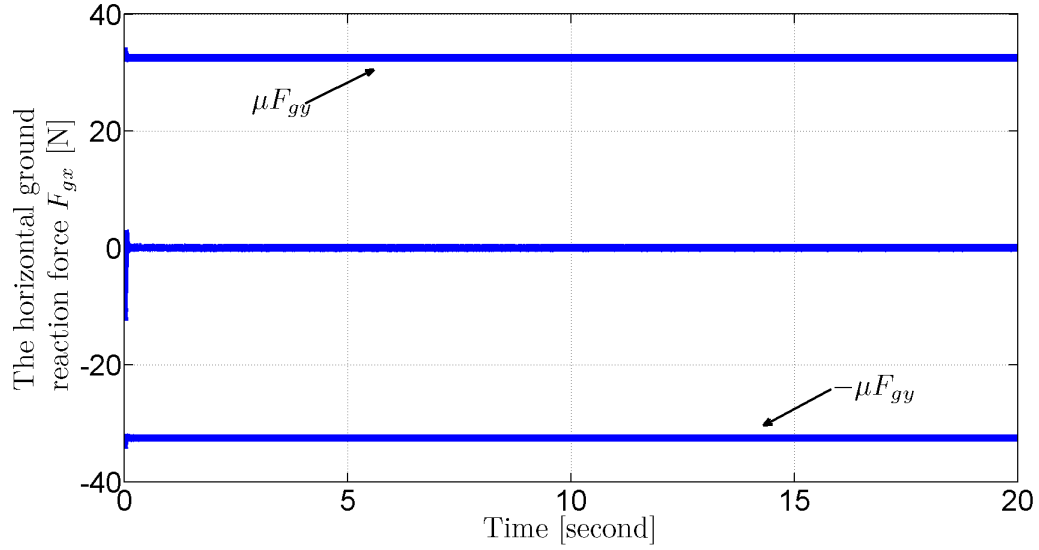
Figure 5.6: Evolution of (A)the angle and the angular speed of the lower link; (B)the angle and the angular speed of the upper link; (C)two control torques applied at the ankle joint and the hip joint respectively; and (D)two voltage signals generated by the PID controllers.



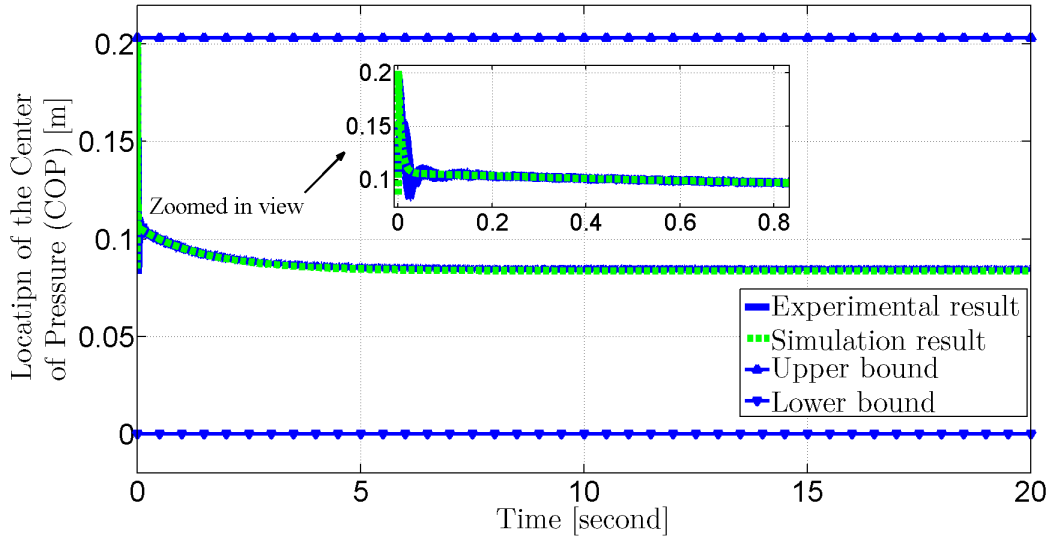
(A)



(B1)



(B2)



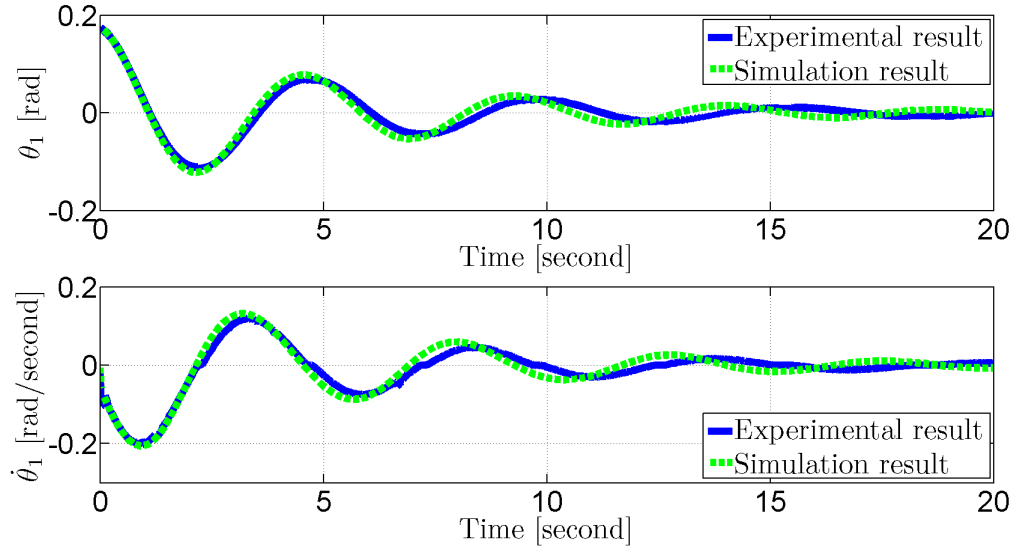
(C)

Figure 5.7: Evolution of (A) the vertical ground reaction force, (B1)&(B2) the horizontal ground reaction force and (C) the location of COP.

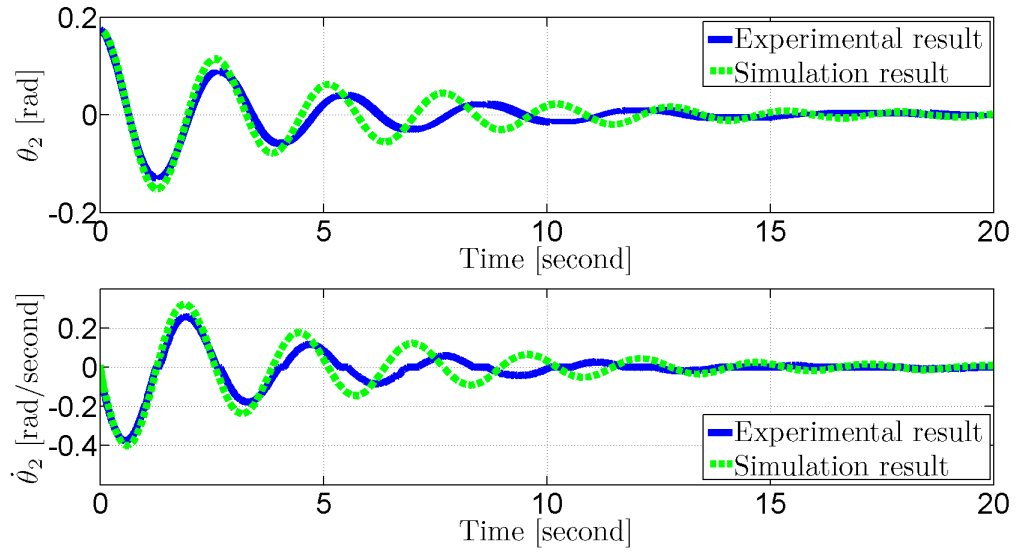
ials. The same fashion can be observed in Figures 5.8 and 5.9, which are displays of an underdamped response corresponding to two other sets of PID gains tuned through

the pole placement approach.

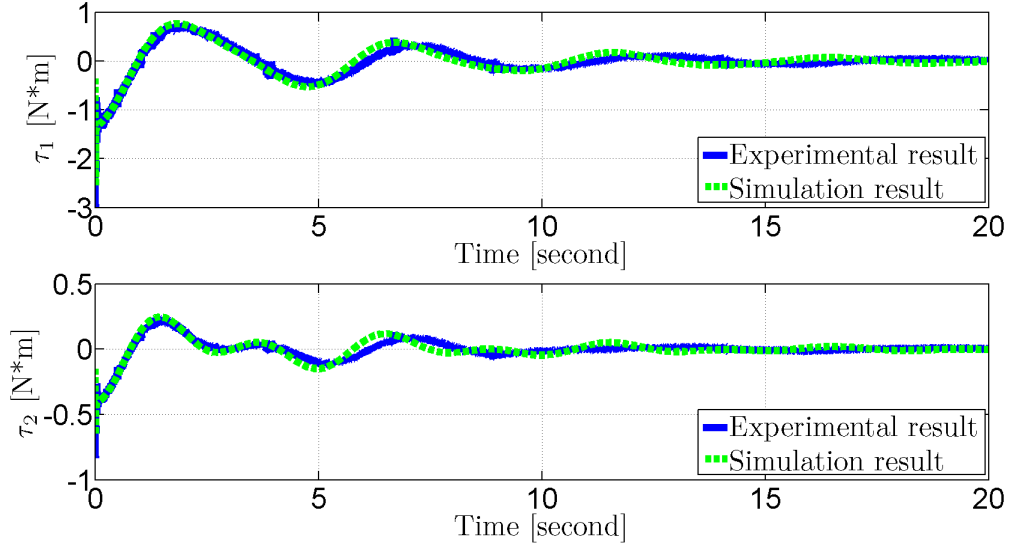
One thing should be noticed here is that, although from Figures 5.6 and 5.7 it can be found the robot could be regulated to the equilibrium without foot motion as well



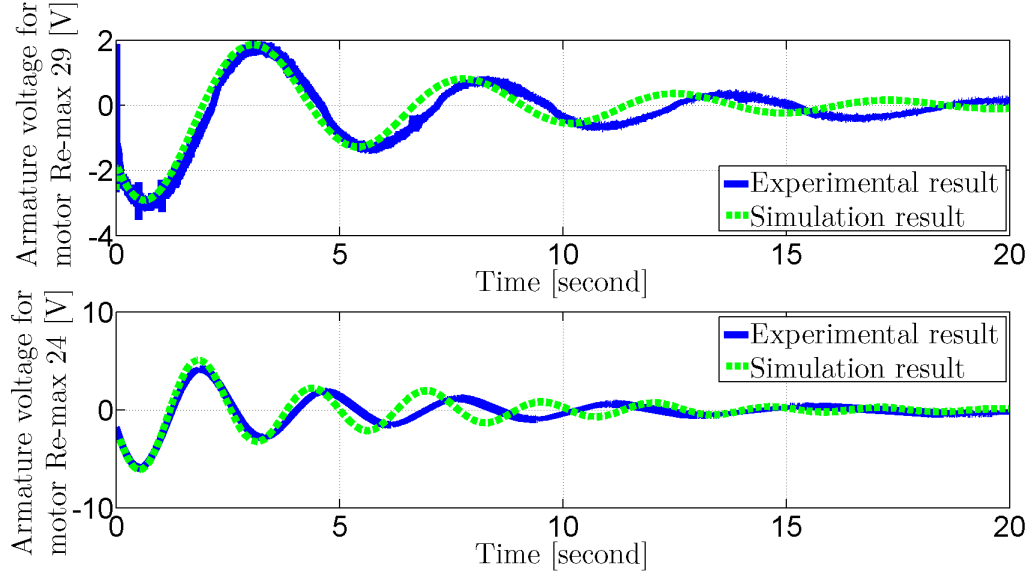
(A)



(B)

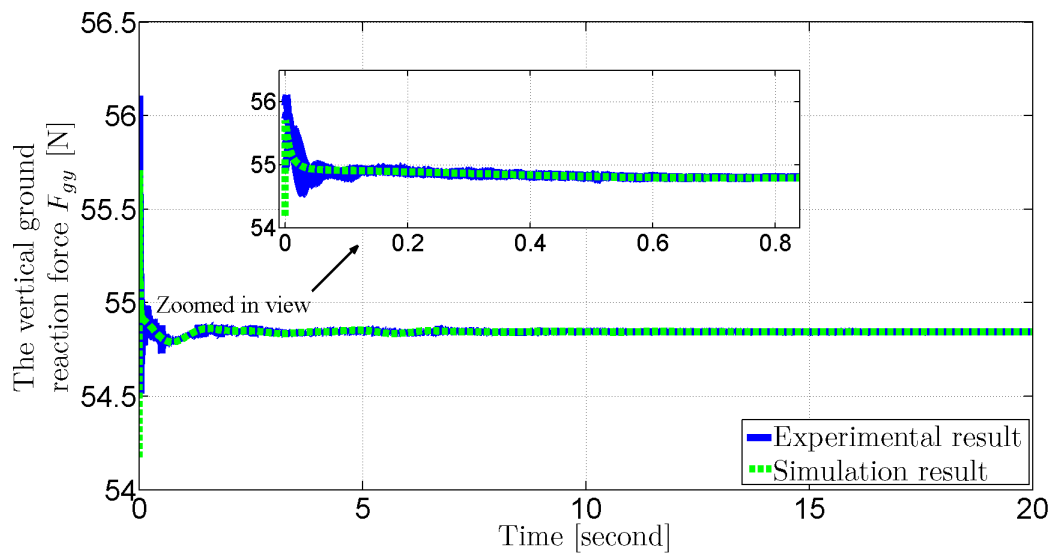


(C)

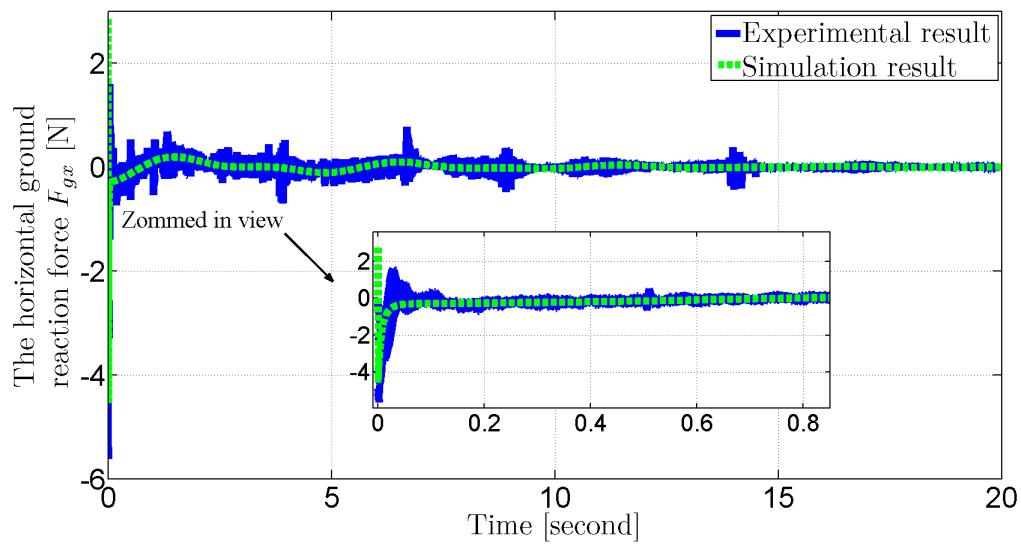


(D)

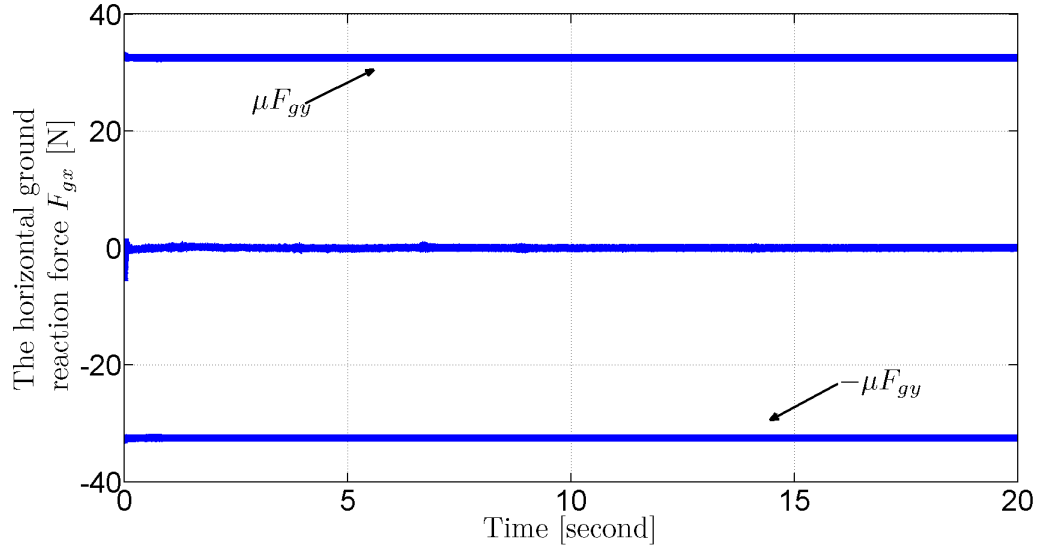
Figure 5.8: Evolution of (A)the angle and the angular speed of the lower link; (B)the angle and the angular speed of the upper link; (C)two control torques applied at the ankle joint and the hip joint respectively; and (D)two voltage signals generated by the PID controllers.



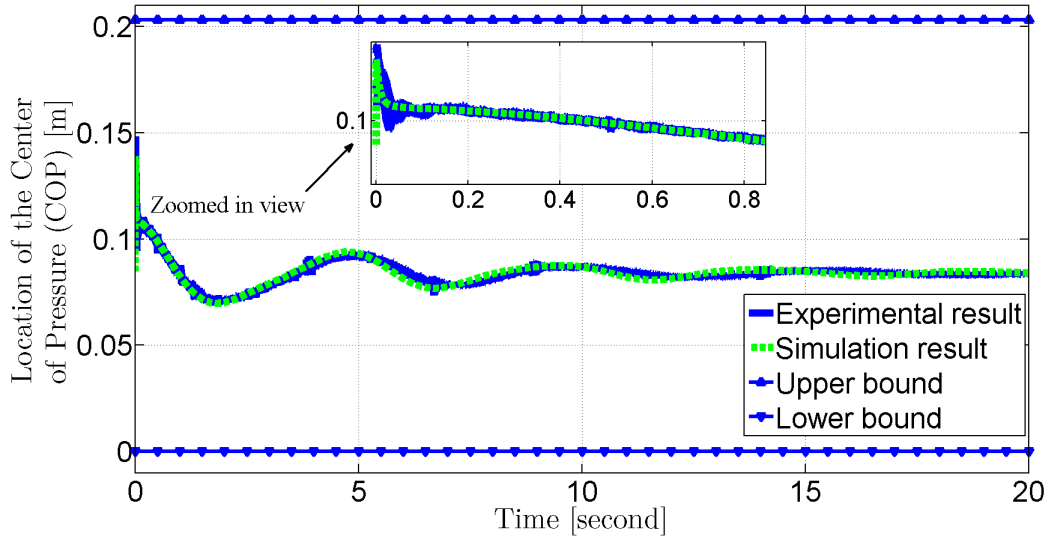
(A)



(B1)



(B2)



(C)

Figure 5.9: Evolution of (A) the vertical ground reaction force, (B1)&(B2) the horizontal ground reaction force and (C) the location of COP.

by the PID gains tuned from the pole placement method, there is no guarantee the constraints can always be satisfied using this method. Derivation of these two sets of

gains was achieved through trial-and-error following no specific rule on the transient performance but the system should settle down to the steady state within 20 seconds. The other two sets of control gains leading to the underdamped response were also obtained obeying the same manner. Besides, one may note that the response differences between the simulation results and the experimental results are more apparent in Figure 5.8 compared with ones in Figure 5.4 and Figure 5.6. This was caused by the fact that the system nonlinearity in terms of backlash hasn't been modeled in the simulation blocks. Basically the backlash from the gearheads and the coupling components between the robot links, and the uncertainties in the system mass parameters and the friction through modeling, are two main sources of the differences between the simulation and the experiment. However, the backlash becomes a dominant factor when the robot possesses an underdamped response as shown in Figure 5.8. In the experiments, the differences induced by the backlash accumulated during the regulation since the robot behaved in an oscillatory motion. As time went by, the differences became more obvious in both senses of magnitude and phase shift. Such differences can be alleviated by resort to adding some nonlinear compensators properly to cope with the backlash. For example, from the viewpoint of control design, a deadband applied to the proportional and integral action, may provide adequate performance in practice. However, it is problematic to set the amount of deadband in a system, and a deadband usually reduces the effective loop gain to zero at small errors, which may cause the controlled motor becoming marginally stable.

Moreover, the numerical values of the index function J for all three cases: the system associated with the GA-based gains, the overdamped system and the under-

damped system with the gains tuned from pole placement method, have been listed in Table 5.3, where for each case, the number shown in the row of experimental result is an average of computational results of ten randomly selected experimental trials. Observations of these selected trials have formerly been demonstrated for each case. The J values of the individual experiment trials are shown in Figure 5.10, which along with Table 5.3, validate that with the GA-based control gains the robot consumes less energy compared with the cases equipped with other PID gains.

Table 5.3: Numerical values of the index J under different cases.

	Case - I ^a	Case - II ^b	Case - III ^c
Simulation result	0.3660	0.7422	1.1208
Experimental result	0.3742	0.7206	0.9734

^aCase - I denotes the system equipped with the GA-based PID gains.

^bCase - II denotes the system with the PID gains leading to an overdamped response.

^cCase - III denotes the system with the control gains such that it features an underdamped response.

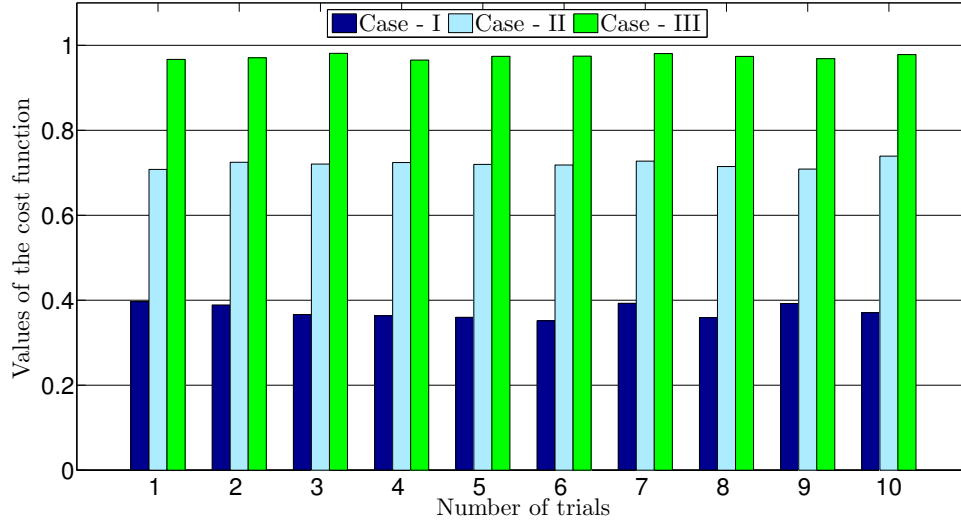


Figure 5.10: J values under different cases.

5.4 Stability Analysis

To investigate the stability of the one legged robot subjected to the proposed motor joint control, the concept of Lyapunov exponents is adopted here to deal with the following two issues: judging the system capability of rejecting disturbances imposed on the states; identifying the shape and size of the region in the state space within which the stability of the system is guaranteed. Taking into consideration that the system response from simulation matches quite well with the experimental observation when the motors were driven by the GA-based PID controller, the system Lyapunov exponents were estimated from the established mathematical model shown in Equation (5.7), following the algorithm developed in (Wolf, Swift, Swinney, and Vastano, 1985). Figure 5.11 shows the evolution of all exponents of the actuated

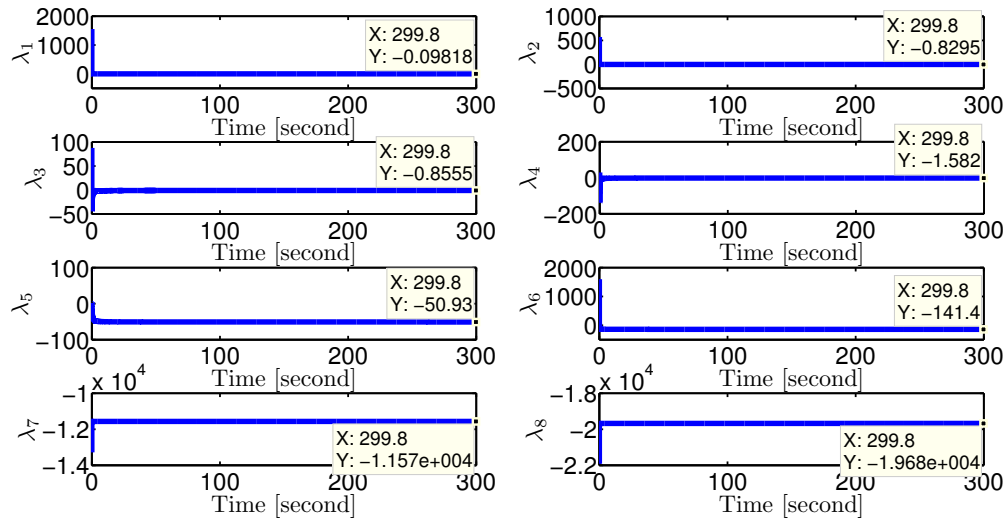


Figure 5.11: Evolution of the system Lyapunov exponents. The converged value of each exponent has been shown as the Y reading of the datatip in the corresponding subplot, while the X reading reflects the time the system has progressed over.

system, from which it can be observed that all the system exponents converge to the negative constants presented as the datatips shown in the individual subplots, suggesting that the actuated robot combined with the GA-based PID controller is exponentially stable about the equilibrium state.

To determine the stability region, again the approach developed by Nusse and Yorke (1994) was employed. Here rather than finding the region in the original 8-dimensional state space, by setting the other states to zeros only the following selected area in a 2-dimensional subspace constituted by θ_1 and θ_2 has been examined:

$$\Gamma = \{\theta_1 \in [-90^\circ, 90^\circ], \theta_2 \in [-90^\circ, 90^\circ]\}.$$

And the area was divided into grid boxes with the size of 1 degree for θ_1 and θ_2 respectively. Figure 5.12 shows the stability region determined by the system largest Lyapunov exponent (LLE), which has been reported in the literatures to dominate the divergence or convergence rate of the hyper-volume in the state space. The black points in Figure 5.12 represent the initial states from which convergent and negative exponents can be obtained using the proposed GA-based PID control gains. In this area all the LLEs are negative, of which the mean value is -0.0982 with a standard deviation less than 1.0×10^{-13} , indicating that all the states colored in black belong to the same stability region as the corresponding LLEs almost value equally. The initial states excluded from this area constitute an unstable region, where convergent LLEs cannot be obtained associated with the GA-based PID control gains. It should be emphasized again that rather than an overall scenery, Figure 5.12 is indeed a projection of the system 8-dimensional stability region on the 2-dimensional $\theta_1 - \theta_2$

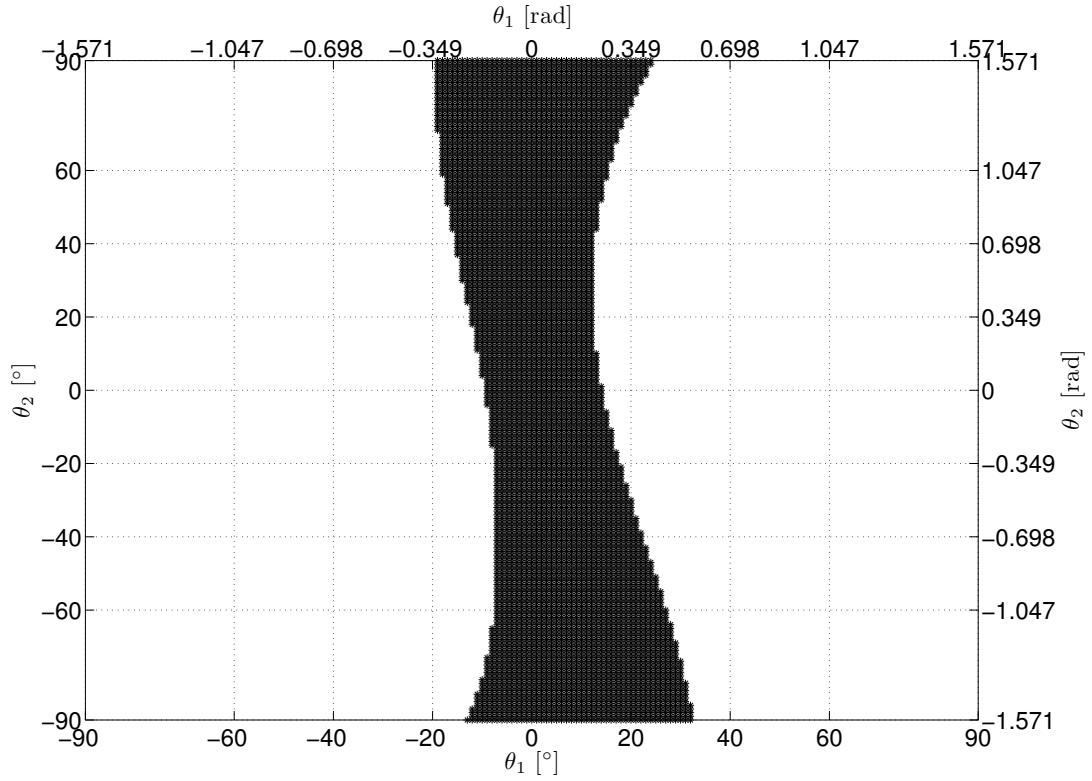


Figure 5.12: A projection of the stability region on the $\theta_1 - \theta_2$ plane of the controlled robot. The black points shown in the figure represent the initial states from which the system trajectory can be regulated to the equilibrium state (the upright posture) without foot motion.

plane.

5.5 Summary

Balance control is important for biped standing. In spite of large efforts, it is very difficult to design balance control strategies satisfying three requirements simultaneously: maintaining postural stability, improving energy efficiency and satisfying the constraints between the biped feet and the ground. To implement such a control, in-

clusion of the actuators' dynamics is necessary, which complicates the overall system, obstructs the control design, and makes the stability analysis more difficult.

In this chapter, a comprehensive study including system modeling, control design, and stability analysis, on the motor joint control of a standing bipedal robot is carried out. As an extension of the work presented in Chapter 4, the overall control system has been modeled as a bipedal robot formed by an inverted double pendulum with one additional foot-link plus the dynamics of the selected actuators for practical implementation, and the correlation between which has been established in the developed model. Instead of directly regulating the robot in computer simulations, the outputs of two PID controllers were used to provide powers to the actuators. With such an experimental protocol, the robot achieved the upright posture associated with a set of control gains developed from GA without foot motion. Comparisons of performances between the systems with different sets of PID control gains have been provided, showing that the system with the GA-based control gains was stabilized at a lower level in terms of energy consumption with respect to those with the gains developed from the pole placement method. Last but not least, the stability of the overall control system has been examined based on the concept of Lyapunov exponents.

The experiment and the simulation results shown in this chapter along with the analysis demonstrate the efficacy of the developed model both from the theoretical sense and the practical sense. As the counterpart of the design method (i.e., the genetic algorithm) used in this work, with the traditional pole placement technique, one can also come up with controllers which could drive the robot to the equilibrium

state. However the constraints on the robot foot cannot be always satisfied, not mention optimization of the energy consumption. The methodology presented here is believed to have the potential of being applied by a wider group of audience with various engineering applications that are inherently highly nonlinear, time-varying, and possibly non-smooth, in the absence of a readily available analytical Lyapunov function needed for implementation of the Lyapunov stability control. The work on the whole, has set up a framework for developing more sophisticated controllers in more complex movement for robot models with less conservative assumptions.

Chapter 6

Conclusions and Future Works

6.1 Conclusions

The contributions of this thesis come from two aspects. The first is that the energy-efficient balance control for a standing biped subject to the foot constraints has been studied via not only the simulation but also the experimentation, with a thorough stability analysis using the concept of Lyapunov exponents. The second contribution lies in the fact that two new methods of stability analysis for potentially stable engineering system based upon the concept of Lyapunov exponents from available time series have been developed. Each contribution is further addressed below.

6.1.1 Balance control and stability analysis of standing bipeds

Standing balance maintenance has been investigated in this thesis. The proposed balancing strategy for a disturbed standing biped considers three important issues

simultaneously: maintaining the postural stability, improving the energy efficiency, and meeting the constraints between the biped feet and the support surface (i.e., no lifting, no sliding, and no rolling about either the toe or the heel is allowed during the regulation phase). An energy-efficient PD controller derived from the GA, has been put forward first and applied in the simulation to a two-link biped model with an additional foot-link. The biped moves only in the sagittal plane, and the foot is assumed to be stationary but not fixed on the ground. Simulation results have shown that the proposed controller can successfully keep the biped upright during standing at a minimum level of energy consumption without foot motion. For such a control system, two different types of stability have been rigorously analyzed using the concept of Lyapunov exponents: system stability with the disturbance imposed on the initial conditions, and structural stability with the disturbance imposed on the control gains. The system ability to reject different kinds of disturbances are quantitatively examined in both the state space and the parametric manifold, based on which the impacts of different control actions within the parametric stability regions on the system performance has been revealed.

Moreover, to physically implement a constrained and energy-efficient balance control to a real bipedal robot, a systematic experimental study including system modeling, control design, and stability analysis has been presented. The established overall system model indicates that inclusion of the actuators' dynamics increases the complexity of the system. The three foot constraints in this scenario are no longer determined by the states of the bipedal robot only. Comparisons of performances between the systems with different sets of PID control gains have been provided via

both the simulations and the experiments. The energy consumption of the system with the GA-based control gains has been found to be less than of those with the gains developed from the traditional pole placement method. The work contributes to bipedal balance control from both the theoretical and practical sense, which is important in the development of bipedal walking machines.

6.1.2 Stability analysis using time-series-based Lyapunov exponents

Adequate stability analysis is crucial for safe and satisfactory performance of robotic systems. One of the classical methods of conducting Lyapunov stability analysis is using Lyapunov's stability theory, of which the key requirement for evaluating the ability of a dynamical system to maintain stable is to construct a Lyapunov function. Since no constructive and general rules were provided for the derivation of such a function, current understanding of Lyapunov's stability theory leaves stability analysis for many complicated engineering systems challenging. Application of this theory thus has been restricted. As the counterpart of the classical Lyapunov's second method, the concept of Lyapunov exponents is believed to be effective for stability analysis of deterministic nonlinear dynamical systems. Methods for calculating the exponents are constructive, which can be categorized into two big streams: the model-based method is applicable only for systems with explicit mathematical models, while the available time-series-based methods are mainly for diagnosing chaotic systems characterized by positive exponents.

In this thesis, new approaches to estimate Lyapunov exponents reliably using the

existing time-series have been proposed for stability analysis of those potentially stable systems with great complexities or implicit models. Two different types of RBF networks are needed in the proposed methods: for a system whose state variables are all practically measurable, a MIMO RBF model should be developed to accurately mimic the original dynamical system. As regards a system with partial states measurable, reconstruction of dynamics using a scalar time series should be fulfilled first, then a MISO RBF model should be tuned appropriately to approximate the embedded system attractor. Consequently in both new approaches, the Jacobian matrices can be derived based on the developed network structural information, from which the system Lyapunov exponents can be estimated. The efficacy of these new methods has been demonstrated through a constrained standing biped subjected to an LQR control, along with several different dynamical systems. The numerical values of the Lyapunov exponents of each system estimated from the newly proposed methods matched quite well with those calculated directly from the system models, even when the observations are contaminated, provided that the measurement noise is controlled under certain level. In one word, the new approaches have a great potential for stability analysis of more general nonlinear dynamical systems, especially for practical engineering systems. They take the advantages of both the model-based method and the traditional time-series-based methods in that (1) no mathematical models are required owing to the involvement of the neural network for system approximation, (2) derivation of Jacobian matrices based on the RBF network is quite straightforward, which makes the proposed method feasible to large dynamic systems, and (3) all Lyapunov exponents, regardless of the signs, can be estimated reliably.

6.2 Future Works

As one of the recommendations made for future works, an advanced biped balancing system with adaptive control needs to be developed for an unknown dynamic environment, rather than a static environment considered in this research. Stability analysis of such control schemes still remains problematic owing to the complexity of the controller. Since the concept of Lyapunov exponents has been demonstrated to be a good option for this analysis, novel control strategies based on the concept of Lyapunov exponents are supposed to be developed for balancing the constrained standing biped such that no additional stability analysis is needed.

Furthermore, methods presented in this research of estimating Lyapunov exponents for potentially stable engineering system only dealt with additive measurement noise of Gaussian white type. However, noise of various other models with different spectral characteristics and the dynamical noise generated within a system itself, are common in many dynamic systems. Investigating the effects of these noise types needs to be carried out, as well as the corresponding determination rule of how much data one needs for obtaining the accurate estimates with different noise levels.

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