

**COMPUTATION OF CORONA FIELDS ASSOCIATED
WITH UNIPOLAR HVDC TRANSMISSION LINES**

BY



MOHAMED M. ABOELSAAD

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A thesis submitted to the Faculty of Graduate Studies of
the University of Manitoba in partial fulfillment of the requirements
of the degree of

DOCTOR OF PHILOSOPHY

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ABSTRACT

Corona performance is a topic of major significance in the design and operation of overhead hvdc transmission lines. This includes such important aspects as corona power loss and possible environmental effects associated with space charge presence in the region under the lines. There exists a need to develop and improve analytical and computational tools that can be used for analyzing and assessing the performance of existing as well as proposed lines.

Previous approaches towards the ionized field solution associated with hvdc corona have usually incorporated simplifying assumptions regarding the interaction between the corona generated space charge and the resulting electric field, and are thus approximate. Two of the most commonly used simplifications are the Deutsch and Kaptzov assumptions. While the former states that only the magnitude and not the direction of the electric field intensity is affected by corona, the latter assumes that the electric field intensity at the coronating conductor surface is always constant at the onset value regardless of the applied voltage level.

In this work, analytical and computational methods, in which these assumptions are waived, are developed. The results, thus, obtained would provide the means for qualitative and quantitative assessment of the influence of such assumptions on unipolar dc corona analysis.

An improved analytical method for determination of the unipolar corona voltage-current characteristics is developed. The method is applied initially to the coaxial cylindrical configuration; then extended to the two dimensional geometry of both smooth and stranded conductors above ground. Universal charts from which the voltage-current relation can be directly obtained are presented. Modified

expressions are derived which can be used for accurate assessment of the right-of-way ground level current density profiles.

The solution of the differential equations describing the unipolar corona problem is also carried out computationally using the finite element method. A finite element based iterative algorithm has been developed and applied successfully to the coaxial cylinder as well as the single conductor above ground geometries. Different configurations and intensities of corona are considered. The influence of Kaptzov's assumption is studied through computations of the corona power loss and spacial distributions of the field quantities at the conductor surface and over the ground plane and compared with available experimental data. The computational aspects regarding the accuracy and the efficiency of the iterative algorithm are also presented. Finally, conclusions and areas for future research are outlined.

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LIST OF SYMBOLS

E	Electric field in the presence of space charge
E'	Electrostatic field (without space charge)
E_0	Corona onset field at the conductor surface
H	Height of the conductor above ground plane
I	Corona current per unit length of conductor
J	Current density
k	Ionic mobility
m	Surface irregularity factor
P	Popkov's empirical constant
r	Radial coordinate of any point
r_g	Radius of outer cylinder
r_c	Radius of conductor
V	Voltage applied to conductor
V_0	Corona onset voltage
x,y	Coordinates
Y	Nondimensional Current
ϵ_0	Permittivity of free space
Φ	Potential in the presence of space charge
ϕ	Space-charge-free potential
ρ	Charge density at any point
ρ_c	Charge density at the conductor
ρ_g	Charge density at the ground

CHAPTER 1

INTRODUCTION

High voltage direct current transmission possesses many operational and economical advantages over conventional ac for bulk transport of electric power over long distances. Consequently, the various problems associated with it are now being studied extensively. One of the important problems in the design of hvdc transmission lines is their corona performance. This is not only of economic importance due to power losses resulting from occurrence of corona on the line conductors, but is also associated with additional undesirable effects such as electromagnetic interference, audible noise and presence of ionic charges in the space surrounding the line. This last mentioned effect, the space charge presence in the interelectrode space, causes significant modification of the static electric field and accordingly has an important environmental impact [1,2,3,4].

Considerable experimental effort has been expended to determine the corona behaviour of hvdc lines on the basis of measurements conducted on test sections of actual lines. Because of the strong influence of weather conditions, collection of experimental data becomes an extremely tedious process and usually requires observations over a long period of time. Accordingly, evaluation of alternate line designs by experimental means becomes both costly and time consuming process. In view of this fact, theoretical as well as computational analysis can provide a useful and significant tool for the study and evaluation of alternative line designs.

Both analytical and computational analysis of the corona problem on hvdc lines has proven over the years to be a difficult task because of the nonlinear character of the differential equations describing it. Some basic assumptions are

usually introduced in order to make the solution of these equations possible. The two fundamental assumptions usually employed by previous researchers are , as referred to in literature, the Deutsch assumption and the Kaptzov assumption. The first assumption, originally introduced by Deutsch [5], states that the space charge affects only the magnitude and not the direction of the electric field. The second assumption, proposed by Kaptzov [6], is that the magnitude of the electric field at the coronating conductor surface remains constant at its onset value regardless of the level of the applied voltage. However, the validity of these assumptions as applied to the analysis of hvdc transmission lines corona has been repeatedly questioned in the literature [7-13]. In fact, it was claimed that these assumptions could have a significant effect on the magnitudes and spatial distributions of the quantities of interest; namely the ionic space charge, the electric field, and the current density in the close vicinity of the lines [7-17].

The objective of this work is towards development of analytical and computational methods, for the solution of the corona equations of unipolar hvdc lines without resorting to simplifying assumptions in the analysis. The results thus obtained; being free from these assumptions, would provide the means for qualitative as well as quantitative assessment of the influence of such assumptions on the unipolar ionized field analysis.

Solution of the unipolar corona equations, carried out in this thesis, consists of several steps. In chapter two, the different physical and mathematical aspects of corona on dc lines are presented. The nonlinear character of the governing equations due to the interaction of the space charge and field is described. Previous published approaches for the solution of the problem are reviewed and their limitations are discussed.

An investigation into the influence and significance of the simplifying assumptions usually employed in the dc ionized field analysis is presented in chapter three. An improved method for solution of the unipolar dc equations that takes into account the influence of Kaptzov's assumption is developed. Application of the method for computing the unipolar power loss is demonstrated for the coaxial as well as the single line-to-ground configurations. The method is also extended to the case of stranded conductors. The influence of Deutsch's assumption on the spacial distributions of ionized field quantities is also investigated in this chapter. Modified analytical expressions that can be used for the assessment of the right-of-way current density profiles are suggested. Results for various cases are compared with available experimental data.

The finite element method, FEM, has been recently recognized as one of the most appropriate numerical techniques for the ionized field analysis because of both its generality and flexibility[7,15,16]. A proposed computational algorithm, that is based on the finite element method, for the iterative solution of the unipolar ionized field problem is described in chapter four. The algorithm does not rely on Deutsch's assumption. Procedures for eliminating Kaptzov's assumption are incorporated in the solution. Application of the algorithm to the solution of the coaxial configuration and comparison with the known analytical solution is carried out. The numerical aspects regarding the accuracy, efficiency and convergence of the iterative procedure are studied using various initial estimates for the space charge distribution.

In chapter five, implementation of the proposed algorithm for the solution of the practical configuration of a single line above a ground plane is presented. Procedures for generating the finite element mesh and the initial space charge distribution are described. The results for the corona power loss are computed with and without Kaptzov's assumption and compared with available measured data. The

various ionized field quantities; namely the charge density, electric field and current density, both at the coronating conductor and along the ground plane are computed and compared with experimental data whenever available.

Finally in chapter 6 conclusions, recommendations and areas for future work are outlined.

CHAPTER 2

ANALYSIS OF DC IONIZED FIELDS

Since hvdc transmission lines generally operate above their corona onset voltage, considerable space charge is generated, which flows towards the ground plane, and results in the electric field being substantially modified from its static counterpart [18]. The corona phenomenon and its related effects depend on a large number of factors such as the applied voltage, line geometry, conductor surface condition and weather variables.

The influence of corona, both on the operation of the power system and on the environment through which a transmission line passes is, generally, undesirable. One of the effects, is the corona power loss which could represent a loss of a few percent of the transmitted energy in steady state operation. Corona generated space charge results also in electric induction effects produced on people and objects located in the vicinity of the lines. This effect is of concern due to its direct environmental impact. Any object located under a dc line intercepts the ion flow between the conductors and the ground. If the object is perfectly insulated from the ground, the magnitude of its potential above ground gradually increases until it approaches the equilibrium space potential or until the insulation breaks down. In practice, the potential to which the object is raised will be limited by its insulation resistance to ground [3,4]. If a grounded person touches the object, he may receive an initial shock of extremely short duration followed by a small steady current. Similar induction effects are also produced when a well-insulated person located under the line touches a grounded object. A well-grounded person standing under a dc line will also experience a steady flow of very small current [4].

In the following sections, the physical and mathematical aspects of corona on dc lines as well as a review of previous methods of theoretical analysis are presented.

2.1 Physical Characteristics of DC Corona.

By the term corona, one generally refers to the phenomenon which occur in a gaseous medium in the vicinity of conductors of small radius of curvature, subjected to high electric fields. Corona is a self sustained discharge, that is, it requires no external source of ionization for the discharge to be maintained. For geometries of practical transmission lines, the electric field in the interelectrode space is highly nonuniform and leads to the formation of corona in the overstressed region immediately surrounding the conductor.

2.1.1 Positive Corona

Corona occurring on a positive electrode is initiated by electrons that are sufficiently accelerated towards the electrode in the high field region. When an electron hits an air molecule in its path, an additional electron is liberated. The original electron as well as that newly released are also accelerated towards the electrode resulting in more ionizing collisions. Due to these ionizing collisions, positive ions and excited molecules are also produced. This ionization process is called an avalanche [19,20]. The positive ions created move out into the gap, while the electrons are neutralized upon contact with the electrode surface. Excited molecules give up their energy in the form of photon radiation. The electrons necessary for maintenance of self sustained discharge are, thus, produced by photoionization of gas molecules.

2.1.2 Negative Corona

In negative corona, avalanches are produced by electrons that are driven in the high field region near the electrode[19,20]. Ultimately, electrons created in the

avalanche arrive in the region of low field intensity, where they attach to neutral molecules and form negative ions. Positive ions left behind in the avalanche move towards the electrode and are neutralized on contact with it. In addition to electrons and positive ions, excited molecules are also present as a result of electron collisions. Excited molecules give up their excess energy in the form of photon radiation. Positive ions as well as photons arriving at the electrode surface create the necessary electrons which help to maintain the self sustained discharge.

2.2 Analysis of Unipolar DC Ionized Fields

As the aim of the present work is towards the assessment of the two main assumptions commonly used by previous researchers; namely, the Deutsch and Kaptzov assumptions, a steady state model of corona is adopted for the purpose of this work.

In both polarities, positive and negative corona, there exists close to the corona site a high concentration of heavy immobile ions, or a space charge of polarity coincident with that of the electrode in corona. This cluster of ions moves away from the electrode as mentioned before, and results in the electric field to be modified from the electrostatic one. In fact, it has been observed that the space charge plays a very important role in reducing the electric field at the surface of the coronating conductor. This has been pointed out by Waters et al[9,10], Abdel-salam et al[13], Popkov et al[21], and Sunaga and Sawada[8]. Accordingly the electric field at the corona site is no longer related in a simple way to the applied potential, but is influenced strongly by other factors.

As the interest is aimed at the global influence of the corona phenomenon on the electrical quantities observed in the interelectrode space rather than on the microscopic behaviour of the phenomenon, the assumption of an average amount of activities is employed. An equivalent average amount of corona activities is

assumed to be distributed along the conductor length with space charge of the same polarity as the conductor, flowing outward from the conductor. Accordingly, the problem is assumed to be two dimensional rather than being three dimensional. In addition, the sheath of corona activities is assumed to be of negligible thickness, in comparison with other dimensions of the line.

The electric field of the space charge interacts with that established by the potential applied to the electrode geometry. Earlier approximate analysis led, originally to the conclusion [6,22], that the entire field intensity at the conductor surface in corona, remains constant at the onset value independent of the applied voltage above corona onset. To illustrate this point further, if a dc voltage applied to a set of electrodes, is increased in magnitude, the electric field intensity at the surface is initially proportional to the voltage. At the point of corona onset, this field reaches the corona onset value. According to the previous statement, then, this field will remain unchanged regardless of what voltage exists on the electrode. This assumption has been employed by almost all past researchers, unless some other less realistic simplifying assumptions are employed [8,13]. However, recent experimental and numerical studies [8,9,10,13,21], revealed that this assumption is not realistic and actually the surface gradient of a conductor in positive corona drops by about 5-10% depending on the level of the applied voltage above corona threshold.

In the conventional methods of describing dc ionized fields, some other simplifying assumptions are usually made; namely, the ionic mobility is constant, not dependent on the electric field, and the diffusion of ions is neglected.

In summary, therefore, the electric field vector E is governed not only by the applied voltage V and electrode geometry, but also by the presence and distribution of the space charge ρ . However, the space charge density ρ is, in turn,

determined by the electric field intensity E and also the applied voltage V . This results in the inherent coupling between the electric field E (or potential Φ), the electrode geometry with potential difference V and the space charge density ρ . It also demonstrates the nonlinear character of the corona problem.

2.3 Equations of Unipolar Corona

Making the general assumptions described above, unipolar dc ionized fields are described by the following equations:

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} , \quad (2.1)$$

$$\nabla \cdot J = 0 , \quad (2.2)$$

$$J = k \rho E , \quad (2.3)$$

$$E = -\nabla \Phi . \quad (2.4)$$

Equation (2.1) is Gauss's law relating the electric field intensity E to the space charge ρ and ϵ_0 is the permittivity of free space. The next equation is the continuity equation for the current density J . Equation (2.3) is the relation between the electric field E and the current density J , the mobility of the ions k is assumed constant. The last equation is the electric field E in terms of the potential Φ .

Combining eqns. 2.1-2.3, the ionized electric field is described by the following equation,

$$E \cdot \nabla (\nabla \cdot E) + (\nabla \cdot E)^2 = 0, \quad (2.5)$$

or, using eq. 2.4, in terms of potential,

$$\nabla \cdot (\nabla \Phi \nabla^2 \Phi) = 0. \quad (2.6)$$

Eqns. 2.5 and 2.6 are the general equations describing a unipolar ionized field. They are nonlinear partial differential equations and there are no known methods available for solving them in the general case. An analytical solution may be obtained only for simple cases like concentric spheres or coaxial cylinders.

2.4 Review of Methods of Calculation

Theoretical investigations of dc ionized fields and of corona losses originated in the classical work of Townsend [23], in which the problem of the simple cylindrical configuration has been analyzed rigorously. Because of the cylindrical symmetry, analysis of the unipolar ionized field reduces to a one dimensional problem. Deutsch [5] extended the analysis of unipolar ionized field to the two dimensional problem of a cylindrical conductor above an infinite ground plane. The basic assumption made by Deutsch is that the space charge affects only the magnitude and not the direction of the electric field. The two dimensional ionized field problems are, thus, reduced to one dimensional problems. He also assumed that the space charge density in the interelectrode space is constant, and that the field at the electrode not in corona is unaffected by the space charge. Popkov [24,25] proposed an improved analysis for the conductor-plane geometry by removing these last two assumptions, while retaining Deutsch's first assumption. Popkov, however, made the additional assumption that the shape of the current density distribution on the ground plane is independent of the geometrical parameters of the configuration or of the intensity of corona. Based on laboratory measurements, Popkov introduced an empirical constant in Townsend's equation for the concentric cylindrical geometry to make it applicable for the conductor-plane geometry. In order to obtain reasonable agreement between experimental data and theoretical results using Popkov's method, Simpson[26,27] suggested that this empirical factor has to be changed according to the line parameters and the corona intensity. Recently, Ciric and Kuffel[11], carried out a rigorous theoretical analysis

of unipolar dc corona losses for a single conductor-to-plane geometry and showed the dependence of this empirical factor on the line parameters. D'Amore et al[28] also proposed an analytical approach for computing unipolar dc corona losses. While using the same assumptions, they assumed that the charge and current densities at the emitter wire can be expressed using a fourier series expansion.

The general feature of the approaches discussed thus far is that they are analytical and produce complicated formulae applicable usually only to simple situations. They are also concerned, only, with corona losses; i.e. the V-I characteristics rather than the knowledge of the different parameters Φ , ρ and E . They, also, retain both Deutsch and Kaptzov assumptions in the analysis.

A detailed investigation of the mathematical aspects of the dc ionized fields has been carried out by Felici [28]. He showed the possibility of obtaining a numerical solution to the corona problem using an iterative technique. The basis for the analysis of the general problem of dc ionized fields is formed by eqns. 2.1-2.3. Combining eqns. 2.1 and 2.3 and substituting for $\nabla \cdot E$ from eq. 2.1, the following relation between the field and charge density distribution is obtained

$$\frac{\rho^2}{\epsilon_0} = -E \cdot \nabla \rho \quad (2.7)$$

If s denotes the distance along the corresponding flux line, then $E \cdot \nabla \rho$ may be replaced by $E \, d\rho/ds$, so that $d\rho/ds$ represents the charge density gradient along the flux line. Eq. 2.7 can then be written as

$$\frac{d\rho}{ds} = -\frac{1}{\epsilon_0} \frac{ds}{E} \quad (2.8)$$

Integration of eq. 2.8 yields

$$\frac{1}{\rho} = \frac{1}{\epsilon_0} \int \frac{ds}{E} + \text{constant} \quad (2.9)$$

Felici [28] showed that the choice of the ground electrode (the collector) as the starting point in the integration is improper as it may lead to some mathematical difficulties. Accordingly, starting at the conductor in corona (the emitter), the expression becomes

$$\frac{1}{\rho} - \frac{1}{\rho_c} = \frac{1}{\epsilon_0} \int_0^s \frac{ds}{E} \quad (2.10)$$

where ρ_c is the charge density at the coronating conductor. Felici[28] suggested that eq. 2.10 together with eq. 2.1 can be solved iteratively to obtain the charge and field distribution. First, the space charge free field is assumed and a first estimate to the charge distribution is obtained using eq. 2.10. This requires the value of ρ_c to be known. The recently calculated charge distribution is then substituted in eq. 2.1 to obtain an improved field distribution and so on. However, there are two limitations to this iterative scheme as pointed out by Sarma and Janischewskyj[30,31]. First, it is generally difficult and time consuming to solve Poisson's equation except in one-dimensional problem. If the given problem can be reduced to an equivalent one-dimensional problem, then significant reduction in the complexity of the numerical solution can be achieved. This can be accomplished by assuming that the space charge affects only the magnitude and not the direction of the electric field, i.e. Deutsch's assumption. The second limitation is in the choice of ρ_c , as Felici considered it to be equal to either a very large or a very small value. From eq. 2.10, it can be seen that when ρ_c is very large, the charge distribution is practically independent of ρ_c . On the other hand, when ρ_c is very small the charge density is very nearly constant throughout the interelectrode space[30]. As pointed out by Sarma and Janischewskyj[30], the consequence of such approximations is the assumption of a uniform charge distribution at the surface of the coronating conductor.

Significant improvement in the numerical analysis of dc ionized fields was achieved by Sarma and Janischewskyj[30,31]. They adopted the concept of iterations, originally suggested by Felici[28], in order to develop an algorithm applicable for corona computation on practical hvdc transmission lines. Although employing the same simplifying assumptions used by other researchers, Sarma and Janischewskyj[30,31] computed, rigorously, the point-to-point variation of the charge density in the field space between the high voltage conductor and the ground plane in the line-to-ground arrangement. Their algorithm is based on expressing the relation between the electric field in presence of space charge E and the space charge free field E' as

$$E = \zeta E' \quad (2.11)$$

where ζ is a scalar point function of the space coordinates depending on charge distribution. A mean charge density along the corresponding flux line is then defined as[30],

$$\rho_m = \epsilon_0 (V - V_0) / \int_0^V \int_{\phi} \frac{d\eta}{(E')^2} d\phi \quad (2.12)$$

where V is the applied voltage, V_0 is the onset voltage and η is a dummy integration variable. Two values of ρ_{c_1} and ρ_{c_2} are then arbitrarily chosen such that $\rho_{c_2} > \rho_{c_1} > \rho_m$. For each of these values, the charge density distribution along the flux line and the corresponding mean charge densities ρ_{m_1}, ρ_{m_2} are evaluated using the following equations[30],

$$\frac{1}{\rho^2} = \frac{1}{\rho_c^2} + \frac{2}{\epsilon_0 \rho_c \zeta_c} \int_{\phi}^V \frac{d\phi}{(E')^2} \quad (2.13)$$

and

$$\rho_m = \int_0^V \int_{\phi} \frac{\rho d\eta}{(E')^2} d\phi \quad / \quad \int_0^V \int_{\phi} \frac{d\eta}{(E')^2} d\phi \quad (2.14)$$

and the distribution of ζ is given by,

$$\zeta^2 = \zeta_c^2 + \frac{2\rho_c \zeta_c}{\epsilon_0} \int_{\phi} \frac{d\phi}{(E')^2} \quad (2.15)$$

Subsequent values ρ_{c_i} are calculated using the following iteration function:

$$\rho_{c_i} = \rho_{c_{i-1}} + \frac{\rho_m - \rho_{m_{i-1}}}{\rho_{m_{i-1}} - \rho_{m_{i-2}}} (\rho_{e_{i-1}} - \rho_{e_{i-2}}), \quad i = 3, 4, \dots \quad (2.15)$$

The corresponding value of ρ_{m_i} is then calculated before proceeding to the next iteration. The procedure is continued until ρ_{m_i} approaches ρ_m to the required precision. Reasonably fast convergence rate was achieved when the two initial values are chosen such that [30]

$$\rho_{c_1} = 1.5 \rho_m, \quad \rho_{c_2} = 3 \rho_m \quad (2.16)$$

Atten [32] developed a finite difference method of solving unipolar dc ionized fields assuming the charge distribution around the conductor surface in corona is known. It is, thus, difficult to extend this analysis to practical dc line configurations. Abdelsalam and Khalifa [14] waived Deutsch's assumption from their analysis, while making additional simplifying assumptions in the process. They assumed the space charge density ρ to vary inversely proportional with the radial distance from the conductor center, and that the equipotential contours are cylindrical surfaces concentric with the conductor. In fact, the ground plane, being an equipotential surface, can not coincide with this assumption. Their analysis was, however, the first attempt at waiving Deutsch's assumption from the unipolar corona analysis. They also showed that the space charge presence in the interelectrode space results in the flux lines being contracted towards the centre line in a

conductor-to-ground configuration[14].

Sunaga and Sawada [8] developed a method to remove Kaptzov's assumption from the analysis while retaining Deutsch's assumption. Their aim was to study corona space charge effects on radio interference of a bundled conductor hvdc transmission line. In their analysis, they introduced empirical constants to be obtained from measurements on a test line; this restricts the generality of their approach.

Gela and Janischewskyj [33,34] proposed a finite element method for solving unipolar dc ionized fields of conductor-plane geometry. While, retaining Kaptzov's assumption, they waived Deutsch's assumption from their analysis. Their assessment of Deutsch's assumption was based on comparing their results, for the corona power loss and the current density distribution on the plane, with those obtained from experimental measurements on a laboratory model of H/r of 446. They showed that corona loss obtained with Deutsch's assumption exceed both experimental values and those obtained without it. Also the maximum value of the current density distribution on the ground was found to be lower when Deutsch's assumption was eliminated. It should be mentioned, here, that the actual effect of Deutsch's assumption, which is the distortion of the flux lines, is not possible to detect through such comparisons. Takuma et al [14] applied the finite element method to the study of unipolar dc ionized fields. In their analysis, however, they assumed the charge density around the conductor surface to be a known constant value; obtained by measurements on an operating line. This limits the application of their analysis. Abdelsalam et al [13] proposed a finite element method to remove Kaptzov's assumption from their analysis. However, they used an additional assumption by accepting Deutsch's assumption to be valid along the centre line in a conductor-plane geometry. In their analysis, they used extrapolation techniques to obtain the current density distributions along the plane, while their

iterative algorithm needed about seventy iterations to achieve a 5% accuracy in the potential values. Their results show that corona losses thus obtained exceed both the experimental values and those obtained using Deutsch's assumption. The same behaviour has been observed for the current density distributions on the ground.

Review of these previous approaches show that, in general, it is extremely difficult to obtain the exact solution to the ionized field problem. This is due to the nonlinearity of the governing equations and the fact that no electrical quantity is precisely known at the coronating conductor except the potential. Rigorous analytical solutions have been obtained for cases of high degree of symmetry such as coaxial cylinders and concentric spheres. In such cases, however, the assumption of the constancy of the electric field at the coronating conductor, i.e. Kaptzov's assumption, has been retained in order to make the solution possible. Few attempts were carried out to determine, analytically, the ionized field solution for practical arrangements such as cylindrical conductors above the ground. However, these earlier techniques were either empirical[24] or dependent on simplifications such as the Deutsch and Kaptzov's assumptions[11,28]. On the other hand, various computational approaches have been proposed in order to obtain an accurate solution of the ionized field problem [8,13,14,32,33,34]. However these previous approaches suffer from lack of generality; e.g. being restricted to a specific line[14], or dependency on the above mentioned simplifying assumptions[33,34]. In addition, no interest have been devoted to the computational aspects regarding the accuracy and efficiency of the iterative process[13,14,33,34].

2.5 Conclusions

In this chapter the different physical and mathematical aspects of corona on dc transmission lines are presented. The inherent coupling between the corona generated space charge and the electric field E or the potential Φ results in the

nonlinear character of the governing equations. As no electrical quantity is known precisely at the coronating conductor, the electric field at its surface has been usually assumed to be constant at the onset value irrespective of the applied voltage level. However, recent experimental and numerical studies showed that this is not the case in reality. In fact the electric field at the energized conductor drops to a value about 5-10% below the onset level depending on the applied voltage level. The various earlier approaches, both analytical and computational, for the solution of the unipolar corona equations are reviewed and the simplifications and limitations involved are discussed.

CHAPTER 3

INFLUENCE OF THE SIMPLIFYING ASSUMPTIONS ON UNIPOLAR DC IONIZED FIELD ANALYSIS (ANALYTIC TREATMENT)

As mentioned before theoretical analysis of the unipolar ionized field associated with dc corona discharges is important for the study and evaluation of alternative designs of hvdc transmission lines. However, one of the main difficulties in the theoretical analysis is the simplifying assumptions usually employed in the solution [8,11,13,14]. In fact, except for the potential on the electrodes, no other quantity is precisely known on the electrodes. For practical arrangements such as a cylindrical conductor above a ground plane, various simplifying assumptions have been incorporated in the analysis of power loss and field quantities of unipolar dc corona [3-8,11-17]. These include a known current density distribution or a known charge density distribution on the coronating electrode [8,14]. Some recent studies assumed a constant charge density distribution at the conductor in corona for calculation of the ionized field quantities [16]. Many researchers have also relied on the Kaptzov assumption that the field intensity at the conductor in corona is always constant at the onset value irrespective of the applied voltage level [12,14,15,24,25,28,30].

In the following sections, investigation into the significance and the impact of the main simplifying assumptions for solution of the unipolar dc corona equations is presented. An improved method for the unipolar ionized field solution which takes into account the influence of Kaptzov's assumption is presented [35]. Initially, the simple geometry of coaxial cylinder is considered. The analysis is then extended to the two dimensional case of a smooth cylinder above a ground plane.

analytical expressions for the practical case of stranded conductors are also developed. The influence of Deutsch's assumption in relation to the distortion of the flux lines pattern is also studied. Modified approximate expressions for the assessment of the right-of-way current density profiles of hvdc lines are derived and compared with other published experimental and numerical data for different geometries and intensities of corona.

3.1 Coaxial Cylinder Configuration

Townsend [23] was the first to solve for the ionized field in cylindrical configuration as shown in fig. 3.1 where r_c and r_g are the inner and outer radii respectively. At voltages above the corona onset level, space charge of polarity similar to the energized conductor is present in the interelectrode gap. For the electric field in the unipolar region, eq. 2.5 reduces to

$$E \frac{d^2 E}{dr^2} + 3 \frac{E}{r} \frac{dE}{dr} + \left(\frac{dE}{dr} \right)^2 = 0 \quad (3.1)$$

where r is the distance in the radial direction. It can be shown by direct substitution that the following expression for the electric field ,

$$E^2 = \frac{A^2 r^2 + B^2}{r^2} \quad (3.2)$$

where A and B are arbitrary constants, satisfies eq. 3.1 . The constants A and B are to be determined from appropriate boundary conditions. The first boundary condition employed is the potential difference between the electrodes while the second is the electric field at the coronating conductor surface. From eq. 3.2, the electric field at the conductor in corona is given by,

$$E_c = \frac{\sqrt{A^2 r_c^2 + B^2}}{r_c} \quad (3.3)$$

where E_c is generally a function of the applied voltage level V and is expressed

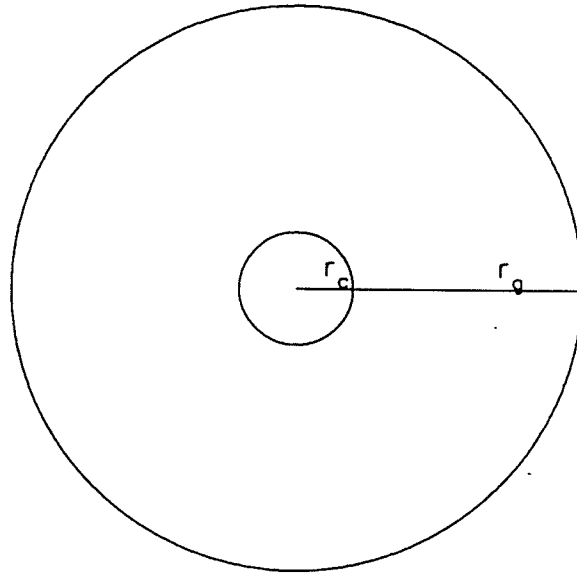


Fig. 3.1 Coaxial cylindrical configuration

as,

$$E_c = E_0 f_1(V/V_0) \quad (3.4)$$

where, E_0 is the mean value of the corona onset gradient around the conductor periphery and V_0 is the corresponding corona onset voltage and is given by,

$$V_0 = r_c E_0 \ln \frac{r_g}{r_c} \quad (3.5)$$

The total corona current from the conductor per unit length I is given by,

$$I = 2\pi r_c J_c \quad (3.6)$$

and at the conductor surface $J_c = k \rho_c E_c$ where ρ_c is the charge density at the conductor. Substituting for J_c in eq. 3.6, one obtains

$$I = 2\pi k r_c \rho_c E_c \quad (3.7)$$

The space charge density at the coronating conductor ρ_c is obtained from eq. 2.1 as,

$$\rho_c = \epsilon_0 \nabla \cdot E \quad (3.8)$$

Substituting for $\nabla \cdot E$, then

$$\rho_c = \frac{\epsilon_0 A^2}{\sqrt{A^2 r_c^2 + B^2}} \quad (3.9)$$

The constant A is obtained by substituting eqs. 3.3 and 3.9 into eq. 3.7,

$$A^2 = \frac{I}{2\pi \epsilon_0 k} = \frac{E_c^2 r_c^2}{r_g^2} \frac{I}{2\pi \epsilon_0 k} \left(\frac{r_g}{E_c r_c} \right)^2 \quad (3.10)$$

and by defining,

$$Y_G = \frac{I}{2\pi \epsilon_0 k} \left(\frac{r_g}{E_c r_c} \right)^2 \quad (3.11)$$

it becomes,

$$A^2 = \frac{E_c^2 r_c^2}{r_g^2} Y_G \quad (3.12)$$

The constant B is obtained by substituting A in eq. 3.3,

$$B^2 = E_c^2 r_c^2 \left[1 - \frac{r_c^2}{r_g^2} Y_G \right] \quad (3.13)$$

Finally, the ionized field E is given by,

$$E = \frac{E_c r_c}{r} \left[\frac{r^2}{r_g^2} Y_G + \left(1 - \frac{r_c^2}{r_g^2} Y_G \right) \right]^{1/2} \quad (3.14)$$

In order to obtain the voltage - current characteristic, eq. 3.14 is integrated between the limits $r=r_c$ and $r=r_g$ and the following equation is obtained,

$$\frac{V - f_1 V_0}{f_1 V_0} \ln \frac{r_g}{r_c} = (1 + Y_G)^{1/2} - 1 + \ln \frac{2}{1 + (1 + Y_G)^{1/2}} \quad (3.15)$$

The formula 3.15 represents the general V-I characteristic for a coaxial cylinder geometry. The relation between the overvoltage level V/V_0 and the nondimensional current Y_G is shown in fig. 3.2 for different values of the ratio r_g/r_c . The function f_1 in eq. 3.15 is assumed to have the following form,

$$f_1 = 1.1339 - 0.1678 \left(\frac{V}{V_0} \right) + 0.03 \left(\frac{V}{V_0} \right)^2 \quad (3.16)$$

where $f_1=1$ at $V=V_0$. The function f_1 is shown in fig. 3.3a to fit on the average other published experimental and numerical data [9,10,13,21,22,31], for the conductor surface gradient as a function of the overvoltage level. Sarma and Janischewskyj[22], for a single conductor above ground geometry, assumed the ionization layer to be a cylindrical surface surrounding the conductor. In their analysis, they showed that the field intensity at the conductor surface is always

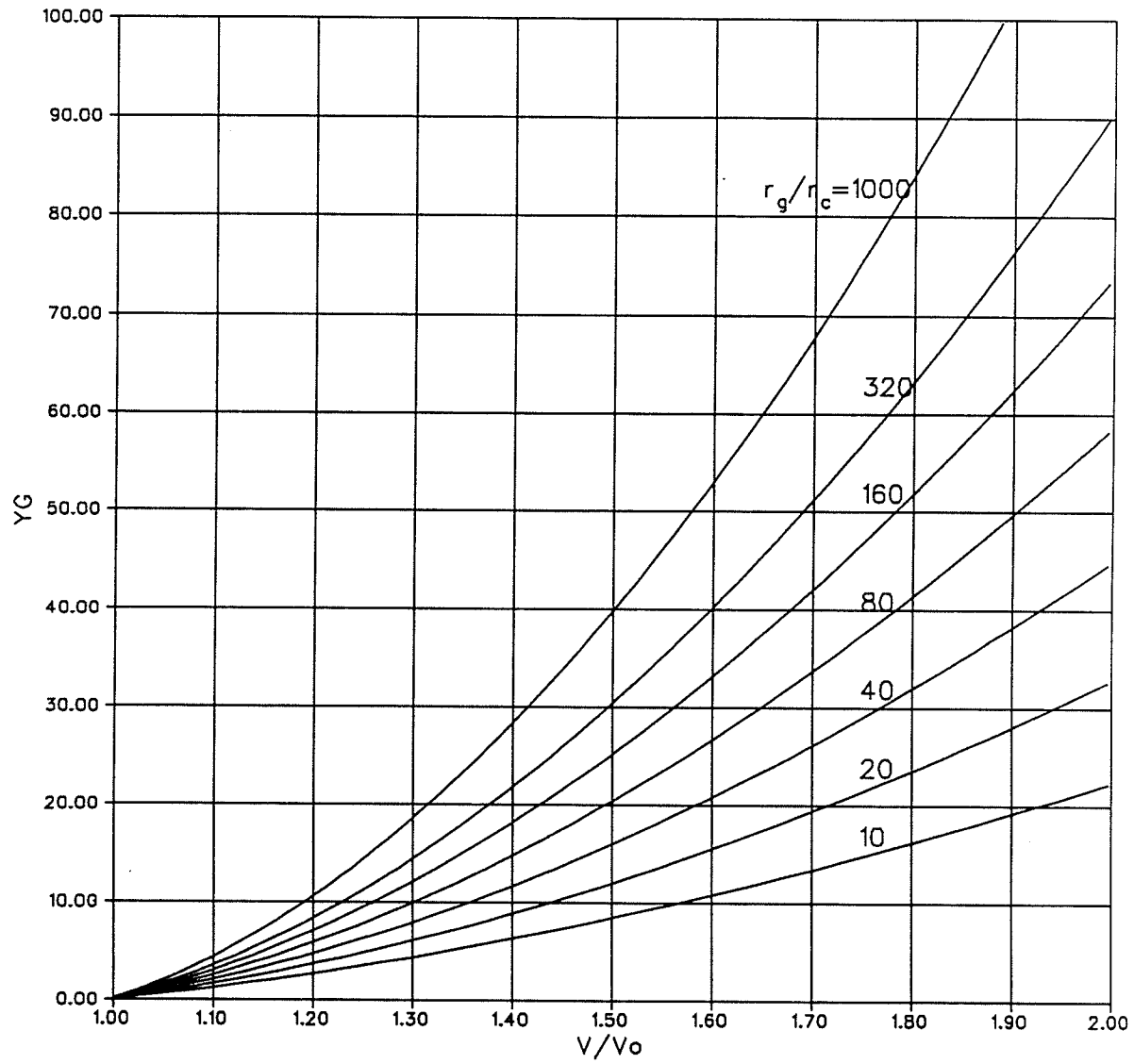


Fig. 3.2 Corona characteristics for cylindrical configuration

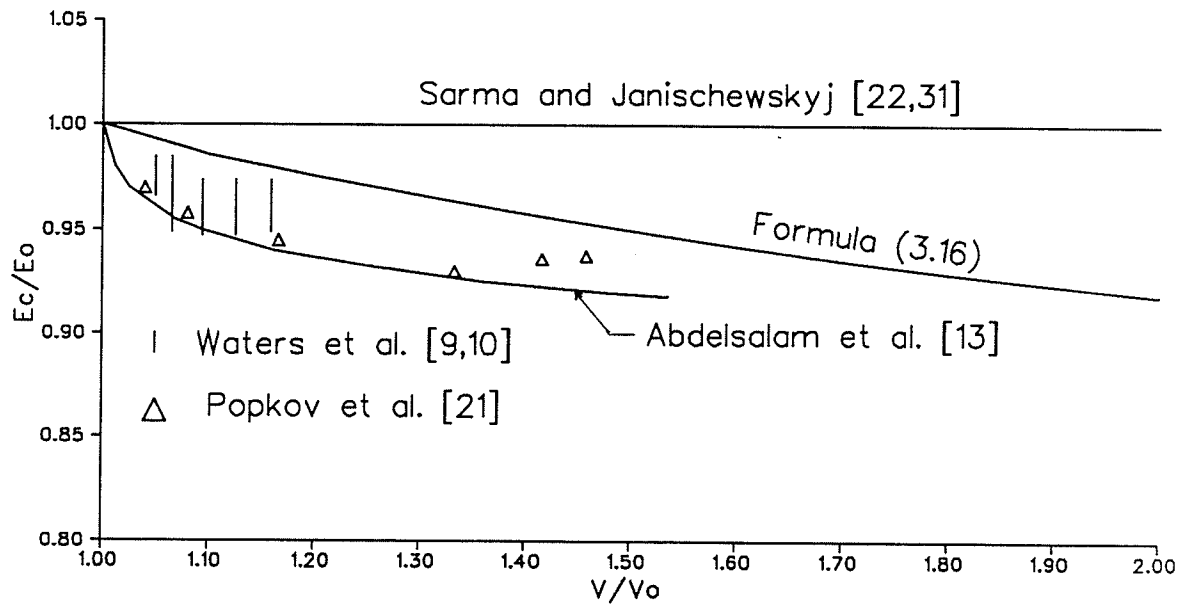


Fig.3.3a Variation of the average electric field at conductor surface against the applied voltage

constant at the onset value regardless of the applied voltage level or the corona intensity. However, Waters et al.[9,10] measured the field intensity at the conductor surface and found that it actually drops below the onset value depending on the level of the applied voltage level. They explained that the reason for this behaviour is the dependence of the electric field on the conductor surface, not only on the charges in the ionization layer but also on charges located further away in the interelectrode space. Popkov et al.[21] and Abdelsalam et al.[13] also obtained similar variation for the electric field dependence on the applied voltage level. Analytical expressions that describe such variation are not actually available. Although eq. 3.16 may not represent the exact relation of the conductor field intensity and the applied voltage level, it provides, nevertheless, an average fit of other published experimental and numerical data[9,10,13,21,22]. As will be shown later, this relation proves to be successful in predicting, with good accuracy, the voltage-current for various geometries and intensities of corona for both smooth and stranded conductors.

Eq. 3.16 is obtained using second order polynomial fitting and accordingly is not exactly equal to one when $V=V_0$. However, all the calculations are carried out with the value of f_1 equals to one at the onset voltage. A modified formula which does not deviate by more than 0.42% from f_1 at twice the onset voltage may also be used and is given by

$$f_2 = 1.1378 - 0.1678\left(\frac{V}{V_0}\right) + 0.03\left(\frac{V}{V_0}\right)^2 \quad (3.16a)$$

which will be, exactly, equal to one at $V=V_0$.

An example of the application of eq. 3.15 for the calculation of the corona V-I characteristic is shown in fig. 3.3b for a conductor of radius $r_c = 0.475\text{cm}$ and for various values of the outer radius r_g . The corona current I is plotted against

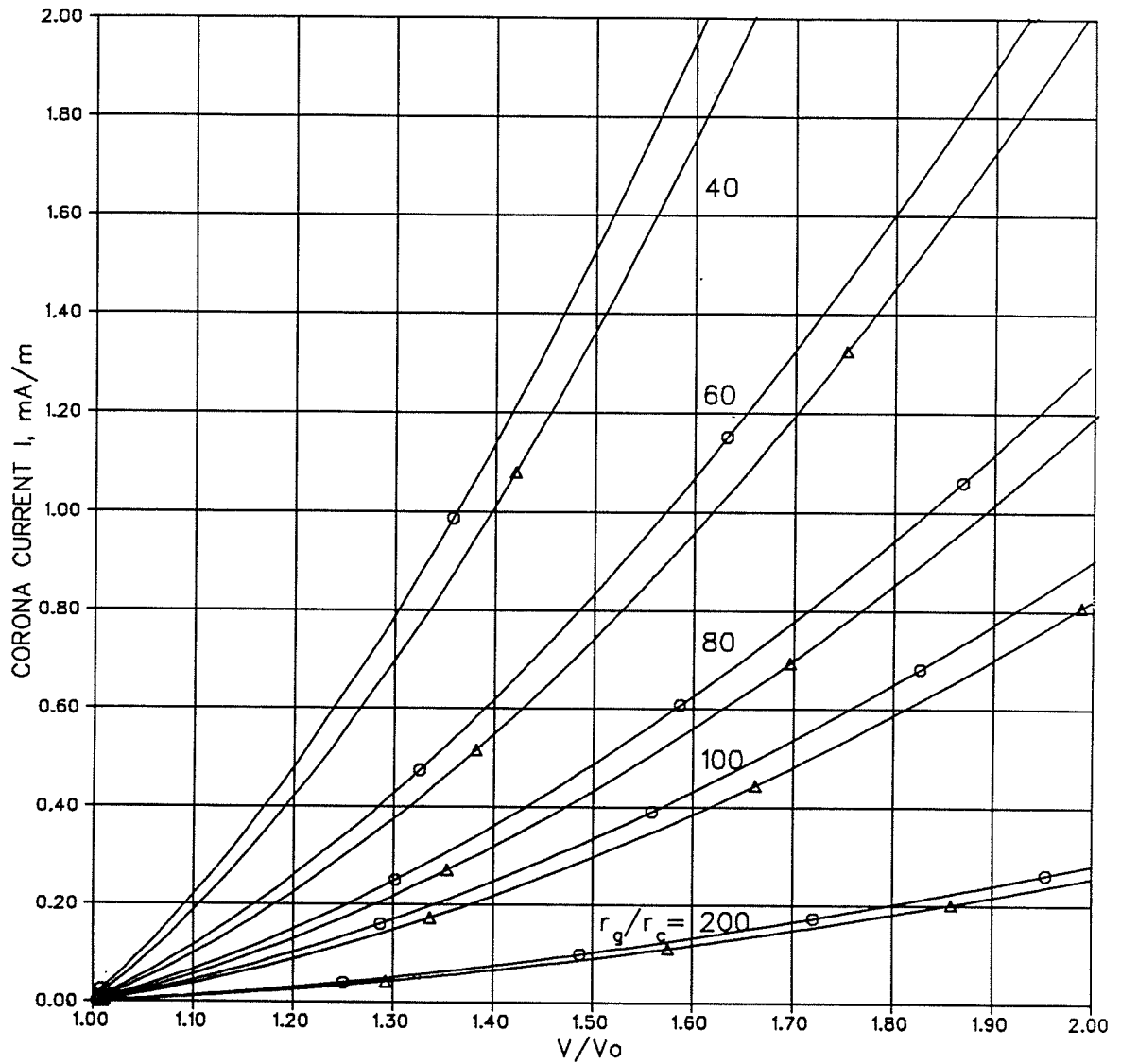


Fig. 3.3b Corona characteristics for cylindrical configuration,
 $r_c=0.475\text{cm}$.

△ $E_c = E_o$

○ $E_c = f_1 E_o$

the applied voltage level for the two cases where the electric field at the coronating conductor surface is assumed to depend on the applied voltage, i.e. without Kaptzov's assumption, and when it is assumed to stay constant at the onset value E_0 , i.e. when Kaptzov's assumption is employed.

Careful measurements were conducted by Waters et al [10] on a coaxial cylinder geometry of inner radius $r_c = 0.475\text{cm}$ and outer radius $r_g = 29.05\text{cm}$. This geometry is considered here, and the results for the two procedures are shown in fig. 3.4, together with the experimental data. The value of the mobility used in the calculations is $1.5 \times 10^{-4} \text{m}^2/\text{V}\cdot\text{sec}$. [26,27,30]. The agreement between the results obtained using the present method, shown in fig. 3.4 as curve A, and the experimental data is satisfactory. Results using the assumption, shown as curve B, tend to underestimate the corona losses and consequently could have a significant influence on the unipolar ionized field solution. Waters et al [10] also pointed out that, for large interelectrode spacings, such an assumption could have more influence than for smaller spacings.

Another relation for the variation of the conductor surface gradient is also attempted based on Water's data alone[9,10]. The results for the corona V-I characteristic are shown in fig. 3.4a. Although, generally, good agreement with measured data was obtained using Water's data alone, closer agreement was observed when eq. 3.16 was used, as shown in fig. 3.4, to describe the relation between the conductor surface gradient and the applied voltage level. The formula f_3 used in fig. 3.4a is given by

$$f_3 = 1.4686 - 0.6888\left(\frac{V}{V_0}\right) + 0.2202\left(\frac{V}{V_0}\right)^2 \quad (3.16b)$$

Although the exact variation of the conductor surface gradient as a function of the applied voltage level is not known, eq. 3.16 was found to result in V-I characteristics that are in good agreement with experimental data not only for the case of

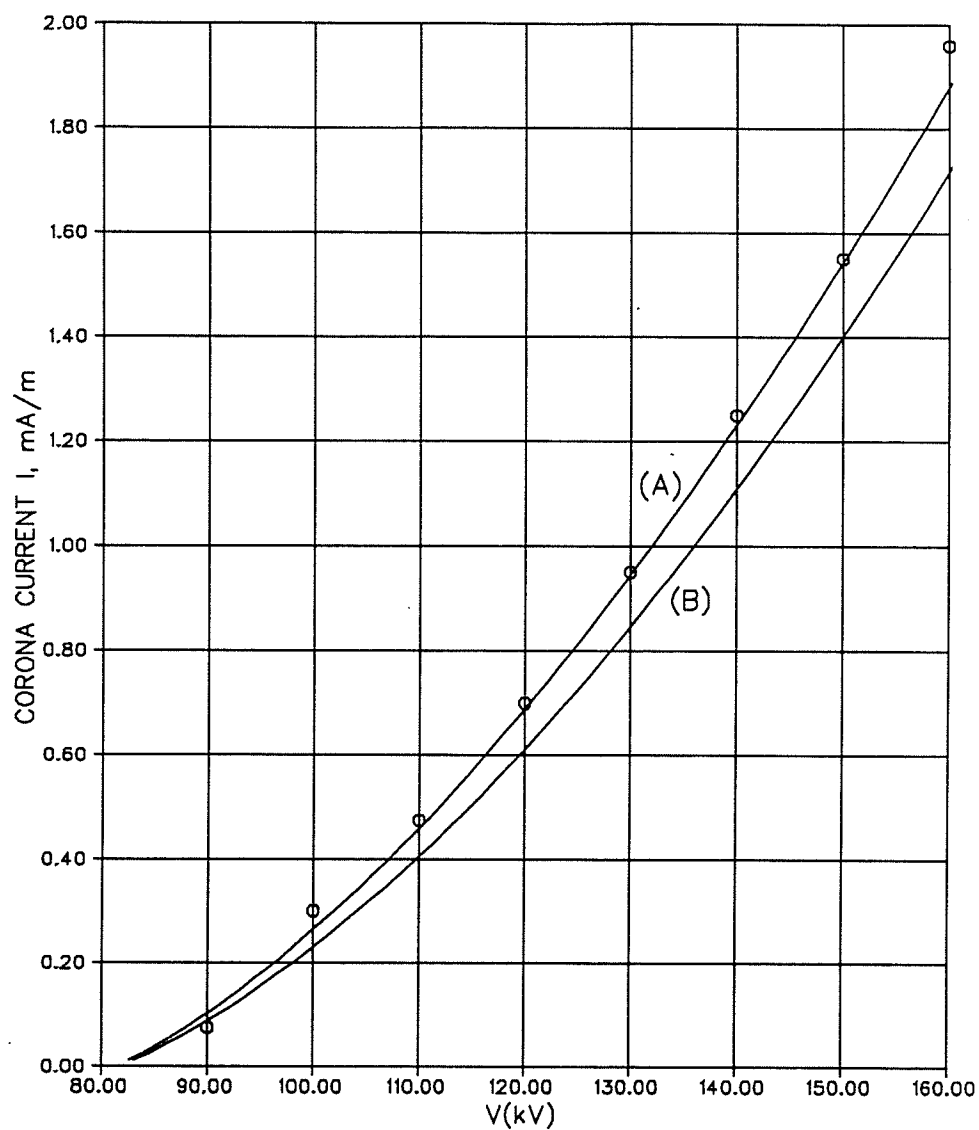


Fig. 3.4 Corona characteristic for cylindrical configuration,

$$r_c = 0.475 \text{ cm}, r_g = 29.05 \text{ cm}$$

$$\text{A} - E_c = f_1 E_o$$

$$\text{B} - E_c = E_o$$

O Experimental data [10]

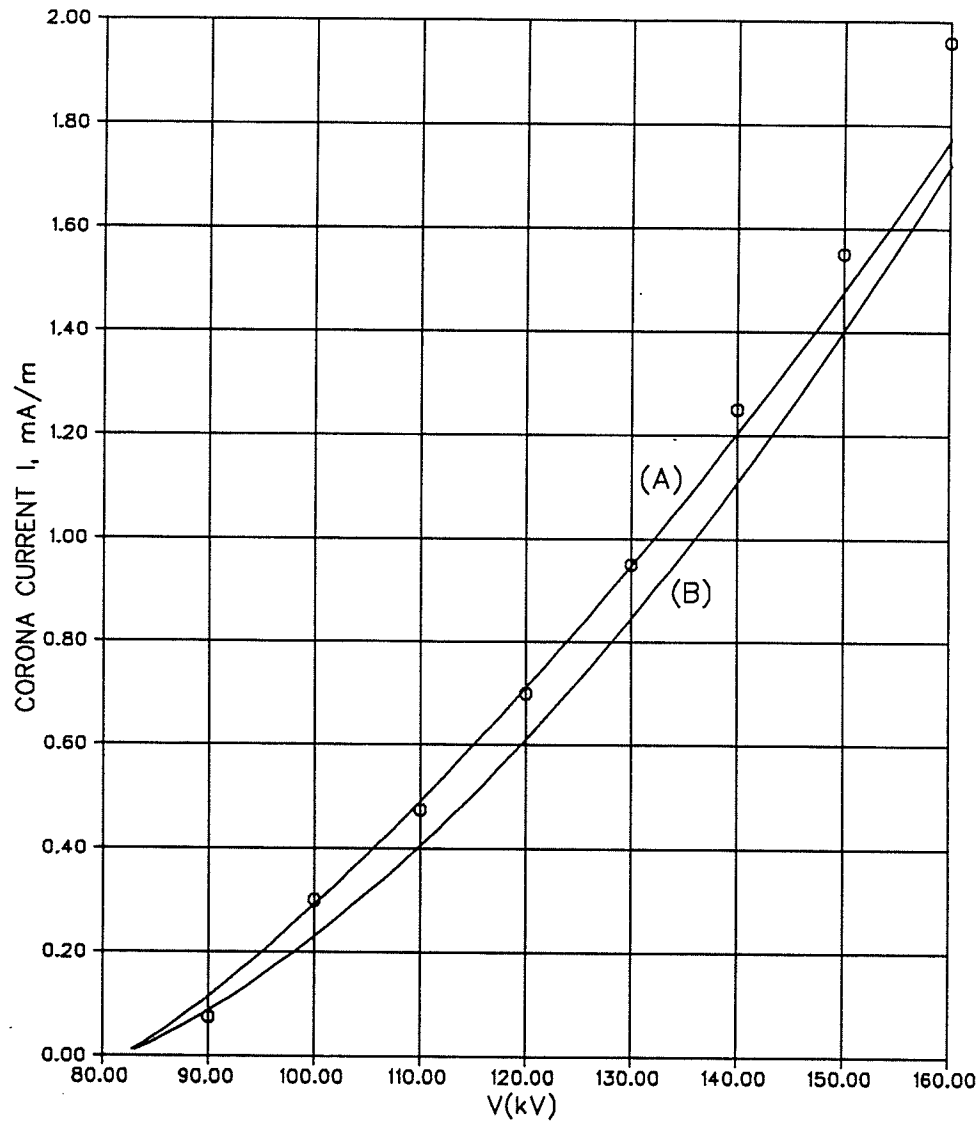


Fig. 3.4a Corona characteristic for cylindrical configuration,

$$r_c = 0.475 \text{ cm}, r_g = 29.05 \text{ cm}$$

$$\text{A} - E_c = f_3 E_0$$

$$\text{B} - E_c = E_0$$

O Experimental data [10]

coaxial cylinder geometry but also for the geometry of a cylinder above a ground plane using both smooth and stranded conductors[13,27,38]. The mean value of the corona onset gradient E_0 used in the calculations is given by the following expression for the positive polarity as suggested by Popkov[24,25,37],

$$E_0 = 31 \left(1 + \frac{0.32}{\sqrt{r_c}} \right) \text{ kV/cm} \quad (3.17)$$

where r_c is the conductor radius in cm.

3.2 Smooth Cylinder Above Ground

Apart from the simple cylindrical configuration discussed before, it is very difficult to obtain analytical solutions without making additional simplifying assumptions. In cylindrical configurations the electric field lines are always radial even in the presence of corona generated space charge. However, for practical arrangements such as a line-to-ground plane configuration, the electric field direction is influenced by the space charge presence and consequently deviates from that of the electrostatic one [13,14,17].

In order to simplify the analysis, Deutsch [5] made use of the assumption, that is the space charge presence does not affect the electric field direction, to solve for the case of the line-to-plane geometry. Following Deutsch's work, Popkov [24,25] in his analysis of the line-to-plane geometry introduced certain improvements. Popkov assumed that the line-to-plane geometry can be approximated by an equivalent cylindrical system and obtained the following equation for the voltage-current characteristic[24],

$$0.41 P \frac{V-V_0}{V_0} \ln \frac{2H}{r_c} = (1+Y_P)^{1/2} - 1 + \ln \frac{2}{1+(1+Y_P)^{1/2}} \quad (3.18)$$

where,

$$Y_P = \frac{PI}{2\pi\epsilon_0 k} \left[\frac{H}{E_0 r_c} \right]^2 \quad (3.19)$$

The empirical constant P is introduced by Popkov in order to take account of the nonuniform current distribution along the ground plane.

The applicability of Popkov's formulas to practical systems has been discussed by Simpson and Morse [26,27]. Accurate measurements were made of corona currents on an outdoor test line using stranded conductors, and reasonable agreement between Popkov's theory and experiment could be obtained only when P was increased from 1.65 (originally suggested by Popkov) to about 5 [27]. Results also indicated that P , contrary to the assumption of Popkov, will vary with the line parameters as well as with the intensity of corona or the overvoltage level. Recent studies by Ciric & Kuffel [12] also indicated the dependence of the empirical factor P on the line parameters.

The configuration considered in the present study is shown in fig. 3.5 and consists of a single smooth cylinder of radius r_c at a height H above the ground plane. In the present analysis the bipolar coordinates α , β , shown in fig. 3.5, are employed which are related to the cartesian coordinates x , y by the following relations[36],

$$y = \frac{a}{s} \sinh \beta, \quad x = \frac{a}{s} \sin \alpha \quad (3.20)$$

where,

$$a = (H^2 - r_c^2)^{1/2}, \quad s = \cosh \beta + \cos \alpha \quad (3.21)$$

The metric coefficients are given by the following expressions:

$$h_\beta = h_\alpha = \frac{a}{s} \quad (3.22)$$

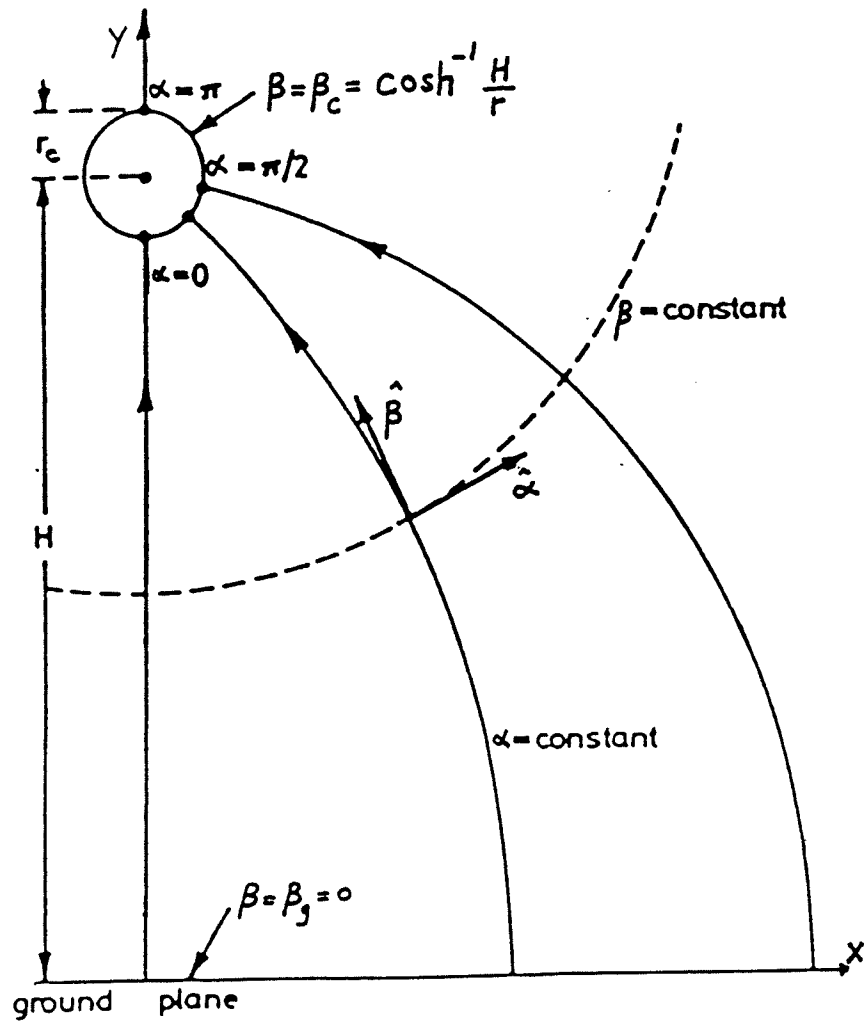


Fig. 3.5 Single conductor above ground .

and the coordinates of the coronating conductor and the ground plane are given by,

$$\beta_c = \cosh^{-1}\left(\frac{H}{r_c}\right), \beta_g = 0 \quad (3.23)$$

For steady state corona, the total current per unit length is,

$$I = \int J \cdot dA \quad (3.24)$$

By using eqs. 2.1-2.4, the expression for the current is,

$$I = \int k \epsilon_0 [\nabla \cdot (\nabla \Phi)] \nabla \Phi \cdot dA \quad (3.25)$$

By transforming to bipolar coordinates and assuming that the elemental surface area dA is on one of the cylindrical surfaces where $\beta = \text{const.}$, then,

$$dA = \hat{a}_\beta a (\cosh \beta + \cos \alpha)^{-1} d\alpha \quad (3.26)$$

where $\hat{a}_\beta$ is a unit vector in the β coordinate direction. Since the electric field is assumed to have the same direction as that of the space charge-free field, then using eq. 2.4,

$$(\nabla \Phi) \cdot \hat{a}_\beta = \frac{\cosh \beta + \cos \alpha}{a} \frac{d\Phi}{d\beta} \quad (3.27)$$

Substituting this expression in eq. 3.25, and integrating between the limits $\alpha = -\pi$ to $\alpha = \pi$, gives,

$$I = -\frac{\pi \epsilon_0 k}{2a^2} (2 \cosh^2 \beta + 1) \frac{d}{d\beta} \left(\frac{d\Phi}{d\beta} \right)^2 \quad (3.28)$$

Integrating eq. 3.28, and imposing the following boundary condition at the conductor surface,

$$\left(\frac{d\Phi}{d\beta} \right)_{\beta=\beta_c} = \frac{f_1 V_0}{\beta_c}, \quad (3.29)$$

the following equation is obtained,

$$\frac{d\Phi}{d\beta} = \left[\left(\frac{f_1 V_0}{\beta_c} \right)^2 + \frac{a^2 I}{\sqrt{3}\pi\epsilon_0 k} \ln \frac{\sqrt{3} + \tanh\beta_c}{\sqrt{3} - \tanh\beta_c} \frac{\sqrt{3} - \tanh\beta}{\sqrt{3} + \tanh\beta} \right]^{1/2} \quad (3.30)$$

In order to obtain the voltage-current relationship, eq. 3.30 is integrated as follows,

$$V = \int_0^{\beta_c} \frac{d\Phi}{d\beta} d\beta \quad (3.31)$$

to give the following expression,

$$\frac{V}{V_0 f_1} \beta_c = (1 + Y_G)^{1/2} \int_0^{\beta_c} \left[1 - \frac{2}{\sqrt{3} G f_1^2} \frac{Y_G}{1 + Y_G} \ln \frac{\sqrt{3} + \tanh\beta}{\sqrt{3} - \tanh\beta} \right]^{1/2} d\beta \quad (3.32)$$

where the parameters β_c , Y_G and G are given by the following expressions,

$$\beta_c = \ln \left[\frac{H}{r_c} + \left(\left(\frac{H}{r_c} \right)^2 - 1 \right)^{1/2} \right], \quad (3.33)$$

$$Y_G = \frac{GI}{2\pi\epsilon_0 k} \left(\frac{H}{E_0 r_0} \right)^2, \quad (3.34)$$

$$G \left(\frac{H}{r_c}, \frac{V}{V_0} \right) = \frac{2}{\sqrt{3} f_1^2} \ln \frac{\sqrt{3} + \left[1 - \left(\frac{r_c}{H} \right)^2 \right]^{1/2}}{\sqrt{3} - \left[1 - \left(\frac{r_c}{H} \right)^2 \right]^{1/2}} \quad (3.35)$$

Eqns. 3.32-3.35 describe the general V-I characteristic for a smooth cylinder above plane configuration. Inspection of eq. 3.35 shows that the function G is dependent on both the line parameters H and r_c and also on the corona intensity level V/V_0 . The variation of G with respect to H/r_c is shown in fig. 3.6 for different overvoltage levels. At a certain overvoltage level the value of G , for $H/r_c \geq 40$, stays almost constant and independent of the line parameters. For a specific ratio H/r_c , G increases as V/V_0 increases. The function G varies generally between $G=0$, for $H/r_c=1$, to an upper limit $G_{\infty, V}$, corresponding to

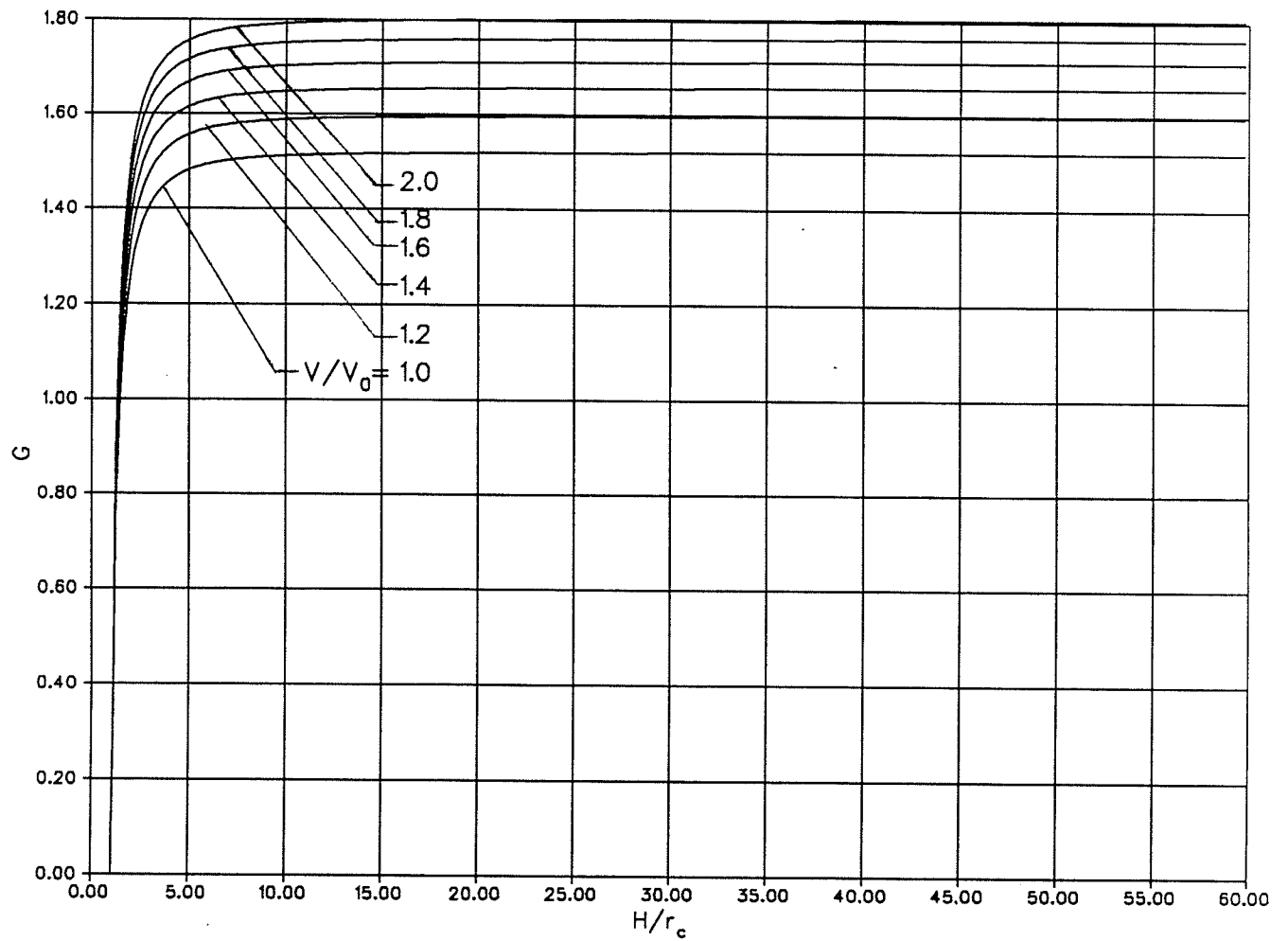


Fig.3.6 Variation of the function G against the line parameter H/r_c for different levels of V/V_0 .

$H/r_c = \infty$, that is determined by the applied voltage level. The upper limit $G_{\infty,V}$ for different overvoltage levels is given in Table 3.1. It is observed that the variation in $G_{\infty,V}$ diminishes as V/V_0 increases, and $G_{\infty,V}$ tends to reach an almost constant value at a voltage level about twice the onset value. The variation of G with respect to V/V_0 for different ratios of H/r_c is shown in fig. 3.7. For large ratios of H/r_c , G becomes less sensitive to the variation in H/r_c and more dependent on the overvoltage level V/V_0 as observed earlier.

In order to determine the corona V-I characteristic, the integral in eq. 3.32 is computed numerically with an accuracy of $10^{-5}\%$ for the two cases when, 1) E_c is held constant at the onset value, and 2) when E_c is allowed to vary as a function of the applied voltage level. An example of the computations is shown in fig. 3.8 for a conductor of radius $r_c = 0.165\text{cm}$ at various heights above the ground. The results are plotted as a relation between the corona current I and the overvoltage level V/V_0 . It is observed that the current I for the case where E_c is variable rises more sharply than when E_c is constant. The deviation between the two cases increases as V/V_0 increases and decreases as H/r_c increases. However, the relative influence of the assumption was found to be more significant for large ratios of H/r_c and voltage levels close to the onset value, i.e. for configurations and voltage levels usually encountered in practical arrangements[10].

The corona characteristics with and without Kaptzov's assumption are computed, using eqns. 3.32-3.35, for different geometries of a smooth cylinder above a ground plane for which experimental data are available [13,38]. The results are shown in figs. 3.9a-3.9d for positive corona. The agreement of the results of present method, shown as curve A, with the measured data is satisfactory. Curve B shows the results when Kaptzov's assumption is employed in the solution. It can be seen that inclusion of this assumption in the analysis results in

Table 3.1 The upper limit $G_{\infty,V}$ of the function G for different overvoltage levels.

V/V_0	$G_{\infty,V}$
1.0	1.52069
1.1	1.56539
1.2	1.59725
1.3	1.62807
1.4	1.65771
1.5	1.68604
1.6	1.71292
1.7	1.73821
1.8	1.76179
1.9	1.78353
2.0	1.80331

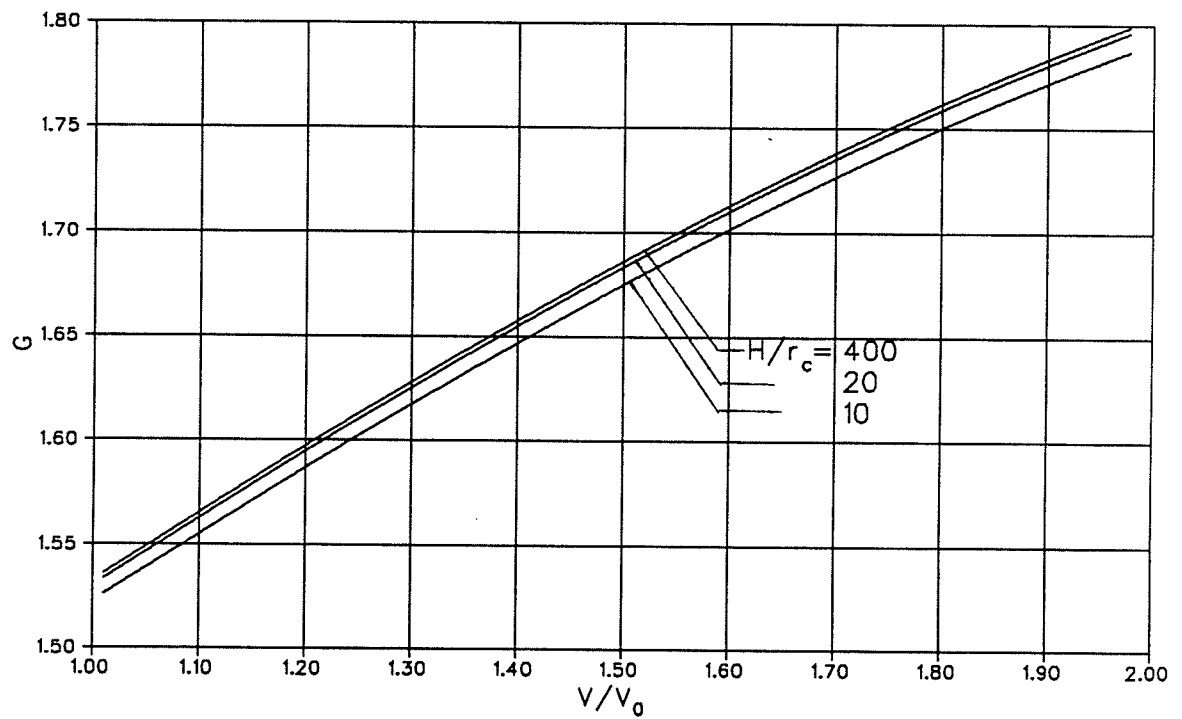


Fig.3.7 Variation of the function G against V/V_0 ,
for different ratios of H/r_c .

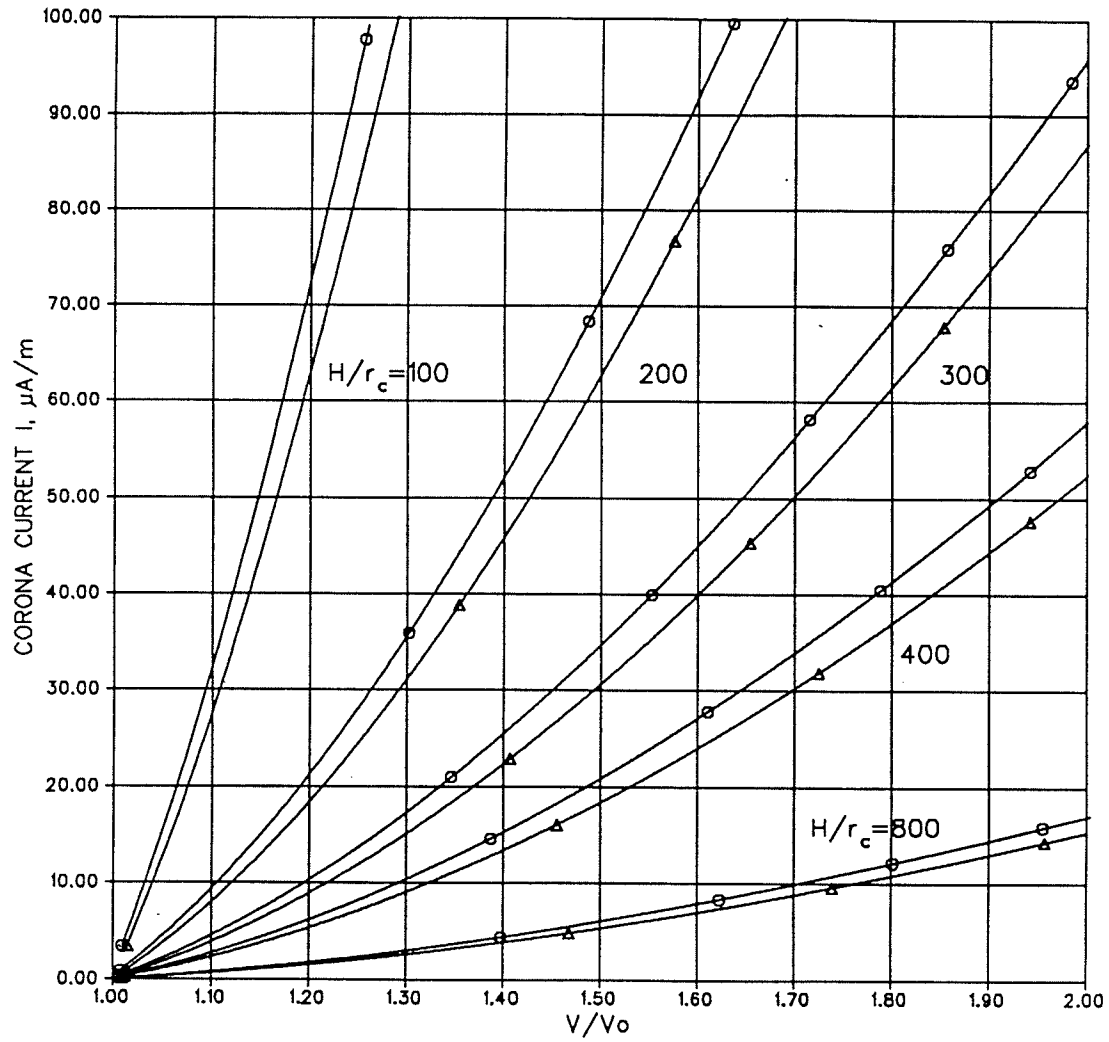


Fig.3.8 Corona characteristics for a smooth cylinder above plane
 $r_c=0.165\text{cm}$ for different ratios of H/r_c

$\Delta E_c = E_0$

$\circ E_c = f_1 E_0$

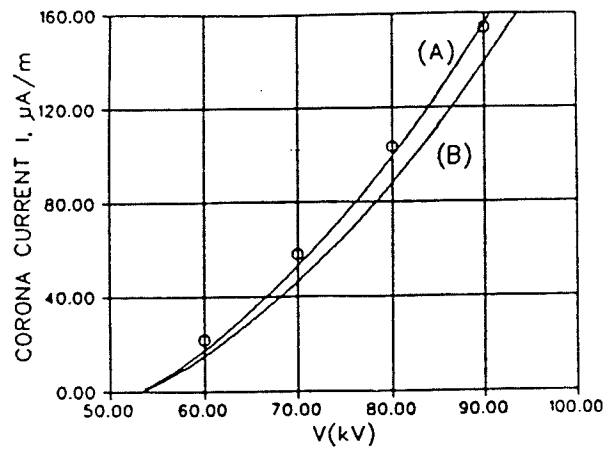
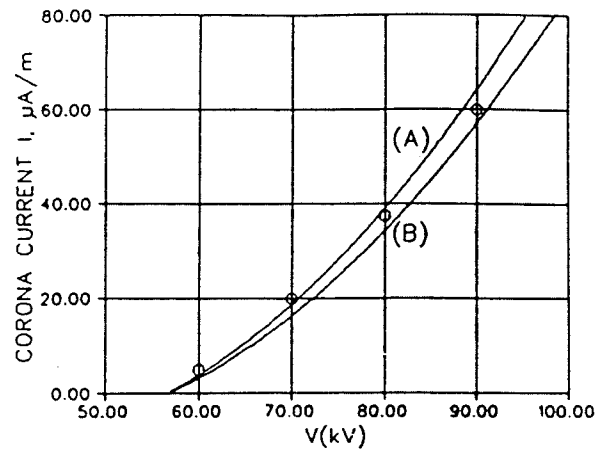
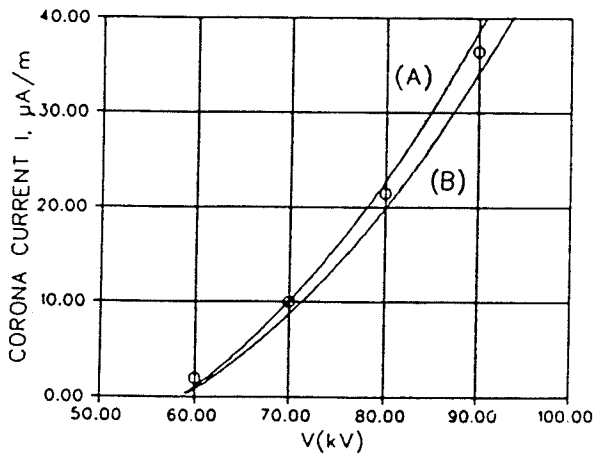
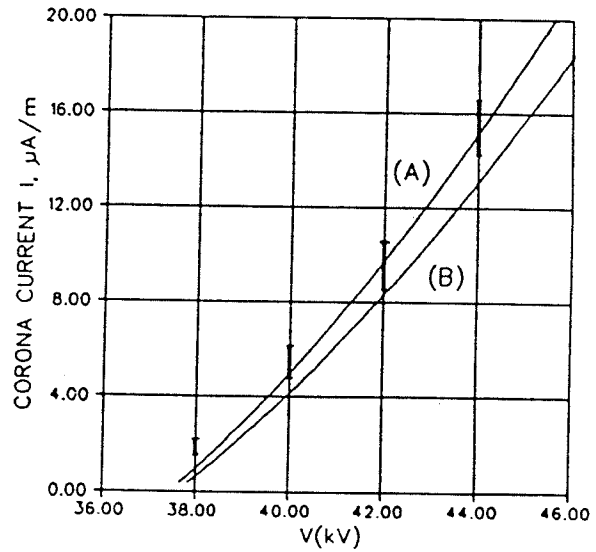
(a) $r_c = 0.165\text{cm}$, $H = 27.5\text{cm}$ (b) $r_c = 0.165\text{cm}$, $H = 40\text{cm}$ (c) $r_c = 0.165\text{cm}$, $H = 49.5\text{cm}$ (d) $r_c = 0.096\text{cm}$, $H = 24\text{cm}$

Fig.3.9 Corona characteristics for smooth cylinder above plane for different configurations.

A- $E_c = f_1 E_o$

B- $E_c = E_o$

O- Experimental data [13]

I- Experimental data [37]

underestimation of the corona losses as observed earlier for the case of cylindrical geometry.

The corona characteristics, with and without the assumption, are also plotted in fig. 3.10 using the generalized coordinate Y_G . The deviation between the two cases is present for different ratios of H/r_c . In order to determine the corona V-I relationship for a smooth cylinder above ground plane configuration, the generalized characteristics, using the present method, are presented in the form of a universal chart shown in fig. 3.11 for H/r_c ranging from 100 to 1000, thus, covering all ranges of practical importance. For a certain overvoltage level V/V_0 and line parameters H and r_c , the nondimensional current Y_G is determined. The corresponding corona current I can then be obtained using eqns. 3.34 and 3.35.

In order to obtain a simple formula for the V-I characteristic, the function in the integrand of eq. 3.32 is replaced by a simpler expression. A good approximation for $H/r_c \geq 100$, at a certain applied voltage level, can be obtained using the following expression[12],

$$\ln \frac{\sqrt{3} + \tanh \beta}{\sqrt{3} - \tanh \beta} \approx \frac{\sqrt{3}}{2} G_{\infty, V} (1 - e^{-\sqrt{2}\beta}) \quad (3.36)$$

Accordingly, the V-I relationship is now given by,

$$\frac{V}{V_0 f_1} \beta_c = (1 + Y_G)^{1/2} \int_0^{\beta_c} \left[1 - \frac{G_{\infty}}{G f_1^2} \frac{Y_G}{1 + Y_G} (1 - e^{-\sqrt{2}\beta}) \right]^{1/2} d\beta \quad (3.37)$$

Eq. 3.37 is integrated analytically and the following form is obtained,

$$\frac{1}{\sqrt{2}} \frac{V}{V_0 f_1} \ln \left[(1+t) \frac{H}{r_c} \right] = (1 + Y_G)^{1/2} \left[1 - q_1 q_2 + \frac{q_1}{2} \ln \frac{q_3 q_4}{q_5 q_6} \right] \quad (3.38)$$

where,

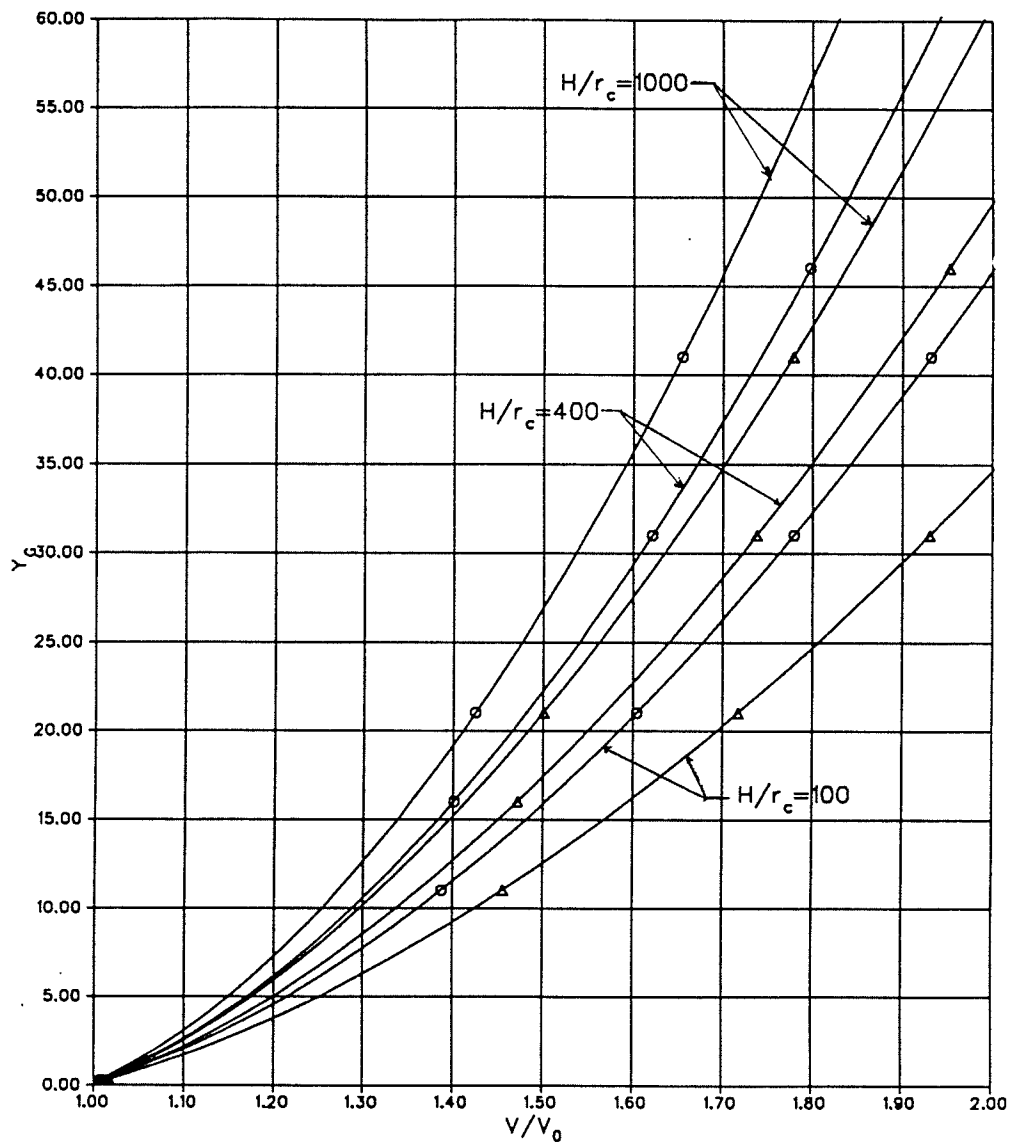


Fig.3.10 Corona characteristics for a smooth cylinder above plane for different configurations

○ $E_c = f_1 E_0$

△ $E_c = E_0$

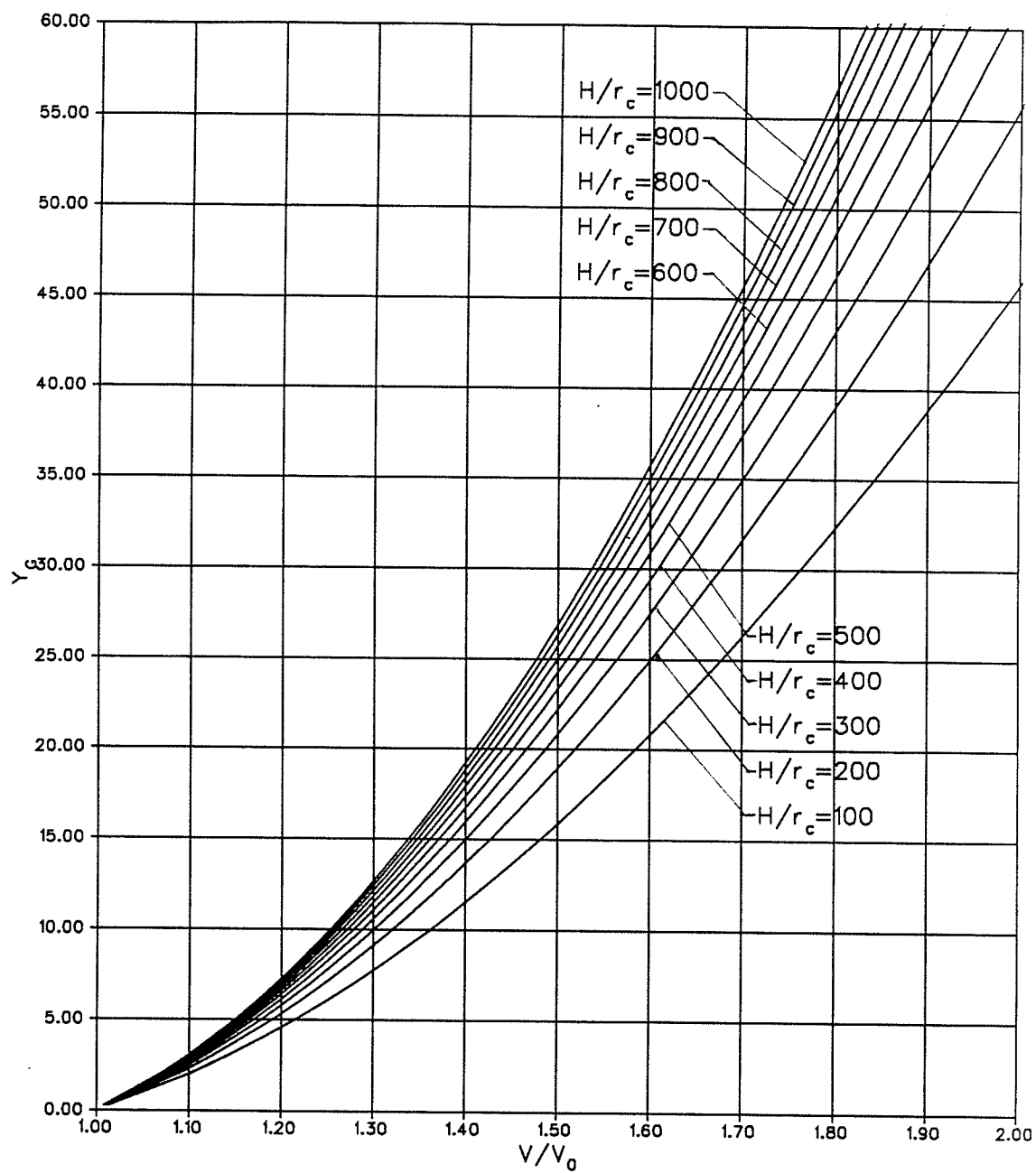


Fig.3.11 Generalized corona characteristics for smooth cylinder above plane for different configurations

$$t = [1 - (\frac{r_c}{H})^2]^{1/2} ,$$

$$q_1^2 = 1 - \frac{G_{\infty,V}}{Gf_1^2} \frac{Y_G}{1+Y_G} ,$$

$$q_2^2 = 1 + \left[\frac{r_c}{(1+t)H} \right]^{\sqrt{2}} \frac{G_{\infty,V}}{Gf_1^2} \frac{Y_G}{1 - (\frac{G_{\infty,V}}{Gf_1^2} - 1)Y_G} ,$$

$$q_3 = 1 - q_1 , \quad q_4 = 1 + q_2 , \quad q_5 = 1 + q_1 , \quad q_6 = q_2 - 1$$

For practical configurations where $H/r_c \gg 1$ and $V/V_0 \leq 3$, eq. 3.38 can be further simplified by considering,

$$t \approx 1, \quad q_1 \approx \frac{1}{(1+Y_G)^{1/2}}, \quad q_2 \approx 1, \quad q_6 \approx \frac{1}{2} \left(\frac{r_c}{2H} \right)^{\sqrt{2}}$$

Accordingly, a modified expression for the voltage-current characteristic which has the same general form as that of Townsend[23], Popkov[24], Ciric and Kuffel[12] is given by,

$$g(V/V_0) \frac{V-V_0}{V_0} \ln \frac{2H}{r_c} = (1+Y_G)^{1/2} - 1 + \ln \frac{2}{1+(1+Y_G)^{1/2}} \quad (3.39)$$

where $g(V/V_0) = \frac{1}{\sqrt{2}} \frac{V/f_1 - V_0}{V - V_0}$ and Y_G is given by eq 3.34 . The accuracy of the expression 3.39 is within $\pm 1\%$ relative to eq. 3.32, for $100 \leq H/r_c \leq 2000$ and $V/V_0 \leq 2.5$.

The function G in eq. 3.34 as mentioned before is dependent on both the line parameters H and r_c and the corona intensity or overvoltage level V/V_0 . The range of variation is between 0 and an upper limit $G_{\infty,V}$ that is dependent on the corona intensity as shown in table 3.1 . In Popkov's expression , eq. 3.19, an empirical constant $P=1.65$ appears instead of the the function G . Also instead of the function $g(V/V_0)$ in the left side of eq. 3.39 , Popkov introduced an empirical

factor of $0.41P$ in the left side of eq. 3.18. These are the reasons for which the numerical value related to P has to be varied, as observed by Simpson [27], for different ranges of the geometry and intensity of corona in order to bring the numerical results closer to the experimental ones.

It is to be noted that the empirical factor $P=1.65$ was introduced by Popkov [24,25] on the basis of studies for geometries of $H/r_c \geq 56$ and voltage levels ranging from $V/V_0 \approx 1.1$ to $V/V_0 \approx 1.8$. As observed earlier for large ratios of H/r_c , the function G becomes almost independent of the line geometry and is only a function of the applied voltage level. When the values of $G_{\infty,V}$, given in Table 3.1, are averaged in the range $V/V_0 = 1.1$ to 1.8 a value of $G \approx 1.66$ is obtained. These are the reasons for empirical constant value of $P = 1.65$ found by Popkov to be suitable for different ranges of geometries and intensities of corona.

3.3 Application to Practical Configurations

Practical transmission-line conductors are normally of stranded construction and often contains surface irregularities such as strand lift, nicks, scratches, grease, dirt, etc. Such irregularities distort the field in the immediate vicinity of the conductor and reduce the corona onset voltage. The reduction in the corona onset voltages due to such irregularities depends on the increase in the conductor surface gradient as well as on the actual field distribution near the conductor. In practice, the influence of the various surface irregularities cannot be separated and it is only possible to obtain an overall reduction factor by testing actual conductor samples.

Stone[39] investigated the reduction in ac corona onset voltage due to stranding by testing aluminum tubes wrapped with a single layer of strands. The stranding factor obtained by Stone ranged from 0.88 to 0.98. Experiments conducted by Janischewskyj [40] on an actual sample of a new stranded conductor in a cylindrical cage under both ac and dc excitation gave values for the stranding factor of

about 0.8. Experiments on test lines carried out by Simpson [27] indicated that the value of the overall reduction factor for the corona onset voltage could be about 0.7 .

In calculation of corona losses, conductor surface irregularities may be taken into account by replacing for the actual conductor an equivalent smooth conductor of the same diameter with a reduced corona onset voltage. Sarma and Janischewskyj [30] modified Popkov's formula, eqns. 3.18 and 3.19, to include the effect of surface irregularities in the calculation of corona losses. They assumed that the coronating conductor surface gradient will also be reduced by the same factor as that of the corona onset voltage. Accordingly, eq. 3.19 is modified to have the form,

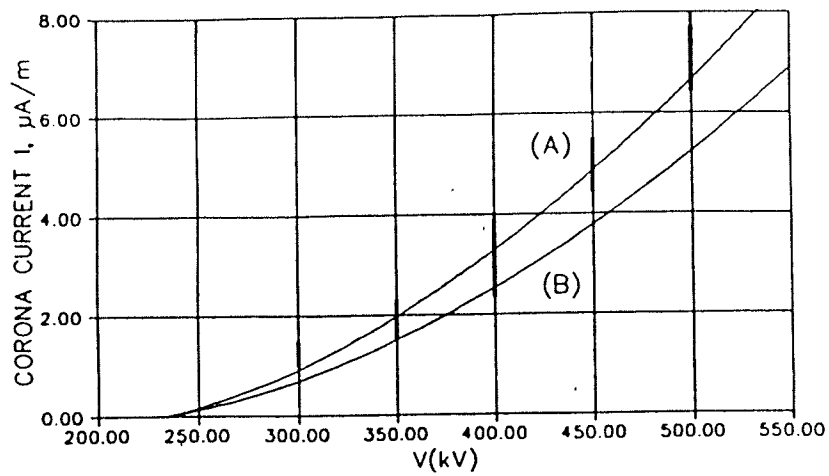
$$Y_P = \frac{PI}{2\pi\epsilon_0 km} \left[\frac{H}{E_0 r_c} \right]^2 \quad (3.40)$$

where m is the factor by which the onset voltage is reduced due to surface irregularities.

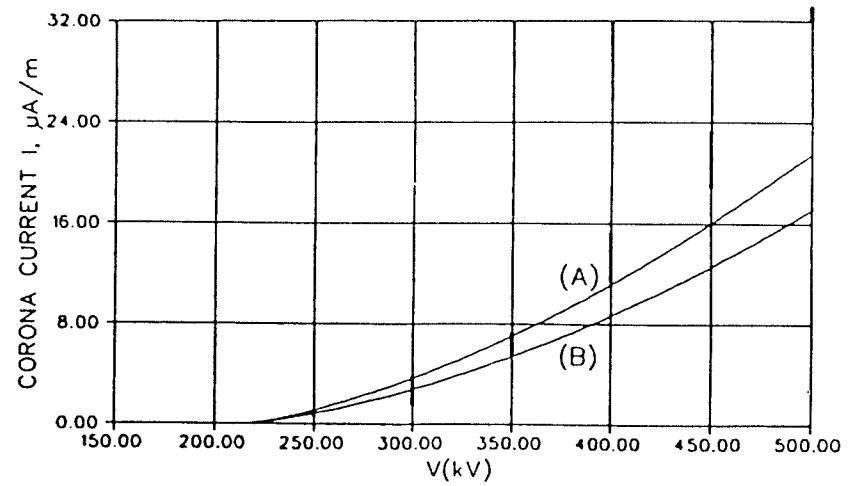
In order to include the influence of surface irregularities in the present analysis, the actual conductor is replaced by an equivalent smooth conductor of the same diameter with the onset voltage reduced by the reduction factor m . Eq. 3.34 is modified accordingly to have the following form,

$$Y_G = \frac{GI}{2\pi\epsilon_0 km} \left(\frac{H}{E_0 r_0} \right)^2, \quad (3.41)$$

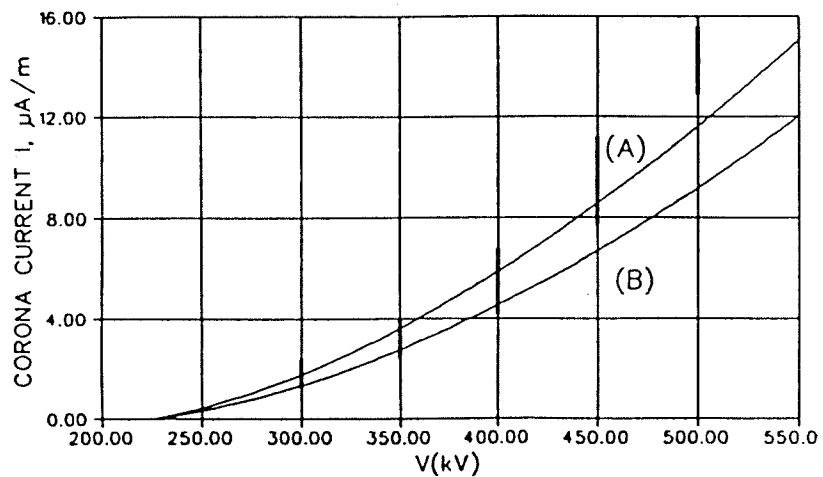
The experimental results for test lines using a Lark conductor[27] are compared with the theoretical calculations using Popkov's modified formula[24,30], eqns 3.18 and 3.40, as well as the method presented, eqns 3.39 and 3.41. Figs. 3.12a-3.12c show the results for different heights and for a positive corona. Fig. 3.12d shows the results for a negative corona. An average value of $m=0.75$ is used



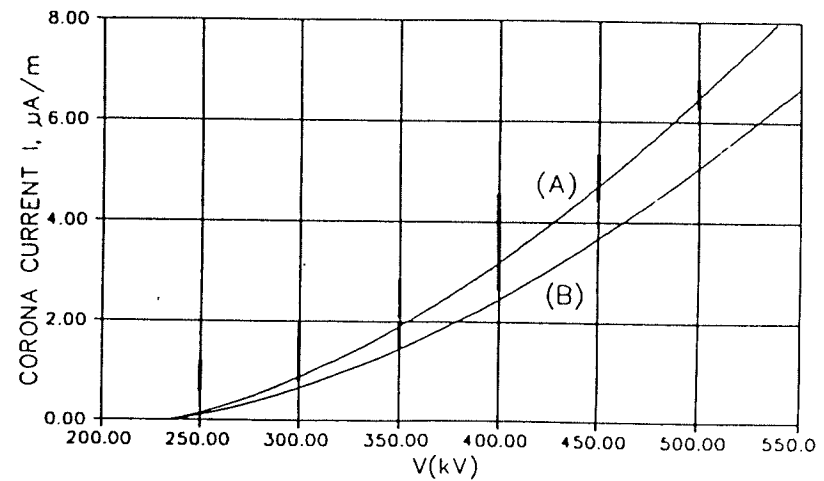
(a) $r_c = 1.02\text{cm}$, $H = 9.217\text{m}$



(c) $r_c = 1.02\text{cm}$, $H = 5.36\text{m}$



(b) $r_c = 1.02\text{cm}$, $H = 7.13\text{m}$



(d) $r_c = 1.02\text{cm}$, $H = 9.35\text{m}$

Fig.3.12 Corona characteristics for lines with a stranded conductor

A- Present method

B- Modified Popkov formula, [25,30]

I - Experimental data [27]

in the calculations based on the previous experimental observations of Simpson[27] and Janischewskyj[38]. The mobility of both positive and negative ions is taken as $1.5 \times 10^{-4} m^2/V.sec.$ [26,27,30]. It is observed that the theoretical results, obtained using the presented method, and experimental data are generally in good agreement for both large and small conductor heights and for both positive and negative coronas.

It should be noted that in modifying Popkov's formula, Sarma and Janischewskyj[30] assumed that the effect of the conductor surface irregularities, rather than the value of P , is responsible for the large differences observed by Simpson[27] between the experimental results and Popkov's formula. Accordingly to account for these differences, they used a value of 0.7 for the reduction factor m , while assuming that P is constant[30]. The present analysis for a smooth cylinder above ground plane showed that, actually, P is not a constant and its value has to be changed, as suggested by Simpson[27], depending not only on the line parameters but also on the corona intensity.

As mentioned before the method developed in the previous sections for computing the unipolar V-I characteristics employs the assumption of Deutsch that only the magnitude and not the direction of the electric field is influenced by the corona generated space charge. In addition, it was assumed that the ionic current continuity be satisfied on average on any cylindrical surface around the conductor periphery and not at any point. Consequently, the expressions for the spacial distributions of the ionized field quantities such as the current density along the ground plane can not be used locally in a correct way; but only in integral form for example to determine the voltage-current relationship as described before.

Sarma and Janischewskyj[30] used Deutsch's assumption, with different set of boundary conditions, to compute the point-to-point variation of the charge and

current density around the conductor and in the interelectrode space. The results of their iterative algorithm showed that the current density distribution around the conductor surface is almost cardioid and varies from a maximum just beneath the conductor surface to a minimum at the top of the conductor. Similar variation has also been suggested by D'Amore et al[28], and Abdelsalam and Khalifa [14].

Determination of the spatial distributions of the ionized field quantities such as the current density along the ground plane is important for hvdc line design due to the associated environmental effects. In the following section investigation into the influence of Deutsch's assumption on the ground level current density profiles is presented. The distortion of the ion flow trajectories due to the corona generated space charge is examined. Modified analytical expressions are suggested for accurate assessment of the right-of-way current density profiles.

3.4 Ground Level Current Density Profiles

Deutsch's assumption, stating that the presence of space charge in the interelectrode space does not alter the shape of the flux lines pattern, was previously employed by many investigators in order to simplify the otherwise cumbersome computational procedure of solving the unipolar corona equations[30,31,42]. However, it has been recognized, recently, that such an assumption could have a significant influence on the distributions of the ionized field parameters even at low overvoltage levels[14,17,41,42,43,44].

In their migration towards the ground plane, the ions will follow the trajectories Ω_{ND} that coincide with those of the electrostatic flux lines pattern Ω_D , in agreement with Deutsch's assumption, only in symmetrical geometries. However, in case of assymetric configurations such as a line above a ground plane, Atten[44] pointed out that the angular deviation between the two patterns may take significant values and Deutsch's assumption would not be actually valid. In

addition, for a given point in space, such deviation may depend on and increase with ionization intensity with the maximum probably occurring for the saturated case[44,45].

If it is assumed that the variation of the current density is cardioid-like around the conductor surface[14,28,43], then,

$$J_{ND}(\theta) = J_{ND}(0)(1 + \cos\theta)/2 \quad (3.42)$$

where θ is the angle between the tangent to the flux line and the center line at the conductor surface. In the following, the subscript D refers to quantities corresponding to Deutsch's assumption while the subscript ND refers to quantities without the Deutsch's assumption.

Consider dI_{ND} and dI_D to be the currents carried by the elementary flux tubes subtended at the conductor surface by $d\theta_{ND}$ and $d\theta_D$ respectively and both intercept the same elementary distance dx on the ground plane. Assuming eq. 3.42 to be valid for both cases, then,

$$\frac{dI_D(x)}{dI_{ND}(x)} = \frac{J_D(x)}{J_{ND}(x)} = \frac{(1 + \cos\theta_D)}{(1 + \cos\theta_{ND})} \frac{d\theta_D}{d\theta_{ND}} \quad (3.43)$$

where $J_D(x)$ and $J_{ND}(x)$ are the current densities along the ground plane.

Along the trajectories $\Omega_D(x)$, it can be shown that[36],

$$\theta_D = 2 \tan^{-1}(x/H) \quad (3.44)$$

Generalizing eq. 3.44, then along the trajectories $\Omega_{ND}(x)$,

$$\theta_{ND} = C \tan^{-1}(x/H) \quad (3.45)$$

The value of the parameter C depends on the degree of distortion of the flux line pattern, and consequently on the intensity of corona. For example, for the electrostatic case $C=2$ which is also the case of Deutsch's assumption. However, in reality θ_{ND} differs from θ_D with the amount of deviation being dependent on the value of

the parameter C or the ionization intensity.

By taking the derivatives of eqns. 3.44 and 3.45 with respect to x , it can be shown that

$$\frac{d\theta_D}{d\theta_{ND}} = \frac{2}{C} \quad (3.46)$$

As $x \rightarrow 0$ when $\theta \rightarrow 0$ then,

$$\frac{J_D(0)}{J_{ND}(0)} = \frac{d\theta_D}{d\theta_{ND}} \quad (3.47)$$

Eq. 3.43 can, thus, be written as

$$\frac{J_D(x)}{J_{ND}(x)} = \frac{1 + \cos[2\tan^{-1}(x/H)]}{1 + \cos[C \tan^{-1}(x/H)]} \frac{J_D(0)}{J_{ND}(0)} \quad (3.48)$$

Gallo and Lama[17] carried out an approximate analysis for the unipolar wire-to-plane corona regarding the lateral distributions of the current along the plane. Using a simple electrostatic model, they assumed the ionization layer surrounding the corona wire to be an equipotential surface that extends a distance about 0.7 of the interelectrode space H . Although they obtained, generally, good agreement with their experimental data, Gallo and Lama[17] pointed out that this large hypothetical equipotential surface can not be taken to correspond to the thin ionization layer around the coronating wire. In their analysis to include the space charge effects, the angular dependence of the space charge density was taken to yield cylindrical equipotential surfaces concentric with the wire. In fact it is not physically possible for it corresponds to a cylindrical equipotential surface touching the ground plane[17]. Their experimental studies revealed that the space charge presence in the interelectrode space seems to contract the lateral distribution of the ground level current density towards the centre line. Aboelsaad[47] computed the ionized field quantities using an isoparametric finite element

algorithm. Mapping of the flux lines , showed that the space charge presence causes distortion of the ions trajectories from those corresponding to the electrostatic case. The ionized flux lines were shown to be pushed away from the centre line close to the conductor and are contracted towards it near the ground plane. Hara et al.[46] observed that the beamwidth of the current density distribution at the ground plane, defined as the total lateral distance subtended at one half of the peak current, is approximately $1.16 H$. On the other hand, Gallo and Lama[17] obtained a value of $1.4 H$ while Popkov[24] reported a value of $1.09 H$. In the following section the variation of the coefficient C , its dependence on the ionization level, and its influence on the ground level current density profiles are studied.

3.4.1 Variation of the Parameter C

As mentioned earlier, the distortion of the ions trajectories would be maximum, as pointed out by Atten[44], in the saturation case. Sigmond[45] carried out an analysis for the unipolar dc ionized field in the saturation case and showed that the current density along the ground plane, for $\theta \leq 120^\circ$, can be approximated by the following expression,

$$\frac{J_{SAT}(x)}{J_{SAT}(0)} = [1 + 2(x/H)^2]^{-3/2} \quad (3.49)$$

where $J_{SAT}(x)$ and $J_{SAT}(0)$ are the current density along the ground plane in the saturation case. On the other hand, Hara et al[46], using Sarma's iterative algorithm that is based on Deutsch's assumption [30], showed that the current density at the ground level will be given by,

$$\frac{J_D(x)}{J_D(0)} = [1 + (x/H)^2]^{-2.04} \quad (3.50)$$

Combining eqs. 3.48, 3.49, and 3.50 yields,

$$\frac{1+\cos\theta_D}{1+\cos\theta_{SAT}} = \frac{[1+(x/H)^2]^{-2.04}}{[1+2(x/H)^2]^{-3/2}} \quad (3.51)$$

Then,

$$\theta_{SAT} = \cos^{-1} \left[\frac{[1+(x/H)^2]^{2.04}}{[1+2(x/H)^2]^{3/2}} (1+\cos\theta_D) - 1 \right] \quad (3.52)$$

Eq. 3.52 represents the relation between the angle θ_{SAT} , for the saturation case, and the angle θ_D , for the Deutsch case, around the conductor periphery. Plot of this relation is shown in fig. 3.13. This proves the angular deviation of the flux lines between the ionized case, corresponding to θ_{SAT} , and the Deutsch's case, corresponding to θ_D . This deviation seems to cause a contraction in the flux lines pattern; i.e. if both Ω_D and Ω_{ND} are to originate from the same point on the conductor surface, then the flux lines corresponding to Ω_{ND} are contracted towards the center line in agreement with previous observations by Abdelsalam and Khalifa[14], Gallo and Lama [17] and Aboelsaad [47]. The space charge presence seems to make the conductor appear larger and that is equivalent to bringing the conductor closer to the ground plane. Consequently, on the basis of eq. 3.44 a contraction of the flux lines towards the centre line would occur. Inspection of fig. 3.13 also shows that the amount of deviation between the two patterns varies around the conductor surface with the maximum approximately given by $\theta_{SAT} = 1.5 \theta_D$. It can be shown, using eqs. 3.44 and 3.45, that this maximum corresponds to a value of the parameter $C \approx 3$. In the present analysis this value of C is taken to represent the upper limit on its variation. The lower limit is assumed to be given by $C=2$ which corresponds to the case when the distortion by space charge is neglected, i.e. the Deutsch's case.

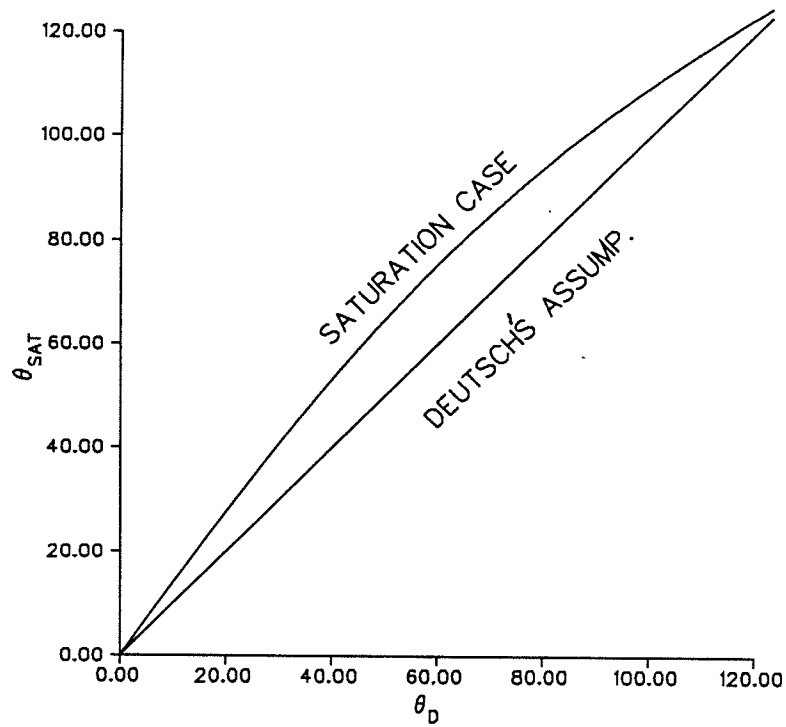


Fig. 3.13 Relation between angular deviations in the saturation case θ_{SAT} and Deutsch's case θ_D .

As mentioned before the degree of distortion of the flux lines pattern , represented by the parameter C , is dependent on the ionization intensity or the overvoltage level V/V_0 . However, due to the nonlinear nature of the governing equations, the functional dependence of C on the overvoltage level V/V_0 can not be determined exactly and can only be expressed in an approximate form.

The current density at the ground level in the Deutsch case, $J_D(0)$, is obtained using the analysis presented in section 3.2 and is given by,

$$J_D(0) = \frac{8 I}{3\pi H} \quad (3.53)$$

where the corona current I can be obtained using eqs. 3.34 and 3.39.

Various studies have been reported in literature regarding the mobility decrease during the ion migration towards the ground plane [24,48,49]. In these studies, it was observed that the aged ions tend to reach a constant mobility beyond a certain lifetime after the emission instant. Accordingly, the ion mobility is considered constant along the ground plane. The results obtained by Popkov[24] were found to fit on the average the values obtained by Arai and Tsunoda[48] and Vereshchagin and Litvinov[49] over all the life time of ions. A value for the ion mobility of $1.3 \text{ m}^2/\text{V}\cdot\text{sec}$ as obtained by Popkov is used in the present calculations.

3.4.2 Profile Assessment

In order to study the ground level current density distributions the following expression, that has been generally accepted for profile assessment[43,45,46], is used,

$$J_{ND}(x) = J_{ND}(0) [1 + (x/H)^2]^{-n} \quad (3.54)$$

where n is determined using eq. 3.48, and is given by,

$$n = 2.04 + \frac{\ln \frac{[1 + \cos 2 \tan^{-1}(x/H)]}{[1 + \cos C \tan^{-1}(x/H)]}}{\ln[1 + (x/H)^2]} \quad (3.55)$$

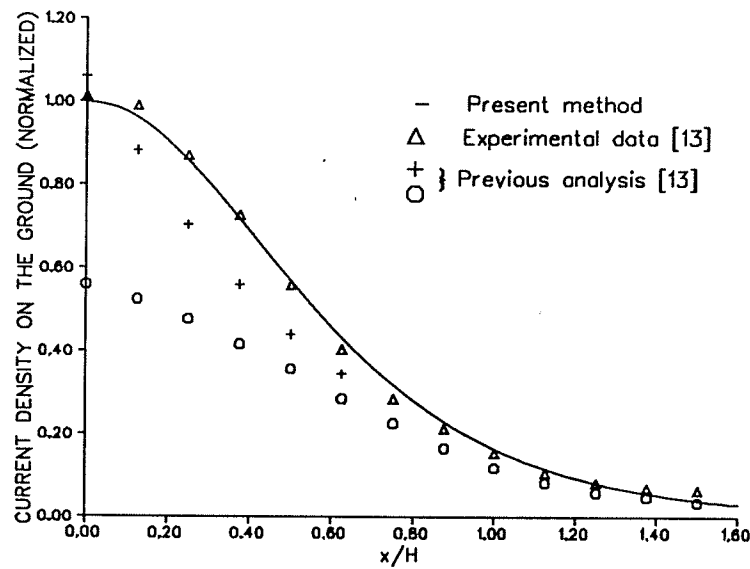
It can be seen that from eq. 3.55 that, for $C > 2$, the value of the exponent n using the present procedure will be larger than the value of $n = 2.04$ obtained previously by Hara et al[46] on the basis of Deutsch's assumption. Hara et al[46] in their analysis showed that a value of $n = 2.40$ will fit on the average their observed experimental data.

Finally, from eqs. 3.46, 3.47 and 3.54 the modified expression for the current density profile along the ground plane is given by,

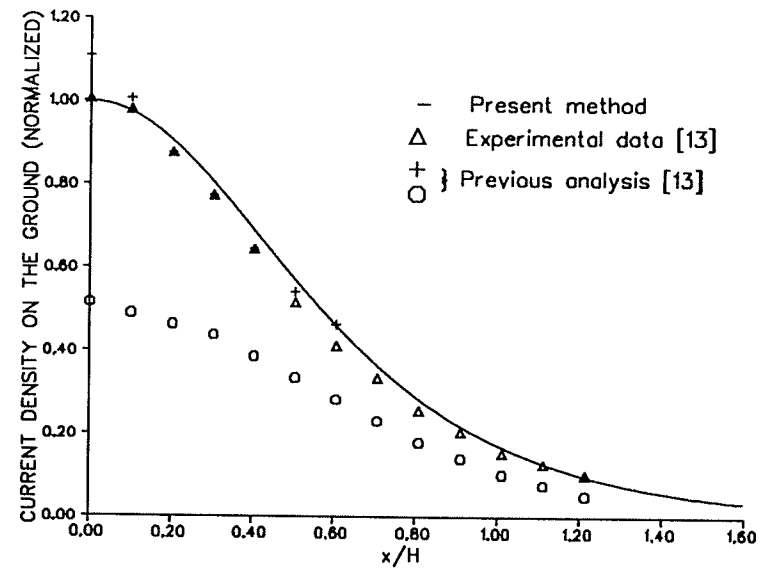
$$J_{ND}(x) = J_D(0) [C/2] [1 + (x/H)^2]^{-n} \quad (3.56)$$

Eq. 3.56 is used for the profile assessment; with n to be obtained from eq. 3.55 corresponding to the value of x/H . The parameter C characterizing the ionization intensity or the degree of distortion of the flux lines pattern is assumed to vary linearly, in the range $V = V_0$ to $V = 2V_0$, between the upper and lower limits given before.

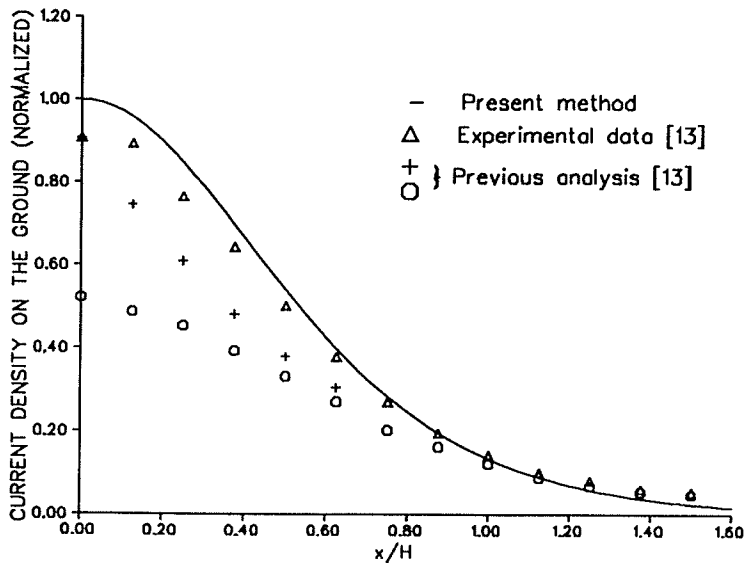
The procedure developed in this section are applied for assessment of the current density profiles of different geometries and intensities of corona. The results for different cases of positive corona are shown in figs. 3.14a-3.14d in normalized form. The theoretically calculated results are generally in good agreement with measured data in comparison with previous computational analysis [13]. It is observed that the agreement is better for lower voltages i.e. lower corona intensity, figs. 3.14a and 3.14c, than at higher voltages, figs. 3.14b and 3.14d. This may be due to the simplification used in the present analysis that the parameter C , characterizing the corona intensity, is a linear function of the applied voltage while it may actually vary in a more complicated form. However, the developed



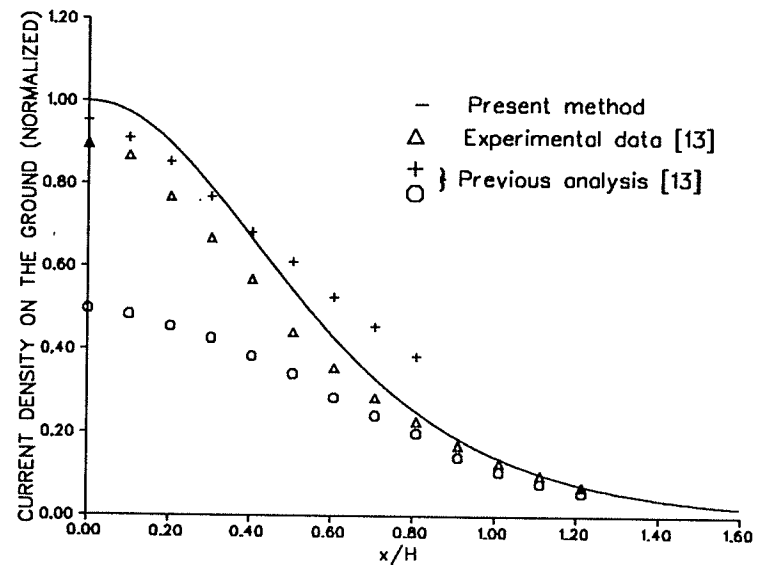
a- $r=0.00165$ m, $H=0.40$ m, $V=80$ kV.



c- $r=0.00165$ m, $H=0.495$ m, $V=80$ kV.



b- $r=0.00165$ m, $H=0.40$ m, $V=90$ kV.



d- $r=0.00165$ m, $H=0.495$ m, $V=90$ kV.

Fig. 3.14 Normalized current density profiles along the ground

approximate expressions can be used with sufficient accuracy, as observed above, for the assessment of the right-of-way current density profiles of hvdc lines.

3.5 Discussion and Conclusions

In this chapter, theoretical analysis into the influence of the main simplifying assumptions; namely the Kaptzov and Deutsch assumptions is presented. An improved method that takes into consideration the influence of Kaptzov assumption is developed. The method is applied, initially, to the case of coaxial cylindrical configuration then extended to the two dimensional case of a smooth cylinder above a ground plane. Improved expressions for the practical case of stranded conductors are also derived. The theoretically calculated results are found to be, generally, in good agreement with available experimental data in all cases. Comparison with the modified Popkov empirical formula shows the results obtained using the present expressions to be in better agreement with experimental data. Study into the influence of Deutsch's assumption in relation to the flux lines pattern distortion is also carried out. Modified approximate expressions are derived that can be used with sufficient accuracy for the assessment of the right-of-way current density profiles of hvdc lines. The agreement of the calculated results with measured data is satisfactory as compared with other published computational results.

CHAPTER 4

ITERATIVE SOLUTION OF THE UNIPOLAR DC CORONA EQUATIONS (NUMERICAL TREATMENT)

The main objective of the present work is, as mentioned earlier, the development of methods for solution of the unipolar ionized field problem, without resorting to simplifying assumptions regarding the interaction among the fundamental field quantities. In chapter 3, analytical procedures were constructed and applied initially to the simple coaxial cylinder geometry. They were then extended to the geometry for a smooth wire above ground, and also to the case of a stranded conductor above ground. In the present chapter, a different approach is followed. The solution of the corona equations is performed using computational rather than analytical procedures. The unipolar corona equations, eqs. 2.1-2.4, are reduced to two second order partial differential equations which are then solved iteratively using the finite element method. The unknowns are the space potential ϕ and the space charge density ρ , both are functions of position in the region of interest.

Although several methods exist for solving the corona problem in the general geometry of a conductor above ground, all of them are approximate; since they employ simplifying assumptions which can significantly affect the spacial distributions of Φ , $\vec{E}$ and ρ . Accordingly, the results obtained by these methods may not reflect the true interactions among the three electrical quantities listed above. It is to be noted that the two most commonly used assumptions are Deutsch's assumption [5] which states that the equipotential-flux line pattern in presence of corona is the same as that which would have existed without corona, and Kaptzov's assumption [6], stating that the conductor surface gradient remains

constant at its onset value regardless of the level of the applied voltage level.

In the following sections, a computational algorithm for the solution of the unipolar corona equations is developed and applied to the geometry of coaxial cylinders. The boundary conditions required to determine the solution are discussed. The algorithm procedures, which are based on the finite element method, are developed and described. Various initial space charge distribution are considered, and the numerical aspects regarding the accuracy, efficiency, and convergence of the iterative process are studied.

4.1 Mathematical Model

As mentioned earlier in chapter 2, unipolar corona is characterized by the flow of ions of one polarity in the interelectrode space, with the ions polarity being the same as the conductor in corona. Eqns. 2.1-2.4 describe the electric field and ionic flow of unipolar corona. From eq. 2.1 and 2.4, the following equation is obtained,

$$-\nabla \cdot (\nabla \Phi) = \frac{\rho}{\epsilon_0} \quad (4.1)$$

and from eqs. 2.2, 2.3, and 2.4,

$$-\nabla \cdot (\rho \nabla \Phi) = 0 \quad (4.2)$$

where the potential Φ and the charge density ρ are functions of the space coordinates. Eqns. 4.1 and 4.2 are coupled differential equations and are equivalent to the systems of equations 2.1-2.4. Thus, simultaneous solution of eqs. 4.1 and 4.2 for the potential Φ and the charge density ρ , subject to proper boundary conditions, provides the solution to the unipolar corona problem.

Inspection of eqs 4.1 and 4.2 shows that they can, in general, be used to determine the potential Φ without great difficulties using proper procedures and boundary conditions. Eq. 4.2 can be used to determine the charge density ρ

provided Φ is known everywhere in the region of interest. However, this approach is not practical since Φ is known usually numerically only at discrete points. Also, determination of ρ from eq. 4.1 requires numerical differentiation which is known to yield significant errors in the results[15,33]. In addition, since eq. 4.2 is homogeneous, it can not be used directly to determine ρ . For example, in the one dimensional case, eq. 4.2 is expressed as,

$$\rho \frac{d\Phi}{dr} = c = \text{const.} \quad (4.3)$$

where the constant c is to be found from boundary values of both the charge density ρ and the electric field E at some point. While the value of E at the conductor surface can be estimated as given in section 3.1, the boundary value of ρ is not known. If it were, then the computation of the corona losses would be trivial. The aim of the present work, as mentioned earlier, is towards assessing the influence of the Deutsch and Kaptzov assumptions on the ionized field characteristics. In view of the above discussion, an approach based on solving eqs. 4.1 and 4.2 for Φ while modifying the charge density ρ continuously after each solution step, depending on the most recent value of Φ , is implemented in the present analysis.

4.2 Boundary Conditions

The equations governing unipolar corona can be reduced to a nonlinear third order differential equation, eq. 2.6, in the potential Φ as shown earlier in chapter 2. Three boundary conditions are, thus, required to determine the solution for the boundary value problem described by eqs. 4.1 and 4.2. The two boundary conditions usually employed are,

- 1- The potential on the coronating conductor is known,

$$\Phi = V \quad (4.4)$$

2- The potential on the noncoronating conductor, ground plane, is known,

$$\Phi = 0 \quad (4.5)$$

An important point in the ionized field computation is the choice of the third boundary condition to be imposed at the coronating conductor surface. In literature, there are four kinds of boundary conditions suggested by various researchers[8,11,13,15,16,24,50]. These are,

- (i) The electric field E_c is assumed constant at the onset value E_0 .
- (ii) The charge density ρ_c is given as a constant.
- (iii) The current density J_c is given as a constant.
- (iv) The current density J_c is given as an empirical function of the electric field at the coronating conductor surface.

Recent studies by Takuma and Kawamoto[50], and Ciric and Kuffel[11] showed that the boundary conditions employed at the conductor surface may have a significant influence on the ionized field characteristics. Waters et al.[9,10], Popkov et al.[21], and Abdelsalam et al.[13], showed that the boundary condition (i) is not actually realistic and that the electric field at the conductor surface drops below the onset value depending on the applied voltage level. The rigorous theoretical analysis carried out earlier in section 3.1 and 3.2 for the coaxial as well as the single line above ground geometries, for smooth conductors, is in agreement with these findings.

Regarding the boundary condition(ii), computations carried out by Sunaga[16] for a single line above ground plane, using the finite difference method, showed that the charge density is constant around the conductor periphery. In another study, Janischewskyj and Gela[15,33], using a finite element algorithm for the single line above ground configuration, obtained a distribution

for the charge density that varies from a maximum just beneath the conductor to a minimum at the top. Although, employing Deutsch's assumption, similar variation has also been obtained by Sarma and Janischewskyj[30,31] and D'Amore et al.[28]. Ciric and Kuffel[12], on the other hand, obtained an analytical expression for the charge density distribution that is very nearly constant around the conductor surface.

The boundary conditions (iii) and (iv) assume the knowledge of the current density at the coronating conductor surface. In reference[11], the boundary condition(iii) was used only for a parametric study of the influence of various boundary conditions on the ionized field quantities. The boundary condition(iv) was proposed by Sunaga et al.[7,8] based on measurements carried out on a test line. As pointed out by Sunaga, the empirical formula used is only valid for lines which have the same configuration and weather conditions as those of the test line. This restricted the applicability of their procedures.

Based on the above discussion, and the results obtained earlier in chapter 3, the third boundary condition employed in the present analysis is as follows,

- 3- The electric field at the coronating conductor surface , E_c , is assumed to decrease as the applied voltage increases. The value of E_c is to be determined from eqs. 3.4 and 3.16 at the respective overvoltage level.

4.3 The Solution Method

Various approaches have been reported in literature to solve the boundary value problem described by eqs. 4.1 and 4.2 . Deutsch's assumption regarding the influence of space charge presence on the flux lines pattern has been employed in many of these approaches. The influence of Deutsch's assumption can be illustrated as follows: The field due to the corona generated space charge at any point in the interelectrode space is decomposed into two components; one along the flux

line passing through the point and another in the normal direction. While the first affects only the magnitude of the electric field, the second component influences its direction. This latter component has a lateral effect on the lines of the electric field causing them to deviate angularly from their trajectories in the absence of the space charge. In approaches based on Deutsch's assumption such as those by Popkov[24,25], and Sarma and Janischewskyj[30,31] the first component was employed in the analysis, while the second component was neglected.

The use of simplifying assumptions; namely the Deutsch and Kaptzov assumptions, in the ionized field analysis has been repeatedly questioned in literature. It was suggested that they could have a significant influence on the ionized field characteristics[8,9,10,11,13,16]. For example, Sunaga et al.[16], using the finite difference method, estimated that the errors in the results caused by inclusion of Deutsch's assumption could reach up to 40% for the corona current. At the ground level, errors up to 10% in the electric field and 40% in the current density were also estimated.

No attempt has yet been fully successful in resolving the problem of corona on unipolar dc lines without resorting to the above mentioned assumptions which could significantly influence the mutual coupling between the space charge density ρ and the electric field E . In the following sections, an algorithm that is based on the finite element method, FEM, is presented to fill this gap.

The usual method of solving numerically the boundary value problem, governed by any of the equations 4.1 or 4.2, is to initially subdivide the region of interest into elements and representing the solution within each element by a relatively simple function. The result is a transformation of the differential equation into a system of algebraic equations which can be solved for the nodal values. Details of the steps carried out to solve eqs. 4.1 and 4.2 using the finite element

method are given in Appendix A.

4.3.1 The Algorithm Procedure

The algorithm is based on the simultaneous solution of eqs. 4.1 and 4.2 for the potential Φ and the space charge density ρ . The algorithm begins by generating a nonzero initial distribution for ρ . Since this distribution is not the one corresponding to the actual configuration, the solutions Φ_1 from eq. 4.1, and Φ_2 from eq. 4.2 will differ. The lack of agreement between Φ_1 and Φ_2 is due to the errors present in ρ . Accordingly, a correction term $\delta\rho$ to ρ should be related to the difference $\delta\Phi = \Phi_1 - \Phi_2$, i.e.

$$\delta\rho = f(\delta\Phi) \quad (4.6)$$

where the function $f(\delta\Phi)$ should be zero when $\delta\Phi$ is zero. Following each solution step, the charge density ρ is modified until convergence is obtained. Janischewskyj and Gela[15], for the coaxial geometry, and Abdelsalam et al.[13], for wire-plane geometry, suggested that a correction term proportional to ρ itself and to $\delta\Phi$ expressed as a fraction of the average of Φ_1 and Φ_2 may generally provide a satisfactory convergence.

In order to provide some guidelines that can be useful in the numerical procedure, the coaxial geometry is discussed first. From chapter 3, it can be shown that all the electrical quantities are functions of the radial coordinate r only. It follows that the distribution of space charge is a monotone decreasing function of the radial coordinate r , i.e. ρ decreases as r increases. This is also in agreement with the physical understanding of the problem provided by Felici[29]. In case the space charge density ρ is assumed constant, then eq. 4.2 reduces to Laplace's equation. The electric field at the conductor surface E_{2c} will then have the electrostatic value. Consequently, the quantity E_{2c} , obtained from the solution of eq. 4.2, can be used to yield information mainly on the shape or the slope of the

distribution of ρ . On the other hand, studies carried out by Gela and Janischewskyj[15,33] as well as in the present work, showed that the electric field E_{1c} of eq. 4.1 will provide information primarily on the magnitude or height of the distribution of ρ . These observations should be useful for correcting the distribution of ρ based on the discrepancies between E_{1c} and E_{2c} from the prespecified third boundary condition E_c .

The correction of ρ , although not based on rigorous mathematical equations relating potential to space charge, can however be established following the above mentioned guidelines. Different approaches can be employed. Abdelsalam et al.[13], for a wire-plane geometry, used an updating formula that is based on the discrepancies between Φ_1 and Φ_2 only. In their analysis, however, they employed a boundary condition that was obtained using both Deutsch and Kaptzov assumptions. Accordingly their analysis is not free from these assumptions. On the other hand, Gela [33], for a similar geometry, used a correction formula that was based only on the deviations of E_{1c} and E_{2c} from the specified boundary condition, which was obtained using Kaptzov's assumption. No intersets was shown, in this case, to the discrepancies between the potentials Φ_1 and Φ_2 . In view of the above discussion, an updating formula for the space charge density is constructed as follows:

$$\rho^{new} = f(E_{1c}, E_{2c}, \rho_c, \rho^{old}) + g(\Phi_1, \Phi_2, \rho^{old}) \quad (4.7)$$

where ρ^{new} is the updated volume charge density at the node and ρ^{old} is the charge density at the same node used in the previous iteration. E_{1c} and E_{2c} are the electric field intensities determined from equations 4.1 and 4.2, respectively, at the node on the conductor surface lying on the same field line as the node considered, and ρ_c is the volume charge density at this node on the coronating conductor surface. For the unipolar DC field case, the functions f and g are chosen to have the

general form as in [13,33] and given by,

$$f = \left[\frac{E_{1c}}{E_c} \right]^a \rho_c \left[\frac{\rho^{old}}{\rho_c^{old}} \right] \left(\frac{E_{2c}}{E_c} \right)^b, \quad (4.8)$$

$$g = c e^{\frac{V - \Phi_{avg}}{dV}} \left[\frac{\Phi_1 - \Phi_2}{\Phi_{avg}} \right]^2 \rho^{old} \quad (4.9)$$

in which V is the applied voltage, Φ_{av} is the arithmetic average of Φ_1 and Φ_2 , E_c is the electric field at the conductor in corona and a, b, c, d are constants to be determined, by trial and error, so that convergence of the iterative process is as rapid as possible. The updating criteria (4.7)-(4.9) reflects the fact that the space charge distribution is not affected by the conductor surface gradient alone or the local potential alone, but is actually affected by their combined effect. This is an improvement over previous techniques [13,33], where only the effect of the corona onset potential gradient alone, or that of the local potential alone, were taken into account. Gela[33], for a wire-plane geometry, used a correction formula that is similar to the function f while neglecting the discrepancies between Φ_1 and Φ_2 expressed by the function g . On the other hand, Abdelsalam et al.[13] used a criteria that is, generally, similar to the function g and based on the potential differences only.

Following the first iteration, which uses the initial distribution of ρ , the FEM solution of equations 4.1 and 4.2 is obtained again by using the new distribution ρ^{new} . The iterative process is performed by continuously inserting into the updating criteria (4.7)-(4.9) the most recent values of all the corresponding quantities in order to obtain the new space charge distribution at a node. It can be seen that upon convergence, i.e. when $\Phi_1 \approx \Phi_2$ and $E_{1c} \approx E_{2c} \approx E_c$, the correction to ρ tends to vanish and $\rho^{new} \approx \rho^{old}$.

Another feature of the updating criteria is that; provided that the initial ρ is monotonically decreasing from the coronating conductor surface, the final distribution will also follow the same pattern. This can be readily seen from eq. 4.8. In order not to influence this pattern, the contribution from eq. 4.9 is kept as small as possible by using a value for the parameter c in the order of 0.1. In fact, it was observed that after the first few iterations, this part of the updating formula plays a minor role in modifying the space charge distribution. In order to ensure an accurate solution for all the field quantities, this process is continued until the following conditions are satisfied simultaneously at all the nodes:

$$|\Phi_1 - \Phi_2| \leq \tau_1 \Phi_{av}, \quad (4.10)$$

$$|\rho^{new} - \rho^{old}| \leq \tau_2 \rho^{old}, \quad (4.11)$$

at the same time, the electric field at the coronating conductor surface is observed during the iterative procedure so that,

$$|E_{1c} - E_{2c}| \leq \tau_3 E_{1c,2c}, \quad (4.12)$$

in which τ_1 , τ_2 and τ_3 are small deviations specified in terms of the desired accuracy. $E_{1c,2c}$ is the average value of E_{1c} and E_{2c} . At a given voltage, the value of E_c in the updating criteria 4.7-4.9 is obtained using eqs. 3.4 and 3.16.

4.4 Application to the Case of Coaxial Cylinder Geometry

The application of the algorithm to the configuration of coaxial cylinder geometry is demonstrated here using a specific example. The arrangement selected is shown in fig. 4.1a, and the finite element mesh is shown in fig. 4.1b. The inner conductor, of radius $r_c = 0.1 \text{ cm}$, is energized to a voltage $V = 2 V_0$, and the outer cylinder, of radius $r_g = 2 \text{ cm}$, is grounded. The electric field at the energized conductor is obtained using eqs. 3.4 and 3.16 and is found to be 62.37 Kv/cm. The exact distributions for the ionized field quantities, subject to the above

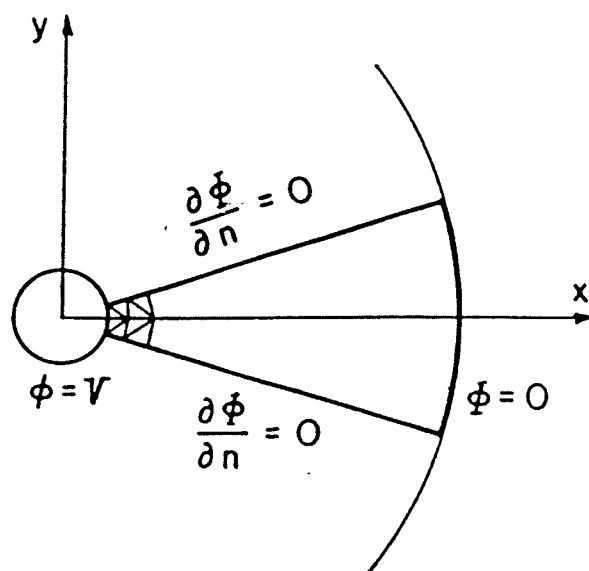


Fig. 4.1a The coaxial cylinder geometry

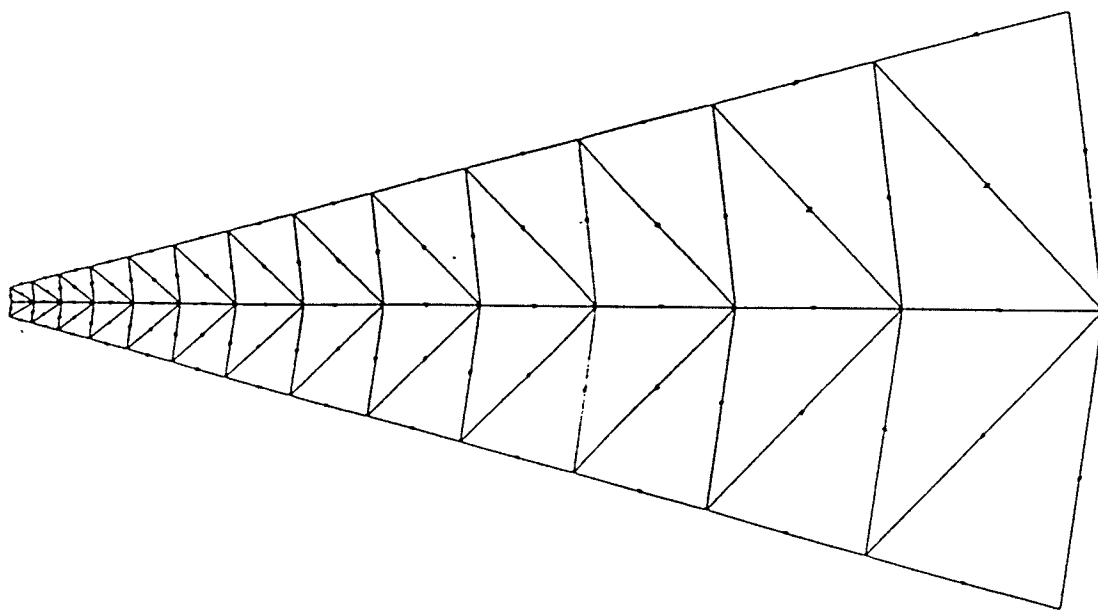


Fig. 4.1b The finite element mesh

conditions are obtained as follows[15]. The governing equations 4.1 and 4.2 when expressed in cylindrical coordinates become,

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{d\Phi}{dr} \right) = \frac{\rho}{\epsilon_0} \quad (4.13)$$

$$\frac{1}{r} \frac{d}{dr} (r \rho \frac{d\Phi}{dr}) = 0 \quad (4.14)$$

Integrating eq. 4.14, and assuming that at the inner cylinder, $E=E_c$ and $\rho=\rho_c$, then,

$$r \frac{d\Phi}{dr} = \frac{r_c \rho_c E_c}{\rho} \quad (4.15)$$

Substituting in eq. 4.13 and integrating, the following expression for the charge density in the radial direction is obtained,

$$\rho(r) = \sqrt{\frac{\epsilon_0 E_c r_c \rho_c^2}{\epsilon_0 E_c r_c + \rho_c (r^2 - r_c^2)}} \quad (4.16)$$

Substituting eq. 4.16 into eq. 4.15, the electric field is obtained as,

$$E(r) = \frac{1}{r} \sqrt{\frac{r_c \rho_c E_c}{\epsilon_0}} \sqrt{\frac{\epsilon_0 E_c r_c}{\rho_c} + r^2 - r_c^2} \quad (4.17)$$

Define,

$$h_1 = \sqrt{\frac{r_c E_c \rho_c}{\epsilon_0}}, \quad h_2 = \sqrt{\frac{r_c \epsilon_0 E_c}{\rho_c}},$$

$$h_3 = \sqrt{h_2^2 - r_c^2}, \quad q(r) = \sqrt{h_2^2 + r^2 - r_c^2}$$

Then eq. 4.17 reduces to

$$E(r) = \frac{h_1}{r} q(r) \quad (4.17a)$$

Integrating eq. 4.17a between the limits $\Phi=0$ and $\Phi=V$ corresponding to $r=r_c$ and $r=r_g$, then

$$\int_0^V d\Phi = h_1 \int_{r_c}^{r_g} \frac{q(r)}{r} dr$$

which reduces to

$$V = h_1 \int_{r_c}^{r_g} \frac{1}{r} (h_3^2 + r^2)^{1/2} dr \quad (4.17b)$$

from[49],

$$\int \frac{(a^2 + x^2)^{1/2}}{x} dx = (a^2 + x^2)^{1/2} - a \ln \frac{a + (a^2 + x^2)^{1/2}}{x}$$

Eq. 4.17a then becomes

$$V = h_1 \left[(h_3^2 + r_g^2)^{1/2} - (h_3^2 + r_c^2)^{1/2} - h_3 \ln \frac{h_3 + (h_3^2 + r_g^2)^{1/2}}{r_g} + h_3 \ln \frac{h_3 + (h_3^2 + r_c^2)^{1/2}}{r_c} \right]$$

which reduces finally to

$$V = h_1 \left[q(r_g) - h_2 + h_3 \left\{ \ln \frac{r_g}{r_c} + \ln(h_2 + h_3) - \ln \left[h_3 + q(r_g) \right] \right\} \right] \quad (4.18)$$

where V is the applied voltage. The value of the charge density ρ_c at the coronating conductor is obtained implicitly from eq. 4.18.

Various initial distributions for the charge density, chosen intentionally to be in error as compared with the exact one, are generated as follows:

case 1 : Using the known solution of the coaxial cylinders geometry with $V=1.7V_0$ and $E_c=E_0$.

case 2 : Same as case 1 but with $V=2.0V_0$ and $E_c=E_0$.

case 3 : same as case 1 but with $V=2.3V_0$ and $E_c=E_0$.

case 4 : In this case the initial space charge distribution is obtained according to the following formula,

$$\rho(r) = \rho_c + \frac{(r-r_c)}{(r_c-r_g)}(\rho_c - \rho_g) \quad (4.19)$$

where ρ_c is obtained as in case 2 and ρ_g is obtained as in case 3.

case 5 : In this case the initial space charge distribution is obtained according to the following expression,

$$\rho(r) = \rho_c \frac{r_c}{r} \quad (4.20)$$

where ρ_c is obtained as in case 2. These distributions are shown in fig. 4.2a as ρ_{i1} , ρ_{i2} , ρ_{i3} , ρ_{i4} , ρ_{i5} respectively, together with the exact distribution ρ_{ex} . The percentage errors in these distributions as compared to the exact solution are shown in fig. 4.2b .

4.5 Computed Results and Analysis

The guidelines mentioned earlier are investigated here in details. The electric fields at the energized conductor from eqs. 4.1 and 4.2, E_{1c} and E_{2c} respectively , are computed following the first iteration for the various initial space charge distributions. The computed values are given in table 4.1 for the five different cases considered.

It can be seen from fig. 4.2a that, in the first case, the whole distribution of ρ_{i1} lies below the exact distribution ρ_{ex} . The value of the electric field at the conductor surface E_{1c} , using eq. 4.1, is 80.92 Kv/cm as shown in table 4.1 . This value is higher than the exact value of 62.37 Kv/cm, indicating that the charge distribution in this case may be weaker than it should be. At the same time, it is observed that the shape of ρ_{i1} is flatter than the exact one, i.e. it has a lesser slope. This is reflected in the fact that E_{2c} , from eq. 4.2, is 73.24 Kv/cm which is higher than the exact value.

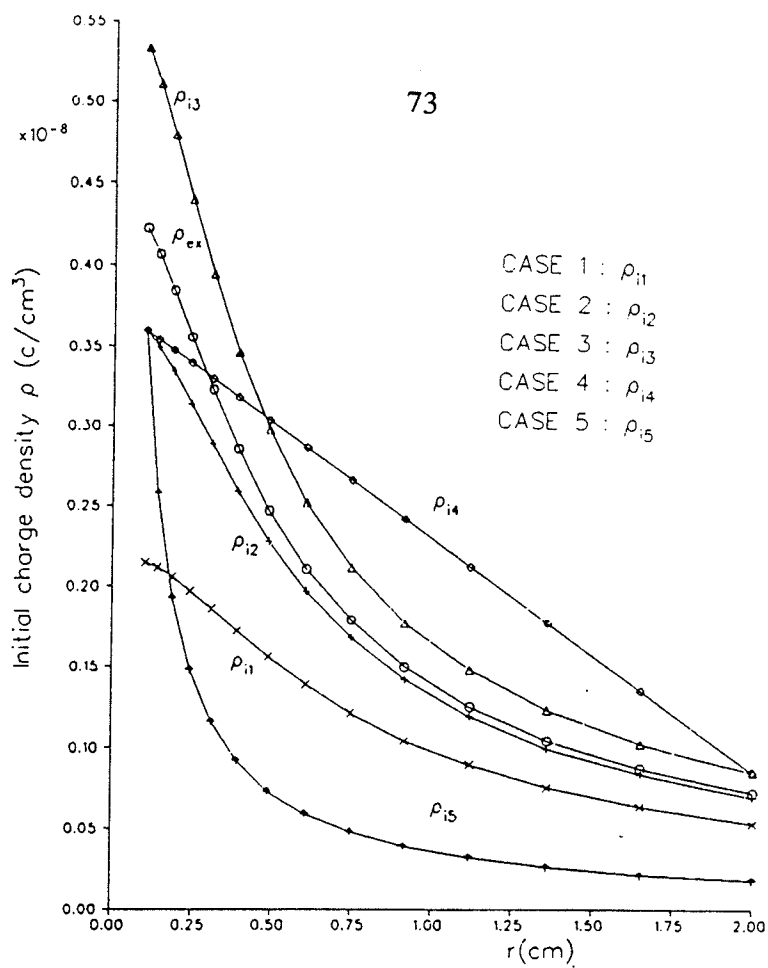


Fig. 4.2a Radial variation of the initial charge distributions

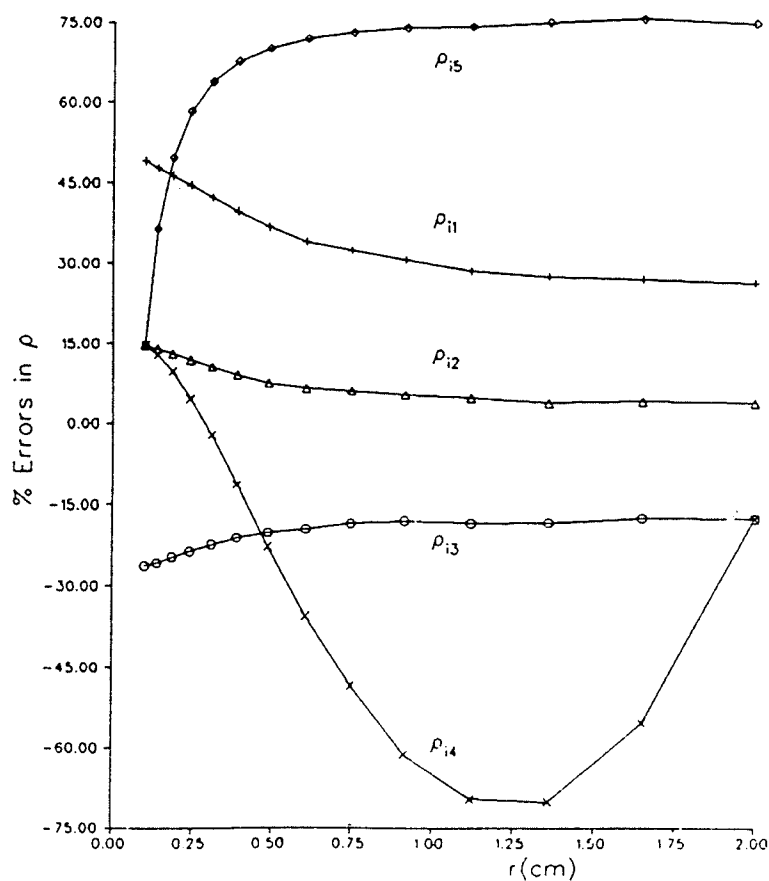


Fig. 4.2b Percentage errors in the initial charge distributions

Table 4.1 The electric field values at the energized conductor for the different initial charge distributions.

Case	Electric field, $E_{1c} (Kv/cm)$, from eq. 4.1	Electric field, $E_{2c} (Kv/cm)$, from eq. 4.2
1	80.92	73.24
2	62.59	62.56
3	44.24	54.70
4	35.60	87.21
5	102.3	21.93

For the second case, the whole distribution of ρ_{i2} lies also below ρ_{ex} but higher than ρ_{i1} . At the same time, the slope of ρ_{i2} is sharper than that of ρ_{i1} but still flatter than the exact one. These trends can be observed by comparing the electric field values E_{1c} and E_{2c} in this case with the exact value. The quantities E_{1c} and E_{2c} are both closer to the exact value as compared with the first case of ρ_{i1} . However, due to the errors present in the magnitude and shape of ρ_{i2} , both quantities are still higher than the exact value E_c .

In contrast to the above two cases, the whole distribution of ρ_{i3} lies above ρ_{ex} with a slope that is sharper than the exact one as shown in fig. 4.2a. As indicated in table 4.1, both quantities E_{1c} and E_{2c} are found to be lower than the exact value. The fourth case, ρ_{i4} , is represented by a linear variation described by eq. 4.19. The shape of the charge distribution in this case is flatter than the exact one, resulting in the value of E_{2c} being higher than the exact one. In this case, ρ_{i4} intersects the exact distribution close to the inner conductor, while almost the whole distribution lies above not only ρ_{ex} but also ρ_{i3} . The quantity E_{1c} , in this case, is less than both the exact value E_c and the quantity E_{1c} corresponding to ρ_{i3} . This demonstrates that not only charges, present in close vicinity of the energized conductor but also those located at larger distances, play a significant role in modifying the electric field at the conductor surface. Sarma and Janischewskyj[22] assumed that the electric field at the energized conductor is always constant at the onset value. They considered that the corona generated space charge is present only in the ionization layer surrounding the conductor. Waters et al.[9,10] measured the electric field strength at the coronating conductor surface and showed that it drops below the onset value depending on the applied voltage level. They attributed these observations to the fact that the space charge located further away from the coronating conductor may have more influence on modifying the field

than previously assumed by Sarma and Janischewskyj[22].

The last case is shown in fig. 4.2a as ρ_{i5} . This distribution is generated using eq. 4.20. The slope of this distribution is very sharp close to the inner conductor while being almost flat away from it. The slope of this distribution is actually larger than any of the other distributions including that of ρ_{ex} . From table 4.1, it can be seen that E_{2c} , in this case, is significantly less than not only E_c but also E_{2c} of all the other distributions. Also E_{1c} is much higher than the exact value E_c . This is a result of ρ_{i5} being very weak in comparison with ρ_{ex} as observed from fig. 4.2a.

The above trends, although generally qualitative, were nevertheless useful in obtaining some guidelines that can be used in constructing the correction formula of the charge distribution ρ as described earlier in this chapter.

The convergence procedure for the different cases considered are shown in figs. 4.3-4.7 respectively. In order to ensure high accuracy of the computed results, the parameters τ_1 , τ_2 and τ_3 in equations 4.7-4.9 are set to 0.05, .01, .015 respectively. The percentage errors in the potential values are expressed as $(\Phi_{ex} - \Phi_{avg})/\Phi_{ex}$, where Φ_{ex} is the exact distribution of the potential. The errors in the electric field values are obtained in the same manner, while the errors in the charge density are obtained as $(\rho_{ex} - \rho)/\rho_{ex}$.

The convergence procedure when using ρ_{i1} is shown in figs. 4.3a-4.3c. The initial errors in the charge distribution vary from about 50% at the inner conductor to about 27% at the outer boundary. These errors and the corresponding errors in the potential and electric field values are shown in fig. 4.3a. Convergence occurred in ten iterations in this case and the residual errors upon convergence are shown in fig. 4.3c. When ρ_{i3} is used as the initial space charge distribution, convergence occurred in four iterations. The initial errors are shown in fig. 4.4a and

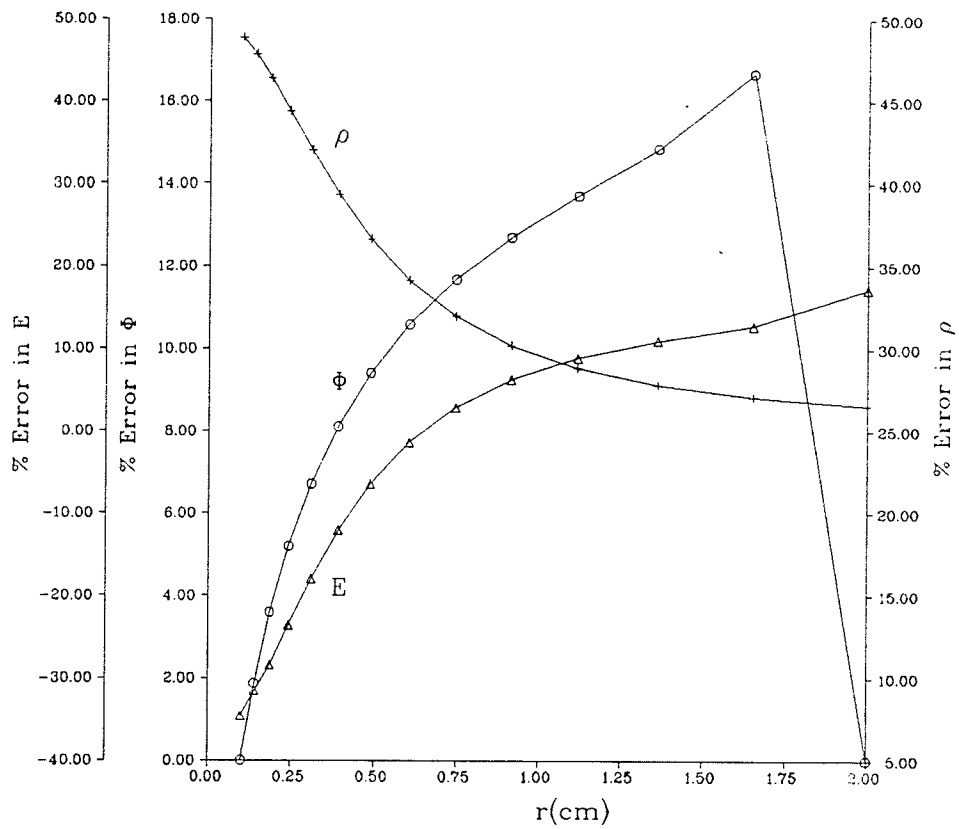


Fig.4.3a % Errors in potential, electric field,
and charge density (initial)

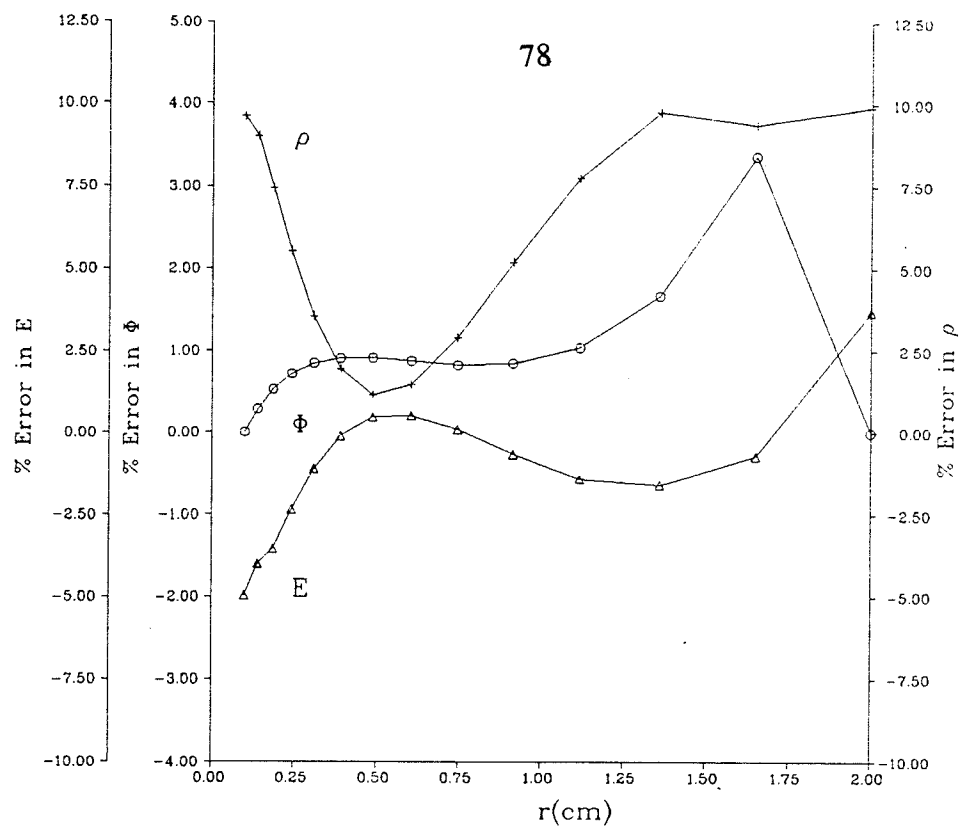


Fig.4.3b % Errors in potential, electric field,
and charge density (3 iterations)

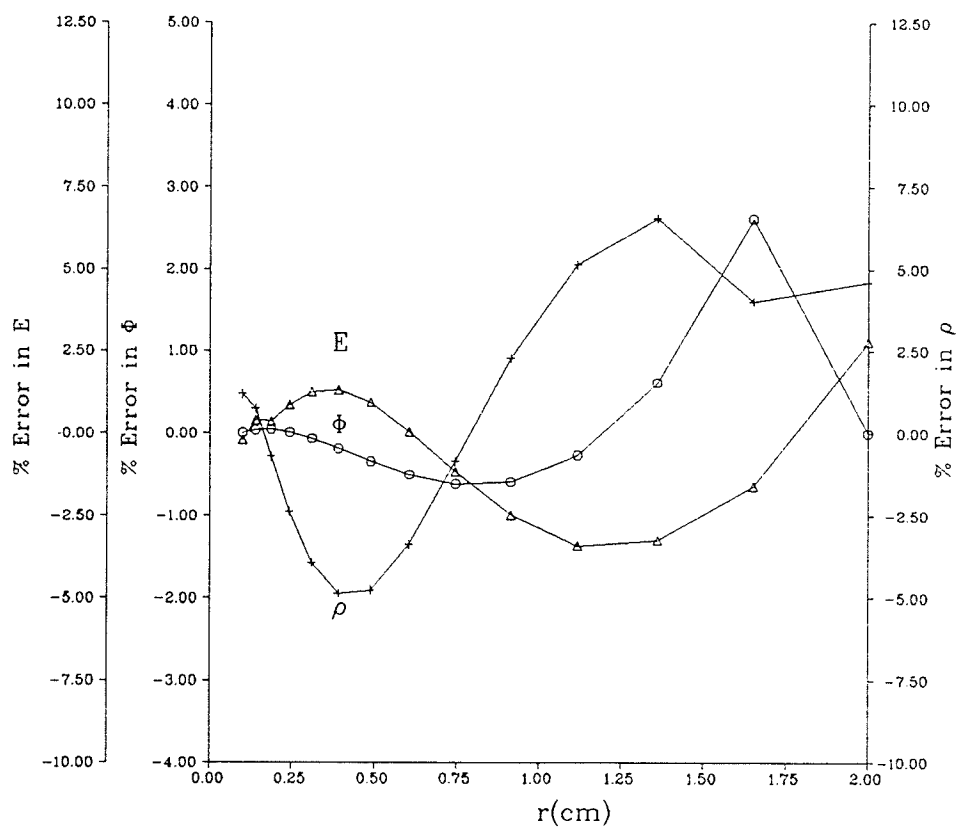


Fig.4.3c % Errors in potential, electric field,
and charge density (10 iterations)

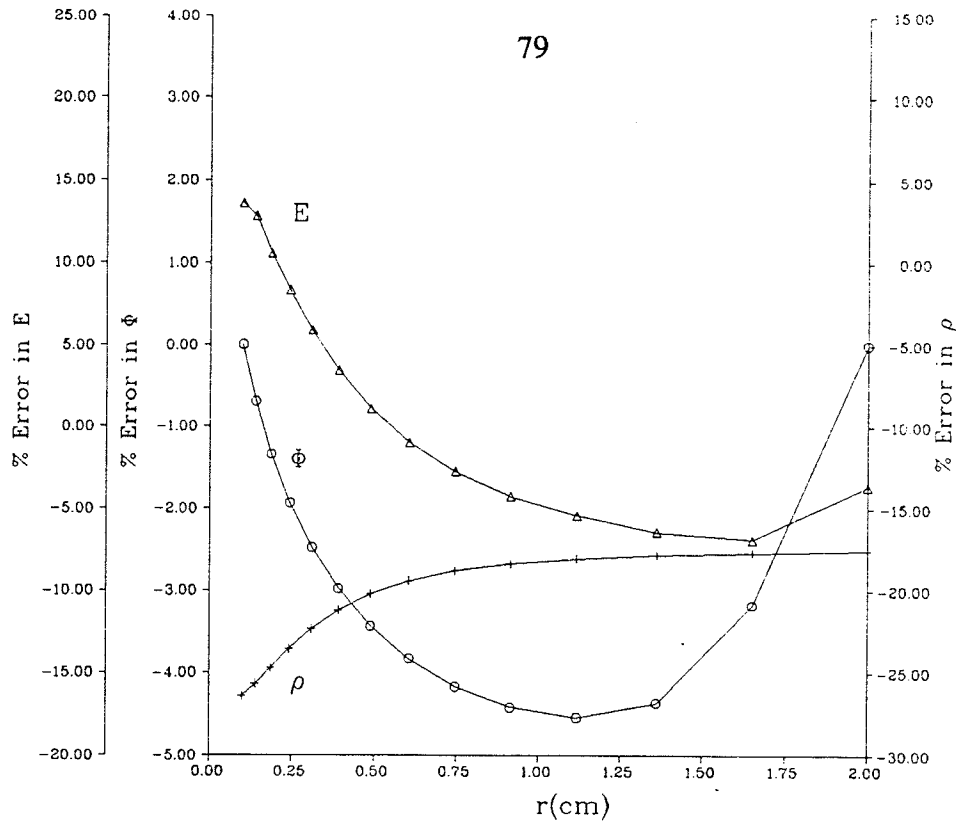


Fig.4.4a % Errors in potential, electric field, and charge density (initial)

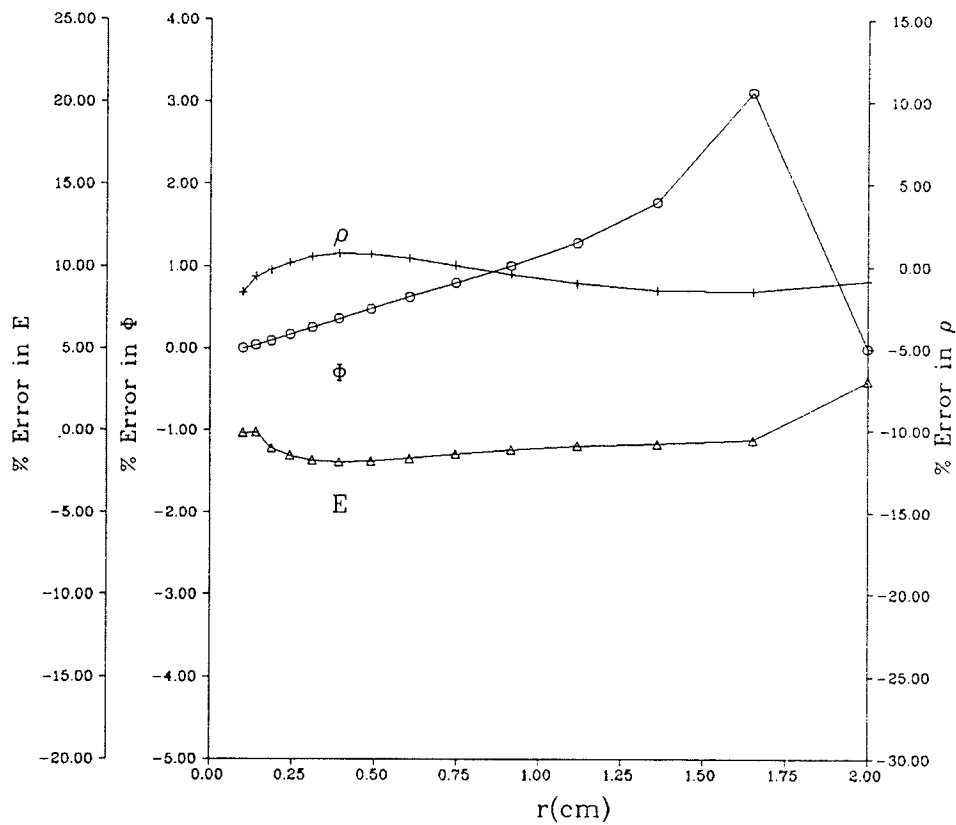


Fig.4.4b % Errors in potential, electric field, and charge density (4 iterations)

the residual errors upon convergence are shown in fig. 4.4b .

As observed earlier, the distribution ρ_{i2} lies close and below ρ_{ex} and above ρ_{i1} . As expected, the initial errors in ρ , Φ , and E shown in fig. 4.5a are less than those corresponding to ρ_{i1} . Only 3 iterations were needed in this case for the iterative algorithm to converge, which is much faster than the case of ρ_{i1} . The residual errors are also smaller in this case as observed in fig. 4.5b compared with the previous two cases. In fact, the absolute final errors in this case were less than 1.5% in the potential, 2.7% in the charge density, and 2.5% in the electric field values. This demonstrates both the accuracy and efficiency of the present computational procedure and the correction formula employed for updating the charge density ρ during the iterative process. Takuma and Kawamoto[50] has recently presented a modified method for the solution of the ionized field problem based on the finite element method. Their results, for coaxial cylinder geometry, showed errors of the current density and electric field of about 30% on the conductor surface and about 12% at the corona sheath[50]. Abdelsalam et al.[13], for a wire-plane geometry, showed that their algorithm needed 70 iterations to achieve a 5% accuracy in the potential values.

For cases 4 and 5, the convergence of the iterative procedure, is shown in figs. 4.6a-4.6d and figs. 4.7a-4.7d respectively. In spite of the significant errors, , not only in magnitude but also in shape, between the initial and exact distributions for these particular cases, the algorithm works its way through to convergence occurred in 19 and 31 iterations respectively.

In general, it can be seen that the closer the initial charge distribution is to the exact one the faster convergence occurs as observed for ρ_{i1} and ρ_{i2} . At the same time, the errors in the final results tend to be smaller when the initial distribution of ρ is close to the correct one. In all cases, the exact distribution is

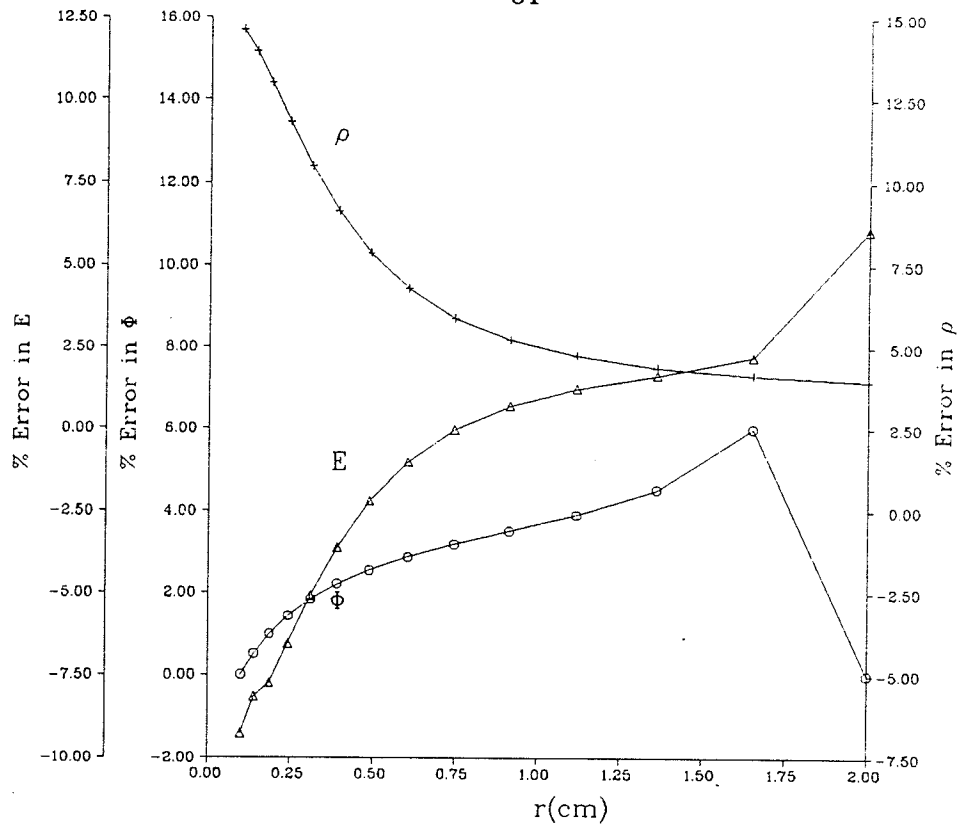


Fig.4.5a % Errors in potential, electric field, and charge density (initial)

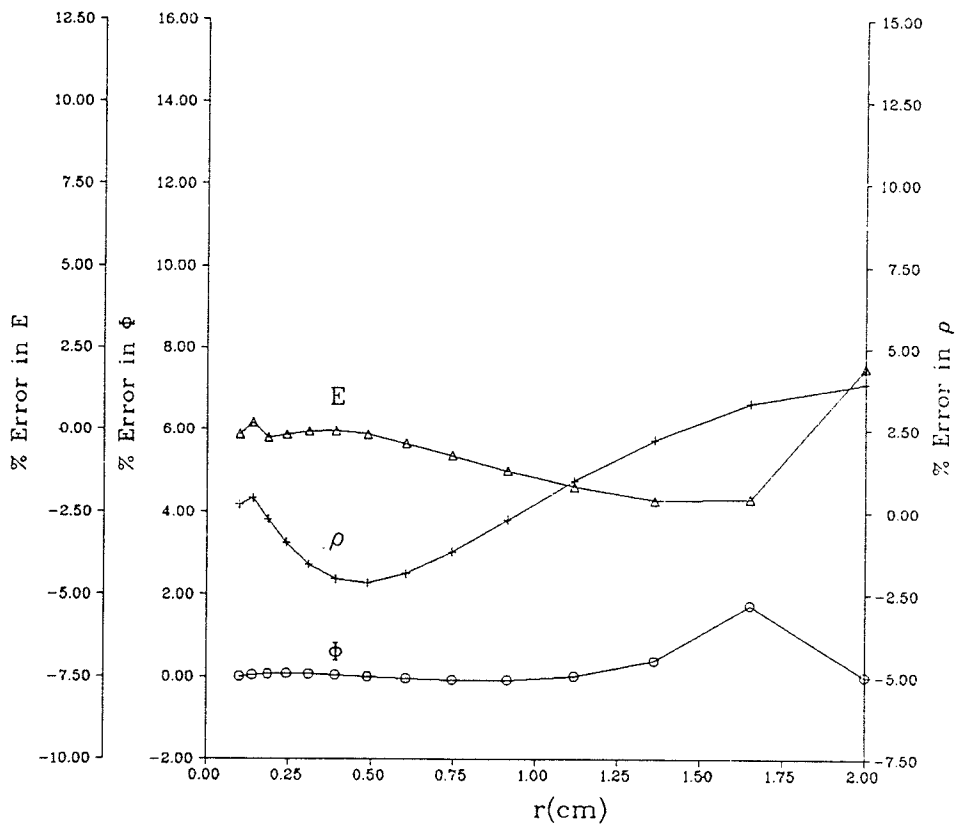


Fig.4.5b % Errors in potential, electric field, and charge density (3 iterations)

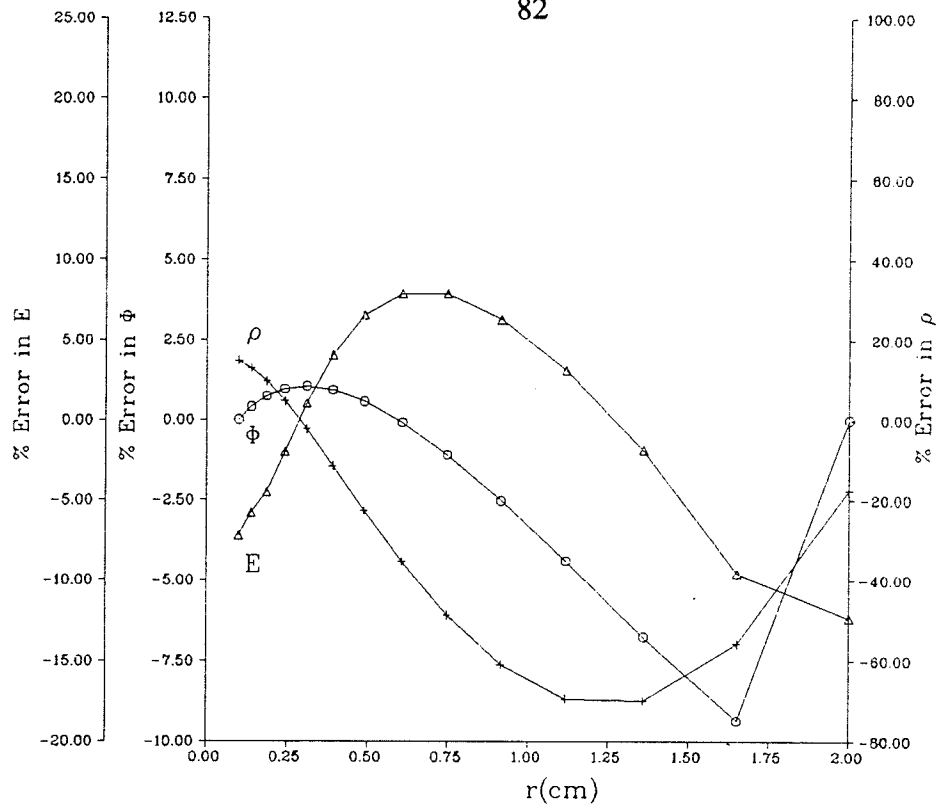


Fig.4.6a % Errors in potential, electric field,
and charge density (initial)

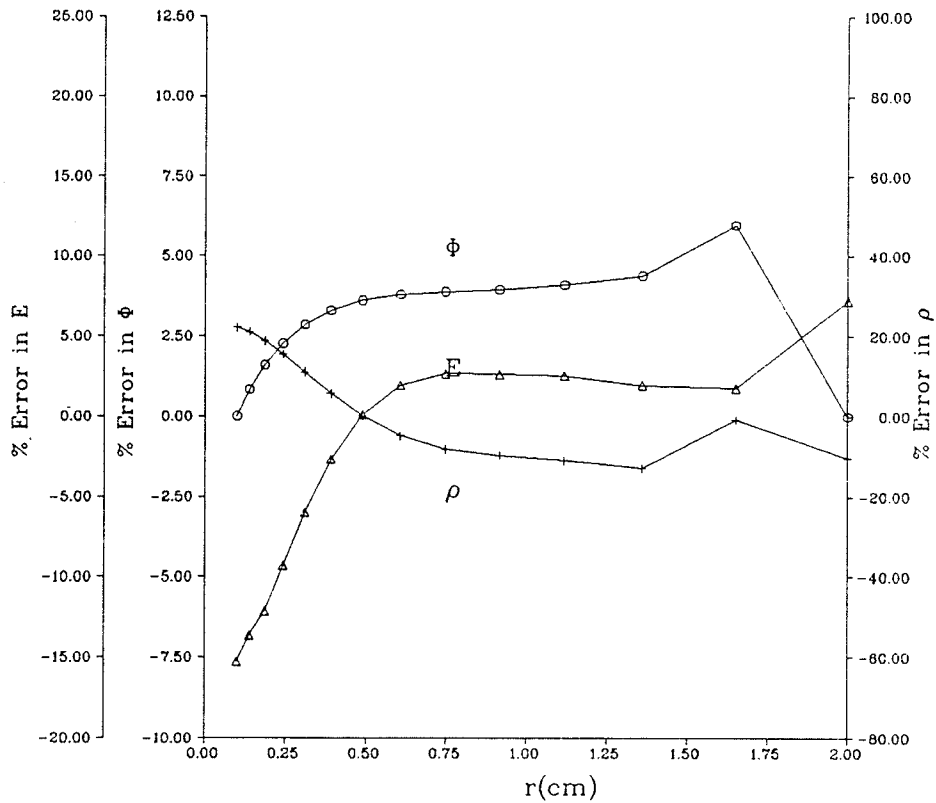


Fig.4.6b % Errors in potential, electric field,
and charge density (5 iterations)

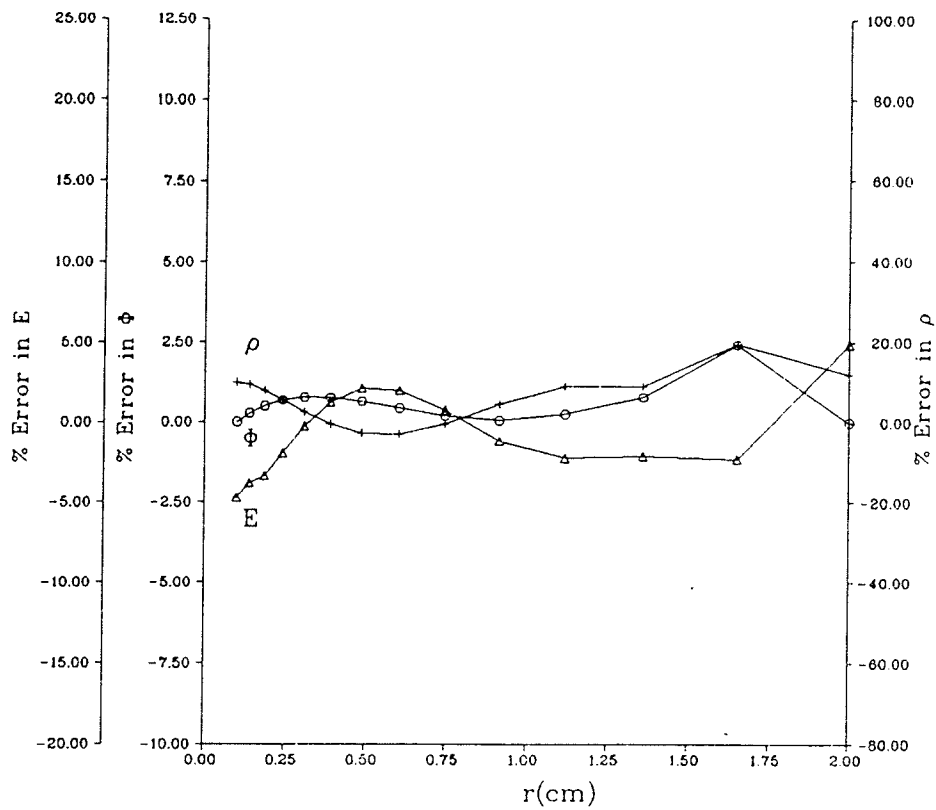


Fig.4.6c % Errors in potential, electric field, and charge density (10 iterations)

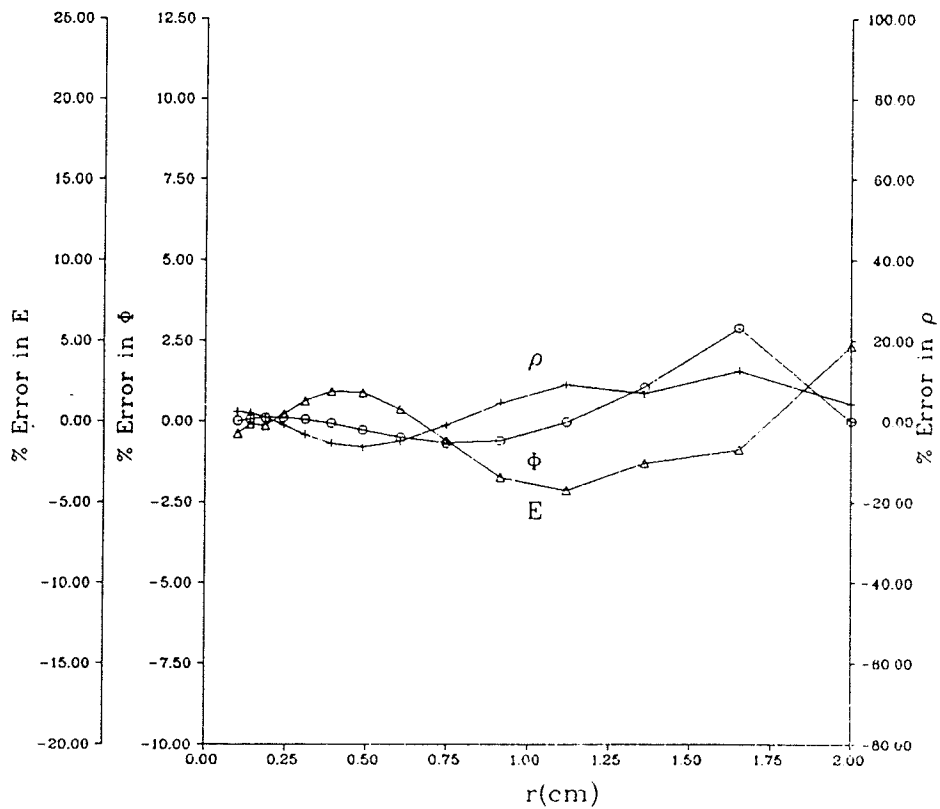


Fig.4.6d % Errors in potential, electric field, and charge density (19 iterations)

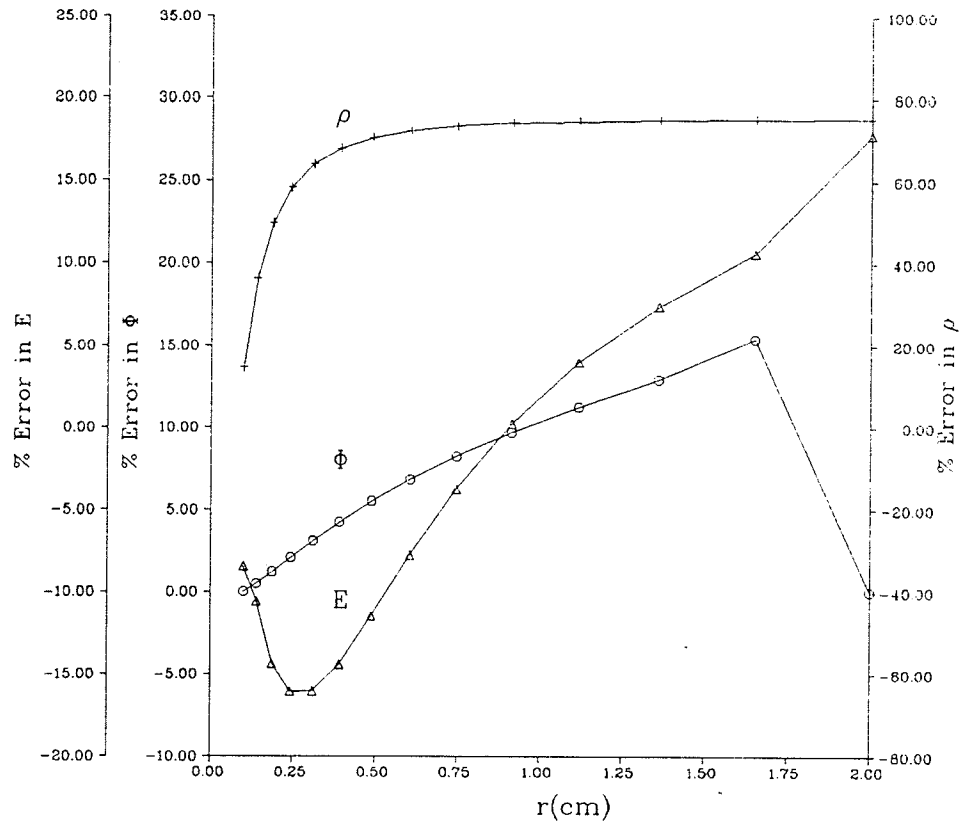


Fig.4.7a % Errors in potential, electric field,
and charge density (initial)

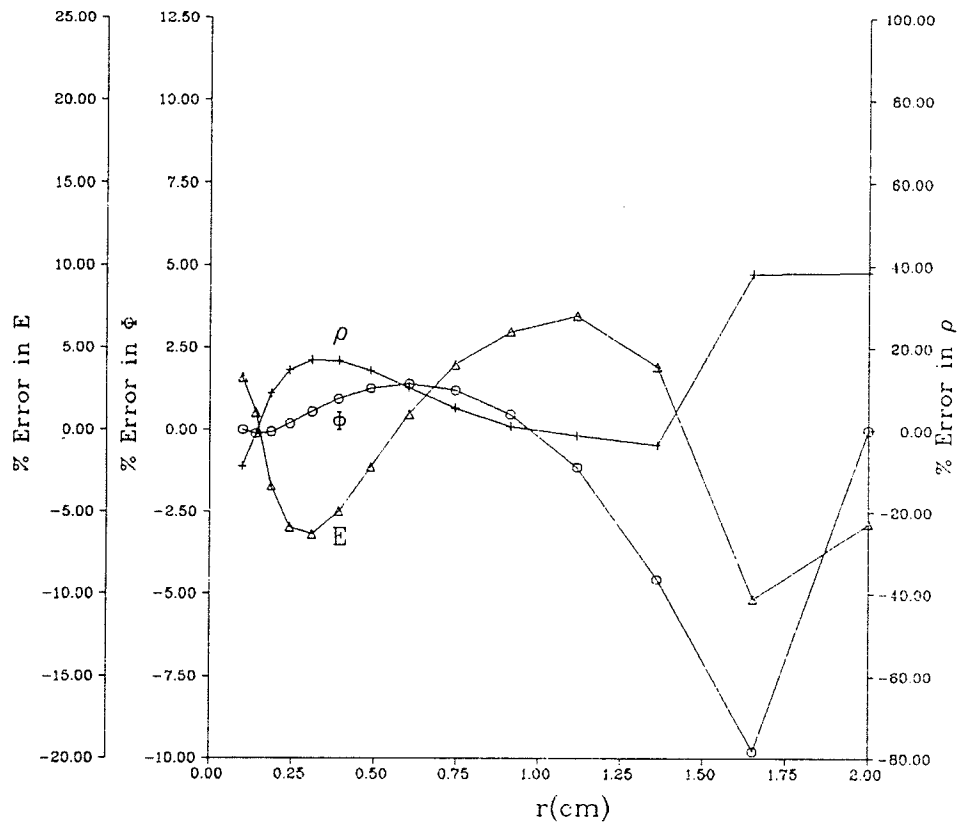


Fig.4.7b % Errors in potential, electric field,
and charge density (5 iterations)

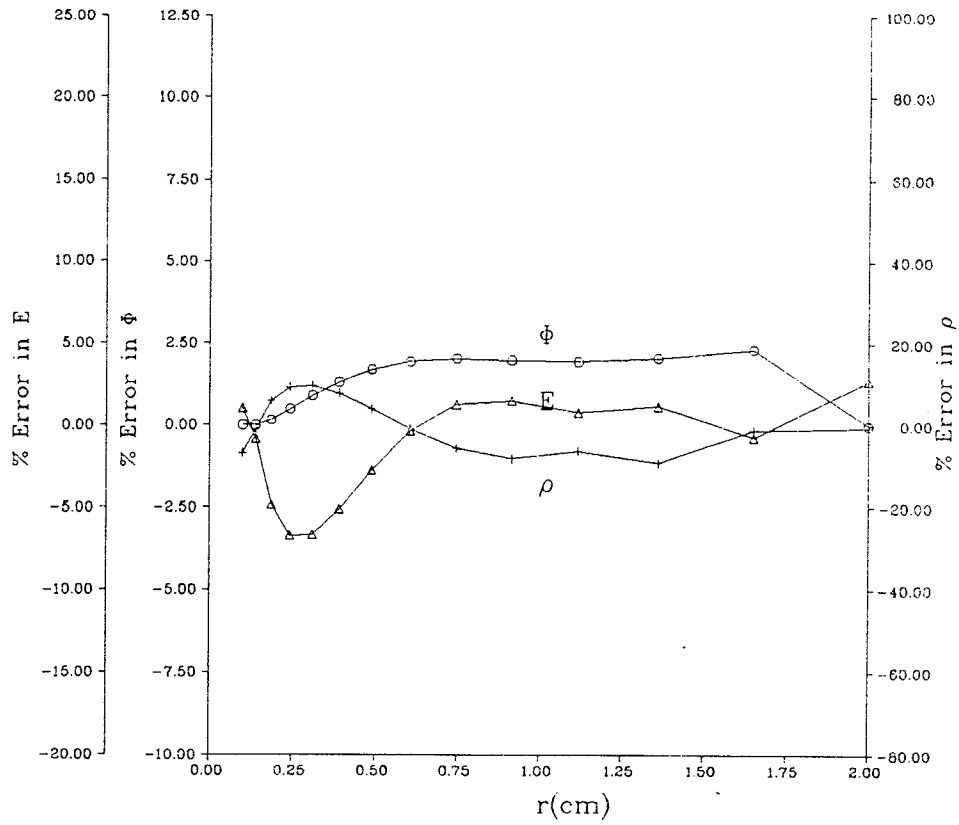


Fig.4.7c % Errors in potential, electric field,
and charge density (20 iterations)

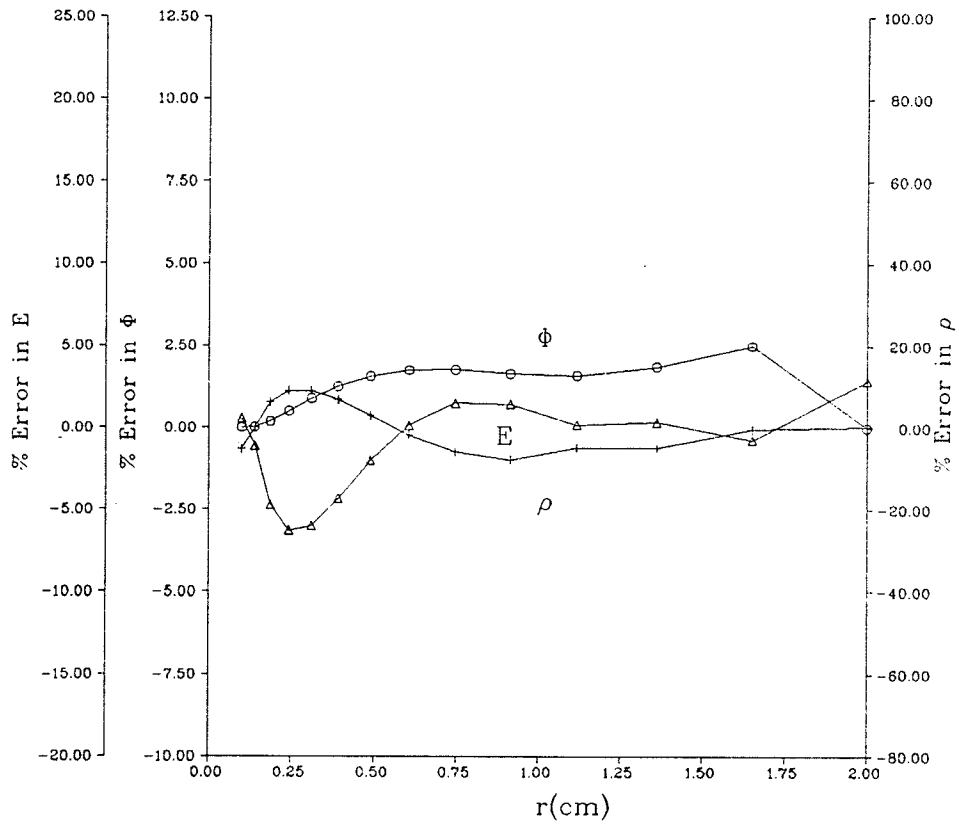


Fig.4.7d % Errors in potential, electric field,
and charge density (31 iterations)

compared with the average values for the potential and electric field values obtained from equations(4.1) and (4.2). Accordingly reliable and accurate estimates of the potential and field values for the solution of both equations may be obtained by taking the average values of the respective quantities upon convergence of the iterative process.

The values of the parameters a, b, c , and d in the updating formula, eqs. 4.7-4.9, are obtained as mentioned earlier by trial and error so that convergence occurs as rapid as possible. These parameters are shown in table 4.2 for the various cases considered. It should be noted that these parameters may not be unique and are probably problem dependent. The criteria used to determine their values is that convergence occurs in the least possible number of iterations. For example, the case corresponding to ρ_{i4} is also solved using the parameters $a=1.0$, $b=1.0$, $c=0.1$, $d=0.26$. Convergence occurred in 30 iterations in this case, with the residual errors being almost the same as before. This set of parameters is chosen to be close to those corresponding to ρ_{i1} , ρ_{i2} , and ρ_{i3} . On the other hand, for the case corresponding to ρ_{i5} , it was not possible to obtain convergence, after many trials, except for the values shown in table 4.2. Abdelsalam et al.[13], in their iterative algorithm, used an updating formula that is dependent on the potential differences only, and similar in form to the function g given by eq. 4.9. They showed that their iterative algorithm converged, with accuracy of 5% in the potential values, in 70 iterations. A parameter similar to c with a value of 0.5 is used in their calculations. No variation of this parameter with respect to the geometry, corona intensity, or initial charge distribution was reported[13]. It should be noted, however, that their algorithm was not applied to the case of cylindrical geometry. Gela[33] used an updating criteria for the wire-plane geometry that is similar to the function f , eq. 4.8, with the parameters $a=b=1.0$. No interest was shown in this case to the deviations between the potential values Φ_1 and Φ_2 expressed here by the

Table 4.2 The values of the parameters used in the updating formula, eqns. 4.8 and 4.9, for the different cases, and the corresponding number of iterations.

Case	a	b	c	d	No. of iterations
1	1.00	1.00	0.10	0.25	10
2	1.00	1.00	0.05	0.25	3
3	1.00	1.00	0.10	0.25	4
4	0.50	0.50	0.20	0.26	19
5	0.40	0.40	0.17	0.26	31

function g of eq. 4.9 . The various parameters used in different cases of in this chapter are of the same order as those used previously by Abdelsalam et al.[13] and Gela[33] . In general, however, it can be observed that if the initial charge distribution is not significantly in error, e.g. ρ_{i1} or ρ_{i3} , with respect to the exact one, a set of parameters that is similar to the one used here, can be employed to obtain convergence.

4.6 Conclusions

In this chapter, an improved algorithm for the iterative solution of the unipolar dc corona equations has been developed and applied successfully to the geometry of coaxial cylinders. The algorithm is based on the finite element method and accordingly does not employ Deutsch's assumption in the solution. The algorithm also does not rely on Kaptzov's assumption as the electric field at the coronating conductor surface is allowed to vary according to the applied voltage level in agreement with previous experimental observations[9,10,21]. The governing equations are modeled as two coupled partial differential equations which are solved simultaneously for the potential Φ and the charge density ρ . A modified updating criteria is employed for continuously modifying the charge distribution during the iterative solution. Various distributions for the initial charge density, chosen intentionally to be in error, are assumed and the algorithm worked its way through to convergence in all cases. Both the accuracy and efficiency of the present algorithm are demonstrated by comparing the computed results for the different quantities with the analytical solution and also with other FEM algorithms. The results of the present chapter should serve as a necessary and useful step for study of the more general case of a conductor above ground geometry which is to be considered in the next chapter.

CHAPTER 5

IONIZED FIELD SOLUTION OF A SINGLE CONDUCTOR ABOVE GROUND GEOMETRY

Corona, as described earlier in chapter 2 is a nonlinear voltage dependent phenomena, consequently results obtained for one voltage level can not be extended easily to other voltage levels. This is because the intensity of corona depends strongly and in nonlinear fashion, on the applied voltage which determines the characteristics of the space charge generated by ionization processes. The space charge distribution is the main factor that determines all other quantities, such as the distribution of the potential, electric field intensity and current density as well as global quantities such as corona power loss. At alternating voltages, the space charge created by corona is constrained to the vicinity of the conductor because of the periodic reversal of the electric field, resulting in the corona losses being dependent on the conductor surface gradient rather than on the actual line configurations. On the other hand, the space charge created by dc corona fills the entire interelectrode space, resulting in a greater dependence of corona on the actual line geometry. Accordingly, recognizing that the space charge may behave differently at various levels of the applied voltage for different configurations, a detailed study and analysis of the computed results at different voltages for different geometries is necessary. This constitutes the main objective of this chapter.

Initially, the electrostatic field solution for the single conductor above ground geometry, using the finite element method, is carried out. This is important as it may provide some insight into the accuracy of the finite element solution of the corresponding ionized field case. Various mesh sizes are considered, and the accuracy of the solution is assessed by comparison with the image method.

In chapter 4, the unipolar corona equations were formulated as two second order nonlinear partial differential equations. These two equations were then solved iteratively using a finite element based algorithm for both the potential Φ and the charge density ρ . The algorithm was applied to the case of cylindrical configuration and the results were compared with analytical solution. In the present chapter, the finite element algorithm is applied to the general case of a cylindrical conductor above a ground plane. Simple procedures for constructing the finite element mesh and generating the initial space charge distribution, for the iterative solution, are developed and described. Different geometries are considered, and the iterative solution is performed at various voltage levels up to twice the onset value. As the aim of the present work is to assess the influence of the simplifying assumptions on the ionized field solution, the iterative solution is carried out, at each voltage level, both with and without Kaptzov's assumption. All quantities of interest such as the charge density, electric field, and current density both at the conductor surface and ground plane as well as the corona losses are computed and compared for both cases. As much interest has been recently given to the possible environmental impact of hvdc lines, the maximum level of the ionized field quantities at the ground level are also computed and compared for both cases. Comparison with available experimental data is also carried out whenever possible. The accuracy and efficiency of the present procedures are also studied and compared with those of other available finite element based algorithms.

5.1 Finite Element Electrostatic Field Solution

The electrostatic field solution in the region under hvdc lines, obtained by application of the finite element method, FEM, is important not only by itself but also because it may be used to assess, qualitatively, the accuracy of the FEM solution of the corresponding ionized field. In fact, both solutions employ the

Laplacian operator; the latter solution is obtained, however, on iterative basis. Methods which are known to yield accurate results in the electrostatic case, such as the image method, are available[52], which makes possible the determination of the accuracy of the corresponding FEM solution. The computations are carried out for a configuration consisting of a single conductor of radius r_c located at height H above the ground plane as shown in fig. 5.1 .

In the fundamental equations of the FEM, unknowns are the potential values at all nodes generated by the division of the region under consideration into elements and, relatively, high accuracy of the calculated potentials can be obtained even if the whole region is divided into comparatively large elements. The basic details of the finite element method are given in Appendix A. The electric field can be obtained from the potential distribution. However, the accuracy of the electric field deteriorates due to the process of differentiation of the potential distribution [8,13]. In addition, no unique value can be assigned to the nodal electric field due to the presence of more than one element in the surrounding of every node.

Tests carried out, initially, for the coaxial cylindrical geometry using second order elements[53] showed that an averaging procedure based on the angles of the corresponding elements at the node can be used to obtain reliable values for the electric field. Takuma et al.[16], using linear elements, reported that this technique gave better results when compared with an area weighted averaging procedures. The angle weighted averaging technique is described as follows: If the angles $\theta_1, \theta_2, \dots, \theta_n$ are used as weighting functions, then the estimate of the electric field at node j is,

$$E'_j = \frac{\sum_{i=0}^n \theta_i E'_i}{\sum_{i=0}^n \theta_i} \quad (5.1)$$

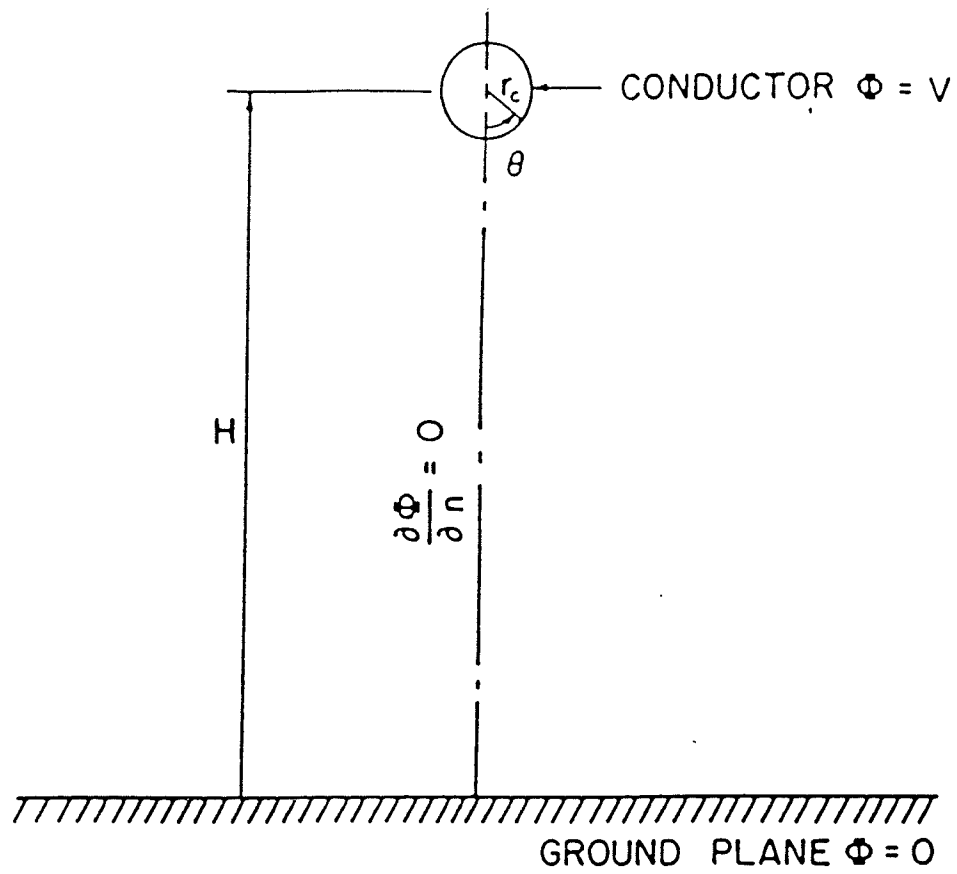


Fig. 5.1 Single conductor above ground geometry

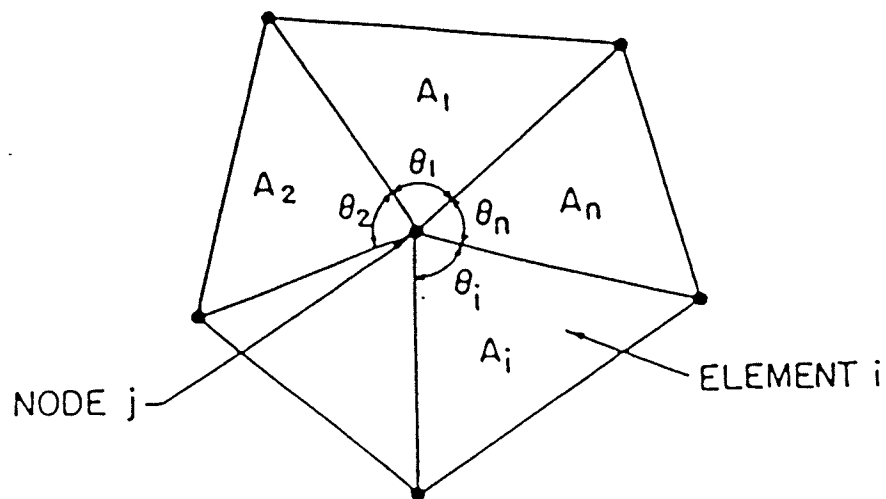


Fig. 5.2 The weighting functions θ_i at node j

where θ_i is the angle shown in fig. 5.2 . E'_i is the electric field obtained using element i and n is the number of elements surrounding the node under consideration. The finite element mesh for the single conductor above ground configuration is generated as described in the following section.

5.1.1 Mesh Generation

A semi-automatic mesh generation routine was developed and used to generate the elements for the single conductor geometry. Initially, a number of equidistant nodes, N_c , is chosen on the conductor surface. Starting from one conductor node, the subsequent nodes are placed approximately on the same field line. The second node is removed from the first one by a distance equal to $r_c \theta$, where θ is the angle subtended at the conductor center by a chord joining two consecutive conductor nodes. Any subsequent node is at a distance equals $l r_n \theta$ from the previous node where r_n is the radial distance of the previous node from the conductor center and l is a parameter that controls the mesh growth pattern and a value of $l = 1.5$ was found to result in good element shapes, generally, throughout the whole mesh.

The configuration considered here consists of a single conductor of radius $r_c = 0.0025\text{m}$ and a height $H=2\text{m}$. The generated mesh for this geometry is shown in fig. 5.3, where the number of elements $NE=333$ and the number of main nodes $NN=195$. The boundary conditions employed in the finite element solution are: $\phi=V$ on the conductor and $\phi=0$ on the ground plane. Only half the region of interest is considered due to the problem symmetry with $\partial\phi/\partial n=0$ along the line of symmetry. An artificial boundary is used with $\phi=\phi_{es}$ on its surface, where ϕ_{es} is the electrostatic potential calculated at nodes on that boundary by replacing the conductor with a filamentary line charge at its center.

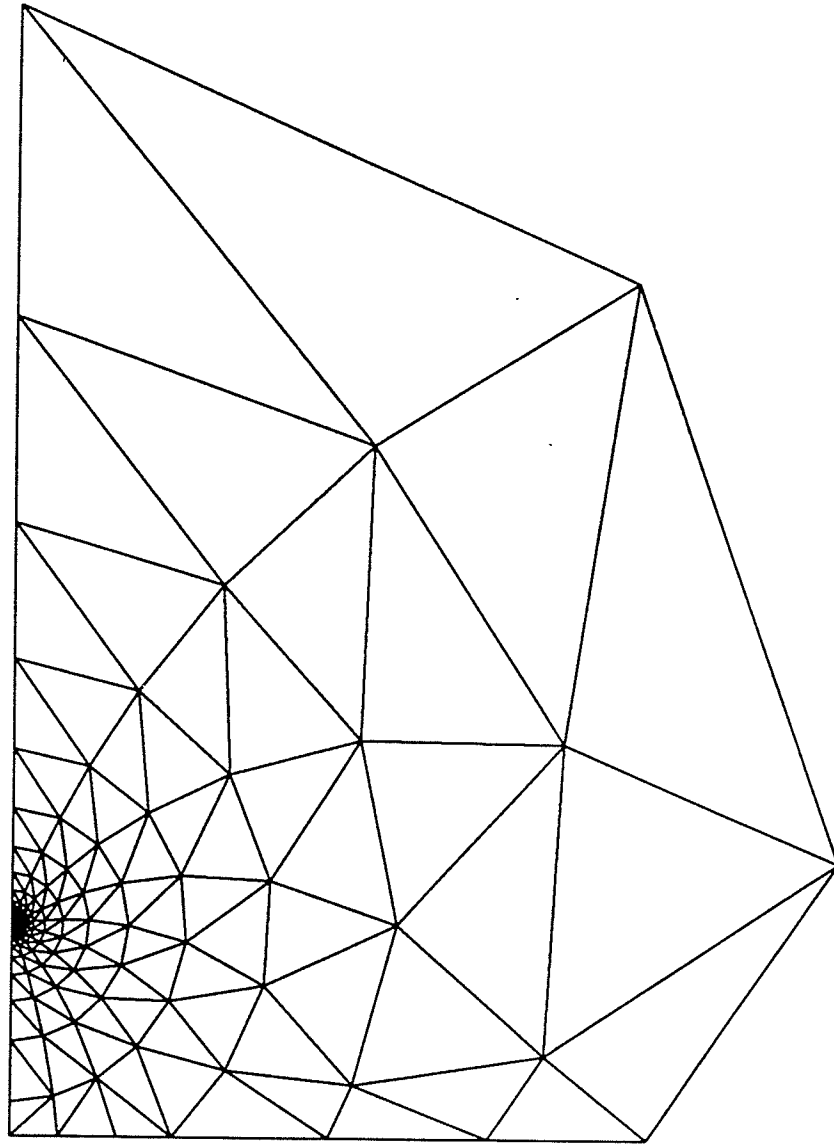


Fig. 5.3 The finite element mesh
 $r_c = 0.0025\text{m}$, $H = 2.0\text{m}$
 $NN = 195$, $NE = 333$

5.1.2 Computed Results

The accuracy of the solution using the finite element method is assessed by comparing the numerically derived results with those obtained using the image method. The potential at any point (x, y) , using the image method is given by,

$$\phi(x,y)=0.067712 \ln \left[\frac{x^2+y^2}{x^2+(y+2H)^2} \right] , \quad (5.2)$$

The electric field is,

$$E'(x,y)=\sqrt{E'_x{}^2+E'_y{}^2} \quad (5.3)$$

where $E'_x = \frac{\partial \phi}{\partial x}$ and $E'_y = \frac{\partial \phi}{\partial y}$, and has the direction,

$$\angle E' = \tan^{-1} \frac{E'_y}{E'_x} .$$

The percentage errors in the potential and electric field values are expressed as $|\phi_I - \phi_F| / \phi_I$ and $|E'_I - E'_F| / E'_I$ where ϕ_I and E'_I are the potential and field values obtained by the image method and ϕ_F and E'_F are those corresponding to the finite element method.

Sunaga et al.[8] reported that one of the important problems in applying the finite element method for the study of ionized fields of dc transmission lines is the accuracy in determining the electric field at the conductor surface. It should be noted that the electric field at the energized conductor is the main factor in determining the ion flow field[8]. Abdelsalam et al.[13] used the finite element method to solve the ionized field problem of a single conductor above ground geometry. In order to truncate the semi-infinite region, they used a lateral artificial surface parallel to the line of symmetry. This boundary intersects the ground plane at distance of $1H$ and extends a distance $1H$ above the energized conductor. They showed that their experimental and numerical results for the current density

distributions along the ground plane becomes almost zero at a distance approximately equals $1.5H$. Gela[33] also used an artificial surface for the single conductor above ground geometry. This surface intersects the ground plane at a distance about $4H$ and extends a distance approximately $6.5H$ above the conductor. The potential on this boundary was assumed zero, while Abdelsalam et al.[13] assumed that it is equal to the electrostatic value. Gela[33] showed that the lateral distributions of the current density along the ground becomes almost zero at a distance about $1.7H$. It should be noted that in both cases no study of the electrostatic field has been reported. A study into the accuracy of the electrostatic field, using the finite element method, of a single conductor monopolar dc line has been carried out in[54]. An artificial surface similar to the one used by Abdelsalam et al.[13] was employed and located at a distance $1.6H$ along the ground plane and $1H$ above the conductor. A comparative study into various element orders showed that second order elements, generally, gave the best results for the electric field at both the conductor and ground surfaces[54].

An example of the influence of the extent of space used in the FEM computations is carried out for a mesh of 343 nodes and 611 elements. The number of nodes on the conductor surface is 14 and along the ground plane is 9. A plot of the mesh is shown in fig. 5.4. The results are shown in tables 5.1a and 5.1b for different cases where the artificial boundary intersects the ground plane at a distance from the centre line $x_g = 3.87, 4.84$ and 6.0 meters which corresponds to $1.94H, 2.42H$ and $3H$ respectively. The average distance of the artificial boundary from the conductor centre is $4.64, 5.79$ and 7.19 meters corresponding to $2.3H, 2.9H$ and $3.6H$ respectively. Table 5.1a shows the results for the electric field values around the conductor surface and the corresponding percentage errors calculated with respect to the image method. It was observed that the accuracy of the electric field at the conductor surface is not significantly influenced by the

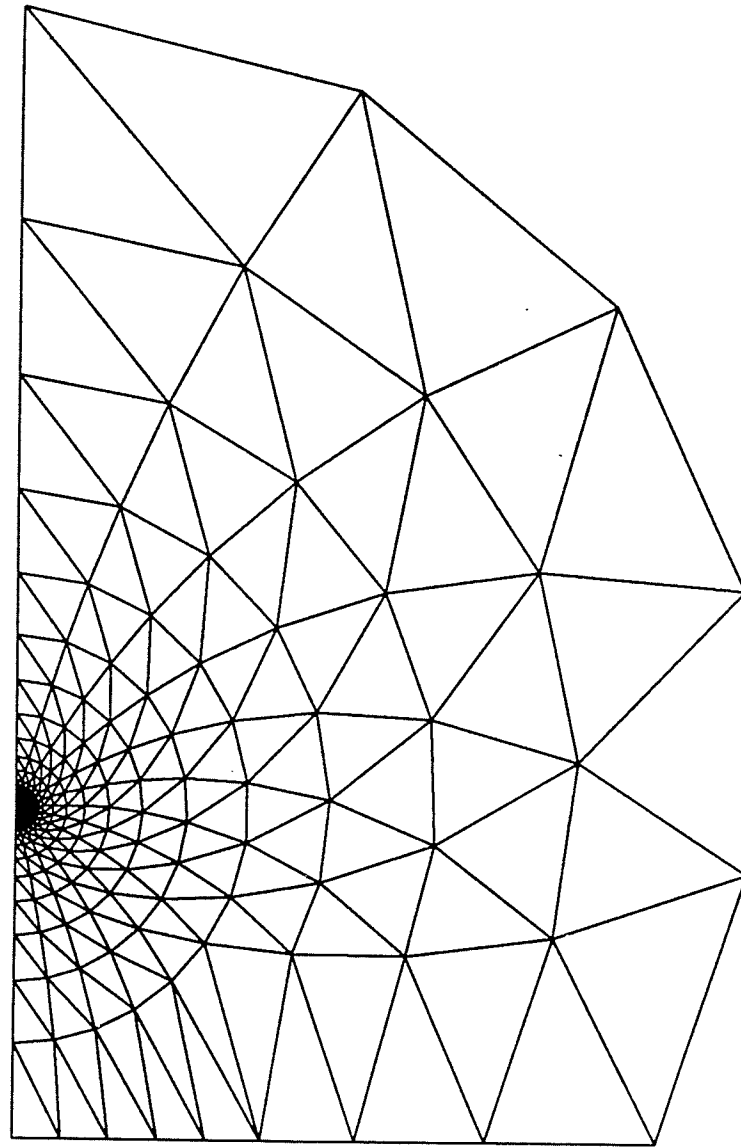


Fig. 5.4 The finite element mesh
 $r_c = 0.0025\text{m}$, $H = 2.0\text{m}$
 $NN = 343$, $NE = 611$

Table 5.1a Electric field (V/m) values around the conductor periphery,
NN=343, NE=611

θ°	E'_I	E'_F $x_g=3.87m$	% Error	E'_F $x_g=4.84m$	% Error	E'_F $x_g=6.0m$	% Error
00.00	54.251	54.325	0.136	54.299	0.088	54.254	0.006
13.85	54.250	54.319	0.127	54.292	0.077	54.247	0.006
27.69	54.247	54.313	0.122	54.285	0.070	54.240	0.013
41.54	54.242	54.297	0.101	54.270	0.052	54.225	0.031
55.38	54.236	54.283	0.087	54.256	0.037	54.211	0.046
69.23	54.229	54.275	0.085	54.248	0.035	54.203	0.048
83.08	54.221	54.255	0.063	54.228	0.013	54.182	0.072
96.92	54.213	54.243	0.055	54.216	0.006	54.170	0.079
110.77	54.205	54.222	0.031	54.194	0.02	54.149	0.103
124.62	54.198	54.213	0.028	54.186	0.022	54.140	0.107
138.46	54.192	54.202	0.018	54.175	0.031	54.129	0.116
152.31	54.187	54.186	0.002	54.160	0.051	54.114	0.135
166.15	54.184	54.172	0.022	54.145	0.072	54.099	0.157
180.00	54.183	54.157	0.048	54.130	0.098	54.084	0.183

Table 5.1b Electric field (V/m) values along the ground plane,
NN=343, NE=611

Node no.	$x(m)$	E'_I	E'_F $x_g=3.87m$	% Error	E'_F $x_g=4.84m$	% Error	E'_F $x_g=6.0m$	% Error
23	0.0000	0.13554	0.13308	1.82	0.13346	1.54	0.13377	1.31
46	0.2910	0.13273	0.12936	2.54	0.12976	2.24	0.13007	2.00
69	0.5845	0.12488	0.12193	2.36	0.12238	2.00	0.12270	1.75
92	0.8803	0.11355	0.11078	2.44	0.11134	1.94	0.11167	1.66
115	1.1721	0.10089	0.09859	2.28	0.09931	1.56	0.09965	1.23
138	1.5000	0.08675	0.08495	2.07	0.08594	0.93	0.08629	0.53
162	2.0642	0.06563	0.06257	4.66	0.06433	1.97	0.06473	1.37
187	2.8462	0.04480	0.03816	14.82	0.04310	3.81	0.04454	0.59

variation in the artificial boundary location, as the average errors were 0.066%, 0.048% and 0.079% for the three cases respectively. The results along the ground plane up to a distance of 2.85 meters, or $1.4H$, from the centre line are given in table 5.1b. The variation in the accuracy of the computed results along the ground is more noticeable than that of the results around the conductor surface. The average errors for the different cases considered are 4.12%, 1.99% and 1.3% respectively.

In order to investigate the influence of the number of conductor surface nodes on the accuracy of the conductor electric field, other mesh densities are considered. Mesh b is generated using 10 conductor nodes with the parameters $NN=195$ and $NE=333$ and was shown earlier in fig. 5.3 . The artificial surface intersects the ground plane at a distance of 6 meters or $3H$ and the average distance from the conductor surface is about 7.84 meters or $3.92H$, thus compatible with the last case of mesh a of 343 nodes and 611 elements when the artificial surface is about 7.19 meters or $3.6H$ from the conductor centre. The results for the electric field around the conductor surface are given in table 5.2a and the average error is about 0.15% . The results along the ground plane are given in table 5.2b and the average error up to a distance of 2.5 meters is about 1.81% . It was observed that in the case of mesh a, the area close to the ground plane is not explored with sufficient number of nodes as in the case of mesh b. In fact the last layer of nodes located laterally parallel to the ground plane is at an average distance of about 1.41 meters from the conductor centre in case of mesh a while this distance is about 1.66 meters in case of mesh b. Mesh a is then used to calculate the field with these nodes relocated at the same average distance from the conductor centre similar to that of mesh b. The results for the electric field along the ground plane are given in table 5.3. The average error in this case is about 0.77%. The results around the conductor were practically the same as before.

Table 5.2a Electric field (V/m) values around the conductor
periphery, NN=195, NE=333.

θ^0	E'_I	E'_F	% Error
00.00	54.251	54.235	0.029
20.00	54.249	54.214	0.065
40.00	54.243	54.193	0.092
60.00	54.234	54.172	0.114
80.00	54.223	54.153	0.129
100.0	54.211	54.128	0.153
120.0	54.200	54.109	0.168
140.0	54.191	54.090	0.186
160.0	54.185	53.054	0.242
180.0	54.183	53.018	0.304

Table 5.2b Electric field (V/m) values along the ground
plane, NN=195, NE=333

Node no.	x(m)	E'_I	E'_F	% Error
18	0.000	0.13554	0.13518	0.266
36	0.450	0.12901	0.12786	0.891
54	1.000	0.10843	0.10676	1.500
72	1.500	0.08675	0.08499	2.024
91	2.500	0.05289	0.05059	4.358

A third mesh, mesh c, is also generated using 6 conductor nodes with the parameters $NN=81$ and $NE=125$ and is shown in fig. 5.5 . The distance of the artificial surface from the centre line is 6 meters or $3H$ and the average distance from the conductor centre is about 11.39 meters or $5.7H$. The accuracy of the electric field values around the conductor surface and along the ground plane using this mesh are given in tables 5.4 and 5.5. The average errors are 0.32% and 1.41% respectively. It can be seen by comparison of tables 5.1a, 5.2a and 5.4 that the accuracy of the electric field around the conductor surface increases as the number of conductor nodes increases. Janischewskyj and Gela[15] investigated the influence of the grid structure on the ionized field solution, in case of coaxial cylinder geometry, using one dimensional linear elements. They observed that not only the number but also the proper position and location of the finite element nodes can have an influence on the solution accuracy. They pointed out, however, that physical understanding of the problem is extremely useful in providing guidelines in this regard. This concludes the analysis of the electrostatic field case, i.e. no corona space charge is assumed present in the interelectrode space. The application of the finite element algorithm to the ionized field solution of a single conductor above ground geometry is considered next.

5.2 Finite Element Ionized Field Solution

As mentioned earlier, corona is a nonlinear phenomenon being dependent on both the line geometry and the applied voltage level. Detailed study and analysis of the computed results for various geometries and voltage levels are thus necessary. In the present study, different geometries ranging from $H/r_c = 300$ to $H/r_c = 1000$ are considered. It should be noted that H/r_c of a typical hvdc line is usually in the range of 700-1000 . For each geometry, the ionized field solution is carried out at different voltage levels, above the corona onset value, up to $V=2V_0$. At a certain applied voltage, the solution is performed for both the cases with and

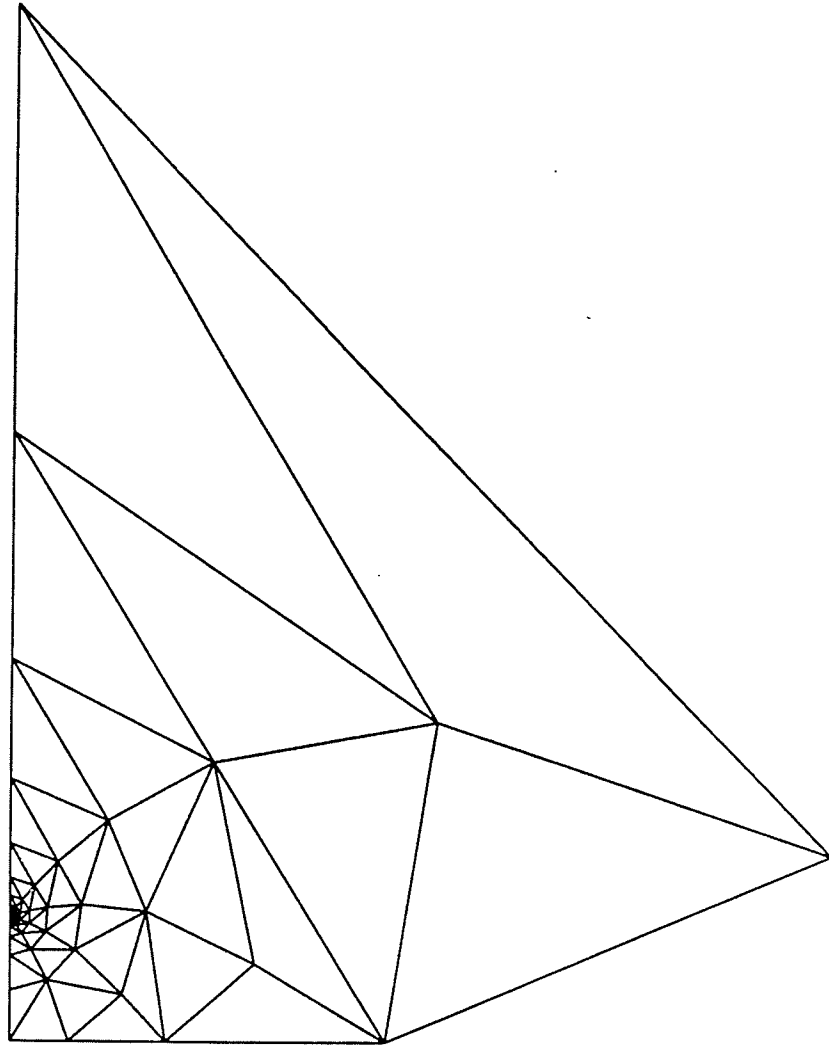


Fig. 5.5 The finite element mesh
 $r_c = 0.0025\text{m}$, $H = 2.0\text{m}$
 $NN = 81$, $NE = 125$

Table 5.3 Electric field (V/m) values along the ground plane,
NN=343, NE=611

Node	x(m)	E'_I	E'_F	% Error
23	0.0000	0.13554	0.13495	0.44
46	0.2910	0.13273	0.13124	1.10
69	0.5845	0.12488	0.12368	0.96
92	0.8803	0.11355	0.11244	0.98
115	1.1721	0.10089	0.10028	0.61
138	1.5000	0.08675	0.08663	0.14
162	2.0642	0.06563	0.06459	1.58
187	2.8462	0.04480	0.04463	0.38

Table 5.4 Electric field (V/m) values around the conductor
periphery, NN=81, NE=125.

θ°	E'_I	E'_F	% Error
00.00	54.251	54.550	0.55
36.00	54.244	54.493	0.46
72.00	54.227	54.436	0.39
108.0	54.207	54.388	0.33
144.0	54.190	54.270	0.15
180.0	54.183	54.152	0.06

Table 5.5 Electric field (V/m) values along the ground plane,
NN=81, NE=125

Node no.	x(m)	E'_I	E'_F	% Error
12	0.0000	0.13554	0.13590	0.26
24	0.9365	0.11117	0.10701	3.74
37	2.5000	0.05289	0.05246	0.81
51	6.0000	0.01355	0.01366	0.81

without Kaptzov's assumption. The results are presented for the spatial distributions of the ionized field quantities around the conductor periphery. The corona current per unit length of the conductor is also computed for both cases. Furthermore, as much interest has been recently directed towards the influence of a dc transmission line on its surroundings[1-4], the distributions of the space charge density, electric field intensity, and current density along the ground plane are computed and presented for both cases. Comparisons with available experimental data are included wherever possible. Accuracy and efficiency of the iterative solution are also studied and demonstrated.

5.2.1 The Initial Space Charge Distribution

The initial distribution of the charge density ρ at the finite element nodes is obtained according to the following procedures. As mentioned before the mesh is generated by tracing the field lines starting from the conductor surface towards the outer boundary. The length of each field line is considered to be equal to the outer radius of a coaxial cylinder system whose inner radius is the conductor radius. The potential gradient on the inner conductor is computed using eqns. 3.4 and 3.16 at the corresponding voltage level. The known analytical solution for the coaxial cylinder geometry, is then applied in order to obtain the charge density distribution along that line. This is repeated for all lines in order to obtain the initial charge distribution at all the nodes. This technique for obtaining the initial charge distribution is simpler and easier to implement than previous techniques[13,33].

5.3 Reduced Scale Model, $H/r = 300$

The geometry considered first, consists of a single smooth conductor of radius $r_c = 0.00165\text{m}$ located at a height $H = 0.495\text{m}$ above a ground plane. This geometry was considered by Abdelsalam et al. [13] and important experimental data for the corona current and the current density distributions along the ground

plane has been reported [13]. The finite element mesh for this geometry is shown in fig. 5.6. The corona onset gradient for this geometry is $E_0 = 55.42$ kV/cm and the corresponding onset voltage is $V_0 = 58.49$ kV. Various voltage levels are considered between 70kV and 120 kV corresponding to $1.2 V_0$ and $2V_0$. At each voltage level, the initial space charge distribution is generated according to the above mentioned procedures. The iterative solution is then carried out at this voltage level until convergence occurs. The updating criteria, eqns. 4.8-4.9, is used in order to modify the charge distribution during the iterative solution.

5.3.1 Computed Results and Analysis:

The spatial distributions of the charge density ρ_c and current density J_c around the conductor periphery are shown in figs. 5.7, 5.8 and 5.9 for different applied voltage levels. The results are computed both with and without Kaptzov's assumption. It is observed that, at all voltages, distributions without the assumption are generally higher than those with the assumption with the maximum deviation being always at the bottom of the conductor, corresponding to $\theta = 0$. The deviations between the distributions, however, tend to decrease as θ increases.

The corona current per unit length of the conductor is computed as follows. At the conductor surface, the corona current density J_c is,

$$J_c = k \rho_c E_c \quad (5.4)$$

Then, the corona current I per unit length of the conductor is,

$$I = \int_{\theta=0}^{\theta=2\pi} J_c r_c d\theta \quad (5.5)$$

where θ is the angle shown in fig 5.1 . Hence the power loss P , per unit length due to corona is,

$$P = IV \quad (5.6)$$

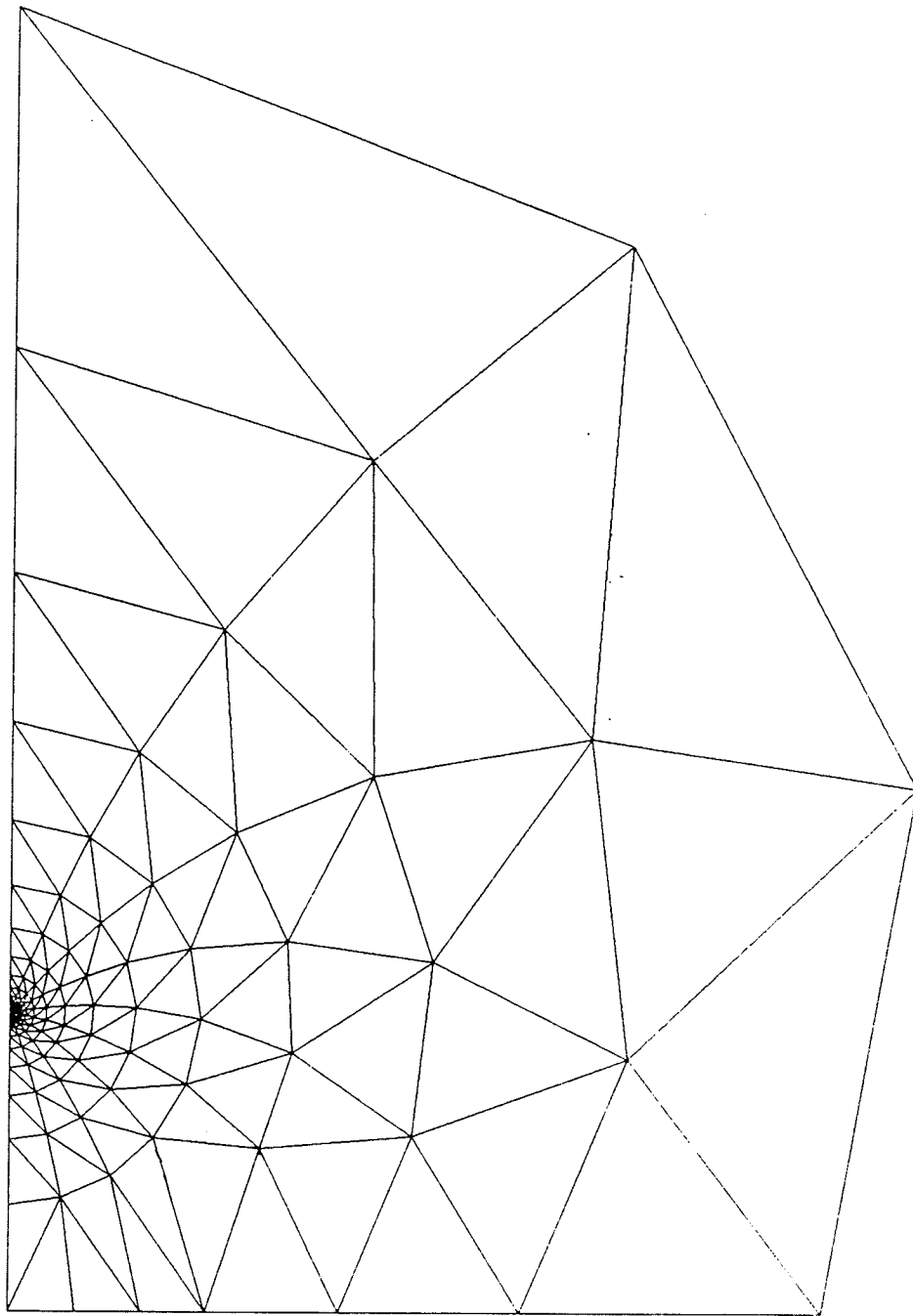
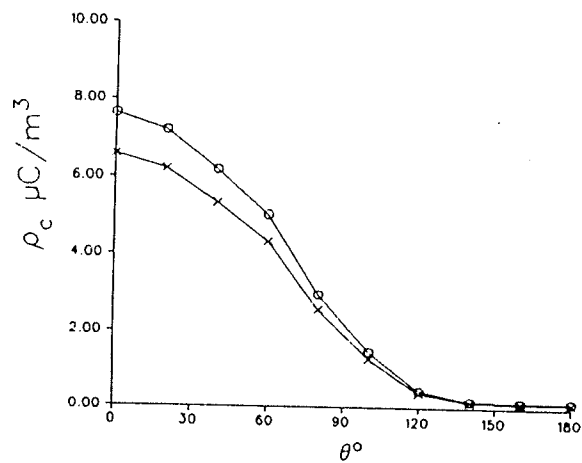
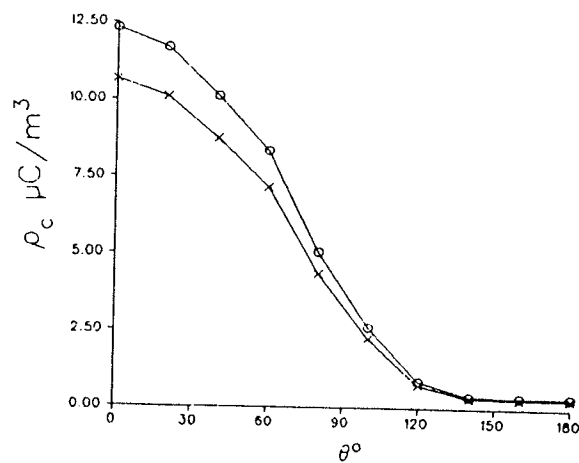


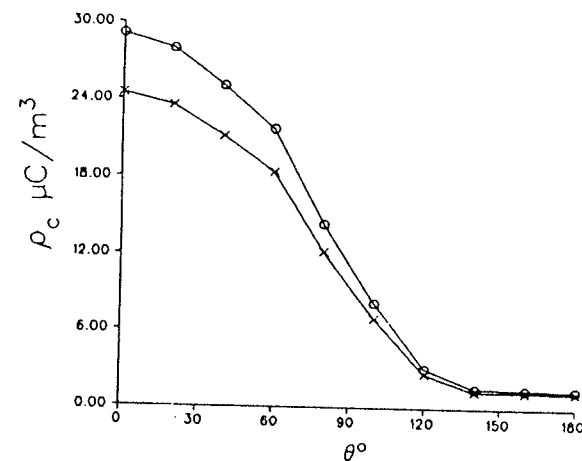
Fig. 5.6 The finite element mesh
 $r_c = 0.00165\text{m}$, $H = 0.495\text{m}$
 $NN = 165$, $NE = 279$



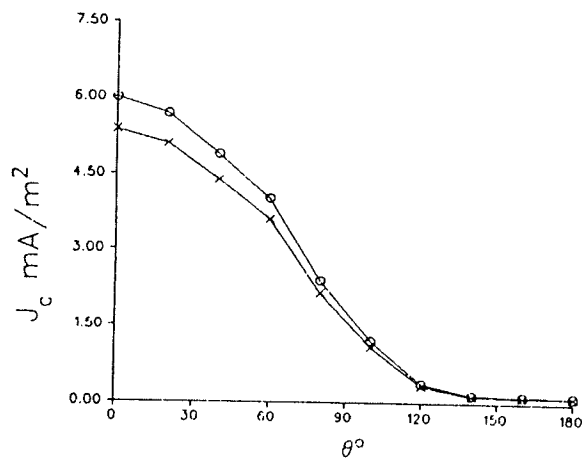
a- Charge density



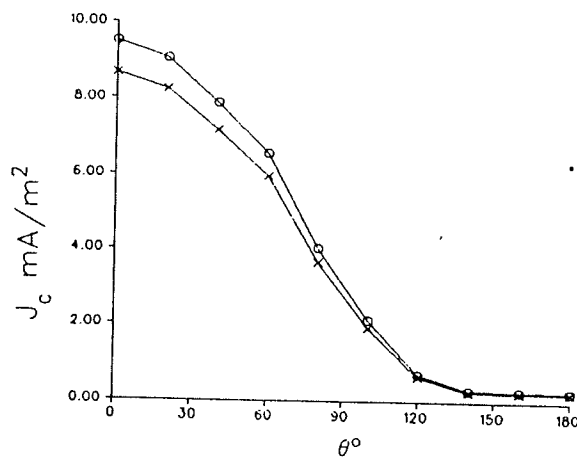
a- Charge density



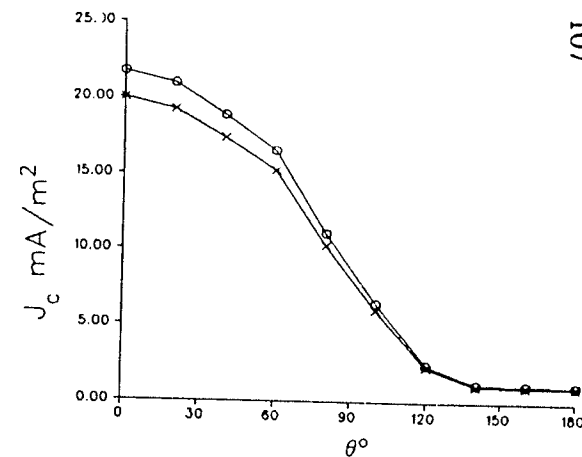
a- Charge density



b- Current density



b- Current density



b- Current density

Fig. 5.7 Results around conductor periphery,
V=80kV.

Fig. 5.8 Results around conductor periphery,
V=90kV.

○ ○ Without Kaptzov's assumption.
× × With Kaptzov's assumption.

Fig. 5.9 Results around conductor periphery
V=120kV.

Table 5.6 Corona current I ($\mu\text{A}/\text{m}$) computed with and without the assumption for $H/r_c = 300$

$V(\text{kV})$	$I^{(K)}$	$I^{(NK)}$	% Difference
70	9.227	10.348	10.84
80	18.968	21.149	10.31
90	31.397	34.546	9.12
110	61.218	67.426	9.21
120	80.539	87.301	7.75

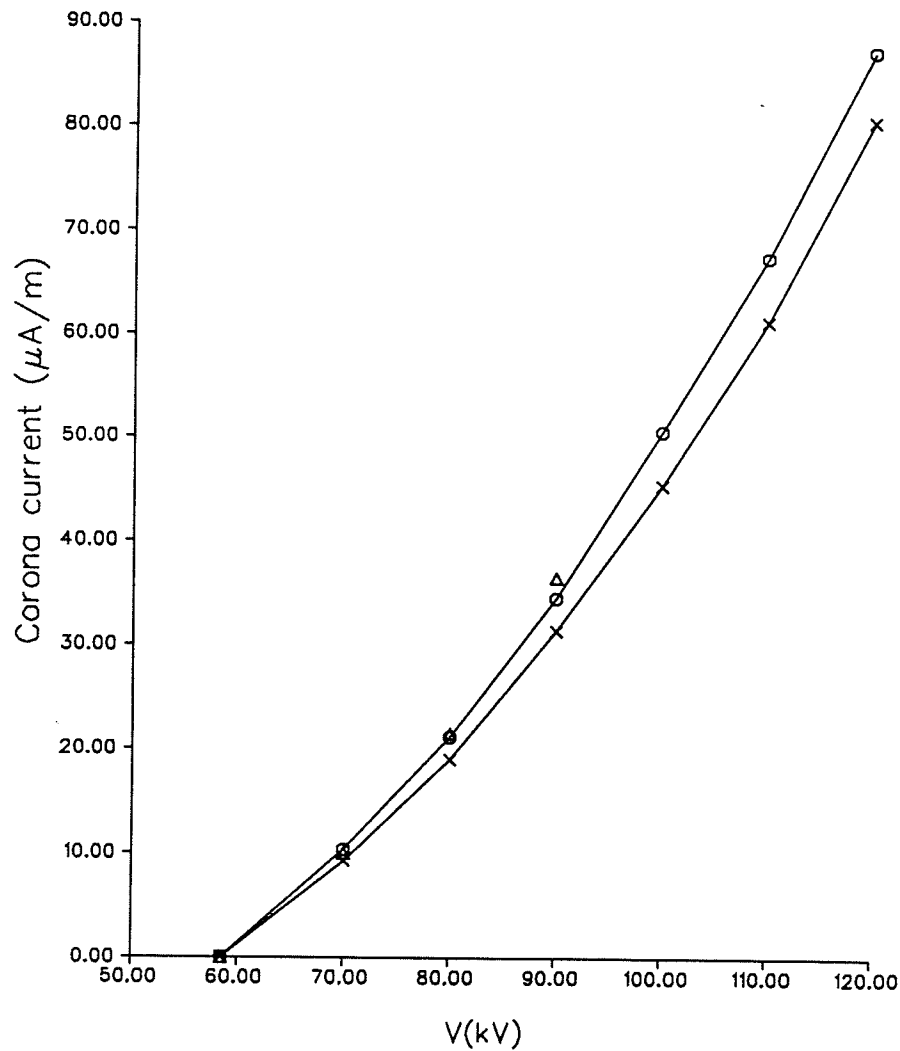


Fig.5.10 Computed V - I characteristics for $H/r_c = 300$

○ Without Kaptzov's assumption

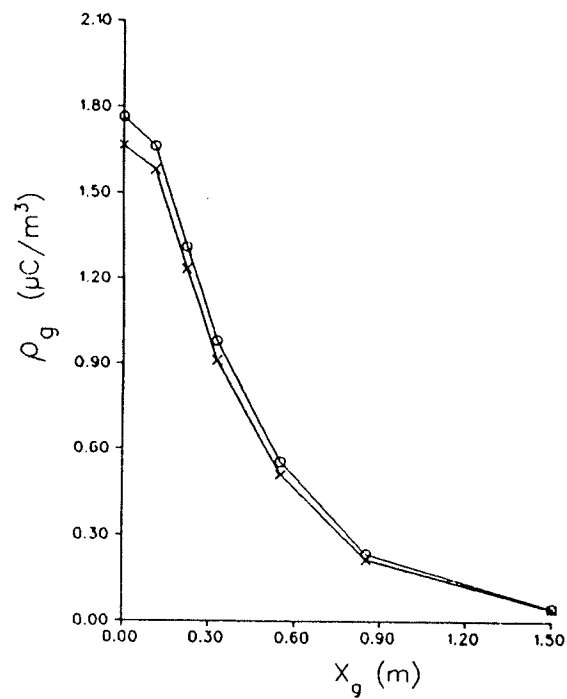
× With Kaptzov's assumption

△ Experimental data [13]

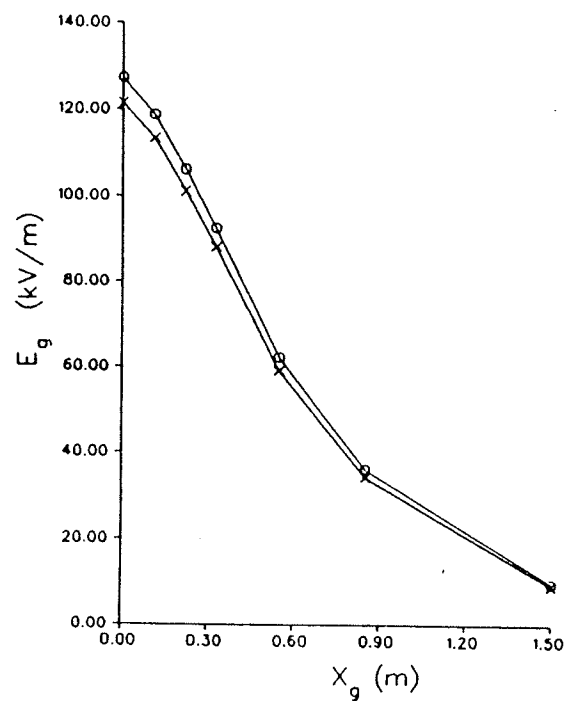
where V is the applied voltage with respect to ground. The corona current is computed without Kaptzov's assumption, $I^{(NK)}$, and with the assumption, $I^{(K)}$ at different voltage levels. The results are given in table 5.6 for both cases and also shown in fig. 5.10 together with measured data. The percentage difference in the results is computed as $(I^{(NK)} - I^{(K)})/I^{(NK)}$. The agreement between the results obtained without the assumption and the measured data[13] is satisfactory as observed in fig. 5.10. The Kaptzov's assumption seems to result in underestimation of the corona losses. This is also observed at all voltage levels in table 5.6. The relative influence of the assumption, however, tends to diminish as the applied voltage increases. The percentage difference varies from 10.84% at $V=70kV$ or $1.2V_0$ to about 7.75% at $V=120kV$ or $2V_0$. It should be noted that, in practical situations, the applied voltage is usually close to the onset level where the the assumption seems to be most significant[9].

The spatial distributions of space charge density ρ_g , electric field E_g , and current density J_g , all over the ground plane are also computed and shown in figs 5.11, 5.12, and 5.13, respectively, for different voltage levels. Similar to the above trends, results without Kaptzov's assumption are higher than those with it and are, generally, in good agreement with the experimental data[13].

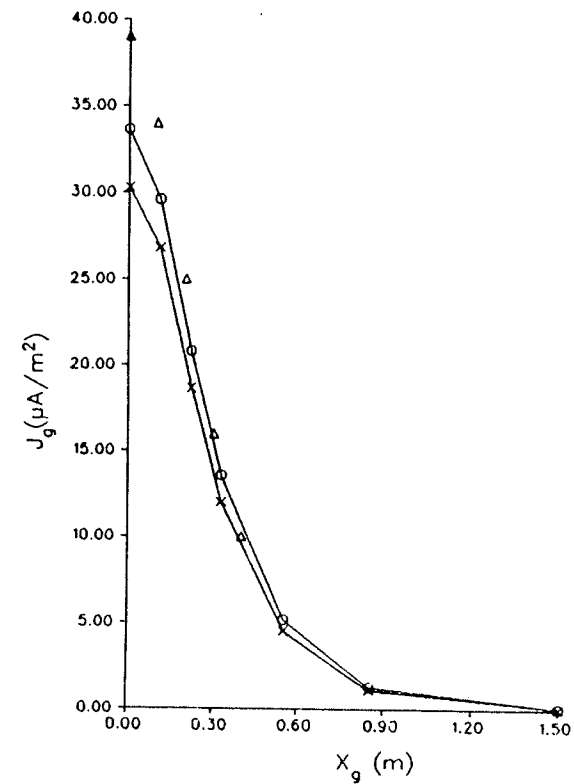
In order to investigate the influence of the assumption on the maximum level of the ionized field quantities at the ground plane, these quantities are also computed with and without the assumption and are given in tables 5.7a-5.7c. The percentage deviations are calculated with respect to the case without the assumption and included in tables 5.7a-5.7c. While the deviations caused by Kaptzov's assumption vary from 7.03% at $1.2 V_0$ to about 4.69% at $2 V_0$ for the charge density, the deviations in the electric field values is practically constant varying between 4.57% to 3.87% as observed from table 5.7b. The influence of Kaptzov's assumption on the current density is, however, more significant as seen from table



a- Charge density

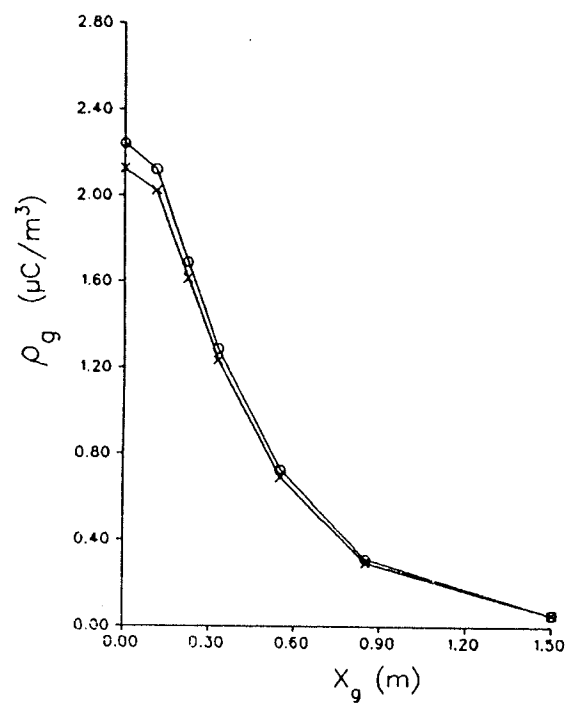


b- Electric field

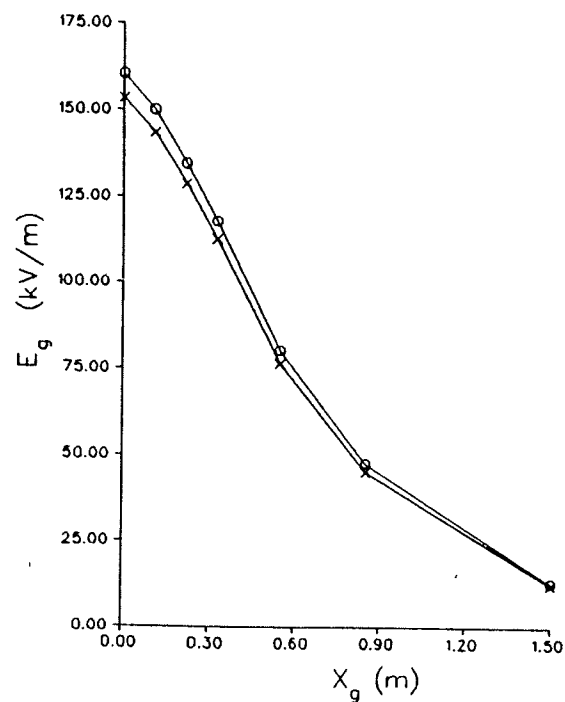


c- Current density

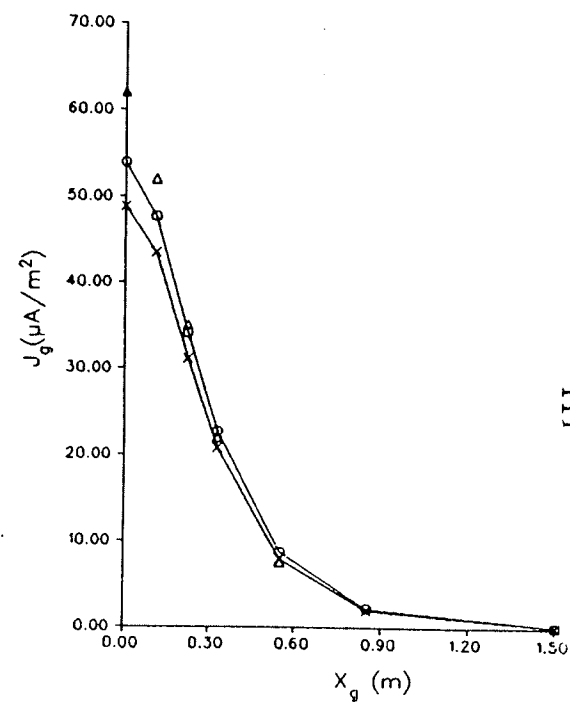
Fig. 5.11 Ionized field quantities along the ground plane, $V=80\text{kV}$
 $\square$ Without Kaptzov's assumption, $\times$ with Kaptzov's assumption
 Δ Experimental data [13]



a- Charge density

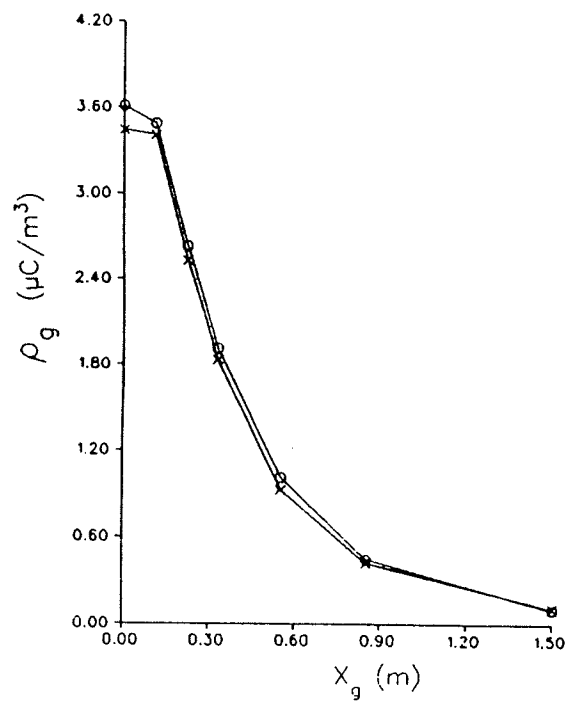


b- Electric field

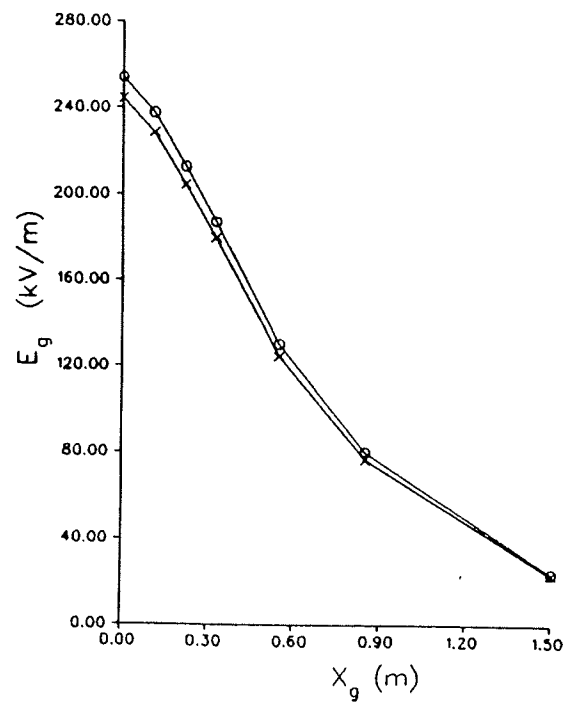


c- Current density

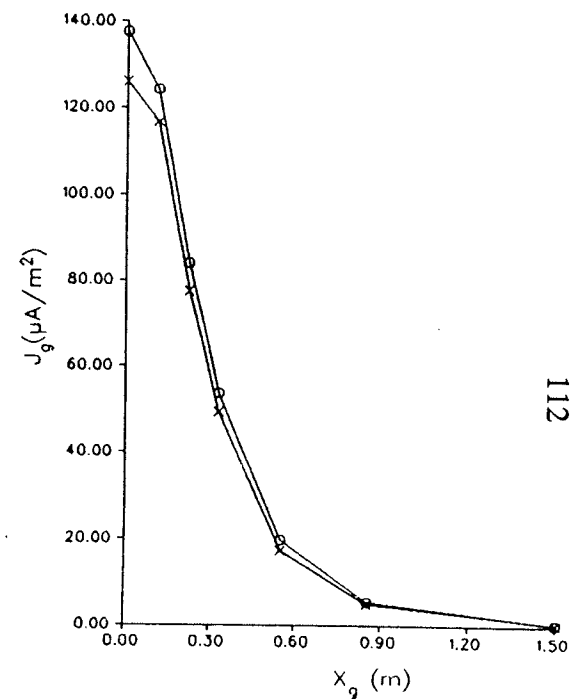
Fig. 5.12 Ionized field quantities along the ground plane, $V=90\text{kV}$
 ○ Without Kaptzov's assumption, × with Kaptzov's assumption
 △ Experimental data [13]



a- Charge density



b- Electric field



c- Current density

Fig. 5.13 Ionized field quantities along the ground plane, $V=120\text{kV}$
 ○ Without Kaptzov's assumption, × with Kaptzov's assumption

Table 5.7a Maximum charge density ($\mu C/m^3$) at ground plane
for $H/r_c = 300$

V(kV)	$\rho_g^{(NK)}$	$\rho_g^{(K)}$	% Difference
70	1.2464	1.1588	7.03
80	1.7629	1.6630	5.67
90	2.2408	2.1237	5.23
110	3.1943	3.0572	4.29
120	3.6092	3.4400	4.69

Table 5.7b Maximum electric field (kV/m) at ground plane
for $H/r_c = 300$

V(kV)	$E_g^{(NK)}$	$E_g^{(K)}$	% Difference
70	92.641	88.732	4.23
80	127.22	121.41	4.57
90	160.31	153.26	4.40
110	223.54	214.00	4.26
120	254.07	244.25	3.87

Table 5.7c Maximum current density ($\mu A/m^2$) at ground plane
for $H/r_c = 300$

V(kV)	$J_g^{(NK)}$	$J_g^{(K)}$	% Difference
70	17.320	15.423	10.95
80	33.642	30.285	9.98
90	53.885	48.822	9.39
110	107.11	98.134	8.38
120	137.55	126.03	8.37

5.7c. It varies from about 11% at 70 kV to about 8.4% at 120 kV. This is may be due to the fact that the current density is dependent on both the charge density and the electric field intensity, and individual variations in their magnitudes are combined to produce a higher deviation in the magnitude of the current density.

In order to study the accuracy and efficiency of the iterative procedures the average electric field at the conductor surface E_{avg} is computed and compared with the imposed value of the boundary condition E_c for the different voltage levels considered. E_{avg} is calculated as the average of the electric field values at all the nodes on the conductor surface upon convergence of the iterative solution. The percentage error in E_{avg} is less than 0.276% at a voltage level of $2 V_0$ as shown in table 5.8a . It should be noted that in previous finite element based algorithms[13,33], no interest was shown as to the accuracy with which that boundary condition is satisfied. The number of iterations needed for the algorithm to achieve convergence are given in table 5.8b for the different cases studied. The number of iterations in the case without the assumption are generally less than those with the assumption. It is to be noted that the initial charge distribution is generated using the value of E_c at the respective voltage level and not the value at onset E_0 . This may be the reason for the lower number of iterations in the case without Kaptzov's assumption. The different parameters used in the updating criteria eqns. 4.8 and 4.9 are given in table 5.8c . These parameters are obtained as usual using trial and error procedures[13,33].

This concludes the analysis of the results computed for the reduced scale model of $H/r_c = 300$. The analysis is carried out at different voltage levels and detailed study of the ionized field quantities as well as global quantities are studied. The accuracy and efficiency aspects of the iterative procedures are also analysed.

Table 5.8a The average electric field (kV/cm) upon convergence of the iterative procedures.

V(kV)	E_{avg}	E_c	% Error
70	54.05	54.09	0.074
80	53.23	53.17	0.113
90	52.46	52.47	0.019
110	51.33	51.23	0.195
120	50.90	50.76	0.276

Table 5.8b The total number of iterations until convergence for different voltages

V(kV)	without Kaptzov's assumption	with Kaptzov's assumption
70	15	16
80	10	12
90	9	9
110	23	39
120	26	33

Table 5.8c The values of the parameters used in the updating criteria.

V(kV)	a	b	c	d
70	1.00	1.00	0.10	0.20
80	1.00	1.00	0.10	0.20
90	0.90	0.90	0.10	0.20
110	0.60	0.60	0.10	0.20
120	0.50	0.50	0.10	0.20

5.4 Practical Model $H/r_c = 1000$

The second geometry considered consists of a single conductor of radius $r_c = 0.01$ m located at a height $H = 10$ m above the ground plane thus H/r_c is 1000. This geometry is considered as it represents a practical unipolar dc transmission line. The same steps used earlier for the reduced scale model of $H/r_c = 300$ are carried out here to obtain the ionized field solution for this configuration. The finite element mesh for this geometry is shown in fig. 5.14 . The corona onset gradient for this geometry is $E_0 = 40.92$ kV/cm and the corresponding onset voltage is $V_0 = 311.03$ kV. Different overvoltage levels are considered between $V = 1.2V_0$ and $V = 2V_0$. At each voltage, the iterative solution is performed until convergence is obtained.

5.4.1 Computed Results and Analysis:

The spatial distributions of the charge density ρ_c and current density J_c around the conductor periphery are shown in figs. 5.15, 5.16 and 5.17 for different applied voltage levels. The results are again computed both with and without Kaptzov's assumption. The same trends observed earlier for the geometry of $H/r_c = 300$ are also present in this case. At all voltages, distributions without the assumption are higher than those with the assumption and the deviations between the distributions tend to decrease as θ increases. The corona current without the assumption $I^{(NK)}$ and with the assumption $I^{(K)}$ are also computed at different voltage levels. The results are given in table 5.9 for both cases and also shown in fig. 5.18. The discrepancies caused by the assumption, as observed from table 5.9, are similar to those obtained earlier for the geometry $H/r_c = 300$. The relative influence of the assumption diminishes as the applied voltage increases. It varies from 11.38% at $V = 372$ kV to about 8.69% at $V = 637.5$ kV . Comparison of tables 5.6 and 5.9 reveals that the discrepancies caused by the assumption in this case

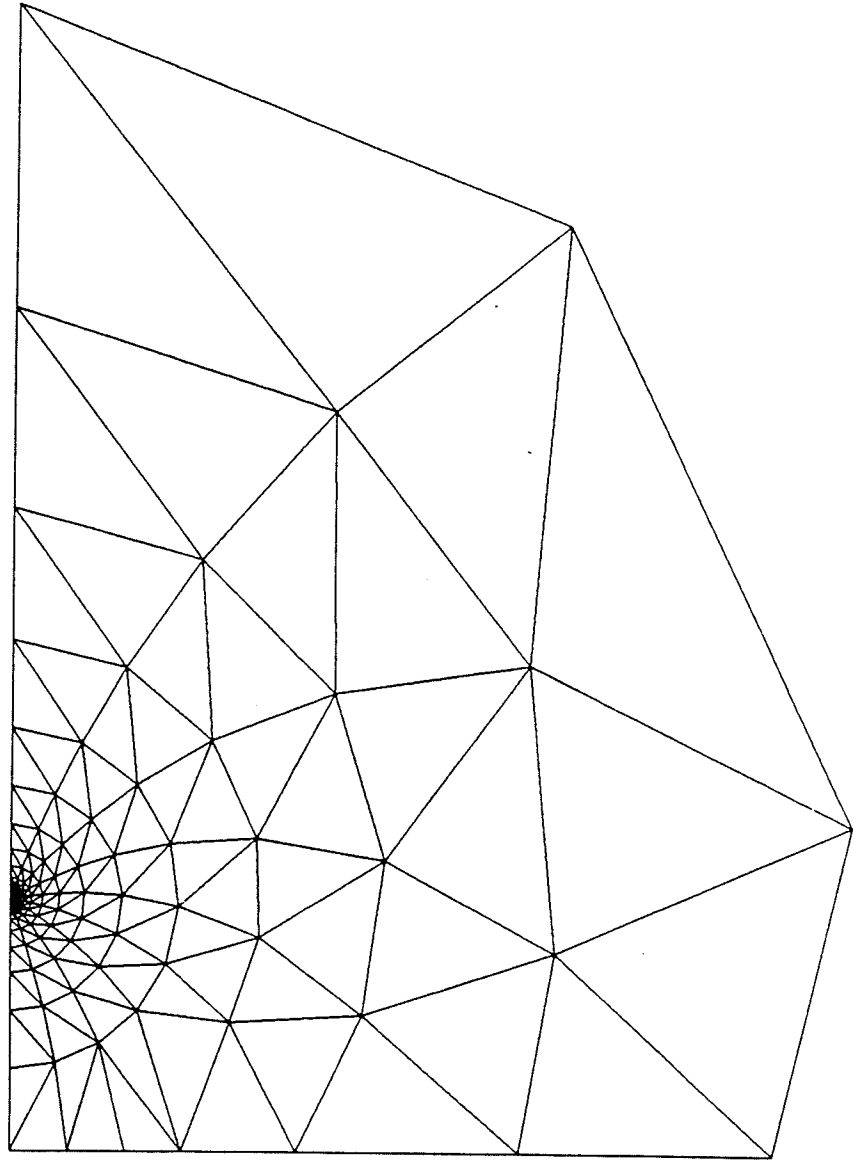
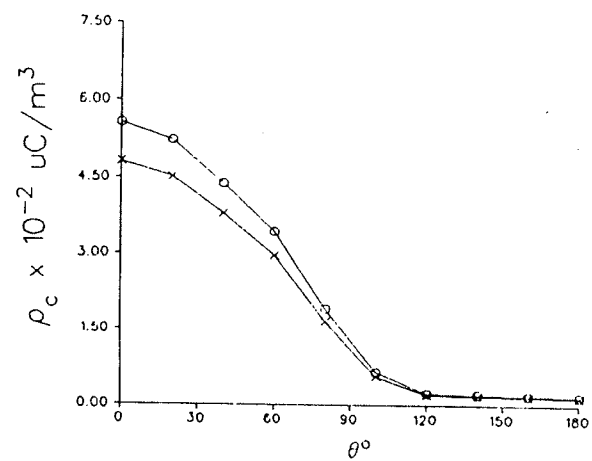
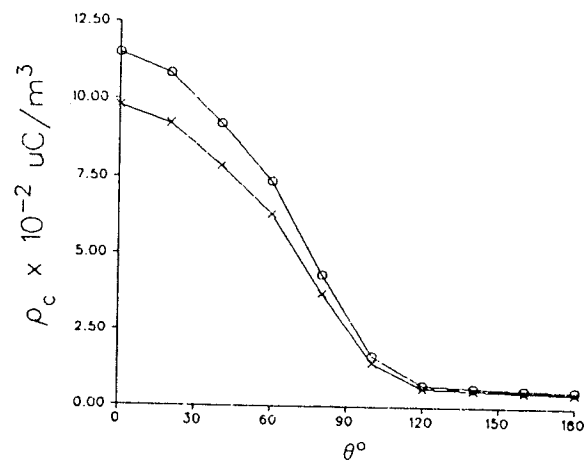


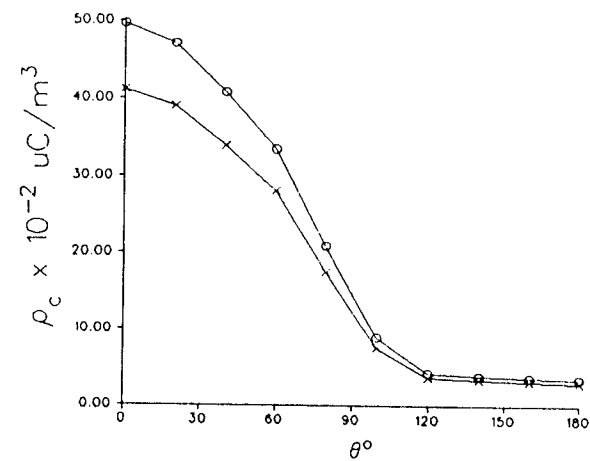
Fig. 5.14 The finite element mesh
 $r_c=0.01\text{m}, H=10\text{m}$
 $NN=195, NE=333$



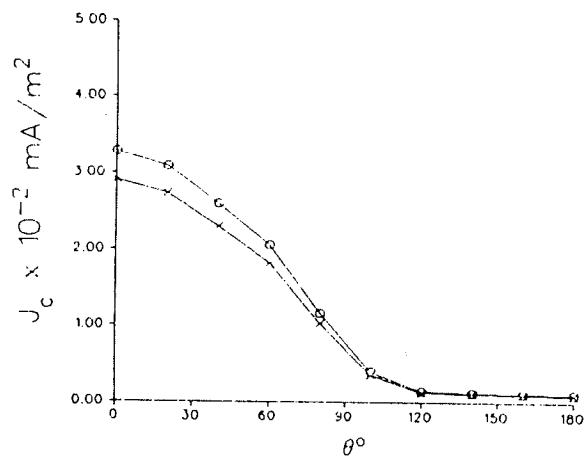
a- Charge density



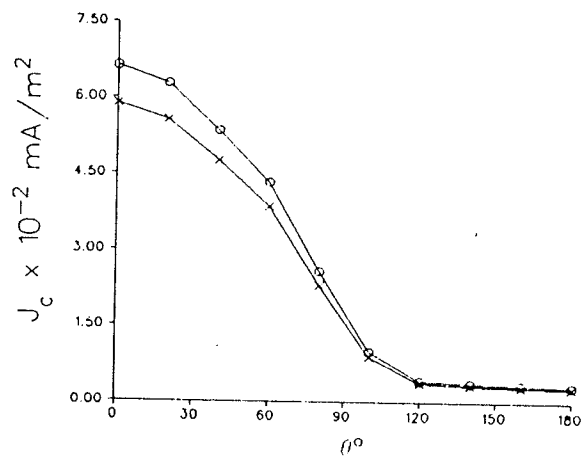
a- Charge density



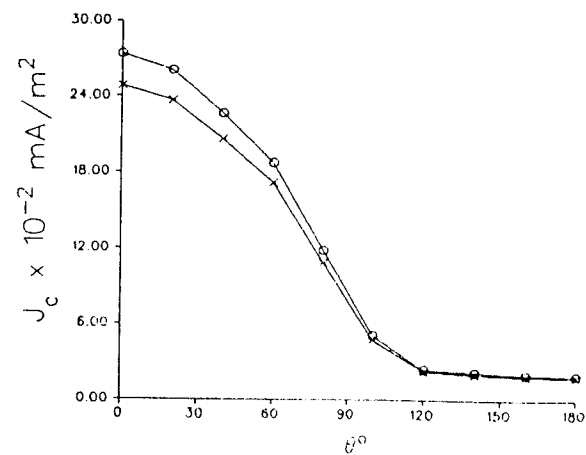
a- Charge density



b- Current density



b- Current density



b- Current density

Fig. 5.15 Results around conductor periphery,
V=372kV.

Fig. 5.16 Results around conductor periphery,
V=425kV.

○ ○ Without Kaptzov's assumption.
× × With Kaptzov's assumption.

Fig. 5.17 Results around conductor periphery,
V=637.5kV.

Table 5.9 Corona current I ($\mu\text{A}/\text{m}$) computed with and without the assumption at different voltages

$V(\text{kV})$	$I^{(K)}$	$I^{(NK)}$	% Difference
372.0	0.58447	0.65949	11.38
425.0	1.2493	1.4037	11.00
478.0	2.0185	2.3184	10.22
585.0	4.2899	4.7253	9.21
637.5	5.6312	6.1669	8.69

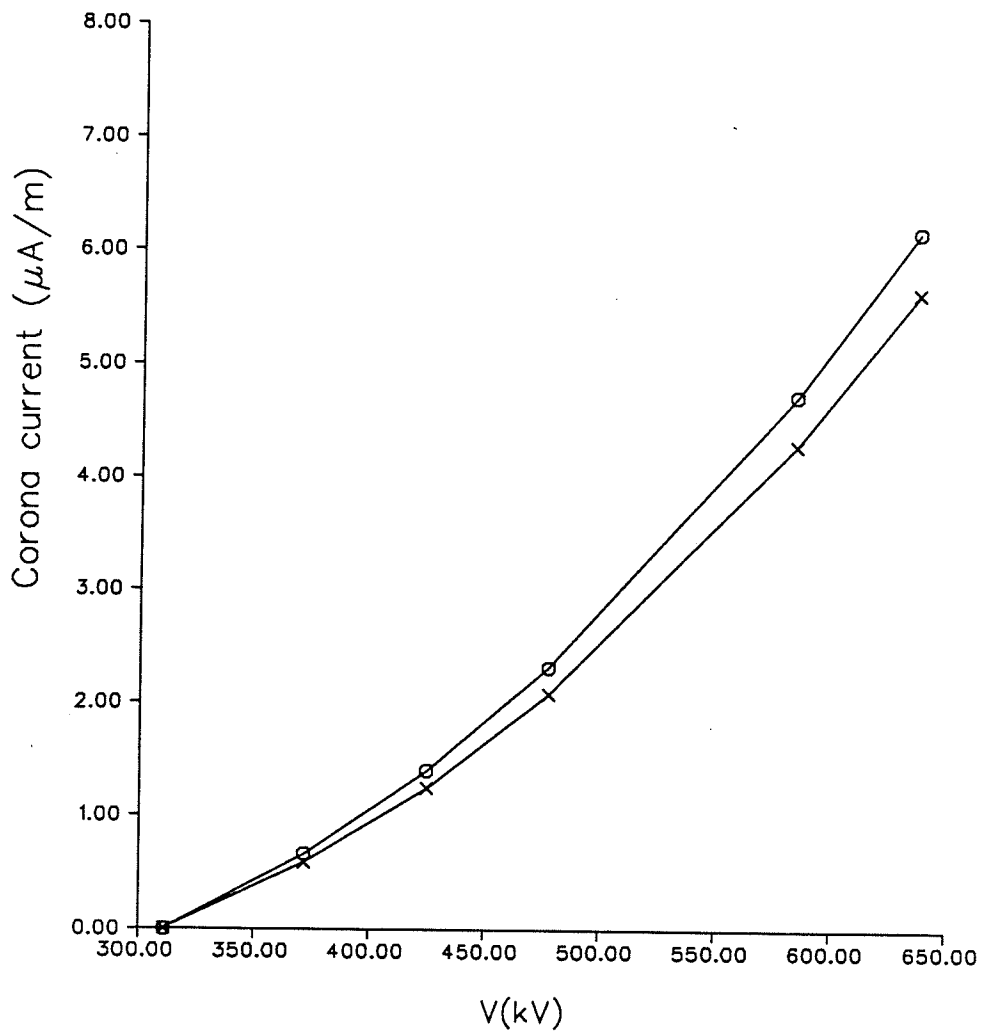


Fig.5.18 Computed V - I characteristics for $H/r_c=1000$

○ Without Kaptzov's assumption

× With Kaptzov's assumption

are, however, slightly larger than those for the geometry $H/r_c = 300$ considered earlier.

The ground level distributions of space charge density ρ_g , electric field E_g , and current density J_g , are computed and shown in figs 5.19, 5.20, and 5.21, respectively, for different voltage levels. Similar to the above trends, results without Kaptzov's assumption are higher than those with it and the deviations tend to decrease as distance from the center line increases.

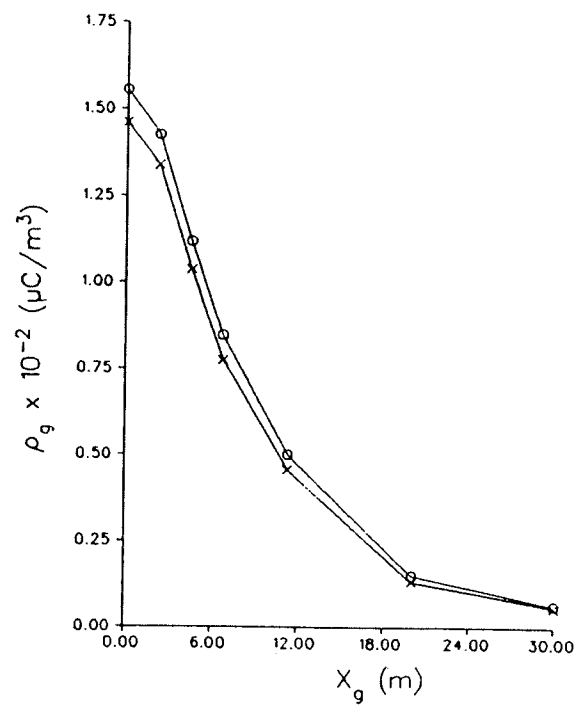
The discrepancies in the computed maximum level of the ionized field quantities at the ground plane due to the assumption are given in tables 5.10a-5.10c. The percentage differences decrease as the voltage increases for all quantities in general as observed earlier. However, comparison with the reduced geometry of $H/r_c = 300$ show that the deviations in the present case are slightly lower.

The average electric field at the conductor surface E_{avg} is computed and compared with the imposed value of the boundary condition E_c for the different voltage levels considered. The percentage error in E_{avg} is less than 0.77% at a voltage level of $1.4 V_0$ as shown in table 5.11a. The number of iterations needed for the algorithm to achieve convergence are given in table 5.11b for the different cases studied. The different parameters used in the updating criteria, eqns. 4.8 and 4.9, is the same as those used for the practical model.

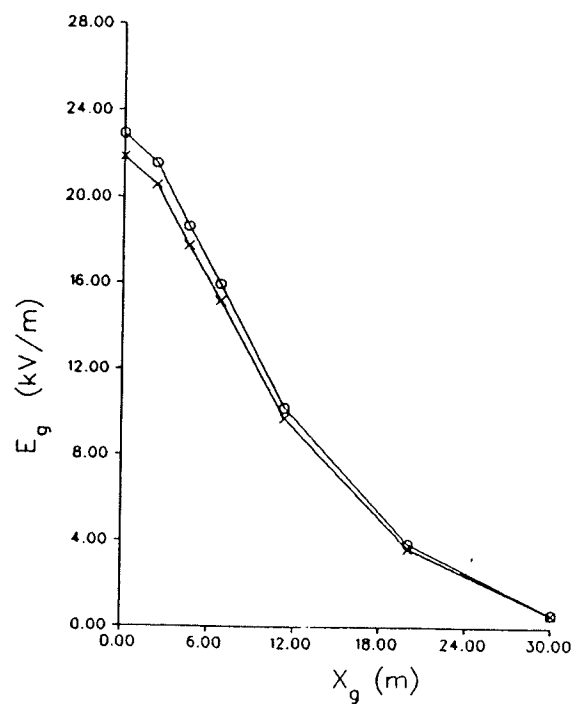
This concludes the analysis of the results computed for the reduced scale model of $H/r_c = 1000$. The analysis is carried out at different voltage levels similar to the case of $H/r_c = 300$ and detailed study of the ionized field quantities as well as global quantities show similar trends in the results.

5.5 Hara's Geometry $H/r = 800$

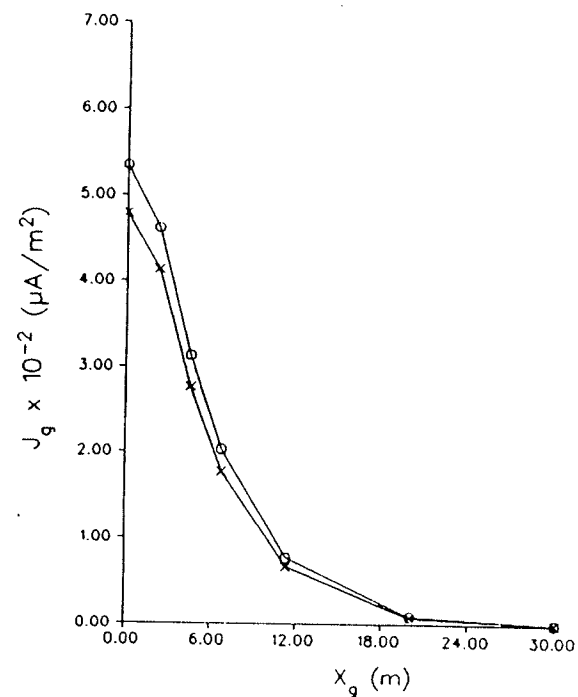
The third geometry considered consists of a single conductor of radius $r_c = 0.0025\text{m}$ located at a height $H = 2\text{ m}$ above a ground plane i.e. $H/r_c = 800$. This



a- Charge density

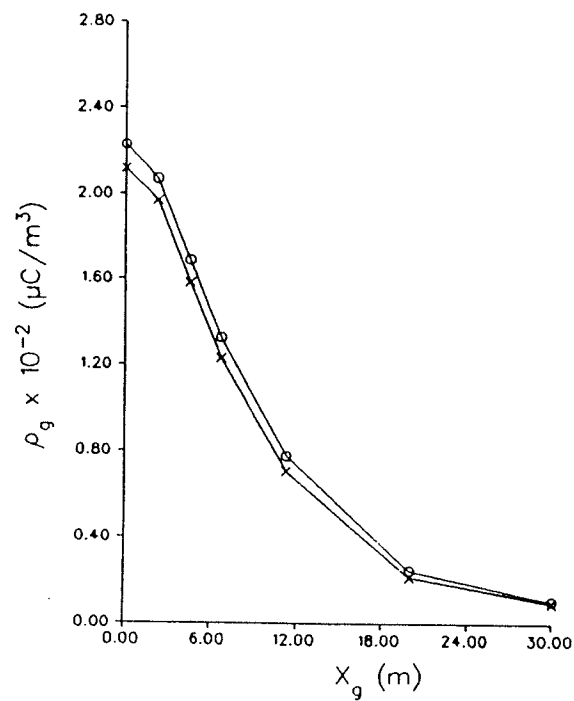


b- Electric field

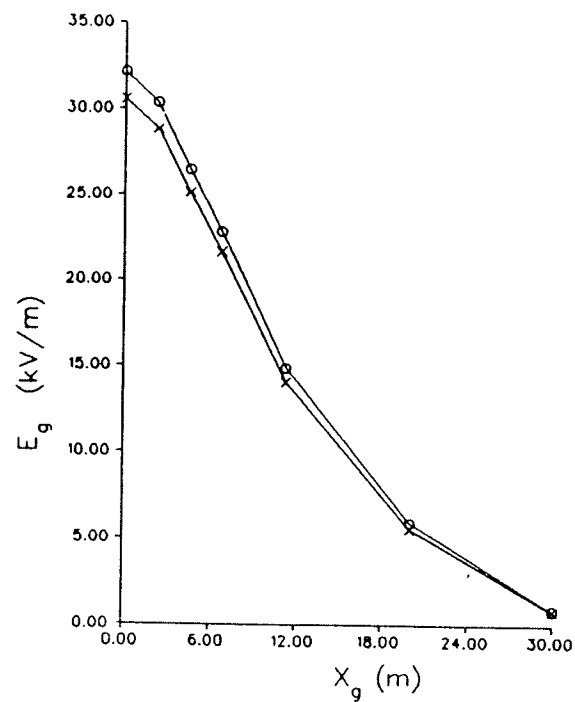


c- Current density

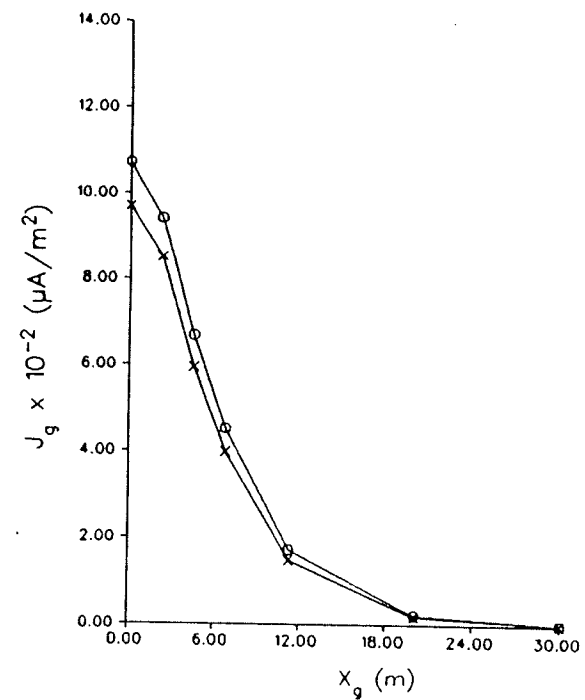
Fig. 5.19 Ionized field quantities along the ground plane, $V=372\text{kV}$
 ○ Without Kaptzov's assumption, x with Kaptzov's assumption



a- Charge density

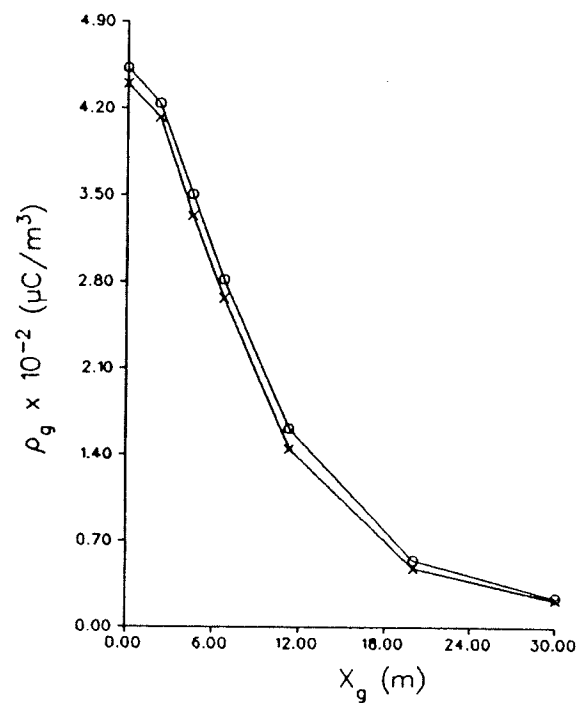


b- Electric field

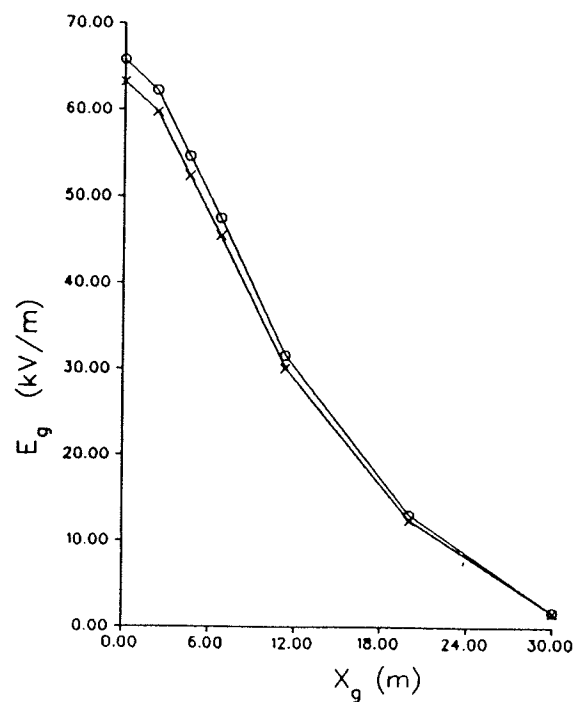


c- Current density

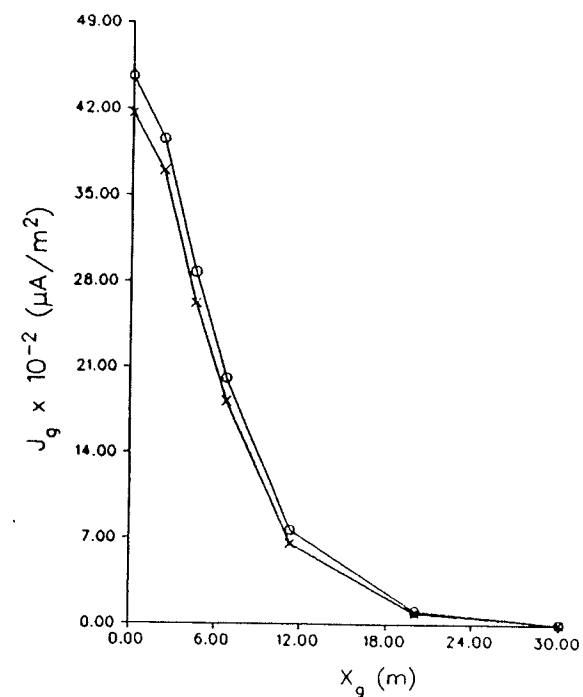
Fig. 5.20 Ionized field quantities along the ground plane, $V=425\text{kV}$
 $\bigcirc$ Without Kaptzov's assumption, $\times$ with Kaptzov's assumption



a- Charge density



b- Electric field



c- Current density

Fig. 5.21 Ionized field quantities along the ground plane, $V=637.5\text{kV}$
 ○ Without Kaptzov's assumption, × with Kaptzov's assumption

Table 5.10a Maximum charge density ($\mu C/m^3$) at ground,
computed at different voltages

V(kV)	$\rho_g^{(NK)}$	$\rho_g^{(K)}$	% Difference
372.0	0.015540	0.014597	6.068
425.0	0.022256	0.021147	4.983
478.0	0.028517	0.027361	4.054
585.0	0.039969	0.038682	3.220
637.5	0.045228	0.043929	2.872

Table 5.10b Maximum electric field (kV/m) at ground,
computed at different voltages

V(kV)	$E_g^{(NK)}$	$E_g^{(K)}$	% Difference
372.0	22.914	21.852	4.635
425.0	32.118	30.540	4.913
478.0	40.769	38.856	4.692
585.0	57.679	55.265	4.185
637.5	65.725	63.156	3.909

Table 5.10c Maximum current density ($\mu A/m^2$) at ground,
computed at different voltages

V(kV)	$J_g^{(NK)}$	$J_g^{(K)}$	% Difference
372.0	0.05341	0.04785	10.423
425.0	0.10723	0.09688	9.654
478.0	0.17439	0.15947	8.556
585.0	0.34580	0.32067	7.267
637.5	0.44589	0.41616	6.668

Table 5.11a The average electric field (kV/cm) upon convergence of the iterative procedures.

V(kV)	E_{avg}	E_c	% Error
372.0	39.74	39.94	0.503
425.0	39.01	39.31	0.769
478.0	38.58	38.75	0.441
585.0	37.69	37.83	0.370
637.5	37.33	37.48	0.400

Table 5.11b The total number of iterations until convergence for different voltages

V(kV)	without Kaptzov's assumption	with Kaptzov's assumption
372.0	10	13
425.0	6	9
478.0	6	9
585.0	8	14
637.5	8	15

geometry was considered by Hara's et al.[46] and important experimental data have been reported not only for the corona losses but also for the spatial distributions of the current density and electric field along the ground plane. The same steps used earlier for the reduced scale model of H/r_c are carried out here to obtain the ionized field solution for this configuration. The finite element mesh for this geometry is similar to the one shown in fig. 5.3 . The corona onset gradient for this geometry is $E_0 = 50.84$ kV/cm and the corresponding onset voltage is $V_0 = 93.77$ kV. Different overvoltage levels are considered and at each voltage, the iterative solution is performed as before until convergence is obtained.

5.5.1 Computed Results and Analysis:

The spatial distributions of the charge density ρ_c and current density J_c around the conductor periphery are shown in figs. 5.22, 5.23 and 5.24 for different applied voltage levels. The results are computed both with and without Kaptzov's assumption. The same trends observed earlier for other geometries are also present in this case. While distributions without the assumption are higher than those with the assumption the deviations between the distributions tend to decrease as θ increases. The corona current without the assumption $I^{(NK)}$ and with the assumption $I^{(K)}$ are also computed at different voltage levels. The results are given in table 5.12 for both cases and also shown in fig. 5.25 together with measured data[46]. The results using the present method in which Kaptzov's assumption is eliminated show good agreement with the measured data. The discrepancies caused by the assumption, as observed from table 5.12, are similar to those obtained earlier for the geometries $H/r_c = 300$ and 1000 . The relative influence of the assumption diminishes as the applied voltage increases. It varies from 11.30% at $V=112$ kV to about 8.79% at $V=200$ kV . The discrepancies caused by the assumption in this case are, however, larger than for those of the geometry $H/r_c =$

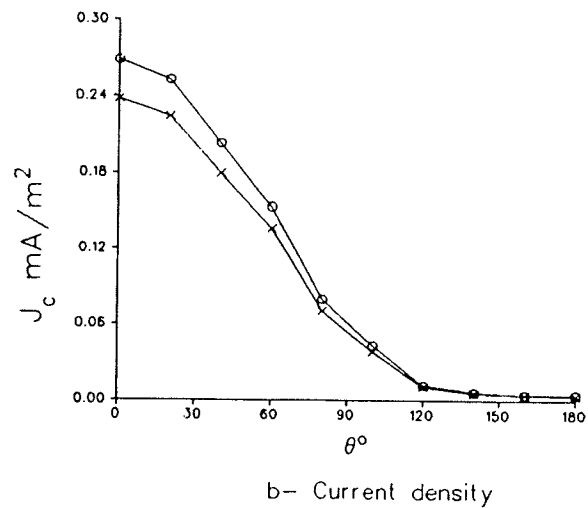
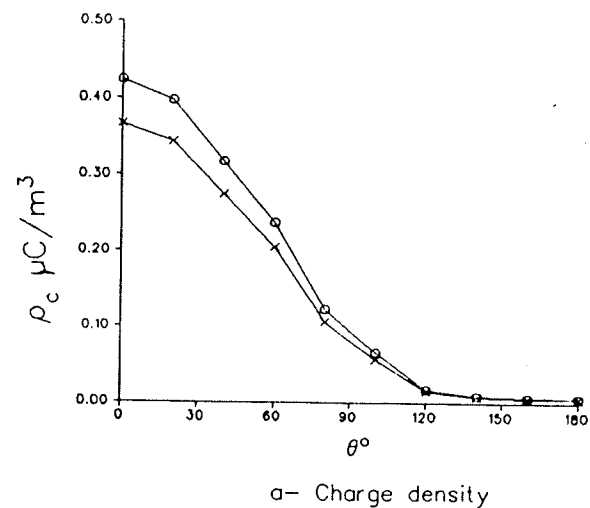


Fig. 5.22 Results around conductor periphery,
V=112kV.

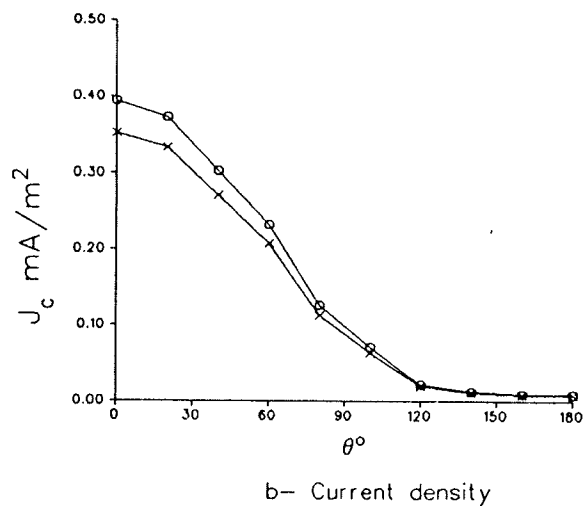
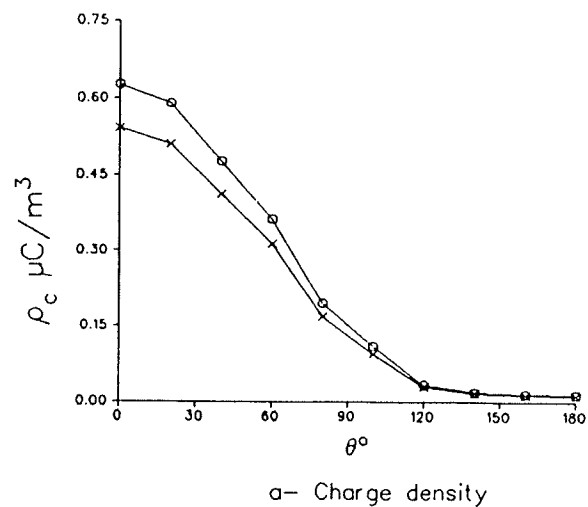


Fig. 5.23 Results around conductor periphery,
V=120kV.

○ ○ Without Kaptzov's assumption.
× × With Kaptzov's assumption.

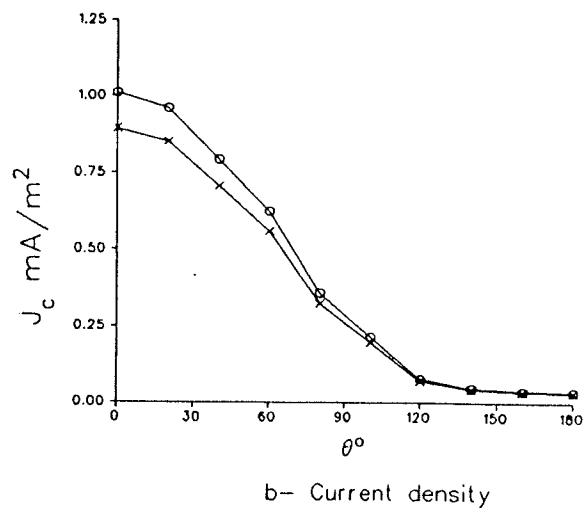
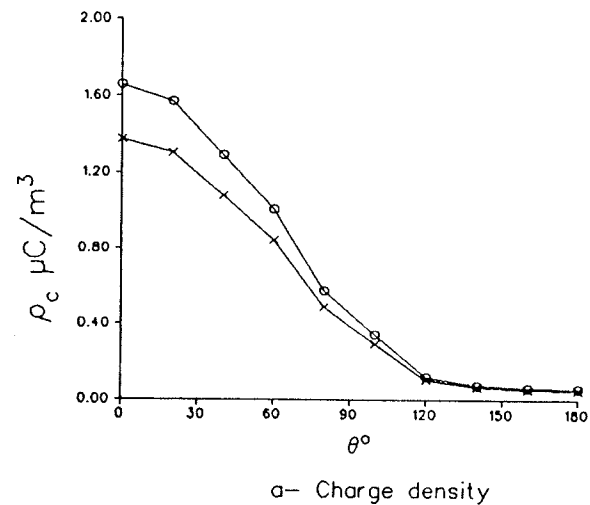


Fig. 5.24 Results around conductor periphery,
V=150kV.

Table 5.12 Corona current I ($\mu\text{A}/\text{m}$) computed with and without the assumption at different voltages

V(kV)	$I^{(K)}$	$I^{(NK)}$	% Difference
112	1.1431	1.2887	11.30
120	1.7468	1.9552	10.66
150	4.7325	5.2953	10.63
200	12.263	13.445	8.79

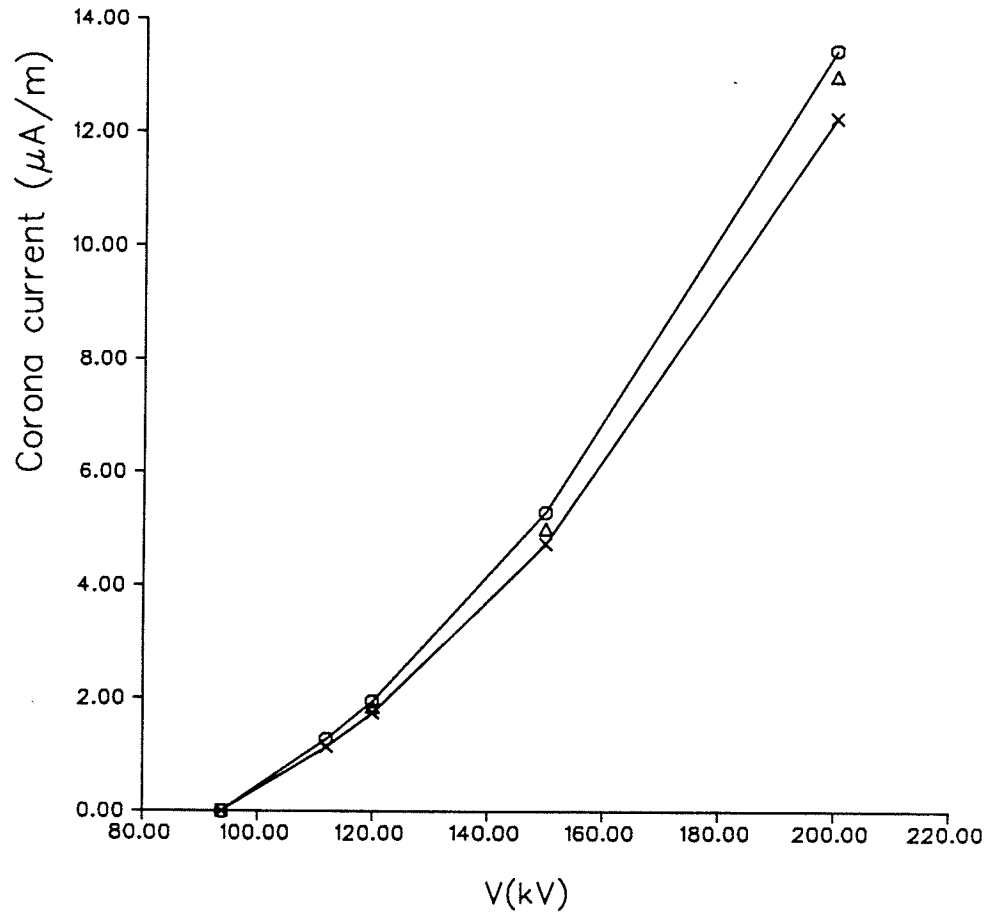


Fig. 5.25 Computed V - I & characteristics, $H/r_c = 800$

○ Without Kaptzov's assumption

× With Kaptzov's assumption

△ Experimental data [44]

300 but less than those of $H/r_c = 1000$, considered earlier, as seen from tables 5.6 5.9 and 5.12.

The ground level distributions of space charge density ρ_g , electric field E_g , and current density J_g , are computed and shown in figs 5.26, 5.27, and 5.28 , respectively, for different voltage levels. Similar to the above trends, results without Kaptzov's assumption are higher than those with it and the deviations tend to decrease as distance from the center line increases. Comparison with experimental data[46] are shown in figs. 5.28b and 5.28c for both the electric field and current density along the ground plane. Satisfactory agreement is generally observed in both cases.

The discrepancies in the computed maximum level of the ionized field quantities at the ground plane due to Kaptzov's assumption are given in tables 5.13a-5.13c . The percentage differences decrease as the voltage increases for all quantities in general. However, comparison with the reduced geometry of $H/r_c = 300$ show that the deviations in the present case are also lower similar to the case of H/r_c of 1000.

The average electric field at the conductor surface E_{avg} is computed and compared with the imposed value of the boundary condition E_c for the different voltage levels considered as shown in table 5.14a. The number of iterations needed for the algorithm to achieve convergence are given in table 5.14b for the different cases studied, while the different parameters used in the updating criteria, eqns. 4.8 and 4.9, are given in table 5.14c.

This concludes the analysis of the results computed for the model of $H/r_c = 800$. The analysis is carried out in a similar manner to the other two geometries. The same trends and observations are also present in this case.

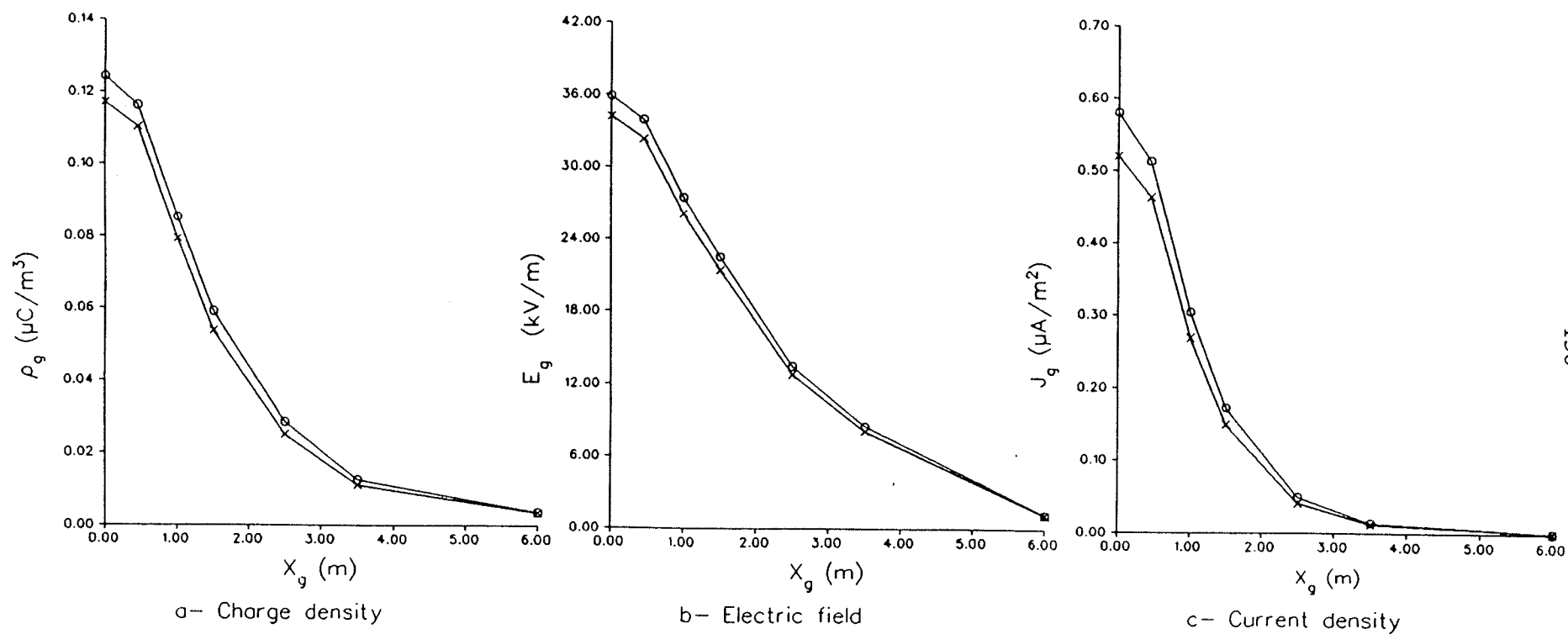


Fig. 5.26 Ionized field quantities along the ground plane, $V=112.0\text{kV}$

○ Without Kaptzov's assumption, × with Kaptzov's assumption

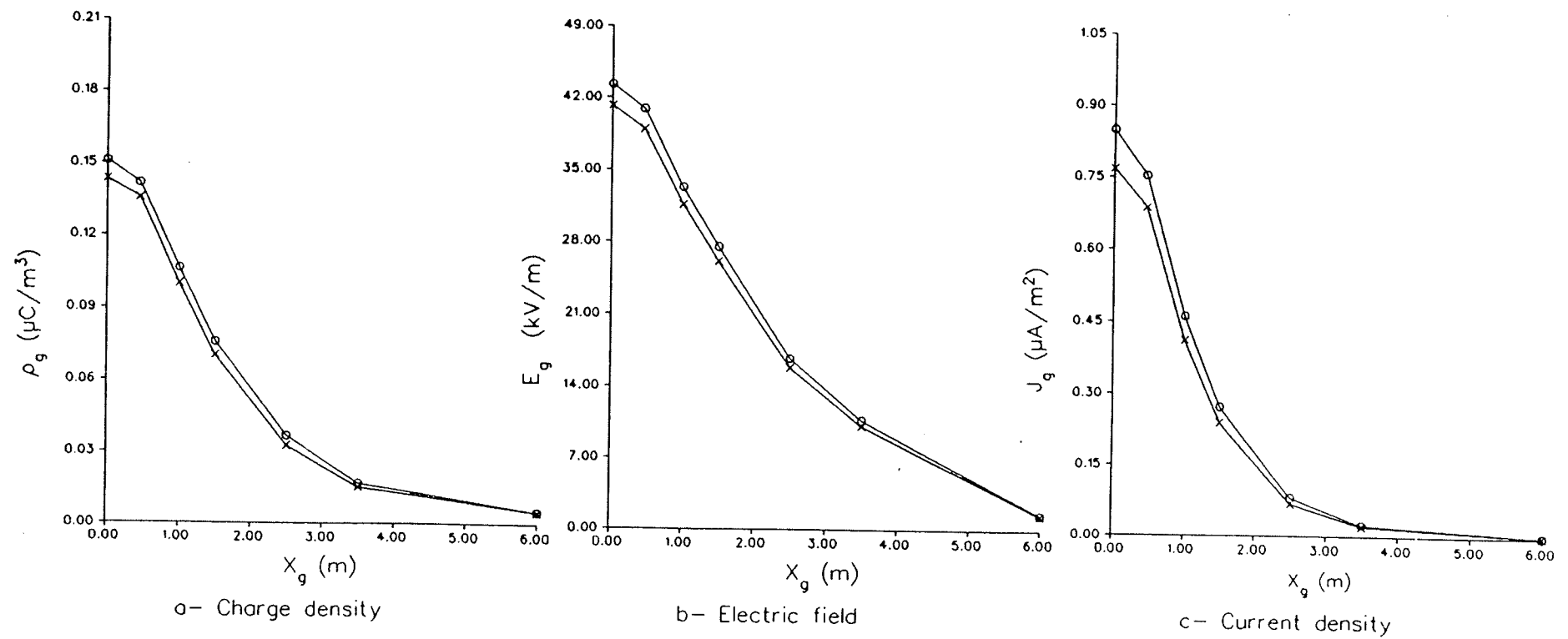


Fig. 5.27 Ionized field quantities along the ground plane, $V=120.0\text{kV}$
 ○ Without Kaptzov's assumption, × with Kaptzov's assumption

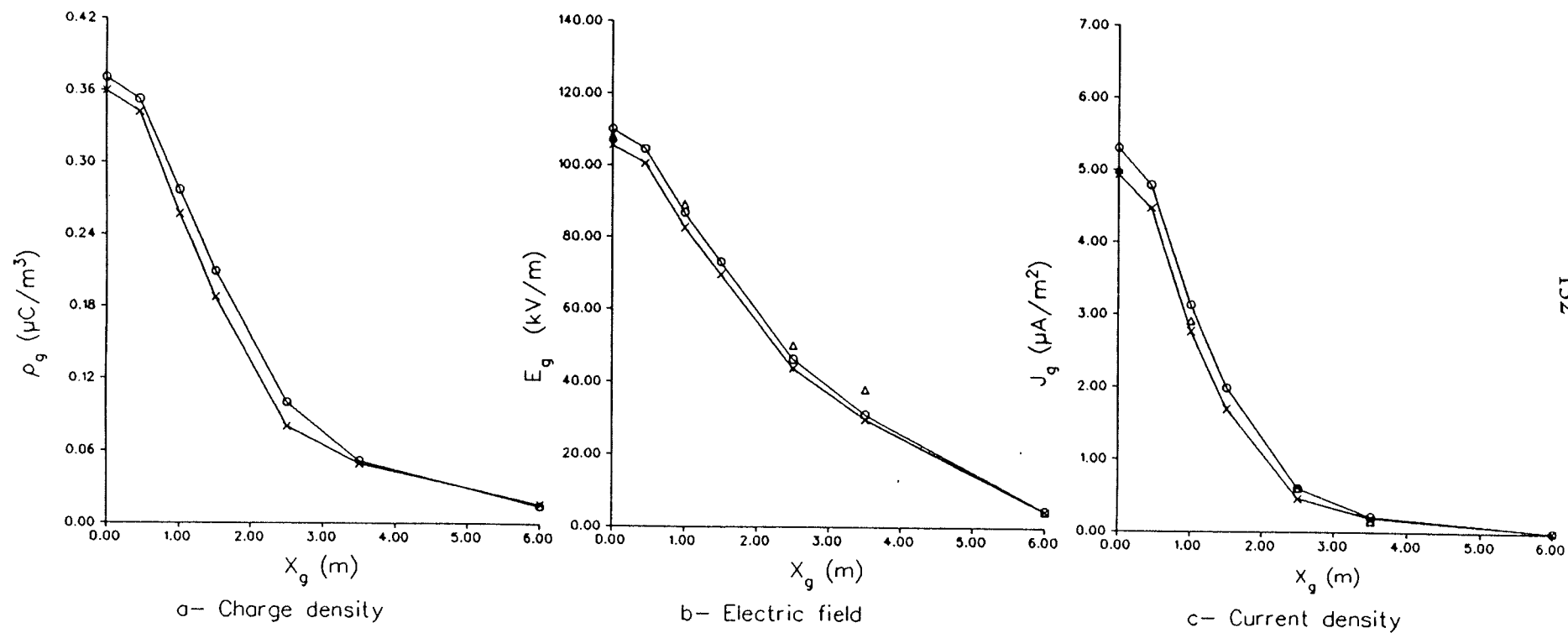


Fig. 5.28 Ionized field quantities along the ground plane, $V=200.0\text{ kV}$

$\circ$ Without Kaptzovs assumption, $\times$ with Kaptzovs assumption

Δ Experimental data [44]

Table 5.13a Maximum charge density ($\mu C/m^3$) at ground,
computed at different voltages

V(kV)	$\rho_g^{(NK)}$	$\rho_g^{(K)}$	% difference
112	0.12423	0.11696	5.852
120	0.15091	0.14330	5.043
150	0.23904	0.23313	2.472
200	0.37032	0.35931	2.973

Table 5.13b Maximum electric field (kV/m) at ground,
computed at different voltages

V(kV)	$E_g^{(NK)}$	$E_g^{(K)}$	% Difference
112	35.873	34.171	4.745
120	43.238	41.162	4.801
150	69.379	66.059	4.785
200	110.02	105.56	4.054

Table 5.13c Maximum current density ($\mu A/m^2$) at ground,
computed at different voltages

V(kV)	$J_g^{(NK)}$	$J_g^{(K)}$	% Difference
112	0.57937	0.51958	10.320
120	0.84826	0.76680	9.603
150	2.1560	2.0020	7.143
200	5.2965	4.9308	6.905

Table 5.14a The average electric field (kV/cm) upon convergence of the iterative procedures.

V(kV)	E_{avg}	E_c	% Error
112	49.53	49.63	0.201
120	49.06	49.22	0.325
150	47.58	47.90	0.668
200	45.99	46.39	0.862

Table 5.14b The total number of iterations until convergence for different voltages

V(kV)	without Kaptzov's assumption	with Kaptzov's assumption
112	13	17
120	10	13
150	8	23
200	13	45

Table 5.14c The values of the parameters used in the updating criteria.

V(kV)	a	b	c	d
112	0.80	0.80	0.10	0.25
120	0.80	0.80	0.10	0.25
150	0.50	0.50	0.10	0.25
200	0.20	0.20	0.11	0.25

5.6 Influence of Mesh Density on the Ionized Field Parameters

In order to study the influence of the mesh parameters on the ionized field solution, meshes of different densities are considered. The iterative solution is carried out in each case until convergence is obtained. The geometry considered is that of $H/r_c = 800$ discussed earlier in section 5.5. The ionized field solution is carried out at voltage $V = 112\text{kV}$. These meshes are denoted as mesh a, mesh b, and mesh c, with the parameters $NN=343, 195, 81$ and $NE= 611, 333, 125$ respectively. These meshes were also shown in figs. 5.4, 5.3 and 5.5.

Janischewskyj and Gela[15] studied the influence of the mesh parameters in the one dimensional case of coaxial cylinder. They used 9, 15 and 25 nodes, along the radial line, and showed that the residual errors, compared with the analytical solution, diminish as the number of nodes was increased. While the errors in the case of 9 nodes were significant, the two cases of 15 and 25 nodes gave practically the same errors.

The finite element algorithm is applied to the solution of the geometry under consideration using the three meshes described above. The results for the three cases are shown in figs. 5.29a-5.29c. Convergence occurred in 15, 13 and 20 iterations respectively. It can be seen that the results obtained using mesh a and mesh b have almost the same behaviour and are generally close to each other. The results obtained using mesh c, however, show almost a linear variation for all distributions. This is probably due to the small number of nodes used in this case. Although no fixed guidelines can be given as to the number or location of the nodes, it can be stated, based on the above observations that the quality of the solution may depend on the discretization of the region of interest; boundaries with large curvature must be approached by a fine grid and the grid size must be smallest where the variation of the solution with position is expected to be very

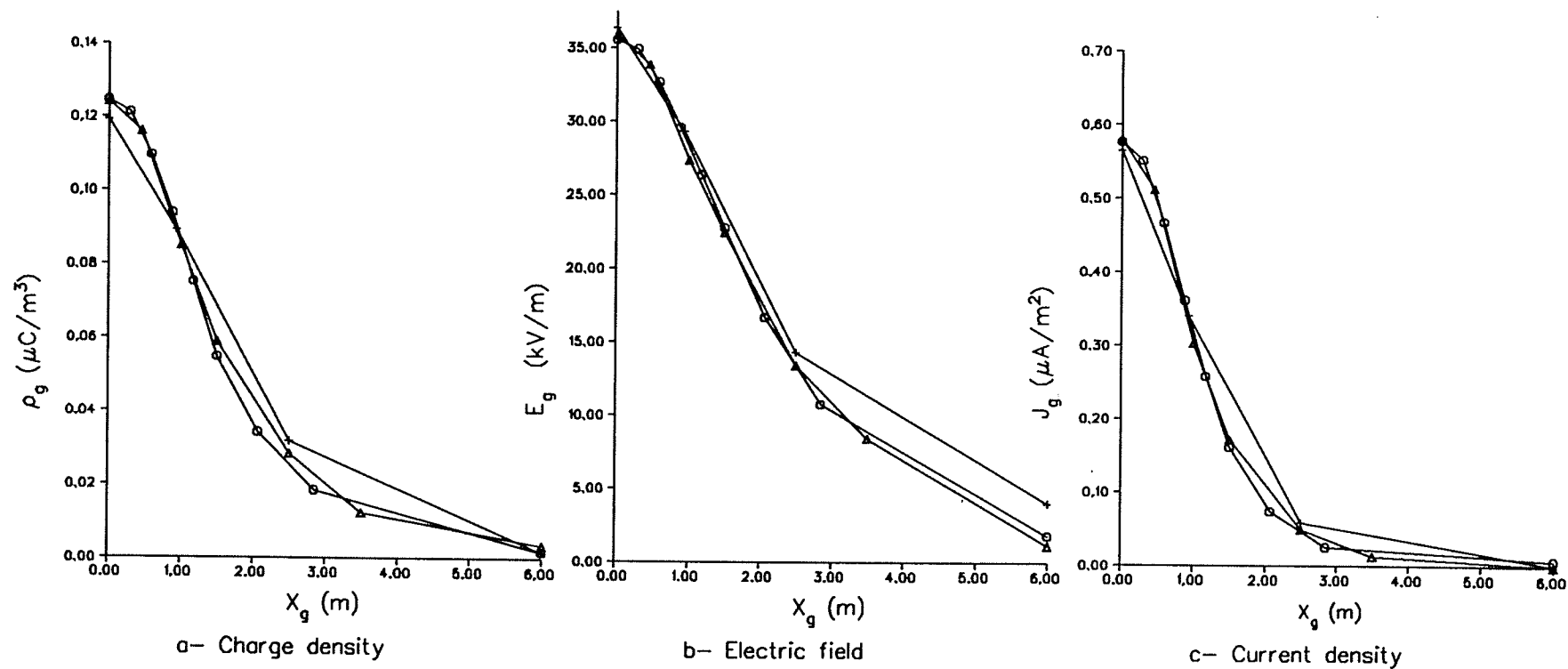


Fig. 5.29 Ionized field quantities along the ground plane for different meshes

- mesh a, NN=343, NE=611
- △ mesh b, NN=195, NE=333
- + mesh c, NN=81, NE=125

significant. Locations where the functions do not vary sharply may be represented by large elements. The finite element grids used in the present work were generated using the procedures described in section 5.1.1. The nodes are located approximately along the electrostatic field lines and, close to the conductor surface, the step used in tracing the lines is small and increases gradually as the distance from the conductor increases. Physical understanding of the problem is extremely useful in providing guidelines in this regard.

5.7 Conclusions

In this chapter the ionized field solution is carried out for the general geometry of a single conductor above a ground plane. The finite element algorithm, developed earlier in chapter 4 for the coaxial cylinder geometry, is applied for the case of a single conductor above ground configuration. Initially, the electrostatic field solution for a geometry of $H/r_c = 800$ is obtained using different mesh densities. The accuracy of the solution is assessed by a comparison with the image method. The ionized field solution is performed for different geometries ranging from $H/r_c = 300$ to 1000 and for voltage levels up to twice the onset value. Simple procedures for constructing the finite element grid and the initial space charge distribution for the different cases are described. Detailed analysis of the results for the corona current as well the spatial distributions of the various ionized field quantities both at the conductor surface and along the ground plane is presented. Investigation of the influence of Kaptzov's assumption on these quantities is also carried out. The numerical aspects regarding the accuracy and efficiency of the iterative procedures were also studied and demonstrated.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The main objective of the research carried out in this thesis has been the development of means that would permit detailed analysis and evaluation of the significance and effects of simplifying assumptions, used by previous researchers, on computations of ionized fields associated with hvdc unipolar corona. The two most commonly used assumptions are the Deutsch and the Kaptzov assumptions. While, the former states that the electric field direction is not affected by space charge presence, the latter assumes that the field intensity at the coronating conductor surface remains constant at the onset value regardless of the level of the applied voltage. Most previous procedures, being dependent on these simplifications, do not reflect a true picture of the corona phenomena and are, thus, approximate. Since, no exact solution have been available for dc transmission line configurations, the assessment of the extent and importance of these simplifications have not been possible. Consequently, in order to achieve the goals of the present work, methods which would be capable of modeling, accurately, the conditions and interactions of the corona generated space charge and the electric field intensity have to be, initially, formulated. Following this step, the subject of comparisons with results based on various assumptions, and assessment of the simplifications involved could, then, be carried out.

An improved analytical approach to the ionized field solution of unipolar dc corona without resorting to these assumptions was developed and described in this work. Initially, the method was applied to the case of coaxial cylindrical configuration; then extended to the two dimensional case of a smooth cylinder above the ground. Various configurations and corona intensities were considered.

Comparison of the theoretically calculated results with available experimental data showed satisfactory agreement. Improved expressions were also derived for the case of stranded conductors. Comparison with the modified Popkov empirical formula showed that the results obtained using the present method were in better agreement with published experimental data.

The influence of Deutsch's assumption on the distortion of the flux lines pattern was also carried out. Modified expressions which can be used for accurate assessment of the right-of-way current density profiles were derived. Comparison of the results, using the developed procedures, with published experimental and numerical data showed reasonably good agreement.

A computational iterative algorithm, based on the finite element method, for the solution of the differential equations of unipolar corona was also developed and described in this work. While, not relying on the above mentioned assumptions, it incorporated an improved criteria for continuously updating the space charge distribution during the solution steps. The algorithm was applied initially to the coaxial cylindrical configuration. Different initial space charge distributions were considered and the algorithm worked its way through to convergence in all cases both accurately and efficiently.

The finite element algorithm was then implemented for the solution of the general geometry of a single conductor above ground. Simple procedures for constructing the finite element mesh and the initial space charge distribution were presented. Different configurations and voltage levels were considered. Assessment of Kaptzov's assumption was established through the analysis of the corona current (per unit length of the conductor) and the spacial distributions of the field quantities both at the conductor surface and over the ground plane. Detailed study of these distributions showed that results obtained without this assumption were

always higher than those obtained using it and were also in better agreement with the experimental data. The relative influence of Kaptzov's assumption on the corona current as well as on the maximum level of the ionized field quantities at ground was found to be more significant at voltages close to the onset level. However, this influence diminished at higher voltage levels. The effect of the assumption on the corona current was found to increase as the ratio of the conductor height H to its radius r_c increased. On the other hand, the influence on the ionized field quantities at the ground level decreased at higher ratios of H/r_c .

Based on the research work carried out in this thesis, it is recommended that the present analysis be also extended to the case of bundled as well as bipolar dc transmission lines. In addition, the weather variables; for example the wind, are known to have a significant influence on the ionized field parameters such as the current density at the ground level. Accurate modeling and simulation of these effects in the theoretical analysis are thus necessary for the assessment of the environmental effects of hvdc transmission lines.

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APPENDIX A

FINITE ELEMENT FORMULATION

In the finite element method, FEM, the solution to a field problem governed by any of the equations (4.1), (4.2) which is a partial differential equation, is approximated by subdividing the region of interest into elements and representing the solution within each element by a relatively simple function. Such a function is called an interpolation function (shape function) and is usually in the form of a polynomial. Mathematically, when a variational formulation is followed, it can be shown by the use of Euler condition [55,56], that the problem can be restated identically as the determination of the minimum value of an appropriate functional subject to Dirichlet boundary conditions. The shapes of the elements and their degree of interconnection (number of nodes) are features by which the accuracy of representation can be improved. In the following sections, a brief review of the finite element method as applied to the solution of above equations; which describe the corona problem of overhead DC transmission lines, is presented.

Equations (4.1) and (4.2) could be expressed, in general, as,

$$\nabla \cdot (\lambda \nabla \Phi) = \gamma, \quad (A-1)$$

where λ and γ are either constants or functions related to ρ .

If λ is ϵ_0 and $\gamma = \rho$; the space charge distribution, then equation(A-1) becomes Poisson's equation for a homogeneous medium, equation(4.1),

$$\nabla \cdot (\epsilon_0 \nabla \Phi) = \rho. \quad (A-2)$$

When $\lambda = \rho$ and $\gamma = 0$, equation(A-1) becomes Laplace's equation for an inhomogeneous medium; equation(4.2),

$$\nabla \cdot (\rho \nabla \Phi) = 0. \quad (\text{A-3})$$

A general computer subroutine capable of solving any of the above two equations, equations (A-2) and (A-3) has been developed, and the solution of a specific equation is performed by appropriate definitions of λ and γ .

In order to solve equation (A-1), the following functional is used as in the usual FEM[55,56,57],

$$F = \iint \left[\lambda \left\{ \left(\frac{\partial \Phi}{\partial x} \right)^2 + \left(\frac{\partial \Phi}{\partial y} \right)^2 \right\} - 2\gamma \Phi \right] dx dy. \quad (\text{A-4})$$

The function Φ that minimizes equation (A-4), while satisfying any prescribed Dirichlet boundary conditions is the solution of equation (A-1).

The interelectrode space is divided into triangle elements and the potential within each element is described by,

$$\Phi = \sum_{j=1}^M \alpha_j \Phi_j, \quad (\text{A-5})$$

where Φ_j are the node potentials, α_j are the shape functions and M is the number of the triangle nodes. Linear and quadratic polynomial shape functions are used, for which M is 3 and 6, respectively.

Equation (A-5) is applied to each element and substituted into equation (A-4). The functional F is thus computed in terms of the node potentials Φ_i where $i=1,2,\dots,K$, and K is the number of nodes. In order to minimize F , the following system of linear equations is solved,

$$\frac{\partial F}{\partial \Phi_i} = 0 \quad i=1,2,\dots,K \quad (\text{A-6})$$

or,

$$\sum_{j=1}^{j=K} s_{ij} \Phi_j = b_i \quad i=1,2,\dots,K \quad (\text{A-7})$$

or in matrix form,

$$S \underline{\Phi} = \underline{b} \quad (\text{A-8})$$

where S and $\underline{b}$ are the finite element matrices, and $\underline{\Phi}$ is the vector of node potentials.

In order to produce S and $\underline{b}$, the contributions from each element in the global coordinates (x, y coordinates), is calculated and these contributions are, then, accumulated into S and $\underline{b}$.

To create S and $\underline{b}$ for a given global triangle, $\Phi = \sum_{i=1}^M \alpha_i \Phi_i$ is, first, substituted into equation (A-4), then by differentiation with respect to Φ_i , the following equation is obtained,

$$\begin{aligned} \frac{\partial F}{\partial \Phi_i} = & 2 \iint \lambda \left[\sum_{j=1}^M \frac{\partial \alpha_j}{\partial x} \Phi_j \right] \frac{\partial \alpha_i}{\partial x} dx dy + 2 \iint \lambda \left[\sum_{j=1}^M \frac{\partial \alpha_j}{\partial y} \Phi_j \right] \frac{\partial \alpha_i}{\partial y} dx dy \\ & - 2 \iint \gamma \alpha_i dx dy \end{aligned} \quad (\text{A-9})$$

A.1 Isoparametric elements (local to global coordinates transformation) :

Isoparametric elements utilize a local coordinate system to formulate the element matrices [55,56,57]. The local coordinates, say, ζ , η are usually dimensionless and range from 0 to 1, or from -1 to 1, thus compatible with the definition of abscissa utilized in numerical integration by Gaussian quadratures. The elements are called isoparametric since the same(iso) local coordinate parametric equations (interpolation functions) used to define any quantity of interest within the elements are also utilized to define the global coordinates of any point within the element in terms of the global spatial coordinates of the nodal points. The global spatial coordinates are denoted by x , y and the number of nodes per element is M . The

value of a scalar quantity of interest, $\Phi(\zeta, \eta)$, at any local point (ζ, η) within the element is assumed to be defined by the values at the M nodal points of the element, and a set of interpolation functions $(\alpha_i, 1 \leq i \leq M)$, That is,

$$\Phi(\zeta, \eta) = \sum_{i=1}^M \alpha_i(\zeta, \eta) \Phi_i. \quad (\text{A-10})$$

Generalizing this concept, the global coordinates are defined as

$$x = \sum_{i=1}^M \alpha_i(\zeta, \eta) x_i \quad (\text{A-11a})$$

$$y = \sum_{i=1}^M \alpha_i(\zeta, \eta) y_i \quad (\text{A-11b})$$

The source of the global element in the local plane is the triangle $(0,0)$, $(1,0)$, $(0,1)$. This is shown in fig.(A.1) for $N=1$ and $N=2$. The shape functions for both cases are given in Table (A.1), where $L_1=\eta$, $L_2=1-\zeta-\eta$, $L_3=\zeta$.

As the shape functions $\alpha_i(\zeta, \eta)$ are defined analytically in terms of the local coordinates, then their partial derivatives with respect to the local coordinate can also be expressed as,

$$\frac{\partial \alpha_i}{\partial \zeta} = \sum_{j=1}^3 \frac{\partial \alpha_i}{\partial L_j} \frac{\partial L_j}{\partial \zeta} \quad (\text{A-12a})$$

$$\frac{\partial \alpha_i}{\partial \eta} = \sum_{j=1}^3 \frac{\partial \alpha_i}{\partial L_j} \frac{\partial L_j}{\partial \eta} \quad (\text{A-12b})$$

In order to calculate the different components of equation (A-9), the values of $\frac{\partial \alpha_i}{\partial x}$ and $\frac{\partial \alpha_i}{\partial y}$ are determined as follows:

$$\begin{bmatrix} \frac{\partial \alpha_i}{\partial x} \\ \frac{\partial \alpha_i}{\partial y} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\partial \alpha_i}{\partial \zeta} \\ \frac{\partial \alpha_i}{\partial \eta} \end{bmatrix}, \quad (\text{A-14})$$

Table A.1 Shape functions $\alpha_i(\xi, \eta)$.

i	$N=1$	$N=2$
1	L_1	$L_1(2L_1-1)$
2	L_2	$L_2(2L_2-1)$
3	L_3	$L_3(2L_3-1)$
4		$4L_1L_2$
5		$4L_2L_3$
6		$4L_3L_1$

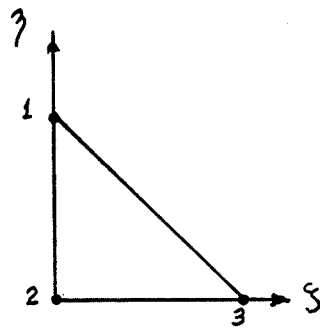
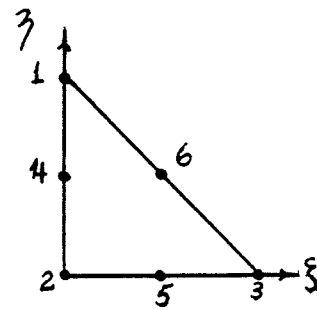
 $N=1, M=3$  $N=2, M=6$

Fig. A.1 The local triangle

where J is the Jacobian matrix and is expressed as,

$$J = \frac{\partial(x,y)}{\partial(\zeta,\eta)} = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad (\text{A-14})$$

In order to evaluate J , it is written in the form,

$$J = \begin{bmatrix} \frac{\partial \alpha_i}{\partial \zeta} & \dots & \frac{\partial \alpha_M}{\partial \zeta} \\ \frac{\partial \alpha_i}{\partial \eta} & \dots & \frac{\partial \alpha_M}{\partial \eta} \end{bmatrix} \begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_M & y_M \end{bmatrix} \quad (\text{A-15})$$

All the integrations are performed in the local plane, using Gaussian quadrature formulas [58].