D-^{16}O SCATTERING AND REACTIONS AS THREE-BODY PROBLEMS

by

TAMER AYTIMUR

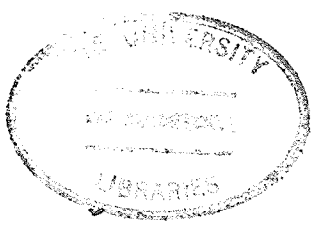
supervised by J. P. SVENNE

submitted at the UNIVERSITY OF MANITOBA

in partial fulfillment of the requirements

for the degree of Doctor of Philosophy

December 1978



$D^{-16}O$ SCATTERING AND REACTIONS AS THREE-BODY PROBLEMS

BY

TAMER AYTIMUR

A dissertation submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

DOCTOR OF PHILOSOPHY

ABSTRACT

In this thesis the elastic and rearrangement scattering processes of the deuteron-^{16}O system are examined and compared as a set of idealized model calculations of a physically relevant problem in the context of the Distorted-Wave Born Approximation (DWBA) and the Faddeev three-body formalisms. The oxygen nucleus is treated on the same footing as the nucleons in the deuteron. Pairwise two-body interactions in the form of parametrized separable potentials were determined by obtaining the closest fits to the two-body scattering phase shifts as taken from the literature and as derived from an optical model approach to nucleon-nucleus scattering. Due to the large size of the three-body calculation, it was necessary to restrict the number of two-body nucleon-nucleus interactions. Coulomb effects were completely left out in both calculations due to the uncertainty and difficulty associated with their inclusion in a three-body formulation. The final results give qualitative agreement between the two model calculations in the case of deuteron elastic scattering. The greater discrepancies found in the rearrangement scattering and polarization results may be attributable to the various limitations imposed on the three-body calculation, the inadequacy of the DWBA approach in treating nuclear reactions in a fundamental way, or the importance of the detailed structure of the ^{16}O nucleus at the projectile energies for which the calculations were carried out-- 20 and 45 MeV deuteron lab energy.

TABLE OF CONTENTS

INTRODUCTION

The objective in this work is to compare, and find relations between, the mathematically exact Faddeev-type equations (Fad61a, b) of the three-body problem and the empirically oriented reaction theories. All quotations of experimental results (which are mostly in the footnotes) will be given in cases where this is helpful in pointing to the resolution of discrepancies between the two approaches, or to make the study more self-contained for a future investigation.

An approach to composite particle reactions via the three-body problem will have to involve various approximations to be manageable, but it is quite possible that an account of some reaction processes thus provided may explain certain empirical and approximate features of the current theories in a more fundamental way. The basis for this supposition is that such an approach retains contact with the well-established and accurate microscopic principles of Quantum Mechanics.

The three-body approach in Nuclear Physics has been extensively developed and applied in three-nucleon structure and scattering problems. (Alt67, Lov63, Lov64, Alt72, Osb67, Dol73) Until recently the Faddeev approach was not quantitatively applied to multi-nucleon systems. The formalism has no intrinsic restrictions in this regard. At present, even though there are recent theories which generalize the three-body formalism to N-particle systems where $N > 3$ (Ben73, Red74), few calculations could be carried out with them due to their complexity. For this reason a numerical investigation of a multi-nucleon system in the

Faddeev formulation can realistically be carried out only if the system is composed of three well-defined entities. In this work such an investigation is performed in the case of certain deuteron-^{16}O processes.

The important ingredients which distinguish the three-cluster problems from the three-nucleon problems are the necessities to construct pairwise effective interactions between clusters, and to take into account their internal structure. The choice of a doubly-magic, hence tightly bound nucleus (first excited state at nearly 6.05 MeV) as the only cluster in this system is assumed to minimize the internal structure effects, especially in the low energy calculation.

An early attempt by Dodd and Greider (Dod67) to solve the three-body problems of nuclear (and atomic) physics was made in the case of one of the particles having a much greater or smaller mass than the other two. A certain class of the solutions thus obtained contains the distorted-wave Born model as part of the lowest order term, while another class does not. A similar work (Alt69) without any restriction on mass obtains an exact rearrangement scattering amplitude, the main term of which approximately reduces to the conventional Distorted-Wave Born Approximation (DWBA, or sometimes abbreviated D.W.) expression, while a remainder term represents multiple exchange processes. A numerical comparison of the DWBA with a three-body calculation using an S-wave separable potential was done by Bouldin and Levin (Bou72) for deuteron stripping reactions. They found good agreement for the distributions, and concluded that for three-body models employing separable potentials the DWBA is a good

approximation. Vanzani (Van73) obtains a distorted-wave-like approximation from the Faddeev-Alt-Grassberger-Sandhas theory via a generalized form of DWBA. In achieving this the implicit channels (i.e. the ones different from the entrance and exit channels) are eliminated, and some contributions from the basic reaction mechanisms are neglected, in addition to a number of other assumptions. Young and Redish (Ycu74) compare a three-body calculation of the reaction $^{4}He(p,2p)^{3}H$ with the distorted-wave impulse approximation (DWIA) at the energies of 65 and 100 MeV. In their calculation the protons were treated as identical bosons, and the p-p interaction consisted of an S-wave term, while the p-t potential was limited to partial waves with L less than or equal to two. The DWIA, as calculated by summing the multiple scattering series, gives comparable results at 100 MeV, but is twice too large at the low energy. The exact, distorted, and plane-wave cross sections were found to be in qualitative agreement.

Among the works along these lines, the present study most closely resembles those by Shanley and Aaron (Sha67), and by Charnomordic et al. (Cha77). Shanley and Aaron carry out a test of the DWBA for (d,p) reactions. All of their potentials are taken to be charge independent, and they consist of and are parametrized in terms of S-wave separable interactions. The optical model potentials they use are generated via a search code from the exact elastic scattering results. Their results are compared with the experimental situation, to which they show resemblance in general features, and are used as a test of the DWBA and some high-energy diffraction models. Charnomordic et

al. calculate the d-α elastic scattering observables, among others, for the A=6 system, with the use of separable potentials for the N-α interaction, and report an improvement over Shanley and Aaron's results. Their N-α interaction was parametrized using the S(1/2), P(1/2) and P(3/2) partial waves. The current status of the application of three-body methods to certain nuclear reactions has recently been reviewed by Redish (Red77).

In this work, in determining the two-cluster interactions a parametric approach is to be followed. The separable two-cluster interaction parameters (to be described later) are fixed by obtaining the closest fit to the experimental or derived phase shifts--see footnote on p.16. These are taken directly from the literature in the n-p case, and they are evaluated from optical model parameters for the N-^{16}O interaction.

The three-body integral equations which use the fitted separable two-particle potentials are then solved by means of the Conjugated Gradient Method (Bcd56), or the Pade Approximant Technique (Sch74).

In what follows all the Coulomb effects will be neglected. Either of the nucleons in the deuteron will then be referred to as N, but these nucleons will be assumed to be distinguishable fermions.

The elastic and rearrangement scattering observables of the deuteron on ^{16}O will be the main points of emphasis in the comparison of the distorted-wave and the three-body calculations. The treatment of the break-up processes was discontinued for the main reason of keeping the scale of the computation to a manageable size.

CHAPTER 1

THE FORMALISM

In the following analysis the Alt-Grassberger-Sandhas (Alt67) formulation of the three-body problem with separable two-cluster potentials is adopted as the starting point.

The three particles are labelled as

$$^{16}O=1, \quad N1=p=2, \quad N2=n=3$$

The goal in this part is to obtain, in the context of the formalism that follows, theoretical predictions for cross-section and polarization observables for the processes given below:

scattering or reaction	symbolic representation
$^{16}O+d \rightarrow {}^{16}O+d$	$1+(23) \rightarrow 1+(23)$
$^{16}O+d \rightarrow {}^{17}O+p$	$1+(23) \rightarrow (13)+2$
$^{16}O+d \rightarrow {}^{17}F+n$	$1+(23) \rightarrow (12)+3$

The first of these describes elastic scattering; the next two are rearrangement reactions. The calculations pertaining to the break-up of the deuteron had to be abandoned due to the excessive amount of additional computing time needed, and the limited amount of insight that could be gained from their less accurate predictions.

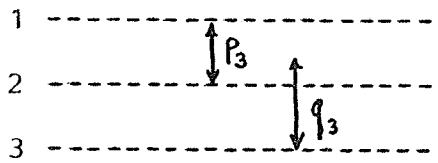
The expressions for the kinematic variables are now given for later convenience. k_i, m_i, σ_i are to stand for momenta, masses and spins of the clusters (particles). p_i is to indicate the relative 'momentum' of the two-cluster subsystem (jk), while

q_i is to be the 'momentum'[1] of the third particle or cluster (i) with respect to the centre of mass of the two-body subsystem (jk). If, for example, the (12)-pair is the subsystem, then p_3 and q_3 are defined as in Lovelace (Lov64):

$$\vec{p}_3 = \frac{m_2 \vec{k}_1 - m_1 \vec{k}_2}{(2m_1 m_2 (m_1+m_2))^{1/2}} \qquad (1.1a)$$

$$\vec{q}_3 = \frac{m_3 (\vec{k}_1 + \vec{k}_2) - (m_1+m_2) \vec{k}_3}{(2m_3 (m_1+m_2) (m_1+m_2+m_3))^{1/2}} \qquad (1.1b)$$

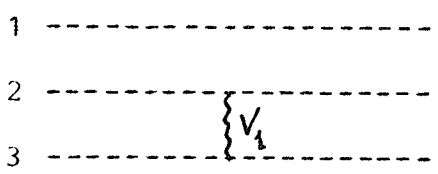
This can be shown schematically as in the diagram



For two particle interactions the usual convention is to write

$$V_k = V_{ij} \quad i,j,k = 1,2,3, \text{ or permutations.} \qquad (1.2)$$

For example, V_1 indicates the nuclear interaction between particles 2 and 3 as in



---

1) These momenta have the dimension $(\text{MeV})^{1/2}$, unlike k .

The three-particle hamiltonian is

$$H = H_0 + \sum_{i=1}^{3} V_i \ . \qquad (1.3)$$

Here H_0 is the free hamiltonian, V_i denotes the two-particle interactions. The potential terms in the hamiltonian contain no Coulomb or three-body interactions. Coulomb effects will completely be left out in both calculations due to the uncertainty and difficulty associated with their inclusion in the three-body formalism. Since the subsequent calculations seek to compare two theories for a model system, this will not be regarded as a deficiency in our approach. Three-body forces will also be neglected, since the basic three-cluster calculations (without three-body forces) have not yet been sufficiently refined and tested without the complexity they would introduce. In three-nucleon problems such effects are estimated to be small.[2)]

In the Alt-Grassberger-Sandhas formulation of the Faddeev equations the reaction

$$(ij)+k \dashrightarrow i+(jk)$$

is described by the solution U_{ik} of the operator equation

$$U_{ik}(s) = (1-\delta_{ik})(s-H_0) + \sum_j (1-\delta_{ij}) T_j(s) G_0(s) U_{jk}(s) \qquad (1.4)$$

where $s=E+i\varepsilon$, $G_0=(s-H_0)^{-1}$, and $T_j(s)=V_j+V_j G_0(s) T_j(s)$. Here E is the three-particle energy, G_0 is the resolvent of the free hamiltonian, and T_j is the two-body operator for the scattering

---

2) In this area, the three-body effects are regarded to be of the same order of importance as the relativistic ones, and are generally ignored along with them. (Van73)

of particles i and k, expressed in the three-body Hilbert space. (In the two particle space the corresponding equation will be written $t=v+vg_o t$.) The U_{ik} are the three-particle transition operators.

In the course of this work separable, real, rank-1, or at most rank-2 potentials are used. Below are given the shortcomings and advantages of such potentials.

The main advantage of separable potentials is that they simplify the calculations by reducing two-dimensional integral equations into one dimensional ones. (Lov64)

On the other hand, separable potentials do not approach the one-pion-exchange potential (OPEP) --which is a local potential-- at large distances, where the latter is more realistic. Particular deficiencies of a 1-term (i.e. rank-1) potential are:

a) it allows for one bound state,

b) the phase shift derived from it cannot change sign --it starts out as positive or negative, and goes to zero asymptotically as k goes to infinity. (Lev74)

One can hope to get around the difficulty of the non-locality of the potential by concentrating on the features of the scattering process for energies higher than, say, 20 MeV, thereby avoiding the low energies where the local, thus long range, behaviour of the interactions would be dominant. This choice of energy region is not only convenient in this regard, but also renders negligible the Coulomb effects, which are henceforth neglected by treating the nucleons in the deuteron to be neutral fermions, and the ^{16}O nucleus as a neutral boson. Thus the model system under investigation consists of two nucleons and a tightly

bound cluster. Also, for the validity of the distorted-wave calculation, in which optical potentials are employed, one must stay away from energies less than about 10 MeV, where the compound nucleus processes are important.

Rank-2 separable potentials will be used whenever there are coupled channels involved, or there are more than one bound state per channel --this is never the case in our problem--, or when the phase shift changes sign in the energy region under consideration.

In operator notation a separable potential of rank N can be written as

$$V = \sum_{n,m=1}^{N} |g_n\rangle \lambda_{nm} \langle g_m| , \tag{1.5}$$

where $\{|g_m\rangle\}$ is a set of orthonormal states. For a separable two-body potential the corresponding two-particle transition matrix t is also separable and has the form (Lev74, Sch74)

$$t(z) = \sum_{m,n=1}^{N} |g_n\rangle \tau_{nm} \langle g_m| . \tag{1.6}$$

We now rewrite the operator equation (1.4) in the representation of the basis states

$$\langle p_i\, q_i\, b_i\, J\, M| = \langle p_i\, q_i\, L_i S_i J_i K_i l_i J\, M| , \tag{1.7}$$

where L_i, S_i, J_i are the relative orbital angular momentum, the total spin, and the total angular momentum of the pair of particles (jk), K_i is the total channel spin, l_i is the orbital angular momentum of the i-th particle with respect to the centre of mass of the (jk)-pair, J is the total angular momentum of the three-body system, and M is the z-component of J. Applying the

bra and ket of the above basis states to equation (1.4) gives

$$\begin{aligned}
&\langle p_i\, q_i\, b_i \,|\, U_{ij}(s) \,|\, p_j\, q_j\, b_j \rangle \\
&\quad = (1-\delta_{ij}) \langle p_i\, q_i\, b_i \,|\, p_j\, q_j\, b_j \rangle (s - p_i^2 - q_i^2) \\
&\quad + \sum_{k=1}^{3} \sum_{b_k k_k'} (1-\delta_{ik}) \int_0^\infty \langle p_i\, q_i\, b_i \,|\, p_k'\, q_k'\, b_k' \rangle \langle p_k'\, q_k'\, b_k' \,|\, T_k(s) \,|\, p_k\, q_k\, b_k \rangle \\
&\quad \times (s - p_k^2 - q_k^2)^{-1} \langle p_k\, q_k\, b_k \,|\, U_{kj}(s) \,|\, p_j\, q_j\, b_j \rangle p_k'^2 q_k'^2 p_k^2 q_k^2 dp_k' dq_k' dp_k dq_k .
\end{aligned} \tag{1.8}$$

The two particle transition operators in the two- and the three-particle space are related by

$$\begin{aligned}
&\langle p_i\, q_i\, b_i \,|\, T_i(s) \,|\, p_i'\, q_i'\, b_i' \rangle \\
&\quad = \langle p_i\, q_i\, L_i\, S_i\, J_i\, K_i\, l_i\, J \,|\, T_i(s) \,|\, p_i'\, q_i'\, L_i'\, S_i'\, J_i'\, K_i'\, l_i'\, J \rangle \\
&\quad = q_i^{-2} \delta(q_i - q_i') \delta_{JJ_i'} \delta_{K_i K_i'} \delta_{l_i l_i'} \delta_{JJ'} \langle p_i\, L_i\, S_i\, J_i \,|\, t_i(s - q_i^2) \,|\, p_i'\, L_i'\, S_i'\, J_i' \rangle .
\end{aligned} \tag{1.9}$$

The separable two-body t-matrix could be represented in the following diagonal form, called the Hilbert-Schmidt expansion: (Wei63 ,Bal68, Sit69)

$$\begin{aligned}
&\langle p_i\, L_i\, S_i\, J_i \,|\, t_i(z) \,|\, p_i'\, L_i'\, S_i'\, J_i' \rangle \\
&\quad = \sum_n \langle p_i\, L_i\, S_i\, J_i \,|\, g_n(z) \rangle\, \tau_n^{J_i}(z)\, \langle g_n(z^*) \,|\, p_i'\, L_i'\, S_i'\, J_i' \rangle .
\end{aligned} \tag{1.10}$$

Following Lovelace (Lov64), we now define the two-cluster three-body "scattering amplitudes" by

$$X_{m_i i_j n_j}(s) = \langle m_i\, b_i\, J \,|\, G_0(s)\, U_{ij}(s)\, G_0(s) \,|\, n_j\, b_j\, J \rangle . \tag{1.11}$$

Here m, n denote different form factors. With this definition and after substituting for the separable T-matrix of Eq. (1.9), Eq. (1.8) assumes the form

$$X_{im_i,jn_j}(s) = Z_{im_i,jn_j}(s) + \sum_{kr_k} Z^{(s)}_{im_i,kr_k} \tau^{J_k}_{r_k}(s)\, X_{kr_k,jn_j}(s) . \qquad (1.12)$$

The definition of $Z_{im_i,jn_j}(s)$ is given below.

Eq. (1.12) is still an operator equation. (Alt67) When it is expressed in the plane-wave $|q_i>$-representation one obtains

$$\begin{aligned} <q_i | X_{im_i,jn_j}(s) | q_j> &= <q_i | Z_{im_i,jn_j}(s) | q_j> \\ &+ \sum_{k=1}^{3} \sum_{r_k} \int_0^\infty <q_i | Z_{im_i,kr_k}(s) | q_k> \tau^{J_k}_{r_k}(s-q^2) <q_k | X_{kr_k,jn_j}(s) | q_j> q_k^2 dq_k , \end{aligned} \qquad (1.13)$$

where

$$\begin{aligned} <q_i | Z_{im_i,jn_j}(s) | q_j> = \\ (1-\delta_{ij}) \int_0^\infty q^*_{m_i} <p_i\, q_i\, m_i | p_j\, q_j\, n_j> (s-p_i^2-q_i^2)^{-1} q_{n_j} p_i^2\, p_j^2\, dp_i\, dp_j . \end{aligned} \qquad (1.14)$$

Eq. (1.13) is a one-dimensional multichannel Lippmann-Schwinger equation. With the definition

$$<q_i\, a_i\, J \mid K_{ij}(s) \mid q_j\, a_j\, J> = \sum_{L_i S_i L_j S_j} <q_i | Z_{im_i,jn_j}(s) | q_j> , \qquad (1.15)$$

where $a_i = m_i\, J_i\, K_i\, l_i$, a simpler integral equation is obtained for the three-particle transition amplitudes, now denoted by $W_{ij}(s)$, and defined in terms of $X_{ij}(s)$ by an equation similar to 1.15:

$$\begin{aligned} <q_i\, a_i\, J | W_{ij}(s) | q_j\, a_j\, J> = <q_i\, a_i\, J | K_{ij}(s) | q_j\, a_j\, J> \\ + \sum_{k=1}^{3} \sum_{a_k} \int_0^\infty <q_i\, a_i\, J | K_{ik}(s) | q_k\, a_k\, J> \\ \times \tau^{J_k}_{a_k}(s-q_k^2) <q_k\, a_k\, J | W_{kj}(s) | q_j\, a_j\, J> q_k^2 dq_k . \end{aligned} \qquad (1.16)$$

The equations describing the elastic and rearrangement

processes on p.5 can be written, in terms of the notation of this last expression, as follows:

$$
\begin{aligned}
W_{11} &= K_{12}\tau_2 W_{21} + K_{13}\tau_3 W_{31} \\
W_{21} &= K_{21} + K_{21}\tau_1 W_{11} + K_{23}\tau_3 W_{31} \\
W_{31} &= K_{31} + K_{31}\tau_1 W_{11} + K_{32}\tau_2 W_{21}
\end{aligned}
\qquad (1.17)
$$

where the sums over the quantum numbers have been left out.

When the particles with the labels 2 and 3 are treated as identical fermions, there arises the necessity to antisymmetrize the amplitudes involving these particles. As a result, the amplitudes W_{21} and W_{31} are not independent, and can be replaced by the single amplitude $W_{23,1}$, where the subscripts indicate that in the outgoing channel the free particle can be either 2 or 3. Hence, in the case of identical particles Eq.(1.8) can be further simplified. This will not be of any use in what follows, however, since it is essential here to regard the two nucleons in the deuteron to be distinguishable.

Eqs.(1.17) are the last set of formal equations to be presented in this section. It is these integral equations that are later solved by numerical techniques, which will be described in Chapter 3.

The definitions of the physical observables which the code at our disposal[3] calculates are given below:

(1) the differential cross section $\sigma(\theta)$

$\sigma(\theta)$ is related to the transition matrix T_{fi} by

---

3) due to P. Doleschall, Central Research Institute for Physics, Budapest.

$$\sigma(\theta) = |f(\theta)|^2 = (2\pi/\hbar)^4 m^2 |T_{fi}|^2. \qquad (1.18)$$

For example, T_{fi} may stand for W_{12} or $W_{23,1}$. The symbol m denotes the reduced mass.

(2) the analyzing power-(Sim73)

Consider the process $\vec{a}+b \rightarrow c+d$. The analyzing power is defined in terms of the density matrix $\rho_{\mu\mu'}$ via the irreducible spherical tensors t_{kq} by

$$w(t_{kq}) = N \sum_{kq} t_{kq} T_{kq}, \qquad (1.19)$$

where w is the counting probability, N is a normalization constant, and

$$t_{kq} = (2s+1)^{1/2} \sum_{\mu\mu'} (-1)^{s-\mu} \langle s\mu' s-\mu | kq \rangle \rho_{\mu\mu'}. \qquad (1.20)$$

In terms of the differential cross section for polarized projectiles we have

$$\sigma(\theta)_p = \sigma(\theta)_o \sum_{kq} t_{kq} T^*_{kq}(\theta, b(\vec{a}, c) d). \qquad (1.21)$$

For spin 1/2 particles, such as nucleons, cartesian tensors are generally used. They are related tc the spherical ones in the usual way. $\vec{A} = (A_x, A_y, A_z)$ denotes the nucleon analyzing power. For s=1/2 we have

$$\sigma(\theta)_p = \sigma(\theta)_o (1 + \vec{p}.\vec{A}), \qquad (1.22)$$

where $\vec{p}$ is the vector polarization. If the scattering takes place in the (x,y)-plane, then

$$A_x = A_y = 0, \qquad (1.23)$$

since parity considerations require that $\vec{A}$ be perpendicular to this plane. Then

$$\sigma(\theta)_p = \sigma(\theta)_0 \ (1 + p_y A_y) . \tag{1.24}$$

Experimentally p_y and A_y are related to each other by

$$p_y A_y = (L-R)/(L+R) , \tag{1.25}$$

where L and R denote the number of particles scattered to the left and to the right respectively. For a spin-1 particle, such as a deuteron, tensor analyzing powers are additional polarization observables. We will later denote these by T_{20}, T_{21}, T_{22}. Here the rank of the tensor is k=2.

(3) the polarization transfer coefficients-(Sim73)

Consider the reaction $\vec{a}+b \dashrightarrow \vec{c}+d$. The polarization transfer coefficients are denoted by

$$t^{k_a q_a}_{k_c q_c}\{\theta; b(\vec{a},\vec{c})\, d\},$$

and are defined by

$$t^{c}_{k_c q_c}(t^{a,in}_{k_a q_a}) = \sum_{k_a q_a} t^{a,in}_{k_a q_a} \ t^{k_a q_a}_{k_c q_c} \ \{\theta; b(\vec{a},\vec{c})\, d\}, \tag{1.26}$$

where $t^{a,in}_{k_a q_a}$ is the polarization of the incoming particle, and $t^{c}_{k_c q_c}(\cdot)$ is that of the outgoing particle expressed in terms of the former. A particular polarization coefficient is referred to by specifying the particles a and c, and by giving the upper (final) and lower (initial) indices of t.

(4) the spin correlation coefficients

These quantities, which are again denoted by T but which carry

subscripts only, are defined by

$$\sigma(\theta)_p = \sigma(\theta)_0 \sum_{k_a q_a k_b q_b} t^{a,in}_{k_a q_a} \; t^{b,in}_{k_b q_b} \; T_{k_a q_a k_b q_b}\{\theta; \vec{b}(\vec{a},c)d\}. \qquad (1.27)$$

Of the above observables only the first two will be used in the comparison of the three-body and the distorted-wave calculations in Chapter 5.

CHAPTER 2

DETERMINATION OF TWO-BODY INTERACTIONS

In this chapter the nucleon-nucleon and nucleon-^{16}O potentials will be completely specified by determining their parameters. Starting from a local optical model parametrization for a nucleon-nucleus potential the real parts of the nuclear phase shifts are derived, and are assumed to represent the on-shell properties of the interaction. For the nucleon-nucleon interaction such a derivation is not necessary since their phase shifts are available in the literature (Mac69). The separable potential parameters are then fixed in a non-unique but mutually consistent way by varying them simultaneously until the scattering phase shifts due to the potential they specify provide a fit[1] to the experimental or the derived phase shifts.

---

1) For the nucleon-nucleon S-waves the scattering length and the effective range, and for the nucleon-nucleon $^3S1$-$^3D1$ interaction the D-state probability and the quadrupole moment were additionally included in the χ^2 minimization. For the N-^{16}O $^2S(1/2)$ interaction the scattering length and the effective range fits were not done, lest the quality of the less accurate phase-shift fits be compromised by the simultaneous fitting of these low-energy quantities. (Their experimental values are given in (Pis63) for the proton-^{16}O system.)

2.1. Nucleon-Nucleon Interactions

The nucleon-nucleon experimental phase shifts (Mac69) [2] cover a wide energy range (0-460 MeV), and include partial waves through L=5. Furthermore, considering the similarity of the three-body part of this work to that carried out by Doleschall for neutron-deuteron scattering and break-up processes (Dol73, 74), the separable interaction parameters he has already obtained by means of phase shift fitting for a number of two-nucleon interactions will be partially adopted here for one one-term, and one two-term interaction.

2.2. Nucleon-^{16}O Interactions

In ^{16}O the eight neutrons, or the eight protons, fill the 1s(1/2), 1p(3/2), 1p(1/2) levels. A nucleon which interacts with the ^{16}O core can therefore occupy any state above the 1p(1/2) level. Hence in the ground state of ^{17}O the last neutron is in the 1d(5/2) state. The higher energy levels form excited states of this nucleus. Since the Coulomb interaction is to be neglected, it is appropriate to select ^{17}O, rather than ^{17}F, as the bound state of either one of the nucleons in the deuteron with ^{16}O. The bound states to be taken into account are then those of ^{17}O. In what follows only the 1D(5/2) and the 2S(1/2) states of the nucleon-^{16}O system are assumed to be bound. The distinguishing feature of these bound states is that they are

---

2) More recent p-p and n-p phase shift sets are given in the reference (Arn77). The 1969 data were used here since the potential parameters providing a fit to them had already been obtained, and since the accuracy of experimental data would not be a crucial factor in a model calculation.