NETWORK RESPONSES DUE TO PARAMETER CHANGES

A THESIS PRESENTED TO

THE FACULTY OF GRADUATE STUDIES AND RESEARCH

THE UNIVERSITY OF MANITOBA

In partial fullfillment

of the requirements for the degree

Master of Science in

Electrical Engineering

by:

Maurice Yunik

January, 1966

ACKNOWLEDGMENTS

The author is grateful to A. Simeon (Assistant Professor of Electrical Engineering, University of Manitoba) for his suggestions in circuit theory and to the University of Manitoba for providing financial assistance towards research done for this thesis.

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ABSTRACT

This thesis discusses the various techniques of obtaining the sensitivity of network due to a parameter change. The idea of sensititivity is extended to approximate a change in a given network function when the change in some parameter is known. Various well established techniques are compared and their merits discussed. A new view is presented by use of three dimensional plots to disucss sensitivities in the real frequency domain.

LIST OF SYMBOLS

Chapter II

2.1

$Z(s)$ – General Network impedance function

$s_1, s_2 \ldots s_k \ldots s_n$ – Zeros of the above network function

$\delta z(s)$ – change at a branch impedance.

A_k – residue of the partial fraction expansion of the reciprocal of network impedance function at the zero in question.

2.2

$H(j\omega)$ – A general network function in the real frequency domain.

ω – radian frequency.

$e_1, e_2 \ldots e_n$ – tolerances of elemental values of network.

P – Numerator of H(jw).

Q – Denominator of H(jw).

δ_i – actual deviation of elemental values at a network.

A – Function of a complex variable.

$|A|$ – magnitude of A

ϕ – phase of A.

$G(s)$ – the voltage gain of a network.

Chapter III

3.1	S_K^T –	Sensitivity of network with respect to a variable K.
	$T(s,x)$ -	Network function.
	K -	parameter of the network function.
	$A(s)+KB(s)$ -	numerator of T(s).
	$C(s)+KD(s)$ -	denominator of T(s).
	x	elemental value of network.
	$H(s,x)$ -	general form of both numerator and denominator of T(s,x).
	$q(s)$ -	polynominal of H(s,x) not containing x.
	$p(s)$ -	polynominal of H(s,x) containing x with x factored from it.
	G -	the gain as defined for root locus by W. R. Evans[6].
3.2	p_i, z_i -	general pole and zero locations.
	$S_x^{z_i}$ -	sensitivity of pole with respect to x.
	g -	the constant appearing in front of T(s) when T(s) is factored form.

V - sum of tree admittance products of a network.

$W(1,2)$ - sum of two tree admittance products.

z_{11}, z_{12}, z_{22} - open circuit impedance parameters of a two-p rt network.

3.3 W - is the closed loop gain of system with negative feedback and an open loop gain of G.

$-k_j$ - residue of W at pole location $S = S_j$.

$\bar{G}(s)$ - voltage gain of a network.

f - superscript, meaning farads.

Ω - superscript, meaning ohms.

h - superscript, meaning henrys.

ΠP - product of pole vectors in the s plane to pole in question.

ΠZ - product of zero vector in the s plane to pole in question.

y_{11}, y_{12}, y_{22} - the short circuit admittance. parameters of a two port network.

x_o - nominal value of the element x.

Δx - small change in the elemental value of x

$\Delta\alpha$ the change in the arguement of x to Δ x.

Chapter IV

4.1

$H(j\omega)$ - network function

$|H|$ - magnitude of H(jw)

$\phi(j\omega)$ - arguement of H(jw)

$S_x^{|H|}$ - sensitivity of $|H(jw)|$ due to a change in x

$\dot{S}_x^{\phi}$ - sensitivity of ϕ due to a change in x

$H(R)$ - a resistive network function.

$a+Rd$ - numerator of H(R)

$c+Rd$ - denominator of H(R).

f_0, f_1, f_2 - complex variable function of w used to describe $H(j\omega, x)$

Chapter V

F - a differentiable function

ΔF - a small change in F due to Δx.

Chapter I

INTRODUCTION AND MOTIVATION

Many articles papers and theses in synthesis are written to realize required network functions and time response phenomena. These give rise to many methods of designing filters, compensators, and other passive network devices. However, few articles describe the difficulties in realizing these networks in practice.

One of the problems that arises is inherent error which occurs in the manufacture of inductors, capacitors, and resistors. For example, the most common types of resistors vary as much as 10% or 20% of their nominal values. Three questions immediately arise from the use of such circuit elements:

1) What will be the resultant response of a network with elements which are not exactly those specified by the designer? What happens to the response when the value of a resistor, inductor or capacitor is 10% or 20% in error from the specified value?

2) How accurate must we choose certain elements in a network? Some may have more effect on the desired response than others. Can certain elements be allowed to be chosen less carefully than others?

3) Which synthesis technique is the best to use in order to realize a desired response? There are many synthesis techniques devised today so that the designer is able to choose from two or more networks. In manufacturing it may be advantageous from an economic point of view to use several more elements which need to be less exact than a simple network which needs very exact values.

Thus the purpose of this thesis is to present techniques already in use to answer the above questions and to propose some new ideas on this matter. The shortcomings of the techniques are illustrated by use of examples.

Chapter II deals with some of the earliest techniques developed. These have many obvious shortcomings. Chapter III uses techniques developed in the field of Control Systems and specializes these for linear, time-invariant, passive, lumped, finite, and bilateral networks. This chapter deals with network functions in the complex frequency domain. Chapter IV is an attempt to solve problems in the real frequency domain by a direct approach. Chapter V extends the idea of sensitivity discussed in the preceding two chapters to more than one element. The author also wishes to present the different techniques established in various papers in such a manner that it can be of use to the reader.

Chapter II

EARLY CONTRIBUTIONS

2.1 Displacement of Zeros Due to Incremental Parameter Changes

Probably the earliest contribution to this problem was made by A. Papoulis in 1955.[1] The following is a review of his work to show the development of this field as well as the need for research in this area.

Consider an impedance function $Z(s)$ with zeros at $s_1, s_2, \cdots s_k \cdots s_n$.

$$\text{i.e.} \quad Z(s)\Big|_{s=s_k} = 0 \qquad \cdots\cdots\cdots (2.1.1)$$

If we now add $\delta Z(s)$ in any branch, the zeros of the impedance function will move to the new positions at $s_1^*, s_2^*, \cdots s_k^* \cdots s_n^*$

Since the zeros of the network are also the roots of the determinant of the system these new zeros can be found by considering the impedance or the admittance looking in at this point where $\delta Z(s)$ is added.

Thus we can write

$$Z(s) + \delta z(s) \Big|_{s=s_k^*} = 0 \qquad \text{.......} \quad (2.1.2)$$

Assume now that zero in question is of multiplicity

Then,

$$\frac{(s-s_k)^m}{Z(s)} \Bigg|_{s=s_k^*} = A_k \neq 0 \qquad \text{.......} \quad (2.1.3)$$

We have from (2.1.2)

$$Z(s) \Big|_{s=s_k^*} = -\delta z(s) \Big|_{s=s_k^*} \qquad \text{.......} \quad (2.1.4)$$

From this we can write the following relationships:

$$(s_k^* - s_k)^m = \frac{-\delta z(s_k^*)\ (s_k^* - s_k)^m}{Z(s_k^*)} \qquad \text{.......} \quad (2.1.5)$$

$$(s_k^* - s_k) = \sqrt[m]{\frac{-\delta z(s_k^*)\ (s_k^* - s_k)^m}{Z(s_k^*)}} \qquad \text{.......} \quad (2.1.6)$$

Define: $$F(s) = \frac{-\delta z(s)\ (s - s_k)^m}{Z(s)} \qquad \text{.......} \quad (2.1.7)$$

Superscripts indicate reference numbers in the Bibliography

Then $F(s_k) = -\delta z(s_k) \quad A_k$ .... (2.1.8)

$F'(s) = \frac{1}{z(s)}$ has a singularity of order m at $s = s_k$.

Thus $F(s)$ will have no singularity at this point since the factors contributing to this singularity have been cancelled. $F(s)$ will therefore be analytic in some region around $s = s_k$.

Thus if s_k^* is close to s_k then $F(s_k^*)$ can be expanded in a Taylor series about s_k as follows:

$$F(s_k^*) = F(s_k) + F'(s_k)(s_k^* - s_k) + \cdots \qquad \text{.......} \; (2.1.9)$$

If $(s_k^* - s_k)$ is sufficiently small then the following approximation can be made:

$$F(s_k^*) \cong F(s_k), \qquad \text{.......} \; (2.1.10)$$

so that

$$(s_k^* - s_k) = \sqrt[m]{\delta z(s) \quad A_k} \qquad \text{.......} \; (2.1.11)$$

A_k is the residue of the partial fraction expansion of $\frac{1}{z(s)}$ at $s = s_k$.

Equation (2.1.11) thus becomes very useful in determining the initial direction of motion of zeros by incremental changes in parameters. However it has a drawback because of tedious calculations necessary for the residue of each zero. Another short-coming is that this technique allows variations only in the small and shows only the initial direction of movement of zeros due to parameter changes.

2.2 Bounds on Frequency Response of a Network

The following development was done as an application to networks by W. C. Yengst[2] based on work done by B. R. Myers[3] in Control Systems in 1959. It enables one to work out the maximum and minimum bounds on $H(j\omega)$ due to many simultaneous changes in parameters of a network.

Let us consider a network where the element values differ at most from those required by the amounts $e_1, e_2, \cdots e_n$. Then the network function can be written as

$$H(s, e_1, e_2 \cdots e_n) = \frac{P(s, e_1, e_2, \cdots e_n)}{Q(s, e_1, e_2, \cdots e_n)} \quad \ldots\ldots\ldots (2.2.1)$$

$$= \frac{P_0(s) + \sum_i e_i P_i(s) + \sum_{i \neq j} e_i e_j P_{ij}(s) + \cdots}{Q_0(s) + \sum e_i Q_i(s) + \sum_{i \neq j} e_i e_j P_{ij}(s) + \cdots} \quad \ldots\ldots\ldots (2.2.2)$$

where the P's and Q's are polynominal in s.

From Myers' Theorem[4] we have

$$P(j\omega, e_1, e_2 \cdots e_n) = P_0(j\omega) + \sum e_i P_i(j\omega) + \sum_{i \neq j} e_i e_j P_{ij}(j\omega)$$

$$+ \cdots + \sum_{i \neq j \neq \cdots \neq n} [e_1, e_2, \cdots e_n] P_{ij \cdots n}(j\omega) \quad \ldots\ldots\ldots (2.2.3)$$

where the e_i's are real numbers.

For a specified $\omega = \omega_0$, $P(j\omega_0, e_1, \cdots e_n)$ falls in the interior of the polygon of P calculated at the extreme values of

$$e_i = \pm \delta_i \quad \text{for} \quad i = 1, 2, \cdots n.$$

If there are n e_i's there will be 2^n extreme values of the polygon.

For example consider $A(j\omega, e_1, e_2)$ with only two variables. We must plot $A(j\omega_0, e_1, e_2)$ in the complex plane for the extreme values of

a) $e_1 = -\delta_1$, $\quad e_2 = -\delta_2$

b) $e_1 = -\delta_1$, $\quad e_2 = \delta_2$

c) $e_1 = \delta_1$, $\quad e_2 = -\delta_2$

d) $e_1 = \delta_1$, $\quad e_2 = \delta_2$

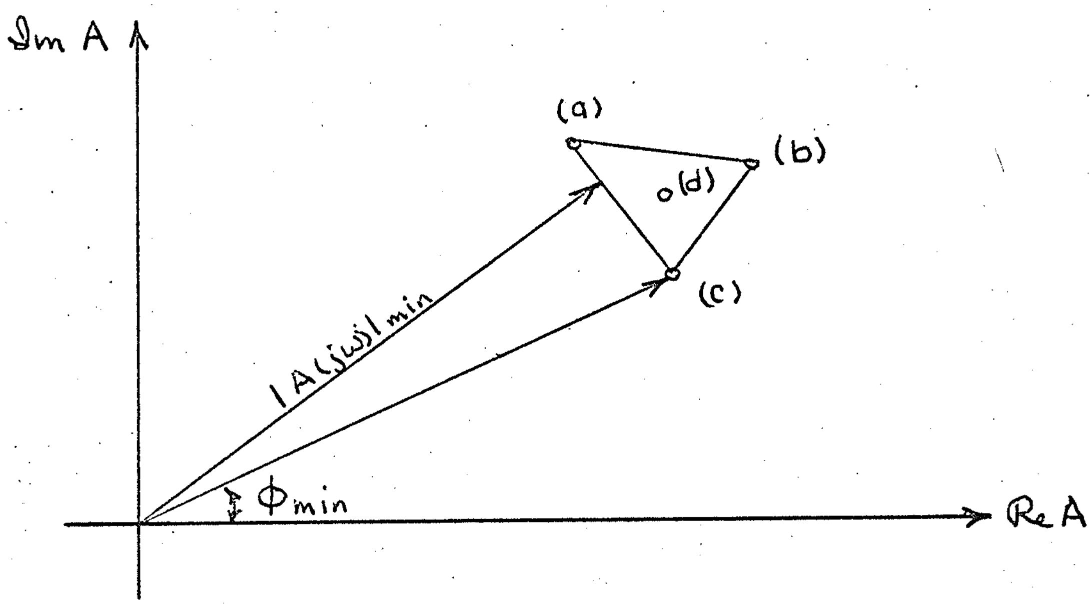

Fig. 2.2.1

Here we can pick out for this example

$|A(j\omega)|_{min}$ and ϕ_{min}

Thus for a ratio of two polynominals as in the case of network functions, the maximum and minimum limits may be found.

i.e. $$|H(j\omega)|_{max} = \frac{|P(j\omega)|_{max}}{|Q(j\omega)|_{min}} \quad \ldots\ldots\ldots (2.2.4)$$

$$|H(j\omega)|_{min} = \frac{|P(j\omega)|_{min}}{|Q(j\omega)|_{max}} \quad \ldots\ldots\ldots (2.2.5)$$

Similarly for the phase of $H(j\omega) = \phi_H(j\omega)$

$$\phi_H(j\omega)_{max} = \phi_P(j\omega)_{max} - \phi_Q(j\omega)_{min} \quad \ldots\ldots\ldots (2.2.6)$$

$$\phi_H(j\omega)_{min} = \phi_P(j\omega)_{min} - \phi_Q(j\omega)_{max} . \quad \ldots\ldots\ldots (2.2.7)$$

Although this seems like a good approach to obtaining errors in network response, it has the following short-comings:

1) The calculations are laborious for any more than two simultaneous variations. Also they must be done for each frequency separately.

2) The maximum and minimum values certainly enclose the limits but in some cases are poor guides since in some cases, these limits seem to be quite large. The following example will illustrate this point:

Consider the voltage gain of the following circuit:

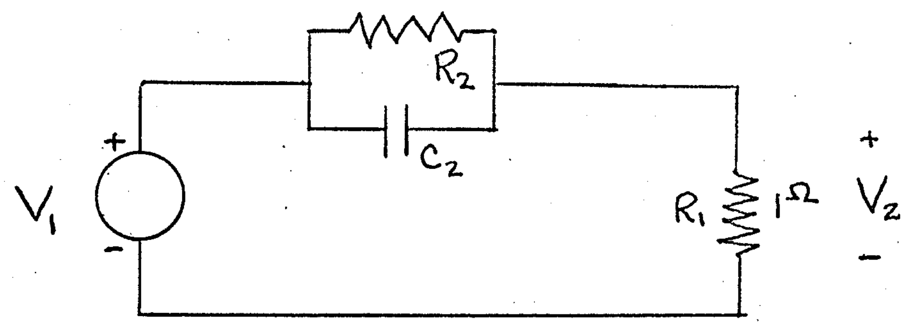

Fig. 2.2.2.

$$G(j\omega) = \frac{V_2}{V_1}(j\omega) = \frac{1+j\omega R_2 C_2}{(R_2+1)+j\omega R_2 C_2} \quad \ldots\ldots\ldots \quad (2.2.8)$$

The nominal values for R_2 and C_2 are:

$R_2 = 0.5$ ohms , $C_2 = 1$ farad

This gives

$$G(s) = \frac{s+2}{s+3} \quad \ldots\ldots\ldots \quad (2.2.9)$$

and the following Bode plot

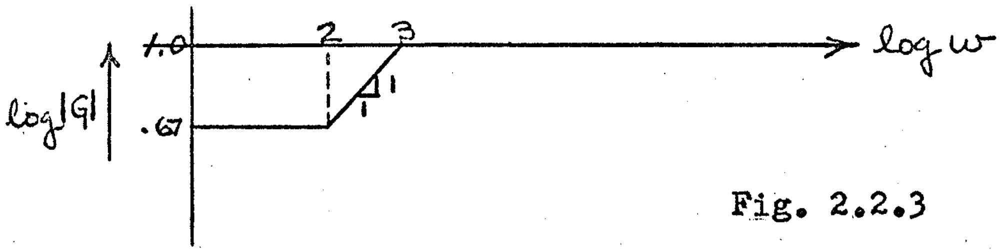

Fig. 2.2.3

Using the technique described on pages 8 - 10 and varying R_2 and C_2 by 10%, the data was plotted in Figure 2.2.4.

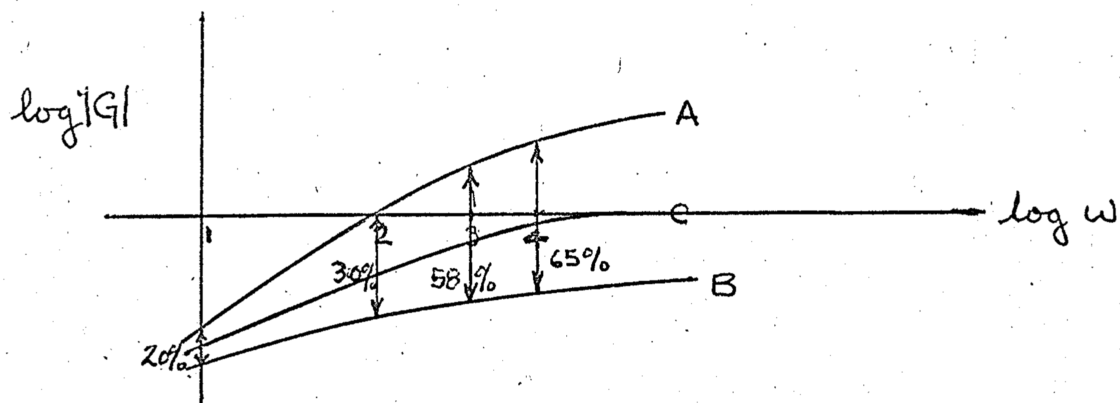

Fig. 2.2.4

C - Required curve

A,B - Bounds given from data.

The percentage error increases with frequency hence becoming quite large for $\omega > 4$. This is due to the fact that the polygon becomes larger with increasing frequency. This gives safe limits in which the resulting gain is to occur but from figure 2.2.4 we see that this does not give a measure of the magnitude of the actual error. We notice that for high frequencies, C_2 shunts out R_2 and the gain becomes unity, independent of any errors in R_2 or C_2.

Chapter III

SENSITIVITY BY ROOT LOCUS METHODS

3.1 General

Bode[5] has defined the term sensitivity as:

$$S_K^T \triangleq \frac{\partial(\ln T(s))}{\partial(\ln K)} = \frac{\partial T/T}{\partial K/K} \qquad \text{.......} \quad (3.1.1)$$

where $T(s)$ is the network function and K is a variable parameter in the system. It could be almost any parameter such as capacitance, resistance, temperature, etc.

Bode has shown that network functions can be written in general as follows:

$$T(s) = \frac{A(s) + K\,B(s)}{C(s) + K\,D(s)} \qquad \text{.......} \quad (3.1.2)$$

where A, B, C and D are polynomials in s. The proof can be simplified somewhat if we let K be an element value in the circuit such as resistance, inductance, or capacitance.

Proof:

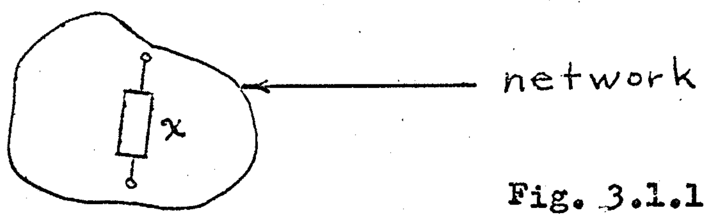

Fig. 3.1.1

$T(s)$ is a ratio of tree admittance or impedance products or of the compliments of the trees considered. Since x either appears once in a tree or not at all and, since these products are summed, x is never raised to any power higher than one. Separating the numerator and denominator into two parts and factoring out the x we obtain the form.

$$T(s) = \frac{A(s) + x\,B(s)}{C(s) + x\,D(s)} \qquad \text{.......} \quad (3.1.3)$$

The following analysis will examine the effect on the poles and zeros of the network function $T(s)$ by varying x . The results of this analysis are useful since many synthesis specifications require prescribed pole-zero configurations.

We note that both the numerator and denominator are of the form

$$H(s,x) = q(s) + x\,p(s) \qquad \text{.......} \quad (3.1.4)$$

By examining the roots of $H(s,\chi)=0$ we can determine the poles and zeros of $T(s)$ using χ as a variable parameter.

i.e. $q(s)+\chi\, p(s)=0$ .... (3.1.5)

or $\chi \dfrac{p(s)}{q(s)} = -1$ .... (3.1.6)

Equation (3.1.6) is the familiar root locus as formulated in detail by W. R. Evans[6] with $G= \chi \dfrac{p(s)}{q(s)}$. Many methods have been established to facilitate the plotting of the locus of $G=-1$.

3.2 Pole and Zero Sensitivity by Root Locus

In general we can write

$$T(s) = g \frac{\prod_{i=1}^{m} (s+z_i)}{\prod_{1}^{n} (s+p_i)} \qquad \text{....... (3.2.1)}$$

noting that $z_i, p_i,$ and g may be functions of the parameter x.

$$S_x^T = \frac{d \ln T}{d \ln x} = x \frac{d \ln T}{dx} \qquad \text{....... (3.2.2)}$$

$$\ln T = \ln g + \sum_1^m \ln(s+z_i) - \sum_1^n \ln(s+p_i) \qquad \text{....... (3.2.3)}$$

$$\frac{d \ln T}{dx} = \frac{1}{g}\frac{\partial g}{\partial x} + \sum_1^m \frac{\frac{\partial z_i}{\partial x}}{s+z_i} - \sum_1^n \frac{\frac{\partial p_i}{\partial x}}{s+p_i} \qquad \text{....... (3.2.4)}$$

so that

$$S_x^T = \frac{x}{g}\frac{\partial g}{\partial x} + \sum_1^m \frac{x\frac{\partial z_i}{\partial x}}{s+z_i} - \sum_1^n \frac{x\frac{\partial p_i}{\partial x}}{s+p_i} \qquad \text{....... (3.2.5)}$$

we now define zero sensitivity as

$$S_x^{z_i} = x\frac{\partial z_i}{\partial x} = \frac{\partial z_i}{\partial \ln x} \qquad \text{....... (3.2.6)}$$

i.e. the change of zero location per relative parameter variation and define pole sensitivity as

$$S_x^{p_i} = x\frac{\partial p_i}{\partial x} = \frac{\partial p_i}{\partial \ln x} \qquad \text{....... (3.2.7)}$$

so that

$$S_x^T = \frac{\partial \ln g}{\partial \ln x} + \sum_1^m \frac{S_x^{z_i}}{s+z_i} - \sum_1^n \frac{S_x^{p_i}}{s+p_i} \qquad \text{....... (3.2.8)}$$

Before we apply these definitions let us consider the following theorem which will allow us to find certain pole zero sensitivities by inspection.

Theorem[7]

If there is a single element of value x connecting two otherwise separate parts of a network, then the sensitivity of the roots of the polynomial V[8] of the overall network with respect to the x is zero.

Proof:

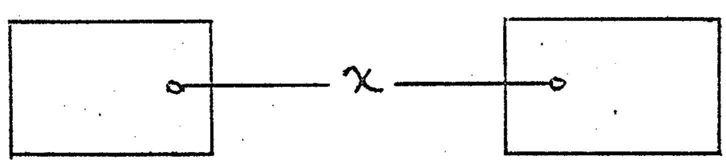

Fig. 3.2.1

- x must appear in all the trees of the network.
- The sum of tree admittance products can be written as

$$V = x\,W(1,2),$$

where $W(1,2)$ is the sum of the 2-tree admittance products not containing x.